Formulario.pdf

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Inferencia Estadística

4º Doble Grado en Ingeniería Informática y Matemáticas

Escuela Técnica Superior de Ingenierías Informática y de Telecomunicación
Universidad de Granada

Formulario

TEMA 1

$F^*_{\xi_1 \dots \xi_n}(x) = \dfrac{\text{nº variables } \xi_i \le x}{n} = \dfrac{\sum I_{(-\infty, x]}(\xi_i)}{n}$

$nF^* \rightarrow B(n, F(x))$, $E[F^*] = F(x)$, $var(F^*) = \dfrac{F(x)(1-F(x))}{n}$

$F^* \xrightarrow[n\to\infty]{} N\left(F(x), \dfrac{F(x)(1-F(x))}{n}\right)$

$\bar{\xi} = \dfrac{\sum \xi_i}{n}$ $\quad M_{\bar{\xi}}(t) = \left(M_\xi(t/n)\right)^n$ $\quad s^2 = \dfrac{\sum(\xi_i - \bar{\xi})^2}{n-1}$

$A_k = \dfrac{\sum \xi_i^k}{n}$ $\quad B_k = \dfrac{\sum(\xi_i - \bar{\xi})^k}{n}$ $\quad M^*(t) = \dfrac{\sum e^{t\xi_i}}{n}$

$c_p = \begin{cases} \dfrac{\xi_{(np)} + \xi_{(np+1)}}{2} & \text{si } np \in \mathbb{N} \\ \xi_{([np]+1)} & \text{si } np \notin \mathbb{N} \end{cases}$

$E[\bar{\xi}] = E[\xi]$, $var(\bar{\xi}) = \dfrac{var(\xi)}{n}$

$E[s^2] = var(\xi)$ $\quad E[B_2] = \dfrac{(n-1)\, var(\xi)}{n}$

$\left(s^2 = \dfrac{n}{n-1} B_2\right)$

$F_{\xi(n)}(x) = (F_\xi(x))^n$ $\quad f_{\xi(n)}(x) = n(F_\xi(x))^{n-1} f_\xi(x)$

$F_{\xi(1)}(x) = 1 - (1 - F_\xi(x))^n$

$f_{\xi(1)}(x) = n(1 - F_\xi(x))^{n-1} f_\xi(x)$

TEMA 2

$\xi \rightarrow \chi^2(n)$

$M_\xi(t) = \dfrac{1}{(1-2t)^{n/2}}$, $t < 1/2$

$E[\xi] = n$ $\quad var(\xi) = 2n$

$\sum \xi_i \rightarrow \chi^2(\sum k_i)$ con $\xi_i \rightarrow \chi^2(k_i)$

$\sum \xi_i^2 \rightarrow \chi^2(n)$ con $\xi_i \rightarrow N(0,1)$

$\chi^2 \xrightarrow[n\to\infty]{} N(n, 2n)$

$T \rightarrow t(n)$ $\quad T = \dfrac{\xi}{\sqrt{Y/n}}$

$\xi \rightarrow N(0,1)$

$Y \rightarrow \chi^2(n)$

$E[\xi] = 0$ $\quad var(\xi) = \dfrac{n}{n-2}$

$F \rightarrow F(m,n)$ $\quad F = \dfrac{\xi/m}{Y/m}$

$\xi \rightarrow \chi^2(m)$

$Y \rightarrow \chi^2(n)$

$E[\xi] = \dfrac{n}{n-2}$

- Inf sobre μ
  - σ_0^2 conocida: $\dfrac{\bar{\xi} - \mu}{\sigma_0/\sqrt{n}} \rightarrow N(0,1)$
  - σ^2 desconocida: $\dfrac{\bar{\xi} - \mu}{s/\sqrt{n}} \rightarrow t(n-1)$
- Inf sobre σ^2
  - μ_0 conocida: $\dfrac{\sum(\xi_i - \mu_0)^2}{\sigma^2} \rightarrow \chi^2(n)$
  - μ desconocida: $\dfrac{(n-1)s^2}{\sigma^2} \rightarrow \chi^2(n-1)$
- Inf sobre $\mu_1 - \mu_2$
  - σ_1^2, σ_2^2 conocidas: $\dfrac{\bar{\xi} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \rightarrow N(0,1)$
  - $\sigma_1^2 = \sigma_2^2 = \sigma^2$ desconocida: $\dfrac{\bar{\xi} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{(n_1-1)s_1^2 + (n_2-1)s_2^2}} \dfrac{\sqrt{n_1+n_2-2}}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \rightarrow t(n_1+n_2-2)$
- Inf sobre σ_1^2/σ_2^2
  - μ_1, μ_2 conocidas: $\dfrac{n_1 \sigma_1^2}{n_2 \sigma_2^2} \dfrac{\sum(Y_i - \mu_2)^2}{\sum(\xi_i - \mu_1)^2} \rightarrow F(n_2, n_1)$
  - μ_1, μ_2 desconocidas: $\dfrac{\sigma_1^2}{\sigma_2^2} \dfrac{s_2^2}{s_1^2} \rightarrow F(n_2-1, n_1-1)$

TEMA 3

TFNF $\rightarrow f_\theta^n = h(x_1 \dots x_n)\, g_\theta(T(x_1 \dots x_n))$

completo $\rightarrow$ si $E_\theta[g(T)] = 0 \Rightarrow P_\theta[g(T) = 0] = 1 \quad \forall\theta$

<u>Fam exponencial</u>

Uniparamétrica

1) $\Theta \subseteq \mathbb{R}$
2) $\mathcal{X}$ es indep de θ $\forall\theta$
3) $f_\theta(x) = \exp\{S(x) + D(\theta) + Q(\theta)T(x)\}$ $\forall\theta$

K-paramétrica

1) $\Theta \subseteq \mathbb{R}^k$
2) $\mathcal{X}$ indep de θ $\forall\theta$
3) $f_\theta(x) = \exp\{S(x) + D(\theta) + \sum^k Q(\theta)T(x)\}$

<u>Teorema</u>

$T = (\sum T_1(\xi_i), \dots, \sum T_k(\xi_i))$ es suf para θ

si $\operatorname{im}(Q(\theta)) \supseteq$ abierto de $\mathbb{R}^k \Rightarrow T$ es completo

TEMA 4

<u>Fam Regular</u>

1) Θ intervalo abierto de $\mathbb{R}$
2) $\mathcal{X}$ indep de θ
3) $E_\theta\left[\dfrac{\partial \ln f_\theta(x)}{\partial\theta}\right] = 0$

$I_\xi(\theta) = Var_\theta\left(\dfrac{\partial \ln f_\theta(x)}{\partial\theta}\right) \geq 0$

$I_{\xi_1 \dots \xi_n}(\theta) = n I_\xi(\theta)$

<u>Estadístico regular</u>

$\dfrac{d}{d\theta} E_\theta[T] = E_\theta\left[T \cdot \dfrac{\partial \ln f_\theta^n}{\partial\theta}\right]$

<u>Eficiente</u>

- Regular
- Insesgado
- $Var_\theta(T) = \dfrac{(g'(\theta))^2}{I_{\xi_1 \dots \xi_n}(\theta)}$

T eficiente $\Leftrightarrow \forall\theta, \exists a(\theta) \neq 0 : \begin{cases} P_\theta\left[\dfrac{\partial \ln f_\theta^n}{\partial\theta} = a(\theta)(T - g(\theta))\right] = 1 \\ I_{\xi_1 \dots \xi_n}(\theta) = a(\theta)\, g'(\theta) \end{cases}$

TEMA 5

$L_{x_1 \dots x_n}(\theta) = f_\theta^n(\underbrace{x_1 \dots x_n}_{\text{fijo}})$

$\hat{\theta} \xrightarrow{cs} \theta$

$\hat{\theta} \xrightarrow[n\to\infty]{L} N\left(\theta, \dfrac{1}{I_{\xi_1 \dots \xi_n}(\theta)}\right)$

<u>Método de los momentos</u>

$\left.\begin{array}{l} m_1 = A_1 \\ \vdots \\ m_k = A_k \end{array}\right\}$ resolver sistema

nº parámetros

$m_j = \begin{cases} \sum x_i^j P[\xi = x_i] \\ \int x^j f_\theta(x) \end{cases}$

<u>Truco</u>: $m_1 = A_1$

$m_2 - m_1^2 = A_2 - A_1^2$

<u>Método mínimos cuadrados</u>

$\xi_i = \varphi(t_i, \theta) + \varepsilon_i$

minimizar $\sum(\xi_i - \varphi(t_i, \theta))^2$

$p = P[F > F_{exp}]$

$\quad\quad\quad\quad \shortparallel$

$\quad\quad\quad F(Y)$

TEMA 6

Intervalo de confianza $\rightarrow \forall\theta$, $P_\theta[I_1 \le \theta \le I_2] \geq \underbrace{1-\alpha}_{nc}$

cota sup $\rightarrow (-\infty, I_2)$: $P_\theta[I_2 \geq \theta] \geq 1-\alpha$

cota inf $\rightarrow (I_1, +\infty)$: $P_\theta[I_1 \le \theta] \geq 1-\alpha$

Int. Chebychev $\rightarrow \left(T - \sqrt{\dfrac{c}{\alpha}}, T + \sqrt{\dfrac{c}{\alpha}}\right)$ con $var(T) \le c$

ξ continuo $\Rightarrow T = -2\sum \ln F_\theta(\xi_i) \rightarrow \chi^2(2n)$

S estadístico continuo $\Rightarrow T = F_\theta^S(S(\xi_1 \dots \xi_n)) \rightarrow U(0,1)$

<u>Normal ó t-Student</u>

$\lambda_1 = -z_{\alpha/2}$

$\lambda_2 = z_{\alpha/2}$

<u>χ^2 o F</u>

$\lambda_1 = \chi^2_{n, 1-\frac{\alpha}{2}}$

$\lambda_2 = \chi^2_{n, \alpha/2}$

Tema 7

$\beta_\varphi(\theta) = E_\theta[\varphi(\xi_1 \dots \xi_n)]$

<u>TNP (simple/simple)</u>

$\varphi(\xi_1 \dots \xi_n) = \begin{cases} 1 & f_1^n > k f_0^n \\ \gamma & f_1^n = k f_0^n \\ 0 & f_1^n < k f_0^n \end{cases}$

<u>TRV</u>

$\varphi(\xi_1 \dots \xi_n) = \begin{cases} 1 & \lambda < c \\ 0 & \lambda \geq c \end{cases}$ con $\lambda = \dfrac{\sup_{\Theta_0} L_{x_1 \dots x_n}(\theta_0)}{\sup_{\Theta} L_{x_1 \dots x_n}(\theta)}$

Dos colas $\rightarrow \alpha/2$

una cola $\rightarrow \alpha$

<u>pValor</u>

RC es cola a la derecha : $p = P_{H_0}[T \geq T_{exp}]$

RC " " " izq: $p = P_{H_0}[T \le T_{exp}]$

RC dos colas:

$p = \begin{cases} 2P_{H_0}[T \geq T_{exp}] & T_{exp} \geq Me \\ 2P_{H_0}[T \le T_{exp}] & T_{exp} \le Me \end{cases}$

TEMA 8

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\hat{\beta}_1 = \dfrac{\sigma_{xy}}{\sigma_x^2}$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$ve = \dfrac{n\sigma_{xy}^2}{\sigma_x^2}$

$vne = (n-2)s_r^2$

$vt = ve + vne = n\sigma_y^2$

$r^2 = \dfrac{ve}{vt} = \dfrac{\sigma_{xy}^2}{\sigma_x^2 \sigma_y^2}$

$ECM = s_r^2\left(1 + \dfrac{1}{n} + \dfrac{(x_p - \bar{x})^2}{n\sigma_x^2}\right)$

$F(Y) = \dfrac{ve}{s_r^2} \rightarrow F_{1, n-2, \alpha}$

$\bar{y}_i = \dfrac{\sum y_{ij}}{n_i}$ $\quad \bar{y} = \dfrac{\sum n_i \bar{y}_i}{n}$

$VE = \sum n_i(\bar{y}_i - \bar{y})^2$

$VT = \sum y_{ij}^2 - n\bar{y}^2$

$F(Y) = \dfrac{ve/k-1}{vne/n-k} = \dfrac{s_e^2}{s_r^2}$

$\rightarrow F_{k-1, n-k, \alpha}$