3 Natural deduction in FOL

1) $(y=\phi)\wedge(y=x)\vdash \phi = x$

1	$(y=\phi)\wedge(y=x)\vdash \phi=x$	premise
2	$y=\phi$	$\wedge e_1$ 1
3	$y=y$	$=i$
4	$\phi=y$	$=e$ 2,3
5	$y=x$	$\wedge e_2$ 1
6	$\phi=x$	$=e$ 5,4

$$\frac{y=\phi \qquad \phi[y/z]}{\phi[\phi/z]}\ {=}e$$
(where ϕ is $z=y$)

$$\frac{\phi=y \qquad \phi[y/z]}{\phi[x/z]}\ {=}e$$
where ϕ is $\phi=z$

2) $t_1=t_2\vdash(t+t_2)=(t+t_1)$

1	$t_1=t_2$	premise	
2	$(t+t_1)=(t+t_1)$	$=i$	
3	$(t+t_2)=(t+t_1)$	$=e$ 1,2	where ϕ is $(t+x)=(t+t_1)$

3 Natural Deduction in FOL

3) $(\forall x\ P(x)) \to (\forall x\ Q(x)) \vdash \forall x((\forall x\ P(x)) \to Q(x))$

1	$(\forall x\ P(x)) \to (\forall x\ Q(x))$	premise
2	x_0	
3	$\forall x_0\ P(x_0)$	assumption
4	$(\forall x_0\ P(x_0)) \to (\forall x_0\ Q(x_0))$	assumption
5	$\forall x_0\ Q(x_0)$	$\to$e 4,3
6	$Q(x_0)$	$\forall x$ e 5
7	$(\forall x_0\ P(x_0)) \to Q(x_0)$	$\to$i 3,6
8	$\forall x((\forall x\ P(x)) \to Q(x))$	$\forall x$ i 2-7

3 Deductions in FOL

4) $\forall x\, P(x, z) \vdash \forall y\, P(y, z)$

1		$\forall x\, P(x, z)$	premise
2	y_0	$P(y_0, z)$	$\forall x$ e 1
3		$\forall y\, P(y, z)$	$\forall y$ 2

3 Natural Deduction in FOL

6) $\exists x \forall y\, P(x,y) \vdash \forall y\, \exists x\, P(x,y)$

see example pg 116 lines 3–6

1	$\exists x \forall y\, P(x,y)$	premise
2	y_0	
3	$x_0 \quad \forall y\, P(x_0,y)$	assumption
4	$P(x_0,y_0)$	$\forall y\, e$ 3
5	$\exists x\, P(x,y_0)$	$\exists x\, i$ 4
6	$\exists x\, P(x,y_0)$	$\exists x\, e$ 1, 3-6
	$\forall y\, \exists x\, P(x,y)$	$\forall y$ 2-6