Question 1

> Consider two random events A and B defined for the same random experiment.
>
> 1. Is it possible that A and B are independent and mutually exclusive (disjoint) at the same time? Explain your answer.
>
> 2. Does the answer change if given that $P(A) > 0$ and $P(B) > 0$? Explain your answer.

1) If two events A and B are independent then

$$P(A \cap B) = P(A) \cdot P(B).$$

On the other hand, if two eventes A and B are mutually exclusive (disjoint) then they never happen together and

$$P(A \cap B) = 0.$$

This gives us the following equivalence:

$$P(A \cap B) = P(A) \cdot P(B) = 0,$$

or simply

$$P(A) \cdot P(B) = 0.$$

In order for the product to be equal to 0, at least one of the multipliers must be equal to 0. Therefore two random events A and B ***can be*** independent and mutually exclusive if and only if $P(A) = 0$ or $P(B) = 0$.

2) In case $P(A) > 0$ and $P(B) > 0$ the events ***cannot be*** independent and disjoint at the same time because the equivalnce $P(A \cap B) = P(A) \cdot P(B) = 0$ never holds.