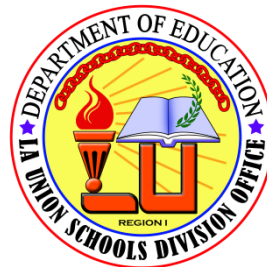


Senior High School



General Mathematics

Module 1: Functions



AIRs - LM

GENERAL MATHEMATICS

Module 1: Functions

Second Edition, 2021

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Region I

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SHS

General Mathematics

Module 1: Functions



Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



Target

A focal concept of extreme importance in modern mathematics is the idea of functions. Functions which deal with corresponding relationship between the elements of two sets, are useful concepts throughout the mathematics curriculum.

Functions explain the input and output effect. Knowing the input means having an output. In this case, you will revisit and apply some basic concepts and ideas about functions, and also learn to represent real life situations using functions including piecewise functions.

In this module, you will be familiarized in evaluating functions and in performing addition, subtraction, multiplication, division and composition of functions. The concept on the operations on algebraic expression are applied in order to perform the said operations. These operations on functions are fully appreciated in solving problems involving functions. This module will also help you visualize the problems in terms of the information given and the data to be found and help you solve problems involving functions.

After going through this module, you are expected to:

1. represent real-life situations using functions including piecewise functions **(M11GM-Ia-1)**,
2. evaluate a function **(M11GM-Ia-2)**,
3. perform addition, subtraction, multiplication, division, and composition of functions **(M11GM-Ia-3)**; and
4. solve problems involving functions **(M11GM-Ia-4)**.

Learning Objectives:

1. recall the concepts of function and relation
2. represent real-life situations using functions including piecewise functions
3. apply the law of substitution in evaluating functions
4. find the sum, difference, product, quotient and composition of functions
5. solve real-life problems involving functions

Pretest

Directions: Read each item carefully and select the correct answer. Write the letter of your choice in separate sheet of paper.

1. Which set of coordinates describes a function?
 - A. $\{(6, 3), (4, 5), (2, 3), (0, 5)\}$
 - B. $\{(2, 4), (2, -4), (4, 8), (6, 2)\}$
 - C. $\{(4, -3), (-4, -6), (4, 3), (-4, 6)\}$
 - D. $\{(-5, -1), (-3, -3), (-1, -5), (-5, -7)\}$
 2. For Saturday's debate tournament, Sarah ordered 3 cookies for each student participant and a tray of 30 cookies for the sponsors' hospitality room. This relationship can be expressed by the function $f(s) = 3s + 30$, where s is the number of student participants. Which is the dependent quantity in this functional relationship?
 - A. The number of sponsors
 - B. The number of trays ordered
 - C. The number of cookies ordered
 - D. The number of student participants
 3. A canned drink will be made using 20 in^2 of aluminum. Let h be the height and r be the radius of the can. Write a function in terms of r that can represent the volume of the can.
 - A. $V = \pi r^2 h$
 - B. $V = r(20 - \pi r^2)$
 - C. $V = h(20 - \pi r^2)$
 - D. $V = r^2(20 - \pi r)$
 4. Evaluate $f(x) = 8x + 2$, $f(3)$.
 - A. 21
 - B. 24
 - C. 26
 - D. 31
 5. Evaluate $f(x) = 2x + 2$, $f(x - 1)$.
 - A. $2x$
 - B. $2x + 1$
 - C. $x + 3$
 - D. $2x - 1$
- For Numbers 6 – 7, refer to the function below:
- $$f(x) = \begin{cases} x^2 + 2, & \text{if } x \geq 12 \\ 2x^2 - 1, & \text{if } x < 12 \end{cases}$$
6. What is $f(5)$?
 - A. 39
 - B. 40
 - C. 44
 - D. 49
 7. What is $f(12)$?
 - A. 140
 - B. 146
 - C. 277
 - D. 287
 8. Given $f(x) = \sqrt{x - 6}$, what is $f(7)$?
 - A. 1
 - B. 2
 - C. 3
 - D. 5

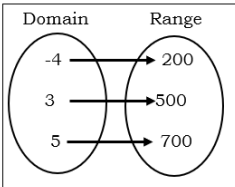
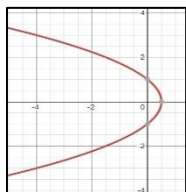
- LU General Mathematics Module1



Jumpstart

Function VS Not Function

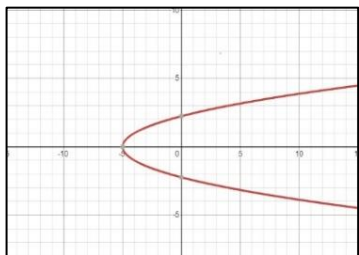
The table below shows how relations and functions are the same and how they are different.

Examples	Relation	Function	Reason										
1. $R = \{(2, a), (9, b), (2, c)\}$	✓	✗	It is a relation but not a function because a function is a set of ordered pairs (x,y) where no two ordered pairs have the same x -value but different y -values. In the example we have two ordered pairs specifically $(2, a)$ and $(2, c)$ that have the same x -value which is 2 but different y -values which are a and c .										
2. $F = \{(1,2), (2,7),(3,8),(4,2)\}$	✓	✓	It is a function because no two ordered pairs have the same x -value though they have the same y -values. It is also a relation because all functions are relation.										
3. <table border="1"><thead><tr><th>YEAR</th><th>EXPENSES</th></tr></thead><tbody><tr><td>2016</td><td>Php 400, 000</td></tr><tr><td>2017</td><td>Php 500, 000</td></tr><tr><td>2018</td><td>Php 400, 000</td></tr><tr><td>2019</td><td>Php 700, 000</td></tr></tbody></table>	YEAR	EXPENSES	2016	Php 400, 000	2017	Php 500, 000	2018	Php 400, 000	2019	Php 700, 000	✓	✓	It is a function because no one value in the year is paired to two values in the expenses. It is a relation since if it is a function then automatically it is a relation.
YEAR	EXPENSES												
2016	Php 400, 000												
2017	Php 500, 000												
2018	Php 400, 000												
2019	Php 700, 000												
4. <div><div>Domain</div><div>Range</div><div></div></div>	✓	✓	It is a function since every domain is mapped to a value in the range. Since it is a function, it is also a relation.										
5. <div></div>	✓	✗	It is not a function because through the vertical line test the line intersects the graph in more than one point.										

Activity 1: Complete Me

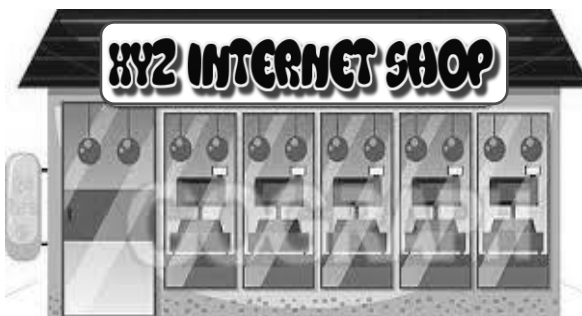
Direction: Complete the table below by determining whether each relation is a function or not. Put an **X** mark on the space that corresponds to your answer then justify your answer by writing your reason.

Justify your answer by writing your reason.

RELATION	FUNCTION	NOT A FUNCTION	REASON								
1. $\{(1, 3), (3, 4), (5, 7)\}$											
2. <table><tr><th>Input</th><th>Output</th></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>9</td></tr><tr><td>5</td><td>10</td></tr></table>	Input	Output	1	3	3	9	5	10			
Input	Output										
1	3										
3	9										
5	10										
3. $\frac{x^2}{9} - \frac{y^2}{25} = 16$											
4. 											
5. Number of minutes spent studying vs. grade on the exam											

Activity 2: Good Deal

Are you aware of the prices by which the different Internet shops charge their clients? You can use your knowledge and skills about linear functions to decide which shop gives really good deal for internet rental.



The two internet shops announced the following promotions, regarding the rental fee of their computers.

ABC Internet Café: Membership fee of ₱40 plus ₱15 per hour or a fraction thereof

XYZ Internet Shop: No membership fee, only ₱25 charge per hour or a fraction thereof

Which of the Internet shop offers a better package?

Complete the table and write a function to show the relationship between the number of hours used and the cost of rent for each computer shop.

Number of Hours Used	ABC Internet Café	XYZ Internet Shop
1		
2		
3		
4		
5		

1. What is the independent quantity in the variations offered by each Internet shop? _____
2. What is the dependent quantity in the conditions offered by each Internet shop? _____
3. What is the equation for the rental fee of using computer in ABC Internet Café?

4. What is the equation for the rental fee of using computer in XYZ Internet Shop?

5. If you will use the computer for 6 hours, which shop is preferable to use? Why? _____



Discover

Representation of Functions

Functions can be represented in five (5) ways:

1. Functions and relations as a set of ordered pairs

A function can be represented by a machine with an “input” and an “output”. But we can also represent these input and output values as a set of ordered pairs. For example, if the input value -1 correspond to output 3, then $(-1, 3)$ is its ordered pairs representation.

Example 1: Which of the following relations are functions?

- (a) $f = \{(1, 4), (4, 4), (3, 6), (6, 7)\}$
- (b) $g = \{(4, 6), (4, 7), (9, 8), (2, 9), (6, 10)\}$
- (c) $h = \{(1, 3), (2, 6), (3, 9), \dots, (n, 3n), (n+1, 3n+3) \dots\}$

Solution: The relations f and h are functions because no two ordered pairs have the same x-value but different y-values, while g is not a function because $(4, 6)$ and $(4, 7)$ are ordered pairs with same x-value but different y-values.

2. Functions and relations as a table of values

In a function, we can use the notation $f(x) = y$ for each ordered pair, (x, y) in f . We can also organize these ordered pair as a table. Thus, a function f can be represented by table of values below:

x	-2	-1	3	6
$f(x)$	-2	-2	6	7

Example 2: The table relates a school (x) to the number of delegates in the IRAA (y) in the year 2020. Does this relation define a one-to-one function?

Solution: The table relating a school (x) to the number of delegates (y), does not defined a one-to-one function because for the value 10 in the range of f , there are two values in the domain namely SJNHS and SGNHS.

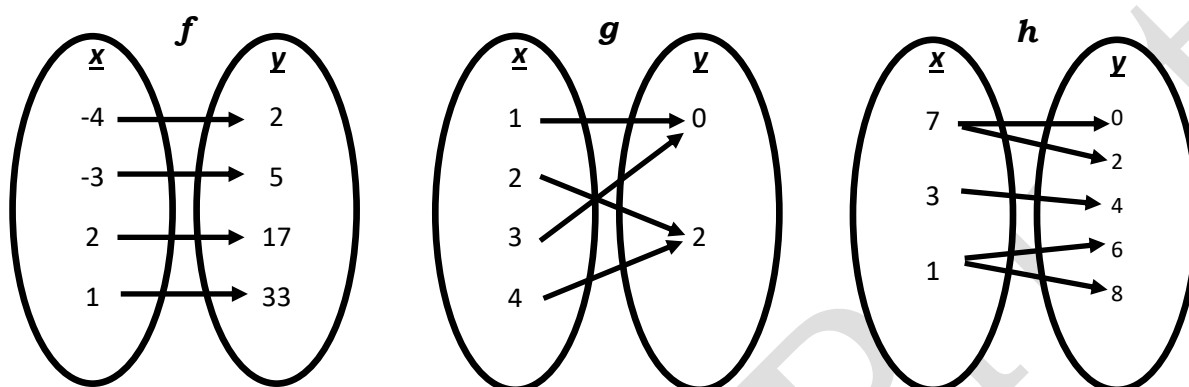
School (X)	Number of Delegates (Y)
CNHS	32
SJNHS	10
SSIS	15
SGNHS	10
WNHS	25

Note: The table represents function but not one-to-one function. One-to-one function is another type of function where each element of the domain corresponds to exactly one element of the range or vice versa.

3. Functions and relations as a mapping diagram

Aside from the table of values, a mapping diagram may also be used to represent the ordered pairs in a relation or function. In this case, the relation or function is represented to the set of all the connections by the arrows.

Example 3: Which of the following mapping diagrams represent function/s?



Solution: The relations f and g are functions because each $x \in X$ corresponds to a unique $y \in Y$. The relation h is not a function because there is at least one element in X for which there is more than one corresponding y -value. For example, $x = 7$ corresponds to $y = 0$ and 2 . Similarly, $x = 1$ corresponds to both $y = 6$ and 8 .

4. Functions as a graph in the Cartesian plane

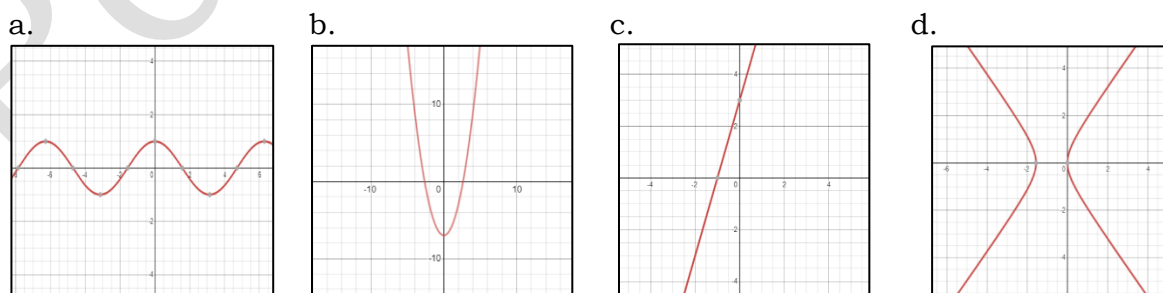
As shown above, ordered pairs can be represented by a table of values or a mapping diagram. A graph in a Cartesian plane is another representation of a relation or function. A quick way to determine whether a given graph represents a function is through the vertical line test.

The Vertical Line Test

A graph represents a function if and only if each vertical line intersects the graph at most once.

If a vertical line $x = a$ intersects a graph twice, say at (a, b) and at (a, c) , then the graph cannot represent a function because two different y -values correspond to $x = a$.

Example 4: Which of the following graphs represent functions?



Solution: Graphs (a), (b), (c) are graphs of functions while (d) is not because the graph did not pass the vertical line test. Each vertical line intersects the graphs at two points.

5. Functions and relations as an equation

Example 5: Which of the following represents a function?

a. $y = 2x + 3$ b. $y = x^2 - 3x + 4$ c. $x^2 + y^2 = 1$ d. $y = \sqrt{x + 1}$

Solution: All are relations but not all are functions. Equation (c) is not a function because we can find an x-value that corresponds to more than one y-values (ex. If $x = 0$, then y can be + 1 or -1).

Functions as Representations of Real-Life Situations

Functions can often be used to model real life situations. If you can identify an appropriate functional model then it will lead you to a better understanding of various phenomena.

Definition.

1. A linear function is a function of the form $f(x) = ax + b$, where $a \neq 0$.
2. A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.
3. A polynomial function of degree n is a function of the form.
 $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x^1 + a_0$ where $n \neq 0$

Below are examples of functions in real life situations:

Example 1: One hundred meters of fencing is available to enclose a rectangular area next to a river (see figure). Give a function A that can represent the area that can be enclosed in terms of x.

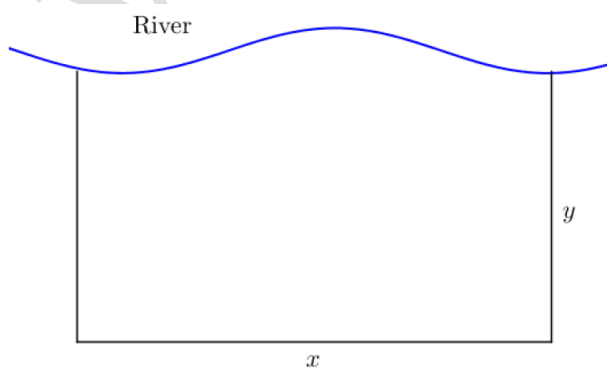


Figure 1. Representation of rectangular area next to a river

Solution: The area of the rectangular enclosure is $A = xy$. We will write this as a function of x. Since only 100 m of fencing is available, then $x + 2y = 100$ or $y = \frac{100-x}{2} = 50 - 0.5x$. Thus,

$$A(x) = x(50 - 0.5x)$$

$$A(x) = 50x - 0.5x^2$$

DEFINITION

A **piecewise function** or a **compound function** is a function built from pieces of different functions over different intervals. It is a function composed of multiple subfunctions where each subfunction applies to a certain interval of the main function domain. We can represent a piecewise function as follows:

$$F(x) = \begin{cases} \text{function 1} & \text{if } x \text{ is in the domain 1} \\ \text{function 2} & \text{if } x \text{ is in the domain 2} \\ \text{function 3} & \text{if } x \text{ is in the domain 3} \end{cases}$$

Example 2: A slice of cassava cake costs ₱30. However, if you buy more than 10 slices of cassava cake, it will be ₱25 per slice. Use a piecewise function to represent the cost in terms of the slices of cassava cake bought.

Solution: If you bought at most 10 slices of cassava cake, you will pay ₱30 per slice. But if you will buy more than 10, you will pay ₱25 per slice of cassava cake, Therefore

$$f(x) = \begin{cases} 25x, & \text{if } x > 10 \\ 30x, & \text{if } 0 < x \leq 10 \end{cases}$$

Example 3: A user is charged P 500 monthly for a particular mobile plan, which includes 200 free text messages. Messages in excess of 200 are charged P1 each. Represent the monthly cost for text messaging using the function $t(m)$, where m is the number of messages sent in a month.

Solution: $t(m) = \begin{cases} 500 & \text{if } 0 < m \leq 200 \\ 500 + (m - 200) & \text{if } m > 200 \end{cases}$

Evaluating Functions

Evaluating function is the process of determining the value of the function at the number assigned to a given variable. Just like in evaluating algebraic expressions, to evaluate function you just need to:

- replace each letter in the expression with the assigned value,
- perform the operations in the expression using the correct order of operations, and
- simplify the resulting numerical expression.

Example 1: Evaluate the function $f(x) = x^2 + 3x - 5$ for each replacement value of x .

- a. -3 b. 0 c. 2 d. 5

Solution: To evaluate the function, simply replace x in $f(x)$ with the given replacement value, then simplify.

a. If $x = -3$ then,

$$\begin{aligned} f(-3) &= (-3)^2 + 3(-3) - 5 \\ &= 9 - 9 - 5 \\ &= -5 \end{aligned}$$

b. If $x = 0$ then,

$$\begin{aligned} f(0) &= (0)^2 + 3(0) - 5 \\ &= 0 - 0 - 5 \\ &= -5 \end{aligned}$$

c. If $x = 2$ then,

$$\begin{aligned} f(2) &= (2)^2 + 3(2) - 5 \\ &= 4 + 6 - 5 \\ &= 5 \end{aligned}$$

d. If $x = 5$ then,

$$\begin{aligned} f(5) &= (5)^2 + 3(5) - 5 \\ &= 25 + 15 - 5 \\ &= 35 \end{aligned}$$

Example 2: If $x = -4$, evaluate the following functions:

- a. $f(x) = 2x - 3$ b. $g(x) = 2x + 1$
 c. $h(x) = \sqrt{25 - x^2}$ d. $k(x) = 2x^2 - 5x + 1$

Solution: To evaluate the function, simply replace x in $f(x)$ with the given replacement value, then simplify.

$$\begin{aligned} \text{a. } f(-4) &= 2x - 3 \\ &= 2(-4) - 3 \\ &= -8 - 3 = -11 \end{aligned}$$

$$\begin{aligned} \text{b. } g(-4) &= 2x + 1 \\ &= 2(-4) + 1 \\ &= -8 + 1 = -7 \end{aligned}$$

$$\begin{aligned} \text{c. } h(-4) &= \sqrt{25 - x^2} \\ &= \sqrt{25 - (-4)^2} \\ &= \sqrt{25 - 16} = \sqrt{9} = \pm 3 \end{aligned}$$

$$\begin{aligned} \text{d. } k(-4) &= 2x^2 - 5x + 1 \\ &= 2(-4)^2 - 5(-4) + 1 \\ &= 2(16) + 20 + 1 = 53 \end{aligned}$$

Example 3: Mark has 280 pesos in his saving account and adding 40 pesos each week. Write an equation relating T (total amount saved) and n (number of weeks Mark has been adding money) and find the total amount Mark saves after 12 weeks.

Solution:

$$\begin{aligned} \text{Equation: } T(n) &= 280 + 40n \\ \text{Total savings: } T(n) &= 280 + 40n \\ &= 280 + 40(12) \\ &= 280 + 480 \\ &= \text{P}760.00 \end{aligned}$$

Operations on Functions

Sum, Difference, Product and Quotient of Functions

Let f and g be any two functions, the sum $(f + g)$, difference $(f - g)$, product $(f \cdot g)$, and quotient $\left(\frac{f}{g}\right)$, are functions whose domains are set of real numbers common to D_f and D_g ($D_f \cap D_g$) and defined as follows:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(f \cdot g)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$.

Example 1: Let $f(x) = 2x^3 - 5x^2 - x + 6$ and $g(x) = 2x^2 - 13x + 15$.

Find a. $(f + g)(x)$ b. $(f - g)(x)$ c. $(f \cdot g)(x)$ d. $\left(\frac{f}{g}\right)(x)$

Solution:

$$\begin{aligned} \text{a. } (f + g)(x) &= (2x^3 - 5x^2 - x + 6) + (2x^2 - 13x + 15) && \text{Apply Sum of a Function} \\ &= 2x^3 - 5x^2 - x + 6 + 2x^2 - 13x + 15 \\ &= 2x^3 - 5x^2 + 2x^2 - x - 13x + 6 + 15 && \text{Arrange terms in descending order} \\ &= 2x^3 - 3x^2 - 14x + 21 && \text{and simplify} \end{aligned}$$

$$\begin{aligned}\text{b. } (f - g)(x) &= (2x^3 - 5x^2 - x + 6) - (2x^2 - 13x + 15) \\ &= 2x^3 - 5x^2 - x + 6 - 2x^2 + 13x - 15 \\ &= 2x^3 - 5x^2 - 2x^2 - x + 13x + 6 - 15 \\ &= 2x^3 - 7x^2 + 12x - 9\end{aligned}$$

Apply Subtraction of a Function

Arrange terms in descending order and simplify

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= (2x^3 - 5x^2 - x + 6) \cdot (2x^2 - 13x + 15) \\ &= 2x^3(2x^2 - 13x + 15) - 5x^2(2x^2 - 13x + 15) - \\ &\quad x(2x^2 - 13x + 15) + 6(2x^2 - 13x + 15) \\ &= 4x^5 - 26x^4 + 30x^3 - 10x^4 + 65x^3 - 75x^2 - 2x^3 + \\ &\quad 13x^2 - 15x + 12x^2 - 78x + 90 \\ &= 4x^5 - 26x^4 - 10x^4 + 30x^3 + 65x^3 - 2x^3 - 75x^2 + \\ &\quad 13x^2 + 12x^2 - 15x - 78x + 90 \\ &= 4x^5 - 36x^4 + 93x^3 - 50x^2 - 93x + 90\end{aligned}$$

Apply Product of a Function
Apply Distributive Property

Arrange terms in descending order and simplify

$$\begin{aligned}\text{d. } \left(\frac{f}{g}\right)(x) &= \frac{(2x^3 - 5x^2 - x + 6)}{(2x^2 - 13x + 15)} \\ &= \frac{(2x - 3)(x + 1)(x - 2)}{(2x - 3)(x - 5)} \\ &= \frac{(x + 1)(x - 2)}{(x - 5)} \text{ or } \frac{x^2 - x - 2}{(x - 5)}\end{aligned}$$

Example 2: Let $f(x) = 2x^3 - 5x^2 - x + 6$ and $g(x) = 2x^2 - 13x + 15$.

Find a. $(f + g)(2)$ b. $(f - g)(-3)$ c. $(f \cdot g)(2)$ d. $\left(\frac{f}{g}\right)(7)$

Solution:

$$\begin{aligned}\text{a. } (f + g)(x) &= (2x^3 - 5x^2 - x + 6) + (2x^2 - 13x + 15) \\ &= 2x^3 - 3x^2 - 14x + 21 \\ (f + g)(2) &= 2(2)^3 - 3(2)^2 - 14(2) + 21 \\ &= 2(8) - 3(4) - 14(2) + 21 \\ &= 16 - 12 - 28 + 21 \\ &= -3\end{aligned}$$

$$\begin{aligned}\text{b. } (f - g)(x) &= (2x^3 - 5x^2 - x + 6) - (2x^2 - 13x + 15) \\ &= 2x^3 - 7x^2 + 12x - 9 \\ (f - g)(-3) &= 2(-3)^3 - 7(-3)^2 + 12(-3) - 9 \\ &= 2(-27) - 7(9) + 12(-3) - 9 \\ &= -54 - 63 - 36 - 9 \\ &= -162\end{aligned}$$

$$\begin{aligned}\text{c. } (f \cdot g)(x) &= (2x^3 - 5x^2 - x + 6) \cdot (2x^2 - 13x + 15) \\ &= 4x^5 - 36x^4 + 93x^3 - 50x^2 - 93x + 90 \\ (f \cdot g)(2) &= 4(2)^5 - 36(2)^4 + 93(2)^3 - 50(2)^2 - 93(2) + 90 \\ &= 4(32) - 36(16) + 93(8) - 50(4) - 93(2) + 90 \\ &= 128 - 576 + 744 - 200 - 186 + 90 \\ &= 0\end{aligned}$$

$$\text{d. } \left(\frac{f}{g}\right)(x) = \frac{(x+1)(x-2)}{(x-5)} \quad \text{or} \quad \left(\frac{f}{g}\right)(x) = \frac{x^2 - x - 2}{(x-5)}$$

$$\left(\frac{f}{g}\right)(7) = \frac{(7+1)(7-2)}{(7-5)} = \frac{8(5)}{2} = \frac{40}{2} = 20 \quad \left(\frac{f}{g}\right)(7) = \frac{(7)^2 - (7) - 2}{(7-5)} = \frac{49 - 7 - 2}{2} = \frac{40}{2} = 20$$

Composition of Functions

The **composition of the function f with g** is denoted by $(f \circ g)$ and is defined by the equation: $(f \circ g)(x) = f(g(x))$

The domain of the composition function $f \circ g$ is the set of all x such that

1. x is in the domain of g and
2. $g(x)$ is in the domain of f

Example 1: Given $f(x) = 4x - 5$ and $g(x) = x^2 + 4$

Find a. $(f \circ g)(x)$ b. $(g \circ f)(x)$

Solution:

$$\begin{aligned}\text{a. } (f \circ g)(x) &= f(g(x)) = 4(g(x)) - 5 \\ &= 4(x^2 + 4) - 5 \\ &= 4x^2 + 16 - 5 \\ &= \mathbf{4x^2 + 11}\end{aligned}$$

$$\begin{aligned}\text{b. } (g \circ f)(x) &= g(f(x)) = (f(x))^2 + 4 \\ &= (4x - 5)^2 + 4 \\ &= 16x^2 - 40x + 25 + 4 \\ &= \mathbf{16x^2 - 40x + 29}\end{aligned}$$

Problem Involving Functions

In solving problems involving functions, George Polya identifies four basic principles of problem solving.

1. **Understand the problem:** Students are often stymied in their efforts to solve problems simply because they don't understand it fully or even in part. In order to show an understanding of the problem, you, of course, need to read the problem carefully. Once the problem is read, you need to list all the components and data that are involved. This is where you will be assigning your variable.
2. **Devise a plan (translate):** There are many ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. Setting up an equation, drawing a diagram, and making a chart are some ways you can go about solving your problem.
3. **Carry out the plan (solve):** This is where you solve the equation you came up with in your 'devise a plan' step.
4. **Look back (check and interpret):** Check if you used all your information and that the answer makes sense.



Modeling real world problems especially those that require optimization is one of the important applications of the study of functions.

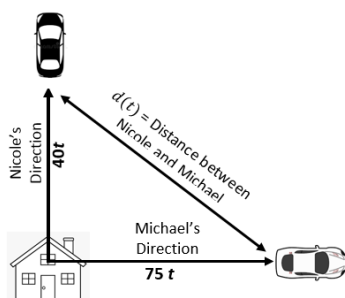
Guidelines in Making Models of Functions:

1. Read the problem carefully. Be sure to understand the problem very well. Write down the given information in the problem in the context that you understand.
2. Determine the independent and dependent variables respectively
3. Write the equation for the relation.
4. Solve the problem, using the equation
5. Check the answer, based on the given information.
6. Make a conclusion.

Example 1: Nicole and Michael drive away from their house. Nicole went North and Michael went East. Nicole's speed is 40 kph and Michael's speed is 75 kph. Express the distance between the cars as a function of time.

Solution:

- a. Every hour, Nicole travels 40 km away from their house while Michael travels 75 km. At any time t from the time they started, Nicole and Michael will be $40t$ km and $75t$ km away from their house, respectively. Since their respective directions are at right angles with each other, then the distance between them can be computed using the Pythagorean Theorem as shown below.



$$\begin{aligned}
 [d(t)]^2 &= (40t)^2 + (75t)^2 \\
 d(t) &= \sqrt{1600t^2 + 5625t^2} \\
 d(t) &= \sqrt{7225t^2} \\
 d(t) &= 85t
 \end{aligned}$$

The distance between them at any time t is given by $d(t) = 85t$. Note that you only consider the positive root of the equation since distance cannot be negative

Example 2: Silangan Network charges ₱500 monthly cable connection fee plus ₱125 for each hour of pay-per-view (PPV) event regardless of a full hour or a fraction of an hour.

- a. Construct table of values that will show a mode of payment for consumers who may spend x number of hours in watching PPV events.
- b. What is the monthly bill of a customer who watched $1/4$ hours of PPV events?
- c. What is the monthly bill of a customer who watched 7.3 hours of PPV events?
- d. What is the monthly bill of a customer who watched 23 hours of PPV events?

Solution:

- a. The table of values that shows the mode of payment for customers who may spend x number of hours in watching PPV events is given below.

<i>Number of Hours Spent in PPV Event (x)</i>	<i>Monthly Cable Connection Fee</i>	<i>Amount of PPV</i>	<i>Total Payment $f(x)$</i>
$0 < x \leq 1$	₱500	(₱125)(1)	₱625
$1 < x \leq 2$	₱500	(₱125)(2)	₱750
$2 < x \leq 3$	₱500	(₱125)(3)	₱875
$3 < x \leq 4$	₱500	(₱125)(4)	₱1000
$4 < x \leq 5$	₱500	(₱125)(5)	₱1125
$n - 1 < x \leq n$	₱500	(₱125)(n)	₱500 + ₱125(n)

- a. The monthly bill of a customer who watched $1/4$ hours of PPV events can be represented by $0 < x \leq 1$. The total payment is $₱500 + ₱125(1) = ₱500 + ₱125 = ₱625$
- b. The monthly bill of a customer who watched 7.3 hours of PPV events can be represented by $7 < x \leq 8$. The total payment is $₱500 + ₱125(8) = ₱500 + ₱1,000 = ₱1,500$.
- c. The monthly bill of a customer who watched 23 hours of PPV events can be represented by $22 < x \leq 23$. The total payment is $₱500 + ₱125(23) = ₱500 + ₱2,875 = ₱3,375$.



Explore

Activity 1

Direction: Write F if the given is a function and NF if it is not a function. Write your solution in a separate paper.

1. $M = \{(-1, 5), (1, -5), (2, -3), (3, -4)\}$

2. $Z = \{(0, 1), (0, -1), (2, 4), (2, 5)\}$

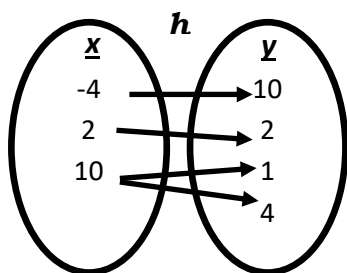
3.

x	-2	-1	0	1	2
y	-2	-1	0	1	2

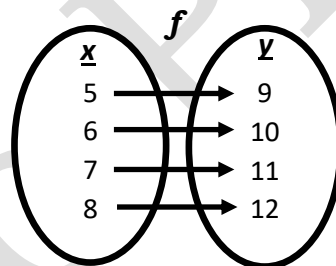
4.

x	0.15	0.20	0.25	0.40	0.15
y	0	3	6	9	0

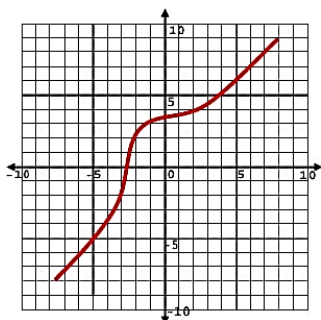
5.



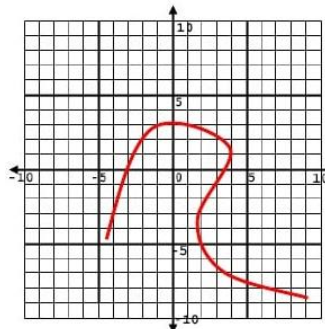
6.



7.



8.



9. $y = 9 - 3(x + 2)$

10. $2x = y^2$

Activity 2

Direction: Evaluate each function. Write your solution in a separate paper.

1. Given the function $f(x) = 4x + 2$ evaluate $f(-1)$.

2. Given the function $h(n) = 3n - 4$ evaluate $h(-6)$

3. Given the function $w(a) = a - 1$ evaluate $w(0)$.

4. Given the function $f(x) = 2x + 1$ evaluate $f(3)$

5. Given the function $g(x) = 3x - 5$ evaluate $g(-2)$.

Activity 3

Direction: Perform the following operations of each given functions. Write your solution in a separate paper.

a. $(f + g)(x)$ b. $(f - g)(x)$ c. $(f \cdot g)(x)$ d. $\left(\frac{f}{g}\right)(x)$ e. $(f \circ g)(x)$

1. $f(x) = x + 1$; $g(x) = 2x - 3$

2. $f(x) = 3x - 1$; $g(x) = 4 - 2x$

3. $f(x) = x^2 + 1$; $g(x) = x - 3$

Activity 4

Direction: Solve what is asked in each item. Write your solution in a separate paper.

- 1. SHOE SIZE:** The following function definitions describe the relationship between the shoe sizes in the Philippines, Korea, and the US

- $f(x) = 3x + 12$ where x is a shoe size in the Philippines and $f(x)$ is a shoe size in Korea.
- $g(x) = \frac{1}{3}x - 3$ where x is a shoe size in Korea and $g(x)$ is a shoe size in US

- If you buy a shoe size of 9 in the Philippines, what will be the corresponding shoe size in Korea?
- If you buy a shoe size of 39 in the Korea, what will be the corresponding shoe size in US?
- What would be the equation of the shoe size from the Philippine to the shoe size of US?
- If you buy a shoe size of 5 in the Philippines, what will be the corresponding shoe size in the US?
- If you buy a shoe size of 11 in the US, what will be the corresponding shoe size in the Philippines?

- 2. BONUS:** Ryan earns a bonus of ₱50.00 for each sale that he makes if the number of sales, s , in a month is 20 or less. He earns an extra ₱40.00 for each additional sale if he makes more than 20 sales in a month.

- Write a piecewise function $B(s)$ that models Ryan's bonus for making " s " sales.
- In August, Ryan made 18 sales. Find his bonus for August.
- In September, Ryan made 25 sales. Find his bonus for September.



Deepen

Additional Activity: Plan Me (Performance Task)

Direction: Read and analyze the situation below and answer the problem in a separate sheet of paper. (20 points)

- Last 2008, the provincial planning department of the Provincial Government of La Union started a tree planting program to beautify the Province of La Union. Below are the data that show the total number of trees (including new trees planted) in the province since the program began.

Year	2008	2009	2010	2011	2012	2015	2017
Number of Trees	600	660	720	780	840	1020	1140

You are a member of the Provincial Planning Department and your job is to decide if the department's plan to continue planting the same number of trees each year until the goal of a total of at least 10,000 trees was reached.

- Write a function rule for the problem.
 - Assuming that no trees die, what should be the total number of trees in La Union in the year 2025?
- CSI La Union charges ₱25 for the first hour or a fraction of an hour as parking fee. An additional ₱15 is charged for every additional hour or a fraction of an hour of parking. The parking area operates from 7 A.M. to 10:30 P.M every day.
 - Write a function rule for the problem.
 - How much will be charged to a car owner who parked a car from 7 A.M. to 1:30 P.M.?
 - A right circular cylinder of height h and a radius r is inscribed in a right circular cone with a height of 10 ft and a base with the radius 6 ft.
 - Express the height h of the cylinder as a function of r .
 - Express the volume V of the cylinder as a function of r .
 - Find two positive integers that satisfy the conditions below.
 - Find the dimensions of the largest rectangular garden that can be enclosed by 60 m of fencing.
 - What is the shape of a rectangle with a given perimeter when its area is a maximum?

Rubrics for the Scoring of the Activity

	5	4	3	2	1
Qualitative Descriptions	Complete and correct solution and answer	Incomplete but correct solution and answer	Correct solution but wrong answer	Correct answer but wrong solution	Wrong solution and wrong answer



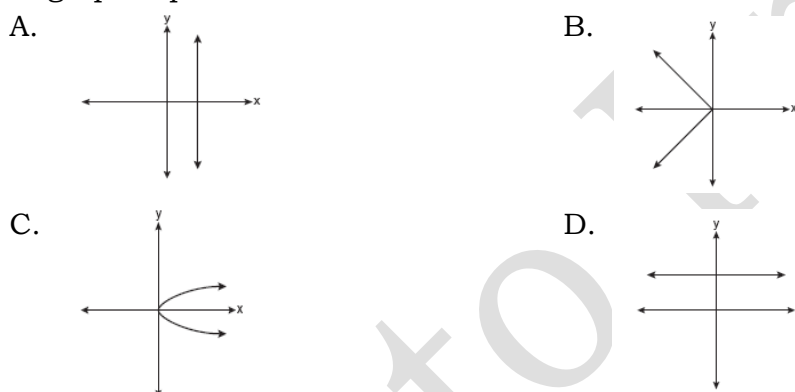
Gauge

Directions: Read each item carefully and select the correct answer. Write the letter of your choice on a separate sheet of paper.

1. What type of function is $f(x) = 25$?

A. Linear function
B. Rational function
C. Quadratic function
D. Constant function

2. Which graph represents a function?



3. Robin kept a log of the number of hours he spent birdwatching during September and the cumulative number of birds he observed.

# of hours (x)	1	3	5	8	10	15
# of birds seen $f(x)$	4	10	16	25	31	46

Which equation best describes the relationship between the number of hours spent birdwatching and the number of birds seen?

A. $f(x) = x + 3$
B. $f(x) = 2x + 2$
C. $f(x) = 3x + 1$
D. $f(x) = 4x$

4. Let x be the annual gross earnings and $T(x)$ be the community tax due defined by the piecewise function:

$$T(x) = \begin{cases} 0 & \text{if } 0 < x \leq 120,000 \\ 0.01(x - 120,000) & \text{if } 120,000 < x \leq 240,000 \\ 0.05(x - 120,000) & \text{if } x > 240,000 \end{cases}$$

What is the community tax due given the annual gross earnings of ₱435,000.00?

A. ₱3,150
B. ₱15,750
C. ₱15,570
D. ₱115,650

5. What is $f(20)$ if $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 10 \\ x + 1 & \text{if } x > 10 \end{cases}$?
- A. 21 B. 23 C. 401 D. 405
6. Evaluate the function $f(x) = x^2 - 5x - 3$ given $x = 3x + 1$
- A. $f(3x + 1) = 9x^2 + 21x + 7$ B. $f(3x + 1) = 9x^2 - 9x - 7$
 C. $f(3x + 1) = 9x^2 - 21x + 7$ D. $f(3x + 1) = 9x^2 + 9x - 7$
7. Given $g(x) = \frac{x^2 - 3}{2}$, determine $g(-3)$
- A. -6 B. -3 C. 3 D. 6
8. Given that $f(x) = x - 3$, and $g(x) = 2x - 3$, what is $\left(\frac{f}{g}\right)(-1)$?
- A. $4/5$ B. $5/4$ C. -1 D. $-4/5$
9. What is $(g + h)(5)$ given $g(x) = 2x + 3$ and $h(x) = 5(x)$?
- A. 23 B. 33 C. 38 D. 43
10. What is $(f \circ g)(5)$ given $f(x) = 5x + 4$ and $g(x) = 3(x)$?
- A. 55 B. 79 C. 85 D. 87
11. What is $(f + g)(3x)$ given $f(x) = x^2 + 3$ and $g(x) = x + 3$
- A. $9x^2 + 3x + 6$ B. $2x^2 + 3x + 6$
 C. $x^2 + 3x + 6$ D. $x^2 + 2x + 6$
12. Villamor photocopy shop charges ₱1.00 per photocopy for orders of 20 or fewer photocopies, ₱0.85 per photocopy for orders of 50 or fewer but more than 20 photocopies and ₱0.75 per photocopy for orders of more than 50 photocopies. How many copies will there be in a ₱100?
- A. 130 B. 132 C. 133 D. 134
13. Baguio Park horseback ride cost ₱150 for the first 5km and for every kilometer added it will cost an additional ₱40. How many kilometer you traveled if your horseback ride cost ₱630?
- A. 12km B. 17km C. 19km D. 21km
14. Rental car charges flat fee of ₱300.00 plus an additional ₱5.00 per miles when the number of miles driven is less than or equal to 100 miles. For any miles over 100, the company reduces the price to ₱2.00. How much will they charge you if you have driven 375 miles?
- A. ₱1350 B. ₱1224 C. ₱1050 D. ₱980
15. Agapito started selling sandwiches in the nearby school, In a day he spends 200 for rent and 25 for each sandwich item he prepares. His expense in a single day can be expressed as $C(x) = 25x + 200$. How much is his expenses if he prepares 100 sandwiches?
- A. ₱1700 B. ₱2700 C. ₱3700 D. ₱4700

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