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Mathematics

Quarter 4- Week 2 Module 2

Proving Inequalities in a Triangle

AIRs - LM

Mathematics 8

Quarter 4- Week 2 Module 2
Proving Inequalities in a Triangle

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Target

This module will help you understand the concept of inequality and prove theorems on triangle inequalities. Your key role is to underscore that the process of answering the essential question on how inequalities in triangles can be justified will;

- improve their attention to details
- shape deductive thinking
- hone reasoning skills; and
- polish mathematical communication

This chapter allows you to see why the sum of the two sides of a triangle must be greater than the third side in order for a triangle to exist.

The introduction, through the essential question serves as a steering mechanism of the lesson and stresses the purpose of studying inequalities in triangles. All activities in the lesson are geared towards the goal of answering it.

Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will enable you to demonstrate your understanding of inequalities in triangles.

After going through this module, you are expected to attain the following objectives:

Learning Competency:

Proves inequalities in a triangle. **M8GEIVc-1**

After going through this module, you are expected to:

1. illustrate the theorems on triangle inequalities, e.g.angle inequality theorem, triangle inequality theorem, hinge theorem
2. apply theorems on triangle inequalities to: (a) determine the possible measures of the angles and sides of triangles and (b) justify claims about the unequal relationships between side and angle measures
3. apply the theorems on triangle inequalities to prove results involving triangle inequalities.

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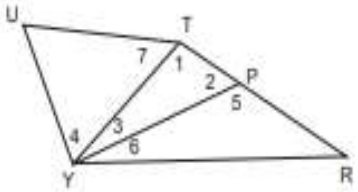
Pretest:

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

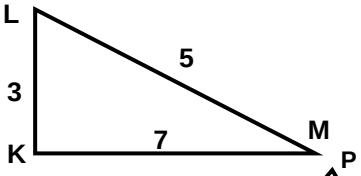
1. Which of the following statements is TRUE. The measure of an exterior angle of a triangle is always _____.
   A. greater than its adjacent interior angle.
   B. less than its adjacent interior angle.
   C. greater than either remote interior angle.
   D. less than either remote interior angle.
2. Each of Xylie, Marie, Angel and Chloe was given an 18-inch piece of stick. They were instructed to create a triangle. Each cut the stick in their own chosen lengths as follows: Xylie—6 in, 6 in, 6 in; Marie—4 in, 5 in, 9 in; Angle—7 in, 5 in, 6 in; and Chloe—3 in, 7 in, 5 in. Who among them was not able to make a triangle?
   A. Xylie B. Marie C. Angel D. Chloe

3.Which of the following angles is an exterior angle of ΔTYP?

A. ∠4
B. ∠5
C. ∠6
D. ∠7

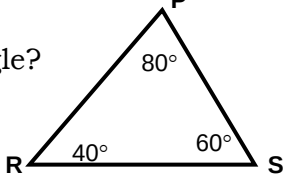


4.Which of the following does not describe the relationship of the angles and the sides of the following triangle?
A. $m\angle L > m\angle M$ B. $LK + LM > KM$
C. $m\angle M > m\angle K$ D. $LM + KM > LK$

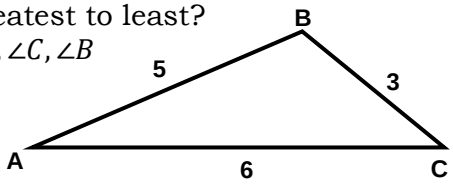


L

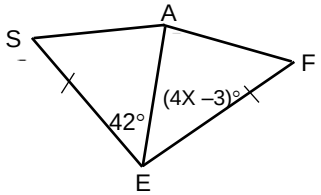
5. Which of the following is the longest side of the given triangle?
   A. PS B. RS C. RP D. can't be determined



6. Which of the following is the list of the angles from greatest to least?
   A. $\angle A, \angle B, \angle C$ B. $\angle B, \angle C, \angle A$ C.$\angle C, \angle B, \angle A$ D. $\angle A, \angle C, \angle B$



1. What are the possible values for x in the figure?



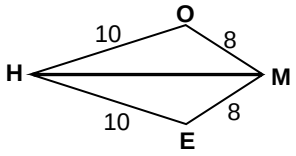
A. x < 11.25 B. x > 11.25
C. x ≤ 11.25 D. x ≥ 11.25

8. In ΔTRU, TR = 8 cm, RU = 9 cm, and TU = 10 cm. What is the order or angles from least to greatest measure?

A. m∠T, m∠R, m∠U B. m∠U, m∠T, m∠R
C. m∠R, m∠T, m∠U D. m∠U, m∠R, m∠T

9. From the inequalities in the triangles shown, a conclusion can be reached using the converse of hinge theorem. Which of the following is the last statement?

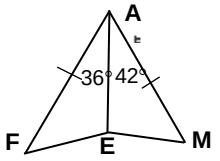
A. HM ≅ HM B. m∠OHM > m∠EHM
C. HO ≅ HE D. m∠EHM > m∠OHM



10. Hikers, Oliver and Ruel who have uniform hiking speed walk in opposite directions. Oliver, eastward whereas Ruel, westward. After walking three kilometers each, both of them take left turns at different angles. Oliver at an angle of 30^0 and Ruel at 40^0. Both continue hiking and cover another four kilometers each before taking a rest. Which of the hikers is farther from their point of origin?

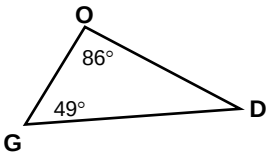
A. Ruel C. It cannot be determined.
B. Oliver D. Ruel is as far as Oliver from the rendezvous.

11. Will you be able to conclude that EM > EF if one of the following statements is not established:
AE ≅ AE, AF ≅ AM, m∠MAE > m∠FAE?



A. Yes, I will. C. It is impossible to decide
B. No, I won't. D. It depends on which statement is left out.

12. Which side of ΔGOD is the shortest?
A. GO B. DG C. DO D. GD



13. A balikbayan chose you to be one of the contractors to design an A-frame house maximizing the size of two square lots with dimensions 18 ft and 24 ft on each side. Which of the following is affected by the dimensions of the lot if the owner would like to spend the same amount of money on the roofs?

I. The width of the base of the house frames
II. Design of the windows
III. The height of the houses
IV. The roof angles

A. I and IV B. II, III and IV C. III and IV d. I, II, III, and IV

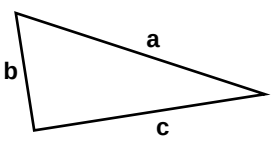
14. Which of the following theorems justifies your response in item no. 13?

I. Triangle Inequality Theorem 1
II. Triangle Inequality Theorem 2
III. Triangle Inequality Theorem 3
IV. Hinge Theorem V. Converse of Hinge Theorem

a. I, II, and III b. IV only c. IV and V d. V only

15. Which of the following is NOT correct?

A. a+b > c B. a+c > b C. c+b > a D. a+b = c

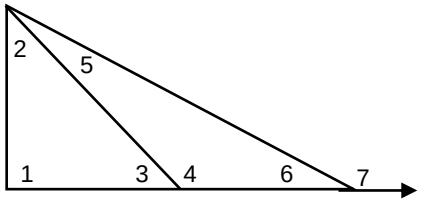


Jumpstart

Directions: Let's start the module by doing the two activities that will reveal your background knowledge on triangle inequalities.

Activity 1: My Symbol!

Write <,> or = to make My Sumbola TRUE statement.



1. m∠7 __ $m\angle 5$
2. m∠1 __ $m\angle 3$
3. m∠2 __ $m\angle 4$
4. m∠6 __ $m\angle 3$
5. m∠5 + $m\angle 4$ __ $m\angle 7$

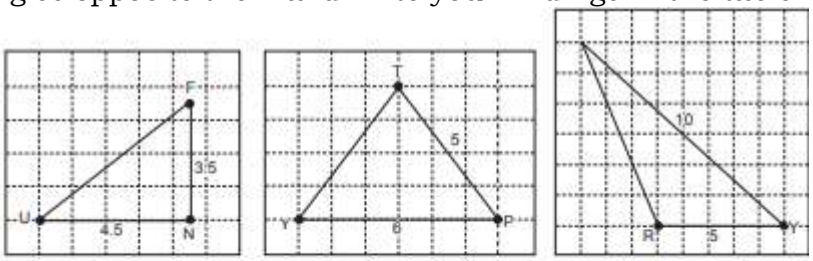
Now that you have already reviewed concepts and skills previously learned that are useful in this module, let us proceed to the main focus of this section—develop, verify, and prove the theorems on inequalities in triangles.

Activity 2: WHAT IF IT'S LONGER?

Materials Needed: protractor, sheet of papers, ruler

Procedures:

1. Replicate each of the triangles on a sheet of paper.
2. Measure using a protractor the angles opposite the sides with given lengths. Indicate the measure in your table.
3. Discover the relationship that exists between the lengths of the sides of triangles and the angles opposite them and write your findings in the table



Triangle	Length of Sides		Measures of Angles Opposite the Sides	
ΔFUN	FN	3.5	$m\angle U$	
	NU	4.5	$m\angle F$	
ΔPTY	TP	5	$m\angle Y$	
	PY	6	$m\angle T$	
ΔRYT	RY	5	$m\angle T$	
	TY	10	$m\angle R$	

1. Is there a relationship between the length of a side of a triangle and the measure of the angle opposite it? Yes, there is. No, there isn't.
2. Making Conjecture: What is the relationship between the sides of a triangle and the angles opposite them? • When one side of a triangle is longer than a second side, the angle opposite the _____.
3. Your findings in no. 2 describe the Triangle Inequality Theorem 1. Write it in if-then form. • If one side of a triangle is longer than a second side, then _____.
4. What is the relationship between the longest side of a triangle and the measure of the angle opposite it?
5. What is the relationship between the shortest side of a triangle and the measure of the angle opposite it?

Discover

From the prior investigations, we have discovered the following theorems on triangle inequalities

Inequalities in One Triangle:

Triangle Inequality Theorem 1 (Ss → Aa) If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.
Triangle Inequality Theorem 2 (Aa → Ss) If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.
Triangle Inequality Theorem 3 (S1 + S2 > S3) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
Exterior Angle Inequality Theorem The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle
Inequalities in Two Triangles:
Hinge Theorem or SAS Inequality Theorem If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Before you go through the process, take a few minutes to review and master again the knowledge and skills learned in previous geometry lessons. The concepts and skills on the following topics will help you succeed in the investigatory and proof-writing activities.

1. **Axioms of Equality**

1.1 **Reflexive Property of Equality** • For all real numbers p, p = p.
1.2 **Symmetric Property of Equality** • For all real numbers p and q, if p = q, then q = p.
1.3 **Transitive Property of Equality** • For all real numbers p, q, and r, if p = q

and q = r, then p = r.

1.4 **Substitution Property of Equality** • For all real numbers p and q, if p = q, then q can be substituted for p in any expression.

2. Properties of Equality

2.1 Addition Property of Equality • For all real numbers p, q, and r, if p = q, then p + r = q + r.

2.2 Multiplication Property of Equality • For all real numbers p, q, and r, if p = q, then pr = qr.

3. Definitions, Postulates and Theorems on Points, Lines, Angles and Angle Pairs

3.1 Definition of a Midpoint • If points P, Q, and R are collinear (P–Q–R) and Q is the midpoint of PR,then PQ ≅ QR.

3.2 Definition of an Angle Bisector • If QS bisects ∠PQR, then ∠PQS ≅ ∠SQR.

3.3 Segment Addition Postulate • If points P, Q, and R are collinear (P–Q–R) and Q is between points P and R, then PQ + QR ≅ PR.

3.4 Angle Addition Postulate • If point S lies in the interior of ∠PQR, then ∠PQS + ∠SQR ≅ ∠PQR.

3.5 Definition of Supplementary Angles • Two angles are supplementary if the sum of their measures is 180°.

3.6 Definition of Complementary Angles • Two angles are complementary if the sum of their measures is 90°.

3.7 Definition of Linear Pair • Linear pair is a pair of adjacent angles formed by two intersecting lines

3.8 Linear Pair Theorem • If two angles form a linear pair, then they are supplementary.

3.9 Definition of Vertical Angles • Vertical angles refer to two non-adjacent angles formed by two intersecting lines.

3.10 Vertical Angles Theorem • Vertical angles are congruent.

3. Definitions and Theorems on Triangles

4.1 The sum of the measures of the angles of a triangle is 180°.

4.2 Definition of Equilateral Triangle • An equilateral triangle has three sides congruent.

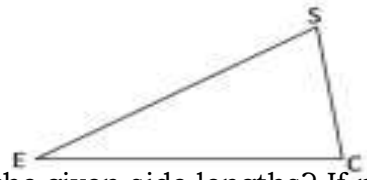
4.3 Definition of Isosceles Triangle • An isosceles triangle has two congruent sides. Base angles of isosceles triangles are congruent. • Legs of isosceles triangles are congruent.

4.4 Exterior Angle of a Triangle • An exterior angle of a triangle is an angle that forms a linear pair with an interior angle of a triangle when a side of the triangle is extended.

4.5 Exterior Angle Theorem • The measure of an exterior angle of a triangle is equal to the sum of the measures of the two interior angles of the triangle.

4.6 Sides and Angles of a Triangle

- ∠S is opposite EC and EC is opposite ∠S.
- ∠E is opposite SC and SC is opposite ∠E
- ∠C is opposite ES and ES is opposite ∠C.



Example 1: Is it possible to form a triangle with the given side lengths? If not, explain why not.

A. 15 yd, 16 yd, 30 yd

Solution:

15 + 16 > 30 15 + 30> 16 16+30>15

31 > 30 45>16 46>15

Since the sum of each pair of side lengths is greater than the third side length, side lengths 15 yd, 16 yd and 30 yd will form a triangle.

B. If 4cm, 8cm and 2cm are the measures of three lines segment. Can it be used to draw a triangle?

Solution: The triangle is formed by three line segments 4cm, 8cm and 2cm, then it should satisfy the inequality theorem.

Hence, let us check if the sum of two sides is greater than the third side.

4 + 8 > 2 ⇒ 12 > 2 ⇒ True

8 + 2 > 4 ⇒ 10 > 4 ⇒ True

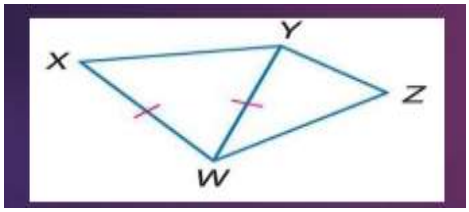
4 + 2 > 8 ⇒ 6 > 8 ⇒ False

Therefore, the sides of the triangle do not satisfy the inequality theorem. So, we cannot construct a triangle with these three line-segments

Example 2: Write a two column proof

Given: $XW \cong YW$

Prove : YZ + ZW > XW

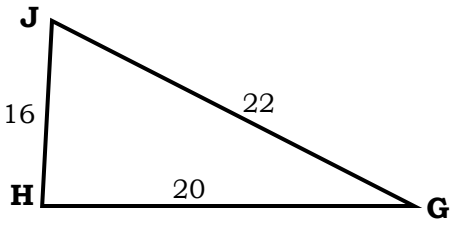


STATEMENTS	REASONS
1. $XW \cong YW$	1. Given
2. $XW = YW$	2. Def. of ≅ segments
3. $YZ + ZW > YW$	3. Triangle Inequality Theorem
4. $YZ + ZW > XW$	4. Substitution

Example 3. In ΔJHG, list the angles in order from least to greatest measure

JH < HG < JG

$m\angle G$ $m\angle J$ $m\angle H$



First, write the segment measures from least to greatest. Then use the theorem on triangle inequality to write the measures of the angles opposite these sides in the same order

The angles in order from least to greatest measure are ∠**G**, ∠**J**, ∠**H**.

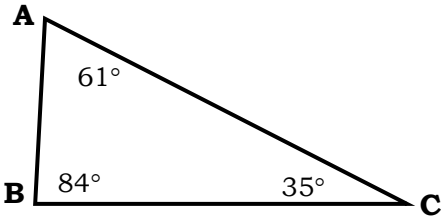
Example 4. Name the sides from longest to shortest.

$m\angle C < m\angle A < m\angle B$

AB < BC < AC

The side in order from longest to shortest are

AC, BC, AB



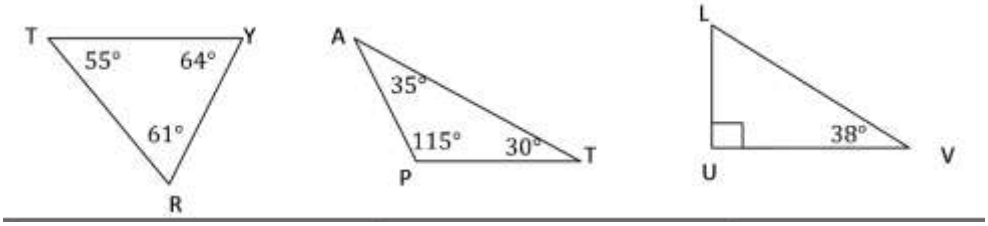
Explore

Work on the following enrichment activities for you to apply your understanding on this lesson.

Activity 3:
Directions:

A. Write your answer on a separate answer sheet.
Note that the diagrams in the exercises are not drawn to scale.

B. Name the shortest side and the longest side of the following triangles:



Complete the table:

	Triangle	Longest Side	Shortest Side
1	**Δ TRY**		
2	**Δ APT**		
3	**Δ LUV**		

True or False : Tell whether the following triples of numbers are possible lengths of the sides of a triangle . If yes write **True** and if it not possible write **False**.

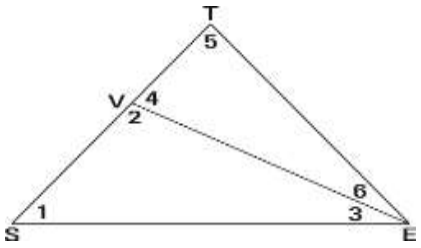
1. 5,4,4 2. 1,-5, 6 3. 2,2,3 4. -1,2,3 5. 4,4,8

Deepen

Activity 4

Complete the following proof below. Write your answers on your answer sheet.

Given : ΔSET , VE *bisects* $\angle SET$
Prove : ET > TV



Statements	Reasons
1. ΔSET , VE *bisects* $\angle SET$	1.Given
2. _____	2. An angle bisector divides an angle into two congruent parts
3. $m\angle 4 = m\angle 1 + m\angle 3$	3.
4. m ____> m < 3	4.The whole is greater than its parts
5. m∠4 > m∠6	5. _____
6. ET > TV	6. _____

Think about this!

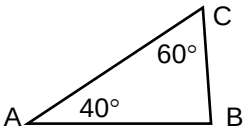
Two sides of a triangle measures 9cm and 15 cm. Write the inequality that represents the range of the values for the possible lengths of the third side.

Gauge

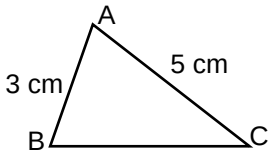
Directions: A. Read and understand the questions below. Select the best answer to each item then write your choice on your answer sheet

1. What theorem states that if one angle is larger than the other angle, the side opposite the larger angle is the longer side?
   A. Hinge Theorem B. Angle -Side Inequality Theorem
   C. Triangle Inequality Theorem D. Exterior Angle Inequality
2. What theorem states that the sum of the measure of any two sides of a triangle is greater than the measure of the third side?
   A. Hinge Theorem B. Exterior Angle Theorem
   C. Triangle Inequality Theorem D. Exterior Angle Inequality Theorem

3. Which theorem states that "If one side is longer than the other side, the angle opposite the longer side is the larger angle"?
   A. Alternate Interior Angle Theorem B. Corresponding Angle
   C. Side -Angle Inequality Theorem D. Same – Side Interior Angles
4. Which of the following combinations could be the sides of a triangle?
   A. 7,9,19 B. 7,12,19 C. 7,7,7 D. 7,7,16
5. Which of the following sides do not represent the lengths of a triangle?
   A. 2cm,6cm,7cm B. 5cm,2cm,5cm
   C. 5cm,5cm,8cm D. 3cm,10cm,15cm
6. Which sides CANNOT be the third side of a triangle given that its first and second sides are 8 cm and 12 cm?
   A.13cm B. 22 cm C. 10 cm D. 5cm
7. What is the ascending order of the sides ofΔABC in the figure?
   A AC<BC<AB B. AC<AB<BC C. AB<BC<AC D. BC<AB<AC

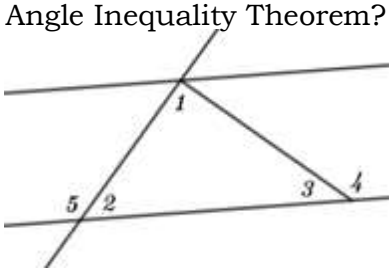


8. If $\angle ABC$ is the largest angle inΔABC, then which of the following sides can be the length of BC?
   A.2 cm B. 6 cm C. 8 cm D. 9 cm



9. Which statement is correct to describe Exterior Angle Inequality Theorem?

   A. $m\angle 4 > m\angle 3 + m\angle 2$
   B. $m\angle 5 > m\angle 2 + m\angle 3$
   C. $m\angle 4 > m\angle 1$ or $m\angle 4 > m\angle 2$
   D. $m\angle 5 > m\angle 1 + m\angle 3$



10. If the two sides of a triangle are 2 and 7. What is the possible lengths of the third side?
   A. 5<x<9 B. 2<x<9 C. 7<x<9 D. 5<x<7
11. Which of the following combinations could be the sides of a triangle?
   A. 3,5,6 B. 7,7,14
   C. 4,4,8 D. 1,2,3
12. In ΔTRU, TR = 8 cm, RU = 9 cm, and TU = 10 cm. List the angles in order from least to greatest measure.
   A. $m\angle T, m\angle R, m\angle U$ B. $m\angle U, m\angle T, m\angle R$
   C. $m\angle R, m\angle T, m\angle U$ D. $m\angle U, m\angle R, m\angle T$

13. The lengths of the sides of a triangle are 16 – k, 16, and 16 + k. What is the range of the possible values of k?
   A. -16 < k < 8 B. 16 < k < 8 C. 16 > k > 8 D. -16 > k > 8

For item 14-15

Given: ΔLMN; $m\angle L > m\angle N$
Prove: $MN > LM$

Indirect Proof:

Assume: $MN \not> LM$

Statements	Reasons
1. $MN = LM$ or $MN < LM$	1. Assumption that $MN \not> LM$
2. Considering $\overline{MN} \cong \overline{LM}$: If $\overline{MN} \cong \overline{LM}$, then ☐	2. Definition of ☐
Consequently, what can you say about $\angle L$ and $\angle N$? ☐	☐ of isosceles triangles are congruent.
The Assumption that $\overline{MN} \cong \overline{LM}$ is ☐ True ☐ False	The conclusion that $\angle L \cong \angle N$ ☐ the given that $m\angle L > m\angle N$.
3. Considering $MN < LM$: If $MN < LM$, then ☐	3. Base angles of isosceles triangles are congruent.
The Assumption that $MN < LM$ is ☐ True ☐ False	The conclusion that $m\angle L < m\angle N$ contradicts the given that ☐
4. Therefore, $MN > LM$ must be ☐ True ☐ False	4. The ☐ that $MN \not> LM$ contradicts the known fact that $m\angle L > m\angle N$.

14. What is the reason for item 2, considering MN ≅ LM : If MN ≅ LM then what kind of triangle is given?
    A. Isosceles triangle
    B. Scalene Triangle
    C. Right Triangle
    D. Straight Angle
15. Considering the statements if MN < LM, then $m\angle L < m\angle N$, what theorem satisfied the given statement?
    A. Hinge Theorem
    B. Angle -Side Inequality Theorem
    C. Triangle Inequality Theorem
    D. Exterior Angle Inequality

References

Books:

Mathematics Grade 8 Learner's Module
Kto12 :Grade 8 :Practical Mathematics by : Evelyn Zara
Grade 8 Mathematics Patterns and Practicalities by: Gladys C. Nivera.PhD.
MSA Geometry by : Merle S. Alferez and Alvin E. Lambino

Links:

https://www.dummies.com/education/math/geometry/proofs-involving-the-triangle-inequality-theorem-practice-geometry-questions/