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DEPARTMENT OF EDUCATION
LA UNION SCHOOLS DIVISION OFFICE
REGION I

AIRs - LM in
Statistics and Probability
Module 4:
Normal Distribution

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Statistics and Probability
Module 4: Normal Distribution
First Edition, 2021

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Target

In the previous lessons, you have used graphs of samples of discrete data to find a probability distribution with the same shape or pattern. The pattern is used to calculate probabilities of a population that will enable us to make predictions or decisions concerning the population.

This module will help you understand the concepts and processes regarding distribution that is commonly known as the normal probability distribution or simply the normal curve. The normal curve is frequently used as a mathematical model in inferential statistics. Through the normal curve, the inferences that will make regarding a population can be visualized.

After going through this module, you are expected to:

1. illustrate a normal random variable and its characteristics. **M11/12SP-IIIc-1**
2. identify regions under the normal curve corresponding to the different standard normal values. **M11/12SP-IIIc -3**

Subtasks

1. understand the concept of the normal curve distribution
2. state and illustrate the properties of a normal distribution
3. sketch the graph of a normal distribution; and
4. recognize the importance of the normal curve in statistical inference

Before going on, check how much you know about the topic. Answer the pretest below in a separate sheet of paper

Pretest

Directions: Read each item carefully, and select the correct answer. Write the letter of your choice in separate sheet of paper.

1. What is a random variable where the data can take infinitely many values?
   A. Continuous random variable
   B. Discrete random variable
   C. Both discrete & continuous
   D. None of these choices

2. Which of following shows a graphical form of the probability distribution for a continuous random variable?
   A. Bell shape
   B. Box shape
   C. Rectangular shape
   D. Circular shape

3. What is the total area under the normal curve?
   A. 1
   B. 2
   C. 3
   D. 6

4. Which of the following is ***NOT*** a characteristic of a normal distribution?
   A. Asymptotic
   B. Symmetrical
   C. The $\overline{X}$, Md and the Mo differ
   D. The area is between 0 to 1

5. Which term defines that the normal curve gets closer and closer to the horizontal axis but never touches it?
   A. Asymptotic
   B. Asymmetrical
   C. Parabolic
   D. Symmetrical

6. What is the skewness of a normal curve?
   A. -1
   B. 0
   C. 1
   D. 3

7. What is the area that corresponds to z-value, $z = 0.3$?
   A. 0.07926
   B. 0.11791
   C. 0.12172
   D. 0.15542

8. What is the area that corresponds to z-value, $z = -0.5$?
   A. 0.07926
   B. 0.12172
   C. 0.15542
   D. 0.19146

9. What is the area that corresponds to z-value, $z = 1.25$?
   A. 0.3531
   B. 0.3749
   C. 0.3944
   D. 0.4115

10. What percentage of the normal curve is $\mu \pm \sigma$?
    A. 50%
    B. 68%
    C. 75%
    D. 95%

11. 95% of the students at school weigh between 62kg and 90 kg. Assuming this data is normally distributed, what is the mean?
    A. 66 kg
    B. 72 kg
    C. 76 kg
    D. 86 kg

12. Refer to item 11, what is the standard deviation?
    A. 2kg
    B. 7 kg
    C. 14 kg
    D. 17kg

13. A machine produces electrical components. 99.7% of the components have lengths between 1.176 cm and 1.224 cm. Assuming this data is normally distributed, what is the mean?

A. 1.190 cm B. 1.200 cm C. 1.211 cm D. 1.219 cm

14. What is the standard deviation of item number 13?

A. 0.001 cm B. 0.008 cm C. 0.019 cm D. 0.123 cm

15. 68% of the marks in a test are between 51 and 64. Assuming this data is normally distributed, what is the mean and standard deviation?

A. 54.25, 3.25 B. 57.5, 4.5 C. 57.5, 6.5 D. 60.75, 9.75

Jumpstart

For you to understand the lesson well, do the activity below. Have fun and good luck!

Activity: TR✓E or F×LSE

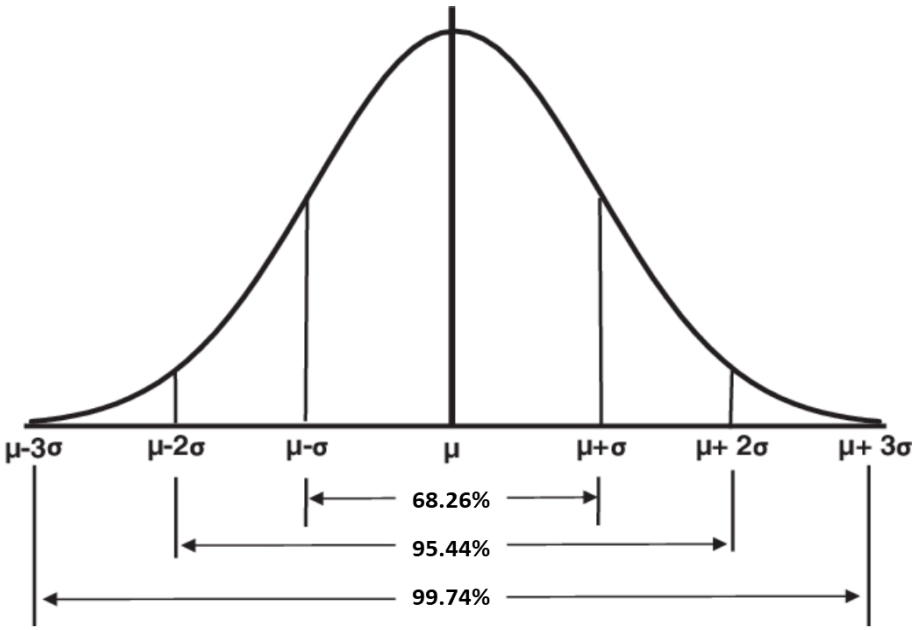
Directions. Determine whether the statement is True or False by checking (✓) the appropriate box.

STATEMENT	TRUE	FALSE
1. Getting the probability distribution in discrete random variable is the same in continuous random variable		
2. In a normal distribution, the mean, median, and mode are equal and located at the center of the distribution.		
3. A normal distribution is a unimodal.		
4. In a normal distribution, the curve is symmetrical to the mean.		
5. The distribution curve is asymptotic to the y-axis.		
6. The normal curve is a bell-shaped probability distribution		
7. The tails of the curve touch the baseline so that the curve can cover 100% of the area under it		
8. The skewness of the normal curve is 1.		
9. The standard deviation is the midpoint of a normal curve		
10. The normal curve for a population distribution is specifically determined by its mean equal to 0 and its standard deviation equal to 1.		

Discover

The ***normal distribution***, also known as ***Gaussian distribution*** is the most important of all distribution because it describes the situation in which very large values are rather rare, very small values are rather rare, but the middle values are rather common. It is symmetric about the mean, showing that data near the mean are more frequent in occurrence than the data far from the mean. In graph form ***normal distribution*** or simply ***normal curve*** will appear as a bell curve.

The normal curve has a very important role in inferential statistics. It provides a graphical representation of statistical values that are needed in describing the characteristics of populations as well as in making decisions. It is defined by an equation that uses the population mean (μ) and the standard deviation (σ). There is no single curve, but rather a whole family of normal curves that have the same basic characteristics but have different mean and standard distribution.



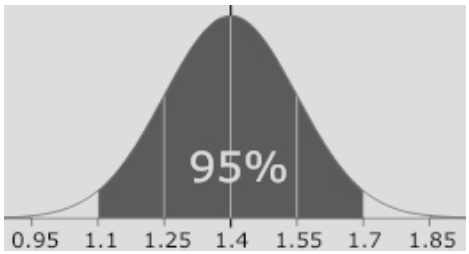
Properties of Normal Probability Distribution

1. The normal distribution is a bell-shaped
2. The mean, median and mode are equal and located at the center of the distribution
3. A normal distribution curve is unimodal
4. The curve is symmetrical about the mean
5. The total area under the normal curve is 1.
   $P(\mu - \sigma < x < \mu + \sigma) \sim$ 0.68 or 68%
   $P(\mu - 2\sigma < x < \mu + 2\sigma) \sim$ 0.95 or 95%
   $P(\mu - 3\sigma < x < \mu + 3\sigma) \sim$ 0.997 or 99.7%
6. The distribution curve is asymptotic to the x-axis.

Illustrative Example 1:

Ninety-five percent (95%) of students at school are between 1.1m and 1.7m tall. Assuming this data is normally distributed, can you calculate the mean and standard deviation?

Solution:



The mean is halfway between 1.1m and 1.7m

Mean (μ) = (1.1m + 1.7m)/2

= 1.4m

95% is 2 standard deviations (σ) on either side of the mean (a total of 4 standard deviations)

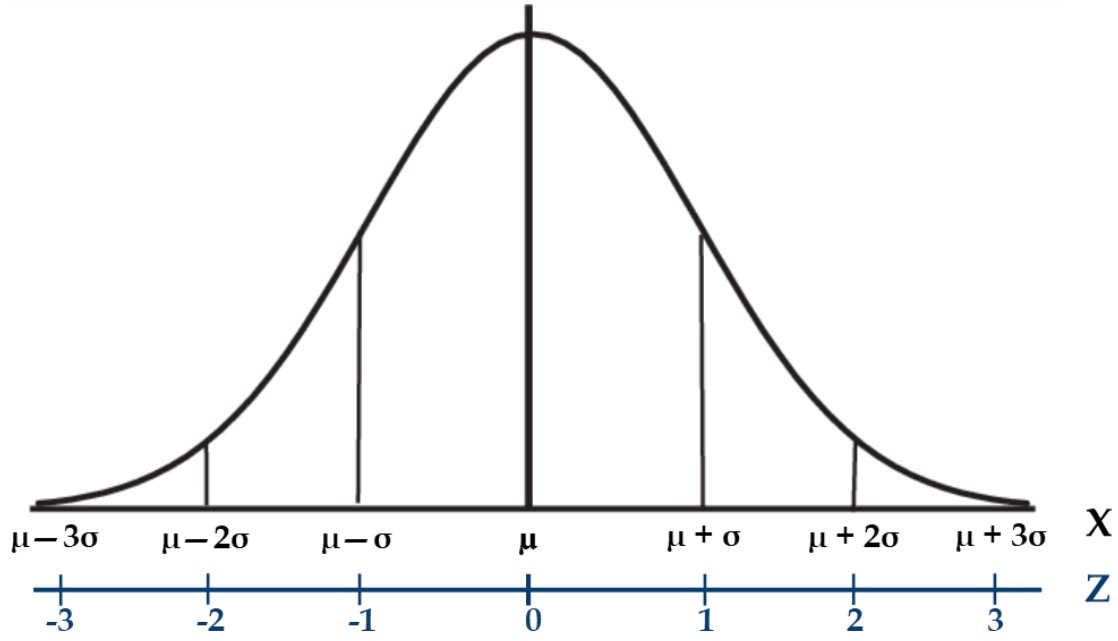
Standard deviations (σ) = (1.7m – 1.1m)/4

= 0.6m / 4

= 0.15m

Understanding the Standard Normal Curve

The standard normal curve is a normal probability distribution that is most commonly used as a model for inferential statistics. It has a mean $\mu = 0$ and a standard deviation $\sigma = 1$.



The **Table of Areas under the Normal Curve** is also known as the **z–Table**. The **z-score** is a measure of relative standing. It is calculated by subtracting $\overline{X}$ (or μ) from the measurement X and dividing the result by s (or σ). The final result, the **z–score** represent the distance between a given measurement X and the mean, expressed in standard deviations. Either the z-score locates X within a sample or within a population.

Table I**: Table of Areas under the Normal Curve**

Z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
0.0	.0000	.0040	.0080	.0120	.0150	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1253	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3288	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3557	.3559	.3621
1.1	.3642	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

For values of z above 3.09, use 0.4999

This table shows the area between zero (the mean of the standard normal variable) and *z*. For example, if $z = 1.61$, look at the row titled **1.6** and then move over to the column titled **.01** to get the result .4463

Four-Step Process in Finding the Areas Under the Normal Curve Given a z-Value

Step 1. Express the given z-value into a three-digit form.

Step 2. Using the z-Table, find the first two digits on the left column.

Step 3. Match the third digit with the appropriate column on the right.

Step 4. Read the area (or probability) at the intersection of the row and the column. This is the required area.

Illustrative Example 2.a:

Find the area that corresponds to z-value, $z = 1$.

Solution:

In the table, find the $z = 1.0$ in the first column

Find the Column with the heading .00

The area is **0.3413**

z	.00	.01
0.8	0.2881	0.291
0.9	0.3159	0.3186
1.0	**0.3413**	0.3438
1.1	0.3642	0.3665
1.2	0.3849	0.3869

Illustrative Example 2.b:

Find the area that corresponds to z-value, $z = 1.36$.

Solution:

Find the $z = 1.3$ in the first column

Find the Column with the heading .06

The area is **0.4131**

z	.05	.06	.07
1.1	0.3749	0.3770	0.3790
1.2	0.3944	0.3962	0.398
1.3	0.4115	**0.4131**	0.4147
1.4	0.4265	0.4279	0.4292
1.5	0.4394	0.4406	0.4418

Illustrative Example 2.c:

Find the area that corresponds to z-value, $z = -2.58$.

Solution:

In the z-table, the area that corresponds to $z = 2.58$ is the same as the area that corresponds to $z = -2.58$. In the graph of this region, it is located on the left of the mean.

Find $z = 2.5$ in the first column

Find the Column with the heading .08

The area is **0.4951**

z	.07	.08	.09
2.4	0.4932	0.4934	0.4936
2.5	0.4949	**0.4951**	0.4952
2.6	0.4962	0.4963	0.4964
2.7	0.4972	0.4973	0.4974
2.8	0.4979	0.498	0.4981

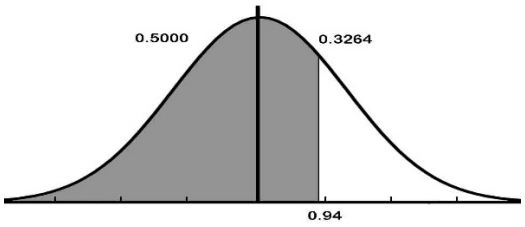
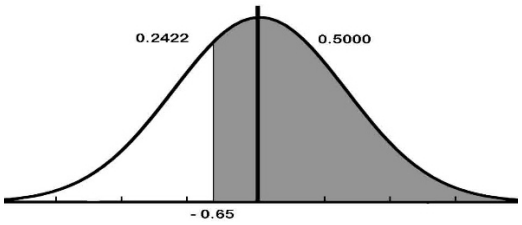
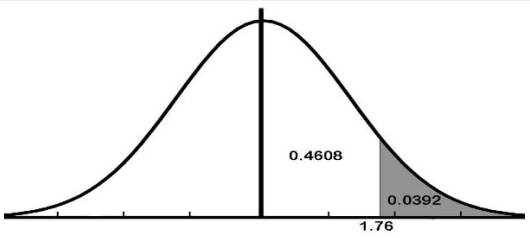
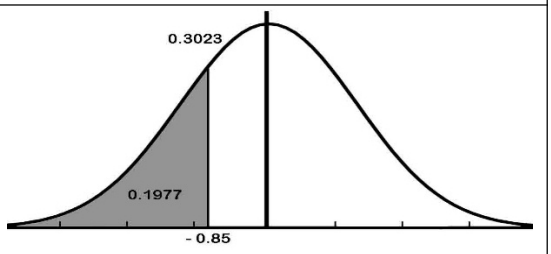
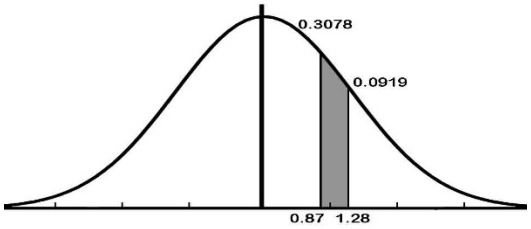
Questions concerning area under normal curves arise in various ways and the ability to find any desired area quickly can be a big help. Although the table gives areas between $z = 0$ and the selected positive values of z, we often have to find areas to the left or to the right of a given positive or negative values of z.

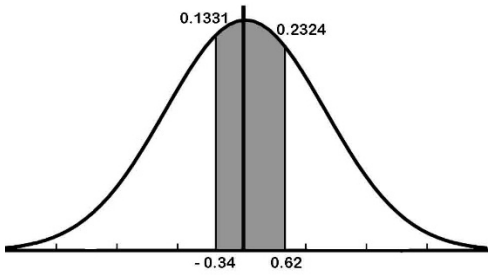
Illustrative Example 3:

Find the area under the standard normal curve which lies

1. to the left of $z = 0.94$
2. to the right of $z = -0.65$
3. to the right of $z = 1.76$
4. to the left of $z = -0.85$
5. between $z = 0.87$ and $z = 1.28$
6. between $z = -0.34$ and $z = 0.62$

Solution:

1. The area to the left of $z = 0.94$ is 0.5000 plus the entry in Table I corresponding to $z = 0.94$, namely, 0.5000 + 0.3264 = **0.8264**	
2. The area to the right of $z = -0.65$ is 0.5000 plus the entry corresponding to $z = -0.65$, namely, 0.5000 + 0.2422 = **0.7422**	
3. The area to the right of $z = 1.76$ is 0.5000 minus the entry corresponding to $z = 1.76$, namely, 0.5000 – 0.4608 = **0.0392**	
4. The area to the left of $z = -0.85$ is 0.5000 minus the entry corresponding to $z = 0.85$, 0.5000 – 0.3023 = **0.1977**	
5. The area between $z = 0.87$ and $z = 1.28$ is the difference between the entry corresponding to $z = 1.28$ and $z = 0.87$, 0.3997 – 0.3078 = **0.0919**	

6. The area between $z = -0.34$ and $z = 0.62$ is the sum between the entries corresponding to $z = 0.34$ and $z = 0.62$, 0.1331 + 0.2324 = **0.3655**	

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Enrichment Activity 1

Directions: Find the corresponding area between z = 0 and each of the following z-value. Use separate sheet of paper for your answers.

1. $z = 0.96$
2. $z = -1.74$
3. $z = 2.18$
4. $z = -2.69$
5. $z = 2.93$

Enrichment Activity 2:

Directions: Fill the blanks with the appropriate word or phrase to make meaningful statements. Use separate sheet of paper for your answers.

1. The curve of a probability distribution is formed by _____.
2. The area under a normal curve is _____.
3. The important values that best describe a normal curve are _____.
4. There are _____ standard deviation units at the baseline of a normal curve.
5. The curve of a normal distribution extends indefinitely at the tails but does not _____.
6. The area under a normal curve may also be expressed in terms of _____ or _____ or _____.
7. The mean, median, and the mode of a normal curve are _____.
8. A normal curve is used in _____.
9. About _____% of a score distribution is between $z = 0$ and $z = 1$.
10. The skewness of a normal curve is _____ because it is symmetrical.

Individual Assessment 1:

Directions: Determine the area under the standard normal curve that lies.

1. between z = 0 and z = 2.47

   Area: _____

2. between z = -1.85 and z = 0

   Area: _____

3. to the right of z = 0.61

   Area: _____

4. to the left of z = -3.02

   Area: _____

5. between z = 1.11 and z = 2.75

   Area: _____

6. between z = -2.06 and z = -0.54

   Area:_____

7. to the left of z = 1.53

   Area:_____

8. to the right of z = -1.34

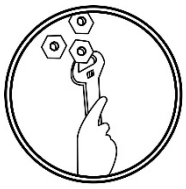
   Area:_____

9. between z = -2.09 and z = 1.72

   Area:_____

10. to the left of z = -1.27 and to the right of z = 2.86

    Area:_____



Deepen

Normal Distribution Activity:

A total of 82 Grade 11 students of ABC Senior High School took the 60-item test in General Mathematics. The result is normally distributed with the mean of 42 and standard deviation of 6. Construct a bell curve to display the data and answer the following questions.

1. How many students in each score interval?

   $\mu - 3\sigma \rightarrow \mu - 2\sigma$ = _____ $\mu \rightarrow \mu + \sigma$ = _____

   $\mu - 2\sigma \rightarrow \mu - \sigma$ = _____ $\mu + \sigma \rightarrow \mu + 2\sigma$ = _____

   $\mu - \sigma \rightarrow \mu$ = _____ $\mu + 2\sigma \rightarrow \mu + 3\sigma$ = _____

2. If the passing score is 36, how many students passed the test? How many of them failed?

3. Only Pamela and three of her classmates got the score of 48. How many students got a score higher than them? How many got a score lower than 48?

You will be graded on the accuracy of their data result paired with the bell curve. You will also be assessed on the quality and neatness of your work using the normal distribution teacher assessment on the next page.

The Normal Distribution Teacher Assessment

ELEMENT	Point Value	Score
1. Has the data been correctly entered into the table?	3	
2. Is the data organized and clear to understand?	3	
3. Is the bell curve neat and organized?	3	
4. Is the bell curve labelled, titled and plotted correctly?	3	
Total	12	

Gauge

Directions: Read each item carefully and select the correct answer. Write the letter of your choice on a separate sheet of paper.

1. Which of the following random variable would you expect to be discrete?
   A. The weights of mechanically produced items
   B. The number of children at a Christmas party
   C. The times, in seconds, for a 100m sprint
   D. The lifetimes of resistors

2. x is a random variable, with mean μ and standard deviation σ . The standardized form of x is $z = (X - \mu)/\sigma$. What are the mean and standard deviation, respectively of z?
   A. 0, 1 B. 1, 0 C. 2, 0 D. 2, 1

3. A distribution of data has a mean of 15 and a standard deviation of 2. How many standard deviations away from the mean is a value of 13?
   A. One standard deviation above the mean
   B. Two standard deviation above the mean
   C. One standard deviation below the mean
   D. Two standard deviation below the mean

4. The average waist size for teenage males is 29 inches with the standard deviation of 2 inches. What is the z-score of a teenage male with a 33 inches waist size?
   A. -2 B. -1 C. 1 D. 2

5. If 99.7% of the teenage males in #4 will be considered, what are the limits of their waist size?
   A. 23 – 33 in B. 23 – 35 in C. 24 - 34 in D. 25 – 35 in

6. What is the area that corresponds to z-value, $z = 1.21$?
   A. 0.34375 B. 0.36650 C. 0.38686 D. 0.40320

7. What is the area that corresponds to z-value, $z = -0.91$?
   A. 0.31859 B. 0.34375 C. 0.36650 D. 0.40320

8. If the area under the normal distribution is 0.44179 what z-value correspond to it?
   A. -1.57 B. -1.47 C. 1.56 D. 1.58

9. What is the area under the standard normal curve to the right of $z = -2.67$?
   A. 0.0038 B. 0.0869 C. 0.1234 D. 0.9962

10. For the standard normal distribution, what is the area between $z = -0.94$ and $z = 0.94$?
    A. 0.1736 B. 0.3264 C. 0.6528 D. 0.8264

11. What value of z has an area of 0.20 to the left of z?
    A. -2.05 B. -0.84 C. 0.84 D. 2.05

12. The distribution of heights of SHS male students is approximately normal with mean 65 inches and standard deviation 2 inches. What percentage of male students are taller than 69 inches?
    A. 1% B. 2.3% C. 5% D. 16%

13. The shelf life of a particular dairy products is normally distributed with a mean of 12 days and a standard deviation of 3 days. About what percent of the products will last between 12 and 15 days?
    A. 2.5% B. 16% C. 34% D. 68%

14. The mean life of a tire is 30,000 km and the standard deviation is 2,000 km. What are the possible limits if 68% of all tires will be considered?
    A. 24,000 km and 34,000 km
    B. 26,000 km and 34,000 km
    C. 27,000 km and 31,000 km
    D. 28,000 km and 32,000 km

15. A company makes parts for a machine. The lengths of the parts must be within a certain limit or they will be rejected. A large number of parts were measured and the mean and the standard deviation were calculated as 3.1m and 0.005m, respectively. What are the limits if 99.7% of the parts were accepted?
    A. Between 3.075m and 3.125 m
    B. Between 3.080m and 3.120 m
    C. Between 3.085m and 3.115 m
    D. Between 3.090m and 3.110 m

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