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MATHEMATICS

Quarter 2 - Module 8:
Solving Linear Equations and Inequalities in One Variable Involving Absolute Value

MATHEMATICS 7
Quarter 2 - Module 8: Solving Linear Equations and Inequalities in One Variable Involving Absolute Value
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MATHEMATICS

Quarter 2 - Module 8:
Solving Linear Equations and Inequalities in One Variable Involving Absolute Value

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

This module was designed and written with you in mind. It is here to help you master your skills in solving linear equation or inequality in one variable involving absolute value and in solving problems involving equations and inequalities in one variable. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course.

After going through this module, you are expected to:

Learning Competencies:

1. solve linear equation or inequality in one variable involving absolute value. by: (a) graphing; and (b) algebraic methods (**M7AL-Iii-j-1**)
2. solve problems involving equations and inequalities in one variable. (**M7AL-Iii-j-1**)

Learning Objectives:

1. Define absolute value.
2. State and follow the steps in solving a word problem.
3. Solve linear equation or inequality in one variable involving absolute value by graphing.
4. Solve linear equation or inequality in one variable involving absolute value by algebraic method.
5. Solve problems involving equations and inequalities in one variable.

Before we start the lesson, find out how much you already know about these topics.

Pre – Assessment

Directions: Read and understand the questions below. Select the letter of the correct answer. Write your answer in a separate sheet of paper.

1. What is the result when you solve for x in the equation $|2x - 1| = 3$?

   A. 0 and 1 B. -1 and 2 C. -2 and 3 D. -4 and 4

2. How do you solve x in $|x - 2| \leq 3$?

   A. $-4 \leq x \leq 2$ B. $-3 \leq x \leq 3$ C. $-2 \leq x \leq 4$ D. $-1 \leq x \leq 5$

3. Which is ***NOT*** part of the solution to the inequality $|x - 3| < 3$?

   A. 0 B. 1 C. 2 D. 3

4. What is the value of x in $|x + 4| > 3$ to make the statement true?

   A. -3 B. -2 C. -1 D. 0

5. How many solution/s does the absolute value equation $|x| = 6$ have?

   A. 1 B. 2 C. 3 D. 4

6. Which value of x will make the expression $|6 - x|$ equal to 3?

   A. 0 B. 3 C. 6 D. 8

7. How many solution/s does the equation $|x + 2| = |x + 4|$ have?

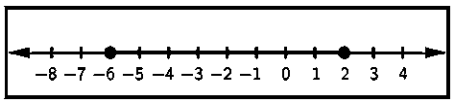
   A. 1 B. 2 C. 3 D. 4

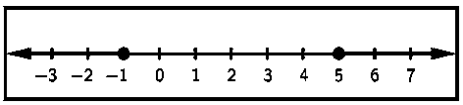
8. How do you solve x in the inequality $|x - 8| + 1 \geq 5$?

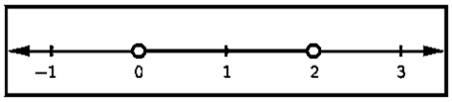
   A. $\{x|x \leq -9 \text{ or } x \geq 2\}$ B. $\{x|x \leq 4 \text{ or } x \geq 12\}$

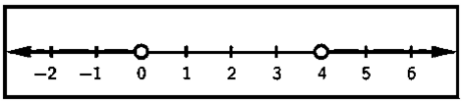
   C. $\{x|x \leq -7 \text{ or } x \geq 5\}$ D. $\{x|x \leq 2 \text{ or } x \geq 10\}$

9. Which shows the graph of the solution of the inequality $|x - 1| + 3 < 4$?

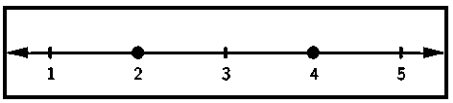
   A. 

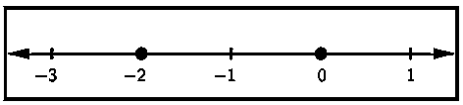
   B. 

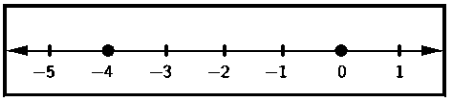
   C. 

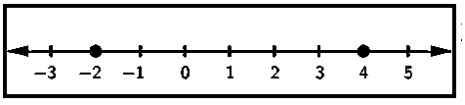
   D. 

10. Which shows the graph of the solution of the equation $|x + 2| + 3 = 5$?

    A. 

    B. 

    C. 

    D. 

11. Kevin can clean the garage in 5 hours. Julie can do the same job in 7 hours. If they work together, how long will it take them to clean the garage?

A. $1\frac{10}{12}\ hours$ B. $2\frac{11}{12}\ hours$ C. $3\frac{3}{7}\ hours$ D. $4\frac{4}{7}\ hours$

12. Three times a number reduced by 2 is less than two times the number added to 4. What is the greatest value of the number?

A. 5 B. 6 C. 7 D. 8

13. A student must gain an average score of at least 90 on five 100-point exams. If the student's scores on the first four examinations are 98, 91, 86, and 92, what must be his score on the fifth exam?

A. at least 83 B. at least 84 C. at least 85 D. at least 86

14. Eden has 3 times as many 25-centavo coins as she has 1-peso coins. How many 1-peso coins does she have if the value of the coins is ₱10.50?

A. 6 B. 8 C. 10 D. 12

15. The sum of three consecutive odd integers is 159. What is the largest integer?

A. 49 B. 51 C. 53 D. 55

Jumpstart

Let us begin this lesson by remembering the different concepts on equations and inequalities previously studied from your mathematics classes. The knowledge and mathematical skill mentioned will help you to solve linear equation or inequality in one variable involving absolute value.

Activity 1: Am I Part of the Solution?

Direction: From the given options, select which is included in the solution set of the given absolute value equations and inequalities. You may have one or more answers in each item.

1. $\|x\| = 17$	A. 7	B. -17	C. 1	D. 17
2. $\|y\| + 3 = 5$	A. 3	B. 5	C. -2	D. -1
3. $\|a - 2\| = 12$	A. 14	B. -10	C. 8	D. -4
4. $2\|m + 1\| = 6$	A. 1	B. 2	C. 3	D. 4
5. $\|x - 2\| < 3$	A. 5	B. -1	C. 4	D. 0
6. $\|y + 4\| \geq 41$	A. -50	B. -20	C. 1	D. 40
7. $2\|a - 3\| > 16$	A. -3	B. -13	C. 7	D. 10
8. $\|2b + 1\| + 3 \leq 6$	A. -4	B. -1	C. 3	D. 1

Activity 2: Find My Values!

Directions: Solve the following problems and choose the letter of the correct answer from the given choices.

1. Find five consecutive odd integers whose sum is 55.

A. 5, 7, 9, 11, 13 B. 7, 9, 11, 13, 15
C. 9, 11, 13, 15, 17 D. 11, 13, 15, 17, 19

2. Margie is 3 times older than Ana. In 15 years, the sum of their ages is 38 years. Find their present ages.

A. 2 and 6 B. 3 and 9 C. 4 and 12 D. 5 and 15

3. A student must gain an average score of at least 90 on five 100-point exams. If the student's scores on the first four examinations are 98, 91, 86, and 92, what must be his score on the fifth exam?

A. at least 90 B. at least 93 C. at least 94 D. at least 97

4. Find a number such that three times the sum of that number and 5 is greater than 33.

A. 4 B. 5 C. 6 D. 7

Discover

Below are some important matters that we need to discuss in order for you to understand how to (1) solve linear equation or inequality in one variable involving absolute value by: (a) graphing; and (b) algebraic methods; and (2) solve problems involving equations and inequalities in one variable. Read carefully and understand all salient points written in this part of the module.

Lesson A:

1. Solving Linear Equation in One Variable involving Absolute Value

(a) by Graphing

The absolute value of a number a is written as |a|.

The absolute value of a number or expression describes its distance from 0 on a number line. Since the absolute value expresses only the distance, not the direction of the number on a number line, it is always expressed as a positive number or 0.

An absolute value equation is an equation that contains an absolute value expression. The equation $|x| = a$ has two solutions $x = a$ and $x = -a$ because both numbers are at the distance a from 0.

To solve linear equation in one variable involving absolute value by graphing, observe and study the following steps in each example:

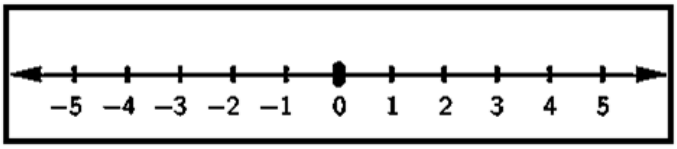
Example 1: $|m| = 4$

Step 1: Equate the expression inside the absolute value sign into 0.

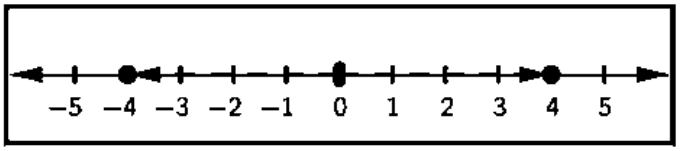
$m = 0$

So, the starting point is 0.

Step 2: Draw a number line and mark the starting point 0.



Step 3: Since the number on the right side of the equation is 4, then count 4 units from the left and right of 0. Plot each end point.



Since the endpoints are -4 and 4, the solutions to the problem are $\boldsymbol{m = 4}$ and $\boldsymbol{m = -4}$.

Example 2: $\boldsymbol{|x + 3| = 2}$

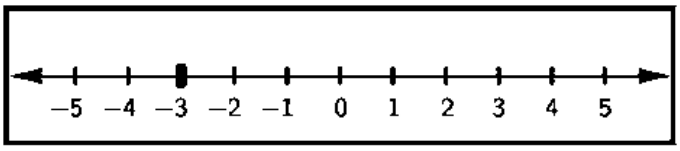
Step 1: Equate the expression inside the absolute value sign into 0 and solve.

$x + 3 = 0$

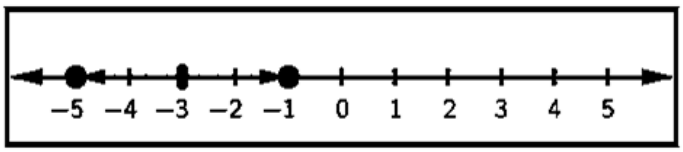
$x = -3$

So, the starting point is -3.

Step 2: Draw a number line and mark the starting point -3.



Step 3: Since the number on the right side of the equation is 2, then count 2 units from the left and right of-3. Plot each end point.



Since the endpoints are -5 and -1, the solutions to the problem are $\boldsymbol{x = -5}$ and $\boldsymbol{x = -1}$.

Example 3: $\boldsymbol{3|y - 1| + 2 = 8}$

Let the expression on one side of the equation consists only of a single absolute value expression.

$3|y - 1| + 2 = 8$

$3|y - 1| = 8 - 2$

$3|y - 1| = 6$

$\boldsymbol{|y - 1| = 2}$

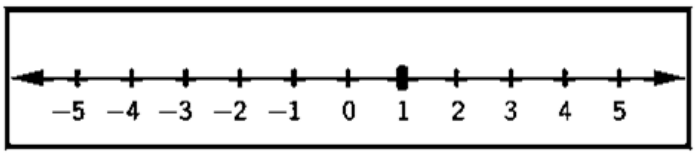
Step 1: Equate the expression inside the absolute value sign into 0 and solve.

$y - 1 = 0$

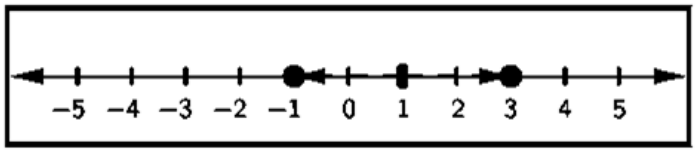
$y = 1$

So, the starting point is 1.

Step 2: Draw a number line and mark the starting point 1.



Step 3: Since the number on the right side of the equation $|\boldsymbol{y-1}| = \boldsymbol{2}$ is 2, then count 2 units from the left and right of 1. Plot each end point.



Since the endpoints are -1 and 3, the solutions to the problem are $\boldsymbol{y = -1}$ and $\boldsymbol{y = 3}$.

(b) by Algebraic Methods

Many absolute value equations are not easy to solve by the guess-and-check method and by graphing. An easier way may be to use the following procedures:

Step 1: Let the expression on one side of the equation consists only of a single absolute value expression.

Step 2: Is the number on the other side of the equation negative? If it is, then the equation has no solution. If it is not, then proceed to step

Step 3: If the absolute value of an expression is equal to a positive number, say x, then the expression inside the absolute value can either be x or $-x$. Equate the expression inside the absolute value sign to x and to $-x$.

Step 4: Solve both equations by algebraic methods.

Example 1: **Solve** $|3x - 4| - 11 = 15$

Step 1: $|3x - 4| - 11 + 11 = 15 + 11$

$|3x - 4| = 26$

Step 2: The number on the other side of the equation is positive, 26, so proceed to step 3.

Step 3: $3x - 4 = 26$; $3x - 4 = -26$

Step 4: $3x - 4 + 4 = 26 + 4$ $\qquad$ $3x - 4 + 4 = -26 + 4$

$3x = 30$ $\qquad$ $3x = -22$

$x = \frac{30}{3}$ $\qquad$ $\boldsymbol{x = \frac{-22}{3}}$

$\boldsymbol{x = 10}$

CHECKING:

$|3x - 4| - 11 = 15$ $\qquad$ $|3x - 4| - 11 = 15$

$|3(10) - 4| - 11 = 15$ $\qquad$ $\left|3\left(\frac{-22}{3}\right) - 4\right| - 11 = 15$

$|30 - 4| - 11 = 15$ $\qquad$ $|-22 - 4| - 11 = 15$

$|26| - 11 = 15$ $\qquad$ $|-26| - 11 = 15$

$26 - 11 = 15$ $\qquad$ $26 - 11 = 15$

$15 = 15$ $\qquad$ $15 = 15$

Example 2: **Solve** $\boldsymbol{|5y + 4| + 12 = 4}$

Step 1: $|5y + 4| + 12 - 12 = 4 - 12$

$|5y + 4| = -8$

Step 2: The number on the other side of the equation is negative, -8, so the equation has no solution.

Example 3: **Solve** $\boldsymbol{|z - 7| = |2z - 2|}$

Step 1: Done, because the expression on the left already consists of a single absolute value expression.

Step 2: $|2z - 2|$ is not negative because the absolute value of a number can never be negative, so proceed to step 3.

Step 3: $z - 7 = 2z - 2$ $\quad$; $\quad$ $z - 7 = -(2z - 2)$

Step 4: $z - 2z - 7 + 7 = 2z - 2z - 2 + 7$ $\qquad$ $z - 7 = -2z + 2$

$-z = 5$ $\qquad$ $z + 2z - 7 + 7 = -2z + 2z + 2 + 7$

$\boldsymbol{z = -5}$ $\qquad$ $3z = 9$

$\boldsymbol{z = 3}$

CHECKING:

$|z-7| = |2z-2|$ $\quad$ $|z-7| = |2z-2|$

$|-5-7| = |2(-5)-2|$ $\quad$ $|3-7| = |2(3)-2|$

$|-12| = |-10-2|$ $\quad$ $|-4| = |6-2|$

$12 = |-12|$ $\quad$ $4 = |4|$

$12 = 12$ $\quad$ $4 = 4$

Example 4: **Solve $|a+2| = |a-3|$**

Step 1: Done, because the expression on the left already consists of a single absolute value expression.

Step 2: $|a-3|$ is not negative because the absolute value of a number can never be negative, so proceed to step 3.

Step 3: $a+2 = a-3$; $a+2 = -(a-3)$

Step 4: $a-a+2-2 = a-a-3-2$ $\quad$ $a+2 = -a+3$

$0 = -5$ $\quad$ $a+a+2-2 = -a+a+3-2$

This is false. $\quad$ $2a = 1$

There is no solution from this equation. $\quad$ $\boldsymbol{a = \frac{1}{2}}$

This problem has only one solution, $\boldsymbol{a = \frac{1}{2}}$**,** that will make the original equation true.

CHECKING:

$|a+2| = |a-3|$

$\left|\frac{1}{2}+2\right| = \left|\frac{1}{2}-3\right|$

$\left|\frac{1+4}{2}\right| = \left|\frac{1-6}{2}\right|$

$\left|\frac{5}{2}\right| = \left|\frac{-5}{2}\right|$

$\frac{5}{2} = \frac{5}{2}$

Example 5: **Solve $|b-4| = |4-b|$**

Step 1: Done, because the expression on the left already consists of a single absolute value expression.

Step 2: $|4 - b|$ is not negative because the absolute value of a number can never be negative, so proceed to step 3.

Step 3: $b - 4 = 4 - b$; $b - 4 = -(4 - b)$

Step 4: $b + b - 4 + 4 = 4 + 4 - b + b$ $b - 4 = -4 + b$

$2b = 8$ $b - b - 4 + 4 = -4 + 4 + b - b$

$b = \frac{8}{2}$ $0 = 0$

$\boldsymbol{b = 4}$ This is true no matter what value b is.

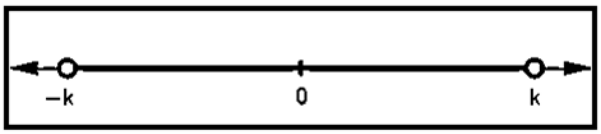
Therefore, the solution to this problem is the set of all real numbers.

2. Solving Linear Inequality in One Variable Involving Absolute Value

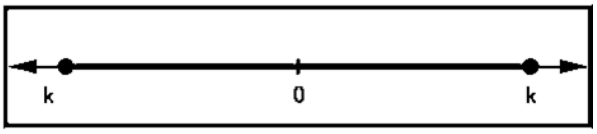
In general, an absolute value inequality may be a "less than" or a "greater than" type of inequality (either $|\boldsymbol{x}| < \boldsymbol{k}$ or $|\boldsymbol{x}| > \boldsymbol{k}$). They result in two different solutions, as discussed below.

1. Let k be a positive number. Given $|\boldsymbol{x}| < \boldsymbol{k}$, then $-\boldsymbol{k} < \boldsymbol{x} < \boldsymbol{k}$.

   The solution may be represented on the number line. Observe that the solution consists of all numbers whose distance from 0 is less than k.

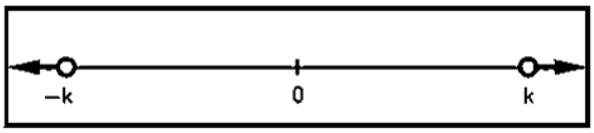


- If the inequality involves $\leq$ instead of $<$, then $\pm k$ will now be part of the solution, which gives $-\boldsymbol{k} \leq \boldsymbol{x} \leq \boldsymbol{k}$. This solution is represented graphically below.

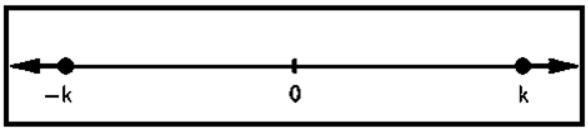


2. Let k be a positive number. Given $|\boldsymbol{x}| > \boldsymbol{k}$, then $\boldsymbol{x} < -\boldsymbol{k}$ **or** $\boldsymbol{x} > \boldsymbol{k}$.

   The solution may be represented on the number line. Observe that the solution consists of all numbers whose distance from 0 is greater than k.



- If the inequality involves ≥ instead of >, then $\pm k$ will now be part of the solution, which gives $\boldsymbol{x \leq -k}$ **or** $\boldsymbol{x \geq k}$. This solution is represented graphically below.



Example 1: $\boldsymbol{|x-2|<1}$

Step 1: This is a "less than" absolute value inequality. Set up a double inequality.

$$-1 < x - 2 < 1$$

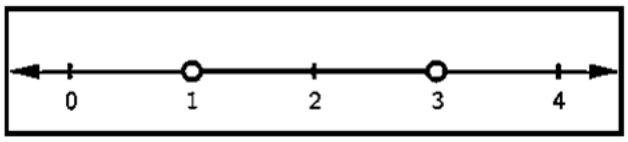
Step 2: Solve the double inequality:

$$-1+2 < x-2+2 < 1+2$$

$$1 < x < 3$$

Therefore, the solution of the inequality is $\{x|1 < x < 3\}$.

Step 3: Graph the solution.



Example 2: $\boldsymbol{|y+3|>2}$

Step 1: This is a "greater than" absolute value inequality. Set up two separate inequalities.

$$y+3 < -2 \qquad \text{or} \qquad y+3 > 2$$

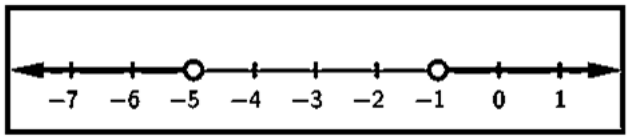
Step 2: Solve the two inequalities:

$$y + 3 - 3 < -2 - 3 \qquad y + 3 - 3 > 2 - 3$$

$$y < -5 \qquad y > -1$$

Therefore, the solution of the inequality is $\{y | y < -5 \text{ or } y > -1\}$.

Step 3: Graph the solution.



Example 3: $|a + 3| - 1 \geq 3$

Step 1: Isolate the absolute value expression on one side.

$$|a + 3| - 1 + 1 \geq 3 + 1$$

$$|a + 3| \geq 4$$

Step 2: This is a "greater than" absolute value inequality. Set up two Separate inequalities.

$$a + 3 \leq -4 \quad \text{or} \quad a + 3 \geq 4$$

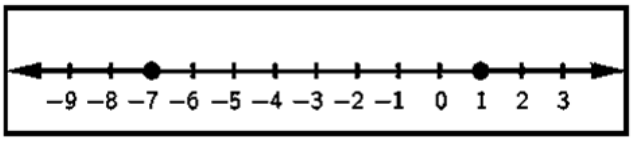
Step 3: Solve the two inequalities:

$$a + 3 - 3 \leq -4 - 3 \qquad a + 3 - 3 \geq 4 - 3$$

$$a \leq -7 \qquad a \geq 1$$

Therefore, the solution of the inequality is $\{a | a \leq -7 \text{ or } a \geq 1\}$.

Step 4: Graph the solution.



Example 4: $|2b - 4| + 3 \leq 5$

Step 1: Isolate the absolute value expression on one side.

$$|2b - 4| + 3 - 3 \leq 5 - 3$$

$$|2b - 4| \leq 2$$

Step 2: This is a "less than" absolute value inequality. Set up a double inequality.

$$-2 \leq 2b - 4 \leq 2$$

Step 3: Solve the double inequality:

$$-2 + 4 \leq 2b - 4 + 4 \leq 2 + 4$$

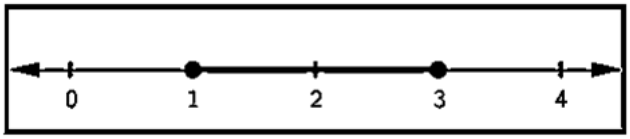
$$2 \leq 2b \leq 6$$

$$\frac{2}{2} \leq \frac{2b}{2} \leq \frac{6}{2}$$

$$1 \leq b \leq 3$$

Therefore, the solution of the inequality is $\{b|1 \leq b \leq 3\}$.

Step 4: Graph the solution.



Lesson B:

1. Solving Problems Involving Equations in One Variable

To solve a word problem, one must know how to write equations. There are steps to follow when writing the equation and finally finding its solution.

1. Read and understand the problem. Identify what is given and what is unknown. Choose a variable to represent the unknown number.
2. Express the other unknowns, if there are any, in terms of the variable chosen in step 1.
3. Write an equation to represent the relationship among the given and unknown/s.
4. Solve the equation for the unknown and use the solution to find the quantities being asked.
5. Check by going back to the original statement.

Illustrative Examples

A. Number Problem

One number is 3 less than another number. If their sum is 49, find the two numbers.

Solution:

Step 1: Let x be the first number.

Step 2: x – 3 is the second number.

Step 3: Write the number sentence, x + (x – 3) = 49

Step 4:
$$x + x - 3 = 49$$
$$2x = 49 + 3$$
$$2x = 52$$
$$x = 26 \quad \text{the first number}$$
$$x + x - 3 = 23 \quad \text{the second number}$$

Step 5: *Check:* The sum of 26 and 23 is 49, and 23 is 3 less than 26.

B. Distance Problem

Two cars leave Manila and travel in opposite directions. One of the cars averages 12 km/h less than the other. After 3 hours, they are 396 km apart. What is the average speed of each car?

Solution:

Note: The distance *d* covered by a moving body is the product of the time *t* spent and the rate *r* of the moving body.

$$d = r \times t$$

Step 1: Let x be the rate of the first car in km/h.
x – 12 is the rate of the second car.

Step 2: 3x is the distance covered by the first car.
3(x – 12) is the distance covered by the second car.

Step 3: $3x + 3(x - 12) = 396$

Step 4:
$$3x + 3(x - 12) = 396$$
$$3x + 3x - 36 = 396$$
$$6x = 396 + 36$$
$$6x = 432$$
$$x = 72 \text{ km/h} \quad \text{rate of the first car}$$
$$x - 12 = 60 \text{ km/h} \quad \text{rate of the second car}$$

Step 5: *Check:* In 3 hours and at the rate of 72 km/h, the first car can travel 3(72) = 216 km. In the same length of time and at the rate of 12 km/h less than the rate of the first car, the second car can travel 3(72 – 12) = 3(60) = 180 km. Since the cars are travelling in opposite directions, they are 216 km + 180 km = 396 km apart after 3 hours.

C. Coin Problem

Mark has four more 5-peso coins than 1-peso coins. If he has 22 coins in all, how much money does he have?

Solution:

Step 1: Let x be the number of 1-peso coins.

x + 4 is the number of 5-peso coins.

Step 2: ₱1.00x is the amount of 1-peso coins.

₱5.00(x + 4) is the amount of 5-peso coins.

Step 3: x + (x + 4) = 22

Step 4: x + x + 4 = 22

2x = 22 – 4

2x = 18

x = 9 number of 1-peso coins

x + 4 = 13 number of 5-peso coins

₱1.00(9) = ₱9.00 amount of 1-peso coins

₱5.00(13) = ₱65.00 amount of 5-peso coins

₱9.00 + ₱65.00 = ₱74.00 total amount of money that Mark has

Step 5: *Check:* The sum of 9 one-peso coins and 13 five-peso coins is 22 coins, and 13 five-peso coins is four more than 9 one - peso coins.

D. Age Problem

Six years ago, Mrs. de la Rosa was 5 times as old as her daughter Leila. How old is Leila now if her age is one-third of her mother's present age?

Solution:

Step 1: Let x be Leila's age now.

3x is Mrs. de la Rosa's age now.

Step 2: x – 6 is Leila's age 6 years ago.

3x – 6 is Mrs. de la Rosa's age 6 years ago.

Step 3: 5(x – 6) = 3x – 6

Step 4: $5(x - 6) = 3x - 6$

$5x - 30 = 3x - 6$

$5x - 3x = -6 + 30$

$2x = 24$

$x = 12$ Leila's age now

$3x = 36$ Mrs. de la Rosa's age now

Step 5: *Check:* Thrice of Leila's present age, 12, is Mrs. de la Rosa's present age, 36. Six years ago, Mrs. de la Rosa was $36 - 6 = 30$ years old which was five times Leila's age, $12 - 6 = 6$.

E. Business Problem

Myrna works as a sales person in a department store. One of her benefits is a 20% discount on all items she buys for herself. If she paid ₱120 for a new blouse, what was the price of the blouse before the discount?

Solution:

Step 1: Let x be the price of the blouse.

Step 2: $0.20x$ is the amount of discount. (20% = 0.20)

Step 3: $x - 0.20x = 120$

Step 4: $x - 0.20x = 120$

$0.80x = 120$

$x = ₱150$ price before the discount

Step 5: *Check:* Twenty percent of ₱150 is ₱30, and ₱150 - ₱30 = ₱120.

F. Work Problem

Christian can wash and wax one car in 3 hours. Jing can do the same job in 4 hours. If they work together, how long will it take them to wash and wax one car?

Solution: (rate of work) × (time) = (work done)

$rt = w$

Step 1: Let t be the time in hours for Christian and Jing to wash and wax one car.

Step 2: Christian can do one-third of the job in 1 hour.
Jing can do one-fourth of the job in hour.

Step 3: $\frac{t}{3} + \frac{t}{4} = 1$ (Together, they complete 1 job.)

Step 4: $4t + 3t = 12$ (Multiply by 12, the LCD.)

$7t = 12$

$t = \frac{12}{7}\ or\ 1\frac{5}{7}\ hours$

Step 5: *Check:* The part of the work that Christian can do in one hour is $\left(\frac{12}{7}\right)\left(\frac{1}{3}\right) = \frac{4}{7}$ while that of Jing is $\left(\frac{12}{7}\right)\left(\frac{1}{4}\right) = \frac{3}{7}$. Working together, $\frac{4}{7} + \frac{3}{7} = 1$, they can finish the work in one hour.

2. Solving Problems Involving Inequalities in One Variable

Word problems involving linear inequalities are solved in a similar way as those involving linear equations.

Illustrative Examples

A. The sum of four consecutive integers decreased by 18 is greater than twice the smallest of the four. What are the four smallest such integers?

Solution:

Step 1: Let x be the smallest of the four integers.

Step 2: The other integers are x + 1, x + 2, and x + 3.

Step 3: x + (x + 1) + (x + 2) + (x + 3) – 18 > 2x

Step 4: $x + x + 1 + x + 2 + x + 3 - 18 > 2x$

$$4x - 12 > 2x$$
$$4x - 2x > 12$$
$$2x > 12$$
$$x > 6$$

Therefore, the smallest of the four integers is at least 7, in which case, the four consecutive integers are 7, 8, 9, 10.

Step 5: *Check:* The sum of the four integers 7, 8, 9, 10 is 34. Eighteen less than 34 is 16, which is larger than twice the smallest, or 14. If we started with 6, then the integers would be 6, 7, 8, 9. Eighteen less than their sum is 6 + 7 + 8 + 9 – 18 = 12, which is not more than two times 6. Therefore, our answer is correct.

B. Rita has five more yellow ribbons than red ribbons. If twice the yellow ribbons is at most four times the red ribbons, at least how many red ribbons does she have?

Solution:

Step 1: Lest x be the number of red ribbons that Rita has.

Step 2: The number of yellow ribbons that Rita has is x + 5.

Step 3: $2(x + 5) \leq 4x$

Step 4: $2x + 10 \leq 4x$

$$2x - 4x \leq -10$$
$$-2x \leq -10$$
$$x \geq 5$$

Therefore, Rita has at least 5 red ribbons.

Step 5: *Check:* If the number of red ribbons is less than 5, then twice the number of yellow ribbons is greater than four times the number of red ribbons. If the number of red ribbons is 5 or greater, then twice the number of yellow ribbons is less than or equal to four times the number of red ribbons. Therefore, our answer is correct.

C. Simon must have an average of 80 on all tests. Suppose Simon has scored 60, 72, 98, and 85 on the first four tests. What score does he need on the fifth test?

Solution:

Step 1: Let x be the score on the fifth test.

Step 2: $\frac{60+72+98+85+x}{5}$ be the average of all tests

Step 3: $\frac{60+72+98+85+x}{5} \geq 80$

Step 4: $60 + 72 + 98 + 85 + x \geq 400$

$315 + x \geq 400$

$x \geq 400 - 315$

$x \geq 85$

Therefore, Simon needs a score of at least 85 in his fifth test.

Step 5: *Check:* If the score of Simon in his fifth test is less than 85, then the average on all tests is less than 80. If the score of Simon in his fifth test is greater than or equal to 85, then the average on all tests is greater than 80. Therefore, our answer is correct.

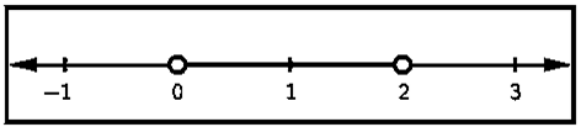
Explore

Activity 3: Find My Partner!

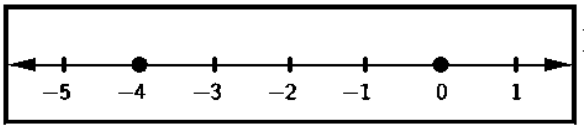
Directions: The graph of the solution of each linear equation and inequality involving absolute value are shown. Solve and match the corresponding absolute value equation or inequality for each graph from the box below.

$\|x+2\|+1 \leq 5$	$\|2x-6\|+1=3$	$\|x-1\|<1$	$\|2x+2\|=\|x-2\|$
$\|x-2\|>2$	$\|x-1\|=3$	$\|x+1\|=1$	$\|5x-10\|-3 \geq 12$

1. _____



2. _____



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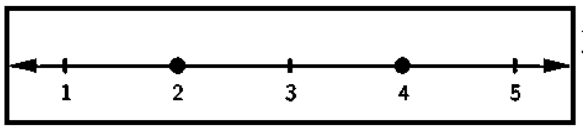

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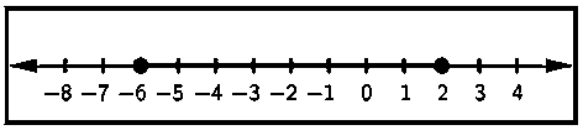

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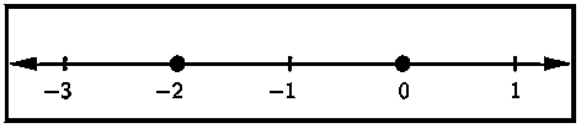
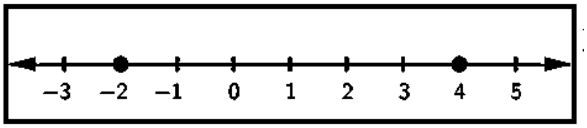


6. _____



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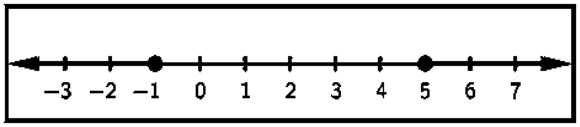
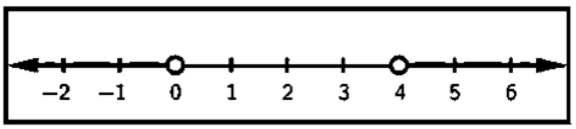


8. _____



How did you find the activity? What mathematical concepts did you use? Now, here is another activity that lets you apply what you have learned about solving problems involving equations and inequalities in one variable.

Activity 4: Solve Me!

Directions: Solve the following problems.

1. The sum of two numbers is 39. One number is 3 less than twice the other. What are the numbers?

2. An item priced at ₱3,600 has a 25% discount. Find the discounted price.

3. The sum of Cora's age and Rose' age is 40. Cora is 4 years older than Rose. How old is Cora?

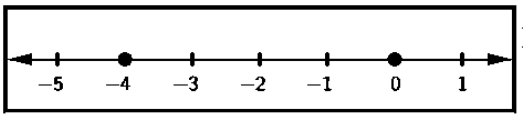
4. The difference between two integers is less than 96. The larger number is 245. Find the smallest value of the other integer.

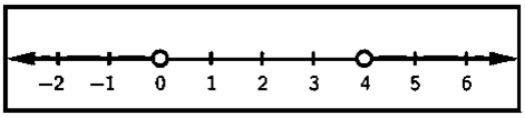
5. Three girls were born in consecutive years. The sum of their ages is more than 39 decreased by the age of the youngest girl. What are the least possible ages of the girls?

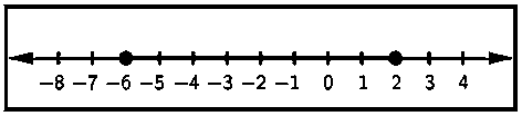
Deepen

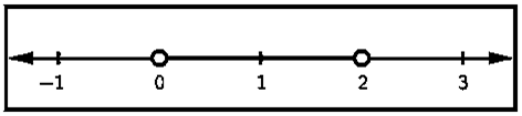
Activity 5: Create Me!

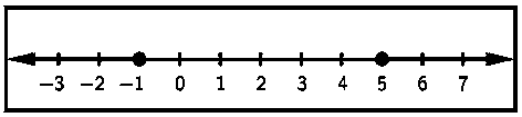
Direction: For each item below, give or create an absolute value equation or inequality that has the same solution as the graph below.

1. 

2. 

3. 

4. 

5. 

Activity 6: Think Deeper!

Direction: Solve the problem. Give your complete solution and explanation.

Beth received grades of 87%, 82%, 96%, and 79% on her last four algebra tests.

A. What grade does she need to make on the fifth test? Assume that the tests are weighted equally and the average of the test grades must be at least 80% but less than 90%.

B. Is it possible for Beth to graduate with honors if it requires an average of at least 90%? Why or Why not?

Rubric for Problem Solving

4	3	2	1
Used an appropriate strategy to come up with a correct solution and arrived at a correct answer	Used an appropriate strategy to come up with a solution, but a part of the solution led to an incorrect answer	Used an appropriate strategy but came up with an entirely wrong solution that led to an incorrect answer	Attempted to solve the problem but used an inappropriate strategy that led to a wrong solution

Gauge

Directions: Read each statement below carefully. Select the letter of the correct answer. Write your answer on a separate sheet of paper.

1. How many solution/s does the absolute value equation $|x| = 8$ have?

   A. 1 B. 2 C. 3 D. 4

2. The absolute value inequality $|x + 3| > 4$ is true at what value of x?

   A. -2 B. -1 C. 1 D. 2

3. Which value of x will make the expression $|3 - x|$ equal to 6?

   A. 0 B. 3 C. 6 D. 9

4. Which is ***NOT*** part of the solution to the inequality $|x - 2| < 2$?

   A. 0 B. 1 C. 2 D. 3

5. How many solution/s does the equation $|x + 3| = |x + 5|$ have?

   A. 1 B. 2 C. 3 D. 4

6. How do you solve x in $|2x - 1| \leq 5$?

   A. $-3 \leq x \leq 2$ B. $-2 \leq x \leq 1$ C. $-2 \leq x \leq 3$ D. $-1 \leq x \leq 2$

7. How do you solve x in the inequality $|x - 9| + 2 \geq 6$?

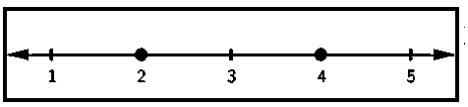
   A. $\{x|x \leq -9 \text{ or } x \geq 2\}$ B. $\{x|x \leq 5 \text{ or } x \geq 13\}$

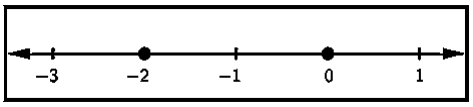
   C. $\{x|x \leq -7 \text{ or } x \geq 5\}$ D. $\{x|x \leq 3 \text{ or } x \geq 10\}$

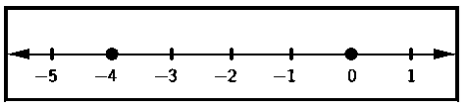
8. Which value of x will satisfy the equation $|2x - 3| = 5$?

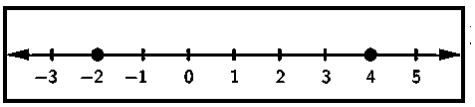
   A. -4 and 1 B. -3 and 2 C. -2 and 3 D. -1 and 4

9. Which shows the graph of the solution of the equation $|x + 1| + 2 = 3$?

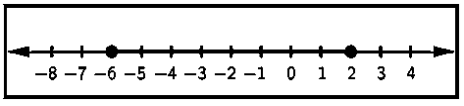
   A. 

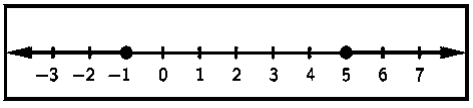
   B. 

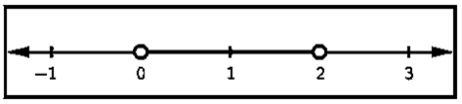
   C. 

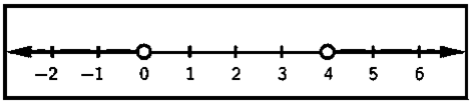
   D. 

10. Which shows the graph of the solution of the inequality $|x + 2| - 3 \leq 1$?

    A. 

    B. 

    C. 

    D. 

11. Four times a number reduced by 3 is less than three times the number added to 5. What is the greatest value of the number?

A. 6 B. 7 C. 8 D. 9

12. The sum of three consecutive even integers is 102. What is the largest integer?

A. 32 B. 34 C. 36 D. 38

13. Nikka has 4 times as many 25-centavo coins as she has 1-peso coins. How many 1-peso coins does she have if the value of the coins is ₱24.00?

A. 8 B. 10 C. 12 D. 14

14. Dan can clean the garage in 6 hours. Bea can do the same job in 8 hours. If they work together, how long will it take them to clean the garage?

A. $1\frac{1}{7}\ hours$ B. $2\frac{2}{7}\ hours$ C. $3\frac{3}{7}\ hours$ D. $4\frac{4}{7}\ hours$

15. A student must gain an average score of at least 92 on five 100-point exams. If the student's scores on the first four examinations are 99, 92, 85, and 90, what must be his score on the fifth exam?

A. at least 91 B. at least 92 C. at least 93 D. at least 94

Great job! You are done with this module.

References

Books:

- Bernabe, Julieta G. Elementary Algebra: Textbook for First Year. Quezon City. SD Publications, InC., 2009
- Miller, Julie, O'Neill, Molly, and Hyde, Nancy. Introductory Algebra: Second Edition. New York. McGraw-Hill Companies, InC., 2009
- Miller, Julie, O'Neill, Molly, and Hyde, Nancy. Intermediate Algebra: Second Edition. New York. McGraw-Hill Companies, InC., 2010
- Learner's Module, K-12 Grade 7 Mathematics (Third Quarter)

Online Resources:

- https://math.libretexts.org/Bookshelves/Algebra
- https://www.mathplanet.com/education/algebra-1/linear-inequalitites/ solving-absolute- value-equations-and-inequalities
- https://courses.lumenlearning.com/cuny-hunter-collegealgebra/chapter/ equations-and-inequalities-with-absolute-value
- https://tutorial.math.lamar.edu/classes/alg/solveabsvalueineq.aspx
- https://www.purplemath.com/modules/absineq.htm
- https://opentextbC.ca/algebratrigonometryopenstax/chapter/linear-inequalities- and-absolute-value-inequalities

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