YOUR NAME
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# Homework 6

1. (1 point) If $X$ and $Y$ are independent exponential($\lambda$) random variables, find the joint density of the polar coordinates $R$ and $\Theta$ of the point $(X, Y)$. Are $R$ and $\Theta$ independent?

2. (1 point) Let $X$ and $Y$ be jointly continuous random variables, with pdf $f(x, y)$. Find an expression for the density of
$$Z = X - Y.$$

3. (1 point) If $T_1$ and $T_2$ are independent exponential random variables, find the density function of
$$R = T_{(2)} - T_{(1)}.$$

4. (1 point) If $X_1$ and $X_2$ are independent Poisson random variables with parameters $\lambda_1$ and $\lambda_2$ respectively, then show that $S_2 = X_1 + X_2$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.

5. (1 point) Use the problem above and induction to argue that if $X_i$ for $i = 1, 2, \ldots, n$ are independent Poisson random variables with parameters $\lambda_i$, respectively, then if we define the sum as
$$S_n = X_1 + X_2 + \ldots + X_n,$$
then $S_n$ is a Poisson random variable with parameter $\sum_i \lambda_i$.

6. Let $T$ be an exponential random variable with parameter $\beta$ and let $W$ be a random variable independent of $T$ which assumes the value 1 with probability 2/3 and the value $-1$ with probability 1/3.

   (a) (1 point) Find the density of $X = WT$.
   **Hint:** I suggest working from first-principles here. Consider $F(x) = P(X \leq x)$, and split up the event $\{X \leq x\}$ as the union of $\{X \leq x, W = 1\}$ and $\{X \leq x, W = -1\}$.

   (b) (1 point) Find $E[X]$

7. A random variable $V$ is said to have a distribution symmetric about 0 if the distriubtion of $V$ is the same as that of $-V$. Let $V$ be a continuous random variable with continuous density function $f$.

   (a) (1 point) Show that $V$ is distributed symmetrically about 0 if and only if $f(t) = f(-t)$, for every $t$. In other words, $f$ is an even function.

   (b) (1 point) Show that $E(X) = 0$.