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COMMON NONSTATIONARY COMPONENTS OF ASSET PRICES*

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Portfolio separation in a multi-period general equilibrium context implies that asset prices are collinear. In economies that do not exhibit separation, but that move close to separation, 'approximate collinearity' or cointegration emerges as a statistical model of actual data. A new test for cointegration is proposed. It is based on time-series canonical correlation analysis and solves the problem of unidentified parameters under the null hypothesis and of identification of the cointegrating vectors when more than two time series are investigated. An empirical analysis of prices of five size-based portfolios and five industry-based portfolios of common stock reveals substantial evidence of cointegration.

1. Introduction

The attractiveness of asset pricing models such as the Capital Asset Pricing Model (CAPM) [Sharpe (1964), Lintner (1965), Mossin (1966)] or the Intertemporal Capital Asset Pricing Model [Merton (1973)] can be ascribed to the fact that they entail separation. A financial market equilibrium is characterized by 'separation' if investors' demands can be summarized by the demand for a limited number of portfolios or 'mutual funds'. Separation allows one to derive asset pricing models in which an individual asset's expected return is determined only by its covariability with the return on the mutual funds. The unattractive part of it, however, is that separation implies perfectly collinear asset prices when cast in an intertemporal general equilibrium model [Rosenberg and Ohlson (1976)]. This implication is readily rejected by the data.

In the CAPM, for instance, investors hold all risky assets in the same proportion, which means that their portfolio demands can be written solely in terms of a risk-free asset and the market portfolio of risky assets. Expected returns on individual assets can then be shown to depend only on the covariation of the asset's return with the return on the market. Let x_i and x_j denote the proportion of his wealth an investor puts in assets i and j,

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respectively. In the CAPM, x_i/x_j is invariable from individual to individual. Let $\phi_{ij} = x_i/x_j$. Since demand for an asset should equal its supply in equilibrium, it follows:

$$p_{it}m_i/p_{jt}m_j = \phi_{ij},$$

where p_{it} and p_{jt} denote the price at t of asset i and j, respectively, and m_i and m_j the number of shares in supply. Rearranging:

$$p_{it} = (m_j/m_i)\phi_{ij}p_{jt}.$$

If the investment opportunity set is constant over time, so that ϕ_{ij} is time-invariant, and if the number of shares in supply does not change either, it must follow from the above equation that prices are perfectly collinear. In particular, the price of any asset is proportional to the price of the market portfolio ($p_{it} = \beta_i p_{wt}$, where p_{wt} denotes the price of the market portfolio). But if that is the case, the CAPM can be rejected immediately, because we know that asset prices are not collinear. The traditional 'solution' to this objection is to let the number of shares outstanding change over time. But it is clear that this will not generate the cross-sectional variability in price changes we actually observe [Rosenberg and Ohlson (1976)].

This paper studies what happens if one relaxes slightly the assumptions that lead to a separating equilibrium. The idea is that, while we lose the separation property and the ensuing asset pricing relationships on the return level, a structure that is more realistic than perfect collinearity may still be present on the level of prices. Indeed, it is conjectured that, if an economy moves close to (but does not reach) the separating equilibrium, prices must be 'approximately collinear', i.e., they can be written as if separation did hold (in which case they are a linear combination of the mutual fund prices), plus a weakly dependent pricing error. If mutual fund prices in the separating economy are nonstationary in the sense that they are weakly dependent only after differencing a certain number of times, then all prices in the approximately separating equilibrium will be 'cointegrated', i.e., they can be expressed as the weighted sum of a limited number of common nonstationary factors (the prices of the mutual funds) and a weakly dependent error:

$$p_{it} = \sum_{l=1}^{k} \beta_{il}p_{lt} + \nu_{it}.$$

Here, p_{lt} denotes the (nonstationary) price on the lth mutual fund ($l = 1, \ldots, k$), and ν_{it} is the weakly dependent pricing error.

Engle and Granger (1987) define the components of a vector $\boldsymbol{p}_t$ to be cointegrated of order d, b [denoted $\boldsymbol{p}_t \sim \mathrm{CI}(d, b)$], if (i) all components of $\boldsymbol{p}_t$

are weakly dependent[1] after differencing d times, and (ii) there exists a vector $\boldsymbol{\alpha}_q$ ($\neq \mathbf{0}$) (called cointegrating vector) so that $z_{qt} = \boldsymbol{\alpha}_q \cdot \boldsymbol{p}_t$ is weakly dependent after differencing $d - b$ times. We will investigate the $d = 1$, $b = 1$ case here. If $\boldsymbol{p}_t$ has n components, then the 'cointegrating rank' of $\boldsymbol{p}_t$ is r if there exist r linearly independent vectors $\boldsymbol{\alpha}_q$ that make z_{qt} weakly dependent. In that case, there exist $n - r$ common nonstationary components that determine the evolution of $\boldsymbol{p}_t$ over time. To illustrate this with the above equation, it is clear that certain linear combinations of p_{it} will not be driven by the common nonstationary components p_{it}, which means that they will be weakly dependent, since they will be a function only of the pricing errors v_{it}. The vectors defining these linear combinations correspond to the cointegrating vectors.

It is not the aim of this paper to derive exact conditions under which asset prices are cointegrated. Instead, this paper should be viewed as an empirical investigation of cointegration in the context of asset prices. Nevertheless, a broad class of economies that generate cointegrated asset prices will be discussed in section 2, to illustrate the generality of the phenomenon.

A new method to test for cointegration is proposed that solves the problem of unidentified parameters under the null hypothesis and of identification of $\boldsymbol{\alpha}_q$ for $n > 2$ [see Engle and Granger (1987)]. It is based on time-series canonical correlation analysis of $\boldsymbol{p}_t$ and $\boldsymbol{p}_{t-1}$, and combines results in Box and Tiao (1977), Tiao and Tsay (1986) and Phillips (1987). Box and Tiao (1977) suggest the use of canonical correlation analysis to investigate patterns in stationary autoregressive vector time series. This paper extends their analysis to vector time series with unit roots. Even in this case, canonical correlation coefficients exist and can be estimated consistently, as shown in Tiao and Tsay (1986). Moreover, as will be shown in section 3, under the null hypothesis that the cointegration rank of $\boldsymbol{p}_t$ equals r, $n - r$ canonical correlation coefficients will be equal to one, whereas the remaining coefficients will be strictly less than one. The canonical variables corresponding to the r coefficients strictly smaller than one define the r cointegrating vectors $\boldsymbol{\alpha}_q$. To distinguish the unit canonical correlation coefficients from those coefficients that are strictly less than one, we can use Phillips' (1987) results on the distribution of the regression coefficient in (nonstationary) AR(1) models with serially correlated errors. Consequently, the proposed procedure provides simultaneously a test of the rank of cointegration and estimates of the cointegrating vectors $\boldsymbol{\alpha}_q$.

Johansen (1987) independently developed an estimation and testing procedure that also uses canonical correlation analysis. His approach, however, differs considerably from the one in this paper. Johansen assumes $\boldsymbol{p}_t$ follows an AR(q) process with independently and identically distributed Gaussian

[1]Engle and Granger (1987) define cointegration in terms of the stationarity of certain linear combinations of the elements in $\boldsymbol{p}_t$. We substitute weak dependence for stationarity. This does not alter the definition conceptually. We shall need weak dependence, for instance, when developing a test statistic for the rank of cointegration.

errors, which allows him to write down the likelihood function for any given sample and extract the dynamics before proceeding with the cointegration part. He comes up with a likelihood ratio test of cointegration, which, if the assumptions are true, will perform better in small samples than the procedure developed in this paper, which is equivalent to Johansen's only asymptotically. Since it is based on functional central limit theory, this paper's procedure, however, allows for more general dynamics and error processes, which is particularly important in the application considered here: changes in asset prices are known to be non-Gaussian and heteroscedastic. Moreover, the order of autoregression of asset price processes is unknown. It is hoped that the sample sizes considered in the application offset to some extent the disadvantage of using tests based on functional central limit theory.

The test proposed by Stock and Watson (1987) is based on arguments similar to the ones advanced in this paper. They use principal components analysis, on the basis that the largest eigenvalues will correspond to common nonstationary components. Calculation of the eigenvalues of the first-order autoregression matrix of the principal components allows one to distinguish between nonstationary and stationary components. The test of this paper, on the other hand, works directly with autoregression matrices.

The test is brought to the data here in an analysis of prices of five size-based common stock portfolios and five industry-based common stock portfolios. The results indicate that both sets of portfolio prices are cointegrated of rank two.

The remainder of the paper is organized as follows. Section 2 elaborates on the economics behind the cointegration of asset prices. Section 3 discusses the test procedure. Section 4 presents the empirical results. The paper ends with a conclusion.

2. Cointegrated asset prices

As an example of an economy that generates cointegrated asset prices, consider a Lucas-type, one-good pure exchange economy with a representative consumer [see Lucas (1979)], where the nonstationary dividend processes $d_{i,t+1}$ $(i = 1,\dots,n)$ are restricted, cross-sectionally, as follows:

$$d_{i,t+1} = \sum_{l} \beta_{il} d_{l,t+1} + \varepsilon_{i,t+1}, \tag{1}$$

where $\varepsilon_{i,t+1}$ is a weakly dependent innovation process and $d_{l,t+1}$ $(l = 1,\dots,k)$ are common nonstationary components. In other words, we assume that dividend processes are cointegrated with cointegration rank $r = n - k$. Assume the infinitely-lived representative consumer has time-additive, von Neumann–Morgenstern preferences with discount factor μ, defined over the single good and represented by the concave function $u(\cdot)$. A necessary (but not sufficient) condition for this economy to exhibit separation (which, as pointed out in the Introduction, will result in prices lying in a lower-dimensional space) is that

$$\mathrm{E}_t\left[\varepsilon_{i,t+1} | d_{l,t+1}, l = 1, \dots k\right] = 0, \tag{2}$$

i.e., the errors in (1), given information at t, are mean-independent of the common nonstationary dividend factors [see Bossaerts and Green (1987)]. This implies, among other things, the stringent condition that the errors in (1) be uncorrelated over time. If the mean-independence condition (2) does not hold, separation will not obtain, and prices will not necessarily lie in a lower-dimensional space. However, our economy will still move close to a separating economy, since we impose the condition that the errors in (1) be weakly dependent. The question is then: will the equilibrium prices also move close to the separating equilibrium, i.e., can we write prices as the sum of the equilibrium prices under separation and a weakly dependent error:

$$p_{it} = \sum_l \beta_{il} p_{lt} + \nu_{it}. \tag{3}$$

If $\mathrm{E}_t[\varepsilon_{i,t+1} | d_{l,t+1}, l = 1, \dots, k] = 0$, prices equal $\sum_l \beta_{il} p_{lt}$, which means that they are spanned by the prices of the common nonstationary dividend processes $d_{l,t+1}$ [Bossaerts and Green (1987), Connor and Korajczyk (1987)]. ν_{it} is the weakly dependent approximation error. Hence, the question is whether asset prices are cointegrated for an economy that moves close to, but does not reach, a separating economy. Cointegration means that the distance between actual asset prices and those that would obtain if the economy were separating remains statistically bounded.

As a simple example of an economy that leads to a weakly dependent ν_{it}, let $k = 1$, and assume the preferences of the representative consumer are exponential: $u(d_t) = -(1/a)\exp\{-a \cdot d_t\}$. Moreover, let $\varepsilon_{i,t+1}$ be a stationary AR(1) process with independently and identically, normally distributed errors $\psi_{i,t+1}$:

$$\varepsilon_{i,t+1} = \theta_i \varepsilon_{it} + \psi_{i,t+1}, \qquad |\theta_i| < 1, \qquad \psi_{i,t+1} \sim \mathrm{N}(0,1).$$

(This clearly violates the necessary condition for separation, namely $\mathrm{E}_t[\varepsilon_{i,t+1} | d_{l,t+1}, l = 1] = 0$.) This implies that $\varepsilon_{i,t+1}$ is weakly dependent [see Assumption 1 below and White (1984, p. 46)]. Let d_{t+1}, the aggregate dividend, be the single common nonstationary factor, and assume it follows a unit root AR(1) model with independently and identically, normally distributed errors ϕ_{t+1}:

$$d_{t+1} = d_t + \phi_{t+1}, \qquad \phi_{t+1} \sim \mathrm{N}(0,1).$$

$\{\psi_{i,t+1}\}$ and $\{\phi_{t+1}\}$ are independent processes, which means that the dynamics of firm i's idiosyncratic component is completely independent of the dynamics of the aggregate dividend. In order to avoid unbounded prices, assume also that $\mu\theta\cdot\exp\{\frac{1}{2}a^2\} < 1$. The price of any dividend process can now be written as follows:

$$p_{it}=\beta_i p_t+\nu_{it},$$

where p_t is the price at t of the aggregate dividend process. It is shown below, in a more general context, that ν_{it} corresponds to the price of an asset that pays $\varepsilon_{i,t+\tau}$, $\tau\geq 1$. Consequently, using the representative consumer's first-order conditions, one obtains the following expression for ν_{it}, the price of the process $\{\varepsilon_{i,t+1}\}$:

$$\begin{aligned}\nu_{it}&=\mathrm{E}_t\left[\sum_\tau \mu^\tau\,\mathrm{e}^{-a(d_{t+\tau}-d_t)}\varepsilon_{i,t+\tau}\right]\\&=\varepsilon_{it}\mathrm{E}_t\left[\sum_\tau(\mu\theta)^\tau\mathrm{e}^{-a\sum_\sigma\phi_{t+\sigma}}\right]\\&=\varepsilon_{it}\sum_\tau(\mu\theta)^\tau\prod_\sigma(\mathrm{e}^{\frac{1}{2}a^2})\\&=\varepsilon_{it}\sum_\tau\left(\mu\theta\mathrm{e}^{\frac{1}{2}a^2}\right)^\tau\\&=\varepsilon_{it}\left(1/\left(1-\mu\theta\mathrm{e}^{\frac{1}{2}a^2}\right)\right).\end{aligned}$$

The second equality arises because $\mathrm{E}_t[\psi_{t+\sigma}|\phi_{t+v},\ v=1,\dots]=0$. Consequently, ν_{it} is weakly dependent, as postulated.

To generalize this example, let us extend the class of economies under consideration and assume that the common nonstationary dividend processes and preferences are such that:

Assumption 1. All prices follow AR(1) processes, possibly with unit roots:

$$p_{it}=\gamma_i p_{i,t-1}+\eta_{it},\qquad -1<\gamma_i\leq 1.$$

The innovation process $\{\eta_{it}\}$ is weakly dependent, i.e.:

(a) $\mathrm{E}(\eta_{it})=0, \forall t$;

(b) $\sup_t\mathrm{E}|\eta_{it}|^\beta<\infty$, for some $\beta>2$;

(c) $\sigma^2=\lim_T \mathrm{E}[T^{-1}(\sum_\tau\eta_{i\tau})^2]$ exists and $\sigma^2>0$;

(d) $\{\eta_{it}\}$ is strong mixing with mixing coefficients α_m that satisfy $\sum_m\alpha_m^{1-2/\beta}<\infty$.

Assumption 2. The one-period risk-free rate, $R_{F,t+1}$, is strictly positive at all times.

In particular, we assume that $\gamma_l = 1$ for the prices of the common nonstationary dividend processes $\{d_{l,t+1}\}$. Assumption 1 at the same time specifies what will be understood by a weakly dependent innovation process in the remainder of this paper. Notice that this condition is weaker than the usual stationarity assumption, but stronger than ergodicity. We need it where we employ functional central limit theorems, such as in the proof of Lemma 1.

ν_{it} represents the price of the error process $\{\varepsilon_{i,t+1}\}$ in eq. (1). To see this, remember the representative consumer's first-order conditions (or stochastic Euler equations):

$$p_{it} = \mathrm{E}_t\left[\left(\mu\frac{u'(d_{t+1})}{u'(d_t)}(p_{i,t+1} + d_{i,t+1}\right)\right].$$

(d_{t+1} denotes the aggregate dividend.) Substituting (1) for d_{it} and (3) for p_{it} gives

$$\begin{aligned}&\sum_l \beta_{il} p_{lt} + \nu_{it}\\ &= \mathrm{E}_t\left[\mu\frac{u'(d_{t+1})}{u'(d_t)}\left(\sum_l \beta_{il} p_{l,t+1} + \nu_{i,t+1} + \sum_l \beta_{il} d_{l,t+1} + \varepsilon_{i,t+1}\right)\right].\end{aligned}$$

Using

$$p_{lt} = \mathrm{E}_t\left[\mu\frac{u'(d_{t+1})}{u'(d_t)}(p_{l,t+1} + d_{l,t+1})\right]$$

in the above Equation leads to the following expression for ν_{it}:

$$\nu_{it} = \mathrm{E}_t\left[\sum_\tau \mu^\tau \frac{u'(d_{t+\tau})}{u'(d_t)}\varepsilon_{i,t+\tau}\right]. \tag{4}$$

In other words, ν_{it} is the price of an asset that pays $\varepsilon_{i,t+\tau}$ at $t+\tau$ ($\tau = 1, \dots$), as postulated. It is possible to construct this payoff pattern by buying asset i and selling short β_{il} units of the common nonstationary dividend processes; ν_{it} is the price of this portfolio.

In order for asset prices to be cointegrated for this class of economies, it is necessary that γ_i^ν in

$$\nu_{it} = \gamma_i^\nu \nu_{i,t-1} + \eta_{it}^\nu$$

is strictly less than one. The following lemma is proved in the appendix.

Lemma 1. Under Assumptions 1 and 2, $\gamma_i^\nu < 1$.

The idea behind the proof is that if $\gamma_i^\nu = 1$, then at some time in the future, there will for sure be arbitrage opportunities. This is so because at a certain moment, ν_{it} may become very large compared to its dividend $(\varepsilon_{i,t+1})$ and its price change over the next time interval $(\eta^\nu_{i,t+1})$, so that constructing a zero-investment portfolio by going short the portfolio that mimicks $\varepsilon_{i,t+1}$ and by going long the one-period risk-free asset, will leave you with positive payoffs with a probability arbitrarily close to one. Of course, the proof depends critically on the weak dependency of the processes $\{\varepsilon_{i,t+1}\}$ and $\{\eta^\nu_{it}\}$. Assumption 2 is needed to assure strictly positive arbitrage profits. Consequently, to avoid arbitrage opportunities, ν_{it} must have $\gamma_i^\nu < 1$, which is a necessary condition for prices to be cointegrated in the class of economies we discussed so far.

So, we shall assume that prices obey the following statistical model:

$$p_{it} = \sum_{l=1}^{k} \beta_{il} p_{lt} + \nu_{it}, \qquad k < n.$$

ν_{it} denotes the weakly dependent pricing error. Hence, $n-k$ linear combinations of asset prices p_{it} will generate weakly dependent random variables. We shall assume, in addition, that p_{lt} $(l = 1, \dots, k)$ is weakly dependent after taking first differences. Combining both assumptions, it follows that the vector consisting of prices (let $\boldsymbol{p}_t$ denote this vector) is cointegrated [in the sense of Engle and Granger (1987)], of order 1, with cointegrating rank equal to $r = n-k$, i.e., $\boldsymbol{p}_t \sim \mathrm{CI}(1,1)$.

To put things into perspective, notice that, unlike Granger and Escribano (1987), we do not claim that evidence of cointegration implies market inefficiency (cointegration namely implies that certain linear combinations of asset prices are expected to add up to zero in the long run; any deviation in the short run is expected to be temporary, hence one can use this information to partially predict future prices). Cointegration arises because of the approximate separation properties of certain economies with a risk-averse representative consumer.

Also, notice how our analysis bears resemblance to the one in Fama and French (1986), where permanent and temporary components of stock prices are studied. Unlike Fama and French, however, we refrain from taking first differences of (4), since we are interested in the number of nonstationary components, as opposed to the existence of stationary components. Indeed, information on the number of nonstationary components would get lost after differencing, a well-known fact from the unit root and cointegration literature.

3. A canonical correlation test of cointegration

A new procedure for detecting cointegration is proposed here. It combines results in Box and Tiao (1977) and Tiao and Tsay (1986) on time series canonical correlation analysis and in Phillips (1987) on unit roots.

The most widely used test of cointegration is based on Engle and Granger (1987). It is a two-step analysis. The first step consists in regressing an element p_{it} onto other elements of $\boldsymbol{p}_t$ (a levels regression). The second step involves a Dickey–Fuller test for unit roots applied to the error from the first-step regression. This testing procedure has two major disadvantages, though. One is the identification of the cointegrating vectors from the first-step regressions for $n > 2$. Indeed, while the number of first-step regressions that can be performed increases dramatically beyond $n = 2$, it is not clear whether each of them gives rise to a different (i.e., linearly independent) cointegrating vector. Further information is needed to solve this identification problem. However, in most cases, such as in the present study, this information is simply not available. The second drawback is theoretical: the step 2 cointegration test uses parameters (from the step 1 regression) that are unidentified under the null hypothesis of no cointegration.

This paper suggests a new procedure as an answer to the above objections to the Engle–Granger test. The idea is the following. Assume $\boldsymbol{p}_t$ is cointegrated with cointegration rank r and cointegrating vectors $\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_r$. In a first step, perform a canonical correlation analysis of $\boldsymbol{p}_t$ and $\boldsymbol{p}_{t-1}$. Let span($\{\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_r\}$) denote the span of the vectors $\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_r$. Also, if $\boldsymbol{x}$ and $\boldsymbol{y}$ are two vectors in R^n, let $\boldsymbol{x} \cdot \boldsymbol{y}$ denote their inner product $(= \sum_i x_i y_i)$. If $\boldsymbol{A}$ is an $n \times n$ matrix, let $\boldsymbol{xAy}$ be the corresponding quadratic form of $\boldsymbol{x}$ and $\boldsymbol{y}$. If D is any subset of R^n, let $D^c = \{ y | y \in R^n \text{ and } y \notin D \}$. Canonical correlation analysis searches for linear combinations of elements of $\boldsymbol{p}_t$ (which define canonical variables $\boldsymbol{e}^+ \cdot \boldsymbol{p}_t$) and linear combinations of $\boldsymbol{p}_{t-1}$ (generating corresponding canonical variables $\boldsymbol{e}^- \cdot \boldsymbol{p}_{t-1}$) that are maximally correlated, subject to a normalization constraint. Let $\boldsymbol{\Sigma}_{++}$ and $\boldsymbol{\Sigma}_{--}$ be the variance–covariance matrix of $\boldsymbol{p}_t$ and $\boldsymbol{p}_{t-1}$, respectively. Let Σ_{+-} be the covariance between $\boldsymbol{p}_t$ and $\boldsymbol{p}_{t-1}$, and let $\boldsymbol{\Sigma}_{-+} = \Sigma'_{+-}$. Canonical correlation analysis solves the following maximization problem:

$$\text{maximize} \quad \left\{ \boldsymbol{e}^+ \boldsymbol{\Sigma}_{+-} \boldsymbol{e}^- \big/ \left[\boldsymbol{e}^+ \boldsymbol{\Sigma}_{++} \boldsymbol{e}^+ . \boldsymbol{e}^- \boldsymbol{\Sigma}_{--} \boldsymbol{e}^- \right]^{1/2} \right\},$$

$$\text{subject to} \quad \boldsymbol{e}^+ \boldsymbol{\Sigma}_{++} \boldsymbol{e}^+ = 1, \ \boldsymbol{e}^- \boldsymbol{\Sigma}_{--} \boldsymbol{e}^- = 1.$$

However, $\boldsymbol{p}_t$ is assumed to be a nonstationary random vector, which implies that the constraints are not well defined, since neither $\boldsymbol{\Sigma}_{++}$ nor $\boldsymbol{\Sigma}_{--}$ exists asymptotically. Consequently, the problem has to be restated in such a way that it only involves quantities that may be well defined even asymptotically. The strategy chosen here is to rewrite the canonical correlations problem in such a way that its solution only involves autoregression matrices. Hence, attention will be restricted to those processes that have an AR(1) representation with unit roots. Ordinary Least Squares estimators of the AR(1) coefficient matrix will be assumed to be consistent. This is proved in Tiao and Tsay (1986) for AR(1) processes with uncorrelated errors. Phillips (1987) shows it for univariate AR(1) processes with weakly dependent errors. This result may be extended to multivariate processes along the lines of Stock (1987), although no proof is attempted here. Hence, we assume that the OLS matrices $\Gamma = \Sigma_{--}^{-1}\Sigma_{-+}$ and $\Gamma^* = \Sigma_{++}^{-1}\Sigma_{+-}$ are consistent estimates of the corresponding AR(1) matrices.

Let us redefine the constraints in the canonical correlations problem as follows:

$$e^+ \cdot e^+ = 1, \qquad e^- \cdot e^- = 1.$$

In words, instead of imposing on the canonical variables that they have unit variance, one requires only that the vectors defining the canonical variables be of unit length. This leads to the following Lagrangean:

$$\begin{aligned}\mathscr{L} = {} & \ln e^+\Sigma_{+-}e^- - \tfrac{1}{2}\ln e^+\Sigma_{++}e^+ - \tfrac{1}{2}\ln e^-\Sigma_{--}e^- \\ & + \lambda(1 - e^+\cdot e^+) + \mu(1 - e^-\cdot e^-).\end{aligned}$$

An analysis of the first-order conditions implies that $\lambda = \mu = 0$, and

$$\Sigma_{--}^{-1}\Sigma_{-+}\Sigma_{++}^{-1}\Sigma_{+-}e^- = \theta e^-, \qquad \Sigma_{++}^{-1}\Sigma_{+-}\Sigma_{--}^{-1}\Sigma_{-+}e^+ = \theta e^+.$$

In words: e^+ and e^- are eigenvectors of $\Gamma^*\Gamma$ and $\Gamma\Gamma^*$, respectively, and both correspond to the same eigenvalue θ. Moreover, θ can be shown to equal the squared correlation coefficient between $e^+ \cdot p_t$ and $e^- \cdot p_{t-1}$. Unlike in traditional canonical correlation analysis, this solution is well defined even in the presence of nonstationary variables, since it entails only autoregression matrices, which, under certain conditions, do exist.

Any two different eigenvectors e_i^+ and e_j^+ of $\Gamma^*\Gamma$, or any two different eigenvectors e_i^- and e_j^- of $\Gamma\Gamma^*$, define canonical variables that are mutually uncorrelated. Indeed, disregarding the scaling factor $[e_i^+\Sigma_{++}e_i^+ \cdot e_j^+\Sigma_{++}e_j^+]^{-1/2}$:

$$\begin{aligned} e_i^+\Sigma_{++}e_j^+ &= (1/\theta_j)e_i^+\Sigma_{++}\Sigma_{++}^{-1}\Sigma_{+-}\Sigma_{--}^{-1}\Sigma_{-+}e_j^+ \\ &= (\theta_i/\theta_j)e_i^+\Sigma_{++}e_j^+. \end{aligned}$$

Since $\theta_i \neq \theta_j$, $e_i^+\Sigma_{++}e_j^+ = 0$, for all i, j. The same holds with respect to $e_i^-\Sigma_{++}e_j^-$.

How does canonical correlation analysis pick cointegrating vectors? It is clear that the k first canonical variables will be nonstationary. Moreover,

$\boldsymbol{e}_i^+ = \boldsymbol{e}_i^-$, $i = 1, \ldots, k$, asymptotically, since that will give maximum correlation (namely unity). The crucial point is that the $n - k$ $(= r)$ remaining canonical variables cannot be defined by vectors $\boldsymbol{e}^+$ and $\boldsymbol{e}^-$ in span($\{\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_r\})^c$, since in that case they would be nonstationary, and cointegrated with the k first canonical variables, which means that they would be correlated with some or all of the first k canonical variables. Such a set of candidate canonical variables does not satisfy the requirement that they be mutually uncorrelated. Consequently, the last $n - k$ canonical variables must be defined by elements from span($\{\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_r)\}$, i.e., they will be cointegrating vectors, since only then will they be weakly dependent and, hence, asymptotically uncorrelated with the first k (nonstationary) canonical variables. So, the last $n - k$ canonical vectors will provide $n - k$ linearly independent cointegrating vectors.

Let $c_{tj}^+ = \boldsymbol{e}_j^+ \cdot \boldsymbol{p}_t$, a canonical variable constructed from $\boldsymbol{p}_t$, and $c_{tj}^- = \boldsymbol{e}_j^- \cdot \boldsymbol{p}_{t-1}$, the corresponding canonical variable constructed from $\boldsymbol{p}_{t-1}$. The second step of our procedure is a test of the rank of cointegration. It is a test that aims at statistically distinguishing canonical correlation coefficients that equal one from those that are less than one. For this purpose, the first-order autoregression coefficient of each canonical variable of $\boldsymbol{p}_t$ and $\boldsymbol{p}_{t-1}$ is computed:

$$c_{tj}^+ = \rho_j^+ \cdot c_{t-1,j}^+ + \nu_{tj}^+,$$

$$c_{tj}^- = \rho_j^- \cdot c_{t-1,j}^- + \nu_{tj}^-.$$

The regression coefficients equal one if the particular linear combination $\boldsymbol{e}_j^+$ or $\boldsymbol{e}_j^-$ does not coincide with a cointegrating vector. This test is obviously a unit root test, and Phillips' (1987) test statistic can be used for verification purposes, since ν_{tj}^+ and ν_{tj}^- are weakly dependent under the assumption that the common nonstationary components are weakly dependent after differencing once.

Of course, ρ_j^+ is not necessarily equal to ρ_j^-, since in general: $\boldsymbol{e}_j^+ \neq \boldsymbol{e}_j^-$. However, eigenvectors $\boldsymbol{e}_j^+$ and $\boldsymbol{e}_j^-$ in span($\{\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_r\})^c$ must converge asymptotically, as mentioned above. Consequently, one can get an idea of the appropriateness of a given sample size by the distance between $\boldsymbol{e}_j^+$ and $\boldsymbol{e}_j^-$ for ρ_j^+ or ρ_j^- statistically insignificantly different from one. This distance should be small. In the application of cointegration of section 4, it is measured by the angle between $\boldsymbol{e}_j^+$ and $\boldsymbol{e}_j^-$.

Notice, in contrast to the Engle–Granger test for cointegration, that the proposed procedure automatically selects the cointegrating vectors, because they will correspond to the canonical variables with canonical correlation strictly less than one, and that the Phillips test can be applied without having to worry about some parameters not being identified under the null hypothesis.

Summarizing, the suggested test of cointegration consists of two steps. In the first one, a canonical correlation analysis is performed to find the (mutually uncorrelated) linear combinations of elements of $\boldsymbol{p}_t$ and of $\boldsymbol{p}_{t-1}$ that are maximally correlated. In the second step, autoregression coefficients of each canonical variable of $\boldsymbol{p}_t$ and $\boldsymbol{p}_{t-1}$ are computed, and the Phillips unit root test is applied. Those canonical variables for which the unit root hypothesis is rejected automatically deliver the cointegrating vectors.

4. Empirical results

The test procedure was brought to the data in an investigation of the number of common nonstationary components of stock prices. Two sets of five value-weighted portfolios were constructed from the CRSP monthly returns file (hence, only NYSE firms were included) for the period January 1926 to December 1984 (708 observations), the first one on the basis of firm size (market value of equity), the other one based on industry codes. The portfolio grouping was done at ten-year intervals, and only securities with continuous price and return histories over the ten years were included. The five size-based portfolios represent the five quintiles of firm size. The five industry-based portfolios are: (1) Mining and Construction (SIC codes 10 to 19), (2) Manufacturing, Transportation, Communications, Electric, Gas and Sanitary Services (SIC codes 20 to 49), (3) Wholesale and Retail Trade (SIC codes 50 to 59), (4) Finance, Insurance and Real Estate (SIC codes 60 to 69), (5) Services (SIC codes 70 to 89). Monthly wealth relatives based on the returns without dividends were computed. These represent the actual values of a portfolio which an investor would have been able to purchase at an initial date and hold to some horizon date with neither intermediate investment nor withdrawals, aside from the dividend payments accruing to the position, which would have to be consumed, just as in the Lucas-type of economies discussed in section 2. Both nominal and real wealth relatives (henceforth called 'prices') were calculated. The consumer price series that was employed to deflate the portfolio prices was taken from Ibbotson (1985).

A canonical correlation analysis was performed on both data sets. First-order autoregression coefficients were computed for each canonical variable and Phillips statistics were computed. The results are displayed in table 1. Notice that this table includes a column with the angle δ_j between $\boldsymbol{e}_j^+$ and $\boldsymbol{e}_j^-$ (in radians), defined as: $\delta_j = \cos^{-1}(\boldsymbol{e}_j^+ \cdot \boldsymbol{e}_j^- / (\|\boldsymbol{e}_j^+\| \; \|\boldsymbol{e}_j^-\|)^{1/2})$ ($\|\boldsymbol{x}\|$ denotes the Euclidean norm of $\boldsymbol{x}$). δ_j provides a rough indication of how close $\boldsymbol{e}_j^+$ and $\boldsymbol{e}_j^-$ are. Ideally, δ_j should not be very different from 0 or π, except for eigenvectors that converge to a cointegrating vector, as discussed in section 3. In these particular samples, however, all $\boldsymbol{e}_j^+$ and $\boldsymbol{e}_j^-$ turn out to be very close.

Table 1 reveals clear evidence of cointegration. Using a cut-off significance level of 0.01, one can conclude that prices are driven by two common nonstationary components. The results do not change as one deflates nominal

Table 1

Test results using monthly prices of five portfolios (1/26 to 12/84, 708 observations).

j[a]	δ_j	ρ_j^+: $\hat{\rho}$[b]	ρ_j^+: Z[c]	ρ_j^-: $\hat{\rho}$[d]	ρ_j^-: Z[c]
A1	3.076	1.007	5.264	1.007	5.256
A2	0.061	0.998	−1.239	0.998	−1.267
A3	0.009	0.984	−11.080	0.985	−10.880
A4	0.038	0.932	−48.349[e]	0.932	−48.344[e]
A5	0.029	0.884	−82.312[e]	0.884	−81.874[e]
B1	0.029	1.004	2.534	1.004	2.544
B2	0.032	0.995	−3.342	0.995	−3.344
B3	3.139	0.986	−9.991	0.986	−9.967
B4	0.020	0.953	−33.239[e]	0.953	−33.314[e]
B5	3.101	0.927	−51.374[e]	0.928	−51.164[e]
C1	0.075	1.002	1.467	1.002	1.470
C2	0.074	0.998	−1.515	0.998	−1.518
C3	0.004	0.982	−12.749	0.982	−12.642
C4	0.006	0.958	−30.040[e]	0.958	−30.042[e]
C5	0.005	0.938	−44.080[e]	0.938	−44.042[e]
D1	0.014	0.997	−2.320	0.997	−2.319
D2	3.135	0.995	−3.212	0.995	−3.208
D3	0.004	0.986	−9.669	0.986	−9.675
D4	3.133	0.975	−17.586[e]	0.975	−17.600[e]
D5	0.007	0.963	−26.208[e]	0.963	−26.205[e]

[a] Canonical variables A1 to A5 computed from nominal prices of size-based portfolios; B1 to B5 from nominal prices of industry-based portfolios; C1 to C5 from real prices of size-based portfolios; and D1 to D5 from real prices of industry-based portfolios.

[b] Estimate of ρ_j^+.

[c] Corresponding Phillips statistic.

[d] Estimate of ρ_j^-.

[e] Significant at the 0.01 level.

prices using the consumer price index. In any event, the null hypothesis of no cointegration can be rejected at very high significance levels. Notice, also, that the results are identical, whether one uses e_j^+ or e_j^- in the test for cointegration. This, of course, is a reflection of the closeness of e_j^+ and e_j^-. The results for the subperiods January 1926 to June 1954 and July 1954 to December 1984 are similar. They are not reported here.[2]

To illustrate cointegration of asset prices in a more intuitive way, the following OLS regressions were performed:

$$p_{it} = \sum_{j \neq i} \omega_j p_{jt} + \xi_{it}.$$

[2] The subperiod results illustrated how unreliable the test procedure becomes as one decreases the sample size: the angles δ_j between the eigenvectors e_j^+ and e_j^- increased considerably in many instances. However, no ambiguity arose when trying to distinguish between roots on and off the unit circle. As mentioned in the Introduction, a test based on the likelihood ratio, such as Johansen's, would have been more appropriate for analyzing these smaller samples.

Table 2

Autocorrelations from the regression: $p_{it} = \Sigma_{j \neq i} \omega_j p_{jt} + \xi_{it}$, using monthly nominal prices of five portfolios (1/26 to 12/84, 708 observations).

	Portfolios				
	1	2	3	4	5
1[a]	0.899	0.899	0.929	0.918	0.963
2	0.799	0.790	0.858	0.838	0.930
3	0.697	0.676	0.807	0.756	0.902
4	0.642	0.586	0.764	0.698	0.876
5	0.608	0.512	0.712	0.651	0.848
6	0.580	0.446	0.656	0.607	0.820
7	0.541	0.370	0.612	0.560	0.797
8	0.502	0.281	0.573	0.515	0.773
9	0.456	0.210	0.527	0.461	0.744
10	0.416	0.163	0.484	0.427	0.722
11	0.390	0.144	0.450	0.408	0.709
12	0.373	0.151	0.426	0.393	0.696
13	0.326	0.140	0.399	0.353	0.672
14	0.279	0.106	0.373	0.308	0.648
15	0.233	0.090	0.360	0.276	0.635
16	0.199	0.089	0.366	0.254	0.624
17	0.157	0.106	0.350	0.239	0.618
18	0.128	0.149	0.323	0.249	0.620
19	0.112	0.194	0.313	0.265	0.623
1[b]	0.929	0.974	0.958	0.936	0.948
2	0.846	0.948	0.910	0.865	0.896
3	0.773	0.930	0.858	0.790	0.857
4	0.706	0.910	0.808	0.721	0.816
5	0.639	0.891	0.760	0.649	0.779
6	0.555	0.878	0.705	0.559	0.749
7	0.477	0.868	0.663	0.478	0.724
8	0.411	0.860	0.615	0.402	0.705
9	0.343	0.846	0.577	0.342	0.675
10	0.279	0.832	0.538	0.283	0.643
11	0.226	0.821	0.501	0.229	0.606
12	0.180	0.814	0.463	0.179	0.579
13	0.136	0.798	0.426	0.132	0.546
14	0.090	0.782	0.400	0.093	0.503
15	0.055	0.769	0.384	0.070	0.465
16	0.027	0.755	0.371	0.056	0.419
17	0.009	0.741	0.364	0.051	0.369
18	−0.017	0.729	0.358	0.042	0.321
19	−0.038	0.719	0.350	0.035	0.271

[a]Autocorrelations (from 1 to 19) of ξ_{it} for size-based portfolios (portfolio 1: smallest quintile, portfolio 5: largest quintile).

[b]Autocorrelations (from 1 to 19) of ξ_{it} for industry-based portfolios.

Table 3

Autocorrelations from the regression: $p_{it} = \sum_{j \neq i} \omega_j p_{jt} + \xi_{it}$, using monthly real prices of five portfolios (1/26 to 12/84, 708 observations).

	Portfolios				
	1	2	3	4	5
1[a]	0.966	0.954	0.957	0.961	0.965
2	0.932	0.905	0.912	0.922	0.929
3	0.897	0.863	0.874	0.887	0.900
4	0.871	0.821	0.836	0.855	0.871
5	0.846	0.777	0.793	0.822	0.839
6	0.822	0.731	0.749	0.786	0.800
7	0.795	0.690	0.709	0.752	0.766
8	0.767	0.652	0.670	0.720	0.734
9	0.740	0.618	0.629	0.683	0.697
10	0.712	0.585	0.586	0.652	0.663
11	0.688	0.563	0.552	0.627	0.637
12	0.666	0.552	0.525	0.608	0.618
13	0.637	0.533	0.494	0.580	0.592
14	0.606	0.506	0.458	0.551	0.564
15	0.579	0.491	0.432	0.529	0.546
16	0.553	0.476	0.412	0.508	0.527
17	0.521	0.464	0.386	0.491	0.513
18	0.490	0.456	0.357	0.482	0.504
19	0.464	0.448	0.338	0.473	0.495
1[b]	0.975	0.989	0.977	0.965	0.981
2	0.948	0.979	0.955	0.932	0.963
3	0.921	0.970	0.928	0.893	0.947
4	0.896	0.958	0.904	0.857	0.931
5	0.868	0.948	0.881	0.819	0.918
6	0.838	0.938	0.854	0.778	0.901
7	0.810	0.928	0.832	0.740	0.886
8	0.785	0.918	0.808	0.703	0.868
9	0.759	0.906	0.789	0.674	0.848
10	0.731	0.892	0.767	0.642	0.830
11	0.708	0.880	0.746	0.614	0.811
12	0.686	0.869	0.724	0.586	0.793
13	0.662	0.856	0.698	0.554	0.773
14	0.635	0.844	0.674	0.521	0.751
15	0.611	0.832	0.655	0.496	0.730
16	0.587	0.819	0.639	0.471	0.706
17	0.565	0.808	0.623	0.445	0.683
18	0.541	0.796	0.605	0.420	0.660
19	0.516	0.785	0.588	0.394	0.637

[a]Autocorrelations (from 1 to 19) of ξ_{it} for size-based portfolios (portfolio 1: smallest quintile, portfolio 5: largest quintile).

[b]Autocorrelations (from 1 to 19) of ξ_{it} for industry-based portfolios.

Autocorrelations of ξ_{it} were calculated for lags one to nineteen. These should die out at least geometrically, if p_{it} is cointegrated with one of the p_{jt}, because in that case ξ_{it} will be stationary. The autocorrelations are reported in tables 2 and 3. Table 2 displays the results for nominal prices of size-based and industry-based portfolios. Table 3 displays the results for real prices of these portfolios. Notice how, in most cases, the autocorrelation function dies out fairly quickly. This should be interpreted as rough evidence of cointegration. The formal test is provided in table 1, using canonical correlations.

A closer examination of the empirical results unveils two interesting facts. First, it is clear from table 1 that the cointegrating vectors define variables that, although stationary, are nevertheless highly autocorrelated. In other words, it may take a substantial lapse of time before prices that deviate from their common nonstationary trend revert to this trend. Further research should clarify (1) what type of economies may have generated this phenomenon, and (2) how this finding is related to the evidence of mean-reverting stock prices that Fama and French (1987) and Poterba and Summers (1987) come up with.

Second, table 2 indicates, roughly, that the large-firm portfolio (the fifth portfolio) is not cointegrated with any other portfolio, because the autocorrelation function of ξ_{it} does not die out geometrically. Table 2 displays a similar result for the second industry portfolio. (This result is not independent of the finding for the large-firm portfolio, however, since there appears to be a substantial overlapping between firms in the second industry portfolio and in the large-firm portfolio.) Further research should clarify this somewhat puzzling result. As may be inferred from table 3, the evidence is less clear-cut for real prices.

4. Conclusion

Portfolio separation in a multi-period general equilibrium context implies that asset prices are collinear. If an economy moves close to, but does not attain, separation, the ensuing pricing error implies that asset prices will be cointegrated, i.e., they will be driven by common nonstationary components and a weakly dependent pricing error.

This paper provided a theoretical illustration of cointegrated asset prices. It also proposed a new test for cointegration that circumvents problems encountered in the traditional Engle–Granger test. Our procedure is based on time series canonical correlation, and combines results in Box and Tiao (1977), Tiao and Tsay (1986) and Phillips (1987).

We used the test in an investigation of the number of common nonstationary components of asset prices in two sets of common stock portfolios. In each case we were able to reject the null hypothesis of no cointegration at very high levels of significance. The results did not change appreciably after deflating the prices using the consumer price index.

Appendix: Proof of Lemma 1

Suppose $\gamma_i^{\nu} = 1$. If that is the case, arbitrage opportunities will exist, in the sense that at some time T a zero investment portfolio can be constructed generating positive payoffs over the next period with a probability arbitrarily close to one. Consider investing in x (> 0) units of the risk-free asset. Since its price equals $1/(1 + R_{F,T+1})$, $x/(1 + R_{F,T+1})$ 'dollars' will have to be put up. Finance this by going short $-x/(1 + R_{F,T+1})$ 'dollars' of the portfolio that mimicks the payoff pattern $\{\varepsilon_{i,t+1}\}$. Since the price of the latter equals ν_{iT}, $-x/((1 + R_{F,T+1})\nu_{iT})$ units of it will have to be sold. At $T+1$, the payoff on this zero-investment portfolio is

$$
\begin{aligned}
&-\frac{x}{1+R_{F,T+1}}\frac{1}{\nu_{iT}}\left(\varepsilon_{i,T+1}+\nu_{i,T+1}\right)+x\frac{1}{1+R_{F,T+1}}\left(1+R_{F,T+1}\right)\\
&=-\frac{x}{1+R_{F,T+1}}\frac{\varepsilon_{i,T+1}+\eta^{\nu}_{i,T+1}+\nu_{iT}}{\nu_{iT}}+x\\
&=x\left(1-\frac{1}{1+R_{F,T+1}}\right)-\frac{x}{1+R_{F,T+1}}\frac{\varepsilon_{i,T+1}+\eta^{\nu}_{i,T+1}}{\nu_{iT}}.
\end{aligned}
\tag{A.1}
$$

By Assumption 2, the first term in the last expression is strictly positive. As to the second term, rewrite it as follows:

$$
\frac{x}{1+R_{F,T+1}}\frac{\varepsilon_{i,T+1}+\eta^{\nu}_{i,T+1}}{\nu_{iT}}=\frac{x}{1+R_{F,T+1}}\frac{\left(\varepsilon_{i,T+1}+\eta^{\nu}_{i,T+1}\right)/\sigma\sqrt{T}}{\nu_{iT}/\sigma\sqrt{T}},
$$

where $\sigma^2 = \lim_T \mathrm{E}[T^{-1}(\sum_\tau \eta^{\nu}_{i\tau})^2]$ ($< \infty$, by Assumption 1). Using Herrndorf's functional central limit theorem [Herrndorf (1984)] and the Continuous Mapping Theorem [see, e.g., Pollard (1984)], we conclude that one over the denominator converges in distribution to the value at 1 of the reciprocal of a standard Wiener variable with sample path defined over the interval $[0,1]$. The numerator obviously converges to zero in probability. Consequently, using a result in White (1984, p. 63), the second term in the last expression of (A.1) will converge to zero in probability, which means that the total payoff on this zero-investment portfolio will become strictly positive as T increases, with probability arbitrarily close to one.

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