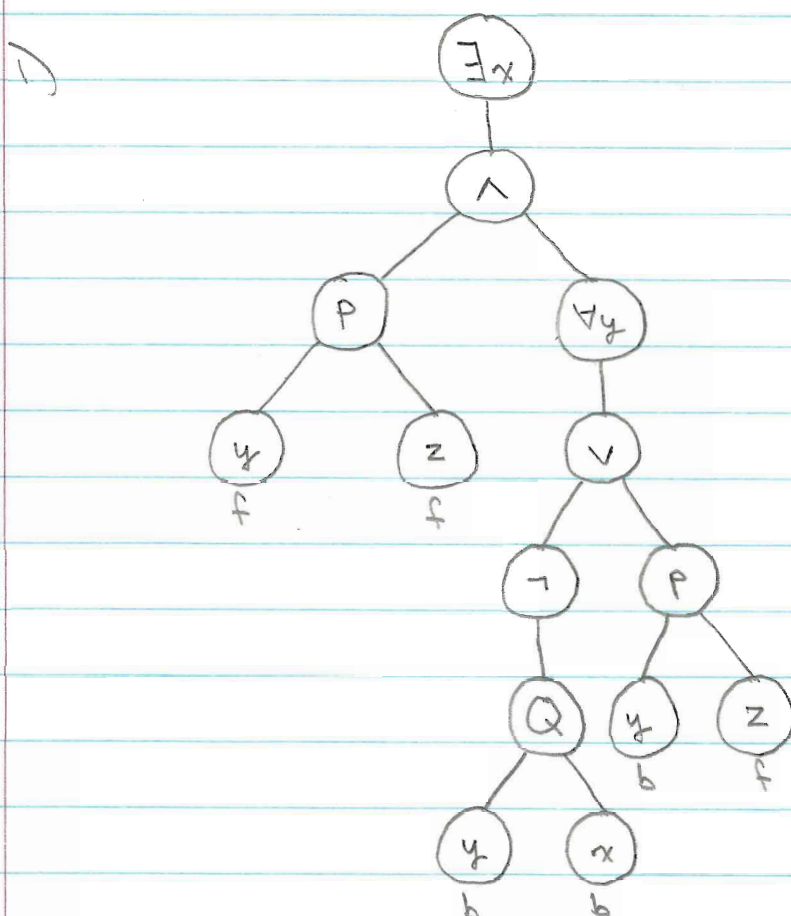


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#2 Parse Trees, Variables and Substitutions

Let the formula Φ be

$$\exists x (P(y, z) \wedge (\forall y (\neg Q(y, x) \vee P(y, z))))$$



2) For each variable occurrence in Φ identify if it is free or bound.

$$\exists x \left(\underset{f}{P}(\underset{f}{y}, \underset{f}{z}) \wedge (\forall y (\neg Q(\underset{b}{y}, \underset{b}{x}) \vee P(\underset{b}{y}, \underset{f}{z}))) \right)$$

$f \equiv \text{free}$

$b \equiv \text{bound}$

#2 Parse Trees, Variables and Substitutions

3) Consider the variable w , the unary function symbol f and the binary function symbol g .

(a) Which of w , $f(x)$, and $g(x, y)$ are free for x in Φ ?
The variable w , $f(x)$, and $g(x, y)$ are free for x in Φ since there are no free occurrences of x .

(b) Which of w , $f(x)$, and $g(x, y)$ are free for y in Φ ?
The variable w and the binary function $g(x, y)$ are free for y in Φ .

(c) Compute $\Phi[w/x]$, $\Phi[f(w)/y]$, and $\Phi[g(w, z)/z]$.

$$\Phi[w/x] \equiv \exists x (P(y, z) \wedge (\forall y (\neg Q(y, x) \vee P(y, z))))$$

$$\Phi[f(w)/y] \equiv \exists x (P(f(w), z) \wedge (\forall y (\neg Q(y, x) \vee P(y, z))))$$

$$\Phi[g(w, z)/z] \equiv \exists x (P(y, g(w, z)) \wedge (\forall y (\neg Q(y, x) \vee P(y, g(w, z)))))$$