AIRs - LM in

Statistics and Probability
Module 2:
Mean and Variance of Discrete Random Variable

Statistics and Probability
Module 2: Mean and Variance of Discrete Random Variable
First Edition, 2021

Target

In your previous Statistics class, you have learned how to compute the mean and variance of ungrouped and grouped data. In these lessons, you will learn how to compute and interpret the mean and variance of discrete random variable.

After going through this learning module, you are expected to:

1. illustrate the mean and variance of discrete random variable **(M11/12SP-IIIb-1)**;

2. calculate the mean and the variance of discrete random variable **(M11/12SP-IIIb-2)**; and

3. interpret the mean and the variance of discrete random variable **(M11/12SP-IIIb-3).**

Specifically, you are going to:

1. define the mean and variance of discrete random variable;

2. identify the formula to be used in finding the mean and variance of discrete random variable;   and

3. analyze, solve and interpret problems involving the mean and variance of discrete random variable.

> *Before going on, check how much you know about this topic. Answer the pretest on the next page in a separate sheet of paper*

Pretest

Directions: Write the CAPITAL LETTER of the best answer on your answer sheet.

1. What do you call the weighted average of the possible values that a random variable can take?
   - A. Variance of discrete random variable
   - B. Simple Variance
   - C. Mean of discrete random variable
   - D. Simple Mean

2. Which tells us how much we can expect results to deviate from expected value?
   - A. Variance of discrete random variable
   - B. Simple Variance
   - C. Mean of discrete random variable
   - D. Simple Mean

3. Which of the following statements is a description of low variance?
   - A. The probability of scores is almost the same as the mean.
   - B. The data are close to the mean.
   - C. Both statements A and B
   - D. None of the above

4. Which of the following statements is a description of high variance?
   - A. The probability of scores is almost the same as the mean.
   - B. The probability of scores are scattered around the mean.
   - C. The data are closed to the mean
   - D. All of the above.

5. What must be the sum of the probabilities P(X) of all the values in the discrete random variable?
   - A. 1%   B. 10%   C. 100%   D. 1 000%

6. What measure should you use to determine what most likely be the probability of an event to happen?
   - A. Variance of discrete random variable
   - B. Simple Variance
   - C. Mean of discrete random variable
   - D. Simple Mean

7. What measure should you use to determine how far or close the probability of events from the center or the mean?
   - A. Variance of discrete random variable
   - B. Simple Variance
   - C. Mean of discrete random variable
   - D. Simple Mean

For numbers 8 - 10, use the given data:

> The number of boxes of face mask being sold by a pharmacy per day are 5, 6, 7, 8, 9, and 10 along with the probabilities of 0.2, 0.1, 0.1, 0.1, 0.2 and 0.3 respectively

8. Based from the data given, more likely, how many boxes of face mask would the pharmacy sell per day?
   - A. 5   B. 6   C. 9   D. 10

9. What is the mean of the discrete random variable?
   - A. 5.9   B. 6.9   C. 7.9   D. 8.9

10. Which of the following statements best interpret the computed mean?
   A. The pharmacy would sell more or less 8 boxes of face mask per day.
   B. The pharmacy would sell more than 7 boxes of face mask per day.
   C. The pharmacy would sell exactly 7 boxes of face mask per day.
   D. The pharmacy would sell 8 boxes of face mask per day.

For numbers 11 - 15, use the information below.

The probabilities of a manufacturing company to produce defective flashlights per day is shown on the table below.

No. of defective flashlights (X)	Probability P(X)
0	0.6
1	0.2
2	0.1
3	0.1

11. What is the mean of the given data?
   A. 0.1   B. 0.2   C. 0.6   D. 0.7

12. Which is the correct entries of $(X - \mu)$?

A.	B.	C.	D.
-0.7	0.7	-0.7	-0.7
0.3	0.3	0.3	0.3
1.3	1.3	0.3	1.3
2.3	2.3	2.3	1.3

13. Which is the correct entries of $(X - \mu)^2$?

A.	B.	C.	D.
-4.9	4.9	0.49	-0.49
0.09	0.09	0.09	0.09
1.69	1.69	1.69	1.69
5.29	5.29	5.29	5.29

14. What is the variance of the given data?
   A. 0.101   B. 1.01   C. 10.1   D. 101

15. Which of the following statements best interpret the computed variance?
   A. The probability of producing the number of defective flashlights is very far from the mean.
   B. The probability of producing the number of defective flashlights is away from the mean.
   C. The probability of producing the number of defective flashlights is close to the mean.
   D. None of the above.

Lesson 1	MEAN OF DISCRETE RANDOM VARIABLE

Jumpstart

For you to understand the lesson well, do the activity below. Have fun and good luck!

Activity 1: MY CHANCE!

Directions: Determine the chance or probability of having the indicated outcome in the following events. Your answers must be in fraction form.

1. The chance of getting 2 heads in tossing 2 coins.
2. The chance of getting an even number in rolling a die.
3. The chance of getting queen of hearts when a card is drawn from a standard deck.
4. The chance of getting two red balls in a box containing 2 red and 2 blue balls when two balls are picked at a time with replacement.
5. The probability of having three girls if a couple has 3 children.

If your answers are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{52}$, $\frac{1}{4}$, and $\frac{1}{8}$, then you did a great start.

Congratulations! Because determining the probability of occurrence is one of the most important steps in finding the mean of discrete random variable.

Discover

Mean of Discrete Random Variable

The mean of discrete random variable **X** is a weighted average of the possible values that the random variable can take. Unlike the sample mean of a group of observations, which gives each observation equal weight, the mean of random variable weights each outcome **X**, according to its possibility, **P(X)** (www.stat.yale.edu.rvmnvar).

Sample Problem 1

1. There are 4 balls in a box, 2 red and 2 yellow. Two balls are picked at a time.   a. What are the possible outcomes?
    b. What is the probability of drawing a red ball?
    c. What is the mean of the given discrete random variable?

Answers:

a. The possible outcomes or sample space = (RR, RY, YR, YY)

b. No. of Red balls **(X)**	Probability **P(X)**
0	$\dfrac{1}{4}$
1	$\dfrac{2}{4} or \dfrac{1}{2}$
2	$\dfrac{1}{4}$

c. To solve for the mean of the discrete random variable, we use the formula:

$$\mu = \sum X \cdot P(X)$$

Where:

μ	-	mean of the discrete random variable
X	-	possible outcome
P(X)	-	the probability of the outcome

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Solution:

$$\mu = \sum X \cdot P(X)$$

$$\mu = 0\left(\frac{1}{4}\right) + 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right)$$

$$= 0 + \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

So, the mean is **1** which implies that the average no. of red balls that can be picked is 1.

Sample Problem 2

The probability that a "balut" vendor can sell 3, 4, 5 baskets of "balut" per day are 0.30, 0.50, 0.20 respectively. Find the mean of the probability distribution.

No. of baskets of "balut" **(X**)	Probability **P(X)**
3	0.30
4	0.50
5	0.20

Solution:

$$\mu = \sum X \cdot P(X)$$

$$= 3(0.30) + 4(0.5) + 5(0.20)$$

$$= 0.9 + 2 + 1$$

$$= 3.9 \text{ or } 4 \text{ baskets}$$

This means that the vendor would probably sell 3 or less **4** baskets of "balut" per day.

Explore

Enrichment Activity: Arithmetic Mean VS. Mean of Discrete Variable

Directions: Make a comparison between Arithmetic Mean and Mean of Discrete Random Variable along the indicated areas by completing the given table below.

| Area	Arithmetic Mean	Mean of Discrete Random Variable
Computation		
Interpretation		

Assessment

Analyze, solve and interpret the following problems correctly. Show your solution and enclose your final answers in your answer sheet.

1. During holiday season, the probability that a family spends Php5 000, Php10 000, Php15 000 and Php20 000 are 0.4, 0.3, 0.2 and 0.1 respectively. Find the average probability of expenditures of a family during the holiday season.

2. The given table below shows the probability that a delivery truck driver encounters *bad order* (BO) every delivery. Find the mean of the given probability distribution.

No. of Bad Orders (X)	Probability P(X)
1	$\dfrac{1}{10}$
2	$\dfrac{2}{10}$
3	$\dfrac{4}{10}$
4	$\dfrac{1}{10}$
5	$\dfrac{2}{10}$
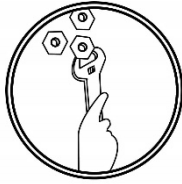
After learning about the mean of discrete random variable, you are now going to apply it in decision-making.

A food delivery services recorded the following data in their business. Can the owner claim that their average daily delivery is 110? Justify your answer.

Day	No. of Deliveries (X)	Probability P(X)
1	100	0.20
2	120	0.20
3	90	0.15
4	35	0.05
5	75	0.15
6	150	0.15
7	125	0.10

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Lesson 2

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VARIANCE OF DISCRETE RANDOM VARIABLE

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Jumpstart

The variance of discrete random variable is a measure of how much the probability mass is spread out around the center which is the mean. (ocw.mit.edu>readings PDF)

Activity 1: Long Distance Relationship

Directions: Describe the distance between the two objects. Write your answers in your answer sheet. Number 1 is done for you.

1. From my house, I need to walk 5 blocks before I reach the pavilion.
   Answer: My house is 5 blocks away from the pavilion.

2. Our students need to walk 1 km from school in order to reach the town plaza.

3. Tourists coming to San Juan, La Union need to travel 13.2 km from town proper to reach the spectacular Tangadan Falls.

4. From Manila, you need to travel 282.6 km bound to Ilocos in order to reach the "Surfing Capital of the North" - San Juan, La Union.

5. *Perseverance,* NASA's 2020 mission to planet Mars need to travel 54 600 000 km from earth to reach its destination.

Discover

The variance tells us how much we can expect the results to deviate from the expected value. Thus, the higher the variance, the farther away the result can be from its expected value (Math Connections in the Digital Age Statistics and Probability - 2016). This implies that if the computed variance is high, the scores are very much scattered around the mean. On the contrary, if the computed variance is low, the data are closely concentrated around the mean.

In this lesson, you will learn to compute and interpret the variance of discrete random variable.

The formula to be used is

$$\sigma^2 = \sum (X - \mu)^2 \cdot P(X)$$

Where:

σ^2	-	variance of the discrete random variable
μ	-	mean of the discrete random variable
X	-	possible outcome
P(X)	-	the probability of the outcome

Sample Problem 1

The probabilities of LED light Corporation to produce 0, 1, 2, 3 and 4 defective lights in a day are 0.80, 0.10, 0.05, 0.03 and 0.02 respectively. Find the variance of the distribution.

Solution:

First you need to solve the mean

$$\mu = 0(0.80) + 1(0.10) + 2(0.05) + 3(0.03) + 4(0.02)$$
$$= 0.37$$

Next, solve for the variance

$\sigma^2 = (0.0.37)^2 (0.80) + (1 - 0.37)^2 (0.10) + (2 - 0.37)^2 (0.05) + (3 - 0.37)^2(0.03) + (4 - 0.37)^2 (0.02)$

$= 0.10952 + 0,0369 + 0.132845 + 0.207507 + 0.263538$

$= 0.75031 \approx 0.75$

So, the variance is 0.75. Since the variance is low, this means that the probability of manufacturing defective LED lights is close to the mean which is 0.37.

Sample Problem 2

The number of COVID - 19 patient admitted per day in a Covid facility along with its corresponding probability is shown on the table below. Solve and interpret the variance of the data.

Number of Covid Patient Admitted (X)	Probability P(X)
0	0.5
1	0.3
2	0.15
3	0.05

Solution:

Step 1: Solve the mean (μ)

$$\mu = 0(0.5)+1(0.3)+2(0.15)+3(0.05)$$
$$= 0.75$$

Step 2: Compute for the $(X - \mu)$

Number of Covid Patient Admitted (X)	Probability P(X)	$(X - \mu)$
0	0.5	(0-0.75)= -0.75
1	0.3	(1-0.75)= 0.25
2	0.15	(2-0.75)=1.25
3	0.05	(3-0.75)=2.25

Step 3: Square (X - μ)

Number of Covid Patient Admitted (x)	Probability (Px)	(X - μ)	(X - μ)^2
0	0.5	(0-0.75)= -0.75	0.5625
1	0.3	(1-0.75)= 0.25	0.0625
2	0.15	(2-0.75)=1.25	1.5625
3	0.05	(3-0.75)=2.25	5.0625

Step 4: Multiply (X - μ)^2 and P(X)

Number of Covid Patient Admitted (x)	Probability (Px)	(X - μ)	(X - μ)^2	(X - μ)^2 · P(X)
0	0.5	(0-0.75)= -0.75	0.5625	0.5625x0.5=0.28125
1	0.3	(1-0.75)= 0.25	0.0625	0.0625x0.3=0.01875
2	0.15	(2-0.75)=1.25	1.5625	1.5625x0.15=0.234375
3	0.05	(3-0.75)=2.25	5.0625	5.0625x0.05=0.253125

Step 5: Get the sum of (X - μ)^2 and P(X)

σ^2 = 0.28125 + 0.01875 + 0.234375 + 0.253125

   = 0.7875 ≈ 0.79

So the variance is 0.79. Since the variance is low, this means that the probability of admitting COVID patient per day is close to the mean which is 0.75.

Explore

Enrichment Activity: What is My Order?

Arrange the given steps in solving for the variance of discrete random variable by writing nos. 1 - 5 on the line before the statement.

_____ Multiply $(x - \mu)^2$ and Px

_____ Get the sum of $(x - \mu)^2 Px$

_____ Solve the mean (μ)

_____ Square $(x - \mu)^2$

_____ Compute for $(x - \mu)$

Assessment

To enhance your understanding of the variance of discrete random variable, answer the problems that follow in your answer sheet.

1. Despite the pandemic, godparents still give cash gifts to their godchildren amounting Php100, Php200, Php500 and Php1 000 with the following probabilities 0.4, 0.3, 0.2, and 0.1 respectively. Compute and interpret the variance of the given data.

2. In retrieving the outputs of the learners in this Modular Distance Learning, a senior high school teacher recorded the following data.

Compute and interpret the variance of the random variable.

No. of students who did NOT submit (x)	Probability (Px)
0	0.4
1	0.3
2	0.2
3	0.1

Deepen

After learning how to compute and interpret the variance of discrete random variable, you are now ready to apply what you've learned by doing the activity below. Your output will be graded using the rubric that follows.

Conduct a survey on the number of social media platforms used by 5 of your friends or classmates and their corresponding probabilities (Note: The sum of P(X) must be equal to 1). Construct a probability distribution and compute the mean and variance then interpret the computed values.

Rubric for Scoring

5	4	3	2	1
Solutions are accurate with correct interpretation	Solutions are accurate but incorrect interpretation	Solutions are NOT so accurate with irrelevant interpretation	Solutions are NOT so accurate with no interpretation at all	Solutions are totally incorrect

Good job! You have already understood the lesson. Are you ready to summarize what you have learned?

Reflect on the activities you have done in this lesson by completing the following sentences. Write your answers in your journal notebook.

I learned that I _____.
I surprised that I _____.
I noticed that I _____.
I discovered that I _____.
I am pleased that I _____.

Directions: Write the CAPITAL LETTER of the best answer on your answer sheet.

1. What measure should you use to determine what most likely be the probability of an event to happen?
   - A. Variance of discrete random variable
   - B. Mean of discrete random variable
   - C. Simple Variance
   - D. Simple Mean

2. What measure should you use to determine how far or close the probability of events from the center or the mean?
   - A. Variance of discrete random variable
   - B. Mean of discrete random variable
   - C. Simple Variance
   - D. Simple Mean

3. What do you call the weighted average of the possible values that a random variable can take?
   - A. Variance of discrete random variable
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4. Which tells us how much we can expect results to deviate from expected value?
   - A. Variance of discrete random variable
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   - C. Simple Variance
   - D. Simple Mean

5. Which of the following statements is a description of low variance?
   - A. The probability of scores is almost the same as the mean.
   - B. The data are close to the mean.
   - C. Both statements A and B
   - D. None of the above

6. Which of the following statements is a description of high variance?
   - A. The probability of scores is almost the same as the mean.
   - B. The probability of scores are scattered around the mean.
   - C. The data are closed to the mean
   - D. All of the above.

7. What must be the sum of the **P(X)** in the mean and variance of discrete random variable?
   - A. 1%   B. 10%   C. 100%   D. 1 000%

For numbers 8 - 10, use the data below.

> The number of boxes of face mask being sold by a pharmacy per day are 5, 6, 7, 8, 9, and 10 along with the probabilities of 0.2, 0.1, 0.1, 0.1, 0.2 and 0.3 respectively

8. Based from the data given, more likely, how many boxes of face mask would the pharmacy sell per day?
  A. 5   B. 6   C. 9   D. 10

9. What is the mean of the discrete random variable?
  A. 5.9   B. 6.9   C. 7.9   D.8.9

10. Which of the following statements best interpret the computed mean?

  A. The pharmacy would sell more or less 8 boxes of face mask per day.
  B. The pharmacy would sell 8 boxes of face mask per day.
  C. The pharmacy would sell exactly 7 boxes of face mask per day.
  D. The pharmacy would sell more than 7 boxes of face mask per day.

For numbers 11 - 15, use the given information below.

The probabilities of a manufacturing company to produce defective flashlights per day is shown on the table below.

No. of defective flashlights (X)	Probability P(X)
0	0.6
1	0.2
2	0.1
3	0.1

11. What is the mean of the given data?
  A. 0.1   B. 0.2   C. 0.6   D. 0.7

12. Which is the correct entries of $(x - \mu)$?

	A.		B.		C.		D.
	-0.7		0.7		-0.7		-0.7
	0.3		0.3		0.3		0.3
	1.3		1.3		0.3		1.3
	2.3		2.3		2.3		1.3

13. Which is the correct entries of $(x - \mu)^2$?

	A.		B.		C.		D.
	-4.9		4.9		0.49		-0.49
	0.09		0.09		0.09		0.09
	1.69		1.69		1.69		1.69
	5.29		5.29		5.29		5.29

14. What is the variance of the given data?
  A. 0.101   B. 1.01   C. 10.1   D. 101

15. Which of the following statements best interpret the computed variance?
    A. The probability of producing the number of defective flashlights is very far from the mean.
    B. The probability of producing the number of defective flashlights is away from the mean.
    C. The probability of producing the number of defective flashlights is close to the mean.
    D. None of the above.

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