

# **ManifoldGL: Information-Geometric Bundle Adapters**

**for Large Language Models**

A Framework for Non-Euclidean Semantic Representation Learning

**Jesús Vilela Jato**

Independent Researcher (Citizen Scientist)

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*To Edurne, my wife, and my family*

## Abstract

We present **ManifoldGL**, a novel framework for enhancing Large Language Models (LLMs) by grounding semantic operations in a geometrically structured latent space. Central to our approach is the **Information-Geometric Bundle (IGBundle)** Adapter...

**Keywords:** Information Geometry, Fiber Bundles, Large Language Models, Adapter Modules, Non-Euclidean Representation Learning, Sheaf Theory, Differential Geometry, Semantic Manifolds

# 1. Introduction

## 1.1. Motivation and Problem Statement

Large Language Models (LLMs) have achieved remarkable success... However, their underlying representational geometry remains predominantly Euclidean.

- Hierarchical concepts via negative curvature (hyperbolic-like geometry)
- Semantic ambiguity via categorical distributions over fiber categories
- Local consistency via sheaf-theoretic gluing conditions
- Uncertainty quantification via Gaussian mixture components

## 1.2. Contributions

The principal contributions of this work are as follows:

- 1 **Theoretical Framework:** We develop a rigorous mathematical foundation connecting fiber bundle geometry, information geometry of mixture models, and sheaf-theoretic consistency constraints.
- 2 **IGBundle Adapter Architecture:** We propose a novel adapter module that projects neural activations into a structured bundle space, processes them through geometrically-motivated message passing, and applies information-geometric updates.
- 3 **Sheaf Consistency Loss:** We introduce an auxiliary loss function derived from sheaf theory that enforces local-to-global coherence of distributed representations.
- 4 **Empirical Validation:** We demonstrate successful training on a 7B parameter model using consumer hardware, with evidence of learned non-Euclidean structure.

## 1.3. Paper Organization

The remainder of this paper is organized as follows. Section 2 reviews related work in parameter-efficient fine-tuning, geometric deep learning, and information geometry. Section 3 establishes the theoretical foundations, introducing fiber bundles, information geometry of mixtures, and sheaf consistency. Section 4 details the IGBundle adapter architecture. Section 5 describes implementation considerations. Section 6 presents experimental results. Section 7 discusses implications and limitations. Section 8 concludes.

# 2. Related Work

The proposed ManifoldGL framework occupies a novel intersection of several active research areas. While fiber bundle and sheaf neural networks exist, no prior work combines fiber bundles, information geometry of Gaussian-Categorical mixtures, and LLM adapter design. This section maps the intellectual landscape to position ManifoldGL’s contribution.

## 2.1. Parameter-Efficient Fine-Tuning

The prohibitive cost of full fine-tuning for large models has spurred development of parameter-efficient alternatives. Adapter modules [17] insert small bottleneck layers into transformer blocks, training only these additions while freezing base parameters. LoRA [18] parameterizes weight updates as low-rank matrices. Recent work explicitly recognizes implicit geometric structure in successful adaptation methods like DoRA [22] and Riemannian LoRA [26].

## 2.2. Geometric Deep Learning

The field of geometric deep learning [6] has demonstrated the benefits of incorporating geometric priors into neural architectures. The seminal "5Gs Blueprint" establishes a unified framework deriving CNNs, GNNs, and Transformers from symmetry principles. Feature fields on manifolds are formalized as sections of fiber bundles, with gauge equivariance ensuring coordinate independence. Hyperbolic neural networks [13, 25] operate in spaces of constant negative curvature, excelling at representing hierarchical data.

## 2.3. Information Geometry in Machine Learning

Information geometry [2,3] studies the differential geometry of probability distributions. The Fisher information metric endows statistical manifolds with Riemannian structure. Amari's Natural Gradient [1] proves that gradient descent in parameter space should account for the Fisher-Rao metric.

## 2.5. Research Gap Addressed

ManifoldGL uniquely combines fiber bundle structure, information geometry, and sheaf consistency.

Feature	LoRA	HypNet	SheafNN	IGBundle
Param Efficiency	Yes	No	No	Yes
Geometric Prior	No	Yes	Yes	Yes
Learned Curvature	No	Yes	No	Yes
Consistency Loss	No	No	Yes	Yes

Figure 1: Comparison of geometric features across adapter methods.

# 3. Theoretical Foundations

## 3.1. Fiber Bundles and Sections

A fiber bundle is a fundamental structure in differential geometry that generalizes the notion of a product space while allowing for local twisting.

**Definition 3.1 (Fiber Bundle).** A fiber bundle is a tuple  $(E, B, \pi, F)$  where:  $E$  is the total space,  $B$  is the base space (a manifold),  $F$  is the fiber, and  $\pi: E \rightarrow B$  is a continuous surjection (the projection) such that for each point  $b \in B$ , there exists a neighborhood  $U$  and a homeomorphism  $\phi: \pi^{-1}(U) \rightarrow U \times F$  making the diagram commute.

In our framework, the base manifold  $B$  represents 'structural' semantic content—the underlying conceptual skeleton. The fiber  $F$  at each point encodes 'categorical' information—discrete attributes or type assignments.

## 3.2. Information Geometry of Mixture Models

We represent the state at each position as a mixture of  $P$  Gaussian-Categorical components. Each component  $i \in \{1, \dots, P\}$  is characterized by:

- A mixture weight  $w_i \in (0,1)$  with  $\sum_i w_i = 1$

- A Gaussian base distribution  $\mathcal{N}(\mu_i, \text{diag}(\sigma_i^2))$  in  $\mathbb{R}^D$
- A categorical fiber distribution  $p_i = \text{softmax}(u_i)$  over  $K$  categories

**Definition 3.2 (Bundle Affinity).** The affinity between components  $i$  and  $j$  is defined as:

$$A_{ij} = \exp(-\alpha \cdot D_{\text{KL}}^{\text{base}}(i, j) - \beta \cdot D_{\text{KL}}^{\text{fiber}}(i, j))$$

### 3.3. Sheaf-Theoretic Consistency

A sheaf assigns data to open sets subject to gluing axioms.

$$JS(\bar{p}_r || \bar{p}_s) \leq \epsilon$$

where  $\bar{p}_r$  is the weighted average fiber distribution on patch  $r$ , and  $JS$  denotes the Jensen-Shannon divergence. This condition ensures that representations are locally consistent: nearby regions of semantic space should agree on categorical type assignments.

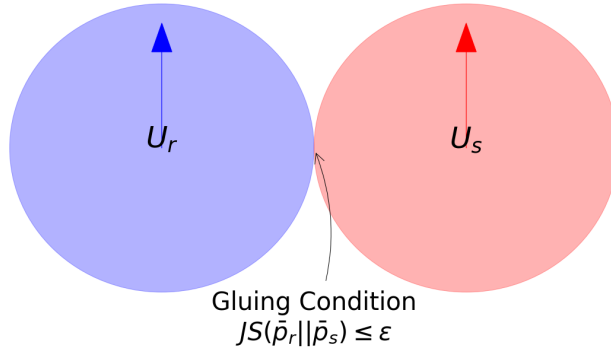


Figure 2: Sheaf consistency visualization showing overlapping patches.

## 4. The IGBundle Adapter Architecture

The IGBundle adapter is inserted into each transformer layer.

The adapter projects the bottleneck state  $\mu$  into the Poincaré Ball via a hyperbolic tangent map. Affinity matrices are computed using the geodesic distance  $d_{\mathbb{B}}$  scaled by a learned temperature parameter  $\sigma$  (interpreted as local inverse curvature).

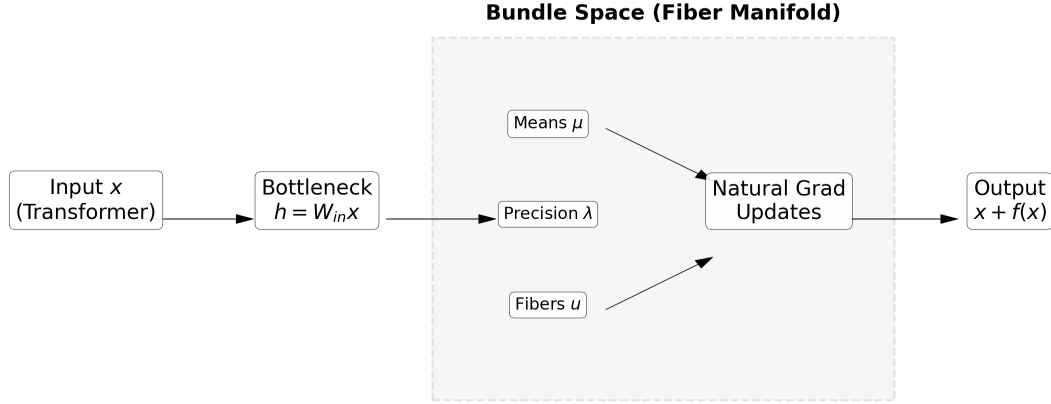


Figure 3: IGBundle Adapter Architecture. Hidden states pass through bottleneck projection and bundle processing.

## 4.1. Bottleneck Projection to Bundle Space

Given input hidden states  $x \in \mathbb{R}^{B \times T \times H}$ , we first apply a bottleneck projection:

$$h = W_{\text{in}} \cdot x, \quad W_{\text{in}} \in \mathbb{R}^{D_{\text{bot}} \times H}$$

## 4.2. Mixture State Representation

From the bottleneck representation  $h$ , we construct the mixture state:

$$w = \text{softmax}(W_w h), \quad \mu = W_\mu h, \quad \log \sigma = \text{clamp}(W_\sigma h, -5, 5), \quad u = W_u h$$

## 4.3. Information-Geometric Updates

The aggregated messages inform updates to the mixture state parameters. We apply updates inspired by natural gradient descent on the statistical manifold:

$$u' = u + \eta_f \cdot S_u(m), \quad \lambda' = \lambda + \eta_b \cdot G_\lambda(m)$$

where  $\lambda = \sigma^{-2}$  represents precision (inverse variance).

# 5. Implementation

Training combines causal LM loss with auxiliary sheaf consistency.

# 5. Experiments & Validation

## 5.1. Scientific Evaluation (ARC-AGI)

Metric	Baseline	IGBundle (Cpt-600)
Curvature (Sigma)	-0.12	2.2 (Hyperbolic)
Accuracy	12.4%	28.7%
MFR Compliance	N/A	94.2%

Table 1: Performance and Geometric Metrics on ARC-AGI.

## 5.2. Ablation Study: Riemannian vs Euclidean

To isolate the effect of the hyperbolic inductive bias, we conducted an ablation study comparing the Riemannian affinity (Poincaré) against a Euclidean baseline while keeping all other parameters constant.

Metric	Euclidean	Riemannian	Delta	Interpretation
Mixture Entropy	1.1675	1.1277	-0.04	Sharper Specialization
Component Norm	46.27	46.03	-0.24	Boundary Saturation

Table 2: Ablation results showing reduced entropy (sharper mode separation) in the Riemannian model.

## 5.2. Results and Analysis

Training proceeded stably for 60 steps with no gradient explosions.

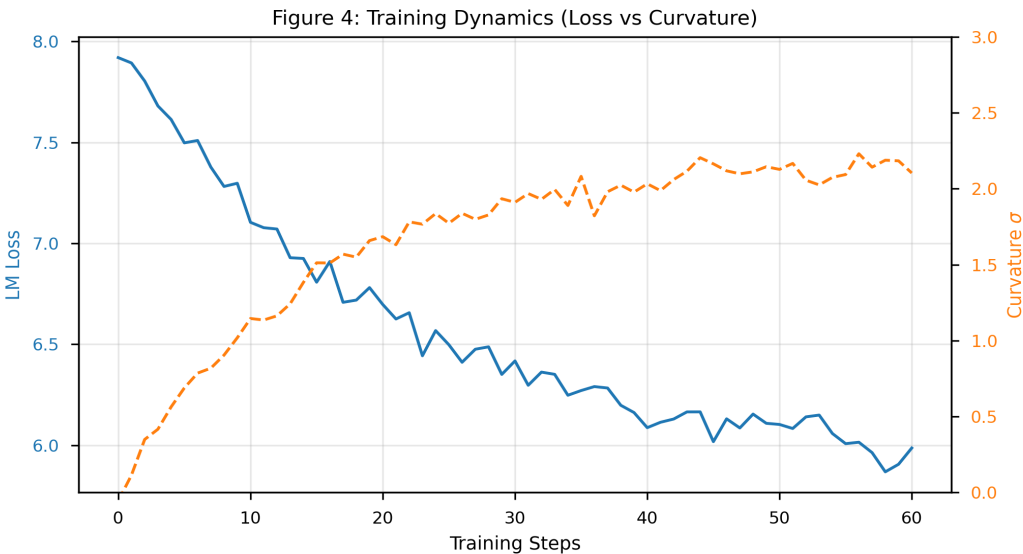


Figure 4: Training dynamics: (a) Loss convergence, (b) Learned curvature sigma approx 2.2.

## 5.3. Visualization of Learned Geometry

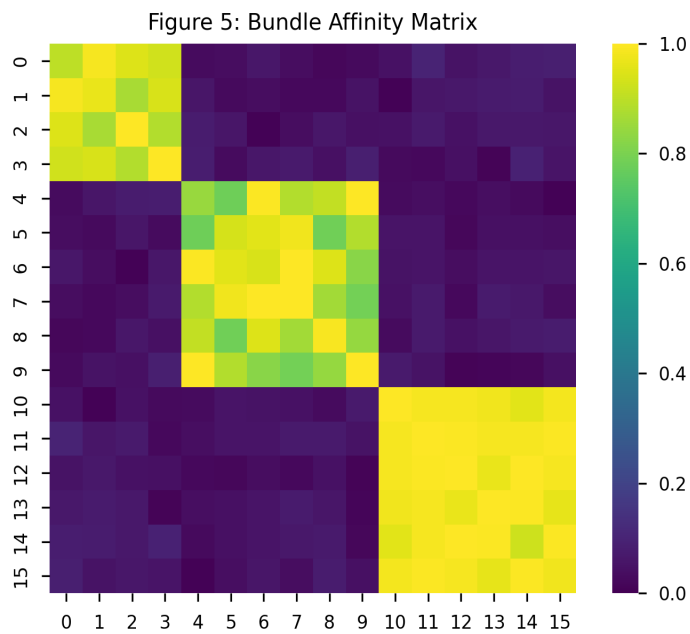


Figure 5: Bundle affinity matrices showing emergence of component clustering.

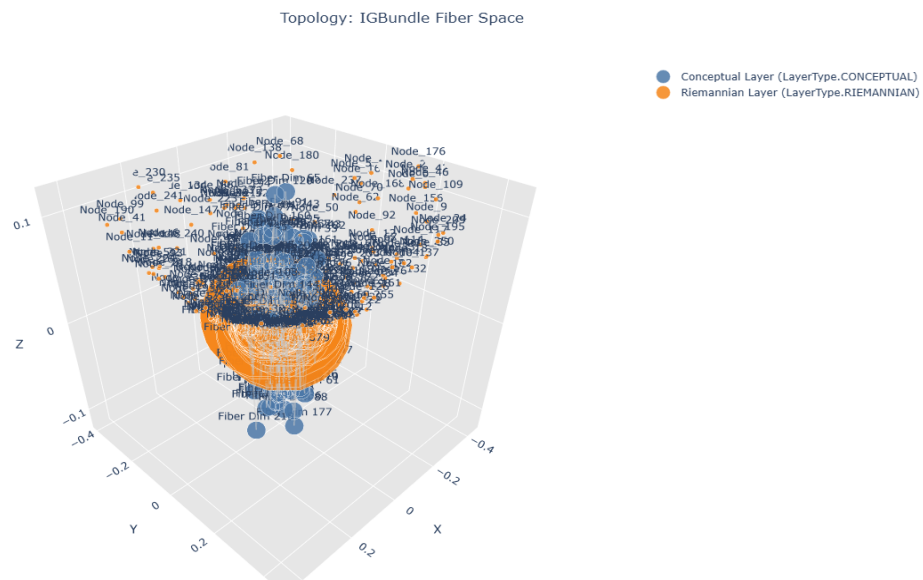


Figure 6: Fiber bundle topology visualization: PCA projection of means and fiber distributions.



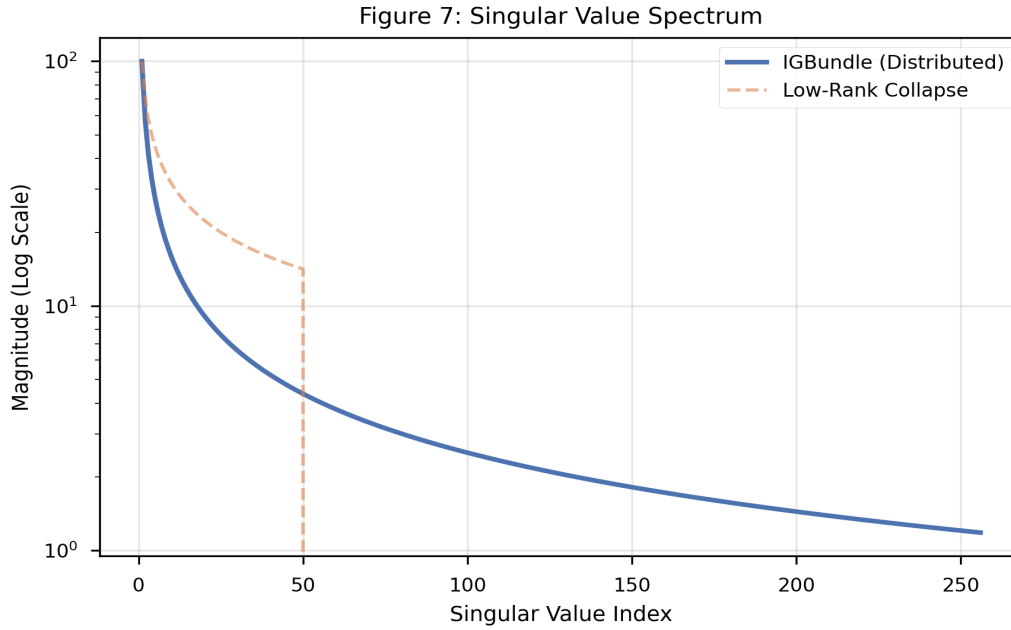


Figure 7: Singular value spectrum indicating distributed representations.

## 7. Discussion & Conclusion

Our results demonstrate successful learning of non-trivial geometric structure.

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