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22C: 188 HW3
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#4 Semantics of FOL

2) $\Phi \stackrel{def}{=} \forall x \exists y \exists z ((P(x,y) \wedge P(z,y) \wedge (P(x,z) \rightarrow P(z,x)))$
Let the universe $A$ be the set of natural numbers $\mathbb{N}$.

a) $P^{m} \stackrel{def}{=} \{(m,n) \mid m < n\}$ ; satisfies $\Phi$

Let's first look at it as $\forall x \exists y \exists z ((x<y) \wedge (z<y) \wedge ((x<z) \rightarrow (z<x)))$
①②③
where all 3 terms must evaluate to True for $\Phi$ to be satisfied.

| $x<z$ | $z<x$ | $(x<z) \rightarrow (z<x)$ | $\forall x \in \mathbb{N}$ and $\exists z \in \mathbb{N}$ |
|---|---|---|---|
| T | T | T | invalid |
| T | F | F | valid but false |
| F | T | T | $\forall x \in \mathbb{N}$, $x<z$ is true $\Rightarrow$ invalid |
| F | F | T | T when $x = z \in \mathbb{N}$ |

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2) b) $p^{m} \overset{def}{=} \{(m, 2m) \mid m \in A\}$; satisfies $\Phi$

The set of natural numbers, $\mathbb{N}$, is the universe A, as given. Every natural number multiplied by 2 is still in the universe of natural numbers.

c) $p^{m} \overset{def}{=} \{(m,n) \mid m < n+1\}$

$\forall x \exists y \exists z ((x < y+1) \wedge (z < y+1) \wedge ((x < z+1) \rightarrow (z < n+1)))$

| $x < z+1$ | $z < x+1$ | $(x < z+1) \rightarrow (z < x+1)$ | $\forall x \in \mathbb{N}$ and $\exists z \in \mathbb{N}$ |
|---|---|---|---|
| T | T | T (if $x = z$) | T when $x = z \in \mathbb{N}$ |
| T | F | F | |
| F | T | T (if $z < x$) | but $z < x$ invalid $\forall x \in \mathbb{N}$ |
| F | F | T | invalid $\forall x \in \mathbb{N}$ |

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# 4 Semantics

3) $\Phi \stackrel{def}{=} \forall x \forall y \exists z \,(R(x,y) \rightarrow R(y,z))$

a) $A \stackrel{def}{=} \{a,b,c,d\}$ and $R^m \stackrel{def}{=} \{(b,c),(b,b),(b,a)\}$

No. For example, let's look at the element $(b,c)$. Then an element $(c,a)$ or $(c,b)$ or even $(c,d)$ should belong to $R^m$ and no such element exists in $R^m$.

b) $A \stackrel{def}{=} \{a,b,c\}$ $R^m \stackrel{def}{=} \{(b,c),(a,b),(c,b)\}$

Yes.

$(b,c) \rightarrow (c,b)$ here $x=z$. ok

$(a,b) \rightarrow (b,c)$ ok

$(c,b) \rightarrow (b,c)$ here $x=z$ again. ok