

Long-Term Care Needs and Savings in Retirement

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Abstract

Self-insurance against long-term-care (LTC) and bequest motives have been identified as two key mechanisms driving savings in retirement. In this paper, I investigate to which extent heterogeneity in access to informal care and health differences when in need of care drive the relative importance of each mechanism. For this purpose, I develop and estimate a model for retired single individuals with heterogeneous LTC needs, endogenous LTC expenses, and differences in access to informal care provision. I find that LTC is relatively more important than bequest motives in explaining the lack of dissaving during retirement for individuals with limited access to informal care. Instead, bequest motives play a larger role than LTC for wealthy individuals with strong informal care support from relatives. Finally, the model shows that states of health implying concurrent cognitive and mental frailty account for 80% of the precautionary savings related to LTC

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1 Introduction

Contrary to the predictions of a standard life-cycle model (Huggett, 1996), many elderly dissave slowly during retirement. Bequests motives and long-term care (LTC) have been identified as key drivers to explain the lack of dissaving of the old (Ameriks, Briggs, Caplin, Shapiro, and Tonetti 2016; Lockwood 2018). In this paper, I show that the importance of these mechanisms depends on the heterogeneous access to informal care from relatives and differences in care needs related to cognitive and/or physical deterioration.

I start by presenting new facts highlighting the importance of modeling heterogeneity in LTC needs, endogenous LTC expenses, and access to informal care in structural models. First, I show that in order to identify the effect of LTC on the savings decision of the old it is important to consider the relation between different levels of frailty and mortality. Using the Health and Retirement Study (HRS), I document that LTC needs can be parsimoniously represented by four latent health states labeled as: healthy, physically frail, mentally frail, and impaired. Healthy individuals do not need help with daily self-care activities. In contrast, physically and mentally frail individuals are in need of assistance with activities related to mobility and cognition, respectively. While these two groups do not show large differences in mortality rates, mentally frail individuals consume much more formal care. Finally, impaired individuals are in need of assistance with both physical and cognitive tasks and are the ones consuming the most formal care and facing larger chances of moving into nursing homes. Crucially, high mortality rates for impaired individuals reduce the risk of being in acute need of care for long periods of time, thus limiting the financial risk implied by LTC.

Second, I document that the observed LTC expenses reflect choices and hence cannot be taken at face value to measure needs. In fact, conditional on health, richer individuals spend significantly more on formal care. An impaired individual in the top quintile of the permanent income distribution consumes three hours more of formal care per day than an impaired individual in the bottom quintile.

Finally, the data reveals that the provision of informal care from relatives affects the consumption of formal care when in need of LTC. Conditional on health, individuals who have limited

access to informal care consume 2.5 times more formal care than those who have strong informal care support. Moreover, the probability of nursing home entry is around 30% higher for those with limited access to informal care. Therefore differences in access to informal care imply a differential exposure to LTC risk.

Motivated by these facts, I develop and estimate a model of single retired individuals allowing for heterogeneity in both LTC needs and family types, as well as in gender, permanent income, medical expenses and nursing home entry, and wealth. Agents in the model derive utility from regular consumption, LTC, and leaving bequests. Family types differ in the hours of informal care provided by children when the old is in need of LTC and drive the probability of nursing home entry. Families provide informal care for free but agents can, additionally buy formal care at a market price. Then, LTC is produced by combining both formal and informal care through a CES production function. The marginal utility of LTC is also allowed to differ depending on the level of LTC need. When individuals enter in a nursing home, they lose most of the informal care support and need to pay a larger price for formal care. The willingness to bestow is modeled using a warm-glow utility function developed by De Nardi (2004) to capture the low dissaving rates of the elderly rich. In order to capture differences in the willingness to bestow, I allow the marginal utility of bequest to vary across family types. Finally, agents have the option to access a government means-tested program that provides a consumption floor and LTC services if necessary.

In order to characterize family types in the model, I investigate socio-economic and demographic characteristics that predict informal care support in the data. Having children (especially a daughter), being African American, and low education for both the parent and children are strong predictors of future available informal care if in need of LTC. Based on these characteristics, I then categorize individuals as being in *close* and *distant* families. In order to estimate differences in the willingness to bestow across family types, I require the model to match the self-reported probability of leaving bequests and Medicaid reciprocity rates for each family type. The model estimates that the willingness to bestow varies little across family types but informal care protects individuals from consuming financial resources when in need of LTC.

I construct counterfactual simulations to identify the relative importance of LTC needs and bequest motives across family types. I find LTC needs to be relatively more important than bequest

for individuals in *distant families* but not for those in close families. In a world without LTC needs, the 75th percentile of wealth holdings at the age of 85 would be 24% and 10% lower than in the benchmark model for individuals in distant and close families respectively. In a world without bequest motives, the 75th percentile of wealth holdings at the age of 85 would be 9% and 26% lower than in the benchmark model for individuals in distant and close families, respectively.

Next, I use the model economy to quantify what fraction of the precautionary savings related to LTC can be attributed to physical difficulties, mental difficulties or both. At the age of 85, 20%, and 80% of the precautionary savings related to LTC can be attributed to cognitive deterioration and both concurrent physical and cognitive deterioration, respectively. Physical care needs playing a negligible role on precautionary savings.

My paper is related to the literature that analyzes the interaction between savings and the provision of informal care during retirement. Barczyk and Kredler (2018) show that individuals whose wealth is higher than \$7.5K are more likely to receive care from children when in need of LTC than those with lower savings. Then, the authors build a model consistent with the idea that accumulating wealth causes informal care provision from children. The authors develop a theoretical framework where provision of informal care is ex-ante available to everyone but constitutes the result from a bargaining process across generations. In their setting, the old have thus an incentive to accumulate wealth to induce children to provide informal care. This mechanism generates that, ex-post, informal care is more likely to occur the wealthier the old generation is. Besides, in a recent paper Barczyk, Fahle, and Kredler (2019) show that illiquid home equity further allows older homeowners to commit to large bequests. In their empirical section, they show that home-owners are 10 p.p. more likely to receive care from children when in need of LTC.

However, the documented positive correlation between wealth and informal care can be established, at least partially, by the reverse causal interpretation made in Barczyk and Kredler (2018). In this paper, I empirically show that a set of demographic characteristics like having a daughter or being African American increase the probability of receiving informal care as much as homeownership. I take this empirical pattern as evidence of heterogeneous access to informal care in the population. In the data healthy individuals hold more wealth than individuals in need of LTC. Interestingly, this difference is larger for those with limited access to informal care provision

which suggest that individuals with limited access to informal care deplete wealth faster in order to cope with LTC expenses. Differences in access to informal care thus can generate the correlation between low wealth holdings and limited informal care provision when in need of LTC.

Lockwood (2018) develops a model with exogenous LTC expenses and finds that strong bequest motives can rationalize the low take-up of LTC insurance. By both endogenizing LTC expenses and requiring the model to match consumption of formal care for different levels of the permanent income distribution and family types, I find that LTC needs play quantitatively a bigger role as determinant of the savings behavior of individuals in *distant families* but are of comparable magnitude for individuals in close families.

In contrast to Lockwood (2018), Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2020) estimate a much smaller intensity of the willingness to bestow. Using a novel dataset with strategic survey questions, the authors document that when faced with hypothetical scenarios between leaving a bequest or consuming when in need of LTC, individuals report a large propensity to spend when in need of LTC. They estimate health transition probabilities using the HRS and define being in LTC need as an individual with one or more difficulties with ADLs. Then, they estimate their model to match answers to the Strategic Survey Questions (SSQ) and find LTC to be the main driver of savings while bequest motives play a relatively minor role. In my paper, I show that in order to study how LTC needs drive savings it is important to account for heterogeneous needs. I find that while physically frail individuals hold marginal propensities of consuming close to healthy, impaired individuals, who represent only one fourth of individuals reporting difficulties with one or more ADLs in the HRS, hold marginal propensities to consume versus bequeath that are in line with the SSQs in Ameriks et al. (2020). However, in my model, both the low likelihood of becoming impaired coupled with large mortality rates in that state, limit the risk of LTC needs when impaired. Thus, in order to match asset holdings of the rich in the data, I estimate a stronger bequest motive.

Dobrescu (2015) estimates a model for a set of European countries in which individuals can produce care by combining formal and informal care. The author assumes that the provision of informal care is exchange motivated by making it a function of bequeathable wealth and ruling out altruistic bequests. In order to match the fact that individuals in southern Europe are more

likely to receive informal care in spite of holding lower wealth, the author allows the elasticity of care to wealth to be larger for children in southern Europe. In contrast, my modeling strategy is to consider bequest as a luxury good modeled as a warm glow following De Nardi (2004) and allowing it to be heterogeneous across families.¹ The benefit of it is twofold. First, I am able to match the large wealth holdings of the elderly rich in the data and second I can require the model to reproduce the fact that wealth poorer individuals are more likely to receive informal care. Through the lens of my model, differences across European countries could potentially be rationalized by variation in socio-economic and demographic factors that drive the share of *close* and *distant families* (e.g. education of parents and children, ethnicity, divorce) as well as cultural differences in family arrangements.

In a life-cycle model including the working life, Kopecky and Koreshkova (2014) estimate the independent contribution of medical and nursing home expenses on aggregate wealth without considering intentional bequests. They find that savings for out-of-pocket health expenses account for 13.5 percent of the aggregate wealth, half of which is due to nursing home expenses. The authors find that in order to determine the risk implied by medical expenses it is important to include the pre-retirement phase as individuals in the decade before retirement are the ones the most exposed to this risk. By focusing exclusively on retired individuals covered through Medicare, in my paper the role played by medical expenses on savings is modest.

On the empirical literature, there is a lack of consensus on how future bequests affect the provision of informal care. On the one hand, Brown (2006) and Groneck (2016) find that end-of-life transfers tend to favor both current and expected caregivers. On the other hand, Mukherjee (2020) uses variation in Social Security benefits and finds little support for exchange motivated transfers. My parameter estimates imply that the intensity of the bequest motive does not significantly vary across family types in spite of the large differences in the provision of informal care and that individuals in close families tend to accumulate lower savings during their working age. These facts point towards a stronger role for altruistic bequest motives even if in my setting I cannot explicitly separate them from exchange motivated transfers.

¹More recently, Lockwood (2012) using the fraction of individuals holding life annuities and Lee and Tan (2019) exploiting the Social Security Notch as an instrument to identify the effect of benefits on bequest, provide evidence that bequest are a luxury good.

The rest of the paper is organized as follows. In Section 2, I explain how I identify different levels of LTC needs from the data and document new facts on LTC expenditure choices. Then, I propose a model that is able to accommodate these facts in Section 3. In Section 4, I present counterfactual experiments to quantify the forces affecting the saving behavior. Section 5 concludes.

2 Heterogeneous Long-Term Care Needs, Provision of Care from Families, and Formal Care Choices

In this section, I first describe how I identify heterogeneity in LTC needs using the HRS. Next, I document determinants of the provision of informal care from family members when individuals are in need of care. Finally, I explain how individuals complement the provision of care provided by relatives using formal care.

2.1 Heterogeneous Needs

The HRS is a longitudinal survey nationally representative of Americans above age 50 conducted by the University of Michigan. It contains a wide array of health variables analyzing the individual's desire for consumption of LTC services. Given its panel structure, the HRS is ideal for analyzing different levels of LTC needs and their dynamics over time which crucially affect individuals' saving and consumption decision.

In order to classify individual health status, I estimate a modified version of the model in Amengual, Bueren, and Crego (2017). Following Amengual et al. (2017), I exploit information contained in 12 dummy variables that characterize individual's reported difficulty with ADLs and IADLs. Each variable is equal to 1 if the individual reports difficulty and 0 otherwise. ADLs were proposed by Katz, Ford, Moskowitz, Jackson, and Jaffe (1963) as a measure of patient's independence with basic personal tasks of everyday life such as being able to get in or out of bed. IADLs, in contrast, consist of activities more closely related to cognition (see Atkinson et al. 2005; Ng, Niti, Chiam, and Kua 2006). Examples of the latter include the ability to use a phone or controlling her medication. Thus, both ADLs and IADLs are related to the individual's LTC

needs.²

I assume that the main source of heterogeneity in I-ADLs³ in the population is represented by a finite number of possible health groups that are not observed by the econometrician. Each individual i at time t belongs to one health group $h_{i,t}$. Health groups differ in the probability of reporting a difficulty with each I-ADL. Given her group g , the probability of facing difficulties with the k 'th I-ADL (out of 12 I-ADLs), say $y_{i,k,t} = 1$ is $\mu_{k,g}$. Under the assumption that I-ADLs are independently distributed conditional on health status, the joint distribution of $\mathbf{y}_{i,t} = (y_{1,i,t}, \dots, y_{12,i,t})$ is characterized by:

$$p(\mathbf{y}_{i,t} | \mu_g) = \prod_{k=1}^{12} \mu_{k,g}^{y_{k,i,t}} (1 - \mu_{k,g})^{1-y_{k,i,t}} \quad (1)$$

On top, dynamics, i.e the probability of moving from one health group to another, and survival rates, are jointly estimated conditioning on individual's cubic in age, gender, gender interacted with age, a quadratic in permanent income decile, and the permanent income decile interacted with age. Permanent income is computed as the individual's average non-asset income over all periods during which she is observed. Non-asset income includes Social Security benefits, defined pensions benefits and annuities.

I estimate the model including all single individuals in the HRS from 1996 to 2014⁴ and who are aged 70 or older. The estimation of the econometric model shows that variation in LTC needs can be parsimoniously represented by four clearly different health states of need: the healthy, the physically frail, the mentally frail, and the impaired.⁵ Figure 1 displays the probability of reporting each I-ADL for each group ($\mu_{k,g}$). The healthy are individuals whose probability of declaring problems with I-ADLs is close to 0 for every I-ADL and thus do not require LTC. The physically frail have problems with physical activities while the mentally frail have problems mainly with ac-

²ADLs: Some difficulty with dressing (DRESS), using the toilet (TOILET), bathing (BATH), getting in or out of bed (BED), to walk across a room (WALK) and eating (EAT). IADLs: Some difficulty with preparing a hot meal (MEALS), shopping for groceries (SHOP), managing money (MONEY), taking medications (MEDS), using a phone (PHONE), and using a map (MAP).

³I use I-ADLs to refer to both ADLs and IADLs

⁴There was a change in the HRS in the number of adls asked prior to 1996, thus I drop observations from earlier waves

⁵The reader is referred to the original paper for details on the estimation procedure

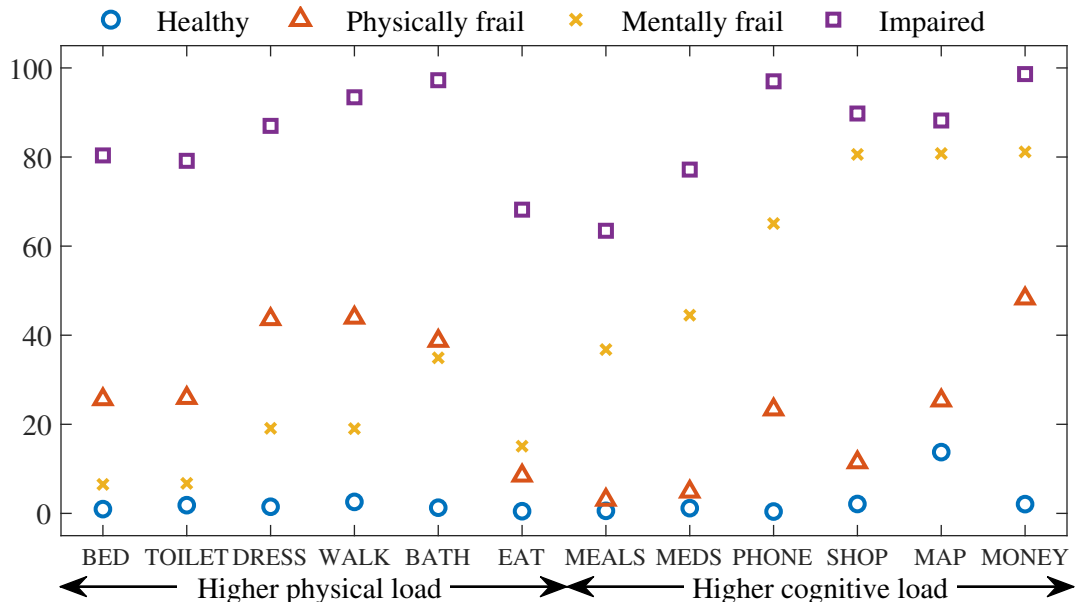


FIGURE 1. PROBABILITY OF REPORTING DIFFICULTY WITH ANY I-ADL BY HEALTH GROUP

tivities related to cognition. Finally, the impaired show problems with both cognitive and physical activities.

However, in order to characterize the risk of LTC needs, one must take into account not only the severity of the different levels of need but also their dynamics. The left panel of Figure 2 shows the estimated two-year mortality rates for women in the median of the permanent income distribution. The figure shows that survival probabilities sharply decrease as individual's health status deteriorates. The differences are salient even after conditioning on being in need of LTC: for example a woman who is impaired faces a mortality rate which is twice as large as a woman who is mentally frail. Differences in mortality rates and transitions across health states characterize the duration that individuals expect to live in different levels of LTC need. Table 1 summarizes the expected duration in each health state at age 70 for the top and bottom PI deciles across gender. The first column sums the expected duration in all possible health states which is equal to the life expectancy at age 70. The table shows dramatic differences in health dynamics across permanent income groups. An individual in the top of the permanent income distribution expects to live around 5 years longer than an individual in the bottom. Moreover, richer individuals live healthier lives with a shorter expected duration (both in years and as a fraction of their remaining life expectancy) in need of LTC. The right panel of Figure 2 compares the share of impaired

TABLE 1. EXPECTED DURATION OF EACH HEALTH STATE AT AGE 70 ACROSS PERMANENT INCOME DECILES AND SEX

Permanent income	Total	=	Healthy	+	Physically frail	+	Mentally frail	+	Impaired
Men									
Bottom	7.9		4.6		1.7		0.6		1.0
Top	12.8		10.4		1.3		0.5		0.6
Women									
Bottom	11.3		5.9		2.6		1.3		1.5
Top	16.5		12.3		2.0		1.0		1.2

Source: HRS 1998-2014. Single and retired individuals in the sample. The second column sums the expected duration in all possible health states which is equal to the life expectancy at age 70.

women conditional on being alive for the top (thick line) and the bottom PI decile. The share of impaired individuals in the top permanent income decile at the age of 80 is around 50% of those in the bottom permanent income decile.

To conclude, I have shown that first, LTC needs can be parsimoniously represented by four different health groups. Second, the estimated transition probabilities imply that as health deteriorates, mortality rates sharply increase. Finally, there is a strong health income gradient with poorer individuals facing larger expected LTC needs in spite of living shorter lives.

2.2 Determinants of Informal Care Provision and Formal Care Consumption Choices

Two thirds of individuals in the HRS, report receiving care from relatives (mainly children) when in need of LTC (physically frail, mentally frail or impaired). Thus, not taking into account the provision of care from family members would overestimate the risk of long-term care. In this section, I document which individual characteristics are informative of the provision of informal care and then how consumption of formal care varies by level of need, permanent income and provision of care from relatives.

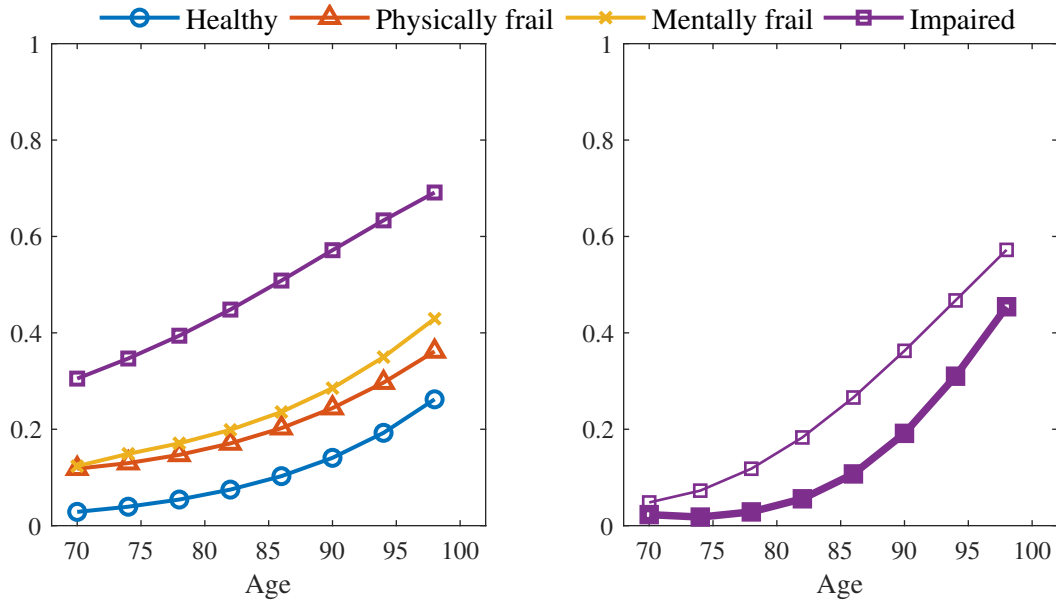


FIGURE 2. TWO-YEAR MORTALITY RATES ACROSS HEALTH GROUPS FOR WOMEN (LEFT PANEL) AND SHARE OF ALIVE INDIVIDUAL IMPAIRED FOR TOP (THICK LINE) AND BOTTOM (THIN LINE) DECILE OF PERMANENT INCOME DISTRIBUTION (RIGHT PANEL)

For this purpose, I make use of the HRS helpers files that contain information on help provided with I-ADLs. This module of the HRS includes information on hours of care provided as well as on the identity of the helpers. I classify informal care received as care provided by relatives or friends and formal care as care provided by a paid helper, a professional or an employee of an institution.

In order to summarize the large heterogeneity in the provision of informal care observed in the data, I group all individuals into two “family types” according to their provision of informal care: *close families* and *distant families*. *Close families* are those whose reported informal care hours lie in the top tercile of the informal care hour distribution conditional on health status. *Distant families* are those whose reported informal care hours are below the top tercile. Individuals belonging to *close families* receive a much stronger support as they receive 3.6, 8.9, and 13.6 hours of care per day from their relatives when physically frail, mentally frail, and impaired. Individuals in *distant families*, on the other hand, receive on average 35 minutes of care per day with little differences across levels of need. The proposed family classification correlates well with having children living nearby. Conditional on having children, the fraction of children within close (distant) families cohabiting is 59% (18%), living within 10 miles is 32% (52%), and living more than 10 miles away

TABLE 2. MARGINAL EFFECTS ON THE PROBABILITY OF BELONGING TO A *Close Family*

Permanent income quintile					
Second	Middle		Fourth	Top	
-0.00	0.02		-0.02	-0.02	
(0.01)	(0.02)		(0.02)	(0.02)	
Female	Any Child	Daughter	N. Children	N. Children ² × 10 ⁻⁴	College (kid)
0.05	0.10	0.11	0.02	-1.17	-0.06
(0.01)	(0.03)	(0.02)	(0.01)	(5.75)	(0.02)
Race		Marital Status		Education	
African Am.	Other	Widowed	Never married	HS	College
0.12	0.05	0.06	0.10	-0.06	-0.08
(0.01)	(0.03)	(0.01)	(0.03)	(0.01)	(0.01)

Source: HRS 1998-2014. Standard errors in parentheses. Single and retired individuals in the sample. Omitted categories: Bottom permanent income quintile, male, no children, Caucasian, Divorced, highschool dropout. N=9,234.

is 9% (29%).

In order to study which individuals are more likely to receive informal care, I run a logistic model on the family type against individual covariates. Table 2 shows the marginal effect of the covariates used. First, the table shows that there is not a clear correlation between permanent income and the provision of informal care as none of the coefficients on quintiles dummies are significantly different from each other. Second, the table shows that having children and especially a daughter increase the chances of receiving strong informal care support by 10 and 11 percentage points, respectively. Moreover, African Americans, never married as opposed to divorced and high-school dropouts are also more likely to belong to a *close* family than Caucasians, divorced and more educated individuals, respectively. Finally, the pseudo r-square of the logistic regression is 5%, showing the difficulty to predict the provision of informal care in the overall population by the observed characteristics.

Now, I document how formal care consumption changes with the level of need, permanent income, and across family types. Table 3 shows average formal care hours consumed across health

status and permanent income quintiles.⁶ As the table shows, individuals adjust their formal care consumption in two ways. First, as health deteriorates, individuals consume more care. While the average consumption on formal care is 24 minutes per day for the physically frail, it rises to 4.3 hours of care per day for the impaired. At 12\$ per hour of care⁷, formal care can constitute a significant financial burden for the old. Nevertheless, as shown in the previous section, the persistence of large LTC needs is limited by high mortality rates. Second, conditional on needs, richer individuals consume more care. This finding suggests that LTC expenses cannot be taken at face value to measure risk. For example, when impaired, an individual in the top decile of the permanent income distribution consumes 3 hours more of formal care per day than an individual in the bottom decile.

Finally, Table 4 shows average formal care and informal care hours per day across family types. When impaired, while an individual in a *close family* receives 13.6 hours of informal care when in need of LTC, an individual who is *on his own*, barely receives any care. Moreover, the table shows that individuals with access to informal care greatly reduce their formal care consumption. For example, an individual in a *distant family* consumes around 6.8 hours of formal care per day when impaired while an individual in a *close family* consumes 2.4 hours per day. Moreover, the last two columns in Table 4 shows that families can reduce the risk of long nursing home stays. Conditional on being impaired, individual in a *close families* has 11.5 p.p. lower probability of nursing home entry. Moreover, the probability of moving out of the institution is also more prevalent among *close families*. Even though nursing home stays are very persistent through time, the probability that an institutionalized impaired individual moves back to the community is 4.8% for those in *distant families* and 6.8% for those in *close families*.

In summary, I document that the provision of informal care is highly heterogeneous across individuals but that there exists a number of characteristics that are helpful as predictors of future reciprocity of informal care. Moreover, individuals greatly adjust their consumption of formal care depending on three characteristics: state of need, financial resources, and access to informal care

⁶I treat reported formal care hours for individuals in nursing homes as missing since they tend to under-report the amount of care received. Indeed, 80% of the institutionalized individuals report zero formal care. Formal care hours consumed for impaired individuals in the top of the permanent income distribution drops to 4.9 hours of care when considering nursing home residents.

⁷Source: Hourly wage for health aide in BLS. Lockwood (2018) finds relatively small differences in the price of care across permanent income groups.

TABLE 3. FORMAL CARE HOURS PER DAY ACROSS PERMANENT INCOME QUINTILES

Health status	Permanent income quintile		
	Bottom	Middle	Top
Physically frail	0.5	0.4	0.6
Mentally frail	1.1	1.5	1.7
Impaired	3.0	4.1	5.9

Source: HRS 1998-2014, single and retired individuals aged over 70. Reported hours of formal care from non-institutionalized individuals.

TABLE 4. FORMAL, INFORMAL CARE HOURS PER DAY AND NURSING HOME ENTRY PROBABILITY ACROSS FAMILY TYPES

Family Types	Health Status	Informal Care	Formal Care	NH entry	NH exit
		Hours per day	Hours per day	Pr (%)	Pr (%)
<i>Distant</i>	Physically frail	0.0	0.5	5.0	21.9
<i>Distant</i>	Mentally frail	0.6	1.8	13.3	15.7
<i>Distant</i>	Impaired	0.6	6.8	37.3	4.8
<i>Close</i>	Physically frail	3.6	0.5	4.3	24.6
<i>Close</i>	Mentally frail	8.9	0.7	11.0	13.5
<i>Close</i>	Impaired	13.6	2.4	25.8	6.8

Source: HRS 1998-2014, single and retired individuals aged over 70. Reported hours of formal care from non-institutionalized individuals.

from relatives. I now develop a structural model that is able to replicate these facts.

3 The Model

Motivated by the previous section, I build a structural model that includes heterogeneity in LTC needs and that can reproduce the main features of the data: (i) as health deteriorates, individuals increase their consumption of formal care, (ii) richer individuals consume more care hours, and (iii) individuals with higher access to informal care, consume less formal care (iv) individuals risk of entry in nursing varies with LTC needs and access to informal care.

The model closely follows De Nardi, French, and Jones (2010) and Dobrescu (2015) but at the same time incorporates three important elements to disentangle LTC choices from LTC needs. First, in order to capture the correlation between frailty and survival probabilities, LTC

needs are heterogeneous. Second, agents in the model suffer health shocks that affect individuals' marginal utility of care allowing individuals to adjust care consumption based on their available financial resources. Third, there are two types of care: formal care bought at a market price and informal care provided by families for free. Agents are ex-ante heterogeneous with regards to their family type, which differs in the amount of care provided. Individuals decide on formal care consumption taking the informal care provided by their families as given and face heterogeneous risk of nursing home entry. Finally, I allow the marginal utility of leaving bequest to depend on the family type, thus allowing for possible compensations for the care provided or differences in the altruistic willingness to bestow.

Timing and Preferences.— Agents start their life at age $a = 70$ and live at most 110 years old. In order to match HRS data, every period lasts for two years of time: $a \in \{70, 72, \dots, 110\}$. Individuals derive utility from regular consumption and care. Care is produced by combining informal and formal care through a CES production function where τ is the share parameter attributed to formal care and ω the substitution parameter. Health status h can take five values: healthy ($h = 1$), physically frail ($h = 2$), mentally frail ($h = 3$), impaired ($h = 4$) and dead ($h = 5$). Furthermore, individuals in need of LTC ($1 < h < 5$), receive informal care hours (l_{ic}) depending on their family type (F), their health, and nursing home state (nh). $\alpha(h)$ captures differences in the marginal utility of care across health states. Following the empirical section, there are two types of families in the model: *distant* ($F = 1$) and *close* ($F = 2$).

Each period their utility flow is given by,

$$u(c, l_{fc}, l_{ic}; h, F) = \frac{c^{1-\sigma}}{1-\sigma} + \exp(\alpha(h)) \frac{\left[\left(\tau \cdot l_{fc}^\omega + (1-\tau) \cdot l_{ic}(h, F, nh)^\omega \right)^{1/\omega} \right]^{1-\nu}}{1-\nu} \quad (2)$$

where, c is regular consumption expressed in dollar values, l_{fc} is hours of care spent in formal care and l_{ic} the informal care provided by relatives. α is the LTC needs shifter, which affects the marginal utility of consuming care hours. σ and ν are the risk aversion parameters of regular consumption and total care hours, respectively. I normalize $\alpha(h = 1) = 0$ so that healthy individuals do not derive utility from consuming care.

When the person dies, individuals derive utility from leaving bequest according to:

$$\phi(k) = \exp(\lambda(F)) \frac{(k + \delta)^{1-\sigma}}{1 - \sigma}, \quad (3)$$

where k denotes savings from the previous period, δ captures the extent to which bequests is a luxury good or a necessity and λ captures the intensity of the bequest motive which might vary across family types.

Medical expenses uncertainty.— Individuals face uncertainty in out-of-pocket medical (m) expenses. I follow French and Jones (2004) and model log medical costs as the sum of a white noise process and a persistent AR(1)⁸. The mean of log medical expenses and the variance of the shock is a function of health status, age, sex, and permanent income.

$$\ln m_{i,t} = m(h, a, s, PI) + \gamma(h, a, s, PI)\psi_{i,t} \quad (4)$$

$$\psi_{i,t} = \xi_{i,t} + \zeta_{i,t}, \quad \zeta_{i,t} \sim N(0, \sigma_\zeta^2) \quad (5)$$

$$\xi_{i,t} = \rho\xi_{i,t-1} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim N(0, \sigma_\epsilon^2) \quad (6)$$

Nursing homes transitions.— Individuals in the model face uncertain entry and exit into nursing homes. Following the empirical section, nursing home transition probabilities exogenously vary by LTC need and family type. Individuals in a nursing home have the same utility function as individuals in the community but need to pay a more expensive price for LTC. Moreover, in order to capture the inability of family members to provide care when the elderly face extreme LTC needs (e.g. advanced states of dementia), I assume that individuals in a close family that move into a nursing home receive the same informal care support as individuals in *distant families*.

Government insurance.— Agents have the option of using a means-tested government program. In case an individual decides to use the government program, consumer's wealth is set to zero⁹ and the government provides a utility floor. The government implements the utility floor by

⁸In the model, medical and nursing home expenses are considered as shocks. Moreover, I do not allow care consumption to affect future LTC needs. A different approach, based on Grossman (1972), is to consider health related expenses as investment (Ozkan 2017; Yogo 2016). However, many studies in the empirical literature have found such effects to be small: Brook et al. (1983), Fisher et al. (2003) or Finkelstein and McKnight (2008).

⁹In reality, Medicaid has an asset disregard threshold whose modal value across states is \$2,000. For simplicity, I set this threshold to zero.

transferring the minimum resources possible $\underline{x}(h, nh)$ such that an individual with limited access to informal care, achieves the floor. I define $G = 1$ if the consumer chooses to use the program and $G = 0$, otherwise. Government transfers are then given by:

$$\max\{0, \underline{c}(h, nh) + p_{fc}(nh)\underline{l}(h, nh) + m - b - (1 + r)k\}, \quad (7)$$

where p_{fc} denotes the market price of an hour of formal care and $\underline{c}(h, nh)$ and $\underline{l}(h, nh)$ denote the level of consumption and care hours, respectively provided by the government, and b is the level of permanent income of the individual. I assume that the government optimally splits $\underline{c}(h, nh)$ and $\underline{l}(h, nh)$ given $\underline{x}(h, nh)$ to maximize the agent's utility.

Solution method.— To save on state variables, I redefine the problem in terms of cash in hand, x :

$$x = (1 + r)k + b(s, PI) - m. \quad (8)$$

Given a set of parameter values, I can solve the model numerically by backward induction starting at age $a = 110$. We can write the model in recursive form in terms of cash in hand. β represents the discount factor. The value function is given by:

$$\begin{aligned} V_a(x, h, \zeta, nh, s, PI, F) = \max_{c, l_{fc}, G} \bigg\{ & u(c, l_{fc}; h, F, nh) \\ & + \beta \pi_{h' \neq 5, h, a, s, PI, nh} E_t[V_{a+2}(x', h', \zeta', nh', s, PI, F)] \\ & + \beta \pi_{h' = 5, h, a, s, PI, nh} \phi(k') \bigg\} \end{aligned} \quad (9)$$

subject to

$$x' = (1 - G) \left[(1 + r) \left(x - c - p_{fc}(nh) \cdot l_{fc} \right) - m' \right] \quad (10)$$

$$G = 1 \Leftrightarrow \begin{cases} c = \underline{c}(h, nh) \\ l_{fc} = \underline{l}(h, nh) \end{cases} \quad (11)$$

3.1 Estimation

I estimate the model using a two steps Method of Simulated Moments (MSM) estimator following Gourinchas and Parker (2002) and Cagetti (2003). In the first step, I estimate all the parameters that can be identified out of the model. In the second stage, I estimate the remaining parameters using the model and taking first-step parameters as given.

First stage parameters include non-asset income levels, health transitions, hours of care received in each type of family, and medical expenses. In the second stage, I estimate the set of parameters $\theta = (\sigma, \nu, \delta, \underline{u}, \alpha(h), \lambda(F), \tau, \omega)$ that minimize the distance between simulated wealth and formal care hours moments with their empirical counterparts.

3.1.1 First Stage Parameters

Permanent income.— To compute permanent income, I consider individual non-asset income in each wave. Non-asset income is the sum of Social Security benefits, defined benefit pension benefits, and annuities. I do not include means-tested government transfers such as Supplemental Security Income or food stamps because agents in the model have access to social insurance. Next, I define permanent income for each individual as average non-asset income across all waves in which she is observed. Finally, I split the distribution into deciles of the permanent income distribution. Each individual in the simulation receives the median non-asset income by gender of her permanent income decile.

Medical expenses and nursing home transition probabilities.— The health expenditure model is estimated using HRS data 1996-2014. Details on the estimation procedure can be found in Appendix D. When estimating medical expenditures, I drop individuals living in nursing homes or in Medicaid since LTC expenditures and government transfers are modeled explicitly. I jointly estimate the mean and variance of log-medical expenditures. The mean is modeled conditional on age, age squared, sex, PI ranking dummies, health dummies and PI interacted with health. I compute nursing home transition probabilities from the data across LTC needs and family types. As explained in the empirical section, nursing home transition probabilities are reported in Table 4.

Hours of care provided by the family.— Following the empirical section, I assume that there exist two family types: *distant* and *close*. Individuals in *distant* and *close* families receive the average value of the two bottom terciles and the top tercile of the informal care hours distribution, respectively. When an individual is in a nursing home, hours of informal care provided by *close* families gets reduced to the hours of informal care provided by *distant* families.

Discount factor and price of formal care.— I set the discount parameter to 0.95 and hourly price of formal care to \$12 per hour for individuals in the community and \$15 per hour for individuals in a nursing home¹⁰.

3.1.2 Second Stage Moments

Empirical wealth moments.— Wealth moments track the evolution of wealth as members of the sample age. I group individuals depending on age at first interview. Individuals belonging to groups 1 to 4 were interviewed for the first time when they were aged 70-74, 75-79, 80-84, and ≥ 85 respectively. For each group and permanent income quintile, I compute the median wealth as individuals age. Thus, there are potentially 160 targeted wealth moments (5 permanent income groups \times 8 waves \times 4 groups).

Formal care moments.— The empirical care moments are average formal care hours across permanent income groups, family types and health status, leaving a total of 21 formal care moments (5 Permanent income groups \times 3 LTC need status + 2 family types \times 3 LTC need status).

Likelihood of leaving bequests.— In order to disentangle the effect of LTC expenses and formal care on savings, I exploit information in the HRS on self-reported probability of leaving large bequests. Given that the probabilities of being in need of care and formal care consumption are known, the self-reported probabilities on leaving large inheritance help disentangle accidental from intentional motives. I thus target the average probability of leaving bequests across family types with a total of 10 moment conditions.

¹⁰According to the BLS the mean hourly wage for health aide in 2019 was \$11.6 and \$14.8 for nursing assistants

3.2 Simulation Procedure

I simulate a large number of artificial individuals. Each of these individuals is endowed with a value of the state vector $(a, s, k, b, h, \zeta, \xi, nh, F)$. (a, s, k, b) are drawn from the data distribution when individuals are first observed. ζ and ξ are Monte Carlo draws from discretized versions of the estimated shock processes.

Sampling family types.— Individuals in the model need to be given a family latent type before the start of the simulation. However, family types are observed only for a fraction of the total sample because in the HRS, only individuals reporting difficulties with I-ADLs answer questions related to the provision of informal care hours. Therefore, I impute the family type for individuals who never report difficulties with I-ADLs. For this purpose, I use the predicted probabilities from the logit from the empirical section. In the simulation, I bootstrap the family type for all individuals whose family type I do not observe.

Moreover, due to differences in reported care hours across waves, an individual might be classified in a *close* family in one wave but in a *distant* one in a different wave. For these individuals I consider the family type as latent and sample the family type based on the frequency a given family type was observed in the data.

Sampling health status.— Following De Nardi et al. (2010), the simulation uses each individual's survival history in 2000-2014 to ensure that individuals contribute to the same wealth moments in the simulation as in the data. Furthermore, given the latent nature of the health classification, I use the Kim smoother proposed by Kim (1994).¹¹

Measurement error in the simulation.— Given that family and health status are latent, moment conditions are estimated with measurement error. In order to take into account this measurement error in the simulation, I sample a latent type which drive the individual policy functions in the simulation and a measured type to construct moment conditions. Given the uncertainty related to the type, the measured and the latent type may not coincide.

Procedure.— Given a guess for my parameter vector θ , I solve the model using discrete value

¹¹Appendix C derives the smoother equations) in order to ensure that the simulated health draws have the same persistence as the estimated health process

function iteration. This yields a set of policy function which allows me to simulate the savings decision and consumption of formal care hours for each artificial individual. The optimal choice of $\hat{\theta}$ is the solution to the criterion function:

$$\hat{\theta} = \arg \min_{\theta} (\mathbf{m}_{\text{data}} - \mathbf{m}_{\text{sim}}(\theta))' \mathbf{W} (\mathbf{m}_{\text{data}} - \mathbf{m}_{\text{sim}}(\theta)) \quad (12)$$

I restrict moment condition to have at least 40 observations. Thus the final estimation of θ is based on 161 moment conditions. The weighting matrix (\mathbf{W}) used in the estimation is the optimal variance-covariance matrix of the moment conditions, meaning, more precisely estimated data moments receive greater weight in the estimation.

3.3 Estimated Parameter values

Table 5 shows the estimated preference parameters. The estimate for σ , the coefficient of risk aversion for regular consumption, is 2.98. De Nardi, French, and Jones (2016) estimate $\sigma = 2.83$ while Ameriks et al. (2020) estimate $\sigma = 5.6$. My estimate of ν , the coefficient of risk aversion for care hours, is 4.52. On top, the estimated care multipliers across different health states imply that, as expected, individual's marginal utility of consuming care increases with deteriorating health conditions. Risk aversion and care multipliers estimates imply that care is an inferior good with richer individuals spending a smaller share of total consumption on formal care than the relatively poor. This share, however, increases as health deteriorates. For example, the share devoted to formal care for an individual spending \$20,000 on total consumption and no access to informal care will be 2%, 22%, and 51% when physically frail, mentally frail and impaired, respectively.

Formal and informal care are estimated to be substitutes although the precision of the estimated parameter is low. I cannot reject the null of a Cobb-Douglas production function. The share parameter attributed to formal care is 0.61. There is currently no consensus on whether informal and formal care are complementary or substitute goods in the micro literature. Langa, Chernew, Kabeto, and Katz (2001) and Liu, Manton, and Aragon (2000) find them to be complements. Van Houtven and Norton (2004) and Bonsang (2009) using instrumental variable estimation find them to be substitutes however they find that this substitution effect tends to disappear as the level

of disability of the elderly person increases. In the model, the lack of substitutability in states of care is exogenously partially captured when individuals in *close families* enter in a nursing home as they lose most of the provision of informal care and must therefore rely on formal care.

The transfer needed to achieve the utility floor when healthy corresponds to \$5,300 per year (\$10,610 per two years) which is on the upper side of the range of the previous estimates. For example, De Nardi et al. (2010) estimate it to be \$2,663 per year and Lockwood (2018) estimate at \$3,000 per year.

Given the small differences in the estimated marginal propensities to consume, I test whether a more restricted version of the model fits the data worse. Column two of Table 5 reports the estimated parameters when restricting bequest utility to be equal across families. Newey and West (1987) shows that the difference in J-statistics under the null that both models fits the data equally well, follows a chi-square distribution with degrees of freedom equal to the number of restrictions. The associated p-value is 0.22, thus I cannot reject the null that both models fit the data equally well at conventional significance levels.

Strength of the bequest motive and willingness to bequeath.—Following Ameriks et al. (2020), I compare the intensity of the bequest motive implied by my estimated preference parameter to those estimated in lead papers. For this purpose, I solve the following maximization problem for different parameter values from the literature¹²:

$$\max_{c, l_{fc}, k} \frac{c^{1-\sigma}}{1-\sigma} + \exp(\alpha(h)) \frac{\left[\left(\tau l_{fc}^\omega + (1-\tau) l_{ic}(h, F)^\omega \right)^{1/\omega} \right]^{1-\nu}}{1-\nu} + \exp(\lambda) \frac{(k+\delta)^{1-\sigma}}{1-\sigma} \quad (13)$$

$$s.t. \ W = c + k + l_{fc} p_{fc}$$

Figure 3 plots the ratio of wealth bestowed to total resources before death (k/W). The left panel displays results from previous literature. There are large differences in the strength of the bequest motives estimated for Ameriks et al. (2020) when healthy (SSQ, healthy), in need of LTC (SSQ, LTC), and in Lockwood (2018) (Lockwood). For individuals holding more than \$200K in

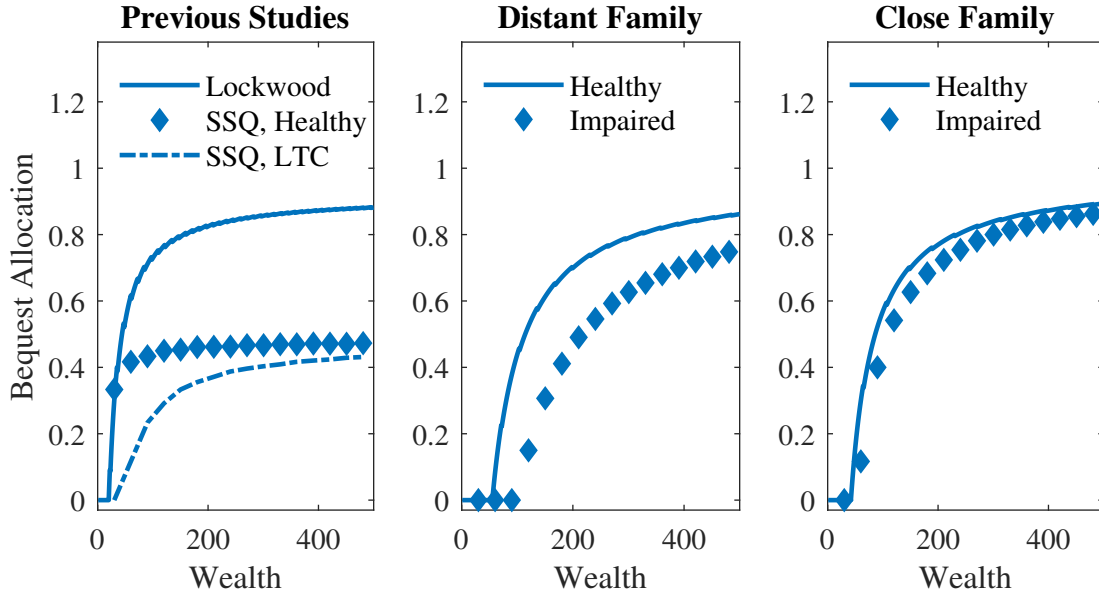
¹²To make my estimates comparable to the previous literature that use annual models, I multiply the utility from regular consumption and care by $2^{-\sigma}$ and $2^{-\nu}$, respectively.

TABLE 5. ESTIMATED PREFERENCE PARAMETERS

	Benchmark (1)	Restricted (2)
Risk Aversion		
σ : Consumption	2.98 (0.27)	2.96 (0.31)
ν : Care hours	4.52 (0.18)	4.51 (0.28)
LTC		
$\alpha(h = 2)$: Physically frail	7.66 (4.07)	7.63 (2.47)
$\alpha(h = 3)$: Mentally frail	16.57 (1.14)	16.53 (1.97)
$\alpha(h = 4)$: Impaired	19.77 (1.80)	20.31 (1.20)
τ : Share formal care	0.66 (0.21)	0.58 (0.19)
ω : substitution formal/informal care	0.05 (1.17)	0.05 (0.47)
Bequest		
δ : curvature	1,691 (9.97)	1,678 (12.72)
$\lambda(F=Distant)$: marginal utility	8.13 (1.38)	8.55 (1.11)
$\lambda(F=Close)$: marginal utility	8.93 (2.42)	8.37 (1.11)
Maximum transfer to achieve utility floor $\times 10^3$		
$\underline{x}(h=1)$: Healthy	10.61 (1.34)	10.54 (1.73)
J-Statistic	356.11	357.58
Moment Conditions	161	161

Notes: Standard errors are reported in parentheses. In the restricted model, the marginal utility of bequests is restricted to be equal across families.

FIGURE 3. BEQUEST ALLOCATION ACROSS STUDIES



Notes: Ameriks et al. (2020): SSQ; Lockwood (2018): Lockwood.

assets, Lockwood (2018) estimates a bequest motive which is around twice as large than the one estimated by Ameriks et al. (2020). The center and right panel of Figure 3 display the results implied by my model across family types when healthy and impaired. When healthy, differences in willingness to bestow are small across family types and line up with the estimates in Lockwood (2018). When individuals in *distant families* become impaired, the share devoted to bequeath falls and the wealth level at which bequest motives become active increases. Given that bequest motives are estimated to be a luxury while LTC is a necessity, the difference in shares devoted to bequest across health states decreases as financial resources increase. The right panel shows that in *close families*, being in LTC implies small changes in the propensity to bestow.

Compared to Ameriks et al. (2016), I find a stronger bequest motive. One important difference that could explain the result is the fact that I allow for heterogeneous LTC needs in my framework. As in Ameriks et al. (2016), impaired individuals in my model report large propensities to consume. However, given that the impaired status is relatively shortly lived due to high mortality rates, my model requires a stronger bequest motive in order to match the savings profiles of the elderly rich.

TABLE 6. MODEL FIT: FORMAL CARE HOURS

LTC Need	Permanent Income			Family Type	
	Bottom	Middle	Top	Distant	Close
Physically Frail	0.5 [0.2]	0.4 [0.3]	0.6 [0.7]	0.5 [0.5]	0.5 [0.2]
Mentally Frail	1.1 [0.8]	1.5 [1.6]	1.7 [2.5]	1.8 [1.9]	0.7 [0.8]
Impaired	3.0 [2.2]	4.1 [2.8]	5.9 [5.6]	6.8 [4.2]	2.4 [2.0]

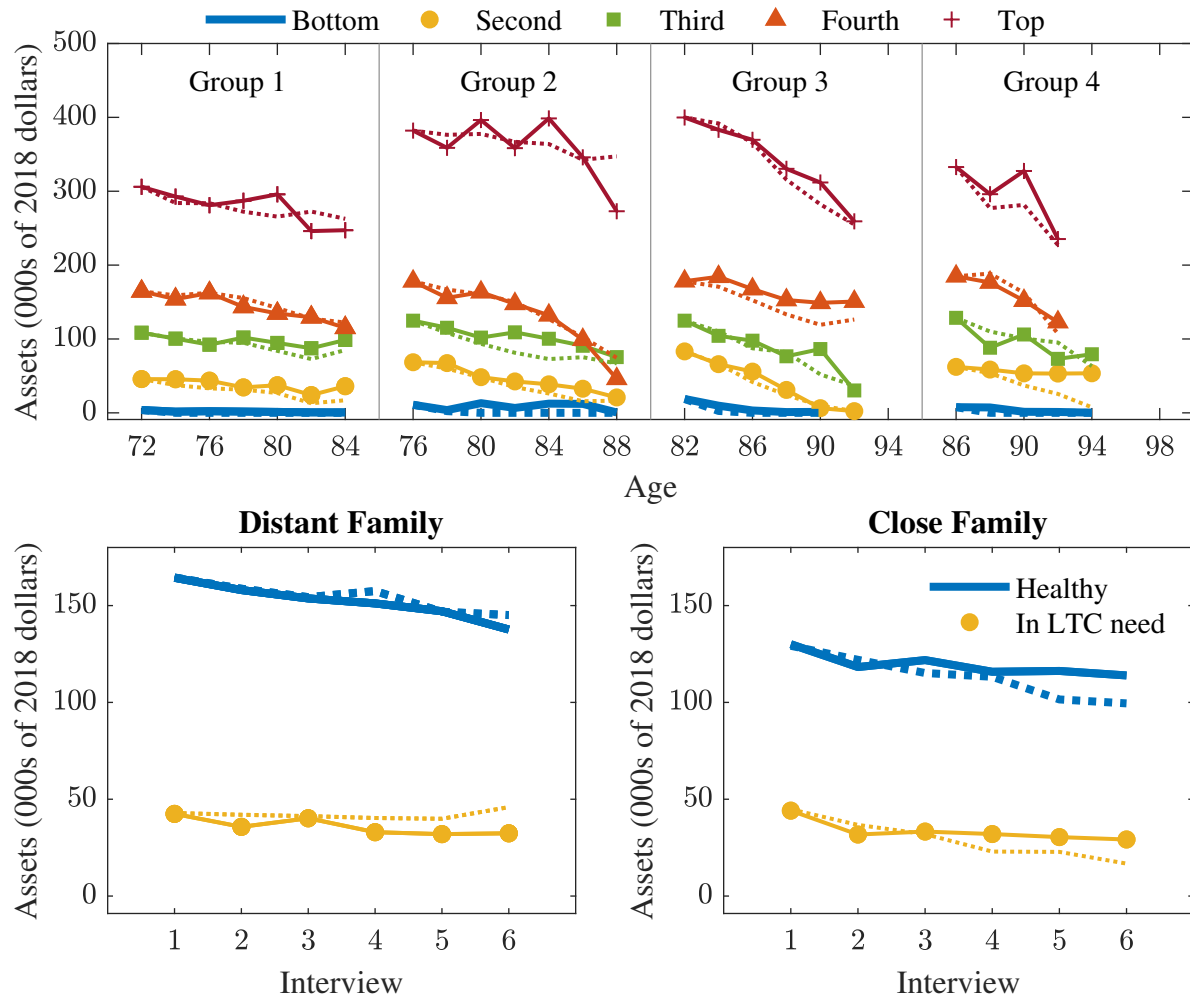
Notes: Numbers in brackets are model moments. HRS 1998-2014, single and retired individuals aged over 70. Reported hours of formal care from non-institutionalized individuals in the data and model.

3.4 Model Fit

The upper panels of Figure 4, Figure 5, and Table 6 report the model fit for the targeted moments in the data. In general, the model is able to quantitatively replicate the key features of the data. The upper panel of Figure 4 shows that the model can generate the lack of dissaving of the elderly rich. Table 6 shows that the model captures the increasing consumption of formal care as health deteriorates, the size of the income gradient of formal care consumption, and the role played by families at reducing the amount of formal care hours bought. The top panel of Figure 5 shows that the model is able to account for the differences in the probability of leaving a positive bequest across levels of the permanent income distribution. Finally, the bottom panel of Figure 5 shows that the model is able to reproduce the fact that while healthy individuals in *close families* are more likely to be in Medicaid than individuals in *close families*, the reverse is true when individuals are impaired.

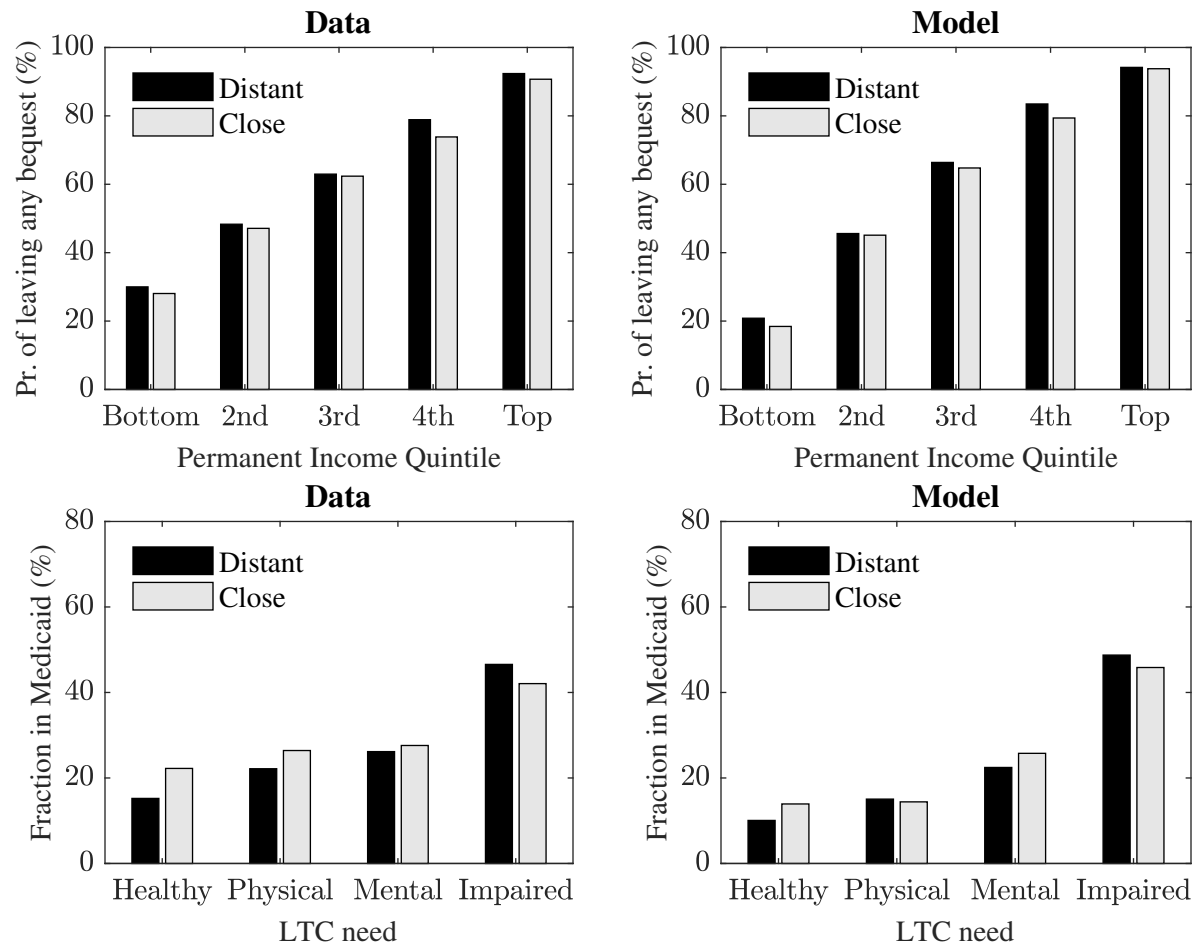
In order to informally validate the model, I assess its accuracy at matching a set of untargeted moments. For this purpose I confront the model predicted dissaving profiles across family types and health status with the data. Given that there are relatively fewer individuals in LTC, I pool individuals across ages and compute median wealth by interview number, family and health status. The lower panel in Figure 4 shows that the model is able to match the fact that the difference in median wealth holdings for individuals who are healthy and in need of LTC is larger for those in

FIGURE 4. MODEL FIT: MEDIAN ASSETS BY PERMANENT INCOME (UPPER PANEL), BY FAMILY TYPE AND HEALTH STATUS (LOWER PANEL)



Notes: Retired single individuals in the HRS 1998-2014. Figure compares data (solid) and simulated (dotted) moments. In the upper panel individuals in groups 1 to 4 were age 70/74, 75/79, 80/84 and 85+ at first interview, respectively. Moments in the lower panel are untargetted in the estimation procedure.

FIGURE 5. MODEL FIT: SELF-REPORTED PROBABILITY OF LEAVING POSITIVE BEQUESTS AND MEDICAID RECIPIENCY RATES ACROSS FAMILY TYPES



distant families even if these are not explicitly targeted in the estimation.

3.5 Identification of Bequest from Long-Term Care Parameters

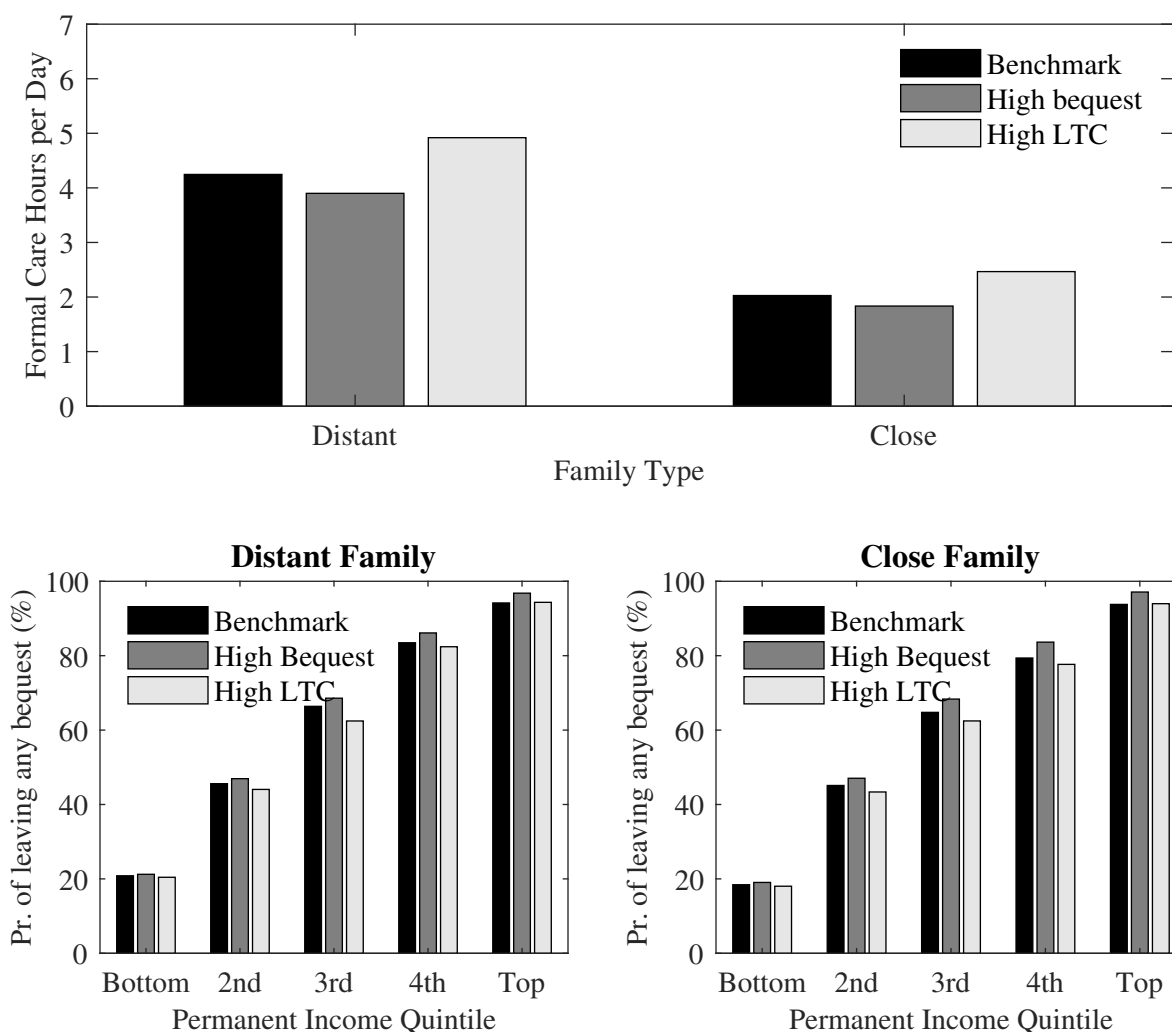
In this section, I discuss how hours of care when impaired and the self-reported probability of leaving bequests allow me to separate savings for intentional bequest from precautionary savings associated to LTC. For this purpose, I illustrate how these moments adjust when I shift the marginal utility of care and bequest, one at a time.

Ex-ante it is hard to say whether hours of care and the probability of leaving bequest contain valuable information for identifying the parameters in the model. More precisely, it could be that both increases in bequest and LTC move these targeted moments in the same direction thus not providing identification. Increases in bequest motives will induce individuals to deaccumulate resources more slowly as they age, driving an increase in their probability of leaving positive bequests. Nevertheless, it is unclear how changes in the bequest motive will drive changes in consumption of formal care: on the one hand, increases in bequest motives will result in higher wealth levels when in need of LTC thus pushing up consumption of formal care. On the other hand, larger willingness to bestow will also result in larger desire for protecting the accumulated savings. The dark grey bars in Figure 6 show that the latter effect dominates and increases in willingness to bequeath result in lower consumption of care when in need.

Second, increases in the marginal utility of care when impaired will increase precautionary savings related to care but the effect on the probability of leaving bequests is unclear. On the one hand, more precautionary savings increase the likelihood of accidental positive bequest. On the other hand, in case the probability of becoming impaired is sufficiently high, individuals will anticipate a decrease in the probability of leaving bequest due to larger spending when in need of LTC. The light grey bars in Figure 6 show that the later effect dominates and that the probability of leaving bequests decreases.

Thus, this exercise shows that increases in the bequest motive and a larger marginal utility of LTC drive formal hours of care and the probability of leaving a bequest in opposite directions, allowing for identification in the model.

FIGURE 6. CHANGE IN FORMAL CARE CONSUMPTION WHEN IMPAIRED AND PROBABILITY OF LEAVING BEQUEST FROM INCREASES IN WILLINGNESS TO BESTOW AND THE MARGINAL UTILITY OF CARE.



4 Results

In order to identify the relative importance of LTC versus bequests as drivers of savings of the elderly for each family type, I use the model to run a set of counterfactual scenarios. For this purpose, I fix the estimated parameters at their benchmark values and change one feature of the model at a time. I then compute the optimal savings decisions, simulate the model, and compare the resulting asset accumulation profile to the asset profile generated by the baseline model for each family type. I display the median assets for individuals who were aged 70-74 at their first interview (Group 1). To focus on underlying changes in saving, I display the asset distribution for individuals who live until age 100.

First, I run two counterfactual simulations. First, to determine the importance of LTC needs, I simulate the model by setting the utility derived from LTC to zero ($\mu(h) = 0 \forall h$). This could be seen as if families were providing enough informal care so that LTC would not distort the savings decisions of the old. Second, to determine the importance of bequests, I set the utility of leaving bequest to zero.

Figure 7 plots the median and the 75th percentile of wealth holdings in the benchmark model and in the two counterfactual scenarios for distant (left panel) and close (right panel) families. The left panel shows that LTC is relatively more important than bequests motives as driver of savings for individuals in *distant families*. As we move along the wealth distribution, the role of bequest increases but still at the 75th LTC plays a quantitatively bigger role. In the absence of LTC needs (bequest motives), the top 75th percentile of the wealth distribution of individuals in *distant families* at age 85 is 24% (9%) lower than in the benchmark. On the other hand, the right panel in 7 shows that wealth trajectories of the rich in *close families* are less affected by LTC while bequest motives are more important. In a world without bequest motives (LTC) the 75th percentile of the wealth distribution of individuals in *distant families* would be 26% (10%) lower.

Second, I run two additional experiments to quantify the role of heterogeneous LTC needs and access to informal care on the savings the decisions of the old. To quantify the effect on savings of each LTC need (physical, mental or impaired) independently, I simulate three counterfactuals by setting the utility derived from LTC to zero ($\mu(h) = 0$) in each state of need, separately. The upper

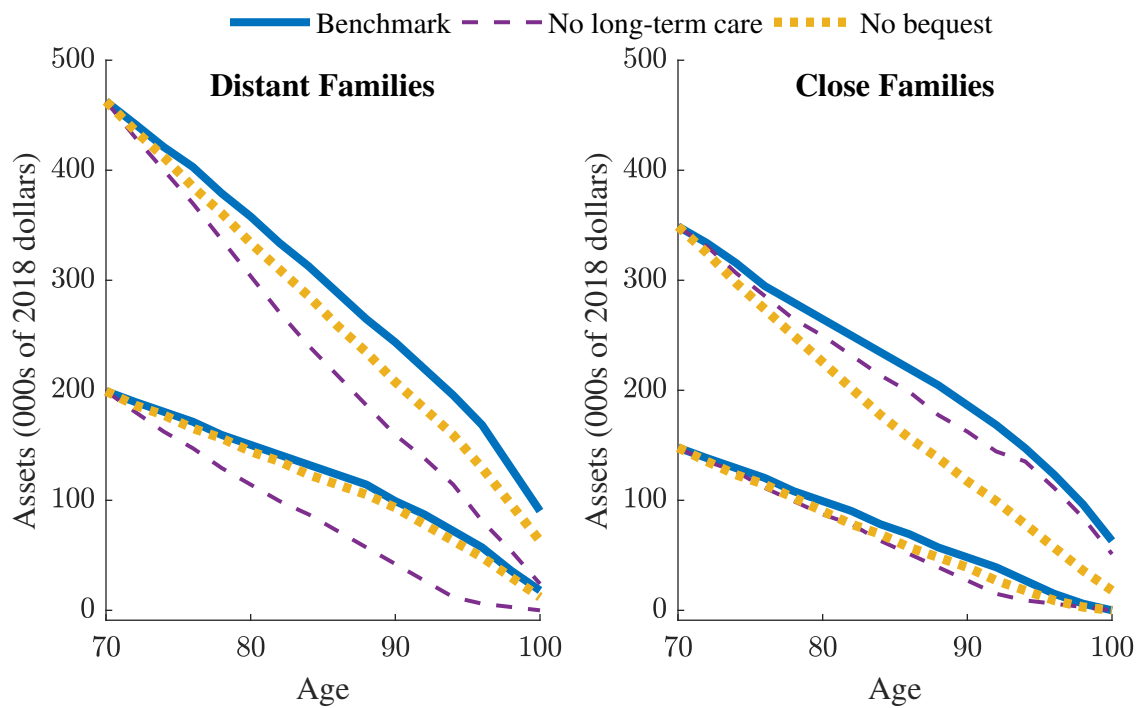
panel of Figure 8 shows in the increase in dissaving in each counterfactual economy as a fraction of the total assets in the benchmark economy at each age. The figure shows that in the absence of all LTC needs, mean assets would be 18% lower at age 85. Most of the increase in dissaving rates arises because of the impaired LTC state. In the absence of LTC needs associated to the impaired status, mean assets at age 85 would be 14% lower or around 80% of the total decrease in savings in the absence of all LTC needs. Cognitive limitation with LTC account for around the remaining 20%. In contrast, physical limitations have a quantitatively negligible effect on the savings decision of the old, despite the fact that physical limitations are the most prevalent state conditional on being in need of LTC.

Second, I quantify to which extent abstracting from care provided by relatives overestimates the importance of LTC in the savings decision of the old. For this purpose, I simulate the economy by setting all family types to *distant families*. The dashed line in Figure 8 shows that an economy without *close families* would overestimate savings related to LTC. In the absence of *close families* and in a world without LTC needs, individuals would dissave faster such that by the age of 85 mean wealth would be 23% lower which corresponds to an increase in the excess savings associated to LTC with respect to the benchmark economy of 25%.

5 Conclusions

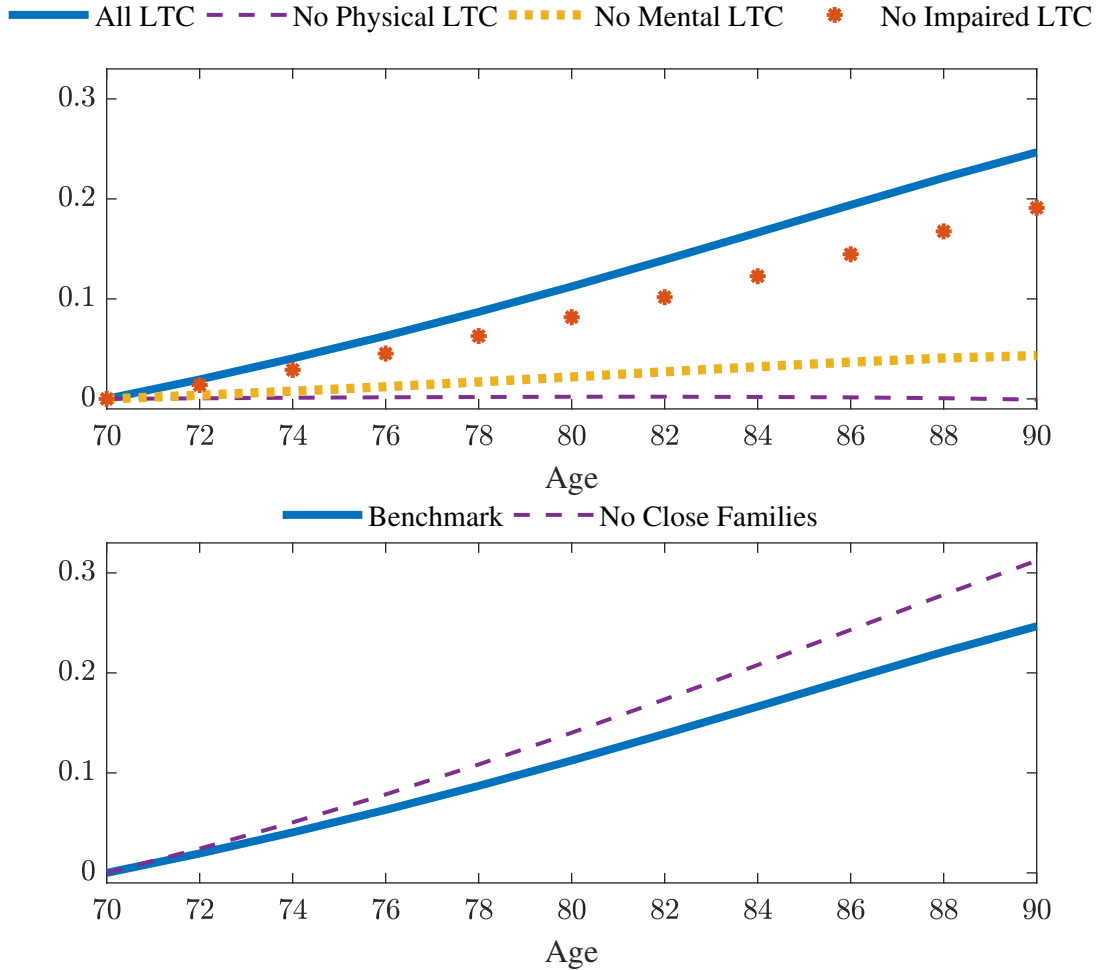
In this paper, I show that differences in access to informal care across individuals and heterogeneous LTC needs are important for understanding the lack of dissaving of the old. First, the savings decisions of individuals with limited access to informal care are mainly driven by LTC needs while strong bequest motives are necessary in order to match the wealth trajectory of rich individuals with large access to informal care. As such, abstracting from the provision of family care would overestimate the role of LTC in explaining the lack of dissaving of the old. Second, the model shows health states with both physical and cognitive limitations account for the largest fraction of precautionary savings related to LTC.

FIGURE 7. COUNTERFACTUAL DISSAVING: 75TH PERCENTILE AND MEDIAN ASSETS.



Notes: Figure shows median assets and the 75th percentile for the simulated benchmark and the counterfactual without long-term-care, and no medical expenses for individuals aged 70/74 at first interview.

FIGURE 8. PRECAUTIONARY SAVINGS RELATED TO LONG-TERM CARE.



Notes: Precautionary savings related to long-term care at each age are computed as the difference in mean assets between an economy with and without long-term care expenses. Figure shows mean difference in assets for individuals in the HRS aged 70/74 at first interview.

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Appendix A Latent Health Model

In this appendix I describe the econometric model used for estimating latent health states and transition probabilities. The model is a slight modification from the original one in Amengual et al. (2017) where transition probabilities differ across deciles of permanent income groups.

The HRS is an unbalanced panel of individuals $i = 1, \dots, N$ followed for $t_i = 1, \dots, T^i$ periods which correspond from ages a_1^i to age $a_{T_i}^i$. We consider that an individual i at time t belongs to a latent health group $h_{i,t}$ out of H possible ones. If the individual belonged to group g , the probability of reporting difficulties with the k 'th I-ADL¹³, say $y_{i,k,t} = 1$, is $\iota_{k,g}$. Under the assumption that I-ADLs are independently distributed conditional on the health status, the joint distribution of $\mathbf{y}_{i,t} = (y_{1,i,t}, y_{2,i,t}, \dots, y_{K,i,t})'$ is characterized by

$$p(\mathbf{y}_{i,t} | \iota_g, h_{i,t} = g) = \prod_{k=1}^K \iota_{k,g}^{y_{k,i,t}} (1 - \iota_{k,g})^{1-y_{k,i,t}}, \quad (1)$$

where $\iota_g = (\iota_{1,g}, \iota_{2,g}, \dots, \iota_{K,g})'$. We take into account health dynamics by explicitly modeling the transition probabilities across groups. In particular, an individual i at time t , with gender s and in PI decile Q who belongs to group g transits to group c with probability

$$\pi_{g,c}(a_{it}, s_i, Q_i) = \frac{\exp[f_{g,c}(a_{it}, s_i, Q_i)]}{1 + \sum_{c \in \mathcal{H}} \exp[f_{g,c}(a_{it}, s_i, Q_i)]} \quad (2)$$

where \mathcal{H} is the set that contains the H health groups. The remaining possible event is that the individual dies, which is an observable state that occurs with probability

$$\pi_{g,D}(a_{it}, s_i, Q_i) = \frac{1}{1 + \sum_{c \in \mathcal{H}} \exp[f_{g,c}(a_{it}, s_i, Q_i)]}.$$

This specification allows health groups to own distinct dynamics as parameters differ according to the current health group. Moreover, to capture within-group heterogeneity, transition probabilities can depend on age, gender, permanent income ranking (I split PI distribution in deciles, $Q = 10$) and interaction terms through the function $f_{g,c}(a, s, Q)$:

$$\begin{aligned} f_{g,c}(a, s, Q) = & \beta_{1,g,c} + \beta_{2,g,c}a + \beta_{3,g,c}a^2 + \beta_{4,g,c}a^3 + \beta_{5,g,c}s + \beta_{6,g,c}(s \times a) + \beta_{7,g,c}Q \\ & + \beta_{8,g,c}Q^2 + \beta_{9,g,c}(Q \times a) \end{aligned} \quad (3)$$

¹³Along the paper I use I-ADLs to denote the set of both ADLs and IADLs

TABLE B1. FRACTION OF INDIVIDUALS BY HEALTH STATUS ACROSS PERMANENT INCOME QUANTILES AND SEX

Category	Healthy	Physically Frail	Mentally Frail	Impaired	Age	Permanent Income	N
women	0.58	0.22	0.10	0.11	81.2	\$16,000	26,939
men	0.64	0.20	0.07	0.09	80.3	\$19,800	7,440
Bottom	0.45	0.26	0.13	0.16	81.1	\$8,900	8,595
Second	0.55	0.23	0.11	0.11	81.4	\$14,200	8,596
Third	0.65	0.20	0.07	0.08	81.1	\$19,500	8,600
Top	0.71	0.16	0.06	0.07	80.0	\$31,700	8,588
All	0.59	0.21	0.09	0.10	81.0	\$16,700	34,379

Source: HRS 1998-2014, single and retired individuals aged over 70. PI is in dollars of 2018.

In practice, I set the number of latent health groups $H = 4$ ¹⁴. Estimation of the econometric model delivers two sets of parameters: $[\hat{\beta}, \hat{\epsilon}]$. $\hat{\epsilon}$ shows that individuals are classified as physically frail, mentally frail, impaired or healthy, represent individuals' LTC needs suitably. Figure 1 shows the probability of reporting difficulty with I-ADLs in each LTC need group. The impaired have physical and cognitive limitations while the healthy have no or light difficulties with I-ADLs. In turn, the physically frail have limited mobility, while the mentally frail have difficulties with more cognitive tasks such as managing money.

Appendix B Descriptive Statistics

Given the estimated parameter values and the latent nature of the health measure used, I compute the probability that each individual in the sample belongs to each health group. Table B1 presents descriptive statistics of the sample used. Women are relatively older, in worse health, and with a permanent income which is 20% lower than males. Poorer individuals have on average higher LTC needs in live with the estimated dynamics. For example, conditioning of permanent income, the fraction of impaired individuals is more than two times higher for the bottom quartile (16 %) than for the top (7 %).

¹⁴For details on the estimation procedure and how we select the optimal number of health groups, the reader is referred to the original paper.

Appendix C Smoothed Probabilities

In this appendix I explain the computation of smoothed probabilities. These are used for computing statistics by health status given our estimates of $\hat{\beta}$ and $\hat{\mu}$. The derivation is split in two parts: the filtered probabilities based on Hamilton (1989) and the smoothed probabilities based on Kim (1994).

Filtered probabilities.— For computing the filtered probabilities, I need first to obtain

$$\begin{aligned} p(\mathbf{x}_{i,t+1}, h_{i,t+1}, h_{i,t} | \mathbf{x}_i^t) &= p(\mathbf{x}_{i,t+1} | \mathbf{x}_i^t, h_{i,t+1}, h_{i,t}) \cdot p(h_{i,t+1} | \mathbf{x}_i^t, h_{i,t}) \cdot p(h_{i,t} | \mathbf{x}_i^t) \\ &= p(\mathbf{x}_{i,t+1} | h_{i,t+1}) \cdot p(h_{i,t+1} | h_{i,t}) \cdot p(h_{i,t} | \mathbf{x}_i^t) \end{aligned}$$

where $p(\mathbf{x}_{i,t+1} | h_{i,t+1})$ is given by equation 1, $p(h_{i,t+1} | h_{i,t})$ is given equation 2 and $p(h_{i,t} | \mathbf{x}_i^t)$ is available by recursion. Then,

$$p(\mathbf{x}_{i,t+1} | \mathbf{x}_i^t) = \sum_{k,l} p(\mathbf{x}_{i,t+1}, h_{i,t+1} = k, h_{i,t} = l | \mathbf{x}_i^t)$$

I can thus compute the filtered probabilities as,

$$p(h_{i,t+1} | \mathbf{x}_i^{t+1}) = \frac{\sum_l p(\mathbf{x}_{i,t+1}, h_{i,t+1}, h_{i,t} = l | \mathbf{x}_i^t)}{p(\mathbf{x}_{i,t+1} | \mathbf{x}_i^t)}$$

Smoothed probabilities.— I observe,

$$\begin{aligned} p(h_{i,t+1}, h_{i,t} | \mathbf{x}_i^T) &= p(h_{i,t+1} | \mathbf{x}_i^T) \cdot p(h_{i,t} | h_{i,t+1}, \mathbf{x}_i^T) = p(h_{i,t+1} | \mathbf{x}_i^T) \cdot p(h_{i,t} | h_{i,t+1}, \mathbf{x}_{i,t}) \\ &= p(h_{i,t+1} | \mathbf{x}_i^T) \cdot \frac{p(h_{i,t+1} | h_{i,t}) \cdot p(h_{i,t} | \mathbf{x}_{i,t})}{\sum_l p(h_{i,t+1} | h_{i,t} = l) \cdot p(h_{i,t} = l | \mathbf{x}_{i,t})} \end{aligned}$$

Therefore, if we sum over all values of $h_{i,t+1}$, I get my target, $p(h_{i,t} | \mathbf{x}_i^T)$.

Sample path for health states, given all the data.— I begin by drawing $h_{i,T}$ from the filtered $p(h_{i,T} | \mathbf{x}_i^T)$, I then draw using:

$$p(h_{i,T-1} | h_{i,T}, \mathbf{x}_i^T) = \frac{p(h_{i,T} | h_{i,T-1}) \cdot p(h_{i,T-1} | \mathbf{x}_i^{T-1})}{\sum_l p(h_{i,T} | h_{i,T-1} = l) \cdot p(h_{i,T-1} = l | \mathbf{x}_i^{T-1})} \quad (4)$$

Appendix D Medical Expenditures: Estimation

Following French and Jones (2004), I estimate the following model:

$$\ln m_{it} = X'_{it}\beta + \sqrt{\exp(X'_{it}\gamma)}(\xi_{it} + \zeta_{it}), \quad \zeta_{it} \sim N(0, \sigma_\zeta^2) \quad (5)$$

$$\xi_{it} = \rho\xi_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_\epsilon^2) \quad (6)$$

X_{it} consists of a cuartic in age, sex, sex interacted with age, a quadratic in permanent income decile, permanent income decile interacted with a, health dummies, and health dummies interacted with income decile. In the estimation of the medical expenditure process I treat health as being observable. Each individual is assigned to the health group with the largest probability. The parameter vector to be estimated is $\theta = (\beta, \gamma, \sigma_\epsilon^2, \sigma_\zeta^2, \rho)$. Given the normality assumptions, I use the kalman filter to evaluate the likelihood function. I then maximize numerically the likelihood function with respect to the parameters.

Appendix E Simulating The Model

This section details step-by-step the simulation procedure:

1. Set the preference parameters for the simulation: θ^g
2. Compute optimal policies using structural model for θ^g .
3. Set a large number of simulations (S=1000)
4. For each single retired individual in the HRS (N=8439):
 - (a) Sample a family type based on individual specific covariates.
 - (b) I give each individual in the simulation health status, mortality history, and the initial level of wealth of the data.
 - (c) Simulate her savings, medicaid, and hours of care decisions.
5. Construct moments for each simulation.
6. Compute the mean of each moment across simulations.

7. Compute the objective objective function using each moment conditions and weighting matrix.
8. Repeat steps 1 to 6 until the minimum objective function is located.

Appendix F Moment Conditions and Parameter Uncertainty

I follow the appendix in De Nardi et al. (2010) for deriving moment conditions. In the model, my estimates $\hat{\theta}$ of the “true” $M \times 1$ preferences parameters θ_0 is the value of θ that minimizes the relative distance between:

- the estimated life cycle profiles for assets.
- Medicaid reciprocity rates and formal care hours.

and the statistics generated by the model.

Asset Moments

For each calendar year $t \in \{1998, \dots, 2014\}$, I match the median assets for $Q = 5$ permanent income quintiles in $P = 4$ groups (groups 1 to 4 correspond to those aged 70 – 74, 75 – 79, 80 – 84, 85+ at first interview as single). In the simulation each individual in the first interview is given her initial wealth level. In addition, I require each group-income-age cell to have at least 30 observations to be included in the GMM criterion.

The conditional wealth moments are defined as:

$$\mathbb{E} \left[\mathbb{1}_{\{a_{it} < a_{P,Q,t}^{0.5}(\theta)\}} - 0.5 \mid p_i = P \wedge q_i = Q \wedge \text{interviewed at } t \right] = 0, \quad (7)$$

where $a_{P,Q,t}^{0.5}$ denotes the model’s implied median asset for group P, permanent income quintile Q in calendar year t and $i \in P, Q, t$ denotes that individual i belongs to P and Q and was observed at t . Following Chamberlain (1992), I write the unconditional moment condition as:

$$\mathbb{E} \left[\left(\mathbb{1}_{\{a_{it} < a_{P,Q,t}^{0.5}(\theta)\}} - 0.5 \right) \times \mathbb{1}_{\{p_i = P \wedge q_i = Q \wedge \text{interviewed at } t\}} \right] = 0 \quad (8)$$

with sample analog,

$$\frac{1}{N} \sum_{i=1}^N \left[\left(\mathbb{I}_{\{a_{it} < a_{P,Q,t}^{0.5}(\theta)\}} - 0.5 \right) \times \mathbb{I}_{\{p_i=P \wedge q_i=Q \wedge \text{interviewed at } t\}} \right]$$

Formal Care hours, Probability of Leaving Positive Bequests, and Medicaid Reciprocity Rats

For all single retired individuals observed in the HRS, I simulate formal care hours decisions, probability of leaving positive bequests, and Medicaid status.

Moment conditions for hours of formal care and probability of leaving positive bequests by permanent income quintiles

$$\mathbb{E} \left[\left(l_{it}(\theta) - \bar{l}_{H,Q} \right) \times \mathbb{I}_{\{h_i=H \wedge q_i=Q \wedge \text{interviewed at } t\}} \right] = 0, \quad (9)$$

$$\mathbb{E} \left[\left(G_{it}(\theta) - \bar{G}_{H,F} \right) \times \mathbb{I}_{\{h_i=H \wedge f_i=F \wedge \text{interviewed at } t\}} \right] = 0, \quad (10)$$

$$\mathbb{E} \left[\left(Pr_{it}(\text{Bequest} > 0)(\theta) - Pr_Q(\text{Bequest} > 0) \right) \times \mathbb{I}_{q_i=Q \wedge \text{interviewed at } t} \right] = 0, \quad (11)$$

where $l_{it}(\theta)$ is the individual i decision on hours of care, Pr_{it} is the individual probability of leaving positive bequests, G_{it} is individual Medicaid status, $\bar{l}_{H,Q}$ is the expected hours of care for individuals with health H and permanent income Q , Pr_Q is the expected probability of leaving bequest for individuals in permanent income Q , $\bar{G}_{H,F}$ is the expected Medicaid reciprocity for individuals in health H and family F .

Asymptotic Distribution

We have data on N independent individuals. Let $\hat{\varphi}(\theta)$ denote the sample analogue of the vector of moment conditions described above. We define,

$$\hat{\theta} = \arg \min_{\theta} \frac{N}{1 + 1/S} \hat{\varphi}(\theta)' W \hat{\varphi}(\theta)$$

The method of simulated moments estimator $\hat{\theta}$ is both consistent and asymptotically normally distributed:

$$\sqrt{N}(\hat{\theta} - \theta_0) \sim N(0, V),$$

where V is given by,

$$V = (1 + 1/S)(D'WD)^{-1}D'W\Omega WD(D'WD)^{-1}$$

with Ω denoting the variance covariance matrix of the moment conditions. D is the Jacobian matrix of the moment conditions with respect to parameter values. To find the derivative of the asset moments with respect to each parameter, I re-write the moment condition as:

$$\frac{1}{N} \sum_{i=1}^N \left[\int_{-\infty}^{a_{P,Q,t}^{0.5}(\theta)} f(a_{it} | p_i = P \wedge q_i = Q \wedge \text{int. at } t) da_{it} \times \mathbb{1}_{\{p_i=P \wedge q_i=Q \wedge \text{int. at } t\}} \right]$$

It follows that the rows of the D matrix associated to asset moments is obtained by applying Leibniz rule for differentiation to the previous equation:

$$\frac{1}{N} \sum_{i=1}^N \left[f(a_{P,Q,t}^{0.5}(\theta) | P \wedge Q \wedge t) \times \mathbb{1}_{\{p_i=P \wedge q_i=Q \wedge \text{int. at } t\}} \times \frac{\partial a_{P,Q,t}^{0.5}(\theta)}{\partial \theta} \right]$$

In practice, I find $f(a_{P,Q,t}^{0.5}(\theta) | P \wedge Q \wedge t)$, the conditional probability density function of assets evaluated at the model's median, by estimating a kernel density estimator on the sample data (I use a Epanechnikov approximation using Silverman (1986) bandwidth's decision rule with a sensitivity parameter equal to 0.5).

I compute $\frac{\partial a_{P,Q,t}^{0.5}(\theta)}{\partial \theta}$ using numerical derivatives.