Name: ____________________ ID: ____________

COMP5201 Assignment 2 Fall 2023
Computer Organization & Design Prof. D.K. Probst
Issued: September 26, 2023 Due: October 19, 2023
Submit electronically to Moodle. No extension will be granted.

1. [24 marks] Digital Logic.

a) 'Z' is the ternary 'odd' connective. 'Zpqr' is true iff an odd number of its arguments is true. 'p + q' is 'xor'. 'F' and 'T' denote the two 0-ary connectives 'false' and 'true'. Put 'sentence' letters (p, q, ...) in alphabetical order and as far to the left as possible..

Using {'Z', 'F', 'T'}, synthesize: ~p |= =| Z ____ ____ ____

Using {'Z', 'F', 'T'}, synthesize: p + q |= =| Z ____ ____ ____

b) '#' is the ternary 'majority' connective. '#pqr' is true iff a majority of its arguments is true. 'F' and 'T' denote the two 0-ary connectives 'false' and 'true'. Put 'sentence' letters (p, q, ...) in alphabetical order and as far to the left as possible.

Using {'#', '~'. 'F', 'T'}, synthesize: p | q |= =| ____ # ____ ____ ____

Using {'#', 'F', 'T'}, synthesize: p \/ q |= =| # ____ ____ ____

2. [16 marks] Binary and Hexadecimal Numbers.

Convert each of the following six binary or hexadecimal natural numbers into decimal. Show work.

a) Binary numbers: 1111, 1111 1011, 1111 1011 1110 0101

b) Hexadecimal numbers: ae, aebb, aebb 51ff

3. [16 marks] Binary Rationals

A number 'a' is a _binary rational_ iff it has a finite binary expansion. The bits to the right of the binary point are called the _fractional part_ of the expansion. When 'a' is nonintegral, the fractional part corresponds to some number 'b' less than one that can be put in the form 'm/2^n', where 'm' is an odd integer less than '2^n' and 'n' is greater than zero. In this case, the fractional part has exactly 'n' bits. Integers obviously qualify as binary rationals but don't interest us now as they have no fractional parts until we normalize their binary expansions.

Binary and decimal expansions of the same number always have the same lengths of their fractional parts. Examples:

```
  bin 0.1    = dec 0.5
  bin 0.01   = dec 0.25
  bin 0.001  = dec 0.125
  bin 0.111  = dec 0.875
  bin 0.1101 = dec 0.8125
```

a) Find the decimal expansion that represents the same number as:

```
   bin 0.1011 1101
```

b) In a _ternary rational_, we would change the form to 'm/3^n' and make 'm' an integer less than '3^n' not divisible by 3. Do ternary and decimal expansions of the same number always have the same lengths of their fractional parts? Try some examples.

4. [24 marks] Fractional Numbers and Floating-Point Blackboard Notation.

Infinite binary expansions of _rational numbers_ are either i) pure recurring or ii) mixed recurring. That is, the cycle starts immediately after the binary point (pure case), or somewhat later (mixed case).

a) Show the infinite binary expansion of 45 1/28 without normalization.
Hint: 28 = 4 * 7.

b) Show the infinite hexadecimal expansion of 45 1/28 without normalization.

c) Show the infinite binary expansion of 45 1/28 with normalization. Write the binary coefficient as an infinite bitstring, but show the scale factor with a decimal exponent. This is a _scaled_ binary expansion. Any binary cycle automatically creates a hexadecimal cycle.

d) Show, in normalized floating-point blackboard notation, an approximation to 45 1/28 for a fixed-size register. You will need to truncate the true fractional part. The register's fractional field is 20 bits. Show these 20 bits in full. Now, show these 20 bits in hexadecimal in full (5 hexits).

5. [20 marks] Integer Adders

a) Show that, regardless of the initial n-bit value of the accumulator, the fused multiply-add result of two n-bit natural-number operands is always representable in '2n' bits. Next, starting from the largest possible FMA result, what is the largest n-bit number that can _still_ be added without producing overflow?

b) A particular modular adder 'M' operates with 16-bit operands and reports its results in 16-bit registers. If you give 'M' two 16-bit natural numbers 'a' and 'b', it will add them correctly, divide the answer by 2^16, keep the quotient 'q' a secret, and publish the remainder 'r'. Hint: Before answering, experiment with some small addition tables.

i) If a = 42,734 and r = 49,529, what are 'b' and 'q'?

ii) If a = 17,345 and r = 13,920, what are 'b' and 'q'?