Uncapacitated lot-sizing problems with remanufacturing and two sale markets

Siao-Leu Phouratsamay, Bernard Penz

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Siao-Leu Phouratsamay
Univ. Grenoble Alpes, CNRS, Grenoble INP G-SCOP, 38000 Grenoble, France
siao.phouratsamay@grenoble-inp.fr

Bernard Penz
Univ. Grenoble Alpes, CNRS, Grenoble INP G-SCOP, 38000 Grenoble, France
barnard.penz@grenoble-inp.fr

Abstract

The remanufacturing process consists in recovering returned products by replacing or repairing components. In this work, we consider an uncapacitated lot-sizing problem with remanufacturing and two types of markets. The demand of new products is only satisfied by manufactured products, whereas the demand of second-hand products is only satisfied by the remanufacturing of returned products. We develop polynomial dynamic programming algorithms for two cases, where the returned products can be disposed or not.

1 Introduction

The remanufacturing is the process of recovering returned products (returns) by replacing or repairing components with a bad quality. The research on production with remanufacturing has been growing over the decades, especially on lot-sizing problems [9].

The lot-sizing problem with remanufacturing was first studied in [5]. The authors propose a polynomial algorithm when the costs are linear and prove that the problem is NP-complete for general concave costs. Polynomial time dynamic programming algorithms has been developed to solve special cases of the problem [1, 6, 10], where other papers focused on formulations of the problem [3, 8]. Recently, Piñeyro and Veira [7] introduce heterogeneous quality for the returns. They provide complexity results, algorithms to solve different cases of the problem and an experimental analysis to evaluate the relevance of using the returns. No paper in the literature distinguishes the demand of new and second-hand items in the lot-sizing problem with remanufacturing.

In the lot-sizing litterature, remanufactured products are considered as equivalent to new products, and can satisfy a part of the demand. However, it is not practical to consider them as new products since the market price, the clients and the marketing strategy are different [2, 4]. This motivates us to study the lot-sizing problem with remanufacturing and two sale markets, one for new products and another one for remanufactured products.

2 Problem definition

The planning horizon includes T discrete periods. The demand of new and second-hand products are denoted by D_t^n and D_t^s respectively. At the beginning of period t, R_t returns arrive and are available for remanufacturing.

Producing a new product (resp. a second-hand product) induces a cost of p_t^n (resp. p_t^s) in period t. Carrying a new product from period t to $t+1$ induces a unit holding cost of h_t^n (h_t^s for second-hand products and h_t^r for returns). At the end of a period t, returns which are not used to produce second-hand products can either be disposed or be placed in the inventory until the next period. The unit cost to dispose the returns is denoted by p_t^r.

We assume that the fixed setup cost f_t in period t is joint by the new and the second-hand products. Note that if the setup costs are not joint, the problem will consist in solving two independent uncapacitated lot-sizing problems.

Moreover, we assume that $D_t^n > 0$, $D_t^s > 0$, $h_t^r < h_t^s < h_t^n$.

The variables x_t^n and x_t^s denote the quantity of new and second-hand items produced in period t. The disposal quantity in period t is given by x_t^r. The inventory level of new products, second hand products and returns at the end of period t are noted s_t^n, s_t^s, s_t^r. The binary variable y_t indicates if a production of new or second-hand products occurs in period t.

The formulation of the uncapacitated lot-sizing problem with remanufacturing and two sale markets is as follows:

$$
\begin{array}{rcll}
\text{Minimize } z & = & \sum_{t=1}^{T}(f_t y_t + p_t^s x_t^s + p_t^n x_t^n + p_t^r x_t^r + h_t^s s_t^s + h_t^n s_t^n + h_t^r s_t^r) & \\
\text{s.t.} & & & \\
s_t^r & = & s_{t-1}^r + R_t - x_t^r - x_t^s & \forall t \in \{1, \cdots, T\} \\
s_t^s & = & s_{t-1}^s + x_t^s - D_t^s & \forall t \in \{1, \cdots, T\} \\
s_t^n & = & s_{t-1}^n + x_t^n - D_t^n & \forall t \in \{1, \cdots, T\} \\
x_t^s + x_t^n & \leq & M y_t & \forall t \in \{1, \cdots, T\} \\
x_t^r, x_t^s, x_t^n, & \in & \mathbb{R}^+ & \forall j \in \{1, \cdots, T\} \\
s_t^r, s_t^s, s_t^n, & \in & \mathbb{R}^+ & \forall j \in \{1, \cdots, T\} \\
y_t & \in & \{0, 1\} & \forall t \in \{1, \cdots, T\}
\end{array}
$$

3 Remanufacturing with joint setups and no disposal (ULSR-2m-nd)

In this section, we assume that the returns cannot be disposed. We propose an $\mathcal{O}(T^2)$ dynamic programming algorithm to solve this problem.

The ZIO property can be generalized to our problem as follows.

Proposition 1 *At each period t, $(x_t^s + x_t^n)(s_{t-1}^s + s_{t-1}^n) = 0$.*

A subplan $S_{u,v}$ is such that the stock is null only in periods $u-1$ and $v-1$ and for each period t such that $u \leq t \leq v-2$, we must have $s_t^s + s_t^n > 0$. Using the definition of a subplan and Proposition 1, the following proposition holds.

Proposition 2 *In a subplan $S_{u,v}$, the period u is the unique production period for both new and second-hand products, i.e. $x_u^s + x_u^n > 0$.*

From Propositions 1 and 2, we develop a polynomial time dynamic algorithm by showing that an optimal solution can be decomposed into a succession of independent subplans.

4 Remanufacturing with joint setups and disposal (ULSR-2m-d)

In this section, the returns can be disposed if they are not used to satisfy the second-hand demand. An $\mathcal{O}(T^6)$ dynamic programming algorithm is developed to solve this problem.

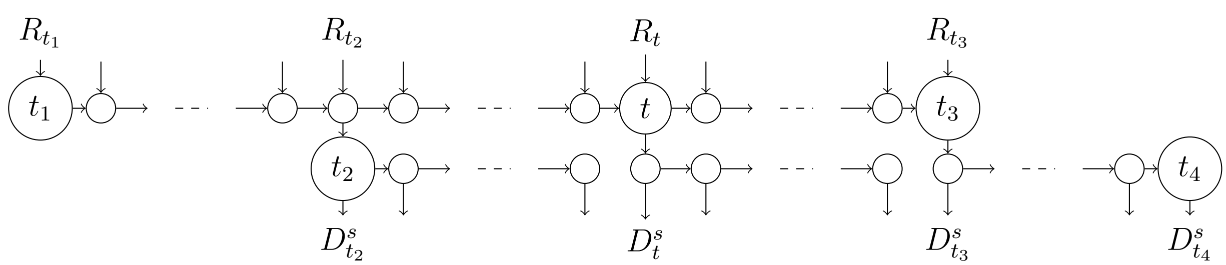

Figure 1: Structure of a separation-block

The idea of the algorithm is to decompose an optimal solution into independent block $\mathcal{B}(t_1, t_2, t_3, t_4)$ (see Figure 1) where t_1 is the unique period before t_3 with a positive disposal quantity, $t_2 \geq t_1$ is the first period with a positive production $(x_{t_2}^s +$

$x^n_{t_2} > 0$), and $t_3 \geq t_2$ is the last period of the block with a positive production quantity. We proved that the cost of an optimal solution for the block $\mathcal{B}(t_1, t_2, t_3, t_4)$ can be calculated in $\mathcal{O}(T^2)$. As the number of blocks are in $\mathcal{O}(T^4)$, the resulting dynamic program is in $\mathcal{O}(T^6)$.

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