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Mathematics

Quarter 1- Module 7: Graphing System of Linear Equations in Two Variables

AIRs - LM

Mathematics 8
Quarter 1- Module 7: Graphing System of Linear Equations in Two Variables

Second Edition, 2021

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Mathematics

Quarter 1- Module 7: Graphing System of Linear Equations in Two Variables

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

In this module, you will learn the concepts of linear equations in two variables. It focuses on the illustrating systems of linear equations in two variables. Moreover, it will help you enhance and master your skills in graphing system of linear inequalities in two variables. You are provided with varied activities to process the knowledge and skills learned and to deepen and transfer your understanding on the lesson.

In this lesson, you will learn to:

1. illustrates a system of linear equations in two variables. **(M8AL-Ih-1)**
2. graphs a system of linear equations in two variables. **(M8AL-Ih-2)**
3. categorizes when a given system of linear equations in two variables has graphs that are parallel, intersecting and coinciding. **(M8AL-Ih-3)**

At the end of this module, you are expected to:

1. define a system of linear equations in two variables;
2. identify the three types of systems of linear equations in two variables: consistent and independent, consistent and dependent, and inconsistent;
3. determine whether an ordered pair satisfies a given pair of linear equations in two variables
4. graphs system of linear equations in two variables; and
5. describe the graph of the systems of linear equations in two variables as parallel intersecting, or coinciding.

Let us find out how much you already know about this module. Answer the pre-assessment in a separate sheet of paper.

PRE-ASSESSMENT

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. What do you call the graph of a system of linear equations in two variables which shows only one solution?
   A. Intersecting B. Coinciding C. Parallel D. Perpendicular
2. Which of the following is true about the system of linear equations in two variables?

   I. Any set of linear equations.
   II. A set of two or more linear equations in two variables.
   III. A system of linear equations may have only one solution, infinitely many solutions, or no solution at all.
   IV. A system of linear equations in two variables can be solve algebraically or graphically.

   A. I and II B. II and III C. I, II, and III D. II, III, and IV
3. How many solutions does a system of linear equations in two variables has if the graphs are interesting?
   A. one B. two C. no solution D. infinitely many
4. How many solutions does a system of linear equations in two variables has if the slopes of the lines are equal and the y-intercepts are also equal?
   A. one B. two C. no solution D. infinitely many
5. What is the first step in graphing system of linear equations in two variables using slope and y-intercept?
   A. Determine the slope and y-intercept of each equation.
   B. Plot the point containing the y-intercepts of each equation.
   C. Use the slopes to locate the other points of each equation.
   D. Write each equation into the slope-intercept from y = mx + b.
6. Which of the following is a system of linear equations in two variables?
   A. $\begin{cases} -y = 4 \\ x - 3y = 4 \end{cases}$ B. $\begin{cases} 2x - y = 4 \\ 2x = -2 \end{cases}$ C. $\begin{cases} 5 - 2y = 4 \\ -3x + y = -2 \end{cases}$ D. $\begin{cases} x + 3 = y \\ 2x + 2y = 6 \end{cases}$
7. Which of the following is a system of equations is consistent and independent?
   A. $\begin{cases} 3x - 2y = 1 \\ x + 2y = 5 \end{cases}$ B. $\begin{cases} x - 2y = 1 \\ x - 2y = 5 \end{cases}$ C. $\begin{cases} x + 3y = 5 \\ 3x + 9y = 15 \end{cases}$ D. $\begin{cases} x - y = 1 \\ x - y = -5 \end{cases}$
8. Which of the following is a system of equations is consistent and dependent?
   A. $\begin{cases} x + 2y = 5 \\ x + 2y = -3 \end{cases}$ B. $\begin{cases} 3x + 2y = 6 \\ 15x - 10y = 30 \end{cases}$ C. $\begin{cases} x + 3y = -5 \\ 3x + 9y = 15 \end{cases}$ D. $\begin{cases} x - y = 1 \\ x + y = 4 \end{cases}$
9. Which of the following is a system of equations is inconsistent?
   A. $\begin{cases} 5x + 2y = 10 \\ 2x + 5y = 10 \end{cases}$ B. $\begin{cases} 4x - 3y = 12 \\ 12x - 9y = -36 \end{cases}$ C. $\begin{cases} 2x + y = 4 \\ 2x + 3y = 2 \end{cases}$ D. $\begin{cases} 5x + y = -3 \\ 15x + 3y = -9 \end{cases}$
10. Is $x + y^2 = 5$ and $x + 4y = 6$ a system of linear equation in two variables?
    A. Yes, because it has two variables, x and y.
    B. Yes, because it is written in standard form and in general form.

C. No, because the constants A, B, and C are all real numbers but A and B are not both zero.

D. No, because the degree one of the equations is not 1.

11. If you will graph the system $\begin{cases} y = 3x - 2 \\ 2x + y = 8 \end{cases}$, then what is the point of intersection of the two graphs?

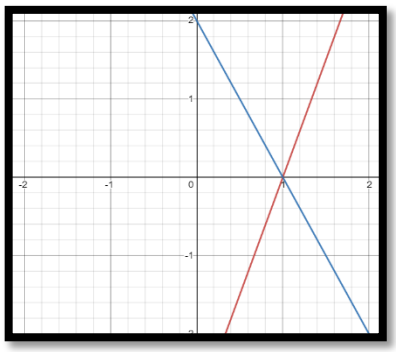
A. (2,4) B. (4,2) C. (3, -2) D. (-6, -20)

12. Which system of equation has intersecting graphs?

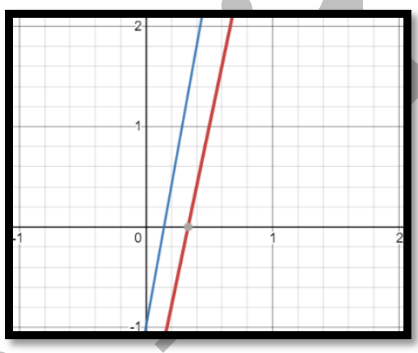
A. $\begin{cases} x + y = 3 \\ x + y = 2 \end{cases}$ B. $\begin{cases} 3x - 2y = 5 \\ 6x - 4y = 10 \end{cases}$ C. $\begin{cases} 2x - y = 2 \\ 3x + y = 2 \end{cases}$ D. $\begin{cases} 3x + y = 2 \\ 3x + y = -2 \end{cases}$

13. Which of the following is the graph of the system $\begin{cases} 3x - y = 3 \\ 2x + y = 2 \end{cases}$?

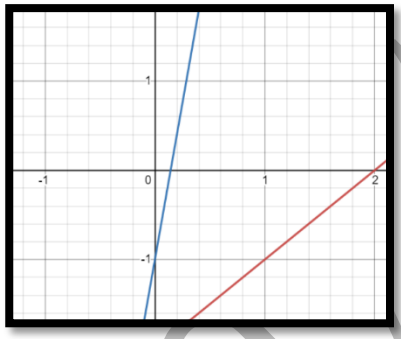
A.



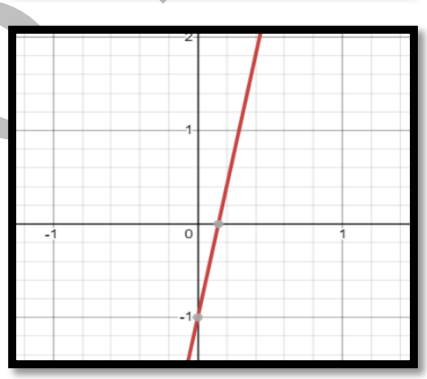
B.



C.



D.



14. Which system of equation has coinciding graphs?

A. $\begin{cases} 2x + 5y = 10 \\ 2x + 5y = -10 \end{cases}$ B. $\begin{cases} 2x - 3y = 5 \\ x + 3y = 1 \end{cases}$ C. $\begin{cases} x + y = -2 \\ 3x = 3y = -6 \end{cases}$ D. $\begin{cases} 5x - 2y = 7 \\ 3x + 2y = 9 \end{cases}$

15. Alyssa was asked by her Mathematics teacher to graph a system of linear equations in two variables. After following all the steps in solving linear equations in by graphing, she was able to draw lines that are parallel to each other. Which of the following can Jane conclude about the solutions of the system?

A. It has no solution. B. It has one solution.

C. It has two solution. D. It has infinitely many solutions.

Lesson 1 Graphing System of Linear Equations in Two Variables

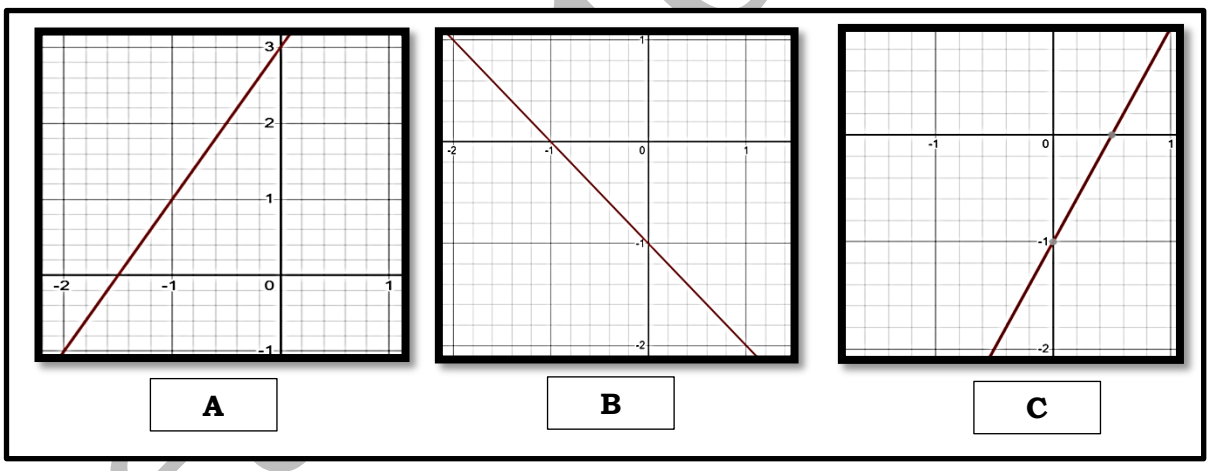
Have you ever asked yourself how our hardworking farmers increase their yiled or harvest? How our wealthy businessmen make profits? How our wise parents budget their income on food, education, clothing and other needs? How mobile phone user wisely choose the best payment plan? Find out the answers to these questions and determine the vast applications of systems of linear equations in two variables through this module.

Jumpstart

Activity 1: Describe Me!

Direction: Graph each of the following linear equations in a Cartesian Coordinate plane. Choose the graph on the box below that corresponds to your answer. Write the letter of the correct answer on the space provided before each number.

_____ **1.)** $x + y = -1$ _____ **2.)** $2x - y = 1$ _____ **3.)** $y = 2x + 3$



Activity 2: Change My Form

Direction: Transform each pair of equations into the slope-intercept from ($y = mx + b$) and identify the slope (m) and y-intercept (b). Write your answers on a separate sheet of paper.

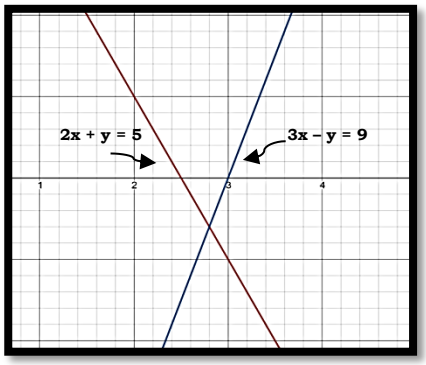
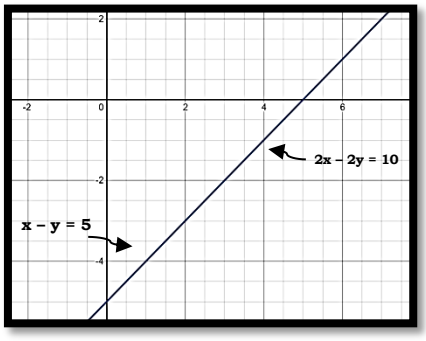
Given	Slope-intercept Form ($y = mx + b$)	Slope (m)	y-intercept (b)
1. $x + y = 5$ and $x - y = 1$			
2. $3x + 2y = 6$ and $6x + 4y = 12$			

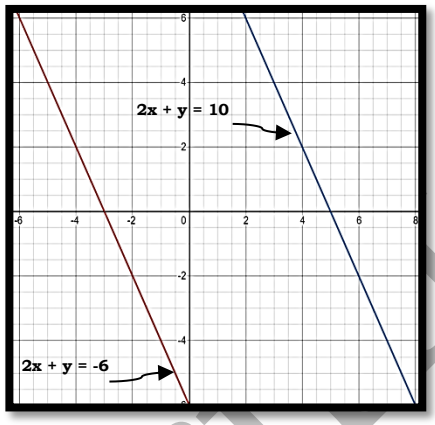
Discover

The equations like x + y = 12 and 3x – y = 2 are called **simultaneous linear equations** or a **system of linear equations** if we want them to be true or the same pairs of numbers. A solution of such equations is an ordered pair of numbers that satisfies both equations. The solution set of a system of linear equations in two variables is the set of all ordered pairs of real numbers that makes every equation in the system true.

The solution of a system of linear equations can be determined algebraically or graphically. To find the solution graphically, graph both equations on a Cartesian plane then find the point of intersection of the graphs, if it exists. The solution of a system of linear equations corresponds to the coordinates of the points of intersections of the graphs of the equation.

There are three kinds of systems of linear equations in two variables according to the number of solutions. The table below summarizes the possible solutions to systems of linear equations.

Type of System	**Number of Solutions**	**Graphs of Equation**	**Example**
Consistent and independent	Exactly one	Intersecting lines *The **slopes** of the lines defined by the equations are **not equal**; their **y-intercepts** could be **equal or unequal**.*	$\begin{cases} 2x + y = 5 \\ 3x - y = 9 \end{cases}$ 
Consistent and dependent	Infinitely many	Coinciding *The **slopes** of the lines defined by the equations are **equal**, their **y – intercepts** are also **equal.***	$\begin{cases} x - y = 5 \\ 2x - 2y = 10 \end{cases}$ 

Inconsistent	None	Parallel lines *The **slopes** of the lines defined by the equations are **equal**, their **y-intercepts** are **not equal**.*	$\begin{cases} 2x + y = -6 \\ 3x - y = 9 \end{cases}$ 

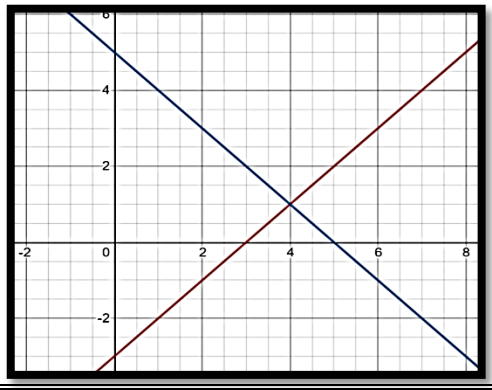
Based on the given graphs of system of linear equations in two variables, we can conclude that:

a. Only **one solution** if the **slopes of the equations are not equal,** that is, $m_1 \neq m_2$;
b. **No solution** if the **slopes of the equations are equal but the y – intercepts are not equal;** that is $m_1 = m_2$, $b_1 \neq b_2$; and
c. **Infinitely many solutions,** if the **slopes are equal and the y-intercepts are equal,** that is, $m_1 = m_2$ and $b_1 = b_2$

Recall that several methods in graphing linear equations in two variables such as the use of any two points, the x and y intercepts, the slope and a point were introduced. In this module, the focus is on the use of the slope and y-intercept to graph the systems of linear equations in two variables.

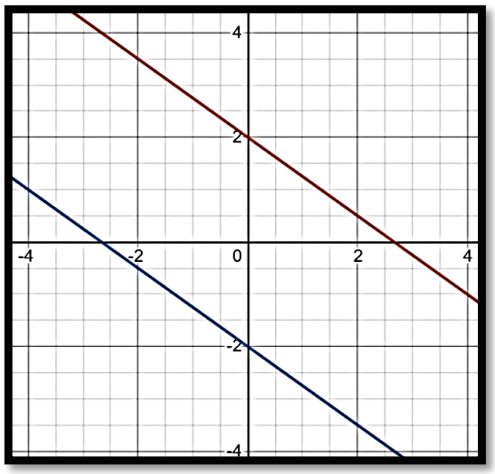
To help you understand better, try to explore and understand the following examples:

1. $\begin{cases} x - y = 3 \\ x + y = 5 \end{cases}$

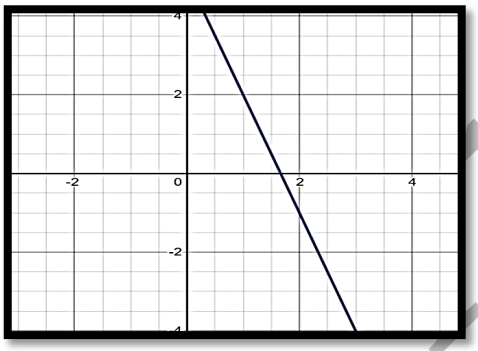
STEP 1	STEP 2	STEP 3
Transform each equation into intercept form. **For x – y = 3;** Set y = 0 Set x = 0 x – y = 3 x – y = 3 x – 0 = 3 0 – y = 3 x = 3 y = -3 x-intercept: 3 y – intercept: - 3 **For x + y = 5** Set y = 0 Set x = 0 x + y = 5 x + y = 5	***Graph each equation in a Cartesian Plane and label the graph. Use the intercept method.*** 	***Identify the point of intersection and test whether it satisfies both the equations.*** The lines intersect at a single point which appears to be (4,1). We can verify that it is the solution by substituting it into the given equations.

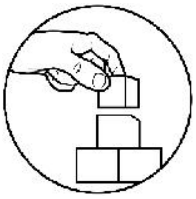
x + 0 = 5 0 + y = 5 x = 5 y = 5 x- intercept: 5 y – intercept: 5	The lines intersect at a single point which appears to be (4,1). We can verify that it is the solution by substituting it into the given equations.	Using the points (4,1), let x = 4 and y =1. x – y = 3 x + y = 5 4 – 1 = 3 4 + 1 = 5 3 $\stackrel{\checkmark}{=}$ 3 5 $\stackrel{\checkmark}{=}$ 5 True True Since substituting the point (4,1) to both equations gives a true statement, then (4,1) is **a solution to the system.**

2. $\begin{cases} 3x + 4y = 8 \\ 15x = -40 - 20y \end{cases}$

STEP 1	STEP 2	STEP 3
Transform each equation into slope-intercept form (y = mx + b) **For 3x + 4y = 8;** 4y = - 3x + 8 y = $-\frac{3}{4}x + 2$ Therefore, Slope(m) = $-\frac{3}{4}$ y – intercept(b) = 2 **For 15x = - 40 – 20y;** 20y = - 15x – 40 y = $-\frac{15}{20}x - 2$ y = $-\frac{3}{4}x - 2$ Therefore, Slope (m) = $-\frac{3}{4}$ y – intercept (b) = -2	***Graph each equation in a Cartesian Plane and label the graph. Use the intercept method.*** The lines appear to be parallel. This can be verified by comparing their slopes and y – intercept. The slopes of 3x + 4y = 8 and -15x = -40 – 20y are both equal to - $\frac{3}{4}$. Their y – intercepts are different.  	***Identify the point of intersection and test whether it satisfies both the equations.*** Since their slopes are **equal** and their y – intercepts are **different**, the lines are **parallel**. This means that the system is **inconsistent** and has **no solution.**

3. $\begin{cases} 6x + 2y = 10 \\ y = -3x + 5 \end{cases}$

STEP 1	STEP 2	STEP 3
Transform each equation into slope-intercept form (y = mx + b) **For 6x + 2y = 10;** 2y = - 6x + 10 y = - 3x + 5 Therefore, slope = - 3 y – intercept = 5 **For y = -3x + 5** Therefore, slope = -3 y – intercept = -5	***Graph each equation in a Cartesian Plane and label the graph. Use the intercept method.*** The equations are the same, so they are dependent. There is an infinite number of solutions. 	***Identify the point of intersection and test whether it satisfies both the equations.*** Since their slopes and y – intercepts are **equal.** The lines **coincide**. This means that the system of equation is **consistent** and **dependent.**



Explore

Activity 3: Relationship Check!

Directions: Identify the slope (m) and y – intercept (b) of each system of linear equations. Write the symbol **=** if they have the same value of slopes and y – intercepts. Otherwise, write the symbol ≠ if their slopes and y – intercepts are different.

1. $\begin{cases} x + 2y = 7 \\ x - y = 4 \end{cases}$

m_1 _____ m_2

b_1 _____ b_2

2. $\begin{cases} x + y = 2 \\ x + y = 4 \end{cases}$

m_1 _____ m_2

b_1 _____ b_2

3. $\begin{cases} 4x + 2y = 8 \\ 6x + 3y = 12 \end{cases}$

m_1 _____ m_2

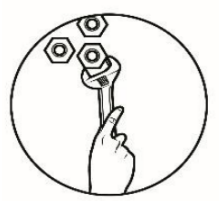
b_1 _____ b_2

Activity 4: What's our Label?

Directions: Examine each system in Column 1 and choose whether the graphs are **intersecting, parallel** and **coinciding** by putting a check mark in Column 2.

EQUATIONS	GRAPHS		
1. $x + y = 6$ $2x + y = 4$	☐ INTERSECTING	☐ PARALLEL	☐ COINCIDING
2. $3x - 2y = 6$ $3x - 2y = 4$	☐ INTERSECTING	☐ PARALLEL	☐ COINCIDING
3. $3x - 6y = 12$ $4x - 8y = 16$	☐ INTERSECTING	☐ PARALLEL	☐ COINCIDING

Now that you know the important ideas about the topic, let us go deeper by moving on to the next section.

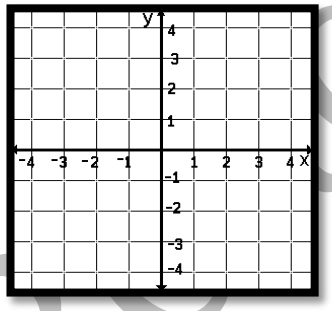


Deepen

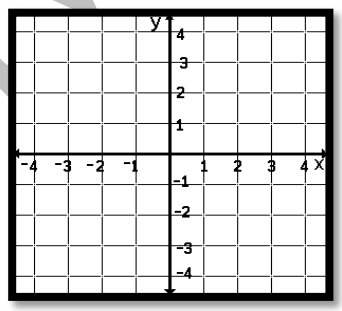
Activity 5: Show me!

Directions: Graph each system of equations and state whether the system is **consistent and dependent, consistent and independent,** or **inconsistent**. Use a graphing paper.

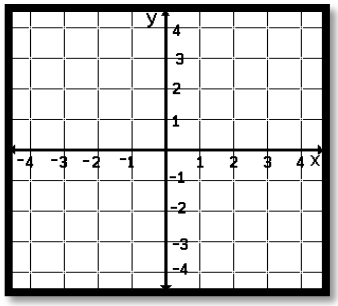
1. $x + 3y = 4$
   $x + 2y = 3$



2. $3x - 9y = 12$
   $x - 3y = 4$



3. $x + y = 6$
   $2x + y = 4$



Activity 6: Story of my Life

Directions: Write a situation in your life which can be represented by systems of linear equations in two variables. Your output will be rated using the following rubrics:

3	2	1
The situation is clear, realistic and the use of the system of linear equations in two variables is properly illustrated	The situation is clear, and the use of the system of linear equations in two variables is not properly illustrated	The situation is not clear, and the use of the system of linear equations in two variables is not properly illustrated

Gauge

Assessment: Post-Test

Direction: Find out how much have you learned from the lesson. Choose the letter which you think best answer to the question. Write your answer in a separate sheet of paper.

1. All of the following are the graphs of the system of linear equations in two variables **EXCEPT**:
   A. coinciding B. intersecting C. parallel D. perpendicular
2. Which of the following is **NOT** a system of linear equations in two variables?
   A. $\begin{matrix} x-y+5 \\ 3x^2=0 \end{matrix}$ B. $\begin{matrix} 3x-y=5 \\ x-11y=-1 \end{matrix}$ C. $\begin{matrix} 4x-8y=12 \\ 2x-\frac{1}{3}y=-12 \end{matrix}$ D. $\begin{matrix} x-4y=5 \\ \frac{3}{5}x-4y=7 \end{matrix}$
3. What system of linear equations in two variables is equivalent to $\begin{matrix} y=\frac{1}{2}x-4 \\ y=4x-5 \end{matrix}$?
   A. $\begin{matrix} x-2y=8 \\ 4x-y=5 \end{matrix}$ B. $\begin{matrix} 4x-y=-2 \\ x+2y=-5 \end{matrix}$ C. $\begin{matrix} x-y=8 \\ 4x-2y=5 \end{matrix}$ D. $\begin{matrix} x+2y=-8 \\ x-4y=-5 \end{matrix}$
4. Which of the following ordered pairs satisfies the system $\begin{matrix} x+4y=-1 \\ x+2y=1 \end{matrix}$?
   A. (-3,1) B. (-3,-1) C. (3,1) D. (3,-1)
5. Which of the following equations can be paired with $6x-3y=24$ to make a consistent and dependent system?
   A. $y=2x-8$ B. $y=2x+8$ C. $y=6x+24$ D. $y=6x-24$

6. Which of the following is a system of a linear equations in two variables?

A. $\begin{matrix} -x = 5 \\ y - 4y = 4 \end{matrix}$ B. $\begin{matrix} 3x - 2y = 4 \\ 4x = -3 \end{matrix}$ C. $\begin{matrix} 3 - y = 8 \\ 6x + y = 6 \end{matrix}$ D. $\begin{matrix} x + 4 = y \\ 2x - 2y = -8 \end{matrix}$

7. All of the following ordered pairs satisfy the system $\begin{matrix} 2x - y = 4 \\ 4x - 2y = 8 \end{matrix}$ **EXCEPT:**

A. (0,-4) B. (2,0) C. (4,4) D. (0,2)

8. Which of the following statements is true about the system $\begin{matrix} 2x - y = 4 \\ 4x - 2y = 8 \end{matrix}$?

A. There is an ordered pair (2,0) that satisfies both equations.
B. There are infinitely many ordered pair that satisfies both equations.
C. The slope of the first equation is equal to the slope of the second equation.
D. The slope and y-intercept of the first equation and second equation are equal.

9. Which statement is true about the system $\begin{matrix} x + y = 15 \\ 3x + 3y = -10 \end{matrix}$?

A. There is one ordered pair that will both equations.
B. There is no ordered pair that will satisfy both equations.
C. There are infinitely many ordered pairs that will satisfy both equations.
D. Equation 2 is obtained by multiplying each term in equation 1 by 3.

For items 10-12. Use the system $\begin{cases} x + y = 2 \\ x - y = 4 \end{cases}$ to answer the questions that follow.

10. What are the slope-intercept form of each equation?

A. $\begin{matrix} y = -x + 2 \\ y = -x + 4 \end{matrix}$ B. $\begin{matrix} y = -x + 2 \\ y = x + 4 \end{matrix}$ C. $\begin{matrix} y = -x + 2 \\ y = -x - 4 \end{matrix}$ D. $\begin{matrix} y = x - 2 \\ y = x + 4 \end{matrix}$

11. What are the slopes and y-intercepts of each equation?

A. $\begin{cases} m_1 = 1;\ b_1 = -2 \\ m_2 = 1;\ b_2 = 4 \end{cases}$ B. $\begin{cases} m_1 = -1;\ b_1 = 2 \\ m_2 = 1;\ b_2 = -4 \end{cases}$ C. $\begin{cases} m_1 = -1;\ b_1 = 2 \\ m_2 = 1;\ b_2 = 4 \end{cases}$ D. $\begin{cases} m_1 = -1;\ b_1 = 2 \\ m_2 = -1;\ b_2 = 4 \end{cases}$

12. What is the point of intersection of the graph?

A. (3, -1) B. (3,1) C. (-3,-1) D. (-3,1)

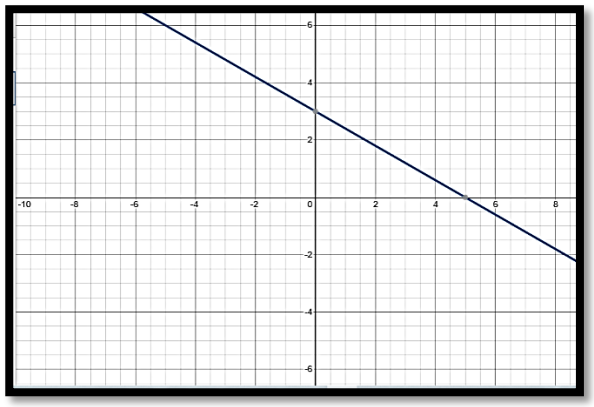
13. Which system of linear equations represent the graph below?

A. $\begin{matrix} 3x - 5y = 15 \\ 6x + 5y = -30 \end{matrix}$

B. $\begin{matrix} 3x + 5y = 15 \\ 6x + 10y = 30 \end{matrix}$

C. $\begin{matrix} -3x - 5y = 15 \\ 6x - 10y = 30 \end{matrix}$

D. $\begin{matrix} 3x + 5y = -15 \\ 6x - 10y = 30 \end{matrix}$



14. Rocky says that the system $\begin{matrix} x+y=10 \\ 3x-y=2 \end{matrix}$ has exactly one solution. Which of the following reasons would support his statement?

I. The graph of the system intersects at (3,7).
II. The graph of the system shows intersecting lines.
III. The graph of the system shows coinciding lines.
IV. The graph of the system shows parallel lines.

A. I only
B. I and II
C. III
D. IV

15. Jeremy Jake was tasked by his teacher to find an ordered pair that satisfies the given system $\begin{cases} 6x-5y=-2 \\ x+2y=-11) \end{cases}$. His answer is point (3,4). Is he correct?

A. Yes, because he substituted the variable of $x = 3$ *and* $y = 4$ to the system of linear equation.
B. Yes, because he yielded two true statements that makes a solution of the system of linear equations.
C. No, because he did not follow the steps in evaluating system of linear equation
in two variables.
D. No, because the point (3,4) does not satisfy the given system of linear equations in two variables.

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