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September 4, 2025

Chapter 1: Discussion Problems

1. A closet contains n pairs of shoes (that is, $2n$ total shoes). If $2r$ shoes are chosen at random ($2r < n$), what is the probability that there will be no matching pair in the sample?

   There are $2n$ shoes total and we are picking $2r$ of these shoes, so there are a total of $\binom{2n}{2r}$ different samples from which we could draw.

   Now I would like to find the number of samples in which there is no matching pair. First, there are n total pairs to choose from. For a sample of $2r$ to have none of the pairs repeated, then there must be $2r$ unique pairs in that sample. Therefore there are $\binom{n}{2r}$ unique sets of $2r$ pairs of shoes. For each pair, I have two shoes to pick from. Therefore I have 2^{2r} different ways of picking which shoe within each chosen pair. Therefore there are $\binom{n}{2r}2^{2r}$ different samples of size $2r$ that do not contain a matching pair.

   Therefore because each sample is equally as likely, the probability that there will be no matching pair in the sample is $\frac{\binom{n}{2r}2^{2r}}{\binom{2n}{2r}}$.