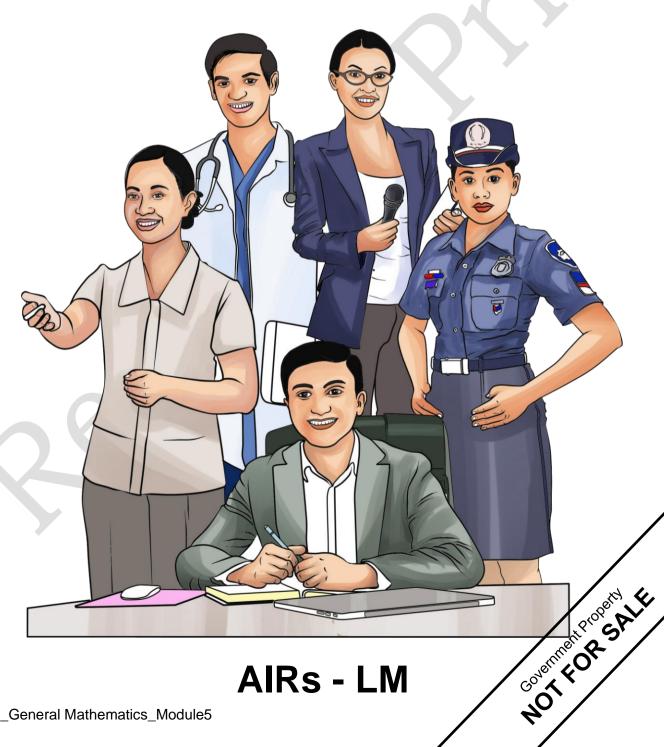


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# RECION DIVIS General Mathematics Module 5: **Exponential Functions, Equations** and Inequalities



AIRs - LM

#### **GENERAL MATHEMATICS**

Module 5: Exponential Functions, Equations and Inequalities Second Edition, 2021

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# SHS

# General Mathematics Module 5: Exponential Functions, Equations and Inequalities



# **Introductory Message**

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge of lessons in each SLM. This will tell you if you need to proceed with completing this module or if you need to ask your facilitator or your teacher's assistance for a better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



In 1683, Jacob Bernoulli discovered the approximate value of the irrational number *e* through a study of compound interest and Johann Bernoulli began the study of the calculus of the exponential function in 1697. (Robertson, 2001)

An exponential function is defined as "a mathematical function in which an independent variable appears in one of the exponents." It is used to define the natural exponential function  $e^x$ , where e is Euler's number approximately equal to 2.718281828...

An exponential equation is an equation in which a variable occurs in the exponent and an exponential inequality is an inequality involving exponential expressions.

In the previous lessons about functions, you were able to learn about Rational Functions, Rational Equations, and Rational Inequalities as well as One-to-One Functions and their Inverse. In this learning material, you will be dealing with Exponential Functions, Exponential Equations, and Exponential Inequalities.

After going through this module, you are expected to:

- 1. represent real-life situations using exponential functions (M11GM-1e-3),
- 2. distinguish among exponential function, exponential equation, and exponential inequality (M11GM ie-4); and
- 3. solve exponential equations and inequalities (M11GM-le-f-1).

#### Learning Objectives:

- 1. name and illustrate the most common applications of exponential functions in real life
- 2. identify exponential functions, exponential equations, and exponential inequalities
- 3. enumerate the steps in solving exponential equations and exponential inequalities
- 4. solve problems involving exponential equations and exponential inequalities

Before going on, check how much you know about this topic by answering the pretest on the next page on a separate sheet of paper.

#### **Pretest**

**Directions:** Read carefully each item. Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

For item numbers 1-3. Use the problem below:

Suppose that a colony of lice grows exponentially. There are 50 lice initially when its population doubles every 6 hours.

1. What is the correct exponential model of the situation?

A. 
$$y = 50(2)^{\frac{t}{6}}$$

B. 
$$y = 50(2)^{\frac{6}{t}}$$

C. 
$$y = 6(2)^{\frac{t}{50}}$$

D. 
$$y = 6(2)^{\frac{50}{t}}$$

2. How many lice are there after 18 hours?

C. 300

D. 400

3. How many lice are there after 2 days?

C. 9,600

For item numbers 4-5. Use the problem below:

Suppose that the half-life of substance K is 50 days and there are initially 2000g.

4. What is the correct exponential model of the situation?

A. 
$$y = 2000 \left(\frac{1}{2}\right)^{\frac{t}{50}}$$

B. 
$$y = 2000 \left(\frac{1}{2}\right)^{\frac{50}{t}}$$

C. 
$$y = 2050 \left(\frac{1}{2}\right)^{\frac{t}{400}}$$

B. 
$$y = 2000 \left(\frac{1}{2}\right)^{\frac{50}{t}}$$
  
D.  $y = 2050 \left(\frac{1}{2}\right)^{\frac{400}{t}}$ 

5. How many grams are left in substance K after 100 years?

6. Which of the following terms is associated to  $f(x) = b^x$ ?

7. What does an expression of the form  $a \cdot b^{x-c} + d$  called?

8. What symbol represents the equality of two terms, values, or expressions?

9. Which of the following is an example of exponential inequality?

A. 
$$3^{2x+1} = 729$$

B. 
$$5^{3x} = 25^{2x-1}$$

$$C. \left(\frac{1}{4}\right)^{3X} \le 16^{X-1}$$

D. 
$$\left(\frac{1}{2}\right)^{x} = 81$$

- 10. For the function  $f(x) = b^x$ , what does **b** stands for?
  - A. Base

C. Input

- D. Output
- 11. What is the first step in solving exponential equations?
  - A. Solve the equation

B. Make the base the same

C. Equate the exponents

D. Use trial and error method

- 12. What is the value of x in the expression  $3^{x+2} = \frac{1}{27}$ ?
  - A. x = -5

B.  $x = \frac{2}{3}$ 

C.  $x = \frac{3}{2}$ 

- D. x = 3
- 13. Which of the following is the solution set of  $25^{x+6} > 125$ ?
  - A. x > -3

B.  $x > -\frac{9}{2}$ 

C.  $x > \frac{2}{9}$ 

- D. x > 2
- 14. When a diesel-electric generator is switched off, the electricity dies away according to the formula  $I(t) = 24(0.25)^t$  amperes, where t is the time in seconds. To have a 0.375 ampere of electricity, how long should be the time elapsed?
  - A. 1 second

B. 2 seconds

C. 3 seconds

- D. 4 seconds
- 15. The weight of a radioactive substance t years after being set aside is given by  $W(t) = 250(0.04)^t$  grams. How many years will you set aside a substance to make its weight be 2 grams?
  - A.  $\frac{2}{3}$

B.  $\frac{3}{2}$ 

C. 3

D. 4



Exponential functions occur in various real-world situations. It is used to model real-life situations such as population growth, carbon dating, growth of an epidemic, loan interest rates, and investments.

This activity will help you to learn the concept of exponential functions.

#### **Activity 1: Fold Me, Count Me!**

**Directions**: Get a whole sheet of paper and follow the steps carefully. Write your answer on the table below for every step you have done.

| No. of<br>Folds | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|-----------------|---|---|---|---|---|---|---|
| No. of          |   |   |   |   |   |   |   |
| Sections        |   |   |   |   |   |   |   |

- Step 1 Count the number of sections created by folds in your paper. (The number of folds is zero)
- Step 2 Fold your paper in half. Make sure to make a crease.
- Step 3 Unfold the paper.
- Step 4 Count the number of sections created by folds in your paper. (The number of folds is 1)
- Step 5 Fold the paper back in half. Then fold it in half again.
- Step 6 Unfold the paper.
- Step 7 Count the number of sections created by folds in your paper. (The number of folds is 2)
- Step 8 Repeat the process until you have 6 folds.
- Step 9 After you fill-up the table, answer the following questions below.

#### Questions:

- a. What did you observe in the number of sections as the number of folds in the paper increases?
- b. Based on the knowledge that you had learned when you were in junior high, what formula can you define to the number of sections as the number of folds increases?

4

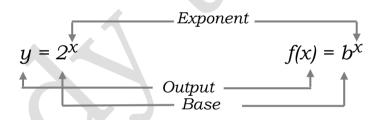


The table below shows the number of sections created. After folding the paper six times, you will be having a total of 64 sections.

| No. of<br>Folds    | 0 | 1 | 2 | 3 | 4  | 5  | 6  |
|--------------------|---|---|---|---|----|----|----|
| No. of<br>Sections | 1 | 2 | 4 | 8 | 16 | 32 | 64 |

The pattern of change in folding the paper in **Activity 1** can be modeled using rules involving exponents. The number of sections created is: 1, 2, 4, 8, 16, 32, 64 ... can be expressed exponentially as:  $2^0$ ,  $2^1$ ,  $2^2$ ,  $2^3$ ,  $2^4$ ,  $2^5$ ,  $2^6$  ..., respectively.

The number of sections is described by the exponential function  $f(x) = 2^x$  or  $y = 2^x$  where **y** represents the number of sections created and **x** represents the number of folds. It can be used to calculate the number of sections created without starting from the first stage.



An exponential function can be written as

$$f(x) = b^x$$

where b > 0,  $b \ne 1$ , and x is any real number.

In the equation  $f(x) = b^x$ , b is a constant called the **base** and x is an independent variable called the **exponent**.

Here are some examples of exponential functions.

1. 
$$f(x) = 5^x$$

$$2. g(x) = 12^x$$

3. 
$$h(x) = 3^{x+1}$$

The following are **not** exponential functions.

1. 
$$F(x) = x^2$$
 2.  $G(x) = 1^x$  3.  $H(x) = x^x$ 

Variable is the The base is a positive base and not constant equal to 1. base and the exponent. exponent are variables.

Exponential Functions can be found in many real-life situations such as growth in population like bacteria and ants, radioactive decay of certain chemical substances, compound interest of loans, and investments.

#### **Exponential Growth**

Scientist often studies bacteria, animals, viruses, observe their behaviors and watch them how their population grows. For example, there's a bacterium in a plate and its population grows every 2 hours, this kind of behavior can be represented using an exponential function.

**Example 1**: Let t = time in days. At t = 0, there were 10 initial bacteria in a specific mug. Suppose that the bacteria double every 20 hours. Give an exponential model for the bacteria as a function of t.

#### Given:

Initially, at t = 0, the number of bacteria is 10.

#### Solution:

| t= 0 | Number of bacteria =10              |   |
|------|-------------------------------------|---|
| t=20 | Number of bacteria $=10(2)^1 = 20$  | where the power of 1 came from (t = 20)/(one cycle=20)    |
| t=40 | Number of bacteria = $10(2)^2 = 40$ | where the power of 2 came from $(t = 40)$ /(one cucle=20) |
| t=60 | Number of bacteria = $10(2)^3 = 80$ | where the power of 3 came from (t = 60)/(one cycle=20)    |

An exponential model for this situation is  $y = 10(2)^{t/20}$ .

# **Exponential Models for Population Growth**

$$y = a_1(2)^{t/T}$$

Where:

y is the quantity that doubles every period of time

 $a_1$  is the initial amount when time is at rest

*t* is the units of time is given

*T* is every unit of time that had been passed when the quantity has doubled.

#### **Exponential Decay (Half-Life)**

The **half-life** of a substance is the time it takes for half of the substance to decay.

**Example 2**: Suppose that the half-life of substance A is 50 days and there are initially 100g, determine the amount of substance A remains after 100 days.

#### Solution:

Initially, at t = 0 amount of substance A = 100g

t = 0Amount of substance A = 100

$$t = 50$$
 Amount of substance  $A = 100 \left(\frac{1}{2}\right)^1 = 50$ 

where the power of 1 came from  $(t = 50)/(one\ cycle = 50)$ 

$$t = 100$$
 Amount of substance  $A = 100 \left(\frac{1}{2}\right)^2 = 50$  where the power of 2 came from  $(t = 100)/(one\ cycle = 50)$ 

An exponential model for this situation is  $y = 100 \left(\frac{1}{2}\right)^{t/50}$ .

#### **Exponential Models for Population Decay**

$$y = a_1 \left(\frac{1}{2}\right)^{t/T}$$

Where:

y is the substance remaining after t units of time

 $a_1$  is the initial amount when time is at rest

t is the units of time is given

*T* is every unit of time that had been passed when the quantity has doubled.

#### **Compound Interest**

A starting amount of money (called the **principal**) can be invested at a certain interest rate that is earned at the end of a given period (such as one year). If the interest rate is **compounded**, the interest earned at the end of the period is added to the principal, and this amount will earn interest in the next period. The same process is repeated for each succeeding period: interest previously earned will also earn interest in the next period.

**Example 3**: Mr. Halog saves his money amounting to Php 50,000 in Landbank that offers 6% interest compounded annually. How much savings does he have after 3 years?

#### Given:

Initial savings is P50,000

Interest rate = 6% = 0.06

#### Solution:

at 
$$t = 1$$
 savings =  $50,000(1.06)^1 = Php 53,000$ 

where the 1.06 came from 1+r where r = 6% = 0.06

at 
$$t = 2$$
 savings =  $Php 50,000(1.06)^2 = Php 56,180$ 

at 
$$t = 3$$
 savings=  $P50,000(1.06)^3 \approx Php 59,550.80$ 

The exponential model for this situation is  $y = 50000(1.06)^t$ .

#### **Compound Interest**

$$y = P(1+r)^t$$

Where:

*y* is the amount after t years

P is the starting amount of money

r is the amount of interest

*t* is the units of time is given

Going back to **Activity 1**, we have the following table which shows the number of sections created after folding a paper six times.

| No. of<br>Folds    | 0 | 1 | 2 | 3 | 4  | 5  | 6  |
|--------------------|---|---|---|---|----|----|----|
| No. of<br>Sections | 1 | 2 | 4 | 8 | 16 | 32 | 64 |

An equation can be formed and can be used to calculate the number of sections created without starting from the first stage. If we let  $\mathbf{y}$  represent the number of sections created and  $\mathbf{x}$  the number of folds, our equation will be  $y = 2^x$ , which is an **exponential function**, where  $\mathbf{x}$  is the independent variable and  $\mathbf{y}$  is the dependent variable.

Note that in the equation formed, the variable **x** appears as an exponent. We can equate the data from the second row of our table as  $1 = 2^x$ ,  $2 = 2^x$ ,  $4 = 2^x$ ,  $8 = 2^x$ , ...,  $64 = 2^x$ . We call each of the listed equations an **exponential equation**.

With the exponential function and exponential equation in mind, we can already define an *exponential inequality* as a form having any among the inequality symbols <, >,  $\le$ ,  $\ge$  and  $\ne$ , in which  $\mathbf{x}$  appears in the exponent.

An **exponential equation** is an equation in which the independent variable occurs in the exponent.

The following equations are examples of exponential equations.

1. 
$$5^x = 125$$

$$2. 12^{x} = 144$$

$$3.3x+1 = 81$$

$$4.4^{2x} = 256$$

$$5. 3^{2x-1} = 243$$

$$6.2^{x-1} = 32$$

Let us know more about the different steps in evaluating exponential equations.

**Example 1:** Find the value of x in the equation  $4^{x+1} = 64$ .

#### Solution:

**Step 1:** Make the bases the same.

$$4^{x+1} = 4^3$$

**Step 2:** Copy the exponents and equate them.

$$x + 1 = 3$$

**Step 3:** Solve the resulting equation.

$$x + 1 = 3$$

$$x = 3 - 1$$

$$x = 2$$

**Step 4:** Check if the obtained value satisfies the given equation.

$$4^{2+1} = 64$$

$$64 = 64$$

Therefore, x = 2.

Some exponential equations can be solved by using the fact that exponential functions are one-to-one.

# One-to-one Property of Exponential Functions

If  $x_1 \neq x_2$ , then  $b^{x_1} \neq b^{x_2}$ . Conversely, if  $b^{x_1} = b^{x_2}$  then  $x_1 = x_2$ .

**Example 2:** Solve the equation  $81^{x-1} = 27^{x+3}$ 

#### Solution:

Both 81 and 27 can be written using 3 as the base.

$$(3^4)^{x-1} = (3^3)^{x+3}$$

$$3^{4(x-1)} = 3^{3(x+3)}$$

$$4(x - 1) = 3(x + 3)$$

$$4x - 4 = 3x + 9$$

$$4x - 3x = 9 + 4$$

$$x = 13$$

**Example 3:** Solve the equation  $4^{x^2} = 2^{x+3}$ 

#### Solution:

Both 4 and 2 can be written using 2 as the base.

$$(2^2)^{x^2} = 2^{x+3}$$

$$2^{2(x^2)} = 2^{x+3}$$

$$2x^2 = x + 3$$

$$2x^2 - x - 3 = 0$$

$$(2x-3)(x+1)=0$$

$$2x - 3 = 0$$
 or  $x + 1 = 0$ 

$$x = \frac{3}{2}$$
 or  $x = -1$ 

An **exponential inequality** is an inequality involving exponential forms where x is in the exponent.

The following are examples of exponential inequalities. 1.  $36^x \ge 6^{x+1}$  2.  $10^{5x-3} > 10000$  3.  $\left(\frac{1}{4}\right)^{3x+5} \le \left(\frac{1}{32}\right)^{x-6}$ 

1. 
$$36^x \ge 6^{x+1}$$

$$2. \ 10^{5x-3} > 10000$$

$$3. \left(\frac{1}{4}\right)^{3x+5} \le \left(\frac{1}{32}\right)^{x-6}$$

4. 
$$\left(\frac{5}{3}\right)^n > \frac{9}{25}$$
 5.  $8^{x-2} < 16$ 

$$5.8x-2 < 16$$

6. 
$$49^x \ge 343$$

Let us know more about the different steps in evaluating exponential inequality.

**Example 1:** Find the possible values of x in  $3^{3x+1} > 243$ .

#### Solution:

**Step 1:** Make the bases the same.

$$3^{3x+1} > 243$$

$$3^{3x+1} > 3^5$$

**Step 2:** Copy the exponents and the relational symbol.

$$3x + 1 > 5$$

**Step 3:** Solve the resulting inequality.

$$3x > 5 - 1$$

$$\frac{3x}{3} > \frac{4}{3}$$

$$x > \frac{4}{3}$$

**Step 4:** Test a value to check if it satisfies the given inequality.

$$x > \frac{4}{3}$$

Let  $x = \frac{5}{3}$  (since this value is greater than  $\frac{4}{3}$ )

$$3^{3\left(\frac{5}{3}\right)+1} > 243$$

$$3^{5+1} > 243$$

Thus, the solution set is  $x > \frac{4}{3}$ .

Exponential inequalities can be solved using the following property.

#### **Property of Exponential Inequalities**

If b > 1, then the exponential function  $y = b^x$  is increasing for all x. This means that  $b^x < b^y$  if and only if x < y.

if 0 < b < 1, then the exponential function  $y = b^x$  is decreasing for all x. This means that  $b^x > b^y$  if and only if x < y.

You should be careful when solving exponential inequalities such as  $b^m < b^n$ . The resulting direction of the inequality (m < n or m > n) is base whether the base b is greater than 1 or less than 1.

**Example 2:** Solve the inequality  $3^x < 9^{x-2}$ .

#### Solution:

Both 3 and 9 can be written using 3 as the base.

$$3^x < (3^2)^{x-2}$$

$$3^x < 3^{2(x-2)}$$

$$3^x < 3^{2x-4}$$

Since the base 3 > 1, then this inequality is equivalent to

x < 2x - 4 (the direction of the inequality is retained)

$$4 < 2x - x$$

Thus, the solution set is  $(4, +\infty]$ .

(You can verify that x = 5 and 6 are solutions, but x = 4 and 3 are not)

$$x = 5$$

$$x = 6$$

$$x = 4$$

$$x = 3$$

$$3^x < 9^{x-2}$$
  $3^x < 9^{x-2}$   $3^x < 9^{x-2}$   $3^x < 9^{x-2}$ 

$$3^x < 9^{x-1}$$

$$2x - 0x - 2$$

$$3^x < 9^{x-2}$$

$$3^5 < 9^{5-2}$$
  $3^6 < 9^{6-2}$   $3^4 < 9^{4-2}$   $3^3 < 9^{3-2}$ 

$$3^6 < 9^{6-2}$$

$$3^4 < 9^{4-2}$$

$$3^3 < 9^{3-2}$$

$$3^{\circ} < 9^{\circ}$$

$$3^5 < 9^3$$
  $3^6 < 9^4$   $3^4 < 9^2$   $3^3 < 9^1$ 

$$3^4 < 9^2$$

$$23 - 01$$

$$243 < 729$$
  $\checkmark$   $729 < 6561$   $\checkmark$   $81 < 81$   $\times$   $27 < 9$   $\times$ 

**Example 3:** Solve the inequality  $\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x}$ .

#### Solution:

Since  $\frac{1}{25} = \left(\frac{1}{5}\right)^2$ , then we write both sides of the inequality with  $\frac{1}{5}$  as the base.

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{3x}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{(6x)}$$

Since the base  $\frac{1}{5}$  < 1, then this inequality is equivalent to

 $x + 5 \le 6x$  (the direction of the inequality is reversed)

$$5 \le 6x - x$$

$$5 \le 5x$$

$$1 \le x$$

Thus, the solution set is  $[1, +\infty)$ .

(You can verify that x = 1 and 2 are solutions, but x = 0 and -1 are not.)

$$x = 1$$

$$x = 2$$

$$x = 0$$

$$x = -1$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x} \qquad \left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x} \qquad \left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x} \qquad \left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{25}\right)^{3x}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)}$$

$$\left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)} \qquad \left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)} \qquad \left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)} \qquad \left(\frac{1}{5}\right)^{x+5} \ge \left(\frac{1}{5}\right)^{2(3x)}$$

$$\left(\frac{1}{5}\right)^{1+5} \ge \left(\frac{1}{5}\right)^{2[3(1)]}$$

$$\left(\frac{1}{5}\right)^{2+5} \ge \left(\frac{1}{5}\right)^{2[3(2)]}$$

$$\left(\frac{1}{5}\right)^{0+5} \ge \left(\frac{1}{5}\right)^{2[3(0)]}$$

$$\left(\frac{1}{5}\right)^{1+5} \geq \left(\frac{1}{5}\right)^{2[3(1)]} \quad \left(\frac{1}{5}\right)^{2+5} \geq \left(\frac{1}{5}\right)^{2[3(2)]} \quad \left(\frac{1}{5}\right)^{0+5} \geq \left(\frac{1}{5}\right)^{2[3(0)]} \quad \left(\frac{1}{5}\right)^{-1+5} \geq \left(\frac{1}{5}\right)^{2[3(-1)]}$$

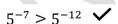
$$\left(\frac{1}{5}\right)^6 \ge \left(\frac{1}{5}\right)^6 \checkmark \qquad \left(\frac{1}{5}\right)^7 \ge \left(\frac{1}{5}\right)^{12}$$

$$\left(\frac{1}{5}\right)^7 \ge \left(\frac{1}{5}\right)^{12}$$

$$0.00032 \ge 1$$
 ×

$$0.00032 \ge 1$$
  $\times$   $\left(\frac{1}{5}\right)^4 \ge \left(\frac{1}{5}\right)^{2[3(-1)]}$ 

$$\frac{1}{625} \ge 15,625$$
 X





Here are some enrichment activities for you to work on to master and strengthen the basic concepts you had learned from this lesson.

## Activity 1: Find My Value.

Directions: Construct the exponential model of every problem. You may use a separate half sheet of paper for your answer.

- 1. At t = 0, there were 30 initial bacteria in dirty clothes. Suppose that the bacteria double every 4 hours. Give an exponential model for the bacteria as a function of t.
- 2. Suppose that the half-life of substance Q is 24 days and there are initially 400g, determine the amount of substance A remains after 96 days.
- 3. Mr. Delos Santos invested his money amounting to Php 80,000 in Robinsons that offers 8% interest compounded annually. How much savings does he have after 6 years?
- 4. Ms. Nica loans Php 200,000 from the Philippine National Bank with an interest of 6.5% compounded annually. How much total money does Philippine National Bank receive after 2 years?
- 5. Philip, a senior high student wants to save his money in Banco de Oro with an interest rate of 9% compounded annually. How much savings does he have after 4 years if he deposits Php 5,000?

# Activity 2: Distinguish the Given

**Directions:** Determine whether each of the given is an exponential equation, an exponential inequality, or an exponential function by placing each of them in their respective column in the table that follows.

A. 
$$3^{x+1} < 81$$

C. 
$$f(x) = 7^{x}$$

C. 
$$f(x) = 7^x$$
  
E.  $4^{2x} = \frac{1}{256}$ 

G. 
$$2^{3x} + 2^{x+1} \ge 0$$

$$I. \left(\frac{1}{9}\right)^{3x-1} \le \left(\frac{1}{81}\right)^{x+5}$$

B. 
$$5^{2x-1} = 125$$

D. 
$$g(x) = 81^{x-2}$$

D. 
$$g(x) = 81^{x-2}$$
  
F.  $h(x) = \left(\frac{1}{9}\right)^{-3x}$ 

H. 
$$6^{2x} - 216^{x-1} = 0$$

| Exponential<br>Equation | Exponential Inequality | Exponential Function |
|-------------------------|------------------------|----------------------|
|                         |                        |                      |
|                         |                        |                      |
|                         |                        |                      |
|                         |                        |                      |

# **Activity 3: Evaluate the Expression**

**Directions:** Evaluate the following exponential equations and inequalities.

A. 
$$4^{x+2} = 8^{2x}$$

B. 
$$81^{2x-3} = 9^{x+2}$$

C. 
$$\left(\frac{1}{4}\right)^{2x} = 4^{3-x}$$

D. 
$$2^{3-x} < 4^{2-x}$$

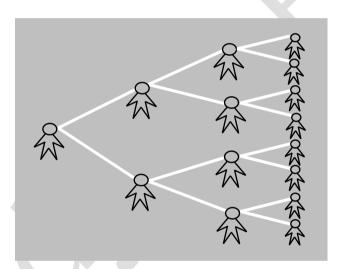
E. 
$$\left(\frac{1}{125}\right)^{-3x-1} \ge 25^{-x-1}$$



# Deepen

In these challenging and difficult times, we all know that God is always there. He is always ready to guide us, to take care of us, and to provide us all the things we need as long as we will ask it to Him diligently and faithfully in our prayers, as the Bible says in Matthew 21:22, "And all things, whatsoever ye shall ask in prayer, believing, ye shall receive."

If the diagram below shows the pattern on how you are going to inform others about this Bible verse, we know that just after the 10<sup>th</sup> stage of the information pattern there will be a total of 2,047 people who will learn and apply it in their daily lives. It just needs a single person, to begin with, and that is nobody but "YOU". (How awesome is that? ©)



At this point, make your artistic tree diagram having the same pattern as the one presented above. Aside from the Bible verse, think of another information that you would like the people in your community to know following the same pattern of sharing presented. Explain also why you want that information to be known by others in the community where you belong by composing a short but substantial essay.

#### What you need

- 1. Pen
- 2. Two Sheets of Papers
- 3. Pencil
- 4. Ruler
- 5. Coloring Materials

#### What you have to do

- 1. Create your artistic tree diagram on a sheet of paper.
- 2. Write your short but meaningful essay on the other sheet of paper.

# **Rubrics for Scoring the Output**

## **Drawing Rubrics**

| Components     | Mastery<br>4              | Accomplished<br>3     | Unacceptable<br>2                 |
|----------------|---------------------------|-----------------------|-----------------------------------|
| Reflection and | The drawing               | The drawing           | The student clearly               |
| Knowledge      | represents a clear        | represents a          | did not understand                |
|                | understanding of the      | somewhat clear        | the information.                  |
|                | information presented.    | understanding of the  | The drawing is not                |
|                | _                         | material.             | representative of                 |
|                |                           |                       | the knowledge                     |
|                |                           |                       | gained.                           |
| Basic Skills   | Drawing exhibits          | The drawing contains  | The drawing lacks                 |
|                | attention to detail and   | many strong elements  | detail. Little effort             |
|                | a variety of drawing      | but needs further     | in work and                       |
|                | techniques                | refinement and fine-  | appears improvised                |
|                |                           | tuning. Only a few    | and looks                         |
|                |                           | drawing techniques    | incomplete. No                    |
|                |                           | were used.            | variety in drawing                |
|                |                           | m : 1 0 : 11          | techniques.                       |
| Originality/   | The student explored      | Tried a few ideas but | Fulfilled the                     |
| Creativity     | several choices,          | based his/her work    | assignment, but                   |
|                | generating many           | on someone else's     | gave no evidence of               |
|                | ideas, unusual            | idea.                 | trying anything unusual. Shows no |
|                | combinations, or          |                       |                                   |
|                | changes on several ideas. |                       | evidence of original              |
| Effort         | The project is            | Worked hard and       | thought. Chose only easy          |
| Elloit         | complete and shows        | completed the project | projects and did the              |
|                | effort far beyond that    | but could have        | work indifferently.               |
|                | which was required.       | improved with more    | Completed with                    |
|                |                           | effort.               | minimum effort or                 |
|                |                           |                       | not finished.                     |
| Performance    | Worked                    | Worked independently  | Made little progress              |
|                | independently.            | most of the time.     | toward goals. Very                |
|                | Followed directions.      | Needed a few          | little was                        |
|                | Used art materials        | reminders on proper   | accomplished.                     |
|                | wisely.                   | care of materials.    | Materials and tools               |
|                |                           |                       | used                              |
|                |                           |                       | inappropriately.                  |

Rubrics Uploaded by Joric Magusara on Sep 02, 2013 https://www.scribd.com/document/164851874/Drawing-Rubrics-General Rubric for Drawing Activity

#### **Rubrics for Essay**

| Criteria     | 5                   | 4                  | 3                 | 2                  |
|--------------|---------------------|--------------------|-------------------|--------------------|
| Focus/ Main  | The essay is        | The essay is       | The essay is      | The essay poorly   |
| Point        | focused,            | focused on the     | focused on the    | addresses the      |
|              | purposeful, and     | topic and          | topic and         | topic and          |
|              | reflects clear      | includes relevant  | includes few      | includes           |
|              | insight and ideas.  | ideas              | loosely related   | irrelevant ideas   |
|              |                     |                    | ideas             |                    |
| Support      | Persuasively        | Supports the       | Supports the      | Provides little or |
|              | supports the        | main point with    | main point with   | no support for     |
|              | main point with     | developed          | some              | the main point     |
|              | well-developed      | reasons and/or     | underdeveloped    |                    |
|              | reasons and/or      | examples           | reasons           |                    |
|              | examples            |                    |                   |                    |
| Organization | Effectively         |                    | Some              | Little or no       |
| & Format     | organizes ideas to  | Organizes ideas    | organization of   | organization of    |
|              | build a logical,    | to build an        | ideas to build an | ideas to build an  |
|              | coherent            | argument           | argument          | argument           |
|              | argument            |                    |                   |                    |
| Originality/ | Distinctive         | Sufficient         |                   |                    |
| Creativity   | experimentation     | experimentation    | Very little       | No                 |
|              | with language       | with language      | experimentation   | experimentation    |
|              | and usage to        | and usage to       | to enhance        | nor enhancement    |
|              | enhance concepts    | enhance concepts   | concepts          | of concepts        |
|              |                     |                    |                   |                    |
|              | Applies higher-     |                    |                   |                    |
|              | order thinking      | Applies basic      | <b>Z</b>          | No adherence to    |
|              | and creative skills | creative skills to | Does not exhibit  | the theme          |
|              | to relay complex    | relay ideas        | creativity        |                    |
|              | ideas               |                    |                   |                    |
|              |                     |                    |                   |                    |

Rubrics downloaded from <u>https://www.kpu.ca</u> > NEVRPDF Web results High School Rubrics

**Grand Total** =  $\frac{\text{(Total Score in Drawing + Total Score in Essay)}}{2}$ 



**Directions:** Read carefully and analyze each item. Use a separate sheet of paper where you will write the letter of the correct answer beside the item number. (Use "CAPITAL" letters)

For numbers 1-3. Use the problem below:

Mrs. Santiago invests her money in Jollibee amounting to Php250,000 that offers 7% interest compounded annually.

1. What is the correct exponential model of the situation?

A. 
$$y = 250.000(1.7t)$$

B. 
$$y = 250,000(1.7)^t$$

C. 
$$y = 250,000(1.07t)$$

D. 
$$y = 250,000(1.07)^t$$

2. How much money does she have after 5 years?

3. How much money does she have after 10 years?

4. There's a colony of termites inside your house. Their initial population is 360. If their population doubles every 2 hours, how many termites are there after 10 hours?

5. Suppose that the half-life of substance K is 50 days and there are initially 2000g. How many grams are left in substance K after 200 days?

6. Each of the following is an example of exponential equations EXCEPT one. Which one is the EXCEPTION?

A. 
$$3^{2x+1} = 729$$

B. 
$$\left(\frac{1}{4}\right)^{3x} \le 16^{x-1}$$
  
D.  $\left(\frac{1}{9}\right)^x = 81$ 

C. 
$$5^{3x} = 25^{2x-1}$$

D. 
$$\left(\frac{1}{2}\right)^x = 81$$

7. Which of the following is an example of exponential inequality?

A. 
$$8^x = 64$$

B. 
$$f(x) = 9^{x-1}$$

C. 
$$7^{x+1} = 343$$

D. 
$$3^{x+5} > 9^{x+2}$$

8. Which among the choices does NOT belong to the group of exponential functions?

A. 
$$g(x) = 4^x$$
  
C.  $j(x) = 2^{x-2}$ 

B. 
$$h(x) = 3^{x+1}$$
  
D.  $k(x) = x^{2x-3}$ 

9. An exponential expression is an expression of the form a  $\cdot$  b<sup>x-c</sup> + d. Which of the variables used should be any real number greater than 0 except 1?

A. a C. c

D. d

- 10. To know if a given function is NOT an exponential function, it may be described by one or more of the following EXCEPT one. Which one is the **EXCEPTION?** 
  - A. Variable is the base and not the exponent.
  - B. Both the base and exponent are variables.
  - C. The variable must be seen as part of the exponent and not the base.
  - D. The base of an exponential function is a positive constant equal to 1.
- 11. What value of x will satisfy the exponential equation  $4^{2x+1} = 8^{1-x}$ ?

A. 
$$x = -5$$

B. 
$$x = \frac{1}{7}$$

C. 
$$x = \frac{3}{2}$$

D. 
$$x = 3$$

12. What value of x will satisfy the exponential equation  $4^x + 2^x - 20 = 0$ ?

A. 
$$-3$$

B. 
$$-2$$

C. 2

13. What solution set will satisfy the exponential inequality  $3^{2-x} < \frac{1}{27}$ ?

A. 
$$x > \frac{2}{3}$$
  
C.  $x > 5$ 

B. 
$$x < \frac{3}{2}$$
  
D.  $x < 5$ 

C. 
$$x > 5^{3}$$

D. 
$$x < \frac{1}{5}$$

- 14. A certain money was invested with a given formula for interest
  - $I(t) = 64(1.25)^t$ , where t is the time in years. What time will it require when the interest is at most ₱125?

A. 
$$t \le 1$$

B. 
$$t \le 2$$

C. 
$$t \le 3$$

D. 
$$t \leq 4$$

15.A certain population of bacterium is tripled every minute. After how many minutes will the number of bacteria be at least 2187?

A. 
$$x \ge 5$$

B. 
$$x \ge 6$$

C. 
$$x \ge 7$$

# References

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