SHS

AIRs - LM in

Statistics and Probability

Quarter 4: Week 1- Module 9

Illustrating Hypothesis Testing

Statistics and Probability
Grade 11 Quarter 4: Week 1 - Module 9: Illustrating Hypothesis Testing
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Target

We make every day decisions in our life. Some of these are very important while others are not. In decision-making, we follow certain processes like weighing alternatives, collecting evidence and making a decision. An appropriate interpretation is form after a decision is made. We follow these steps in testing hypothesis in Statistics. Hypothesis testing is a way for you to test the results of a survey or experiment to see if you have meaningful results.

After going through this lesson, you are expected to:

1. illustrates: (a) null hypothesis; (b) alternative hypothesis; (c) level of significance; (d) rejection region; and (e) types of errors in hypothesis testing. (**M11/12SP-IVa-1**)

Subtasks:
1. distinguish null hypothesis from alternative hypothesis.
2. determine whether a hypothesis test is non-directional or directional.
3. determine whether a directional test is left-tailed or right-tailed
4. locate critical values under the normal curve.
5. determine critical values for the hypothesis testing; and
6. understand the concept of Type I and Type II error.

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the decision-making process for evaluating claims about a population based on the characteristics of a sample purportedly coming from the population?
   A. Decision Testing B. Null Testing
   C. Alternative Testing D. Hypothesis Testing
2. What hypothesis states that there is no difference between a parameter and a specific value?
   A. Left-Tailed Hypothesis B. Null Hypothesis
   C. Alternative Hypothesis D. Right-Tailed Hypothesis

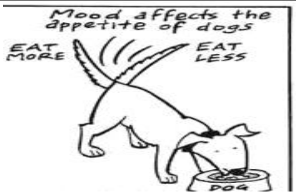
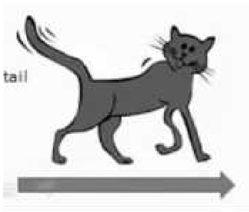
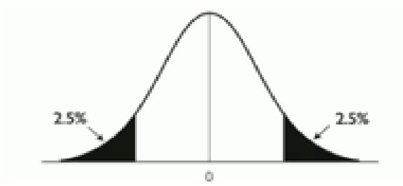
3. What hypothesis states that there is a difference between a parameter and a specific value?
   A. Left-Tailed Hypothesis
   B. Null Hypothesis
   C. Alternative Hypothesis
   D. Right-Tailed Hypothesis
4. Which of the following is the probability of the study rejecting the null hypothesis, given that the null hypothesis was assumed to be true?
   A. Level of Error
   B. Level of Values
   C. Level of Critical
   D. Level of Significance
5. Which of the following is the interval measured in the sampling distribution of the statistic under study that leads to rejection of the null hypothesis in a hypothesis test?
   A. Critical Region
   B. Rejection Region
   C. Acceptance Region
   D. Significance Region
6. What type of error is occurred in decision making when the true hypothesis is rejected?
   A. Type I error
   B. Type IV error
   C. Type II error
   D. Type III error
7. What type of error is occurred in decision making when the false hypothesis is accepted?
   A. Type I error
   B. Type IV error
   C. Type II error
   D. Type III error
8. The following mathematical symbols or inequalities utilize the alternative hypothesis. What inequality utilize right-tailed test?
   A. $<$ B. $>$ C. $=$ D. $\neq$
9. The following mathematical symbols or inequalities utilize the alternative hypothesis. What inequality utilize left-tailed test?
   A. $<$ B. $>$ C. $=$ D. $\neq$
10. The following mathematical symbols or inequalities utilize the alternative hypothesis. What inequality utilize two-tailed test?
    A. $<$ B. $>$ C. $=$ D. $\neq$
11. What level of confidence has the value of type I error of 0.05?
    A. 90% B. 95% C. 99% D. 100%
12. What is the level of confidence if the significance level is 0.01?
    A. 90% B. 95% C. 99% D. 100%
13. What level of confidence has the value of type I error of 0.10?
    A. 90% B. 95% C. 99% D. 100%
14. What is the rejection region of a two-tailed test with a 95% level of confidence?
    A. ± 1.28 B. ± 1.56 C. ± 1.65 D. ± 1.96
15. What is the rejection region of a left-tailed test with a 95% level of confidence?
    A. -1.96 B. -1.65 C. $+1.65$ D. $+1.96$

Jumpstart

For you to understand the lesson well, do the following activities. Have fun and good luck!

Activity 1: Identify Me!

Directions: Identify the following illustration whether it is non-directional or directional, two-tailed or one tailed by checking the appropriate box.

Illustrations	Directional	Non-Directional	One-Tailed	Two-Tailed
1. 				
2. 				
3. 				
4. 				
5. 				

Activity 2: What mistakes do people make?

Directions: Read the following statements and identify the phrase/s that makes the statement wrong.

1. Bryan thinks that he is a six-footer. His actual height is 156 cm.
2. On a moonlit night, a young man declares that there are two moons.
3. Mark says "I am virtuous!" In the next moment, he finds himself in jail.
4. Thousands of years ago, Ptolemy declared that the earth is flat.
5. On a beachfront, a signage reads, "No littering of plastic wrappers, empty bottles and cans." A few yards away, environmentalists are picking up the rubbish left behind by the picnic lovers.
6. The doctor says "Congratulations. You are pregnant" to the man with a stomach ache.
7. Angela says "I don't have any allergy in seafood". In the next moment, she finds herself itching all over her body.
8. Today is not my friend's birthday but I will wish her happy birthday.
9. Back in the day, scientist believes that Earth is at the center of the universe.
10. Hannah said to her friend "People's intelligence is measured through their proficiency in English".

Discover

Hypothesis testing is a decision-making process for evaluating claims about a population based on the characteristics of a sample purportedly coming from the population. We get a random sample from the population, collect data from the sample, and use this data to make a decision as to whether the hypothesis is acceptable or not.

Two types of hypothesis

Null hypothesis, denoted by H_0, is a statement that there is no difference between two parameters It can be written as $H_0\colon \mu_1 = \mu_2$.

Alternative hypothesis, denoted by H_1 or H_a, is a statement that there is a difference two between a parameters. It can be written as $H_1\colon \mu_1 \neq \mu_2$, $H_1\colon \mu_1 < \mu_2$, or $H_1\colon \mu_1 > \mu_2$

Let's formulate the null and alternative hypothesis for each of the following examples.
Example 1: The average TV viewing time of all six-year old children is 4 hours daily.

Answer:

In words, the hypotheses are:

H_0: The average TV viewing time of six-year old children is 4 hours.

H_1 or H_a: The average TV viewing time of six-year old children is less than 4 hours.

In symbols, we write:

$H_0: \mu = 4$

H_1 or H_a: $\mu < 4$

Example 2: A college librarian claims that 25 story books on the average are borrowed daily.

Answer:

In words, the hypotheses are:

H_0: The average story books borrowed in the library is 25.

H_1 or H_a: The average story books borrowed in the library is more than 25.

In symbols, we write:

$H_0: \mu = 25$

H_1 or H_a: $\mu > 25$

When the alternative hypothesis utilizes the < or the > symbol, the test is said to be ***directional.*** A directional test may either be left-tailed or right-tailed. In problems that involve hypothesis testing, there are words like greater, efficient, improves, effective, increases and so on that suggest a ***right-tailed*** direction in the formulation of the alternative hypothesis. Words like decrease, less than, smaller, and so on suggest a ***left-tailed*** direction.

Example 3: The inventor of a new kind of light bulb claims that all such bulbs last as long as 3000 hours.

Answer:

In words, the hypotheses are:

H_0: The new kind of light bulb will last as long as 3000 hours.

H_1 or H_a: The new kind of light bulb will not last as long as 3000 hours.

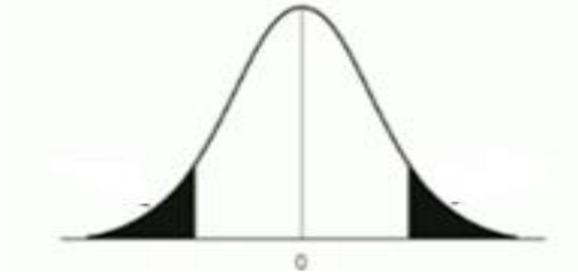
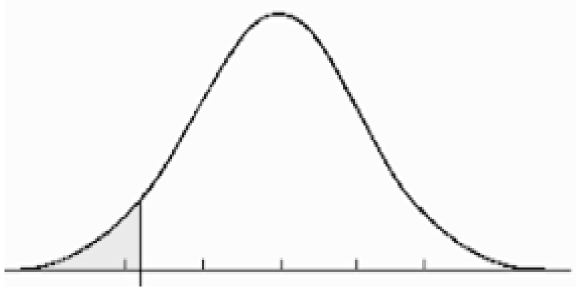
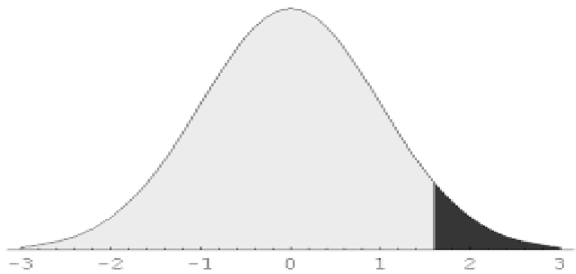
In symbols, we write:

$H_0: \mu = 3000$

H_1 or H_a: $\mu \neq 3000$

When the alternative hypothesis utilizes the $\neq$ symbol, the test is said to be ***non-directional.*** A non-directional test is also called a ***two-tailed test.***

These are the graphical representations of the two-tailed test and the one-tailed test.

Non-directional (two-tailed) The probability is found on both tails of the distribution.	 *Figure 1*
Directional (one-tailed, left-tailed) The probability is found at the left tail of the distribution.	 *Figure 2*
Directional (one-tailed, right-tailed) The probability is found at the right tail of the distribution.	 *Figure 3*

Let's consider the following examples. Determine whether the given situation is a directional or non-directional test. If it is a non-directional, identify whether it is left-tailed or right tailed test.

Example 4: The owner of a factory that sells a particular bottled water claims that the average capacity of a bottle of their product is 200 ml. Is the claim true?

Solution:

In words, the hypotheses are:

H_0: The bottled water contains 200ml per bottle.

H_1 or H_a: The bottled water does not contain 250 ml per bottle.

In symbols, the hypotheses are:

$H_0: \mu = 50$

H_1 or Ha: $\mu \neq 50$

Since the alternative hypothesis utilizes $\neq$, then we can say that the test is non-directional and two-tailed.

Example 5. A rice farmer believes that using organic fertilizer on his plants will yield greater income. His average income from the past was Php200,000 per year. State the hypothesis in symbols.

Solution:

In words, the hypotheses are:

H_0: The rice farmer yields an income of Php200,000.

H_1 or H_a: The rice farmer yields an income greater than Php200,000.

In symbols, the hypotheses are:

H_0: μ=200,000

H_1 or H_a: μ >200,000

The phrase 'greater income' is a clue as to the direction of the investigation. In addition, the alternative hypothesis utilizes > symbol, therefore we can say that the test is directional and right-tailed.

In hypothesis testing, we make decisions about the null hypothesis. Of course, there are risks when we make decisions. There are four possible outcomes when conducting hypothesis testing. The following table shows these four outcomes.

Table 1. Four Possible Outcomes in Decision-Making

		Decisions about H_0	
		Reject	**Do not Reject H_0 (or Accept H_0)**
Reality	**H_0 is true**	Type I error	Correct Decision
	H_0 is false	Correct Decision	Type II error

The table shows that if null hypothesis is true and accepted, or if it is false and rejected, the decision is correct. If the null hypothesis is true and rejected, the decision is incorrect and this is *Type I error*. If the null hypothesis is false and accepted, the decision is incorrect and this is *Type II error*.

Example 6: Understanding Errors

A. Maria insists that she is 30 years old when, in fact, she is 35 years old. What error is Maria committing?

Solution:

Maria is rejecting the truth. She is committing a Type I error.

B. A man plans to go hunting the Philippine monkey-eating eagle believing that it is a proof of his mettle. What type of error is this?

Solution:

Hunting the Philippine monkey-eating eagle is prohibited. Thus, it is not a good sport. It is a Type II error.

In decisions that we make, we form conclusions and these conclusions are the bases of our actions. The probability of committing a Type I error is denoted by the Greek letter α(alpha) while the probability of committing Type II error is denoted by β(beta).

The following table shows the probability with which decisions occur.

Table 2. Types of Errors

Error in Decision	Type	Probability	Correct Decision	Type	Probability
Reject a true H_0	I	α	Accept a true H_0	A	$1-\alpha$
Accept a false H_0	II	β	Reject a false H_0	B	$1-\beta$

We can control the errors by assigning small probability values to each of them. The most frequently used probability values for α *and* β are 0.10,0.05 and 0.01. The probability assigned to each depends on its seriousness. The symbols α *and* β are each probabilities of error, each under separate conditions, and they cannot be combined. As can be seen in table 2, 1-α is the probability of a correct decision when the null hypothesis is true, and $1-\beta$ is the probability of a correct decision when the null hypothesis is false.

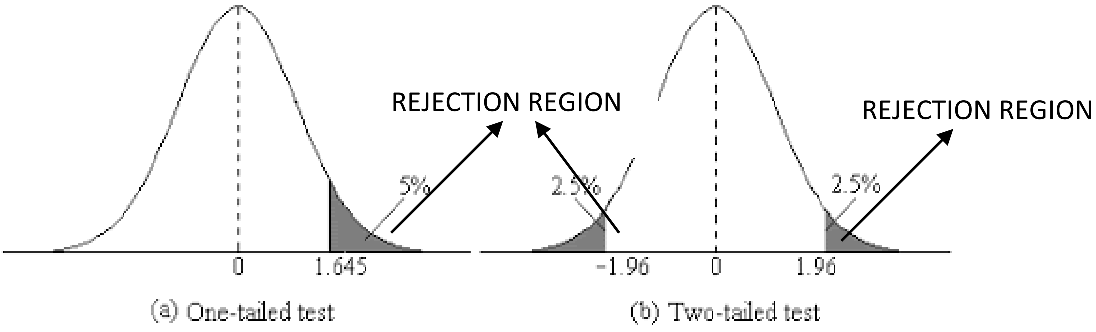
Table 3. Level of Significance

Level of Significance	Confidence Level
1% or 0.01	1-α=1-0.01=0.99 or 99%
5% or 0.05	1-α=1-0.05=0.95 or 95%
10% or 0.1	1-α=1-0.1=0.9 or 90%

Level of Significance (denoted as alpha or α) is a measure of the strength of the evidence that must be present in a sample before rejecting the null hypothesis and conclude that the effect is statistically significant. It is the probability of rejecting the null hypothesis when it is true. The commonly used level of significance are 1%, 5% and 10%. In table 3, it shows the confidence level for every level of significance that are commonly used in researches.

Under the normal curve, the ***rejection region*** refers to the region where the value of the test statistic lies for which we will *reject the null hypothesis*. This region is also called ***critical region.*** So, if your computed statistic is found in the rejection region, then you reject H_0. If it is found outside the rejection region you accept H_0.

Graphically, we can show the decision errors under normal curve.



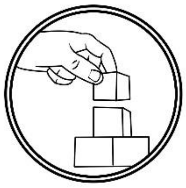
In figure 4, it shows the line that separates the rejection region from the non-rejection region (1- α). This line passes through the confidence coefficients, which also called **critical values**. The critical values can be obtained from the critical values table of the test statistic. For example, for a 95% confidence level if the test statistic is a z and it is a non-directional test, it can be determined by having this equation $\frac{0.95}{2} = 0.4750$ (expressed up to four decimal places so that we can identify an area in the normal curve table as close as possible to this value).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	**0.4515**	0.4525
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	**0.4608**	0.4616
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	**0.4686**	0.4693
1.9	**0.4713**	**0.4719**	**0.4726**	**0.4732**	**0.4738**	**0.4744**	**0.4750**	**0.4756**
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	**0.4803**	0.4808

In the z-table, the area 0.4750 corresponds to z = 1.96 so the critical values for a non-directional test or two-tailed are -1.96 and +1.96. This can be written as $z_{a/2} = \pm\ 1.96$. When the confidence level is 95% and the test statistic is a z and it is a directional test or one-tailed. The critical values can be determined by changing 95% to 0.9500 (expressed up to four decimal places so that we can identify an area in the normal curve table as close as possible to this value).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633

In the z-table, there are two areas close to this value: 0.9495 that corresponds to z = 1.64 and 0.9505 that corresponds to z =1.65. Then we get the average of the z-values. This results to 1.645. In practice, we use the z-values of -1.65 for left-tailed and +1.65 for right tailed



Explore

Activity 1: Illustrating Hypothesis

Directions: Explicate a null hypothesis and its alternative hypothesis in (a) words and in (b) symbols for each of the following. Tell whether the test is directional and non-directional.

1. A librarian of a school claims that all their senior high school students read an average of 10 books a month. A random sample of senior high students read an average 12 books. The confidence statement is 95%.
2. According to a factory employer, the mean working time of workers in the factory is 6. A researcher interviewed 50% of the employees and found out that their mean working time is 8 hours. The α level is 0.05.
3. A random sample of 200 students got a mean score of 62 in a knowledge test in mathematics. In the standardization of the test, $\mu = 50$.

Activity 2: Type of Error

Directions: Determine what type of error is committed in every statement.

1. Punishing a person who is truly innocent and putting them wrongly in jail.
2. Criminals gets away with crimes and perhaps thinks he always can.
3. Bryan thinks that he is a six-footer. His actual height is 156 cm.
4. Angela says "I don't have any allergy in seafood". In the next moment, she finds herself itching all over her body.
5. Hannah said to her friend "People's intelligent are measured through their proficiency in English".

Activity 3: Fill me up

Directions: Complete the summary table of critical values.

Confidence Level	**Two-tailed**	**One-tailed**	
		Left-tailed	Right-tailed
$90\%(1-\alpha)$	$z\alpha/_2$=__________	z=__________	z=__________
$95\%(1-\alpha)$	$z\alpha/_2$=__________	z=__________	z=__________
$99\%(1-\alpha)$	$z\alpha/_2$=__________	z=__________	z=__________

Deepen

At these points, you are going to use internet or any reference to collect data worth investigating from the government agencies (e.g., mean age of high school students' dropouts, mean salaries of a specific employee), Formulate the null hypothesis and the alternative hypothesis for your data at hand. The scoring rubric will be used in assessing your performance.

What you need:

Bond paper, printed copy or photocopy of the data collected

What you have to do:

1. Formulate the null and alternative hypothesis of the data you collected.
2. Identify whether the test to be administer in the data you have is directional or non-directional.
3. Tell the possible type I and type II error if a mistake in decision will be made.

Rubrics in Scoring

Indicators	**5**	**4**	**3**	**2**	**1**
Content	Data collected is accurate and information's are complete.	Data collected is accurate but some information's are missing.	Data collected is accurate but only few information's are found.	Data collected is questionable and information's are difficult to understand.	Data collected is inaccurate and information's are difficult to understand
Process and Strategies	Demonstrate an excellent application of skills	Demonstrate a clear application of skills	Demonstrate a general application of skills	Demonstrate a limited application of skills	Demonstrate a little application of skills
Mechanics	Formulated hypothesis is in author's words.	Formulated hypothesis is in author's words with few grammar errors	Formulated hypothesis is in author's words with some grammar errors	Most of the formulated hypothesis is in author's words with some grammar errors	Text is copied.
Score					/15

Gauge

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What statement states that there is no difference between two parameters?
   A. Alternate Hypothesis B. Null Hypothesis
   C. Alternative Hypothesis D. Void Hypothesis
2. What statement states that there is a difference between two parameters?
   A. Alternate Hypothesis B. Null Hypothesis
   C. Alternative Hypothesis D. Void Hypothesis
3. Which of the following is the probability of the study rejecting the null hypothesis, given that the null hypothesis was assumed to be true?
   A. Level of Error B. Level of Values
   C. Level of Critical D. Level of Significance
4. Which of the following is the interval measured in the sampling distribution of the statistic under study that leads to rejection of the null hypothesis in a hypothesis test?
   A. Critical Region B. Rejection Region
   C. Acceptance Region D. Significance Region
5. What type of error is occurred in decision making when the true hypothesis is rejected?
   A. Type I B. Type II C. Type A D. Type B
6. What type of error is occurred in decision making when the false hypothesis is accepted?
   A. Type I B. Type II C. Type A D. Type B
7. The net weight of a packet of a snack is 130 g. A sample of 80 packets yielded a sample mean weight of 112 g with a standard deviation of 15 g. What is the null hypothesis of the problem?
   A. $H_{0:} \mu = 15$ B. $H_{0:} \mu = 112$ C. $H_{0:} \mu = 80$ D. $H_{0:} \mu = 130$
8. The average height of senior high female student is 160 cm. The mean height of a sample of 100 female students is 163 cm with a standard deviation of 6 cm. What is the alternative hypothesis of the problem?
   A. $H_{1:} \mu \neq 160$ B. $H_{1:} \mu < 160$ C. $H_{1:} \mu = 160$ D. $H_{1:} \mu > 160$
9. In a graduate college, the average length of registration time during a semester is 120 minutes with a standard deviation of 25 minutes. With the introduction of a new registration procedure, a random sample of 50 student's system took a lower average of 80 minutes with a standard deviation of 12 minutes. What is the alternative hypothesis of the problem?
   A. $H_{1:} \mu \neq 120$ B. $H_{1:} \mu < 120$ C. $H_{1:} \mu = 120$ D. $H_{1:} \mu > 120$

10. Given the problem "Bryan administered a mathematics achievement test to a random sample of 50 graduating pupils. In this sample, $\bar{x} = 90$ *and* $s = 10$. The population parameters are $\mu = 83$ *and* $\sigma = 15$. Is the performance of the sample above the average?" What is the alternative hypothesis of the problem?
    A. $H_{1:} \mu \neq 83$ B. $H_{1:} \mu < 83$ C. $H_{1:} \mu = 83$ D. $H_{1:} \mu > 83$
11. What is the rejection region of a two-tailed test with a 99% level of confidence?
    A. ± 1.65 B. ± 1.96 C. ± 2.33 D. ± 2.58
12. What is the rejection region of a left-tailed test with a 99% level of confidence?
    A. -2.58 B. -2.33 C. $+2.33$ D. $+2.58$
13. What is the rejection region of a right-tailed test with a 99% level of confidence?
    A. -2.58 B. -2.33 C. $+2.33$ D. $+2.58$
14. What is the rejection region of a two-tailed test with a 95% level of confidence?
    A. ± 1.65 B. ± 1.96 C. ± 2.33 D. ± 2.58
15. What is the rejection region of a right-tailed test with a 99% level of confidence?
    A. -1.96 B. $-1/65$ C. $+1.65$ D. $+1.96$

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z-table for one-tailed

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

z-table for two-tailed

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999