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Adverbs of Quantification

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Cast of Characters

The adverbs I wish to consider fall into six groups of near-synonyms, as follows.

(1) Always, invariably, universally, without exception
(2) Sometimes, occasionally, [once]
(3) Never
(4) Usually, mostly, generally, almost always, with few exceptions, [ordinarily], [normally]
(5) Often, frequently, commonly
(6) Seldom, infrequently, rarely, almost never

Bracketed items differ semantically from their list-mates in ways I shall not consider here; omit them if you prefer.

First Guess: Quantifiers over Times?

It may seem plausible, especially if we stop with the first word on each list, that these adverbs function as quantifiers over times. That is to say that *always*, for instance, is a modifier that combines with a sentence Φ to make a sentence *Always* Φ that is true iff the modified sentence Φ is true at all times. Likewise, we might guess that *Sometimes* Φ, *Never* Φ, *Usually* Φ, *Often* Φ, and *Seldom* Φ are true, respectively, iff Φ is true at some times, none, most, many, or few. But it is easy to find various reasons why this first guess is too simple.

First, we may note that the times quantified over need not be moments of time. They can be suitable stretches of time instead. For instance,

(7) The fog usually lifts before noon here

means that the sentence modified by *usually* is true on most days, not at most moments. Indeed, what is it for that sentence to be true at a moment?

Second, we may note that the range of quantification is often restricted. For instance,

(8) Caesar seldom awoke before dawn

is not made true by the mere fact that few of all times (past, present, or future) are times when Caesar was even alive, wherefore fewer still are times when he awoke before dawn. Rather it means that few of all the times when Caesar awoke are times before dawn; or perhaps that on few of all the days of his life did he awake before dawn.

Third, we may note that the entities we are quantifying over, unlike times,[1] may be distinct although simultaneous. For instance,

(9) Riders on the Thirteenth Avenue line seldom find seats

may be true even though for 22 hours out of every 24 – all but the two peak hours when 86% of the daily riders show up – there are plenty of seats for all.

Second Guess: Quantifiers over Events?

It may seem at this point that our adverbs are quantifiers, suitably restricted, over events; and that times enter the picture only because events occur at times. Thus (7) could mean that most of the daily fog-liftings occurred before noon; (8) could mean that few of Caesar's awakenings occurred before dawn; and (9) could mean that most riders on the Thirteenth Avenue line are seatless. So far, so good; but further difficulties work both against our first guess and against this alternative.

Sometimes it seems that we quantify not over single events but over enduring states of affairs. For instance,

(10) A man who owns a donkey always beats it now and then

means that every continuing relationship between a man and his donkey is punctuated by beatings; but these continuing relationships, unlike the beatings, are not events in any commonplace sense. Note also that if *always* were a quantifier over times, the sentence would be inconsistent: it would say that the donkey-beatings are incessant and that they only happen now and then. (This sentence poses other problems that we shall consider later.)

We come last to a sweeping objection to both of our first two guesses: the adverbs of quantification may be used in speaking of abstract entities that have no location in time and do not participate in events. For instance,

(11) A quadratic equation never has more than two solutions
(12) A quadratic equation usually has two different solutions

mean, respectively, that no quadratic equation has more than two solutions and that most – more precisely, all but a set of measure zero under the natural measure on the set of triples of coefficients – have two different solutions. These sentences have nothing at all to do with times or events.

Or do they? This imagery comes to mind: someone is contemplating quadratic equations, one after another, drawing at random from all the quadratic equations there are. Each one takes one unit of time. In no unit of time does he contemplate a quadratic equation with more than two solutions. In most units of time he contemplates quadratic equations with two different solutions.

For all I know, such imagery may sustain the usage illustrated by (11) and (12), but it offers no hope of a serious analysis. There can be no such contemplator. To be more realistic, call a quadratic equation *simple* iff each of its coefficients could be specified somehow in less than 10,000 pages; then we may be quite sure that the only quadratic equations that are ever contemplated are simple ones. Yet

(13) Quadratic equations are always simple

is false, and in fact they are almost never simple.

Third Guess: Quantifiers over Cases

What we can say, safely and with full generality, is that our adverbs of quantification are quantifiers over cases. What holds always, sometimes, never, usually, often, or seldom is what holds in, respectively, all, some, no, most, many, or few cases.

But we have gained safety by saying next to nothing. What is a case? It seems that sometimes we have a case corresponding to each moment or stretch of time, or to each in some restricted class. But sometimes we have a case for each event of some sort; or for each continuing relationship between a man and his donkey; or for each quadratic equation; or – as in the case of this very sentence – for each sentence that contains one of our adverbs of quantification.

Unselective Quantifiers

It will help if we attend to our adverbs of quantification as they can appear in a special dialect: the dialect of mathematicians, linguists, philosophers, and lawyers, in which variables are used routinely to overcome the limitations of more colloquial means of pronominalization. Taking m, n, p as variables over natural numbers, and x, y, z as variables over persons, consider:

(14) Always, p divides the product of m and n only if some factor of p divides m and the quotient of p by that factor divides n
(15) Sometimes, p divides the product of m and n although p divides neither m nor n
(16) Sometimes it happens that x sells stolen goods to y, who sells them to z, who sells them back to x
(17) Usually, x reminds me of y if and only if y reminds me of x

Here it seems that if we are quantifying over cases, then we must have a case corresponding to each admissible assignment of values to the variables that occur free in the modified sentence. Thus (14) is true iff every assignment of natural numbers as values of m, n, and p makes the open sentence after *always* true – in other words, iff all triples of natural numbers satisfy that open sentence. Likewise (15) is true iff some triple of numbers satisfies the open sentence after *sometimes*; (16) is true iff some triple of persons satisfies the open sentence after *sometimes*; and (17) is true iff most pairs of persons satisfy the open sentence after *usually*.

The ordinary logicians' quantifiers are selective: $\forall x$ or $\exists x$ binds the variable x and stops there. Any other variables y, z, ... that may occur free in its scope are left free, waiting to be bound by other quantifiers. We have the truth conditions:

(18) $\forall x\Phi$ is true, under any admissible assignment f of values to all variables free in Φ except x, iff for every admissible value of x, Φ is true under the assignment of that value to x together with the assignment f of values to the other variables free in Φ;
(19) $\exists x\Phi$ is true, under any admissible assignment f of values to all variables free in Φ except x, iff for some admissible value of x, Φ is true under the assignment of that value to x together with the assignment f of values to the other variables free in Φ;

and likewise for the quantifiers that select other variables.

It is an entirely routine matter to introduce *unselective quantifiers* $\forall$ and $\exists$ that bind all the variables in their scope indiscriminately. Without selectivity, the truth conditions are much simpler; with no variables left free, we need not relativize the truth of the quantified sentence to an assignment of values to the remaining free variables.

(20) $\forall\Phi$ is true iff Φ is true under every admissible assignment of values to all variables free in Φ;
(21) $\exists\Phi$ is true iff Φ is true under some admissible assignment of values to all variables free in Φ

These unselective quantifiers have not deserved the attention of logicians, partly because they are unproblematic and partly because strings of ordinary, selective quantifiers can do all that they can do, and more besides. They have only the advantage of brevity. Still, brevity *is* an advantage, and it should be no surprise if unselective quantifiers are used in natural language to gain that advantage. That is what I claim; the unselective $\forall$ and $\exists$ can show up as the adverbs *always*, and *sometimes*.[2] Likewise *never*, *usually*, *often*, and *seldom* can serve as the unselective analogs of the selective quantifiers *for no x*, *for most x*, *for many x*, and *for few x*.[3]

To summarize, what we have in the variable-using dialect is roughly as follows. Our adverbs are quantifiers over cases; a case may be regarded as the 'tuple of its participants; and these participants are values of the variables that occur free in the open sentence modified by the adverb. In other words, we are taking the cases to be the admissible assignments of values to these variables.

But matters are not quite that simple. In the first place, we may wish to quantify past our adverbs, as in

(22) There is a number q such that, without exception, the product of m and n divides q only if m and n both divide q

So our adverbs of quantification are not entirely unselective: they can bind indefinitely many free variables in the modified sentence, but some variables – the ones used to quantify past the adverbs – remain unbound. In (22), m and n are bound by *without exception*; but q is immune, and survives to be bound by *there is a number q such that*, a selective quantifier of larger scope.

In the second place, we cannot ignore time altogether in (16)–(17) as we can in the abstract cases (11)–(15); (16)–(17) are not confined to the present moment, but are general over time as well as over 'tuples of persons. So we must treat the modified sentence as if it contained a free time-variable: the truth of the sentence depends on a time-coordinate just as it depends on the values of the person-variables, and we must take the cases to include this time coordinate as well as a 'tuple of persons. (Indeed, we could go so far as to posit an explicit time-variable in underlying structure, in order to subsume time-dependence under dependence on values of variables.) Our first guess about the adverbs is revived as a special case: if the modified sentence has no free variables, the cases quantified over will include nothing but the time coordinate. As noted before, the appropriate time-coordinates (accompanied by 'tuples or not, as the case may be) could either be moments of time or certain stretches of time, for instance days.

Sometimes we might prefer to treat the modified sentence as if it contained an event-variable (or even posit such a variable in underlying structure) and include an event-coordinate in the cases. The event-coordinate could replace the time-coordinate, since an event determines the time of its occurrence. If so, then our second guess also is revived as a special case: if there are no free variables, the cases might simply be events.

In the third place, not just any 'tuple of values of the free variables, plus perhaps a time- or event-coordinate, will be admissible as one of the cases quantified over. Various restrictions may be in force, either permanently or temporarily. Some standing restrictions involve the choice of variables: it is the custom in mathematics that λ is a variable that can take only limit ordinals as values (at least in a suitable context). I set up semi-permanent restrictions of this kind a few paragraphs ago by writing

(23) Taking m, n, p as variables over natural numbers, and x, y, and z as variables over persons ...

Other standing restrictions require the participants in a case to be suitably related. If a case is a 'tuple of persons plus a time-coordinate, we may take it generally that the persons must be alive at the time to make the case admissible. Or if a case is a 'tuple of persons plus an event-coordinate, it may be that the persons must take part in the event to make the case admissible. It may also be required that the participants in the 'tuple are all different, so that no two variables receive the same value. (I am not sure whether these restrictions are always in force, but I believe that they often are.)

Restriction by If-Clauses

There are various ways to restrict the admissible cases temporarily – perhaps only for the duration of a single sentence, or perhaps through several sentences connected by anaphoric chains. If-clauses seem to be the most versatile device for imposing temporary restrictions. Consider:

(24) Always, if x is a man, if y is a donkey, and if x owns y, x beats y now and then

A case is here a triple: a value for x, a value for y, and a time-coordinate (longish stretches seem called for, perhaps years). The admissible cases are those that satisfy the three if-clauses. That is, they are triples of a man, a donkey, and a time such that the man owns the donkey at the time. (Our proposed standing restrictions are redundant. If the man owns the donkey at the time, then both are alive at the time; if the participants are a man and a donkey, they are different.) Then (24) is true iff the modified sentence

(25) x beats y now and then

is true in all admissible cases. Likewise for

(26) Sometimes / (27) Usually / (28) Often } if x is a man, if y is a donkey, and if x owns y, x beats y now and then

(29) Never / (30) Seldom } if x is a man, if y is a donkey, and if x owns y, does x beat y now and then

The admissible cases are the triples that satisfy the if-clauses, and the sentence is true iff the modified sentence (25) – slightly transformed in the negative cases (29)–(30) – is true in some, most, many, none, or few of the admissible cases.

It may happen that every free variable of the modified sentence is restricted by an if-clause of its own, as in

(31) Usually, if x is a man, if y is a donkey, and if z is a dog, y weighs less than x but more than z

But in general, it is best to think of the if-clauses as restricting whole cases, not particular participants therein. We may have any number of if-clauses – including zero, as in (14)–(17). A free variable of the modified sentence may appear in more than one if-clause. More than one variable may appear in the same if-clause. Or it may be that no variable appears in an if-clause; such if-clauses restrict the admissible cases by restricting their time-coordinates (or perhaps their event-coordinates), as in

(32) Often if it is raining my roof leaks

(in which the time-coordinate is all there is to the case) or

(33) Ordinarily if it is raining, if x is driving and sees y walking, and if y is x's friend, x offers y a ride

It makes no difference if we compress several if-clauses into one by means of conjunction or relative clauses. The three if-clauses in (24) or in (26)–(30) could be replaced by any of:

(34) if x is a man, y is a donkey, and x owns y ...
(35) if x is a man and y is a donkey owned by x ...
(36) if x is a man who owns y, and y is a donkey ...
(37) if x and y are a man and his donkey ...

Such compression is always possible, so we would not have gone far wrong to confine our attention, for simplicity, to the case of restriction by a single if-clause.

We have a three-part construction: the adverb of quantification, the if-clauses (zero or more of them), and the modified sentence. Schematically, for the case of a single if-clause:

$$(38) \quad \left\{ \begin{array}{c} \text{Always} \\ \text{Sometimes} \\ \cdot \\ \cdot \\ \cdot \end{array} \right\} + \text{if } \Psi + \Phi$$

But could we get the same effect by first combining Ψ and Φ into a conditional sentence, and then taking this conditional sentence to be the sentence modified by the adverb? On this suggestion (38) is to be regrouped as

$$(39) \quad \left\{ \begin{array}{c} \text{Always} \\ \text{Sometimes} \\ \cdot \\ \cdot \\ \cdot \end{array} \right\} + \text{if } \Psi, \Phi$$

Sentence (39) is true iff the conditional *If* Ψ, Φ is true in all, some, none, most, many, or few of the admissible cases – that is, of the cases that satisfy any permanent restrictions, disregarding the temporary restrictions imposed by the if-clause. But is there any way to interpret the conditional *If* Ψ, Φ that makes (39) equivalent to (38) for all six groups of our adverbs? No; if the adverb is *always* we get the proper equivalence by interpreting it as the truth-functional conditional $\Psi \supset \Phi$, whereas if the adverb is *sometimes* or *never*, that does not work, and we must instead interpret it as the conjunction Φ & Ψ. In the remaining cases, there is no natural interpretation that works. I conclude that the *if* of our restrictive if-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. The *if* in *always if* ..., ..., *sometimes if* ..., ..., and the rest is on a par with the non-connective *and* in *between* ... *and* ..., with the non-connective *or* in *whether* ... *or* ..., or with the non-connective *if* in *the probability that* ... *if* ... It serves merely to mark an argument-place in a polyadic construction.[4]

Stylistic Variation

Sentences made with the adverbs of quantification need not have the form we have considered so far: adverb + if-clauses + modified sentence. We will find it convenient, however, to take that form – somewhat arbitrarily – as canonical, and to regard other forms as if they were derived from that canonical form. Then we are done with semantics: the interpretation of a sentence in canonical form carries over to its derivatives.

The constituents of the sentence may be rearranged:

(40) If x and y are a man and a donkey and if x owns y, x usually beats y now and then
(41) If x and y are a man and a donkey, usually x beats y now and then if x owns y
(42) If x and y are a man and a donkey, usually if x owns y, x beats y now and then
(43) Usually x beats y now and then, if x and y are a man and a donkey and x owns y

All of (40)–(43), though clumsy, are intelligible and well-formed.

Our canonical restrictive if-clauses may, in suitable contexts, be replaced by when-clauses:

(44) When m and n are positive integers, the power m^n can always be computed by successive multiplications

Indeed, a when-clause may sound right when the corresponding if-clause would be questionable, as in a close relative of (8):

$$(45) \quad \text{Seldom was it before dawn} \left\{ \begin{array}{c} \text{when} \\ \text{? if} \end{array} \right\} \text{Caesar awoke}$$

Or we may have a where-clause or a participle construction, especially if the restrictive clause does not come at the beginning of the sentence:

(46) The power m^n, where m and n are positive integers, can always be computed by successive multiplications
(47) The power m^n (m and n being positive integers) can always be computed by successive multiplications

Always if – or is it *always when*? – may be contracted to *whenever*, a complex unselective quantifier that combines two sentences:

(48) Whenever m and n are positive integers, the power m^n can be computed by successive multiplications

(49) Whenever x is a man, y is a donkey, and x owns y, x beats y now and then
(50) Whenever it rains it pours

Always may simply be omitted:

(51) (Always) When it rains, it pours
(52) (Always) If x is a man, y is a donkey, and x owns y, x beats y now and then
(53) When m and n are positive integers, the power m^n can (always) be computed by successive multiplications

Thus we reconstruct the so-called "generality interpretation" of free variables: the variables are bound by the omitted *always*.

Our stylistic variations have so far been rather superficial. We turn next to a much more radical transformation of sentence structure – a transformation that can bring us back from the variable-using dialect to everyday language.

Displaced restrictive terms

Suppose that one of our canonical sentences has a restrictive if-clause of the form

(54) if α is $\tau \ldots$,

where α is a variable and τ is an indefinite singular term formed from a common noun (perhaps plus modifiers) by prefixing the indefinite article or *some*.
Examples:

(55) if x is a donkey ...
(56) if x is an old, grey donkey ...
(57) if x is a donkey owned by y ...
(58) if x is some donkey that y owns ...
(59) if x is something of y's ...
(60) if x is someone foolish ...

(Call τ, when so used, a *restrictive term*.) Then we can delete the if-clause and place the restrictive term τ in apposition to an occurrence of the variable α elsewhere in the sentence. This occurrence of α may be in the modified sentence, or in another if-clause of the form (54), or in an if-clause of some other form. Often, but not always, the first occurrence of α outside the deleted if-clause is favoured. If τ is short, it may go before α; if long, it may be split and go partly before and partly after; and sometimes it may follow α parenthetically. The process of displacing restrictive terms may – but need not – be repeated until no if-clauses of the form (54) are left. For instance:

(61) Sometimes, if x is some man, if y is a donkey, and if x owns y, x beats y now and then
$\Rightarrow$
Sometimes if y is a donkey, and if some man x owns y, x beats y now and then
$\Rightarrow$
Sometimes, if some man x owns a donkey y, x beats y now and then

(62) Often, if x is someone who owns y, and if y is a donkey, x beats y now and then
$\Rightarrow$
Often, if x is someone who owns y, a donkey, x beats y now and then
$\Rightarrow$
Often, someone x who owns y, a donkey, beats y now and then

Instead of just going into apposition with an occurrence of the variable α, the restrictive term τ may replace an occurrence of α altogether. Then all other occurrences of α must be replaced as well, either by pronouns of the appropriate case and gender or by terms *that* ν or *the* ν, where ν is the principal noun in the term τ. For instance,

(63) Always, if y is a donkey and if x is a man who owns y, x beats y now and then
$\Rightarrow$
Always, if x is a man who owns a donkey, x beats it now and then
$\Rightarrow$
Always, a man who owns a donkey beats it now and then

Now it is a small matter to move *always* and thereby derive the sentence (10) that we considered earlier. Sure enough, the canonical sentence with which the derivation (63) began has the proper meaning for (10). It is in this way that we return from the variable-using dialect to an abundance of everyday sentences.

I conclude with some further examples.

(64) Always, if x is someone foolish, if y is some good idea, and if x has y, nobody gives x credit for y
$\Rightarrow$
Always, if y is some good idea, and if someone foolish has y, nobody gives him credit for y
$\Rightarrow$
Always, if someone foolish has some good idea, nobody gives him credit for that idea

(65) Often, if y is a donkey, if x is a man who owns y, and if y kicks x, x beats y
$\Rightarrow$
Often, if y is a donkey, and if y kicks a man who owns y, he beats y
$\Rightarrow$
Often, if a donkey kicks a man who owns it, he beats it

(66) Often, if y is a donkey, if x is a man who owns y, and if y kicks x, x beats y
$\Rightarrow$
Often, if x is a man who owns a donkey, and if it kicks x, x beats it
$\Rightarrow$
Often, if it kicks him, a man who owns a donkey beats it

(67) Usually, if x is a man who owns y and if y is a donkey that kicks x, x beats y
$\Rightarrow$
Usually, if x is a man who owns a donkey that kicks x, x beats it
$\Rightarrow$
Usually, a man who owns a donkey that kicks him beats it

(68) Usually, if x is a man who owns y and if y is a donkey that kicks x, x beats y
$\Rightarrow$
Usually, if y is a donkey that kicks him, a man who owns y beats y
$\Rightarrow$
Usually, a man who owns it beats a donkey that kicks him

Notes

1 Unlike genuine moments or stretches of time, that is. But we may truly say that Miles the war hero has been wounded 100 times if he has suffered 100 woundings, even if he has been wounded at only 99 distinct moments (or stretches) of time because two of his woundings were simultaneous.

2 It is pleasing to find that Russell often explained the now-standard selective quantifiers by using an unselective adverb of quantification to modify an open sentence. For instance in *Principia* 1, *9, we find the first introduction of quantifiers in the formal development: "We shall denote 'Φx *always*' by the notation (x). Φx ... We shall denote 'Φx *sometimes*' by the notation $(\exists x)$. Φx."

3 It is customary to work with assignments of values to all variables in the language; the part of the assignment that assigns values to unemployed variables is idle but harmless. But for us this otherwise convenient practice would be more bother than it is worth. In dealing with *usually*, *often*, and *seldom* we must consider the fraction of value-assignments that satisfy the modified sentence. Given infinitely many variables, these fractions will be ∞/∞ (unless they are 0 or 1). We would need to factor out differences involving only the idle parts of assignments.

4 What is the price of forcing the restriction-marking *if* to be a sentential connective after all? Exorbitant: it can be done if (1) we use a third truth value, (2) we adopt a far-fetched interpretation of the connective *if*, and (3) we impose an additional permanent restriction on the admissible cases. Let *If* Ψ, Φ have the same truth value as Φ if Ψ is true, and let it be third-valued if Ψ is false or third-valued. Let a case be admissible only if it makes the modified sentence either true or false, rather than third-valued. Then (39) is equivalent to (38) for all our adverbs, as desired, at least if we assume that Ψ and Φ themselves are not third-valued in any case. A treatment along similar lines of if-clauses used to restrict ordinary, selective quantifiers may be found in Belnap (1970).

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8

A Theory of Truth and Semantic Representation

Hans Kamp

1 Introduction

Two conceptions of meaning have dominated formal semantics of natural language. The first of these sees meaning principally as that which determines conditions of truth. This notion, whose advocates are found mostly among philosophers and logicians, has inspired the disciplines of truth-theoretic and model-theoretic semantics. According to the second conception meaning is, first and foremost, that which a language user grasps when he understands the words he hears or reads. This second conception is implicit in many studies by computer scientists (especially those involved with artificial intelligence), psychologists and linguists – studies which have been concerned to articulate the structure of the representations which speakers construct in response to verbal inputs.

It appears that these two conceptions, and with them the theoretical concerns that derive from them, have remained largely separated for a considerable period of time. This separation has become an obstacle to the development of semantic theory, impeding progress on either side of the line of division it has created.

The theory presented here is an attempt to remove this obstacle. It combines a definition of truth with a systematic account of semantic representations. These two components are linked in the following manner. The representations postulated here are (like those proposed by others; cf. e.g. Hendrix (1975) or Karttunen (1976)) similar in structure to the models familiar from model-theoretic semantics. In fact, formally they are nothing other than partial models, typically with small finite domains. Such similarity should not surprise; for the representation of, say, an indicative sentence ought to embody those conditions which the world must satisfy in order that the sentence be true; and a particularly natural representation of those conditions is provided by a partial model with which the (model describing the) real world will be compatible just in case the conditions are fulfilled.

Interpreting the truth-conditional significance of representations in this way we are led to the following characterization of truth: A sentence S, or discourse D, with representation m is true in a model M if and only if M is compatible with m; and