Supplementary material of the paper Furtado, Munari and Morabito (2017)

In this supplementary material, we present the Appendix mentioned in [1]. Please refer to the mentioned paper for further information.

Appendix

In this Appendix, we present the mixed-integer programming formulations of the two compact models addressed in Section 2 of [1], namely the classical three-index formulation and the two-index formulation proposed in [2]. Following the notation and decision variables defined in Sections 2.1 and 2.2 of [1], we can state the three-index vehicle flow formulation of the PDPTW as [3, 4]:

$$
\begin{aligned}
\min \quad & \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij}x_{ijk} & & (1)\\
\text{s.t.} \quad & \sum_{k\in K}\sum_{j\in N} x_{ijk} = 1 & \forall i \in P \quad & (2)\\
& \sum_{j\in N} x_{ijk} - \sum_{j\in N} x_{n+i,j,k} = 0 & \forall i \in P; k \in K \quad & (3)\\
& \sum_{j\in N} x_{0jk} = 1 & \forall k \in K \quad & (4)\\
& \sum_{i\in N} x_{i,2n+1,k} = 1 & \forall k \in K \quad & (5)\\
& \sum_{j\in N} x_{jik} - \sum_{j\in N} x_{ijk} = 0 & \forall i \in P \cup D; k \in K \quad & (6)\\
& B_{jk} \geq B_{ik} + t_{ij} - M\left(1 - x_{ijk}\right) & \forall i \in N; j \in N; k \in K \quad & (7)\\
& Q_{jk} \geq Q_{ik} + q_j - M\left(1 - x_{ijk}\right) & \forall i \in N; j \in N; k \in K \quad & (8)\\
& B_{ik} + t_{i,n+i} \leq B_{n+i,k} & \forall i \in P; k \in K \quad & (9)\\
& e_i \leq B_{ik} \leq l_i & \forall i \in N; k \in K \quad & (10)\\
& \max\left\{0, q_i\right\} \leq Q_{ik} \leq \min\left\{Cap, Cap + q_i\right\} & \forall i \in N; k \in K \quad & (11)\\
& x_{ijk} \in \{0,1\} & \forall i \in N; j \in N; k \in K \quad & (12)
\end{aligned}
$$

The objective function (1) minimizes the total routing costs. Constraints (2) and (3) guarantee that each customer is visited once and that the pickup and delivery nodes are visited by the same vehicle. Constraints (4) and (5) ensure that each route begins in the initial depot and finishes in the final depot. It can happen that not all vehicles are used, but in this case the vehicle leaves the initial depot 0 and travels (with no cost and travel time) to the final depot $2n + 1$, which results in $x_{0,2n+1,k} = 1$. Constraints (6) guarantee that the same vehicle that enters in node i must leave this node. Time and load constraints are guaranteed by (7) and (8), respectively, where the constant M is defined as a suffciently large number. Constraints (9) ensure that, for each request i, the pickup node is visited before the delivery node. Time windows and vehicle capacity are imposed by constraints (10) and (11), respectively. Finally, constraints (12) impose the integrality of variables.

Consider now the decision variables defined in Section 2.3 of [1]. The two-index formulation proposed in [2] can be stated as follows:

$$
\begin{aligned}
\min \quad & \sum_{i\in\tilde{N}}\sum_{j\in\tilde{N}} c_{ij}x_{ij} && && (13)\\
\text{s.t.} \quad & \sum_{(i,j)\in\tilde{A}} x_{ij} = 1 && \forall i \in \tilde{N}\backslash\{2n+m+1\} && (14)\\
& \sum_{(i,j)\in\tilde{A}} x_{ij} = 1 && \forall j \in \tilde{N}\backslash\{2n+1\} && (15)\\
& b_{ki} \le b_{kj} + (1-x_{ij}) && \forall\,(i,j) \in \tilde{A}\backslash\{(2n+m+1, 2n+1)\}\,; k \in \tilde{N}\backslash\{i\} && (16)\\
& b_{kj} \le b_{ki} + (1-x_{ij}) && \forall\,(i,j) \in \tilde{A}\backslash\{(2n+m+1, 2n+1)\}\,; k \in \tilde{N}\backslash\{i\} && (17)\\
& x_{ij} \le b_{ij} && \forall\,(i,j) \in \tilde{A} && (18)\\
& b_{ii} = 0 && \forall i \in \tilde{N} && (19)\\
& b_{n+i,i} = 0 && \forall i \in P && (20)\\
& b_{i,n+i} = 1 && \forall i \in P && (21)\\
& b_{i,2n+j} = b_{n+i,2n+j} && \forall i \in P; 2n+j \in N_0 && (22)\\
& q_j + \sum_{i\in\tilde{N}} q_i b_{ij} \le Cap && \forall j \in P && (23)\\
& b_{i,2n+1} = 0 && \forall i \in \tilde{N} && (24)\\
& b_{2n+k,2n+j} = 1 && \forall k < j; 2n+k \in N_0; 2n+j \in N_0 && (25)\\
& b_{2n+j,2n+k} = 0 && \forall k < j; 2n+k \in N_0; 2n+j \in N_0 && (26)\\
& b_{i,2n+m+1} = 1 && \forall i \in \tilde{N}\backslash\{2n+m+1\} && (27)\\
& T_i + t_{ij} \le T_j + M\,(1-x_{ij}) && \forall i,j \in P\cup D \text{ with } (i,j)\in\tilde{A} && (28)\\
& E_i + t_{2n+i,j} \le T_j + M\,(1-x_{2n+i,j}) && \forall i = 1,\dots,m,\ j \in P \text{ with } (2n+i,j)\in\tilde{A} && (29)\\
& T_i + t_{i,2n+j} \le L_{j-1} + M\,(1-x_{i,2n+j}) && \forall i \in D,\ j = 2,\dots,m+1 \text{ with } (i,2n+j)\in\tilde{A} && (30)\\
& e_i \le T_i \le l_i && \forall i \in P\cup D && (31)\\
& x_{ij}, b_{ij} \in \{0,1\} && \forall i,j \in \tilde{N} && (32)
\end{aligned}
$$

The objective function (13) minimizes the total routing costs. Constraints (14) and (15) ensure that all nodes are visited. Constraints (16) and (17) copy the value of b_{ki} to b_{kj} when a route goes from node i directly to node j. Constraints (18) enforce that $b_{ij} = 1$ when $x_{ij} = 1$, while constraints (19) guarantee that a node never precedes nor succeeds itself. The pickup node must be visited before the delivery node, as imposed by constraints (20) and (21). The same vehicle must visit the pickup and its corresponding delivery node, as stated by (22). Constraints (23) ensure that vehicle capacity is satisfied. As presented in [2], these constraints can be easily adapted to cases in which vehicles have different capacities. Constraints (24) impose that $2n+1$ is the first node in the Hamiltonian tour. Constraints (25) and (26) guarantee that the depot nodes are visited in the correct sequence so that each vehicle departs from and returns to the right depot. Constraint (27) imposes that $2n+m+1$ is the last node in the Hamiltonian tour. Time windows are ensured by constraints (28)–(31), where E_i and L_i are the earliest start time and the latest return time, respectively, for vehicle $i = 1,\dots,m$. Finally, (32) impose the integrality constraints.

References

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