

Mathematics 9

Quarter 3-Week 2 – Module 2: Theorems on the Different Kinds of Parallelogram



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Mathematics 9

Quarter 3- Week 2 – Module 2: Theorems on the Different Kinds of Parallelogram

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Region I

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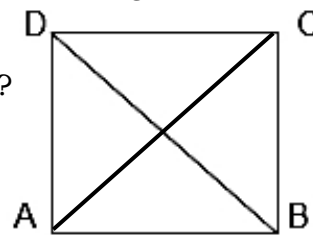
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Use figure 3 for numbers 13 – 15.

Figure 3



13. ABCD is a square with diagonals AC and BD.
If $AC = 10x - 10$ and $BD = x + 71$. How long is each diagonal?
A. 9 B. 80 C. 90 D. 100
14. ABCD is a square. If $m\angle ABD = 3(x + 10)$, what is x ?
A. 1 B. 3 C. 5 D. 7
15. If $DC = 4m$, what is the length of each diagonal?
A. $2\sqrt{2}$ B. $3\sqrt{2}$ C. $4\sqrt{2}$ D. $5\sqrt{2}$

You've just answered the pre – assessment of this module. This time, let us test your prior knowledge about the properties of a parallelogram before proceeding to the main topic of this module by answering the given activities below. Goodluck!



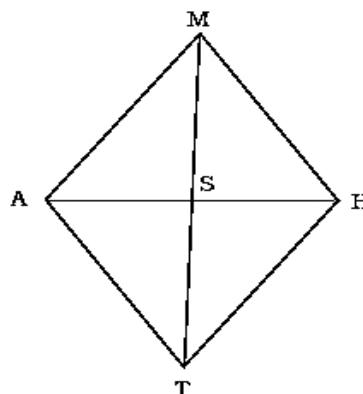
Jumpstart

Activity 1: YES, YOU CAN!

Refer to the given figure at the right and answer the following using properties of a parallelogram.

GIVEN: \square MATH is a parallelogram

1. $\overline{MA} \cong$ _____
2. $\overline{MS} \cong$ _____
3. $\triangle THM \cong$ _____
4. $\angle ATH \cong$ _____
5. If $m\angle MHT = 100$, then $m\angle MAT =$ _____
6. If $m\angle AMH = 70$, then $m\angle MHT =$ _____
7. If $MS = 7$, then $MT =$ _____
8. If $AH = 14$, then $SH =$ _____
9. If $MT = 21$, then $ST =$ _____
10. If $AT = 34$, then $MH =$ _____



Activity 2: FIND MY VALUE!

Answer the following by applying the different properties of a parallelogram.

1. Given: $HE = 2x$

$OR = x + 7$

Find: a) x

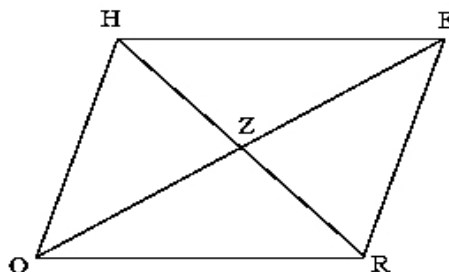
b) HE

2. Given: $m\angle HER = 5y - 3$

$m\angle ROH = 2y + 9$

Find: a) y

b) $m\angle ROH$



KEY POINTS

Properties of Parallelogram

1. Any two opposite sides are congruent
2. Any two opposite angles are congruent
3. Any two consecutive angles are supplementary
4. The diagonals of a parallelogram bisect each other
5. A diagonal of a parallelogram divides the parallelogram into two congruent triangles

You are doing great! Now, you are ready to learn more about theorems on the different kinds of a parallelogram from a deeper perspective.

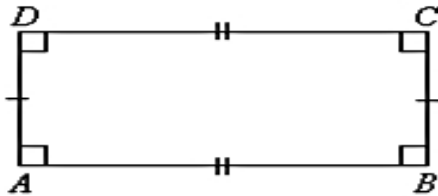


Discover

Before you proceed to the different proofs of the theorems on parallelogram, you first study the definition and properties of rectangle, rhombus and square and its relationships to one another.

Definition

Rectangle



Example:

$$m\angle DAB = m\angle ABC$$

$$mDC = mAB$$

If $AD = 7\text{cm}$, then $BC = 7\text{cm}$

Rhombus

Example:

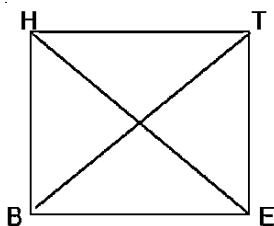
$$m\angle UCE = m\angle ETU$$

$$mCU = mTE$$

$$m\angle CUE = m\angle TUE$$

Square

- **Rectangle + Rhombus = Square**



Example:

1. $m\angle HBE = 90$, then what is $m\angle BET = ?$

Solution: $m\angle BET = 90$ since all angles of a square measures 90°

2. $HE = BT$

Solution: Diagonals of a square are equal

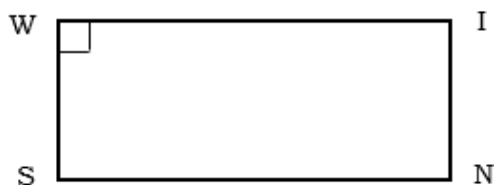
Theorems on Rectangle

Theorem 1: If a parallelogram has a right angle, then it has four right angles and the parallelogram is a rectangle.

Given: \square WINS is a parallelogram

$\angle W$ is a right angle

Prove: $\angle I, \angle N$ and $\angle S$ are right angles



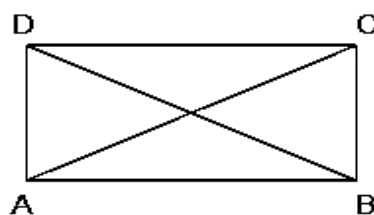
PROOF:

Statements	Reasons
1. WINS is a parallelogram with $\angle W$ is a right angle	1. Given
2. $\angle W = 90$	2. Definition of right angle
3. $\angle W \cong \angle N$ & $\angle I \cong \angle S$	3. In a parallelogram, opposite angles are congruent
4. $m\angle W = m\angle N$ $m\angle I = m\angle S$	4. Definition of congruent angles
5. $m\angle N = 90$	5. Substitution (SN 2 & 4)
6. $m\angle W + m\angle I = 180$	6. Consecutive angles are supplementary
7. $90 + m\angle I = 180$	7. Substitution (SN 2 & 6)
8. $90 = 90$	8. Reflexive Property
9. $m\angle I = 90$	9. Subtraction Property (SN 7 & 8)
10. $m\angle S = 90$	10. Substitution (SN 4 and 9)
11. $\angle I, \angle N$ and $\angle S$ are right angles.	11. If the measure of an angle is 90, then it is a right angle.
12. WINS is a rectangle.	12. Definition of rectangle.

Theorem 2: The diagonals of a rectangle are congruent.

Given: Rectangle ABCD with diagonals \overline{AC} and \overline{BD}

Prove: $\overline{BD} \cong \overline{AC}$



PROOF:

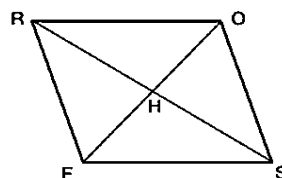
Statements	Reasons
1. Rectangle ABCD with diagonals \overline{AC} and \overline{BD}	1. Given
2. $\overline{AD} \cong \overline{BC}$	2. Opposite sides of a parallelogram are congruent (Remember, a rectangle is a parallelogram)
3. $\angle DAB$ and $\angle CBA$ are right angles	3. A rectangle is a parallelogram with four right angles
4. $\angle DAB \cong \angle CBA$	4. Any two right angles are congruent
5. $\overline{AB} \cong \overline{AB}$	5. Reflexive Property of Congruence
6. $\triangle DAB \cong \triangle CBA$	6. SAS Congruence
7. $\overline{BD} \cong \overline{AC}$	7. Corresponding Parts of Congruent Triangles are Congruent

Theorems on Rhombus

Theorem 3: The diagonals of a rhombus are perpendicular.

Given: Rhombus ROSE

Prove: $\overline{RS} \perp \overline{OE}$



PROOF:

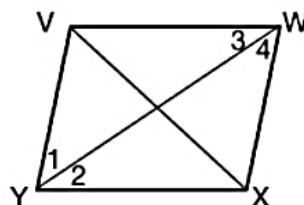
Statements	Reasons
1. Rhombus ROSE	1. Given
2. $\overline{OS} \cong \overline{RO}$	2. Definition of rhombus
3. \overline{RS} and \overline{EO} bisect each other	3. The diagonals of a parallelogram bisect each other.
4. H is the midpoint of RS	4. \overline{EO} bisects \overline{RS} at H
5. $\overline{RH} \cong \overline{HS}$	5. Definition of midpoint
6. $\overline{OH} \cong \overline{OH}$	6. Reflexive Property
7. $\triangle RHO \cong \triangle SHO$	7. SSS Congruence Postulate
8. $\angle RHO \cong \angle SHO$	8. CPCTC
9. $\angle RHO$ and $\angle SHO$ are right angles	9. $\angle RHO$ and $\angle SHO$ form a linear pair and are congruent.
10. $\overline{RS} \perp \overline{OE}$	10. Perpendicular lines meet to form right angles.

Theorem 4: Each diagonal of a rhombus bisects opposite angles.

Given: Rhombus VWXY

Prove: $\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$



PROOF:

Statements	Reasons
1. Rhombus VWXY	1. Given
2. $\overline{YV} \cong \overline{VW} \cong \overline{WX} \cong \overline{XY}$	2. Definition of rhombus
3. $\overline{WY} \cong \overline{YW}$	3. Reflexive Property
4. $\triangle YVW \cong \triangle WXY$	4. SSS Congruence Postulate
5. $\angle 1 \cong \angle 2$	5. CPCTC

Remember:

The theorems true to a rectangle and the theorems true to a rhombus are both **true to a square**.

Were you able to follow and understand the discussion of the proofs presented? Let's continue exploring!



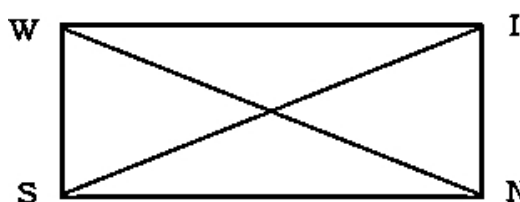
Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3: COMPLETE ME!

Complete the proof of the following theorem. Choose your answer on the word bank below.

Given: Rectangle WINS with diagonals \overline{WN} and \overline{SI}



Prove: $\overline{WN} \cong \overline{SI}$

Statements	Reasons
1. Rectangle WINS with diagonals \overline{WN} and \overline{SI}	1.
2. $\overline{WS} \cong \overline{IN}$	2.
3. $\angle WSN$ and $\angle INS$ are right angles	3.
4.	4. Any two right angles are congruent
5. $\overline{SN} \cong \overline{SN}$	5.
6.	6. SAS Congruence
7. $\overline{WN} \cong \overline{SI}$	7.

Word Bank

Given

CPCTC

Theorem 1

$\angle WSN \cong \angle INS$

$\Delta WSN \cong \Delta INS$

Reflexive Property

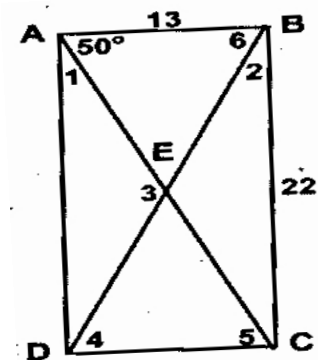
Opposite sides of a parallelogram are congruent

Now that you have learned how to complete the proof, you can now proceed to the next activity.

Activity 4: Find My Measure!

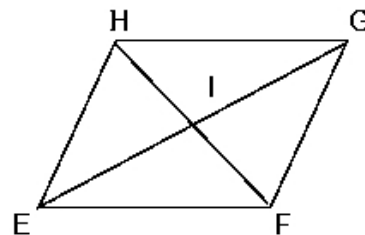
A. Given: $ABCD$ is a rectangle.

1. What is $m\angle 1$? _____
2. What is $m\angle E$? _____
3. What is $m\angle 5$? _____
4. What is the Perimeter of $ABCD$? _____



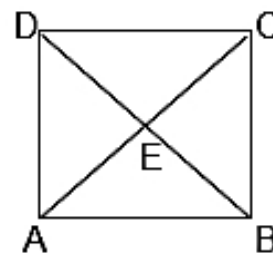
B. Given: *Rhombus EFGH*.

5. If $m\angle HGE = 34$. Find $m\angle FGE$.
6. If $m\angle GHE = 110$. Find $m\angle GHF$.
7. If $m\angle HGF = 80$. Find $m\angle FHG$.
8. If $m\angle IGF = 12$ and $m\angle IGH = 3x$, what is x ?



C. Given: *Square ABCD*.

9. If ABCD is a square, then what is $m\angle ACB$?
10. If ABCD is a square, then what is $m\angle DEC$?



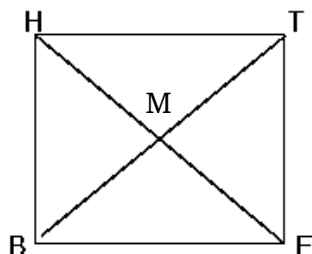
How was the activity? Did you enjoy applying your knowledge on the different theorems presented? Now let's go deeper!



Deepen

At this point, you are going to apply the mathematical concepts learned from this module.

BETH is a rhombus with diagonals BT and HE intersect each other at M.



1. If $HM = x + 15$ and $HE = 40$, what is x ?
2. If $m\angle HBT = 5x - 5$ and $m\angle EBT = 2x + 25$.
 - a. What is x ?
 - b. What is the measure of $m\angle HBT$?