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Mathematics

Quarter 2 – Module 6:
Operations on Radical Expressions

AIRs - LM

Mathematics 9
Quarter 2 - Module 6: Operations on Radical Expressions
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Activity 2: Translate Me

Directions: Translate or convert radical expressions to exponential expressions and vice versa. Write your answer on your answer sheets.

A. Radical to Exponential

1.) $\sqrt[3]{2}$ = ________

2.) $\sqrt{3}$ = ________

B. Exponential to Radical

3.) $5^{\frac{1}{2}}$ = ________

4.) $7^{\frac{1}{3}}$ = ________

You have learned in the previous lessons on how to write expressions with rational exponents to radicals and vice versa and you will need these skills to succeed in the next activities as we apply the different operations in simplifying radicals.

Discover

Operations on Radicals

Adding and subtracting radical expressions works like adding and subtracting expressions involving variables. Just as we need like terms when combining expressions involving variables, we need like radicals in order to combine radical expressions.

Two radical expressions are said to be **like radicals** if they have the same indices and the same radicands.

Examples:

1.) $3\sqrt{5}$ *and* $8\sqrt{5}$ → like radicals, *they have the same indices and the same radicands.*

2.) $2\sqrt[3]{6}$ *and* $\sqrt[3]{7}$ → unlike radicals, *they have the same indices but different radicands.*

3.) $4\sqrt{10}$ *and* $8\sqrt[5]{10}$ → unlike radicals, *they have the same radicands but different indices.*

4.) $5\sqrt[3]{6}$ *and* $11\sqrt[4]{9}$ → unlike radicals, *they have different radicands and different indices.*

Other Examples:

1.) $7\sqrt{11}$ *and* $3\sqrt{11}$ → like radicals

2.) $5\sqrt{3b}$, $-2\sqrt{3b}$ *and* $4\sqrt{3b}$ → like radicals

There are terms which may not look like they are similar or like radicals, but when some simplifications are done, they are actually like radicals.

Example:

$\sqrt{2}$ *and* $\sqrt{8}$ *at first look, it seems that they are not like radicals*

$\sqrt{4 \bullet 2}$ *by simplifying radicals*

$\boldsymbol{\sqrt{2}}$ ***and*** $\boldsymbol{2\sqrt{2}}$ *are now like radicals*

More Examples: *Simplify to make them like radicals*

1.) $2\sqrt{54}$ *and* $5\sqrt{24}$

$2\sqrt{9 \bullet 6}$ *and* $5\sqrt{4 \bullet 6}$

$6\sqrt{6}$ *and* $10\sqrt{6}$

like radicals

2.) $\sqrt[3]{16}$ *and* $7\sqrt[3]{250}$

$\sqrt[3]{8 \bullet 2}$ *and* $7\sqrt[3]{125 \bullet 2}$

$2\sqrt[3]{2}$ *and* $35\sqrt[3]{2}$

like radicals

3.) $\sqrt{x^2y}$ *and* $\sqrt{x^4y^3}$

$x\sqrt{y}$ *and* $x^2y\sqrt{y}$

like radicals

Adding and Subtracting Radical Expressions

To add or subtract radicals, the indices and what is inside the radical (called the radicand) must be exactly the same. If the indices and radicands are the same, then add or subtract the terms in front of each like radical. If the indices or radicands are not the same, then you cannot add or subtract the radicals.

Examples:

1.) $3\sqrt{5} + 8\sqrt{5}$ $= (3+8)\sqrt{5}$ *add 3 & 8, then copy* $\sqrt{5}$

$\mathbf{= 11\sqrt{5}}$

2.) $8\sqrt{13} - 2\sqrt{13}$ $= (8-2)\sqrt{13}$ *subtract 8 to 2, then copy* $\sqrt{13}$

$\mathbf{= 6\sqrt{13}}$

3.) $10\sqrt[3]{7} - 4\sqrt[3]{7} + \sqrt[3]{7} - 2\sqrt[3]{7}$ $= (10-4+1-2)\sqrt[3]{7}$

$\mathbf{= 5\sqrt[3]{7}}$

Other Examples:

1.) $\underbrace{2\sqrt{11} + 5\sqrt{11}} - \underbrace{5\sqrt{3} + 8\sqrt{3}}$ *combine similar terms or like radicals*

$7\sqrt{11} \quad + 3\sqrt{3} \quad = \quad \mathbf{7\sqrt{11} + 3\sqrt{3}}$ *this is now the final answer because you cannot add/ subtract unlike radicals*

2.) $\sqrt[3]{6} - 16\sqrt{2} + 5\sqrt[3]{6} + 10\sqrt{2}$ *rearrange and*

$\underbrace{\sqrt[3]{6} + 5\sqrt[3]{6}} - \underbrace{16\sqrt{2} + 10\sqrt{2}}$ *combine similar terms or like radicals*

$6\sqrt[3]{6} \quad - 6\sqrt{2} \quad = \quad \mathbf{6\sqrt[3]{6} - 6\sqrt{2}}$ *this is now the final answer because you cannot add/ subtract unlike radicals*

3.) $-5\sqrt{2} + 7\sqrt{3} \quad = \quad \mathbf{-5\sqrt{2} + 7\sqrt{3}}$ just copy, *cannot add/ subtract unlike radicals*

Remember that we can only combine like radicals.

Examples:

1.) $2\sqrt{54} + 5\sqrt{24}$ *simplify each radical*

Add or subtract the radicals.

$2\sqrt{9 \bullet 6} + 5\sqrt{4 \bullet 6}$

$6\sqrt{6} + 10\sqrt{6} = \mathbf{16\sqrt{6}}$ *add 6 & 10, then copy* $\sqrt{6}$

2.) $\sqrt[3]{16} - 7\sqrt[3]{250}$

$\sqrt[3]{8 \bullet 2} - 7\sqrt[3]{125 \bullet 2}$

$2\sqrt[3]{2} - 35\sqrt[3]{2} = \mathbf{-33\sqrt[3]{2}}$

3.) $-2\sqrt{3} + 3\sqrt{27} - \sqrt{12} + 3\sqrt{3}$

$-2\sqrt{3} + 3\sqrt{9 \bullet 3} - \sqrt{4 \bullet 3} + 3\sqrt{3}$

$-2\sqrt{3} + 9\sqrt{3} - 2\sqrt{3} + 3\sqrt{3}$

$= (-2 + 9 - 2 + 3)\sqrt{3} = \mathbf{8\sqrt{3}}$

4.) $-\sqrt{45} + 2\sqrt{5} - \sqrt{20} - 2\sqrt{6}$ *simplify each radical*

$-\sqrt{9 \bullet 5} + 2\sqrt{5} - \sqrt{4 \bullet 5} - 2\sqrt{6}$

$\underbrace{-3\sqrt{5} + 2\sqrt{5} - 2\sqrt{5}} - 2\sqrt{6}$ *combine similar terms or like radicals*

$\underbrace{(-3 + 2 - 2)\sqrt{5}}$

$\mathbf{= -3\sqrt{5} - 2\sqrt{6}}$ *this is now the final answer because you cannot add/ subtract unlike radicals*

5.) $-3\sqrt{3} + \sqrt{8} - 3\sqrt{3} \quad \rightarrow \quad -3\sqrt{3} + 2\sqrt{2} - 3\sqrt{3} \quad = \quad \mathbf{-6\sqrt{3} + 2\sqrt{2}}$

6.) $2x\sqrt[3]{24x} + 5\sqrt[3]{81x^4} \quad \rightarrow \quad 2x\sqrt[3]{8 \bullet 3x} + 5\sqrt[3]{27 \bullet 3x^4}$

$= 4x\sqrt[3]{3x} + 15x\sqrt[3]{3x} \;=\; \mathbf{19x\sqrt[3]{3x}}$

7.) $3\sqrt{20} - 6\sqrt{125} + 5\sqrt{45} \;\rightarrow\; 3\sqrt{4 \bullet 5} - 6\sqrt{25 \bullet 5} + 5\sqrt{9 \bullet 5}$

$= 6\sqrt{5} - 30\sqrt{5} + 15\sqrt{5} \quad = \; \mathbf{-9\sqrt{5}}$

Multiplying Radical Expressions

You can multiply any two radicals that have the same indices together. If the radicals do not have the same indices, manipulate the equation until they do.

A. Multiply Radicals without Coefficients

1.) $(\sqrt{18})(\sqrt{2})$ Step 1: *Make sure that the radicals have the same index*

$\sqrt{(18)(2)} = \sqrt{36}$ Step 2: *Multiply the numbers under the radical signs*

$= \sqrt{36} = \mathbf{6}$ Step 3: *Simplify the radical expressions.*

2.) $\sqrt{10} \bullet \sqrt{5} = \sqrt{50} = \sqrt{25 \bullet 2} = \mathbf{5\sqrt{2}}$

3.) $(\sqrt[3]{3})(\sqrt[3]{9}) = \sqrt[3]{27} = \mathbf{3}$

4.) $5 \bullet \sqrt{6} = \mathbf{5\sqrt{6}}$

B. Multiply Radicals with Coefficients

1.) $(3\sqrt{2})(\sqrt{10})$ Step 1: *Multiply the coefficients.*

$(3\sqrt{2})(\sqrt{10})$ Step 2: *Multiply the numbers under the radical signs*

$3\sqrt{20} = 3\sqrt{4 \bullet 5} = \mathbf{6\sqrt{5}}$ Step 3: *Simplify the radical expressions.*

2. $(4\sqrt{3})(3\sqrt{6}) \;\rightarrow\; (4\sqrt{3})(3\sqrt{6}) \;=\; 12\sqrt{18} \;=\; 12\sqrt{9 \bullet 2} \;= \mathbf{36\sqrt{2}}$

C. Multiply Radicals with Different Indices

1.) $(\sqrt[3]{5})(\sqrt{2})$ — Step 1: *Convert radical expression to exponential expression*

$5^{\frac{1}{3}} \bullet 2^{\frac{1}{2}}$ — Step 2: *Find the LCD of the exponents $\frac{1}{3}$ and $\frac{1}{2}$ which is 6 and became $\frac{2}{6}$ and $\frac{3}{6}$ respectively (similar fractions)*

$5^{\frac{2}{6}} \bullet 2^{\frac{3}{6}}$ — Step 3: *Convert exponential expression to radical expression Notice that the radicals have the same indices.*

$(\sqrt[6]{5^2})(\sqrt[6]{2^3})$ — Step 4: *Simplify the radicand and you can now multiply radicals with the same indices*

$(\sqrt[6]{25})(\sqrt[6]{8}) = \boldsymbol{\sqrt[6]{200}}$

2.) $(\sqrt[3]{2})(\sqrt[4]{3}) \quad = 2^{\frac{1}{3}} \bullet 3^{\frac{1}{4}} \quad = 2^{\frac{4}{12}} \bullet 3^{\frac{3}{12}} \quad = (\sqrt[12]{2^4})(\sqrt[12]{3^3}) = (\sqrt[12]{16})(\sqrt[12]{27}) = \boldsymbol{\sqrt[12]{432}}$

D. Multiplying Radicals with More Than One Term

1.) $\sqrt{2}(\sqrt{3}+\sqrt{5}) \quad \rightarrow \quad \sqrt{2}(\sqrt{3}+\sqrt{5})$ *Distributive Property*

$$= \underbrace{\sqrt{2} \bullet \sqrt{3}} + \underbrace{\sqrt{2} \bullet \sqrt{5}}$$

$$= \boldsymbol{\sqrt{6} + \sqrt{10}}$$

2.) $\sqrt{3}(\sqrt{6}-\sqrt{3}) \quad \rightarrow \quad \sqrt{3}(\sqrt{6}-\sqrt{3}) \quad = \sqrt{18} - \sqrt{9}$

$$= \sqrt{9 \bullet 2} - 3$$

$$= \boldsymbol{3\sqrt{2} - 3}$$

E. Multiplying Binomials Involving Radicals

1.) $(2-\sqrt{7})(5-\sqrt{3}) \quad \rightarrow \quad (2-\sqrt{7})(5-\sqrt{3})$ FOIL method

$$= \underset{\mathbf{F}}{\underline{2 \bullet 5}} \quad \underset{\mathbf{O}}{\underline{2 \bullet -\sqrt{3}}} \quad \underset{\mathbf{I}}{\underline{-\sqrt{7} \bullet 5}} \quad \underset{\mathbf{L}}{\underline{-\sqrt{7} - \sqrt{3}}}$$

$$= \boldsymbol{10 - 2\sqrt{3} - 5\sqrt{7} + \sqrt{21}}$$

2.) $(3-\sqrt{2})(4+\sqrt{2})$ → **F**: $3(4) = 12$

O: $3(\sqrt{2}) = 3\sqrt{2}$

I: $-\sqrt{2}(4) = -4\sqrt{2}$

L: $-\sqrt{2}(\sqrt{2}) = -2$

$= \mathbf{12 + 3\sqrt{2} - 4\sqrt{2} - 2}$

$= \mathbf{10 - \sqrt{2}}$

3.) $(4+\sqrt{5})(3-\sqrt{7})$ → **F**: $4(3) = 12$

O: $4(-\sqrt{7}) = -4\sqrt{7}$

I: $\sqrt{5}(3) = 3\sqrt{5}$

L: $\sqrt{5}(-\sqrt{7}) = -\sqrt{35}$

$= \mathbf{12 - 4\sqrt{7} + 3\sqrt{5} - \sqrt{35}}$

F. Multiplying Radicals with Its Conjugate

A conjugate is a binomial formed by negating the second term of a binomial. An example is, the conjugate of (x + y) is (x - y).

1.) $(3-\sqrt{5})(3+\sqrt{5})$ → $(3-\sqrt{5})(3+\sqrt{5})$

$= 9 - \sqrt{25}$
$= 9 - 5$
$= \mathbf{4}$

2.) $(\sqrt{6}+2)(\sqrt{6}-2)$ → $(\sqrt{6}+2)(\sqrt{6}-2)$ $= \sqrt{36} - 4$ $= 6 - 4$ $= \mathbf{2}$

3.) $(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})$ → $(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})$ $= \sqrt{4} - \sqrt{49}$ $= 2 - 7$ $= \mathbf{-5}$

4.) $(3\sqrt{11}+5\sqrt{2})(3\sqrt{11}-5\sqrt{2})$ → $(3\sqrt{11}+5\sqrt{2})(3\sqrt{11}-5\sqrt{2})$ $= 9\sqrt{121} - 25\sqrt{4}$

$= 99 - 50$ $= \mathbf{49}$

Dividing Radical Expressions

In the previous activity, you were able to simplify the radicals by rationalizing the denominator. Rationalization is a process where you simplify the expression by making the denominator free from radicals. This skill is necessary in the division of radicals.

A. Dividing Radicals with the Same Indices

1.) $\sqrt{12} \div \sqrt{3} \quad = \sqrt{\frac{12}{3}} \quad = \sqrt{4} \quad = \mathbf{2}$

2.) $\sqrt{\frac{9}{25}} \quad = \frac{\sqrt{9}}{\sqrt{25}} \quad = \mathbf{\frac{3}{5}}$

Rationalizing the Denominator

3.) $\sqrt{\frac{7}{5}} \quad = \frac{\sqrt{7}}{\sqrt{5}} = \frac{\sqrt{7}}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{35}}{\sqrt{25}} = \mathbf{\frac{\sqrt{35}}{5}}$

4.) $\frac{4}{\sqrt{3}} \quad = \frac{4}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} \quad = \mathbf{\frac{4\sqrt{3}}{3}}$

B. Dividing Radicals with Different Indices

1.) $\underbrace{\sqrt[3]{3} \div \sqrt{3}}$ Step 1: *Convert radical expression to exponential expression*

$\frac{\sqrt[3]{3}}{\sqrt{3}} = \frac{3^{\frac{1}{3}}}{3^{\frac{1}{2}}}$

$= \frac{3^{\frac{2}{6}}}{3^{\frac{3}{6}}}$ Step 2: *Find the LCD of the exponents $\frac{1}{2}$ and $\frac{1}{3}$ which is 6 and became $\frac{2}{6}$ and $\frac{3}{6}$ respectively (similar fractions)*

$= \frac{\sqrt[6]{3^2}}{\sqrt[6]{3^3}}$ Step 3: *Convert exponential expression to radical expression Notice that the radicals have the same indices.*

$= \frac{\sqrt[6]{3^2}}{\sqrt[6]{3^3}} \bullet \frac{\sqrt[6]{3^3}}{\sqrt[6]{3^3}}$ Step 4: *Rationalize the denominator*

$= \frac{\sqrt[6]{3^5}}{\sqrt[6]{3^6}} \quad = \mathbf{\frac{\sqrt[6]{243}}{3}}$

C. Rationalizing the Denominator Using Its Conjugate

1.) $\frac{3}{\sqrt{3}-\sqrt{2}} \rightarrow \frac{3}{\sqrt{3}-\sqrt{2}} \bullet \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3(\sqrt{3}+\sqrt{2})}{\sqrt{9}-\sqrt{4}} = \frac{3\sqrt{3}+3\sqrt{2}}{3-2} = \frac{3\sqrt{3}+3\sqrt{2}}{1} = \mathbf{3\sqrt{3}+3\sqrt{2}}$

2.) $\frac{6-\sqrt{3}}{2+\sqrt{7}} \rightarrow \frac{6-\sqrt{3}}{2+\sqrt{7}} \bullet \frac{2-\sqrt{7}}{2-\sqrt{7}} = \frac{12-6\sqrt{7}-2\sqrt{3}+\sqrt{21}}{4-\sqrt{49}} = \frac{12-6\sqrt{7}-2\sqrt{3}+\sqrt{21}}{4-7}$

$= \frac{12-6\sqrt{7}-2\sqrt{3}+\sqrt{21}}{-3}$ *or*

$= \frac{\mathbf{-12+6\sqrt{7}+2\sqrt{3}-\sqrt{21}}}{\mathbf{3}}$

3.) $\frac{6-\sqrt{2}}{4-3\sqrt{2}} \rightarrow \frac{6-\sqrt{2}}{4-3\sqrt{2}} \bullet \frac{4+3\sqrt{2}}{4+3\sqrt{2}} = \frac{24+18\sqrt{2}-4\sqrt{2}-3\sqrt{4}}{16-9\sqrt{4}} = \frac{24+14\sqrt{2}-6}{16-18}$

$= \frac{18+14\sqrt{2}}{-2}$

$= \mathbf{-9-7\sqrt{2}}$

Explore

Activity 3: Like or Unlike Radicals

A. Identify whether the following are ***Like*** radicals or ***Unlike*** radicals. For like radicals write **LR** otherwise write **UR** on your answer sheets

1) $\sqrt{2}$ *and* $\sqrt{5}$ → ____________________
2) $2\sqrt{7y}$ *and* $3\sqrt{7y}$ → ____________________
3) $\sqrt{3}$ *and* $\sqrt[3]{3}$ → ____________________
4) $5\sqrt[6]{3x}$ *and* $5\sqrt[6]{2x}$ → ____________________
5) $2\sqrt[7]{m^2n^3}$ *and* $-\sqrt[7]{n^3m^2}$ → ____________________

B. Simplify to make each pair of radical expression like radicals. An example is provided for you. Write your answer on your answer sheets.

Example $\sqrt{2}$ *and* $\sqrt{8}$ $=\sqrt{2}$ *and* $\sqrt{4 \cdot 2}$ $=\underline{\sqrt{2} \text{ and } 2\sqrt{2}}$

6) $\sqrt{2}$ *and* $\sqrt{18}$ → _______ *and* ________
7) $5\sqrt{3}$ *and* $\sqrt{27}$ → _______ *and* ________

8) $2\sqrt{28}$ *and* $5\sqrt{7}$ $\rightarrow$ ________ *and* ________

9) $\sqrt{20a^2}$ *and* $a\sqrt{45}$ $\rightarrow$ ________ *and* ________

10) $2\sqrt{3a^3}$ *and* $5a\sqrt{3a}$ $\rightarrow$ ________ *and* ________

Activity 4: Adding & Subtracting Radicals

Directions: Add or subtract the following radical expressions. Make sure your answer is in simplest radical form. Write your answer on your answer sheets.

1.) $5\sqrt{2} + 3\sqrt{2}$ = ________________________

2.) $4\sqrt[3]{6} - \sqrt[3]{6}$ = ________________________

3.) $10\sqrt{7} + 12\sqrt{7} - 8\sqrt{7} + \sqrt{7}$ = ________________________

4.) $\sqrt{12} + \sqrt{27} - \sqrt{3}$ = ________________________

5.) $\sqrt{50} + \sqrt{32} - \sqrt{8}$ = ________________________

Deepen

Activity 5: Multiplying and Dividing Radicals

Performance Task: Simplify each radical expression by multiplying or dividing the radicals. Write your answer on your answer sheets. Please show your *SOLUTIONS*. The rubric below will guide you in the presentation of your solutions.

1.) $\sqrt{5} \bullet \sqrt{10}$

2.) $-3\sqrt{3}(2 + \sqrt{6})$

3.) $(7 + \sqrt{12})(7 - \sqrt{12})$

4.) $\sqrt{6} \div \sqrt{5}$

5.) $\dfrac{5}{5+\sqrt{2}}$