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Mathematics

Quarter 4- Week 1- Module 1:
Illustrates the Six Trigonometric Ratios: Sin, Cosine, Tangent, Secant, Cosecant, and Cotangent

AIRs - LM

Mathematics 9
Quarter 4- Week 1 Module 1: Illustrates the six trigonometric ratios: sin, cosine, tangent, secant, cosecant, and cotangent

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Development Team of the Module

Writer: Jocelyn G. Lopez
Editor: SDO La Union, Learning Resource Quality Assurance Team
Layout Artist: Ernesto F. Ramos Jr., *P II*

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent

Vivian Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent

German E. Flora, PhD, *CID Chief*

Virgilio C. Boado, PhD, *EPS in Charge of LRMS*

Erlinda M. Dela Peña, EdD, *EPS in Charge of Mathematics*

Michael Jason D. Morales, *PDO II*

Claire P. Toluyen, *Librarian II*

Module 1

The Six Trigonometric Ratios: Sin, Cosine, Tangent, Secant, Cosecant, and

In this module, the lesson starts with assessing your prior knowledge of the diverse mathematics principles and concepts studied previously, and enhancing your skills in performing mathematical operations. All these skills and knowledge may help you in applying the solutions and processes to real-life problems.

Jumpstart

Let's start by doing this activity. Have fun learning!

Activity 1: Triangles of Different Sizes

This activity helps you recall the concepts of similar triangles.

Investigate the following triangles:

1. Draw three similar right triangles ABC, DEF, and GHI in different sizes in such a way that m∠C = m∠F = m∠I = 63.
2. Measure the second acute angle in each of the triangles.
3. Use a ruler to measure the sides of the triangles to the nearest tenths in centimeters. Then find each of the following ratios for all the three triangles.

Record your findings in the given table.

Measures	in Δ ABC	in Δ DEF	in ΔGHI
Leg opposite the 63˚ angle			
Leg adjacent to the 63˚ angle			
hypotenuse			
$\frac{\text{Leg opposite } 63^\circ \text{ angle}}{\text{hypotenuse}}$			
$\frac{\text{Leg adjacent to } 63^\circ \text{ angle}}{\text{hypotenuse}}$			
$\frac{\text{Leg opposite } 63^\circ \text{ angle}}{\text{Leg adjacent to } 63^\circ \text{ angle}}$			

From Activity 1, you have discovered the different ratios derived from the sides of a right triangle having an acute angle. Let's discuss also the importance of the use of scientific calculator in determining the values of the trigonometric ratios and their equivalent angle measure.

Using the Calculator to Find Trigonometric Ratios

A. Finding a ratio given the angle

Example: To find the value of sin 38˚, ensure that your calculator is operating in degrees.

Solution: Press $\boxed{\text{sin}}$ 38 $\boxed{=}$ 0.615661475

The calculator should give 38˚ = 0.616, correct to three decimal places.

B. Finding an angle given the ratio

In finding the size of the angle to the nearest minute, given the value of the trigonometric ratio, just follow the steps in the examples below.

Example: sin θ = 0.725 , find θ to the nearest minute

Solution: Press $\boxed{\text{2ndF}}$ $\boxed{\text{sin}}$ 0.725 $\boxed{=}$ 46.46884783

To convert this to degrees/minutes/seconds mode,

Press $\boxed{\text{2ndF}}$ $\boxed{\text{D°M'S}}$

The calculator gives you 46˚28'(nearest minute)

C. Degrees and minutes

Example: Write 54.46˚ in degree and minute, giving an answer correct to the nearest minute.

Solution: Press 54.46˚ $\boxed{\text{2ndF}}$ $\boxed{\text{D°M'S}}$

The calculator gives 54˚27'36", or 54˚28' (nearest minute)

Activity 2: TRY THIS!!!

1. Use your calculator to find the value of the following, correct to two decimal places.
   a. cos 85˚ b. sin 7˚ c. tan 35˚ d. cos 34˚

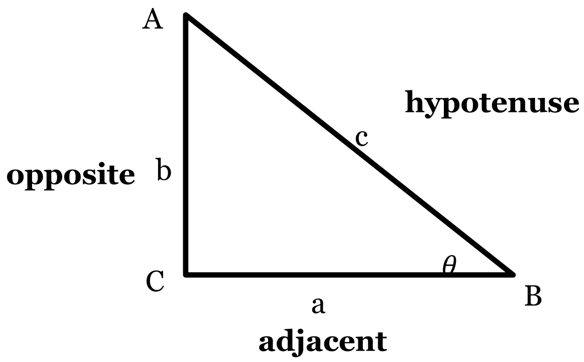
2. Using the degrees/minutes/seconds button on your calculator, write each of the following in degrees and minutes, given answers to the nearest minute.
   a. 17.8˚ b. 48.52˚ c. 63.7˚ d. 108.33˚

3. Find the size of the angle θ (to the nearest degree) where θ is acute.
   a. sin θ = 0.529 c. tan θ = 1.8
   b. cos θ = 0.493 d. sin θ = 0.256

4. Find m∠θ, to the nearest minute, given that θ is acute.
   a. sin θ = 0.9 c. tan θ = 0.958
   b. cos θ = 0.013 d. tan θ = 2.3

Discover

In a right triangle, we can define actually six trigonometric ratios. Consider the right triangle ABC below. In this triangle we let θ represent $\angle$ B. Then the leg denoted by a is the side adjacent to θ, and the leg denoted by b is the side opposite to θ.



> We will use the convention that angles are symbolized by capital letters, while the side opposite each angle will carry the same letter symbol, in lowercase.

sine of θ = sin θ = $\frac{\boldsymbol{opposite}}{\boldsymbol{hypotenuse}}$

cosecant of θ = cos θ = $\frac{\boldsymbol{hypotenuse}}{\boldsymbol{opposite}}$

cosine of θ = cos θ = $\frac{\boldsymbol{adjacent}}{\boldsymbol{hypotenuse}}$

secant of θ = sec θ = $\frac{\boldsymbol{hypotenuse}}{\boldsymbol{adjacent}}$

tangent of θ = tan θ = $\frac{\boldsymbol{opposite}}{\boldsymbol{adjacent}}$

cotangent of θ = cot θ = $\frac{\boldsymbol{adjacent}}{\boldsymbol{opposite}}$

SOH-CAH-TOA is a mnemonic used for remembering the equations.

A. Solving a right triangle given the measure of the two parts; the length of the hypotenuse and the length of one leg

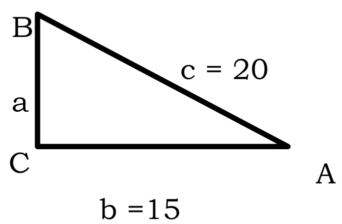
Solving a right triangle means finding the measure of the remaining parts.

Example: Triangle BCA is right-angled at C. If c = 20 and b = 15, find $\angle$A, $\angle$B and a.

Solution: Sketch a figure:

a. Side b is the adjacent side of $\angle$A; c is the hypotenuse of right triangle BCA. Use CAH, that is

$$\cos\theta = \frac{adjacent}{hypotenuse}$$



$$\cos A = \frac{b}{c}$$

$$\cos A = \frac{15}{20}$$

$$\cos A = 0.75$$

We can use our scientific calculator to find an angle whose cosine value is 0.75.

Using a scientific calculator, A = 41.41˚

b. Since in part (a), it was already found that ∠ A = 41.41˚,

then $\angle B = 90^\circ - 41.41^\circ$

$\angle B = 48.59^\circ$

c. Using the Pythagorean theorem:

$a^2 + b^2 = c^2$

$a^2 + (15)^2 = (20)^2$

$a^2 + 225 = 400$

$a^2 = 400 - 225$

$a^2 = 175$

$a = \sqrt{175}$

$a = 13.23$

B. Solving a Right Triangle Given the Length of the Hypotenuse and the Measure of One Acute Angle

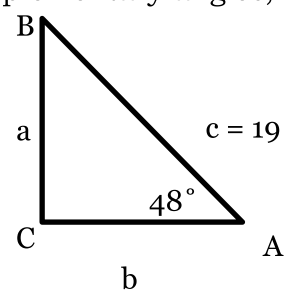
Example: Triangle BCA is right-angled at C if c = 19 and ∠A = 48˚, find ∠B, *b*, and *a*.
Solution:

a. To find ∠B, since ∠B and ∠A are complementary angles, then

$\angle B + \angle A = 90^\circ$

$\angle B = 90^\circ - 48^\circ$

$\angle B = 42^\circ$



b. To find *b*, since *b* is the adjacent side of ∠A and *c* is the hypotenuse of right triangle BCA, then use CAH.

$\cos \theta = \frac{adjacent}{hypotenuse}$

$\cos A = \frac{b}{c}$

$\cos 48^\circ = \frac{b}{19}$

$b = 19\cos 48^\circ$

$b = 19(0.6691)$

$b = 12.71$

c. To find a, since *a* is the opposite side of ∠A and *c* is he hypotenuse of right triangle BCA, then use SOH.

$\sin \theta = \frac{opposite}{hypotenuse}$

$\sin A = \frac{a}{c}$

$\sin 48^\circ = \frac{a}{19}$

$a = 19 \sin 48^\circ$

$a = 19\ (0.7431)$

$a = 14.12$

C. Solving a Right Triangle Given the Length of One Leg and the Measure of One Acute Angle

Example: Triangle ACB is right-angled at C. If ∠ A = 65˚ and a = 12 cm, find ∠B, *b* and *c*.

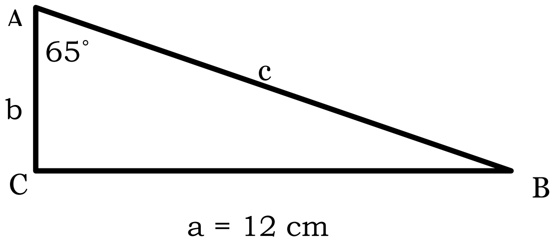
Solution:

a. To find ∠B, take note that ∠B and ∠A are complementary angles. Then,

$\angle B + \angle A = 90^\circ$

$\angle A = 90^\circ - 65$

$\angle A = 25^\circ$



b. To find b, since *b* is the adjacent side and *a* is the opposite side SOH.
of ∠A, then use TOA.

$\tan \theta = \frac{opposite}{adjacen}$

$\tan A = \frac{a}{b}$

$\tan 65^\circ = \frac{12}{b}$

$b \tan 65^\circ = 12$

$b(2.1445) = 12$

c. To find *c*, since *c* is the hypotenuse and *a* is the opposite side of ∠A, then use

$\sin \theta = \frac{opposite}{hypotenuse}$

$\sin A = \frac{a}{c}$

$\sin 65^\circ = \frac{12}{c}$

$c \sin 65^\circ = 12$

$c(0.9063) = 12$

$b = \frac{12}{2.1445}$

$b = 5.60$ cm

$c = \frac{12}{0.9063}$

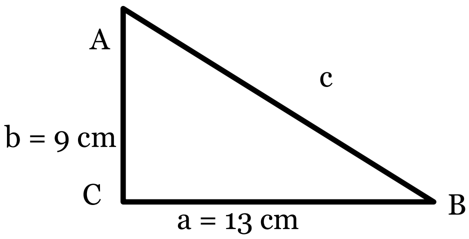
$c = 13.24$ cm

D. Solving a Right Triangle Given the Length of the Two Legs

Example: Triangle ACB is right-angled at C. If a = 13 cm and b = 9 cm, find c, ∠A, and ∠B.

Solution:

To find *c,* use the Pythagorean theorem:



$c^2 = a^2 + b^2$

$c^2 = (13)^2 + (9)^2$

$c^2 = 169 + 81$

$c^2 = 250$

$c = \sqrt{250}$

$c = 15.81$

a. To find ∠A, since a and b are opposite and adjacent side of ∠A respectively, then use TOA.

$\tan \theta = \frac{opposite}{adjacent}$

$\tan A = \frac{a}{b}$

$\tan A = \frac{13}{9}$

$\tan A = 1.4444$

We can use our scientific calculator to find an angle whose tangent is 1.4444.
$A = 55.30^\circ$

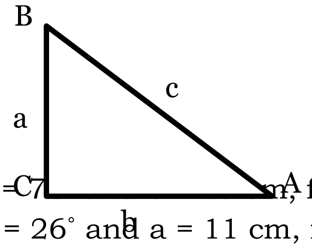
c. Based on the fact that ∠A and ∠B are complementary, the measure of angle ∠B is $90^\circ - 55.30^\circ = 34.7^\circ$

You have learned the definition of the six trigonometric ratios. Make sure that you will be able to use these in the succeeding activities.

Explore

Activity 3: Find My Parts!!!

Directions: Using the given figure, find the unknown in each number. Express your answers to the nearest hundredths.



1. If A = 7[illegible]m, find b.
2. If B = 26˚ and a = 11 cm, find c.
3. If A = 49˚ and a = 10 cm, find c.
4. If a = 7 c and b = 12 cm , find A.
5. If a = 8 cm and c = 12 cm, find B.

> Now that you know the important ideas about the topic, let's go deeper by moving on to the next activity.

Deepen

Activity 4: Draw Me Then Solve!

Directions: Sketch a figure and solve the remaining parts of each right triangle ABC with right angle at C, given that:

1. A = 15˚ and c = 37 cm
2. B = 30˚ and b = 11 cm
3. a = 7 cm and b = 15 cm
4. A = 48˚ and b = 22 cm