Parameters & Notation

Let

$$
\begin{aligned}
y_t &= \text{load at time } t \\
n &= 1 \text{ (One observation at each time point)} \\
m &= \text{Number of covariates at each time point} \\
D_t &= \{y_t, \ldots, y_0, D_0, F_t, G\} \text{ (leaving } V, W_t \text{ out for notational clarity)} \\
G &= I_m \\
F_t &= \begin{bmatrix} 1 & time_t & time_t^2 & bushr_t & y_{t-1} & hr.1_t & \ldots & hr.23_t \end{bmatrix}
\end{aligned}
$$

Where

$$
\begin{aligned}
bushr_t &= \text{Indicator for business operating hour; 1 = business hour, 0 = not business hour} \\
hr.1_t &= \text{Indicator for time-point t being during hour 1 of the day}
\end{aligned}
$$

Model

$$
\begin{aligned}
(y_t|\theta_t, D_t, v, W_t) &\sim N_1\left(F_t'\theta_t, v\right) \\
(\theta_t|\theta_{t-1}, D_t, W_t) &\sim N_m\left(G\theta_{t-1}, W_t\right) = N\left(\theta_{t-1}, W_t\right)
\end{aligned}
$$

Model assumes v scalar and constant in time, and W_t is an (mxm) covariance matrix which varies in time. We assume W_t is diagonal.

Prior for v

$$
\begin{aligned}
v|D_{t-1} &\sim IG\left(\frac{n_{t-1}}{2}, \frac{n_{t-1}S_{t-1}}{2}\right) \\
v|D_t &\sim IG\left(\frac{n_t}{2}, \frac{n_t S_t}{2}\right)
\end{aligned}
$$

where

$$S_0 = \text{hyperparameter; point estimate for observational variance v at time 0}$$

Prior for W_t

Assume W_t diagonal, so $W_t = diag\left(w_{1t}, \ldots, w_{mt}\right)$.

$$
\begin{aligned}
w_{jt}|D_{t-1} &\sim IG\left(\frac{n_{t-1}}{2}, \frac{n_{t-1}T_{j,t-1}}{2}\right) \\
v|D_t &\sim IG\left(\frac{n_t}{2}, \frac{n_t T_{j,t}}{2}\right)
\end{aligned}
$$

where

$$T_{j,0} = \text{hyperparameter; point estimate for system variance } w_j \text{ at time 0}$$

Initial Information

$$
\begin{aligned}
\theta_0|D_0, v) &\sim N(m_0, VC_0^*) \\
(v|D_0) &\sim IG(\frac{n_0}{2}, \frac{n_0 S_0}{2}) \\
w_{j0} &\sim IG(\frac{n_0}{2}, \frac{n_0 T_{j0}}{2}) \text{ for } j = \{1 \dots m\}
\end{aligned}
$$

Quantities which must be specified:

$$m_0, C_0^*, n_0, S_0, T_{10}, \dots, T_{m0}$$

Inverse Gamma Parameterization

IG indicates the inverse-gamma, s.t.

$$x \sim IG(a, b) \quad \rightarrow \quad p(x|a, b) = \frac{b^a}{\gamma a} x^{-a-1} \exp\left[-\frac{b}{x}\right]$$