Mathematics: Number Systems

Duration: $2\frac{1}{2}$ *hours* 30 *Points*

THIS QUESTION PAPER CONTAINS SIX QUESTIONS ON ONE PAGE. PLEASE ANSWER ALL QUESTIONS. EACH QUESTION CARRIES 5 POINTS.

Some instructions and guidelines:

- A *set* is a collection of items having a specific property. For example, $\mathbb{Q}$ is the set of all rational numbers.
- To denote that a number x belongs to a specific set $\mathbb{X}$ we use the notation $x \in \mathbb{X}$, which is read as "x belongs to or is in the set $\mathbb{X}$".
- A set $\mathbb{A}$ is a *subset* of set $\mathbb{B}$ if every element of $\mathbb{A}$ is also an element of $\mathbb{B}$. In addition to this, $\mathbb{B}$ may or may not have any other elements. We write this as: $\mathbb{A} \subseteq \mathbb{B}$.
- The *cardinality* of a set is the number of elements in it. It is notationally written as $|\mathbb{S}|$.
- These are some set notations that might come in handy to you:
  - $\mathbb{R}$: Set of all real numbers.
  - $\mathbb{Q}$: Set of all rational numbers.
  - $\mathbb{N}$: Set of all natural numbers.
  - $\mathbb{Z}$: Set of all integers.
  - $\mathbb{P}$: Set of all prime numbers.
  - We may use a superscript $\pm$ to denote a set that contains only positive or negative numbers, respectively.

ALL THE BEST!

1. Suppose you are given a number a. Given that $a < 0$ and $a^2 = 2$, justify if $a \in \mathbb{Q}^-$.
2. Order the following sets as subsets of one another (if it is possible at all, of course - if not, state why). $\mathbb{N}$, $\mathbb{Q}^+$,$\mathbb{R}^+$ $\mathbb{Z}^+$. Can you find three rational numbers between -1 and 1 such that five numbers you have (including the two given) are at exactly the same difference from one another, taken in order? This is also called an *arithmetic progression*, when the difference between any two consecutive numbers in a sequence is the same.
3. Let us take the number $b = 0.9999\ldots$. Which of the above sets does b belong to? We know that π is irrational. That means - we do not the value of π very accurately, and we use approximations like $\pi = 3.14159\ldots$. Then, how can we use it if its value keeps varying? Or is this statement wrong? Justify.
4. Solve for c : $\sqrt{c} + \frac{1}{\sqrt{c}} = 4$. and $\sqrt{d} - \frac{1}{\sqrt{d}} = 0$. Let, $e = d^2$ and $f = \sqrt{e}$. Find the possible value(s) of $\left(\frac{f}{c}\right)^{f^{-1}}$.
5. Given that $a^{a^{-1}} = b^{b^{-1}} = c^{c^{-1}}$ and $a^{bc} + b^{ca} + c^{ab} = 729$, show that $a \notin \mathbb{Q}$ but $a \in \mathbb{R}$. What is the value of a? What is the value of $|\mathbb{R}|$ - $|\mathbb{Q}|$.?
6. Find the value of: $\sqrt{p + \sqrt{p + \sqrt{p + \ldots}}}$, if $p = 2$. If $q = 1 + \sqrt[3]{5} + \sqrt[3]{25}$, find the value of $q^3 - 3q^2 - 12q + 6$.

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