1 Fixed Income

Periodicity conversion

$$\left(1+r^{(k)}\right)^k = \left(1+r^{(m)}\right)^m$$

$$r^{(1)_k} := r^{(k)} \times k \iff r^{(k)} = \frac{r^{(1)_k}}{k}$$

Current Yield

$$CY_t = \frac{c_t^{(1)}}{P_t}$$

where P_t is the price of the bond, and $c_t^{(1)}$ is the coupon (stated as an annual rate). Cash flow structures:

$$\begin{array}{lllllll}
\text{Bullet Bond} & : & [-P_0 & I_1, & I_2, & ..., & I_n + A] \\
\text{Fully Amortizing Loan} & : & [-P_0 & I_1 + A_1, & I_2 + A_2, & ..., & I_n + A_n] \\
\text{Partially Amortizing Loan} & : & [-P_0 & I_1 + A_1, & I_2 + A_2, & ..., & I_n + \frac{A}{2}] \quad \textit{Balloon payment} = \frac{A}{2}
\end{array}$$

Loan

Periodic payment of a loan:

$$a = \frac{r \times A}{1-(1+r)^{-n}}$$

where a is the periodic payment, A is the principal, r is the market interest rate per period and n are the total number of periods.

1.1. Yield Spread Measures for Fixed-Rate Bonds

G-Spread

$$\text{G-Spread} = YTM^{(1)\{H\}} - G^{\{H\}}$$

where $YTM^{(1)\{H\}}$ is the yield-to-maturity of a bond expressed in anual terms and for a horizon/-maturity of H years, and $G^{\{H\}}$ is the actual or interpolated government bond yield that matches the horizon H of the bond coupon of which we are calculating the YTM.

Modus Operandi: For a bond that has B years until settlement, and given government bond yields for A and C horizons ($G^{\{A\}}$ and $G^{\{C\}}$), with $A < B < C$.

1) Compute $YTM^{(1)\{B\}}$ $\quad YTM^{(1)\{B\}} = YTM^{(k)\{B\}} \cdot k \ : \ P_t = \sum_t^{t+kB} \frac{c_t^{(k)}}{\left(1+YTM^{(k)\{B\}}\right)^t} + \frac{100}{\left(1+YTM^{(k)\{B\}}\right)^{t+kB}}$

2) Find $G^{\{B\}}$ $\quad G^{\{B\}} = \frac{B-A}{C-A} \times G^{\{A\}} + \frac{C-B}{C-A} \times G^{\{C\}}$

3) Compute G-Spread $\quad \text{G-Spread} = YTM^{(1)} - G^{\{B\}}$

I-Spread

$$\text{I-Spread} = YTM^{(1)\{H\}} - SSR^{\{H\}}$$

where $SSR^{\{H\}}$ is the Standard Swap Rate in the same currency and with the same tenor (horizon) as the bond.

Z-Spread

Z-Spread, aka "*zero-volatility spread*"

$$Z \quad : \quad PV_t = \sum_t^{t+T} \frac{PMT}{(1+z_t+Z)^t} + \frac{FV}{(1+z_T+Z)^T}$$

OAS

OAS, aka "*Option-Adjusted Spread*" on a callable bond

$$OAS = \text{Z-Spread} - \text{Option value in basis points per year}$$

1.2. Yield Measures for Money Market Instruments

Discount Rates (DR)

$$PV = FV\left(1 - DR \times \frac{\text{Days}}{\text{Year}}\right)$$

where PV is the Present Value, FV is the Final Value, DR is the Discount Rate, Days represents the number of days between settlements and maturity, and Year is the number of days in the year.

Add-On Rates (AOR)

$$PV = \frac{FV}{1 + AOR \times \frac{\text{Days}}{\text{Year}}} \iff PV\left(1 + AOR \times \frac{\text{Days}}{\text{Year}}\right) = FV$$

where PV is the Present Value, FV is the Final Value, AOR is the Add-On Rate Days represents the number of days between settlements and maturity, and Year is the number of days in the year.

"*Bond Equivalent Yield*" or "*Investment Yield*"

It's a money market rate quoted on a 365-day add-on rate basis.

- Going from discount rates to bond equivalent yields

  1) Compute FV/PV using DR $\quad \frac{FV}{PV} = \frac{1}{1 - DR \times \frac{\text{Days}}{\text{Year}}}$

  2) Solve for the 365-day AOR $\quad AOR = \left(\frac{FV}{PV} - 1\right) \times \frac{365}{\text{Days}}$

- From add-on rates to bond equivalent yields

  1) Compute FV/PV using AOR $\quad \frac{FV}{PV} = \left(1 + AOR \times \frac{\text{Days}}{\text{Year}}\right)$

  2) Solve for the 365-day AOR $\quad AOR = \left(\frac{FV}{PV} - 1\right) \times \frac{365}{\text{Days}}$

Important: The periodicity in Money Market Instruments is $k = \frac{\text{Year}}{\text{Days}}$, hence, in order to convert periodicities:

$$\left(1 + \frac{APR^{(m)}}{m}\right)^m = \left(1 + \frac{APR^{(n)}}{n}\right)^n \implies \left(1 + \frac{APR^{\left(\left(\frac{\text{Year}}{\text{Days}}\right)\right)}}{\left(\frac{\text{Year}}{\text{Days}}\right)}\right)^{\left(\frac{\text{Year}}{\text{Days}}\right)} = \left(1 + \frac{APR^{(n)}}{n}\right)^n$$

1.3. Floating Rate Note

Price of a T-year FRN with periodicity k

$$P = \sum_{t=1}^{kT} \frac{PMT_t^{(k)}}{\left(1 + r_t^{(k)}\right)^t}$$

with:

$$\begin{aligned} PMT_t^{(k)} &:= MRR_t^{(k)} + QM^{(k)} &= \frac{MRR_t^{(1)}}{k} + \frac{QM^{(1)}}{k} \\ r_t^{(k)} &:= MRR_t^{(k)} + DM_t^{(k)} &= \frac{MRR_t^{(1)}}{k} + \frac{DM_t^{(1)}}{k} \end{aligned}$$

where MRR is the market reference rate, QM is the quoted margin and DM is the discount margins. Usually they are quoted in annual terms: $MRR_t^{(1)}, QM^{(1)}, DM_t^{(1)}$
On each periodic reset date, the FRN is priced at par. Between coupon dates, its flat price will be at premium or discount to par values.

$$\text{Discount / Premium} = \sum_{t=1}^{kT} \frac{QM^{(k)} - DM^{(k)}}{\left(1 + MRR_t^{(k)} + DM^{(k)}\right)^t} \implies \begin{cases} > 0 & \Rightarrow \quad \text{Premium} \\ = 0 & \Rightarrow \quad \text{At Par} \\ < 0 & \Rightarrow \quad \text{Discount} \end{cases}$$

1.4. Spot, Forward, Par rates

Spot curve (zero or strip curve)

- Plot spot rates against all maturities
- Ideal rates = default-risk-free zero-coupon bonds for a full range of maturities
- in practice, use developed-market sovereing bonds

No arbitrage bond pricing with spot rates. Given a sequence of (annual) spot rates $\{z_t^{(1)_k}\}_{t=1}^T$

$$P_0 = \sum_{t=1}^{kT} \frac{c_t^{(k)}}{(1 + z_t^{(k)})^t}$$

Par rate

Hypothetical coupon implied by the spot curve. Given a sequence of (annual) spot rates $\{z_t^{(1)_k}\}_{t=1}^T$, solve for $\{c_\tau^{(k)}\}_{\tau=1}^T$.

$$c_\tau^{(k)} : 100 = \sum_{t=1}^{k\tau} \frac{c_\tau^{(k)}}{(1 + z_t^{(k)})^t} + \frac{100}{(1 + z_t^{(k)})^{k\tau}}$$

Forward Rates

Implied Forward Rate from par rates: "*(B-A)y(A)y*" = *(B-A)* year into A year rate $\implies$ A is the forward period, *B-A* is the tenor of the forward yield.

$$(1 + Z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + Z_B)^B$$

Spot Curve Shape	Par Curve	Forward Curve
Upward Sloping	$<$ Spot curve	$>$ Spot curve
Flat	$=$ Spot curve	$=$ Spot curve
Downward sloping	$>$ Spot curve	$<$ Spot curve