Delft University of Technology

Networked and Distributed Control Systems

SC42100

Assignment 1

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1 The rise of the Medici

This MATLAB assignment is only party covered in this rapport, the full results can be found in the attached m-file, namely Assignment1_Q1.
The nodes have been named as given in figure 1.

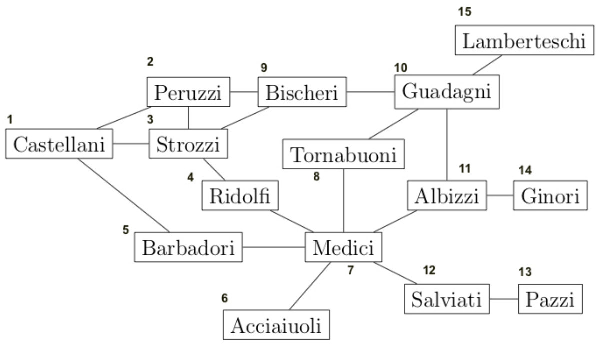

Figure 1: Node naming

a)
Bonacich centrality:

$$\begin{aligned} \text{Medici} &:= 0.5388 \\ \text{Strozzi} &:= 0.3592 \\ \text{Tornabuoni} &:= 0.1796 \end{aligned}$$

b)
Closeness centrality:

$$\begin{aligned} \text{Medici} &:= 0.600 \\ \text{Strozzi} &:= 0.455 \\ \text{Tornabuoni} &:= 0.484 \end{aligned}$$

c)

	δ=0.25	δ=0.50	δ=0.75
Medici	1.859	1.273	1.016
Strozzi	0.653	0.435	0.275
Tornabuoni	13.751	13.673	13.692

For low values of δ decay centrality puts much more weight on closer nodes, thus becoming proportional to degree centrality, whereas for high values of δ it measures the size of the component a node lies in. The centralities converge to 0 for $\delta \to 0$, and converge to 14 (ie number of nodes $-$ 1) for $\delta \to 1$.

d)
Betweennes centrality:

$$\begin{aligned} \text{Medici} &:= 0.1904 \\ \text{Strozzi} &:= 0.0664 \\ \text{Tornabuoni} &:= 0.0710 \end{aligned}$$

e)
From a visual inspection one would expect the order of centrality to be lead by the Medici, followed by Strozzi and Tornabuoni. Of course this depends on the way centrality is defined. The expectation is largely verified by the determined centralities from this assignment, only using the closeness centrality, the order of Strozzi and Tornabuoni is swapped.

2 Graphs and Opinions

a)
A graph is balanced if $w^- = w$ wherein w is the out-degree and w^- is the in-degree. In this case both are 15 and hence the graph is balanced.

For the graph to be regular $w_i = w_i^- = \bar{w}$ for all i. Since this does not hold because $\bar{w} = 2.5$ the graph is not regular. This could also have been seen at first sight since not all nodes have the same amount of links.

The diameter of G is the maximum of the shortest distances from i to j. In this case the diameter is 3, caused by going from 1 to 6 or from 4 to 3 or for both the other way around.

The graph is aperiodic because it contains a self-loop.

b)
The weight matrix W is given as:

$$W = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{1}$$

The outdegree vector is given as:

$$w = W1 = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \\ 3 \\ 2 \end{bmatrix} \tag{2}$$

The normalized weight matrix is given as:

$$P = D^{-1}W = \begin{bmatrix} 1/3 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 \end{bmatrix} \tag{3}$$

with the degree matrix:

$$D = diag(w) = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{4}$$

The Laplacian matrix is given as :

$$L = D - W \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -2 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \tag{5}$$

c)
The normalised Bonacich centrality vector π is the corresponding non negative eigenvector of

$$\pi = P'\pi \tag{6}$$

with eigenvalue one. Hence π is found by solving

$$(T' - I)\pi = 0 \tag{7}$$

However, the given graph is a balanced graph. As discussed in the lectures of course SC42100 2018/2019 the node centrality then is the normalized degree given as

$$\pi_i = \frac{w_i}{n\bar{w}} \tag{8}$$

with $\bar{w}$ as the average degree.

$$\begin{aligned} \bar{w} &= \frac{1}{n} * \mathbb{1}'W\mathbb{1} \\ &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \frac{15}{6} \end{aligned} \tag{9}$$

From here the normalized Bonacich centrality vector is given as

$$\begin{aligned}\pi_i &= \frac{w_i}{15} \\ &= \begin{bmatrix}\frac{3}{15} & \frac{3}{15} & \frac{2}{15} & \frac{2}{15} & \frac{3}{15} & \frac{2}{15}\end{bmatrix}\end{aligned} \tag{10}$$

d)
The degree centrality nodes of node 1 & node 5 means simply comparing the nodes by their degree. In this case in- or out degree does not matter since $w = w^{-}$. The degrees of the nodes are $w_1 = 3$ and $w_5 = 3$ and hence are the same. Since w_1 has a self link, this can be seen as less important than the third link of w_5. From this it is concluded that w_5 is more central according to the degree of centrality.

The closeness centrality of nodes w_1 and w_2 are given by

$$\begin{aligned}\pi_i &= \frac{n}{\sum j \in vdist(i,j), i \in v} \\ \pi_1 &= \frac{6}{1+2+1+2+3} = \frac{2}{3} \\ \pi_2 &= \frac{6}{1+1+2+1+2} = \frac{6}{7}\end{aligned} \tag{11}$$

Hence node w_2 is more central according to the closeness centrality measure.

For betweenness centrality all maximum distance paths from i to j are considered for every two nodes i and j. Let g_{ij}^k be the fraction of such paths passing over node k.

$$\begin{aligned}\text{betweenness}(k) &= \frac{1}{n^2}\sum_{i,j} g_{ij}^k, k \in v \\ \text{betweenness}(2) &= \frac{1}{36}(\frac{4}{7} + \frac{4}{7} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6}) = \frac{15}{252} \\ \text{betweenness}(3) &= \frac{1}{36}(\frac{1}{6} + \frac{1}{6} + \frac{2}{7}) = \frac{26}{1512}\end{aligned} \tag{12}$$

From this it can be concluded that node w_2 is more central according to the betweennes centrality measure.

e)
The centrality vector means a normalized π so that $\sum_i \in v\pi_i = 1$ such that π becomes a probability vector. If $x(t)$ converges it's limit must be a consensus vector .

As concluded in **a**, the graph is aperiodic and connected. Given this, the opinion vector converges.

f)
To minimize the De Groot opinion dynamics the smallest values are assigned to the largest nodes. The largest values are assigned to the smallest nodes. This results in the

limit given below with the second vector representing the chosen initial opinions.

$$\lim_{t\to+\infty}\sum_i x_i(0)^- = \begin{bmatrix}\frac{3}{15} & \frac{3}{15} & \frac{2}{15} & \frac{2}{15} & \frac{3}{15} & \frac{2}{15}\end{bmatrix}\begin{bmatrix}1\\1\\2\\2\\1\\2\end{bmatrix} = \frac{21}{15} \tag{13}$$

To maximize the De Groot opinion dynamics the smallest values are now assigned to the smallest nodes and the largest values to the largest nodes. This results in the limit given below with the second vector being the initial opinions.

$$\lim_{t\to+\infty}\sum_i x_i(0)^+ = \begin{bmatrix}\frac{3}{15} & \frac{3}{15} & \frac{2}{15} & \frac{2}{15} & \frac{3}{15} & \frac{2}{15}\end{bmatrix}\begin{bmatrix}2\\2\\1\\1\\2\\1\end{bmatrix} = \frac{24}{15} \tag{14}$$

g)
The asymptotic opinion value of the remaining nodes is given by

$$y = \begin{bmatrix}y2\\y3\\y4\\y5\end{bmatrix} = (I-Q)^{-1}Bu, y \in \mathbb{R}^{\mathcal{R}} \tag{15}$$

With remaining nodes the set of regular agents $\mathcal{R} = \{2, 3, 4, 5\}$ is meant. The stubborn agents are in $\mathcal{S} = \{1, 6\}$. The needed matrix are given below after which the asymptotic opinion values are calculated.

$$\begin{aligned} Q &= \begin{bmatrix}0 & \frac{1}{3} & 0 & \frac{1}{3}\\ \frac{1}{2} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{2}\\ \frac{1}{3} & 0 & \frac{1}{3} & 0\end{bmatrix}, Q \in \mathbb{R}^{\mathcal{R}x\mathcal{R}}\\ B &= \begin{bmatrix}\frac{1}{3} & 0\\ 0 & \frac{1}{2}\\ \frac{1}{2} & 0\\ 0 & \frac{1}{3}\end{bmatrix}, B \in \mathbb{R}^{\mathcal{R}x\mathcal{S}}\\ u &= \begin{bmatrix}0\\1\end{bmatrix}, u \in \mathbb{R}^{\mathcal{S}}\\ y &= \begin{bmatrix}\frac{2143}{5000}\\ \frac{7143}{10000}\\ \frac{2857}{100000}\\ \frac{2857}{5000}\end{bmatrix} \end{aligned} \tag{16}$$

h)
The connected components in $\mathcal{G}_{\mathcal{A}}$ are nodes 1,2 and the group 3, 4 and 5. The condensation graph is given in figure 2.
help

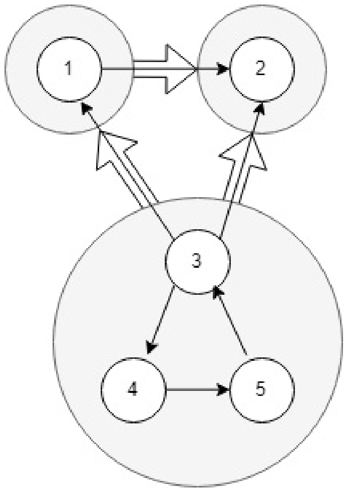

Figure 2: Condensation graph of $\mathcal{G}_{\mathcal{A}}$ consisting of three super nodes, depicted in grey.

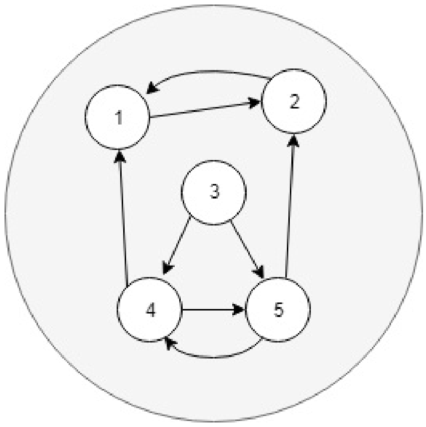

Figure 3: The condensation graph of $\mathcal{G}_{\mathcal{B}}$ consisting of one super node, depicted in grey.

For $\mathcal{G}_{\mathcal{B}}$ we find that each node can be reached from each node, meaning that the graph is connected. Therefore the condensation graph is given by a single super node depicted in figure 3.

3 Influence on Twitter

This MATLAB assignment is only party covered in this rapport, the full results can be found in the attached m-file, namely Assignment1_Q3.
a)
Before the iteration can take place, the adjacency matrix must be made square for the problem to be solvable. To prevent the added columns to act as virtual users and sinks, all are given a self-loop, such that the degree matrix has no zero's. The iterative computation gave the following Twitter users to have the highest page rank centrality.

User	**PageRank**
2	0.0175
11	0.0163
35	0.0064
26	0.0063
112	0.0062

b
Below firstly in figure 4, the change of opinions of the nodes [500 : 500 : 5000] are shown, for when the stubborn nodes are chosen to be the two most PageRank central nodes. This means that all the opinions of the nodes adapt to the two stubborn nodes.

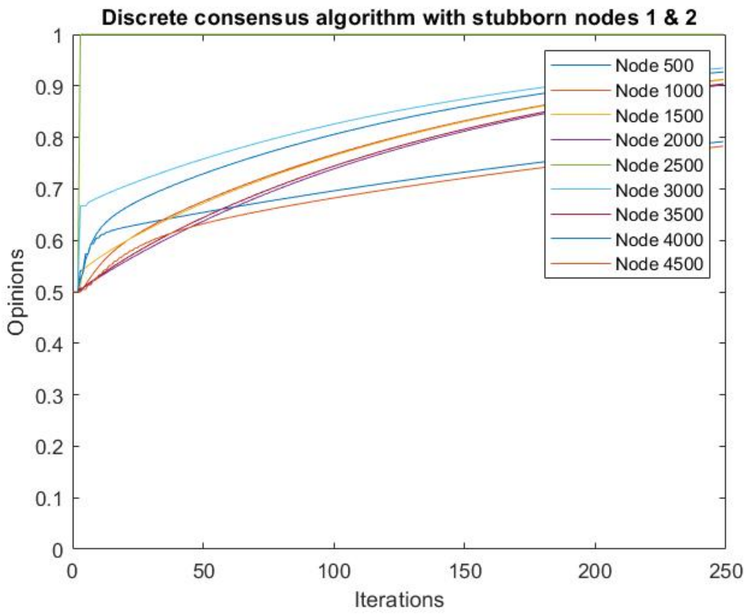

Figure 4: Discrete Simulation, stubborn nodes with high centrality

c
In figure 5, the change of opinions of the same nodes is given with respect to stubborn nodes 11 and 35 which have low centrality. It can now be seen that some of the nodes do not converge to 0 or 1 and hence are not influenced by the opinion of the stubborn nodes.

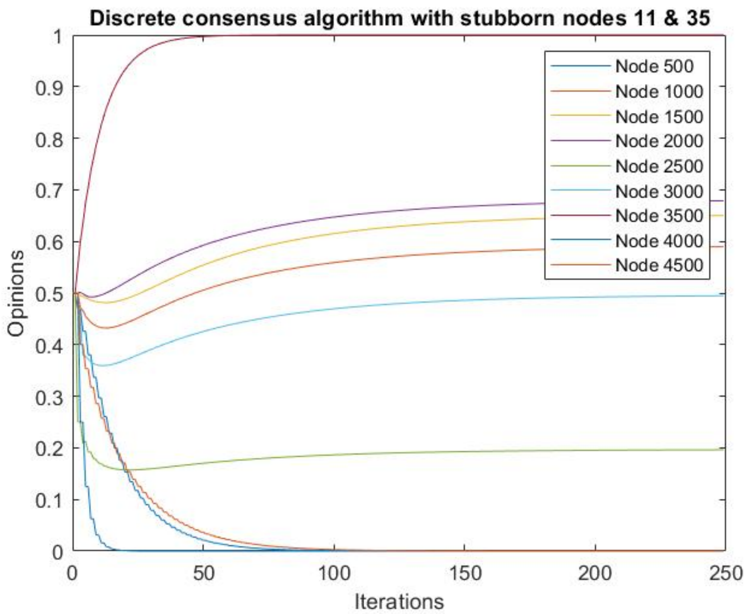

Figure 5: Discrete Simulation, stubborn nodes with low centrality

4 Flows, flows, flows

a)
There are nine origin destination cuts. The capacity of these cuts and the links they are

cutting through are given in table 1. The min-cut capacity is $Cu_3 = 14$. The cuts are displayed in figure 6 and table **??**.

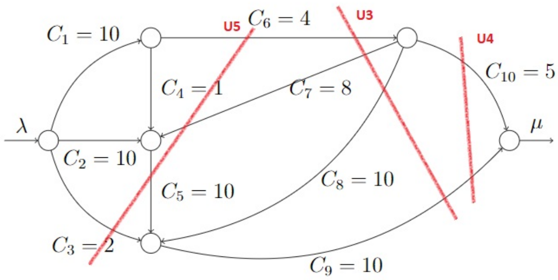

Figure 6: Flow network with cuts U3, U4 and U5 displayed.

Cut	**Including**	**Capacity**
U1	C1 + C2 + C3	22
U2	C1 + C9	20
U3	C6 + C9	**14**
U4	C9 + C10	**15**
U5	C3 + C5 + C6	16
U6	C2 + C3 + C4 + C_6	17
U7	C1 + C9	20
U8	C3 + C5 + C8+ C10	27
U9	C2 + C3 + C4 + C7 + C8 + C10	36

Table 1: Possible cuts and their capacities

b)
A possible flow to facilitate a throughput of $\lambda = 11$ is given in figure 7. To show this clearly the capacities are set to the equivalent numbers of flow going through each link.

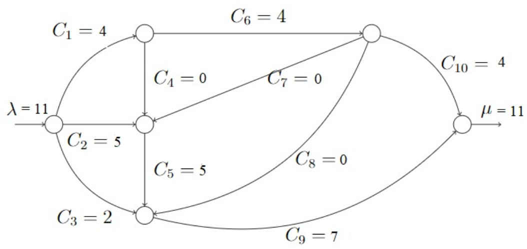

Figure 7: A throughput of $\lambda = 11$ and their corresponding capacities

c)
The minimum total capacity that has to be removed from the network so that no feasible throughput of $\lambda = 8$ can exists is 7 from C9 making the capacity there 3.

d)
The two extra units are distributed across C6 and C9. The minimum capacity then becomes 16 instead of 14. U3, U4 and U5 then all have the minimum capacities.

e)
The Wardrop equilibrium describes a flow network were all the agents make selfish choices, neglecting the social optimum. The Wardrop equilibrium flow vector is given by

$$f^{(0)} = Az \tag{17}$$

Wherein A is the link-path incidence matrix given by

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \in \mathbb{R}^{\mathcal{E}x\mathcal{P}} \tag{18}$$

$z \in \mathcal{R}^{\mathcal{P}}$ such that $z \geq 0, \infty' z = \tau$ and for every path $p \in \mathcal{P}$

$$z_p > 0 \rightarrow \sum_{e \in \mathcal{E}} A_{ep} d_e(f_e^{(0)}) \leq \sum_{e \in \mathcal{E}} A_{eq} d_e(f_e^{(0)}) \tag{19}$$

In this case our graph $\mathcal{G}$ has three paths, $p^{(1)}, p^{(2)}, p^{(3)}$ and a distribution $z = (z_{p(1)}, z_{p(2)}, z_{p(3)})$ such that $z > 0$ and $z' \infty = 1$ is a vector of flow assignments to origin-destination paths. The flow vector is then given as

$$f = \begin{bmatrix} z_{p(1)} + z_{p(3)} \\ z_{p(2)} \\ z_{p(3)} \\ z_{p(1)} \\ z_{p(2)} + z_{p(3)} \end{bmatrix} \tag{20}$$

Whereby $z_{p(1)} + z_{p(2)} + zp(3) = 1$ The delays per path are given as

$$\begin{aligned} \textbf{Total delay p(1)} &= de1(fe1) + de4(fe4) \\ &= 6z(p1) + z(p3) + 2 \\ \textbf{Total delay p(2)} &= de2(fe2) + de5(fe5) \\ &= 6z(p2) + z(p3) + 2 \\ \textbf{Total delay p(2)} &= de1(fe1) + de3(fe3) + de5(fe5) \\ &= z(p1) + z(p2) + 2z(p3) + 3 \end{aligned} \tag{21}$$

Solving this gives $z_{p(1)} = z_{p(2)} = z_{p(3)} = \frac{1}{3}$. With a total delay of $4\frac{1}{3}$ for each route. Filling the found distribution back in in equation 20 gives a flow vector of

$$f^{(0)} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \tag{22}$$

f)

The social optimum makes sure that the average delay for all agents is as low as possible. In order to solve this optimization problem equation XX is used.

$$M(v) = \min_{f \geq 0, Bf = v} \sum_{e \in \mathcal{E}} c_e(f_e) \tag{23}$$

Now ignoring the constraint that all total path delays must be equal, $z_{p(1)} = z_{p(2)} = 0$ and $z_{p(3)} = 1$. This gives a total delay of 11, and an average delay per path of $3\frac{2}{3}$. This results in the flow vector

$$\begin{aligned} f^* &= \begin{bmatrix} z_{p(1)} + z_{p(3)} \\ z_{p(2)} \\ z_{p(3)} \\ z_{p(1)} \\ z_{p(2)} + z_{p(3)} \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \end{aligned} \tag{24}$$

g)

The price of anarchy is given as

$$PoA(\omega) = \frac{\sum_{e\in\rceil} f_e^{(0)} d_e(f_e^{(0)}}{\sum_{e\in\rceil} f_e^* d_e(f_e^*} = \frac{4\frac{1}{3}}{3\frac{2}{3}} = 1\frac{2}{11} \tag{25}$$

h) When the Price of Anarchy is one, the Wardrop equilibrium delay must be equal to the delay of the social optimum. In order to achieve that the wardrop equilibrium flow vector becomes the flow vector as in **4f**, a toll of two must be added to link 2 and link 4. This gives a total delay per path of 5 for both the Wardrop equilibrium as the social optimum. Since the delay at the user optimum is then equal to the delay at the social optimum, the price of anarchy becomes one.

5 Traffic in LA

This MATLAB assignment is only party covered in this rapport, the full results can be found in the attached m-file, namely Assignment1_Q5.

a)
Searching for the path with respect to traveling time and assuming empty streets yields:

$$1 \to 2 \to 3 \to 9 \to 13 \to 17.$$

b)
The maximum flow through the network of streets with respect to the given capacities is:

$$\text{Maxflow} = 22448.$$

c)
The following table shows the external net inflow and outflow of the nodes. Logically, the minus sign means the net flow is outbound, positive inbound.

Node	In-/Outflow
1	16806
2	8570
3	19448
4	4957
5	-746
6	4768
7	413
8	-2
9	-5671
10	1169
11	-5
12	-7131
13	-380
14	-7412
15	-7810
16	-3430
17	-23544

d)
The objective function is

$$\sum_{e\in\mathcal{E}} \frac{l_e C_e}{1 - f_e/C_e} - l_e C_e$$

which translates in CVX to

```
sum(((l.*C).*inv_pos(1-(f./C)))-(l.*C))
```

The constraints, namely that the flow must be smaller than the capacities and greater than zero, and the external inflow is $\tau_1 = 16806$ at node 1, which is equal to the outflow at node 17 ($\tau_{17} = -16806$), translate into:

```
        0 <= f <= C;
        B(1,:)*f == 16806;
        B(17,:)*f == -16806;
        B(2:16,:)*f == 0;
```

These constraints will also be used for the coming sub questions. The flow vector can be found in Appendix A.

e)
The cost function for finding the Wardrop equilibrium f^W is determined by solving the integral

$$\sum_{e\in\mathcal{E}} \int_0^{f_e} d_e(x)dx = l_e C_e \log(C_e) - l_e C_e \log(C_e - f_e).$$

The flow vector can be found in Appendix A.

f)
The toll matrix is

$$\omega_e = f_e^* d_e'(f_e^*) = f_e^* \frac{C_e l_e}{(C_e - f_e^*)^2} \tag{26}$$

The optimum flows for the Wardrop equilibrium and the system optimum are equal, i.e. $f^* = f^{(\omega^*)}$. (Though, probably because of the solvers accuracy the numerical result is $f^* \approx f^{(\omega^*)}$.)

g)
With the new given cost function, the toll matrix can be set up using

$$c_e'(f_e^*) = f_e^* d'(f_e^*) + d(f_e^*) - l_e$$

and $d'(f_e^*)$ as in Eq (26), to become

$$\omega_e^* = f_e^* \frac{C_e l_e}{(C_e - f_e^*)^2} - l_e.$$

The flow vectors can be found in Appendix A.

6 The little chemist

a)
The compartmental system is open due to the external inflow λ into $x1$. In order to model the system correctly a sink $x0$ is added underneath flow $k20$. According to , the desired flow dynamics are given by:

$$\dot{x} = -L'x(t) \tag{27}$$

To find L the weight matrix W is set up. The dynamics of the dummy node $x0$ ar found in the first column of W . Multiplying by a vector of ones gives the weight vector w.

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k14 \\ k20 & k21 & 0 & 0 & 0 \\ 0 & 0 & k32 & 0 & 0 \\ 0 & 0 & 0 & k43 & 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ k14 \\ k20 \\ k21 \\ k32 \\ k43 \end{bmatrix} \tag{28}$$

Putting the weight on the diagonal of a unit matrix, forming matrix D

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k14 & 0 & 0 & 0 \\ 0 & 0 & k20 + k21 & 0 & 0 \\ 0 & 0 & 0 & k32 & 0 \\ 0 & 0 & 0 & 0 & k43 \end{bmatrix} \tag{29}$$

A step further the sought after Laplacian matrix is found.

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k14 & 0 & 0 & -k14 \\ -k20 & -k21 & k20 + k21 & 0 & 0 \\ 0 & 0 & -k32 & k32 & 0 \\ 0 & 0 & 0 & -k43 & k43 \end{bmatrix} \tag{30}$$

Filling the found L in equation 27 gives the desired flow dynamics.

b)
Now the inflows are known, the limit densities for each stage can be found by calculating the limit of the subsystem

$$\lim_{t\to\infty} y(t) = (M')^{-1}\lambda \tag{31}$$

M represents the subsystem of L without the dummy sink and is given by

$$M = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, (M')^{-1} = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix} \tag{32}$$

This is can be done because the system is open. The vector λ displays the external inflows at the nodes and $\lim_{t\to\infty} y(t)$ the limit densities at each stage.

$$\lambda = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \lim_{t\to\infty} y(t) = \begin{bmatrix} 20 \\ 10 \\ 20 \\ 20 \end{bmatrix} \tag{33}$$

7 Bibliography

All fundamental information of this document is described by the sources below.

Giacomo Como, Chapter 1 Networks as graphs Network Centrality and Connectivity, course SC42100 2018/2019, University of Technology

Giacomo Como, Chapter 2 Deterministic Linear Network Dynamics, course SC42100 2018/2019, University of Technology

Giacomo Como, Chapter 4 Network Flow Optimization, course SC42100 2018/2019, University of Technology

8 Appendix A

Flow vectors for questions 5.d, 5.e, 5.f, 5.g respectively.

Link	Flow
1	6642
2	6059
3	3132
4	3132
5	10164
6	4638
7	3006
8	2543
9	3132
10	583
11	0
12	2927
13	0
14	3132
15	5525
16	2854
17	4886
18	2215
19	464
20	2338
21	3318
22	5656
23	2373
24	0
25	6414
26	5505
27	4886
28	4886

Link	Flow
1	6716
2	6716
3	2367
4	2367
5	10090
6	4645
7	2804
8	2284
9	3418
10	0
11	177
12	4171
13	0
14	2367
15	5445
16	2353
17	4933
18	1842
19	697
20	3036
21	3050
22	6087
23	2587
24	0
25	6919
26	4954
27	4933
28	4933

Link	Flow
1	6642
2	6059
3	3132
4	3132
5	10164
6	4638
7	3006
8	2543
9	3132
10	583
11	0
12	2927
13	0
14	3132
15	5525
16	2854
17	4886
18	2215
19	464
20	2338
21	3318
22	5656
23	2373
24	0
25	6414
26	5505
27	4886
28	4886

Link	f^*	$f^{(\omega^*)}$
1	6653	6653
2	5775	5775
3	3420	3420
4	3420	3420
5	10153	10153
6	4643	4643
7	3106	3106
8	2662	2662
9	3009	3009
10	878	878
11	0	0
12	2355	2355
13	0	0
14	3420	3420
15	5510	5510
16	3044	3044
17	4882	4882
18	2415	2415
19	444	444
20	2008	2008
21	3487	3487
22	5495	5495
23	2204	2204
24	0	0
25	6301	6301
26	5624	5623
27	4882	4882
28	4882	4882