# MEGR 2810
# LECTURE 12-

# NUMERICAL METHODS FOR BOUNDARY VALUE PROBLEMS IN ODES

# DEFINITION AND APPLICATIONS OF BVPS

### Definition of BVPs

BVPs are differential equations with conditions specified at multiple boundary points.

### Difference from IVPs

BVPs differ from IVPs by requiring constraints at interval endpoints, not just initial points.

### Applications in Science

BVPs appear in physics, engineering, fluid dynamics, and electrostatics applications.

### Numerical Methods Necessity

Complex BVPs often require numerical methods due to difficulty in analytical solutions.

# COMPARISON BETWEEN IVPS AND BVPS

**Definition of IVPs**

IVPs specify all necessary conditions at a single initial point in the domain.

**Definition of BVPs**

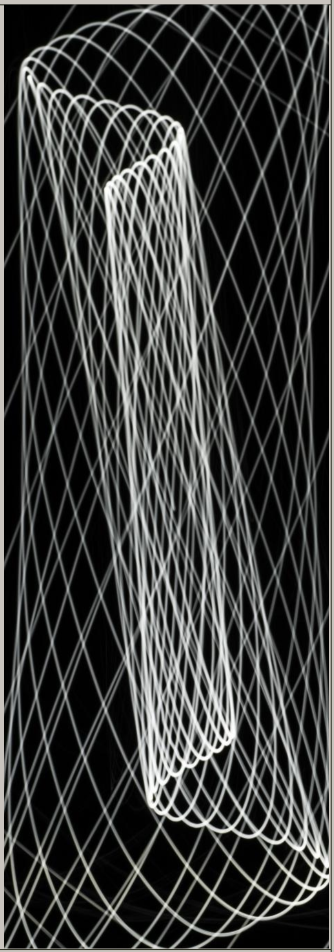
BVPs require solutions to meet conditions at two or more boundary points.

**Numerical Methods for IVPs**

IVPs are easier to solve numerically using methods like Euler's or Runge-Kutta.

**Complexity of BVPs**

BVPs need iterative or discretization methods due to constraints at both ends.



# NUMERICAL METHODS FOR SOLVING BVPS

# FINITE DIFFERENCE METHOD: ALGORITHM AND IMPLEMENTATION

Discretize the domain and approximate derivatives using finite differences.

| STEP | DESCRIPTION |
|---|---|
| 1 | Discretize domain into N points |
| 2 | Apply central difference to approximate derivatives |
| 3 | Formulate linear system Ax = b |
| 4 | Incorporate boundary conditions |
| 5 | Solve using NumPy's linear solver |

# SHOOTING METHOD: ALGORITHM AND IMPLEMENTATION

The shooting method converts a BVP into an IVP by guessing the initial slope and iteratively adjusting it until the boundary condition at the other end is satisfied.

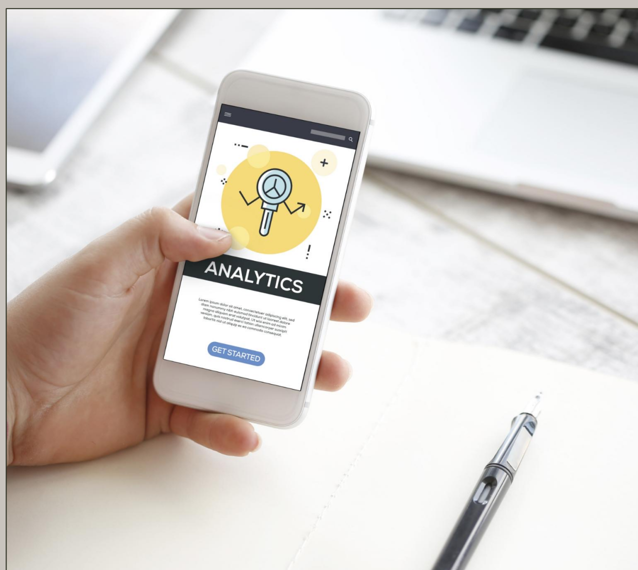
| STEP | DESCRIPTION |
|---|---|
| 1 | Convert BVP to IVP |
| 2 | Guess missing initial condition |
| 3 | Integrate using Euler or Runge-Kutta |
| 4 | Check boundary condition at endpoint |
| 5 | Adjust guess using root-finding |

$$x_{n+1} = x_n - f(x_n)\,\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

# COMPARISON OF SHOOTING AND FINITE DIFFERENCE METHODS

| ASPECT | SHOOTING METHOD | FINITE DIFFERENCE METHOD |
|---|---|---|
| **Approach** | Convert BVP to IVP and iterate | Discretize and solve linear system |
| **Ease of Implementation** | Simple for basic problems | Requires matrix formulation |
| **Stability** | Less stable for stiff problems | More robust |
| **Accuracy** | Depends on integration and root-finding | O(h^2) with central difference |
| **Flexibility** | Limited for complex BCs | Handles complex BCs well |

# ERROR ANALYSIS AND STABILITY CONSIDERATIONS




### Errors in Shooting Method

Errors stem from initial guess, numerical integration, and root-finding steps in the shooting method.

### Finite Difference Errors

Truncation and round-off errors arise from derivative approximations and matrix operations.

### Stability and Step Size

Step size affects accuracy and computational cost; stability prevents oscillations and divergence.

### Method Comparison

Shooting method is intuitive but less suited for stiff problems; finite difference is robust for complex cases.