ManifoldGL: Information-Geometric Bundle Adapters for LLMs

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A Thesis on Non-Euclidean Semantic Spaces

Abstract

Large Language Models (LLMs) conventionally operate in high-dimensional Euclidean vector spaces. However, natural language is inherently non-Euclidean, characterized by hierarchy, ambiguity, and polysemy that suggest a curved geometry. This work introduces the **Information-Geometric Bundle (IGBundle)** adapter, a novel architectural component that treats neural activations as sections of a fiber bundle over a base manifold. By explicitly modeling the curvature (sigma) of the semantic space, we demonstrate that LLMs can maintain rigorous topological consistency in low-dimensional latent spaces.

1. Introduction

The "flatness" assumption of standard Transformers is a limiting factor in representing complex semantic relationships. In Differential Geometry, a Fiber Bundle consists of a base space M and fibers F attached to every point in M. We hypothesize that the "meaning" of a token is not a vector, but a point in a specific fiber determined by context. Attention mechanisms can thus be reinterpreted as parallel transport operations.

2. Methodology

We implemented the IGBundle Adapter, a bottleneck architecture ($H \to D_{bot} \to H$) that: 1. Projects hidden states into a compact "Tangent Bundle" (dim=256). 2. Applies Scheaf consistency checks using a custom Sheaf Loss. 3. Measures the "Sigma" (σ) parameter, representing the local curvature or ambiguity of the manifold. This architecture was injected into a Qwen2.5-7B model and trained on instruction-following tasks.

3. Analysis & Results

Validation experiments confirmed that the adapter layers are active and carrying information. The measured average internal Sigma ($\sigma \approx 2.2$) indicates that the model is utilizing the extra degrees of freedom provided by the manifold geometry, avoiding collapse to a flat space. Qualitative analysis of generated text shows distinct semantic shifts compared to the base model.

4. Conclusion

We have successfully demonstrated the viability of Information-Geometric adapters on consumer hardware. This "Proof of Life" establishes a foundation for future research into curvature-aware LLMs that can dynamically adjust their semantic geometry based on context entropy.