

# Mathematics

**Quarter 3- Week 4 Module 4**

**Solving Problems Involving  
Parallelograms, Trapezoids and Kites**



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## **Mathematics 9**

QUARTER 3-WEEK 4 MODULE 4: Solving Problems Involving  
Parallelograms, Trapezoids and Kites

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Region I

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## Target

This module acquires knowledge and skills needed to solve problems involving shapes and geometric relationships, and use their properties in solving problems on geometry.

After going through this module, you are expected to:

Solve problems involving parallelograms, trapezoids and kites. **(M9AL-IIIe-1)**

### Subtask:

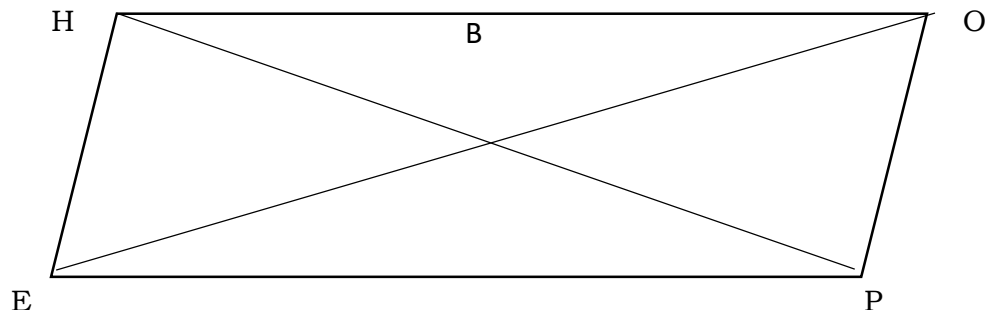
- Use properties of parallelograms, isosceles and kites in finding measures of angles, sides and other quantities involving parallelograms, isosceles and kites.

### Pre-Test

Let's find out how much you already know about this module.

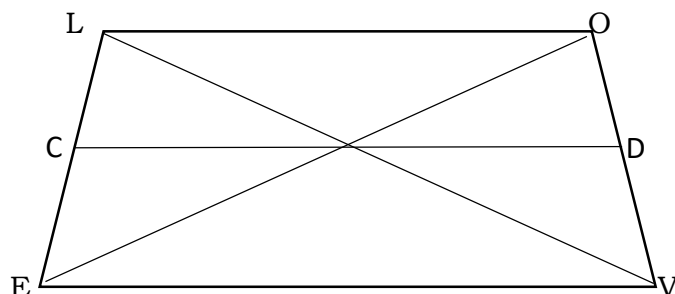
Directions: Choose the letter of the best answer. Write your answer on a separate sheet of paper.

- A. Refer to the figure below. Given: HOPE is a parallelogram.



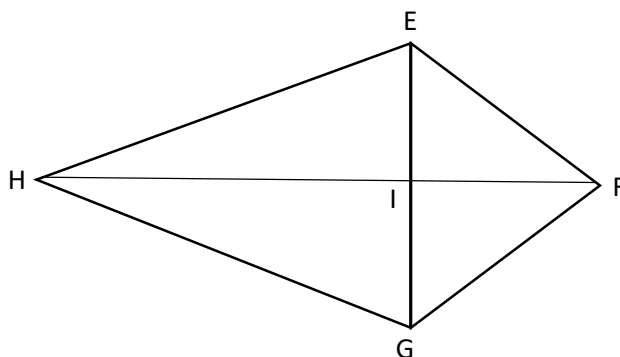
- \_\_\_\_ 1. If  $HB = 7\text{ cm}$ , then  $HP =$  \_\_\_\_.
- A) 7 cm                      B) 8 cm                      C) 14 cm                      D) 15 cm
- \_\_\_\_ 2. If  $OE = 24\text{ cm}$ , then  $HP =$  \_\_\_\_.
- A) 7 cm                      B) 12 cm                      C) 14 cm                      D) 24 cm
- \_\_\_\_ 3. If  $HO = 12.5\text{ cm}$ , then  $EP =$  \_\_\_\_.
- A) 12.5 cm                      B) 14 cm                      C) 24 cm                      D) 25 cm
- \_\_\_\_ 4. If  $m\angle OHE = 120$ , then  $m\angle OPE =$  \_\_\_\_.
- A) 60                      B) 120                      C) 140                      D) 180
- \_\_\_\_ 5. If  $m\angle HOP = 72$ , then  $m\angle OPE =$  \_\_\_\_.
- A) 108                      B) 120                      C) 140                      D) 144

B. Refer to the figure below. Given: Trapezoid LOVE with median  $CD$ .



- \_\_\_\_\_ 6. If  $CD = 7\text{ cm}$ , then  $EV = 10\text{ cm}$ , then  $LO =$ \_\_\_\_\_.
- A) 4 cm                      B) 7 cm                      C) 10 cm                      D) 14 cm
- \_\_\_\_\_ 7. If  $LO = 4.25\text{ cm}$ , then  $EV = 5.75\text{ cm}$  then  $CD =$ \_\_\_\_\_.
- A) 10 cm                      B) 7 cm                      C) 5 cm                      D) 4 cm
- \_\_\_\_\_ 8. If  $LC = 24.7\text{ cm}$ , then  $CE =$ \_\_\_\_\_.
- A) 12.31 cm                      B) 12.5 cm                      C) 24.7 cm                      D) 48.4 cm
- \_\_\_\_\_ 9. If trapezoid LOVE is isosceles, then  $\overline{LV} \cong$ \_\_\_\_\_.
- A)  $\overline{OE}$                       B)  $\overline{LV}$                       C)  $\overline{CD}$                       D)  $\overline{VL}$
- \_\_\_\_\_ 10. If trapezoid LOVE is isosceles, then  $\angle OVE \cong$ \_\_\_\_\_.
- A)  $\angle EVO$                       B)  $\angle ELO$                       C)  $\angle LOV$                       D)  $\angle OVL$

C. Refer to the figure below. Quadrilateral EFGH is a kite.  $\overline{HF}$  and  $\overline{EG}$  are diagonals. Complete each statement.



- \_\_\_\_\_ 11.  $\triangle EFG$  is a/an \_\_\_\_\_ triangle.  
 A) equiangular      B) equilateral      C) scalene      D) isosceles
- \_\_\_\_\_ 12. The measure of  $\angle FIG$  is \_\_\_\_\_.  
 A) 30      B) 60      C) 90      D) 180
- \_\_\_\_\_ 13.  $m\angle EIH + m\angle EIF$  \_\_\_\_\_.  
 A) 60      B) 90      C) 120      D) 180
- \_\_\_\_\_ 14. If  $m\angle HEI$  is 60, then  $m\angle EHG$  is \_\_\_\_\_.  
 A) 30      B) 45      C) 60      D) 70
- \_\_\_\_\_ 15. If  $GH$  is 5 cm then  $EH$  is \_\_\_\_\_.  
 A) 5 cm      B) 10 cm      C) 15 cm      D) 20 cm

**Lesson****4****Solving Problems Involving Parallelogram,  
Trapezoids and Kites**

In the previous lessons, you have learned about the three types of quadrilaterals: the parallelograms, trapezoids and the kite. You have also learned about each of their properties. It is important to remember the properties since they are useful in solving problems on geometry.

**Activity 1: FACT OR BLUFF!**

A review on the properties of parallelograms, trapezoids and kites. Read carefully the direction.

**DIRECTION:** Write **FACT** if the statement identifies the properties of parallelogram, trapezoids and kites. **BLUFF** if it is not a property of a parallelograms, trapezoid, and kites.

1. In a parallelogram, any two opposite angles are supplementary.
2. In a parallelogram, any two opposite sides are congruent.
3. In a parallelogram, any two consecutive angles are complementary.
4. A diagonal of a parallelogram forms two congruent triangles.
5. The base angles of a an isosceles trapezoid are congruent.
6. The diagonals of an isosceles trapezoid are not equal.
7. If the base angles of a trapezoid are not equal, then the trapezoid is isosceles.
8. If a quadarilateral is a kite, then its diagonals are perpendicular.
9. If exactly one diagonal of a quadrilateral is the perpendicular bisector of the other diagonal, then the quadrilateral is a kite.
10. The area of a kite is half the product of the lengths of its diagonals.

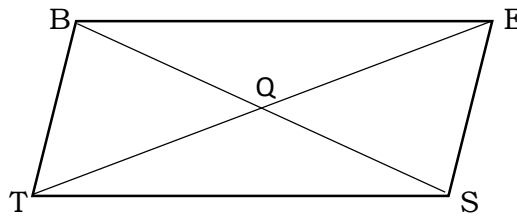


## Jumpstart

In the preceding lesson, you have studied some properties of parallelograms, trapezoids and kites.

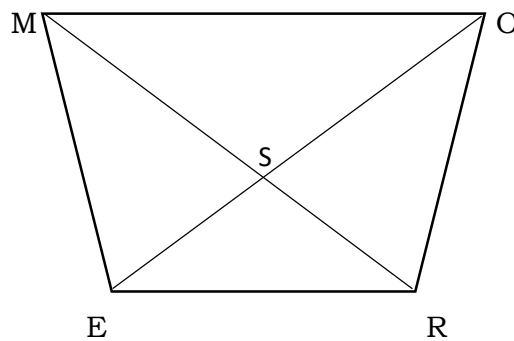
### Activity 2: Tests for Parallelograms, Trapezoids and Kites

- A. For each of the following, state the condition that supports why BEST is a parallelogram.



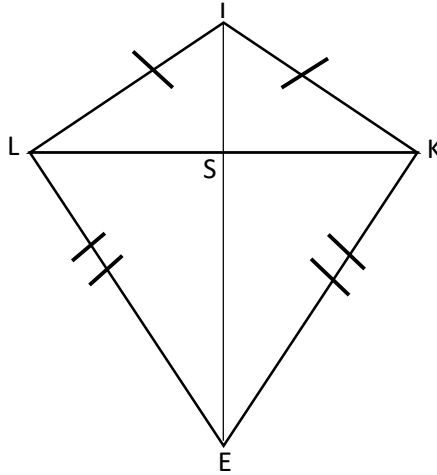
1.  $\overline{BQ} \cong \overline{SQ}$ ,  $\overline{TQ} \cong \overline{EQ}$  \_\_\_\_\_
2.  $\overline{BT} \parallel \overline{ES}$ ,  $\overline{BT} \cong \overline{ES}$  \_\_\_\_\_
3.  $\overline{BE} \parallel \overline{TS}$ ,  $\overline{BT} \parallel \overline{ES}$  \_\_\_\_\_

- B. MORE is an isosceles trapezoid with bases ER and MO. With this figure and information, find the indicated measures and state the definition or theorem that supports your answer.



4.  $ME = 7.8$  cm, find  $OR =$  \_\_\_\_\_, \_\_\_\_\_
5.  $MR = 5.9$  cm, find  $EO =$  \_\_\_\_\_, \_\_\_\_\_
6.  $m\angle EMO = 78$ , find  $m\angle ROM =$  \_\_\_\_\_, \_\_\_\_\_

- C. Consider kite LIKE below. Complete the each statement. Give the reason that justifies the statement.



7. If  $LI=6$ , what is  $IK=$  \_\_\_\_\_,  
8. If  $EL=10.5$ , what is  $EK=$  \_\_\_\_\_,  
9. If  $LK=7$  cm and  $IE= 13$  cm, what is the area? \_\_\_\_\_,  
10. If the area  $96 \text{ cm}^2 = LK=8$  cm, what is  $IE?$  \_\_\_\_\_,



## Discover

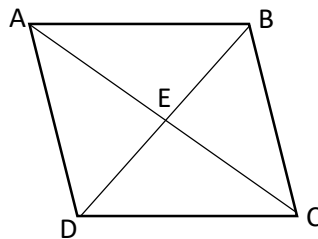
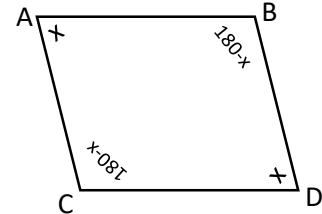
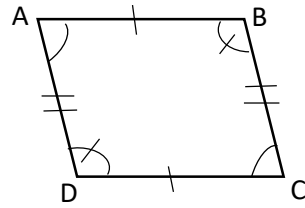
To build your understanding in solving problems on properties of parallelograms, trapezoids and kites, let's discover more of their properties. Let's start.

### A. PROPERTIES OF PARALLELOGRAMS

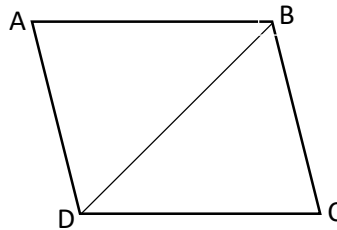
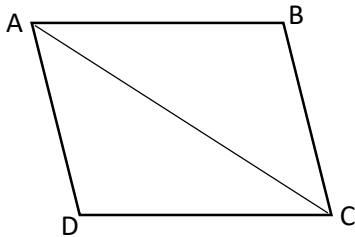
1. The opposite sides of a parallelogram are parallel and congruent.
2. The opposite angles of a parallelogram are congruent.
3. The consecutive angles of a parallelogram are supplementary
4. The diagonals of a parallelogram bisect each other.
5. Each diagonal divides a parallelogram into two congruent triangles.

**Example 1:** ABCD is parallelogram.

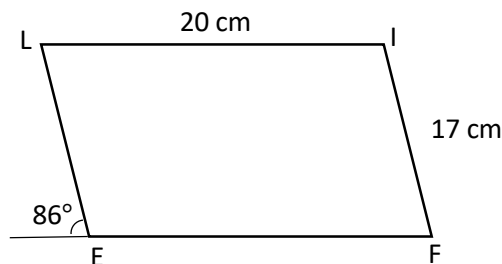
- The opposite sides are parallel and congruent.  
 $\overline{AB} \parallel \overline{DC}$  and  $\overline{AD} \parallel \overline{BC}$   
 $\overline{AB} \cong \overline{DC}$  and  $\overline{AD} \cong \overline{BC}$
- The opposite angles are congruent.  
 $\angle A \cong \angle C$  and  $\angle B \cong \angle D$
- The consecutive angles are supplementary.  
 $m\angle A + m\angle B = 180$   
 $m\angle D + m\angle C = 180$   
 $m\angle A + m\angle D = 180$   
 $m\angle B + m\angle C = 180$
- The diagonals bisect each other.  
 Diagonal  $\overline{AC}$  bisects  $\overline{BD}$ , this means  $\overline{BE} \cong \overline{DE}$ .  
 Diagonal  $\overline{BD}$  bisects  $\overline{AC}$ , this means  $\overline{AE} \cong \overline{CE}$ .



- Each diagonal bisects the parallelogram into two congruent triangles.  
 For diagonal  $\overline{AC}$ ,  $\triangle ABC \cong \triangle CDA$   
 For diagonal  $\overline{BD}$ ,  $\triangle ABD \cong \triangle CDB$

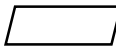


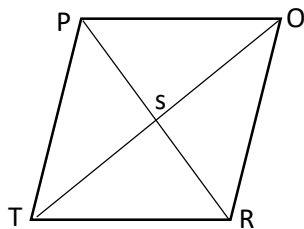
**Example 2:** Use the properties to solve for the parallelogram LIFE.



Solution:  $LE = IF = 17$  cm and  $m\angle LEF = 180 - 86 = 94$ . Also  $m\angle I = m\angle LEF = 94$   
 $EF = LI = 20$  cm  
 $m\angle L = 180 - m\angle LEF = 86$   
 $m\angle F = m\angle L = 86$



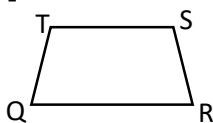
**Example 3:**  PORT is a parallelogram. If OT is 6 more than PR, and PS=9, then ST= \_\_\_\_\_.



Solution: Since  $\overline{PR}$  and  $\overline{OT}$  bisect each other, and PS=9, then PR=18. This further implies that  $OT=PR+6=18+6=24$  units and ST=12 units.

## B. Definition of Trapezoid

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides.

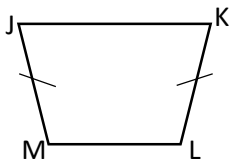


The parallel sides are called the **bases of the trapezoid**. The nonparallel sides are called **legs**. The angles formed by a base and the legs are called **base angles**.

Quadrilateral QRST is a trapezoid.  $\overline{QR}$  and  $\overline{TS}$  are its bases and  $\overline{QT}$  and  $\overline{RS}$  are its legs.  $\angle Q$  and  $\angle R$  are its base angles.  $\angle T$  and  $\angle S$  are also base angles.

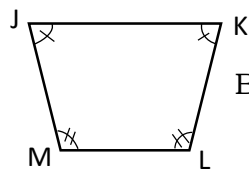
### Definition of an Isosceles Trapezoid

An *isosceles trapezoid* is trapezoid with congruent legs.



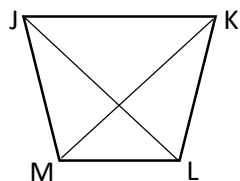
Trapezoid JKLM is an isosceles trapezoid. The consequence of this definition is stated in the next theorems.

**Theorem 1:** The base angles of an isosceles trapezoid are congruent.



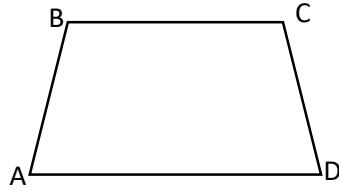
Base angles are  $\angle J \cong \angle K$  and  $\angle M \cong \angle L$

**Theorem 2:** The diagonals of an isosceles trapezoid are congruent



Diagonals are  $\overline{JL} \cong \overline{KM}$

**Example 4:** In an isosceles trapezoid ABCD,  $m\angle A = 3x+40$  and  $m\angle D = x+60$ . Find  $m\angle B$ .



Solution: By definition,  $\angle A \cong \angle D$

$$m\angle A = m\angle D$$

$$3x+40=x+60$$

$$2x=20$$

$$x=10$$

$$m\angle A = 3x+40$$

$$m\angle A = 3(10) + 40$$

$$m\angle A = 70$$

With AB as transversal to parallel  $\overline{BC}$  and  $\overline{AD}$ ,  $\angle A$  and  $\angle B$  are supplementary.

$$m\angle A + m\angle B = 180$$

$$70 + m\angle B = 180$$

$$m\angle B = 180 - 70$$

$$m\angle B = 110$$

Definition of Median of a Trapezoid

The *median* of a trapezoid is the segment joining the midpoints of the legs.



In the trapezoid MNOP,  $\overline{QR}$  is the median.

**Theorem 3:** The median of the trapezoid is parallel to its bases.

Trapezoid MNOP with median  $\overline{QR}$ .

$$\overline{QR} \parallel \overline{MN} \text{ and } \overline{QR} \parallel \overline{PO}$$

**Theorem 4:** The median of the trapezoid is half the sum of the lengths of the bases.

Trapezoid MNOP with median  $\overline{QR}$ .

$$QR = \frac{1}{2} (MN + PO)$$

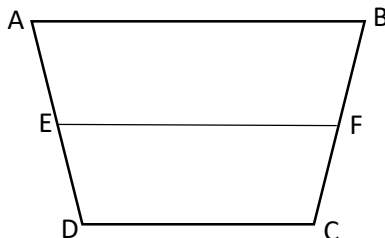
**Example 5:** The length of the bases of a trapezoid are 17 cm and 11 cm. How long is its median?

Solution: By the Theorem 4, the median's length is half the sum of bases.

The sum of the bases is 17 cm+ 11 cm=28 cm.

Thus, the median is 28 cm divided by 2 is equal to 14 cm.

**Example 6:** ABCD is a trapezoid , with EF as median. If EF=16 and AB=22, find DC.



Solution:  $EF = \frac{1}{2} (AB + DC)$

$$16 = \frac{1}{2} (22 + DC)$$

$$16(2) = 22 + DC$$

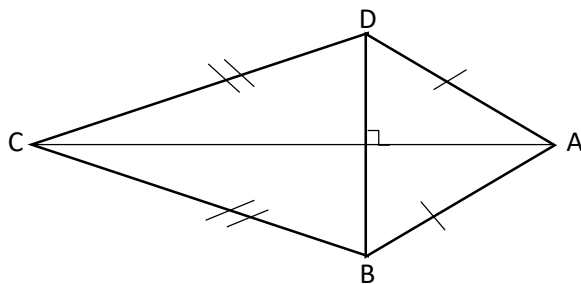
$$32 = 22 + DC$$

$$DC = 32 - 22$$

$$DC = 10$$

### C. Definition of a Kite

A *kite* is another type of a quadrilateral. It has two distinct pairs of adjacent congruent sides. Quadrilateral ABCD below is a kite.



In kite ABCD,  $\overline{AB} \cong \overline{AD}$  and  $\overline{BC} \cong \overline{DC}$ . The two segments  $\overline{AC}$  and  $\overline{BD}$  are its diagonals.

Some properties of kites are stated in the next theorems.

**Theorem 5:** If a quadrilateral is a kite, then its diagonals are perpendicular.

$$\overline{AC} \perp \overline{DB}$$

**Theorem 6:** The area of a kite is half the product of the length of its diagonals.

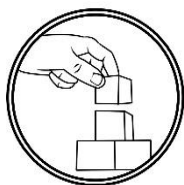
$$\text{Area of kite ABCD} = \frac{1}{2} (\overline{AC} + \overline{BD})$$

**Example 7:** The diagonals of a kite have lengths of 13 cm and 9 cm. Find the area of a kite?

$$\begin{aligned}\text{Solution: Area of a Kite} &= \frac{1}{2} (d_1) (d_2) \\ &= \frac{1}{2} (13 \text{ cm}) (9 \text{ cm}) \\ &= \frac{1}{2} (117 \text{ cm}^2) \\ &= 58.5 \text{ cm}^2\end{aligned}$$

**Example 8:** The area of a kite is  $180 \text{ cm}^2$  and the length of the diagonal is 36 cm. How long is the other diagonal?

$$\begin{aligned}\text{Solution: Area of a Kite} &= \frac{1}{2} (d_1) (d_2) \\ 180 \text{ cm}^2 &= \frac{1}{2} (36 \text{ cm}) (d_2) \\ 2(180 \text{ cm}^2) &= (36 \text{ cm}) (d_2) \\ 360 \text{ cm}^2 &= (36 \text{ cm}) (d_2) \\ \frac{360 \text{ cm}^2}{36 \text{ cm}} &= \frac{36 \cancel{\text{cm}}}{36 \cancel{\text{cm}}} (d_2) \\ d_2 &= 10 \text{ cm}\end{aligned}$$

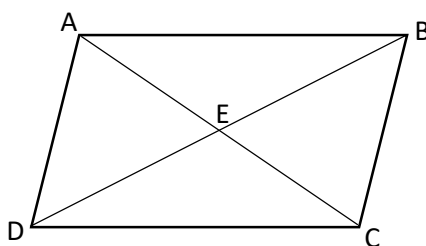


## Explore

Now, you are equipped with knowledge on applying the properties of parallelograms, trapezoids, and kites in solving problems, it's about time to find out what you can do. Enjoy and have fun!

### Activity 3: YES! YOU CAN DO IT!

Below is parallelogram ABCD. Consider each given information and answer the questions that follow.

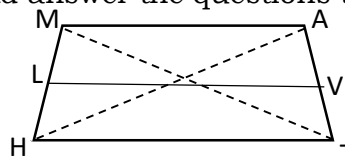


1. Given:  $AB = (3x - 5) \text{ cm}$ ,  $BC = (2y - 7) \text{ cm}$ ,  $CD = (x + 7) \text{ cm}$  and  $AD = (y + 3) \text{ cm}$ .
  - a. What is the value of  $x$ ? \_\_\_\_\_
  - b. How long is  $AB$ ? \_\_\_\_\_
  - c. What is the value of  $y$ ? \_\_\_\_\_
  - d. How long is  $AD$ ? \_\_\_\_\_
  - e. What is the perimeter of parallelogram ABCD? \_\_\_\_\_
2.  $\angle BAD$  measures  $(2a + 25)^\circ$  while  $\angle BCD$  measures  $(3a - 15)^\circ$ .
  - a. What is the value of  $a$ ? \_\_\_\_\_
  - b. What is  $m\angle BAD$ ? \_\_\_\_\_
  - c. What is  $m\angle CBA$ ? \_\_\_\_\_

3. Diagonals  $\overline{AC}$  and  $\overline{BD}$  meet at E. DE is 8 cm and AC is 13 cm.
  - a. How long is BD? \_\_\_\_\_
  - b. How long is AE? \_\_\_\_\_

#### Activity 4: SHOW WHAT YOU'VE GOT!

Consider the figure on the right and answer the questions that follow.  
 Given: Quadrilateral MATH is an \_\_\_\_\_  
 Isosceles trapezoid with bases MA  
 and HT, LV is a median.

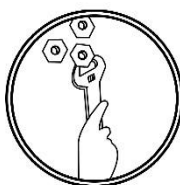
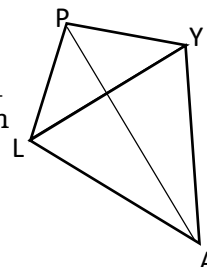


1. Given:  $MA = 3y - 2$ ;  $HT = 2y + 4$ ;  $LV = 8.5$  cm
  - a. What is the value of y? \_\_\_\_\_
  - b. What are MA and HT? \_\_\_\_\_
2. Given:  $\angle HMA = 115^\circ$ 
  - a. What is  $m\angle TAM$ ? \_\_\_\_\_
3. Given:  $m\angle MHT = 3x + 10$ ;  $m\angle MAT = 2x - 5$ 
  - a. What is the value of x? \_\_\_\_\_
  - b. What are the measures of the two angles? \_\_\_\_\_
4. Given:  $AH = 4y - 3$ ;  $MT = 2y + 5$ 
  - a. What is the value of y? \_\_\_\_\_
  - b. How long is each diagonal? \_\_\_\_\_

#### Activity 5: PLAY A KITE

Consider the figure that follow and answer the given questions.  
 Given: Quadrilateral PLAY is a kite.

1. Given:  $PA = 12$  cm;  $LY = 6$  cm
  - a. What is the area of kite PLAY? \_\_\_\_\_
2. Given: Area of kite  $PLAY = 135 \text{ cm}^2$ ;  $LY = 9$  cm
  - a. How long is PA? \_\_\_\_\_



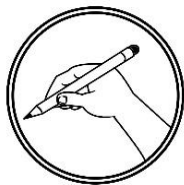
### Deepen

#### Activity 6: YOU CAN DO IT AGAIN!

Directions: Solve each problem completely and accurately. You may illustrate each given, to serve as your guide.

- A. Given: Given Quadrilateral WISH is a parallelogram.
  - a. If  $m\angle W = x + 15$  and  $m\angle S = 2x + 5$ , what is  $m\angle W$ ?
  - b. If  $WI = 3y + 3$  and  $HS = y + 13$ , how long is HS?
- B. Given: Quadrilateral POST is an isosceles trapezoid with  $\overline{OS} \parallel \overline{PT}$ ,  $\overline{ER}$  is its median.
  - a. If  $OS = 3x - 2$ ,  $PT = 2x + 10$  and  $ER = 14$ , how long is each base?
  - b. If  $\angle P = 2x + 5$  and  $m\angle O = 3x - 10$ , what is  $m\angle T$ ?

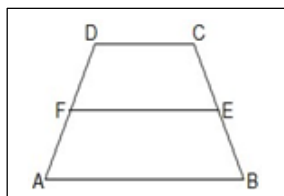
- C. Given: Quadrilateral LIKE is a kite with  $\overline{LI} \cong \overline{IK}$  and  $\overline{LE} \cong \overline{KE}$ .
- What is its area if one of the diagonals is 4 more than other and  $IK + LK = 16$  in?
  - If  $IE = (x - 1)$  ft and  $LK = (x + 2)$  ft. If its area is  $44 \text{ ft}^2$ , how are IE and LK?



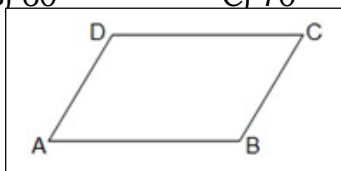
## Gauge

**Directions:** Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

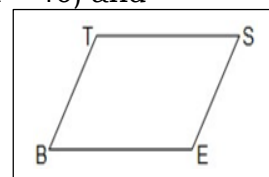
- Consecutive angles of a parallelogram are \_\_\_\_\_.  
A) adjacent    B) complementary    C) congruent    D) supplementary
- A quadrilateral with exactly one pair of parallel sides.  
A) rectangle    B) rhombus    C) square    D) trapezoid
- In the figure below,  $DC = 20$  cm and  $AB = 36$  cm. What is FE?  
A) 16 cm    B) 28 cm    C) 46 cm    D) 56 cm



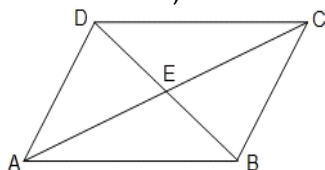
- The figure to the right is a parallelogram. If  $AD = 2x - 10$  and  $BC = x + 30$ , then  $BC =$  \_\_\_\_\_.  
A) 50    B) 60    C) 70    D) 80



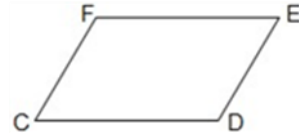
- Quadrilateral BEST is a parallelogram. If  $m\angle B = (x + 40)$  and  $m\angle E = (2x + 20)$ , what is the  $m\angle B$ ?  
A) 50    B) 60    C) 70    D) 80



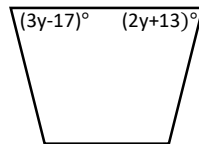
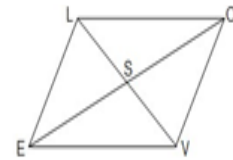
- The figure below is a parallelogram. The diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect at E. if  $AE = 2x$  and  $EC = 12$ , what is  $x$ ?  
A) 5    B) 6    C) 7    D) 8



- \_\_\_\_ 7. Quadrilateral CDEF is a parallelogram. If  $m\angle C = y$  and  $m\angle E = (2y - 40)$ , then  $m\angle D$  is  
 A) 80      B) 110      C) 140      D) 170



- \_\_\_\_ 8. How many congruent triangles are formed when a diagonal of parallelogram is drawn?  
 A) 1      B) 2      C) 3      D) 4
- \_\_\_\_ 9. Base angles of an isosceles trapezoid are \_\_\_\_\_.  
 A) adjacent      B) complementary      C) congruent      D) supplementary
- \_\_\_\_ 10. All the following are properties of a parallelogram EXCEPT:  
 A) diagonals bisect each other.      B) opposite angles are congruent  
 C) opposite sides are congruent.      D) opposite sides are not parallel
- \_\_\_\_ 11. If LOVE is a parallelogram and  $SE = 6$ , what is  $SO$ ?  
 A) 3      B) 6      C) 12      D) 15
- \_\_\_\_ 12. Find the length of a diagonal of a kite whose area is 176 sq.cm and other diagonal is 16 cm long.  
 A) 22 cm      B) 24 cm      C) 26 cm      D) 28 cm
- \_\_\_\_ 13. The diagonals of an isosceles trapezoid are represented by  $4x - 47$  and  $2x + 31$ . What is the value of  $x$ ?  
 A) 37      B) 39      C) 107      D) 109
- \_\_\_\_ 14. Find the value of  $y$  in the figure below.



- A) 24      B) 30      C) 35      D) 80
- \_\_\_\_ 15. Draw the figure to complete the statement for parallelogram SEND, if  $m\angle SDN = 28$  then  $m\angle E =$  \_\_\_\_\_.  
 A) 152      B) 124      C) 56      D) 28

# References

## **Books**

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Grade 9 Mathematics Patterns and Practicalities by Salesiana BOOKS by Don Bosco Press, Inc. Copy Right 2013

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