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ECE 6390
Computer Project #1

+10 using python

The original state-space model of a gas turbine has 12 states, 2 inputs, and 2 outputs. State-space matrices A, B, C, and D are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.202 & -1.150 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.360 & -13.60 & -12.80 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.620 & -9.400 & -9.150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -188.0 & -111.6 & -116.4 & -20.80 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1.0439 & 0 & 0 & -1.794 & 0 & 0 & 1.0439 & 0 & 0 & 0 & -1.704 \\ 0 & 4.1486 & 0 & 0 & 2.6775 & 0 & 0 & 4.1486 & 0 & 0 & 0 & 2.6775 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 0.2640 & 0.8060 & -1.420 & -15.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.9000 & 2.1200 & 1.9500 & 9.3500 & 25.800 & 7.1400 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

1)

A reduced model, $\hat{A}$, was developed using the second Cauer method. All computations were completed in Python v3.4 with the Numpy and Scipy libraries. The code and numerical results are included in the Appendix.

The 2nd order reduced model (4x4) from the second Cauer method has the form below. Results from Python computations are also shown.

$$\hat{A} = \begin{bmatrix} -H_1H_2 & -H_1H_4 \\ -H_1H_2 & -(H_1+H_3)H_4 \end{bmatrix} = \begin{bmatrix} 0.1988 & -0.0494 & -2.1189 & 0.3359 \\ -0.0966 & -0.1764 & 0.5585 & -0.0419 \\ 0.1988 & -0.0494 & -1.2867 & 0.1363 \\ -0.0966 & -0.1764 & 0.0611 & -0.4851 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} I_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{C} = [H_2 \quad H_4] = \begin{bmatrix} -0.1178 & 0.7930 & 3.0498 & -0.6611 \\ 0.6139 & 2.3887 & -0.5713 & -0.5007 \end{bmatrix}$$

The state equation for the reduced model is

$$\dot{\hat{x}} = \hat{A}\hat{x}(t) + \hat{B}u(t)$$
$$y(t) = \hat{C}\hat{x}(t)$$

The eigenvalues of the reduced model are

$$s_{1,2} = -0.2197 \pm 0.0014j$$
$$s_3 = -0.4482$$
$$s_4 = -0.8617$$

For comparison, the eigenvalues of the original model are listed below:

$$s_1 \text{ through } s_{12} =$$
$$\begin{gathered} -0.2164 + 0.j \\ -0.2172 + 0.j \\ -0.2172 + 0.j \\ -0.3700 + 1.3226j, \\ -0.3700 - 1.3226j, \\ -0.9325 + 0.j \\ -0.9328 + 0.j \\ -0.9336 + 0.j \\ -8.0004 + 0.j \\ -9.0644 + 0.j \\ -10.9955 + 0.j \\ -11.6500 + 0.j \end{gathered}$$

A mixed reduced model was attempted, using residualization to reduce down to a 6x6 matrix, A_r, and then using the second Cauer method to further reduce down to a 4x4 matrix $\hat{A}_r$. However, some issues were not resolved before the deadline.

2)

Numerical results were exported from Python to text files, which were loaded into Matlab for LTI simulation. Python's Scipy library is only compatible with SISO and SIMO systems.

(a) Using Matlab, the impulse response of the original and reduced order models is shown in Figure 1.

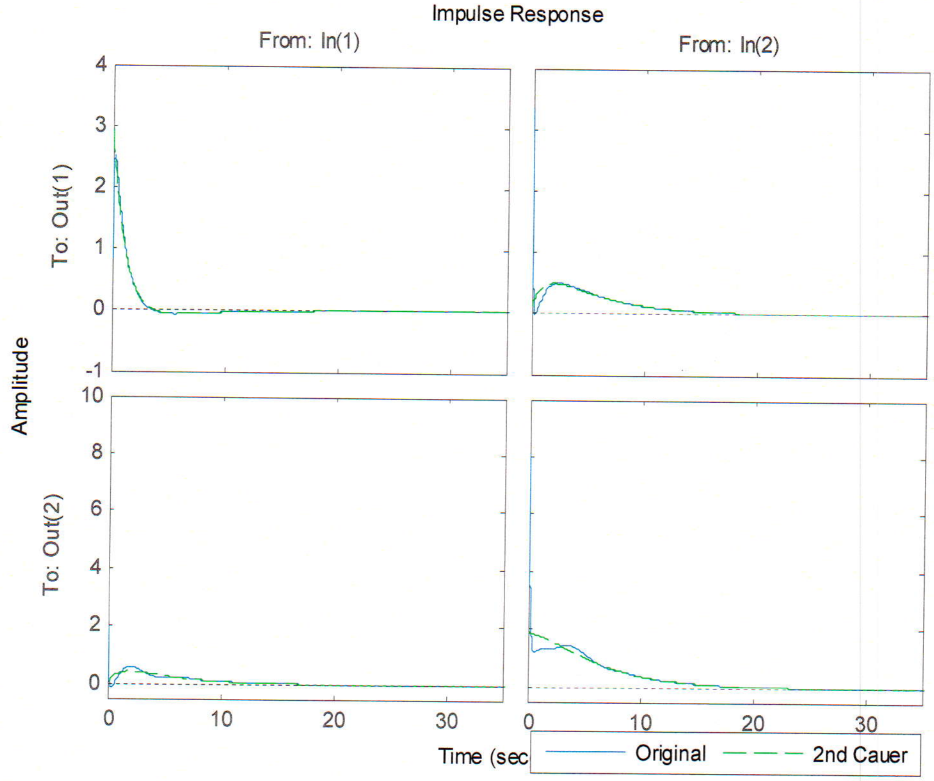

Figure 1. Impulse response of the original and reduced order models.

(b) Using Matlab, the step response of the original and reduced order models is shown in Figure 2.

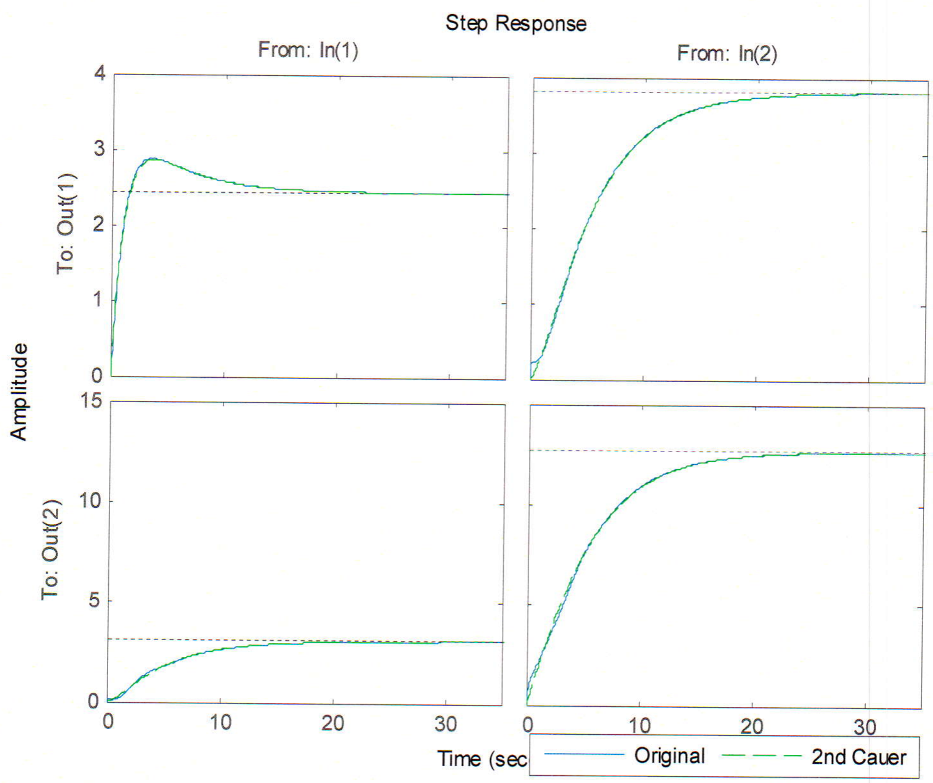

Figure 2. Step response of the original and reduced order models.

Based on the two figures, the reduced model is a good approximation of the original system.

3)

Two sinusoidal inputs were generated using Matlab's *gensig* function. They each had unit amplitude, frequency of 0.04 [Hz], and duration of 50 [s]. One sine wave was -1 times the other. The original and reduced order models were simulated and plotted in Matlab using *lsimplot*. Simulation results are shown in Figure 3.

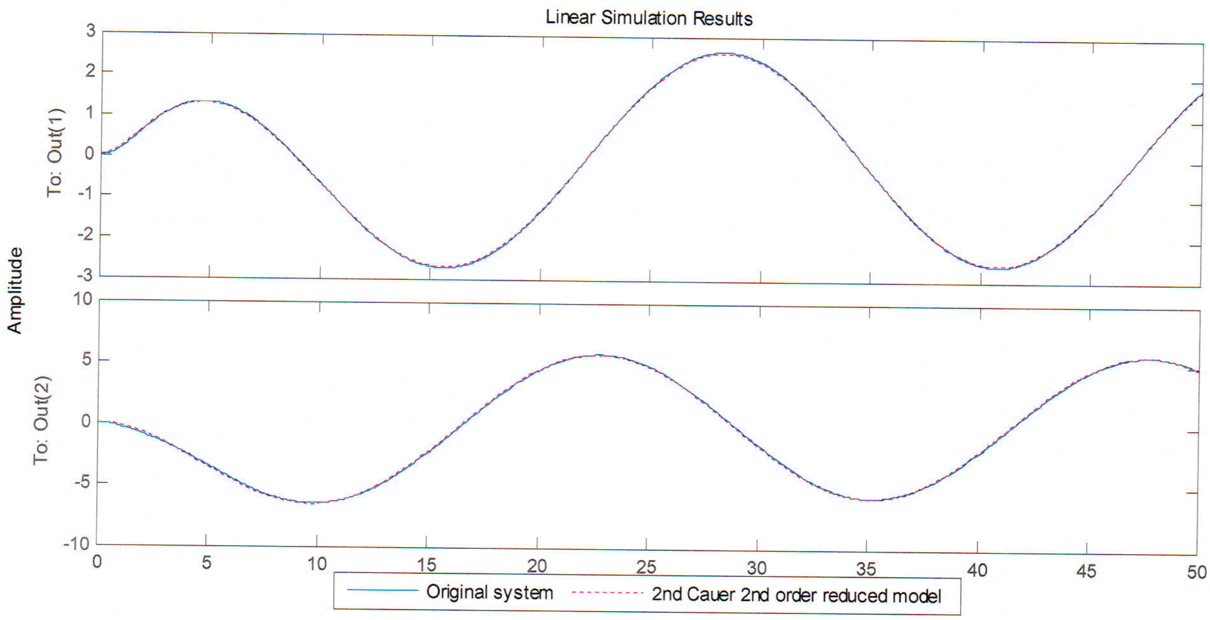

Figure 3. Sinusoidal response of the original and reduced order models.

Based on the figure, the reduced model is a good approximation of the original system.

Appendix

Python code and results are attached.

Matlab code is attached after Python code.