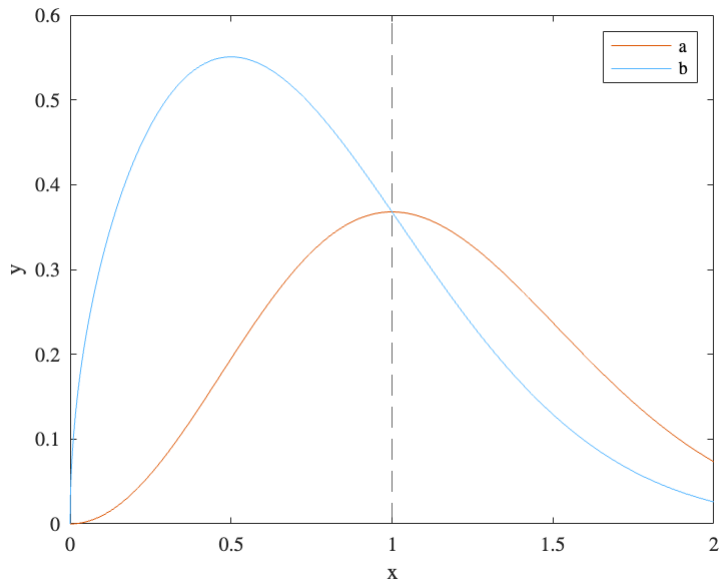


# Lab 5

Jessie Li // October 25, 2023

## 1. Numerical integration (Romberg & trapezoidal)

```
[r_a01, r_a12, r_b01, r_b12] = main1()
```

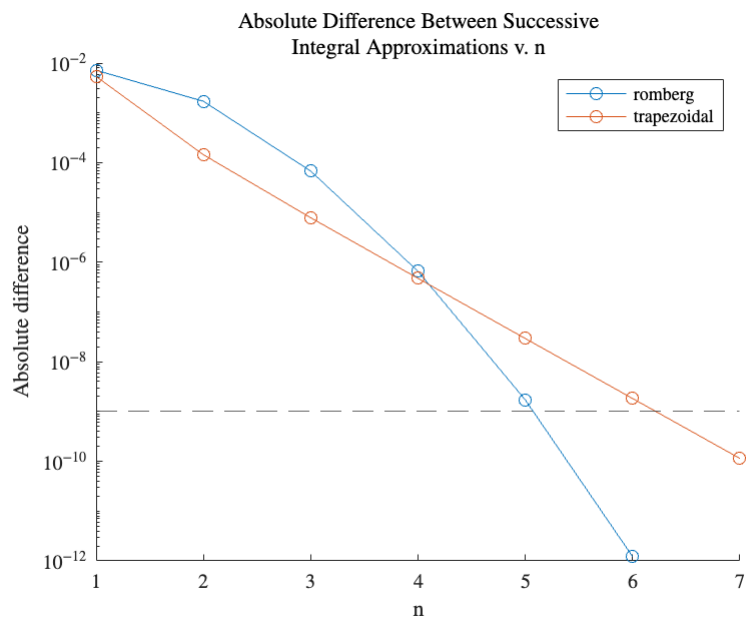
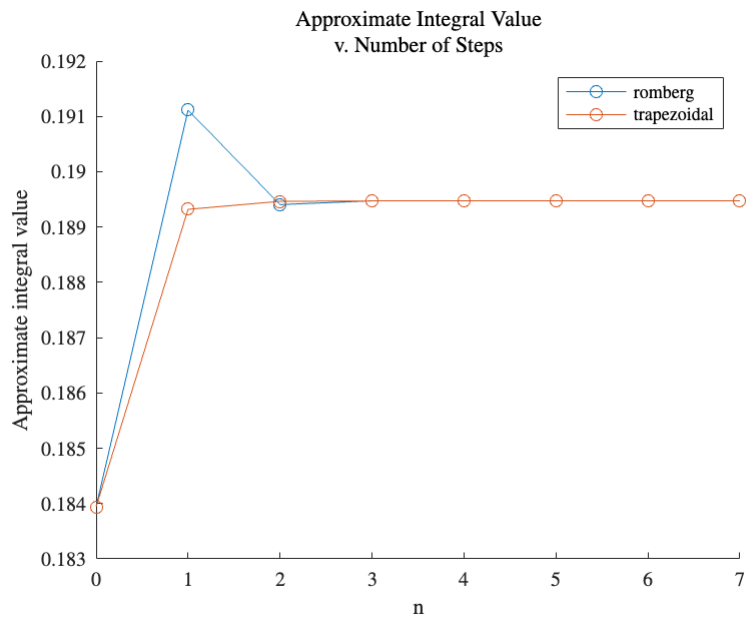


```
-----  
function (a) with limits of integration from 0 to 1  
----- romberg -----
```

```
n: 6  
y = 0.1894723457  
function evaluations: 134
```

```
----- trapezoidal -----
```

```
n: 7  
y = 0.1894723458  
function evaluations: 129  
-----
```



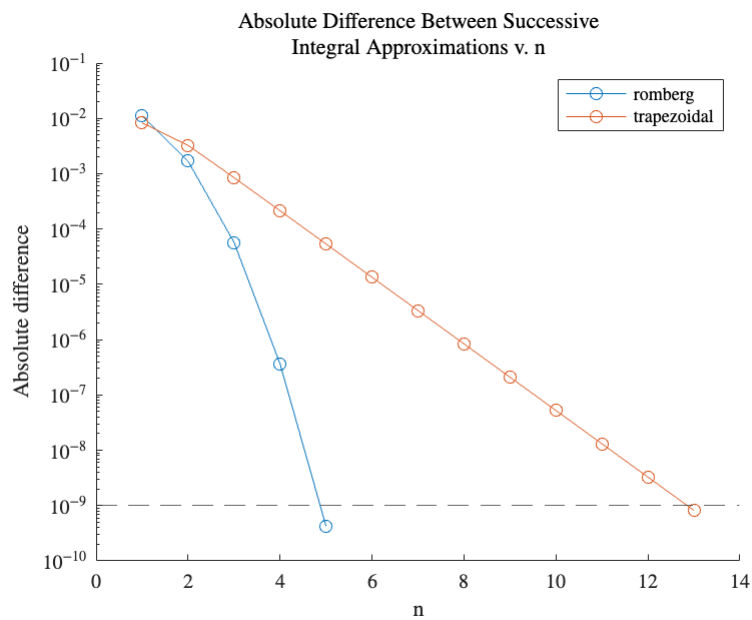
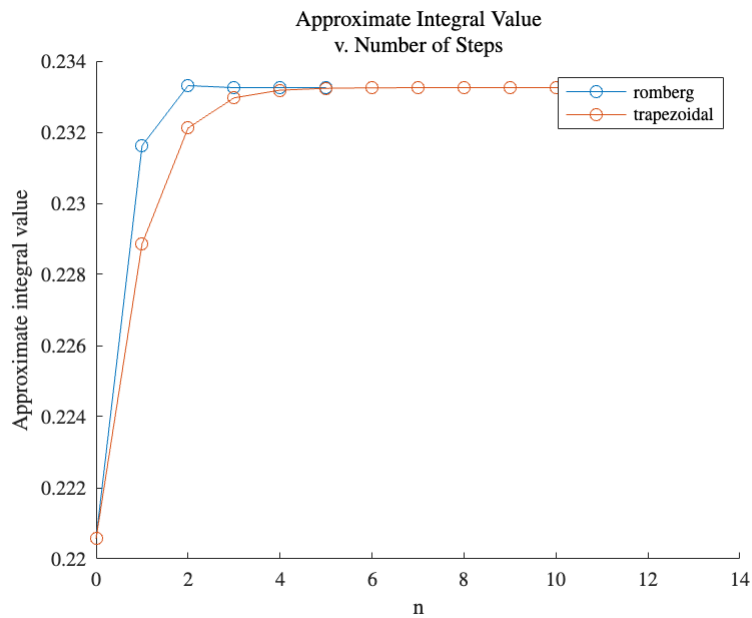
-----  
function (a) with limits of integration from 1 to 2

----- romberg -----

n: 5  
y = 0.2332348266  
function evaluations: 69

----- trapezoidal -----

n: 13  
y = 0.2332527104  
function evaluations: 8193



-----  
function (b) with limits of integration from 0 to 1

----- romberg -----

n: 18

y = 0.4533919429

function evaluations: 524306

-----

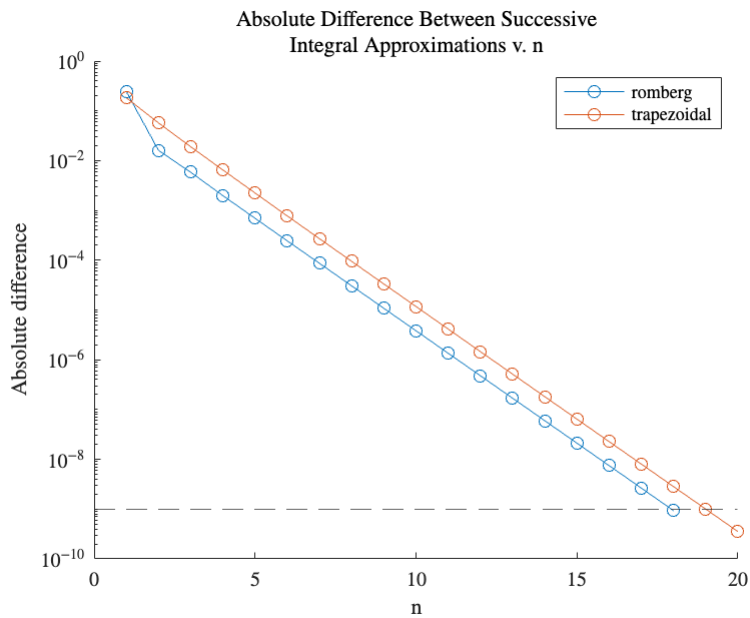
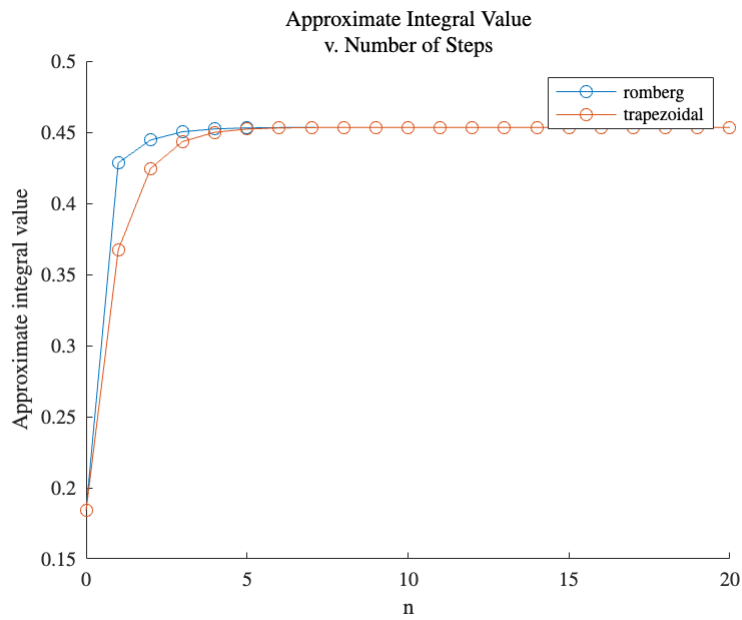
----- trapezoidal -----

n: 20

y = 0.4533919443

function evaluations: 1048577

-----



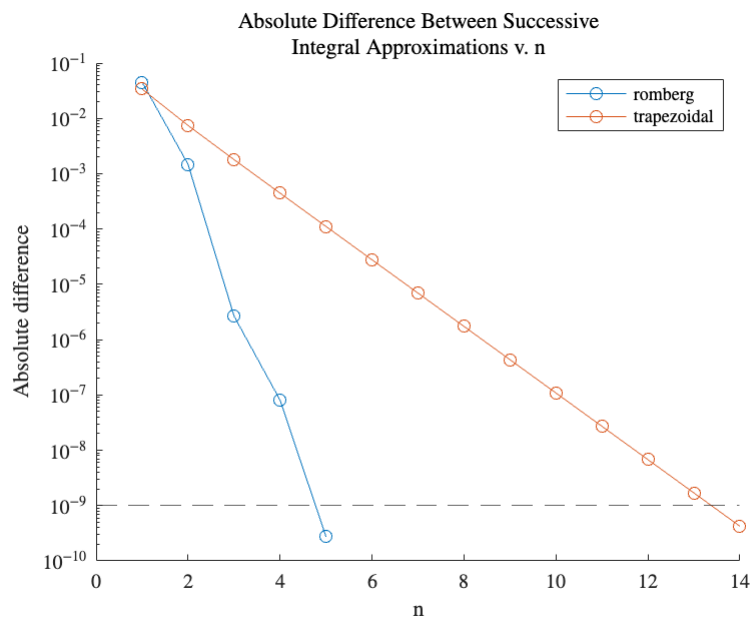
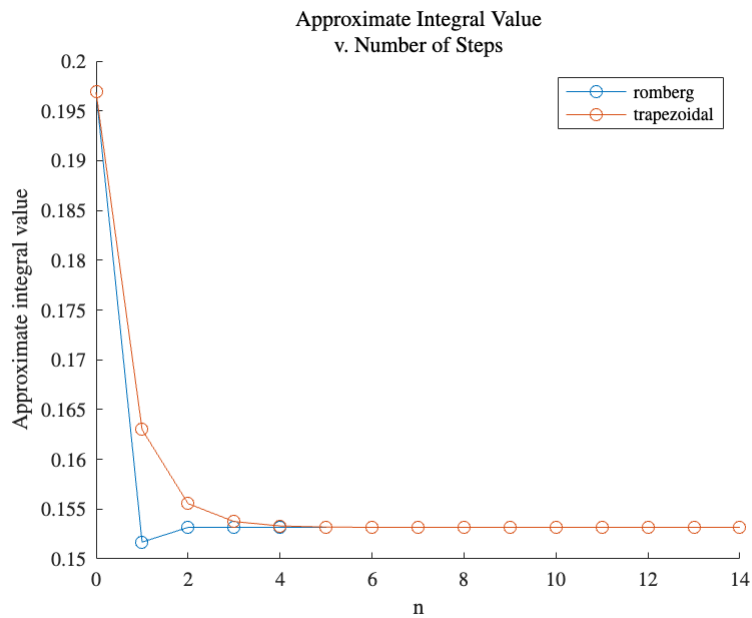
function (b) with limits of integration from 1 to 2

----- romberg -----

n: 5  
y = 0.1531975722  
function evaluations: 69

----- trapezoidal -----

n: 14  
y = 0.1531605646  
function evaluations: 16385



$r_{a01} = 0.1895$   
 $r_{a12} = 0.2333$   
 $r_{b01} = 0.4534$   
 $r_{b12} = 0.1532$

## 2. Gaussian quadrature

```
main2(r_a01, r_a12, r_b01, r_b12)
```

```

----- n = 1 -----
----- gaussian -----
y = 0.1947001958
function evaluations: 1
-----
----- gaussian -----
y = 0.2371482553
function evaluations: 1
-----

```

```

----- gaussian -----
y = 0.5506953149
function evaluations: 1
-----

----- gaussian -----
y = 0.1290871597
function evaluations: 1
-----

----- n = 2 -----
----- gaussian -----
y = 0.1883211725
function evaluations: 2
-----

----- gaussian -----
y = 0.2343887541
function evaluations: 2
-----

----- gaussian -----
y = 0.4581994573
function evaluations: 2
-----

----- gaussian -----
y = 0.1541447597
function evaluations: 2
-----

----- n = 3 -----
----- gaussian -----
y = 0.1895387663
function evaluations: 3
-----

----- gaussian -----
y = 0.2331969968
function evaluations: 3
-----

----- gaussian -----
y = 0.4559023862
function evaluations: 3
-----

----- gaussian -----
y = 0.1531581855
function evaluations: 3
-----

----- n = 4 -----
----- gaussian -----
y = 0.1894704161
function evaluations: 4
-----

----- gaussian -----
y = 0.2332537495
function evaluations: 4
-----

----- gaussian -----
y = 0.4545472518
function evaluations: 4
-----

----- gaussian -----
y = 0.1531603246
function evaluations: 4
-----

----- n = 5 -----
----- gaussian -----
y = 0.1894723827
function evaluations: 5
-----

```

```

----- gaussian -----
y = 0.2332527023
function evaluations: 5
-----

----- gaussian -----
y = 0.4540208302
function evaluations: 5
-----

----- gaussian -----
y = 0.1531605702
function evaluations: 5
-----

----- n = 6 -----
----- gaussian -----
y = 0.1894723453
function evaluations: 6
-----

----- gaussian -----
y = 0.2332527106
function evaluations: 6
-----

----- gaussian -----
y = 0.4537713472
function evaluations: 6
-----

----- gaussian -----
y = 0.1531605644
function evaluations: 6
-----

----- n = 7 -----
----- gaussian -----
y = 0.1894723458
function evaluations: 7
-----

----- gaussian -----
y = 0.2332527107
function evaluations: 7
-----

----- gaussian -----
y = 0.4536382313
function evaluations: 7
-----

----- gaussian -----
y = 0.1531605645
function evaluations: 7
-----

----- n = 8 -----
----- gaussian -----
y = 0.1894723458
function evaluations: 8
-----

----- gaussian -----
y = 0.2332527107
function evaluations: 8
-----

----- gaussian -----
y = 0.4535608041
function evaluations: 8
-----

----- gaussian -----
y = 0.1531605645
function evaluations: 8
-----

----- n = 9 -----

```

```

----- gaussian -----
y = 0.1894723458
function evaluations: 9
-----
----- gaussian -----
y = 0.2332527107
function evaluations: 9
-----
----- gaussian -----
y = 0.4535127280
function evaluations: 9
-----
----- gaussian -----
y = 0.1531605645
function evaluations: 9
-----
----- n = 10 -----
----- gaussian -----
y = 0.1894723458
function evaluations: 10
-----
----- gaussian -----
y = 0.2332527107
function evaluations: 10
-----
----- gaussian -----
y = 0.4534813088
function evaluations: 10
-----
----- gaussian -----
y = 0.1531605645
function evaluations: 10
-----
----- n = 11 -----
----- gaussian -----
y = 0.1894723458
function evaluations: 11
-----
----- gaussian -----
y = 0.2332527107
function evaluations: 11
-----
----- gaussian -----
y = 0.4534599117
function evaluations: 11
-----
----- gaussian -----
y = 0.1531605645
function evaluations: 11
-----

```

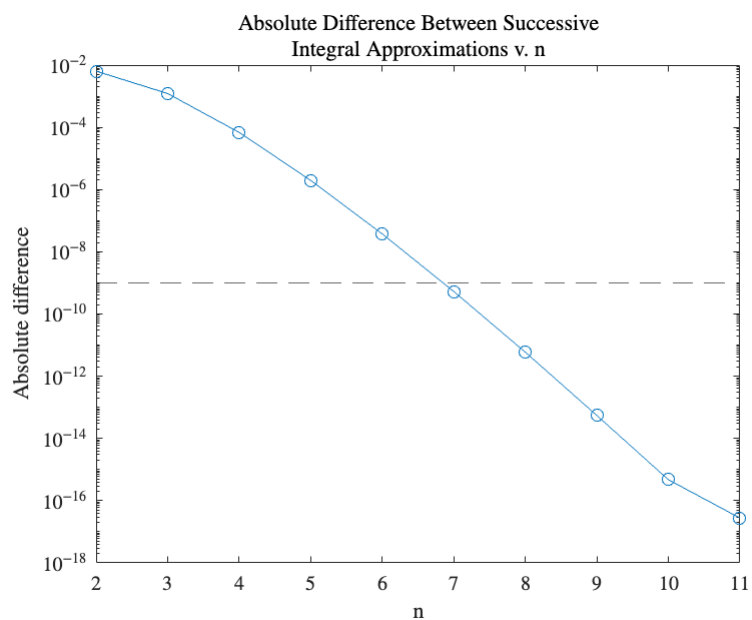
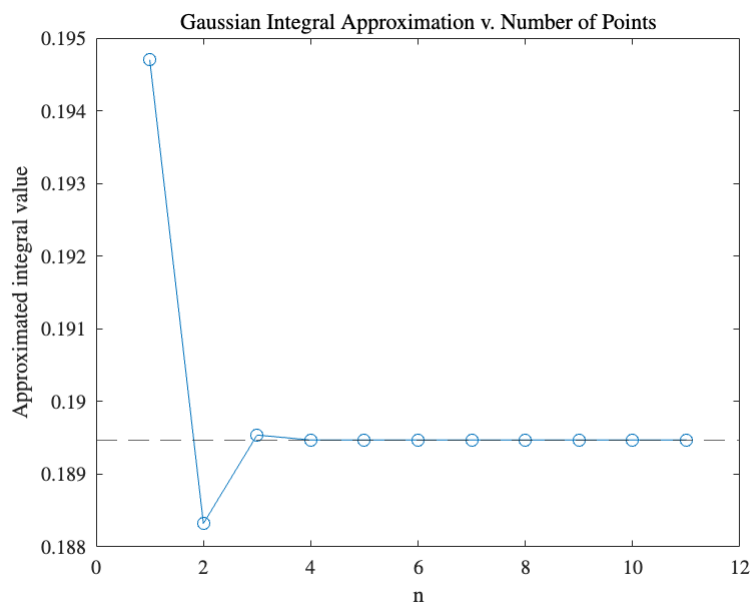
ans = 11×5 table

	Var1	a01	a12	b01	b12
1	1	0.1947	0.2371	0.5507	0.1291
2	2	0.1883	0.2344	0.4582	0.1541
3	3	0.1895	0.2332	0.4559	0.1532
4	4	0.1895	0.2333	0.4545	0.1532
5	5	0.1895	0.2333	0.4540	0.1532
6	6	0.1895	0.2333	0.4538	0.1532

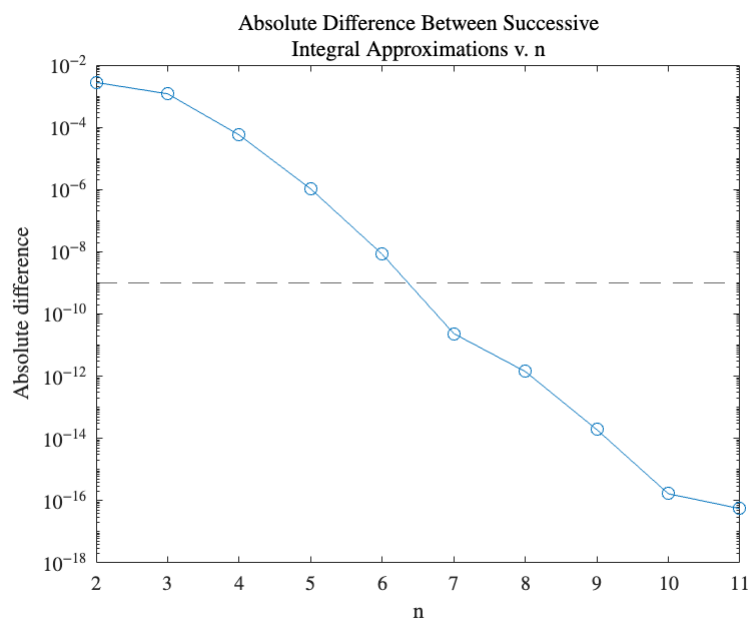
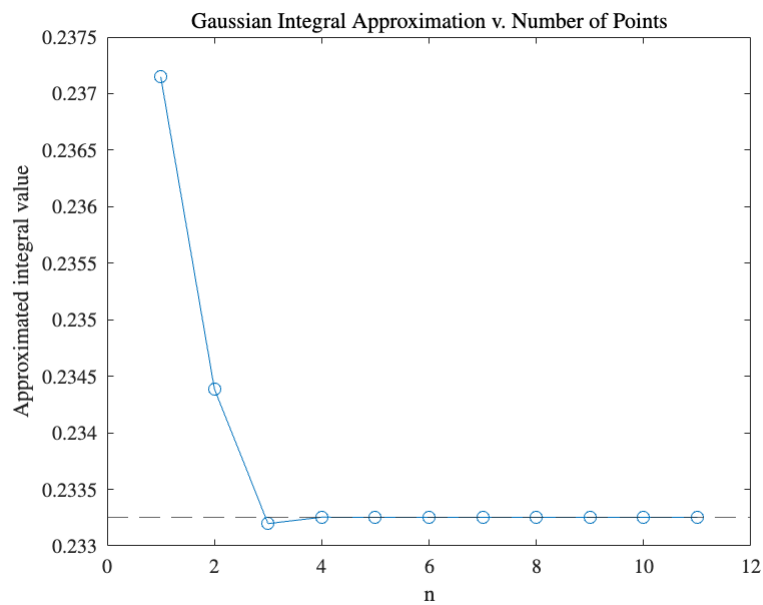


	Var1	a01	a12	b01	b12
7	7	0.1895	0.2333	0.4536	0.1532
8	8	0.1895	0.2333	0.4536	0.1532
9	9	0.1895	0.2333	0.4535	0.1532
10	10	0.1895	0.2333	0.4535	0.1532
11	11	0.1895	0.2333	0.4535	0.1532

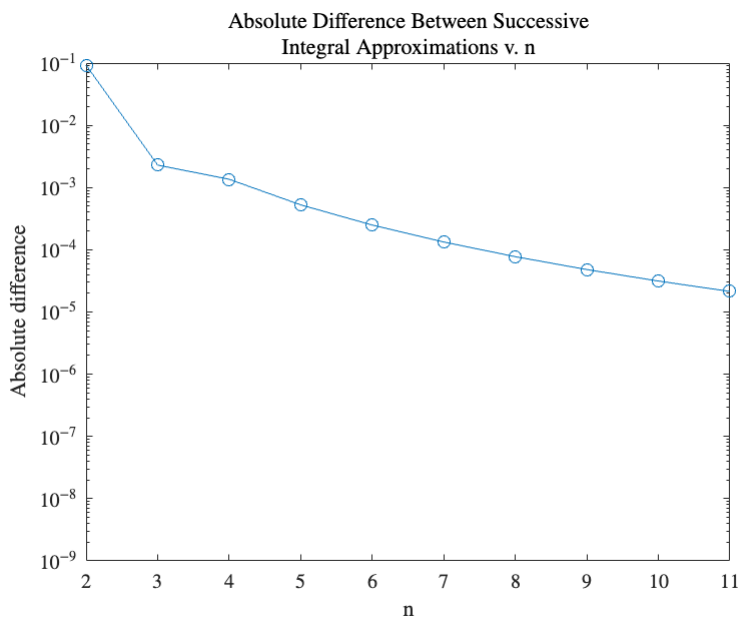
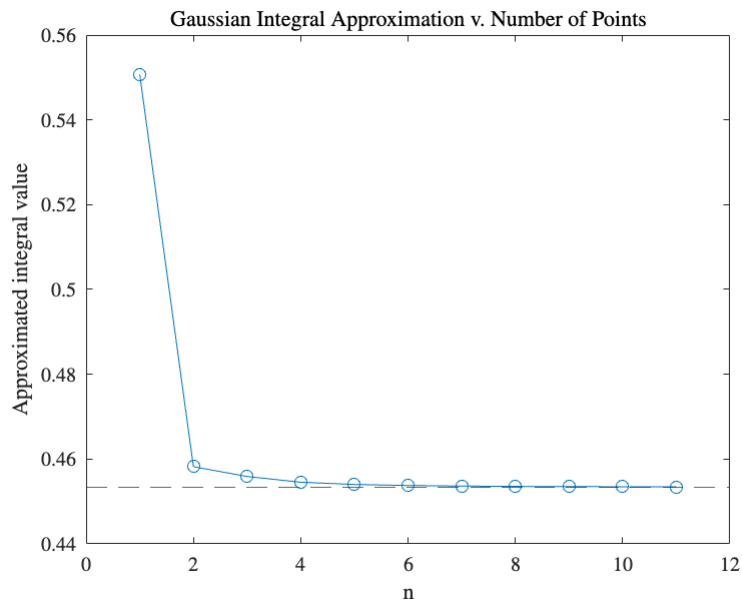
function (a) with limits of integration from 0 to 1



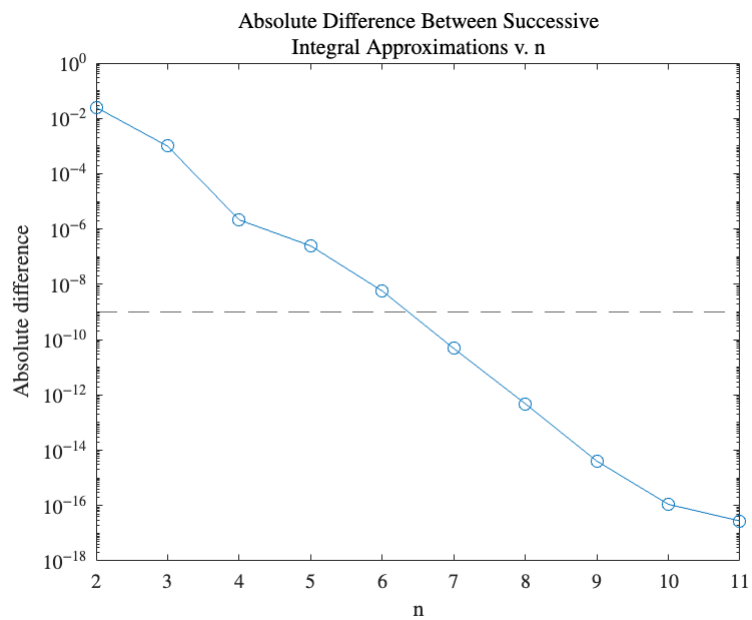
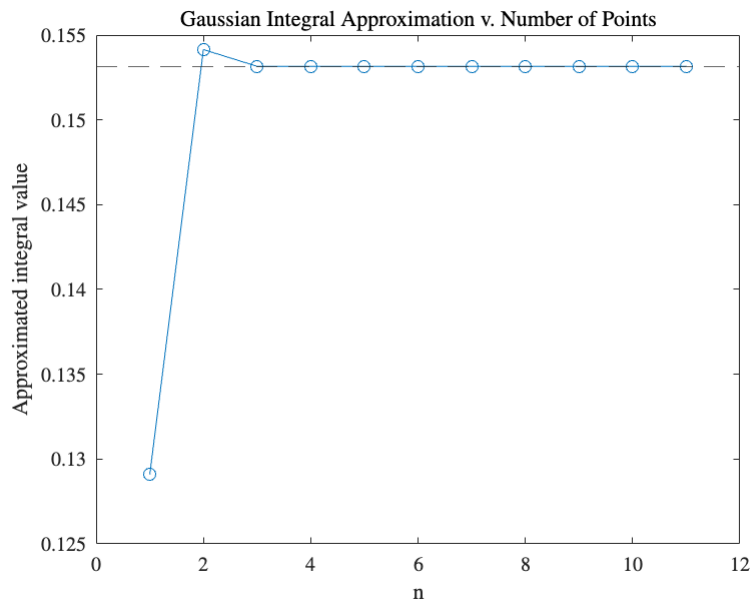
function (a) with limits of integration from 1 to 2



function (b) with limits of integration from 0 to 1



function (b) with limits of integration from 1 to 2



### 3. IVP with Euler's, Midpoint Rule (RK2), Modified Euler's, 2-Step A-B/A-M Predictor-Corrector, and RK4

```
main3
```

