SHS

AIRs - LM in

Statistics and Probability

Module 6:

Sampling and Sampling Distribution

Statistics and Probability
Module 6: Sampling and Sampling Distribution

First Edition, 2021

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Target

Researchers in different fields want to learn about a population and describe its properties and characteristics. However, it may be impossible or at least impractical to obtain data from the whole population.

In your earlier study of Statistics, you have learned about frequency distribution. You also learned about probability distribution. In this learning material, you will learn another kind of distribution, which is sampling distribution of the means.

After going through this AIRs-LM, you are expected to:

1. illustrate random sampling **(M11/12SP–IIId-2)**
2. distinguish between parameter and statistic **(M11/12SP–IIId-3)**
3. identify sampling distributions of statistics (sample mean) **(M11/12SP–IIId-4)**
4. find the mean and variance of the sampling distribution of the sample mean **(M11/12SP–IIId-5)**
5. define the sampling distribution of the sample mean for normal population when the variance is: (a) known; (b) unknown **(M11/12SP–IIIe-1)**

Subtasks:

1. define random sampling, parameter, and statistic
2. identify sample means
3. enumerate the steps in finding the mean and variance of sampling distribution of sample means.
4. determine the formula to be used in defining sampling distribution of sample mean if variance is known or unknown.

Before going on, check how much you know about this topic. Answer the pretest on the next page on a separate sheet of paper.

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which word refers to the totality of subjects under consideration?
   A. Parameter B. Population C. Sample D. Statistic

2. Which of the following refers to a portion taken from the total number of subjects?
   A. Parameter B. Population C. Sample D. Statistic

3. What do we call the descriptive measures computed from the population?
   A. Parameter B. Population C. Sample D. Statistic

4. Which of the following refers to the calculated value from a sample?
   A. Parameter B. Population C. Sample D. Statistic

5. Which sampling technique is illustrated when each member of the population is given equal chance to be chosen as part of the sample?
   A. Random Variable B. Random Sampling
   C. Sampling Distribution D. Sample Statistic

6. Which of the following terms is the same as the word average?
   A. Mean B. Variance
   C. Sample Size D. Standard Deviation

7. How many different samples of size $n = 3$ can be selected from a population with the size $N = 4$?
   A. 2 B. 3 C. 4 D. 5

8. What do we call the difference between the sample mean and the population mean?
   A. Correction Factor B. Probability Distribution
   C. Sampling Error D. Sampling Distribution

9. A group of modules has the following number of pages: 12, 16, 20, 24 and 28. Consider samples of size 3 that can be drawn from this population. The following could possibly be a sample **EXCEPT ONE**. Which is the exception?
   A. 12, 16, 20 B. 16, 20, 28 C. 20, 24, 28 D. 16, 20, 22

10. A jar contains the number 1, 3 and 5. If you got samples of size 2 from this population and compute the mean of each sample, what are the sample means that you will be able to get?
    A. 0, 1, 2 B. 1, 2, 3 C. 2, 3, 4 D. 3, 4, 5

11. Which of the following is true for the mean of the population and the mean of the
sample?
   A. The mean of the population is equal to the mean of the sample.
   B. The mean of the population is less than the mean of the sample.
   C. The mean of the population is greater than the mean of the sample.
   D. The mean of the population is different from the mean of the sample.

12. In a research conducted, there are 30 participants, what type of statistical test should be used for the study?
   A. t-table   B. t-test   C. z-table   D. z-test

13. If a random sample of size 16 is taken with replacement from a population with $\mu = 12$ and $\sigma^2 = 4$, what is the value of the variance ${\sigma^2}_{\bar{x}}$?
   A. $\frac{1}{12}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

14. Determine the value of t based from the given below:
   $n = 23$ $\bar{x} = 49$
   $\mu = 54$ $s = 9$
   A. –2.64   B. –2.65   C. –2.66   D. 2.67

15. Assume that Filipino teenagers spend an average of ₱500.00 on shopping per month with standard deviation of ₱150.00. If the expenses are normally distributed, and 50 Filipino teenagers are randomly selected, what is the probability that their mean expenses on shopping per month is less than ₱450?
   A. 0.62%   B. 0.91%   C. 0.94%   D. 1.07%

Jumpstart

Activity 1: Try Me!

Directions: Read and analyze the given activity. Answer and do what is needed for the following.

A. Computing for the Mean of a Sample

Find the mean of the following sets of numbers.

Given	Mean
3, 4, 9, 11, 5	
7, 11, 10, 13, 5, 6	
13, 3, 6, 8, 12, 9, 5	
15, 11, 16, 15, 18, 20, 20	
21, 23, 25, 19, 15, 26, 27, 29	

B. Constructing Frequency Distribution

The following are the answers of the students to the first item of a multiple-choice test with options A, B, C and D. Construct a frequency distribution for their different answers.

A	D	B	A	C
B	D	B	D	A
B	B	D	D	D
C	C	A	D	B
D	B	D	C	A

Frequency Distribution

Options	Frequency
A	
B	
C	
D	

If the mean that you solved in test A are 6.4, 8.667, 8, 16.429 and 23.125 and the frequency in test B are 5, 7, 4, and 9 then you are correct. You did a great start.

Congratulations! You are now ready to learn more.

Discover

Sampling and Sampling Distribution

The totality of subjects (people, animals or objects) under consideration is called ***population***. The portion chosen from a population is called ***sample*** and the process of taking samples is called ***sampling***.

Random Sampling is a sampling technique in which each member of the population is given equal chance to be chosen as part of the sample. The lottery method, drawing lots, or the use of random numbers can be used to accomplish random sampling.

Examples of population:

- All likely voters in the next election
- All parts produced today
- All sales receipts for November

Examples of sample:

- 1000 voters selected at random for interview
- A few parts selected for destructive testing
- Random receipts selected for audit

The measurement or quantity that describes the population is called ***parameter*** while the measurement or quantity that describes the sample is called ***statistic***.

<u>Example 1:</u>

In order to test the effect of the new drug against the corona virus to humans, 20 patients were given the dose. After a minute, it was found that the body temperature in average, decreased by 2^0C. Answer the following:

a) Are the 20 patients mentioned above population or sample?

b) Is the 2^0C decrease in the body temperature considered parameter or statistic?

<u>Answer:</u>

a) The 20 patients taken are considered **sample**.

b) Since the measurement 2^0C refers to the average decrease of the 20 patients (sample), it is therefore considered as **statistic**.

<u>Example 2:</u>

The average score of the whole class of Grade 11 – GAS from their first performance task in General Mathematics is 28.2 having 30 points as the total score. Answer the following:

a) Is the “whole class” stated population or sample?

b) Is the 28.2 average score considered parameter or statistic?

<u>Answer:</u>

a) The whole class is considered **population**.

b) Since the 28.2 refers to the average score of the whole class (population), it

is therefore considered as **parameter**.

<u>Example 3:</u>

47 out of the 100 athletes were checked on their height. The variance of their height is 3.4cm. Answer the following:

a) Are the 47 students a population or sample?

b) Is the 3.4cm variance considered parameter or statistic?

<u>Answer:</u>

a) The 47 students are considered **sample**.

b) Since the 3.4cm refers to the variance of the height of the 47 students (sample), it is therefore considered as **statistic**.

Sampling Distribution of Sample Means

Sampling distribution of sample means is a frequency distribution using the means computed from all possible random samples of a specific size taken from a population. It is also known as the probability distribution of the sample means.

Steps in Constructing the Sampling Distribution of the Sample Means

1. Determine the number of possible samples that can be drawn from the population
using the formula:

$$_N C_n$$

where: N = size of the population
n = size of the sample

2. List all the possible samples and compute the mean of each sample.
3. Construct a frequency distribution of the sample means obtained in step 2.

Example 1: A population consists of the numbers 2, 4, 5, 9 and 10. Take three numbers as samples from the population. Present the frequency distribution of all the possible samples taken and draw the histogram.

Step 1: Determine the number of samples that can be drawn from the population considering the sample size.

The number of samples of size n that can be drawn from a population of size N is given by $_N C_n$.

$$_N C_n = \frac{N!}{(N-n)!n!}$$

3. Therefore, the total number of samples

$$_N C_n = {}_5 C_3 = \frac{5!}{(5-3)!3!} = \frac{5 \cdot 4 \cdot 3 \cancel{2 \cdot 1}}{(2 \cdot 1) 3 \cancel{2 \cdot 1}} = \frac{20}{2} = 10.$$

Step 2: Let us list all possible samples of size 3 from this population and compute the mean of each sample.

The samples and their means can be presented by the following table:

Sample	Mean
2,4,5	3.67
2,4,9	5.00
2,4,10	5.33
2,5,9	5.33
2,5,10	5.67
2,9,10	7.00
4,5,9	6.00
4,5,10	6.33
4,9,10	7.67
5,9,10	8.00

If we denote the means as random variable $\overline{X}$, then:

$$\overline{X} = \{3.67, 5.00, 5.33, 5.67, 6.00, 6.33, 7.00, 7.67, 8.00\}$$

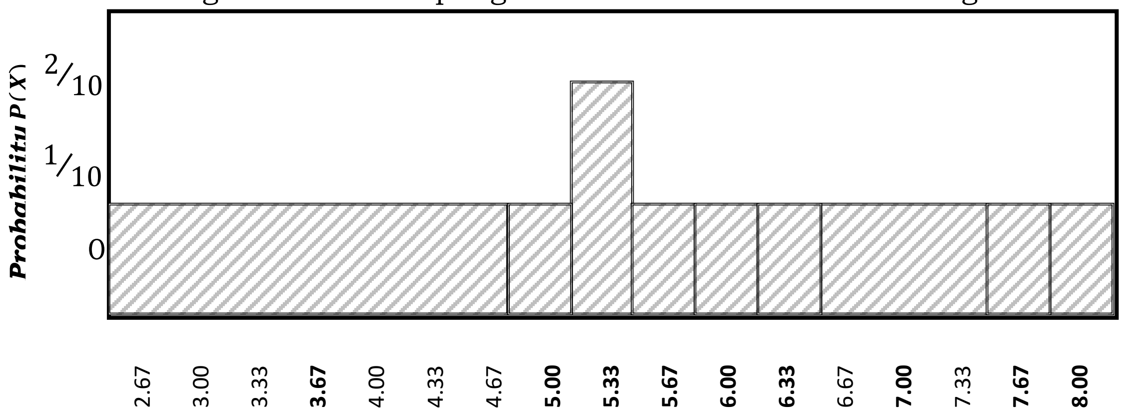
Observe that the means vary from sample to sample. Thus, any mean based on the sample drawn from a population is expected to assume different values for the samples. So this leads us to a conclusion that sample mean is a random variable, which depends on a particular sample. Being a random variable, it has a probability distribution.

Step 3: Therefore, the probability distribution of $\overline{X}$ is:

Sample Mean $\overline{X}$	Frequency	Probability $P(\overline{X})$
3.67	1	$1/10 = 0.10$
5.00	1	$1/10 = 0.10$
5.33	2	$2/10 = 0.20$
5.67	1	$1/10 = 0.10$
6.00	1	$1/10 = 0.10$
6.33	1	$1/10 = 0.10$
7.00	1	$1/10 = 0.10$
7.67	1	$1/10 = 0.10$
8.00	1	$1/10 = 0.10$
Total	**10**	**1**

The probability distribution above represents the means of the samples, that's why the distribution is now called ***Sampling Distribution of the Sample Mean***

The histogram of the sampling distribution will be the following:



<u>Example 2:</u> Samples of four cards are drawn at random and without replacement from a population of six cards numbered from 1 to 6. Present the frequency distribution of all the possible samples taken and draw the histogram.

Step 1: How many possible samples can be drawn?

Steps	**Solution**
Identify the given.	Here N = 6 and n = 4.
Use the formula ${}_NC_n$ to find the number of possible samples that can be drawn.	${}_6C_4 = \frac{6!}{(6-4)!4!} = \frac{6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1}}{(2\cdot1)\cancel{4\cdot3\cdot2\cdot1}} = \frac{30}{2} = 15$

So there are 15 possible samples that can be drawn.

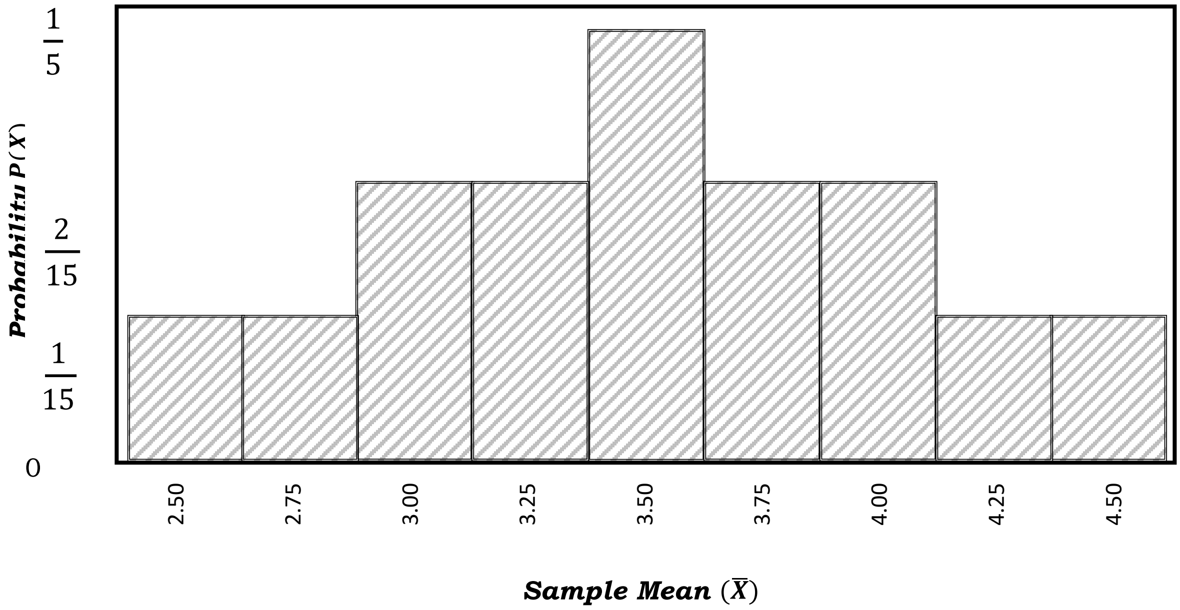
Step 2: List all the possible samples and compute the mean of each sample.

Sample	**Mean**
1,2,3,4	2.50
1,2,3,5	2.75
1,2,3,6	3.00
1,2,4,5	3.00
1,2,4,6	3.25
1,2,5,6	3.50
1,3,4,5	3.25
1,3,4,6	3.50
1,3,5,6	3.75
1,4,5,6	4.00
2,3,4,5	3.50
2,3,4,6	3.75
2,3,5,6	4.00
2,4,5,6	4.25
3,4,5,6	4.50

Step 3: Construct a frequency distribution of the sample means obtained in step 2.

Sample Mean $\bar{X}$	**Frequency**	**Probability** $P(\bar{X})$
2.50	1	$^1/_{15}$
2.75	1	$^1/_{15}$
3.00	2	$^2/_{15}$
3.25	2	$^2/_{15}$
3.50	3	$^3/_{15}$ or $^1/_5$
3.75	2	$^2/_{15}$
4.00	2	$^2/_{15}$
4.25	1	$^1/_{15}$
4.50	1	$^1/_{15}$
Total	**15**	**1**

Construct a histogram of the sampling distribution of the means.



Mean and Variance of the Sampling Distribution

Problem 1. Consider a population consisting of 1,2,3,4, and 5. Suppose samples of size 2 are drawn from this population. Describe the sampling distribution of the sample means.

- What is the mean and variance of the sampling distribution of the sample means?
- Compare these values to the mean and variance of the population.
- Draw the histogram of the sampling distribution of the population mean.

Steps	***Solution***
1. Compute the mean of the population (μ).	$\mu = \frac{\sum X}{N}$ $= \frac{1+2+3+4+5}{5}$ $= 3.00$ So, the mean of the population is 3.00.
2. Compute the variance of the population (σ^2)	see table below $\sigma^2 = \frac{\sum (X-\mu)^2}{N}$

X	$X - \mu$	$(X - \mu)^2$
1	- 2	4
2	-1	1
3	0	0
4	1	1
5	2	4
		$\sum (X-\mu)^2 = 10$

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$$

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$$= \frac{10}{5}$$
$$= 2$$

So, the variance of the population is 2.
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<tr>
<td>3. Determine the number of possible samples of size n=2.</td>
<td>

Use the formula ${}_{N}C_{n}$. Here N=5 and n=2.

$${}_{N}C_{n} = 10$$

So, there are 10 possible samples of size 2 that can be drawn.
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</tr>
<tr>
<td>4. List all possible samples and their corresponding means.</td>
<td>
<table>
<tr><th>Samples</th><th>Mean</th></tr>
<tr><td>1,2</td><td>1.50</td></tr>
<tr><td>1,3</td><td>2.00</td></tr>
<tr><td>1,4</td><td>2.50</td></tr>
<tr><td>1,5</td><td>3.00</td></tr>
<tr><td>2,3</td><td>2.50</td></tr>
<tr><td>2,4</td><td>3.00</td></tr>
<tr><td>2,5</td><td>3.50</td></tr>
<tr><td>3,4</td><td>3.50</td></tr>
<tr><td>3,5</td><td>4.00</td></tr>
<tr><td>4,5</td><td>4.50</td></tr>
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</tr>
<tr>
<td>5. Construct the sampling distribution of the sample means.</td>
<td>
<table>
<tr><th colspan="3">Sampling Distribution of Sample Means</th></tr>
<tr><th>Sample Mean $\overline{X}$</th><th>Probability $P(\overline{X})$</th><th>$\overline{X} \cdot P(\overline{X})$</th></tr>
<tr><td>1.50</td><td>1</td><td>$\frac{1}{10}$</td></tr>
<tr><td>2.00</td><td>1</td><td>$\frac{1}{10}$</td></tr>
<tr><td>2.50</td><td>2</td><td>$\frac{1}{5}$</td></tr>
<tr><td>3.00</td><td>2</td><td>$\frac{1}{5}$</td></tr>
<tr><td>3.50</td><td>2</td><td>$\frac{1}{5}$</td></tr>
<tr><td>4.00</td><td>1</td><td>$\frac{1}{10}$</td></tr>
<tr><td>4.50</td><td>1</td><td>$\frac{1}{10}$</td></tr>
<tr><td>Total</td><td>10</td><td>1.00</td></tr>
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</tr>
</table>

6. Compute the mean of the sampling distribution of the sample means ($\mu_{\bar{x}}$). Follow these steps:

a. Multiply the sample mean by the corresponding probability.

b. Add the results

Sample Mean $\bar{X}$	**Probability $P(\bar{X})$**	**$\bar{X} \cdot P(\bar{X})$**
1.50	$\frac{1}{10}$	0.15
2.00	$\frac{1}{10}$	0.20
2.50	$\frac{1}{5}$	0.50
3.00	$\frac{1}{5}$	0.60
3.50	$\frac{1}{5}$	0.70
4.00	$\frac{1}{10}$	0.40
4.50	$\frac{1}{10}$	0.45
Total	**1.00**	3.00

$$\mu_x = \bar{X} \cdot P(\bar{X})$$
$$= 3.00$$

So, the mean of the sampling distribution of the means is 3.00.

7. Compute the variance ($\sigma^2{}_{\bar{x}}$) of the sampling distribution of the sample means. Follow these steps:

a. Subtract the population mean (μ) from each sample mean ($\bar{X}$). Label this as $\bar{X} - \mu$.

b. Square the difference. Label this as $(\bar{X} - \mu)^2$.

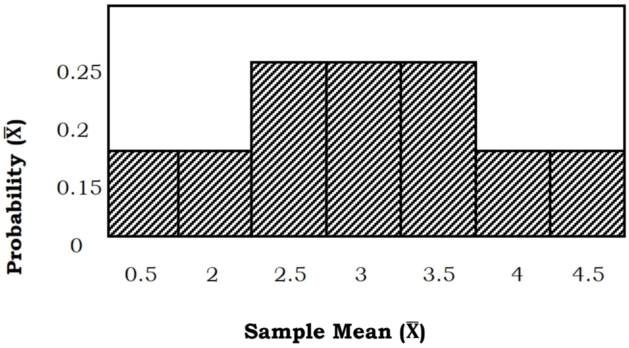
c. Multiply the results by the corresponding probability. Label this as $P(\bar{X}) \cdot (\bar{X} - \mu)^2$

d. d. Add the results.

$\bar{X}$	$P(\bar{X})$	$\bar{X} - \mu$	$(\bar{X} - \mu)^2$	$P(\bar{X}) \cdot (\bar{X} - \mu)^2$
1.50	$\frac{1}{10}$	-1.50	2.25	0.225
2.00	$\frac{1}{10}$	-1.00	1.00	0.100
2.50	$\frac{1}{5}$	-0.50	0.25	0.050
3.00	$\frac{1}{5}$	0.00	0.00	0.000
3.50	$\frac{1}{5}$	0.50	0.25	0.050
4.00	$\frac{1}{10}$	1.00	1.00	0.100
4.50	$\frac{1}{10}$	1.50	2.25	0.225
Total	**1.00**			0.750

$$\sigma^2{}_x = \sum P(\bar{X}) \cdot (\bar{X} - \mu)^2$$
$$= 0.75$$

So, the variance of the sampling distribution is 0.75.

<table>
<tr>
<td>8. Construct the histogram for the sampling distribution of the sample means.</td>
<td>
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</table>

Our comparison for both the mean and variance of the sample means and the mean and variance of the population respectively will follow after our further discussion for the problem below.

Problem 2. Let us take our example 1 in our previous discussion about Sampling Distribution of the Sample Mean. What is the mean and variance of the sampling distribution of the sample means? Compare these values to the mean and variance of the population.

A population consists of the numbers 2, 4, 5, 9 and 10. Take three numbers as samples from the population.

<table>
<tr><th>Solution for the Population</th><th>Solution for the Sample</th></tr>
<tr>
<td>Compute the mean of the population (μ).

$$\mu = \frac{\sum X}{N}$$
$$= \frac{2+4+5+9+10}{5}$$
$$= 6.00$$

So, the mean of the population is **6.00**.</td>
<td>Compute the mean of the sampling distribution of the sample means ($\mu_{\bar{x}}$). Follow these steps:

- Multiply the sample mean by the corresponding probability.
- Add the results

<table>
<tr><th>Sample Mean $\bar{X}$</th><th>Probability $P(\bar{X})$</th><th>$\bar{X} \cdot P(\bar{X})$</th></tr>
<tr><td>3.67</td><td>$1/10$</td><td>0.367</td></tr>
<tr><td>5.00</td><td>$1/10$</td><td>0.500</td></tr>
<tr><td>5.33</td><td>$1/5$</td><td>1.066</td></tr>
<tr><td>5.67</td><td>$1/10$</td><td>0.567</td></tr>
<tr><td>6.00</td><td>$1/10$</td><td>0.600</td></tr>
<tr><td>6.33</td><td>$1/10$</td><td>0.633</td></tr>
<tr><td>7.00</td><td>$1/10$</td><td>0.700</td></tr>
<tr><td>7.67</td><td>$1/10$</td><td>0.767</td></tr>
<tr><td>8.00</td><td>$1/10$</td><td>0.800</td></tr>
<tr><td>Total</td><td>1.00</td><td>6.00</td></tr>
</table></td>
</tr>
</table>

$$\mu_x = \overline{X} \cdot P(\overline{X})$$
$$= 6.00$$

So, the mean of the sampling distribution of the means is **6.00**.

Compute the variance of the population (σ^2)

X	$\boldsymbol{X - \mu}$	$\boldsymbol{(X - \mu)^2}$
2	- 4	16
4	- 2	4
5	-1	1
9	3	9
10	4	16
		$\sum (X - \mu)^2 = 46$

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$
$$= \frac{46}{5}$$
$$= 9.2$$

So, the variance of the population is **9.2**.

Compute the variance ($\sigma^2_{\bar{x}}$) of the sampling distribution of the sample means. Follow these steps:

- Subtract the population mean (μ) from each sample mean ($\overline{X}$). Label this as $\overline{X} - \mu$.
- Square the difference. Label this as $(\overline{X} - \mu)^2$.
- Multiply the results by the corresponding probability. Label this as $P(\overline{X}) \cdot (\overline{X} - \mu)^2$
- Add the results.

$\overline{X}$	$P(\overline{X})$	$\overline{X} - \mu$	$(\overline{X} - \mu)^2$	$P(\overline{X}) \cdot (\overline{X} - \mu)^2$
3.67	$1/10$	- 2.33	5.4289	0.54289
5.00	$1/10$	- 1	1	0.1
5.33	$1/5$	- 0.67	0.4489	0.08978
5.67	$1/10$	- 0.33	0.1089	0.01089
6.00	$1/10$	0	0	0
6.33	$1/10$	0.33	0.1089	0.01089
7.00	$1/10$	1	1	0.1
7.67	$1/10$	1.67	2.7889	0.27889
8.00	$1/10$	2	4	0.4
Total	**1.00**			1.5333

$$\sigma^2_x = \sum P(\overline{X}) \cdot (\overline{X} - \mu)^2$$
$$= 1.5333$$

So, the variance of the sampling distribution is **1.5333**.

Let us summarize what we have done for the preceding problems by comparing the means and variances of the population and the sampling distribution of the means.

	Problem 1		**Problem 2**	
	Population (N = 5)	Sampling Distribution of the Sample Means (n = 2)	Population (N = 5)	Sampling Distribution of the Sample Means (n = 3)
Mean	$\mu = 3.00$	$\mu_x = 3.00$	$\mu = 6.00$	$\mu_x = 6.00$
Variance	$\sigma^2 = 2.00$	$\sigma^2_x = 0.75$	$\sigma^2 = 9.2$	$\sigma^2_x = 1.5333$
Standard Deviation	$\sigma = 1.41$	$\sigma_x = 0.87$	$\sigma = 3.033$	$\sigma_x = 1.238$

Observe that the mean of the sampling distribution of the sample means is always equal to the mean of the population. The variance of the sampling distribution is obtained by using the formula, $\sigma^2{}_{\bar{x}} = \frac{\sigma^2}{n}\ (\frac{N-n}{N-1})$. This formula holds when the population is finite. The problems shown are all finite population.

We summarize the properties of the sampling distribution below.

Theorem
If all possible random samples of size n are taken *with replacement (independent)* from a population with a mean μ and variance σ^2, then the mean ($\mu_{\bar{x}}$), variance ($\sigma^2{}_{\bar{x}}$) and standard deviation ($\sigma_{\bar{x}}$) of the sampling distribution of the sample mean are: FOR INFINITE POPULATION $\boldsymbol{\mu_{\bar{x}} = \mu}$ (mean) $\boldsymbol{\sigma^2{}_{\bar{x}} = \frac{\sigma^2}{n}}$ (variance) $\boldsymbol{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$ (standard deviation or standard error) If all possible samples of size n are taken *without replacement (dependent)* from a finite population of size N with a mean μ and variance σ^2, then the mean ($\mu_{\bar{x}}$), variance ($\sigma^2\bar{x}$) and standard deviation ($\sigma_{\bar{x}}$) of the sampling distribution of the sample mean are: FOR FINITE POPULATION $\boldsymbol{\mu_{\bar{x}} = \mu}$ (mean) $\boldsymbol{\sigma^2{}_{\bar{x}} = \frac{\sigma^2}{n}(\frac{N-n}{N-1})}$ (variance) $\boldsymbol{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}$ (standard deviation or standard error) *Note:* The factor $\frac{N-n}{N-1}$ is called ***correction factor*** for finite population. It will be close to 1 and can be safely ignored when n is small compared to N.

Note that as **we increase the sample size,** the **variance of the sample mean decreases.**

Example 1:

A population has a mean of 60 and a standard deviation of 5. A random sample of 16 measurements is drawn from this population. Describe the sampling distribution of the sample means by computing its mean and standard deviation.

We shall assume that the population is infinite.

Steps	Solution
1. Identify the given information	Here $\mu = 60$, $\sigma = 5$ and $n = 16$
2. Find the mean of the sampling distribution. Use the property that $\mu_{\bar{x}} = \mu$	$\mu_{\bar{x}} = \mu$ $= \mathbf{60}$
3. Find the standard deviation of the sampling distribution. Use the property that $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{16}} = \frac{5}{4}$ $= \mathbf{1.25}$

Example 2:

The heights of male college students are normally distributed with mean of 68 inches and standard deviation of 3 inches. If 80 samples consisting of 25 students each are drawn from the population, what would be the expected mean and standard deviation of the resulting sampling distribution of the means?

We shall assume that the population is infinite.

From the problem, the given are $\mu = 68$, $\sigma = 3$ and $n = 25$.

To find the mean of the sampling distribution, we need to use the formula $\mu_{\bar{x}} = \mu$. Solving the mean, we have $\boldsymbol{\mu_{\bar{x}}} = \mu = \mathbf{68}$.

In getting the value of the standard deviation of the sampling distribution, the formula $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ will be used. That is $\boldsymbol{\sigma_{\bar{x}}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = \frac{3}{5} = \mathbf{0.6}$.

Sampling Distribution of Large Sample Size ($n \geq 30$)

Large Sample Size

Statisticians consider a sample of size 30 or more as large. If this large sample size is taken from a population with mean μ and standard deviation σ, then the sampling distribution of the sample mean approaches the normal distribution with a mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma/\sqrt{n}$, thus can be standardized as:

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

Caution
The formula $z = \frac{\bar{x}-\mu}{\sigma}$ is for individual score/observation, while $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ is for sample mean. Both formulas use the z-table/normal distribution table.

As stated above, the formula can only be used if the **population variance or standard deviation is known.** If the population variance or standard deviation is not known, the formula above still holds when the sample size is large ($n \geq 30$). That is the sample standard deviation s can be used to replace σ. This is known as the **Central Limit Theorem.**

Theorem
If random samples of size n are taken from a population with a mean μ and standard deviation σ, then the sampling distribution of the sample mean $\bar{x}$ approaches normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, thus can be standardized as: $$\mathbf{z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}}$$ As the n increase, the sampling distribution of the sample mean gets nearer and nearer to the normal distribution. *Note:* ➢ If σ in unknown, compute the sample standard deviation s then use

it to replace σ in the formula provided that $n \geq 30$.

- Even if $n \geq 30$, the formula can still be used provided that the population is approximately normal and the **population standard deviation σ is known.**

<u>Example:</u>

The height of pupils in Luna Elementary School has a mean of 121 cm with standard deviation of 5 cm. If 50 of them are taken as samples, what is the probability that their mean height is less than 120 cm?

<u>Answer:</u>

From the problem, $\mu = 121$, $\sigma = 5$, $\bar{x} = 120$, and $n = 50$. Using the formula: $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{120-121}{5/\sqrt{50}} = \frac{-1}{0.707} = -1.41$

Using the z-table on the page after the GAUGE ICON, the area below $z = -1.41$ is 0.0793. Thus the probability that the mean height of the sample is less than 120 cm is **0.0793** or **7.93%**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	$\boxed{.0793}$	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

Sampling Distribution of Small Sample Size ($n < 30$)
Small Sample Size

If the sample size is less than 30 ($n < 30$) it is considered small, thus, even if the variance of the population is given, the formula for standardizing the sampling distribution of the sample mean cannot be used. For this is small sample, the normality of the distribution sample mean cannot be guaranteed, thus, the *z*-table cannot be used.

For a special case where the small sample ($n < 30$) is known to be from a normally distributed population, even when the variance is unknown, another type of distribution is used. This special case is stated in the theorem that follows.

Theorem
If $\bar{x}$ and s are the mean and standard deviation, respectively, of a random sample of size n taken from a ***normally distributed*** population with a mean μ, can be standardized as $$\mathbf{t} = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$ a value of a random variable ***t*** following the t-distribution. *Note:* ➢ The formula is used when ($n < 30$) and the ***population standard deviation is unknown.*** *Recall:* The sample standard deviation is computed as: $$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

Example:

Ten (10) "*Taklobo*" or giant clams have an average of 45 inches across its shell with standard deviation of 4 inches. What is the standardized score for the average measure across the shell of the whole population of "*taklobo*" equal to 43 inches?

Answer:

From the problem, $\bar{x} = 45$, $s = 4$, $\mu = 43$, and $n = 10$. Using the formula:

$$\mathrm{t} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{45-43}{4/\sqrt{10}} = \frac{2}{1.26} = \mathbf{1.59}$$

Explore

Here are some activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 1. Distinguish the Given!

Directions: Determine if the given subject is ***population*** or ***sample***, then describe the given quantity as ***parameter*** or ***statistic***:

1. The average grade of the whole class under study is 82.15.

   Whole class: ________________ *Average grade (82.15):* ________________

2. 50 out of the 200 animals in the zoo were taken and checked on their weight. The variance of their weight is 12.5 kg.

   50 animals: __________________ *Variance (12.5 kg):* __________________

3. The standard deviation of the life span of a specie endemic in the Philippines is 2.3 years

A specie endemic in the Philippines: ________ *Standard deviation (2.3 years):* _________

Activity 2. Construct Me!

A population consists of the five numbers 2, 3, 6, 8 and 11. Consider samples of size 2 that can be drawn from this population.

a. List all the possible samples and the corresponding mean.

Sample	Mean

b. Construct the sampling distribution of the sample means.

Sample Mean $\bar{x}$	Frequency	Probability $P(\bar{x})$

c. Draw a histogram of the sampling distribution of the means.

Activity 3. Solve Me!

Problem 1. The scores of individual students on a national test have a normal distribution with mean 18.6 and standard deviation 5.9. At Federico Ramos Rural High School, 76 students took the test. If the scores at this school have the same distribution as national scores, what are the mean and standard deviation of the sample mean for 76 students?

Problem 2. Assume that the height of adult women were normally distributed with a mean of 63 inches and standard deviation of 2.5 inches. If 36 women are randomly selected, what is the probability that their mean height is less than 62 inches?

Problem 3. Find the t-value considering the given below:

$\mu = 41$ $\bar{x} = 43$

s= 3 n =20

Great job! You have understood the lesson.
Are you now ready to summarize?

Deepen

Take into account that you are now a researcher in your chosen field and you are tasked to solve for a certain kind of distribution, which is sampling distribution of the means. What will be your research topic? Give also a certain reason why you wanted to conduct the study.

Create an **ESSAY** explaining your topic while giving its possible contribution to your work and to the society.

Examples:

Field	***Possible Topic***
Education	*Describe the Mathematics Performance of your students to plan and prepare instructional activities and materials appropriate for them*
Business	*Find out the most wanted brand of cellphone in the market*
Medicine	*Determine the most effective and most affordable vaccine against Covid-19*
Politics	*Find out the acceptability and popularity ratings of prominent leaders in the country*

Rubrics for the Constructed Essay

Criteria	5	4	3	2
Focus/ Main Point	The essay is focused, purposeful, and reflects clear insight and ideas.	The essay is focused on the topic and includes relevant ideas	The essay is focused on topic and includes few loosely related ideas	The essay poorly addresses topic and includes irrelevant ideas
Support	Persuasively supports main point with well-developed reasons and/or examples	Supports main point with developed reasons and/or examples	Supports main point with some underdeveloped reasons	Provides little or no support for the main point
Organization & Format	Effectively organizes ideas to build a logical, coherent argument	Organizes ideas to build an argument	Some organization of ideas to build an argument	Little or no organization of ideas to build an argument
Originality	Distinctive experimentation with language and usage to enhance concepts	Sufficient experimentation with language and usage to enhance concepts	Very little experimentation to enhance concepts	No experimentation nor enhancement of concepts

	Applies higher order thinking and creative skills to relay complex ideas	Applies basic creative skills to relay ideas	Does not exhibit creativity	No adherence to the theme

Rubrics downloaded from https://www.kpu.ca > NEVRPDF Web results High School Rubrics

Gauge

Directions: Read carefully and analyze each item. Use a separate sheet of paper where you will write the letter of the correct answer beside the item number. (Use "CAPITAL" letters)

1. What word refers to the totality of subjects under consideration?
   A. Parameter B. Population C. Sample D. Statistic

2. Which of the following refers to a portion taken from the total number of subjects?
   A. Parameter B. Population C. Sample D. Statistic

3. What do we call the descriptive measures computed from the population?
   A. Parameter B. Population C. Sample D. Statistic

4. Which of the following refers to the calculated value from a sample?
   A. Parameter B. Population C. Sample D. Statistic

5. Which sampling technique is illustrated when each member of the population is given equal chance to be chosen as part of the sample?
   A. Random Variable B. Random Sampling
   C. Sampling Distribution D. Sample Statistic

6. Which of the following terms is the same as the word average?
   A. Mean B. Variance
   C. Sample Size D. Standard Deviation

7. How many different samples of size $n = 3$ can be selected from a population with the size $N = 4$?
   A. 2 B. 3 C. 4 D. 5

8. What do we call the difference between the sample mean and the population mean?
   A. Correction Factor B. Probability Distribution
   C. Sampling Error D. Sampling Distribution

9. A group of modules has the following number of pages: 12, 16, 20, 24 and 28. Consider samples of size 3 that can be drawn from this population. The following could possibly be a sample **EXCEPT ONE**. Which is the exception?

   A. 12, 16, 20 B. 16, 20, 28 C. 20, 24, 28 D. 16, 20, 22

10. A jar contains the number 1, 3 and 5. If you got samples of size 2 from this population and compute the mean of each sample, what are the sample means that you will be able to get?

   A. 0, 1, 2 B. 1, 2, 3 C. 2, 3, 4 D. 3, 4, 5

11. Which of the following is true for the mean of the population and the mean of the sample?

   A. The mean of the population is equal to the mean of the sample.
   B. The mean of the population is less than the mean of the sample.
   C. The mean of the population is greater than the mean of the sample.
   D. The mean of the population is different from the mean of the sample.

12. In a research conducted, there are 30 participants, what type of statistical test should be used for the study?

   A. t-table B. t-test C. z-table D. z-test

13. If a random sample of size 16 is taken with replacement from a population with $\mu = 12$ and $\sigma^2 = 4$, what is the value of the variance $\sigma^2{}_{\bar{x}}$?

   A. $\frac{1}{12}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

14. Determine the value of t based from the given below:

   $n = 23$ $\bar{x} = 49$
   $\mu = 54$ $s = 9$

   A. –2.64 B. –2.65 C. –2.66 D. 2.67

15. Assume that Filipino teenagers spend an average of ₱500.00 on shopping per month with standard deviation of ₱150.00. If the expenses are normally distributed, and 50 Filipino teenagers are randomly selected, what is the probability that their mean expenses on shopping per month is less than ₱450?

   A. 0.62% B. 0.91% C. 0.94% D. 1.07%

Nice work! Continue to work on your own but you can still ask guidance from your loved ones. Stay safe!

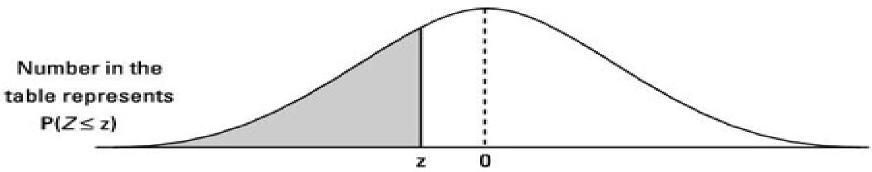
Additional Activity:

Form a group of six members from your family/relatives. Get the weight of each member of the group. Consider samples of size 4 that can be drawn from this population.

1. How many possible samples can be drawn?
2. List all possible samples and the corresponding means.
3. Construct the sampling distribution of the sample means.
4. Draw the histogram for the sampling distribution of the sample means.

STANDARD NORMAL PROBABILITIES

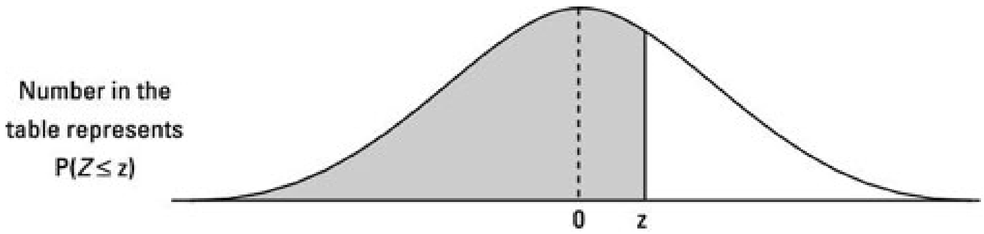
The table entry for *z* is the area under the standard normal curve to the left of *z*.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

STANDARD NORMAL PROBABILITIES

The table entry for *z* is the area under the standard normal curve to the left of *z*.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

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