Question 2

> Consider Bernoulli random variable, i.e. random variable that takes value 1 with probability p and value 0 with probability $1 - p$. Find its variance. Then find variance of binomial random variable with probability of success (i.e. head) p and number of trials (i.e. tosses) n.
>
> Hint. We discussed the relation between Bernoulli and binomial random variables in programming assignment on Week 2. Use properties of variance here; be careful with your explanations: do not forget all the conditions that you need.

1) We can use the formula for variance $Var(X) = E\left[X^2\right] - E[X]^2$ to find the variance of a Bernoulli random variable. Let X be a Bernoulli random variable with parameter p.

We have:

$$E[X] = 1 \cdot P(X = 1) + 0 \cdot P(X = 0) = p.$$

To calculate $E\left[X^2\right]$, we note that X^2 can only take on values of 0 or 1. Thus:

$$E\left[X^2\right] = 0^2 \cdot P(X = 0) + 1^2 \cdot P(X = 1) = 0 \cdot (1 - p) + 1 \cdot p = p.$$

Therefore, from the formula for variance, we get:

$$Var(X) = E\left[X^2\right] - E[X]^2 = p - p^2 = p(1 - p)$$

So the variance of a Bernoulli random variable with parameter p is $p(1 - p)$.

2) A binomial random variable with parameters n and p can be thought of as the sum of n independent and identically distributed Bernoulli random variables, each with parameter p. Let X be a binomial random variable with parameters n and p. Then we have:

$$X = X_1 + X_2 + ... + X_n,$$

where X_i is a Bernoulli random variable with parameter p, for $i = 1, 2, ..., n$.

As discussed above, the expected value of X_i is:

$$E[X_i] = 1 \cdot P(X_i = 1) + 0 \cdot P(X_i = 0) = p,$$

which, by the way, basing on linearity of expectation, gives the expected value of X:

$$E[X] = E[X_1 + X_2 + ... + X_n] = E[X_1] + E[X_2] + ... + E[X_n] = np,$$

but this is not a part of the task.

Again, from the formula for variance, we get:

$$Var(X_i) = E\left[X_i^2\right] - E[X_i]^2 = p - p^2 = p(1 - p).$$

Now, **since the X_i's are independent**, we can use the property of the variance of a sum of n Bernoulli random variables to find the variance of X:

$$Var(X) = Var(X_1 + X_2 + ... + X_n) = Var(X_1) + Var(X_2) + ... + Var(X_n) = np(1 - p).$$

Therefore, the variance of a binomial random variable with parameters n and p is $np(1 - p)$.