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Homework 1

1. Using the axioms of probability, prove that for any sets $A_1, A_2, \ldots, A_n \subset \Omega$,

$$P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i).$$

   (Hint: consider the sets $B_1 = A_1$, $B_2 = A_2/A_1$, $B_3 = A_3/A_2 \cup A_1, \ldots$.

2. A deck of 52 cards is shuffled thoroughly. What is the probability that the four aces are all next to each other?

3. How many ways are there to place n indistinguishable balls into n boxes so that exactly one box is empty?

4. If n balls are distributed randomly into k boxes, what is the probability that the last box contains n balls?

5. How many unique ways are there to encode a 26-letter alphabet into 8-bit binary strings?

6. Suppose a monkey has a typewriter, and types each of the 26 letters of the alphabet randomly, exactly once[1].

   - What's the probability that the word "random" appears somewhere in the string of letters?
   - How many independent monkey typists would you need in order that the probability that the word appears is at least 0.9?

7. Show that if $A, B,$ and E are events defined on the same sample space Ω, and $P(A|E) \geq P(B|E)$ and $P(A|E^c) \geq P(B|E^c)$, then $P(A) \geq P(B)$

8. Suppose there is a coin that has probability of heads occurring $0 < p < 1$. Two players, A and B, alternately and independently flip a coin and the first player to obtain a head wins.

   - Assume player A flips first. What is the probability that player A wins?

[1] This question is related to the Library of Babel