General Mathematics

Module 2:

Rational Functions, Equations, and Inequalities

AIRs - LM

GENERAL MATHEMATICS
Module 2: Rational Functions, Equations, and Inequalities
Second Edition, 2021

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General Mathematics
Module 2:
Rational Functions, Equations, and Inequalities

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Rational functions can model several real-life situations. One example is the help that is extended by the government to the citizen during the time of pandemic. Majority of our fellow citizens experienced hardship and required help coming from the government. As a response, they provided a particular amount to a certain percentage of the population that can be represented as rational function to determine how much either in cash or kind an individual may receive. However, it is not enough that only the government would take part to solve this crisis everyone can be part of the solution if we played our role properly. Real-life situations that involve rational functions is mostly seen in Economics and Science however other disciplines also incorporate this concept.

In this lesson, you shall explore more about solving rational equations and inequalities by carefully studying the step-by-step methods of solutions. You will first start from the easiest procedures in solving this type of equation and as you progress you will gain and learn more techniques and concepts that will help you solve more complex problems related to this topic. This will also help you master the domain and range of a rational function. The scope of this module permits it to be used in many different learning situations.

If you wonder how rational functions can help, you need to explore this module. After going through this module, you are expected to:

1. represent real-life situations using rational functions **(M11GM-Ib-1),**
2. distinguish rational function, rational equation, and rational inequality **(M11GM-Ib-2),**
3. solve rational equations and inequalities **(M11GM-Ib-3),**
4. represent a rational function through its: (a) table of values, (b) graph, and (c) equation **(M11GM-Ib-4)**; and
5. find the domain and range of a rational function **(M11GM-Ib-5).**

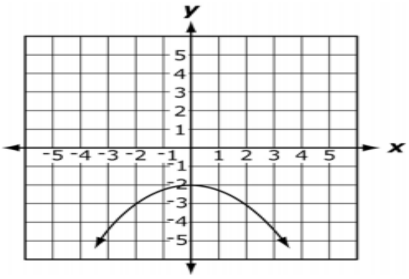
Learning Objectives:
1. define rational function, rational equation and rational inequality
2. represent real-life situations using rational functions
3. distinguish rational function, rational equation, and rational inequality
4. analyze the steps in solving rational equations and inequalities
5. solve rational equations and inequalities
6. represent a rational function through its: (a) table of values, (b) graph, and (c) equation
7. find the domain and range of a rational function

Pretest

Directions: Read each item carefully and answer what is being asked. Write your answer on a separate sheet paper.

1. Which function is express in forms of ratios and quotient of polynomials?
   A. Ratio function B. Quotient function
   C. Rational function D. Irrational function
2. Which of the following is NOT an example of rational equation?
   A. $\frac{5x-1}{x+3} = \frac{x}{5}$ B. $\frac{5x}{x^{\frac{3}{4}}-3} = \frac{3}{x+1}$
   C. $\frac{1}{x-2} = \frac{2}{3x-9}$ D. $\frac{x^2-5}{x-2} = \frac{x-3}{x+1}$
3. What symbol is used to mark the number line if the value of x is included in the solution?
   A. An arrow B. Hollow circle C. Letter x D. Shaded circle
4. Which of the following involves rational expression and the relational symbols ≤, <, ≥, >?
   A. Rational equation B. Rational inequality
   C. Rational function D. Algebraic expression
5. Which of the following is defined as set of all possible values that the x variable can take?
   A. Domain B. Ordinate C. Range D. Set
6. Which of the following is defined as set of all possible values that the y or f(x) variable can take?
   A. Domain B. Ordinate C. Range D. Set
7. Which of the following is NOT a part of the procedures in solving rational equations?
   A. Checking the answer B. Simplifying the equation
   C. Determining the LCD D. Identifying the critical areas
8. Which of the following should be determined when adding and subtracting rational expressions with different denominators?
   A. Lowest common factor B. Least common denominator
   C. Greatest common factor D. Greatest common denominator
9. What is the Least Common Denominator (LCD) of the given rational equation below?
   $$\frac{4}{x} = \frac{9}{x-2}$$
   A. x^2 - 2 B. $x(x-2)$ C. $x - 2x$ D. x^2 + 2
10. Which of the following represents -4 and 1 as part of the solution?
    A. (-4,1) B. (-4, 1] C. [-4,1) D. [-4,1]
11. Which symbol denotes that the endpoints of the interval is NOT included in the solution set?
    A. ( ) B. [ ] C. { } D. none of the above
12. Given the inequality: $\frac{x+1}{x-5} \leq 0,$ what are the critical values?
    A. 1 and 5 B. -1 and 5 C. 1 and -5 D. -1 and -5

13. Which function represents the graph below?

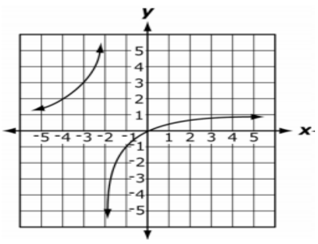


A. $f(x) = \frac{-1}{4}\boldsymbol{x^2 - 2}$ B. $f(x) = \frac{1}{4}x^2 - 2$

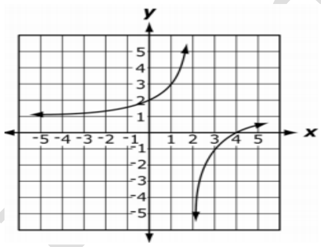
C. $f(x) = -4x^2 - 2$ D. $f(x) = 4x^2 - 2$

14. What is the graph of the function $f(x) = \frac{x}{x-2}$?

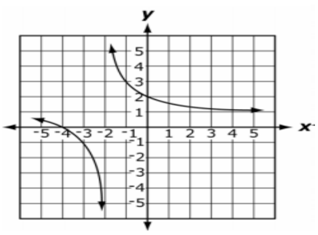
A.



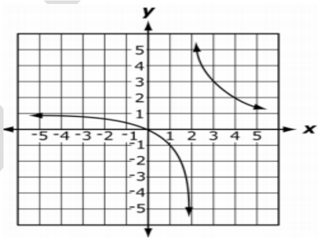
B.



C.



D.



15. What is the domain and range of the rational function $f(x) = \frac{-22+x}{25+x}$?

A. D $(-\infty, -22) \cup (-22, \infty)$; R $(-\infty, 1) \cup (1, \infty)$

B. D $(-\infty, -25) \cup (-25, \infty)$; R $(-\infty, 1) \cup (1, \infty)$

C. D $(-\infty, -25) \cup (-25, \infty)$; R $(-\infty, -25) \cup (-25, \infty)$

D. D $(-\infty, -22) \cup (-25, \infty)$; R $(-\infty, -25) \cup (-22, \infty)$

Jumpstart

Activity 1. Do the Jogging Exercise

Source: https://www.vectorstock.com/royalty-free-vector/boy-running-kid-marathon-runner-or-a-boy-running-vector-25869498

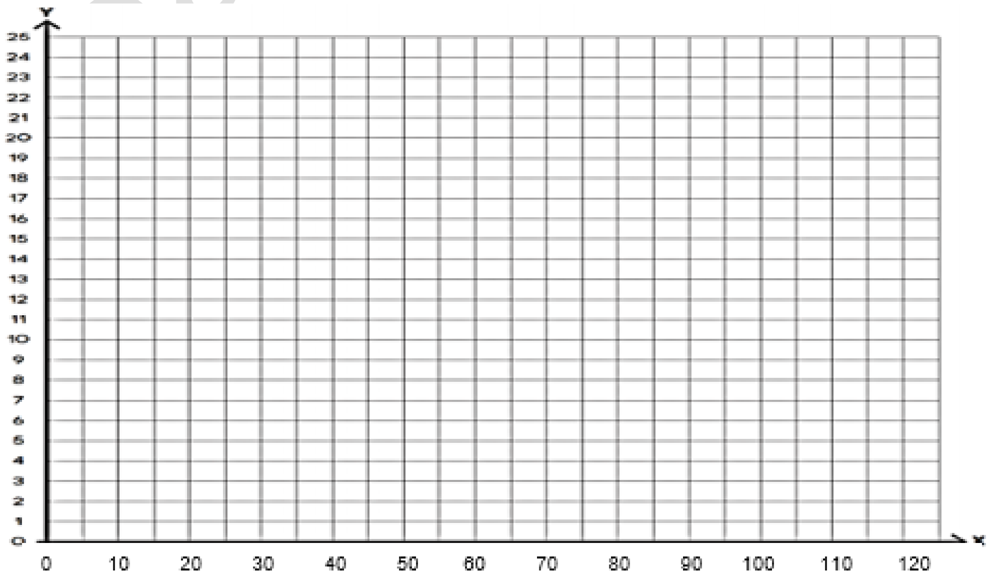
In the morning, as you wake up. I want you to have an exercise. You will jog in your community (following health protocols of the DOH and IATF). I want you to mark and record your time (in minutes) as you jog for at most 100 meters. You will write down your progress in a piece of paper following the table below.

Remember that d is the distance you will travel, and t will be the time you will travel the d-mark.

d *(meters)*	10	20	40	50	60	80	100
t (minutes)							

Did you feel energized?

After recording your distance and time, I want you to plot this in a Cartesian plane.



Discover

Represent real-life situations using rational functions

A **polynomial function** p of degree n is a function that can be written in the form

$$\mathbf{p(x)} = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdot\cdot\cdot + a_1 x + a_0$$

where $a_0, a_1, \ldots, a_n \in \mathbb{R}$, $a_n \neq 0$, and n is a positive integer. Each summand is a term of the polynomial function. The constants $a_0, a_1, a_2 \ldots, a_n$ are the **coefficients**. The **leading coefficient** is a_n. The leading term is $a_n x^n$ and the constant term is a_0.

Here are some examples of polynomial functions of particular degree together with their names.

Polynomial function	**Degree**	**Special name**
$f(x) = 4$	0	Constant function
$f(x) = 3x - 1$	1	Linear function
$f(x) = 2x^2 + 4x - 6$	2	Quadratic function
$f(x) = x^3 + 2x^2 - x + 1$	3	Cubic function

A **rational function** is a function of the form $\mathbf{f(x)} = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions, and q(x) is not the zero function. The domain of f(x) is all values of x where **q(x) ≠ 0**.

Real-world relationships that can be modeled by rational functions. Unlike polynomial functions, **rational functions may contain a variable in the denominator**.

Example 1: An object is to travel 10 meters. Express velocity v as a function *v(t)* of travel time *t*, in seconds.

Solution: The following table of values shows v for various values of *t*.

The function $v(t) = \frac{10}{t}$ can represent v as a function of *t*.

t (seconds)	1	2	4	5	10
v (meters per second)	10	5	2.5	2	1

Example 2: Suppose that $\mathbf{c(t)} = \frac{5t}{t^2+1}$ (in mg/mL) represents the concentration of a drug in a patient's bloodstream t hours after the drug was administered. Construct a table of values for c(t) for t = 1, 2, 5, 10. Round off answers to three decimal places.

Solution: The following table of values shows c for various values of t.

t	0	1	2	5	10
c(t)	0	2.5	2	0.962	0.495

Rational function, equation and inequality

Rational expression is an expression that can be written as a ratio of two polynomials. A **polynomial** is an expression consisting of variables (such as x and y) and coefficients with one or more than one term and variables, examples of it are 1, x^3, $3x^2 - x + 1$ and $x^3 + 2xyz^2 - yz + 1$.

These are the conditions when an expression is not considered as a polynomial:

1. The variable of any term has a negative exponent. ($\boldsymbol{4x^{-3}}+2x^2 - 5$)

2. The variable of any term is inside the radical symbol. ($4x^2 - \sqrt{\boldsymbol{x}}$)

3. The variable of any term has a fraction as exponent ($\boldsymbol{x^{\frac{2}{3}}}$ + 3x - 1)

Therefore, if an expression (whether the numerator/denominator) is not a polynomial, then it is not a rational expression.

A **rational equation** is an equation involving rational equations. It only uses = symbol.

A **rational inequality** is a rational expression combines with any of these inequality symbols; ≤ ,≥ ,< ,or >.

A **rational function** is a function of the form *f(x)* = $\frac{p(x)}{q(x)}$ where *p(x)* and *q(x)* are polynomial functions, and *q(x)* is not a zero function. The domain of *f(x)* is all values of x where ***q(x) ≠ 0***.

Use the table below to show how to distinguish among rational equations, rational inequalities and rational function.

	Rational Equation	**Rational Inequality**	**Rational Function**
Examples	$\frac{3}{x} - \frac{2}{3x} = \frac{1}{x}$ $\frac{x^3 + 3x - 2}{x} = \frac{3}{x+2}$	$\frac{4}{x^2 - 3x + 4} \geq \frac{3}{x}$ $\frac{x-2}{4x} < 5$	$f(x) = \frac{x^2-2x-3}{x+2}$ or $y = \frac{x^2-2x-3}{x+2}$
Explanation	The given examples are rational equations because it involves the equal sign (=) that shows the equality of two rational expressions.	The given examples are rational inequalities because it involves the inequality signs (≥, <) that shows the inequality of two rational expressions.	The given examples are rational functions because these are functions function of the form of $f(x) = \frac{p(x)}{q(x)}$ and $y = \frac{p(x)}{q(x)}$ where *p(x)* and *q(x)* are polynomials, and *q(x)* is not the zero function.

Solving Rational Equations

To solve an equation or inequality in one variable such as x, means to find all values of x for which the equation or inequality is true.

Procedure in Solving Rational Equations

1. Determine the least common denominator
2. Eliminate denominators by multiplying each term of the equation by the least common denominator. Note that eliminating denominators may introduce extraneous solutions. An extraneous solution yields an undefined expression when substituted to the equation.
3. Check the solutions of the transformed equations with the original equation.

Example1: Solve for x: $\frac{3}{4} = \frac{x}{4}$

Since the denominators of each expression is the same, the numerators must be equivalent also. This means that x = 3.

Therefore, the answer is **x = 3**.

Example 2: Solve for x: $\frac{2}{x}$ - $\frac{3}{2x} = \frac{1}{5}$

Step 1. Determine the LCD since it has different denominators. LCD is the least common number/expression divisible to all the denominators.

LCD: **10x**

Step 2. Eliminate the denominators by multiplying each term of the equation by the LCD.

$10x\left(\frac{2}{x}\right) - 10x\left(\frac{3}{2x}\right) = 10x\left(\frac{1}{5}\right)$

20 - 15 = 2x

5 = 2x

$\boldsymbol{x = \frac{5}{2}}$

Step 3. Check the solutions of the transformed equations with the original equation.

$$\frac{2}{x} - \frac{3}{2x} = \frac{1}{5} \quad \rightarrow \quad \frac{2}{(\frac{5}{2})} - \frac{3}{2(\frac{5}{2})} = \frac{1}{5} \quad \rightarrow \quad \frac{4}{5} - \frac{3}{5} = \frac{1}{5} \quad \rightarrow \quad \frac{1}{5} = \frac{1}{5}$$

Therefore, **x =** $\frac{5}{2}$ is a solution.

Example 3: Solve for x: $\frac{x}{x+2}$ - $\frac{1}{x-2} = \frac{8}{x^2-4}$

Step 1. Determine the LCD since it has different denominators. LCD is the least common number/expression divisible to all the denominators.

LCD: $\boldsymbol{x^2}$ **- 4** or **(x - 2) (x + 2)**

Step 2. Eliminate the denominators by multiplying each term of the equation by the LCD

$$(x-2)(x+2)\cdot\frac{x}{x+2}-(x-2)(x+2)\cdot\frac{1}{x-2}-[(x-2)(x+2)]\left(\frac{8}{(x-2)(x+2)}\right)$$

$$(x-2)x-(x+2)=8$$

$$x^2-3x-10=0$$

$$x^2-3x-10=0$$

$$(x-5)\ (x+2)=0$$

$$x-5=0\ /\ x+2=0$$

x = 5 or x = -2

Step 3. Check the solutions of the transformed equations with the original equation.

If $x=-2$, then

$\frac{x}{x+2}$ - $\frac{1}{x-2}=\frac{8}{x^2-4}\rightarrow\frac{-2}{-2+2}$ - $\frac{1}{-2-2}=\frac{8}{(-2)^2-4}$ $\rightarrow$ **undefined** because the denominator of the first term will be 0. Therefore, -2 is an example of extraneous solution.

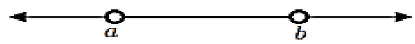
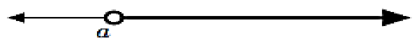
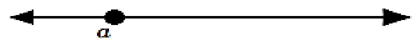
If $x=5$, then

$\frac{x}{x+2}$ - $\frac{1}{x-2}=\frac{8}{x^2-4}\rightarrow\frac{5}{5+2}$ - $\frac{1}{5-2}=\frac{8}{5^2-4}\rightarrow\frac{5}{7}$ - $\frac{1}{3}=\frac{8}{21}\rightarrow\frac{8}{21}=\frac{8}{21}$

Therefore, **x = 5** is a solution.

Solving Rational Inequalities

Solving inequalities is like solving equations. The solution set of an inequality consists of a range of values. The set of all solutions can be represented using set notations or interval notation. These notations are presented in the table below:

Interval	Set Notation	Graph
(a,b)	$\{x \mid a<x<b\}$	
$[a,b]$	$\{x \mid a\le x\le b\}$	
$[a,b)$	$\{x \mid a\le x<b\}$	
$(a,b]$	$\{x \mid a<x\le b\}$	
(a,∞)	$\{x \mid a<x\}$	
$[a,\infty)$	$\{x \mid a\le x\}$	
$(-\infty,b)$	$\{x \mid x<b\}$	
$(-\infty,b]$	$\{x \mid x\le b\}$	
$(-\infty,\infty)$	$\mathbb{R}$ (set of all real numbers)	

Procedure in solving rational inequalities

1. Rewrite the inequality as a single fraction on one side of the inequality symbol and 0 on the other side.

2. Determine over what intervals the fraction takes on positive and negative values. Locate the x-values for which the rational expression is zero or undefined (factoring the numerator and denominator is a useful strategy)

3. Mark the numbers found in (i) on a number line. Use a shaded circle to indicate that the value is included in the solution set and hallow circle to indicate that the value is excluded. These numbers partition the number line into intervals.

4. Select a test point within the interior of each interval in step 3. The sign of the rational expression at this test point is also the sign of the rational expression at each interior point in the interval.

5. Summarize the intervals containing the solutions.

TAKE NOTE: It is **NOT** valid to multiply both sides of an inequality by a variable. Recall that

- Multiplying both sides of an inequality by a positive number ***retains*** the direction of the inequality, and
- Multiplying both sides of an inequality by a negative number ***reverses*** the direction of the inequality.

Since the sign of a variable is unknown, then it is not valid to multiply both sides of an inequality by a variable.

Example 1: Solve the inequality $\frac{2x}{x+1} \geq 1$

Step 1. Rewrite the inequality as a single fraction on one side of the inequality symbol and 0 on the other side.

To make the one side equal to zero, we will subtract 1 on both side

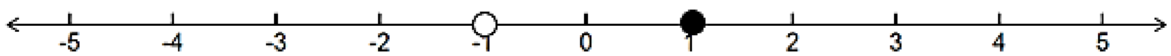
$$\frac{2x}{x+1} - 1 \geq 1 - 1$$

$$\frac{2x}{x+1} - 1 \geq 0 \quad \rightarrow \quad \frac{2x-(x+1)}{x+1} \geq 0 \quad \rightarrow \quad \frac{\boldsymbol{x-1}}{\boldsymbol{x+1}} \geq \mathbf{0}$$

Step 2. Determine over what intervals the fraction takes on positive and negative values. Find the critical values which will be used in locating intervals by setting the numerator and denominator to 0.

Numerator: $x - 1 = 0 \rightarrow \mathbf{x = 1}$
Denominator: $x + 1 = 0 \rightarrow \mathbf{x = -1}$

Step 3. Mark the numbers found in step 2 on a number line. Use a **shaded circle** to indicate that the value is included in the solution set and **hollow circle** to indicate that the value is excluded. These numbers partition the number line into intervals.



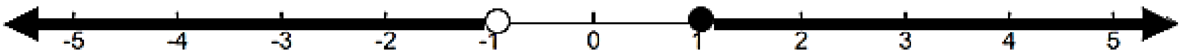
- The value x = 1 is included in the solution since it makes the fraction equal to zero and the inequality true because $0 \geq 0$.
- The value x = -1 is excluded in the solution since it makes the inequality undefined.

Step 4. Select a test point within the interior of each interval in step 3. The sign of the rational expression at this test point is also the sign of the rational expression at each interior point in the interval.

Interval	$x < -1$	$-1 < x < 1$	$x > 1$
Test point	$x = -2$	$x = 0$	$x = 2$
$x - 1$	$-$	$-$	$+$
$x + 1$	$-$	$+$	$+$
$\frac{x-1}{x+1}$	$+$	$-$	$+$

Step 5. Summarize the intervals containing the solutions.

Since we are looking for the intervals where the fraction is zero or positive, we determine the solution intervals to be x < 1 and x ≥ 1. Plot these intervals on the number line.



The solution set is $\{\mathbf{x} \in \mathbf{R} \mid \boldsymbol{x} < -\mathbf{1} \text{ } \boldsymbol{or} \text{ } \boldsymbol{x} \geq \mathbf{1}\}$. It can also be written using interval notation: $(-\infty, -\mathbf{1}) \cup [\,\mathbf{1}, +\infty\,)$.

Example 2: Solve the inequality $\frac{3}{x-2} < \frac{1}{x}$

Step 1. Rewrite the inequality as a single fraction on one side of the inequality symbol and 0 on the other side.

To make the one side equal to zero, we will subtract $\frac{1}{x}$ on both sides.

$$\frac{3}{x-2} < \frac{1}{x} \rightarrow \frac{3}{x-2} - \frac{1}{x} < 0 \rightarrow \frac{3x-(x-2)}{x(x-2)} < 0 \rightarrow \frac{2x+2}{x(x-2)} < 0 \rightarrow \boldsymbol{\frac{2x+2}{x(x-2)} < 0}$$

Step 2. Determine over what intervals the fraction takes on positive and negative values. Find the critical values which will be used in locating intervals by setting the numerator and denominator to 0.

Numerator: 2x + 2 = 0 → **x = -1**
Denominator: **x = 0**
Denominator: x - 2 = 0 → **x = 2**

Step 3. Mark the numbers found in step 2 on a number line. Use a **shaded circle** to indicate that the value is included in the solution set and **hollow circle** to indicate that the value is excluded. These numbers partition the number line into intervals.

- The value x = -1 is excluded in the solution since it makes the fraction equal to zero. The inequality is false because 0 is not lesser than 0.
- The value x = 0 is excluded in the solution since it makes the inequality undefined.

- The value x = 2 is excluded in the solution since it makes the inequality undefined.

Step 4. Select a test point within the interior of each interval in step 3. The sign of the rational expression at this test point is also the sign of the rational expression at each interior point in the interval.

Interval	$x < -1$	$-1 < x < 0$	$0 < x < 2$	$x > 2$
Test point	$x = -2$	$x = -\frac{1}{2}$	$x = 1$	$x = 3$
$2(x+1)$	−	+	+	+
x	−	−	+	+
$x-2$	−	−	−	+
$\frac{2(x+1)}{x(x-2)}$	−	+	−	+

Step 5. Summarize the intervals containing the solutions.

Since we are looking for the intervals where the fraction is negative, we determine the solution intervals to be x < -1 and 0 < x < 2. Plot these intervals on the number line.

The solution set is $\{\mathbf{x} \in \mathbf{R} \mid \boldsymbol{x} < -\mathbf{1}\ \boldsymbol{or}\ \mathbf{0} < \boldsymbol{x} < \mathbf{2}\}$. It can also be written using interval notation: $(-\infty, -\mathbf{1}) \cup (\mathbf{0}, \mathbf{2})$.

Representing a rational function through its: (a) table of values, (b) graph, and (c) equation

A **rational function** is a function of the form ***f(x)*** **=** $\frac{p(x)}{q(x)}$ where *p(x)* and *q(x)* are polynomial functions, and *q(x)* is not a zero function. The domain of *f(x)* is all values of x where ***q(x) ≠ 0***.

To fully understand rational functions, let's look on these examples.

Example 1: Average speed (or velocity) can be computed by the formula $s = \frac{d}{t}$. Consider a 100-meter track used for foot races. The speed of a runner can be computed by taking the time for him to run the track and applying it to the formula $s = \frac{100}{t}$, since the distance is fixed at 100 meters.

A. Represent the speed of a runner as a function of the time it takes to run 100 meters in the track.

Solution: Since the speed of a runner depends on the time it takes to run 100 meters; we can represent speed as a function of time.

Let x represent the time it takes to run 100 meters. Then the speed can be represented as a function s(x) as follows:

$$s(x) = \frac{100}{\mathrm{x}}$$

Observe that it is like the structure to the formula $s = \frac{\mathrm{d}}{\mathrm{t}}$ relating speed, distance, and time. Continuing the scenario above, construct a table of values for the speed of a runner against different run times.

A table of values can help us determine the behavior of a function as the variable changes.

The current world record (as of October 2015) for the 100-meter dash is 9.58 seconds set by the Jamaican Usain Bolt in 2009. We start our table of values at10 seconds.

Let x be the run time and *s(x)* be the speed of the runner in meters per second, where $s(x) = \frac{100}{\mathrm{x}}$. The table of values for run times from 10 seconds to 20 seconds is as follows:

x	10	12	14	16	18	20
	$s(x) = \frac{100}{\mathrm{x}}$ $s(x) = \frac{100}{10}$	$s(x) = \frac{100}{\mathrm{x}}$ $s(x) = \frac{100}{12}$	$s(x) = \frac{100}{\mathrm{x}}$ $s(x) = \frac{100}{14}$	$s(x) = \frac{100}{\mathrm{x}}$ $s(x) = \frac{100}{16}$	$s(x) = \frac{100}{\mathrm{x}}$ $s(x) = \frac{100}{18}$	$s(x) = \frac{100}{\mathrm{x}}$ $s(x) = \frac{100}{20}$
s(x)	10	8.33	14	6.25	5.56	5

B. From the table above we can observe that the speed decreases with time. Plot the points on the table of values on a Cartesian plane. Determine if the points on the function $s(x) = \frac{100}{\mathrm{x}}$ follow a smooth curve or a straight line.

Solution: Assign points on the Cartesian plane for each entry on the table of values above:

A (10,10) B (12,8.33) C (14, 7.14) D (16, 6.25) E (18,5.56) F (20,5)

Plot these points on the Cartesian plane and observe that it follows a smooth curve.

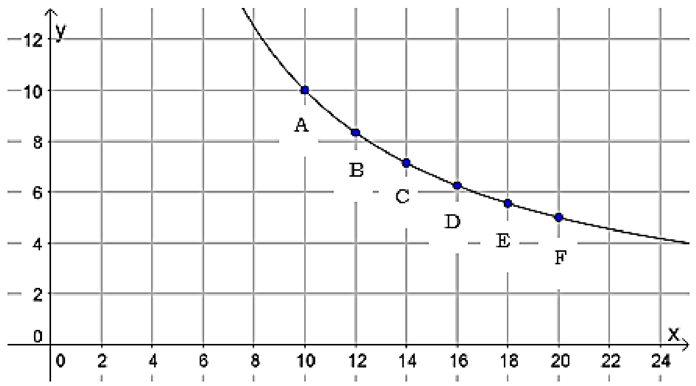


Figure 1. Graph of $s(x) = \frac{100}{\mathrm{x}}$

For the 100-meter dash scenario, we have constructed a function of speed against time, and represented our function with a table of values and a graph. The previous example is based on a real-world scenario and has limitations on the values of the x-variable. For example, a runner cannot have negative time (which would mean he is running backwards in time!), nor can he exceed the limits of human physiology (can a person run 100-meters in 5 seconds?). However, we can apply the skills of constructing tables of values and plotting graphs to observe the behavior of rational functions

Example 2: Represent the rational function given by $f(x) = \frac{x-1}{x+1}$ using a table of values and plot a graph of the function by connecting points.

Solution: Since we are now considering functions in general, we can find function values across more values of x. Let us construct a table of values for some x-values from -10 to 10:

x	-10	-8	-6	-4	-2	0	2	4	6	8	10
f(x)	1.22	1.29	1.4	1.67	3	-1	0.33	0.6	0.71	0.78	0.82

Plotting the points in cartesian plane and connecting the points we get:

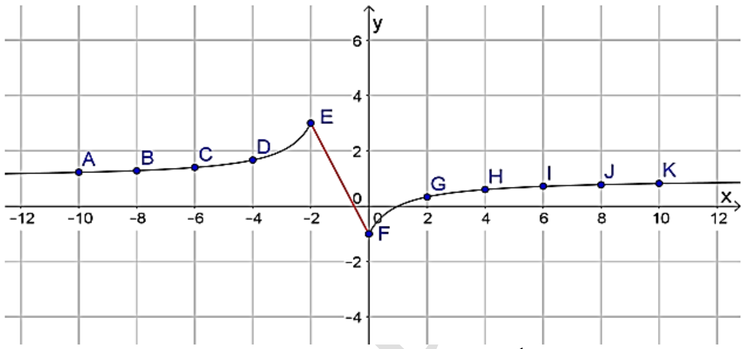


Figure 2. Graph of $f(x) = \frac{x-1}{x+1}$

Why would the graph unexpectedly break the smooth curve and jump from point E to point F? The answer is that it doesn't break! Let us look at the function again:

$$f(x) = \frac{x-1}{x+1}$$

Observe that the function will be undefined at x = -1. This means that there cannot be a line connecting point E and point F as this implies that there is a point in the graph of the function where x = -1. We will cover this aspect of graphs of rational functions in a future lesson, so for now we just present a partial graph for the function above as follows:

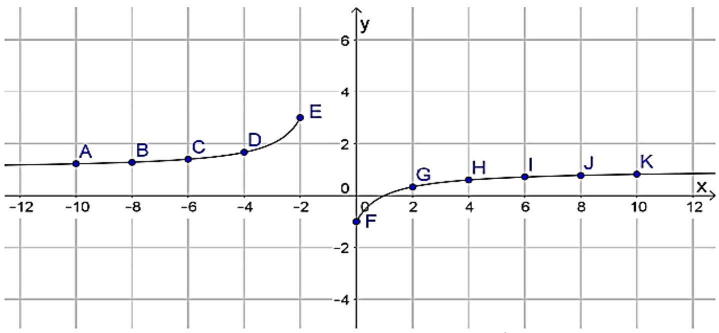


Figure 3. Graph of $f(x) = \frac{x-1}{x+1}$

Domain and Range of Rational Functions

The **domain** of a function is the set of all values that the variable x can take. The **range** of a function is the set of all values that the variable *f(x)* can take.

There are two main ways to write domains: **interval notation** and **set notation.**

Interval notation used parenthesis or brackets to imply where the function is defined. In the case of our example, we would write our domain using interval notation in the following way:

D: $(-\infty, 0) \cup (0, \infty)$, read as:

"The domain is from negative infinity to less than 0 and from greater than 0 to positive infinity."

It means that from negative infinity up to 0 we can plug anything into our function and (the ∪ is called a union and it means 'and') from greater than 0 to positive infinity we can plug in anything.

Set notation uses sets to say explicitly where the function is or isn't defined. For instance, we would use set notation in the following way:

D: $\{x \mid x \neq 0\}$, read as:

"The domain is set of all real numbers x, such that x is not equal to 0,"

The vertical line in $\{x \mid x \neq 0\}$ means 'such that'.

Before we look at the given example, let's talk for a little bit about range. Range is a little trickier to find than domain. Most of the time, we're going to look at the graph of the function to determine its range.

Example 1: Consider the function $f(x) = \frac{x-2}{x+2}$. Find its a. domain and b. range.

Solution: Observe that the function is undefined at x = -2. This means that x = -2 is not a part of the domain of *f(x).* In addition, o other values of x will make the function undefined.

a. Domain:

Set notation: $D\!: \{\mathbf{x} \in \mathbf{R} \mid x \neq -2\}$.

("The domain is set of all real numbers x, such that x is not equal to -2")

Interval notation: D: **$(-\infty, -2)$ U $(-2, +\infty)$.**

("The domain is from negative infinity to less than -2 and from greater than -2 to positive infinity.")

b. Range: To find the range, let's graph the given function first.

x	-5	-4	-3	-2	-1	0	1	2	3
f(x)	2.33	-3	-5	Und	-3	-1	0.33	0	0.20

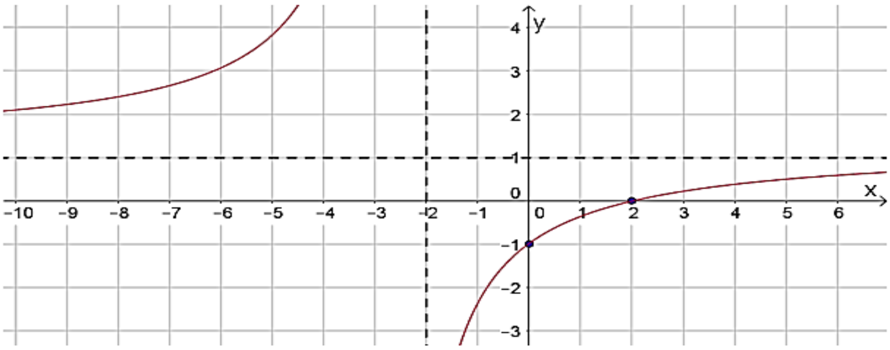
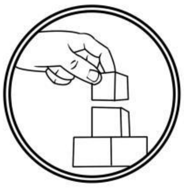


Figure 4. Graph of $f(x) = \frac{x-2}{x+2}$

Since the graph didn't touch the line y=1, then the range of *f(x)* is $(-\infty, \mathbf{1}) \cup (\mathbf{1}, +\infty)$.



Explore

Activity 1

Direction: Determine whether the given expression is a rational function, a rational equation, a rational inequality, or none of these. Write RF, RE, RI and NT. Write your solution in a separate paper.

_____1. $\frac{1+x}{x-2} = 4$

_____2. $5x \geq \frac{3}{x+5}$

_____3. $g(x) = \frac{2x-1}{x-2}$

_____4. $\frac{4-x}{3x} = \sqrt{x-4}$

_____5. $y = \frac{2+6x^2}{x-2}$ *y is represented by f(x)*

_____6. $\frac{3x^2+x-4}{x-2} = 1$

_____7. $\frac{10}{3x+1} < \frac{1}{2}$

_____8. $\sqrt{x+9} = 1$

_____9. $9x \geq \frac{9}{9x-5}$

_____10. $2m^{-3} + 1 = \frac{3}{4}$

Activity 2

Direction: Solve the value of x of the given rational equations and rational inequalities. Write your solution on a separate sheet of paper.

1. $\frac{x+2}{3} = \frac{2x-4}{2}$

2. $\frac{2x}{x+1} + \frac{5}{2x} = 2$

3. $\frac{(x+3)(x-2)}{(x+2)(x-1)} \geq 0$

4. $\frac{x+3}{x^2-5x+4} \geq 0$

Activity 3

Direction: Answer the following using the given rational function below. Write your solution on a separate sheet of paper.

a. Complete the table of values

b. Graph the given function. You can confirm your work using graphing software.

c. Find the domain

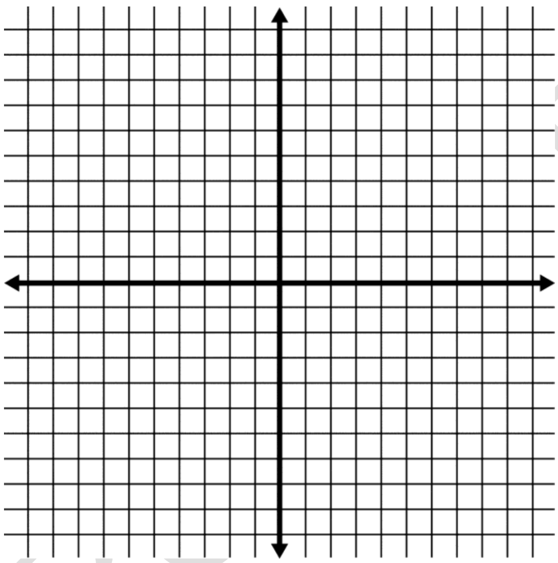
d. Find the range

$$f(x) = \frac{x^2 - 3x - 4}{x+1}$$

A. Table of values

x	-4	-3	-2	-1	0	1	2	3	4
y									

B. Graph



C. Domain

D. Range

Deepen

At this point, you are task to apply what you have learned about rational functions. Read each statement and answer what is being asked.

Activity 1

Mario produces facemasks for his friend's store. He has realized that, because of his fixed costs, his average cost per facemask depends on the number of facemasks he produces. The cost of producing x facemasks is given by

$$C(x) = 2.5 + 1.5x$$

a. Mario wants to figure out how much to charge his friend for the facemasks. He's not trying to make any money on the venture, but he wants to cover his costs. Suppose Mario made 100 facemasks. What is the cost of producing this number of facemasks? How much per facemask?
b. Mario is hoping to make more than 100 facemasks for his friends. Complete the table showing his costs at different levels of production.

Number of facemasks	10	100	200	300	500	1,000
Total Cost						
Cost per facemask						

c. Explain why the average cost per facemask levels off.
d. Find an equation for the average cost per facemask of producing x facemasks.
e. Find the domain of the average cost function.
f. Using the data points from your table above, sketch the graph of the average cost function. How does the graph reflect that the average cost levels off?

Rubrics for the Scoring of the Activity

CRITERIA	5	4	3	2
Understanding the problem	Identifies all the elements of the problem	Identifies most of the element of the problem	Identifies few elements of the problem	Response does not fit the given problem task
Applies procedure/ techniques	Apply all the procedure and techniques appropriately and in sequence	Apply all the procedure and techniques but not in sequence	Apply some of the procedure and techniques	Apply wrong procedure/ technique
Accuracy	Show correct computation and arrive to an accurate answer	Show computation with minimal error but generally correct	Show computation with many errors arriving an incorrect answer	Does not show any computation

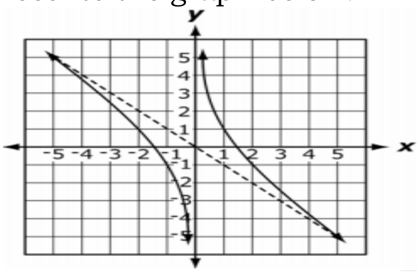
Gauge

Direction: Read each item carefully and answer what is being asked. Write your answer on a separate sheet of paper.

1. What symbol is used to mark the number line if the value of x is excluded in the solution?
   A. An arrow B. Hollow circle C. Letter x D. Shaded circle
2. Which of the following situations represent a rational function?
   A. The velocity of the jeep
   B. The speed of a runner to finish the race
   C. The subsidy given by the government to its people
   D. All of the above
3. Which of the following should be determined when adding and subtracting rational expressions with different denominators?
   A. Lowest common factor B. Least common denominator
   C. Greatest common factor D. Greatest common denominator
4. Which of the following symbols is NOT used in rational inequality?
   A. > B. < C. ≥ D. =
5. Which of the following is NOT example of rational inequality?
   A. $\frac{2x+1}{x} \leq \frac{2x}{5}$ B. $\frac{x+1}{x-5} = \frac{8}{5x+1}$
   C. $\frac{x+1}{x+7} < 1$ D. $\frac{x^2-5}{x-2} \geq 0$
6. Which of the following is an example of rational equation?
   A. $\frac{\mathbf{5x-1}}{\mathbf{x+3}} = \frac{\mathbf{x}}{\mathbf{5}}$ B. $\frac{5x}{x^{\frac{3}{4}}-3} = \frac{3}{x+1}$
   C. $\frac{1}{x-2} > \frac{2}{3x-9}$ D. $\frac{x^2-5}{x-2} = f(x)$
7. Which of the following steps is/are included in solving rational inequality?
   I. Simplifying equation
   II. Factoring the equation
   III. Finding the critical values
   IV. Checking the answer from the original equation

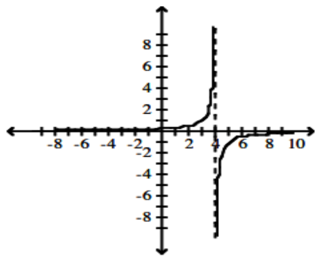
   A. I only B. III & IV C. I, II & III D. I, II, III & IV
8. Which of the following is NOT a part of the procedures in solving rational equations?
   A. Checking the answer B. Simplifying the equation
   C. Determining the LCD D. Identifying the critical values
9. What is the Least Common Denominator (LCD) of the given rational equation below?
$$\frac{4-2x}{2x} = \frac{2+5x}{x-2}$$
   A. $2x^2$ – 2x B. $2x^2$ – 4x C. $2x^2$ + 4x D. $2x^2$ + 2x

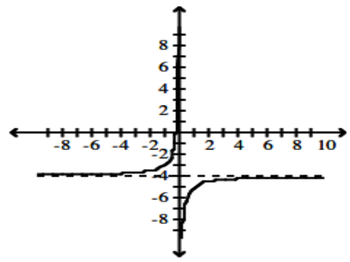
10. Which of the following represents -3 and 2 as part of the solution?
    A. (-3, 2) B. (-3, 2] C. [-3, 2) D. [-3, 2]
11. Which of the following is TRUE about [-1, 5)?
    A. -1 is a solution and 5 is not
    B. 5 is a solution and -1 is not
    C. -1 is not a solution and 5 is a solution
    D. 5 is not a solution and -1 is a solution
12. Given the inequality: $\frac{x+3}{x-1} \geq 0$, what are the critical values?
    A. -3 and -1 B. -3 and 1 C. 3 and -1 D. 3 and 1
13. Which function represents the graph below?

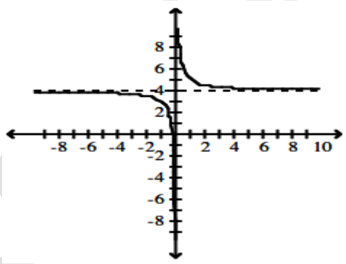


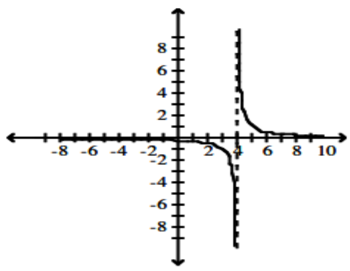
A. $f(x) = \frac{-2+x^2}{x}$ B. $f(x) = \frac{-2-x^2}{x}$
C. $f(x) = \frac{2-x^2}{x}$ D. $f(x) = \frac{2+x^2}{x}$

14. What is the graph of the rational function $f(x) = = \frac{4x+1}{x}$?

A.


B.


C.


D.


15. What is the domain and range of the rational function $f(x) = \frac{x^2-x-6}{x+2}$?
    A. D $(-\infty,-2) \cup (-2,\infty)$; R $(-\infty,3) \cup (3,\infty)$
    B. D $(-\infty,-2) \cup (-2,\infty)$; R $(-\infty,-3) \cup (-3,\infty)$
    C. D $(-\infty,-2) \cup (-2,\infty)$; R $(-\infty,-2) \cup (-2,\infty)$
    D. D $(-\infty,-2) \cup (-2,\infty)$; R $(-\infty,-5) \cup (-5,\infty)$

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Image

Rommeo79, "Boy running kid marathon runner or a boy running vector image," VectorStock, 2021, https://www.vectorstock.com/royalty-free-vector/boy-running-kid-marathon-runner-or-a-boy-running-vector-25869498.

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