9

Mathematics

Quarter 2 - Module 5:
Laws of Radicals and Simplifying Radical Expressions

AIRs - SLMs

Mathematics 9
Quarter 2 - Module 5: Laws of Radicals and Simplifying Radical Expressions
Second Edition, 2021

Development Team of the Module

Author: Cleofe M. Lacbao
Editor: SDO La Union, Learning Resource Quality Assurance Team
Content Reviewers: Philip R. Navarette and Jocelyn G. Lopez
Language Reviewers: Teresa A. Villanueva and Cleofe M. Lacbao
Illustrators: Ernesto F. Ramos, Jr. and Christian Bautista
Design and Layout: Dana Kate J. Pulido

Management Team:

Atty. Donato D. Balderas Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent
German E. Flora, PhD, *CID Chief*
Virgilio C. Boado, PhD, *EPS in Charge of LRMS*
Erlinda M. dela Peña, EdD, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

Printed in the Philippines by: ________________________

Department of Education – SDO La Union

Office Address: Flores St. Catbangen, San Fernando City, La Union

Telefax: 072 – 205 – 0046

Email Address: launion@deped.gov.ph

Jumpstart

Let's do it. Have fun learning!

What is the connection between expressions with rational exponents and radicals? Why do we need to know how to simplify radicals? Are radicals really needed in life outside math studies? How do we derive the laws of radicals? How do we simplify radicals using the different laws?

In this lesson we will address these questions and look at some important real-life applications of radicals.

Activity 1: Who Am I?

The First Man In The Universe

In 1961, this Russian cosmonaut orbited the earth in a spaceship. Who was he?
Directions: To find the answer to the question, evaluate the radical expressions. Write the letter of the correct answer on your answer sheets to spell out the name of the Russian cosmonaut. Have fun!

____	____	____	____		____	____	____	____	____	____	____
1	2	3	4		5	6	7	8	9	10	11

1. $\sqrt{3} \bullet \sqrt{4}$	Y. $\sqrt{12}$	Z. $\sqrt{3}$
2. $\sqrt{\sqrt[3]{8}}$	O. $\sqrt{7}$	U. $\sqrt{2}$
3. $-(7^2)^{\frac{1}{2}}$	Q. 25	B.-5
4. $\sqrt[3]{216}$	E. 16	I.6
5. $\sqrt[4]{5^4}$	G. 5	H.14
6. $\sqrt{5} \bullet \sqrt{2}$	A. $\sqrt{10}$	M.-9
7. $\sqrt[3]{125}$	F. -4	X. 4
8. $\sqrt{\frac{20}{2}}$	S. 81	J. -81
9. $\sqrt[3]{-7^3}$	R. -7	S. 7
10. $\sqrt{36}$	L. -16	D.16
11. $\frac{\sqrt{24}}{\sqrt{2}}$	N. $2\sqrt{3}$	P. $\sqrt[3]{5}$

Source (Modified) : EASE Modules, Year 2- Module 2 Radical Expressions, page 9-10

Process Questions:

a. How did you solved the given activity?
b. What mathematical concepts are important in simplifying expressions with rational exponents?
c. Did you encounter any difficulties while solving? If yes, what are your plans to overcome those difficulties?

Now that you have applied the general rule;

If $\sqrt[n]{a}$ is a real number, then $a^{\frac{1}{n}} = \sqrt[n]{a}$.

You are now capable of writing expressions with rational exponents to radicals which is very useful as we now learn how to simplify radical expressions using the laws of radicals.

Activity 2: Complete Me

Directions: Carefully analyze the first two examples below then fill in the rest of the exercises with the correct answer. Write your answer on your answer sheets.

$3^{2/4}$	$\sqrt[4]{3^2}$	$\sqrt[4]{9}$
$4^{2/3}$	$\sqrt[3]{4^2}$	$\sqrt[3]{16} = \sqrt[3]{8 \bullet 2} = 2\sqrt[3]{2}$
$5^{3/4}$	1.	2.
$3b^{1/2}$	3.	4.
$(2x)^{2/5}$	5.	6.

You have learned in the previous lessons on how to write expressions with rational exponents to radicals and vice versa and you will need these skills to succeed in the next activities as we apply the different laws in simplifying radicals.

Discover

Laws of Radicals

The laws for radicals are obtained directly from the laws of exponents by means of the definition $\sqrt[n]{a^m} = a^{\frac{m}{n}}$. If n is even, assume a, b ≥ 0.

1. Product Rule for Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$$

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$are real numbers and n is a natural number, then $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$ (Remember that the product rule can only be used when the indexes are the same).

Examples:

1. $\sqrt{5} \cdot \sqrt{7} = \sqrt{35}$
2. $\sqrt[3]{3} \cdot \sqrt[3]{12} = \sqrt[3]{36}$
3. $\sqrt[6]{10m^4} \cdot \sqrt[6]{5m} = \sqrt[6]{50m^5}$
4. $\sqrt[4]{2} \cdot \sqrt[5]{2}$ cannot be simplified, different indexes

2. Quotient Rule for Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$are real numbers and n is a natural number, then $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ *Examples:*

1. $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$
2. $\sqrt[3]{-\frac{8}{125}} = \sqrt[3]{\frac{-8}{125}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{125}} = \frac{-2}{5} = -\frac{2}{5}$
3. $\sqrt[3]{\frac{m^6}{125}} = \frac{\sqrt[3]{m^6}}{\sqrt[3]{125}} = \frac{m^2}{5}$

3. $\left(\sqrt[n]{a}\right)^n = a$

Examples:

1. $\left(\sqrt[3]{9}\right)^3 = 9$
2. $\left(\sqrt[3]{27}\right)^3 = 27$
3. $\sqrt{81} = \sqrt{9^2} = 9$

4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Examples:

1. $\sqrt[6]{9} = \sqrt[3]{\sqrt{3^2}} = \sqrt[3]{3}$
2. $\sqrt[3]{\sqrt{8}} = \sqrt{\sqrt[3]{8}} = \sqrt{\sqrt[3]{2^3}} = \sqrt{2}$

Were you able to analyze each example? If YES, then you are ready perform the next activity.

Simplifying Radical Expressions

When is a radical expression considered simplified?
The following are the three conditions:

1. The radicand has no factor raised to a power greater than or equal to the index.
2. The radical does not contain a fraction or the denominator of the expression does not contain a fraction.
3. The index of the radical is in the lowest possible form

Let us illustrate each condition as we show how it is simplified by applying the laws of radicals.

CONDITION 1: The radicand has no factor raised to a power greater than or equal to the index.

Example 1: Simplify $\sqrt{24}$

To simplify $\sqrt{24}$, first check to see if the radicand is divisible by a perfect square (the square of a natural number) such as 4, 9, …. Choose the largest perfect square that divides in to 24, which is 4. Write 24 as the product of 4 and 6, then use the product rule.

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$$

Example 2: Simplify $\sqrt{108}$

It may not be obvious that 108 is divisible by the perfect square 36. In such a case, factor the radicand into its prime factors to aid in identifying perfect squares.

$$\sqrt{108} = \sqrt{2^2 \cdot 3^3} = \sqrt{2^2 \cdot 3^2 \cdot 3} = \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{3} = 2 \cdot 3 \cdot \sqrt{3} = 6\sqrt{3}$$

Example 3: Simplify $\sqrt[3]{16}$

Look for the largest perfect cube root that divides into 16.

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

Examples using radicals with variables:

Example 4: Simplify $\sqrt{16x^3} = \sqrt{16x^2} \cdot \sqrt{x}$ factor(square root; look for ways to factor squares)

$= 4x\sqrt{x}$

Example 5: Simplify $\sqrt{200x^7q^8}$

$= \sqrt{200} \cdot \sqrt{x^7} \cdot \sqrt{q^8}$ separate the individual elements

$= \sqrt{100 \cdot 2} \cdot x \cdot \sqrt{(q^4)^2}$ factor (square root; look for ways to factor squares)

$=10 \cdot \sqrt{2} \cdot x^3 \cdot \sqrt{x} \cdot$ take the perfect squares "outside" the radical sign

$= 10x^3q^4\sqrt{2x}$ commutative property

CONDITION 2: The radical does not contain a fraction or the denominator of the expression does not contain a fraction.

Example 6: Simplify $\sqrt{\frac{1}{2}}$

Multiply the radicand by $\frac{2}{2}$ to make the denominator a perfect square

$$\sqrt{\frac{1}{2} \cdot \frac{2}{2}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$$

Another solution is $\sqrt{\frac{1}{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

Example 7: Simplify $\frac{2}{\sqrt[3]{4}}$

Multiply the radicand by $\frac{\sqrt[3]{4^2}}{\sqrt[3]{4^2}}$ to make the denominator a perfect square

$\frac{2}{\sqrt[3]{4}} \bullet \frac{\sqrt[3]{4^2}}{\sqrt[3]{4^2}} = \frac{2\sqrt[3]{16}}{\sqrt[3]{4^3}}$ apply the law of radicals $\left(\sqrt[n]{a}\right)^n = a$

$$\frac{2\sqrt[3]{16}}{\sqrt[3]{4^3}} = \frac{2\sqrt[3]{16}}{4} = \frac{\sqrt[3]{16}}{2}$$

Example 8: Simplify $\sqrt[3]{\frac{4}{x}}$

Multiply the radicand by $\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$ to make the denominator a perfect square

$$\sqrt[3]{\frac{4}{x}} \bullet \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{\sqrt[3]{4x^2}}{\sqrt[3]{x^3}} = \frac{\sqrt[3]{4x^2}}{x}$$

> *The process of eliminating the radicals in the denominator of a fraction is called* ***rationalization***

CONDITION 3: The index of the radical is in the lowest possible form

Simplify Radicals by Using Smaller Indexes

Sometimes we can write a radical using rational exponents and then simplify the rational exponent to lowest terms. Then we write the answer as a radical.

Example 9: Simplify $\sqrt[9]{5^6} = 5^{\frac{6}{9}} = 5^{\frac{2}{3}} = \sqrt[3]{5^2} = \sqrt[3]{25}$

Transform the radical into rational expressions then reduce the exponent to lowest term

Example 10: Simplify $\sqrt[4]{25} = (25)^{\frac{1}{4}} = (5^2)^{\frac{1}{4}} = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} = \sqrt{5}$

Example 11: Simplify$\sqrt[3]{24x^4y^8}$= $\sqrt[3]{24x^4y^8}$= $\sqrt[3]{8 \bullet 3x^3xy^5y^3}$

$$= \sqrt[3]{2^3 \bullet 3x^3xy^6y^2}$$
$$= (2^3x^3y^6 \bullet 3xy^2)^{\frac{1}{3}}$$
$$= \ 2^{\frac{3}{3}}x^{\frac{3}{3}}y^{\frac{6}{3}} \bullet 3^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{2}{3}}$$
$$= 2xy^2\sqrt[3]{3xy^2}$$

Explore

You have learned how to simplify radicals by applying the laws of radicals and now let us put those skills in higher level through the different activities.

Activity 3: Try this out!

Directions: Use the laws of radicals to answer the following problems. Write your answer on your answer sheets.

1. $\sqrt{18} =$ _____________
2. $\sqrt{12} =$ _____________
3. $\sqrt[3]{16} =$ _____________
4. $\sqrt{\frac{2}{3}} =$ _____________
5. $\sqrt[6]{64} =$ _____________

Activity 4: Who Am I?

Directions: Use the laws of radicals to answer the following problems. Write your answer on your answer sheets.

WHO IS THIS MATHEMATICIAN

This Polish mathematician was the first to use the symbol $\sqrt{\ }$ for square root. Born in 1499, he studied algebra at the University of Vienna between 1517 and 1521. He was a German mathematician whose book *Coss* is the first German Algebra book in 1525. He introduced the radical symbol (√) for the square root. It is believed that this was because it resembled a lowercase "r" (for "radix").

To find out:

1. Find the answer to each number.
2. Write the letter under its matching number in the DECODER.

$\sqrt[3]{16}$	$\sqrt{80}$	$-\sqrt{28}$	$\sqrt{9x^3}$	$\sqrt{75x^2}$	$\sqrt[3]{\frac{1}{8}}$	$\sqrt{9x^4}$	$-\sqrt{144x}$	$\sqrt[3]{24x^4}$	$\sqrt{50x^{11}}$	$\sqrt{\frac{1}{3}}$
I	**O**	**U**	**C**	**H**	**D**	**S**	**R**	**T**	**F**	**L**

$3x\sqrt{x}$	$5x\sqrt{3}$	$-12\sqrt{x}$	$2\sqrt[3]{2}$	$3x^2$	$2x\sqrt[3]{3x}$	$4\sqrt{5}$	$5x^5\sqrt{2x}$	$5x^5\sqrt{2x}$

$-12\sqrt{x}$	$-2\sqrt{7}$	$\frac{1}{2}$	$4\sqrt{5}$	$\frac{\sqrt{3}}{3}$	$5x^5\sqrt{2x}$	$5x^5\sqrt{2x}$

Deepen

Activity 5: Guess What?

What goes up when the rain comes down?

Directions: Perform and simplify the following radicals. Write the letter of the correct answer on your answer sheets. Keep working and you will discover the answer the questions

$2x$	$\sqrt{5}$		$\sqrt{10x}$	$3\sqrt{2x}$	$\frac{2\sqrt{5}}{5}$	$\frac{10\sqrt{3x}}{3x^3}$	$3x\sqrt{5x}$	$x\sqrt[3]{x}$	$x\sqrt[3]{x}$	$2x$

1. $\sqrt{2x} \bullet \sqrt{5} = U$
2. $\sqrt[3]{x^4} = L$
3. $\frac{10}{\sqrt{3x^5}} = R$
4. $\sqrt{45x^3 = E}$
5. $\frac{\sqrt{4}}{\sqrt{5}} = B$
6. $\sqrt{18x} = M$
7. $\sqrt[4]{16x^4} = A$
8. $\sqrt[4]{25} = N$

Activity 6: Do you know?

Why are Oysters greedy?

Directions: Perform and simplify the following radicals. Copy the table on your answer sheets and write the letter in the box above its correct. Decode the answer to the question.

$\frac{x^2}{3}$	$10\sqrt{x}$	$5\sqrt[3]{x}$	$x\sqrt[3]{x}$		$6\sqrt{6}$	3	$5\sqrt[3]{x}$

$5x\sqrt{3}$	$10\sqrt{x}$	$5\sqrt[3]{x}$	3x	3x	$14\sqrt{x}$	$2\sqrt[3]{6}$	$5x\sqrt{3}$	$10\sqrt{x}$

1. $\sqrt[3]{125x} = E$
2. $\sqrt{75x^2} = S$
3. $\frac{\sqrt{x^4}}{\sqrt{9}} = T$
4. $\sqrt{100x} = H$
5. $\sqrt[5]{243} = R$
6. $\sqrt{216} = A$
7. $2\sqrt{49x^3} = F$
8. $\sqrt[3]{48} = I$
9. $\sqrt[4]{81x^4} = L$
10. $\sqrt[3]{x^4} = Y$