Normalized Taylor diagrams

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A Taylor diagram is a polar coordinate plot that summarizes multiple aspects of model performance in a single diagram (Taylor, 2001). An even powerful normalized version (Kärnä and Baptista, 2016) is used in this project. This section describes the theoretical aspects behind it.

Let o_i and m_i, $i = 1, \ldots, N$ be observed and modeled time series of any scalar variable (e.g. water level, velocity component, salinity), respectively. Their mean ($\bar{o}$ and $\bar{m}$, respectively) and standard deviation (σ_o and σ_m, respectively) are defined as

$$\bar{o} = \frac{1}{N}\sum_{i=1}^{N} o_i, \tag{1}$$

$$\bar{m} = \frac{1}{N}\sum_{i=1}^{N} m_i, \tag{2}$$

$$\sigma_o^2 = \frac{1}{N}\sum_{i=1}^{N} (o_i - \bar{o})^2, \tag{3}$$

$$\sigma_m^2 = \frac{1}{N}\sum_{i=1}^{N} (m_i - \bar{m})^2. \tag{4}$$

The bias (BIAS), root mean square error (RMSE), centered root mean squared

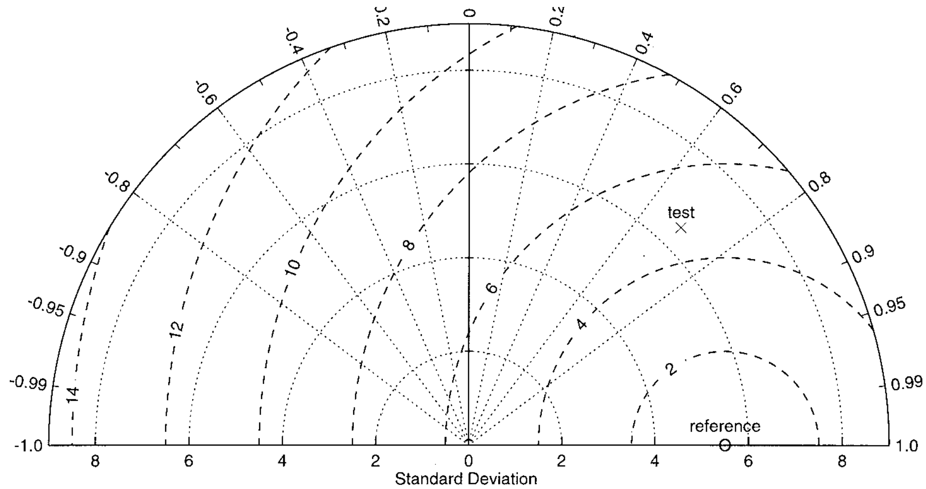

Figure 1: Taylor diagram where the reference $(\sigma_o, 0)$ and test $\big(\sigma_m, \arccos(R)\big)$ points represent the observed and modeled time series, respectively. Figure by Taylor (2001).

error (CRMSE) and correlation coefficient (R) are given by

$$\begin{aligned} \text{BIAS} &= \bar{m} - \bar{o}, && (5) \\ \text{RMSE}^2 &= \frac{1}{N}\sum_{i=1}^{N}(m_i - o_i)^2, && (6) \\ \text{CRMSE}^2 &= \frac{1}{N}\sum_{i=1}^{N}\big((m_i - \bar{m}) - (o_i - \bar{o})\big)^2, && (7) \\ R &= \frac{1}{\sigma_o \sigma_m}\frac{1}{N}\sum_{i=1}^{N}(m_i - \bar{m})(o_i - \bar{o}). && (8) \end{aligned}$$

CRMSE is related to σ_o, σ_m and R as follows (Taylor, 2001):

$$CRMSE^2 = \sigma_o^2 + \sigma_m^2 - 2\sigma_o\sigma_m R. \qquad (9)$$

Making use of the law of cosines, equation (9) can be visualized in a Taylor diagram (Taylor, 2001), a polar coordinate plot where the radial coordinate is the standard deviation and the angular coordinate is $\arccos(R)$. CRMSE then appears as the distance from the position of the observed time series $(\sigma_o, 0)$, as illustrated in Figure 1.

Equation (9) has the dimension of the considered variable squared. Dividing it by σ_o^2 leads to dimensionless quantities and the normalized Taylor diagram (Kärnä and Baptista, 2016):

$$CRMSE'^2 = 1 + \sigma_m'^2 - 2\sigma_m' R, \qquad (10)$$

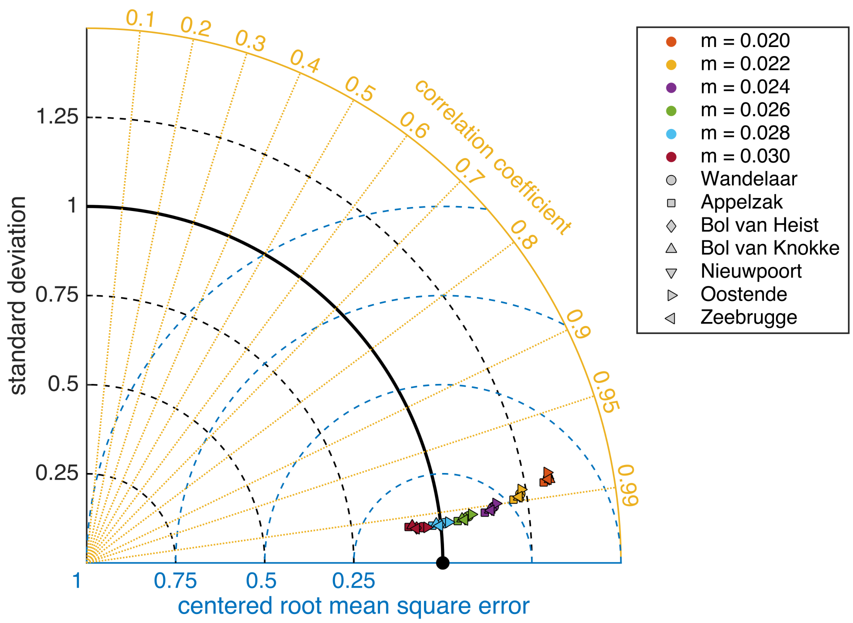

Figure 2: Normalized Taylor diagrams displaying water elevation time series at different locations of the *Meetnet Vlaamse Banken* (Flemish Banks Monitoring Network), simulated with different values of the Manning coefficient m.

with

$$CRMSE' = \frac{1}{\sigma_o} CRMSE, \tag{11}$$

$$\sigma'_m = \frac{\sigma_m}{\sigma_o}. \tag{12}$$

In the normalized diagram, the observed time series always lie at $(1, 0)$.

With Taylor diagrams, one (as in Figure 1) or several model runs are compared to only one reference data set. Indeed, two different observed time series would have different standard deviations and therefore lie on a difference target point $(\sigma_o, 0)$. A different diagram is therefore needed for each available observed time series. It is not the case with normalized Taylor diagrams, as the target point is always $(1, 0)$. Time series at different locations or even from different variables can be displayed on a single diagram (Figure 2).

References

Kärnä, T. and Baptista, A. M. (2016). Evaluation of a long-term hindcast simulation for the Columbia River estuary. *Ocean Modelling*, 99:1–14.

Taylor, K. E. (2001). Summarizing multiple aspects of model performance in a single diagram. *Journal of Geophysical Research*, 106(D7):7183–7192.