Transportation Research Part B 111 (2018) 395–421

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Review

A survey of dial-a-ride problems: Literature review and recent developments

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ARTICLE INFO

Article history:
Received 21 May 2017
Revised 29 December 2017
Accepted 1 February 2018
Available online 15 February 2018

Keywords:
Dial-a-ride
Heuristics
Exact methods
Survey
Share-a-ride
Pickup
Delivery

ABSTRACT

There has been a resurgence of interest in demand-responsive shared-ride systems, motivated by concerns for the environment and also new developments in technologies which enable new modes of operations. This paper surveys the research developments on the Dial-A-Ride Problem (DARP) since 2007. We provide a classification of the problem variants and the solution methodologies, and references to benchmark instances. We also present some application areas for the DARP, discuss some future trends and challenges, and indicate some possible directions for future research.

1. Introduction

1.1. History of the problem

Public transit systems always face the conflicting objectives of cost-efficient operations and high quality service – in delivering customers from/to their desired origin/destination at the desired time. Scheduled bus or train services can carry a large number of passengers (and thus are cost efficient), but travel on fixed routes at scheduled times to which passengers must adjust their travel plans accordingly. Scheduled bus services are often not provided (or very infrequent) for rural communities because the cost of running the service cannot be justified by the low demand. Taxi services offer door-to-door services on request, but the cost of this service is high, both in monetary terms and in impact to the environment. Thus, there has been much interest in on-demand public transit services that combine cost-efficiency and customizable

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https://doi.org/10.1016/j.trb.2018.02.001
0191-2615/

Table 1
A comparison between three public transportation services.

	Bus	On-demand transit	Taxi
Route	fixed	flexible	customized
Schedule	fixed	by request	by request
Speed	slow	medium	fast
Cost	low	medium	high
Mode	shared	shared	non-shared
Capacity	high	medium	low
Reservation	not needed	often needed	not needed

service. On-demand transit vehicles do not have fixed routes or schedules but are dispatched based on the transport requests received; unlike taxi services, passengers may share the use of the vehicles and thus may not be taken along the most direct route from their origin to destination. A comparison of these three types of public transportation services is given in Table 1.

The first trial of an on-demand public transit service, called Dial-A-Ride (since customers phone in their transport requests) was offered in Mansfield, Ohio, USA in 1970. The first demand-responsive service in the United Kingdom was offered in Abingdon in 1972 by the City of Oxford Motor Services. The feasibility of such Dial-a-Ride (DAR) services was demonstrated and similar schemes sprang up elsewhere (see Oxley, 1980). DAR services are particularly valuable to disabled persons and the elderly who may have difficulties using standard public transit services. The Americans with Disabilities Act (ADA), signed into law in 1990, requires all public transport agencies to provide specialized transportation comparable to public transit bus services (sometimes called paratransit) for individuals with disabilities. As a consequence, many demand-responsive systems have evolved from general public service to focused paratransit services. The complexities of operating DAR services (e.g., tight time-windows, last-in-first-out due to vehicle layout) mean that computerized planning and scheduling is necessary for systems of realistic size.

Early solution approaches for the planning and scheduling of DAR systems were heuristic methods, e.g., those developed at MIT for the DAR systems in Rochester, New York, USA (see Wilson et al., 1971). Stein (1978) presented the first models for the planning and scheduling problem of DAR systems, i.e., the Dial-a-Ride Problem (DARP), and obtained bounds for both the static and dynamic versions. Psaraftis (1980) developed a dynamic-programming exact algorithm for both the static and dynamic versions of the DARP with a single vehicle. In the past 40 years, research into the DARP has been growing steadily. For a survey of the models and algorithms developed up to 2007, the reader is referred to Cordeau and Laporte (2007).

1.2. Applications

DARPs are always motivated by real-life applications. Each addresses various realistic features that lead to specific constraints or objectives and yields further insights. Below, we highlight several major application areas since 2007.

A traditional application is non-profit DAR services for the elderly and disabled, which often have cost minimization as the objective. Operational constraints include ride and waiting time, pickup/delivery time-windows, vehicle capacity, and equipment layout within the vehicle (e.g., Karabuk, 2009; Qu and Bard, 2013; Qu and Bard, 2015). Some DAR systems use heterogeneous fleets (e.g., Häll and Peterson, 2013; Häll et al., 2015). Others may allow transfers from one vehicle to another, e.g., for mentally disabled but ambulant passengers (Masson et al., 2014). With different stakeholders, DAR systems often have multiple (and sometimes conflicting) goals, necessitating multi-criteria models (e.g., Paquette et al., 2013; Lehuédé et al., 2014).

Many airports offer dedicated transportation for injured, elderly, weak, and disabled passengers with reduced mobility (PRMs). There are very tight time-windows for pick-ups (exactly when alighting upon arrival) and drop-offs (seated in aircraft well before departure when boarding), and the PRMs may not be left unsupervised. These constraints often originate from a service contract among the service provider, the airport, and the airlines (Reinhardt et al., 2013).

Another major application area is in health care. In this application, time urgency and equipment/staff compatibilities are important. Staff and maintenance scheduling concerns also add considerable complexity. For intra-hospital transportation, which involves the movement of patients, supplies, and equipment for diagnostic or therapeutic reasons, additional constraints may include non-sharing of ambulances of isolation patients, accompanying staff/equipment, specific pickup and delivery sequence of doctors and patients, and prioritization (urgent vs. normal) of requests (Hanne et al., 2009; Beaudry et al., 2010). For non-urgent patient transportation to/from hospitals, the vehicle may be re-configured to provide for staff seats, patient seats, stretchers, and wheelchairs. Constraints include mode-dependent capacities, driver-vehicle assignments, maximum shift lengths, and mandatory driver breaks. Parragh (2011), Parragh et al. (2012), and Schilde et al. (2011, 2014) studied the Austrian Red Cross in Graz. For the Hong Kong Hospital Authority (HKHA), each ambulance interior must be disinfected between consecutive trips to avoid the spread of disease. The choice not to serve some clients is allowed (Zhang et al., 2015; Liu et al., 2015; Lim et al., 2017). In Molenbruch et al. (2017c), restrictions on particular user–user and user–driver combinations are considered. In the application in Tuscany studied by Detti et al. (2017), a patient can choose the transport provider among different non-profitable organizations.

An emerging application area is in public transportation. When scheduled public transit is unavailable for low demand periods (e.g., night time) or locations (e.g., rural areas), it may be replaced by demand responsive transportation (DRT). In a pilot project serving young people to/from night clubs in Porto (Parragh et al., 2015), the DRT was operated by a private company and therefore the objective is to maximize profit. Moreover, group requests can be split. For some systems, the parties involved – Transportation Authorities, local taxi companies, subcontractor hauliers, and passengers – may have conflicting concerns. The on-demand transportation system in the rural Doubs region in France adopted the objective of maximizing occupancy rate to encourage people meeting during transportation for social cohesion (Garaix et al., 2011). In an integrated system for the elderly and disabled in Sweden, regular public transportation services are used as the trunk services of their journeys and flexible dial-a-ride services are used as feeders for the first and last mile (Häll et al., 2009; Posada et al., 2017). Integrated service can reduce the operation cost and increase utilization of the dial-a-ride vehicles, but transfers may lead to long waits and passenger discomfort. This type of mixed-mode operation is being considered by many public transit authorities, and also points to new research directions for the DARP. See Section 6 for further discussions.

1.3. Motivation and objectives of this survey

In the last few years, there has been a resurgence of interest in demand-responsive shared-ride systems for the general public. This has been fueled in part by concerns for the environment; each commuter using a separate car leaves a large carbon footprint and also causes congestion in central business districts. Thus, the notion of a sharing economy that advocates a shift from car ownership to "mobility as a service" is gaining popularity. Technological developments (e.g., web and mobile communications, cloud computing, data analytics), enabling new ways of operating DAR systems, have also contributed to the revitalization. Coming full circle, the Ford Motor Company – being a partner of some of the earliest DAR systems in the1970's – launched the *Dynamic Shuttle* system in 2015, which offers on-demand ride-sharing for employees in Dearborn, Michigan, USA. Other car manufacturers are also investing in various forms of car-sharing systems.

The resurgence of DAR systems has also provided much impetus into research investigations into the DARP. This paper provides a classification and summary of 86 papers published since 2007. The key contributions are the following:

1. a comprehensive survey of the journal papers published since 2007,
2. an overview of application areas of DARPs,
3. a detailed taxonomy of the problem variants, and discussion on subtleties of the classification,
4. a systematic review of exact and meta-heuristic solution methodologies,
5. full references to benchmark instances, with valid hyperlinks and computational comparisons,
6. identification of potential research gaps, and
7. discussion of emerging technologies and their impacts on future research directions.

Compared with the most recent review by Molenbruch et al. (2017d), our taxonomy and identified research directions are different (see Section 4.1 and the last section, respectively) and we have more emphasis on emerging technologies (information and communications technologies, autonomous vehicles, electric vehicles), societal changes (sharing economy, changing travel patterns, promotion of green transport), computational results (instances and comparisons), and applications. We focus more on the first two aspects as they enable new modes of operations, leading to new research directions. We also focus more on the third aspect because the latest summary of computation results allow researchers to evaluate the performance of their developed algorithms. We emphasize on applications because one of the new directions can be formulating and solving new applications. These four important aspects have not been mentioned in the previous review.

In the remainder of this paper, we begin with the research methodology in Section 2 and a description of the DARP in Section 3. Section 4 gives a classification of the many variants of the problem studies in the literature. Section 5 surveys the solution methods and references to benchmark instances. In the last section, we discuss some future trends, challenges, and opportunities for future research.

2. Research methodology

We first searched Scopus for journal publications using the keyword *dial-a-ride*. In addition, the databases of ISI Web of Science and Google Scholar were also used to identify any other relevant publications that were missed from Scopus. This survey includes mainly publications since 2007, with a few publications published earlier to recall the history and for comparison purposes. A total of 86 publications since 2007 are considered with around half of these published in *Computers & Operations Research* (nine papers), *Transportation Research Part B* (eight papers), *Transportation Science* (eight papers), *European Journal of Operational Research* (seven papers), *Operations Research Letters* (four papers), *Public Transport* (four papers), and *Transportation Research Part C* (four papers). Fig. 1 summarizes the number of papers published since 2007.

3. Problem description

3.1. Problem features

In the DARP, multiple users make their requests for transportation from their specific origins to destinations (known as *pick-up* and *drop-off/delivery points*, respectively). The transportation service provider receives the requests and then arranges

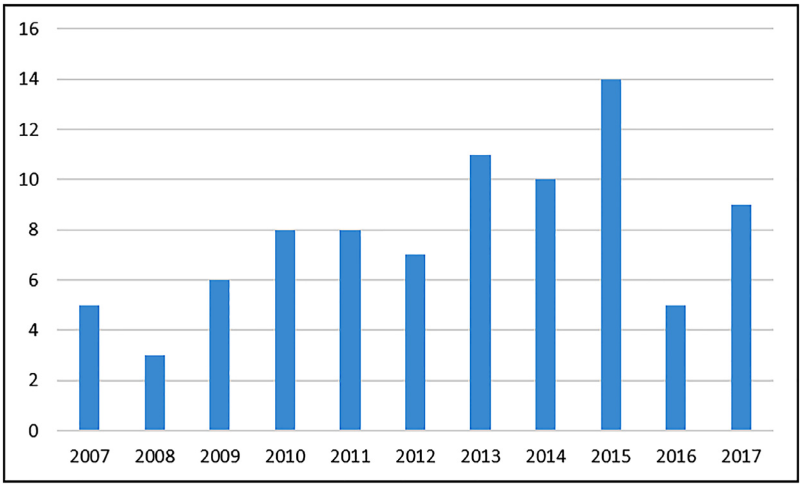

Fig. 1. Number of papers per year on the Dial-a-Ride Problem since 2007.

with its fleet of vehicles for the delivery service. The transportation service is shared in the sense that multiple users (with different requests) may be in the same vehicle at the same time.

The typical features of the DARP include the following:

1. Visit: Each user has to be delivered from the origin to the destination if rejections of requests are not allowed. If rejections are allowed, the service provider can make *selective visits* and decides which requests to accommodate.
2. Time window: Each user can specify the earliest and latest times of pick-up and/or drop-off (i.e., the departure time from the origin and the arrival time at the destination).
3. Depot(s): The starting and ending location(s) of a trip (or *route*) of a vehicle.
4. Trip: A vehicle finishes a trip once it returns to the depot. A vehicle finishes *multiple trips* if it leaves from and returns to a depot more than once in a single day.
5. Vehicle capacity: The maximum number of users in a vehicle at the same time. The number of users in the vehicle is known as *load*.
6. Ride time: The time a user spends in a vehicle (i.e., the difference between the scheduled times of pick-up and delivery).
7. Route duration: The time a vehicle travels for a trip (i.e., the difference between the times of leaving from and returning to a depot).

A typical DARP assigns the vehicles to the requests and determines the vehicle routes for transportation services, with the above features taken into account. The objective of the problem depends on the application. Typical objectives adopted are from the operator's perspective and/or the users' perspective. However, the objectives from the two perspectives can be conflicting to each other – improving users' experience (e.g., reducing the total ride time and waiting time) may need to increase the operating cost (e.g., hiring more vehicles). The goal is to optimize the objective function (which can consist of just a single measure or multiple measures) subject to constraints related to the above features such as capacity or time window constraints. A more detailed discussion on DARP objectives will be provided in Section 3.2.

We refer the reader to Cordeau (2006) for the basic mathematical model of the DARP, which has been studied extensively by other authors as well and has formed the basis for other problem extensions. For tighter formulations, we refer the reader to Ropke et al. (2007). In the literature, several variants of DARPs have been studied, as summarized in Tables 5–8 in the Appendix according to the typical features considered (i.e., time window, vehicle capacity, ride time, route duration, and selective visits), the degree of fleet heterogeneity (i.e., homogenous vs. heterogeneous fleet), the number of vehicles used (i.e., single vs. multiple vehicles), and the number of objective functions considered (i.e., single vs. multiple objectives), the number of depots considered, and numbers of trips allowed in a single day. More detailed discussions on these variants will also be provided in Section 4 of this survey, based on the classification introduced in that section.

3.2. Objective functions of DARPs

The most popular objectives of DARPs are to minimize service provider's operating costs (e.g., total transportation time, total distance traveled by the vehicles, total route duration, the number of vehicles required, and driver working time) and/or users' inconvenience metrics (e.g., total ride time, user waiting time, and deviations from requested pick-up/drop-off time windows). A small proportion of work considers vehicle emissions as well (e.g., Atahran et al., 2014; Chevrier et al., 2012). Other more problem-specific objectives include optimizing passenger occupancy rate (Garaix et al., 2011), cost-effectiveness

Table 2
Taxonomy of Dial-a-Ride Problems.

		Information known with certainty (at time of decision)	
		Yes	No
Decisions can be modified in response to new information received after time 0?	No	Static and deterministic	Static and stochastic
	Yes	Dynamic and deterministic	Dynamic and stochastic

metric (D'Souza et al., 2012), operator's profit (Parragh et al., 2015), staff workload (Lim et al., 2017), and the reliability of the system (Pimenta et al., 2017).

As shown in Tables 5–8, while a significant number of DARPs optimize only a single objective, some consider multiple objectives where the decision-maker has to determine an optimal solution among different goals. The research that deals with multi-objective DARPs can be mainly categorized into three types. The first type is to treat the multiple objectives as a weighted sum of different measures (e.g., Jorgensen et al., 2007; Kirchler and Wolfler Calvo, 2013; Mauri et al., 2009; Melachrinoudis et al., 2007). The weighted sum of objectives is appropriate for problems that have well-defined and straightforward evaluations of the weights of the different objectives. For example, one may consider the monetary value of each measure per unit as its weight, such as cost per vehicle and cost per mile traveled. If only a single solution is required for implementation (particularly for those applications that require a prompt single solution for a real-time recommendation), this approach takes advantage of reducing the post-solution efforts to making the final decision. However, this approach is not applicable to problems where the relative importance of each objective is unknown or unquantifiable. Furthermore, the solutions are also highly sensitive to the objective weights.

The second type considers lexicographic objective functions in the order of the importance, where a higher-level objective must be optimized first and a lower-level objective is then further optimized if possible. For example, Garaix et al. (2010) first minimized the operating cost and then maximized the service quality measure; Schilde et al. (2011, 2014) considered three levels of objectives: first maximizing the service quality measure and then minimizing operating costs at two different levels. The approach of having lexicographic objective functions applies to problems having one objective significantly dominating the others but the relative importance of the different measures cannot be represented by the same unit. This approach is particularly suitable for DARPs where the operating cost is substantially more important than the users' experience, or vice versa. However, the hierarchical structure prevents the decision maker from examining the tradeoffs between different objectives.

The third type aims to obtain the Pareto frontier of the problem. The Pareto frontier consists of the solutions that are not dominated by any of the other solutions, in terms of the concerned criteria. Similar to the lexicographic approach, the assignment of a weight to each objective and the conversion of the different objectives to the same unit are not required (Zidi et al., 2012). In addition, the full set of non-dominated solutions can be generated for the decision maker to choose the right plan for the final implementation (e.g., Núñez et al., 2014; Parragh et al., 2009). The Pareto solution approach provides the decision maker with the full picture of all the possible optimal solutions, which is especially favorable when he/she is uncertain about the relative importance of each criterion. It also helps to analyze the tradeoffs between opposing objectives at a tactical level (Paquette et al., 2013). However, this approach requires computing the full set of optimal solutions and human beings to make the final decision from this generally huge set. Thus, it may not be suitable for online DARPs where transportation plans are required to be automatically revised in a timely and frequent fashion (e.g., the applications where user requests can arrive unexpectedly).

The aforementioned multi-objective approaches consider different performance metrics independently. Lehuédé et al. (2014) is the only paper in this survey that considers the importance of each metric and the interaction between each pair of metrics.

4. Problem classification

4.1. Taxonomy

We classify DARPs broadly according to two aspects: (1) whether decisions are made a priori (i.e., *static*) or if the decision maker is allowed to modify existing plans in response to new information received (after the start of operations, i.e., time 0) as execution proceeds (i.e., *dynamic*), and (2) whether the information (when received) is known with certainty (i.e., *deterministic*) or still undetermined when decisions are made (i.e., *stochastic*). This classification leads to four basic DARP categories – *static-deterministic, static-stochastic, dynamic-deterministic*, and *dynamic-stochastic* (see Table 2). This taxonomy is similar to that of Pillac et al. (2013) for vehicle routing problems.

The difference between a static and dynamic DARP is as follows. If all information relevant to decision making is provided to the decision maker prior to the start of operations, a DARP is static. In this case, even if the information available for decision making does evolve as time passes, we assume that the decision maker develops a grand plan for a predetermined number of users for the entire planning horizon prior to the start of operations. This plan is either a routing policy or a set of specific routes and schedules, and it cannot be changed at a later time. On the other hand, if some actionable information is revealed while operations are ongoing *and* if the decision maker is allowed to respond to this new information, the problem is dynamic. For example, if the decision maker is allowed to modify existing plans in response to (i) the sudden appearance of new users, (ii) updated information concerning existing users, or (iii) unexpected disturbances such as delays and/or vehicle breakdowns as operations unfold, the problem is dynamic.

We now discuss the distinction between deterministic and stochastic DARPs. In a deterministic DARP, information is known with certainty at the time of decision. A stochastic DARP is one where information is unknown or uncertain at the time when decisions are made, although information about the uncertainty (e.g., range of values and probability distributions) may be available to the decisions maker. The distinction between deterministic and stochastic DARPs can also be viewed in terms of perfect vs. imperfect information. A DARP is deterministic if decisions are made in the context of perfect information. In a *static and deterministic* DARP, the decision maker has, at time 0, perfect information concerning all current and future operations. That is, at time 0 the decision maker knows (a) the set of all potential users; (b) whether or not each potential user will actually show up; (c) the exact needs of users; and (d) the exact duration of every operation – e.g., vehicle journey, user pick-up, and customer drop-off – that could potentially take place in the future. This type of problem can approximate the actual outcomes well if the information is not largely distorted from the expected. A DARP is considered *dynamic and deterministic* if, at every instant from time 0 onwards, the decision maker has perfect information concerning all current and future operations except for the appearance of new users and cancellations of users. That is, at every instant from time 0 onwards, the decision maker has perfect information regarding (b)–(d) for the users who have already appeared. In this DARP, the appearance of a new user and cancellations of users come as a surprise to the decision maker, but the decision maker has perfect information regarding each the appearance and cancellations and can modify previously planned (i.e., existing) routes accordingly.

A DARP is considered stochastic if decisions are made in the context of imperfect information. A DARP is *static and stochastic* if the decision maker must decide everything at time 0 based on imperfect information (e.g., uncertainty), regarding items such as (b)–(d). The decisions made at time 0 either consist of a set of vehicle routes or a routing policy, and they cannot be changed at a later time. A DARP is considered *dynamic and stochastic* if, at every instant from time 0 onwards, the decision maker has imperfect information regarding (a) and at least one other item such as (b)–(d). In this type of DARP, the decision maker is continually confronted with uncertainty regarding not only the appearance of future users and cancellations of users but also the operations concerning users who have already appeared. In this case, the decision maker can change previously planned routes in response to new information, but such information may still be imperfect at the time of the change decision.

Note that real-world DARPs are mostly stochastic because the processes are often unpredictable due to human vagaries, changing circumstances, and other externalities; the exact duration of each process (e.g., the pick-up of a handicapped passenger) remains unknown until the process is completed. In a static and stochastic DARP, the (a priori) decisions may involve routing plans and planned arrival times. As the operations proceed, actual travel and arrival times will be realized that may be different than planned. If the sequence of requests served is not changed (even though arrival times differ from expected), the problem is still considered a static one. Only if routing, holding or vehicle assignment decisions are revised based on new information would the problem be considered a dynamic one.

It should be noted that our classification differs slightly from that provided by some other authors such as Pillac et al. (2013). The difference is that we allow for an environment with imperfect information in which the decision maker is not provided with information regarding the nature of the stochasticity. A DARP with such feature is classified as a stochastic DARP in our classification but does not fall into any category in the classification of Pillac et al. (2013). It should also be noted that unlike main classifiers used by Molenbruch et al. (2017d), the main classifiers used in this paper are static-dynamic and deterministic-stochastic because these classifiers define the key features of the problems, and may lead to very different methodological approaches to obtain solutions. For example, dynamic problems allow to accept or reject requests in real time. Dynamic problems also allow readjustment of plans in response to new information received. Dynamic problems, therefore, require fast solution algorithms, such as heuristics, for real-time operations. Stochastic problems do not require that all information is known with certainty. Extra solution procedures, such as Monte Carlo Sampling, are often needed to obtain solutions. We now discuss the articles belonging to each of the four basic DARP categories – *static-deterministic, static-stochastic, dynamic-deterministic*, and *dynamic-stochastic*.

4.2. Static and deterministic DARPs

The majority of articles in this survey consider static and deterministic DARPs. Among these articles, the typical setting considers a homogeneous fleet of vehicles, passengers' pick-up and drop-off time windows, maximum passenger ride time, maximum route duration, and vehicle capacity. This setting is particularly popular among those papers having their focus on algorithmic advancement (e.g., Parragh et al., 2010; Parragh and Schmid, 2013; Chassaing et al., 2016; Ritzinger et al., 2016). To impose the requirements, some papers include them as hard constraints in their optimization models (e.g., Chassaing et al., 2016; Ritzinger et al., 2016), while others may allow violations but such violations are penalized in the objective function so as to be avoided as much as possible (e.g., Urra et al., 2015). Other than those papers focusing on algorithmic advancement, most of the remaining papers focus on the modeling issues regarding new problem features motivated by new applications of DARPs or real needs in practice. Very few focus on drawing insights derived from computational experiments to assist DAR operations. In the following four sections, we highlight different key modeling aspects of the problem variants, followed by the section on the insights.

4.2.1. Heterogeneity of users and vehicles

User heterogeneity is mainly motivated by the real need for the specific application. For example, users may have their individual requirement or expectation on the service provided (e.g., Ilani et al., 2014) or some combinations of users and drivers are more (or less) favorable (e.g., Molenbruch et al., 2017c).

The existence of heterogeneous passengers has led to a growing number of papers considering heterogeneous vehicles. Interestingly, we observe that most of the studies that consider heterogeneous vehicles are motivated by real-world applications of transferring people with limited mobility (e.g., patients and the elderly). This is mainly because the transportation service providers require various combinations of equipment (e.g., wheelchairs and stretchers) for different types of passengers. In these problems, the vehicles are differentiated by their equipment and capacities. The joint consideration of heterogeneous users and heterogeneous vehicles imposes further challenges on solving the problem. A typical setting is the necessity of respecting user-vehicle compatibility (e.g., Parragh, 2011; Parragh et al., 2012; Carnes et al., 2013; Braekers et al., 2014; Detti et al., 2017). For example, a patient can only be transferred by a vehicle which is equipped with the full set of requested equipment. More specific challenges of individual DARPs with both heterogeneous users and vehicles include the sequencing of picking up and dropping off different types of users in accordance with the layout of the vehicle (e.g., Karabuk, 2009) and decisions on the vehicle configuration for each trip (e.g., Qu and Bard, 2013; Qu and Bard, 2015). These problem features may require the introduction of additional indices to the formulation (e.g., Qu and Bard, 2013; Braekers et al., 2014; Qu and Bard, 2015; Braekers and Kovacs, 2016) and, therefore, can increase the problem complexity and the time to obtain an optimal solution extensively.

We also notice that some studies consider regular and extra vehicles but the users are homogeneous. One of their goals is to minimize the fixed cost of extra vehicles needed (e.g., Guerriero et al., 2013). However, these studies do not consider the user–vehicle compatibility since the users are homogeneous.

4.2.2. Passenger transfers

In the conventional DARP, a user is transported in the same vehicle for the whole journey. Some recent papers consider passengers' transfers from one vehicle to another during their trip. These papers were motivated by the needs in real-world applications such as transporting passengers with reduced mobility at airports (e.g., Reinhardt et al., 2013). In many of these applications, an additional set of constraints is needed to ensure that a user is dropped off at a transfer point before he/she is picked up by another vehicle (e.g., Schönberger, 2017). The main challenge of allowing passenger transfers in DARPs is to ensure the synchronization when a passenger transfers between vehicles. More specifically, this area of research aims to minimize the impacts of passenger transfers on users' inconvenience (e.g., users' waiting time).

Although users' trips become less direct and may have to wait at transfer points, a number of studies show the benefits of allowing such transfers. Cortés et al. (2010) showed that the flexibility of having passenger transfers could improve the overall efficiency of the system, for example, by reducing the total ride time. Masson et al. (2014) presented a generalization of the DARP where passengers could make transfers at intermediate points and showed that significant savings, in terms of total distance traveled, could be achieved when transfers were allowed.

While the DARP focuses on demand-responsive transportation services, some research (e.g., Posada et al., 2017) allows the use of a fixed route and scheduled public transit service during the user's journey. One of the benefits of the integration of on-demand transportation and public transit services is that it can help reduce the operating cost of the overall system because public transit is less expensive. The main challenge is, again, the synchronization of the demand-responsive and the public transit services. If the public transit service has a very high frequency, one may neglect the synchronization (Häll et al., 2009). However, as pointed out by Ronald et al. (2015), the ignorance of the synchronization can be an issue if the frequency of public transit services is low. To address the issue, Posada et al. (2017) proposed two models for an integrated DARP that take into account public transit timetables.

4.2.3. Manpower requirements

The majority of DARPs that consider manpower requirements are motivated by the transportation of people with limited mobility. In these problems, there might be the preference of assigning the same driver to a user in multiple periods (e.g., Braekers and Kovacs, 2016) or the users might request to have accompanying staff members on the vehicle (e.g., Parragh, 2011; Parragh et al., 2012). Additional concerns about manpower requirements in DARPs include the loads of the accompanying staff members (e.g., Parragh et al., 2012; Zhang et al., 2015; Lim et al., 2017), the schedules of meal breaks (e.g., Parragh et al., 2012; Liu et al., 2015; Zhang et al., 2015; Lim et al., 2017), and the synchronization of vehicles and assistants (e.g., Liu et al., 2015). In addition to the challenges arising from the modeling issues, the joint problem of the DARP and staff scheduling, which are both $\mathcal{NP}$-hard, imposes further complexity in finding optimal solutions.

4.2.4. Horizontal cooperation

Almost all existing DARP studies implicitly assume that only one DAR service provider exists in a single operating area. In reality, multiple DAR service providers can exist in one operating area. This characteristic was considered by Molenbruch et al. (2017a) who are the pioneers to examine the operational benefit of the cooperation between multiple

DAR service providers. Based on a real-life case study, they found that the benefit was mainly due to the empty trip reduction. However, because the variable cost associated with travel distance only explained a small portion of the total cost (including driver wage) incurred by the provider, the percentage of the reduction in the total cost was small.

4.2.5. Insights derived from DARPs

In addition to the contribution from modeling aspects, some papers draw managerial insights from DARPs using computational experiments. With a simulation study of the ADA paratransit services in Houston, Shen and Quadrifoglio (2013) reported that a centralized strategy (i.e., the entire service region is a single zone) could reduce the number of routes required and the total distance traveled by empty vehicles and increase the passenger trips per revenue hour, while a decentralized strategy (i.e., the entire service region is divided into multiple zones to manage) could decrease the average deviation time between requested and actual pickup times. Feng et al. (2014) found that by allowing the same airport shuttle to drop off outbound and pick up inbound passengers during the trip, significant savings (in terms of total number of vehicles required, vehicle idle time, distance traveled, and total cost) could be achieved, compared with the policies of (i) having two separate sets of vehicles to handle outbound and inbound trips and (ii) restricting that all outbound passengers must get off before the same vehicle picks up the inbound passengers. Molenbruch et al. (2017b) found that the reduction of the service quality did not lead to significant cost savings when the service quality was bad to a certain extent, and the variations in service level requirements had a greater impact on operating costs for larger service providers.

Some papers aim to study the practical impacts of the modeling techniques of DARPs. Garaix et al. (2010) discussed the problem of the graph representation of the road network where arcs are computed according to only a single criterion, for example, travel time. This practice could eliminate some possible routes from consideration. They introduced a multigraph representation whose arcs were characterized by multiple attributes and demonstrated the cost savings resulting from the multigraph model. Hu and Chang (2015) considered the fact that travel times were time-dependent and used a traffic simulation model to examine DARPs with such travel times. They reported that the increase in the length of the time window could reduce the total travel time and the number of vehicles used, and increase the CPU time, average pickup/delivery delay time, and the average actual/direct ride time.

4.3. Static and stochastic DARPs

The research on static and stochastic DARPs is relatively inadequate, compared with the other three categories. Only three papers in this line of research have been found. All the three papers in this categories consider the stochasticity regarding the user arrivals – the number of user requests follows a Poisson process (Hyytiä et al., 2010), a certain user requests the service with a given probability (Ho and Haugland, 2011), and the users' arrival times at the pick-up points are stochastic (Heilporn et al., 2011). The common objective of the research is to optimize the expectation of the objective function in the anticipation of future events (e.g., Ho and Haugland, 2011; Heilporn et al., 2011) or to investigate the system performance under a static and stochastic environment (e.g., Hyytiä et al., 2010).

4.4. Dynamic and deterministic DARPs

The papers in this survey on dynamic and deterministic DARPs can be categorized as theoretical or experimental. Theoretical research on these DARPs has typically been characterized by the presentation of (1) an online algorithm that has a proven competitiveness ratio versus its offline counterpart and/or (2) a new lower bound for such a competitiveness ratio that is higher than the previously established lower bound (e.g., Waisanen et al., 2008).

Experimental research on dynamic and deterministic DARPs has typically been characterized by the presentation of a simulation or other dynamic model in which decisions that are made in response to new information are fed back into the model so as to affect the future evolution of the system state tracked by the model (e.g., Häll et al., 2015; Quadrifoglio et al., 2008). The majority of research in this category considers new user requests as the events that trigger the replanning procedure (e.g., Hanne et al., 2009; Berbeglia et al., 2012a; Häll and Peterson, 2013; Häll et al., 2015; Wong et al., 2014; Marković et al., 2015). The goal is to determine how the new request is accommodated, if not rejected. Some studies incorporate pricing decisions when considering new user requests (e.g., Santos and Xavier, 2015; Sayarshad and Chow, 2015). The research on dynamic and deterministic DARPs is mostly restricted to the consideration of accommodation of new user requests. While, in reality, there could be other types of events that may trigger a revision in the transportation plan, only one paper (Beaudry et al., 2010) in this survey considered events such as vehicle breakdowns and unexpected rest breaks.

4.5. Dynamic and stochastic DARPs

Different types of stochasticity are considered in this category of papers, such as future user requests (e.g., Xiang et al., 2008; Hyytiä et al., 2012; Schilde et al., 2011), stochastic travel times (e.g., Xiang et al., 2008; Schilde et al., 2014), user no-shows (e.g., Xiang et al., 2008), and desired drop-off times (e.g., Maalouf et al., 2014). The stochastic information could be used to predict the scenarios that may happen in the future for optimal control (e.g., Núñez et al., 2014; Muñoz-Carpintero et al., 2015). With the use of stochastic information about unknown future events for decision-making in response to the recently realized events (also known as a non-myopic approach, Hyytiä et al., 2012), the solutions are expected to have a higher quality than those produced by myopic methods.

Among the four categories of DARPs, dynamic and stochastic DARPs appear to be the most challenging, regarding the difficulties arising from the modeling of a combination of stochastic components, the evaluations of outcomes under a large number of scenarios, the integration of stochastic processes and optimization algorithms, and the timely provision of high-quality solutions for recovery.

5. Solution methods

Different solution approaches have been proposed for the DARP and its variants. Some solution approaches can apply to more than one problem type. For example, heuristics or metaheuristics can be applied to solve both static-deterministic DARPs and dynamic-deterministic DARPs. Therefore, in this section, we classify different solution approaches by techniques.

5.1. Exact methods

Exact algorithms for DARPs are developed mainly based upon the concept of branch-and-bound (B&B). These algorithms can be classified as branch-and-cut (B&C), branch-and-price (B&P), and branch-and-price-and-cut (B&P&C) algorithms. For specific small DARPs, a reduction approach was proposed.

The development of exact methods for DARPs is heavily inclined towards deterministic and static problems. All the papers, except one, reviewed in this survey that developed exact methods for DARPs considered a deterministic and static environment; the remaining one considered a stochastic and static setting. The main reasons are that many of the stochastic programming problems are computationally intractable for realistic DARP instances and exact methods may not be capable of providing timely solutions for dynamic DARPs. Moreover, exact methods can provide solutions of the highest quality, which is the most concern in the planning phase (i.e., static problems). Another observation of exact methods reviewed is that they are for single objective DARPs. Exact algorithms for multi-objective DARPs have not been found in the reviewed period.

Table 9 summarizes the largest instances that have been solved to optimality by exact methods. The largest instances are up to 8 vehicles and 96 requests for the basic DARP. For other variants, the sizes are smaller because the variants are more complicated.

5.1.1. Branch-and-cut algorithms

A B&C algorithm is based on the B&B procedure, where cutting planes are added to the problems in the B&B tree. The addition of cuts tightens the LP-relaxations in the B&B tree, leading to a higher chance of finding integer solutions and also providing stronger bounds for verifying optimality. To the best of our knowledge, the first B&C algorithm for the DARP was introduced by Cordeau (2006) who applied several families of valid inequalities as cuts for the three-index mixed integer programming formulation. These are derived from well-known inequalities for the traveling salesman problem and the vehicle routing problem (VRP). Ropke et al. (2007) presented tighter DARP formulations for a two-index mixed integer programming model, by introducing three new classes of valid inequalities – strengthened capacity, strengthened infeasible path, and fork constraints – and adopting some previously identified cuts, including subtour elimination and generalized order constraints from Cordeau (2006) and reachability constraints from the VRP with time windows. Some of these inequalities were also applied in other studies (e.g., Parragh, 2011; Braekers et al., 2014; Braekers and Kovacs, 2016). Other types of valid inequalities are derived for problem-specific features such as trip number (e.g., Liu et al., 2015), lunch breaks (e.g., Liu et al., 2015), symmetry breaking (e.g., Braekers and Kovacs, 2016), and driver consistency (e.g., Braekers and Kovacs, 2016).

Another approach to solving deterministic DARPs in addition to the use of cuts was proposed by Cortés et al. (2010), who developed a B&C method to solve the DARP that allows passengers to transfer from one vehicle to another at specific locations. The method uses Benders Decomposition (Benders, 1962) that applies the Combinatorial Benders Cuts introduced by Codato and Fischetti (2004). In this method, the set of constraints is decomposed into pure integer and mixed constraints, and then a B&C procedure is applied to the resulting pure integer problem, by using real variables and constraints related as cut generators (Cortés et al., 2010).

For a stochastic version of the DARP, Heilporn et al. (2011) incorporated an integer-L-shaped algorithm within the B&C framework proposed by Cordeau (2006), where the non-linear term of the stochastic customer delays at pick-up locations in the objective function is captured through the construction of optimality cuts during the B&B procedure.

The effects of cuts on the efficiency of a B&C procedure heavily depends on (i) the strength of the cuts and (ii) the choices of the cuts to be included at different nodes in the B&B tree. For (i), while the majority of research aims to identify new or strengthened cuts and to prove their validity, it appears that there is a lack of theoretical research on studying the conditions as to which of the cuts are strong (e.g., whether they are facet-defining under certain conditions). Such theoretical investigation will contribute to a deeper understanding of the polyhedral structure of the DARP, thereby leading to more compact DARP formulations and further strengthened inequalities. For (ii), the numbers of valid inequalities of many of the classes can grow exponentially as the problem size increases. The inclusion of all these inequalities in the base formulation can render the problem unmanageable. Thus, separation procedures were developed for the determination of effective cuts to be included during the B&B procedure (e.g., Ropke et al., 2007; Parragh, 2011; Liu et al., 2015) to ensure that the cuts are generated and applied only when needed. At each node of the B&B tree, these separation procedures aim to promptly identify those inequalities that are violated by the current fractional solution and determine if they should be included in the formulation. When the separation procedure requires the enumeration of a huge number of possibilities, it is a common practice to develop separation heuristics to speed up the process.

5.1.2. Branch-and-price algorithms

Unlike B&C algorithms, B&P algorithms focus on column generation rather than generating cuts for LP relaxations in a B&B procedure. B&P algorithms require the reformulation of the problem into a restricted master problem and a pricing subproblem. In the restricted master problem, a set of columns (i.e., variables) is excluded from the LP relaxation to reduce computational efforts. At each node of the B&B tree, columns may be generated by solving a pricing subproblem and added to the restricted master problem to tighten (improve) the LP relaxation. The advantage of this method over the B&B method is the ability to handle larger mixed-integer programs, while convergence to a global optimal solution can still be guaranteed. For the DARP, the main difficulty is how to reformulate and decompose the original problem to obtain the pricing subproblem that can be solved efficiently by existing algorithms, while the global convergence of the B&P algorithm is assured. For the studies that developed B&P algorithms for DARPs, the master problems were mainly to optimize the objective function subject to the request constraints and the subproblem aimed to generate routes for the vehicles. There are three main approaches to solve the subproblems: exact, heuristic, and a hybrid of exact and heuristic.

The exact approach was used by Garaix et al. (2010, 2011). They solved the pricing subproblem, referred to as the elementary shortest path problem with resource constraints, by dynamic programming. The dynamic programming algorithm associated partial paths with labels and extended these labels taking into account the resource constraints and the reduced cost of the subproblem. It is worth noting that the B&P algorithm developed in Garaix et al. (2011) is able to handle a linear fractional objective function. Feng et al. (2014) generated columns through a constraint programming framework to guide the search procedure for the routing decision in a reduced search space. While the global convergence of the solutions is guaranteed when applying an exact method to solve the pricing subproblem, the major drawback is that the computational time can be long for identifying an optimal solution and verifying its optimality for each subproblem throughout the procedure.

The heuristic approach was used by Hu and Chang (2015) who developed a B&P algorithm to solve the DARP with the consideration of time-dependent travel times. The pricing subproblem was solved by large neighborhood search. While the heuristic could identify good-quality solutions for the subproblem in a much faster fashion, the limitation of this approach is that these solutions may not be optimal and hence the global convergence of the overall B&P algorithm is not guaranteed.

The hybrid approach was used by Parragh et al. (2015) who solved the subproblem by both dynamic programming algorithm and heuristics as in the algorithm of Ropke and Cordeau (2009) for the pick-up and delivery problem with time windows. The hybrid was incorporated into their B&P framework for solving the DARP with split requests and profits, in which a given transportation request may be served by multiple vehicles or by multiple trips of the same vehicle. The hybrid approach can guarantee the global convergence while the computation efficiency is enhanced compared with a standalone exact approach.

5.1.3. Branch-and-price-and-cut algorithms

B&P&C algorithms are B&P algorithms with cutting planes added to the LP relaxations throughout the procedure. B&P&C algorithms can take advantage of both B&P and B&C methods by (i) tackling the reduced problem of a significantly smaller size with columns generated through solving the subproblem and (ii) tightening the bounds for the LP relaxation. In Qu and Bard (2015) and Gschwind and Irnich (2015), dominance rules were used to reduce the number of paths to be enumerated for solving the subproblems. Qu and Bard (2015) applied subset-row inequalities in their B&P&C algorithm and showed the benefits of these cuts on reducing the number of nodes to be visited in the B&B tree and the overall solution time compared with only B&P. Gschwind and Irnich (2015) considered intra-route synchronization in their problem and derived an effective column-generation formulation with the inclusion of these intra-route constraints. They adopted several classes of inequalities from Cordeau (2006), Ropke et al. (2007), and Ropke and Cordeau (2009) for their B&P&C procedure. They showed that their algorithm outperforms (i) the B&C algorithm proposed by Ropke et al. (2007), and (ii) the B&P&C algorithm by Ropke (2005) that is an earlier version of Ropke and Cordeau (2009) for the pickup and delivery problem with time windows. As reported by Qu and Bard (2015), while the inclusions of cuts are effective in reducing the number of nodes to be explored in the B&B tree, the majority of the overall runtime of the B&P&C is spent solving the subproblems. This suggests that further research on faster algorithms to solve the subproblems are needed to enhance the overall efficiency of B&P&C algorithms.

5.1.4. Reduction approach

A reduction approach was used by Ilani et al. (2014) who examined a two-campus transport problem. They proposed an algorithm based on a reduction of an ordered-set partition problem (Chakravarty et al., 1982) to a shortest-path problem, which can be solved in polynomial time. However, this problem considers only one route (in two directions) and may not be applicable to general DARPs.

5.1.5. Lessons learned

Most of the exact methods for the DARP are based on a B&B framework. B&B is designed for general discrete and combinatorial optimization problems. It enumerates all possible solutions for the problem. Thus, the computational time of a B&B procedure can be exponential in the worst case. There has been much work on improving the bounds for the optimal objective value by the use of cutting planes (which has been discussed in Section 5.1.1) and the generation of promising feasible solutions using heuristics or metaheuristics (e.g., Parragh, 2011; Braekers et al., 2014) (which will be discussed in detail in Section 5.2) in the hope of completing the enumeration at an earlier stage. The recent development of B&P&C suggested that an integration of problem decomposition techniques and cutting planes can potentially significantly reduce computational efforts for solving DARPs.

Other techniques that have considerable impact on the speed of a B&B framework include preprocessing (e.g., Ropke et al., 2007; Parragh, 2011; Braekers et al., 2014; Liu et al., 2015; Braekers and Kovacs, 2016) and branching rules (e.g., Ropke et al., 2007; Garaix et al., 2010; Parragh, 2011; Qu and Bard, 2015; Gschwind and Irnich, 2015; Braekers and Kovacs, 2016). The preprocessing steps adopted in these studies are mainly based on those proposed by Dumas et al. (1991) and Cordeau (2006), while the branching schemes vary among applications. The reasons are that DARPs share common features such that preprocessing routines can be similar (e.g., time window tightening and arc elimination) but the benefits of branching priority can be different according to their objective functions and problem-specific constraints.

The development of these exact methods based on a B&B framework benefits from the ease of the implementation through off-the-shelf mixed-integer programming solvers such as IBM ILOG CPLEX (which was adopted by almost all studies reviewed in this section as the solver, except those without mentioning the solver used). The solvers can take charge of the B&B setting, for example, branching schemes (e.g., Braekers et al., 2014; Liu et al., 2015). The performance of these exact algorithms can also be enhanced by the advancement of the general discrete optimization techniques (e.g., more effective pre-solve procedures and branching schemes in a B&B framework).

The main advantage of the adoption of exact methods for solving the DARP is that solution optimality is guaranteed. This is particularly important for problems in the planning phase where no adjustment is allowed once the plan is implemented. Thus, the exact methods were applied to static DARPs. Computational times of hours may also be acceptable for planning problems, where decisions are made infrequently (e.g., daily) and the information is provided some hours in advance. Even if an optimal solution cannot be identified at termination, the planner still has an idea of how good the solution is by measuring the optimality gap.

For problems that are not manageable by exact methods, for example, due to memory issues or unacceptable solution time, heuristics and metaheuristics (which will be discussed in the next section) are essential. In this case, the bounds derived from exact methods can also serve as a quality measure for solutions produced by heuristics and metaheuristics.

5.2. Heuristics and metaheuristics

As exact methods can only solve small-sized instances to optimality using a considerable amount of computing time, and due to the $\mathcal{NP}$-hardness of DARPs, the focus of much research has been on developing efficient and effective heuristic techniques.

5.2.1. Construction insertion heuristics

Simple construction/insertion heuristics have been proposed in the last decade for solving the basic DARP and its extensions. Although metaheuristics are more effective, construction heuristics are useful when there is a need of quickly finding feasible solutions, e.g., in dynamic DARPs (Xiang et al., 2008; Wong et al., 2014; Marković et al., 2015), to initialize a more complex method (e.g. Braekers et al., 2014; Masmoudi et al., 2016), or to evaluate various operational policies/strategies (Wong et al., 2014; Feng et al., 2014). These heuristics are usually inspired from the greedy insertion heuristic by Jaw et al. (1986), where each request is assigned to a position in the vehicle route by the cheapest insertion criterion. This heuristic is simple and fast. For the details of more advanced construction heuristics, please see Luo and Schonfeld (2007), Wolfler Calvo and Colorni (2007), and Häme (2011).

5.2.2. Tabu search

Tabu search (TS) follows the principle of local search that avoids revisiting previously visited solutions by recording the search history in a tabu list. To avoid getting stuck in local optima, non-improving solutions are accepted. Cordeau and Laporte (2003) were among the first ones to present a TS algorithm for the DARP. Besides using a simple neighborhood operator (i.e., relocating a request from one route to another) to generate the neighborhood, Cordeau and Laporte have also incorporated several diversification strategies; penalizing frequently made moves and temporarily accepting infeasible solutions. This heuristic has shown to be effective and efficient. For that reason, many of the recent studies of TS on DARPs (Beaudry et al., 2010; Ho and Haugland, 2011; Guerriero et al., 2013; Paquette et al., 2013; Kirchler and Wolfler Calvo, 2013; Detti et al., 2017) are in fact inspired from Cordeau and Laporte (2003) TS. These studies are typically on DARPs with more complicated and real-life constraints. The authors adapted Cordeau and Laporte (2003) TS to handle the more complex DARPs. Usually, the most time-consuming task with the TS is the evaluation of the neighborhood. To speed up the evaluation, some may only consider moves within a certain threshold (Kirchler and Wolfler Calvo, 2013) while others do a random sampling (Detti et al., 2017). TS works well as a stand-alone method, but it also shows to work well when incorporating into a multi-start heuristic (Guerriero et al., 2013) or a multi-criteria framework (Paquette et al., 2013).

5.2.3. Simulated annealing

Simulated annealing (SA) is a stochastic local search based metaheuristic inspired by the physical annealing process. A neighbor of the current solution is selected at each iteration. Usually, this solution is randomly selected. To avoid getting stuck in local optima, non-improving solutions are accepted with a probability. SA has not been as widely used to solve DARPs as the other metaheuristic approaches. A few authors (Mauri et al., 2009; Zidi et al., 2012; Reinhardt et al., 2013) have implemented a standard SA using simple neighborhood operators and obtained reasonable results.

Recently, a highly effective and efficient variant of SA was proposed by Braekers et al. (2014). It is a deterministic variant of SA, which is named deterministic annealing (DA), where non-improving solutions are accepted as long as the deterioration of the objective value is smaller than a deterministic threshold. Braekers et al. used more complicated neighborhood operators than previous authors, and utilized a restart strategy whenever the search is stranded in unattractive regions of the solution space. This combination turns out to be beneficial as the heuristic provides very good results for different DARP variants.

5.2.4. Variable neighborhood search

Variable neighborhood search (VNS) proposed by Mladenović and Hansen (1997) is a metaheuristic based on a systematic change of neighborhoods in the descent and perturbation phases of the local searches. Parragh et al. (2009) proposed the first VNS heuristic for the DARP. They implemented a basic VNS for a bi-objective DARP, where four different neighborhood operators are used in the shaking step. The shaking step of VNS involves introducing randomly generated solutions from the neighborhoods. In the shaking step, the focus was to move requests between the routes whereas the focus in the local search was to re-assign the requests to different positions within the route. To avoid getting stuck in local optima, they employed a simulated annealing acceptance criterion. Improving solutions are always accepted while non-improving ones are accepted with a probability. This version of VNS lays the groundwork for the recent studies of VNS on DARPs (Parragh et al., 2010; 2012; 2015; Parragh, 2011; Schilde et al., 2011; 2014; Muelas et al., 2013; 2015; Detti et al., 2017).

Due to the great results achieved from the VNS for both the single-objective DARP (Parragh et al., 2010) and the bi-objective DARP (Parragh et al., 2009), other authors have since adapted their VNS to tackle richer DARPs. While a majority of the studies used the same operators as Parragh et al. (2009) in the shaking phase, some have also employed other operators such as greedy worst origin move and greedy best destination move (see Muelas et al., 2013; Muelas et al., 2015). Besides performing well on its own, VNS also works well under a simulation framework (Schilde et al., 2011; 2014) or a distributed algorithm framework (Muelas et al., 2015).

5.2.5. Large neighborhood search

At each iteration of the large neighborhood search (LNS), a part of the solution is destroyed (e.g., in DARPs, q requests are removed from the solution) and then this partial solution is rebuilt into a complete solution (e.g., the q requests are reinserted back to the partial solution). The removals are done by selecting one of the removal heuristics and the insertions are done by choosing one of the insertion heuristics. The changes that are made to the solution are larger than those by the typical neighborhood operators employed in other previously described metaheuristics. Ropke and Pisinger (2006) published an article on an adaptive large neighborhood search heuristic (ALNS) for the pickup and delivery problem with time windows. Ropke and Pisinger used simple and fast heuristics that already existed in the literature for removing and inserting requests. The removal heuristics include those based on Shaw (1997) and random and worst removal, while the insertion heuristics include greedy and regret heuristics. The selection of the heuristic is guided by the heuristic's past performance. In addition, they used a simulated annealing acceptance criterion to determine whether a solution should be accepted or not. This method is highly effective and efficient, and lays the foundation for the recent studies of LNS on DARPs (Häll and Peterson, 2013; Qu and Bard, 2013; Lehuédé et al., 2014; Masson et al., 2014; Braekers and Kovacs, 2016; Gschwind and Drexl, 2016; Masmoudi et al., 2016; Molenbruch et al., 2017a).

Besides adapting Ropke and Pisinger (2006) operators to tailor for more complicated DARPs, several authors (Masson et al., 2014; Braekers and Kovacs, 2016; Masmoudi et al., 2016) also used operators from the pickup and delivery problem with transfers, the consistent vehicle routing problem, and the pollution-routing problem. (A)LNS works well as a stand-alone procedure, but it can also be incorporated under the multi-criteria framework (Lehuédé et al., 2014) as well as into a multi-start heuristic (Qu and Bard, 2013).

Gschwind and Drexl (2016) adopted Ropke and Pisinger (2006) ALNS and added three more removal operators that were proposed by Masson et al. (2013) and Parragh et al. (2010) for the pickup and delivery problem with transfers and the DARP, respectively. This version of ALNS outperformed all algorithms in terms of solution quality except for the hybrid GA by Masmoudi et al. (2017) on the instances for the standard DARP. To further improve the solutions, after a solution is repaired, promising solutions are improved by the Balas-Simonetti neighborhood (Balas and Simonetti, 2001). In addition, solutions are further improved by solving a set-covering problem (with a limited running time of two minutes) at the end of the ALNS. With the addition of two new improvement components, the algorithm even outperformed the hybrid GA by Masmoudi et al. (2017).

5.2.6. Genetic algorithms

Genetic algorithms (GA) are population-based metaheuristics and are inspired by the evolution of species. GAs start with an initial population of individuals (i.e., solutions). At each iteration, individuals are chosen to be the parents where individuals with better fitness have a higher probability from being chosen. New individuals (i.e., offsprings) are created by applying crossover and mutation operators on the parents. Then, some of the existing individuals may be replaced by the new offsprings.

Both Jorgensen et al. (2007) and Cubillos et al. (2009) presented a cluster-first, route-second approach where the method alternates between GA for constructing clusters of requests and a greedy heuristic to construct the routes. Results indicate the latter is slightly better. GA have also been successfully incorporated within a multi-objective framework (Atahran et al., 2014) and a hybrid predictive control framework (Núñez et al., 2014; Muñoz-Carpintero et al., 2015).

5.2.7. Hybrid algorithms

Combining metaheuristics with other types of metaheuristics, mathematical programming, etc. is a growing trend. Many of the state-of-the-art algorithms for solving combinatorial optimization problems are indeed hybrid algorithms (Talbi, 2002; Jourdan et al., 2009). In the literature, we find recent studies of hybrids of metaheuristics (Parragh et al., 2009; Chevrier et al., 2012; Santos and Xavier, 2015; Zhang et al., 2015; Chassaing et al., 2016; Masmoudi et al., 2016; 2017; Molenbruch et al., 2017c; Pimenta et al., 2017; Lim et al., 2017; Schönberger, 2017), hybrids of metaheuristics with mathematical programming approaches (Parragh et al., 2012; Parragh and Schmid, 2013; Gschwind and Drexl, 2016; Ritzinger et al., 2016), and hybrids of metaheuristics with constraint programming (Berbeglia et al., 2012a).

Hybrids of metaheuristics. Hybrids of metaheuristics usually come in two forms: (1) each metaheuristic is executed sequentially, and (2) a metaheuristic is executed within another metaheuristic. Parragh et al. (2009), Santos and Xavier (2015), and Molenbruch et al. (2017c) are examples of (1) where path relinking is applied after a VNS, GRASP, and a multi-directional local search algorithm, respectively. Elite solutions are collected from the respective searches, and the idea is to get even better solutions by exploring the trajectories between the elite solutions.

A popular way to integrate metaheuristics is to embed a single-solution based metaheuristic (e.g., local search, SA, TS, VNS, LNS) into a population-based metaheuristic (e.g., GA, bee algorithms) (Chassaing et al., 2016; Chevrier et al., 2012; Zhang et al., 2015; Masmoudi et al., 2016; 2017; Schönberger, 2017) due to the population-based metaheuristic's ability for exploration and the single-solution based metaheuristic's ability for exploitation. In these publications, GA is the most widely used population-based method while local search is the most used single-solution based method. For example, Masmoudi et al. (2017) presented a hybrid GA algorithm that employed two crossover operators and four mutation operators. The local search consists of five well-known operators from the routing literature and is applied to the newly generated offsprings. This algorithm is highly effective as it is currently the best method for solving the heterogeneous DARP. Two uncommon examples follow. Chassaing et al. (2016) presented an evolutionary local search with elements from both evolutionary algorithms and local search. At each iteration, several individuals are created by mutation and each individual is improved by local search. The best of these individuals is used to restart the search. This method is efficient and effective. Two hybrid bee algorithms are presented by Masmoudi et al. (2016). Unlike the previous algorithms, a DA (Braekers et al., 2014) or SA is embedded within a bee algorithm. These methods provide competitive results to the heterogeneous DARP with multiple depots.

Another way to make hybrids is to replace the local search within a metaheuristic with a ruin-and-recreate method in GRASP (Pimenta et al., 2017) or with a variable neighborhood descent in iterated local search (Lim et al., 2017). The latter has shown to be effective as it also provides competitive results for the vehicle routing problem with multi trips.

Hybrids of metaheuristics with mathematical programming approaches. A popular way to combine a metaheuristic with a mathematical based approach is to embed a metaheuristic into a mathematical based approach or vice versa. Parragh et al. (2012) and Parragh and Schmid (2013) presented two hybrid column-generation approaches where the pricing of columns is carried out by VNS. To further improve the solution, Parragh and Schmid (2013) also applied LNS to a feasible solution that is obtained by solving a restricted set-covering problem, and the routes generated are transformed into columns and added to the master problem. This hybrid method is effective as a few new best solutions are identified using much less computing time.

Ritzinger et al. (2016) and Gschwind and Drexl (2016) presented a different approach where dynamic programming (DP) is used as building blocks within a LNS. Like Parragh and Schmid (2013), they also used some (or all) operators from Ropke and Pisinger (2006). While Ritzinger et al. (2016) presented some new removal/insertion operators based on DP, Gschwind and Drexl (2016) presented an operator based on DP for intensification purposes and is currently the best method for solving the standard DARP.

Hybrids of metaheuristics with constraint programming. Berbeglia et al. (2012a) presented a hybrid algorithm of tabu search and constraint programming (CP) for a dynamic DARP. They adapted Cordeau and Laporte (2003) tabu search and used the CP algorithm by Berbeglia et al. (2011). CP is a programming paradigm based on reasoning and search techniques. TS is used for scheduling new requests and for improving the current feasible solution while CP is used to determine whether the addition of a new request will result in feasibility or not. Both TS and CP are run in parallel when a new request is received. The request is accepted if either TS or CP finds a feasible solution. This hybrid method outperforms both TS and CP on its own.

5.2.8. Lessons learned

Every heuristic exhibits elements from both diversification (e.g., multiple neighborhood operators, perturbation, temporarily accepting infeasible solutions, randomness, and penalizing frequently made moves) and intensification (e.g., neighborhood search, restarting the search from the best solution, post-optimization). The diversification and intensification obtained depends on the metaheuristic framework used. Clearly, single-solution based methods focus more on intensification while population-based methods focus more on diversification. Hence, it is important to balance the diversification and intensification search strategies. Here, we list a few common search strategies in the DARP literature where authors have used to compensate for the weaknesses in their chosen metaheuristic framework.

While employing *multiple neighborhood operators* is the main feature of VNS and ALNS, other researchers have also utilized this in their SA, GA, and hybrid algorithms (e.g., Chassaing et al., 2016; Zhang et al., 2015; Masmoudi et al., 2016; Masmoudi et al., 2017; Ritzinger et al., 2016; Lim et al., 2017; Molenbruch et al., 2017c) in order to explore different regions of the solution space. With the rise of using multiple neighborhood operators and ALNS, more emphasis is also put on more *complex neighborhood operators* that make big changes to a solution rather than only using simple moves. The VNS, (A)LNS, DA, and hybrid algorithms reviewed in this paper employ several operators that make big moves, while the TS and SA algorithms utilize more simple operators. Another trait of all reviewed algorithms except TS algorithms is the use of *randomness*. Random numbers are used for different purposes, for example, (1) the shaking phase of VNS; (2) the Metropolis condition (typically applied in SA, VNS, and (A)LNS algorithms); (3) the choice of removal and insertion operators in (A)LNS; (4) the order the neighborhood operators (e.g., Braekers et al., 2014; Chassaing et al., 2016; Lim et al., 2017; Masmoudi et al., 2016); and (5) mutation and crossover operations in GA (e.g., Jorgensen et al., 2007; Masmoudi et al., 2017; Zhang et al., 2015). A common strategy used in numerous algorithms is to *temporarily accept infeasible solutions* by adding a penalty term to the objective function. With this strategy, the search switches back and forth between two solution spaces to conduct a wider search. Most of the reviewed TS and VNS algorithms as well as Braekers and Kovacs (2016), Chassaing et al. (2016), and Masmoudi et al. (2016) use this strategy.

The above strategies are widely used because it is easy to implement them and their effects can be seen relatively fast under the development of the methods. In the following, we present a few search strategies that are only common for certain solution methods, but are rarely seen in the reviewed DARP literature.

Discouraging frequently made moves is a long-term diversification strategy introduced in connection with tabu search. This is however only used in the TS algorithms. Another uncommon strategy is the *addition of a noise term* to the objective function, whose purpose is to make the neighborhood search less myopic. Only a few algorithms applied this strategy (see Lehuédé et al., 2014; Braekers and Kovacs, 2016; Gschwind and Drexl, 2016; Lim et al., 2017). A simple intensification strategy is to *restart the search with elite solutions*. However, this is one of the neglected search strategies (Glover and Laguna, 1997) and is only applied by Braekers et al. (2014) and Chassaing et al. (2016).

5.3. Other methods

5.3.1. Algorithms for feasible solutions

The determination of the solution feasibility of a DARP is an important issue in real situations. Such determination saves time for schedule planners from spending several hours for finding solutions that do not exist (in a static setting), or helps to reject user requests in a much faster fashion if the acceptance of such request results in problem infeasibility (in a dynamic setting). In this line of research, the solution quality, in terms of objective value, is not a concern. The most important aspects are the computational speed and the worst case complexity. A few existing studies focus on developing algorithms to obtain feasible solutions, in terms of route or schedule. For example, to determine a feasible route, Berbeglia et al. (2011) developed a constraint-programming algorithm whereas Häme and Hakula (2013) introduced a modified version of hyperlink-induced topic search. Häme and Hakula (2015) developed a maximum cluster algorithm based upon dynamic programming for this purpose. Meanwhile, to determine whether a feasible schedule for a given route exists, Tang et al. (2010) proposed a revised algorithm in an $\mathcal{O}(n^2)$ worst-case time, where n is the number of users' requests, to correct the flaw of the three-pass algorithm proposed by Hunsaker and Savelsbergh (2002). Haugland and Ho (2010) introduced a correct algorithm for the same purpose in linearithmic time. Firat and Woeginger (2011) analyzed a more general setting compared with Hunsaker and Savelsbergh (2002) and presented a simple linear time algorithm. While these studies do not consider solution quality, but only feasibility, they can play an important role in optimizing DARPs. For example, the feasible solutions generated can provide bounds for optimal values in exact methods and construct the solution pool in heuristics and metaheuristics. In particular, Gschwind and Drexl (2016) proposed a constant-time feasibility test for the DARP and adopted the test for request insertions in an adaptive large neighborhood search. They demonstrated that their approach outperforms other state-of-the-art DARP heuristics. A shortcoming of this line of research is that the algorithms developed may not be applicable to DARPs with problem-specific features (e.g., manpower requirements) since the solution feasibility depends on the set of constraints. Thus, additional efforts may be needed when moving from one application to another.

Table 3
Benchmark instances.

Problem type[a]	First reference	Number of instances	Size[b]	Link
DARP	Cordeau and Laporte (2003)	20	3–13/24–144	http://neumann.hec.ca/chairedistributique/data/darp/tabu/
DARP	Cordeau (2006)	24	2–4/16–48	http://neumann.hec.ca/chairedistributique/data/darp/branch-and-cut/
DARP	Ropke et al. (2007)	24	5–8/40–96	http://neumann.hec.ca/chairedistributique/data/darp/
HDARP	Parragh (2011)	24[c]	2–4/16–48	http://prolog.univie.ac.at/research/DARP/
HDARP	Braekers et al. (2014)	24[c]	5–8/40–96	http://alpha.uhasselt.be/kris.braekers/
MD-HDARP	Braekers et al. (2014)	72	2–8/16–96	http://alpha.uhasselt.be/kris.braekers/
R-DARP	Liu et al. (2015)	42	2–2/16–23	http://www.computational-logistics.org/orlib/topic/R-DARP/
DARPSRP	Parragh et al. (2015)	75	2–6/8–72	http://prolog.univie.ac.at/research/DARP/
MTDARP	Zhang et al. (2015)	80	2–11/29–185	http://www.computational-logistics.org/orlib/topic/MTDARP/
DC-HDARP	Braekers and Kovacs (2016)	1296	2–186/16–888[d]	http://alpha.uhasselt.be/kris.braekers/
MTPDPTW-MP	Lim et al. (2017)	365	3–17/26–214	http://www.computational-logistics.org/orlib/topic/MTPDPTWMP/

[a] HDARP: Heterogeneous DARP; MD-HDARP: HDARP with multiple depots; R-DARP: realistic DARP; DARPSRP: DARP with split requests and profits; MTDARP: multi-trip DARP; DC-HDARP: HDARP with driver consistency; MTPDPTW-MP: multitrip pickup and delivery problem with time windows and manpower planning.
[b] min-max number of vehicles/requests.
[c] Data sets E and I.
[d] Number of requests over the multi-period.

5.3.2. Approximation algorithms and approximate reasoning

Because static and deterministic DARPs have all the required information for decisions being given, researchers could derive some nice theoretical results about their computational complexities from this aspect and then developed approximated algorithms from them. For example, Gupta et al. (2010) studied the approximability of the DARP and proved that, when there are k vehicles, if there exists an α-approximation algorithm for the k-forest problem (whose goal is to find a minimum cost subgraph that connects at least k node pairs), then there is an $\mathcal{O}(\alpha \log^2 n)$-approximation algorithm for the DARP. They then provided an $\mathcal{O}(\min\{\sqrt{n}, \sqrt{k}\} \log^2 n)$-approximation algorithm for the DARP. Their results are comparable with those provided by Charikar and Raghavachari (1998) and are even better when the vehicle capacity is larger. The development of approximation algorithms provides bounds for the solution quality and the ideas of conditions under which the algorithms are effective. However, for the sake of theoretical results, this line of research typically considers the basic DARP where the features of variants are not taken into account and the objective function is fixed. When additional features or objectives are considered, the approximability may not be guaranteed. Furthermore, since these algorithms consider the worst-case scenarios, exact or heuristic approaches may have better average performance for general instances.

To the best of our knowledge, only a minority of studies focused on developing approximate reasoning approaches to obtain solutions to DARPs. For example, Maalouf et al. (2014) developed a fuzzy logic algorithm to solve a dynamic capacitated DARP with multiple vehicles, impression on customers' request, and uncertainty in travel time. Their approach also takes advantage of the flexibility of fuzzy rules to be incorporated into the DARP. While there is lack of the theoretical development of approximate reasoning approaches to solving DARPs, they can have more practical applications as many realistic factors can be captured. Due to the inadequacy of research in this area, there is much room to explore how approximate reasoning approaches can be further applied to DARPs where uncertainty or imperfect information arises in the system.

5.4. Benchmark instances

Researchers have used both artificially-generated and real-life data to evaluate the quality of the developed algorithms. Some of the artificially-generated data are publicly accessible on the Internet. These are listed in Table 3 by problem type and in chronological order. The instances by Ropke et al. (2007) are an extension of the instances by Cordeau (2006). These two sets divide the instances into two categories: *a* (small vehicle capacities) and *b* (large vehicle capacities).

Only benchmark instances solved by at least two algorithms are compared with each other in Table 4. Hence, a comparison of the last five sets of benchmark instances is not included in the table. *Gap* denotes the average deviation (in %) from the best known solution (see Tables 10–13), *Gap** denotes the minimal deviation (in %) from the best known solution, and *CPU* denotes the average computing times (in minutes) for running the respective algorithms. The algorithms are run on different machines using different programming languages, compilers, parameter settings, etc., and hence it is not possible to directly compare CPU times (and hence efficiency) of the algorithms. The average results were obtained by running the algorithms five times on each instance, except for the one by Ritzinger et al. (2016) and the one by Molenbruch et al. (2017a) whose results were obtained with ten and twenty runs, respectively. Currently, the hybrid ALNS by Gschwind and Drexl (2016) is the most efficient algorithm for solving the DARP instances, with Masmoudi et al. (2017) hybrid GA as the first runner-up and Gschwind and Drexl (2016) ALNS as the second runner-up. However, the hybrid GA is not tailor-made for solving the basic DARP, but rather for solving a more complex DARP. The hybrid GA by Masmoudi et al. (2017) is currently the most efficient and effective algorithm for solving the heterogeneous DARP, while Masmoudi et al. (2016) hybrid bee algorithms obtained the best results for the heterogeneous DARP with mul-

Table 4
A comparison of recent algorithms on the benchmark instances.

	DARP[a]			DARP[b]			DARP[c]			HDARP[d]			HDARP[e]			MD-HDARP[e]		
	Gap	Gap*	CPU	Gap	Gap*	CPU	Gap	Gap*	CPU	Gap	Gap*	CPU	Gap	Gap*	CPU	Gap	Gap*	CPU
VNS (Parragh et al., 2010)	1.77	1.00	133.30	–	–	–	–	–	–	–	–	–	–	–	–	-	-	-
VNS (Parragh, 2011)	–	–	–	0.34[f]	0.15[f]	1.86[f]	–	–	–	0.37	0.18	1.85	–	–	–	-	-	-
CG+LNS (Parragh and Schmid, 2013)	1.77	0.82	21.84	0.17	0.06	2.26	0.12	0.06	2.25	–	–	–	–	–	–	-	-	-
MS-ALNS (Qu and Bard, 2013)	–	–	–	–	–	–	–	–	–	0.25[f]	0.11[f]	2.52[f]	–	–	–	-	-	-
DA (Braekers et al., 2014)	1.16	0.81	1.39	0.01	0.00	0.52	0.01	0.00	0.51	0.01	0.00	0.28	0.21[g]	0.17[g]	0.68[g]	0.21	0.14	0.46
ALNS (Masson et al., 2014)	1.34	0.72	40.12	–	–	–	–	–	–	–	–	–	–	–	–	-	-	-
ELS (Chassaing et al., 2016)	1.04	0.64	9.87	0.07	0.00	1.02	0.05	0.02	1.02	–	–	–	–	–	–	-	-	-
ALNS[h] (Gschwind and Drexl, 2016)	0.88	0.50	0.91	0.28	0.15	0.28	0.21	0.11	0.27	–	–	–	–	–	–	-	-	-
H-ALNS[i] (Gschwind and Drexl, 2016)	0.50	0.32	3.49	0.05	0.01	0.63	0.05	0.01	0.65	–	–	–	–	–	–	-	-	-
LNS+DP (Ritzinger et al., 2016)	3.64	2.35	66.72	0.21	0.10	5.01	0.30	0.17	4.51	–	–	–	–	–	–	-	-	-
ALNS (Masmoudi et al., 2016)	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.15	0.07	2.92
BA-DA (Masmoudi et al., 2016)	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.07	0.00	6.19
BA-SA (Masmoudi et al., 2016)	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.09	0.03	6.83
GA+LS (Masmoudi et al., 2017)	0.65	0.37	1.77	0.01	0.00	0.74	–	–	–	0.01	0.00	0.43	0.07	0.01	1.06	-	-	-
LNS (Molenbruch et al., 2017a)	1.19	0.46	88.44	0.03	0.00	6.23	–	–	–	–	–	–	–	–	–	0.15[j]	0.06[j]	5.92[j]

[a] Cordeau and Laporte (2003).
[b] Cordeau (2006) and Ropke et al. (2007): Category *a*.
[c] Cordeau (2006) and Ropke et al. (2007): Category *b*.
[d] Parragh (2011).
[e] Braekers et al. (2014).
[f] They only solved 12 of the instances.
[g] The results are available from http://hdarp-results.e-monsite.com.
[h] Number of ALNS iterations: 50,000.
[i] Number of ALNS iterations: 75,000. Also used two improvement procedures.
[j] They only solved 24 of the instances.

tiple depots and trips. Tables 3, 4, 10–13 are also available at https://sites.google.com/site/darpsurvey, and these tables will be updated when necessary.

6. Future challenges and opportunities

The previous sections have given a synopsis of the research on DARPs since 2007. There have been substantial advances in the research on DARPs – in the richness of the problem and model variations investigated, in the development of new solution methods (especially meta-heuristics), and in the breadth of applications. In this final section, we indicate some promising areas for future research on DARPs. We begin by discussing some technological trends that might impact how DAR systems operate, leading to new research directions.

6.1. New technologies and trends

6.1.1. Developments in information and communications technologies

The explosion of Big Data and Telemetrics has definitely changed the landscape of DAR systems and DARP research. The technical advances in remote sensing and communications channels have made the quantity and quality of information available substantially different from a decade ago. DAR systems have traditionally depended on centralized planning requiring advanced booking of trips; yet often the planned schedule must be adjusted "on-the-fly" due to unexpected delays and other events. In the era of Big Data, travel times can be estimated much more accurately by, for example, machine learning and deep learning approaches and updated dynamically; faster algorithms and more powerful hardware will allow "real-time" planning and dispatching. This may strengthen the need for dynamic and deterministic DARP models, especially when the impacts of the sources of other uncertainties are small and negligible. This trend may shift research on solution methodologies to focus more on fast on-line algorithms for dynamic and deterministic models to facilitate "real-time" re-optimization. When optimization is done periodically using "snapshot" data, deterministic models may suffice. This may also raise the need for algorithms that not just minimize costs but minimize deviation from a "pre-set" schedule when new requests are incorporated. Meanwhile, large volumes of different types of historical data allow researchers to consider and capture the critical sources of stochasticity in the systems during real-time planning and dispatching, enabling the development of better dynamic and stochastic DARP models and algorithms for practical applications. On the other hand, the advancement of information and communications technologies can impose a tremendous computational burden on the delivery of solutions due to several reasons: the growing number of users across diverse DAR platforms, the very frequent re-optimization necessitated by continuous information updates from multiple sources, and the high granularity of spatio-temporal stochastic modeling.

The keys to bridging the research and the practice include effective modeling (an optimal selection of the most suitable components to capture the problem characteristics) and the determination of the events that should trigger re-optimization. The focus for stochastic DARP research may be directed towards incorporating more realistic aspects into the models and towards the use of stochastic models for performance evaluation and for longer-term manpower planning purposes.

6.1.2. The sharing economy and changing travel patterns

Millenials own fewer cars than previous generations. As evidenced by the meteoric rise of Uber, Lyft, and similar service platforms, and on-demand mobility (especially coupled with ease of payment), DAR systems will be much more prevalent in the future. However, the modus operandi might be very different than before. The DAR vehicle fleet need not be centrally owned, and individual drivers can bid to take on the trip assignments (Uber and some taxi services operate in this way). This distributed and game-theoretic mode of operation opens up exciting new avenues for DARP research.

Travel patterns may also change as more people (especially non-car-owners) rely on public transit for commuting. As indicated in Section 1.2, commuting trips in the future may be of mixed-mode consisting of travel on different transport modes (see Häll et al., 2009; Posada et al., 2017). DAR systems may serve as feeders to long-haul public transit (e.g., trains). The co-ordination of the schedule and capacity between scheduled public transit and DAR feeders is a research topic that has not been much explored. An emerging trend is also for commuters to bring their personal mobility devices (PMDs) – e-bikes, Segways, e-scooters – onto public transit. Since December 2016, Singapore commuters can carry foldable bicycles and other PMDs onto the public transit system (Today, 2017). This may change the model of the DAR feeder systems, since the trip may not be point-to-point but zone-to-zone, because the commuter has the flexibility to travel to meet up with the DAR vehicle. Such zone-to-zone DARP model may be a new direction of research.

6.1.3. Green transport and autonomous vehicles

For large cities, switching from individualized transport (e.g., cars) to public mass transit is better for the environment due to higher energy efficiency (person-km per kW) and reduced congestion. Thus, concern for the environment is also an impetus for the trend of integrating DAR systems with public transport, even for elderly (but relatively mobile) passengers. As noted above, this trend leads to new research directions for the DARP where synchronization of schedules and capacities between the public transit and the DAR system becomes important. Districting and location of transfer points are also issues of concern.

With the global concern for the environment, the world of transport is under pressure to develop greener technologies and energy-efficient operations. 2015 was a significant year; the number of electric vehicles in use in the world exceeds one million in that year. The use of electric vehicles in transport fleets (including DAR systems) requires new models and algorithms. With the limited range of electric vehicles, visits to charging stations (for battery swapping or charging) must be included in the route planning and scheduling. For DAR systems, there is the added concern that passengers would prefer their trips not to be interrupted by stops at or detours to charging stations.

Although the technology for autonomous vehicles is still in the experimental stage, many expect that the use of driverless cars will be prevalent in the not too distant future. New technologies will bring new ways of operating. Transport systems with electric and/or autonomous vehicles may be zone-based for the ease of control (for learning and adaption for the vehicles). This brings the new problem of districting and assignment of vehicles to zones. The use of a mixed fleet (using traditional petrol-based, electric and hybrid propulsion, manually and/or autonomously controlled) will require developments in integrated models for DARPs.

Autonomous vehicles may bring increased mobility to the elderly or the disabled, thus reducing the need for the many DAR systems currently in use for elderly/disabled transport. On the other hand, might the Internet of Things develop to a stage where autonomous vehicles can “self-organize” to provide “car-pooling” for their users? That would certainly be exciting and opens up entirely new areas for DARP research. There may also be more research interests in developing multi-objective models that include environmental concerns in addition to mobility as goals.

6.2. Research gaps and opportunities

In this section, we discuss possible extensions and new directions for DARP research.

6.2.1. Models

Analysis of mixed-mode operations under a stochastic and dynamic setting.. As indicated, many DAR systems have evolved from the canonical modus operandi. There is a growing interest in mixed-mode operations, with DAR systems acting as feeders to regularly-scheduled public transit. For example, a DAR vehicle may pick up several passengers from diverse remote locations and bring them to a transit point where the passengers continue their journey on regularly-scheduled public transit, and finally complete the last part of their journey on a shared-ride DAR vehicle.

A major challenge of the mixed mode operations is the synchronization of DAR systems with public transit. The existing research on mixed-mode operations typically considers a static and deterministic environment. However, when travel times are stochastic, a user may be left behind at the transfer point, for example, due to the delay in drop-off time. This will be a more common and important issue for an integrated DAR service with the use of public transit that has infrequent service. The schedule planner may need to work out a more robust plan, for example, by reserving sufficient waiting times at the transfer points. In case of transfers not being realized as planned, it is essential to recover the plan by deploying additional vehicles or making adjustments in the plans of other vehicles. This will be a more practical and interesting direction to investigate how decisions are made in a stochastic environment and adjusted dynamically for DARPs with passenger transfers.

Another way of mixing is the so-called “share-a-ride” problem, where passenger transport is combined with the pickup and delivery of goods in the same vehicle. There may be a difference in priority and urgency in servicing passenger rather than goods delivery trips. Therefore, new models will be needed to investigate such integrated systems.

Stochastic and dynamic modeling for disruption management. As every transport operator knows, things never go according to plan. For DAR systems, as for other transport systems, disruption management is essential. In Section 4.4, we observe that most DARPs consider only new user requests under a dynamic and deterministic environment; these problems rarely consider other types of events (e.g., long delays in vehicle arrival times and accidents) that require modifications of existing plans or affect the synchronization of vehicles. New modeling techniques and frameworks should be proposed to capture these factors in disruption management.

Consideration of staff rostering. We observe that there has been a growing number of papers which consider manpower requirements in their DARPs, as suggested in Section 4.2.3. While these applications assumed that the staff work shift is given, an integrated DARP that simultaneously optimizes both staff rostering decisions across multiple days and routing decisions within the day can potentially lead to a more efficient roster, reduced cost, and enhanced service quality, in particular for problems with high variations in daily user demands. The determination of the joint decisions will require the consideration of a more complicated and larger integrated problem. Therefore, research on efficient modeling for problem size reduction is needed.

Modeling of worst-case scenarios and multiple sources of stochasticity under a static environment. As indicated by the small number of papers in the category of static and stochastic DARPs in Section 4.3, there is much room for research in this direction. The existing research on static and stochastic DARPs was developed heavily based on the assumptions of certain probability distributions or very specific problem settings. In particular, only one dimension of stochasticity was considered in the three papers reviewed in this category; all considered stochastic arrivals only. Another type of stochasticity that

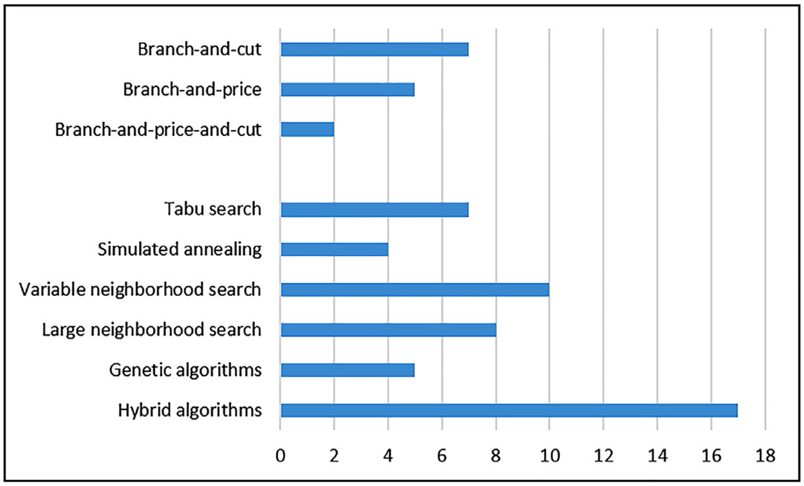

Fig. 2. Number of publications per solution method.

commonly arises in practice is the uncertain travel times, not to mention the added challenge due to the interaction of the multiple dimensions of stochasticity – the number of user requests, the delays in the user arrival times at the pick-up locations and the travel times. Moreover, when most models use an expected value as the objective, some realized outcomes can be unacceptably bad. The exploration of robust models that capture worse-case scenarios and multiple sources of stochasticity using general probability distributions is, therefore, an important research direction.

Modeling the competition between multiple DAR service providers. As mentioned in Section 4.2.4, multiple DAR service providers can exist in a single operating area in reality. Instead of cooperation, they can compete with each other, by improving the level of service provided or reducing the fares, to attract more customers. This realistic situation has not been modeled and analyzed in the existing DARP studies, leading to one research gap for future research.

6.2.2. Algorithms

Theoretical investigation into the strengths of different classes of valid inequalities.. As discussed in Section 5.1.1, there is a lack of theoretical investigation into the strength of the classes of valid inequalities for DARPs, in particular, the conditions whereby they are facet-defining. This line of research can help design better separation procedures in a B&C algorithm and derive strengthened inequalities.

Effectiveness of various branching schemes in B&B-based algorithms. It appears that there is inadequate research on the performances of branching schemes in a B&B-based algorithm, as suggested in Section 5.1.5. Most existing results on branching schemes focused on the problem-specific applications. More comprehensive computational experiments may need to be conducted for the examination of the effectiveness of these schemes with different DARP features.

Faster algorithms for solving subproblems. As discussed in Sections 5.1.2 and 5.1.3, B&P and B&P&C are two of the most popular exact algorithms to solve DARPs. The essence of B&P and B&P&C algorithms is the reformulation of the overall problem into a master and subproblems. Their performance highly depends on the effectiveness of this decomposition. In particular, the majority of the overall runtime is observed to be spent on solving the subproblems (Qu and Bard, 2015). Research on new decomposition methods and efficient algorithms to solving the subproblems will enhance the computational performance of these exact algorithms.

Integration of decomposition techniques into multi-objective exact algorithms. As mentioned in Section 5.1, the exact algorithms reviewed are for single objective DARPs. However, multi-objective DARPs are considered in the literature as shown in Tables 5–8 in the Appendix but were only solved by meta-heuristics. A future research direction is, therefore, to integrate decomposition techniques into multi-objective exact algorithms, to provide a benchmark for the results obtained by the multi-objective heuristics.

New metaheuristics and hybrids of exact methods and heuristics. It is no doubt that using meta-heuristics to solve the DARP and its variants is a popular choice among the research community. This is because models are becoming more complex and rich due to the need to capture more and more realistic operational features and the increasing number of user requests. There is also a need to react fast as users expect to get an answer/solution relatively quickly. Fig. 2 shows the number of publications per method published since 2007. Please note that a publication might belong to more than one category if the publication describes more than one solution method. We observe from the figure that using hybrid methodologies to solve DARPs is the most popular in the survey period. According to the survey by Cordeau and Laporte (2007), there were no publications on using hybrid algorithms to solve DARPs, implying that using hybrid algorithms to solve DARPs is becoming increasingly popular. We also see from the figure that variable neighborhood search comes in second. However, seven out of the ten publications originate from the same research group. Moreover, no new meta-heuristics (e.g., artificial bee colony methods, chemical reaction optimization, etc.) have been used to solve DARPs according to the figure. Certainly, we expect the body of research of applying meta-heuristics to the DARP and its variants to continue to grow, particularly in the exploration of new meta-heuristics and in hybrid methods combining exact and heuristic approaches to deal with the specific features of DARPs.

The utilization of more simple neighborhood moves. As mentioned in Sections 5.2.5 and 5.2.7, the hybrid ALNS algorithm by Gschwind and Drexl (2016) is currently the best algorithm for solving the standard DARP. This outstanding performance is due to the two new improvement components. However, these additional components rely on dynamic-programming and solving a set-covering problem. According to Cordeau et al. (2002), a good heuristic should also be simple besides being accurate and fast. This leads to the question: Can these two components be replaced by more simple neighborhood moves and the algorithm still exhibit a similar performance?

New algorithms for feasible solutions to DARP variants. As stated in Section 5.3.1, only a few studies focus on this line of research and solution feasibility depends on the set of constraints. Different DARP variants have their own set of specific constraints that make existing algorithms for feasible solutions cannot be directly used for specific applications. Additional efforts are required to modify these existing algorithms for different DARP applications. A natural and potential future research direction is therefore to develop new algorithms or modifying existing algorithms for determining feasible solutions to DARP variants, especially for those with specific constraints such as manpower requirement and transfers.

Novel approximation algorithms for new DARP variants. As shown in Section 5.3.2, approximation algorithms receive much less attention than heuristics and metaheuristics and focus on solving the basic DARP. More research can be done on this line of research, especially those approximation algorithms for new problem variants with special features.

Approximate reasoning approaches to solving more practical stochastic DARP applications. As reflected in Section 5.3.2, only very little research on approximate reasoning approaches to solving DARPs. A lot more practical stochastic applications can be solved using these approaches in the future. Moreover, these approaches can be further improved to suit specific applications and compared with algorithms for stochastic DARPs.

Development of more algorithms using stochastic information. One interesting finding in our survey of the literature is that relatively few papers – only seven by our count – have proposed solution methods to stochastic and/or dynamic DARPs in which stochastic information is utilized by the decision maker. Two such studies – by Ho and Haugland (2011) and Heilporn et al. (2011) – considered static and stochastic DARPs. Two studies – by Schilde et al. (2011, 2014) – considered dynamic and stochastic DARPs. Interestingly, we also found three studies – by Hyytiä et al. (2012), Sayarshad and Chow (2015), and Muñoz-Carpintero et al. (2015) – that considered dynamic and deterministic DARPs in which the proposed solution methods utilized stochastic information concerning the appearance of future users. With more auto-sensed data available, more detailed information will be available on the stochasticity of travel times and other aspects of the DARP. Thus, we encourage future researchers to develop additional DARP solution methods that utilize stochastic information.

Effective and efficient solution methods for disruption management under a stochastic and dynamic environment. To support effective disruption management, new research is needed to develop robust solution methods for stochastic and dynamic models for DARPs. Stochastic optimization algorithms can be developed to mitigate the risk of service disruptions. Fast recourse methods are needed for real-time recovery after accidents and service interruptions. Hybrid methods that combine exact and heuristic approaches may be particularly pertinent.

Unified methods for solving different DARP variants. Each DAR system may have problem-specific constraints due to its underlying motivating application. DARP algorithms may need to be adaptable to different problem variants. For example, modeling the DARP problem with transfers of mental health patients requires precedence constraints related to transfers and solving this problem requires a procedure for checking the feasibility of routes and heuristic operators dedicated to transfers (Masson et al., 2014). It would be interesting to explore unified DARP algorithms that work well on a variety of DARP models (and possibly other routing problems as well). Thus far, the majority of research on the DARP has been on static and deterministic models as well as their methods, it would be very interesting to explore extensions of these models and methods to the dynamic and/or stochastic settings.

Development of fast on-line algorithms based on parallel computing and distributed bidding. With advances in communications technologies, DAR system will evolve from being based on advanced reservation to being based on “real-time” dispatch. Thus, fast on-line algorithms are needed to be able to insert new trips into routes that are already on-going. To achieve the computational speeds necessary, parallel computation (perhaps using meta-heuristics) is a promising research direction. Further, as discussed above, for future DAR systems operating in a distributed bidding mode, on-line competitive algorithms will be an exciting new research area.

6.3. Conclusions

In this paper, we summarized the research on the DARP since 2007. We provided a taxonomy of the problem variants studied and the algorithms developed. We also described the diverse areas of applications for the DARP. Finally, with the recent emergence of new technologies, exciting new avenues of research for the DARP are opening up, and we look forward to significant advances in DARP research in meeting the challenges in the years to come.

Acknowledgments

This work was partially supported by two grants from the Research Grants Council of Hong Kong (414313 and 17201217). This support is gratefully acknowledged. The authors are also grateful to the two reviewers for their constructive comments.

Appendix A. Problem overview

Table 5
Static and deterministic DARPs.

Reference	Single (S)/Multi (M) depots	Single (S)/Multi (M) trips	Single (S)/Multi (M) vehicles	Homogeneous (HO)/ Heterogeneous (HE) fleet	Vehicle capacity	Time windows	Ride time	Route duration	Selective visits	Single (S)/Multi (M) objectives
Jorgensen et al. (2007)	S	S	M	HO	✓	✓	✓	✓		M
Luo and Schonfeld (2007)	S	S	M	HO	✓	✓	✓			S
Melachrinoudis et al. (2007)	M	S	M	HE	✓	✓				M
Ropke et al. (2007)	S	S	M	HO	✓	✓	✓	✓		S
Wolfler Calvo and Colorni (2007)	S	S	M	HO	✓	✓			✓	S
Cubillos et al. (2009)	S	S	M	HO	✓	✓				M
Häll et al. (2009)	S	S	M	HO	✓	✓	✓	✓		S
Karabuk (2009)	M	S	M	HE	✓	✓	✓	✓		S
Mauri et al. (2009)	M	S	M	HE	✓	✓	✓	✓		M
Parragh et al. (2009)	S	S	M	HO	✓	✓	✓	✓		M
Cortés et al. (2010)	M	S	M	HO	✓	✓				S
Garaix et al. (2010)	M	S	M	HE	✓	✓	✓			M
Gupta et al. (2010)	S	S	S	HO	✓					S
Haugland and Ho (2010)	S	S	S	HO	✓	✓	✓			S
Parragh et al. (2010)	S	S	M	HO	✓	✓	✓	✓		S
Tang et al. (2010)	S	S	S	HO	✓	✓	✓			S
Berbeglia et al. (2011)	S	S	M	HO	✓	✓	✓			S
Firat and Woeginger (2011)	S	S	S	HO	✓	✓	✓			S
Garaix et al. (2011)	M	S	M	HE	✓	✓	✓			S
Häme (2011)	S	S	S	HO	✓	✓	✓	✓		M
Parragh (2011)	S	S	M	HE	✓	✓	✓	✓		S
Berbeglia et al. (2012b)	M	S	M	HO	✓		✓			S
Chevrier et al. (2012)	S	S	M	HE	✓	✓	✓	✓		M
D'Souza et al. (2012)	S	S	S	HO				✓		M
Parragh et al. (2012)	S	S	M	HE	✓	✓	✓	✓		S
Zidi et al. (2012)	S	S	M	HE	✓	✓				M
Carnes et al. (2013)	M	S	M	HE	✓	✓	✓	✓		S
Guerriero et al. (2013)	S	S	M	HE	✓	✓	✓	✓		S
Häme and Hakula (2013)	S	S	M	HO	✓	✓	✓	✓		S
Kirchler and Wolfler Calvo (2013)	S	S	M	HO	✓	✓	✓	✓	✓	M

(*continued on next page*)

Table 5 (*continued*)

Reference	Single (S)/Multi (M) depots	Single (S)/Multi (M) trips	Single (S)/Multi (M) vehicles	Homogeneous (HO)/ Heterogeneous (HE) fleet	Vehicle capacity	Time windows	Ride time	Route duration	Selective visits	Single (S)/Multi (M) objectives
Muelas et al. (2013)	S	S	M	HO	✓	✓	✓			S
Paquette et al. (2013)	S	S	M	HE	✓	✓	✓	✓		M
Parragh and Schmid (2013)	S	S	M	HO	✓	✓	✓	✓		S
Qu and Bard (2013)	S	S	M	HE	✓	✓				S
Reinhardt et al. (2013)	M	M	M	HE	✓	✓		✓	✓	M
Shen and Quadrifoglio (2013)	M	S	M	HE	✓	✓	✓			S
Atahran et al. (2014)	S	S	M	HE	✓	✓	✓			M
Braekers et al. (2014)	M	S	M	HE	✓	✓	✓	✓		S
Feng et al. (2014)	S	M	M	HE	✓	✓	✓	✓		S
Ilani et al. (2014)	M	M	M	HE	✓	✓	✓			S
Masson et al. (2014)	M	S	M	HO	✓	✓	✓	✓		S
Lehuédé et al. (2014)	M	S	M	HO	✓	✓	✓	✓		M
Gschwind and Irnich (2015)	S	S	M	HO	✓	✓	✓	✓		S
Hu and Chang (2015)	S	S	M	HO	✓	✓	✓			S
Liu et al. (2015)	S	M	M	HE	✓	✓	✓	✓		S
Muelas et al. (2015)	S	S	M	HO	✓	✓	✓			S
Parragh et al. (2015)	S	M	M	HO	✓	✓	✓	✓	✓	S
Qu and Bard (2015)	S	S	M	HE	✓	✓	✓	✓		M
Urra et al. (2015)	S	S	M	HO	✓	✓	✓	✓		M
Zhang et al. (2015)	S	M	M	HO	✓	✓	✓	✓	✓	M
Braekers and Kovacs (2016)	S	S	M	HE	✓	✓	✓	✓		S
Chassaing et al. (2016)	S	S	M	HO	✓	✓	✓	✓		S
Gschwind and Drexl (2016)	S	S	M	HO	✓	✓	✓	✓		S
Masmoudi et al. (2016)	M	M	M	HE	✓	✓	✓	✓		S
Ritzinger et al. (2016)	S	S	M	HO	✓	✓	✓	✓		S
Detti et al. (2017)	M	S	M	HE	✓	✓	✓			M
Lim et al. (2017)	S	M	M	HE	✓	✓	✓	✓	✓	M
Masmoudi et al. (2017)	S	S	M	HE	✓	✓	✓	✓		S
Molenbruch et al. (2017a)	M	S	M	HE	✓	✓	✓	✓		S
Molenbruch et al. (2017b)	S	S	M	HO	✓	✓	✓	✓		S
Molenbruch et al. (2017c)	S	S	M	HO	✓	✓	✓	✓		M
Pimenta et al. (2017)	S	M	M	HO	✓		✓			S
Posada et al. (2017)	S	S	M	HE	✓	✓	✓			S
Schönberger (2017)	M	M	M	HE	✓		✓	✓		S

Table 6
Static and stochastic DARPs.

Reference	Single (S)/Multi (M) depots	Single (S)/Multi (M) trips	Single (S)/Multi (M) vehicles	Homogeneous (HO)/Heterogeneous (HE) fleet	Vehicle capacity	Time windows	Ride time	Route duration	Selective visits	Single (S)/Multi (M) objectives
Hyytiä et al. (2010)	S	S	S	HO	✓				✓	
Heilporn et al. (2011)	S	S	S	HO	✓	✓	✓	✓		S
Ho and Haugland (2011)	S	S	M	HE	✓	✓	✓			S

Table 7
Dynamic and deterministic DARPs.

Reference	Single (S)/Multi (M) depots	Single (S)/Multi (M) trips	Single (S)/Multi (M) vehicles	Homogeneous (HO)/Heterogeneous (HE) fleet	Vehicle capacity	Time windows	Ride time	Route duration	Selective visits	Single (S)/Multi (M) objectives
Quadrifoglio et al. (2008)	M	S	M	HE	✓	✓	✓	✓		M
Hanne et al. (2009)	M	S	M	HE	✓	✓	✓			M
Beaudry et al. (2010)	M	S	M	HE	✓	✓	✓	✓		M
Berbeglia et al. (2012a)	S	S	M	HO	✓	✓	✓	✓	✓	S
Häll and Peterson (2013)	M	S	M	HE	✓	✓	✓			S
Wong et al. (2014)	S	S	M	HO		✓	✓		✓	M
Häll et al. (2015)	M	S	M	HE	✓	✓	✓			M
Häme and Hakula (2015)	S	S	M	HO	✓	✓	✓	✓		S
Marković et al. (2015)	S	S	M	HE	✓	✓	✓	✓		M
Santos and Xavier (2015)	M	S	M	HE	✓	✓	✓	✓	✓	M

Table 8
Dynamic and stochastic DARPs.

Reference	Single (S)/Multi (M) depots	Single (S)/Multi (M)trips	Single (S)/Multi (M) vehicles	Homogeneous (HO)/Heterogeneous (HE) fleet	Vehicle capacity	Time windows	Ride time	Route duration	Selective visits	Single (S)/Multi (M)objectives
Waisanen et al. (2008)	M	S	M	HO						S
Xiang et al. (2008)	S	S	M	HE	✓	✓	✓	✓	✓	S
Schilde et al. (2011)	S	S	M	HO	✓	✓	✓	✓		M
Hyytiä et al. (2012)	S	S	M	HO						M
Maalouf et al. (2014)	S	S	M	HO	✓	✓	✓		✓	M
Núñez et al. (2014)	M	S	M	HO	✓					M
Schilde et al. (2014)	S	S	M	HO	✓	✓	✓	✓		M
Muñoz-Carpintero et al. (2015)	M	S	M	HO	✓					S
Sayarshad and Chow (2015)	S	S	M	HO	✓					S

Appendix B. The largest solved instances by exact methods

Table 9
The largest solved instances by exact methods.

Problem type[a]	Solution methodology & reference	Size of the largest solved instance[b]	CPU (s)
DARP	B&P&C (Gschwind and Irnich, 2015)	8/96	898.8
HDARP	B&C (Braekers et al., 2014)	8/96[c]	NA[d]
MD-HDARP	B&C (Braekers et al., 2014)	8/64[e]	1331
DARPSRP	B&P (Parragh et al., 2015)	4/40; 5/20[e]	5605.18; 312.89
HPDP-CVC	B&P&C (Qu and Bard, 2015)	11/38[e] ; NA[d] /50[e]	885.22; 893.08
DC-HDARP	B&C (Braekers and Kovacs, 2016)	4/40	578[f]
R-DARP	B&C (Liu et al., 2015)	2/22[e]	2094
PDPT	B&C (Cortés et al., 2010)	2/6	119.531
S-DARP	Integer-L-shaped algorithm (Heilporn et al., 2011)	1/26[e]	6913

[a] HDARP: Heterogeneous DARP; MD-HDARP: HDARP with multiple depots; DARPSRP: DARP with split requests and profits; HPDP-CVC: Heterogeneous pickup and delivery problem with configurable vehicle capacity; DC-HDARP: HDARP with driver consistency; R-DARP: Realistic DARP; PDPT: Pickup and delivery problem with transfers; S-DARP: Single-vehicle DARP with stochastic customer delays.
[b] Number of vehicles/requests.
[c] The results are available from http://hdarp-results.e-monsite.com.
[d] Not reported in the paper/website.
[e] There were some smaller instances unable to solve to optimality.
[f] This is the running time for solving one of the 108 instances to optimality.

Appendix C. Benchmark results

Table 10
Best known results for the DARP instances by Cordeau and Laporte (2003).

Instance	BKS	Instance	BKS
R1a	190.02	R1b	164.46
R2a	301.34	R2b	295.66
R3a	532.00	R3b	484.83
R4a	570.25	R4b	529.33
R5a	625.64	R5b	573.56
R6a	783.78	R6b	725.22
R7a	291.71	R7b	248.21
R8a	487.84	R8b	458.73
R9a	653.94	R9b	592.23
R10a	845.47	R10b	783.81

Table 11
Optimal results for the DARP instances by Cordeau (2006) and Ropke et al. (2007).

Instance	Optimal	Instance	Optimal
a2-16	294.25	b2-16	309.41
a2-20	344.83	b2-20	332.64
a2-24	431.12	b2-24	444.71
a3-18	300.48	b3-18	301.64
a3-24	344.83	b3-24	394.51
a3-30	494.85	b3-30	531.44
a3-36	583.19	b3-36	603.79
a4-16	282.68	b4-16	296.96
a4-24	375.02	b4-24	371.41
a4-32	485.50	b4-32	494.82
a4-40	557.69	b4-40	656.63
a4-48	668.82	b4-48	673.81
a5-40	498.41	b5-40	613.72
a5-50	686.62	b5-50	761.40
a5-60	808.42	b5-60	902.04
a6-48	604.12	b6-48	714.83
a6-60	819.25	b6-60	860.07
a6-72	916.05	b6-72	978.47
a7-56	724.04	b7-56	823.97
a7-70	889.12	b7-70	912.62
a7-84	1033.37	b7-84	1203.37
a8-64	747.46	b8-64	839.89
a8-80	945.73	b8-80	1036.34
a8-96	1229.66	b8-96	1185.55

Table 12
Best known results for the HDARP instances by Parragh (2011) and Braekers et al. (2014).

Instance	Best known	Instance	Best known
E		*I*	
a2-16	331.16	a2-16	294.25
a2-20	347.03	a2-20	355.74
a2-24	450.25	a2-24	431.12
a3-18	300.63	a3-18	302.17
a3-24	344.91	a3-24	344.83
a3-30	500.58	a3-30	494.85
a3-36	583.19	a3-36	618.15
a4-16	285.99	a4-16	299.05
a4-24	383.84	a4-24	375.02
a4-32	500.24	a4-32	486.93
a4-40	580.42	a4-40	557.69
a4-48	670.52	a4-48	670.72
a5-40	500.06	a5-40	507.18
a5-50	693.77	a5-50	690.99
a5-60	828.90	a5-60	816.15
a6-48	614.36	a6-48	604.12
a6-60	847.58	a6-60	829.23
a6-72	949.17	a6-72	936.32*
a7-56	740.63	a7-56	727.20
a7-70	946.32	a7-70	916.06*
a7-84	1092.90	a7-84	1035.11
a8-64	762.81	a8-64	748.04
a8-80	982.71	a8-80	956.98*
a8-96	1265.36	a8-96	1222.03*

Results with * are the best feasible results, while results without * are the optimal results.
The best lower bounds for a6-72, a7-70, a8-80, and a8-96 (Set *I*) are 935.52, 903.43, 954.52, and 1218.84, respectively.

Table 13
Best known results for the MD-HDARP instances by Braekers et al. (2014).

Instance	Best known	Instance	Best known	Instance	Best known
U		*E*		*I*	
a2-16	284.18	a2-16	327.67	a2-16	284.18
a2-20	343.43	a2-20	345.59	a2-20	358.88
a2-24	427.17	a2-24	445.88	a2-24	439.29
a3-18	289.67	a3-18	289.67	a3-18	292.41
a3-24	348.30	a3-24	348.61	a3-24	348.54
a3-30	469.16	a3-30	471.43	a3-30	486.04
a3-36	592.42	a3-36	593.84	a3-36	626.96
a4-16	262.44	a4-16	262.44	a4-16	285.4
a4-24	355.72	a4-24	365.54	a4-24	357.51
a4-32	461.65	a4-32	476.59	a4-32	471.54
a4-40	540.34	a4-40	562.86	a4-40	542.56
a4-48	631.75	a4-48	633.49	a4-48	637.58
a5-40	482.19	a5-40	483.84	a5-40	496.36
a5-50	664.54	a5-50	674.19	a5-50	669.30
a5-60	789.87	a5-60	813.96	a5-60	800.10
a6-48	586.08	a6-48	599.76	a6-48	586.08
a6-60	776.63	a6-60	802.49	a6-60	789.40
a6-72	883.78	a6-72	915.03	a6-72	910.24
a7-56	680.08	a7-56	703.62	a7-56	688.51
a7-70	854.22	a7-70	910.91	a7-70	867.47*
a7-84	1007.33	a7-84	1059.12	a7-84	1006.32*
a8-64	713.11	a8-64	731.11	a8-64	713.11
a8-80	885.91*	a8-80	925.72	a8-80	908.30*
a8-96	1172.98*	a8-96	1215.87*	a8-96	1172.90*

Results with * are the best feasible results, while results without * are the optimal results. The best lower bounds for Set *U*: a8-80: 885.45; a8-96: 1170.91 Set *E*: a8-96: 1215.38 Set *I*: a7-70: 865.36; a7-84: 1005.13; a8-80: 904.73; a8-96: 1169.75

References

Atahran, A., Lenté, C., T'kindt, V., 2014. A multicriteria dial-a-ride problem with an ecological measure and heterogeneous vehicles. J. Multi-Criteria Decis. Anal. 21 (5–6), 279–298.
Balas, E., Simonetti, N., 2001. Linear time dynamic-programming algorithms for new classes of restricted TSPs: a computational study. INFORMS J. Comput. 13 (1), 56–75.
Beaudry, A., Laporte, G., Melo, T., Nickel, S., 2010. Dynamic transportation of patients in hospitals. OR Spectr. 32 (1), 77–107.
Benders, J.F., 1962. Partitioning procedures for solving mixed-variables programming problems. Numer. Math. 4 (1), 238–252.
Berbeglia, G., Cordeau, J.-F., Laporte, G., 2012a. A hybrid tabu search and constraint programming algorithm for the dynamic dial-a-ride problem. INFORMS J. Comput. 24 (3), 343–355.
Berbeglia, G., Pesant, G., Rousseau, L.-M., 2011. Checking the feasibility of dial-a-ride instances using constraint programming. Transp. Sci. 45 (3), 399–412.
Berbeglia, G., Pesant, G., Rousseau, L.-M., 2012b. Feasibility of the pickup and delivery problem with fixed partial routes: a complexity analysis. Transp. Sci. 46 (3), 359–373.
Braekers, K., Caris, A., Janssens, G.K., 2014. Exact and meta-heuristic approach for a general heterogeneous dial-a-ride problem with multiple depots. Transp. Res. Part B: Methodol. 67, 166–186.
Braekers, K., Kovacs, A.A., 2016. A multi-period dial-a-ride problem with driver consistency. Transp. Res. Part B: Methodol. 94, 355–377.
Carnes, T.A., Henderson, S.G., Shmoys, D.B., Ahghari, M., MacDonald, R.D., 2013. Mathematical programming guides air-ambulance routing at ornge. Interfaces 43 (3), 232–239.
Chakravarty, A.K., Orlin, J.B., Rothblum, U.G., 1982. A partitioning problem with additive objective with an application to optimal inventory groupings for joint replenishment. Oper. Res. 30 (5), 1018–1022.
Charikar, M., Raghavachari, B., 1998. The finite capacity dial-a-ride problem. In: Proceedings 39th Annual Symposium on Foundations of Computer Science, pp. 458–467.
Chassaing, M., Duhamel, C., Lacomme, P., 2016. An ELS-based approach with dynamic probabilities management in local search for the dial-a-ride problem. Eng. Appl. Artif. Intell. 48, 119–133.
Chevrier, R., Liefooghe, A., Jourdan, L., Dhaenens, C., 2012. Solving a dial-a-ride problem with a hybrid evolutionary multi-objective approach: application to demand responsive transport. Appl. Soft Comput. 12 (4), 1247–1258.
Codato, G., Fischetti, M., 2004. Combinatorial benders' cuts. In: Bienstock, D., Nemhauser, G. (Eds.), Integer Programming and Combinatorial Optimization. Springer, Berlin, pp. 178–195.
Cordeau, J.-F., 2006. A branch-and-cut algorithm for the dial-a-ride problem. Oper. Res. 54 (3), 573–586.
Cordeau, J.-F., Gendreau, M., Laporte, G., Potvin, J.-Y., Semet, F., 2002. A guide to vehicle routing heuristics. J. Oper. Res. Soc. 53 (5), 512–522.
Cordeau, J.-F., Laporte, G., 2003. A tabu search heuristic for the static multi-vehicle dial-a-ride problem. Transp. Res. Part B: Methodol. 37 (6), 579–594.
Cordeau, J.-F., Laporte, G., 2007. The dial-a-ride problem: models and algorithms. Ann. Oper. Res. 153 (1), 29–46.
Cortés, C.E., Matamala, M., Contardo, C., 2010. The pickup and delivery problem with transfers: formulation and a branch-and-cut solution method. Eur. J. Oper. Res. 200 (3), 711–724.
Cubillos, C., Urra, E., Rodríguez, N., 2009. Application of genetic algorithms for the DARPTW problem. Int. J. Comput. Commun. Control 4 (2), 127–136.
Detti, P., Papalini, F., de Lara, G.Z.M., 2017. A multi-depot dial-a-ride problem with heterogeneous vehicles and compatibility constraints in healthcare. Omega 70, 1–14.
D'Souza, C., Omkar, S.N., Senthilnath, J., 2012. Pickup and delivery problem using metaheuristics techniques. Expert Syst. Appl. 39 (1), 328–334.
Dumas, Y., Desrosiers, J., Soumis, F., 1991. The pickup and delivery problem with time windows. Eur. J. Oper. Res. 54 (1), 7–22.
Feng, L., Miller-Hooks, E., Schonfeld, P., Mohebbi, M., 2014. Optimizing ridesharing services for airport access. Transp. Res. Rec.: J. Transp. Res. Board 2467, 157–167.

Firat, M., Woeginger, G.J., 2011. Analysis of the dial-a-ride problem of hunsaker and savelsbergh. Oper. Res. Lett. 39 (1), 32–35.
Garaix, T., Artigues, C., Feillet, D., Josselin, D., 2010. Vehicle routing problems with alternative paths: an application to on-demand transportation. Eur. J. Oper. Res. 204 (1), 62–75.
Garaix, T., Artigues, C., Feillet, D., Josselin, D., 2011. Optimization of occupancy rate in dial-a-ride problems via linear fractional column generation. Comput. Oper. Res. 38 (10), 1435–1442.
Glover, F., Laguna, M., 1997. Tabu Search. Springer, Boston, MA, US.
Gschwind, T., Drexl, M., 2016. Adaptive Large Neighborhood Search with a Constant-Time Feasibility Test for the Dial-a-Ride Problem. Technical Report LM-2016-08. Johannes Gutenberg University, Mainz. http://logistik.bwl.uni-mainz.de/Dateien/LM-2016-08.pdf.
Gschwind, T., Irnich, S., 2015. Effective handling of dynamic time windows and its application to solving the dial-a-ride problem. Transp. Sci. 49 (2), 335–354.
Guerriero, F., Bruni, M.E., Greco, F., 2013. A hybrid greedy randomized adaptive search heuristic to solve the dial-a-ride problem. Asia-Pac. J. Oper. Res. 30 (01), 1250046.
Gupta, A., Hajiaghayi, M., Nagarajan, V., Ravi, R., 2010. Dial-a-ride from k-forest. ACM Trans. Algorithms (TALG) 6 (2), 41.
Häll, C.H., Andersson, H., Lundgren, J.T., Värbrand, P., 2009. The integrated dial-a-ride problem. Public Transp. 1 (1), 39–54.
Häll, C.H., Lundgren, J.T., Voß, S., 2015. Evaluating the performance of a dial-a-ride service using simulation. Public Transp. 7 (2), 139–157.
Häll, C.H., Peterson, A., 2013. Improving paratransit scheduling using ruin and recreate methods. Transp. Plan. Technol. 36 (4), 377–393.
Häme, L., 2011. An adaptive insertion algorithm for the single-vehicle dial-a-ride problem with narrow time windows. Eur. J. Oper. Res. 209 (1), 11–22.
Häme, L., Hakula, H., 2013. Routing by ranking: a link analysis method for the constrained dial-a-ride problem. Oper. Res. Lett. 41 (6), 664–669.
Häme, L., Hakula, H., 2015. A maximum cluster algorithm for checking the feasibility of dial-a-ride instances. Transp. Sci. 49 (2), 295–310.
Hanne, T., Melo, T., Nickel, S., 2009. Bringing robustness to patient flow management through optimized patient transports in hospitals. Interfaces 39 (3), 241–255.
Haugland, D., Ho, S.C., 2010. Feasibility testing for dial-a-ride problems. In: Chen, B. (Ed.), Algorithmic Aspects in Information and Management. Springer, Berlin, pp. 170–179.
Heilporn, G., Cordeau, J.-F., Laporte, G., 2011. An integer l-shaped algorithm for the dial-a-ride problem with stochastic customer delays. Discret. Appl. Math. 159 (9), 883–895.
Ho, S.C., Haugland, D., 2011. Local search heuristics for the probabilistic dial-a-ride problem. OR Spectr. 33 (4), 961–988.
Hu, T.-Y., Chang, C.-P., 2015. A revised branch-and-price algorithm for dial-a-ride problems with the consideration of time-dependent travel cost. J. Adv. Transp. 49 (6), 700–723.
Hunsaker, B., Savelsbergh, M., 2002. Efficient feasibility testing for dial-a-ride problems. Oper. Res. Lett. 30 (3), 169–173.
Hyytiä, E., Aalto, S., Penttinen, A., Sulonen, R., 2010. A stochastic model for a vehicle in a dial-a-ride system. Oper. Res. Lett. 38 (5), 432–435.
Hyytiä, E., Penttinen, A., Sulonen, R., 2012. Non-myopic vehicle and route selection in dynamic DARP with travel time and workload objectives. Comput. Oper. Res. 39 (12), 3021–3030.
Ilani, H., Shufan, E., Grinshpoun, T., Belulu, A., Fainberg, A., 2014. A reduction approach to the two-campus transport problem. J. Sched. 17 (6), 587–599.
Jaw, J.-J., Odoni, A.R., Psaraftis, H.N., Wilson, N.H., 1986. A heuristic algorithm for the multi-vehicle advance request dial-a-ride problem with time windows. Transp. Res. Part B: Methodol. 20 (3), 243–257.
Jorgensen, R.M., Larsen, J., Bergvinsdottir, K.B., 2007. Solving the dial-a-ride problem using genetic algorithms. J. Oper. Res. Soc. 58 (10), 1321–1331.
Jourdan, L., Basseur, M., Talbi, E.-G., 2009. Hybridizing exact methods and metaheuristics: a taxonomy. Eur. J. Oper. Res. 199 (3), 620–629.
Karabuk, S., 2009. A nested decomposition approach for solving the paratransit vehicle scheduling problem. Transp. Res. Part B: Methodol. 43 (4), 448–465.
Kirchler, D., Wolfler Calvo, R., 2013. A granular tabu search algorithm for the dial-a-ride problem. Transp. Res. Part B: Methodol. 56, 120–135.
Lehuédé, F., Masson, R., Parragh, S.N., Péton, O., Tricoire, F., 2014. A multi-criteria large neighbourhood search for the transportation of disabled people. J. Oper. Res. Soc. 65 (7), 983–1000.
Lim, A., Zhang, Z., Qin, H., 2017. Pickup and delivery service with manpower planning in Hong Kong public hospitals. Transp. Sci. 51 (2), 688–705.
Liu, M., Luo, Z., Lim, A., 2015. A branch-and-cut algorithm for a realistic dial-a-ride problem. Transp. Res. Part B: Methodol. 81 (Part 1), 267–288.
Luo, Y., Schonfeld, P., 2007. A rejected-reinsertion heuristic for the static dial-a-ride problem. Transp. Res. Part B: Methodol. 41 (7), 736–755.
Maalouf, M., MacKenzie, C.A., Radakrishnan, S., Court, M., 2014. A new fuzzy logic approach to capacitated dynamic dial-a-ride problem. Fuzzy Sets Syst. 255, 30–40.
Marković, N., Nair, R., Schonfeld, P., Miller-Hooks, E., Mohebbi, M., 2015. Optimizing dial-a-ride services in maryland: benefits of computerized routing and scheduling. Transp. Res. Part C: Emerg. Technol. 55, 156–165.
Masmoudi, M.A., Braekers, K., Masmoudi, M., Dammak, A., 2017. A hybrid genetic algorithm for the heterogeneous dial-a-ride problem. Comput. Oper. Res. 81, 1–13.
Masmoudi, M.A., Hosny, M., Braekers, K., Dammak, A., 2016. Three effective metaheuristics to solve the multi-depot multi-trip heterogeneous dial-a-ride problem. Transp. Res. Part E: Logist. Transp. Rev. 96, 60–80.
Masson, R., Lehuédé, F., Péton, O., 2013. An adaptive large neighborhood search for the pickup and delivery problem with transfers. Transp. Sci. 47 (3), 344–355.
Masson, R., Lehuédé, F., Péton, O., 2014. The dial-a-ride problem with transfers. Comput. Oper. Res. 41, 12–23.
Mauri, G., Antonio, L., Lorena, N., 2009. Customers' satisfaction in a dial-a-ride problem. IEEE Intell. Transp. Syst. Mag. 1 (3), 6–14.
Melachrinoudis, E., Ilhan, A.B., Min, H., 2007. A dial-a-ride problem for client transportation in a health-care organization. Comput. Oper. Res. 34 (3), 742–759.
Mladenović, N., Hansen, P., 1997. Variable neighborhood search. Comput. Oper. Res. 24 (11), 1097–1100.
Molenbruch, Y., Braekers, K., Caris, A., 2017a. Benefits of horizontal cooperation in dial-a-ride services. Transp. Res. Part E: Logist. Transp. Rev. 107, 97–119.
Molenbruch, Y., Braekers, K., Caris, A., 2017b. Operational effects of service level variations for the dial-a-ride problem. Central Eur. J. Oper. Res. 25 (1), 71–90.
Molenbruch, Y., Braekers, K., Caris, A., Berghe, G.V., 2017c. Multi-directional local search for a bi-objective dial-a-ride problem in patient transportation. Comput. Oper. Res. 77, 58–71.
Molenbruch, Y., Braekers, K., Caris, A., 2017d. Typology and literature review for dial-a-ride problems. Ann. Oper. Res. 259 (1-2), 259–325.
Muelas, S., LaTorre, A., Peña, J.-M., 2013. A variable neighborhood search algorithm for the optimization of a dial-a-ride problem in a large city. Expert Syst. Appl. 40 (14), 5516–5531.
Muelas, S., LaTorre, A., Peña, J.-M., 2015. A distributed VNS algorithm for optimizing dial-a-ride problems in large-scale scenarios. Transp. Res. Part C: Emerg. Technol. 54, 110–130.
Muñoz-Carpintero, D., Sáez, D., Cortés, C.E., Núñez, A., 2015. A methodology based on evolutionary algorithms to solve a dynamic pickup and delivery problem under a hybrid predictive control approach. Transp. Sci. 49 (2), 239–253.
Núñez, A., Cortés, C.E., Sáez, D., Schutter, B.D., Gendreau, M., 2014. Multiobjective model predictive control for dynamic pickup and delivery problems. Control Eng. Pract. 32, 73–86.
Oxley, P., 1980. Dial/a/ride: a review. Transp. Plan. Technol. 6 (3), 141–148.
Paquette, J., Cordeau, J.-F., Laporte, G., Pascoal, M.M.B., 2013. Combining multicriteria analysis and tabu search for dial-a-ride problems. Transp. Res. Part B: Methodol. 52, 1–16.
Parragh, S.N., 2011. Introducing heterogeneous users and vehicles into models and algorithms for the dial-a-ride problem. Transp. Res. Part C: Emerg. Technol. 19 (5), 912–930.

Parragh, S.N., Cordeau, J.-F., Doerner, K.F., Hartl, R.F., 2012. Models and algorithms for the heterogeneous dial-a-ride problem with driver-related constraints. OR Spectr. 34 (3), 593–633.
Parragh, S.N., Doerner, K.F., Hartl, R.F., 2010. Variable neighborhood search for the dial-a-ride problem. Comput. Oper. Res. 37 (6), 1129–1138.
Parragh, S.N., Doerner, K.F., Hartl, R.F., Gandibleux, X., 2009. A heuristic two-phase solution approach for the multi-objective dial-a-ride problem. Networks 54 (4), 227–242.
Parragh, S.N., Schmid, V., 2013. Hybrid column generation and large neighborhood search for the dial-a-ride problem. Comput. Oper. Res. 40 (1), 490–497.
Parragh, S.N., de Sousa, J.P., Almada-Lobo, B., 2015. The dial-a-ride problem with split requests and profits. Transp. Sci. 49 (2), 311–334.
Pillac, V., Gendreau, M., Guéret, C., Medaglia, A.L., 2013. A review of dynamic vehicle routing problems. Eur. J. Oper. Res. 225 (1), 1–11.
Pimenta, V., Quilliot, A., Toussaint, H., Vigo, D., 2017. Models and algorithms for reliability-oriented dial-a-ride with autonomous electric vehicles. Eur. J. Oper. Res. 257 (2), 601–613.
Posada, M., Andersson, H., Häll, C.H., 2017. The integrated dial-a-ride problem with timetabled fixed route service. Public Transp. 9 (1–2), 217–241.
Psaraftis, H.N., 1980. A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem. Transp. Sci. 14 (2), 130–154.
Qu, Y., Bard, J.F., 2013. The heterogeneous pickup and delivery problem with configurable vehicle capacity. Transp. Res. Part C: Emerg. Technol. 32, 1–20.
Qu, Y., Bard, J.F., 2015. A branch-and-price-and-cut algorithm for heterogeneous pickup and delivery problems with configurable vehicle capacity. Transp. Sci. 49 (2), 254–270.
Quadrifoglio, L., Dessouky, M.M., Ordóñez, F., 2008. A simulation study of demand responsive transit system design. Transp. Res. Part A: Policy Pract. 42 (4), 718–737.
Reinhardt, L.B., Clausen, T., Pisinger, D., 2013. Synchronized dial-a-ride transportation of disabled passengers at airports. Eur. J. Oper. Res. 225 (1), 106–117.
Ritzinger, U., Puchinger, J., Hartl, R.F., 2016. Dynamic programming based metaheuristics for the dial-a-ride problem. Ann. Oper. Res. 236 (2), 341–358.
Ronald, N., Thompson, R., Winter, S., 2015. Simulating demand-responsive transportation: a review of agent-based approaches. Transp. Rev. 35 (4), 404–421.
Ropke, S., 2005. Heuristic and Exact Algorithms for Vehicle Routing Problems. University of Copenhagen, Copenhagen Ph.D. thesis.
Ropke, S., Cordeau, J.-F., 2009. Branch and cut and price for the pickup and delivery problem with time windows. Transp. Sci. 43 (3), 267–286.
Ropke, S., Cordeau, J.-F., Laporte, G., 2007. Models and branch-and-cut algorithms for pickup and delivery problems with time windows. Networks 49, 258–272.
Ropke, S., Pisinger, D., 2006. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. Transp. Sci. 40 (4), 455–472.
Santos, D.O., Xavier, E.C., 2015. Taxi and ride sharing: a dynamic dial-a-ride problem with money as an incentive. Expert Syst. Appl. 42 (19), 6728–6737.
Sayarshad, H.R., Chow, J.Y.J., 2015. A scalable non-myopic dynamic dial-a-ride and pricing problem. Transp. Res. Part B: Methodol. 81 (Part 2), 539–554.
Schilde, M., Doerner, K.F., Hartl, R.F., 2011. Metaheuristics for the dynamic stochastic dial-a-ride problem with expected return transports. Comput. Oper. Res. 38 (12), 1719–1730.
Schilde, M., Doerner, K.F., Hartl, R.F., 2014. Integrating stochastic time-dependent travel speed in solution methods for the dynamic dial-a-ride problem. Eur. J. Oper. Res. 238 (1), 18–30.
Schönberger, J., 2017. Scheduling constraints in dial-a-ride problems with transfers: a metaheuristic approach incorporating a cross-route scheduling procedure with postponement opportunities. Public Transp. 9 (1–2), 243–272.
Shaw, P., 1997. A New Local Search Algorithm Providing High Quality Solutions to Vehicle Routing Problems. Technical Report. Department of Computer Science, University of Strathclyde, Scotland.
Shen, C.-W., Quadrifoglio, L., 2013. Evaluating centralized versus decentralized zoning strategies for metropolitan ADA paratransit services. J. Transp. Eng. 139 (5), 524–532.
Stein, D.M., 1978. Scheduling dial-a-ride transportation systems. Transp. Sci. 12 (3), 232–249.
Talbi, E.-G., 2002. A taxonomy of hybrid metaheuristics. J. Heuristics 8 (5), 541–564.
Tang, J., Kong, Y., Lau, H., Ip, A.W.H., 2010. A note on "efficient feasibility testing for dial-a-ride problems". Oper. Res. Lett. 38 (5), 405–407.
Today, 2017. PMDs, Foldable Bikes, Allowed on Public Transport After Trial. http://www.todayonline.com/singapore/personal-mobility-devices-foldable-bikes-allowed-public-transport-after-trial. Accessed: 27 December 2017.
Urra, E., Cubillos, C., Cabrera-Paniagua, D., 2015. A hyperheuristic for the dial-a-ride problem with time windows. Math. Problems Eng. 2015, 707056.
Waisanen, H.A., Shah, D., Dahleh, M.A., 2008. A dynamic pickup and delivery problem in mobile networks under information constraints. IEEE Trans. Autom. Control 53 (6), 1419–1433.
Wilson, N.H.M., Sussman, J.M., Wong, H.K., Higonnet, T., 1971. Scheduling Algorithms for a Dial-a-ride System. Technical Report TR-70-13. Massachusetts Institute of Technology, USA.
Wolfler Calvo, R., Colorni, A., 2007. An effective and fast heuristic for the dial-a-ride problem. 4OR 5 (1), 61–73.
Wong, K.I., Han, A.F., Yuen, C.W., 2014. On dynamic demand responsive transport services with degree of dynamism. Transp. A: Transp. Sci. 10 (1), 55–73.
Xiang, Z., Chu, C., Chen, H., 2008. The study of a dynamic dial-a-ride problem under time-dependent and stochastic environments. Eur. J. Oper. Res. 185 (2), 534–551.
Zhang, Z., Liu, M., Lim, A., 2015. A memetic algorithm for the patient transportation problem. Omega 54, 60–71.
Zidi, I., Mesghouni, K., Zidi, K., Ghedira, K., 2012. A multi-objective simulated annealing for the multi-criteria dial a ride problem. Eng. Appl. Artif. Intell. 25 (6), 1121–1131.