Question 1

Space Company Z is preparing to launch a spaceship. If the launch is successful, the company earns \$100 million in profit. In case of failure, the company loses \$200 million (i.e. negative profit). Probability of failure is 1/10. The company can buy insurance for this launch. Cost of insurance is \$30 million (paid before the launch). In case of failure the insurer will pay Space Company Z \$200 million (thus compensating all the damages). Consider two cases: 1. Space Company Z decided not to buy insurance. 2. Space Company Z decided to buy insurance. Denote its profit in the first case by X and in the second case by Y. (In the second case profit includes payments to/from the insurer taken with appropriate sign.) Find expected values and variances of X and Y. Describe how buying of insurance affects the profit, its expected value and variance? Does buying insurance is cost-efficient in the long run? In which case Space Company Z can decide to buy insurance and why?

1) Let us write initial data and calculations in a table form.

million dollars

(*) spaceship launch outcome events: success and failure	S	F
(1) probabilities of events	0.9	0.1
(2) event caused net profit / loss (X)	100	-200
additional cash flows driven by insurance and resulting profit / loss:		
- insurance cost	-30	-30
- insurance coverage	0	200
(3) event caused net profit / loss, insurance employed (Y)	70	-30

2) Now we can calculate expected values and variance of X and Y using formulas

$$E(X) = \sum_k x_k \cdot p(x_k)$$

and

$$Var(X) = E(X - E(X))^2 = E\left(X^2\right) - E(X)^2.$$

For variable X:

million dollars

(*) spaceship launch outcome events: success and failure	S	F
(4) expected profit / loss per event	90	-20
(5) expected value of X	70	
(6) variance of X	8100	

EXPLANATIONS:
Line (4) is a product of lines (1) and (2) above.

Line (5) is a total of values on line (4).

Line (6) was calculated as per variance formula above, where $E\left(X^2\right) = \sum_k x_k^2 \cdot p(x_k)$.

For variable Y:

million dollars

(*) spaceship launch outcome events: success and failure	S	F
(7) expected profit / loss per event, insurance employed	63	-3
(8) expected value of Y	60	
(9) variance of Y	900	

EXPLANATIONS:
Line (7) is a product of lines (1) and (3) above.
Line (8) is a total of values on line (7).

Line (9) was calculated as per variance formula above, where $E\left(Y^2\right) = \sum_k y_k^2 \cdot p(y_k)$.

3) Buying insurance noticeably lowers probable profits but even more noticeably lowers probable losses. Insurance purchase also decreases expected value of profit but allows to reach much lower variance.

Abstract comparison of profit expected values for non-insured and insured cases leads to a conclusion that buying insurance decreases cost-efficiency of the business in the long run.

Again, abstractly, in terms of given figures Space Company Z can decide to buy insurance if probability of failure increases to $p_f > 0.15$, which can be derived from

$$
\begin{gathered}
100 \cdot (1 - p_f) + (-200) \cdot p_f < 70 \cdot (1 - p_f) + (-30) \cdot p_f \\
100 - 100p_f - 200p_f < 70 - 70p_f - 30p_f \\
200p_f > 30 \\
p_f > 3/20 = 0.15.
\end{gathered}
$$

In this case profit expectation with insurance employment is larger than without and buying insurance becomes cost-efficient in the long run.

In terms of more-or-less real business application Space Company Z needs no insurance if it has access to sufficient volumes of money, which is cheaper than insurance, does not have to be paid back and can freely be used to cover sudden damages.
Or if it can decrease failure probability and associated losses close to 0.

This means that Space Company Z needs to buy insurance in any case if it seeks to lowering variance of profit expected value in order to have predictable cash flow and minimized loss risks.
Otherwise, one or two failed launches may collapse the business.