Principal Component Analysis for Risk Management

A. Loading Libraries

In [2]:
```
pip install fredapi
```

```
Defaulting to user installation because normal site-packages is not writeable
Collecting fredapi
  Obtaining dependency information for fredapi from https://files.pythonhosted.org/packages/73/64/1db43417cf7ed430f104a347126b5260a1724ee9a1b7d0b1622262c9c4df/fredapi-0.5.2-py3-none-any.whl.metadata
  Downloading fredapi-0.5.2-py3-none-any.whl.metadata (5.0 kB)
Requirement already satisfied: pandas in c:\programdata\anaconda3\lib\site-packages (from fredapi) (1.5.3)
Requirement already satisfied: python-dateutil>=2.8.1 in c:\programdata\anaconda3\lib\site-packages (from pandas->fredapi) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in c:\programdata\anaconda3\lib\site-packages (from pandas->fredapi) (2022.7)
Requirement already satisfied: numpy>=1.21.0 in c:\programdata\anaconda3\lib\site-packages (from pandas->fredapi) (1.24.3)
Requirement already satisfied: six>=1.5 in c:\programdata\anaconda3\lib\site-packages (from python-dateutil>=2.8.1->pandas->fredapi) (1.16.0)
Downloading fredapi-0.5.2-py3-none-any.whl (11 kB)
Installing collected packages: fredapi
Successfully installed fredapi-0.5.2
Note: you may need to restart the kernel to use updated packages.
```

In [11]:
```
import pandas as pd
from fredapi import Fred
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
```

B. Loading US Treasury Yields Data

In [4]:
```
fred = Fred(api_key='5079f41d061a4037d81f3da69e018803')

# List of Treasury yield series IDs
series_ids = ['DGS1MO', 'DGS3MO', 'DGS6MO', 'DGS1', 'DGS2', 'DGS3', 'DGS5', \
              'DGS7', 'DGS10', 'DGS20', 'DGS30']

# Function to get data for a single series
def get_yield_data(series_id):
    data = fred.get_series(series_id, observation_start="1975-01-01", observation_end="2024-05-03")
    return data

# Get data for all series
yields_dict = {series_id: get_yield_data(series_id) for series_id in series_ids}

# Combine into a single DataFrame
yields = pd.DataFrame(yields_dict)

# Rename columns for clarity
yields.columns = ['1 Month', '3 Month', '6 Month', '1 Year', '2 Year', '3 Year', '5 Year', \
                  '7 Year', '10 Year', '20 Year', '30 Year']
```

In [5]:
```
# Make datetime as the index
yields.index = pd.to_datetime(yields.index)
```

In [6]:
```
# Drop NaN in the dataset
yields = yields.dropna()
```

In [12]:
```
yields
```

Out[12]:

	1 Month	3 Month	6 Month	1 Year	2 Year	3 Year	5 Year	7 Year	10 Year	20 Year	30 Year
2001-07-31	3.67	3.54	3.47	3.53	3.79	4.06	4.57	4.86	5.07	5.61	5.51
2001-08-01	3.65	3.53	3.47	3.56	3.83	4.09	4.62	4.90	5.11	5.63	5.53
2001-08-02	3.65	3.53	3.46	3.57	3.89	4.17	4.69	4.97	5.17	5.68	5.57
2001-08-03	3.63	3.52	3.47	3.57	3.91	4.22	4.72	4.99	5.20	5.70	5.59
2001-08-06	3.62	3.52	3.47	3.56	3.88	4.17	4.71	4.99	5.19	5.70	5.59
...	...	...	...	...	...	...	...	...	...	...	...
2024-04-29	5.48	5.45	5.43	5.20	4.97	4.80	4.65	4.64	4.63	4.86	4.75
2024-04-30	5.48	5.46	5.44	5.25	5.04	4.87	4.72	4.71	4.69	4.90	4.79
2024-05-01	5.47	5.46	5.43	5.21	4.96	4.79	4.64	4.64	4.63	4.85	4.74
2024-05-02	5.51	5.46	5.42	5.16	4.87	4.71	4.57	4.57	4.58	4.82	4.72
2024-05-03	5.51	5.45	5.41	5.12	4.81	4.63	4.48	4.49	4.50	4.75	4.66

5693 rows × 11 columns

C. Covariance & Correlation Matrix

In [7]:
```
# Calculate covariance matrix for US Treasury yields in the dataset
covariance_matrix = yields.cov()
print("Covariance Matrix:")
print(covariance_matrix)
```

```
Covariance Matrix:
          1 Month   3 Month   6 Month    1 Year    2 Year    3 Year    5 Year  \
1 Month  2.909927  2.946278  2.948709  2.812408  2.499031  2.251515  1.850674
3 Month  2.946278  3.004493  3.018967  2.887166  2.571375  2.317856  1.901841
6 Month  2.948709  3.018967  3.051413  2.931246  2.624882  2.372696  1.951042
1 Year   2.812408  2.887166  2.931246  2.836161  2.569272  2.340278  1.944752
2 Year   2.499031  2.571375  2.624882  2.569272  2.388302  2.216908  1.897526
3 Year   2.251515  2.317856  2.372696  2.340278  2.216908  2.092766  1.842915
5 Year   1.850674  1.901841  1.951042  1.944752  1.897526  1.842915  1.714385
7 Year   1.565717  1.605385  1.647697  1.654011  1.648671  1.634697  1.582433
10 Year  1.315260  1.344596  1.381242  1.395294  1.418752  1.435157  1.447139
20 Year  1.078020  1.098030  1.128773  1.154327  1.214425  1.267258  1.350980
30 Year  0.862766  0.872645  0.893530  0.919017  0.988536  1.052167  1.161403

           7 Year   10 Year   20 Year   30 Year
1 Month  1.565717  1.315260  1.078020  0.862766
3 Month  1.605385  1.344596  1.098030  0.872645
6 Month  1.647697  1.381242  1.128773  0.893530
1 Year   1.654011  1.395294  1.154327  0.919017
2 Year   1.648671  1.418752  1.214425  0.988536
3 Year   1.634697  1.435157  1.267258  1.052167
5 Year   1.582433  1.447139  1.350980  1.161403
7 Year   1.504196  1.416231  1.371829  1.204537
10 Year  1.416231  1.378491  1.383803  1.238816
20 Year  1.371829  1.383803  1.455676  1.324389
30 Year  1.204537  1.238816  1.324389  1.229380
```

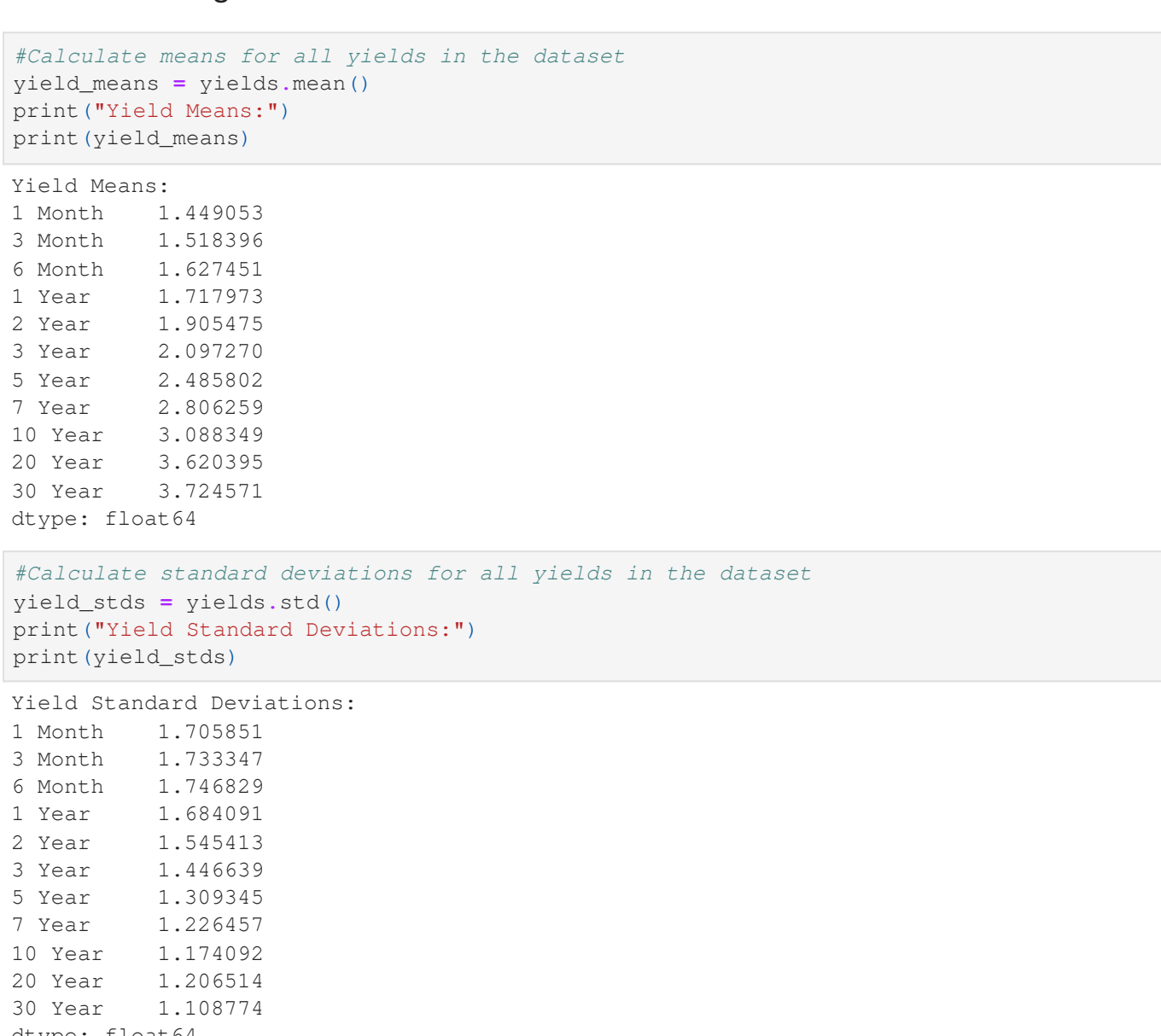
In [8]:
```
#Make a heatmap for covariance matrix
plt.figure(figsize=(8, 6))
sns.heatmap(covariance_matrix, annot=True, cmap='coolwarm', fmt=".1f")
plt.title('Covariance Heat Map of Treasury Bond Yields')
plt.show()
```

In [9]:
```
# Calculate correlation matrix for US Treasury yields in the dataset
correlation_matrix = yields.corr()
print("Correlation Matrix:")
print(correlation_matrix)
```

```
Correlation Matrix:
          1 Month   3 Month   6 Month    1 Year    2 Year    3 Year    5 Year  \
1 Month  1.000000  0.996431  0.989556  0.978975  0.947951  0.912375  0.828580
3 Month  0.996431  1.000000  0.997062  0.989056  0.959920  0.924359  0.837981
6 Month  0.989556  0.997062  1.000000  0.996406  0.972332  0.938926  0.853025
1 Year   0.978975  0.989056  0.996406  1.000000  0.987188  0.960598  0.881951
2 Year   0.947951  0.959920  0.972332  0.987188  1.000000  0.991614  0.937754
3 Year   0.912375  0.924359  0.938926  0.960598  0.991614  1.000000  0.972950
5 Year   0.828580  0.837981  0.853025  0.881951  0.937754  0.972950  1.000000
7 Year   0.748376  0.755164  0.769085  0.800794  0.869836  0.921350  0.985414
10 Year  0.656702  0.660700  0.673469  0.705664  0.781916  0.844962  0.941356
20 Year  0.523785  0.525045  0.535579  0.568108  0.651319  0.726060  0.855190
30 Year  0.456151  0.454056  0.461334  0.492170  0.576905  0.655966  0.799992

           7 Year   10 Year   20 Year   30 Year
1 Month  0.748376  0.656702  0.523785  0.456151
3 Month  0.755164  0.660700  0.525045  0.454056
6 Month  0.769085  0.673469  0.535579  0.461334
1 Year   0.800794  0.705664  0.568108  0.492170
2 Year   0.869836  0.781916  0.651319  0.576905
3 Year   0.921350  0.844962  0.726060  0.655966
5 Year   0.985414  0.941356  0.855190  0.799992
7 Year   1.000000  0.983513  0.927076  0.885778
10 Year  0.983513  1.000000  0.976877  0.951616
20 Year  0.927076  0.976877  1.000000  0.990011
30 Year  0.885778  0.951616  0.990011  1.000000
```

In [10]:
```
# Make a heatmap for correlation matrix
plt.figure(figsize=(8, 6))
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm', fmt=".2f")
plt.title('Correlation Heat Map of Treasury Bond Yields')
plt.show()
```



D. Feature Extractions & Principal Component Analysis

1. Normalizing the Variables

In [13]:
```
#Calculate means for all yields in the dataset
yield_means = yields.mean()
print("Yield Means:")
print(yield_means)
```

```
Yield Means:
1 Month    1.449053
3 Month    1.518396
6 Month    1.627451
1 Year     1.717973
2 Year     1.905475
3 Year     2.097270
5 Year     2.485802
7 Year     2.806259
10 Year    3.088349
20 Year    3.620395
30 Year    3.724571
dtype: float64
```

In [14]:
```
#Calculate standard deviations for all yields in the dataset
yield_stds = yields.std()
print("Yield Standard Deviations:")
print(yield_stds)
```

```
Yield Standard Deviations:
1 Month    1.705851
3 Month    1.733347
6 Month    1.746829
1 Year     1.684091
2 Year     1.545413
3 Year     1.446639
5 Year     1.309345
7 Year     1.226457
10 Year    1.174092
20 Year    1.206514
30 Year    1.108774
dtype: float64
```

In [15]:
```
# Now create a standardized US Treasury yield dataset
standardized_data = (yields - yield_means) / yield_stds
print("Standardized Yield (first 5 rows):")
print(standardized_data.head())
```

```
Standardized Yield (first 5 rows):
             1 Month   3 Month   6 Month    1 Year    2 Year    3 Year  \
2001-07-31  1.301958  1.166300  1.054796  1.075968  1.219431  1.356751
2001-08-01  1.290234  1.160531  1.054796  1.093781  1.245314  1.377489
2001-08-02  1.290234  1.160531  1.049071  1.099719  1.284139  1.432789
2001-08-03  1.278510  1.154762  1.054796  1.099719  1.297080  1.467352
2001-08-06  1.272647  1.154762  1.054796  1.093781  1.277668  1.432789

              5 Year    7 Year   10 Year   20 Year   30 Year
2001-07-31  1.591786  1.674532  1.687817  1.649052  1.610273
2001-08-01  1.629973  1.707147  1.721886  1.665629  1.628311
2001-08-02  1.683435  1.764222  1.772989  1.707071  1.664387
2001-08-03  1.706348  1.780529  1.798541  1.723647  1.682425
2001-08-06  1.698710  1.780529  1.790023  1.723647  1.682425
```

2. Calculating Covariance Matrix of the Normalized Data

In [16]:
```
import numpy as np
from numpy import linalg as LA
```

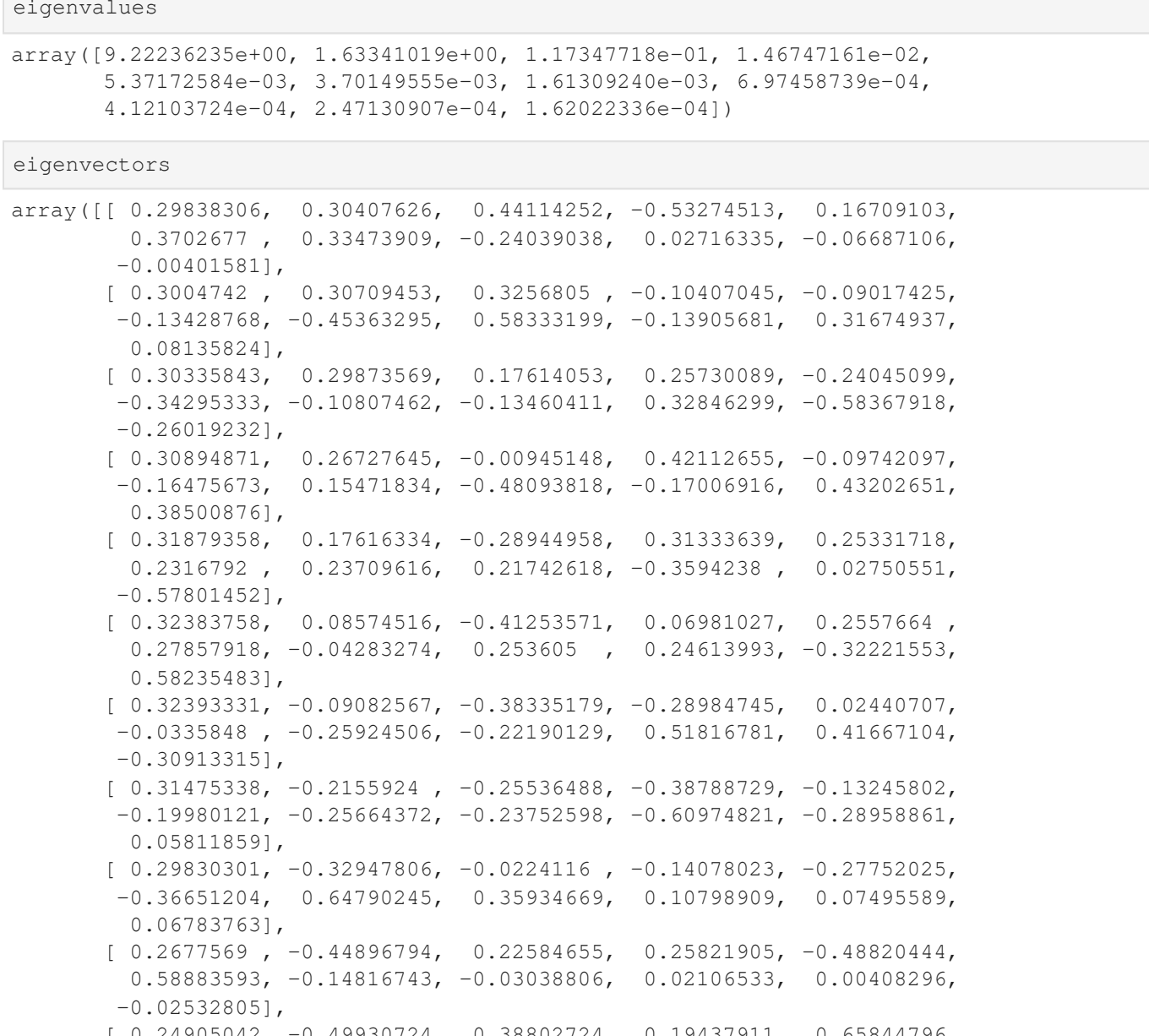
In [17]:
```
# Calculate covariance matrix of the standardized dataset
std_data_cov = standardized_data.cov()
```

In [19]:
```
std_data_cov
```

Out[19]:

	1 Month	3 Month	6 Month	1 Year	2 Year	3 Year	5 Year	7 Year	10 Year	20 Year	30 Year
1 Month	1.000000	0.996431	0.989556	0.978975	0.947951	0.912375	0.828580	0.748376	0.656702	0.523785	0.456151
3 Month	0.996431	1.000000	0.997062	0.989056	0.959920	0.924359	0.837981	0.755164	0.660700	0.525045	0.454056
6 Month	0.989556	0.997062	1.000000	0.996406	0.972332	0.938926	0.853025	0.769085	0.673469	0.535579	0.461334
1 Year	0.978975	0.989056	0.996406	1.000000	0.987188	0.960598	0.881951	0.800794	0.705664	0.568108	0.492170
2 Year	0.947951	0.959920	0.972332	0.987188	1.000000	0.991614	0.937754	0.869836	0.781916	0.651319	0.576905
3 Year	0.912375	0.924359	0.938926	0.960598	0.991614	1.000000	0.972950	0.921350	0.844962	0.726060	0.655966
5 Year	0.828580	0.837981	0.853025	0.881951	0.937754	0.972950	1.000000	0.985414	0.941356	0.855190	0.799992
7 Year	0.748376	0.755164	0.769085	0.800794	0.869836	0.921350	0.985414	1.000000	0.983513	0.927076	0.885778
10 Year	0.656702	0.660700	0.673469	0.705664	0.781916	0.844962	0.941356	0.983513	1.000000	0.976877	0.951616
20 Year	0.523785	0.525045	0.535579	0.568108	0.651319	0.726060	0.855190	0.927076	0.976877	1.000000	0.990011
30 Year	0.456151	0.454056	0.461334	0.492170	0.576905	0.655966	0.799992	0.885778	0.951616	0.990011	1.000000

In [18]:
```
# Draw a heatmap of the covariance matrix
plt.figure(figsize=(8, 6))
sns.heatmap(std_data_cov, annot=True, cmap='coolwarm', fmt=".2f")
plt.title('Covariance Heat Map of Standardized Treasury Bond Yields')
plt.show()
```



3. Calculating Eigenvectors and Eigenvalues

In [20]:
```
# Calculate eigenvectors and eigenvalues of the covariance matrix of standardized yield dataset
eigenvalues, eigenvectors = LA.eig(std_data_cov)
eigenvalues
```

Out[20]:
```
array([9.22236235e+00, 1.63341019e+00, 1.17347718e-01, 1.46747161e-02,
       5.37172584e-03, 3.70149555e-03, 1.61309240e-03, 6.97458739e-04,
       4.12103724e-04, 2.47130907e-04, 1.62022336e-04])
```

In [21]:
```
eigenvectors
```

Out[21]:
```
array([[ 0.29838306,  0.30407626,  0.44114252, -0.53274513,  0.16709103,
         0.3702677 ,  0.33473909, -0.24039038,  0.02716335, -0.06687106,
        -0.00401581],
       [ 0.3004742 ,  0.30709453,  0.3256805 , -0.10407045, -0.09017425,
        -0.13428768, -0.45363295,  0.58333199, -0.13905681,  0.31674937,
         0.08135824],
       [ 0.30335843,  0.29873569,  0.17614053,  0.25730089, -0.24045099,
        -0.34295333, -0.10807462, -0.13460411,  0.32846299, -0.58367918,
        -0.26019232],
       [ 0.30894871,  0.26727645, -0.00945148,  0.42112655, -0.09742097,
        -0.16475673,  0.15471834, -0.48093818, -0.17006916,  0.43202651,
         0.38500876],
       [ 0.31879358,  0.17616334, -0.28944958,  0.31333639,  0.25331718,
         0.2316792 ,  0.23709616,  0.21742618, -0.3594238 ,  0.02750551,
        -0.57801452],
       [ 0.32383758,  0.08574516, -0.41253571,  0.06981027,  0.2557664 ,
         0.27857918, -0.04283274,  0.253605  ,  0.24613993, -0.32221553,
         0.58235483],
       [ 0.32393331, -0.09082567, -0.38335179, -0.28984745,  0.02440707,
        -0.0335848 , -0.25924506, -0.22190129,  0.51816781,  0.41667104,
        -0.30913315],
       [ 0.31475338, -0.2155924 , -0.25536488, -0.38788729, -0.13245802,
        -0.19980121, -0.25664372, -0.23752598, -0.60974821, -0.28958861,
         0.05811859],
       [ 0.29830301, -0.32947806, -0.0224116 , -0.14078023, -0.27752025,
        -0.36651204,  0.64790245,  0.35934669,  0.10798909,  0.07495589,
         0.06783763],
       [ 0.2677569 , -0.44896794,  0.22584655,  0.25821905, -0.48820444,
         0.58883593, -0.14816743, -0.03038806,  0.02106533,  0.00408296,
        -0.02532805],
       [ 0.24905042, -0.49930724,  0.38802724,  0.19437911,  0.65844796,
        -0.21614344, -0.11702391, -0.07221199,  0.03078012, -0.01338036,
         0.00323551]])
```

In [22]:
```
# Transform standardized data with Loadings
principal_components = standardized_data.dot(eigenvectors)
principal_components.columns = ["PC_1","PC_2","PC_3","PC_4","PC_5","PC_6","PC_7","PC_8","PC_9","PC_10","PC_11"]
principal_components
```

Out[22]:

	PC_1	PC_2	PC_3	PC_4	PC_5	PC_6	PC_7	PC_8	PC_9	PC_10	PC_11
2001-07-31	4.608202	-0.918184	0.138746	-0.223360	0.013690	0.063247	0.008572	0.006278	0.002652	0.017945	-0.002084
2001-08-01	4.665170	-0.950595	0.102492	-0.220182	0.013318	0.054096	0.014505	0.002266	0.000391	0.027474	-0.006491
2001-08-02	4.766161	-1.009754	0.054519	-0.230244	0.021179	0.064157	0.017119	0.011726	-0.002439	0.025895	-0.003628
2001-08-03	4.807092	-1.038602	0.027695	-0.224234	0.025575	0.063723	0.018644	0.020406	0.009374	0.017296	0.002344
2001-08-06	4.781112	-1.044855	0.048161	-0.228694	0.013594	0.051783	0.009102	0.010319	0.003817	0.021904	-0.007042
...	...	...	...	...	...	...	...	...	...	...	...
2024-04-29	5.826095	1.299385	0.346345	-0.111170	0.030878	0.032624	0.014876	0.024558	-0.004012	-0.018347	-0.011511
2024-04-30	5.937236	1.256035	0.301150	-0.109479	0.037070	0.028763	0.020369	0.024325	-0.012125	-0.012149	-0.009569
2024-05-01	5.816673	1.308147	0.347863	-0.110336	0.023314	0.022601	0.014241	0.025915	-0.009769	-0.014609	-0.006476
2024-05-02	5.717150	1.341993	0.420558	-0.125210	0.020940	0.027305	0.013387	0.020816	0.002778	-0.018245	-0.004162
2024-05-03	5.583922	1.416405	0.462381	-0.122794	0.019738	0.027403	0.011454	0.019110	-0.000008	-0.024613	-0.007920

5693 rows × 11 columns

In [23]:
```
# Put data into a DataFrame
df_eigval = pd.DataFrame({"Eigenvalues":eigenvalues}, index=range(1,12))

# Work out explained proportion
df_eigval["Explained proportion"] = df_eigval["Eigenvalues"] / np.sum(df_eigval["Eigenvalues"])
#Format as percentage
df_eigval.style.format({"Explained proportion": "{:.2%}"})
```

Out[23]:

	Eigenvalues	Explained proportion
1	9.222362	83.84%
2	1.633410	14.85%
3	0.117348	1.07%
4	0.014675	0.13%
5	0.005372	0.05%
6	0.003701	0.03%
7	0.001613	0.01%
8	0.000697	0.01%
9	0.000412	0.00%
10	0.000247	0.00%
11	0.000162	0.00%

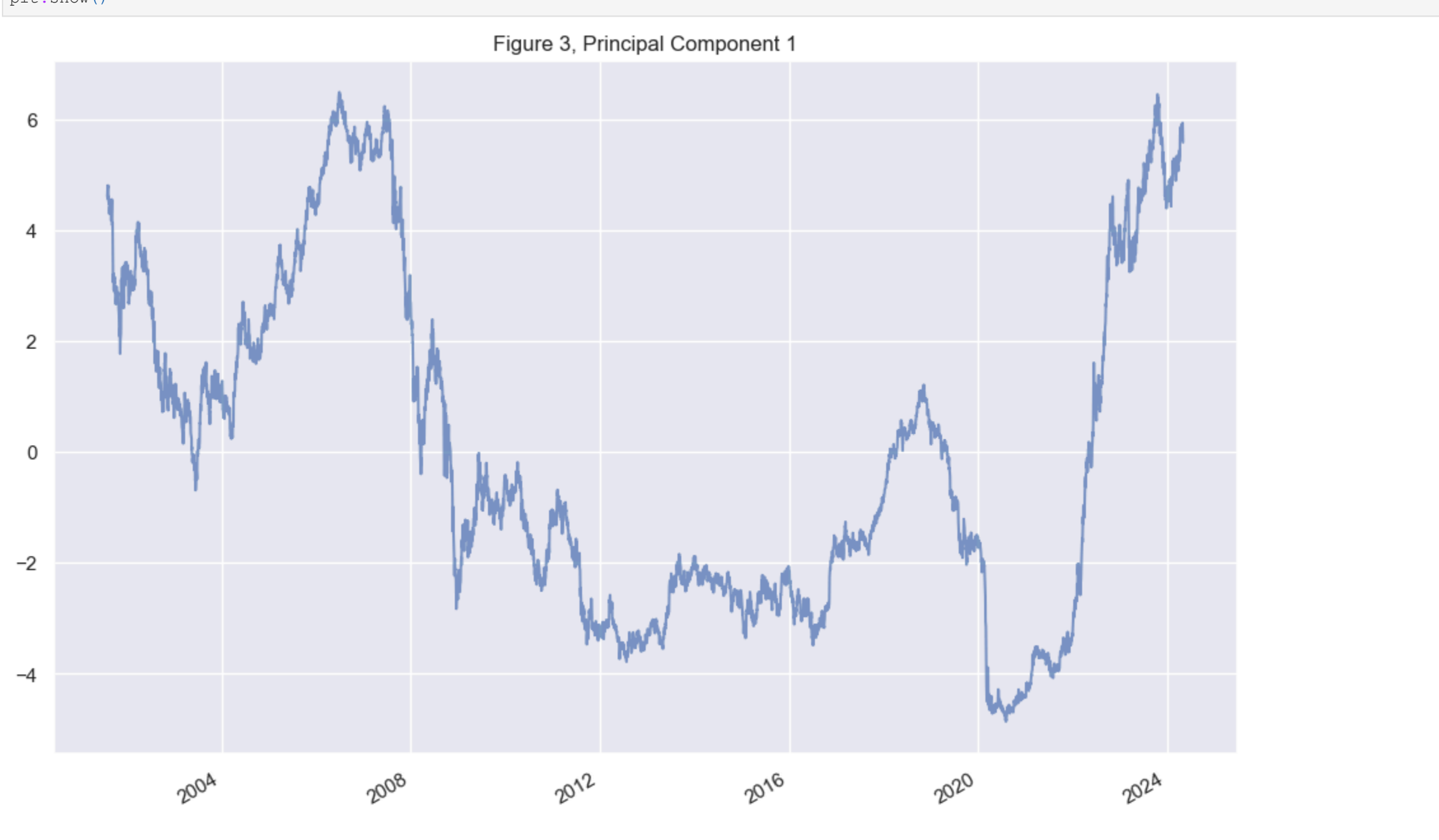
From the above table, we can see PC_1 can explain almost 84% of the variance in the standardized data. PC_2 can explain almost 15% of the variance in the standardized data. The first two leading principal components can explain almost 99% of the variance in the dataset. The rest of the 9 principal components only explain 1% of the variance of the data.

Hence, to make data analysis more efficient without losing too much information, we can just use the first two principal components for analysis instead of all 11 principal components.

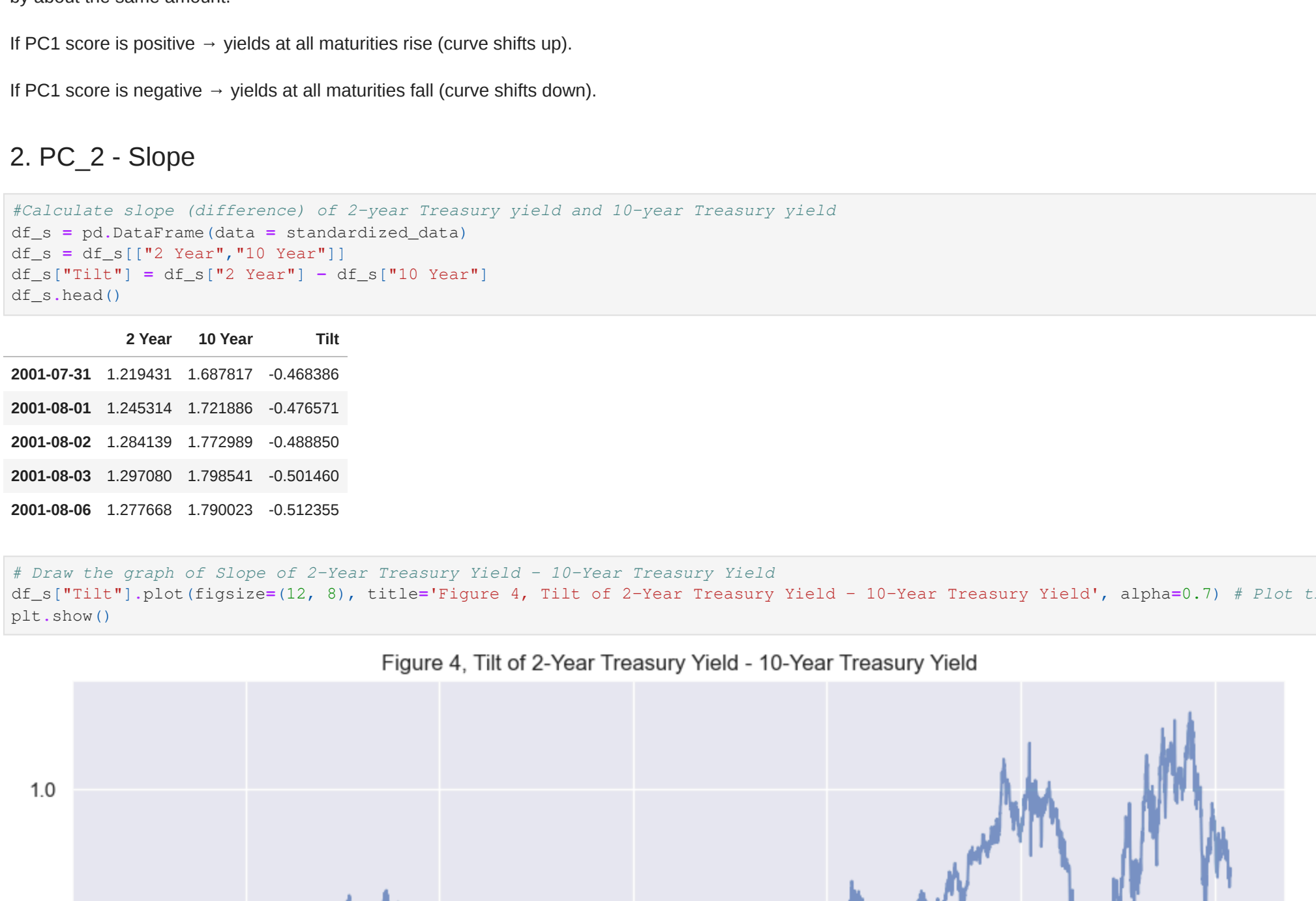
E. Principal Component Analysis

1. PC_1 Parallel Shift

In [24]:
```
# Treasury Yield Curve
yields.plot(figsize=(12, 8), title='Figure 2, Treasury Yields', alpha=0.7) # Plot the yields
plt.legend(bbox_to_anchor=(1.03, 1))
plt.show()
```



In [25]:
```
# Plot PC_1
principal_components["PC_1"].plot(figsize=(12, 8), title='Figure 3, Principal Component 1', alpha=0.7)
plt.show()
```



This PC_1 is used to represent the yield curve parallel shift. It explains the largest share of variance in yields (often 70–80%). When this factor goes up, all yields move up together by about the same amount.

If PC1 score is positive → yields at all maturities rise (curve shifts up).

If PC1 score is negative → yields at all maturities fall (curve shifts down).

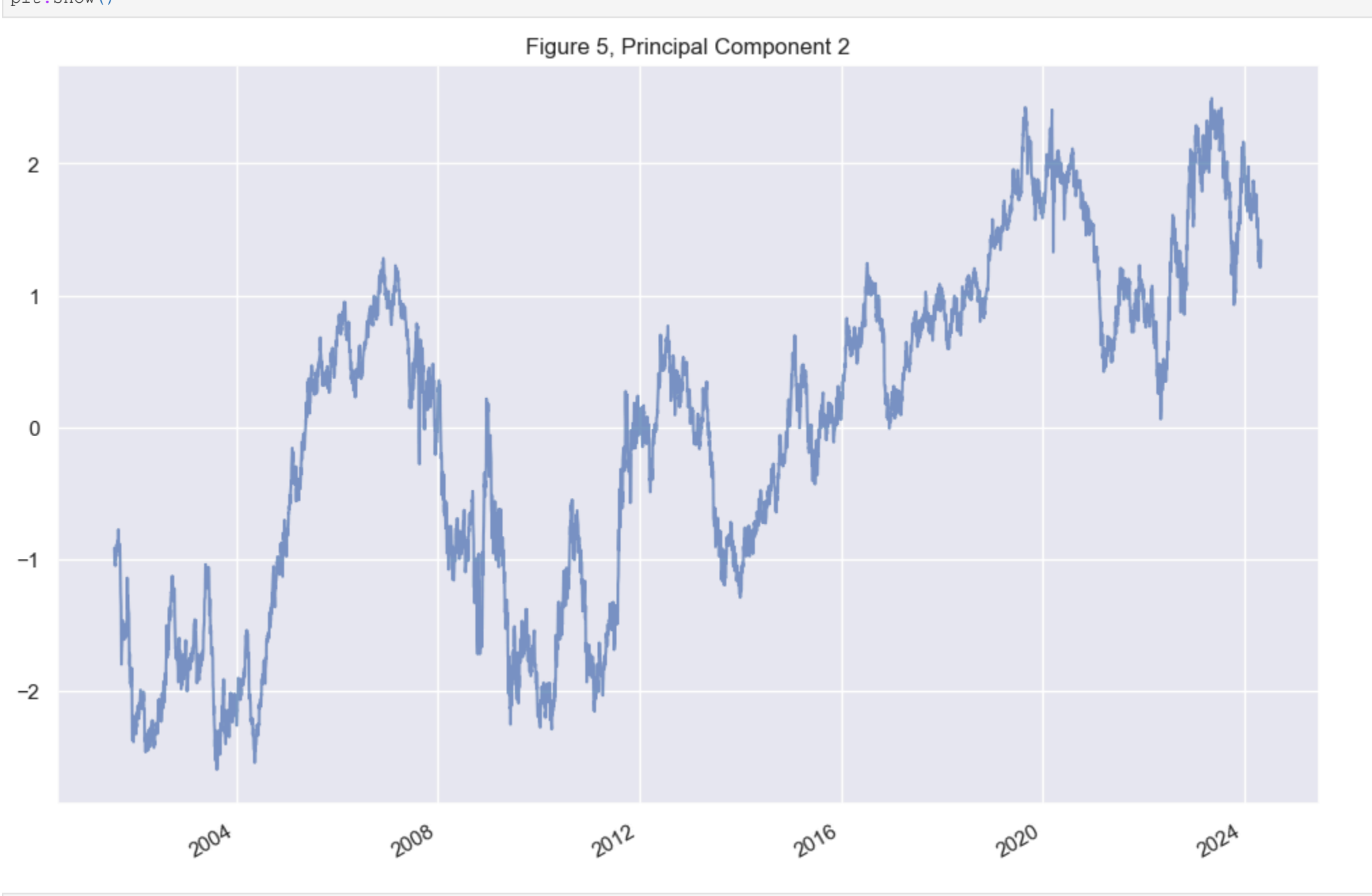
2. PC_2 - Slope

In [26]:
```
#Calculate slope (difference) of 2-year Treasury yield and 10-year Treasury yield
df_s = pd.DataFrame(data = standardized_data)
df_s = df_s[["2 Year","10 Year"]]
df_s["Tilt"] = df_s["2 Year"] - df_s["10 Year"]
df_s.head()
```

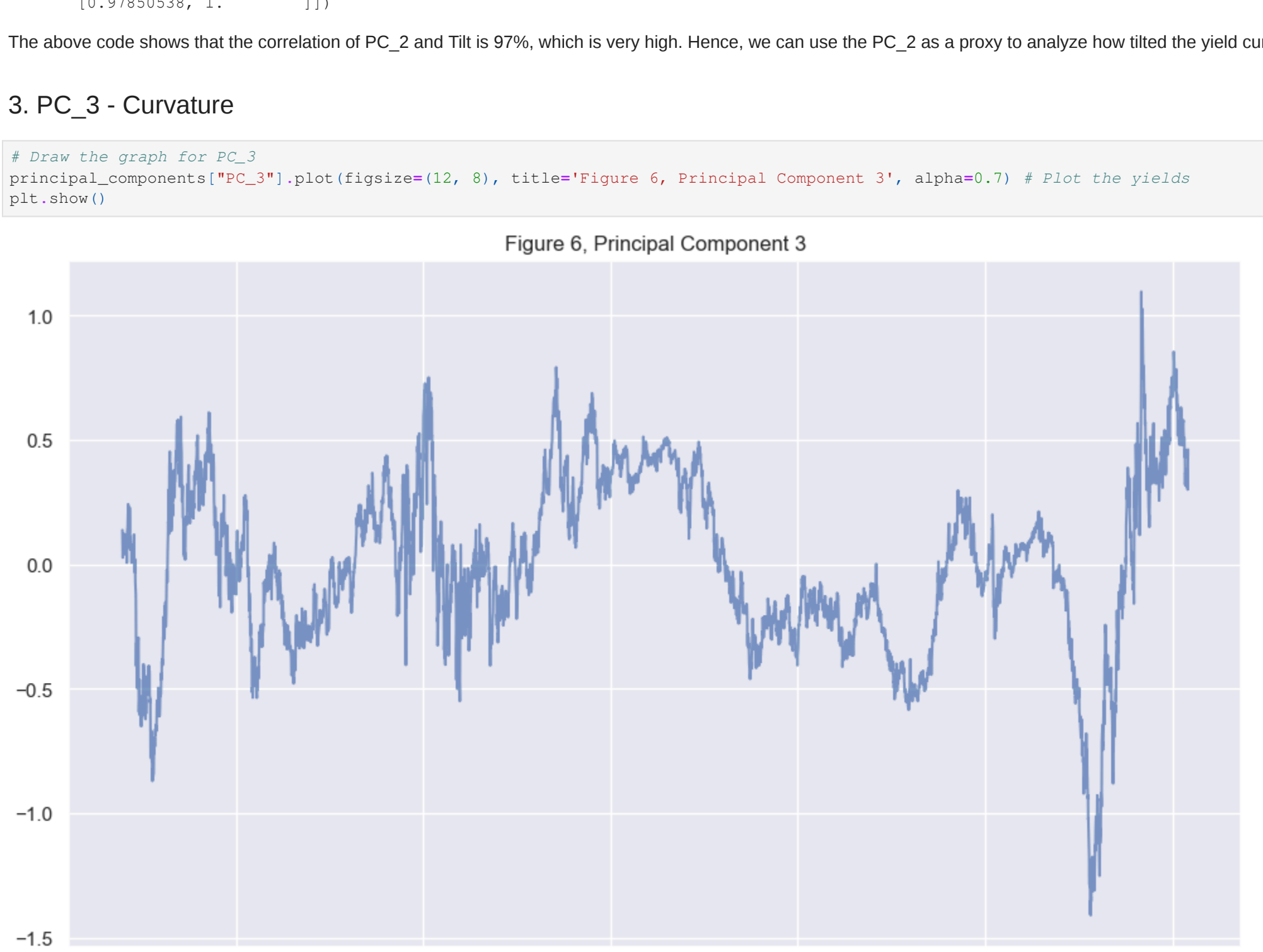
Out[26]:

	2 Year	10 Year	Tilt
2001-07-31	1.219431	1.687817	-0.468386
2001-08-01	1.245314	1.721886	-0.476571
2001-08-02	1.284139	1.772989	-0.488850
2001-08-03	1.297080	1.798541	-0.501460
2001-08-06	1.277668	1.790023	-0.512355

In [27]:
```
# Draw the graph of Slope of 2-Year Treasury Yield - 10-Year Treasury Yield
df_s["Tilt"].plot(figsize=(12, 8), title='Figure 4, Tilt of 2-Year Treasury Yield - 10-Year Treasury Yield', alpha=0.7) # Plot the yields differenc
plt.show()
```



In [28]:
```
# Draw the graph for PC_2
principal_components["PC_2"].plot(figsize=(12, 8), title='Figure 5, Principal Component 2', alpha=0.7) # Plot the yields
plt.show()
```



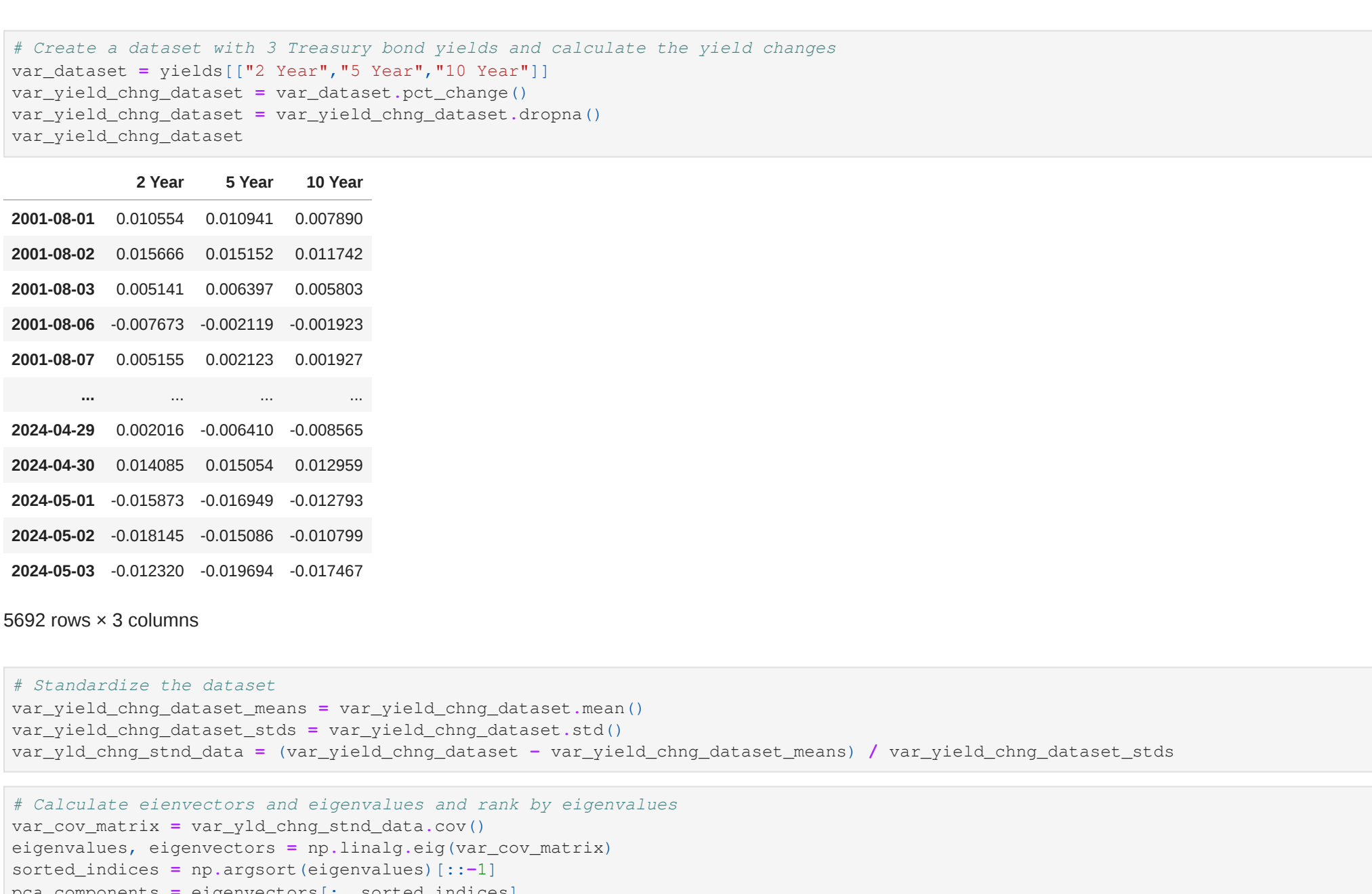
In [29]:
```
np.corrcoef(principal_components["PC_2"], df_s["Tilt"])
```

Out[29]:
```
array([[1.        , 0.97850538],
       [0.97850538, 1.        ]])
```

The above code shows that the correlation of PC_2 and Tilt is 97%, which is very high. Hence, we can use the PC_2 as a proxy to analyze how tilted the yield curve is.

3. PC_3 - Curvature

In [30]:
```
# Draw the graph for PC_3
principal_components["PC_3"].plot(figsize=(12, 8), title='Figure 6, Principal Component 3', alpha=0.7) # Plot the yields
plt.show()
```



We can see that the change in curvature of the yield curve oscillates around 0.

F. Value at Risk for a Fixed Income Portfolio

In [32]:
```
# Create a dataset with 3 Treasury bond yields and calculate the yield changes
var_dataset = yields[["2 Year","5 Year","10 Year"]]
var_yield_chng_dataset = var_dataset.pct_change()
var_yield_chng_dataset = var_yield_chng_dataset.dropna()
var_yield_chng_dataset
```

Out[32]:

	2 Year	5 Year	10 Year
2001-08-01	0.010554	0.010941	0.007890
2001-08-02	0.015666	0.015152	0.011742
2001-08-03	0.005141	0.006397	0.005803
2001-08-06	-0.007673	-0.002119	-0.001923
2001-08-07	0.005155	0.002123	0.001927
...	...	...	...
2024-04-29	0.002016	-0.006410	-0.008565
2024-04-30	0.014085	0.015054	0.012959
2024-05-01	-0.015873	-0.016949	-0.012793
2024-05-02	-0.018145	-0.015086	-0.010799
2024-05-03	-0.012320	-0.019694	-0.017467

5692 rows × 3 columns

In [34]:
```
# Standardize the dataset
var_yield_chng_dataset_means = var_yield_chng_dataset.mean()
var_yield_chng_dataset_stds = var_yield_chng_dataset.std()
var_yld_chng_stnd_data = (var_yield_chng_dataset - var_yield_chng_dataset_means) / var_yield_chng_dataset_stds
```

In [35]:
```
# Calculate eienvectors and eigenvalues and rank by eigenvalues
var_cov_matrix = var_yld_chng_stnd_data.cov()
eigenvalues, eigenvectors = np.linalg.eig(var_cov_matrix)
sorted_indices = np.argsort(eigenvalues)[::-1]
pca_components = eigenvectors[:, sorted_indices]
```

In [36]:
```
# Put data into a DataFrame
df_eigval = pd.DataFrame({"Eigenvalues":eigenvalues}, index=range(1,4))

# Work out explained proportion
df_eigval["Explained proportion"] = df_eigval["Eigenvalues"] / np.sum(df_eigval["Eigenvalues"])
#Format as percentage
df_eigval.style.format({"Explained proportion": "{:.2%}"})
```

Out[36]:

	Eigenvalues	Explained proportion
1	2.545146	84.84%
2	0.384763	12.83%
3	0.070091	2.34%

From the above table, we can see the first two eigenvectors account for 97% of the variance in the dataset. Hence, we are going to select the first two eigenvectors for analysis.

In [37]:
```
# Choose number of components (e.g., 2)
n_components = 2
selected_components = pca_components[:, :n_components]
```

In [38]:
```
# Define a simple portfolio
portfolio = {
    2: 2000000,  # $2M in 2-year bond
    5: 2000000,  # $2M in 5-year bond
    10: 1000000  # $1M in 10-year bond
}
```

In [39]:
```
# Calculate portfolio sensitivities (assuming duration = maturity for simplicity)
sensitivities = np.array([maturity * amount for maturity, amount in portfolio.items()])

# Calculate portfolio value changes
portfolio_changes = (var_yield_chng_dataset*sensitivities) @ selected_components

# Calculate VaR
confidence_level = 0.95  # 95% VaR
var = -np.percentile(portfolio_changes, 100 * (1 - confidence_level))

print(f"1-day 95% VaR: ${var:,.2f}")

# Display summary statistics
print("\nSummary Statistics:")
print(f"Portfolio Value: ${sum(portfolio.values()):,.2f}")
print(f"VaR as % of Portfolio Value: {var / sum(portfolio.values()) * 100:.3f}%")
```

```
1-day 95% VaR: $458,248.59

Summary Statistics:
Portfolio Value: $5,000,000.00
VaR as % of Portfolio Value: 9.165%
```

The above result shows that the 1-day VaR at 95% confidence level for our simple Treasury bond portfolio is $458,249. It is about 9% of the total portfolio value.

In [ ]: