7

Mathematics

Quarter 4 - Week 7: Module 7

MEASURES OF VARIABILITY

AIRs - LM

Mathematics Grade 7
Quarter 4 - Week 7: Module 7 - **Measures of Variability**

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Target

In this lesson, you will learn to interpret, draw conclusions and make recommendations.

After going through this module, you are expected to:

Learning Competency

calculate the measure of variability of grouped and ungrouped data
(M7SP-IVh-i-1)

Before going on, check how much you know about this topic.

Pre – Assessment

Directions: Read and understand the questions below. Select the best answer to each item then write your choice on your answer sheet.

1. What is the measure of the spread of a data set around a mean?
   A. Measures of Average B. Measures of Position
   C. Measures of Central Tendency D. Measures of Variability
2. What measure of variability is the simplest?
   A. Mean Deviation B. Range
   C. Standard Deviation D. Variance
3. Which measure is the mean of the square deviations from the mean of a frequency distribution?
   A. Mean Deviation B. Range
   C. Standard Deviation D. Variance
4. Which measure is considered the best indicator of the degree of dispersion among the measures of variability?
   A. Mean Deviation B. Range
   C. Standard Deviation D. Variance
5. What to do in getting the range of grouped data?
   A. Subtract the highest and the lowest value
   B. Subtract the upper boundary of the lowest class interval and the lower boundary of the lowest interval.
   C. Subtract the upper boundary of the highest class interval and the lower boundary of the lowest interval.
   D. Subtract the upper boundary of the highest class interval and the lower boundary of the highest interval.
6. Which is true about the measures of variability?
   A. The smaller the standard deviation, the less reliable the scores.
   B. The higher the standard deviation, the more reliable the scores.
   C. The smaller the standard deviation, the less spread the distribution.
   D. The smaller the standard deviation, the more spread the distribution
7. If two classes have the same measures of variability, what do you need to compare the two classes?
   A. Determine the average of the two classes.
   B. Determine which class got the highest scores.
   C. Determine which class who got more high scores.
   D. Determine which class have more scores around the mean.
8. What will Mr. Padilla use if he wants to know who among his 3 agents is consistent in their sales in one year?
   A. Range, to know how far is the highest sale to lowest sale of each agent.
   B. Standard Deviation, to know the average variability of the sales of each agent.
   C. Variance, to know the square of the deviations to the average sales of each agent.
   D. Mean Deviation, to get the deviation of the sales to the average sales of each agent.

9. The Barangay captain wants to know how is his constituents' income differ from the average income of each family, What should he do?
   A. Get the standard deviation of all the family's income.
   B. Get the standard deviation of the richest and the poorest family's income.
   C. Get the range of the richest and poorest family's income in the barangay.
   D. Get the range of the richest family's income to the fund of the barangay for each family.
10. Given the following data (Jose: average = 90 and SD = 2.5) (Pedro: average = 90 and SD = 5), what can you conclude between Jose and Pedro?
   A. Pedro has better grades than Jose.
   B. Jose has better grades than Pedro.
   C. Jose and Pedro are equal in performances.
   D. It cannot be determined since they have the same average.
11. What is the range (ungrouped data) of the heights (cm) of the five siblings: 100, 121, 152, 154, 160?
   A. 40 B. 50 C. 60 D. 70
12. Given the interval of the member of Grade 7 students per classroom: 60 – 62, 57 – 59, 54 – 56, 51 – 53, 48 – 50, what is the range of the grouped data?
   A. 15 B. 16 C. 17 D. 18
13. If $\sum|x - \bar{x}|$ is 12 and N is 5, what is the mean deviation?
   A. 1.05 B. 2.5 C. 2.4 D. 1.4
14. If $\sum(x - \bar{x})^2$ is 10 and N is 5, what is the standard deviation (ungrouped data)?
   A. 1.40 B. 1.41 C. 1.42 D. 1.43
15. If $\sum f(x - \bar{x})^2$ is 750 and $\sum f$ = 24, what is the standard deviation (grouped data)?
   A. 5.69 B. 5.70 C. 5.71 D. 5.72

Jumpstart

Activity 1: **Which Taste Better?**

A house wife surveyed canned ham for a special family affair. She picked 5 cans each from two boxes packed by company A and company B. Both boxes have the same weight. Consider the following weights in kilograms of the canned ham packed by the two companies (sample A and sample B).

Sample A: 0.97, 1.00, 0.94, 1.03, 1.11
Samples B : 1.06, 1.01, 0.88, 0.90, 1.14

Help the housewife choose the best sample by doing the following procedures.

a. Arrange the weights in numerical order.
b. Frind the mean weight of each sample.
c. Analyze the spread of the weights of each sample from the mean.
d. Which sample has weights closer to the mean?
e. If you were to choose from these two samples, which would you prefer? Why?

Discover

Measures of variability refer to the spread of the values about the mean.Smaller dispersion of scores arising from the comparison often indicates more consistency and more reliability.

UNGROUPED DATA

A. RANGE

The **range** is the simlplest measure of variability. It is the difference between the longest value and the smallest value.

R = H – L, where R = Range, H = Highest Value, L = Lowest Value

Examples:

1. Test scores of 10, 9, 8, 7, 5 and 3 will give us a range of 7. The range 7 is obtained by subtracting 3 from 10 (R = 10 – 3 = 7).
2. Compute the range of the given set of numbers : $\{12, 13, 17, 22, 22, 23, 25, 26\}$.

   R = H – L
   = 26 – 12
   = **14**

B. MEAN DEVIATION

The dispersion of a set of data about the average of these data is the **average deviation or mean deviation.**

To compute the mean deviation of an ungrouped data, we use the formula

$$\text{M.D.} = \frac{\sum|x-\bar{x}|}{N}$$

where: M.D. = mean deviation

x = individual score

$\bar{x}$ = mean

N = number of scores

$|x-\bar{x}|$ = absolute value of the deviation from the mean

Procedure in computing the mean deviation:

1. Find the mean for all the cases.
2. Find the absolute difference between each score and the mean.
3. Find the sum of the differences and divide it by N.
4. Solve for the mean deviation by dividing the result in step 3 by N.

Example 1: Find the mean deviation of the following data: 12, 17, 13, 18, 18, 15, 14, 17, 11.

Step 1: Find the mean ($\bar{x}$).

$$\bar{x} = \frac{\sum x}{N} = \frac{12+17+13+18+18+15+14+17+11}{9} = \frac{135}{9} = 15$$

Step 2: Find the absolute difference between each score and the mean.

$$\begin{aligned}|x-\bar{x}| &= |12-15| = 3\\ &= |17-15| = 2\\ &= |13-15| = 2\\ &= |18-15| = 3\\ &= |18-15| = 3\\ &= |15-15| = 0\\ &= |14-15| = 1\\ &= |17-15| = 2\\ &= |11-15| = 4\end{aligned}$$

Step 3: Find the sum of the absolute differences.

$$\sum|x-\bar{x}| = 3+2+2+3+3+0+1+2+4 = 20$$

This can be represented in tabular form as shown below.

x	$\bar{x}$	$\|x-\bar{x}\|$
12	15	3
17	15	2
13	15	2
18	15	3
18	15	3
15	15	0
14	15	1
17	15	2
11	15	4
N = 9		$\sum\|x-\bar{x}\|$=20

Step 4: Solve for the mean deviation by dividing the result in step 3 by N.

$$\text{M.D.} = \frac{\sum|x-\bar{x}|}{N} = \frac{20}{9} = \mathbf{2.22}$$

Example 2: Solve for the mean deviation of the weights in kilogram of 10 students: 52, 55, 50, 55, 43, 45, 40, 48, 45, 47.

$$\bar{x} = \frac{\sum x}{N} = \frac{52 + 55 + 50 + 55 + 43 + 45 + 50 + 48 + 45 + 47}{9} = \frac{480}{10} = 48$$

x	$\bar{x}$	$\lvert x - \bar{x}\rvert$
52	48	4
55	48	7
50	48	2
55	48	7
43	48	5
45	48	3
40	48	8
48	48	0
45	48	3
47	48	1
N = 10		$\sum\lvert x - \bar{x}\rvert$=40

$$\text{M. D.} = \frac{\sum|x-\bar{x}|}{N} = \frac{40}{10} = \mathbf{4}$$

C. VARIANCE

The **variance** of a set of data is devoted by the symbol σ^2. To find the variance (σ^2), we use the formula:

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{N-1}$$

where: N = total number data
x = raw score
$\bar{x}$ = mean

Example: Find the variance 6, 8, 8, 9, 10, 11, 11, 11, 14, 16.

Score	$\lvert x - \bar{x}\rvert$	$(x - \bar{x})^2$
6	4.4	19.36
8	2.4	5.26
8	2.4	5.26
9	1.4	1.96
10	0.4	0.16
11	0.6	.036
11	0.6	0.36
11	0.6	0.36
14	3.6	12.96
16	5.6	31.36
N = 10		$\sum(x-\bar{x})^2 = 78.4$

Solution:

$$\bar{x} = \frac{\sum x}{N} = \frac{6+8+8+9+10+11+11+11+14+16}{10} = \frac{104}{10} = 10.4$$

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{N-1} = \frac{78.4}{10-1} = \frac{78.4}{10-9} = \mathbf{8.71}$$

D. STANDARD DEVIATION

Like the mean deviation, the **standard deviation** differentiates sets of scores with equal averages. But the advantage of standard deviation over mean deviation is that it has several applications in the inferential statistics.

Formula:

$$S.D. = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}}$$

where: S.D. = mean deviation
x = individual score
$\bar{x}$ = mean
N = number of scores

Example: Compare the standard deviation of the scores of the three students in their Mathematics quizzes.

Student A	97, 92, 96, 95, 90
Student B	94, 94, 92, 94, 96
Student C	95, 94, 93, 96, 92

Solution:

Student A:

Step 1. Compute the mean score.

$$\bar{x} = \frac{\sum x}{N} = \frac{97+92+96+95+90}{5} = \frac{470}{5} = 94$$

Step 2. Complete the table below

x	$\bar{x}$	$(x-\bar{x})^2$
97	3	9
92	-2	4
96	2	4
95	1	1
90	-4	16
$N = 5$		$\sum(x-\bar{x})^2 = 34$

Step 3. Compute for the standard deviation.

$$S.D. = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}} = \sqrt{\frac{34}{5-1}} = \sqrt{\frac{34}{4}} = \sqrt{8.5} = 2.92$$

Student B:

Step 1. Compute the mean score.

$$\bar{x} = \frac{\sum x}{N} = \frac{94+94+92+94+96}{5} = \frac{470}{5} = 94$$

Step 2. Complete the table below

x	$\bar{x}$	$(x-\bar{x})^2$
94	0	0
94	0	0
92	-2	4
94	0	0
96	2	4
$N = 5$		$\sum(x-\bar{x})^2 = 8$

Step 3. Compute for the standard deviation.

$$S.D. = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}} = \sqrt{\frac{8}{5-1}} = \sqrt{\frac{8}{4}} = \sqrt{2} = 1.41$$

Student C:

Step 1. Compute the mean score.

$$\bar{x} = \frac{\sum x}{N} = \frac{95+94+93+96+92}{5} = \frac{470}{5} = 94$$

Step 2. Complete the table below

x	$\bar{x}$	$(x-\bar{x})^2$
95	1	1
94	0	0
93	-1	1
96	2	4
92	-2	4
$N = 5$		$\sum(x-\bar{x})^2 = 10$

Step 3. Compute for the standard deviation.

$$S.D. = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}} = \sqrt{\frac{10}{5-1}} = \sqrt{\frac{10}{4}} = \sqrt{2.5} = 1.58$$

The result of the computation of the standard deviation of the scores of the three students can be summarized as:

SD (A) = 2.92
SD (B) = 1.41
SD (C) = 1.58

Interpretation:

- Student B has the set of scores with ***least spreadness*** to the mean because it has the smallest standard deviation which is 1.41.
- Student A has the set of scores with the ***most spreadness*** to the mean because it has a largest standard deviation which is 2.92.

GROUPED DATA

A. RANGE

The **range** is the simplest measure of variability. The range of a frequency distribution is simply the difference between the upper class boundary of the top interval and the lower class boundary of the bottom interval.

Range = Upper Boundary of the Highest Class Interval – Lower Boundary of the Lowest Class Interval

Example: Solve for the range.

Scores in the Second Periodical Test of 7 – Faith in Mathematics 7

Scores	Frequency
46 -50	1
41 – 45	10
36 – 40	10
31 – 35	9
26 – 30	9
21 – 25	4

Solutions:
Upper Class Limit of the Highest Interval = 50
Upper Class Boundary of the Highest Interval = 50 + 0.5 = 50.5

Lower Class Limit of the Lowest Interval = 21
Lower Class Boundary of the Lowest Interval = 21 – 0.5 = 20.5

Range = Upper Boundary of the Highest Class Interval – Lower Boundary of the Lowest Class Interval

= 50.5 – 20.5
Range = **30**

B. VARIANCE

Variance is the mean of the square of the deviations from the mean of a frequency distribution. For large quantities, the variance is computed using frequency distribution with columns for the midpoint value, the product of the frequency and midpoint value for each interval, the deviation and its square, and the product of the frequency and the squared deviation.

To find the variance of a groupe data, use the formula:

$$\sigma^2 = \frac{\sum f(x-\bar{x})^2}{\sum f - 1}$$

where: f = class frequency
x = class mark
$\bar{x}$ = class mean
$\sum f$ = total number of frequency’

In calculating the variance, do the following:

1. Prepare a frequency distribution with appropriate class intervals and write the corresponding frequency (f).
2. Get the midpoint (x) of each class interval in column 2.
3. Multiply the frequency (f) and midpoint (x) of each class interval to get *fx*.
4. Add *fx* of each interval to get $\sum fx$.
5. Compute for the meanusing $\bar{x} = \frac{\sum fx}{\sum f}$
6. Calculate the deviation $(x - \bar{x})$ by subtracting the mean from each midpoint.
7. Square the deviation of each interval to get $(x - \bar{x})^2$.
8. Multiply (f) to $(x - \bar{x})^2$. Find the sum of each product to get $\sum f(x - \bar{x})^2$.
9. Calculate the variance using the formula: $\sigma^2 = \frac{\sum f(x-\bar{x})^2}{\sum f - 1}$

Example: Find the following variance of the given data set:

Scores in the Seond Periodical Test of 7 – Faith in Mathematics 7

Scores	Frequency
46 -50	1
41 – 45	10
36 – 40	10
31 – 35	16
26 – 30	9
21 – 25	4

Solution:

Scores	**Frequency (f)**	**Class Mode (x)**	***fx***	$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
46 -50	1	48	48	13.4	179.56	179.56
41 – 45	10	43	430	8.4	70.56	705.6
36 – 40	10	38	380	3.4	11.56	115.6
31 – 35	16	33	528	-1.6	2.56	40.96
26 – 30	9	28	252	-6.6	43.56	392.04
21 – 25	4	23	92	-11.6	134.56	538.24
i = 5	$\sum f = 50$		$\sum fx$ = 1730			$\sum f(x - \bar{x})^2$ = 1972

$$\text{Mean } \bar{x} = \frac{\sum fx}{\sum f} = \frac{1730}{50} = 34.60$$

$$\sigma^2 = \frac{\sum f(x - \bar{x})^2}{\sum f - 1} = \frac{1972}{50 - 1} = \frac{1972}{49} = 40.2448 \approx \mathbf{40.24}$$

Therfore, the variance (σ^2) is 40.24.

C. STANDARD DEVIATION (s)

The **standard deviation** is considered the best indicator of the degree of disperssion among the measures of variability because it represents an average variability of the distribution. Given the set of data, the smaller the standard deviation, the less spread is the distribution.

To get the value of the standard deviation (s), get the square root of the variance (σ^2):

$$s = \sqrt{\sigma^2}$$

where: s = standard deviation
σ^2 = variance

Example: Refer to the previous example. Get the square root of the value of the variance.

$$s = \sqrt{\sigma^2}$$
$$s = \sqrt{40.24}$$
$$s = 6.34$$

Therefore the standard deviation of the scores in the Second Periodical Test of 7 – Faith in Mathematics 7 is 6.34

Explore

Activity 2:

A. Write TRUE, if the statement is true and FALSE if the statement is fales.

1. The measures of variability allows us to determine the spread of the data.
2. The greater the variability, the more consistent the scores.
3. In finding the ranged of the ungrouped data, we will subtract the highest score to the lowest score.
4. In finding the ranged of the grouped data, we will subtract the highest class mark to the lowest class mark.
5. The first step in getting the variance of grouped data is to determine the class boundary
6. In finding the mean deviation , it is important to get the absolute value of mean (x).
7. In finding the variance, we need to square root the result of the mean (x).
8. Standard deviation is the square root of the variance.

B. Complete the table then solve of the Variance and Standard Deviation.

Class	f	(x)	fx	$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
18 – 20	2	19	38	10	100	200
15 – 17	3		48	7		
12 – 14	4	13			16	
9 – 11	2	10		1		2
6 – 8	5		35	-2		20
3 – 5	5			-5	25	
0 – 2	3	1				192
	$\sum f =$		$\sum fx =$			$\sum f(x-\bar{x})^2 =$

Deepen

Activity 3: Below are the scores of 65 students in a Mathematics Test.

Score	f	(x)	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
55 – 58						
51 – 54						
47 – 50						
43 – 46						
39 – 42						
35 – 38						
31 – 34						
27 – 30						
23 – 26						
19 – 22						
15 – 18						
11 – 14						
	$\sum f =$		$\sum fx =$			$\sum f(x-\bar{x})^2 =$

Find the following:

1. $\sum f$ = _______
2. $\sum fx$ = _______
3. Mean ($\bar{x}$) = _______
4. $\sum f(x-\bar{x})^2$ = _______
5. Variance (σ^2) = _______
6. Standard Deviation (s) = _______

GAUGE

Assessment

Directions: Read and understand the questions below. Write the letter of the correct answer on your answer sheet.

1. What measure of variability is the simplest?
   A. Mean Deviation
   B. Range
   C. Standard Deviation
   D. Variance
2. Which is true about the measures of variability?
   A. The smaller the standard deviation, the less reliable the scores.
   B. The higher the standard deviation, the more reliable the scores.
   C. The smaller the standard deviation, the less spread the distribution.
   D. The smaller the standard deviation, the more spread the distribution.
3. If two classes have the same measures of variability, what do you need to compare the two classes?
   A. Determine the average of the two classes.
   B. Determine which class got the highest scores.
   C. Determine which class who got more high scores.
   D. Determine which class have more scores around the mean.

4. Which measure is the mean of the square deviations from the mean of a frequency distribution?
   A. Mean Deviation B. Range
   C. Standard Deviation D. Variance
5. Which measure is considered the best indicator of the degree of dispersion among the measures of variability?
   A. Mean Deviation B. Range
   C. Standard Deviation D. Variance
6. Find the range of the given data: 3, 8, 15, 12, 6, 5, 9, 16.
   (NOTE: R = H – L)
   A. 10 B. 11 C. 12 D. 13

For items 7 – 8. Calculate the mean deviation of 2, 4, 6, 8, 10.

NOTE: Mean Deviation = $\frac{\sum|x-\bar{x}|}{N}$

7. What is the value of the Mean?
   A. 3 B. 4 C. 5 D. 6
8. What is the value of the Mean Deviation?
   A. 2. 4 B. 2.5 C. 2.6 D. 2.7

For items 9 – 10. Find the variance of the following: 5, 6, 2, 3, 1, ,7, 4, 8.

NOTE: $\sigma^2 = \frac{\sum(x-\bar{x})^2}{N-1}$

9. What is the value of the Mean?
   A. 3 B. 3.5 C. 4 D. 4.5
10. What is the value of the variance?
   A. 4 B. 5 C. 6 D. 7

For items 11 – 12. Calculate the standard deviation of the following scores 5, 4, 3, 6, 2.

NOTE: $S.D. = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}}$

11. What is the value of the Mean?
   A. 3 B. 4 C. 5 D. 6
12. What is the value of the standard deviation?
   A. 1.58 B. 2.58 C. 3.58 D. 4.58

For items 13 – 15, use the table of the distribution of mistakes of 50 students in their Mathematics Quiz.

Numbers of Mistakes	f	(x)	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
18 – 20	2		38	10.24	104.86	209.72
15 – 17	5	16	80	7.24	52.42	262.1
12 – 14	6	13	78	4.24	17.78	106.68
9 – 11	10	10	100	1.24	1.54	15.4
6 – 8	15	7	105	-1.76	3.1	46.5
3 – 5	8	4	32	-4.76	22.66	181.28
0 – 2	4	1	5	-7.76	60.22	240.88
	$\sum f$ = **50**		$\sum fx$ =**438**			$\sum f(x-\bar{x})^2$ = **1062.56**

13. What is the midpoint (x) in the interval 18 -20?

A. 17.5 B. 18 C. 19 D. 20.5

14. What is the mean? NOTE: $\bar{x} = \frac{\sum fx}{\sum f}$

A. 8.75 B. 8.76 C. 8.77 D. 8.78

15. What is the standard deviation? NOTE: $S.D. = \sqrt{\frac{\sum(x-\bar{x})^2}{\sum f - 1}}$

A. 4.55 B. 4.66 C. 4.77 D. 4.88

Great job! You made it. Congratulations!

References:

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Oba*ñ*a, G. G. & Mangaldan, E.R. Making Connections in Mathematics, 2004

Links:

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