9

# Mathematics

## Quarter 4 - Module 5
## Illustrates Laws of Sine and Cosines

## AIRs - LM

Mathematics 9
Quarter 2: Week 6-7, Module 5: Illustrates Law of Sines and Cosines
First Edition, 2021



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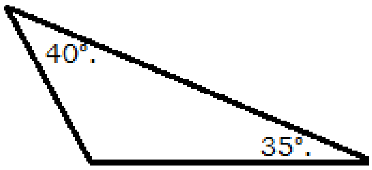
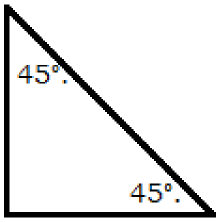
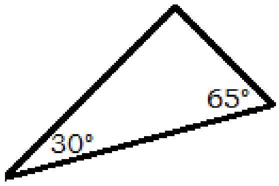
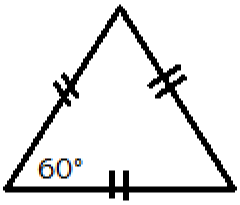
**Activity 1: Missing Me!**

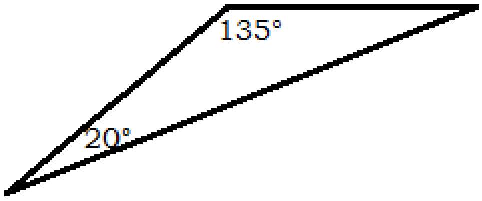
**Directions:** Find the missing angle/s and identify whether the triangle is ***acute***, ***obtuse***, or ***neither***. Write your answers on a separate sheet of paper.

1. 


2. 


3. 


4. 


5. 


## *Discover*

To solve problems involving radicals, you must answer the question asked. Well-labeled diagrams and pictures will help you understand the problem.

# Lesson 1.1 Illustrating Law of Sines and Its Application

Start this module's lesson by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. This knowledge and skills will illustrate the laws of sines and cosines. If you find any difficulty in answering the activities, seek the assistance of your teacher or refer to the modules you have gone over earlier.

The Law of Sines is easy to follow and very useful in solving oblique triangles when you know the following information:

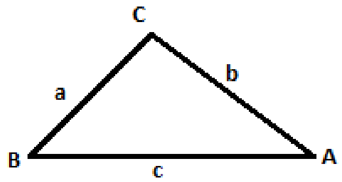
- Two angles and one side (SAA Case & ASA Case)
- Two sides and an angle opposite one of these sides (SSA Case)

The Law of Sines is described by the relation,

Law of Sines

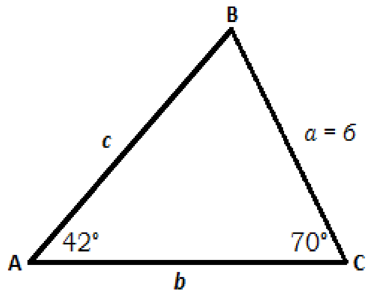
In any ΔABC,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$




**Example 1: SAA Case**

Find the missing parts of ΔABC on the right.




**Given: two angles and one side**

$\angle A = 42°$

$\angle C = 70°$

$a = 6$

**Solutions:**

Since, side b and $\angle$ B are unknown, we can use the formula

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

| | |
|---|---|
| $\frac{\sin A}{a} = \frac{\sin C}{c}$ | *Formula to use to solve for c* |
| $\frac{\sin 42°}{6} = \frac{\sin 70°}{c}$ | *Substitute the given values* |
| $c \sin 42° = 6 \sin 70°$ | *Cross multiply* |
| $0.6691\ c = 6\ (0.9397)$ | *Compute for the values of sin 42° and sin 70° using a scientific calculator* |
| $0.6691\ c = 5.6382$ | *Simplify the resulting equation* |
| $c = \frac{5.6382}{0.6691}$ | *Solve for c* |
| $c = 8.43$ | |

To solve for *b*, the formula to be used is $\frac{\sin A}{a} = \frac{\sin B}{b}$. Notice that $\angle$ B is unknown. You have learned in Mathematics that in any triangle, the sum of the measures of the three angles is 180°. Using this concept, $\angle A + \angle B + \angle C = 180°$, we have

$$\begin{aligned} 42° + \angle B + 70° &= 180° \\ 112° + \angle B &= 180° \\ \angle B &= 180° - 112° \\ \angle \mathbf{B} &= \mathbf{68°} \end{aligned}$$

Thus, we can now solve for side *b*.

Following the steps used earlier in solving for *c*, we now have

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\qquad$ $0.6691\ b = 5.5632$

$$\frac{\sin 42°}{6} = \frac{\sin 68°}{b}$$

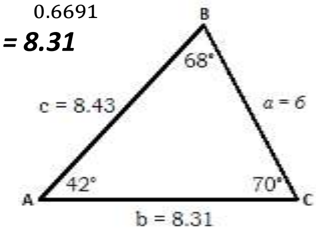
*b sin 42° = 6 sin 68°*

*0.6691 b = 6(0.9272)*

$$b = \frac{5.5632}{0.6691}$$

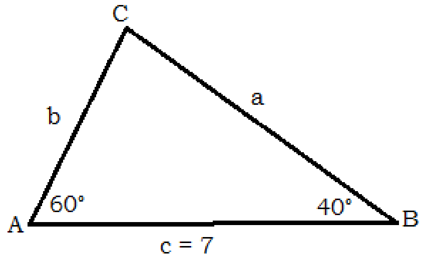
***b = 8.31***

*Thus, the triangle with the measures of its parts is shown at the right.*




**Example 2: ASA Case**

Determine the measure of the missing parts of ΔABC on the right.




**Given: two angles and one side**

∠ *A = 60°*

∠ *B = 40°*

*c = 7*

**Solutions:**

Since the measures of the two angles of the triangle are known, the measure of the third angle can be determined using the concept that the sum of the angles of the triangle is 180°.

$$\begin{aligned}
\angle A + \angle B + \angle C &= 180° \\
60° + 40° + \angle C &= 180° \\
100° + \angle C &= 180° \\
\angle C &= 180° - 100° \\
\mathbf{\angle C} &= \mathbf{80°}
\end{aligned}$$

To solve for side a, we can use the formula $\frac{\sin A}{a} = \frac{\sin C}{c}$.

| | |
|---|---|
| $\frac{\sin A}{a} = \frac{\sin C}{c}$ | *Formula to use to solve for c* |
| $\frac{\sin 60°}{a} = \frac{\sin 80°}{c}$ | *Substitute the given values* |
| *a sin 80° = 7 sin 60°* | *Cross multiply* |
| *0.9848 a = 7(0.8660)* | *Compute for the values of sin 80° and sin 60° using a scientific calculator* |
| *0.9848 a = 6.062* | *Simplify the resulting equation* |
| $a = \frac{6.062}{0.9848}$ | *Solve for a* |
| *a = 6.16* | |

To solve for side b, use the formula $\frac{\sin B}{b} = \frac{\sin C}{c}$ and follow the steps earlier.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 40°}{b} = \frac{\sin 80°}{7}$$

*b sin 80° = 7 sin 40°*

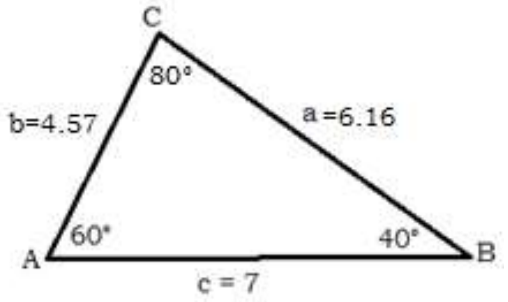
*0.9848 b = 7(0.6428)*

*0.9848 b = 4.4996*

$$b = \frac{4.4996}{0.9848}$$

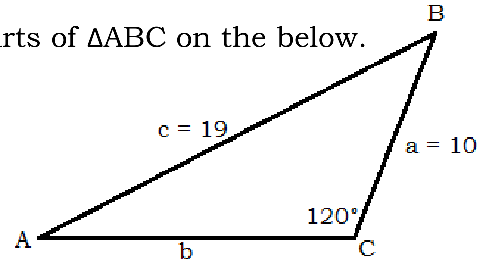
***b = 4.57***

Shown below is the triangle with its complete parts.




**Example 3: SSA Case**

Solve for the missing parts of ΔABC on the below.




**Given: two sides and an angle opposite of these sides**

a = 10

c = 19

∠ *A = 120°*

**Solutions:**

∠C is an obtuse angle and *c* > *a*, thus there is exactly one solution.

Since *a, c* and ∠C are known, we can use the formula formula $\frac{sin\ A}{a} = \frac{\sin C}{c}$.

| | |
|---|---|
| $\frac{sin\ A}{a} = \frac{\sin C}{c}$ | *Formula to use to solve for A* |
| $\frac{sin\ A}{10} = \frac{\sin 80°}{19}$ | *Substitute the given values* |
| *19 (sin A) = 10 (sin 120°)* | *Cross multiply* |
| *19 (sin A) = 10(0.8660)* | *Compute for the values of sin 120° and sin 60° using a scientific calculator* |
| *19 (sin A) = 8.66* | *Simplify the resulting equation* |
| $\sin A = \frac{8.66}{19} = 0.4558$ | *Solve for A* |
| *A = 27.12°* | |

Using the concept that the sum of the angles of a triangle is 180°, we have

| | |
|---|---|
| ∠A + ∠ B + ∠C= 180° | ∠ B = 180° - 147.12° |
| 27.12° +∠ B + 120° = 180° | ∠ **B = 32.88°** |
| ∠B + 147.12° **=** 180° | |

Following the steps used earlier in solving for *c*, we can now solve for *b*.

$\frac{sin\ B}{b} = \frac{sin\ C}{c}$ *b(0.8660) = 19(0.5429)*

$\frac{\sin 32.88°}{b} = \frac{\sin 120°}{19}$

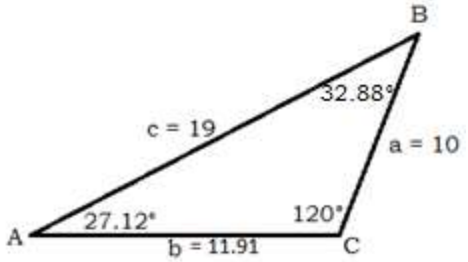
*b sin 120° = 19 sin 32.88°*

*0.9848 b = 7(0.6428)*

*0.8660 b = 10.3151*

$b = \frac{10.3151}{0.8660} = \mathbf{11.91}$

Shown below is the triangle with its complete parts.




# Lesson 1.2

# Illustrating Law of Cosines and Its Application

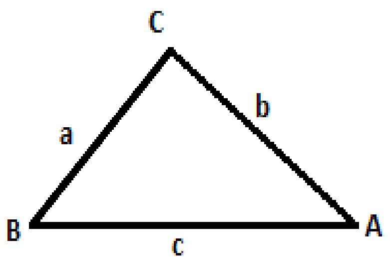
Oblique triangles can also be solved using the Law of Cosines. This law states the following:

The square length of one side is equal to the sum of the other two sides minus the product of twice the two sides and the cosine of the angle between them.

Law of Cosines

In any ΔABC,

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$b^2 = a^2 + c^2 - 2ac(\cos B)$
$c^2 = a^2 + b^2 - 2ab(\cos C)$

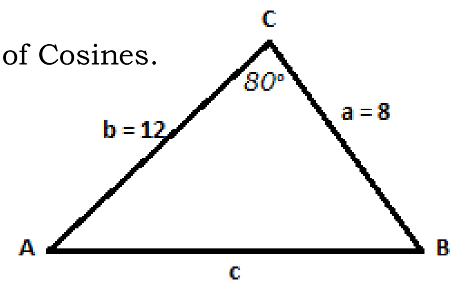



The Law of Cosines can be used in the following situations:

- Two sides and the included angle are known
- Three sides known

**Example 1:**

Let us use ΔABC to illustrate the Law of Cosines.




**Given: two sides and the included angle**

*∠ C = 80°*

*a* = *8*
*b* = *12*

**Solutions:**

To solve for c,

$c^2 = a^2 + b^2 - 2ab(\cos C)$
$c^2 = 8^2 + 2^2 - 2(8)(2)(\cos 80°)$
$c^2 = 64 + 144 - 192(0.1736)$
$c^2 = 208 - 33.3312$
$c^2 = 174.6688$
$\boldsymbol{c^2 = 13.22}$

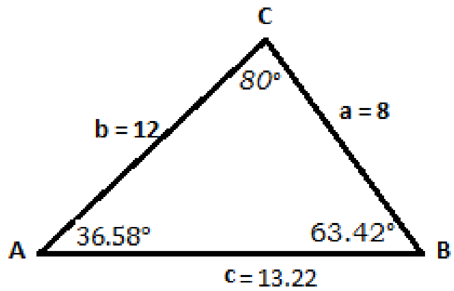
To determine the measure of ∠A,

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$8^2 = 12^2 + 13.22^2 - 2(12)(13.22)(\cos A)$
$64 = 144 + 174.7684 - (37.22)cos\ A$
$64 = 318.7684 - (317.28)(cos\ A)$
$(317.28)(cos\ A) = 318.7684 - 64$
$(317.28)(cos\ A) = 254{,}7684$
$cosA = \frac{2547684}{317.28} = 0.8030$
$\boldsymbol{A = 36.58°}$

Since the measure of ∠C is given, and the measure of ∠ A is now known, the measure of ∠B can be computed using the equatior∠ A +∠ B +∠C = 180°.

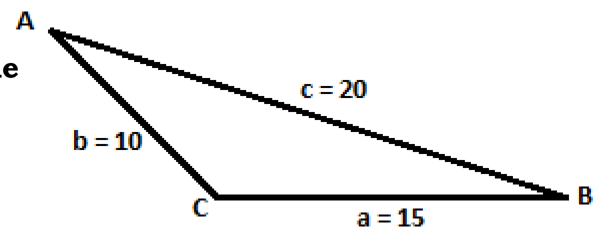
∠A + ∠B + ∠C = 180°
36. 58° +∠ B + 80° = 180°
116.58 + ∠B = 180°
∠B = 180° - 116.58°
∠ **B = 63.42°**

The triangle with its complete parts is shown below.




**Example 2:**

Determine the missing parts of ΔABC.




**Given: two sides and the included angle**

*a* = *15*
*b* = *10*
*c* = *20*

**Solutions:**

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$b^2 = a^2 + c^2 - 2ac(\cos B)$
$c^2 = a^2 + b^2 - 2ab(\cos C)$

Let us solve for the measure of ∠A.

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$15^2 = 10^2 + 20^2 - 2(10)(20)(cosA)$

$225 = 100 + 400 - 400(cosA)$

$-275 = -400(cosA)$

$\frac{-275}{-400} = \frac{-400(cosA)}{-400}$

$0.6875 = cosA$

$\boldsymbol{A = 46.57°}$

Using the formula $b^2 = a^2 + c^2 - 2ac(\cos B)$ and following the steps used above, let's find the measure of ∠B.

$b^2 = a^2 + c^2 - 2ac(\cos B)$

$10^2 = 15^2 + 20^2 - 2(15)(20)(cosB)$

$100 = 225 + 400 - 600(cosB)$

$-525 = -600(cosB)$

$\frac{-525}{-600} = \frac{-600(cosB)}{-600}$

$0.8750 = cosB$

$\boldsymbol{B = 28.96°}$

Since two angles are already known, substitute their values in the equation ∠A + ∠B + ∠C = 180° to solve for ∠C.

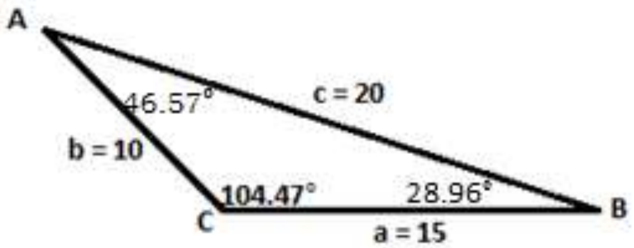
46.57° + 28.96 + ∠C = 180°

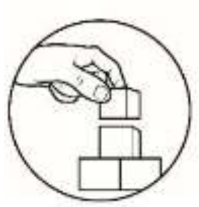
75.53° + ∠C = 180°

∠C = 180° - 75.53°

∠C = 104.47°

Below is the triangle with its complete parts.



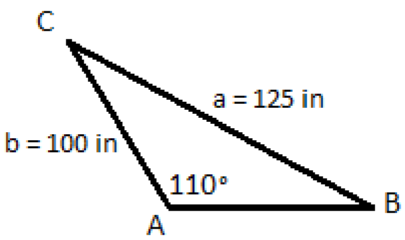


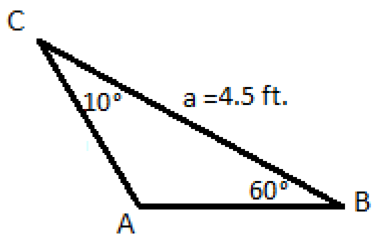
## *Explore*

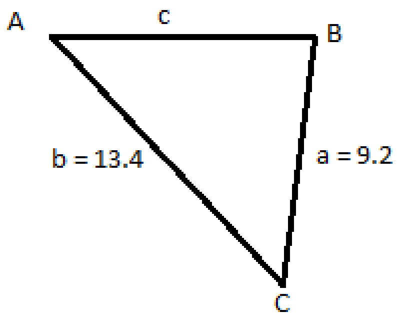
*Here are some enrichment activities to master and strengthen the basic concepts you have learned from this lesson.*

**Activity 2: Practice makes perfect!**
**Directions:** Find all the missing parts of each oblique triangle.

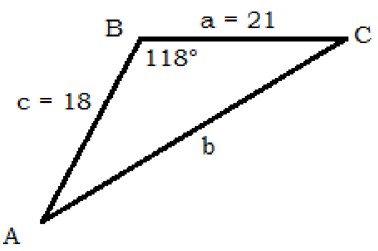
A. Use the Law of Sines to find the all the missing parts of ΔABC.

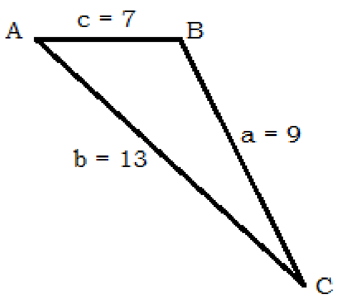
1. 


2. 


3. 

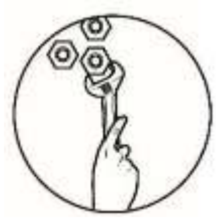

B. Use the Law of Cosines to find the all the missing parts of ΔABC.

4. 


5. 


*In the previous activity, you were able to apply your understanding of the illustrating law of sine and cosine.*

*Let us put that understanding to the test by answering the next activity.*



## *Deepen*

**Activity 3: Illustrate then Solve!**
**Directions:** Illustrate the oblique triangle then find its missing parts using the Law of Sine and Law of Cosine.

1. Given: a = 3; b = 2; $\angle$ A = 50°
   Find: $\angle$ B, $\angle$ C and side c

2. Given: b = 4; c = 6; $\angle$ B = 20°
   Find: $\angle$ A, side a and $\angle$ C

3. Given: a = 29; b = 39; $\angle$ C = 49°
   Find: $\angle$ A, $\angle$ B and side c

4. Given: a = 3; b = 7; c = 9
   Find: $\angle$ A, $\angle$ B and $\angle$ C

In this activity, the discussion was about your understanding of illustrating the law of sines and cosines. Let us see what you had learned on this module by filling – out the next activity.

**Activity 4: Synthesis Journal**

**Directions**: Fill in the table by answering the given questions. Use a separate sheet of paper for your answer.

| Synthesis Journal | | |
|---|---|---|
| **What interest me.** | **What I learned.** | **How can the knowledge of the law of sine and cosine help us in solving oblique triangles?** |