$$S(k) = S(k-1) + (k-1)$$

Given S(1) = 0

$$n = 1$$
; $S(k) = S(k-1) + (k-1)$

$$S(1) = S(1-1) + (1-1) = 0 + 0 = 0$$

$$n = 2$$
; $S(k) = [S(k-2) + (k-2)] + (k-1)$

$$S(2) = S(2-2) + (2-2) + (2-1) = 0 + 0 + 1 = 1$$

$$n = 3$$
; $S(k) = [S(k-3) + (k-3)] + (k-2) + (k-1)$

$$S(3) = S(3-3) + (3-3) + (3-2) + (3-1) = 0 + 0 + 1 + 2 = 3$$

$$n = 4$$
; $S(k) = [S(k-4) + (k-4)] + (k-3) + (k-2) + (k-1)$

$$S(4) = S(4-4) + (4-4) + (4-3) + (4-2) + (4-1) = 0 + 0 + 1 + 2 + 3 = 6$$

$$S(k) + 0 + 1 + \cdots + (k-1)$$

Sum of first n terms given by n=k-1, so $\frac{n(n+1)}{2}=\frac{(k-1)(k-1+1)}{2}=\frac{k(k-1)}{2}$