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Practice quiz on Probability Concepts

PUNTOS TOTALES DE 9

1. If $x = \text{"It is raining,"}$ what is $\sim (\sim x)$?

1 / 1 puntos

- ☒ "It is raining"
- ☐ "It is not raining"
- ☐ "It is always raining"
- ☐ "It is never raining"

✓ Correcto

The second negation cancels out the first one.

Similarly $\sim (\sim (\sim x)) = \sim x$

2. If the statement "I am 25 years old" is assigned probability 0, what probability is assigned to the statement "I am not 25 years old"?

1 / 1 puntos

- ☐ 0
- ☐ -1
- ☐ Unknown
- ☒ 1

✓ Correcto

It is always the case that $p(x) + p(\sim x) = 1$.

3. If I assign to the statement $x = \text{"it will rain today"}$ a probability of $p(x) = 0.35$, what probability must I assign to the statement "it will not rain today?"

1 / 1 puntos

- ☐ .35
- ☒ .65
- ☐ .5
- ☐ 0

✓ Correcto

$$p(x) + p(\sim x) = 1$$

4. Is the following collection of statements a probability distribution?

1 / 1 puntos

1. I own a Toyota pickup truck
2. I do not own a Toyota pickup truck
3. I own a non-Toyota pickup truck
4. I do not own a non-Toyota pickup truck

- ☒ No
- ☐ Yes

✓ Correcto

The statements are not *exclusive*: 1 and 4 could both be true, 2 and 3 could both be true, 2 and 4 could both be true, and even (1) and (3) could both be true (if I owned more than one pickup truck).

5. I don't know what it means to be "ingenuous." What probability would I assign to the statement, "I am ingenuous OR I am not ingenuous"?

0 / 1 puntos

- ☒ 0
- ☐ .5
- ☐ 1
- ☐ -1

! Incorrecto

A statement and its negation form a probability distribution, and their probabilities must therefore sum to 1.

6. A friend of mine circumscribes a circle inside a square, so that the diameter of the circle and the edge of the square are the same length. He asks me to close my eyes and pick a point at random inside the square. He says the probability that my point will also be inside the circle is $\frac{\pi}{4}$

1 / 1 puntos

Is this correct?

- ☒ Yes
☐ No

✓ Correcto

Probabilities can be any real number between 0 and 1. They do not need to be rational numbers – a numerator that is a transcendental number like π is acceptable.

Note that the correct probability does not depend on the length r of the circle's radius. For a circle with any radius r to be circumscribed inside a square, the square must have sides each of length $2r$. The area of the circle is πr^2 and the area of the square is $(2r)^2 = 4r^2$. The probability of landing in a circle of area πr^2 when it is known that one is in the area of the square is equal to the ratio of the area of the circle to the area of the square in which it is circumscribed, or $\pi r^2 / 4r^2$, which equals $\pi/4$.

7. The probability of drawing a straight flush (including a Royal Flush) in a five-card poker hand is 0.0000153908

1 / 1 puntos

What is the probability of **not** drawing a straight flush?

- ☐ .9996582672
☒ .9999846092
☐ .9999745688
☐ .9967253809

✓ Correcto

$$p(\sim x) = 1 - p(x)$$

8. What is the probability that a fair, six-sided die will come up with a prime number? (Recall that prime numbers are positive integers other than 1 that are divisible only by themselves and 1)

1 / 1 puntos

- ☐ $\frac{1}{6}$
☐ $\frac{2}{3}$
☒ $\frac{1}{2}$
☐ $\frac{1}{3}$

✓ Correcto

The faces with 2, 3 and 5 satisfy the condition – which makes 3 relevant outcomes out of the “universe” of 6 outcomes = $\frac{3}{6} = \frac{1}{2}$

9. The joint probability p (the die will come up 5, the next card will be a heart) is equal to the joint probability: 1 / 1 puntos

- ☒ p (the next card will be a heart, the die will come up 5)
- ☐ p (the die will **not** come up 5, the next card will **not** be a heart)
- ☐ p (the next card will **not** come up 5, the next card will be a heart)
- ☐ p (the next card will be a heart, the die will **not** come up 5)

✓ Correcto

In joint probabilities, the order does not change the probability: $p(A, B) = p(B, A)$

Practice quiz on Problem Solving

PUNTOS TOTALES DE 9

1. I am given the following 3 joint probabilities:

1 / 1 puntos

$p(\text{I am leaving work early, there is a football game that I want to watch this afternoon}) = .1$

$p(\text{I am leaving work early, there is not a football game that I want to watch this afternoon}) = .05$

$p(\text{I am not leaving work early, there is not a football game that I want to watch this afternoon}) = .65$

What is the probability that there is a football game that I want to watch this afternoon?

- ☐ .2
- ☐ .1
- ☐ .35
- ☒ .3

✓ Correcto

Getting the answer is a two-step process. First, recall that the sum of probabilities for a probability distribution must sum to 1. So the "missing" joint distribution

$p(\text{I am not leaving work early, there is a football game I want to watch this afternoon})$ must be $1 - (0.1 + 0.05 + 0.65) = 0.2$

By the sum rule, the marginal probability $p(\text{there is a football game that I want to watch this afternoon})$ = the sum of the joint probabilities

$P(\text{I am leaving work early, there is a football game that I want to watch this afternoon}) + P(\text{I am not leaving work early, there is a football game I want to watch this afternoon}) = .1 + .2 = .3$

2. The joint probability of my summiting Mt. Baker in the next two years AND publishing a best-selling book in the next two years is .05. If the probability of my publishing a best-selling book in the next two years is 10%, and the probability of my summiting Mt. Baker in the next two years is 30%, are these two events dependent or independent?

1 / 1 puntos

- ☒ Dependent
- ☐ Independent

✓ Correcto

We know this because the joint distribution of 5% does not equal the product distribution of $(0.1) \times (0.3) = 3\%$. If I summit Mt. Baker, I am more likely to publish a best-selling book, and vice versa.

3. The Joint probability of my summiting Mt. Baker in the next two years AND my publishing a best-selling book in the next two years is .05.

1 / 1 puntos

If the probability of my publishing a best-selling book in the next two years is 10%, and the probability of my summiting Mt. Baker in the next two years is 30%, what is the probability that (sadly) in the next two years I will neither summit Mt. Baker nor publish a best-selling book?

- ☐ .9
- ☒ .65
- ☐ .95
- ☐ .25

✓ Correcto

Set A = I will summit Mt. Baker in the next two years

Set B = I will publish a best-selling book in the next two years.

Since $p(A) = 0.3$ and $p(A, B) = 0.05$, by the SUM RULE we know that $p(A, \sim B) = (0.3 - 0.05) = 0.25$

Since $p(B) = 0.1$, $p(\sim B) = 0.9$

Since $p(\sim B) = 0.9$ and $p(A, \sim B) = 0.25$ and again by the SUM RULE, $p(\sim A, \sim B) = 0.9 - 0.25 = .65$

4. I have two coins. One is fair, and has a probability of coming up heads of .5. The second is bent, and has a probability of coming up heads of .75. If I toss each coin once, what is the probability that *at least* one of the coins will come up heads?

1 / 1 puntos

- ☐ 1.0
- ☒ .875
- ☐ .625
- ☐ .375

✓ Correcto

We apply the rule $p(A \text{ or } B \text{ or both})$

$$= 1 - (p(-A)p(-B))$$

$$= 1 - ((1 - .5)(1 - .75))$$

$$= 1 - .125$$

$$=.875$$

5. What is $\frac{11!}{9!}$?

1 / 1 puntos

- ☐ 110,000
- ☐ 554,400
- ☒ 110
- ☐ 4,435,200

✓ Correcto

$$\frac{11!}{9!} = 11 \times 10 = 110$$

6. What is the probability that, in six throws of a die, there will be exactly one each of "1" "2" "3" "4" "5" and "6"?

1 / 1 puntos

- ☐ .01432110
- ☒ .01543210
- ☐ .01176210
- ☐ .00187220

✓ Correcto

There are $6! = 720$ permutations where each face occurs exactly once.

There are $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656$ total permutations of 6 throws.

The probability is therefore $\frac{720}{46656} = 0.01543210$

7. On 1 day in 1000, there is a fire and the fire alarm rings.

1 / 1 puntos

On 1 day in 100, there is no fire and the fire alarm rings (false alarm)

On 1 day in 10,000, there is a fire and the fire alarm does not ring (defective alarm).

On 9,889 days out of 10,000, there is no fire and the fire alarm does not ring.

If the fire alarm rings, what is the (conditional) probability that there is a fire?

Written $p(\text{there is a fire} \mid \text{fire alarm rings})$

- ☒ 9.09%
- ☐ 1.1%
- ☐ 90.9%
- ☐ 1.12%

✓ Correcto

10 days out of every 10,000 there is fire and the fire alarm rings.

100 days out of every 10,000 there is no fire and the fire alarm rings.

110 days out of every 10,000 the fire alarm rings.

The probability that there is a fire, given that the fire alarm rings, is $\frac{10}{110} = 9.09\%$

8. On 1 day in 1000, there is a fire and the fire alarm rings.

1 / 1 puntos

On 1 day in 100, there is no fire and the fire alarm rings (false alarm)

On 1 day in 10,000, there is a fire and the fire alarm does not ring (defective alarm).

On 9,889 days out of 10,000, there is no fire and the fire alarm does not ring.

If the fire alarm does not ring, what is the (conditional) probability that there is a fire?

$p(\text{there is a fire} \mid \text{fire alarm does not ring})$

- ☐ .10011%
- ☐ 1.0001%
- ☒ 0.01011%
- ☐ .01000%

✓ Correcto

On $(1 + 9,889) = 9,890$ days out of every 10,000 the fire alarm does not ring.

On 1 of those 10,000 days there is a fire.

$$\frac{1}{9890} = 0.01011\%$$

9. A group of 45 civil servants at the State Department are newly qualified to serve as Ambassadors to foreign governments. There are 22 countries that currently need Ambassadors. How many distinct groups of 22 people can the President promote to fill these jobs?

1 / 1 puntos

- ☐ 8.2334×10^{12}
- ☐ $= 2.429 \times 10^{13}$
- ☒ 4.1167×10^{12}
- ☐ $= 1.06 \times 10^{35}$

✓ Correcto

$$\binom{45}{22}$$

$$= 45! / (23!)(22!)$$

$$= \frac{45!}{23! \times 22!}$$

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100 %

Practice quiz on Bayes Theorem and the Binomial Theorem

PUNTOS TOTALES DE 9

1. A jewelry store that serves just one customer at a time is concerned about the safety of its isolated customers.

1 / 1 puntos

The store does some research and learns that:

- 10% of the times that a jewelry store is robbed, a customer is in the store.
- A jewelry store has a customer on average 20% of each 24-hour day.
- The probability that a jewelry store is being robbed (anywhere in the world) is 1 in 2 million.

What is the probability that a robbery will occur while a customer is in the store?

- ☐ $\frac{1}{500000}$
- ☐ $\frac{1}{2000000}$
- ☒ $\frac{1}{4000000}$
- ☐ $\frac{1}{5000000}$

✓ Correcto

What is known is:

A : "a customer is in the store," $P(A) = 0.2$

B : "a robbery is occurring," $P(B) = \frac{1}{2,000,000}$

$P(\text{a customer is in the store} \mid \text{a robbery occurs}) = P(A \mid B)$

$P(A \mid B) = 10\%$

What is wanted:

$P(\text{a robbery occurs} \mid \text{a customer is in the store}) = P(B \mid A)$

By the product rule:

$$P(B \mid A) = \frac{P(A, B)}{P(A)}$$

and $P(A, B) = P(A \mid B)P(B)$

Therefore:

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} = \frac{(0.1)\left(\frac{1}{2000000}\right)}{0.2} = \frac{1}{4000000}$$

2. If I flip a fair coin, with heads and tails, ten times in a row, what is the probability that I will get exactly six heads?

1 / 1 puntos

- ☐ 0.021
- ☐ 0.187
- ☒ 0.2051
- ☐ 0.305

✓ Correcto

By Binomial Theorem, equals

$$\begin{aligned} & \binom{10}{6} (0.5^{10}) \\ &= \left(\frac{10!}{4! \times 6!} \right) \left(\frac{1}{1024} \right) \\ &= 0.2051 \end{aligned}$$

3. If a coin is bent so that it has a 40% probability of coming up heads, what is the probability of getting *exactly* 6 heads in 10 throws?

1 / 1 puntos

- ☐ 0.0974
- ☐ 0.1045
- ☒ 0.1115
- ☐ 0.1219

✓ Correcto

$$\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$$

4. A bent coin has 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

1 / 1 puntos

- ☐ 0.0132
- ☒ 0.0123
- ☐ 0.0213
- ☐ 0.0312

✓ Correcto

The answer is the sum of three binomial probabilities:

$$\begin{aligned} & \left(\binom{10}{8} \times (0.4^8) \times (.6^2) \right) + \left(\binom{10}{9} \times (0.4^9) \times (0.6^1) \right) + \\ & \left(\binom{10}{10} \times (0.4^{10}) \times (0.6^0) \right) \end{aligned}$$

5. Suppose I have a bent coin with a 60% probability of coming up heads. I throw the coin ten times and it comes up heads 8 times.

1 / 1 puntos

What is the value of the "likelihood" term in Bayes' Theorem -- the conditional probability of the data given the parameter.

- ☐ 0.122885
- ☐ 0.043945
- ☐ 0.168835
- ☒ 0.120932

✓ Correcto

Bayesian "likelihood" --- the $p(\text{observed data} \mid \text{parameter})$ is

$p(8 \text{ of } 10 \text{ heads} \mid \text{coin has } p = .6 \text{ of coming up heads})$

$$\binom{10}{8} \times (0.6^8) \times (0.4^2) = 0.120932$$

6. We have the following information about a new medical test for diagnosing cancer.

1 / 1 puntos

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer. The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of "Negative" for cancer. The other 10% get a false test result of "Positive" for cancer.

What is the conditional probability that I have Cancer, if I get a "Positive" test result for Cancer?

**Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.

- ☐ 67.9%
- ☒ 32.1% probability that I have cancer
- ☐ 4.5%
- ☐ 9.5%

✓ Correcto

I still have a more than $\frac{2}{3}$ probability of not having cancer

Posterior probability:

$p(\text{I actually have cancer} \mid \text{receive a "positive" Test})$

By Bayes Theorem:

$$= \frac{(\text{chance of observing a PT if I have cancer})(\text{prior probability of having cancer})}{(\text{marginal likelihood of the observation of a PT})}$$

$$= \frac{p(\text{receiving positive test} \mid \text{has cancer})p(\text{has cancer} \mid \text{before data is observed})}{p(\text{positive} \mid \text{has cancer})p(\text{has cancer}) + p(\text{positive} \mid \text{no cancer})p(\text{no cancer})}$$

$$= (90\%)(5\%) / ((90\%)(5\%) + (10\%)(95\%))$$

$$\approx 32.1\%$$

7. We have the following information about a new medical test for diagnosing cancer.

1 / 1 puntos

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer.

The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative" for cancer.

The other 5% get a false test result of "Positive" for cancer.

What is the conditional probability that I have cancer, if I get a "Negative" test result for Cancer?

- ☒ 0.9%
- ☐ 99.1%
- ☐ .80%
- ☐ 88.2%

✓ Correcto

$p(\text{cancer} \mid \text{negative test}) =$

$$\frac{p(\text{negative test} \mid \text{Cancer}) p(\text{Cancer})}{p(\text{negative test} \mid \text{cancer}) p(\text{cancer}) + p(\text{negative test} \mid \text{no cancer}) p(\text{no cancer})}$$

$$\frac{(10\%)(8\%)}{(10\%)(8\%) + (95\%)(92\%)}$$

$$\frac{0.8\%}{0.8\% + 87.4\%}$$

$$\frac{0.8\%}{88.2\%}$$

$$= 0.9\%$$

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

1 / 1 puntos

You are not told whether the draw was done “with replacement” or “without replacement.”

What is the probability that the draw was done with replacement?

- ☐ 1
- ☐ 13.98%
- ☒ 12.27%
- ☐ 87.73%

✓ Correcto

$p(40 \text{ blue and } 10 \text{ white} \mid \text{draws without replacement}) = 1$ [this is the only possible outcome when 50 draws are made without replacement]

$p(40 \text{ blue and } 10 \text{ white} \mid \text{draws with replacement})$

$S = 40$

$N = 50$

$P = .8$ [for draws with replacement] because 40 blue of 50 total means $p(\text{blue}) = 40/50 = .8$

$$\binom{50}{40} (0.8^{40}) (0.2^{10})$$

$$= 13.98\%$$

By Bayes' Theorem:

$p(\text{draws with replacement} \mid \text{observed data}) =$

$$\frac{13.98\%(.5)}{(13.98\%(.5) + (1)(.5)}$$

$$= \frac{0.1398}{1.1398}$$

$$= 12.27\%$$

9. According to Department of Customs Enforcement Research: 99% of people crossing into the United States are not smugglers.

1 / 1 puntos

The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- ☐ 7.92%
- ☐ 92.42%
- ☒ 7.58%
- ☐ 8.57%

✓ Correcto

By Bayes' Theorem, the answer is

$$\frac{(.65)(.01)}{((.65)(.01) + (.08)(.99))}$$

$$= 7.58\%$$

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Probability (basic and Intermediate) Graded Quiz

CALIFICACIÓN DEL ÚLTIMO ENVÍO

83.33%

1. What additional statement, added to the three below, forms a probability distribution?

1 / 1 puntos

- (1) I missed only my first class today
- (2) I missed only my second class today
- (3) I missed both my first and second class today

✓ Correcto

2. My friend takes 10 cards at random from a 52-card deck, and places them in a box. Then he puts the other 42 cards in a second, identical box. He hands me one of the two boxes and asks me to draw out the top card. What is the probability that the first card I draw will be the Ace of Spades?

1 / 1 puntos

✓ Correcto

3. I will go sailing today if it does not rain. Are the following two statements Independent or dependent?

1 / 1 puntos

(1) "I will go sailing today"

(2) "It will not rain today"

✓ Correcto

4. The probability that I will go sailing today AND the fair six-sided die will come up even on the next roll is .3.

0 / 1 puntos

If these events are independent, what is the probability that I will go sailing today?

! Incorrecto

5. I have two coins. One is fair, and has a probability of coming up heads of .5.
The second is bent, and has a probability of coming up heads of .75.

1 / 1 puntos

If I toss each coin once, what is the probability that at least one of the coins will come up tails?

✓ Correcto

6. What is the probability, when drawing 5 cards from a fair 52-card deck, of drawing a "full house" (three of a kind and a pair) in the form AAABB?

1 / 1 puntos

✓ Correcto

7. If it rains, I do not go sailing. It rains 10% of days; I go sailing 3% of days.

1 / 1 puntos

If it does not rain, what is the (conditional) probability that I go sailing?

Written " $p(\text{I go sailing} \mid \text{it does not rain})$ "?

✓ Correcto

8. I am at my office AND not working 2% of the time. I am at my office 10% of the time. What is the conditional probability that I am not working, if I am at my office?

1 / 1 puntos

✓ Correcto

9. The factory quality control department discovers that the conditional probability of making a manufacturing mistake in its precision ball bearing production is 4% on Tuesday, 4% on Wednesday, 4% on Thursday, 8% on Monday, and 12% on Friday.

1 / 1 puntos

The Company manufactures an equal amount of ball bearings (20%) on each weekday. What is the probability that a defective ball bearing was manufactured on a Friday?

✓ Correcto

10. An Urn contains two white marbles and one black marble. A marble is drawn from the Urn without replacement and put aside without my seeing it. Then a second marble is drawn, and it is white.

0 / 1 puntos

What is the probability that the unknown removed marble is white, and what is the probability that it is black?

! Incorrecto

11. What is the probability, if I flip a fair coin with heads and tails ten times in a row, that I get at least 8 heads?

1 / 1 puntos

✓ Correcto

12. Suppose I have either a fair coin or a bent coin, and I don't know which. The bent coin has a 60% probability of coming up heads.

1 / 1 puntos

I throw the coin ten times and it comes up heads 8 times. What is the probability I have the fair coin vs. the probability I have the bent coin?

Assume at the outset there is an equal (.5, .5) prior probability of either coin.

*Please note that in order to fit the entire formula in the feedback, probability has been abbreviated to "prob."

✓ Correcto