

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Given:

$$m_1 = (a + d)(e + h) = ae + ah + de + dh$$

$$m_2 = (c - a)(e + f) = ce + cf - ae - af$$

$$m_3 = (b - d)(g + h) = bg + bh - dg - dh$$

Form:

$$m_1 + m_2 = ah + de + dh + ce + cf - af$$

$$m_1 + m_3 = ae + ah + de + bg + bh - dg$$

Need:

$$ae + bg = m_1 + m_3 - ah - de - bh + dg$$

$$af + bh = ?$$

$$ce + dg = ?$$

$$cf + dh = m_1 + m_2 - ah - de - ce + af$$

Find:

$$m_4 = -ah - bh = -h(a + b)$$

$$m_5 = -de + dg = -d(e - g)$$

$$m_6 = -ah + af = -a(h - f)$$

$$m_7 = -de - ce = -e(d + c)$$

So:

$$ae + bg = m_1 + m_3 + m_4 + m_5$$

$$af + bh = -ah + af - (-ah - bh) = m_6 - m_4$$

$$ce + dg = -de + dg - (-de - ce) = m_5 - m_7$$

$$cf + dh = m_1 + m_2 + m_6 + m_7$$

Compute:

$$\begin{bmatrix} 5 & 7 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 9 & 1 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} m_1 + m_3 + m_4 + m_5 & m_6 - m_4 \\ m_5 - m_7 & m_1 + m_2 + m_6 + m_7 \end{bmatrix}$$

$$m_1 = (5)(9) + (5)(6) + (2)(9) + (2)(6) = 45 + 30 + 18 + 12 = 105$$

$$m_2 = (4)(9) + (4)(1) - (5)(9) - (5)(1) = 36 + 4 - 45 - 5 = -10$$

$$m_3 = (7)(3) + (7)(6) - (2)(3) - (2)(6) = 21 + 42 - 6 - 12 = 45$$

$$m_4 = -(5)(6) - (7)(6) = -30 - 42 = -72$$

$$m_5 = -(2)(9) + (2)(3) = -18 + 6 = -12$$

$$m_6 = -(5)(6) + (5)(1) = -30 + 5 = -25$$

$$m_7 = -(2)(9) - (4)(9) = -18 - 36 = -54$$

$$\begin{bmatrix} 105 + 45 - 72 - 12 & -25 + 72 \\ -12 + 54 & 105 - 10 - 25 - 54 \end{bmatrix} = \begin{bmatrix} 66 & 47 \\ 42 & 16 \end{bmatrix}$$