Pairs-Trading a Sparse Synthetic Control

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Abstract

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1. Introduction

Pairs trading is widely recognized as a cornerstone of statistical arbitrage, offering a market-neutral investment approach that exploits temporary divergences in the prices of historically correlated or economically linked assets. By simultaneously taking a long position in the relatively undervalued asset and a short position in the relatively overvalued one, pairs traders aim to profit from the eventual convergence of these prices. This strategy has garnered enduring prominence among quantitative researchers and practitioners, attributing its appeal to both conceptual simplicity–focusing on the relative mispricing of two assets–and the potential for stable returns independent of broader market movements.

While pairs trading is conceptually straightforward, its effective implementation faces notable complexities in practice. Traditional approaches often rely on simple distance measures or cointegration-based criteria to identify pairs and establish entry and exit rules. However, these methods can be hampered by strict parametric assumptions, sensitivity to transient noise, and an inability to adapt to evolving market conditions. Structural breaks, non-linear dependencies, and time-varying correlation patterns often violate the assumptions of classical linear models, increasing the risk of identifying spurious relationships and making it difficult to achieve stable performance over diverse market regimes.

To address these challenges, recent research has explored more flexible frameworks that combine advanced econometric tools with statistical learning. In particular, incorporating synthetic control methodologies and copula-based dependence modeling aims to better capture the dynamic interactions between assets. By abandoning the sole reliance on fixed, potentially fragile pair relationships, such approaches promise to more robustly uncover the underlying economic or statistical linkages that drive temporary mispricings, thus laying the groundwork for improved performance and risk control in pairs trading strategies.

Building on the challenges and limitations outlined above, this paper proposes a novel pairs trading framework that integrates sparse synthetic control methods with copula-based dependence modeling. The primary research question we aim to answer is: "*Can the integration of sparse synthetic control and copula-based dependence modeling improve the performance of pairs trading strategies?*" To address this question, we design a methodology that overcomes several shortcomings of traditional pairs trading.

First, rather than relying on a fixed or pre-specified partner asset, we construct a *synthetic asset* through a sparse linear combination of assets from a larger donor pool. This allows the framework to discover the most influential contributors to the target asset's behavior, effectively automating pair selection. By enforcing sparsity in the weight vector, we reduce computational complexity and enhance interpretability, while mitigating overfitting risks in thinner markets.

Second, we incorporate copula-based dependence modeling to capture potentially complex, non-linear relationships and tail dependencies that can arise in financial returns. Unlike correlation- or cointegration-based strategies, which often impose strict distributional assumptions, copulas decouple the marginal distributions from the joint dependence structure, thereby offering a more nuanced view of how assets co-move. This feature is especially important in periods of market stress, when returns frequently exhibit heightened correlations and non-linearities.

Finally, we adapt and extend the Mispricing Index (MI) strategy of Xie et al. (2016) by introducing a Cumulative Mispricing Index (CMI) that resets upon trade closure, ensuring that stale signals do not accumulate across different trading episodes. As in Rad et al. (2016), we adopt an "*AND-OR*" logic for opening and closing positions, requiring persistent mispricing signals from both the target and synthetic assets to initiate a trade and closing positions promptly when either market correction or stop-loss conditions are met.

The remainder of this paper proceeds as follows. In Section 1 we begin by reviewing the relevant literature on pairs trading, synthetic control methods, and copula-based dependence modeling. In Section 2 we present our methodological framework, detailing how sparse synthetic control and copula families are jointly employed to construct a robust trading signal, and introduce the mispricing index (MI) strategy adapted to incorporate copula-driven signals. Subsequently, in Section 3 we conduct an empirical evaluation using real-world market data, illustrating the performance and practical implications of our approach. We conclude in Section 4 by summarizing key insights, discussing limitations, and outlining prospective directions for future research.

1.1 Literature Review

"*Pairs Trading Classics*"

- Gatev et al. (2006)
- Elliott et al. (2005)

"*Cointegration-based*"

- Vidyamurthy (2004)
- Caldeira and Moura (2013)
- Huck and Afawubo (2014)

- Cartea and Jaimungal (2015)
- Lintilhac and Tourin (2016)

"*Empirical investigations of pairs trading*"

- Chen et al. (2019)
- Do and Faff (2010)
- Bowen and Hutchinson (2014)
- Krauss (2016)
- Rad et al. (2016)

"*Didactic sources*"

- Thames (2024)
- Alexander (2008)

"*Copula-based pairs trading*"

- Min and Czado (2010)
- Stander et al. (2013)
- Liew and Wu (2013), Xie et al. (2016)
- Lau et al. (2016)
- Krauss and Stübinger (2017)
- Zhi et al. (2017)
- Chu and Chan (2018)
- Sabino da Silva et al. (2023)
- Wang and Ding (2023)
- He et al. (2024)
- Tadi and Witzany (2025)

"*Pairs Trading: other approaches*"

- Do et al. (2006)
- Zeng and Lee (2014)
- Sarmento and Horta (2020),
- Johansson et al. (2024)
- Han et al. (2023)
- Qureshi and Zaman (2024)
- Roychoudhury et al. (2023)
- Rotondi and Russo (2025)

"*Synthetic Controls / Index-tracking*"

- Alexander (1999)
- Alexander and Dimitriu (2002)
- Alexander and Dimitriu (2005a)
- Alexander and Dimitriu (2005b)
- Shu et al. (2020)
- Bradrania et al. (2021)

2. Literature Review

Pairs Trading Classics. Early foundational work on pairs trading can be traced to studies that pioneered the strategy's empirical validation and theoretical underpinnings. In particular, Gatev et al. (2006) proposed a simple yet effective framework in which stocks are paired according to minimized distance between normalized historical prices, thus generating substantial excess returns. Another key reference, Elliott et al. (2005), introduced a mean-reverting Gaussian Markov chain model for the spread, providing one of the first analytical frameworks that bridges theory and practice in pairs trading.

Cointegration-Based Approaches. A popular way to identify and exploit persistent relationships in pairs trading has involved cointegration analysis. Vidyamurthy (2004) stands out as a seminal reference, detailing how cointegration can be applied to detect mean-reverting spreads in equity markets. Subsequent work (e.g., Caldeira and Moura (2013), Huck and Afawubo (2014), Cartea and Jaimungal (2015), Lintilhac and Tourin (2016)) consistently investigates how cointegration frameworks can isolate stable long-term relationships, offering systematic procedures for portfolio construction, parameter estimation, and risk management within the pairs trading context.

Empirical Investigations of Pairs Trading. Several studies have examined the profitability and practical nuances of pairs trading in different markets and time periods. For instance, Chen et al. (2019) reported large abnormal returns driven by short-term reversals and pairs momentum effects, while Do and Faff (2010) showed that simple pairs trading remains viable in turbulent periods despite a general profitability decline in later years. In a UK-centric study, Bowen and Hutchinson (2014) recorded moderate annual returns once risk and liquidity were accounted for. Large-scale assessments in Krauss (2016) and Rad et al. (2016) confirmed that distance, cointegration, and copula-based strategies can yield significant alpha but exhibit important differences regarding convergence speed and trading frequencies.

Didactic Sources. Practical guidance and pedagogical discussions on pairs trading can be found in Thames (2024), which provides a broad compendium of methods, from classical cointegration to machine learning-based selection. In a related vein, Alexander (2008) offers a detailed overview of cointegration techniques in Chapter II.5 and introduces fundamental principles of copula modeling for financial applications in Chapter II.6, thus serving as an essential reference for new entrants to the field.

Copula-Based Pairs Trading. An evidently growing strand of research leverages copulas to model more general dependencies beyond linear correlation. Early foundational methods include Min and Czado (2010), which highlights Bayesian pair-copula constructions, and Stander et al. (2013), offering a copula-based approach for detecting relative mispricing but noting potential challenges posed by transaction costs. Extensions in Liew and Wu (2013) and Xie et al. (2016) underscore that copulas outperform distance-of-prices rules in capturing tail dependencies. Multi-dimensional variants have been proposed (e.g., Lau et al. (2016)) to incorporate three or more assets into a single framework. Further refinements, like those introduced in Krauss and Stübinger (2017) and Zhi et al. (2017), combine t-copulas or dynamic copula-GARCH models with individualized thresholds for improved risk-adjusted returns. In the high-frequency domain, Chu and Chan (2018) showed that copula-based mispricing indices can be coupled with deep learning for profitability enhancements. Recent efforts also explore mixed copulas (Sabino da Silva et al. (2023)), ARMA-GARCH approaches (Wang and Ding (2023)), and copulas specialized for cointegrated assets (He et al. (2024)), culminating in improved alpha extraction. Finally, Tadi and Witzany (2025) proposes reference-asset-based copula trading specifically for cryptocurrencies, aligning well with the expanding scope of digital assets.

Pairs Trading: Other Approaches. Beyond cointegration or copula methodologies, several innovative techniques have surfaced. New approaches to modeling and parameter estimation for pairs trading appear in Do et al. (2006) and Zeng and Lee (2014), with the latter introducing threshold-based mean-reversion strategies. In more recent research, Sarmento and Horta (2020) incorporates machine learning (OPTICS clustering) to constrain search space, while Johansson et al. (2024) leverages convex-concave optimization for multi-asset statistical arbitrage. Reinforcement learning is featured in Han et al. (2023) for automated pair selection, and Qureshi and Zaman (2024) employs a graphical matching approach to reduce overlap among chosen pairs. Further, Roychoudhury et al. (2023) couples clustering with deep RL for equity indices, whereas Rotondi and Russo (2025) applies a partial correlation-based distance to cluster promising trading candidates.

Synthetic Controls / Index-Tracking. The method of replicating a target asset's returns by constructing a portfolio of contributor assets is reminiscent of index-tracking procedures. Classic treatments connecting cointegration analysis and hedging tasks (e.g., Alexander (1999) and Alexander and Dimitriu (2002)) lay theoretical groundwork for such an approach. Subsequent refinements in Alexander and Dimitriu (2005a) and Alexander and Dimitriu (2005b) investigate how cointegration outperforms traditional techniques in crafting robust index trackers and exploiting time-varying market regimes. Complementary research (e.g., Shu et al. (2020)) shows that sparse solutions across a large universe can reduce transaction costs, an idea further corroborated in Bradrania et al. (2021), where machine learning identifies dynamic selection methods for index constituents. These frameworks illustrate how synthetic control concepts provide a flexible foundation for building market-neutral positions or tracking assets with fewer assumptions.

3. Methodology

3.1 Sparse Synthetic Control

The core component of our pairs trading strategy involves constructing a synthetic asset that replicates the price behavior of a target security (e.g: AAPL) using a combination of assets from a donor pool. Let $\mathbf{y} = [y_t]_{t=1}^T \in \mathbb{R}^T$ denote the log-price time series of a target asset and $\mathbf{X} = [x_{1t}, ..., x_{Nt}]_{t=1}^T \in \mathbb{R}^{T\times N}$ denote the log-price time series of a donor pool of assets. We construct a synthetic asset $\mathbf{y}^*$ through a sparse linear combination

$$y_t^* = \sum_{i=1}^N w_i^* x_{it}.$$

The weights $\mathbf{w}^* = [w_1^*, ..., w_N^*]$ are determined via a cardinality-constrained quadratic program

$$\mathbf{w}^* = \arg\min_{\mathbf{w}\in\mathbb{R}^N} \sum_{t=1}^T \left(y_t - \sum_{i=1}^N w_i x_{it} \right)^2 \quad \text{s.t.} \quad \left| \begin{array}{ll} \mathbf{1}^\top \mathbf{w} & = 1 \\ \|\mathbf{w}\|_0 & \leq K \end{array} \right.$$

where $\|\mathbf{w}\|_0 := \sum_{i=1}^N \mathbb{I}(w_i \neq 0)$ counts the non-zero elements in $\mathbf{w}$. The goal is to enforce sparsity so that only a limited number of assets receive a nonzero weight. The NP-hard cardinality constraint is approximated by the following procedure:

1. Solve the full least squares problem

$$\mathbf{w}^{(1)} = \arg\min_{\mathbf{w}\in\mathbb{R}^N} \|\mathbf{y} - \mathbf{X}\mathbf{w}\|_2^2 \quad \text{s.t.} \quad \mathbf{1}^\top \mathbf{w} = 1.$$

2. Select the K largest weights (in absolute value) from $\mathbf{w}^{(1)}$ into

$$\mathcal{I} := \{i : |w_i^{(1)}| \text{ among } K \text{ largests}\}$$

3. Solve the restricted program on support $\mathcal{I}$

$$\mathbf{w}^{(2)} = \arg\min_{\mathbf{w}_\mathcal{I}\in\mathbb{R}^K} \|\mathbf{y} - \mathbf{X}_\mathcal{I}\mathbf{w}_\mathcal{I}\|_2^2 \quad \text{s.t.} \quad \mathbf{1}^\top \mathbf{w}_\mathcal{I} = 1$$

   where $\mathbf{X}_\mathcal{I} \in \mathbb{R}^{T\times K}$ is the resricted donor matrix and $\mathbf{w}_\mathcal{I} \in \mathbb{R}^K$ is the restricted weight vector for the selected assets.

4. Construct the full weight vector $\mathbf{w}^* \in \mathbb{R}^N$ by embedding the optimized restricted weights back into the original N-dimensional space.

$$w_i^* = \begin{cases} w_j^{(2)} & \text{if } i = \mathcal{I}_j \\ 0 & \text{otherwise} \end{cases}$$

4. Copula-Based Dependence Modeling

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let $R, R^* : \Omega \to \mathbb{R}$ be real-valued random variables representing the target and synthetic log-returns, respectively, Let F_R and F_{R^*} denote their respective cumulative distribution functions (CDFs).

Definition 1 (Copula)**.** *A bivariate copula is a function $C : [0,1]^2 \to [0,1]$ satisfying:*

1. *$C(u,0) = C(0,v) = 0$ and $C(u,1) = u$, $C(1,v) = v$ for all $u, v \in [0,1]$ (boundary conditions)*
2. *$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geq 0$ for all $u_1 \leq u_2$, $v_1 \leq v_2$ in $[0,1]$ (2-increasing)*

The fundamental relationship between copulas and joint distributions is established by Sklar's theorem:

Theorem 1 (Sklar (1959))**.** *Let F_{R,R^*} be the joint CDF of (R, R^*). Then there exists a copula $C : [0,1]^2 \to [0,1]$ such that*

$$F_{R,R^*}(r, r^*) = C(F_R(r), F_{R^*}(r^*)) \quad \forall r, r^* \in \mathbb{R}. \tag{1}$$

If F_R and F_{R^*} are continuous, then C is unique. Conversely, if C is a copula and F_R, F_{R^*} are CDFs, then F_{R,R^*} defined above is a joint CDF with margins F_R and F_{R^*}.

When uniqueness holds, the copula can be expressed through the probability integral transform:

$$C(u,v) = \mathbb{P}(F_R(R) \leq u, F_{R^*}(R^*) \leq v) \quad \text{for} \quad (u,v) \in [0,1]^2.$$

The corresponding copula density $c : [0,1]^2 \to \mathbb{R}_+$, when it exists, is given by

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v},$$

and the joint density can be expressed as

$$f_{R,R^*}(r, r^*) = c(F_R(r), F_{R^*}(r^*)) f_R(r) f_{R^*}(r^*),$$

where f_{R,R^*} is the joint density and f_R and f_{R^*} are the marginal densities.

This decomposition provides a framework for modeling the dependence structure between the target and synthetic returns independently of their marginal distributions. The implementation involves three stages: (1) nonparametric estimation of the marginal CDFs F_R, F_{R^*} , (2) copula calibration from parametric classes $\mathcal{C} = \{C_\theta : \theta \in \Theta\}$ via maximum likelihood estimation, (3) selection of an appropriate copula family

4.1 Marginal Distribution Estimation

The foundation of copula modeling lies in the accurate estimation of marginal distributions for both target and synthetic asset returns. To maintain flexibility and avoid restrictive parametric assumptions, we adopt a non-parametric approach through empirical cumulative distribution functions (ECDFs). First, we construct logarithmic return series for both assets. Let y_t and y_t^* denote the log-prices of the target and synthetic assets at time t, respectively. The log-returns are computed via

$$r_t = y_t - y_{t-1} \quad \text{and} \quad r_t^* = y_t^* - y_{t-1}^* \quad \text{for } t = 2, \ldots, T,$$

establishing stationary time series $\{r_t\}$ and $\{r_t^*\}$ that form the basis for distributional analysis.

Next, we estimate the marginal distributions through linearly interpolated ECDFs. For any $r \in \mathbb{R}$, the empirical distribution functions are given by

$$\hat{F}_R(r) = \frac{1}{T-1} \sum_{t=2}^{T} \mathbb{I}(r_t \leq r) \quad \text{and} \quad \hat{F}_{R^*}(r^*) = \frac{1}{T-1} \sum_{t=2}^{T} \mathbb{I}(r_t^* \leq r^*),$$

where $\mathbb{I}(\cdot)$ denotes the usual indicator function. Linear interpolation between observed returns ensures continuity of the distribution functions across their support. To mitigate numerical instabilities during subsequent copula estimation, we constrain the ECDF outputs within $[\epsilon, 1-\epsilon]$ where $\epsilon = 10^{-5}$, thereby avoiding boundary effects at the distribution tails.

The final stage applies the probability integral transform to obtain uniform marginals. Specifically, we compute pseudo-observations

$$u_t = \hat{F}_R(r_t) \quad \text{and} \quad v_t = \hat{F}_{R^*}(r_t^*) \quad \text{for } t = 2, \ldots, T,$$

yielding paired realizations $(\mathbf{u}, \mathbf{v}) = \{(u_t, v_t)\}_{t=2}^{T}$ that reside in the unit square $[0,1]^2$. This transformation, justified by Sklar's Theorem, effectively decouples the marginal distributions from the dependence structure. The resulting uniform variates serve as canonical inputs for copula specification while preserving the essential dependence characteristics between target and synthetic returns.

4.2 Copula calibration from parametric classes

The goal of copula fitting is to find the best copula that describes the dependence structure between the returns of the target and synthetic assets. This is done by maximizing the likelihood of the observed data under different copula models. We consider parametric copula families $\mathcal{C} =$

$\{C_\theta : \theta \in \Theta\}$ where each copula C_θ has density $c_\theta(u,v) = \frac{\partial^2 C_\theta}{\partial u \partial v}(u,v)$. For each candidate copula family, we estimate parameters via constrained maximum likelihood:

$$\hat{\theta} = \arg\max_{\theta \in \Theta} \ell(\theta|\mathbf{u},\mathbf{v}) \quad \text{where} \quad \ell(\theta|\mathbf{u},\mathbf{v}) = \sum_{t=2}^{T} \ln c_\theta(u_t, v_t). \tag{2}$$

The optimization is subject to parameter constraints Θ specific to each copula family:

- **Elliptical Copulas:**
  - Gaussian: $\Theta = \{\rho \in (-1,1)\}$ with density

    $$c_\rho^{Gauss}(u,v) = \frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{\zeta_u^2 + \zeta_v^2 - 2\rho\zeta_u\zeta_v}{2(1-\rho^2)} + \frac{\zeta_u^2 + \zeta_v^2}{2}\right)$$

    where $\zeta_u = \Phi^{-1}(u)$, $\zeta_v = \Phi^{-1}(v)$ and Φ is the standard normal CDF.
  - Student-t: $\Theta = \{\rho \in (-1,1), \nu > 2\}$ with density

    $$c_{\rho,\nu}^{t}(u,v) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}{\sqrt{1-\rho^2}\Gamma\left(\frac{\nu+1}{2}\right)^2} \frac{\left(1 + \frac{\zeta_u^2+\zeta_v^2-2\rho\zeta_u\zeta_v}{\nu(1-\rho^2)}\right)^{-(\nu+2)/2}}{\prod_{i\in\{u,v\}}\left(1+\frac{\zeta_i^2}{\nu}\right)^{-(\nu+1)/2}}$$

    where $\zeta_u = t_\nu^{-1}(u)$, $\zeta_v = t_\nu^{-1}(v)$ and t_ν is the Student-t CDF.
- **Archimedean Copulas:** For generator function ψ_θ,

  $$C_\theta(u,v) = \psi_\theta(\psi_\theta^{-1}(u) + \psi_\theta^{-1}(v))$$

  - Clayton: $\Theta = (0,\infty)$ with $\psi_\theta(t) = (1+t)^{-1/\theta}$
  - Gumbel: $\Theta = [1,\infty)$ with $\psi_\theta(t) = \exp(-t^{1/\theta})$
  - Frank: $\Theta = \mathbb{R} \setminus \{0\}$ with $\psi_\theta(t) = -\frac{1}{\theta}\ln\left(1-(1-e^{-\theta})e^{-t}\right)$
  - Joe: $\Theta = [1,\infty)$ with $\psi_\theta(t) = 1-(1-e^{-t})^{1/\theta}$
- **Mixed Copulas:**
  - N14: Rotated Clayton-Gumbel mixture with $\Theta \subset \mathbb{R}_+^2$

4.3 Selection of an appropriate copula family

After estimating parameters for each candidate copula family $\mathcal{C} = \{C_\theta : \theta \in \Theta\}$, we select the optimal model using information criteria that balance goodness-of-fit against model complexity. Let $\ell(\hat{\theta}|\mathbf{u}, \mathbf{v}) = \max_{\theta\in\Theta} \sum_{t=2}^{T} \ln c_\theta(u_t, v_t)$ be the maximized log-likelihood for a copula with parameter estimate $\hat{\theta}$, where T is the sample size and k is the number of parameters. We evaluate the following information criterions:

$$
\begin{array}{lll}
\textit{Akaike} & \text{AIC} & = 2k - 2\ell(\hat{\theta}|\mathbf{u}, \mathbf{v}) \\
\textit{Schwarz/Bayesian} & \text{SIC} & = k\ln(T-1) - 2\ell(\hat{\theta}|\mathbf{u}, \mathbf{v}) \\
\textit{Hannan-Quinn} & \text{HQIC} & = 2k\ln(\ln T - 1) - 2\ell(\hat{\theta}|\mathbf{u}, \mathbf{v})
\end{array}
$$

The copula family with the lowest value for a chosen criterion is selected as optimal. These criteria penalize overfitting through the k term while rewarding better fit through the log-likelihood.

TABLE 1: Copula Model Selection Criteria

Copula	SIC	AIC	HQIC
Student-t	-138.78	-145.51	-143.18
Gumbel	-73.85	-80.58	-78.25
Clayton	-53.64	-60.37	-58.04
Joe	-50.61	-57.34	-55.01
Gaussian	-44.21	-50.93	-48.60
Frank	-38.37	-45.10	-42.76
N14	-	-	-

Table 1 presents the fitting results for different copula families.

5. Pairs Trading Strategy via Mispricing Indices (MI)

In this section, we adapt the mispricing index (MI) strategy from Xie et al. (2016) to our setting, wherein we trade a target asset (with returns R_t) against its synthetic counterpart (with returns R_t^*). While the strategy might initially appear unconventional, it hinges on interpreting conditional probabilities of daily returns as an evolving measure of relative mispricing. Below, we detail the essential components of the approach and how trading positions are opened and closed.

5.1 Mispricing Index (MI), Flags and Cumulative Mispricing Index (CMI)

On each trading day t, let r_t and r_t^* respectively denote the realized returns for the target and synthetic assets. We define two conditional mispricing indices,

$$MI_t^{R|R^*} = \mathbb{P}(R_t \leq r_t \mid R_t^* = r_t^*) = \frac{\partial C_{\hat{\theta}}(F_R(r_t), F_{R^*}(r_t^*))}{\partial F_{R^*}(r_t^*)},$$

$$MI_t^{R^*|R} = \mathbb{P}(R_t^* \leq r_t^* \mid R_t = r_t) = \frac{\partial C_{\hat{\theta}}(F_R(r_t), F_{R^*}(r_t^*))}{\partial F_R(r_t)}.$$

The quantity $MI_t^{R|R^*}$ measures how "mispriced" the target asset appears when conditioned on that day's synthetic return, whereas $MI_t^{R^*|R}$ does the same for the synthetic asset when conditioned on the target return. Since a single day's mispricing index reflects only an instantaneous view, we accumulate daily signals over time to gauge how much the returns have gradually driven prices apart (or together). We define a *flag* series for each asset, defined as a running sum of daily deviations from 0.5[1]. Let $\text{Flag}_R(0) = \text{Flag}_{R^*}(0) = 0$, then, for $t = 1, ..., T$ we have

$$\begin{aligned}
\text{Flag}_t^R &= \text{Flag}_{t-1}^R + (MI_t^{R|R^*} - 0.5) &= \textstyle\sum_{s=1}^t (MI_s^{R|R^*} - 0.5), \\
\text{Flag}_t^{R^*} &= \text{Flag}_{t-1}^{R^*} + (MI_t^{R^*|R} - 0.5) &= \textstyle\sum_{s=1}^t (MI_s^{R^*|R} - 0.5).
\end{aligned}$$

Similar to plotting cumulative returns, these raw flags track the net effect of mispricing signals over time. To prevent the compounding of stale mispricing signals, we formally define a Cumulative Mispricing Index (CMI) as the reset-adjusted flag series through the recursive relationship:

$$\text{CMI}_t^R = \begin{cases} \text{CMI}_{t-1}^R + \left(MI_t^{R|R^*} - 0.5\right), & \text{if no position reset occurs at time } t, \\ 0, & \text{if a position is closed at } t, \end{cases}$$

$$\text{CMI}_t^{R^*} = \begin{cases} \text{CMI}_{t-1}^{R^*} + \left(MI_t^{R^*|R} - 0.5\right), & \text{if no position reset occurs at time } t, \\ 0, & \text{if a position is closed at } t, \end{cases}$$

where $\text{CMI}_0^R = \text{CMI}_0^{R^*} = 0$. Unlike the raw flags that accrue continuously, each CMI absorbs daily mispricing signals only until a trade is exited, at which point it is reset to zero. This mechanism ensures that any fresh mispricing accumulates from a "clean slate," thereby preventing the influence of past, already-traded mispricing from compounding future signals.

[1] The subtraction of 0.5 centers the cumulative sum so that deviations from zero reflect mispricing.

5.2 Trading Logic

We implement a dollar-neutral trading strategy that capitalizes on relative mispricing signals between the target and synthetic assets. The trading rule (TR) we employ builds upon the frameworks of Xie et al. (2016) and Rad et al. (2016), incorporating their key insights about signal combination logic. While Xie et al. (2016) originally proposed an "OR-OR" framework-where trades are initiated when either asset shows mispricing and closed when either asset exhibits correction-Rad et al. (2016) demonstrated that a more conservative "AND-OR" approach yields more robust performance. This latter approach requires concurrent mispricing signals from both assets to open positions while maintaining a sensitive exit strategy where correction in either asset triggers position closure.

Let D_l and D_u denote the lower and upper thresholds for opening positions, and S_l and S_u the lower and upper stop-loss boundaries. Starting with $TR_0 = 0$, for $t = 1, ..., T$, the trading rule evolves as follows:

$$TR_t(\mathrm{CMI}_t^R, \mathrm{CMI}_t^{R^*}, TR_{t-1}; D_l, D_u, S_l, S_u) = \tag{3}$$

$$\begin{cases} +1 & \text{if } (\mathrm{CMI}_t^R \le D_l \text{ and } \mathrm{CMI}_t^{R^*} \ge D_u) \\ -1 & \text{if } (\mathrm{CMI}_t^R \ge D_u \text{ and } \mathrm{CMI}_t^{R^*} \le D_l) \\ 0 & \text{if either} \begin{cases} \left\{ TR_{t-1} = 1 \quad \text{and } \left[(\underbrace{\mathrm{CMI}_t^R \ge 0 \text{ or } \mathrm{CMI}_t^{R^*} \le 0}_{\text{take profit}}) \text{ or } (\underbrace{\mathrm{CMI}_t^R \le S_l \text{ or } \mathrm{CMI}_t^{R^*} \ge S_u}_{\text{stop loss}}) \right] \right\}, \text{or} \\ \left\{ TR_{t-1} = -1 \text{ and } \left[(\underbrace{\mathrm{CMI}_t^R \le 0 \text{ or } \mathrm{CMI}_t^{R^*} \ge 0}_{\text{take profit}}) \text{ or } (\underbrace{\mathrm{CMI}_t^R \ge S_u \text{ or } \mathrm{CMI}_t^{R^*} \le S_l}_{\text{stop loss}}) \right] \right\} \end{cases} \\ TR_{t-1} & \text{otherwise} \end{cases} \tag{4}$$

That is, at the beginning of each trading day t, observe the current values of both mispricing indicators, CMI_t^R (for the target asset) and $\mathrm{CMI}_t^{R^*}$ (for the synthetic). The trading rule TR_t can take one of three values: $+1$, -1, or 0, indicating a "*long-short*", "*short-long*", or "*flat*" position, respectively. When no position is open (i.e., $TR_{t-1} = 0$), the rule opens a position only if there is simultaneous mispricing in both assets according to the thresholds D_l and D_u. Specifically,

- "*Long target/Short synthetic*" (+1): Entered when both CMIs indicate the target asset is underpriced relative to the synthetic ($\mathrm{CMI}_t^R \le D_l$ **and** $\mathrm{CMI}_t^{R^*} \ge D_u$).
- "*Short target/Long synthetic*" (-1): Entered when both CMIs indicate the target asset is overpriced relative to the synthetic ($\mathrm{CMI}_t^R \ge D_u$ **and** $\mathrm{CMI}_t^{R^*} \le D_l$).

Once a position is open (either $TR_{t-1} = +1$ or $TR_{t-1} = -1$), the logic checks each day whether the mispricing has corrected enough to trigger a take-profit condition or crossed critical boundaries that trigger a stop-loss. These checks apply to either of the two mispricing indices, so if correction or a stop-loss occurs in any one of them, the entire position is closed. Mathematically, this is captured by the "*OR*" clauses in the formula, which evaluate whether CMI_t^R or $\mathrm{CMI}_t^{R^*}$ has crossed the zero line (for take-profit) or moved beyond the (S_l, S_u) band (for stop-loss). If one of these events occurs, then TR_t is set to 0, and the mispricing indices are both reset to zero for the next trading day. If neither a take-profit nor a stop-loss threshold is met, then the position remains unchanged, meaning TR_t simply inherits the previous value TR_{t-1}.

Intuitively, when both indicators are simultaneously misaligned (one significantly high and the other significantly low), the strategy deems it a strong signal to open a dollar-neutral position that is long the "*undervalued*" side and short the "*overvalued*" side. As soon as either index crosses back toward zero (suggesting partial correction of that asset's mispricing) or breaches a stop-loss boundary (indicating that the trade is moving unfavorably), the position is liquidated. This "*AND-OR*" logic helps filter out noise in the daily movements and more reliably captures episodes in which both assets appear to be drifting apart (opening a trade) and then swiftly catches at least one side reverting (closing the trade).

As in Xie et al. (2016), we set $(D_l, D_u) = (-0.6, 0.6)$ and $(S_u, S_l) = (-2, 2)$ and we will explore other parametric choices in the robustness checks.

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A. Online Appendix

A.1 Algorithms

Algorithm 1. Sparse Synthetic Control

Require:

Target asset log-prices $\mathbf{y} = [y_t]_{t=1}^{T} \in \mathbb{R}^T$

Donor pool log-prices $\mathbf{X} = [x_{1t}, ..., x_{Nt}]_{t=1}^{T} \in \mathbb{R}^{T \times N}$

Maximum number of assets $K \in \mathbb{N}$ with $K \leq N$

Ensure: Sparse weight vector $\mathbf{w}^* \in \mathbb{R}^N$

function SyntheticControl($\mathbf{y}, \mathbf{X}, K$)

  # Stage 1: Unrestricted optimization

  $\mathbf{w}^{(1)} = \arg\min_{\mathbf{w} \in \mathbb{R}^N} \|\mathbf{y} - \mathbf{X}\mathbf{w}\|_2^2$ s.t. $\mathbf{1}^\top \mathbf{w} = 1$ ▷ Solve full least squares problem

  # Stage 2: Support selection

  $\mathcal{I} \leftarrow \{i : |w_i^{(1)}|$ among K largest$\}$ ▷ Select K largest weights

  $\mathbf{X}_{\mathcal{I}} \leftarrow [\mathbf{x}_{\mathcal{I}_1}, \ldots, \mathbf{x}_{\mathcal{I}_K}]$ ▷ Restricted donor matrix

  # Stage 3: Restricted optimization

  $\mathbf{w}^{(2)} = \arg\min_{\mathbf{w}_{\mathcal{I}} \in \mathbb{R}^K} \|\mathbf{y} - \mathbf{X}_{\mathcal{I}}\mathbf{w}_{\mathcal{I}}\|_2^2$ s.t. $\mathbf{1}^\top \mathbf{w}_{\mathcal{I}} = 1$ ▷ Solve restricted program

  for each $i \in \{1, \ldots, N\}$ **do**

    $w_i^* \leftarrow w_j^{(2)}$ if $i = \mathcal{I}_j$, else 0 ▷ Construct full weights

  end for

  return $\mathbf{w}^*$

end function

Algorithm 2. Copula Fitting

Require:

Target returns $\mathbf{r} = [r_t]_{t=2}^{T} \in \mathbb{R}^{T-1}$

Synthetic returns $\mathbf{r}^* = [r_t^*]_{t=2}^{T} \in \mathbb{R}^{T-1}$

Parametric copula families $\mathcal{C} = \{C_\theta : \theta \in \Theta\}$

Numerical tolerance $\epsilon = 10^{-5}$

Ensure: Marginal ECDFs $\hat{F}_R, \hat{F}_{R^*}$ and fitted copulas $\{C_{\hat{\theta}}\}_{C_\theta \in \mathcal{C}}$

function CopulaFit($\mathbf{r}, \mathbf{r}^*$)

  # Construct linearly interpolated ECDFs

  for each return series $\mathbf{x} \in \{\mathbf{r}, \mathbf{r}^*\}$ **do**

    Sort unique values: $x_{(1)} < \cdots < x_{(m)}$

    $p_i \leftarrow \frac{1}{T-1} \sum_{t=2}^{T} \mathbb{I}(x_t \leq x_{(i)})$ ▷ Compute empirical probabilities

    $\hat{F}_X(x) \leftarrow p_i + (p_{i+1} - p_i) \frac{x - x_{(i)}}{x_{(i+1)} - x_{(i)}}$ for $x \in [x_{(i)}, x_{(i+1)}]$: ▷ Piecewise linear interpolation

  end for

  # Apply probability integral transform

  for $t \in \{2, \dots, T\}$ **do**

    $u_t \leftarrow \max\{\epsilon, \min\{\hat{F}_R(r_t), 1 - \epsilon\}\}$ ▷ Adjust ECDF outputs to tolerance level ϵ

    $v_t \leftarrow \max\{\epsilon, \min\{\hat{F}_{R^*}(r_t^*), 1 - \epsilon\}\}$

  end for

  # Fit each copula family & Obtain information criterion

  for each copula family $C_\theta \in \mathcal{C}$ **do**

    $\hat{\theta} \leftarrow \arg\max_{\theta \in \Theta} \sum_{t=2}^{T} \ln c_\theta(u_t, v_t)$ ▷ Estimate parameters via maximum likelihood

    $\ell(\hat{\theta}) \leftarrow \sum_{t=2}^{T} \ln c_{\hat{\theta}}(u_t, v_t)$ ▷ Obtain maximized likelihood

  end for

  return $\hat{F}_R, \hat{F}_{R^*}, \{C_{\hat{\theta}}\}_{C_\theta \in \mathcal{C}}$

end function

Algorithm 3. Mispricing Indices Calculation

Require:

Target return r_t, synthetic return r_t^*

Optimal copula $C_{\hat{\theta}}$

Marginal ECDFs $\hat{F}_R, \hat{F}_{R^*}$

Ensure: Mispricing indices $MI_t^{R|R^*}, MI_t^{R^*|R}$

function MispricingIndices($r_t, r_t^*, C_{\hat{\theta}}, \hat{F}_R, \hat{F}_{R^*}$)

  $u_t \leftarrow \hat{F}_R(r_t)$, $v_t \leftarrow \hat{F}_{R^*}(r_t^*)$ ▷ Compute uniform marginals (pseudo-observations)

  $MI_t^{R|R^*} \leftarrow \dfrac{\partial C_{\hat{\theta}}(u_t, v_t)}{\partial v_t}$ ▷ Compute target-synthetic MI

  $MI_t^{R^*|R} \leftarrow \dfrac{\partial C_{\hat{\theta}}(u_t, v_t)}{\partial u_t}$ ▷ Compute synthetic-target MI

  return $MI_t^{R|R^*}, MI_t^{R^*|R}$

end function

Algorithm 4. Update Cumulative Mispricing Index (CMI)

Require:

Mispricing indices: $(\mathrm{MI}_t^{R|R^*}, \mathrm{MI}_t^{R^*|R})$

Previous CMIs: $(\mathrm{CMI}_{t-1}^{R}, \mathrm{CMI}_{t-1}^{R^*})$

Reset flag: `reset`

Ensure: Updated CMIs: $(\mathrm{CMI}_t^{R}, \mathrm{CMI}_t^{R^*})$

function UpdateCMI($\mathrm{MI}_t^{R|R^*}, \mathrm{MI}_t^{R^*|R}, \mathrm{CMI}_{t-1}^{R}, \mathrm{CMI}_{t-1}^{R^*}$, `reset`)

  if `reset` **then**

    $\mathrm{CMI}_t^{R} \leftarrow 0$, $\mathrm{CMI}_t^{R^*} \leftarrow 0$ ▷ Reset the CMIs to 0

  else

    $\mathrm{CMI}_t^{R} \leftarrow \mathrm{CMI}_{t-1}^{R} + (\mathrm{MI}_t^{R|R^*} - 0.5)$ ▷ Update target CMIs with new realization of MI

    $\mathrm{CMI}_t^{R^*} \leftarrow \mathrm{CMI}_{t-1}^{R^*} + (\mathrm{MI}_t^{R^*|R} - 0.5)$

  end if

  return $(\mathrm{CMI}_t^{R}, \mathrm{CMI}_t^{R^*})$

end function

Algorithm 5. Trading Rule

Require: Mispricing indices $\text{CMI}_t^R, \text{CMI}_t^{R^*}$ and thresholds D_l, D_u, S_l, S_u

Ensure: Trading position $TR_t \in \{-1, 0, +1\}$

```
function TradingRule( $\text{CMI}_t^R, \text{CMI}_t^{R^*}$, $D_l$, $D_u$, $S_l$, $S_u$ )
    if $TR_{t-1} = 0$ then                                                        ▷ No existing position
        if $\text{CMI}_t^R \leq D_l$ and $\text{CMI}_t^{R^*} \geq D_u$ then
            $TR_t \leftarrow +1$                                                  ▷ Long target, short synthetic
        else if $\text{CMI}_t^R \geq D_u$ and $\text{CMI}_t^{R^*} \leq D_l$ then
            $TR_t \leftarrow -1$                                                  ▷ Short target, long synthetic
        else
            $TR_t \leftarrow 0$                                                   ▷ Remain flat
        end if
    else if $TR_{t-1} = +1$ then                                                  ▷ Currently long target, short synthetic
        if ($\text{CMI}_t^R \geq 0$ or $\text{CMI}_t^{R^*} \leq 0$) or ($\text{CMI}_t^R \leq S_l$ or $\text{CMI}_t^{R^*} \geq S_u$) then
            $TR_t \leftarrow 0$                                                   ▷ Close position (take profit or stop-loss)
            Reset $\text{CMI}_t^R \leftarrow 0$ and $\text{CMI}_t^{R^*} \leftarrow 0$
        else
            $TR_t \leftarrow +1$                                                  ▷ Maintain current position
        end if
    else if $TR_{t-1} = -1$ then                                                  ▷ Currently short target, long synthetic
        if ($\text{CMI}_t^R \leq 0$ or $\text{CMI}_t^{R^*} \geq 0$) or ($\text{CMI}_t^R \geq S_u$ or $\text{CMI}_t^{R^*} \leq S_l$) then
            $TR_t \leftarrow 0$                                                   ▷ Close position (take profit or stop-loss)
            Reset $\text{CMI}_t^R \leftarrow 0$ and $\text{CMI}_t^{R^*} \leftarrow 0$
        else
            $TR_t \leftarrow -1$                                                  ▷ Maintain current position
        end if
    end if
    return $TR_t$
end function
```

Algorithm 6. Main. "*Pairs-trading a Sparse Synthetic Control*"

Require:

Target asset log-prices $\mathbf{y} = [y_t]_{t=1}^{T}$

Donor pool log-prices $\mathbf{X} = [x_{1t}, ..., x_{Nt}]_{t=1}^{T}$

Maximum number of assets $K \in \mathbb{N}$ with $K \leq N$

Entry thresholds (D_l, D_u), stop-loss thresholds (S_l, S_u)

Parametric copula families $\mathcal{C} = \{C_\theta : \theta \in \Theta\}$

Ensure: Trading signals $\{TR_t\}_{t=1}^{T}$

procedure $\textsc{Main}(\mathbf{y}, \mathbf{X}, K, D_l, D_u, S_l, S_u, \mathcal{C})$
    $\mathbf{w}^* \leftarrow \textsc{SyntheticControl}(\mathbf{y}, \mathbf{X}, K)$ ▷ Construct synthetic asset
    $\mathbf{y}^* \leftarrow \mathbf{X}\mathbf{w}^*$
    $\mathbf{r} \leftarrow \text{diff}(\mathbf{y})$, $\mathbf{r}^* \leftarrow \text{diff}(\mathbf{y}^*)$ ▷ Compute returns
    $C_{\hat{\theta}}, \hat{F}_R, \hat{F}_{R^*} \leftarrow \textsc{CopulaFit}(\mathbf{r}, \mathbf{r}^*)$ ▷ Fit copula
    Initialize $TR_0 \leftarrow 0$, $\text{CMI}_0^R \leftarrow 0$, $\text{CMI}_0^{R^*} \leftarrow 0$
    for $t = 1$ to T **do**
        $\text{MI}_t^{R|R^*}, \text{MI}_t^{R^*|R} \leftarrow \textsc{MispricingIndices}(r_t, r_t^*, C_{\hat{\theta}}, \hat{F}_R, \hat{F}_{R^*})$
        $TR_t \leftarrow \textsc{TradingRule}(\text{CMI}_{t-1}^R, \text{CMI}_{t-1}^{R^*}, TR_{t-1}, D_l, D_u, S_l, S_u)$
        `reset` $\leftarrow (TR_t = 0$ and $TR_{t-1} \neq 0)$ ▷ Reset CMI if position closed
        $\text{CMI}_t^R, \text{CMI}_t^{R^*} \leftarrow \textsc{Update}\text{CMI}(MI_t^{R|R^*}, \text{MI}_t^{R^*|R}, \text{CMI}_{t-1}^R, \text{CMI}_{t-1}^{R^*}, \texttt{reset})$
    end for
    return $\{TR_t\}_{t=1}^{T}$
end procedure