

Mathematics

Quarter 3 – Week 1 - Module 1: Properties and Measures of Parallelogram



AIRs - LM

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NOT FOR SALE

Subject

Quarter 3 – Week 1 – Module 1: Properties and Measures of Parallelogram
First Edition, 2021

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Region I

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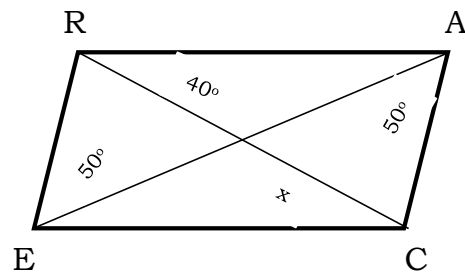
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14. What is the value of x that will make quadrilateral RACE a parallelogram in figure 6?

- A. 40°
B. 50°
C. 60°
D. 70°

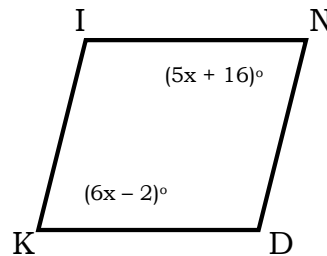
figure 6



15. What is the measure of $\angle D$ in parallelogram KIND in figure 7?

- A. 18°
B. 74°
C. 106°
D. 110°

figure 7



Did you do well in the pretest? Are there items that you were not sure of your answers? No worries! You can go back to those items as you gain new knowledge and skills in this module. But, don't forget to list ideas and concepts in the lessons. Enjoy!

Lesson

1

Properties of Parallelogram

There are many kinds of quadrilaterals. Some quadrilaterals are parallelograms, some are not. For example, kites and trapezoids are quadrilaterals but they are not parallelograms. Rectangles, squares, and rhombuses are parallelograms. What makes these quadrilaterals parallelograms? To answer this question, perform activity 1.



Jumpstart

Activity 1: Fantastic Four!

Materials: Protractor, graphing paper, ruler, pencil, compass

Directions: Follow the given procedures below and answer the processing questions.

1. Draw each of following quadrilaterals on a graphing paper.
 - a. Parallelogram OBEY
 - b. Rectangle GIVE
 - c. Rhombus THNX
 - d. Square LOVE
2. Measure the sides and the angles and record your findings in your own table similar to what is shown below.
3. Draw the diagonals and measure the segments formed by the intersecting diagonals. Again, record your findings in the table.

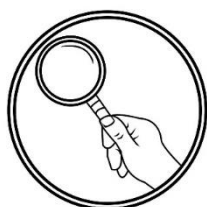
Drawing	In your drawing, identify the following	Measurement	Are the measurements equal or not equal?
	Opposite sides		
	Opposite angles		
	Consecutive angles		
	Segments formed by intersecting diagonals		

Processing Questions:

1. Based on the table above, what is true about the following?
 - a. Pairs of opposite sides
 - b. Pairs of opposite angles
 - c. Pairs of consecutive angles
 - d. Pairs of segments formed by intersecting diagonals
2. What does each diagonal do to a parallelogram?

3. Make a conjecture about the two triangles formed when a diagonal of a parallelogram is drawn. Explain your answer.
4. Do the findings apply to all kinds of parallelogram? Why?

Your answers to the questions show the conditions that guarantee that a quadrilateral is a parallelogram.



Discover

A parallelogram is a special type of quadrilateral. It is a four-sided polygon with two pairs of opposite sides that are parallel. To write the name of a parallelogram, we use the symbol \square . When we mark diagrams of quadrilaterals, use matching arrowheads to indicate which sides are parallel. In Figure 1.2, quadrilateral ABCD is a parallelogram where segment AD is parallel to segment CB and segment AB is parallel to segment CD.

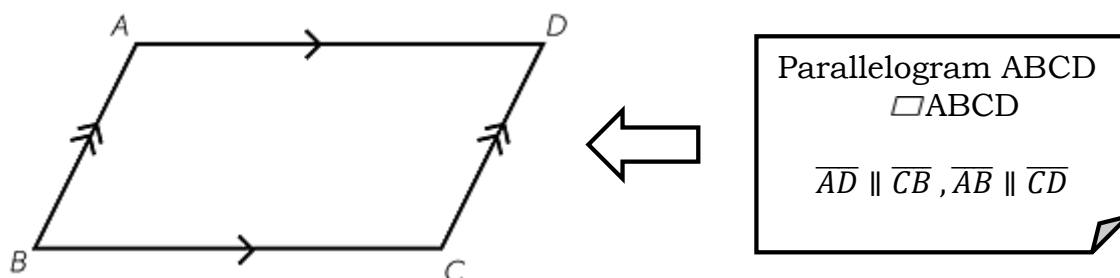
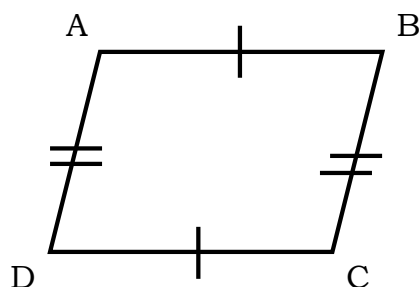


Figure 1.2

In Activity 1, you have just discovered the properties of parallelogram. Now, let's finalize these properties and be ready to use these properties to answer the succeeding activities.

Properties of Parallelogram

1. In a parallelogram, any two opposite sides are congruent.



In figure 1.3, segment AB is congruent to segment CD, and segment AD is congruent to segment CB.

$$\overline{AB} \cong \overline{CD} \text{ and } \overline{AD} \cong \overline{CB}$$

Figure 1.3

2. In a parallelogram, any two opposite angles are congruent.

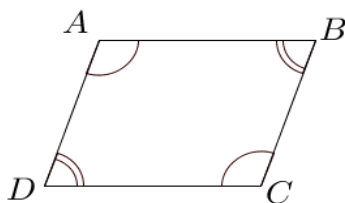


Figure 1.4

In figure 1.4, angle A is congruent to angle C, and angle B is congruent to angle D.

$$\angle A \cong \angle C$$

$$\angle B \cong \angle D$$

3. In a parallelogram, any two consecutive angles are supplementary.

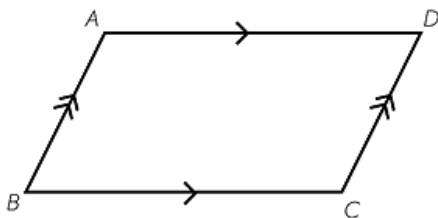


Figure 1.5

In figure 1.5,

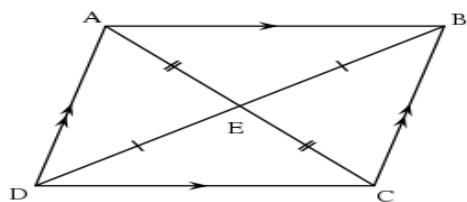
$$m\angle A + m\angle B = 180$$

$$m\angle B + m\angle C = 180$$

$$m\angle C + m\angle D = 180$$

$$m\angle D + m\angle A = 180$$

4. The diagonals of a parallelogram bisect each other.



$$\overline{AE} \cong \overline{CE} \text{ and } \overline{DE} \cong \overline{BE}$$

Figure 1.6

In figure 1.6,

The diagonals \overline{AC} and \overline{BD} bisect each other.

Segment AE is congruent to segment CE, and segment DE is congruent to segment BE.

5. A diagonal of a parallelogram forms two congruent triangles.

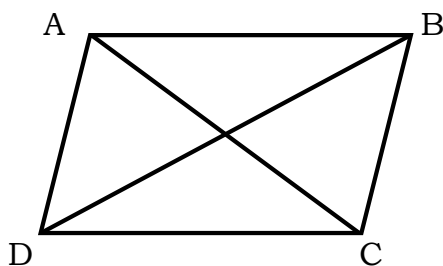


Figure 1.7

In figure 1.7,
Triangle DAB is congruent to triangle DCB.

Triangle ABC is congruent to triangle ADC.

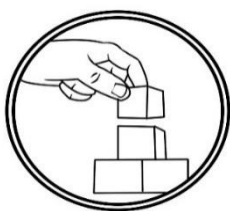
$$\triangle DAB \cong \triangle DCB, \text{ and}$$

$$\triangle ABC \cong \triangle ADC$$

Now that you already know the properties of parallelograms, we can already come up with the different conditions that guarantee that a quadrilateral is a parallelogram.

Conditions that Guarantee that a Quadrilateral is a Parallelogram

1. A quadrilateral is a parallelogram if both pairs of opposite sides are congruent.
2. A quadrilateral is a parallelogram if both pairs of opposite angles are congruent.
3. A quadrilateral is a parallelogram if both pairs of consecutive angles are supplementary.
4. A quadrilateral is a parallelogram if the diagonals bisect each other.
5. A quadrilateral is a parallelogram if each diagonal divides a parallelogram into two congruent triangles.
6. A quadrilateral is a parallelogram if one pair of opposite sides are both parallel and congruent.



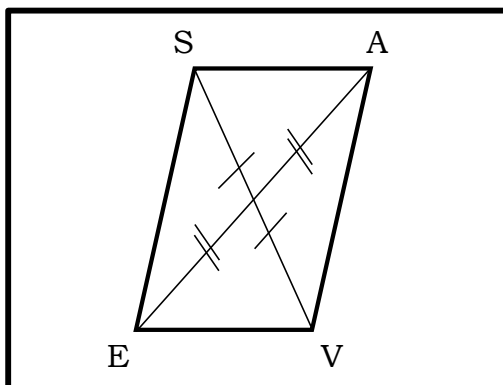
Explore

Let's master and strengthen the basic concepts you have learned from this lesson by answering activity 2. Have fun and enjoy!

Activity 2: State My Condition!

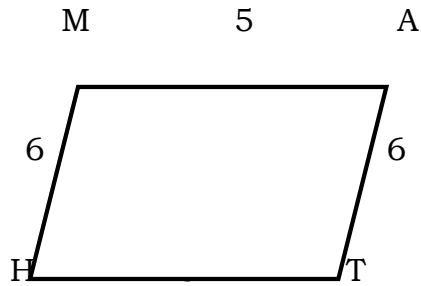
Directions: Study the markings in the parallelograms below then determine the condition that will make each of the given figure a parallelogram. The first item is done for you.

Example:

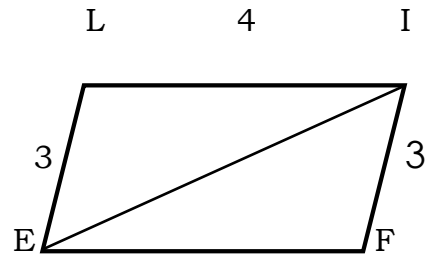


Answer: In the figure, \overline{EA} and \overline{SV} are the diagonals of quadrilateral SAVE. Based from the markings, \square SAVE is a parallelogram because the diagonals bisect each other.

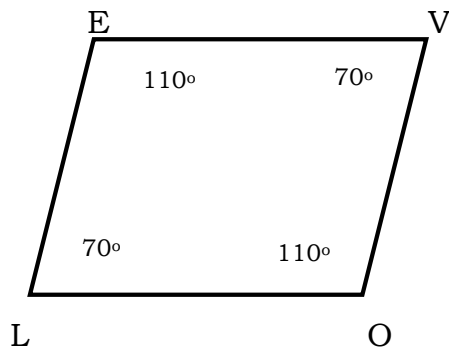
1.



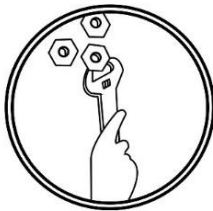
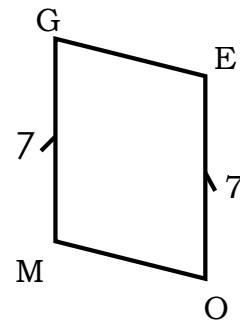
2.



3.



4.



Deepen

Activity 3: Is it a Parallelogram?

Directions: Read and analyze the situation below then answer the question that follows.

Jomari concluded that if one pair of sides is congruent, and the other pair of sides is parallel then it is sufficient to prove that the given quadrilateral is a parallelogram. Is he correct? Justify your answer.

Lesson 2

Find Measures of Angles, Sides, and Other Quantities Involving Parallelograms

Now that you know the properties of parallelogram, let's put this to use by answering the next activity.



Jumpstart

Activity 1: TRUE or FALSE

Directions: Read and analyze each statement, then write whether it is **TRUE** or **FALSE**.

1. In parallelogram JUMP, $\overline{JU} \cong \overline{MP}$ and $\overline{UM} \cong \overline{JP}$.
2. If $\angle A$ is 70° , then $\angle Y$ is also 70° in parallelogram RAYS.
3. In parallelogram CUTE, $\overline{CT} \cong \overline{UE}$.
4. \overline{ML} and \overline{IE} bisect each other in parallelogram MILE.
5. In parallelogram DAYS, \overline{AS} divides it into two congruent triangles.

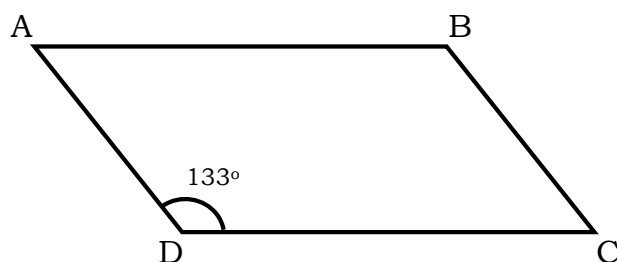
Did you get all the items correctly? Since you have already mastered the properties of parallelograms, you are now ready to apply these properties to uncover missing sides, angles and other quantities of a given parallelogram. Let's get started!



Discover

If one angle in a parallelogram is a right angle, then all angles are right angles. This means that if we know the properties of parallelograms, we can easily identify missing angles and sides. Let's have some illustrative examples.

Example 1. What is the measure of $\angle A$, $\angle B$, $\angle C$ in the parallelogram below?



You know that the opposite angles are congruent and the consecutive angles are supplementary.

Solution:

$\angle B = 133^\circ$ since it is opposite $\angle D$ and opposite angles are congruent.

Since consecutive angles are supplementary,

$$m\angle C + m\angle D = 180$$

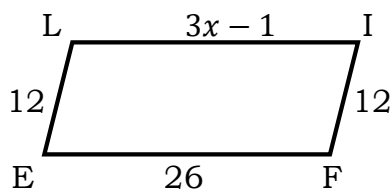
$$m\angle C + 133 = 180$$

$$m\angle C = 180 - 133$$

$$m\angle C = 47$$

$\angle A = 47^\circ$ since it is opposite $\angle C$.

Example 2. What is the value of x in the parallelogram below?



Solution:

Since \overline{LI} and \overline{FE} are opposite sides they are congruent.

$$\overline{LI} \cong \overline{FE}$$

$$LI = FE$$

$$3x - 1 = 26$$

$$3x - 1 + 1 = 26 + 1$$

$$3x = 27$$

$$x = 9$$

Opposite sides are congruent.

Substitution Property

Addition Property of Equality

Division Property of Equality)

(Divide both sides by 3

Simplify

Check:

If we substitute

$x = 9$ in \overline{LI} ,

$$LI = 3x - 1$$

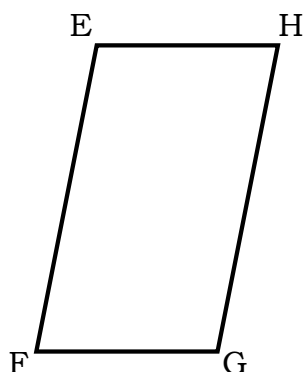
$$LI = 3(9) - 1$$

$$LI = 27 - 1$$

$$LI = 26$$

$\therefore \overline{LI} \cong \overline{FE}$ since they are opposite sides of $\square LIFE$.

Example 3. Find the measures of all the angles of parallelogram EFGH.



Given:

$$m\angle E = 6x - 2$$

$$m\angle F = 2x + 6$$

Solution:

Since $\angle E$ and $\angle F$ are consecutive angles they are supplementary.

$$m\angle E + m\angle F = 180$$

$$6x - 2 + 2x + 6 = 180$$

$$8x + 4 = 180$$

$$8x + 4 - 4 = 180 - 4$$

$$8x = 176$$

$$x = 22$$

Consecutive angles are supplementary.

Substitute $m\angle E$ and $m\angle F$ with their measures

Combine similar terms

Subtract 4 from both sides

Divide both sides by 8

Simplify

To find the measures of the angles, substitute $x = 22$ in $m\angle E$ and $m\angle F$.

a. $m\angle E = 6x - 2$

$$m\angle E = 6(22) - 2$$

$$m\angle E = 132 - 2$$

$$m\angle E = 130$$

c. $\angle G$ is 130° since it is opposite $\angle E$ and opposite angles are congruent.

b. $m\angle F = 2x + 6$

$$m\angle F = 2(22) + 6$$

$$m\angle F = 44 + 6$$

$$m\angle F = 50$$

d. $\angle H$ is 50° since it is opposite $\angle F$ and opposite angles are congruent.

Check:

If we substitute the measures of $\angle E$ and $\angle F$, we have,

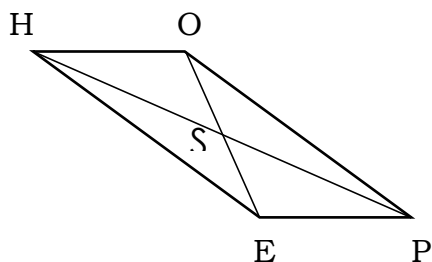
$$m\angle E + m\angle F = 180$$

$$130^\circ + 50^\circ = 180^\circ$$

$$180^\circ = 180^\circ$$

\therefore Since $\angle E$ and $\angle F$ are consecutive angles they are supplementary.

Example 4. What is the length of \overline{HS} and \overline{PS} in $\square HOPE$ below?



Given:

$\square HOPE$ with diagonals \overline{HP} and \overline{OE} intersecting at point S.

$$HS = x + 10$$

$$PS = 2x - 14$$

Solution:

Since the diagonals bisect each other, $\overline{HS} \cong \overline{PS}$.

$$HS = PS$$

$$x + 10 = 2x - 14$$

$$10 + 14 = 2x - x$$

$$24 = x$$

Diagonals bisect each other.

Substitution Property

Combine similar terms

Simplify

Check:

If we substitute $x = 22$ in \overline{HS} and \overline{PS} , we have

$$HS = x + 10$$

$$HS = 24 + 10$$

$$HS = 34$$

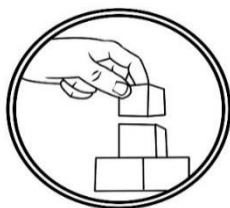
$$PS = 2x - 14$$

$$PS = 2(24) - 14$$

$$PS = 48 - 14$$

$$PS = 34$$

$\therefore \overline{HS} \cong \overline{PS}$ since diagonals bisect each other.



Explore

Activity 2: Challenge Time!

Directions: Use the properties of parallelogram to do what is asked in each item. Consider $\square WXYZ$ in Figure 2.1 below.

Given: Quadrilateral WXYZ is a Parallelogram. The diagram is not drawn to scale.

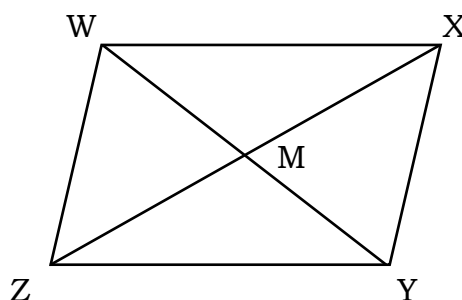


Figure 2.1

1. If $WX = 18$ and $WM = 10$, find: a. $YZ = \underline{\hspace{2cm}}$ b. $YM = \underline{\hspace{2cm}}$	2. If $WM = 9$ and $WZ = 5$, find: a. $WY = \underline{\hspace{2cm}}$ b. $XY = \underline{\hspace{2cm}}$
3. If $XZ = 14$ and $WX = 10$, find: a. $XM = \underline{\hspace{2cm}}$ b. $YZ = \underline{\hspace{2cm}}$	4. If $m\angle Z = 42$, find: a. $m\angle X = \underline{\hspace{2cm}}$ b. $m\angle W = \underline{\hspace{2cm}}$
5. If $WX = 2x - 1$ and $YZ = 5x - 10$, find: a. $x = \underline{\hspace{2cm}}$	6. If $XM = 6x + 1$ and $ZM = 4x + 31$, find: a. $x = \underline{\hspace{2cm}}$
7. If $m\angle WXY = 3x - 7$ and $m\angle XYZ = x + 15$, find: a. $x = \underline{\hspace{2cm}}$ b. $m\angle WXY = \underline{\hspace{2cm}}$ c. $m\angle XYZ = \underline{\hspace{2cm}}$	8. If $m\angle Z = 7x - 3$ and $m\angle X = 5x + 7$, find: a. $x = \underline{\hspace{2cm}}$ b. $m\angle W = \underline{\hspace{2cm}}$ c. $m\angle X = \underline{\hspace{2cm}}$



Deepen

Activity 3: X and Y in Parallelogram

Directions: Find the values of x and y that will make the given quadrilateral a parallelogram.

