In a market consisting of N stocks, we denote the dividend-adjusted return on stock i at trading day t by $r_{i,t}$. We adopt a factor model for stock return,

$$r_t - r_f = \beta_t F_t + \epsilon_t, \quad t = 1, 2, \ldots, T \tag{1}$$

Here, $r_t = \{r_{i,t}\}_{i=1}^{N} \in \mathbb{R}^N$ are the dividend-adjusted daily return, $r_f \in \mathbb{R}$ is the risk-free rate, $F_t \in \mathbb{R}^{K\times 1}$ are the underlying factors, $\beta_t \in \mathbb{R}^{N\times K}$ are the corresponding loadings on K factors, and $\epsilon_t \in \mathbb{R}^N$ are the residual returns. Factor candidates varies widely, ranging from economical-driven factors such as the Fama-French factors, to statistically-driven factors derived from PCA. In our approach, factors are selected as the leading eigenvectors in PCA. The number of factors K is chosen based on the eigenvalue spectrum of the empirical correlation of daily returns.
Without loss of generality, these factors can be interpreted as portfolios of stocks,

$$F_t = \omega_t \left(r_t - r_f\right) \tag{2}$$

where $\omega_t \in \mathbb{R}^{K\times N}$ contains corresponding portfolio weights. Eq. 2.1.1 and Eq. 2.1.2 give

$$r_t - r_f = \beta_t \omega_t \left(r_t - r_f\right) + \epsilon_t \Rightarrow \epsilon_t = \left(I - \beta_t \omega_t\right)\left(r_t - r_f\right) := \Phi_t \left(r_t - r_f\right)$$

Here,

$$\Phi_t := \left(I - \beta_t \omega_t\right)$$

defines a linear transformation from r_t to ϵ_t. More importantly, $\epsilon_{i,t}$ can be viewed as the return of a tradable portfolio with weights specified by the i-th row of Φ_t. Consequently, the investing universe spanned by r_t is termed as name equity space, and that spanned by ϵ_t as name residual space.
We denote the portfolio weights in name equity space as $w_t^{R,\text{ name}}$ and portfolio weights in name residual space as $w_t^{\epsilon,\text{ name}}$. These weights are related by

$$w_t^{R,\text{ name}} = \Phi_t^T w_t^{\epsilon,\text{ name}}$$

, directly following the equality in portfolio return,

$$\left(w_t^{\epsilon\text{ name}}\right)^T \epsilon_t = \left(w_t^{\epsilon,\text{ name}}\right)^T \Phi_t \left(r_t - r_f\right) = \left(w_t^{R,\text{ name}}\right)^T \left(r_t - r_f\right)$$

For factors derived by PCA, we have

$$\Phi_t \beta_t = 0 \Longrightarrow \left(w_t^{R,\text{ name}}\right)^T \beta_t = \left(w_t^{\epsilon,\text{ name}}\right)^T \Phi_t \beta_t = 0, \quad \forall w_t^{\epsilon,\text{ name}}$$

, with proof given in the appendix. It means that for any $w_t^{\epsilon,\text{name}}$, the $w_t^{R,\text{name}}$ calculated by Eq. 2.1.5 satisfy,

$$\left(w_t^{R,\text{ name}}\right)^T (r_t - r_f) = \left(w_t^{\epsilon,\text{ name}}\right)^T \Phi_t (\beta_t F_t + \epsilon_t) = \left(w_t^{\epsilon,\text{ name}}\right)^T \Phi_t \epsilon_t = \left(w_t^{R,\text{ name}}\right)^T \epsilon_t$$

It suggests that the return of our statistical arbitrage portfolios is independent of market factors and relies solely on residual returns, a property usually termed as market neutrality. Ideally, portfolios are also desired to have a zero net value, known as dollar neutrality. Empirical evidence suggests that market-neutral portfolios are also approximately dollarneutral.