

# Mathematics

## Quarter 3: Week 3 - Module 3: The Midline Theorem, Theorems on Trapezoids and Theorems on Kites



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## **Mathematics 9**

Quarter 3: Week 3 - Module 3: The Midline Theorem, Theorems on Trapezoids and Theorems on Kites

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Region I

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## Target

After applying the different theorems on rectangle, rhombus and square you are now ready to prove the Midline Theorem, theorems on trapezoid and theorems on kites.

At the end of this module, you are expected to attain the following objectives:

### Learning Competencies:

Proves the Midline Theorem (**M9GE-IIIId1**)

Proves Theorems on trapezoids and kites (**M9GE-IIId-1**)

### Subtasks:

- solve problem using the midline theorem, theorems on trapezoids and theorems on kites

*Before you start doing the activities in this lesson, find out how much you already know about this module. Answer the pre-test on the next page in a separate sheet of paper. Write the letter that corresponds to the best answer.*

### Pre-Test

**Directions:** Choose the letter of the correct answer. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. Which of the following parallelograms has diagonals that do not bisect each other?  
A. Rectangle      B. Rhombus      C. Square      D. Trapezoid
2. What type of quadrilateral has two distinct pairs of congruent adjacent sides?  
A. Kite      B. Parallelogram      C. Square      D. Trapezoid
3. What do you call the segment that joins the midpoints of the legs of a trapezoid?  
A. Altitude      B. Base      C. Midline      D. Midpoint

For numbers 4-7, please refer to  $\triangle ABC$  with  $\overline{DE}$  as the Midline Segment.

4. If the length of  $AD = 14$  cm, what is the length of  $BD$ ?

- A. 7 cm      B. 14 cm      C. 21 cm      D. 28 cm

5. If the length of  $AC = 26$  cm, what is the length of  $AE$ ?

- A. 13 cm      B. 26 cm      C. 39 cm      D. 52 cm

6. If the length of  $DE = 14$  cm, what is the length of  $BC$ ?

- A. 14 cm      B. 21 cm      C. 28 cm      D. 52 cm

7. If the length of  $BC = 42$  cm, what is the length of  $DE$ ?

- A. 10.5 cm      B. 21 cm      C. 42 cm      D. 84 cm

8. What do you call a quadrilateral with exactly one pair of parallel sides?

- A. Rectangle      B. Rhombus      C. Square      D. Trapezoid

Refer to the figure at the right for numbers 9-12.

9. The diagonals of an isosceles trapezoid are congruent.

Which segment is congruent to  $\overline{HP}$ ?

- A.  $\overline{EO}$       B.  $\overline{HE}$       C.  $\overline{OP}$       D.  $\overline{PE}$

10. Which theorem satisfy the given equation

$$m\angle HEP = m\angle EPO?$$

- A. The legs are congruent.  
B. The diagonals are congruent.  
C. The base angles are congruent.  
D. Opposite angles are supplementary.

11. Opposite angles of an isosceles trapezoid are supplementary. Which of the following statement is true?

- A.  $m\angle HEP + m\angle EPO = 180$       B.  $m\angle HEP + m\angle HOP = 180$   
C.  $m\angle EHO + m\angle HOP = 180$       D.  $m\angle HEP + m\angle HEO = 180$

12. Which segment is parallel to  $\overline{EP}$ ?

- A.  $\overline{EO}$       B.  $\overline{HE}$       C.  $\overline{HO}$       D.  $\overline{OP}$

13. Which of the following statements is TRUE?

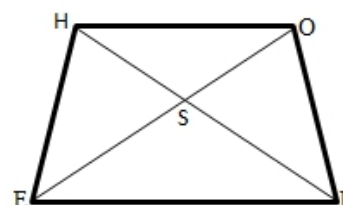
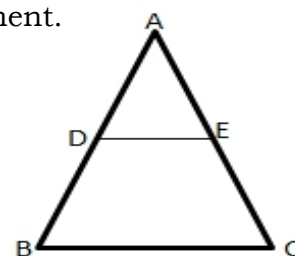
- A. A trapezoid can have four equal sides.  
B. A trapezoid can have three right angles.  
C. The base angles of an isosceles trapezoid are congruent.  
D. The diagonals of an isosceles trapezoid bisect each other.

14. What statement/s is/are TRUE about isosceles trapezoids?

- A. The diagonals are congruent.      B. The median is parallel to the bases.  
C. Both A and B.      D. Neither A and B.

15. Which of the following statement is **incorrect** about kites?

- A. The area is half the product of the lengths of its diagonals.  
B. The perpendicular bisector of at least one diagonal is the other diagonal.  
C. It has two pairs of congruent and adjacent sides.  
D. The diagonals are congruent.



## Lesson 1

# The Midline Theorem, Theorems on Trapezoids and Theorems on Kites



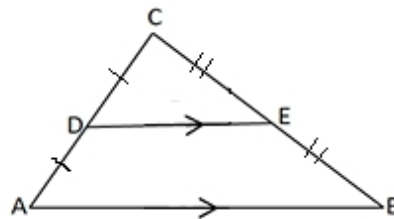
## Jumpstart

Let us start this lesson by doing the Activity 1 and 2. These statements will serve as your guide as you go through the lesson.

### Activity 1. Am I Right or Wrong?

**Directions:** Use the figure at the below, write **R** if the statement is correct and **W** if incorrect.

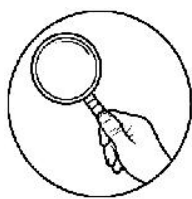
- \_\_\_\_\_ 1. D is the midpoint of  $\overline{AC}$
- \_\_\_\_\_ 2. E is the midpoint of  $\overline{CB}$
- \_\_\_\_\_ 3.  $\overline{DE} \parallel \overline{AB}$
- \_\_\_\_\_ 4.  $\overline{DE}$  is the mid-segment
- \_\_\_\_\_ 5.  $DE = AB$



### Activity 2. True or False

**Directions:** Write T if your guess on the statement is true; otherwise, write F.

- \_\_\_\_\_ 1. The median of the trapezoid is parallel to the bases and its length is equal to half the sum of the lengths of the bases.
- \_\_\_\_\_ 2. The legs of an isosceles trapezoid are parallel and congruent.
- \_\_\_\_\_ 3. The bases of an isosceles trapezoid are congruent.
- \_\_\_\_\_ 4. The segment that joins the midpoints of the legs of a trapezoid is called the median.
- \_\_\_\_\_ 5. The area of a kite is half the product of the lengths of its diagonals.



## Discover

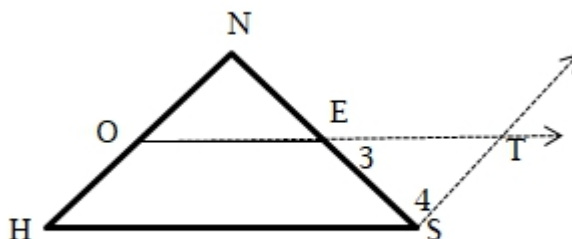
### The Midline Theorem

**Theorem 5.** The segment that joins the midpoint of two sides of a triangle is parallel to the third side and half as long.

Let us prove this theorem by two-column proof.

Given:  $\triangle HNS$ , O is the midpoint of  $\overline{HN}$   
E is the midpoint of  $\overline{NS}$

Prove:  $\overline{OE} \parallel \overline{HS}$ ,  $OE = \frac{1}{2} HS$



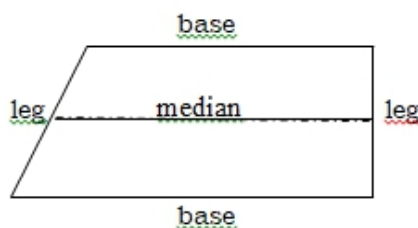
Proof:

Statements	Reasons
1. In $\triangle HNS$ , O is the midpoint of $\overline{HN}$ E is the midpoint of $\overline{NS}$	1. Given
2. In a ray opposite $\overrightarrow{EO}$ , there is a point T such that $OE = ET$	2. In a ray, point at a given distance from the endpoint of a ray.
3. $\overline{EN} \cong \overline{ES}$	3. Definition of Midpoint
4. $\angle 2 \cong \angle 3$	4. Vertical Angle Theorem
5. $\triangle ONE \cong \triangle TSE$	5. SAS Congruence Postulate
6. $\angle 1 \cong \angle 4$	6. Corresponding parts of congruent triangles are congruent
7. $\overline{HN} \parallel \overline{ST}$	7. If Alternate Interior Angles are Congruent, then the lines are parallel.
8. $\overline{OH} \cong \overline{ON}$	8. Definition of Midpoint
9. $\overline{ON} \cong \overline{TS}$	9. CPCTC (SN 5)
10. $\overline{OS} \cong \overline{ST}$	10. Transitive Property
11. Quadrilateral HOTS is a parallelogram.	11. Definition of parallelogram.
12. $\overline{OE} \parallel \overline{HS}$	12. $\overline{OE}$ is on the side of $\overline{OT}$ of HOTS
13. $OE + ET = OT$	13. Segment Addition Postulate (SAP)
14. $OE + OE = OT$	14. Substitution (SN 2)
15. $2OE = OT$	15. Addition
16. $\overline{HS} \parallel \overline{OT}$	16. Parallelogram Property
17. $2OE = HS$	17. Substitution
18. $OE = \frac{1}{2} HS$ (The segment joining the midpoints of two sides of a triangle is half as long as the third side.)	18. Substitution (SN 14 and 15)

You've just completed the proof of The Midline Theorem. Let us continue with the next theorem.

Given

But before that, let us recall the different parts of a trapezoid. All trapezoids have two main parts: bases and legs. The opposite sides of a trapezoid that are parallel to each other are the bases and the remaining sides are called the legs of the trapezoid. The segment that connects the midpoints of the legs of trapezoid is called the mid-segment or the median.



### The Mid-segment Theorem of Trapezoid

**Theorem 6.** The median of a trapezoid is parallel to each base and its length is one-half the sum of the lengths of the bases.

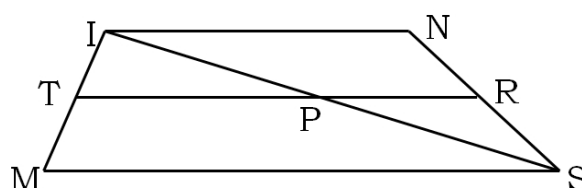
$$\text{Mid-segment} = \frac{1}{2} (\text{base}_1 + \text{base}_2)$$

Let us now prove the theorem using trapezoid MINS.

Given: Trapezoid MINS with median  $\overline{TR}$

Prove:  $\overline{TR} \parallel \overline{IN}$ ,  $\overline{TR} \parallel \overline{MS}$

$$TR = \frac{1}{2} (MS + IN)$$



Proof:

Statements	Reasons
1. Trapezoid MINS with median $\overline{TR}$	1. Given
2. Draw $\overline{IS}$ , with P as it's midpoint.	2. Line Postulate
3. $TP = \frac{1}{2} MS$ and $\overline{TP} \parallel \overline{MS}$	3. Theorem 5 (Midline theorem), on $\Delta IMS$
4. $PR = \frac{1}{2} IN$ and $\overline{PR} \parallel \overline{IN}$	4. Theorem 5 (Midline Theorem) on $\Delta INS$
5. $\overline{MS} \parallel \overline{IN}$	5. Definition of trapezoid
6. $\overline{TP} \parallel \overline{IN}$	6. Definition of parallel, $\overline{TP} \parallel \overline{MS}$ and $\overline{MS} \parallel \overline{IN}$
7. $\overline{TP}$ And $\overline{PR}$ are both parallel to $\overline{TP} \parallel \overline{IN}$ . Thus, T, P and R are collinear.	7. $\overline{TP}$ and $\overline{PR}$ are either parallel or the same line (definition of parallel). Since they contain a common point P, then $\overline{TP}$ and $\overline{PR}$ are contained in the same line.
8. $TR = TP + PR$	8. Segment Addition Postulate
9. $TR = \frac{1}{2}MS + \frac{1}{2}IN$	9. Substitution
10. $TR = \frac{1}{2} (MS + IN)$	10. Distributive Property of Equality

We've just proven Theorem 6. Now what if the legs of the trapezoid become congruent? What must be true about its base angles and its diagonals? To answer this, we will study the different theorems on isosceles trapezoid.

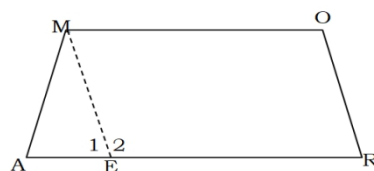
**Theorem 7.** The base angles of an isosceles trapezoid are congruent.

Given: Isosceles Trapezoid AMOR

$$\overline{MO} \parallel \overline{AR}$$

Prove:  $\angle A \cong \angle R$ ,  $\angle AMO \cong \angle O$

Proof:



Statements	Reasons
1. Isosceles Trapezoid AMOR	1. Given
2. $\overline{AM} \cong \overline{OR}$ ; $\overline{MO} \parallel \overline{AR}$	2. Definition of Isosceles Trapezoid
3. From M, draw $\overline{ME} \parallel \overline{AR}$ where E lies on $\overline{AR}$ .	3. Parallel Postulate
4. MORE is a parallelogram	4. Definition of a parallelogram
5. $\overline{ME} \cong \overline{OR}$	5. Parallelogram Property 1
6. $\overline{OR} \cong \overline{ME}$	6. Symmetric Property
7. $\overline{AM} \cong \overline{ME}$	7. Transitive Property (SN 2 and 6)
8. $\triangle AME$ is an isosceles triangle.	8. Definition of an Isosceles Triangle
9. $\angle 1 \cong \angle A$	9. Base angles of an isosceles triangle are congruent.
10. $\angle 1 \cong \angle R$	10. Corresponding angles are congruent.
11. $\angle R \cong \angle A$	11. Substitution
12. $\angle A \cong \angle R$	12. Symmetric Property
13. $\angle A$ and $\angle AMO$ are supplementary angles. $\angle O$ and $\angle R$ are supplementary angles.	13. Same side interior angles are supplementary
14. $\angle AMO \cong \angle O$	14. Supplements of congruent angles are congruent.

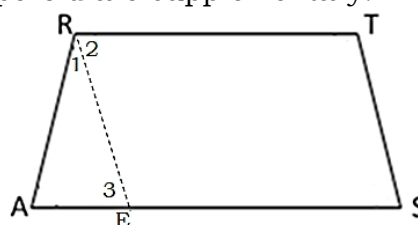
We have just proven Theorem 7. Let us now proceed to the next theorem.

**Theorem 8.** Opposite angles of an isosceles trapezoid are supplementary.

Given: Isosceles Trapezoid ARTS.

Prove:  $\angle A$  and  $\angle T$  are supplementary.

$\angle R$  and  $\angle S$  are supplementary.



Proof:

Statements	Reasons
1. Isosceles Trapezoid ARTS	1. Given
2. $\overline{AR} \cong \overline{TS}$ ; $\overline{RT} \parallel \overline{AS}$	2. Definition of Isosceles Trapezoid

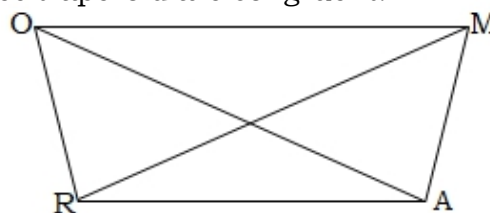


3. From R, draw $\overline{RE} \parallel \overline{TS}$ where E lies on $\overline{AS}$ .	3. Parallel Postulate
4. REST is a parallelogram.	4. Definition of parallelogram
5. $\overline{TS} \cong \overline{RE}$	5. Parallelogram Property 1
6. $\overline{AR} \cong \overline{RE}$	6. Transitive Property
7. $\triangle ARE$ is an isosceles triangle	7. Definition of isosceles triangle
8. $\angle 3 \cong \angle A$	8. Isosceles Triangle Theorem
9. $m\angle 1 + m\angle 3 + m\angle A$	9. Interior Angle Sum Theorem on Triangle
10. $\angle 3 \cong \angle 2$	10. Alternate Interior Angles are Congruent
11. $\angle A \cong \angle S$	11. Theorem 7
12. $m\angle 1 + m\angle 2 + m\angle S$	12. Substitution (SN 9, 10 & 11)
13. $\angle 1 + \angle 2 \cong \angle ART$	13. Angle Addition Postulate
14. $m\angle ART + m\angle S$	14. Substitution
15. $m\angle S + m\angle T$	15. Same Side Interior Angles are Supplementary
16. $m\angle A + m\angle T$	16. Substitution
17. $\angle ART$ and $\angle S$ are supplementary. $\angle A$ and $\angle T$ are supplementary.	17. Definition of Supplementary Angles

**Theorem 9.** The diagonals of an isosceles trapezoid are congruent.

Given: Isosceles Trapezoid ROMA

Prove:  $\overline{RM} \cong \overline{AO}$



Proof:

Statements	Reasons
1. Isosceles Trapezoid ROMA	1. Given
2. $\overline{OR} \cong \overline{MA}$	2. Definition of Isosceles Trapezoid
3. $\angle ROM \cong \angle AMO$	3. Theorem 7
4. $\overline{OM} \cong \overline{MO}$	4. Reflexive Property
5. $\triangle ROM \cong \triangle AMO$	5. SAS Congruence Postulate
6. $\overline{RM} \cong \overline{AO}$	6. CPCTC

We've just proven the different theorems concerning trapezoids. Now, you will prove another set of theorems, this time concerning kites.

Kites are quadrilaterals with two pairs of congruent and adjacent sides. Note that rhombus is a special kind of kite where all adjacent sides are equal. There are two theorems related to kite.

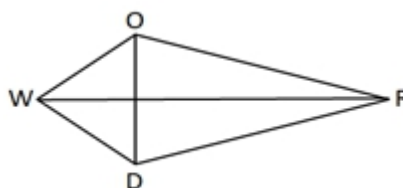
**Theorem 10.** In a kite, the perpendicular bisector of at least one diagonal is the other diagonal.

Given: Kite WORD with diagonals

$\overline{WR}$  and  $\overline{OD}$

Prove:  $\overline{WR}$  is the perpendicular bisector of  $\overline{OD}$ .

Proof:

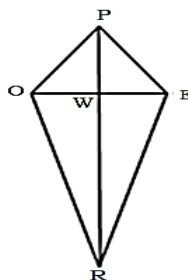


Statements	Reasons
1. Kite WORD with diagonals $\overline{WR}$ and $\overline{OD}$	1. Given
2. $\overline{WO} \cong \overline{WD}$ ; $\overline{OR} \cong \overline{DR}$	2. Definition of kite
3. $WO = WD$ ; $OR = DR$	3. Definition of Congruent segments
4. $\overline{WR} \perp \overline{OD}$	4. If a line contains two points each of which is equidistant from the endpoints of a segment, then the line is the perpendicular bisector of the segment.

**Theorem 11.** The area of a kite is half the product of the lengths of its diagonals.

Given: Kite ROPE

Prove: Area of kite ROPE =  $\frac{1}{2}(OE)(PR)$



Proof:

Statements	Reasons
1. Kite ROPE	1. Given
2. $\overline{PR} \perp \overline{OE}$	2. The diagonals of a kite are perpendicular to each other.
3. Area of kite ROPE = Area of $\triangle OPE$ + Area of $\triangle ORE$	3. Area Addition Postulate
4. Area of $\triangle OPE = \frac{1}{2}(OE)(PW)$ Area of $\triangle ORE = \frac{1}{2}(OE)(WR)$	4. Area Formula for Triangles
5. Area of kite ROPE = $\frac{1}{2}(OE)(PW) + \frac{1}{2}(OE)(WR)$	5. Substitution
6. Area of kite ROPE = $\frac{1}{2}(OE)(PW + WR)$	6. Distributive Property of Equality
7. $PW + WR = PR$	7. Segment Addition Postulate
8. Area of kite ROPE = $\frac{1}{2}(OE)(PR)$	8. Substitution

It's amazing that the area of a kite has been derived from the formula in finding for the area of a triangle.

Now that you have proven the midline theorem and the different theorems on trapezoid and kites, you are now ready to apply it in doing the next activity

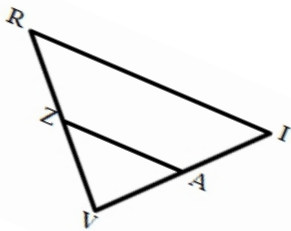


## Explore

Work on the following enrichment activities for you to apply your understanding on this lesson.

### Activity 3: Fill Me!

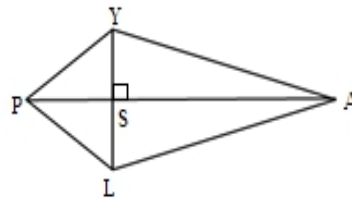
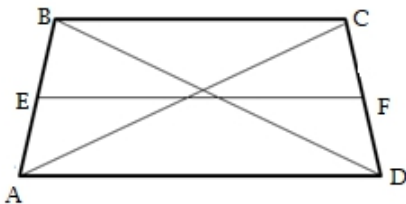
**Directions:** Use the Triangle Mid-segment Theorem to name parts of  $\triangle RIV$ . Z and A are the midpoints of  $\overline{RV}$  and  $\overline{IV}$ , respectively.



1. \_\_\_\_\_ is a mid-segment of  $\triangle RIV$ .
2. \_\_\_\_\_ is a segment parallel to  $\overline{AZ}$ .
3.  $\overline{RZ}$  is a segment that has the same length as \_\_\_\_\_.
4. \_\_\_\_\_ is half as long as  $\overline{RI}$ .
5. \_\_\_\_\_ is twice as long as  $\overline{AZ}$ .

### Activity 4: Complete Me!

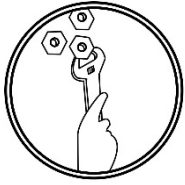
**Directions:** Use the figures below to complete the sentences that follow.



1. In trapezoid ABCD, \_\_\_\_\_ is the midline.
2.  $\overline{AD}$  is parallel to \_\_\_\_\_.
3.  $\overline{BD}$  is congruent to \_\_\_\_\_.
4.  $\angle BAD$  is the supplement of \_\_\_\_\_.
5. In kite PLAY, \_\_\_\_\_ is the perpendicular bisector of  $\overline{LY}$ .

Did you apply the knowledge and concepts of Midline Theorem and the different theorems on trapezoid and kites? If you did then you are now ready for the next activity.

Answer the next activity to deepen more your understanding.

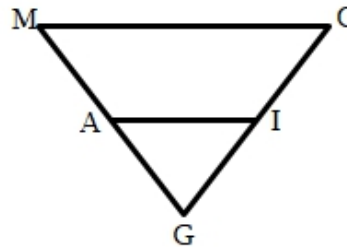


## Deepen

### Activity 5: Go For It!

**Directions:** Consider each given information and answer the questions that follow.

In  $\triangle MCG$ , A and I are the midpoints of  $\overline{MG}$  and  $\overline{GC}$ , respectively.

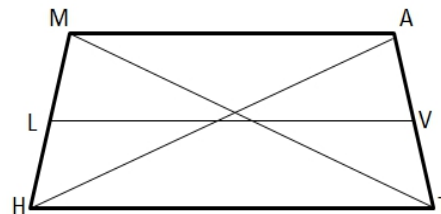


1. If  $AI = 10$ , what is  $MC$ ?
2. If  $MC = 32$ , what is  $AI$ ?
3. If  $MG = 12$ , what is  $AG$ ?

### Activity 6: Show More What You've Got!

**Directions:** Consider the figure that follows and answer the given questions.

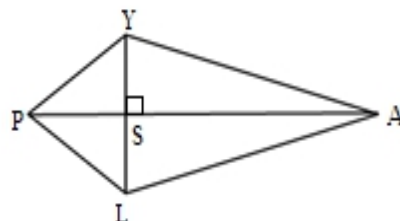
- A. Quadrilateral MATH is an isosceles trapezoid with bases  $\overline{MA}$  and  $\overline{HT}$ ,  $\overline{LV}$  is the median.



1. If  $MA = 10$  and  $HT = 14$ , what is  $LV$ ?
2. If  $MT = 25$ , what is  $AH$ ?
3. If  $m\angle MHT = 65$ , what is  $m\angle ATH$ ?

- B. Quadrilateral PLAY is a kite.

1. If  $m\angle SPL = 30$ , what is  $m\angle SLP$ ?
2. If  $PA = 12\text{cm}$  and  $LY = 6\text{cm}$ , what is the area of kite PLAY?



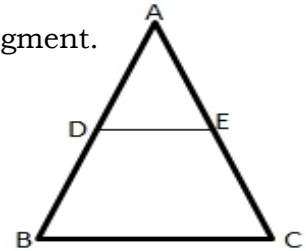
The activities you did above clearly reflect your deeper understanding of the lessons taught to you in this module. Now you are ready to put your knowledge and skills to practice and be able to answer the questions you've instilled in your mind from the very beginning of this module.



## Gauge

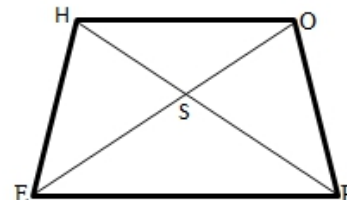
**Directions:** Choose the letter of the correct answer.

For numbers 1-3, please refer to  $\triangle ABC$  with  $\overline{DE}$  as the Midline Segment.



- Which segment is parallel to  $\overline{DE}$ ?  
A.  $\overline{AB}$       B.  $\overline{BC}$       C.  $\overline{AC}$       D.  $\overline{AE}$
- Which segment is half as long as  $\overline{BC}$ ?  
A.  $\overline{AB}$       B.  $\overline{BC}$       C.  $\overline{AC}$       D.  $\overline{DE}$
- What is the length of BD if the length of AD = 16 cm  
A. 7 cm      B. 16 cm      C. 20 cm      D. 28 cm
- Which of the following is a quadrilateral that has one pair of parallel sides?  
A. Kite      B. Parallelogram      C. Rhombus      D. Trapezoid
- What do you call the non-parallel sides of a trapezoid?  
A. Base      B. Diagonals      C. Legs      D. Midline
- What do you call the segment that joins the midpoints of the legs of a trapezoid?  
A. Altitude      B. Base      C. Midline      D. Midpoint
- What can you say about the legs of an isosceles trapezoid?  
A. Congruent      B. Parallel      C. Perpendicular      D. Skew
- Which statement is TRUE about isosceles trapezoids?  
A. The diagonals are congruent.  
B. The median is perpendicular to the bases.  
C. Base angles are supplementary,  
D. Opposite angles are congruent.
- Which of the following is a quadrilateral with two pairs of congruent and adjacent sides?  
A. Kite      B. Parallelogram      C. Rectangle      D. Trapezoid
- Which of the following quadrilateral is considered special kind of kite since all its adjacent sides are equal?  
A. Square      B. Parallelogram      C. Rhombus      D. Trapezoid

Refer to the figure at the right for numbers 11-13.



- Which theorem satisfy the given equation  
 $m\angle HEP + m\angle HOP = 180$ ?  
A. The legs are congruent.  
B. The diagonals are congruent.  
C. The base angles are congruent.  
D. Opposite angles are supplementary.
- Base angles of an isosceles trapezoid are supplementary. Which of the following statement is true?  
A.  $m\angle HEP = m\angle EPO$       B.  $m\angle HEP = m\angle HOP$   
C.  $m\angle EHO = m\angle HOE$       D.  $m\angle HEP = m\angle HEO$
- Which segment is equal to  $\overline{HP}$ ?  
A.  $\overline{EO}$       B.  $\overline{HE}$       C.  $\overline{HO}$       D.  $\overline{OP}$

14. Which of the following statement is **TRUE** about kites?
- A. The diagonals are congruent.
  - B. It has two pairs of parallel sides.
  - C. The area is equal the product of the lengths of its diagonals.
  - D. The perpendicular bisector of at least one diagonal is the other diagonal.
15. The area of a kite is half the product of the lengths of the diagonals. What is the area of a kite with diagonals that measure 20 cm and 10 cm respectively?
- A. 100 sq. cm                      B. 80 sq. cm                      C. 60 sq. cm    D. 50 sq. cm