8

Mathematics

Quarter 4 – Week 4; Module 4: Parallelism and Perpendicularity

AIRs - LM

First Edition, 2021

Development Team of the Module

Writer: Mary Ann M. Diaz
Editor: SDO La Union, Learning Resource Quality Assurance Team
Layout Artist: Ernesto F. Ramos Jr., *P II*

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent

Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent

German E. Flora, PhD, *CID Chief*

Virgilio C. Boado, PhD, *EPS in Charge of LRMS*

Erlinda M. Dela Peña, EdD, *EPS in Charge of Mathematics*

Michael Jason D. Morales, *PDO II*

Claire P. Toluyen, *Librarian II*

Target

We were created in different shapes and angles . . . but we are still congruent one way or another. No matter which sides, even if there are terms undefined; may we always remain parallel with what God wants us to become and by the time our paths perpendicularly crossed, we could say with confidence that He continuously showers us with unending love and providence.

After going through this module, you are expected to:
- determine the conditions under which lines and segments are parallel and perpendicular. **(M8GE-IVe-1)**

Subtasks:
1. give the differences among parallel lines, perpendicular lines, and intersecting lines.
2. state the converses of the theorems on parallel lines cut by a transversal.

Before you start learning this module, check how much you already know about the topic by answering the pre – test:

Pre-Test:

Direction: Choose the letter that corresponds to the best answer. Write your answers on a separate sheet of paper.

*For items 1-3; refer to the characteristics below.
   i.   Lines are coplanar
   ii. Lines are non-coplanar
   iii. Lines do not intersect
   iv. Lines intersect and form right angles

1. Which characteristic(s) is/are true for parallel lines?
   A. i only   B. i and iii only
   C. iii only   D. ii and iii only
2. Which characteristic(s) is /are true for perpendicular lines?
   A. ii only   B. i and ii only
   C. i and iii only   D. i and iv only
3. Which characteristic(s) is/are true for skew lines?
   A. i and iii only   B. i and iv only
   C. ii and iii only   D. ii and iv only

* For items 4-11; refer to the given figure:
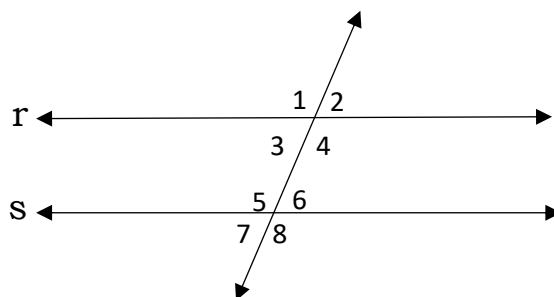
4. What pair of angles are corresponding angles?
   A. ∠1 and ∠5   B. ∠2 and ∠3
   C. ∠4 and ∠7   D. ∠6 and ∠8
5. Which pair of angles are alternate exterior angles?
   A. ∠1 and ∠4   B. ∠2 and ∠7
   C. ∠3 and ∠6   D. ∠4 and ∠5
6. Give the pair of angles which are on the same side interior angles.
   A. ∠1 and ∠2   B. ∠1 and ∠7
   C. ∠3 and ∠8   D. ∠4 and ∠6
7. If r∥s, which pair of angles are congruent?
   A. ∠1 and ∠5   B. ∠2 and ∠4
   C. ∠6 and ∠8   D. ∠7 and ∠5
8. What angles are supplementary when r∥s?
   A. ∠1 and ∠7   B. ∠2 and ∠3
   C. ∠3 and ∠5   D. ∠4 and ∠8
9. Which statement is true if r∥s?
   A. ∠2 ≅ ∠4   B. ∠7 ≅ ∠8
   C. m∠4 + m∠6 =180°   D. m∠3 + m∠7=180°
10. Find x if r∥s and m<2=4x+5 and m<6=5x-15?
   A. 20°   B. 35°   C. 40°   D. 80°

11. If m∠3=70°, what is the m∠5?
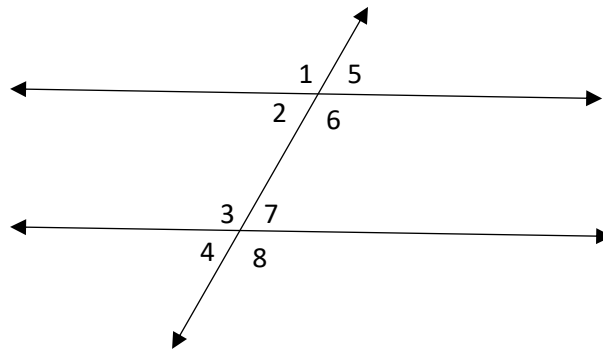   A. 20°   B. 40°   C. 70°   D. 110°
  *For items 12-15; use the given figure to find the measurement of the given angles
     and/or the value of x.

12. The m∠4= 120°, what is the m∠3?
     A. 30°   B. 60°   C. 90°   D. 120°
13. If m∠2 = 60°, what is the m∠7?
     A. 30°   B. 60°   C. 70°   D. 120°
14. Find x if m∠1 = 3x+15 and m∠5 = 2x-20.
     A. 35°   B. 37   C. 75°   D. 105°
15. Solve for the measurement of ∠1?
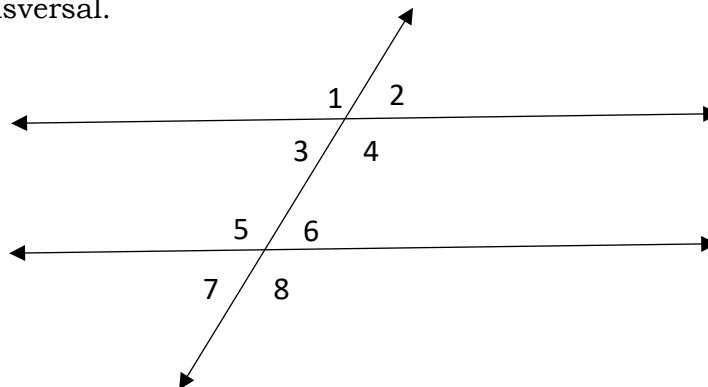   A. 95   B. 110°   C. 126°   D. 135°

Jumpstart

Activity: Name It... A Recall!

Directions: Analyzed the figure and answer the questions that follow as
            accurately as you can. The figure below shows m ‖ n with t
            as transversal.

   I. Identify the following:

      A. Corresponding angles   B. Alternate interior angles
         1. ∠1 and ∠ _____   1. ∠1 and ∠ _____
         2. ∠2 and ∠ _____   2. ∠2 and ∠ _____
         3. ∠3 and ∠ _____
         4. ∠4 and ∠ _____   D. Same-side interior angles
                                                  1. ∠3 and ∠ _____
      C. Alternate exterior angles   2. ∠4 and ∠ _____

1. ∠1 and ∠ ＿＿＿＿

2. ∠2 and ∠ ＿＿＿

E. Same-side exterior angles
   1. ∠1 and ∠ ＿＿＿＿
   2. ∠2 and ∠ ＿＿＿＿

II. Find the measurements of the other angles:
  1. m∠1 = 100°
  2. m∠2 = ＿＿＿
  3. m∠3 = ＿＿＿
  4. m∠4 = ＿＿＿

  5. m∠5 = ＿＿＿＿
  6. m∠6 = ＿＿＿＿
  7. m∠7 = ＿＿＿＿
  8. m∠8 = ＿＿＿＿

III. Based from your observations on the measurements of the angles formed by parallel lines cut by a transversal, complete the following phrases.
 1. Corresponding angles are ＿＿＿＿＿＿.
  2. Alternate interior angles are＿＿＿＿＿＿.
  3. Alternate exterior angles are＿＿＿＿＿＿.
  4. Exterior angles of the same side of transversal are＿＿＿＿＿＿.
  5. Interior angles of the same side of transversal are ＿＿＿＿＿＿.

Discover

We see parallel and perpendicular lines everywhere . . . railroads, pedestrian lane, edges and intersection of doors or windows, fences, etc... these things are just some representations of parallel and perpendicular lines

Activity 1: Road Map
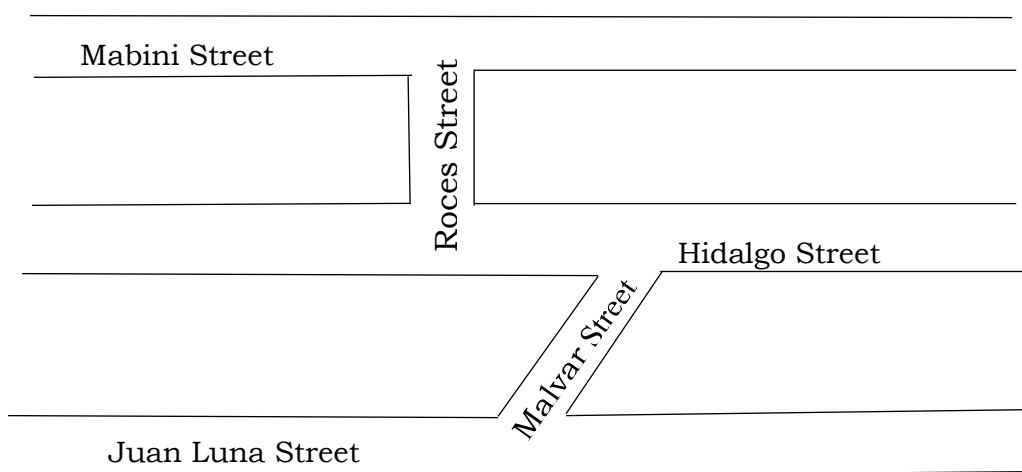
Directions: Base your answers from the given road map above.

Questions:
  1. Which roads intersect?
  2. Which roads are parallel?
  3. Which roads are perpendicular?

Answers:
1. Roces Street intersects Mabini Street
   Roces Street intersects Hidalgo Street
   Malvar Street intersects Hidalgo Street
   Malvar Street intersects Juan Luna Street

2. Mabini Street is parallel to Hidalgo Street
   Mabini Street is parallel to Juan Luna Street

3. Roces Street is perpendicular to Mabini Street
   Roces Street is perpendicular to Hidalgo Street

From the activity the following terms could be differentiated:

Parallel lines are coplanar lines that do not intersect.
Intersecting lines are coplanar lines that have a point in common.
Perpendicular lines are lines that intersect and form right angles.

Activity 2: Checking Parallelism
1. Draw two intersecting lines q and r. Label an angle at their intersection as ∠1.
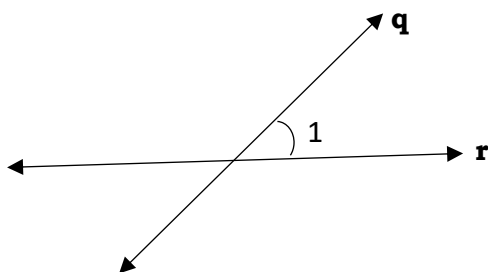
2. Select a point P on line q. Construct an alternate exterior angle at P congruent to ∠1. Extend the side of the new angle and label it lines. Name the other angle as ∠2.
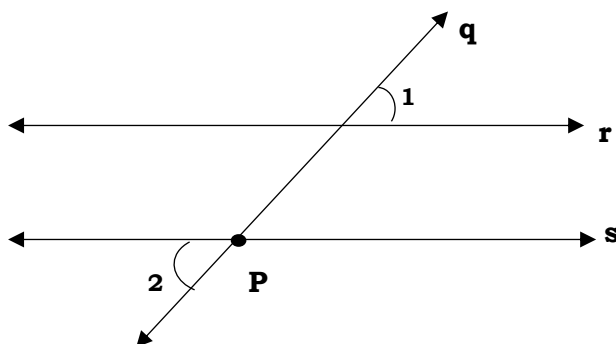
*Lines r and **s** form parallel lines.*

The conclusion in the activity summarizes the converse of the corresponding angles postulate.

Converse of the Corresponding Angles Postulate
If two lines are cut by a transversal and a pair of corresponding angles are

congruent, then the lines are parallel.

Now, let us discuss the theorems that prove that two lines are parallel .These theorems are the converses of the theorems presented on the previous lessons.

Converse of the Alternate Interior Angles Theorem
If two lines are cut by a transversal and a pair of alternate interior angles are congruent , then the lines are parallel.

Converse of the Alternate Exterior Angles Theorem
If two lines are cut by a transversal and a pair of alternate exterior angles are congruent , then the lines are parallel.

Converse of the Same-side Interior Angles Theorem
If two lines are cut by a transversal and the same-side interior angles are supplementary, then the lines are parallel.

Examples:
1. Decide which lines are parallel given the figure and the following conditions.
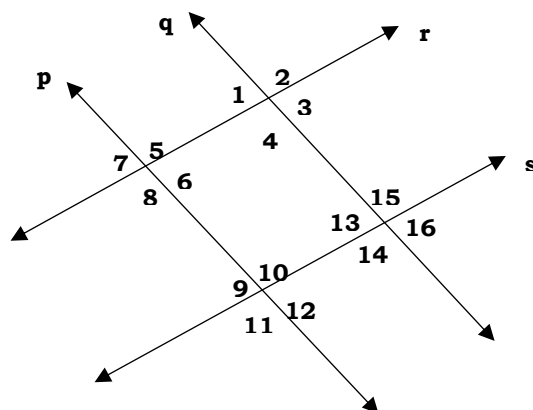
a. $\angle 1 \cong \angle 13$ b. $\angle 6 \cong \angle 1$ c. $\angle 9 \cong \angle 16$

d. $\angle 9 \cong \angle 12$ e. $m\angle 3 + m\angle 15 = 180$ f. $m\angle 5 + m\angle 7 = 180$

Solutions:

a. Lines r and s
    $\angle$ and $\angle 13$ are corresponding angles with **q** being the transversal. Line **q** intersects line **r** and **s**. using the converse of the corresponding angles postulate since $\angle 1 \cong \angle 13$, lines **r** and **s** are parallel.

b. Lines **p** and **q**
    Since $\angle 6 \cong \angle 1$ and $\angle 6$ and $\angle 1$ are alternate interior angles with line **r** being the transversal that intersects lines **p** and **q**, the converse of the alternate interior angles theorem will justify that lines **p** and **q** are parallel.

c. Lines **p** and **q**

With line **s** being a transversal that cuts lines **p** and **q**, ∠9 and ∠16 are alternate exterior angles by definition. Because ∠9 ≅ ∠16 are alternate exterior angles theorem, lines **p** and **q** are parallel.

d. None

∠9 and ∠12 are vertical angles, and congruent verical angles do not guarantee parallelism.

e. Lines **r** and **s**

With line **q** being a transversal intersecting lines **r** and **s**, ∠3 and ∠15 are same-side interior angles. Because ∠3 and ∠15 are supplementary, which satisfies the converse of the same-side interior angles theorem, lines **r** and **s** are parallel.

f. None

Linear pairs do not guarantee parallelism.

p   **q**   **r**

Two Other Theorems Involving Parallel Lines.

1. In a plane, two coplanar lines perpendicular to the same line are parallel.
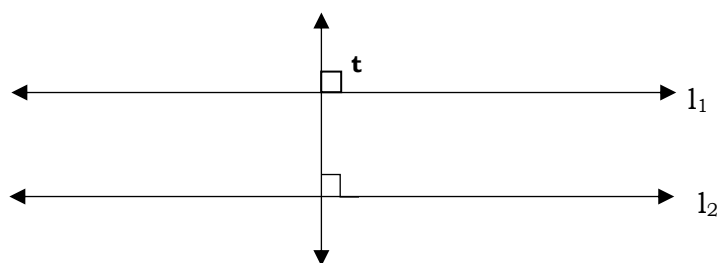
If $l_1 \perp t$ and $l_2 \perp t$ then $l_1 \parallel l_2$

2. Two lines parallel to a third line are parallel to each other.
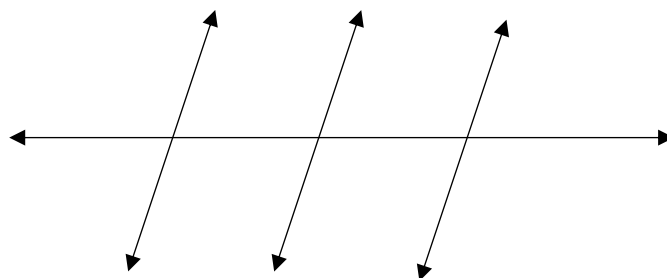
If $p \parallel r$ and $q \parallel r$ then $p \parallel r$

Activity 3: Am I Perpendicular? Let's Find Out!

Given any two distinct lines on a plane, the lines either intersect or parallel. If two lines intersect , then they form four angles. Consider the figures below to answer the questions that follow.

| Figure 1 | Figure 2 | Figure 3 | Figure 4 |
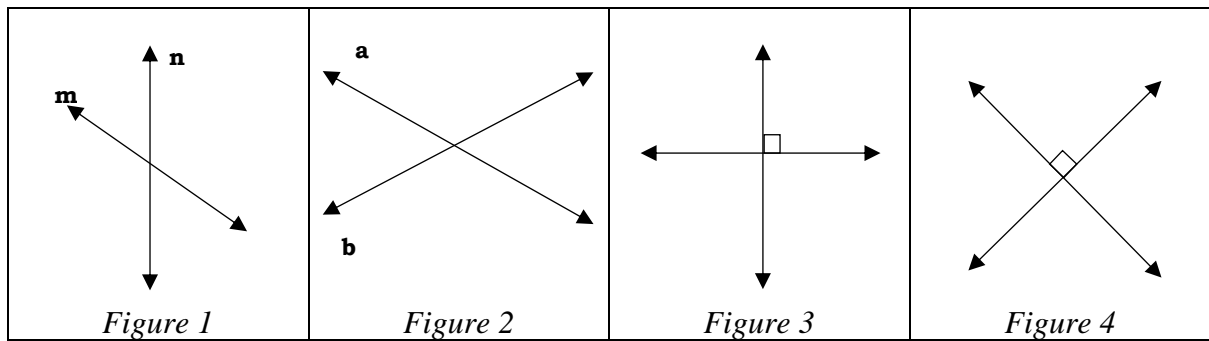
*The small square drawn on inersecting lines indicates a right angle. The symbol ⊥ indicates perpendicularity.

Questions:
1. What is common in the four figures given above? _____
2. What makes figure 3 and 4 different from the first two figures? _____
3. Which among the four figures show perpendicularity? _____
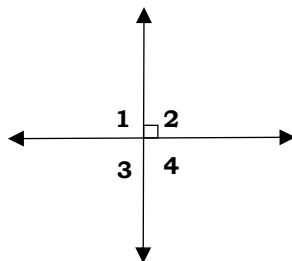4. When are lines said to be perpendicular to each other? _____

To prove that two lines are perpendicular, the following theorems must be true.

1. If two lines are perpendicular to each other , then they form four right angles.
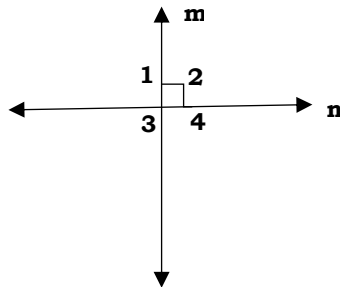
If m ⊥ n, then ∠1, ∠2, ∠3, and ∠4 are right angles.

2. If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.

If ∠1 and ∠2 form a linear pair, and ∠1 ≅ ∠2 then $l_1 ⊥ l_2$

3. If two angles are adjacent and complementary, the non-common sides are perpendicular.
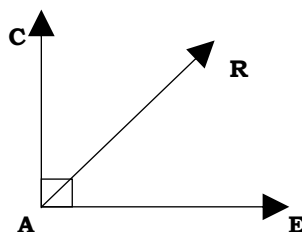
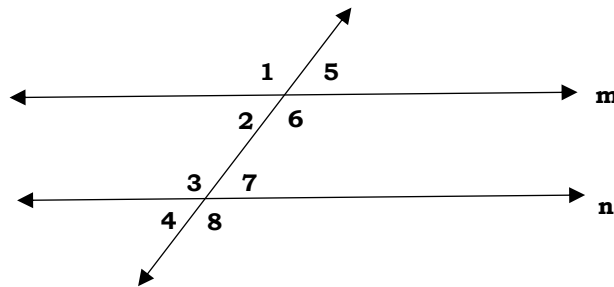If ∠CAR and ∠EAR are complementary and adjacent, then $\overline{AC} \perp \overline{AE}$

Explore

Here are some enrichment activities for yo to answer in order to master and strengthen the basic concepts you have learned from this module.
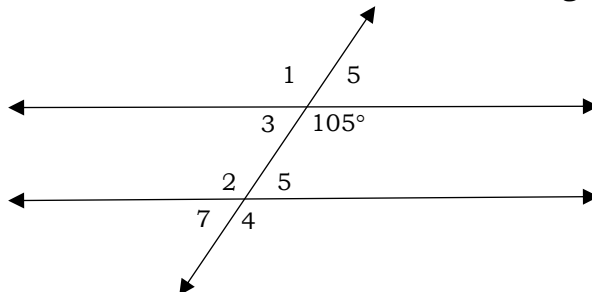
Activity1: Identify Me.

Given lines m and n are parallel, identify whether the following pairs of angle are congruent or supplementary.

1. ∠1 and ∠8 _____	6. ∠3 and ∠6 _____
2. ∠1 and ∠6 _____	7. ∠4 and ∠7 _____
3. ∠2 and ∠3 _____	8. ∠4 and ∠8 _____
4. ∠2 and ∠5 _____	9. ∠5 and ∠6 _____
5. ∠3 and ∠1 _____	10.∠5 and ∠7 _____

Activity 2: Find My Measurement:

Determine the measures of the numbered angles in the figure; given m∥n.

|	
1. ∠1 = _____	6. ∠6 = _____
2. ∠2 = _____	7. ∠7 = _____
3. ∠3 = _____	8. m∠2 + m∠3 = _____
4. ∠4 = _____	9. m∠1 + m∠6 = _____
5. ∠5 = _____	10.m∠2 + m∠5 – m∠4 = _____

Deepen

Activity 1: Agree or Disagree

Now see how well you understood the lesson presented. Write **A** if you agree with the statement and write **D** if you disagree.

_____1. Lines on the same plane that do not intersect are parallel lines.
_____2. Transversal lines are lines that intersect two or more lines.
_____3. Perpendicular lines are intersecting lines.
_____4. If two lines are perpendicular to the same line, then the two lines are parallel
_____5. If two lines are parallel to the same line, then the two lines are parallel.
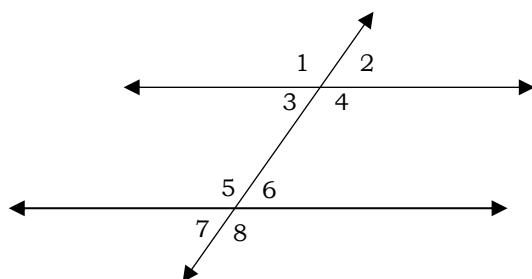   *For items 6-10 refer to the given figure:

Given: $\overline{MI} \cong \overline{IL}$
       $\overline{SE} \cong \overline{EL}$
       m $\angle SEI = 90°$
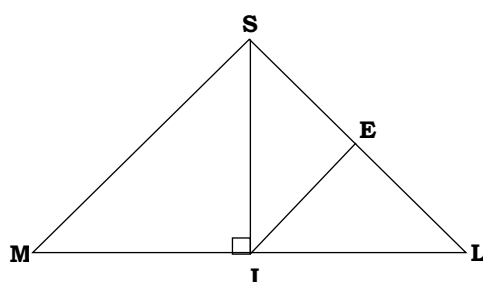
_____6. $\overline{ML} \perp \overline{IS}$
_____7. $\angle MSI$ and $\angle ISL$ are complementary angles.
_____8. $\overline{IE}$ is a perpendicular bisector of $\overline{SL}$.
_____9. $\angle MIS$ and $\angle SIL$ form a linear pair.
_____10. m$\angle MIS = 90°$?

Activity 2: Find My X

Find the value of x for which r‖s and solve for the measurement of the given angle.

1. m$\angle 1$ = 3x + 5 and m$\angle 5$ = 4x – 20
    x = _____
    m$\angle 1$ = _____
    m$\angle 5$ = _____

2. m ∠2 = 5x – 24 and m ∠7 = 3x + 16

   x = _____

   m ∠2 = _____

   m ∠5 = _____

3. m ∠4 = 3x – 34 and m ∠6 = x + 22

   x = _____

   m ∠4 = _____

   m ∠6 = _____

4. m ∠3 = 2x +1 and m ∠6 = 3x – 29

   x = _____

   m ∠3 = _____

   m ∠6 = _____

5. m ∠3 = 5x -1 and m ∠5 = 13x + 1

   x = _____

   m∠3 = _____

   m∠5 = _____
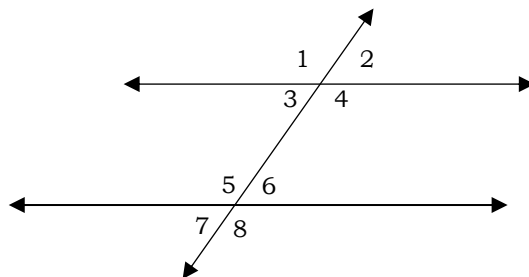
Gauge

Post-Test:

Directions: Choose the letter of the correct answer. Answers must be written on a separate piece of paper.

1. Lines that are coplanar and do not intersect.
   A. Intersecting lines   B. Parallel lines
   C. Perpendicular lines   D. Skew lines

2. Lines that are coplanar and have a point in common.
   A. Intersecting lines   B. Parallel lines
   C. Perpendicular lines   D. Skew lines

3. Lines that intersect and form right angles.
   A. Intersecting lines   B. Parallel lines
   C. Perpendicular lines   D. Skew lines

*For items 4-6 refer to the figure given thatv l₁ ∥ l₂ and t is a transversal;
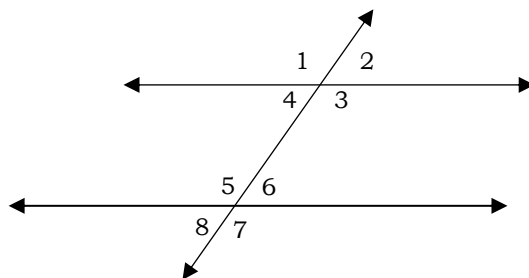
4. Which of the folowing are corresponding angles.
   A. ∠1 and ∠5, ∠2 and ∠6   B. ∠1 and ∠7, ∠2 and ∠8
   C. ∠4 and ∠5, ∠3 and ∠6   D. ∠4 and ∠6, ∠3 and ∠5

5. Which of the folowing are alternate interior angles.
      A. ∠1 and ∠2, ∠5 and ∠6   B. ∠1 and ∠7, ∠2 and ∠8
      B. ∠3 and ∠5, ∠4 and ∠6   D. ∠4 and ∠5, ∠3 and ∠6
6. Which of the folowing are alternate exterior angles.
      A. ∠1 and ∠2, ∠5 and ∠6   B. ∠1 and ∠7, ∠2 and ∠8
      C. ∠3 and ∠5, ∠4 and ∠6   D. ∠4 and ∠5, ∠3 and ∠6
7. All of thefollowing are characteristics of a parallel lines except:
      A. Lines are coplanar   B. Lines do not intersect
      C. Lines never meet   D. Lines have point in common
8. Lines m and n are parallel cut by a transversal t, which is also perpendicular
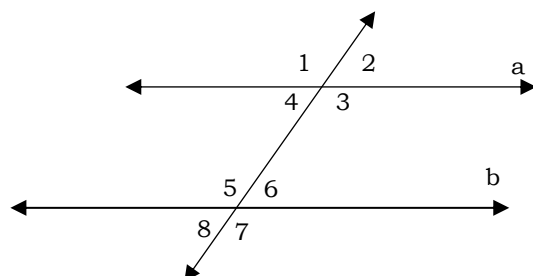   to m and n. Which statement is not correct?
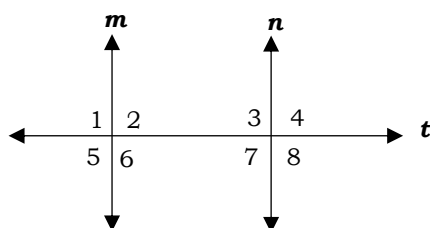
      A. ∠1 and ∠6 are congruent   B. ∠2 and ∠3 are complementary
      C. ∠3 and ∠5 are congruent angles   D. ∠1 and ∠4 form a linear pair
9. In the figure below, which of the following does not guarantee that m∥n?
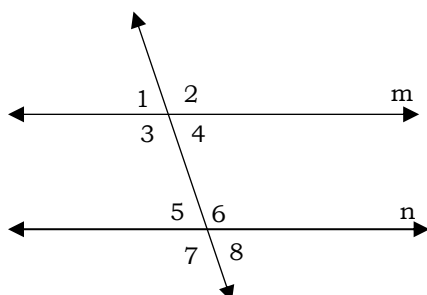
  A. ∠1 ≅ ∠7   B. ∠2 ≅ ∠6
 C. ∠4 ≅ ∠5   D. ∠5 ≅ ∠8
*For items 10-12: Parallel lines a and b are cut by a transversal t.

10. If m∠1= 85°, what is the measureof ∠5?
  A. 5°   B. 85°   C. 95°   D. 275°
11. If m∠1= 85°, what is the measure of ∠6?
  A. 5°   B. 85°   C. 95°   D. 175°
12. Which angles are congruent to ∠5?
  A. ∠1, ∠2, ∠3   B. ∠1, ∠3, ∠8
  C. ∠1, ∠3, ∠7   D. ∠5, ∠6, ∠7

13. Which of the following statement is always true.
   A. Lines that do not intersect are parallel lines.
   B. Lines that form right angles are parallel lines.
   C. Skew lines are parallel lines.
   D. Two coplanar lines that do not intersect are parallel lines.
*For items 14-15, given: r∥s
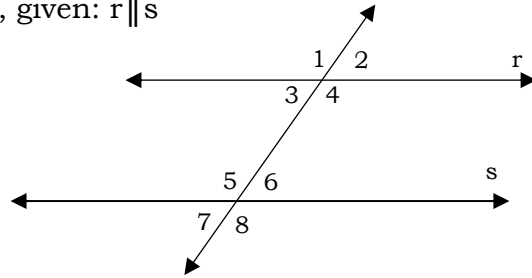
14. Find for the value of x if: m ∠4 = 5x-2 and m ∠8 = 2x +10.

   A. 4°   B. 8°   C. 16°   D. 32°

15. Solve for the measure of ∠8.

   A. 18°   B.36°   C. 42°   D. 64°

REFERENCES:

Books

Mathematics Grade 8 Learner's Module, First Edition,2013
Published by the Department of Education

Grade 8 Mathematics Patterns and Practicalities
By Gladys C. Nivera, Ph.D.,
Published by Salesiana Books by Don Bosco Press Inc., 2014

Practical Math 8
Published by Diwa Learning Systems, Inc., 2018

Next Century Mathematics 8
By Phoenix Publishing House, Inc.,2017

Exploring Math 8 textbook ,
By Orlando A. Oronce and Marilyn O. Mendoza
Published by Rex Book Store, Inc. First Edition 2018

Soaring 21st Century Mathematics Grade 7 (K to12),2014
Published by Phoenix Publishing House

Links:

www.mathplanet.Com/education/algebra-1

formulating-line_equations/parallel-and-perpendicular-lines

math-aids.com/Geometry/ParallelLines/Lines_PPI.html