PyDICE2016: A Python solution for DICE 2016 model

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July 9, 2019

1 Introduction

Climate change affects the economy at the short run. Direct and indirect losses are often discussed when a natural disaster hits a country (see as example Hallegatte and Przyluski [2010]). Direct losses are "easily" counted, i.e. damages to the built environment and manufactured goods, health impacts, loss of lives, natural asset damages and ecosystem losses. Indirect losses constitute a debate in the literature of economic climate change. Indirect losses include all losses that are not caused by the disaster, but by its consequences, i.e. shortage of supply, interruption of electricity or water, etc. These losses recover mainly within one year.

Studying the effects of climate changes within one year (short run effect), make policies inefficient to reduce the social cost of climate change as discussed in Nordhaus [2017]. Natural disasters are the consequence of climate change, and this latter is the consequence of the high carbon emission in the atmosphere. Therefore, in order to model a realistic effect of natural disasters on the economy, we need to model the long run relationship between carbon emission and economic growth.

The Dynamic Integrated Climate-Economy (DICE) model by William Nordhaus[1] is one of the mainstream Integrated Assessment Models (IAMs) for evaluation of optimal climate policy. DICE models the endogenous process of the atmospheric temperature growth, which depends on the world carbon emission and affects the world gross product via damages caused by the natural disasters.

In this documents, I will write all the equations of the DICE 2016 model. The equations are similar to those of DICE 2013 model (see Nordhaus and Sztorc [2013]) with some small differences. Then, I will propose a Python approach to solve the model, which is inspired by the R solution of the model[2]. Finally, I will expose some of the results to validate my implementation.

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[1] $https://en.wikipedia.org/wiki/William_Nordhaus$

[2] $https://github.com/olugovoy/climatedice/tree/master/R$

2 Equations of the DICE 2016 model

2.1 Objective function and preferences

The model tries to find the policy that maximizes the social welfare, W which is defined as the discounted sum of the population-weighted utility of per capita consumption.

$$W = \sum_{t=0}^{T_{max}} U[c(t), L(t)].R(t). \tag{1}$$

$c(t), L(t)$, and $R(t)$ denote respectively the per capita consumption, the labor input (total population), and the discount factor. The utility function can be written as follows:

$$U[c(t), L(t)] = L(t) \cdot [\frac{c(t)^{1-\alpha}}{1-\alpha}]. \tag{2}$$

α is a constant elasticity of the marginal utility of consumption. Finally, discount on the economic well-being of future generations is given by:

$$R(t) = (1+\rho)^{-t}. \tag{3}$$

2.2 The dynamics of economic variables

Labor equation satisfies:

$$L(t) = L(t-1) \cdot [1 + g_L(t)]. \tag{4}$$

$g_L(t)$ is the population growth rate, which declines with an exogenous rate δ_L, as follows:

$$g_L(t) = g_L(t-1) \cdot \frac{1}{1+\delta_L}. \tag{5}$$

The dynamics of the total factor of productivity (TFP) is given by:

$$A(t) = A(t-1) \cdot [1 + g_A(t)]. \tag{6}$$

where,

$$g_A(t) = g_A(t-1) \cdot \frac{1}{1+\delta_A}. \tag{7}$$

Production function is supposed to be constant-returns-to-scale Cobb-Douglas production function in capital, labor, and Hicks-neutral technological change. It is given by:

$$Q(t) = [1 - \Lambda(t)] \cdot A(t) \cdot K(t)^{\gamma} \cdot L(t)^{1-\gamma} \cdot \frac{1}{1+\Omega(t)}. \tag{8}$$

$\Lambda(t)$ and $\Omega(t)$ are climate related effects to the economy. They represent respectively the climate change damages and the abatement cost (the cost of the reduction of CO2 emission).

$$\Omega(t) = \psi_1 \cdot T_{AT}(t) + \psi_2 \cdot T_{AT}(t)^2. \tag{9}$$

$$\Lambda(t) = \theta_1 \cdot \mu(t)^{\theta_2}. \tag{10}$$

The gross product includes consumption and investment, as follows:

$$Q(t) = C(t) + I(t). \tag{11}$$

The investment is given by a saving rate policy:

$$I(t) = S(t) \cdot Q(t). \tag{12}$$

The per capita consumption is given by:

$$c(t) = \frac{C(t)}{L(t)}. \tag{13}$$

The dynamics of the capital is given by:

$$K(t) = I(t) - \delta_k \cdot K(t-1). \tag{14}$$

Because of the production, the industrial emission of carbon is given by:

$$E_{ind}(t) = \sigma(t) \cdot [1 - \mu(t)] \cdot A(t) \cdot K(t)^{\gamma} \cdot L(t)^{1-\gamma}. \tag{15}$$

The carbon intensity is given by:

$$\sigma(t) = \sigma(t-1) \cdot [1 - g_{\sigma}(t)]. \tag{16}$$

$$g_{\sigma}(t) = g_{\sigma}(t-1) \cdot \frac{1}{1 + \delta_{\sigma}}. \tag{17}$$

A limit of carbon emission is given as a constraint:

$$CCum \geq \sum_{t=0}^{T_{max}} E_{ind}(t). \tag{18}$$

Finally, the real interest rate is given by:

$$r(t) = (1 + \rho) \cdot \left(\frac{c(t+1)}{c(t)} \right)^{\alpha} - 1. \tag{19}$$

2.3 Geophysical equations

The total carbon emission is the industrial emission plus the land emission.

$$E(t) = E_{ind}(t) + E_{land}(t). \tag{20}$$

Carbon concentration is a mixing between the concentration in the atmosphere, $M_{AT}(t)$, concentration in the upper oceans, $M_{up}(t)$, and concentration in the deep oceans. The biophysical equations are given by:

$$M_{AT}(t) = E(t) + \phi_{11} \cdot M_{AT}(t-1) + \phi_{21} \cdot M_{up}(t-1). \tag{21}$$

$$M_{up}(t) = \phi_{12} \cdot M_{AT}(t-1) + \phi_{22} \cdot M_{up}(t-1) + \phi_{32} \cdot M_{lo}(t-1). \tag{22}$$

$$M_{lo}(t) = \phi_{23} \cdot M_{up}(t-1) + \phi_{33} \cdot M_{lo}(t-1). \tag{23}$$

The radiative forcing is given by:

$$F(t) = \eta \left\{ \mathrm{Ln} \left[\frac{M_{AT}(t)}{M_{AT}(1750)} \right] \right\} + F_{Ex}(t). \tag{24}$$

$M_{AT}(1750)$ is the carbon concentration in the atmosphere in 1750, and $F_{Ex}(t)$ is an exogenous radiative forcing. The atmosphere and oceans temperatures are modeled as follows:

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1 \cdot \{F(t) - \xi_2 \cdot T_{AT}(t-1) - \xi_3 \left[T_{AT}(t-1) - T_{lo}(t-1)\right]\}. \tag{25}$$

$$T_{lo}(t) = T_{lo}(t-1) + \xi_4 \cdot \{T_{AT}(t-1) - T_{lo}(t-1)\}. \tag{26}$$

$$\Delta T_{AT}(t) = \frac{\Delta F(t)}{\xi_2}. \tag{27}$$

3 Python solution

The solution proposed in the Python code implements the whole equations as functions. Then, all function are called in an objective function that returns a scalar, the total social welfare. Then, a nonlinear optimization solver is used to find the control variables that maximize the social welfare. The two control variables are the emissions control rate and the saving rate (the investment as a fraction of net product, i.e. gross product after climate losses). Three main python libraries are used: Numpy for arrays, Scipy for optimization, and Numba to get the python speed.

Another approach could be used to solve DICE with Python by using the Pyomo library. It uses the algebraic modeling languages approach as GAMS (GAMS solution for DICE is proposed in $https://github.com/olugovoy/climatedice/tree/master/GAMS$). For reference on Pyomo, I invite readers to see Hart et al. [2017].

4 Some results

The results I found are identical to the original GAMS solution, and to the R solution proposed hereafter: $https : //github.com/olugovoy/climatedice/blob/master/R/DICE2016inR_standalone.pdf$. I show in the following the plots related to the economic equations. Users may get the plot of all equations.

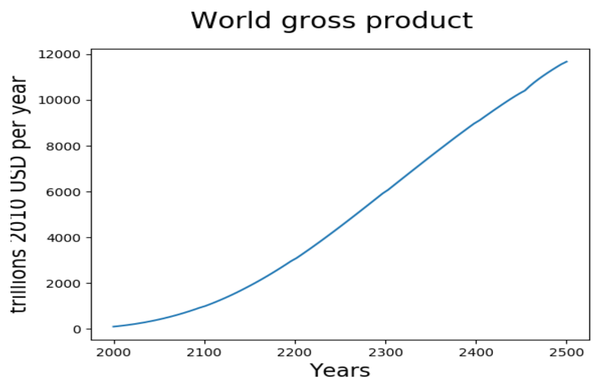

Fig. 1. The gross product before climate losses, Eq. 8.

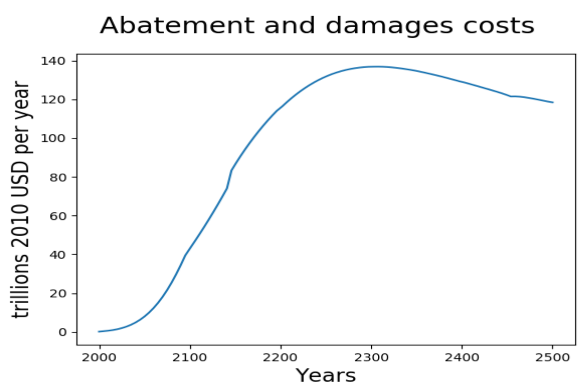

Fig. 2. The climate cost, i.e. the difference between the gross production and the net product: the abatement and damages cost, Eq. 8.

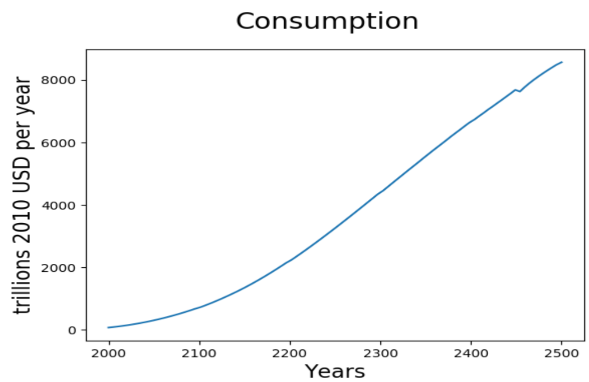

Fig. 3. The consumption dynamics, Eq. 11.

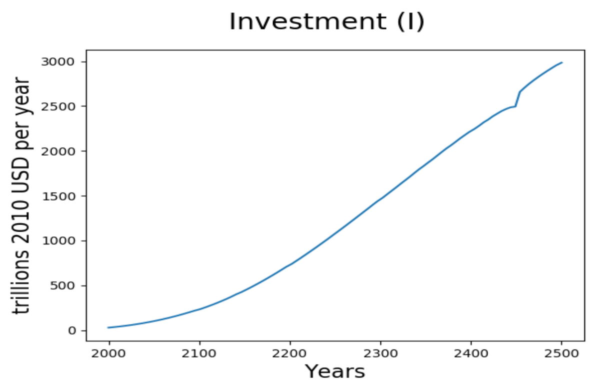

Fig. 4. The investment dynamics, Eq. 11.

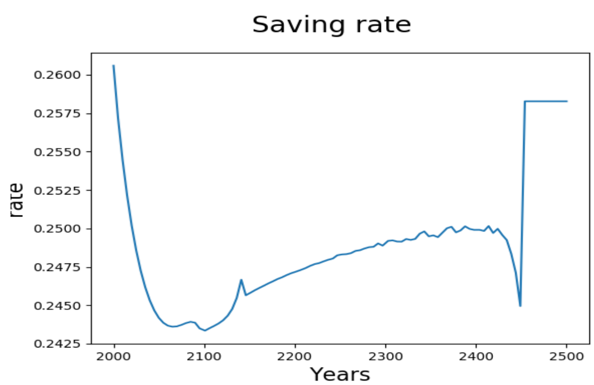

Fig. 5. The optimal saving rate, Eq. 12.

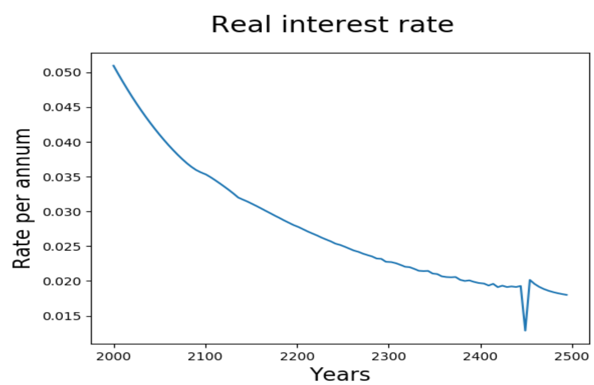

Fig. 6. The dynamics of the real interest rate, Eq. 19.

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