SHS

AIRs - LM in
Statistics and Probability
Quarter 4 – Week 6, Module14
Test of Hypothesis on Population Proportion

Statistics and Probability

Quarter 4 – Week 6 Module 14: Test of Hypothesis on Population Proportion

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The Test of Hypothesis on Population Proportion

Target

Advertisements make use of population proportion to promote their products. They would often endorse that 8 out of 10 people has switched to their product over their competitors or 3 out of 4 cool people use this product. However, they should be able to test their hypothesis so that they wouldn't be accused of false advertisement.

In order for us to fully grasp the concept of hypothesis testing, we must practice on a lot of problems. With enough practice, the steps in hypothesis testing will be second nature to us.

After going through this module, you are expected to:

1. computes for the test-statistic value (population proportion) (M11/12SP-IVf-1)
2. draws conclusion about the population proportion based on the test-statistic value and the rejection region (M11/12SP-IVf-2)
3. solves problems involving test of hypothesis on the population proportion (M11/12SP-IVf-g-1)

Subtask:

1. formulate null and alternate hypotheses of population proportion
2. recall Central Limit Theorem (CLT), one-tailed and two-tailed test
3. apply the steps in test of hypothesis on population proportion

Before you start doing the activities in this lesson, find out how much you already know about this module. Answer the pretest below in a separate sheet of paper. Write the letter that corresponds to the best answer.

Goodluck and enjoy the lesson!

Pre-test: Write the letter of the correct answer on a separate sheet of paper.

1. The test statistic value for the population proportion can be solved using the ____.
   A. Means of proportion
   B. T-test for proportion
   C. Variance of proportion
   D. Z-test of proportion
2. What is the formula for z-test for proportion?
   A. $z = \frac{\hat{p}-p_0}{\sqrt{\frac{pq}{n}}}$ B. $z = \frac{p_0-\hat{p}}{\sqrt{\frac{pq}{n}}}$ C. $z = \frac{\hat{p}-p_0}{\sqrt{\frac{n}{pq}}}$ D. $z = \frac{p_0-\hat{p}}{\sqrt{\frac{n}{pq}}}$
3. A certain population has a sample proportion of 85%, population proportion is 81%, and the sample size is 175. How are you going to plot the given in the formula for the test statistic value of the population proportion?
   A. $z = \frac{0.81-0.85}{\sqrt{\frac{(0.85)(0.15)}{175}}}$ B. $z = \frac{0.81-0.85}{\sqrt{\frac{(0.81)(0.19)}{175}}}$ C. $z = \frac{0.85-0.81}{\sqrt{\frac{(0.81)(0.19)}{175}}}$ D. $z = \frac{0.85-0.81}{\sqrt{\frac{(0.85)(0.15)}{175}}}$
4. The sample proportion is 28%, population proportion is 30%, and the sample size is 81. How are you going to plot the given in the formula for the test statistic value of the population proportion?
   A. $z = \frac{0.30-0.28}{\sqrt{\frac{(0.28)(0.72)}{81}}}$ B. $z = \frac{0.30-0.28}{\sqrt{\frac{(0.30)(0.70)}{81}}}$ C. $z = \frac{0.28-0.30}{\sqrt{\frac{(0.30)(0.70)}{81}}}$ D. $z = \frac{0.28-0.30}{\sqrt{\frac{(0.28)(0.72)}{81}}}$
5. It is believed that at least 40% of the residents in a certain area is in favor of abolishing the death penalty. Out of the 500 residents surveyed, 210 are in favor of the issue. What is the value of the test statistic? Round your answer to the nearest hundredths.
   A. 0.90 B. 0.91 C. 0.92 D. 0.93
6. The goal of hypothesis testing is to make a judgement about the difference between the _______ and a _______.
   I. hypothesized population parameter
   II. population mean
   III. sample mean
   IV. sample statistics
   A. I and II B. II and III C. III and IV D. I and IV
7. When is the null hypothesis rejected?
   A. If the test-statistic value falls within the rejection region
   B. If the test-statistic value does not fall within the rejection region
   C. If the test-statistic falls to the left of the rejection region
   D. If the test-statistic falls to the right of the rejection region
8. A cashier in a supermarket claims that at least 60% of the shoppers pay in cash. What is the correct hypothesis test?
   A. z-test statistic B. two-tailed test C. right-tailed test D. left-tailed test
9. A basketball coach claims that at most 25% of his players are honor students. What is the correct hypothesis test?
   A. z-test statistic B. two-tailed test C. right-tailed test D. left-tailed test
10. Consider a screening test for prostate cancer that its maker claims will detect the cancer in 80% of the men that actually have the disease. 175 men who have been previously diagnosed with prostate cancer are given the screening test, and 141 of the men are identified as having the disease. Using 0.05 level of significance, what is the appropriate decision rule?
    A. Do not reject the alternative hypothesis.
    B. Do not reject the null hypothesis.
    C. Reject the alternative hypothesis.

D. Reject the null hypothesis.

11. A large city's Department of Motor Vehicles claimed that 80% of candidates pass the driving test, but newspaper reports that out of 90 randomly selected local teens who had taken the test, only 60 passed. Can the *z*-test for proportion be used?

A. Yes, because $np \geq 5$ and $nq \geq 5$ C. No, because $np \geq 5$ and $nq \geq 5$
B. Yes, because $np \leq 5$ and $nq \leq 5$ D. No, because $np \leq 5$ and $nq \leq 5$

12. What is the value of the test statistic of a sample proportion of 32% given that the population proportion is 30% and the sample size is 520? Round your answer to the nearest thousandths.

A. 0.990 B. 0.995 C. 0.999 D. 1.000

13. Records show that at most 10% of the patients afflicted with a certain disease die from it. Out of 190 patients afflicted with the said disease, 45 did not recover. What is the value of the test statistic? Round your answer to the nearest hundredths.

A. 6.40 B. 6.41 C. 6.42 D. 6.43

14. Some boxes of a certain brand of breakfast cereal include a voucher for a free video rental inside the box. The company that makes the cereal claims that a voucher can be found in 20% of the boxes. However, based on their experiences eating this cereal at home, a group of students believe that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company`s clam. The students found a total of 11 vouchers for free video rentals in the 65 boxes. What is the value of the test statistic? Round your answer to the nearest hundredths.

A. -0.60 B. -0.65 C. -0.70 D. -0.75

15. In a sample of 60 teens, 21 reported they had tried surfing at least once. In a particular region, it is believed that 45% have tried surfing at least once. What is the value of the test statistic? Round your answer to the nearest hundredths.

A.-1.52 B. -1.54 C. -1.56 D. -1.58

How was your performance in the pre-assessment? Were you able to answer all the problems? Did you find difficulties in answering them? Are there questions familiar to you? Keep yourself on track as we learn new concepts in this module.

Jumpstart

You have learned how to construct hypothesis testing involving means. In this lesson, you will learn how to conduct tests involving count data. But first, how ready are you for this lesson? Let us work first with Activity 1 to test your readiness.

Activity 1: True or False

A. Determine whether the statement is True or False by ticking the appropriate box.

	True	False
1. Percentages can be expressed as proportions.		
2. A proportion is obtained when a frequency of desired events is multiplied by the sum of events		
3. If n=25, the Central Limit Theorem applies.		
4. If the confidence level is 95%, then $\frac{\mu}{2}$ *is* 0.025.		
5. When x =124 and n=260, then $\frac{X}{n}$ = 0.48.		
6. The p-value of z=2 is 0.4772.		
7. The p-value of z≤-2 is 0.0228.		
8. When H0 is rejected, it means that a significant difference does not exist.		
9. When the evidence is not enough, do not reject the null hypothesis.		
10. When the evidence is sufficient to reject the null hypothesis, a significant difference exists.		

There are certain situations when the data to be analyzed involve population proportions or percentages.

You will encounter more of them as you continue with the lesson.

Discover

In interpreting the hypothesis of a population proportion, the test statistic is used to interpret whether a data is accepted or rejected. The test-statistic must lie on a certain region for it to be accepted, much like the die has to give a result of 3 or 4 for it to be counted. Otherwise, it will be rejected.

The test statistic value for the population proportion can be solved using the z-test for proportion given by the following formula.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

or $$z = \frac{\acute{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$$

where:

z = z-value
$\acute{p}$ = sample proportion
p_0 = hypothesized population proportion,
$q_0 = 1 - p$
n = sample size.

The expression $\sqrt{\frac{pq}{n}}$ is the standard deviation of the sampling distribution, denoted by a variable σ.

The goal of hypothesis testing is to make a judgment about the difference between the sample statistic and a hypothesized population parameter. Thus, it is important to begin a testing with a clear null and alternative hypothesis.

Example 1: Using the 0.05 level of significance, run a z-test given the following:

$n = 74$; $\acute{p} = \frac{5}{74}$; $p_0 = 10\%$

Use both the traditional method and the p-value method.

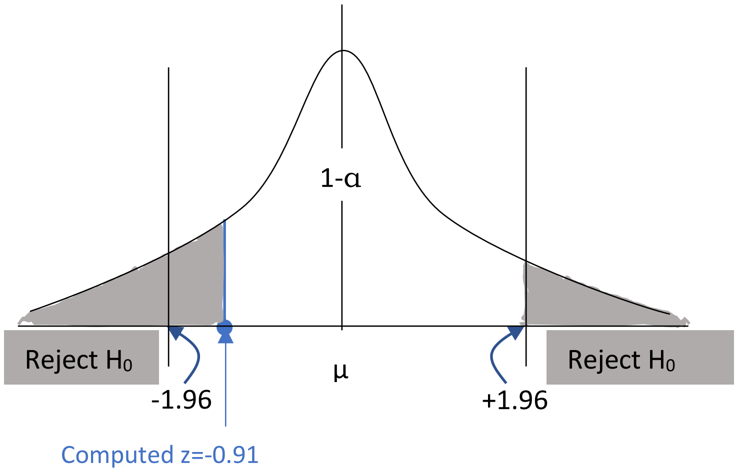
Solution:

STEPS	SOLUTION
1. Describe the population parameter of interest.	The parameter of interest is the population proportion p.
2. Formulate the hypothesis: the null hypothesis and the alternative hypothesis. That is, state a null	H_0: p = p_0 H_0: p = .10 H_1: p ≠ .10

hypothesis, H_0, in such a way that a Type I error can be calculated.	
3. Check the assumptions. • Is the sample size large enough for the Central Limit Theorem (CLT) to apply?	With n = 74, the CLT applies
4. Choose a significance level size for α. Make α small when the consequences of rejecting a true H_0 is severe. • Is the test two-tailed or one-tailed?	α = .05 Two-tailed. (The problem does not suggest a direction)
5. Select the appropriate test statistic. • Compute the z statistic	z-statistic $z = \frac{ṕ - p_o}{\sqrt{\frac{p_o q_o}{n}}}$ $ṕ = \frac{5}{74} = 0.068$ p_o = population proportion (given in the null hypothesis) $q_o = 1 - p_o = 1 - 0.10 = 0.9$ $z = \frac{0.068 - 0.1}{\sqrt{\frac{(0.1)(0.9)}{74}}}$ $z = \frac{-0.032}{\sqrt{\frac{0.09}{74}}}$ $z = \frac{-0.032}{\sqrt{0.001}}$ $z = \frac{-0.032}{0.035}$ $z = -0.91$ In the p=value approach, we compute the probability value to the left of -0.91 That is, the area between z=0 and z=0.91 is given in the z-table as 0.3186 Therefore, the observed probability value is 0.500-0.3186=0.1814. Since the test is two-tailed, the p-value is multiplied by 2. So, p-value =0.1814x2=0.3628.
6. State the decision rule for rejecting or not rejecting the null hypothesis.	In the traditional method, α=0.05, reject if H_0 if the computed z-value ≤-1.96 or if the computed value is ≥1.96. Do not reject H_0 if the computed z-value is >-1.96 or if the computed value is <1.96. In the p-value method, reject H_0 if the computed probability value is ≤0.05. Do not reject (or accept) H_0 if the computed probability value is >0.05.

7. Compare the computed values • Interpret the result.	• Traditional Method: We know that -0.19>-1.96 • p-value method: We know that 0.3628>0.05. Thus, based on the evidence at hand, we cannot reject the null hypothesis H_0. There is no significant difference between the sample proportion and the population proportion.

The figure shows that the computed *z* statistic lies outside the rejection region.



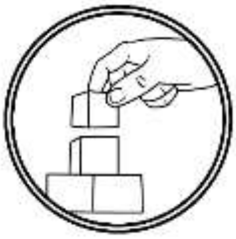
Example 2: Defective Bulbs

Mr. Caberto asserts that fewer than 5% of the bulbs that he sells are defective. Suppose 300 bulbs are randomly selected, each are tested, and 10 defective bulbs are found. Does this provide sufficient evidence for Mr. Caberto to conclude that the fraction of defective bulbs is less than 0.05? use α=0.01 and p-value approach.

Solution:

STEPS	SOLUTION
1. Describe the population parameter of interest.	The parameter of interest is the population proportion p_0.
2. Formulate the hypothesis: the null hypothesis and the alternative hypothesis. That is, state a null hypothesis, H_0, in such a way that a Type I error can be calculated.	H_0: $p = p_0$ H_0: $p = 0.05$ H_1: $p < 0.05$
3. Check the assumptions. • Is the sample size large enough for the Central Limit Theorem (CLT) to apply?	With n = 300, the CLT applies

4. Choose a significance level size for α. Make α small when the consequences of rejecting a true H_0 is severe. • Is the test two-tailed or one-tailed?	α = .01 One-tailed. (The clue word: fewer than)
5. Select the appropriate test statistic. • Compute the probability value (p-value)	z-statistic $z = \frac{\acute{p}-p_o}{\sqrt{\frac{p_o q_o}{n}}}$ $\acute{p}=\frac{10}{300} = 0.033$ p_o = 0.05 (given in the null hypothesis) q_o = 1- p_o = 1 – 0.05 = 0.95 $z = \frac{0.033 - 0.05}{\sqrt{\frac{(0.05)(0.95)}{300}}}$ $z = \frac{-0.017}{\sqrt{\frac{0.0475}{300}}}$ $z = \frac{-0.017}{\sqrt{0.00016}}$ $z = \frac{-0.017}{0.0126}$ $z = -1.35$ So, p-value = P(z≤-1.35) The area between z=0 and z=-1.35 is 0.4115. So, P(z≤-1.35) = 0.05-0.4115 = 0.0885 That is, p-value = 0.0885
6. State the decision rule for rejecting or not rejecting the null hypothesis.	Reject H_0 if the computed probability value is ≤ 0.01. Do not reject (or accept) H_0 if the computed probability value is >0.01.
7. Compare the computed probability value and α. • Based on the decision rule, decide whether to reject or not to reject H_0. • Interpret the result. • Take a course of action. (optional)	We know that 0.0855>0.01. Thus, based on the evidence at hand, we cannot reject the null hypothesis H_0. There is no sufficient evidence to reject Mr. Caberto's statement.



Explore

Activity 2: Examining Canteen Service: Supply the missing parts of the solution to the given problem.

A school administrator claims that less than 50% of the students of Sinapangan National High School are dissatisfied by the community canteen services. Test this claim by using sample data obtained from a survey of 500 students of the school where 54% indicated their dissatisfaction of the community canteen service. Use α=0.05.

STEPS	SOLUTION
1. Describe the population parameter of interest.	The parameter of interest is the population proportion p of dissatisfied students.
2. Formulate the hypothesis: the null hypothesis and the alternative hypothesis. That is, state a null hypothesis, H_0, in such a way that a Type I error can be calculated.	H_0: p = p_0 H_0: p = 0.05 H_1: p < 0.05
3. Check the assumptions. • Is the sample size large enough for the Central Limit Theorem (CLT) to apply?	With n = ________, the CLT applies
4. Choose a significance level size for α. Make **α** small when the consequences of rejecting a true H_0 is severe. • Is the test two-tailed or one-tailed?	α = .05 ______________.
5. Select the appropriate test statistic. • Compute the probability value (p-value)	z-statistic $z = \frac{\acute{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$ $z = \frac{0.54 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{500}}}$ $z = \frac{\_\_}{\sqrt{\_\_}}$

	$z = \frac{\quad}{\sqrt{\quad}}$ $z = \frac{\quad}{}$ $z = \_\_\_\_\_\_$ (Complete the solution.)
6. State the decision rule for rejecting or not rejecting the null hypothesis.	Reject H_0 if the absolute value of the computed probability value is ≤ 0.05. Do not reject (or accept) H_0 if the computed probability value is >0.05.
7. Compare the computed probability value and α. • Based on the decision rule, decide whether to reject or not to reject H_0. • Interpret the result.	We know that ____________________. Thus, based on the evidence at hand, we (can/cannot) reject the null hypothesis H_0. ____________________________________ ____________________________________

Deepen

Now that you have learned the steps of computing the test-statistic value of population proportion, drawing conclusions and interpreting results of a real-life problem, it's time to apply it on your own.

Activity 3: Discover Your Barangay: Decide for a situational problem you would like to discover in your barangay, example, you claim that 75% preferred drinking water during this time of pandemic rather than soft drinks. Make a survey by deciding the number of respondents, you can asked them through messenger, text or call for physical distancing during this time of pandemic, then formulate your hypothesis and choose a significance level size for α. After having all the data needed, show the complete step by step solution for the problem.

STEPS	SOLUTION

Rubrics for scoring the output:

Evaluation Method	3 points	2 points	1 point	Score
Problem Focus	Students demonstrate full understanding of the problem chosen.	Students demonstrate partial understanding of the problem chosen.	Students demonstrate unclear and inaccurate understanding of the problem chosen.	
Output Focus	Students provide a complete and accurate solution of the key subject matter and followed the step by step procedure.	Students provide a partial but mostly accurate solution of the key subject matter and followed the step by step procedure.	Students provide an incomplete, unclear and inaccurate solution of the key subject matter and followed the step by step procedure.	
TOTAL POINTS				

Gauge

Read each item carefully and write the correct answer in your activity notebook.

1. What kind of test is used to solve the test statistic value for the population proportion?
   A. Z-test of proportion C. T-test for proportion
   B. Variance of proportion D. Means of proportion
2. In the formula for z-test for proportion, which variable represents the hypothesized mean?
   A. p_o B. $\acute{p}$ C. pq D. h_m
3. How are you going to plot the given in the formula for the test statistic value of the population proportion given that a certain population has a sample proportion of 80%, population proportion is 85%, and the sample size is 170?
   A. $z = \frac{0.85-0.8}{\sqrt{\frac{(0.8)(0.15)}{170}}}$ B. $z = \frac{0.85-0.8}{\sqrt{\frac{(0.8)(0.19)}{170}}}$ C. $z = \frac{0.8-0.85}{\sqrt{\frac{(0.85)(0.19)}{170}}}$ D. $z = \frac{0.85-0.81}{\sqrt{\frac{(0.85)(0.15)}{170}}}$

4. A study found that 68% of the population owns a home. In a random sample of 150 households, 92 owned a home. At the α = 0.01 level, is there enough evidence to reject the claim? Choose the correct hypotheses.

   A. H_0: p = 0.68 H_a: p ≠ 0.68
   B. H_0: μ = 0.68 H_a: μ ≠ 0.68
   C. H_0: p = 0.68 H_a: p > 0.68
   D. H_0: p = 0.68 H_a: p < 0.68

5. What is the value of the test statistic rounded to the nearest hundredths if it is believed that at least 40% of the residents in a certain area is in favor of abolishing the death penalty and out of the 500 residents surveyed, 210 are in favor of the issue?

   A. 0.89 B. 0.90 C. 0.91 D. 0.92

6. Which of the following do we need to differentiate in order to make a judgement which is the goal of hypothesis testing?

   I. hypothesized population parameter
   II. population mean
   III. sample mean
   IV. sample statistics

   A. I and II B. I and III C. I and IV D. II and IV

7. If the test-statistic value falls within the rejection region, what should we do with the null hypothesis?

   A. Accept the hypothesis mean.
   B. Reject the alternative hypothesis.
   C. Accept the null hypothesis.
   D. Reject the null hypothesis.

8. What is the correct hypothesis test if a cashier in a supermarket claims that at least 60% of the shoppers pay in cash?

   A. left-tailed B. right-tailed C. two-tailed D. *z*-test

9. A study found that 68% of the population owns a home. In a random sample of 150 households, 92 owned a home. At the α = 0.01 level, is there enough evidence to reject the claim? What is the correct conclusion?

   A. Fail to reject the null hypothesis.
   B. Reject the null hypothesis.
   C. Fail to reject the alternative hypothesis.
   D. Reject the alternative hypothesis.

10. On average, 86% of all enrolled college students are undergraduates. A random sample of 500 college students revealed that 420 were undergraduates. At α = 0.10 level, is there enough evidence to conclude that the proportion is lower than the national average? What is the correct conclusion?

   A. Do not reject the alternative hypothesis.
   B. Do not reject the null hypothesis.
   C. Reject the alternative hypothesis.
   D. Reject the null hypothesis.

11. Can the *z*-test for proportion be used if in a large city's Department of Motor Vehicles claimed that 80% of candidates pass the driving test, but newspaper reports that out of 90 randomly selected local teens who had taken the test, only 60 passed?

   A. No, because $np \geq 5$ and $nq \geq 5$
   B. No, because $np \leq 5$ and $nq \leq 5$
   C. Yes, because $np \geq 5$ and $nq \geq 5$
   D. Yes, because $np \leq 5$ and $nq \leq 5$

12. A study found that 68% of the population owns a home. In a random sample of 150 households, 92 owned a home. At the α = 0.01 level, is there enough e vidence to reject the claim? Which value is closest to the p-value?

   A. 0.2 B. 0.4 C. 0.6 D. 0.8

13. What is the value of the test statistic rounded to the nearest hundredths if records show that at most 10% of the patients afflicted with a certain disease die from it, out of 190 patients afflicted with the said disease and 45 did not recover?

A. 6.42 B. 6.43 C. 6.44 D. 6.45

14. A cigarette manufacturer claims that 35% of the cigarette smokers prefer the ir leading brand. If out of a random sample of 50 smokers, 15 prefer the said brand, what conclusion could we draw using a 0.05 level of significance?
    A. There is a sufficient evidence to accept the claim of the manufacturer that 30% of the cigarette smokers prefer Bataang Matamis.
    B. There is a sufficient evidence to accept the claim of the manufacturer that 35% of the cigarette smokers prefer Bataang Matamis.
    C. There is no sufficient evidence to accept the claim of the manufacturer that 30% of the cigarette smokers prefer Bataang Matamis.
    D. There is no sufficient evidence to accept the claim of the manufacturer th at 35% of the cigarette smokers prefer Bataang Matamis.
15. A study found that 68% of the population owns a home. In a random sample of 150 households, 92 owned a home. At the α = 0.01 level, is there enough evidence to reject the claim?
    A. There is not enough evidence to reject the claim that 68% of the population owns a home.
    B. There is enough evidence to reject the claim that 68% of the population owns a home.
    C. There is not enough evidence to accept the claim that 68% of the population owns a home.
    D. There is enough evidence to accept the claim that 68% of the population owns a home.

Congratulations for a great job well done!

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