Macroeconomics I
Homework 2

This homework is due on November the 21^{st} at 5 pm.

Exercise 1

In this exercise you will solve the the social planner problem of the Neoclassical growth model following a detailed set of instructions. As we saw in class the planner's recursive problem can be written as:

$$V(k) = \max_{k' \in \Gamma(k)} \{u(f(k) + (1-\delta)k - k') + \beta V(k')\} \tag{1}$$

We are going to assume $f(k) = k^\alpha$ with $\alpha = 0.36$, $u(c) = log(c)$, $\beta = 0.9$, $\delta = 0.025$

a Define parameter values (β, α, δ).

b Given the parameter values compute the steady state level of capital.

c Define an equally spaced grid of capital around the steady state level of capital: $k \in [0.9k^*, 1.1k^*]$ with 500 points in the grid ($nkk = 500$)

d For each level of capital in the grid, set a guess for your value function $V^g(k)$. V^g is a $nkk \times 1$ vector (e.g. $V^g(k) = 0\ \forall k$).

e Set today's level (k) to the first position in the grid of capital i.e. $i = 1$ or $k_i = k_1 = 0.9k^*$.

   i Create a $nkk \times 1$ vector containing $\log(k_i^\alpha + (1-\delta)k_i - k'_j) + \beta V^g(k'_j)$ for each possible level of capital tomorrow k'_j. In case $k'_j \geq k_i^\alpha + (1-\delta)k_i$, set $\log(k_i^\alpha + (1-\delta)k_i - k'_j) + \beta V^g(k'_j) = -Inf$

   ii Define $V^{g+1}(k_i) = TV^g(k_i) = \max_{k'_j}\{\log(k_i^\alpha + (1-\delta)k_i - k'_j) + \beta V^g(k'_j)\}$ where T is the operator defined by equation (1).

   iii Set $i = i + 1$ and go back to (i.) until $i = nkk$.

f At this stage you will have a new vector V^{g+1}. Compute the distance between V^{g+1} and V^g as $\epsilon = \max_{k \in \{k_1, \ldots, k_{nkk}\}} |V^{g+1}(k) - V^g(k)|$

   i If $\epsilon >$ tolerance error, then, set $V^g = V^{g+1}$ and go back to e).

   ii Otherwise, you have already found V that solves the functional equation.

g Given V, you can find the policy function for capital as

$$\pi(k_i) = \arg\max_{k'_j} \log(k_i^\alpha + (1-\delta)k_i - k'_j) + \beta V(k'_j)$$

h Given an initial level of capital $k_{t=0} = 0.9k^*$ and the policy function π, simulate the path of assets for 50 periods.

i You should report:

- Number of iteration over the value function until convergence and computing time.
- Plot of your final value function as a function of current capital.
- Plot of the policy function k' as a function of capital with a reference 45 degree line. Plot $k' - k$ as a function of capital.
- Path of capital over time.

Exercise 2

Consider the following economy populated by a continuum of identical agents. Time is discrete and the horizon is infinite. Each individual owns a set of trees at time 0, denoted by s_0. In each period t, the individual has to cop some trees to produce (and consume) fruits. The technology to produce fruits is denoted by

$$y_t = f(\theta_t, n_t)$$

where θ_t is the amount of trees cut in period t; and n_t is the labor used to cut trees. Each individual is endowed with 1 unit of time each period. Agents decide how much to work and how many trees to cut in each period in order to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

with $\beta \in (0,1)$ where c_t is consumption of fruits at time t.

a let X be the state space, $F : X \times X \to R$ be the return function, and let $\Gamma : X \twoheadrightarrow X$ be the correspondence between the state variable and the feasible set. Specify these objects for the above problem.

b Write down the dynamic programming problem associated with this problem.

c Let $f(\theta_t, n_t) = \theta_t^{\alpha} n_t^{1-\alpha}$, with $\alpha \in (0,1)$ and $u(c_t) = \dfrac{c_t^{1-\sigma}}{1-\sigma}$ with $\sigma \in (0,1)$. Show that the sequence of chopped trees will follow the following equation:

$$\theta_{t+1} = \beta^{1/(1-\alpha+\alpha\sigma)}\ \theta_t$$

d Argue that the relationship between the initial set of trees s_0 and the sequence of chopped trees $\{\theta_t\}_{t=0}^{\infty}$ can be used to pin down θ_0, and hence to define the whole sequence $\{\theta_t\}_{t=0}^{\infty}$.