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¡presentation¿ $\boxed{\text{Outline}}$

χ^2 distributions

[allowframebreaks]Introduction

This material comes primarily from [][Chapter 6]rice07.

Here, we introduce several important distributions that arise from transformations applied to normal distributions.

Many of these distributions form the basis of traditional statistical inference procedures that are taught in introductory

They are very useful in practice due to the central limit theorem: with enough observations, the limiting behavior of nea

[allowframebreaks]χ^2_ν Distribution

The first distribution we will consider is the χ^2_1 (Chi-square with 1 degree of freedom).

Definition: χ^2_1 distribution If Z is a standard normal random variable, then $X = Z^2$ is called the chi-square distribution

We typically use the notation $X \sim \chi^2_1$ (in LaTeX: `\chi`).

The pdf of χ^2_1 Let X follow a χ^2_1 distribution. Then, the pdf of X is given by

¡article¿ There are a few ways to show this is the case, and was one of the early examples we saw in Chapter 2. For pra

By definition, X has the same distribution of Z^2, where Z is a standard normal.

Recall the standard normal density is:

Lemma 6.1: Independent Normal RVs Let X and Y be normally distributed random variables. Then X and Y are inde

The above statement can be proved using the factorization theorem, and considering the MGF or pdf of a bivariate nor
Recall that for most distributions, independence implies $\text{Cov}(X,Y)=0$, but not the other way around.
It turns out that the normal distribution is the only distribution that has this property.
Theorem 6.1: Independence of Deviations Let $X_1,\ldots,X_n$ be iid $N(\mu,\sigma^2)$ random variables. Then, $\bar{X}_n$ is independent o
¡article¿ First, note that $\bar{X}_n$ is normally distributed. Using Lemma 6.1, all we need to do is argue that the deviations a
$= \text{Cov}\big(\sum_{j=1}^n \frac{1}{n}X_j, X_i\big) - \text{Cov}\big(\sum_{i=1}^n \frac{1}{n}X_i, \sum_{j=1}^n \frac{1}{n}X_j\big)$
$= \frac{1}{n}\text{Cov}(X_i,X_i) - \sum_{i=1}^n\sum_{j=1}^n \frac{1}{n^2}\text{Cov}(X_i,X_j)$
$= \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=j}^n \text{Cov}(X_i,X_j)$
$= \frac{\sigma^2}{n} - \frac{1}{n^2}(n\sigma^2) = 0$. Thus, by Lemma 6.1, $\bar{X}_n$ is independent of $X_i - \bar{X}_n$ for all i.
¡presentation¿ *Proof.*
Corollary 6.1 If the X_i are iid $N(\mu,\sigma^2)$, then $\bar{X}_n$ is independent of the sample variance S^2, defined by

¡article¿ First recall from one of our homework problems that S^2 is an unbiased estimate of σ^2: $E[S^2]=\sigma^2$ (Note that
From Theorem 6.1, if the X_i are normally distributed, then $\bar{X}_n$ is independent of the vector $\big(X_i - \bar{X}_n\big)_{i=1}^n$. Because S^2
Theorem 6.2 If the X_i are iid normal, then $(n-1)S^2/\sigma$ has a chi-square distribution with $n-1$ degrees of freedom.
¡article¿ First recall how we defined the χ^2_n distribution. First, if we square a standard normal $Z\sim N(0,1)$, then $Z^2 \sim$

and therefore

$\boxed{\text{The sample variance}}$
¡presentation¿ allowframebreaks=0.8]References and Acknowledgements

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