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22C:188 HW3
Spring 2014

HW3

#1 Formalization in FOL

1) $R(x)$: x is a red thing
$B(x)$: x is in the box

a) All red things are in the box.
$\forall x(R(x) \rightarrow B(x))$

b) Only red things are in the box
$\forall x(B(x) \rightarrow R(x))$

c) No animal is both a cat and a dog.
$A(x)$: x is an animal
$C(x)$: x is a cat
$D(x)$: x is a dog

$\forall x(A(x) \rightarrow \neg(C(x) \wedge D(x)))$
$\forall x(A(x) \rightarrow (\neg C(x) \vee \neg D(x)))$

d) Every prize was won by a boy.
$P(y)$: y is a prize
$B(x)$: x is a boy
$W(x,y)$: x won y (Adam won the lottery)

$\forall y(P(y) \rightarrow \exists x(B(x) \wedge W(x,y)))$

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1) e) A boy won every prize.

$P(y)$: y is a prize
$B(x)$: x is a boy
$W(x,y)$: x won y

$$\exists x \forall y \, (B(x) \wedge P(y) \rightarrow W(x,y))$$

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2) a) $\forall x \forall y (x = y)$ (where all quantifiers have the same domain)

- Every pair of elements consists of two equal elements.

b) $\exists x \exists y (x \neq y)$ (where all quantifiers have the same nonempty domain)

- There exists at least one pair of elements such that the elements are not equal.

c) $\exists x \exists y (x \neq y \wedge \forall z (z \neq x \rightarrow z = y))$ (where all quantifiers have the same nonempty domain)

- There exists a model with only two distinct elements.

Note:		$z \neq x$	$z = y$	$z \neq x \rightarrow z = y$	$x \neq y \wedge \forall z (z \neq x \rightarrow z = y$
2 distinct elements	$(x, y = z, y = z)$	T	T	T	T
3 distinct elements	(x, y, z)	T	F	F	F
1 distinct element	$(x = z, y = z)$	F	T	T	F
2 distinct elements	$(x = z, y)$	F	F	T	T

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#1 Formalization in FOL

3) If there are any tax payers, then all politicians are tax payers. If there are any philanthropists, then all tax payers are philanthropists. Therefore, if there are any tax-paying philanthropists, then all politicians are philanthropists.

$T(x)$: x is a tax payer
$P(x)$: x is a politician
$H(x)$: x is a philanthropist

$$\exists x \left(T(x) \rightarrow \left(\forall x \left(P(x) \rightarrow T(x) \right) \right) \right),$$

$$\exists x \left(H(x) \right) \rightarrow \forall x \left(T(x) \rightarrow H(x) \right),$$

$$\vdash \quad \exists x \left(T(x) \wedge H(x) \right) \rightarrow \forall x \left(P(x) \rightarrow H(x) \right)$$

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#1 Formalizing in FOL

4) It is possible to specify a formula for models with a specified concrete number of elements; for example a model with exactly three distinct elements or even at most three distinct elements. When not specifying a concrete bound as in the previous two examples, it is possible to specify an upper bound with the "$\bigwedge_{i=1}^{k}$" operator. Within this operator, the upper bound is 'k', where 'k' is an arbitrary natural number. A subset bounded by k is a subset of infinitely many elements. A subset of an infinite set is still an infinite set. Therefore, there can be no formula in first-order logic that is true only for models with finitely many distinct elements.