Precision matters: From quantum thermometry to the quantum estimation of scales, and back

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Key works:

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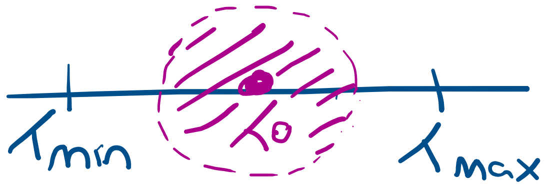
Our plan for today

I. Precision in quantum technologies: a brief review

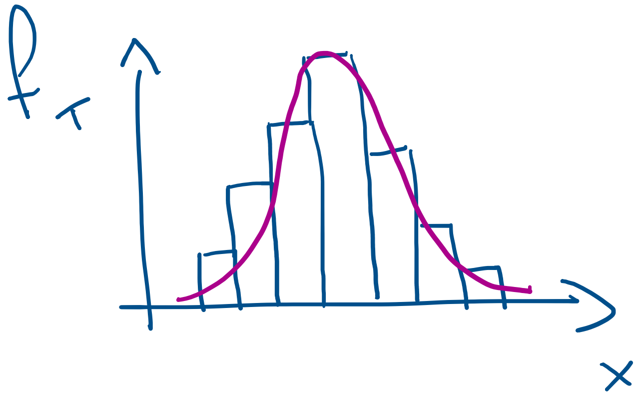
II. Global quantum thermometry: a Bayesian upgrade



III. Quantum metrology of scale parameters

> Designing efficient protocols for quantum scale estimation
> Example: optimal POVM with no *a priori* knowledge
> Application: optimal cold atom thermometry

IV. Many quantum thermometries?

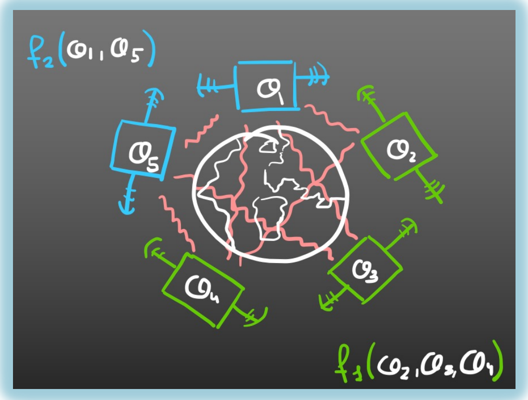


V. Conclusions and outlook

I. Precision in quantum technologies: a brief review

- Measurand x, unknown parameter Θ, known parameters $\boldsymbol{y} = (y_1, y_2, \dots)$

- Prior probability $p(\theta)$ and quantum state $\rho_{\boldsymbol{y}}(\theta)$



- POVM $M_{\boldsymbol{y}}(x)$

- Outcome statistics via the Born rule $p(x|\theta, \boldsymbol{y}) = \mathrm{Tr}[M_{\boldsymbol{y}}(x)\rho_{\boldsymbol{y}}(\theta)]$

- Estimator $\tilde{\theta}_{\boldsymbol{y}}(x)$

- Uncertainty

$$\bar{\epsilon}_{\boldsymbol{y}}(x) = \int d\theta\, p(\theta|x, \boldsymbol{y})\, \mathcal{D}[\tilde{\theta}_{\boldsymbol{y}}(x), \theta]$$

(for experiments)

or

$$\bar{\epsilon}_{\boldsymbol{y}} = \int dx\, p(x|\boldsymbol{y})\, \bar{\epsilon}_{\boldsymbol{y}}(x)$$

(to formulate the optimization problem)

II. Global quantum thermometry: a Bayesian upgrade

- Scale invariance: standard definition

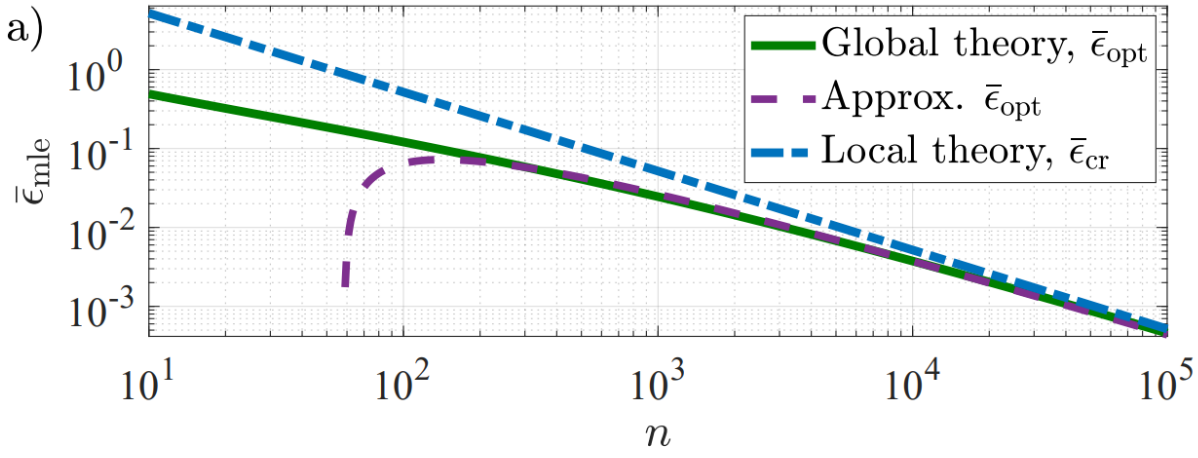
$$p(E|\theta)dE = \frac{f\,[E/(k_B\theta)]}{\int d\hat{E}\, f[\hat{E}/(k_B\theta)]} dE \quad \mapsto \quad \bar{\epsilon}_{\rm mle} = \int dE d\theta\, p(E,\theta) \log^2 \left[\frac{\tilde{\theta}(E)}{\theta}\right]$$

- **Optimal rule to post-process measurements into a temperature reading**

$$\frac{k_B\tilde{\vartheta}(E)}{\varepsilon_0} = \exp\left[\int d\theta\, p(\theta|E) \log\left(\frac{k_B\theta}{\varepsilon_0}\right)\right]$$

- **Minimum** uncertainty (not just a bound)

$$\bar{\epsilon}_{\rm mle} \geq \bar{\epsilon}_p - \mathcal{K} \longrightarrow$$



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III. Quantum metrology of scale parameters

- **Generalised scale invariance**

$$\begin{cases} Y \text{ is `large' when } Y/\Theta \gg 1 \\ Y \text{ is `small' when } Y/\Theta \ll 1 \end{cases} \mapsto \quad \Theta \text{ is a \underline{scale parameter}}$$

$$\begin{cases} Y \mapsto Y' = \gamma Y \\ \Theta \mapsto \Theta' = \gamma \Theta \end{cases} \mapsto \quad Y'/\Theta' = Y/\Theta \text{ is the key symmetry}$$

Examples:

- temperature: $\frac{E}{k_B T}$
- (Inverse of) rate: $kt = \frac{t}{1/k}$

- Scale-invariant probability models (a particular subset)

$$p(x|\theta, \boldsymbol{y}) = \mathrm{Tr}[M_{\boldsymbol{y}}(x)\rho_{\boldsymbol{y}}(\theta)] = h\left(x, \frac{\boldsymbol{y}}{\theta}\right)$$

- **Goal**: to optimise the mean logarithmic error <u>wrt to POVM + estimator</u>

$$\min_{\tilde{\theta}(x),\, M(x)} \mathrm{Tr}\left\{\int dx\, M(x)\, W[\tilde{\theta}(x)]\right\} = \bar{\epsilon}_{\min}$$

arXiv:2111.11921

Designing efficient protocols for quantum scale estimation

1. Calculate: $\varrho_{\boldsymbol{y},k} := \int d\theta\, p(\theta)\rho_{\boldsymbol{y}}(\theta) \log^k\left(\frac{\theta}{\theta_u}\right)$

2. Solve for $\mathcal{S}_{\boldsymbol{y}}$: $\mathcal{S}_{\boldsymbol{y}}\varrho_{\boldsymbol{y},0} + \varrho_{\boldsymbol{y},0}\mathcal{S}_{\boldsymbol{y}} = 2\varrho_{\boldsymbol{y},1}$

3. Calculate the spectral decomposition of $\mathcal{S}_{\boldsymbol{y}}$

4. The eigenstates give the elements of the optimal POVM

5. The spectrum $\{s\}$ leads to the optimal estimates as $\tilde{\vartheta}_{\boldsymbol{y}}(s) = \theta_u \exp(s)$

Basic recipe to solve *any* quantum scale estimation problem

arXiv:2111.11921

Example: optimal POVM with no a priori knowledge

Is it a scale estimation problem?

$$\left\{\begin{aligned} \rho_{\boldsymbol{y}}(\theta) &= \frac{\exp[-H/(k_B\theta)]}{\mathrm{Tr}\{\exp[-H/(k_B\theta)]\}} = \sum_n |n\rangle\langle n| \frac{\exp(-y_n/\theta)}{\sum_m \exp(-y_m/\theta)}, \\ M_{\boldsymbol{y}}(x) &= \sum_{nm} |n\rangle\langle m|\, M_{\boldsymbol{y},nm}(x) = M_{\boldsymbol{y}'}(x) \end{aligned}\right. \mapsto \boxed{\begin{aligned} p(x|\boldsymbol{y},\theta) &= h(x|\boldsymbol{y}/\theta) \\ \boldsymbol{y} &= (\varepsilon_0, \varepsilon_1, \dots)/k_B \end{aligned}}$$

Solution:

$$S_{\boldsymbol{y}} = \sum_n |n\rangle\langle n| \frac{\chi^{n,1}_{\boldsymbol{y}}}{\chi^{n,0}_{\boldsymbol{y}}}, \longrightarrow$$

Energy measurements are optimal with both global and local prior information

arXiv:2111.11921

Application: optimal cold atom thermometry

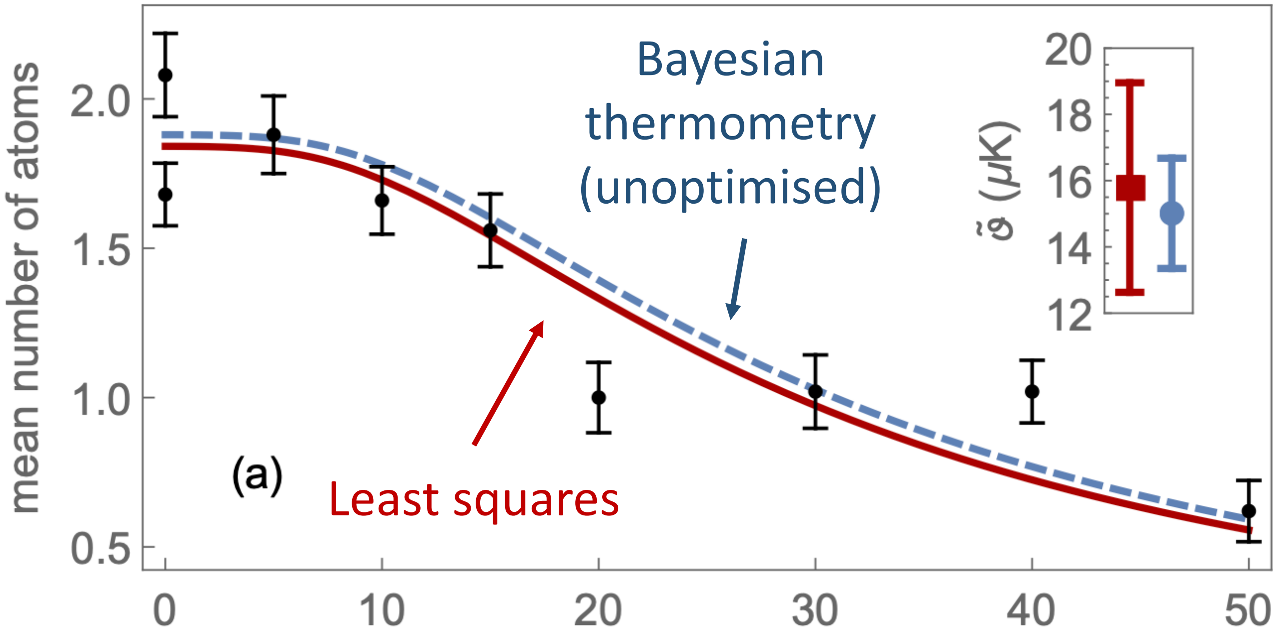
Fraction of recaptured atoms: $f(T,t)$

$$\mapsto \quad P(\text{‘recapture’}|\theta, \boldsymbol{y}) = h\left(\frac{\boldsymbol{y}}{\theta}\right), \quad \boldsymbol{y} = (y_1, y_2) = \left(\frac{U_0}{k_B}, \frac{m\omega_0^2}{k_B t^2}\right)$$

Unknown temperature

Known parameters

Experimental verification of the validity of both GQT and QSE



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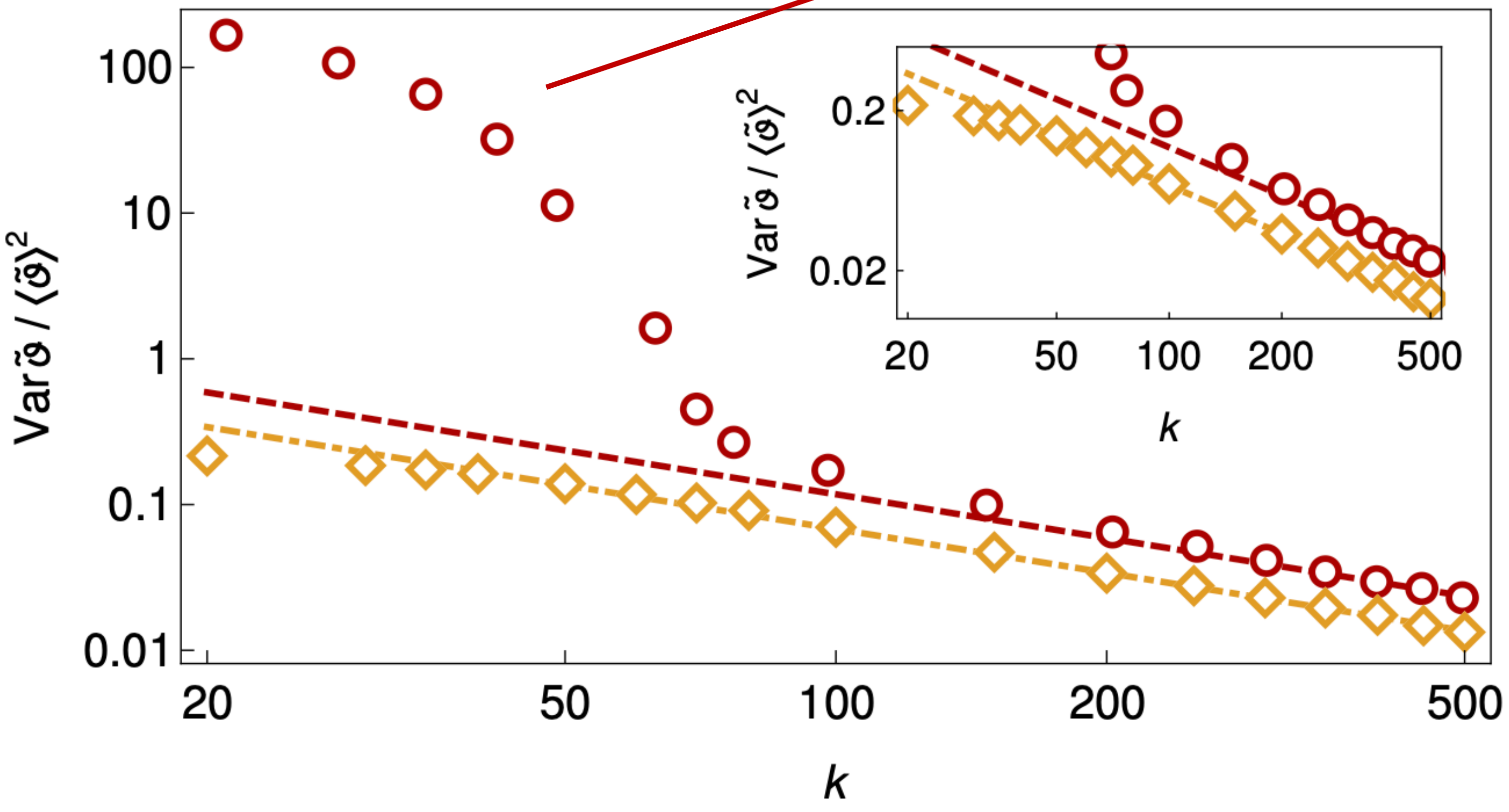
Application: optimal cold atom thermometry

Least squares



Bayesian thermometry (single-shot optimised)

Twice as much precision with half of the measurement data

arXiv:2204.11816

New J. Phys. **21**, 043037 (2019)

IV. Many quantum thermometries?

- **MSE-based**
  - Easy to use, practical
  - Ignores relevant symmetries
  - Valid in the limited-data regime?

Optimal Probes for Global Quantum Thermometry

Wai-Keong Mok,[1] Kishor Bharti,[1] Leong-Chuan Kwek,[1,2,3,4] and Abolfazl Bayat[5]
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Bayesian estimation for collisional thermometry

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Non-informative Bayesian Quantum Thermometry

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(Dated: August 18, 2021)

- **Scale invariant** (this talk)
  - Easy to use, practical
  - Minimal assumptions
  - Physically sound

Global Quantum Thermometry

Jesús Rubio,[1,*] Janet Anders,[1,2] and Luis A. Correa[1]
...nited Kingdom.
...any.

Fundamental limits in Bayesian thermometry and attainability via adaptive strategies

Mohammad Mehboudi,[1,*] Mathias R. Jørgensen,[2,†] Stella Seah,[1]
Jonatan B. Brask,[2] Jan Kołodyński,[3] and Martí Perarnau-Llobet[1,‡]
...oland

Optimal cold atom thermometry using adaptive Bayesian strategies

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- **Fully invariant**
  - Elegant
  - Are all transformations physically equivalent?
  - $p(\theta) \propto \sqrt{F(\theta)}$

Bayesian quantum thermometry based on thermodynamic length

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(Dated: August 19, 2021)

V. Conclusions and outlook

- Global (Bayesian) quantum thermometry is a particular realisation of a more general estimation framework: **quantum scale estimation**

- Quantum scale estimation …
  - … enables the most precise estimation that the laws of quantum mechanics allow for scale parameters
  - … closes an important gap in quantum metrology
  - … provides a fundamental picture : **phases, locations and scales**

Type of parameter	phase	location	scale
General support	$0 \leq \theta < 2\pi$	$-\infty < \theta < \infty$	$0 < \theta < \infty$
Symmetry	$\theta \mapsto \theta' = \theta + 2\gamma\pi,\ \gamma \in \mathbb{Z}$	$\theta \mapsto \theta' = \theta + \gamma,\ \gamma \in \mathbb{R}$	$\theta \mapsto \theta' = \gamma\theta,\ \gamma \in \mathbb{R}_*^+$
Maximum ignorance	$p(\theta) = 1/2\pi$	$p(\theta) \propto 1$	$p(\theta) \propto 1/\theta$
Deviation function $\mathcal{D}[\tilde{\theta}(x), \theta]$	$4\sin^2\{[\tilde{\theta}(x) - \theta]/2\}$	$[\tilde{\theta}(x) - \theta]^2$	$\log^2[\tilde{\theta}(x)/\theta]$

- Next steps: application of quantum scale estimation to non-equilibrium quantum thermometry, as well as for the measurement of scales other than temperatures (e.g., rates).

Thank you for your attention

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If you have any question or comment:

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