Question 2

> Let X and Y be two independent normally distributed random variables with expected value 0 and variance 1. Find their joint PDF. Plot its level curves.

Since X and Y are independent, their joint PDF is simply the product of their individual $PDFs$:

$$pdf_{X,Y}(x,y) = pdf_X(x) \cdot pdf_Y(y) = \left(\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2}{2}}\right) = \frac{1}{2\pi} \cdot e^{-\frac{x^2+y^2}{2}}$$

This is the joint PDF of a bivariate normal distribution with mean (expectation) $\mu = (0,\ 0)$ and covariance matrix: $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

We can plot level curves and the shape of the joint PDF using this code snippet:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import multivariate_normal

mean = [0, 0]
cov_matrix = [[1, 0], [0, 1]]
data = multivariate_normal(mean, cov_matrix)

x, y = np.mgrid[-2:2:.01, -2:2:.01]
xy_position = np.dstack((x, y))

plt.contourf(x, y, data.pdf(xy_position))
plt.colorbar()

plt.figure().add_subplot(projection='3d')
plt.contourf(x, y, data.pdf(xy_position), 200);
```

The resulting plots themselves will have the following look: