A Problem about Convergence of Tensor Factorizations

Shangsi Wang

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Set up:
Consider a 3-D array $P \in \mathbb{R}^{m \times n \times n}$. Assume P is a list of rank-1 symmetric matrices, that is $P[i,,] = \lambda_i h h^T$. Here, λ_i is a scalar and h is a norm-1 vector in $\mathbb{R}^n$. If we put λ_i into a vector $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)$ and write P in terms of tensor product, we have

$$P = \lambda \otimes h \otimes h$$

Instead of P, we observe a random noisy version of it : A, which satisfies for any i, j and k

$$E(A[i,j,k]) = P[i,j,k] \text{ (unbiased)}$$

$$|A[i,j,k] - P[i,j,k]| < M \text{ (bounded)}$$

$$A[i,j,k] \perp A[i',j',k'] \text{ when } \{j,k\} \neq \{j',k'\} \text{ (independent)}$$

$$A[i,j,k] = A[i,k,j] \text{ (symmetirc)}$$

To estimate λ and h, we try to minimize the reconstruction error and get our estimator $\hat{\lambda}$ and $\hat{h}$.

$$(\hat{\lambda}, \hat{h}) = \arg\min_{\lambda, h} ||A - \lambda \otimes h \otimes h||$$

Here, $||\cdot||$ denotes the L_2 norm.

If we assume λ_i are drawn from a distribution F (F can be a point mass distribution for simplicity), we can grow P and hence A in its first dimension. However, $\hat{h}$ remains in $\mathbb{R}^n$ as m increases.

Question:
We have done some analysis which makes us believe $\hat{h} \not\Rightarrow h$, but
1. Can we have a non-trivial bound on $||\hat{h} - h||$ as $m \to \infty$?
2. Can we prove $\hat{h}$ converges to a vector h' as $m \to \infty$?