8

Mathematics
Quarter 1- Module 2:
Rational Algebraic Expressions

AIRs - LM

LU_Q1_Mathematics 8_ Module 1

MATH 8
Quarter 1 - Module 2: Rational Algebraic Expressions
First Edition, 2021

Development Team of the Module

Author: John Michael L. Ordinario
Editor: SDO La Union, Learning Resource Quality Assurance Team
Content Reviewer: Virginia G. Acas
Language Reviewer: Lorna M. Tampos
Illustrator: Ernesto F. Ramos Jr.
Design and Layout: Jerome P. Medriano

Management Team:

Atty. Donato D. Balderas Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent
German E. Flora, PhD, *CID Chief*
Virgilio C. Boado, PhD, *EPS in Charge of LRMS*
Belen C. Aquino, PhD, *EPS in Charge of English*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

Printed in the Philippines by: ________________________

Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

8

Mathematics

Quarter 1- Module 2:
Rational Algebraic Expressions

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

This module will help you understand the key concepts of Rational Algebraic Expressions. Moreove, you' ll find out how these mathematics concepts are used in solving real-life problems. In all lessons, you are given the opportunity to use your prior knowledge and skills in linear inequalities in one variable. Activities are also given to process your knowledge and skills acquired, deepen and transfer your understanding of the different lessons. The scope of this modules enables you to use it in many different learning situations. The lesson are arranged to follow the standard sequence of the course. But in order in which you read them can be changed to corresponds with the textbooks you are using.

This module contains the following lessons:

Lesson 1: Illustrating Rational Algebraic Expressions
Lesson 2: Simplifying Rational Algebraic Expressions
Lesson 3: Performing Operations on Rational Algebraic Expressions
Lesson 4: Solving Problems involving Rational Algebraic Expressions

After going through this module, you are expected to

1. Illustrates rational algebraic expressions **(M8AL-Ic-1)**;
2. Simplifies rational algebraic expressions **(M8AL-Ic-2)**;
3. Performs operations on rational algebraic expressions **(M8AL-Ic-d-1)**; and
4. Solves problems involving rational algebraic expressions **(M8AL-Id-2)**.

Pre - test

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following expressions equates the denominator to 0?

   A. $\frac{x^4-2x}{m}$ B. $\frac{4y}{3^0}$ C. $\frac{p}{m-m}$ D. $\frac{x^2-2x+4x}{x+2}$

2. What value of the x makes the expression $\frac{x^4-2}{x}$ undefined?

   A. 0 B. 1 C. 2 D. 3

3. When is a rational algebraic expression in lowest term?

   A. If the numerator and denominator are both of degree one.
   B. If either the numerator or the denominator is factored completely.
   C. If the numerator and denominator have no common factor other than 1.
   D. If the numerator and denominator have no common factor other than -1.

4. Which of the following is the simplified form of the rational expression $\frac{y+5}{5+y}$?

   A. -1 B. 0 C. 1 D. $\frac{y+5}{5+y}$

5. Which of the following is the simplest form of $\frac{x^2-1}{x-1}$?

   A. $\frac{1}{x}$ B. $x+1$ C. $\frac{x+1}{x^2+x+1}$ D. $\frac{x+1}{x^2-x+1}$

6. What are the common factors in the numerators and denominators of

$$\left(\frac{6xy^5}{yz^2}\right) * \left(\frac{xy^3z}{x^3yz}\right)?$$

   A. xyz B. x^2yz C. xy^2z D. x^2y^2z

7. What is the product of $\left(\frac{4}{6-2x}\right)\left(\frac{3-x}{2}\right)$?

   A. 1 B. 2 C. 3 D. 4

8. What is the quotient of $\frac{2y+5}{4} \div \frac{2y+5}{6}$?

   A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

9. Below are the steps in finding the quotients of rational algebraic expressions.

   I. Divide out common factors.
   II. Simplify the remaining factors.
   III. Find the reciprocal of the divisor and proceed to multiplication.
   IV. Determine the dividend and divisor of the given expression.

   Which of the following is in correct order?

   A. I, II, III B. II, III, IV C. IV, III, I, II D. IV, I, II, III

10. What is the sum of $\frac{3y-5}{2} + \frac{y+3}{2}$?

    A. 2y-1 B. 2y-2 C. 2y-3 D. 2y-4

11. Given $\frac{y+1}{3}$ as one addend of the sum $\frac{8y-7}{3}$, what is the other addend?

    A. $\frac{7y-4}{3}$ B. $\frac{7y-6}{3}$ C. $\frac{7y-8}{3}$ D. $\frac{7y-10}{3}$

12. What is the difference of $\frac{y-2}{3} - \frac{y+2}{2}$?

    A. $\frac{y+1}{6}$ B. $\frac{y+5}{6}$ C. $\frac{y-6}{6}$ D. $\frac{-y-10}{6}$

13. What is the difference when you subtract $\frac{x+9}{x-4}$ from $\frac{3x+1}{x-4}$?

    A. 2 B. 4 C. 6 D. 8

For items 14 and 15. Let x be a number. Double the number. Subtract 6 from the result and divide the answer by 2, the quotient will be 20.

14. What is the correct rational equation to represent the problem?

A. $2x - \frac{6}{2} = 20$ C. $\frac{2x-6}{2} = 20$

B. $\frac{2x}{2} = \frac{x-6}{20}$ D. $\frac{1}{2}x = \frac{2x-6}{20}$

15. What is the value of x?

A. 15 B. 18 C. 20 D. 23

Lesson 1 Illustrating Rational Algebraic Expressions

Jumpstart

Activity 1: **Pair Appear!**

Directions: Below are the list of expressions grouped into columns. Pair expressions in column A and column B to illustrate a ratio of two expressions. The answer of the first item is provided.

	Column A	***Column B***	$\frac{\textbf{Column A}}{\textbf{Column B}}$
1	5	$x-2$	$\frac{5}{x-2}$
2	x^2-2	x^3+3	
3	$9y$	y^2+y-2	
4	$3a - b$	$5a$	
5	$4-m^3$	m^2+3	

Discover

A **rational algebraic expression** is a ratio of two polynomials provided that the denominator is not equal to zero. In symbols: $\frac{P}{Q}$, where P and Q are polynomials and $Q \neq 0$.

The ratio of two polynomials which the denominator is not equal to zero is an algebraic expression.

Let's take some examples.

1. $\frac{x}{2} + 7$

Notice that neither of the denominators of the expression is zero. The sum of the expression will not also equate to zero. Therefore, it is a rational algebraic expression.

2. $\frac{x+6}{0}$

It is very visible that the denominator of the ration is zero, thus it is not a rational algebraic expression.

3. $\frac{x^6}{p-p}$

Though there is no zero quantity seen on the denominator of the expression, but this is not a rational algebraic expression. Why? Because $p - p = 0$.

4. $x^2 - 16$

Do you remember what factoring method can factor this expression? Difference of two squares.

What is the denominator? Is it 0? No. the denominator is not 0.

The denominator of our expression is 1. This would look like $\frac{x^2 - 16}{1}$.

5. $\frac{b}{2-0}$

You can see zero in the denominator, but this expression is a rational algebraic expression. $2 - 0 = 2$ thus, the simplified form of this is $\frac{b}{2}$.

Why is it that if the denominator is equal to zero, the expression is not rational algebraic expression? Because the denominator zero will make the expression undefined or meaningless.

Explore

Activity 2: Where do I belong?

Directions: From the given expressions, write in column A, the rational algebraic expressions and in column B, the non-rational algebraic expressions. Use separate sheet of paper for your answers.

$\frac{a}{2}$ $\quad$ $\frac{5b}{2c}$ $\quad$ $\frac{d}{5+7e}$ $\quad$ $\frac{f-2}{f}$ $\quad$ $\frac{1-j}{j-1}$

$\frac{k^6}{l-l}$ $\quad$ $\frac{m^2-9}{m}$ $\quad$ $\frac{n}{2-0}$ $\quad$ $\frac{p+6}{0}$ $\quad$ $\frac{x^4-2x+4}{x}$

Rational Algebraic Expressions	Not Rational Algebraic Expressions

Deepen

Activity 3: **All about rational!**

Directions: Determine whether the following statements on rational algebraic expressions are correct. Write "True" if the statement is true and write "False" if the statement is false. Use separate sheet of paper for your answer.

1. $\frac{x+2y}{x}$ is a rational algebraic expression.
2. $\left(\frac{x^2}{y^3}\right)^0$ is equal to 1
3. $\frac{P}{Q}$ is a rational expression if $P \neq 1, Q = 0$
4. (x+4) is the factor of $\frac{x^2-16}{x-4}$.
5. $\frac{1}{y^{-9}}$ is equal to y^9.

Lesson 2 — Simplifying Rational Algebraic Expressions

Jumpstart

Activity 4: Let's Go and Be Unique?

Directions: Complete the table below. In each item, a pair of polynomials is given. The third column is the factored form of each polynomial, the fourth column is the factor/s common to each pair of polynomials, and the last column is the factor/s not common to each pair of polynomials. Write your answers on a separate sheet of paper. The first item is done to serve as an example, you may start in the second item.

Item No.	Given	Factored form	Let Go	Be Unique
1	$y^2 + y - 6$	(y-2)(y+3)	y+3	y-2
	$y^2 - 9$	(y-3)(y+3)		y-3
2	15a			
	$12a^2b$			
3	$3y^2 - 12y$			
	$6y^2 + 3y$			
4	$y^2 + 4y + 3$			
	$y^2 - 3y - 4$			
5	$2z^2 + 11z + 5$			
	$z^2 + 6z + 5$			

Discover

A fraction is said to be in simplified form when all pair of factors common to the numerator and denominator have been removed.

To simplify a fraction, we remove a factor equal to 1. Similarly, a rational expression is said to be in simplified form when its numerator and denominator have no common factor other than 1. The process of simplifying rational algebraic expressions is similar to simplifying fractions. That is, we write the rational algebraic expressions so that the numerator and denominator have no common factors other than 1.

Steps on Simplifying Rational Expression

1. Factor completely the numerator and denominator.
2. Separate and divide out common factor/s if there is/are any.
3. Multiply the remaining factors.

Examples:

1. $\frac{16}{24}$ 2. $\frac{14y^2}{28y}$ 3. $\frac{x^2-49}{2x-14}$ 4. $\frac{z^2+z-42}{z^2-36} =$

Solutions:

1. $\frac{16}{24} = \frac{(8)(2)}{(8)(3)}$ Factoring by Common Monomial Factor

$= 1 * \frac{2}{3}$ Dividing common factors (8÷8=1)

$= \frac{2}{3}$ Simplified Form

2. $\frac{14y^2}{28y} = \frac{(14)(y)(y)}{(14)(2)(y)}$ Factoring by Common Monomial Factor

$= 1 * \frac{y}{2}$ Dividing common factors (14y÷14y=1)

$= \frac{y}{2}$ Lowest Term

3. $\frac{x^2-49}{2x-14} = \frac{(x+7)(x-7)}{(2)(x-7)}$ Factoring by Difference of Two Squares

Factoring by Common Monomial Factor

$= \frac{(x+7)}{2} * \frac{(x-7)}{(x-7)}$ Grouping Common Factors

$= \frac{(x+7)}{2} * \frac{\cancel{(x-7)}}{\cancel{(x-7)}}$ Cancelling common factors

$= \frac{(x+7)}{2}$ Simplified Form

4. $\frac{z^2+z-42}{z^2-36} = \frac{(z+7)(z-6)}{(z+6)(z-6)}$ Factoring General Trinomials and

Factoring by Difference of Two Squares

$= \frac{(z+7)(z-6)}{(z+6)(z-6)}$ Cancellation Method

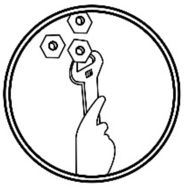
$= \frac{(z+7)(z-6)}{(z+6)(z-6)}$ Simplified Form

Explore

Activity 5: **How Simple I am!**

Directions: Write the simplest form of the given rational algebraic expressions.

Rational Algebraic Expression	**Simplified Expression**
1. $\frac{x^2+6x+5}{x+1}$	
2. $\frac{2y^2-y}{y}$	
3. $\frac{18xy}{6y}$	



Deepen

Activity 6: **Low, Low, Low!**

Directions: Reduce the following rational algebraic expressions to its simplest form.

		Hint:
1.	$\frac{2xy}{4wxy}$	**Common monomial factor (CMF)**
2.	$\frac{-20x^2y^2}{8x^3y^3}$	**Common monomial factor (CMF)**
3.	$\frac{b+1}{b^2-1}$	**Difference of two squares**
4.	$\frac{c^2-9}{c^2-3c}$	**Difference of two squares and CMF**
5	$\frac{a^2+a-20}{a+5}$	**Factor by general trinomial**

Lesson 3: Performing Operations on Rational Algebraic Expressions

Jumpstart

Activity 7: Let's Perform!

Directions. Perform the operation on the following fractions. Use separate sheet of paper for your answers.

1. $\frac{2}{7} \cdot \frac{7}{8}$
2. $\frac{3}{4} \cdot \frac{2}{3}$
3. $\frac{8}{11} \cdot \frac{33}{40}$
4. $\frac{1}{2} \div \frac{3}{4}$
5. . $\frac{5}{2} \div \frac{9}{4}$
6. $\frac{9}{2} \div \frac{3}{4}$
7. $\frac{1}{2} + \frac{3}{2}$
8. $\frac{9}{4} - \frac{5}{4}$
9. $\frac{10}{13} + \frac{5}{2}$
10. $\frac{5}{6} - \frac{2}{3}$

Discover

<u>A. Multiplying Rational Algebraic Expressions</u>

Just as we can multiply fractions, we can multiply rational expressions, or fractions that include polynomials. In fact, we use the same processes for multiplying rational expressions as we use for multiplying numeric fractions.

The product of two rational expressions is the product of the numerators divided by the product of the denominators. In symbols,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, bd \neq 0.$$

The concept of multiplying the numerators and the denominators of fractions also applies to multiplying rational algebraic expressions. For example, the distance in the previous activity is solved by multiplying the numerators and denominators of the speed and time of travel. In addition to just multiplying the numerators and denominators, there are methods of reducing the product into its lowest forms. Multiplying rational algebraic expressions and methods of reducing the product in lowest form will be discussed using series of examples.

The first two examples will be using monomials to illustrate multiplication of rational algebraic expressions.

Method 1: Divide Out Greatest Common Monomial Factor (GCMF) After Multiplying

Example 1:

$$\frac{17a}{2b} * \frac{b}{4}$$

Solution:

Step 1. Multiply the numerators and denominators of the given rational algebraic expressions.

(17a)(b)	=	17ab	Multiply the numerators.
(2b)(4)	=	8b	Multiply the denominators.

Step 2: Find the GCMF of the product of numerators and denominators.

17ab = (17)(a)(b)	Look for prime factors.
8b = (2)(2)(2)(b)	Look for prime factors
GCF = b	Look for the common factors from the two groups of factors.

Step 3: Divide out the GCMF of the product of numerators and denominators.

$\frac{17ab}{8b}$ product (not yet reduced)

$\frac{17a}{8}$ Divide out GCMF.

Step 4: Simplify the remaining factors.

$\frac{17a}{2b} * \frac{b}{4} = \frac{17ab}{8b}$ Product in reduced form.

Method 2: Divide Out Greatest Common Monomial Factor (GCMF) Before Multiplying

Example 2:

$$\frac{15x}{2x} * \frac{2x}{5y^2}$$

Solution:

Step 1. Find the GCMF of the of numerators and denominators

Look for prime factors of the two numerators.

$$(15x)(2x) = (3)(5)(x)(2)(x)$$

Look for prime factors of the two denominators.

$$(2x)(5y^2) = (2)(x)(5)(y)(y)$$

Look for common factors from the numerators and

$$GCF = (2)(y)(5)$$

Step 2: Divide out the GCMF of the numerators and denominators.

Factorization of the numerators and denominators.

$$\frac{15x}{2x} * \frac{2x}{5y^2} = \frac{(3)(5)(x)(2)(x)}{2)(x)(5)(y)(y)}$$

Divide out GCMF.

$$\frac{15x}{2x} * \frac{2x}{5y^2} = \frac{(3)(x)}{(y)(y)}$$

Step 3: Simplify the remaining factors.

Remaining factors

$$\frac{15x}{2x} * \frac{2x}{5y^2} = \frac{(3)(x)}{(y)(y)}$$

Multiply the numerators and multiply the denominators.

$$\frac{15x}{2x} * \frac{2x}{5y^2} = \frac{3x}{y^2}$$

Product in reduced form.

The methods illustrated by the previous examples can also be used to rational algebraic expressions involving polynomials.

Method 1: Divide Out Greatest Common Monomial Factor (GCMF) After Multiplying

Example 1:

$$\frac{y^2-4}{2} * \frac{4}{y-2}$$

Solution:

Step 1. Multiply the numerators and denominators of the given rational algebraic expressions.

Distributive Property

$$(y^2-4)(4) = 4y^2 - 16$$

$$2(y-2) = 2y-4$$

Step 2: Find the GCMF of the product of numerators and denominators.

Factoring Difference of Two Squares

$$4y^2 - 16 = (2y+4)(2y-4)$$

Factoring the Greatest Common Monomial Factor (GCMF)

$$4y^2 - 16 = (2)(y+2)(2)(y-2)$$

Factoring the Greatest Common Monomial

$$2y - 4 = 2(y-2)$$

Look for the common factors from the numerators and denominators.

$$GCF = 2(y-2)$$

Step 3: Divide out the GCMF of the product of numerators and denominators.

Product (not yet reduced)

$$\frac{y^2-4}{2} * \frac{4}{y-2} = \frac{4y^2-16}{2y-4}$$

Factored form of the numerator and denominator.

$$\frac{y^2-4}{2} * \frac{4}{y-2} = \frac{(2)(y+2)(2)(y-2)}{2(y-2)}$$

Divide out the GCMF.

$$\frac{y^2-4}{2} * \frac{4}{y-2} = \frac{(2)(y+2)}{1}$$

Step 4: Simplify the remaining factors.
Remaining factors

$$\frac{y^2-4}{2} * \frac{4}{y-2} = (2)(y+2)$$

Distributive Property

$$\frac{y^2-4}{2} * \frac{4}{y-2} = 2y+4)$$

Product in reduced form.

Method 2: Divide Out Greatest Common Monomial Factor (GCMF) Before Multiplying
Example 2:

$$\frac{y-5}{y^2-7y+10} * \frac{y^2+y-6}{5}$$

Solution:
Step 1. Find the GCMF of the of numerators and denominators

$$(y-5)(\,y^2+y-6)$$
$$(y^2-7y+10)(5)$$

Retain the other factor and write the factors of the trinomial.

$$(y-5)(\,y^2+y-6) = (y-5)(y+3)(y-2)$$
$$(y^2-7y+10)(5) = (y-5)(y-2)(5)$$

Look for common factors from the numerators and denominators.

$$\text{GCF} = (y-5)(y-2)$$

Step 2: Divide out the GCMF of the numerators and denominators.

$$\frac{y-5}{y^2-7y+10} * \frac{y^2+y-6}{5} = \frac{(y-5)(y+3)(y-2)}{(y-5)(y-2)(5)}$$

Divide out GCMF.

$$\frac{y-5}{y^2-7y+10} * \frac{y^2+y-6}{5} = \frac{(y+3)}{5}$$

Step 3: Simplify the remaining factors (if possible).

$$\frac{y-5}{y^2-7y+10} * \frac{y^2+y-6}{5} = \frac{(y+3)}{5}$$

Product in reduced form.

In multiplying rational algebraic expressions, either before or after multiplying, always divide out the common factors to attain the product in reduced form.

B. Dividing Rational Algebraic Expressions

The quotient of two rational algebraic expressions is the product of the dividend and the reciprocal of the divisor.In symbols,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, b, c, d \neq 0$$

To help you understand how to divide rational expressions, examine the following examples.

Example 1.

$$\frac{x+y}{\mathrm{x-y}} \div \frac{\mathrm{x}}{y}$$

Solution:

Step 1: Determine the dividend and the divisor. Then find the reciprocal of the divisor.

The divisor is $\frac{\mathrm{x}}{y}$ and its reciprocal is $\frac{\mathrm{y}}{x}$.

Step 2: Multiply the dividend with the reciprocal of the divisor by following the steps on how to multiply rational expressions.

$$\frac{x+y}{\mathrm{x-y}} * \frac{\mathrm{y}}{x} = \frac{(\mathrm{x+y})(\mathrm{y})}{(x-y)(x)}$$

Distributive property

$$= \frac{(\mathrm{x})(\mathrm{y}) + (\mathrm{y})(\mathrm{y})}{(x)(x) - (y)(x)}$$

Simplify

$$= \frac{\mathrm{xy} + y^2}{x^2 - xy}$$

Quotient in reduced form.

$$\frac{x+y}{\mathrm{x-y}} \div \frac{\mathrm{x}}{y} = \frac{\mathrm{xy} + y^2}{x^2 - xy}$$

Example 2:

$$\frac{y^2+5y+6}{y^2+4y+4} \div \frac{\mathrm{y}+1}{y+3}$$

Solution:

Step 1: Determine the dividend and the divisor. Then find the reciprocal of the divisor.

The divisor is $\frac{y+1}{y+3}$ and its reciprocal is $\frac{y+3}{y+1}$.

Step 2: Multiply the dividend with the reciprocal of the divisor by following the steps on how to multiply rational expressions.

$$\frac{y^2+5y+6}{y^2+4y+4} * \frac{\mathrm{y}+3}{y+1} = \frac{(y^2+5y+6)(\mathrm{y}+3)}{(y^2+4y+4)(y+1)}$$

Factoring trinomial

$$= \frac{(y+2)(y+3)(\mathrm{y}+3)}{(\mathrm{y}+2)(y+2)(y+1)}$$

Divide out GCMF (y+2)

$$= \frac{(y+3)(y+3)}{(y+2)(y+1)}$$

Simplify the remaining factors using FOIL method

$$= \frac{y^2+6y+9}{y^2+3y+2}$$

Quotient in reduced form.

$$\frac{y^2+5y+6}{y^2+4y+4} \div \frac{y+1}{y+3} = \frac{y^2+6y+9}{y^2+3y+2}$$

C. Addition and Subtraction of Similar Rational Algebraic Expressions

In adding or subtracting similar rational expressions, add or subtract the numerators and write the answer in the numerator of the result over the common denominator. In symbols,

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}, b \neq 0$$

Example 1:

$$\frac{3x}{y} + \frac{2x}{y}$$

Solution:

Step 1. Write the given as one expression.

$$\frac{3x+2x}{y}$$

Step 2. Combine like terms in the numerator by the given operation.

$$\frac{3x}{y} + \frac{2x}{y} = \frac{3x+2x}{y}$$

$$= \frac{5x}{y}$$

Step 3. Express the answer in reduced form.

$$\frac{3x}{y} + \frac{2x}{y} = \frac{5x}{y}$$

Example 2.

$$\frac{2y-3}{3y^2+y+2} - \frac{-5y-1}{3y^2+y+2}$$

Solution:

Step 1. Write the given as one expression.

$$\frac{2y-3-(-5y-1)}{3y^2+y+2}$$

Step 2. Combine like terms in the numerator by the given operation.

$$\frac{2y-3-(-5y-1)}{3y^2+y+2} = \frac{2y-3+5y+1)}{3y^2+y+2}$$

$$= \frac{7y-2}{3y^2+y+2}$$

Step 3. Express the answer in reduced form.

$$\frac{2y-3}{3y^2+y+2} - \frac{-5y-1}{3y^2+y+2} = \frac{7y-2}{3y^2+y+2}$$

D. Addition and Subtraction of Similar Rational Algebraic Expressions

In adding or subtracting dissimilar rational expressions, change the rational algebraic expressions into similar rational algebraic expressions using the least common denominator or LCD and proceed as in adding similar fractions.

Example 1.

$$\frac{x+y}{x} + \frac{x+y}{y}$$

Solution:

Step 1. Find the LCD of the expressions.

The LCD of the expressions is xy.

Step 2. Find the equivalent expression of each of the given using the LCD as denominator.

(first term)

$$\frac{x+y}{x} = \frac{?}{xy}$$

2a. Divide the LCD by the original denominator

$$\frac{xy}{x} = y$$

2b. Multiply the result in 2a with the original numerator.

$$y(x+y) = xy + y^2$$

2c. The answer in 2b is the missing numerator of the equivalent expression.

$$\frac{x+y}{x} = \frac{?}{xy}$$

$$\frac{x+y}{x} = \frac{xy+y^2}{xy}$$

(second term)

$$\frac{x+y}{y} = \frac{?}{xy}$$

2a. Divide the LCD by the original denominator

$$\frac{xy}{y} = x$$

2b. Multiply the result in 2a with the original numerator.

$$x(x+y) = x^2 + xy$$

2c. The answer in 2b is the missing numerator of the equivalent expression.

$$\frac{x+y}{y} = \frac{?}{xy}$$

$$\frac{x+y}{y} = \frac{x^2+xy}{xy}$$

Step 3. Proceed to perform the operation using the equivalent fractions and using the steps in similar algebraic expressions.

$$\frac{x+y}{x}+\frac{x+y}{y}=\frac{xy+y^2}{xy}+\frac{x^2+xy}{xy}$$

Write as one expression

$$=\frac{xy+y^2}{xy}+\frac{x^2+xy}{xy}$$

$$=\frac{xy+y^2+x^2+xy}{xy}$$

Determine the like terms in the numerator

$$=\frac{2xy+y^2+x^2}{xy}$$

Simplify the numerator.

$$=\frac{x^2+2xy+\ y^2}{xy}$$

Factoring the numerator (trinomial)

$$=\frac{(x+y)(x+y)}{xy}$$

Sum in expanded form

$$=\frac{x^2+2xy+\ y^2}{xy}$$

Example 2:

$$\frac{2}{y^2-2y-3}-\frac{2}{y^2-y-2}$$

Solution:

Step 1. Find the LCD of the expressions.

Factor the denominators.

$$y^2-2y-3=(y-3)(y+1)$$

$$y^2-y-2=(y-2)(y+1)$$

The factors are (y-3), (y+1), (y-2) and (y+1). We will be combining all the factors to get the LCD. If one factor is repeated, we will only use one.

Therefore, the LCD is $(y-3)(y+1)(y-2)$

Step 2. Find the equivalent expressions of each of the given using the LCD as denominator.

(first term)

$$\frac{2}{y^2-2y-3}=\frac{2}{(y-3)(y+1)}$$

$$\frac{2}{(y-3)(y+1)}=\frac{?}{(y-3)(y+1)(y-2)}$$

2a. Divide the LCD by the original denominator in factored form.

$$\frac{(y-3)(y+1)(y-2)}{(y-3)(y+1)} = (y-2)$$

2b. Multiply the result in 2a with the original numerator

$$(y-2)(2) = 2y-4$$

2c. The answer in 2b is the missing numerator of the equivalent expression.

$$\frac{2}{(y-3)(y+1)} = \frac{2y-4}{(y-3)(y+1)(y-2)}$$

(second term)

$$\frac{2}{y^2-y-2} = \frac{2}{(y-2)(y+1)}$$

$$\frac{2}{(y-2)(y+1)} = \frac{?}{(y-3)(y+1)(y-2)}$$

2a. Divide the LCD by the original denominator in factored form.

$$\frac{(y-3)(y+1)(y-2)}{(y-2)(y+1))} = (y-3)$$

2b. Multiply the result in 2a with the original numerator

$$(y-3)(2) = 2y-6$$

2c. The answer in 2b is the missing numerator of the equivalent expression.

$$\frac{2}{(y-2)(y+1)} = \frac{2y-6}{(y-3)(y+1)(y-2)}$$

Step 3. Proceed to perform the operation using the equivalent fractions and using the steps in similar algebraic expressions.

$$\frac{2}{(y-3)(y+1)} + \frac{2}{(y-2)(y+1)} = \frac{2y-4}{(y-3)(y+1)(y-2)} + \frac{2y-6}{(y-3)(y+1)(y-2)}$$

Write as one expression

$$= \frac{2y-4}{(y-3)(y+1)(y-2)} + \frac{2y-6}{(y-3)(y+1)(y-2)}$$

$$= \frac{2y-4+2y-6}{(y-3)(y+1)(y-2)}$$

Determine the like terms in the numerator and simplify

$$= \frac{4y-10}{(y-3)(y+1)(y-2)}$$

Factoring the numerator (trinomial)

$$= \frac{2(2y-5)}{(y-3)(y+1)(y-2)}$$

Sum in expanded form

$$= \frac{4y-10}{(y-3)(y+1)(y-2)}$$

Explore

Activity 8: **Find my Match!**

Directions: Perform the given operation in the following rational algebraic expressions and simplify. Match column A to column B. Use separate sheet of paper for your answers.

Column A	**Column B**
1. $\frac{x}{x-6} + \frac{6}{6-x}$	A. 3x+5
2. $\frac{9x^2}{3x-5} - \frac{25}{3x-5}$	B. $x^2 + 2x - 8$
3. $\frac{3x}{3x-12} * \frac{4x-16}{12x^2}$	C. 1
4. $\frac{x^2-16}{x+2} \div \frac{x-4}{x^2-4}$	D. $\frac{x^2-7x-19}{(x+6)(x+2)}$
5. $\frac{x-7}{x+6} - \frac{2x+5}{x^2+8x+12}$	E. $\frac{1}{3x}$

Deepen

Activity 9: **Can you perform?**

Directions: Perform the given operation in the following rational algebraic expressions and simplify. Use separate sheet of paper for your answers.

1. $\frac{2ab}{3c^2} \cdot \frac{c}{a^2}$

2. $\frac{3d}{3f-12} \cdot \frac{4f-16}{12d}$

3. $\frac{x^2-16}{x+2} \div \frac{x-4}{x^2-4}$

4. $\frac{a^2+a-12}{a^2+3a-4} \div \frac{3-a}{1-a}$

5. $\frac{h^2+8h-15}{3h^2-9h} - \frac{4h+6}{3h^2-9h}$

6. $\frac{2a}{a^2-16} + \frac{3}{a^2+a-12}$

Lesson 4 Solving Problems Involving Rational Algebraic Expressions

Jumpstart

Activity 10: Translate!
Directions: Match the verbal phrase in column A with the corresponding algebraic expression in column B. Use separate sheet of paper for your answers.

A	B
1. A number x subtracted from 2	a. x+2
2. Twice the product of 2 and x	b. 2(2x)
3. A number x divided by 2	c. 2x
4. Two times a number x	d. 2 - x
5. A number x increased by 2	e. $\frac{x}{2}$

Discover

To solve word problems on rational algebraic expressions, we will know how to write equations. There are steps to follow in writing equations and finding the solutions.

Step 1. Read and understand the problem. Identify what is given and what is being unknown. Choose a variable to represent the unknown number.

Step 2. Express the other unknowns, if there are any in terms of the variable chosen in step 1.
Step 3. Write an equation to represent the relationship among the given and the unknown/s.
Step 4. Solve the equation for the unknown and use the solution to find the quantities being asked.
Step 5. Check.

Example 1. Number Problem

If the same number is added to both numerator and denominator of the fraction $\frac{1}{2}$, the result is $\frac{3}{4}$.

Solution:

Step 1. Let x be the number.
Step 2. 1+x = numerator
2+x = denominator
Step 3. Equation

$$\frac{1+x}{2+x}=\frac{3}{4}$$

Step 4.

$$\frac{1+x}{2+x}=\frac{3}{4}$$

Cross multiply

$$4(1+x)=3(2+x)$$

Distributive property

$$4+4x=6+3x$$

Addition property of equality

$$4+(-4)+\ 4x+(-3x)=6+(-4)+3x+(-3x)$$

Combine like terms

$$x=2$$

Hence, the number is 2.
Step 5. Check

$$\frac{1+x}{2+x}=\frac{3}{4}$$
$$\frac{1+2}{2+2}=\frac{3}{4}$$
$$\frac{3}{4}=\frac{3}{4}$$

Example 2. Age Problem
Age problems are algebraic problems that deal with the ages of people currently, in the past or in the future.

Prolem. Five years ago, John's age was half of the age he will be in 8 years. How old is he now?
Solution:
Step 1. Let x be John's age
Step 2. x-5 is John's age five years ago
$\frac{1}{2}(x+8)$ is half of the gae he will be in 8 years
Step 3. $x-5=\frac{1}{2}(x+8)$
Step 4. Solve

$$x-5=\frac{1}{2}(x+8)$$

Cross multiply

$$2(x-5)=x+8$$

Distributive property

$$2x-10=x+8$$

Addition property of Equality

$$2x+(-x)-10+10=x+(-x)+8+10$$

Combine like terms

$$x=18$$

Hence, John's age now is 18 years old.

Step 5. Check

$$x-5=\frac{1}{2}(x+8)$$
$$18-5=\frac{1}{2}(18+8)$$
$$13=\frac{1}{2}(26)$$
$$13=13$$

Example 3. Work Problem
The formula for work problem that involves two person is

$$\frac{1}{t_1}+\frac{1}{t_2}=\frac{1}{t_3}$$

where

t_1 is time taken by the first person
t_2 is time taken by the second person
t_3 is the time taken by both

Problem. Mark can wash the car in 40 minutes and Lawrence can wash the car in 60 minutes. How long will it take for them to wash the car together?

Solution:

Step 1. Assign variables

Let x = time to wash the car together

Step 2. Mark = $\frac{1}{40}$, Lawrence =$\frac{1}{60}$, Mark and Lawrence will wash together = $\frac{1}{x}$

Step 3. Write the equation

$$\frac{1}{40}+\frac{1}{60}=\frac{1}{x}$$

Step 4. Solve.

$$\frac{1}{40}+\frac{1}{60}=\frac{1}{x}$$

Multiply both sides by 120x, the LCM of 40, 60 and x

$$120x\left(\frac{1}{40}+\frac{1}{60}\right)= 120x\left(\frac{1}{x}\right)$$

Distributive property

$$\frac{120x}{40}+\frac{120x}{60}=\frac{120x}{x}$$

Simplify

$$3x+2x=120$$

Combine like terms

$$5x=120$$

Divide both sides by 5

$$\frac{5x}{5}=\frac{120}{5}$$

Simplify

$$x=24$$

Hence, it will take 24 minutes for both of them to wash the car together.

Step 5.Check

$$\frac{1}{40}(24)+\frac{1}{60}(24)=\frac{24}{40}+\frac{24}{60}=\frac{3}{5}+\frac{2}{5}=\frac{5}{5}=1$$

Example 4. Speed/Travel Problem

An object is said to be in uniform motion when it moves without changing its speed or rate.

$$distance = rate\ x\ time$$
$$d=rt$$

Problem. Kenneth won a two-day motorcycle race. He travelled 60 km each day and his average on the second day was doubled than that of the first day. If Kenneth rode for a total of 6 hours, what was his average speed each day?

Solution:

Step 1. Assign variables

Let x = speed on the first day

Step 2.

	distance	Speed (rate)	time
Day 1	60	x	$\frac{60}{x}$
Day 2	60	2x	$\frac{60}{2x}$
Total			6 hours

Step 3. Write the equation

$$\frac{60}{x}+\frac{60}{2x}=6$$

Step 4. Solve

$$\frac{60}{x}+\frac{60}{2x}=6$$
$$2x\left(\frac{60}{x}+\frac{60}{2x}\right)=2x(6)$$
$$\frac{120x}{x}+\frac{120x}{2x}=12x$$
$$120+60=12x$$
$$180=12x$$
$$\frac{180}{12}=\frac{12x}{12}$$
$$15=x$$

Hence, the speed on Day 1 is 15 km/h and on Day 2 is 2x = 2(15) = 30 km/h.

Step 5. Check

$$\frac{60}{15}+\frac{60}{2(15)}=6$$
$$4+2=6$$

Explore

Activity 11: **Complete the Solution!**
Directions: Analyze and solve the following problem by filling in the correct answer. Use separate sheet of paper for your answer.

Problem. One-half of Justine's age two years from now plus one-third of his age three years ago is twenty years. How old is he now?

Solution:

Step 1. Let x = Justine's age now

Step 2. Create an equation using the problem.

(1.)________________ = Justine's age two years from now
(2.)________________ = Justine's age three years ago
(3.)________________ = one-half of age 2 years from now
(4.)________________ = one-third of age 3 years ago

Step 3. Write the equation

(5.)________________ Equation based on the problem

Step 4. Solution:

$$\frac{1}{2}(x+2)+\frac{1}{3}(x-3)=20$$

$$\left(\frac{1}{2}x+1\right)+\left(\frac{1}{3}-1\right)=20$$

Combine like terms and simplify

(6.)________________

Multiply both sides by the LCM

$$6\left(\frac{1}{2}x+\frac{1}{3}x\right)=6(20)$$

Distributive property

(7.)________________

Simplify

$$3x+2x=120$$

Combine like terms

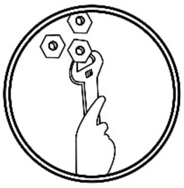
(8.)________________

Divide both sides by 5

(9.)________________

Final answer

(10.)________________



Deepen

Activity 12: **I can solve it!**

Directions: Solve the following problems. Follow the 5 steps in solving word problems. Use separate sheet of paper for your answers.

Problem 1.

The denominator of a fraction is one more than the numerator. If 3 is subtracted to the numerator and to the denominator, the resulting fraction is equivalent to $\frac{1}{2}$. What is the original fraction?

Problem 2.

Jemuel and Lemuel are asked to paint a house. Jemuel can paint the house by himself 16 hours and Lemuel can paint by himself in 12 hours. How long would it take to paint the house if they worked together?

Gauge

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following expressions equates the denominator to 0?

   A. $\frac{x^2+2}{-y+y}$ B. $\frac{3x}{2^0}$ C. $\frac{p}{q}$ D. $\frac{y^2-2y+4y}{y+2}$

2. What value of the **x** makes the expression $\dfrac{x^4-1}{x-1}$ undefined?

   A. 0 B. 1 C. 2 D. 3

3. Which of the following is the correct order of simplifying rational algebraic expressions?

   I. Multiply the remaining factors.
   II. Factor completely the numerator and denominator.
   III. Separate and divide out common factor/s if there is/are any.

   A. I, II, III B. I, III, II C. II, III, I D. III, II, I

4. Which of the following is a rational expression in simplest form?

   A. $\frac{2x}{4y}$ B. $\frac{2y-6}{24}$ C. $\frac{y^2-1}{y^3+1}$ D. $\frac{2a^2+7a-4}{a+2}$

5. Which of the following is the simplest form of $\dfrac{y^3-2y^2+y}{y^3-y}$

A. $\frac{y+1}{y-1}$ B. $\frac{y+1}{1-y}$ C. $\frac{1-y}{y+1}$ D. $\frac{y-1}{y+1}$

6. What are the common factors in the numerator and denominator of $\left(\frac{y^2-9}{y^2+y-20}\right)\left(\frac{y^2-8y+16}{3y-9}\right)$?

   A. y+3 B. y+4 C. (y+3)(y-4) D. (y-3)(y-4)

7. Given $\frac{m+3}{3}$ as one factor of the rational algebraic expression $\frac{m^2-9}{6}$, what is the other factor?

   A. $\frac{m-2}{1}$ B. $\frac{m-3}{2}$ C. $\frac{m-4}{3}$ D. $\frac{m-5}{4}$

8. What factor or factors can be divided out in $\frac{2y-5}{4} \div \frac{y+3}{4}$?

   A. 4 B. y+3 C. 2y-5 D.(y+3)(2y-5)

9. Below are the steps in finding the product of rational expressions. Which is the correct order?

   I. Divide out the GCMF of the numerators and denominators

   II. Find the GCMF of the numerators and denominators

   III. Simplify the remaining factors.

   A. I, II, III B. I, III, II C. II, III, I D. II, I, III

10. What is the least common denominator of $\frac{4}{2xy^2}$ *and* $\frac{5}{4xy}$?

    A. xy^2 B. $2xy^2$ C. $4xy^2$ D. $6xy^2$

11. What is the difference of $\frac{2}{x^2+x} - \frac{3}{x+1}$?

    A. $\frac{2}{x+1}$ B. $\frac{-3}{x+1}$ C. $\frac{2-3x}{x}$ D. $\frac{2-3x}{x(x+1)}$

12. What is the sum of $\frac{3}{x} + \frac{5}{x-1}$?

    A. $\frac{8x-1}{x(x+1)}$ B. $\frac{8x-2}{x(x+1)}$ C. $\frac{8x-3}{x(x+1)}$ D. $\frac{8x-4}{x(x+1)}$

13. John's father is twice as old as Jay. In 20 years, John will be two-thirds as old as his father. How old is Jay now?

    A. 10 years old B. 15 years old C. 20 years old D. 25 years old

14. Michael can do his project in x hours, what part of the job can be completed after 2 hours?

    A. x-2 B. x+2 C. $\frac{2}{x}$ D. $\frac{x}{2}$

15. It took six hours for Christian to clean the attic while Lester took 12 hours to do the same job. If Christian and Lester worked together, how long had it taken them to complete the job?

    A. 3 B. 4 C. 6 D. 11

Great job! You are done with this module.

References

A. Books

Emmanuel P. Abuzo, Merden L.Bryant, Jem Boy B.Cabrera, Belen P. Caldez, Melvin M. Callanta, Anastacia Proserfina I. Castro, Alicia R. Halabaso, Sonia P. Javier, Roger T. Nocom, and Concepcion S. Ternida, (2013), Mathematics Grade 8 Learner's Module, Quezon City, Book Media Press, Inc., and Printwell, Inc.

Orlando A. Oronce, Marilyn O. Mendoza. (2015). Exploring Math 8, Worktext in Mathematics. Sampaloc, Manila: Rex Book Store, Inc.

B. Other References

Hand-outs, DepEd Regional Mass Training on Critical Content on Mathematics 8, May 21-June 1, 2018.

For inquiries or feedback, please write or call:

Department of Education – SDO La Union
Curriculum Implementation Division
Learning Resource Management Section
Flores St. Catbangen, San Fernando City La Union 2500
Telephone: (072) 607 - 8127
Telefax: (072) 205 - 0046
Email Address:
launion@deped.gov.ph
lrm.launion@deped.gov.ph