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Mathematics

Quarter 1 - Module 1:
Illustrating and Solving Quadratic Equations

AIRs - LM

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Quarter 1 - Module 1: Illustrating and Solving Quadratic Equations
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11. Find the solutions of quadratic equation $b^2 - 8b - 9 = 0$ by completing the square.
    A. 1, -9
    B. -1, 9
    C. 2, -4
    D. -2, 4
12. What are the roots of the quadratic equation $x^2 + 5x = -6$?
    A. 2, 3
    B. -1, 6
    C. 3, -4
    D. -2, -3
13. Given the quadratic equation $x^2 + 9x = 12$, what is the value of c?
    A. 9
    B. -9
    C. 12
    D. -12
14. In solving $2x^2 = -3(x + 2)$ using quadratic formula, what is the first thing to consider?
    A. Identify the value of a, b and c.
    B. Rewrite the equation in general form.
    C. Divide both sides by a common factor.
    D. Substitute the values of a, b, and c in the quadratic formula.
15. What are the roots of $x^2 - 6x + 2 = 0$ using quadratic formula?
    A. $2 \pm\sqrt{5}$
    B. $3 \pm\sqrt{7}$
    C. $\pm\sqrt{5}$
    D. $\pm\sqrt{7}$

Were you able to answer all the questions? If not, don't worry because the next activity will help you better understand the lesson.

Jumpstart

Activity 1: Put Me In!

Directions: Below are different equations. If the equation is linear, write LE otherwise write NL.

1. $x^2 - 2x + 3 = 0$
2. $2s - 5 = 0$
3. $3r^2 + 5r + 12 = 0$
4. $8k - 6 = 1$
5. $6t + 7 = 0$

Process Questions:

a. Which of the given equations are linear?
b. How do you describe linear equations?
c. Which of the equations are not linear, why?
d. What common characteristics do these equations have?

Activity 2: Solve Me!

Direction: Find the solution/s of each of the following equations.

1. $x + 7 = 12$
2. $t - 4 = 10$
3. $-5x = 35$
4. $x - 10 = -2$
5. $x^2 - 4 = 0$

Process Questions:

a. What type of equations are the following?
b. How did you solve each equation?
c. What mathematics concepts or principles did you apply to come up with the solution of each equation?
d. Which equations did you find difficult to solve? Why?

Discover

Quadratic comes from the Latin word "quadratus" which means "square". A quadratic equation is any equation that can be rearranged in standard form as $ax^2 + bx + c = 0$ where a, b, and c represent known numbers, and $a \neq 0$.

A quadratic equation is an equation of the second degree, which contains at least one term that is squared.

In the equation $ax^2 + bx + c = 0$, $\mathbf{ax^2}$ is the *quadratic term*, **bx** is the *linear term*, and **c** is the *constant term*. For example, in the equation $3x^2 + 4x + 6 = 0$, the *quadratic term* is $\mathbf{3x^2}$, the *linear term* is **4x** and the **constant term** is **6** where the value **of a = 3, b = 4 and c = 6.**

Example 1:

$2x^2 + 6x + 4 = 0$ is a quadratic equation in standard form with a = 2, b = 6, and c = 4.

Example 2:

$x^2 - 5x = 0$ is a quadratic equation. However, it is not written in standard form.

This one is a little tricky:

- What is the value of **a**?
  Well, **a=1** as we don't usually write "$1x^2$"
- **b** = -5
- And where is **c**?
  Well, **c = 0**, so it is not shown

The **Standard form** of a quadratic equation is $\mathbf{ax^2 + bx + c = 0}$ like examples 1 and 2**.** But sometimes, a quadratic equation doesn't look like that.

Example 3:

$4x^2 - 3x = -1$ is a quadratic equation. However, it is not written in standard form. To transform it to its standard form, we can use transposition method by transferring the constant term to the other side of the equation as shown below.

$$4x2 - 3x = -1 \longrightarrow 4x2 - 3x + 1 = 0$$

The standard form of $4x^2 - 3x = -1$ *is* $4x^2 - 3x - 1 = 0$*, where* $a = 4$*,* $b = -3$*, and* $c = -1$.

Solving Quadratic Equations Using the Four Methods

A. Extracting Square Roots

Quadratic equations that can be written in the form $x^2 = k$ can be solved by applying the following properties:

1. If $k > 0$, then $x^2 = k$ has two real solutions or roots: $x = \pm\sqrt{k}$.
2. If $k = 0$, then $x^2 = k$ has one real solution or root: $x = 0$
3. If $k < 0$, then $x^2 = k$ has no real solutions or roots.

The method of solving quadratic equation $x^2 = k$ is called extracting square roots.

Example 1: Find the solutions of the equations $x^2 - 16 = 0$

Write the equation in the form $x^2 = k$

$$x^2 - 16 = 0 \longrightarrow x^2 - 16 + 16 = 0 + 16$$
$$x^2 = 16$$
$$x^2 = \pm\sqrt{16}$$
$$x = \pm 4$$

To check, substitute these values in the original equation.

For $x = 4$:

$$x^2 - 16 = 0$$
$$4^2 - 16 = 0$$
$$16 - 16 = 0$$
$$0 = 0$$

For $x = -4$:

$$x^2 - 16 = 0$$
$$(-4)^2 - 16 = 0$$
$$16 - 16 = 0$$
$$0 = 0$$

Both values of x satisfy the given equation. So the equation $x^2 - 16 = 0$ is true when $x = 4$ or when $x = -4$.

Answer: The equation $x^2 - 16 = 0$ has two solutions: $x = 4$ or $x = -4$

Example 2: Solve the equation $m^2 = 0$

Since m^2 equals 0, then the equation has only one solution.
That is, $m^2 = \sqrt{0}$
$m = 0$

To check: $m^2 = 0$
$0^2 = 0$
$0 = 0$

Answer: The equation $m^2 = 0$ has one solution: $m = 0$

Example 3: Find the roots of the equation $s^2 + 9 = 0$

Write the equation in the form $x^2 = k$.
$s^2 + 9 = 0 \longrightarrow s^2 + 9 - 9 = 0 - 9$
$s^2 = \sqrt{-9}$

Since $k = -9$ and it is less than 0, then the equation $s^2 = -9$ has no real solution or root. There is no real number when squared gives – 9.

B. Factoring

To understand more about factoring, let's study the following examples.

Steps on how to solve a quadratic equation by factoring:

1. Move all the terms of the equation in the left side if necessary. In this case, the other side must be zero.
2. Combine the similar terms in the left side.
3. Factor the left side of the equation.
4. Equate each factor that holds the unknown variable to zero.
5. Solve the equated form.

Example 1: $4x^2 = 6x$

Solution:

$4x^2 = 6x$
$4x^2 - 6x = 6x - 6x$ Subtract 6x from both sides
$4x^2 - 6x = 0$
$2x(2x - 3) = 0$ Factor by GCF (GCF is 2x)

$2x = 0$ $2x - 3 = 0$ Use the Zero Product Property
$x = 0$ $x = \frac{3}{2}$

*The solutions are **0** and **3/2**.*

Example 2: $x^2 - 3x = 18$

Solution:

$x^2 - 3x = 18$

$x^2 - 3x - 18 = 18 - 18$

$(x + 3)(x - 6) = 0$ — Factor the trinomial.

$x + 3 = 0$ $\quad x - 6 = 0$ — Use the Zero Product Property

$x = -3$ $\quad x = 6$ — Solve each equation

*The solutions are **-3** and **6**.*

Example 3: $9x^2 - 4 = 0$

Solution:

$9x^2 - 4 = 0$

$(3x + 2)(3x - 2) = 0$ — Factor the equation

$3x + 2 = 0$ $\quad 3x - 2 = 0$ — Use the Zero Product Property

$x = -2/3$ $\quad x = 2/3$ — Solve each equation

*The solutions are **-2/3** and **2/3**.*

C. Completing the Square

These are the steps in completing the square.

1. Place the constant term on the right side of the equation. All the terms with unknowns are on the left side.
2. The numerical coefficient of x^2 should be 1. Divide each term of the equation with the numerical coefficient of x^2 if necessary.
3. To get the constant term needed to complete the square, get the numerical coefficient of x, divide it by 2 and square it. Add the result to both sides of the equation.
4. Factor the perfect square trinomial. Values will be obtained for the right side
5. Extract the square root from both sides. Two values will be obtained for the right side of the equation.
6. Equate the linear expressions to each of the two values.
7. Solve each of the resulting linear equations.
8. Check your answer by substituting to the original equation.

Example: Solve $x^2 - 8x - 9 = 0$ *by completing the square.*

Solution:

$x^2 - 8x - 9 = 0$ — Original equation

$x^2 - 8x - 9 + 9 = 0 + 9$ — Add 9 to both sides

$x^2 - 8x = 9$

$x^2 - 8x + (4)^2 = 9 + (4)^2$

$x^2 - 8x + 16 = 9 + 16$ — Add 16 to both sides. The right side is a perfect square trinomial.

Note: To get the constant that will complete the square, take the coefficient of x, divide it by 2, then square it.

$x^2 - 8x + 16 = 25$

$(x - 4)^2 = 25$ — Factor the trinomial square.

$x - 4 = \sqrt{25}$ — Extract the square root of both sides.

$x - 4 = \pm 5$

$x - 4 = 5$ $x - 4 = -5$ — Equate the linear expressions to each of the two values. Solve each of the resulting linear equation.

$x = 9$ $x = -1$

*The solutions are **9** and -**1**.*

Checking:

For x = -1	For x = 9
$x^2 - 8x - 9 = 0$	$x^2 - 8x - 9 = 0$
$(-1)^2 - 8(-1) - 9 = 0$	$(9)^2 - 8(9) - 9 = 0$
$1 + 8 - 9 = 0$	$81 - 72 - 9 = 0$
$0 = 0$	$0 = 0$

D. Using the Quadratic Formula

In solving quadratic equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$ use the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1: Find the solution of the equation $x^2 + 2x - 8 = 0$.

$x^2 + 2x - 8 = 0 \longrightarrow a = 1;\ b = 2;\ c = -8$

Substitute the values of a, b, and c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-8)}}{2(1)}$$

Simplify the result.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(9)}}{2(1)} \longrightarrow x = \frac{-2 \pm \sqrt{4 + 32}}{2}$$

$$x = \frac{-2 \pm \sqrt{36}}{2}$$

$$x = \frac{-2 \pm 6}{2}$$

$$x = \frac{-2 + 6}{2} = \frac{4}{2} \quad ; \quad x = \frac{-2 - 6}{2} = \frac{-8}{2}$$

$$x = 2 \qquad\qquad x = -4$$

*The equation $x^2 + 2x - 8 = 0$ has two solutions: **x = 2** and **x = -4***

Example 2: Find the solutions of the equation $2x^2 + 3x = 27$ using the quadratic formula.

Write the equation in standard form.

$$2x^2 + 3x = 27 \longrightarrow 2x^2 + 3x - 27 = 0$$

Determine the values of a, b and c.

$$2x^2 + 3x - 27 = 0 \longrightarrow a = 2;\ b = 3;\ c = -27$$

Substitute the values of a, b, and c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-27)}}{2(2)}$$

Simplify the result.

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-27)}}{2(2)} \longrightarrow x = \frac{-3 \pm \sqrt{9 + 216}}{4}$$

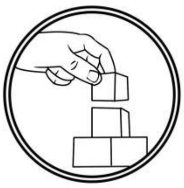
$$x = \frac{-3 \pm \sqrt{225}}{4}$$

$$x = \frac{-3 \pm 15}{4}$$

$$x = \frac{-3 + 15}{4} = \frac{12}{4} \quad ; \quad x = \frac{-3 - 15}{4} = \frac{-18}{4}$$

$$x = 3 \qquad\qquad x = \frac{-9}{2}$$

The equation $2x^2 + 3x - 27 = 0$ has two solutions: $\boldsymbol{x = 3}$ *and* $\boldsymbol{x = \frac{-9}{2}}$.



Explore

Activity 3: Set Me to Your Standard!

Directions: Write each quadratic equation in standard form, $\mathbf{ax^2 + bx + c = 0}$ then identify the values of **a, b and c.**

1. $5x^2 + 6x = 3$ a = _______ b= ______ c=______
2. $3x(x + 2) = 6$ a = _______ b= ______ c = _____
3. $(x + 4)(x - 6) = 15$ a = _______ b= ______ c = _____

Activity 4:

Direction: Find the solutions of the following quadratic equations using the best method.

1. $x^2 + 7x = 0$
2. $6s^2 + 18s = 0$
3. $x^2 - 2x = 3$
4. $s^2 - 121 = 0$
5. $x^2 - 10x + 25 = 0$

Activity 5: Let's Explore to Discover!

Directions: Read the situation carefully, then answer the questions below.

We are facing a great challenge nowadays because of the COVID-19 pandemic. So your adviser took advantage of the situation. She planned to have an activity entitled "Gulayan sa Bakuran." She asked each of you to make a layout of a rectangular garden whose area is 2000 cm^2. She specified that the length of the rectangular garden must be 100 cm more than its width.

Length = 100 cm more than its width

Area = 2000cm^2 Width = x

Questions:

1. If x represents the width of the rectangular garden, how would you represent the length?
   Answer: Width = x
   Length = ____
2. How would you represent the area of the rectangular garden? (Hint: formula for the area of a rectangle is **A = lw**).
   Given: A = 2000
3. What are the dimensions of the rectangular garden?

Deepen

Activity 6: Dig Deeper!

Direction: Answer the following questions.

1. How are quadratic equations different from linear equations?
2. How do you write quadratic equations in standard form? Give at least three (3) examples.
3. These are the values of a, b, and c that Maria and Juan got when they expressed $6 - 5x = 2x^2$ in standard form.

Maria:	a = 2;	b = 5;	c = -6
Juan:	a = -2;	b = -5;	c = 6

   Who got the correct values of a, b and c? Justify your answer.
4. Do you agree that the equation $4 - 5x = 2x^2$ can be written in standard form in two different ways? Justify your answer?