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Mathematics
Quarter 2 - Module 1:
Variations

AIRs - LM

MATHEMATICS 9
Quarter 2 - Module 1: Variations
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Mathematics
Quarter 2 - Module 1:
Variations

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Do you know that an increasing demand of paper contributes to the destruction of trees from which papers are made? If waste papers were recycled regularly, it would help prevent the cutting down of trees, global warming and other adverse effects that would destroy the environment. This is one situation where question such as "Will a decrease in production of paper contribute to the decrease in the number of trees being cut?" can be answered using the concepts of variations. In this module, you will found out the relation between quantities.

Learning Competencies:

After going through this module, you are expected to:
- Illustrates situations that involve the following variations: (a) direct; (b) inverse; (c) joint; (d) combined **(M9AL-IIa-1)**
- Translates into variation statement a relationship between two quantities given by: (a) a table of values; (b) a mathematical equation; (c) a graph, and vice versa **(M9AL-IIa-b-1)**

Subtasks:
1. Define direct, inverse, joint, and combined variations;
2. Find the constant of variation and the equation of relation;
3. Appreciate the uses of variations in real-life situations;
4. Illustrate situations that involve the following variations: (a) direct; (b) inverse; (c) joint; (d) combined;
5. Translate into variation statement a relationship between two quantities given by: (a) a table of values; (b) a mathematical equation; (c) a graph, and vice versa.

Pre-Assessment

Directions. Find out how much you already know about this module. Choose the letter that you think best answers the question. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module. Write your answer on a separate sheet of paper.

1. Which of the following variation occurs whenever a situation produces pairs of numbers whose product is constant?
   A. Combined variation
   B. Direct variation
   C. Inverse variation
   D. Joint variation

2. Which of the following expression illustrates the statement: *The volume* **V** *of a gas varies directly as the temperature* **T** *and inversely as the pressure* **P**?
   A. $V = \frac{kT}{P}$
   B. $V = \frac{T}{kP}$
   C. $V = kTP$
   D. $V = TP$

3. Which kind of variation involves both the direct and inverse variations?
   A. Combined Variation
   B. Direct Variation
   C. Inverse Variation
   D. Joint Variation

4. Which is an example of direct variation?
   A. y = 3x
   B. $y = \frac{4}{x}$
   C. y = 5xz
   D. xy = 6

5. Which expression is read as *"The area* **A** *of a triangle varies jointly as the base* **b** *and the altitude* **h** *of the triangle*?
   A. A = kbh
   B. $A = \frac{k}{bh}$
   C. $\frac{bh}{k} = A$
   D. k = Ab

6. What kind of variation relates **n** to **t** in this statement: *An encoder can type* **n** *words in* **t** *hours.* (Hint: The formula that relates **n** to **t** is $n = \frac{k}{t}$)?
   A. Combined variation
   B. Direct variation
   C. Inverse variation
   D. Joint variation

7. Which of the following describes an inverse variation?

A.

x	2	3	4	5
y	5	$\frac{10}{3}$	$\frac{10}{4}$	2

B.

x	40	30	20	10
y	8	6	4	2

C.

x	1	2	3	4
y	5	10	15	20

D.

x	4	8	10	12
y	2	4	5	6

8. What happens to **T** when **h** is doubled in the equation **T = 4h?**
   A. T is halved
   B. T is doubled
   C. T is doubled twice
   D. T becomes zero

9. Which mathematical equation best describe the table?
   A. x = 12y
   B. x = $\frac{4}{y}$
   C. x = $\frac{12}{y}$
   D. x = $\frac{y}{4}$

X	4	3	2	1
y	3	4	6	12

10. Which of the following words complete the statement "P = $\frac{kXY}{Z}$ means P varies _____ as X and Y and _____ as Z?
    A. directly, inversely
    B. inversely, jointly
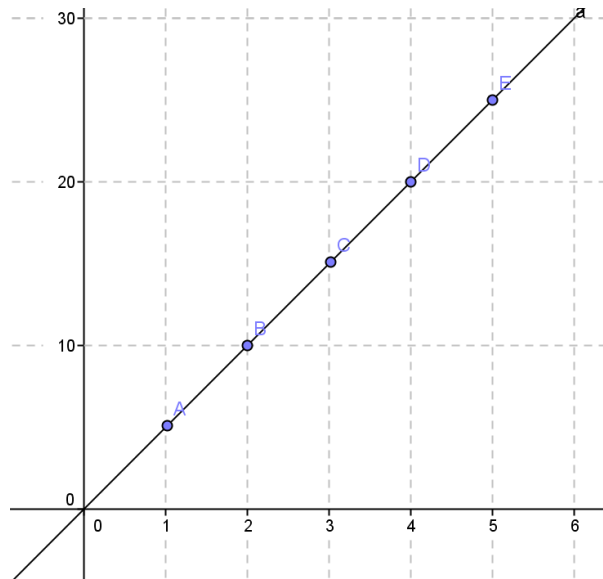   C. jointly, directly
    D. jointly, inversely

11. What mathematical statement describes the graph below?
    A. y = $\frac{1}{5}$ x
    B. y = $\frac{1}{2}$ x
    C. y = x
    D. y = 5x

12. What is the constant of variation if y varies directly as x, and y = 10 when x =5?
    A. 50
    B. 15
    C. 2
    D. $\frac{1}{2}$

13. What happens to y when x is tripled in the relation y = $\frac{k}{x}$?
    A. y is tripled
    B. y is halved
    C. y is doubled
    D. y is divided by 3

14. What kind of variation relates **d** to **t** in this situation *"A car travels a distance of* **d** *km in* **t** *hours?* (Hint: formula is d = kt)
    A. Direct
    B. Inverse
    C. Joint
    D. Combined

15. Which is an example of combined variation?
    A. y = 3x
    B. y = $\frac{4x}{z}$
    C. y = 5xz
    D. xy = 10

> *Were you able to answer all the questions? If not, don't worry because the next activity will help you better understand the lesson.*

Jumpstart

Let's start this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you understand variations. As you go through this lesson, think of this important question: "How are concepts of variations used in solving real-life problems and in making decisions? If you find any difficulty in answering the exercises, seek assistance of your teacher or refer to the modules you have gone over earlier.

Activity 1: TRANSLATE ME!

Directions: Rewrite the following expressions into verbal statements.
1. 3x
2. 10x + y
3. P = $2l + 2w$
4. A = $\frac{1}{2}bh$
5. Z = 2xy

Activity 2. MAKE ME SIMPLE!

Directions. Express the following ratios in lowest terms:

1. $\frac{25}{50}$

2. 10:15

3. $(-9)\left(\frac{2}{3}\right)$

4. $(-100)(-4)$

5. $(-3)(5)^2$

6. $(9^2)(1^2)$

7. $\frac{-45}{(9)(5)}$

8. $\frac{-20}{-4}$

9. $\frac{(6)(-1^2)}{18}$

10. $\frac{(5)(8)}{50}$

How did you find the activity? I am sure you did not find any difficulty in answering the questions. The next activity will help you fully understand the concepts behind this activity. But before doing the next activity, you have to read thoroughly and understand first some important notes about variations.

Discover

Lesson 1
Direct Variation

There is ***direct variation*** whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:
"y varies directly as x"
"y is directly proportional to x"
"y is proportional to x"

Imply direct variation and are used in many situations. Likewise the statements translate mathematically as $y = kx$. Where k is often referred to as the constant of proportionality or the constant of variation.

For two quantities x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Examples 1:
"The amount that a family (f) gives a charity varies directly as its income (i)" is translated as $f = ki$

Example 2:
"T varies directly as M" is translated as $T = kM$

Example 3:
If y varies directly as x and y = 50 when x = 10, find the constant of variation and the equation of variation.

Solution:
a. Express the statement "y varies directly as x" as y =kx.
b. Solve for k by substituting the given values in the equation.

y = kx

50 = 10k

$k = \frac{50}{10}$

k = 5

Therefore the **constant of variation** is **5**.

c. Form the equation of variation by substituting 5 in the statement, y = kx
So we have   **y = 5x**

Example 4:

The table shows that the distance (*d*) varies directly as the time (*t)*. Find the constant of variation and the equation which describes the relation, then graph.

Time (hr)	1	2	3	4	5
Distance(km)	10	20	30	40	50

Solution:

Since the distance *d* varies directly as the time *t,* then
$$d = kt$$
Using one of the pairs of values (2, 20) from the table, substitute the values of *d* and *t* in *d =kt* and solve for *k*.
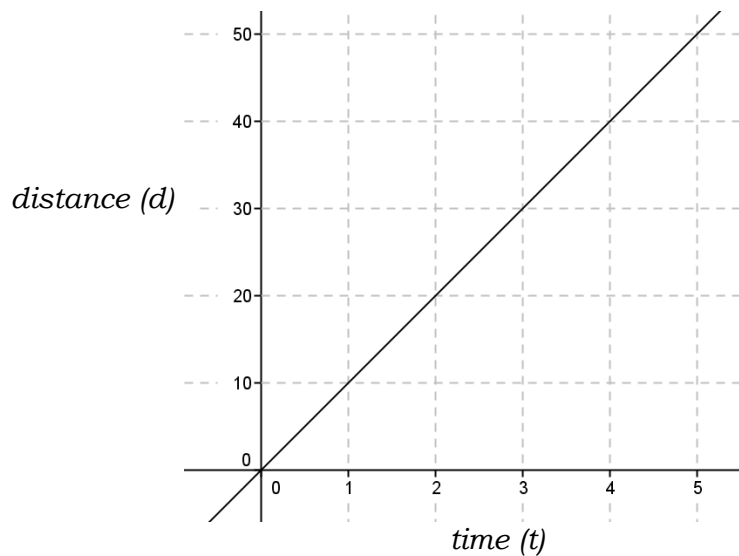
$$d = kt$$

$$20 = 2k$$

$$k = \frac{20}{2}$$

$$k = 10$$

Therefore, the constant of variation is 10

The equation of the variation is d = 10t

Below is the graph of the relation d = 10t.

time (t)

As the time *(t)* increases, the distance *(d)* being travelled also increases.

Lesson 2
Inverse Variation

This lesson deals with the relation of two quantities where one value increases as the other value decreases and vice versa. Inverse variation concepts can also be used to solve problems in other fields of mathematics.

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or $y = \dfrac{k}{x}$

Example 1:
   "The number of notebooks sold **(N)** varies inversely as the price per notebook **(P)**" is translated as $\mathbf{N = \dfrac{k}{P}}$

Example 2:
   Find the variation constant and the equation of variation if y varies inversely x, and y = 5 when x = 10.

   Solution: The relation y varies inversely as x translates to $y = \dfrac{k}{x}$ **,** then substitute the values to find k (constant of variation).

$$y = \frac{k}{x}$$

$$5 = \frac{k}{10}$$

$$k = (5)(10)$$

$$k = 50$$

   Therefore, the equation of variation is $y = \dfrac{50}{x}$

Example 3:
   The distance **(d)** from the center of a seesaw varies inversely as the weight of the child **(w)**. Daniel, who weighs 100 lb., sits 10 feet from the fulcrum.

   Solution:

$$d = \frac{k}{w}$$

$$10 = \frac{k}{w} \qquad \text{by Substitution}$$
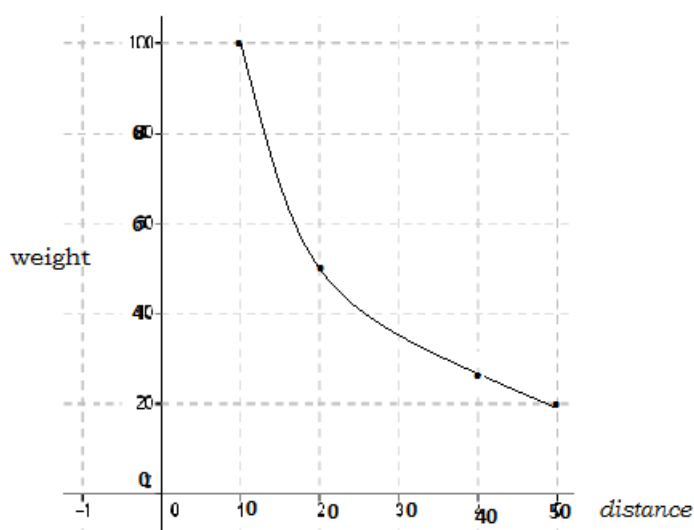
$$k = (100)(10)$$

$$k = 1000$$

8

The constant of variation is 1000; therefore, the equation of variation is

$$y = \frac{1000}{x}$$

In table form the relation between the distance (d) and the weight of the child is shown

Weight (w)	100	50	40	20
Distance (d)	10	20	25	50

The graph of the relation looks like this:

Lesson 3
Joint Variation

Joint variation is just like direct variation but involves more than one other variable. All the variables are directly proportional, taken one at a time.

The statement "*a varies jointly as b and* c" means a = kbc, or k = $\frac{a}{bc}$ where *k* is the constant of variation

Example 1:
The lateral surface area **(A)** of a cylindrical jar varies jointly as the diameter **(d)** and the height **(h)** of the jar.

Translated as: *A = kdh*

Example 2.

Find an equation of variation where a varies jointly as b and c, and a = 24 when b = 3 and c = 4.

Solution: $a = kbc$

$24 = k(3)(4)$ substitute the set of given data to find k

$k = \dfrac{24}{12}$ apply the properties of equality

$k = 2$

Therefore, the required equation of variation is: $a = 2bc$

Example 3:

z varies jointly as x and y. If $z = 16$ when $x = 4$ and $y = 6$, find the constant of variation and the equation of the relation.

Solution: $z = kxy$

$16 = k(4)(6)$ Substitute the set of given data to find k

$k = \dfrac{16}{24}$ Apply the properties of equality

$k = \dfrac{2}{3}$

The equation of the variation is: $z = \dfrac{2}{3}xy$

Lesson 4
Combined Variation

Combined variation is another physical relationship among variables. This is the kind of variation that involves both the direct and inverse variations.

Combined variation describes a situation where a <u>variable</u> depends on two (or more) other variables, and <u>varies directly</u> with some of them and <u>varies inversely</u> with others (when the rest of the variables are held constant).. These equations are a little more complicated.

> The statement "z varies directly as x and inversely as y" means $z = \dfrac{kx}{y}$, or $k = \dfrac{zy}{x}$, where k is the constant of variation.

This relationship among variables will be well illustrated in the following examples:

Examples:

1. Translating statements into mathematical equations using k as the constant of variation.

   a. T varies directly as a and inversely as b.
   $$T = \frac{ka}{b}$$

   b. Y varies directly as x and inversely as the square of z.

   $$Y = \frac{kx}{z^2}$$

2. If z varies directly as x and inversely as y , and z = 9 when x = 6 and y = 2, find the constant of variation and the equation of the variation.

   Solution:

   The equation is $z = \dfrac{kx}{y}$

   $$9 = \frac{6k}{2}$$

   $$k = \frac{9}{3}$$

   $$k = 3$$

   The constant of variation is 3, therefore the equation is $z = \dfrac{3x}{y}$

3. t varies directly as m and inversely as the square of n. If $t = 16$ when $m = 8$ and $n = 2$, find the constant of variation k

   Solution:

   The equation of the variation: $t = \dfrac{km}{n^2}$

   To find k, where $t = 16$, $m = 8$ and $n = 2$, substitute the given values

   $$16 = \frac{k(8)}{(2)^2}$$

   $$k = \frac{16(2)^2}{8}$$

   $$k = \frac{(16)(4)}{8}$$

   $$k = \frac{64}{8}$$

   $$k = 8$$

   Therefore, the constant of variation is 8 and the equation is $t = \dfrac{8m}{n^2}$

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Explore

Activity 3: It's Your Turn!

A. Identify whether the following illustrates direct variation **(DV)**, *i*nverse **(IV)**, joint **(JV)** or combined variation **(CV)**. Write only the symbol.

1. $y = \dfrac{25}{x}$

2.

x	4	8	10	12
y	2	4	5	6

3.

x	6	4	3	2	1
y	2	3	4	6	12

4. $A = kbh$

5. $P = \dfrac{kx^2}{s}$

B. Write an equation for the following statements:
1. The perimeter **(P)** of a square varies directly as the distance **(d).**
2. The weight **(W)** of an object is directly proportional to its mass **(m).**
3. Air pressure **(P)** varies inversely as its altitude **(h)**
4. The volume **(V)** of a pyramid varies jointly as the base area **(b)** and the altitude **(a)**
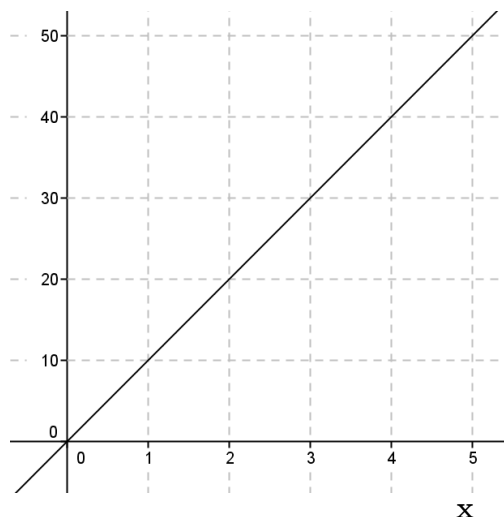5. **(U)** varies directly as **(c)** and inversely as **(d)**

Activity 4

Directions: Find the constant of variation and write the equation representing the relationship between the quantities in each of the following:

1.

x	1	2	3	4
y	3	6	9	12

2. y

x

3. y varies inversely as x and y = 12 when x = 4
4. z varies jointly as x and y if z =27, x = 3 and y = 3
5. "p varies directly as q and the square of r and inversely as s"
    if p = 40 when q = 5, r = 4 and s = 6.

> *Now that you have understood the concept and ideas of this topic, let's now deepen your understanding by moving on to the next activity.*

 Deepen

Activity 5: How Well Do You Understand?

Direction: Find the constant of variation of the following situations below.

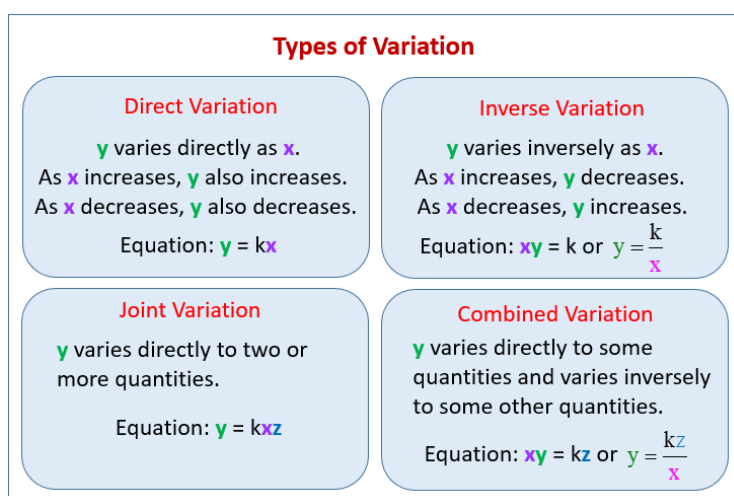1. The current **I** varies directly as the electromotive force **E** and inversely as the resistance **R**. In a system a current of 20 amperes flows through a resistance of 20 ohms with an electromotive force of 100 volts.
    a. Equation: _____
    b. Current (I) : _____ amperes
    c. Resistance (R): _____ ohms
    d. Electromotive force (E): _____ volts
    e. Find the constant of variation (k)

2. Lory is asking for donations for a charity walk-a-thon. She wants to raise Php100, 000.00 for the charity. Write an inverse variation equation to calculate the amount she should ask each donor for based on the number of donors she plans on asking.

CRITERIA	4	3	2	1
Solution	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.
Mathematical Work and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it easy to understand what was done.	Correct terminology and notation are used, making it easy to understand what was done.	There is little use, or a lot of inappropriate use of terminology and notation.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat, clear, organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.

Generalization

In your notebook, summarize what you have learned from this lesson. Provide one real life example. Then illustrate using table, graph or mathematical equation showing relationship of quantities.

Types of Variation

Direct Variation

y varies directly as x.
As x increases, y also increases.
As x decreases, y also decreases.

Equation: $y = kx$

Inverse Variation

y varies inversely as x.
As x increases, y decreases.
As x decreases, y increases.

Equation: $xy = k$ or $y = \dfrac{k}{x}$

Joint Variation

y varies directly to two or more quantities.

Equation: $y = kxz$

Combined Variation

y varies directly to some quantities and varies inversely to some other quantities.

Equation: $xy = kz$ or $y = \dfrac{kz}{x}$

https://www.onlinemathlearning.com/joint-variation.html

Gauge

Post-Assessment:

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following variations occurs whenever a situation produces pairs of numbers whose ratio is constant?
   - A. Combined variation
   - B. Direct variation
   - C. Inverse variation
   - D. Joint variation

2. Which of the following expressions illustrates the situation *"The area* **(A)** *of the surface of a sphere varies directly as the square of its radius* **(r)**?
   - A. $A = k^2r$
   - B. $A = \frac{k}{r^2}$
   - C. $A = kr$
   - D. $A = kr^2$

3. Which of the following variations belong to a combined variation?
   - A. Direct and Joint
   - B. Joint variation
   - C. Inverse variation
   - D. Direct and Inverse

4. Which is an example of direct variation?
   - A. $m = 7n$
   - B. $m = \frac{9}{n}$
   - C. $m = 15xz$
   - D. $mn = 10$

5. What expression is read as *"The area* **(A)** *of a trapezoid varies jointly as the sum of its bases* **(b)** *and its height* **(h)**?
   - A. $A = kbh$
   - B. $A = \frac{1}{2}(b + h)$
   - C. $\frac{bh}{2k} = A$
   - D. $k = Ab$

6. What kind of variation relates the expression $n = kp$?
   - A. Combined variation
   - B. Direct variation
   - C. Inverse variation
   - D. Joint variation

7. Which of the following describes an inverse variation?

   A.

x	2	3	4	6
y	4	$\frac{16}{6}$	2	$\frac{16}{3}$

   B.

x	40	30	20	10
y	8	6	4	2

   C.

x	1	2	3	4
y	4	8	15	20

   D.

x	4	8	10	12
y	2	4	5	6

8. What happens to **x** when **y** is doubled in the equation x = 4y?
   A. x is halved
   B. x is doubled
   C. x is tripled
   D. x becomes zero

9. Which mathematical equation best describe the table?
   A. x = 12y
   B. x = $\frac{4}{y}$
   C. x = $\frac{12}{y}$
   D. x = $\frac{y}{4}$

X	4	3	2	1
y	3	4	6	12

10. Which of the following words complete the statement "D = $\frac{kCS}{n}$ means D varies _____ as C and S and _____ as n.
    A. directly, inversely
    B. inversely, jointly
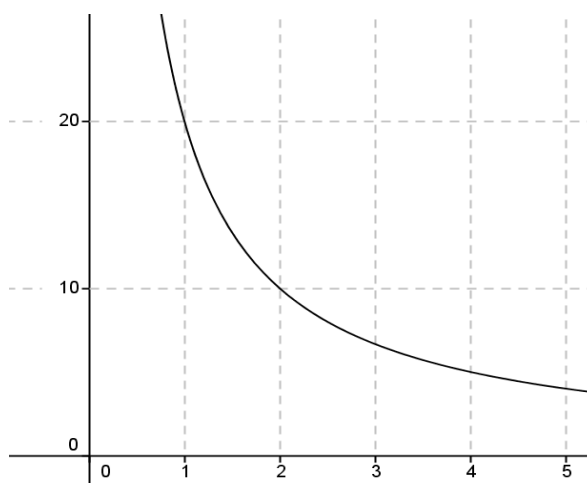   C. jointly, directly
    D. jointly, inversely

11. What mathematical statement describes the graph below?
    A. y = $\frac{20}{x}$
    B. y = $\frac{1}{20}x$
    C. y = x
    D. y = 20x

12. What is the constant of variation if **y** varies directly as **x**, and y = 20 when x =10?
    A. 50
    B. 15
    C. 2
    D. $\frac{1}{2}$
13. What happens to **T** when **P** is tripled in the relation **T = ?**
    A. T is tripled
    B. Ty is halved
    C. T is doubled
    D. T is divided by 3.
14. What kind of variation relates d to t in this statement "*A car travels a distance of* **d** *km in* **t** *hours*? (Hint: The formula that relates d to t is d = kt).
    A. Combined
    B. Direct
    C. Inverse
    D. Joint
15. Which is an example of combined variation?
    A. P = 3T
    B. m = $\frac{2n}{r}$
    C. y = 8xz
    D. mn = 14

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