9

# Mathematics

## Quarter 2 – Module 4:
## Simplifying Expressions with Rational Exponents

**AIRs - LM**

**MATHEMATICS 9**
**Quarter 2 - Module 4: Simplifying Expressions with Rational Exponents**
**Second Edition, 2021**



**Development Team of the Module**

**Author:** Miriam C. Fajardo
**Editor:** SDO La Union, Learning Resource Quality Assurance Team
**Content Reviewers**: Philip R. Navarette and Jocelyn G. Lopez
**Language Reviewers**: Teresa A. Villanueva and Cleofe M. Lacbao
**Illustrators:** Ernesto F. Ramos, Jr. and Christian Bautista
**Design and Layout:** Dana Kate J. Pulido

**Management Team:**

Atty. Donato D. Balderas Jr.
*Schools Division Superintendent*
Vivian Luz S. Pagatpatan, PhD
*Assistant Schools Division Superintendent*
German E. Flora, PhD, *CID Chief*
Virgilio C. Boado, PhD, *EPS in Charge of LRMS*
Erlinda M. dela Peña, EdD, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

Printed in the Philippines by: ________________________

**Department of Education – SDO La Union**

Office Address: Flores St. Catbangen, San Fernando City, La Union

Telefax: 072 – 205 – 0046

Email Address: launion@deped.gov.ph

## *Jumpstart*

### Activity 1: Follow Me!

*Directions:* Fill in the missing parts of the solution in simplifying expressions with rational exponents. The first one is done for you.

1. $(m^{\frac{2}{3}})(m^{\frac{4}{3}})$ = m $^{2/3 + 4/3}$ = m $^{6/3}$ = m$^2$

2. $(k^{\frac{1}{4}})(k^{\frac{2}{3}})$ = k $^{?/12 + ?/12}$ = k $^{?/12}$

3. $\frac{a^{\frac{5}{7}}}{a^{\frac{3}{2}}}$ = a$^{10/? - 21/?}$ = a $^{-11/?}$ = $\frac{1}{a^{\frac{11}{14}}}$

4. (r$^{12}$s$^{9}$)$^{1/3}$ = r $^{?/3}$ s$^{?/3}$ = r$^4$ s$^3$

Process Questions:

a. Based on the activity, how do you simplify expressions involving rational exponents?

b. What are the necessary skills in simplifying expressions with rational exponents?

### Activity 2. Find My Partner!

*Directions:* Write each rational exponent to radical form and radical form to exponential form. Choose the letter of the correct answer and write in a separate sheet of paper.

| Exponential Form | Radical Form |
|---|---|
| 1. $16^{\frac{1}{2}}$ | a. $\sqrt[3]{x^2}$ |
| 2. $x^{\frac{2}{3}}$ | b. $\sqrt{16}$ |
| 3. $x^{\frac{1}{3}}$ | c. $3\sqrt{x^3}$ |
| 4. $3x^{\frac{3}{2}}$ | d. $\sqrt{(3x)^3}$ |
| 5. $(3x)^{\frac{3}{2}}$ | e. $\sqrt[3]{x}$ |

| Radical Form | Exponential Form |
|---|---|
| 6. $\sqrt[3]{27^2}$ | f. $16^{\frac{3}{4}}$ |
| 7. $\sqrt[4]{16^3}$ | g. $27^{\frac{2}{3}}$ |
| 8. $\sqrt[3]{\sqrt{25}}$ | h. $25^{\frac{1}{6}}$ |
| 9. $\frac{1}{\sqrt{25}}$ | i. $8x^{\frac{3}{2}}$ |
| 10. $8\sqrt{x^3}$ | j. $25^{\frac{-1}{2}}$ |

# *Discover*

The previous activities enabled you to realize that the laws of exponents for integral exponents may be used in simplifying expressions with rational exponents.

Use the Properties of Exponents to simplify expressions with rational exponents.

The same properties of exponents that we have already used also apply to rational exponents. We will list the Properties of Exponents here to have them for reference as we simplify expressions.

**Properties of Exponents**

If $a$ and $b$ are real numbers and $m$ and $n$ are rational numbers, then

1. Product Property $(a^m)(a^n) = a^{m+n}$
2. Power Property $(a^m)^n = a^{mn}$
3. Product to a Power $(ab)^n = a^m b^n$
4. Quotient Property $\frac{a^m}{a^n} = a^{m-n}$ , $a \neq 0$
5. Zero Exponent Definition $a^0 = 1$ ; $a \neq 0$
6. Quotient to a Power Property $(\frac{a}{b})^m = \frac{a^m}{b^m}$ ; $b \neq 0$
7. Negative Exponent Property $a^{-n} = \frac{1}{a^n}$; $a \neq 0$

We will apply these properties in the next examples.

Examples:

1. Simplify: $(x^{\frac{1}{3}})(x^{\frac{1}{3}})$
   Solution:

$(x^{\frac{1}{3}})(x^{\frac{1}{3}}) = x^{\frac{1+1}{3}}$ The bases are the same, so we add the exponents.

$= x^{\frac{2}{3}}$ Add the fractions

*Therefore,* $(x^{\frac{1}{3}})(x^{\frac{1}{3}}) = x^{\frac{2}{3}}$

- *The Power Property tells us that when we multiply the same base, we add the exponents.*

2. Simplify: $(x^{\frac{1}{4}})^8$
Solution:

$(x^{\frac{1}{4}})^8 = x^{(\frac{1}{4})(8)}$ — To raise a power to a power, we multiply the exponents.

$= x^{\frac{8}{4}}$

$= x^2$ — Simplify

*Therefore,* $(x^{\frac{1}{4}})^8 = x^2$

- *The Power Property tells us that when we raise a power to a power, we multiply the exponents.*

3. Simplify: $27^{\frac{2}{3}}$
Solution:

$27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}$ — Express 27 to exponential form

$= 3^{(\frac{2}{3})(3)}$ — Definition of laws of exponent

$= 3^2$ — Simplify

$= 9$

*Therefore,* $27^{\frac{2}{3}} = 9$

- *The Power Property tells us that when we raise a power to a power, we multiply the exponents.*

4. Simplify: $\frac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}}$
Solution:

$\frac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}} = x^{\frac{1-5}{3}}$ — To divide with the same base, we subtract the exponents.

$= x^{\frac{-4}{3}}$

$= \frac{1}{a^{\frac{4}{3}}}$ — Simplify

*Therefore,* $\frac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}} = \frac{1}{a^{\frac{4}{3}}}$

- *The Quotient Property tells us that when we divide with the same base, we subtract the exponents.*

**Transforming Rational Exponents to Radical Form and Vice versa**

We define $x^{\frac{1}{n}}$ as the $n^{th}$ root of x. We use the symbol $\sqrt[n]{x}$ to mean $x^{\frac{1}{n}}$ where $n$ is the **index** of the radical, $x$ is the **radicand** and $\sqrt[n]{x}$ is itself the **radical**. If the index is not indicated, then it is understood to be 2.

The symbol $\sqrt[n]{a^m}$ is called **radical**. A **radical expression** or a **radical** is an expression containing the symbol $\sqrt{\phantom{x}}$ called **radical sign**. In the symbol $\sqrt[n]{a^m}$ , $n$ is called the **index** or **order** which indicates the degree of the radical such as square root $\sqrt{\phantom{x}}$ , cube root $\sqrt[3]{\phantom{x}}$ , and fourth root $\sqrt[4]{\phantom{x}}$, $a^m$ is called the **radicand** which is a number or expression inside the radical symbol and $m$ is the power or exponent of the radicand.

If $\frac{m}{n}$ is a rational number and $\boldsymbol{a}$ is a positive real number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ provided that $\sqrt[n]{a^m}$ is a real number. The form $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$ is called the principal $n$th root of $a^m$. Through this, we can write expressions with rational exponents as radicals.

Examples:

*A. Write as a radical expression.*

1. $x^{\frac{1}{2}}$ 2. $x^{\frac{1}{3}}$ 3. $x^{\frac{3}{2}}$

We want to write each expression in the form $\left(\sqrt[n]{a}\right)$

| | |
|---|---|
| 1. $x^{\frac{1}{2}} = \sqrt{x}$ | The denominator of the rational exponent is 2, so the index of the radical is 2. We do not show the index when it is 2. |
| 2. $x^{\frac{1}{3}} = \sqrt[3]{x}$ | The denominator of the exponent is 3, so the index is 3. |
| 3. $x^{\frac{2}{3}} = \sqrt[3]{x^2}$ | The denominator of the exponent is 3, so the index is 3. The numerator 2 becomes the exponent. |

*B. Write as a rational exponent.*

1. $\sqrt{y^3}$ 2. $\left(\sqrt[3]{2x}\right)^4$ 3. $\sqrt{\left(\frac{3a}{4b}\right)^3}$

We want to use $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ to write each radical in the form $a^{\frac{m}{n}}$.

| | |
|---|---|
| 1. $\sqrt{y^3} = y^{\frac{3}{2}}$ | The numerator of the exponent is the exponent *3*.<br>The denominator of the exponent is the index of the radical *2*. |
| 2. $\left(\sqrt[3]{2x}\right)^4 = (2x)^{\frac{4}{3}}$ | The numerator of the exponent is the exponent *4*.<br>The denominator of the exponent is the index of the radical *3*. |
| 3. $\sqrt{\left(\frac{3a}{4b}\right)^3} = \left(\frac{3a}{4b}\right)^{\frac{3}{2}}$ | The numerator of the exponent is the exponent *3*.<br>The denominator of the exponent is the index of the radical *2*. |

# Explore

## Activity 3: Fill-Me-In!

*Directions:* Simplify the following expressions with rational exponents by filling in the boxes with solutions. Number 1 is done for you.

| | | | | |
|---|---|---|---|---|
| 1. | $(4^2)^{\frac{1}{2}}$ → | $4^{(\frac{1}{2})(2)}$ → | $4^{\frac{2}{2}}$ → | $4$ → |
| 2. | $(x^6)^{\frac{-1}{2}}$ → | → | → | $\frac{1}{x^3}$ → |
| 3. | $(n^4)^{\frac{3}{2}}$ → | → | → | $n^6$ → |
| 4. | $(p^{\frac{3}{2}})^{-2}$ → | → | → | $\frac{1}{p^3}$ → |
| 5. | $(x^{12}y^{10})^{\frac{-1}{2}}$ → | → | → | $\frac{1}{x^6y^5}$ → |

## Activity 4. Transformers I!

*Directions:* Transform the given radical form into exponential form and exponential form into radical form. Assume that all the letters represent positive real umbers.

| Radical Form | Exponential Form |
|---|---|
| 1. $\sqrt{6}$ | |
| 2. | $26^{\frac{1}{4}}$ |
| 3. $\sqrt[4]{x}$ | |
| 4. | $9^{\frac{1}{3}}$ |
| 5. | $x^{\frac{3}{5}}$ |
| 6. $\sqrt[5]{a^3}$ | |
| 7. $\sqrt[4]{(xy)^3}$ | |
| 8. | $(5a^3b^2)^{\frac{2}{3}}$ |
| 9. $\sqrt{3m^2}$ | |
| 10. | $(4r^2s^3)^{\frac{2}{5}}$ |

## Deepen

### Activity 5: Make Me Simple!

*Directions:* Using your knowledge of rational expressions, simplify the following,

| Given | Final Answer |
|---|---|
| 1. $c^{\frac{1}{4}}\, c^{\frac{5}{8}}$ | $c^{\frac{7}{8}}$ |
| 2. $(p^{12})^{\frac{3}{4}}$ | |
| 3. $\frac{r^{\frac{4}{5}}}{r^{\frac{9}{5}}}$ | |
| 4. $y^{\frac{1}{2}}\, y^{\frac{3}{4}}$ | |
| 5. $(x^{16}y^{20}z^{8})^{\frac{1}{4}}$ | |

### Activity 6. Transform Me!.

A. Express the following in radical form.

1. $x^{\frac{4}{5}}$
2. $(5x^2y)^{\frac{2}{5}}$
3. $(a+b)^{\frac{1}{3}}$
4. $(4xy)^{\frac{3}{4}}$
5. $(2x-5y)^{\frac{1}{3}}$

B. Change the following radicals to exponential form.

1. $\sqrt[3]{1000}$
2. $\sqrt[5]{32^3}$
3. $\sqrt{16xy^3}$
4. $\sqrt[3]{(a+b)^2}$
5. $\sqrt[4]{\sqrt[5]{x}}$