Pricing Rainfall Derivatives at the CME

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Motivation


U.S. Drought Monitor
July 17, 2012
D0 Abnormally Dry
D1 Drought - Moderate
D2 Drought - Severe
D3 Drought - Extreme
D4 Drought - Exceptional
USDA
Released Thursday, July 19, 2012
Author: Richard Heim/Liz Love-Brotak, NOAA/NESDIS/NCDC
http://droughtmonitor.unl.edu/
IF YOU'VE PRAYED FOR RAIN, YOU CAN STOP NOW
NOW ENROLLING
LEARNING TREE
PRESCHOOL
FREE-VPK-FREE
737-8842

Motivation

Situation

- Rainfall risk affects many economic sectors.
- Rainfall risk can be insured with rainfall derivatives.
- The CME started trading rainfall derivatives in 2011.
  - → Prices of exchange-traded rainfall derivatives are available for the first time.

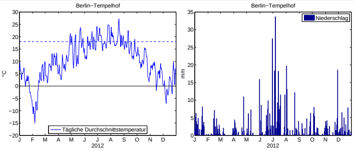
CME monthly rainfall futures

Index	Monthly sum of rainfall (inches)
Tick size	$500 per index point
Contract months	Mar, Apr, May, Jun, Jul, Aug, Sep, Oct
Reference stations	Chicago, Dallas, Des Moines, Detroit, Jacksonville, Los Angeles, New York City, Portland, Raleigh

Motivation

Pricing rainfall derivatives

- Benth et al. (2007), Härdle/López Cabrera (2011): Pricing models for temperature futures including the Market Price of Risk (MPR)
  → But: Rainfall different from temperature



- Cao et al. (2004): Fair premium for rainfall futures, no MPR
- Leobacher/Ngare (2011): Indifference prices
- Lee/Oren (2010), Härdle/Osipenko (2011): Equilibrium prices

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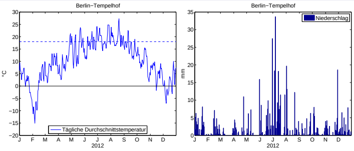

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Goal

- Pricing model for CME rainfall futures including the MPR

Methods

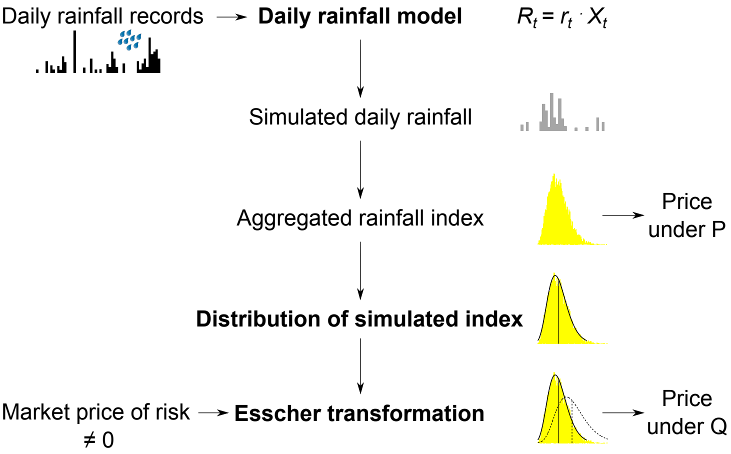
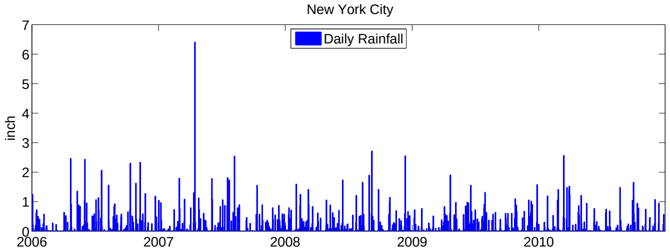


Figure: Model for pricing rainfall derivatives

Daily Rainfall Model



Daily rainfall model

(Wilks 1998, Cao et al. 2004, Odening et al. 2007, Ritter et al., 2012)

The daily rainfall amount R_t at time t is described as the product of a rainfall amount process r_t and a rainfall occurrence process X_t.

$$R_t = r_t \cdot X_t, \qquad X_t = \begin{cases} 0 \text{ if day } t \text{ is dry} \\ 1 \text{ if day } t \text{ is wet} \end{cases}$$

Daily Rainfall Model

X_t is modelled as a first-order, two-state Markov process.

(Todorovic/Woolhiser 1975, Katz 1977, Wilks 1998/1999, Odening et al. 2007)

$$\text{Transition probabilities: } p_t^{01} = \Pr\{X_t = 1 | X_{t-1} = 0\}$$
$$p_t^{11} = \Pr\{X_t = 1 | X_{t-1} = 1\}$$

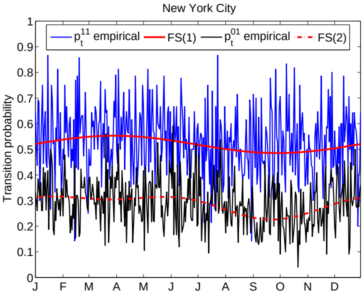


Figure: Empirical and estimated transition probabilities for New York City

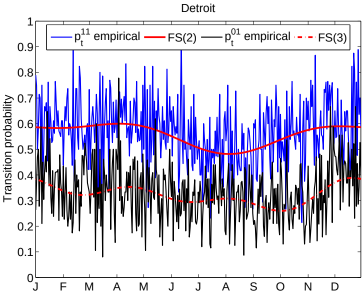


Figure: Empirical and estimated transition probabilities for Detroit

Daily Rainfall Model

The process r_t follows a mixed exponential distribution.

(Woolhiser/Roldán 1982, Foufoula-Georgiou/Lettenmaier 1987, Wilks 1998/1999, Odening et al. 2007)

$$f[r_t] = \frac{\alpha_t}{\beta_t} \exp\left[\frac{-r_t}{\beta_t}\right] + \frac{1-\alpha_t}{\gamma_t} \exp\left[\frac{-r_t}{\gamma_t}\right]$$

with $\beta_t \geq \gamma_t > 0$ and $0 < \alpha_t < 1$.

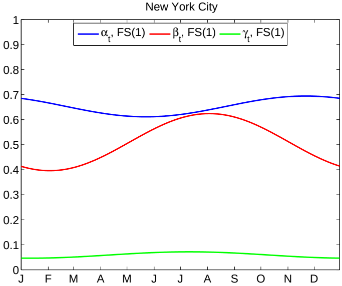


Figure: Parameters of the mixed exponential distribution for New York City

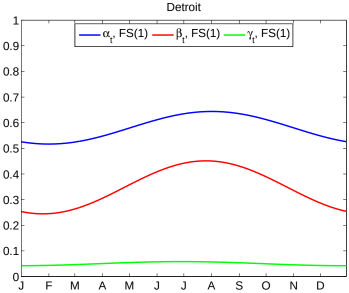


Figure: Parameters of the mixed exponential distribution for Detroit

Rainfall simulation

Occurrence process

Recursive simulation with starting value X_0 via uniform random variable $u_{1,t} \sim \mathcal{U}(0,1)$:

$$X_t^{\text{sim}} = \begin{cases} 1 \text{ if } u_{1,t} \leq p_t^{\text{X1}}, \\ 0 \text{ otherwise.} \end{cases}$$

Amount process

Simulation via uniform random variables $u_{2,t}$ and $u_{3,t} \sim \mathcal{U}(0,1)$, independent from $u_{1,t}$:

$$r_t^{\text{sim}} = r_{\min} - \delta_t \ln\left[u_{2,t}\right],$$

$$\delta_t = \begin{cases} \beta_t \text{ if } u_{3,t} \leq \alpha_t, \\ \gamma_t \text{ if } u_{3,t} > \alpha_t. \end{cases}$$

Methods

Esscher transform

- The Esscher transform allows to get an equivalent martingale measure for Lévy processes.
- It corresponds to a Girsanov transform for the Brownian motion.
- The density under the equivalent martingale measure is defined by

$$f_t(x;\theta) = \frac{e^{\theta x} f_t(x)}{\int_{-\infty}^{\infty} e^{\theta y} f_t(y) \mathrm{d}y}$$

- Certain distributions retain their original form under the Esscher transform, e. g., the normal-inverse Gaussian distribution.

Methods

Normal-inverse Gaussian distribution NIG$(\alpha, \beta, \mu, \delta)$

- Flexible distribution with 4 parameters
- Density

$$f_X(x) = \frac{\alpha\delta \exp(\delta\sqrt{\alpha^2 - \beta^2} + \beta(x-\mu))}{\pi\sqrt{\delta^2 + (x-\mu)^2}} \cdot K_1\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)$$

  - μ location
  - α tail heaviness
  - β asymmetry parameter
  - δ scaling parameter
  - K_1 modified Bessel function of second kind
- After an Esscher transform with parameter θ, an NIG$(\alpha, \beta, \mu, \delta)$ distributed random number is NIG$(\alpha, \beta + \theta, \mu, \delta)$ distributed.

Data

Rainfall data

- Daily rainfall amount 1980–2011
- Detroit, Jacksonville, New York City
- National Climatic Data Center (NCDC)

Market data

- Daily CME prices of futures on the monthly sum of rainfall (in inches)
- All contracts 2011 (March–October)
- Detroit, Jacksonville, New York City
- From Bloomberg via the RDC of SFB649

Example

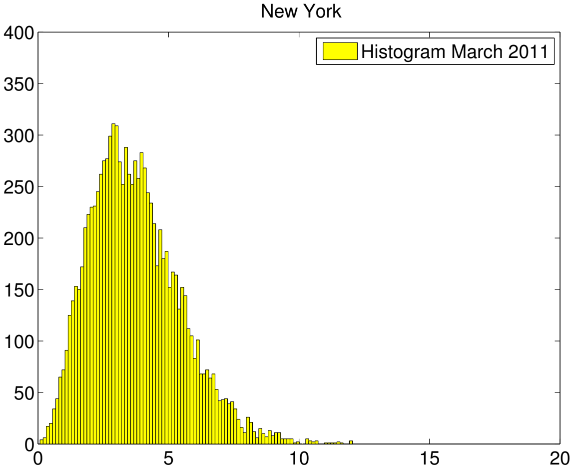


Fig.: Histogram of the simulated rainfall index, 03.01.2011

Example

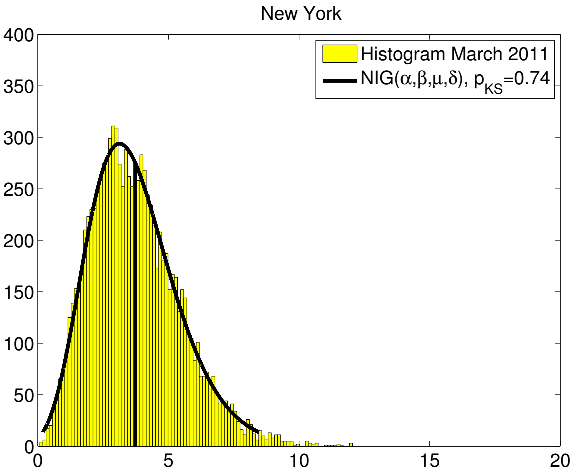


Fig.: Histogram of the simulated rainfall index, as well as the fitted NIG distribution, 03.01.2011

Example

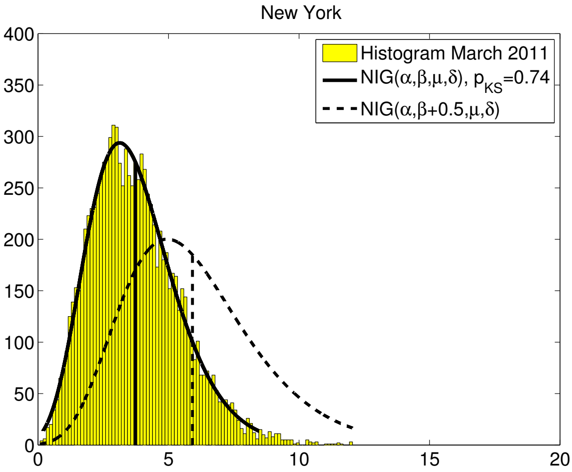


Fig.: Histogram of the simulated rainfall index, as well as the fitted and the transformed NIG distributions, 03.01.2011

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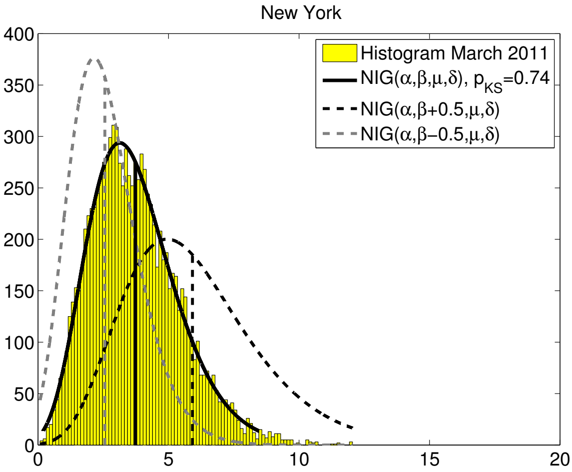

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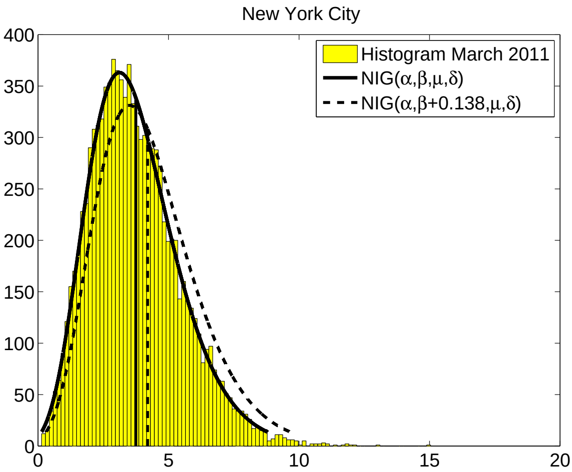
Prices

New York City (03.01.2011)

Methode	MPR θ	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
CME	–	4.20	4.40	3.20	5.00	4.50	4.30	4.20	4.60
DRM	-2.00	0.92	0.76	0.83	0.69	0.42	0.34	0.42	0.64
	-1.00	1.82	1.80	1.97	1.87	1.74	1.55	1.40	1.58
	-0.50	2.54	2.62	2.90	2.87	2.86	2.64	2.30	2.43
	-0.30	2.97	3.07	3.46	3.50	3.50	3.21	2.84	2.92
	-0.15	3.34	3.47	3.94	3.98	4.10	3.85	3.35	3.40
	0.00	3.75	3.96	4.54	4.69	4.85	4.53	3.99	3.98
	0.15	4.28	4.50	5.31	5.55	5.81	5.43	4.78	4.83
	0.30	4.87	5.23	6.31	6.78	7.13	6.77	6.11	5.89
	0.50	6.09	6.64	8.30	9.55	10.21	9.77	9.24	8.66
	0.75	8.71	9.77	14.55	21.64	23.20	24.29	70.04	35.24

Fig.: Theoretical Prices for New York City, calculated on 03.01.2011, as well as the CME market prices the same day

Prices



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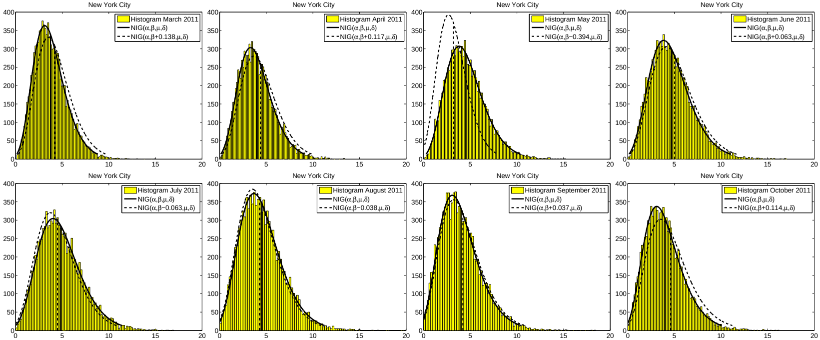
Prices



MPR $\widehat{\theta}$	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
Detroit	-0.232	-0.216	0.198	0.014	-0.024	-0.235	-0.038	-0.282
Jacksonville	-0.052	-0.165	-0.119	0.203	-0.054	-0.124	0.091	-0.334
New York City	0.138	0.117	-0.394	0.063	-0.063	-0.038	0.037	0.114

Tab.: Estimated MPR for different cities and contracts, 03.01.2011

Problem

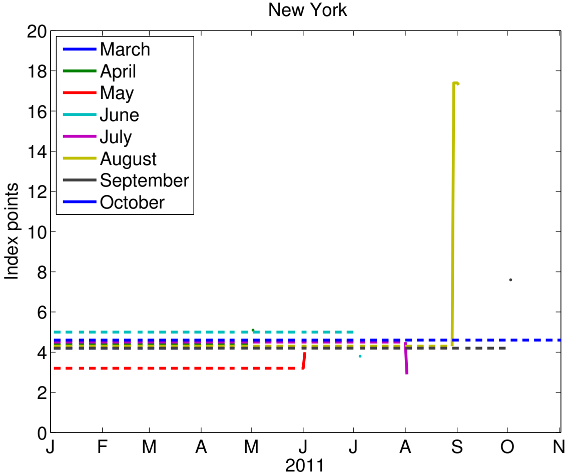


Fig.: CME prices 2011 for rainfall futures, New York City

Conclusion

Summary

- Flexible and practicable model for pricing rainfall futures
- Approach not limited to NIG distribution
- Fitting to actual market prices via the market price of risk

Discussion

- CME prices not (yet) real market prices
- Weather forecasts

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