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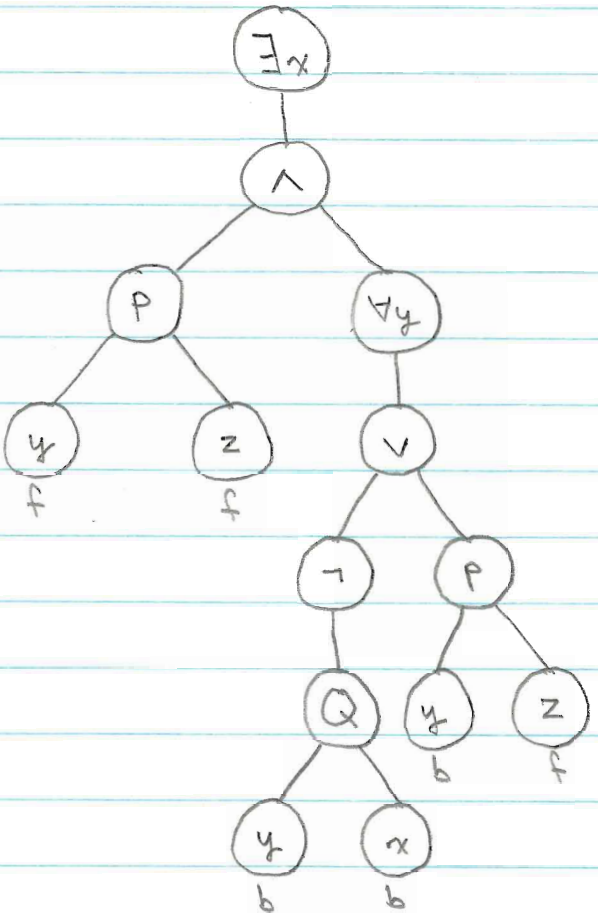
22C:188 Hw3
Spring 2014

2 Parse Trees, Variables and Substitutions

Let the formula Φ be

$$\exists x \left(P(y,z) \wedge \left(\forall y \left(\neg Q(y,x) \vee P(y,z) \right) \right) \right)$$

1)



2) For each variable occurrence in Φ identify if it is free or bound.

$$\exists x \left(P(y,z) \wedge \left(\forall y \left(\neg Q(y,x) \vee P(y,z) \right) \right) \right)$$
$$\quad\ f\ f \qquad\qquad\qquad\qquad b\ b \qquad\quad b\ f$$

f ≡ free

b ≡ bound

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3) Consider the variable ω, the unary function symbol f and the binary function symbol g.

(a) Which of ω, $f(x)$, and $g(x,y)$ are free for x in Φ?
The variable ω, $f(x)$, and $g(x,y)$ are free for x in Φ since there are no free occurences of x.

(b) Which of ω, $f(x)$, and $g(x,y)$ are free for y in Φ?
Only the variable ω is free for y in Φ.
Both $f(x)$ and $g(x,y)$ would be captured by '$\exists x$'.

(c) Compute $\Phi[\omega/x]$, $\Phi[f(\omega)/y]$, and $\Phi[g(\omega,z)/z]$.

$$\Phi[\omega/x] \equiv \exists x\left(P(y,z) \wedge \left(\forall y\left(\neg Q(y,x) \vee P(y,z)\right)\right)\right)$$

$$\Phi[f(\omega)/y] \equiv \exists x\left(P(f(\omega),z) \wedge \left(\forall y\left(\neg Q(y,x) \vee P(y,z)\right)\right)\right)$$

$$\Phi[g(\omega,z)/z] \equiv \exists x\left(P(y,g(\omega,z)) \wedge \left(\forall y\left(\neg Q(y,x) \vee P(y,g(\omega,z))\right)\right)\right)$$