Production

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Exercise 1. The Robot Uprising: Perfect Substitutes Production

Consider an economy where robots and workers are perfect substitutes in production. Assume that the production function is given by:

$$Y = A\Big[\alpha(zK) + (1-\alpha)L\Big],$$

where:

- Y is total output in the economy.
- $A > 0$ is total factor productivity, scaling how effectively inputs generate output.
- K denotes the amount of physical capital (robots or machines).
- $z > 1$ captures the productivity of robots relative to workers; a higher z means each unit of capital is more effective due to advances in AI.
- L is the amount of labor supplied by workers.
- $\alpha \in (0, 1)$ determines the weight on capital in production, while $1-\alpha$ determines the weight on labor.

Unlike the simple case with fixed factors, this economy has **elastic factor supply**:

- Capital supply: $K^s = \gamma_K r$, where $\gamma_K > 0$ (more robots available when rental rates rise)
- Labor supply: $L^s = \gamma_L w$, where $\gamma_L > 0$ (more workers available when wages rise)

(a) Compute the marginal product of capital (MPK) and marginal product of labor (MPL). How do they compare to the Cobb–Douglas case? What happened to diminishing returns?

(b) Write down the firm's profit-maximization problem and derive the first-order conditions. Under perfect competition, what must r and w equal?

(c) **Defining equilibrium:** Write down all five equations that characterize the general equilibrium of this economy (capital demand, labor demand, capital supply, labor supply, and production function). List all endogenous and exogenous variables. Do you have as many equations as unknowns?

(d) **Solving the model:** Find the equilibrium values r^*, w^*, K^*, L^*, and Y^* in terms of the exogenous parameters $(A, \alpha, \gamma_K, \gamma_L)$.

Hint: Start by using the first-order conditions to pin down factor prices, then use the supply curves to find quantities.

(e) Compute the labor share of output, $s_L = \frac{w^* L^*}{Y^*}$, in equilibrium. Express your answer in terms of K^* and L^*.

(f) **Technology shock:** Suppose productivity A increases by 20%. What happens to:

- The rental rate r^* and wage w^*?
- Capital K^* and labor L^*?
- Total output Y^*?

Are workers better or worse off after this shock?

(g) **Rise of the robots:** Now suppose robots become more productive relative to workers, so z increases. What happens to:

- The rental rate r^* and wage w^*?
- Capital K^* and labor L^*?
- The labor share s_L?

Show mathematically that the labor share falls when z rises.

- How does it compare with the Cobb-Douglas case where $Y = A(zK)^\alpha L^{1-\alpha}$