

MATHEMATICS

Quarter 2- Module 8

Using Inductive or Deductive Reasoning And Writing a Proof (Both Direct and Indirect)



AIRs - LM

MATHEMATICS 8

Quarter 2 - Module 8: Using Inductive or Deductive Reasoning and Writing a proof (Both Direct and Indirect)

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Region I

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MATHEMATICS

Quarter 2- Module 8

Using Inductive or Deductive Reasoning And Writing a Proof (Both Direct and Indirect)

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



Target

The importance of geometry is to be able to show what you have learned through constructed written judgement.

This module was designed and written with you in mind. It is here to help you master how to work from a conjecture to a conclusion. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course.

After going through this module, you are expected to attain the following objectives:

Learning Competency:

- Uses inductive or deductive reasoning in an argument. **M8GE- IIh-1**
- Writes a proof (both deductive and inductive). **M8GE-III-j-1**

Subtasks:

- Distinguish between inductive and deductive reasoning.
- Write formal arguments as a series of statements that make up a proof (both direct and indirect)

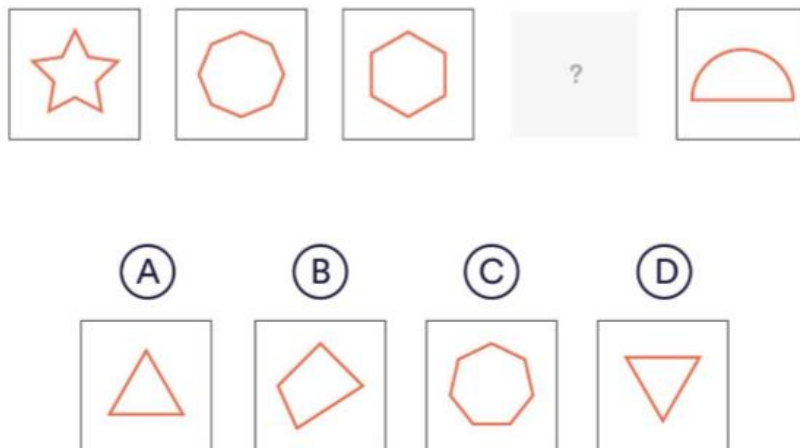


Pretest

DIRECTIONS: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Using the distributive property, which of the following expressions will make the statement $5(a + b) = \underline{\hspace{1cm}}$ true?
A. $5a + b$ B. $B + 5a$ C. $5a + 5b$ D. $5 + a + b$
2. What is the valid conclusion for the given hypothesis: If \overline{OM} bisects $\angle LON$ then
A. $\angle LOM \cong \angle NOM$ B. $\angle LOM \cong \angle LON$
C. $\angle MON \cong \angle NOL$ D. $m\angle LON = m\angle LOM + m\angle MON$
3. What is the next term in the sequence 3, 6, 9, 12, ...?
A. 15 B. 16 C. 20 D. 24
4. What is the next term in the series 1, 2, 4, 7, 11, 16, ...?
A. 18 B. 20 C. 21 D. 22
5. What the next term in 400, 200, 100, 50, ...?
A. 30 B. 25 C. 20 D. 10
6. "Coplanar points are points on the same plane. L, M, N are coplanar". Using deductive reasoning, what is the correct conclusion for these statements?
A. Points L, M, and N are collinear.
B. Points L, M, and N are not coplanar.
C. Points L, M, and N are non – collinear.
D. Points L, M, and N are points on the same plane
7. What property is illustrated in the statement: "If $\angle X \cong \angle Y$, $\angle Y \cong \angle Z$, then $\angle X \cong \angle Z$."?
A. Reflexive Property B. Symmetric Property
C. Transitive Property D. Addition Property
8. A quadrilateral is a polygon of four sides. A rectangle is a quadrilateral. What is the correct proof in the given statement?
A. A rectangle is a quadrilateral.
B. A parallelogram is a quadrilateral.
C. A polygon with four sides is a rectangle.
D. A polygon with four sides is a quadrilateral.
9. What is the next term in 1, 8, 27, 64, ___?
A. 125 B. 136 C. 145 D. 155
10. What is the next term in 2, 6, 18, 54, ...?
A. 160 B. 162 C. 164 D. 166

11. Filipinos are hospitable people. John is a Filipino. What can you conclude from the given statements?
- John is not a Filipino.
 - John is a hospitable person.
 - A person John is not hospitable.
 - John is not a person that is hospitable.
12. Which of the following is an example of a deductive argument?
- There are 25 CDs on the top shelf of my bookcase and 14 on the lower shelf. There are no other CDs in my bookcase. Therefore, there are 39 CDs in my bookcase.
 - Topeka is either in Kansas or Honduras. If Topeka is in Kansas, then Topeka is in North America. If Topeka is in Honduras, then Topeka is in Central America. Therefore, Topeka is in Kansas.
 - No one got an A on yesterday's test. Jimmy wasn't in school yesterday. Jimmy will make up the test today and get an A.
 - All human beings are in favor of world peace. Terrorists don't care about world peace. Terrorists bring about destruction
13. Complete the sequence



Source: http://www.education.com/reference/article/deductive-reasoning_answer/

For items number 14-15, refer on the given below.

Write a paragraph proof that this is true. If $5(x+1) = 25$, then $x = 4$

- Since $5(x + 1) = 25$, it follows the distributive Property that $5x+5 = 25$.
- Using the Additional Property, $5x = 20$.
- Then applying the Division Property, it can be concluded that $x = 4$

14. What is the first statement of your proof?
- I only
 - II only
 - III only
 - use I, II, and III
15. What do you think the last process use in writing a proof?
- I only
 - II only
 - III only
 - use I , II, and III



Jumpstart

This chapter introduces you to the language of inductive and deductive reasoning. You will learn how to form statements and decide which statements can be assumed without proof.

Logic and reasoning are very useful tools in decision making. Reasoning is very important skill in mathematics. Now you will learn more about inductive and deductive reasoning.

One of the tools used in proving is reasoning, specifically deductive reasoning. Deductive reasoning is a type of logical reasoning that uses accepted facts (in our case: undefined terms, definitions, properties, postulates, and previously proven theorems) to a reason in a step-by-step manner until we arrive at the arrive statements.

Since you have learned the two types of reasoning, you will also how to write deductive proofs that are characterized by rigorous geometric thought. It is used in formal proofs, and you will often be resorted to in proving theorems and corollaries for the remainder of your study in Geometry.

Activity 1:

- A. Justify each statement by giving the Property of Equality or Property of Congruence used. Choose your answers above and write it on your answer sheet.

Substitution Property of Equality	Division Property of Equality (DPE)
Addition Property of Equality (APE)	Distributive Property
Subtraction Property of Equality (SPE)	Reflexive Property
Multiplication Property of Equality (MPE)	Symmetric Property
Transitive Property	

- _____ 1. If $\overline{TX} = BK$, then $BK = TX$
_____ 2. $8(m + n) = 8m + 8n$
_____ 3. If $\overline{CT} = 12$ and $\overline{PR} + \overline{CT} = 20$, then $\overline{PR} + 12 = 20$
_____ 4. $m\angle HIT = m\angle HIT$
_____ 5. If $\angle S \cong \angle P$, $\angle B \cong \angle S$, then $\angle P \cong \angle B$

- B. Find the next term of the following sequences, then identify the reasoning used (inductive or deductive).

- _____ 6. 2, 4, 6, 8, 10, ____
_____ 7. 1, 4, 9, 16, 25, ____
_____ 8. A, B, C, D, E, ____

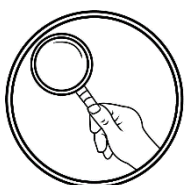
- C. Use deductive reasoning to give another statement that must also be true. Select the best statement.

- ___ 9. All snakes are cold blooded. A cobra is a snake.
A. A cobra is not a snake
B. A cobra is cold – blooded
C. A cold snake is blooded
D. A snake is cold
- ___ 10. No human can fly. I am a human.
A. All human can fly.
B. All human can't fly.
C. All can fly except human.
D. I cannot fly because I am human.

Activity 2

Directions: Use the given property to draw a conclusion. Write your answers on your answer sheet.

1. Reflexive Property: Given: \overline{AB} is real number, then _____.
2. Symmetric Property: Given: $\overline{AB} = \overline{CD}$, then _____.
3. Transitive Property: Given: $\overline{AB} = \overline{CD}$, and $\overline{CD} = \overline{EF}$, then _____.
4. Addition Property: Given: If $a = b$ and $c = d$, then _____.
5. Division Property: Given: If $a = b$ and $c \neq 0$, then _____.



Discover

In Activity 1, we discovered a pattern and found the next number in the sequence by assuming that the pattern will continue. This is inductive and deductive reasoning. Inductive reasoning is the process of reasoning that a rule or statement is true because specific cases are true. The statement believed to be true based on inductive reasoning is called **conjecture**.

One of the tools used in proving is reasoning, specifically deductive reasoning. Deductive reasoning is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

Deductive Reasoning derives general principles from specific observation.

Examples of Deductive Reasoning are as follows:

1. If $\angle A$ is acute, $m\angle A < 90^\circ$, then we can say that $\angle A$ is acute angle.
2. Suppose $3x = 12$, we conclude $x = 4$.
3. The length of a rectangle is 6 and its side is 5. We conclude its area is 30.

Inductive Reasoning is a reasoning that is based on patterns you observed.

Examples of Inductive Reasoning are as follows:

1. 2, 4, 6, 8, 10, 12, 14, 16, ... What is the 19th term in the pattern?
2. 1, 4, 16, 64, ... What is the next term?
Solution The following number is four times the previous number
 $64(4) = 256$
3. -5, -2, 4, 13, ...
Solution: Add 3 then 6, then 9, so the next number would add is 12
 $13 + 12 = 25$

A **proof** is a logical argument in which each statement you make is supported/justified by given information, definitions, axioms, postulates, theorems, and previously proven statement.

Remember:

- Postulate is a statement that is accepted without proof.
- Theorem is a statement accepted after it is proved deductively.

WRITING PROOFS / PROVING THEOREMS

In proving theorems, the properties of equality and congruence are the bases for reasoning.

Properties of Equality

- *Addition Property of Equality (APE)*
For all real numbers a , b , c and d , if $a = b$ and $c = d$, then $a + c = b + d$
- *Subtraction Property of Equality (SPE)*
If $a = b$ and $c = d$, then $a - c = b - d$.
- *Multiplication Property of Equality (MPE)*
If $a = b$, then $ac = bc$
- *Division Property of Equality (DPE)*
If $a = b$ then $a/c = b/c$
- *Substitution Property of Equality*
If $m\angle A = 60$, $m\angle B = 60$ then $m\angle A = m\angle B$
- *Distributive Property*
 $a(b + c) = ab + ac$

Properties of Congruence:

Reflexive Property	$AB \cong AB$
Symmetric Property	If $\angle A \cong \angle B$ then $\angle B \cong \angle A$
Transitive Property	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$ then $\angle A \cong \angle C$

Aside from the properties of equality and congruence, you should be equipped with the knowledge about undefined terms, definitions, and postulates in geometry. These are necessary to successfully support the statement of a proof.

Illustrative Examples:

Justify each statement by giving the Property of Equality or Property of Congruence used.

1. If $TX = BK$, then $BK = TX$
2. $8(m + n) = 8m + 8n$
3. If $CT = 12$ and $PR + CT = 20$, then $PR + 12 = 20$.
4. $m\angle HIT = m\angle HIT$
5. If $\angle S \cong \angle P$, $\angle B \cong \angle S$, then $\angle P \cong \angle B$

Answers:

1. Symmetric Property
2. Distributive Property
3. Substitution Property of Equality
4. Reflexive Property
5. Transitive Property

Proofs can be written in different ways

1. Paragraph Form/ Informal Proof

The paragraph or informal proof is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

2. Two-Column Form/ Formal Proof

3. Flowchart Form

A flowchart-proof organizes a series of statements in a logical order, starting with the given statements. Each statement together with its justification is written in a box. Arrows are used to show how each statement leads to another. It can make one's logic visible and help others follow the reasoning.

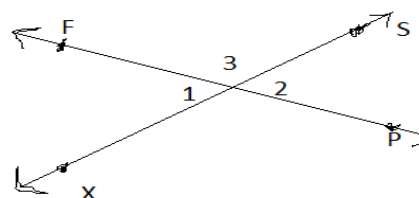
4. Indirect Proof

An indirect proof usually is paragraph form, the opposite of the statement to be proven is assumed true until the assumption leads to contradiction.

Example 1: Prove: Vertical angles are congruent

Solution: Rewrite: If the angles are vertical, then the angles are congruent.

DRAW:



STATEMENT:

Given: FP intersects XS at O

Prove: $\angle 1$ and $\angle 2$ are vertical angles

PLAN: First, show that $\angle 1$ and $\angle 2$ form a linear pair. Also, $\angle 2$ and $\angle 3$ form a linear pair. Then to establish that $\angle 1 \cong \angle 2$, use the fact that the angles in a linear pair are supplementary and supplements of congruent angles are congruent.

DEMONSTRATE:

Statements

Reasons

1. FP intersects XS at O
 $\angle 1$ and $\angle 2$ are vertical angles
2. $\angle 1$ and $\angle 2$ form a linear pair
 $\angle 2$ and $\angle 3$ form a linear pair
3. $\angle 1$ and $\angle 2$ are supplementary
 $\angle 2$ and $\angle 3$ are supplementary
4. $\angle 1$ is a supplement of $\angle 3$
 $\angle 2$ is a supplement of $\angle 3$
5. $\angle 1 \cong \angle 2$

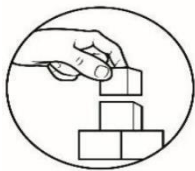
1. Given
2. Definition of Linear Pair
3. If two angles form a linear pair, then the angles are supplementary
4. Definition of Supplement of an Angle
5. Supplements of the same angles are congruent.

Example 2. Write a paragraph proof that this is true

If $6(2x-1) = 30$, then $x = 3$

Solution:

Proof : Since $6(2x - 1) = 30$, it follows the distributive Property that $12x - 6 = 30$. Using the Additional Property, $12x = 36$, Then applying the Division Property, it can be concluded that $x = 3$



Explore

Work on the following enrichment activities for you to apply your understanding on this lesson.

Activity 3:

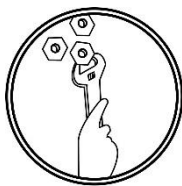
DIRECTIONS: Draw conclusion from each given situation and identify the kind of reasoning used. Write your answers on your answer sheet.

1. 5, 10, 15, 20. The next number is ____.
2. Coplanar points are points on the same plane. X, Y, Z are coplanar.
Therefore _____.
3. Regular polygon is equilateral. BELEN is a regular pentagon.
Therefore _____.
4. A child's teacher in preschool was a female, in his grades 1 and 2 his teachers were both female. The child may say _____.
5. Filipinos are peace-loving people. Julia is a Filipino.
Therefore _____.

Activity 4

Complete the following proof below. Write your answers on your answer sheet.

STATEMENTS	REASONS
$\angle 1$ and $\angle 3$ are vertical angles	1. Given
$\angle 1$ and $\angle 2$ form a linear pair $\angle 2$ and $\angle 3$ form a linear pair	2.
$\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	3.
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ $\angle 2$ is a supplement of $\angle 3$	4.
$m\angle 2 \cong m\angle 2$	5.
$m\angle 1 \cong m\angle 3$	6.



Deepen

Activity 5: CUBRA CUBE

Complete the table below. Write your answers on your answer sheet

STATEMENT	CONCLUSION
1. Filipinos are hospitable. Bonifacio is a Filipino.	
2. If points are collinear, then they lie on the same plane . Points R,M,and N are collinear	
3. A quadrilateral is a polygon of four sides. A parallelogram is a quadrilateral.	
4. Smoking can cause cancer. Tomas is smoking	
5. An angle is acute if its measure is between 0 and 90. Angle B is acute.	

Activity 6. On Your Own

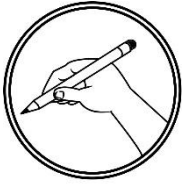
A. Copy each proof, then name the property that justifies each statements. Write your answers on your answer sheet

STATEMENTS	REASONS
1. If $3 = x$, then $x = 3$	1.
2. If $1/3x = 8$, then $x = 24$	2.
3. If $AB = 5$ and $AB = CD$ then $CD = 5$	3.

B. Write a paragraph proof to prove that each conditional is true.

4. $5x - 8 = 12$, then $x = 4$

5. If $-2(x+) + 4 = -2$, then $x = 0$

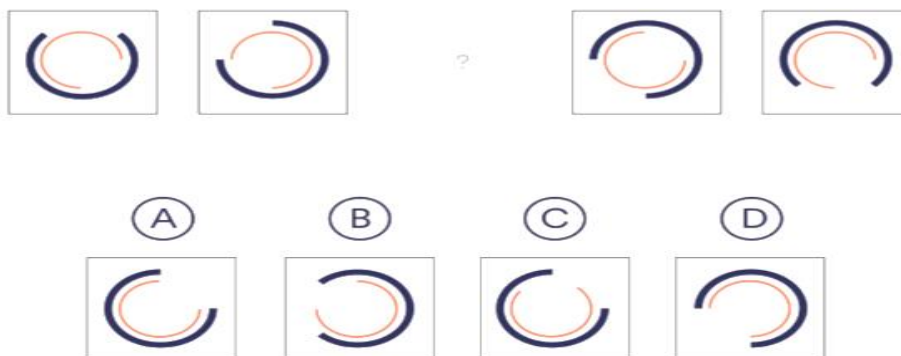


Gauge

A. Directions: Read and understand the questions below. Select the best answer to each item then write your choice on your answer sheet.

1. What property is illustrated in statement: If $\angle A \cong \angle B$, $\angle B \cong \angle C$ then $\angle A \cong \angle C$?
A. Reflexive Property
B. Symmetric Property
C. Transitive Property
D. Addition Property
2. Using the distributive property, which of the following expressions will make the statement, $4(a + b) = \underline{\hspace{1cm}}$ true?
A. $4a + b$
B. $B + 4a$
C. $4a + 4b$
D. $4 + a + b$
3. What is the valid conclusion for the given hypothesis, If \overline{OM} bisects $\angle LON$?
A. $\angle LOM \cong \angle NOM$
B. $\angle MON \cong \angle NOL$
C. $\angle LOM \cong \angle LON$
D. $m\angle LON = m\angle LOM + m\angle MON$
4. Using the inductive reasoning, what is the next term of the sequence 4, 8, 12, 16, ...?
A. 18
B. 19
C. 20
D. 24
5. Using inductive reasoning, what is the next term of the sequence 1, 2, 4, 7, 11, 16...?
A. 18
B. 20
C. 21
D. 22
6. Using inductive reasoning, what is the next term of the sequence 400, 200, 100, 50...?
A. 30
B. 25
C. 20
D. 10
7. "Coplanar points are points on the same plane. X, Y, Z are coplanar". Using deductive reasoning, what can you conclude from these statements?
A. Points X, Y, and Z are points on the same plane.
B. Points X, Y, and Z are collinear.
C. Points X, Y, and Z are not coplanar.
D. Points X, Y, and Z are non – collinear.
8. If the measure of $m\angle A$ is less than 90° , what can say about angle A?
A. $\angle A$ is acute angle.
B. $\angle A$ is a right angle.
C. $\angle A$ is an obtuse angle.
D. $\angle A$ is a right and acute angle.

9. Which of the following will complete the sequence?

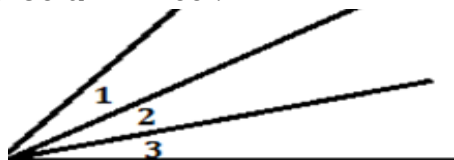


Source: http://www.education.com/reference/article/deductive-reasoning_answer/

For item numbers 10 – 11, refer to the given Two-Column Proof.

Given: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

Prove: $m\angle 1 = m\angle 3$



STATEMENT	REASON
1. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	1. Given
2. $m\angle 2 = m\angle 2$	2. _____
3. $m\angle 1 = m\angle 3$	3. _____

10. Which Property of Equality reasons out Statement 2?

A. Addition PE B. Reflective PE C. Subtraction PE D. Transitive PE

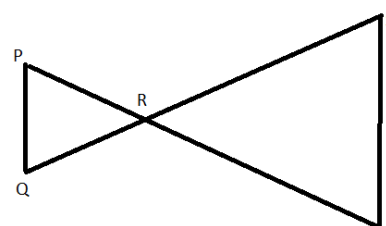
11. Which property of Equality reasons out Statement 3?

A. Addition PE B. Reflective PE C. Subtraction PE D. Transitive PE

B. Directions: Use the following figure to answer the questions regarding this indirect proof.

Given : $\overline{ST} \neq \overline{PQ}$, \overline{QS} and \overline{PT}

Prove : \overline{QS} and \overline{PQ} do not bisect each other.



STATEMENTS	REASONS
$\overline{ST} \neq \overline{PQ}$, \overline{QS} and \overline{PT}	Given
\overline{QS} and \overline{PQ} do not bisect each other.	Assumption
$\overline{PR} \cong \overline{RT}$ and $\overline{QR} \cong \overline{RS}$	$\angle PQR$ and $\angle SRT$

12. _____	Intersecting lines form vertical angles
$\angle PRQ \cong \angle SRT$	13. _____
$\triangle PQR \cong \triangle SRT$	14. _____
$ST \cong PQ$	15. _____
\overline{QS} and \overline{PQ} bisect each other.	Contradiction-step 7 contradicts step 1, therefore, the opposite must be true.

References

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Links

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