¡article¿
¡presentation¿ Outline
χ^2 distributions
allowframebreaks]Introduction
This material comes primarily from [][Chapter 6]rice07.
Here, we introduce several important distributions that arise from transformations applied to normal distributions.
Many of these distributions form the basis of traditional statistical inference procedures that are taught in introductory
They are very useful in practice due to the central limit theorem: with enough observations, the limiting behavior of ne
allowframebreaks]χ^2_ν Distribution
The first distribution we will consider is the χ^2_1 (Chi-square with 1 degree of freedom).
Definition: χ^2_1 distribution If Z is a standard normal random variable, then $X = Z^2$ is called the chi-square distribution
We typically use the notation $X \sim \chi^2_1$ (in LaTeX: `\chi`).
The pdf of χ^2_1 Let X follow a χ^2_1 distribution. Then, the pdf of X is given by

¡article¿ There are a few ways to show this is the case, and was one of the early examples we saw in Chapter 2. For pra
By definition, X has the same distribution of Z^2, where Z is a standard normal.
Recall the standard normal density is:

Using the CDF method, we write $F_X(x) = P(X \leq x)$
$= P(Z^2 \leq x)$
$= P(-\sqrt{x} \leq Z \leq \sqrt{x})$
$= P(Z \leq \sqrt{x}) - P(Z \leq -\sqrt{x})$
$= \Phi(\sqrt{x}) - \Phi(-\sqrt{x})$,
Where $\Phi(z)$ is the cdf of Z.
Taking the derivative of both sides of the equation, the chain rule gives us $f_X(x) = \frac{1}{2}x^{-1/2}\phi(\sqrt{x}) + \frac{1}{2}x^{-1/2}\phi(-\sqrt{x})$
$= x^{-1/2}\phi(\sqrt{x})$,
where the last step is a result of the symmetry of $\phi(x)$, noting $\phi(-x) = \phi(x)$ for all $x \in R$.
Thus, replacing $\phi(\sqrt{x})$ with the definition,

In Chapter 2, we previously noted that that $f_X(x)$ is an example of a Gamma distribution.
Specifically, the *kernel* of the Gamma density is x raised to some power, and e raised to some multiple of x:

Thus, ignoring the constant for a moment, if $\alpha = 1/2$, $\lambda = 1/2$, then the pdf of $X \sim \chi^2_1$ is just this Gamma density:

Since both functions are proper probability density functions, they have to integrate to one, so the normalizing constant
This is also easily verified. The normalizing constant of the Gamma distribution is $\lambda^\alpha/\Gamma(\alpha)$.
With our specific values of $\lambda = \alpha = 1/2$, and recalling that $\Gamma(1/2) = \sqrt{\pi}$,

MGF of χ^2_1 We previously derived the MGF of a Gamma(α, λ) distribution: $M(t) = \big(\lambda/(\lambda - t)\big)^\alpha$. Thus, the MGF of a

Definition If $U_1, U_2, \ldots, U_n$ are n independent χ^2_1 random variables, then

then the distribution of V is called the Chi-square distribution with n degrees of freedom, denoted χ^2_n.
There are a few different ways of deriving the pdf of a χ^2_n random variable. Here, we will use the MGF uniqueness theor
Let $M_i(t)$ denote the MGF of U_i, where $U_i \sim \chi^2_1$. Then, due to independence, $M_V(t) = M_{\sum_i U_i}(t) = \prod_{i=1}^n M_i(t) = \big(M$
Compare this to the Gamma MGF: $M(t) = \big(\lambda/(\lambda - t)\big)^\alpha$. Then, setting $\lambda = 1/2$, $\alpha = n/2$, we see that V has a Gamma
Thus, the pdf of V is given by:

The expected value and variance of the χ^2_n distribution can easily be found then by using the fact that it is a special cas
The t and F distributions
allowframebreaks]The Student's t distributions The Student's t distribution If $Z \sim N(0, 1)$ and $U \sim \chi^2_n$, and Z and U a

is called the Student's t distribution (or simply the t distribution) with n degrees of freedom, which is often denoted t_n
Students often forget the make sure that Z and U in the definition of the t distribution are independent.
The t distribution is the distribution used to perform the famed "t-test".
The density of the t_n distribution The pdf of the t distribution with n degrees of freedom is:

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