Senior High School

General Mathematics
Module 8:
Problem Solving Involving Logarithmic Functions, Equations, and Inequalities

Government Property
NOT FOR SALE

AIRs - LM

General Mathematics
Module 8: Problem Solving Involving Logarithmic Functions, Equations, and Inequalities
Second Edition, 2021

Copyright © 2021
La Union Schools Division
Region I

All rights reserved. No part of this module may be reproduced in any form without written permission from the copyright owners.

Development Team of the Module

Authors: Vener Dan Apilado
Olivia A. Cagujas
Raymart Gaco
Remedios Labiano
Editor: SDO La Union, Learning Resource Quality Assurance Team
Content Reviewers: Catherine F. Carbonell
Lovely G. Tadije
Language Reviewer: Sherlyn A. De la Peña
Reviewers: Ryan V. Espiritu
Jiezyl Jamaica M. Aquino
Reynan O. Ibay
Illustrator: Ernesto F. Ramos, Jr.
Christian Bautista
Design and Layout: Angela Pauline C. Ganuelas

Management Team:

Atty. Donato D. Balderas Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, Ph.D
Assistant Schools Division Superintendent
German E. Flora, Ph.D, *CID Chief*
Virgilio C. Boado, Ph.D, *EPS in Charge of LRMS*
Erlinda M. Dela Peña, Ed.D, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

Printed in the Philippines by: ________________________

Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

Ready to Print

SHS

General Mathematics
Module 8:
Problem Solving Involving Logarithmic Functions, Equations, and Inequalities

Ready to Print

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Ready to Print

Target

In the previous module, you already have an idea about logarithmic function which is the inverse of exponential function.

Logarithmic functions can be written as $\boldsymbol{f(x) = log_b\, x}$ (*read as f of x is equal to logarithm of x to the base b*). The variable **b,** which represents any positive number not equal to 1, is called the base of the logarithmic function.

In this module, you will learn more about the different properties of logarithmic functions which can be presented in table of values, graphs, and equation. Moreover, you will know how to determine the domain and range as well as the intercepts, zeroes, and asymptotes of logarithmic functions. There will also be a discussion on solving problems involving logarithmic functions, equations, and inequalities.

After going through this learning material, you are expected to:

1. represent logarithmic functions through: (a) table of values, (b) graph, and (c) equation **(M11GM-Ii-2)**,
2. find the domain and range of logarithmic function **(M11GM-Ii-3)**,
3. determine the intercepts, zeroes, and asymptotes of logarithmic function (**M11GM-Ii-4**); and
4. solve problems involving logarithmic functions, equations and inequalities **(M11GM-Ij-2)**.

Learning Objectives:

1. identify the equation of a logarithmic function
2. describe the graph of a logarithmic function
3. write the domain and range of a logarithmic function
4. identify the intercepts, zeroes, and asymptotes of logarithmic functions
5. show step-by-step process on solving logarithmic functions, equations and inequalities

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper

Ready to Print

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

_____1. Which of the following is true about the domain of logarithmic function?
The domain of logarithmic function is the set of…

A. Real numbers. B. Rational numbers.
C. All positive real numbers. D. Negative real numbers.

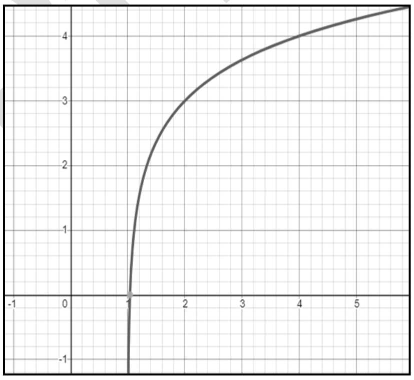
_____2. Which of the following is true about the range of logarithmic function?
The range of logarithmic function is the set of…

A. Real numbers. B. Rational numbers.
C. All positive real numbers. D. Negative real numbers.

_____3. Which of the following logarithmic function has this domain, $D = \{x|x < 5\}$?

A. $f(x) = 2\log_3(10 - 2x) + 5$ B. $f(x) = 2\log_3(2x + 10) + 5$
C. $f(x) = 2\log_3(x + 5)$ D. $f(x) = 2\log_3(x - 5)$

_____4. Given the graph on the right, what is the range?



A. $R = \{y|y > 1\}$
B. $R = \{y|y < 1\}$
C. $R = \{y|y > 0\}$
D. $R = \{y|y \in \mathbb{R}\}$

_____5. Given the same graph, what is the domain?

A. $D = \{x|x > 0\}$
B. $D = \{x|x > -1\}$
C. $D = \{x|x > 1\}$
D. $D = \{x|x \in \mathbb{R}\}$

_____6. Which of the following refers to a line on the graph of a function representing a value toward which the function may approach but does not reach?

A. Intercepts B. Slope
C. Zeros of function D. Asymptotes

For numbers 7 – 10, consider the function $\boldsymbol{f(x) = log_3(x - 6) - 2}$.

_____7. What is the vertical asymptote of the function?

A. x = 3 B. x = 6
C. x = –3 D. x = –6

_____8. What is the x - intercept of the function?

A. (0, 15) B. (1, 0)
C. (15, 0) D. (0, 1)

_____9. What is the y - intercept of the function?

A. (0, 7) B. (1, 0)
C. (7, 0) D. None

_____10. What is the zero of the function?

A. 0 B. 1
C. 2 D. 15

_____11. How much more intense is $10^3\ watts/m^2$ sound than the least audible sound a human can hear?

A. 1 000 000 times B. 1 000 000 000 times
C. 1 000 000 000 000 times D. 1 000 000 000 000 000 times

Ready to Print

_____12. An earthquake released approximately 10^5 *joules*. How much more energy does this earthquake release than the reference earthquake?

A. 9.38 times B. 8.39 times

C. 3.98 times D. 3. 89 times

_____13. If the exponential function $2^x = y$ is translated into logarithmic function. Which of the following is the correct translation?

A. $x = \log_2 y$ B. $y = \log_2 x$

C. $2 = \log_x y$ D. $x = \log_2 xy$

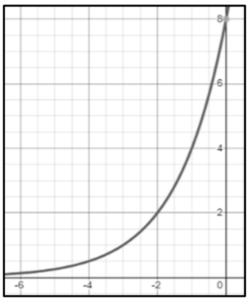
_____14. Given the table of values below to the logarithmic function $f(x) = \log_2 x$, what will be the value on the missing part of the table?

x	1	2	4	8
***f* (x)**	**?**	1	2	3

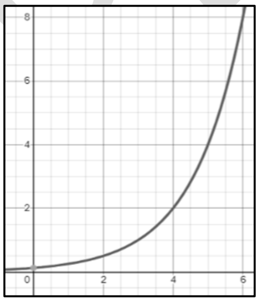
A. 0 B. 1 C. 2 D. 3

_____15. Which of the following is the correct graph of the function, $\boldsymbol{f(x) = \log_2 x}$?

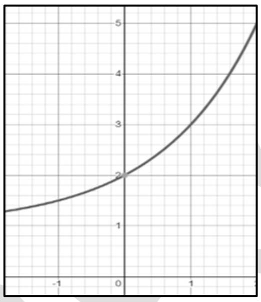
A.



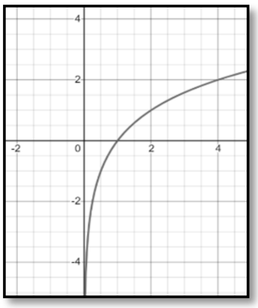
B.



C.



D.



Ready to Print

Jumpstart

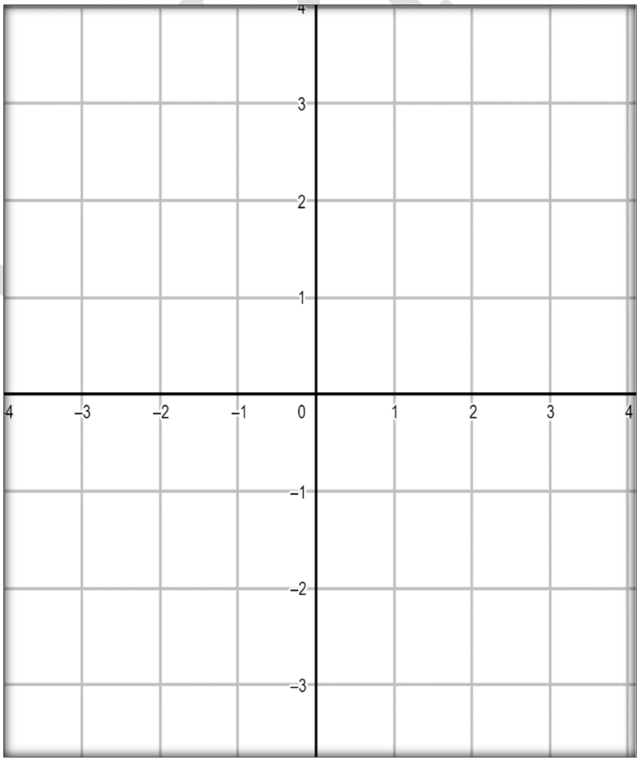
For you to understand the lesson well, do the following activities. Have fun and good luck!

Activity 1: YOU COMPLETE ME!

Directions: Construct a table of values for the given function below and sketch its graph. Then, answer the questions that follow.

Given: $f(x) = \log_2 x$

X	f(x)
$\frac{1}{8}$	$f\left(\frac{1}{8}\right) = \log_2\left(\frac{1}{8}\right) = -3$
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	
4	



Process Questions:

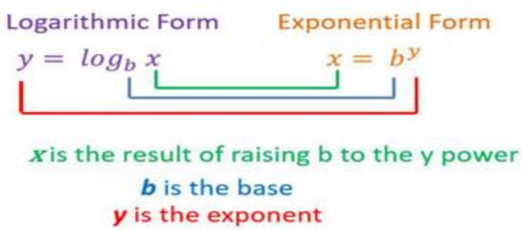
1. What is the equation of the logarithmic function? ___________________________
2. What are the values of f(x) given the values of x? _______________________________
3. What is the domain and range of the function? __________________________
4. At what point does the graph intersect the x – axis? _____________________
5. What happens to the graph of the function as x approaches 0 from the right? ____

Discover

Representation of Logarithmic Function Through Table of Values, Graph and Equation

A useful family of functions that is related to exponential functions is the logarithmic function. You have been calculating the result of b^x, and this gave us the exponential function. A logarithm is a calculation of the exponent in the equation $y = b^x$. Put another way, finding a logarithm is the same as finding the exponent to which the given base must be raised to get the desired value. The exponent becomes the output rather than the input.

A **logarithmic function** is a function of $\boldsymbol{y = \log_b x}$, $x > 0$ where $b > 0$ and $b \neq 1$.



The equation of the logarithmic function is read as "y is the logarithm of x to the base b". Take note that in the notation, b is the base, x is the result of raising b to the exponent y.

Example 1: The logarithmic equation $\mathbf{2 = \log_7 49}$ is read as "2 is the logarithm of 49 to the base 7" or "the logarithm of 49 to the base 7 is 2" which means that the exponent of 7 in order to get 49 is 2.

Example 2: The logarithmic equation $4 = \log_2 16$ is read as "the logarithm of 16 to the base 2 is 4" or "4 is the logarithm of 16 to the base 2" meaning the exponent of 2 is 4 to get 16.

Notice that the notation above is equivalent to $\boldsymbol{x = b^y}$. Thus, an equation in exponential form can be expressed in logarithmic form and vice versa. Study the examples that follow.

Example 3: Transform the following equations in logarithmic form.

a. $2^6 = 64$ b. $3^{-2} = \frac{1}{9}$ c. $(128)^{\frac{1}{7}} = 2$

Ready to Print

Solutions:

1. In $2^6 = 64$, the base is 2, the exponent is 6 and the power is 64. Thus, $2^6 = 64$ is equivalent to $6 = \log_2 64$ or $\log_2 64 = 6$

2. In $3^{-2} = \frac{1}{9}$, the base is 3, the exponent in -2 and the power is $\frac{1}{9}$. Hence, $3^{-2} = \frac{1}{9}$, is equivalent to $-2 = \log_3 \frac{1}{9}$ or $\log_3 \frac{1}{9} = -2$

3. In $128^{\frac{1}{7}} = 2$, the base is 128, the exponent is $\frac{1}{7}$ and the power is 2. Therefore $128^{\frac{1}{7}} = 2$ is equivalent to $\frac{1}{7} = \log_{128} 2$ or $\log_{128} 2 = \frac{1}{7}$

From the previous module, you have learned about the graphs of inverse functions, that is, the graphs of inverse functions are reflections of each other and that they are symmetrical about line y = x. Thus, the graph of a logarithmic function $y = \log_b x$ can be obtained from the graph of $y = b^x$ along the line y = x.

But before doing so, let's solve first for the value of x (as for the table of values) and then plot the points.

Example 1: Draw the graph of $y = \log_2 x$. Make a table of values.

Solution:

To draw the graph of $y = \log_2 x$, recall the graph of $y = 2^x$. Flip the graph of $y = 2^x$ about the line $y = x$. You should be able to observe that the two graphs contain the following integral values.

X	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

X	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y = \log_2 x$	-3	-2	-1	0	1	2	3

For example, the point (1,2) is on graph of $y = 2^x$ and the point (2,1) is on the graph of $y = \log_2 x$. Observe now that the graph of $y = \log_2 x$ is a reflection of the graph of exponential function $y = 2^x$ along the line $y = x$ which is the axis of symmetry. This can be seen in the given figure on the next page.

Ready to Pri

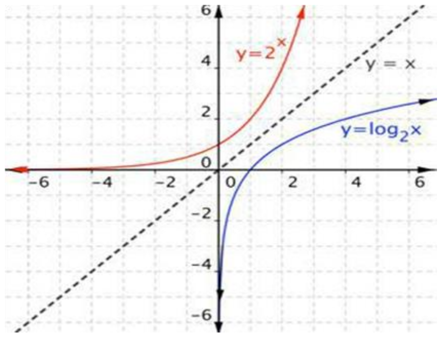


Figure 1. Graph of y = 2^x and y = $\log_2 x$

You can see from the graph that the *range* (y-values) of the exponential function (graph at the top) is the set of positive real numbers. Since the input and output have been switched, the *domain* (x-values) of the logarithmic function (graph at the bottom) is the set of positive real numbers.

Similarly, the *domain* of the exponential function is the set of all real numbers. The *range* of the logarithmic function is the set of all real numbers.

Example 2. Graph $\boldsymbol{f(x) = \log_3 x}$. Make a table of values when y is (-2, -1, 0, 1, 2).

Solution:

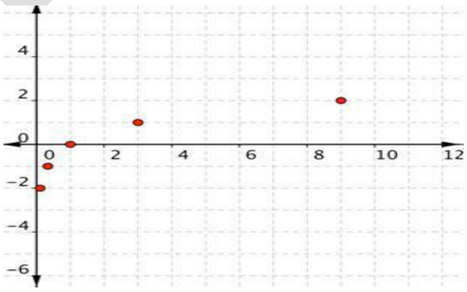
$f(x) = \log_3 x$ or $y = \log_3 x$ is the same as $3^y = x$. To make the calculations easier, convert the logarithm into an exponential equation.

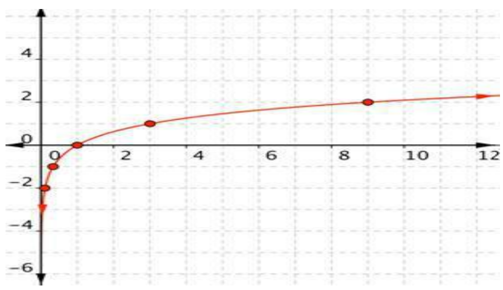
Start with a table of values. With logarithmic functions, it's usually easier to choose the y values instead of the x values. Be sure to include some negative values for y.

$3^y = x$					
y	-2	-1	0	1	2

$3^y = x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
y	-2	-1	0	1	2

Be careful with the negative exponent. Use the table as ordered pairs. Remember that the graph of the function will show all x-y correspondences, so any pair that could be in the table must be on the graph.





Ready to Print

Domain and Range of Logarithmic Functions

Recall your lesson about the domain and range. The **domain** is the ***set of all x-coordinates*** or the ***input***, while the **range** is the ***set of all y-coordinates*** or the ***output***.

For you to familiarize in finding the domain and range of logarithmic function, let us first create a table of values of the exponential function and observe the respective values of x and f(x) or y and you will connect it with logarithmic function.

TABLE OF VALUES

- $\boldsymbol{f(x) = 2^x}$

X	-3	-2	-1	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Observe that the set of x, the **domain,** is $\{-3,-2,-1,0,1,2,3\}$. The domain is composed of **real numbers**. Therefore, you can conclude that the domain of exponential function $\boldsymbol{f(x) = 2^x}$, is the ***set of real numbers***. You can also use set builder notation, $\mathbf{D} = \{\mathbf{x}|\mathbf{x} \in \mathbb{R}\}$**,** (*reads as "the set of all **x**'s such that **x** is an element/member of real numbers"*)**.**

Another is the set of $f(x)$ or y, the range, which is $\{\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 0, 1, 2, 4, 8\}$. The range is composed of **<u>only</u> positive real numbers**. Therefore, you can conclude that the range of exponential function $\boldsymbol{f(x) = 2^x}$, is the ***set of positive real number***. In set builder notation, $\mathbf{R} = \{\mathbf{y}|\mathbf{y} \in \mathbb{R}^+\}$, (*reads as "the set of all **y**'s such that **y** is an element/member of positive real numbers"*).

Since <u>*exponential function is the inverse of logarithmic function*</u>, you can use this idea for you to identify the domain and range of logarithmic function.

- $\boldsymbol{f(x) = \log_2 x}$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
	$f(x) = \log_2 x$ $f\left(\frac{1}{8}\right) = \log_2 \frac{1}{8}$	$f(x) = \log_2 x$ $f\left(\frac{1}{4}\right) = \log_2 \frac{1}{4}$	$f(x) = \log_2 x$ $f\left(\frac{1}{2}\right) = \log_2 \frac{1}{2}$	$f(x) = \log_2 x$ $f(1) = \log_2 1$	$f(x) = \log_2 x$ $f(2) = \log_2 2$	$f(x) = \log_2 x$ $f(4) = \log_2 4$	$f(x) = \log_2 x$ $f(8) = \log_2 8$
f(x)	-3	-2	-1	0	1	2	3

Notice that the domain and range of exponential function just **<u>interchanged</u>** in the domain and range of the logarithmic function.

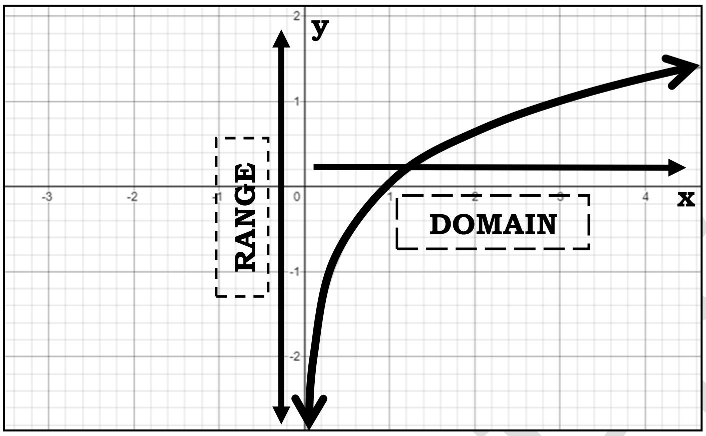
Therefore, we can conclude that the **domain** of logarithmic function is the ***set of positive real numbers*** while the **range** is the ***set of real numbers***.

Ready to Print

GRAPH

Another way to identify the domain and range of logarithmic function is through its graph.

Let us try to graph the logarithmic function $\boldsymbol{f(x) = \log_3 x}$



Based on the graph of the logarithmic function, the **domain**, which is the value of x, is only limited on the **positive side of the x-axis**, then the **domain** is the **set of positive real numbers**.

On the other hand, the **range**, which is the value of f(x) or y, **extends both in positive and negative side of the y-axis**, therefore the **range** is the **set of real numbers.**

EQUATION

If you need to identify immediately the domain and range of logarithmic function without looking on its table of values or graphing, you just need to use its equation.

The rule here is to always set to greater than 0 (**> 0**) the part/term (also called argument) of the logarithmic function that is enclosed with parenthesis.

Example 1: Given the logarithmic function $\boldsymbol{f(x) = \log_3(x+2)}$, find the domain and range.

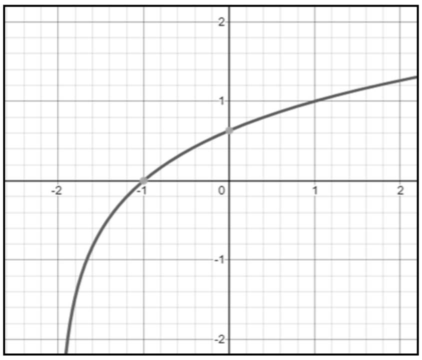
Get the term/part which is enclosed with parenthesis and set it to greater than (> 0). In this case $x + 2$.

$x + 2 > 0$	Add -2 both sides.
$x + 2 - 2 > 0 - 2$	Simplify.
$x > -2$	

Therefore, the **domain** of the given logarithmic function is $\boldsymbol{D = \{x | x > -2\}}$. The **range** is still the set of real numbers $\boldsymbol{R = \{y | y \in \mathbb{R}\}}$

Ready to Print

Look at the graph of $\boldsymbol{f(x) = \log_3(x+2)}$, the **domain** is the set of x-values at the right of $\boldsymbol{x = -2}$.

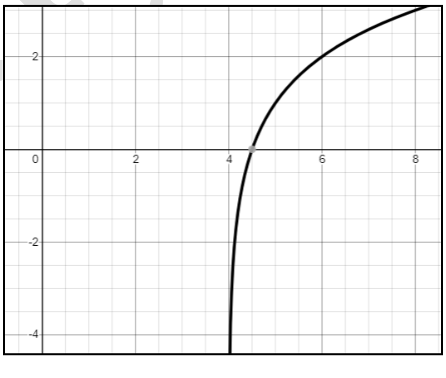


Example 2: Given the logarithmic function $\boldsymbol{f(x)} = \boldsymbol{\log_3(2x-8)}$, find the domain and range.

Get the term/part which is enclosed with parenthesis and set it to greater than (>0). In this case $2x - 8$

$2x - 8 > 0$	Add +8 both sides.
$2x - 8 + 8 > 0 + 8$	Simplify.
$2x > 8$	Divide both side with 2.
$\frac{2x}{2} > \frac{8}{2}$	Simplify.
$x > 4$	

Therefore, the **domain** of the given logarithmic function is $\boldsymbol{D} = \{\boldsymbol{x}|\boldsymbol{x} > \boldsymbol{4}\}$. The **range** is still the set of real numbers $\boldsymbol{R} = \{\boldsymbol{y}|\boldsymbol{y} \in \mathbb{R}\}$**.**



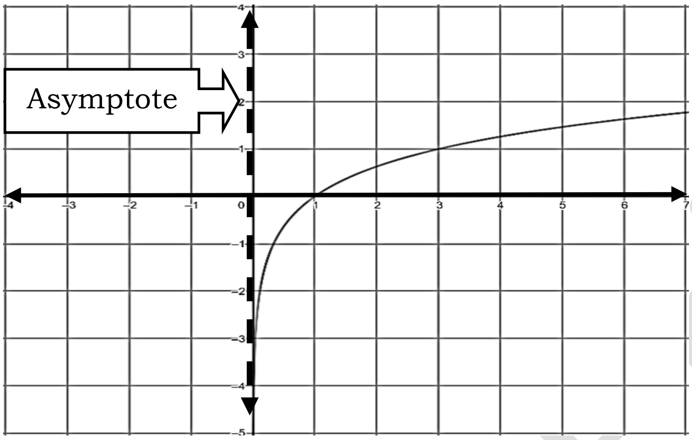
Look at the graph of the function, the **domain** is the set of x-values at the right side of $\boldsymbol{x = 4}$.

Ready to Print

ASYMPTOTES OF LOGARITHMIC FUNCTIONS

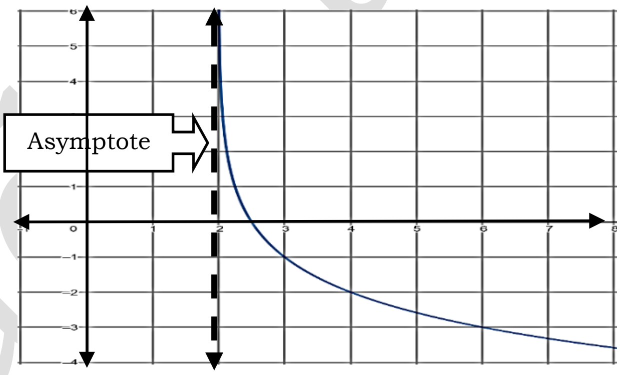
An **asymptote** is a line on the graph of a function representing a value toward which the function may approach but does not reach.

Example 1: Consider the graph of $\boldsymbol{f(x) = log_3\, x}$ below.



By looking at the graph, you could see that the function is asymptotic to the y – axis. Meaning, the graph gets closer and closer to the y – axis but then it does not cross or touch it. With this, you can say that the graph has a vertical asymptote at x = 0. On the other hand, the graph does not have any horizontal asymptote.

Example 2: Consider the graph of $\boldsymbol{f(x) = log_{1/2}(2x - 4)}$ below.



By looking at the graph, you could see that the function is asymptotic to x = 2. Meaning, the graph gets closer and closer to x = 2 but then it does not cross or touch it. With this, you can say that the graph has a vertical asymptote at x = 2. On the other hand, the graph does not have any horizontal asymptote.

How to determine the vertical asymptote of a logarithmic function without sketching its graph?

To determine the vertical asymptote of a logarithmic function, simply copy the argument of the logarithmic function then set it equal to zero. Then, solve for the value of the variable.

Ready to Print

Example 1: What is the vertical asymptote of $\boldsymbol{f}(\mathbf{x}) = \mathbf{log_3(5-2x)}$?

Solution: $5 - 2x = 0$ Copy the argument then set it equal to zero.

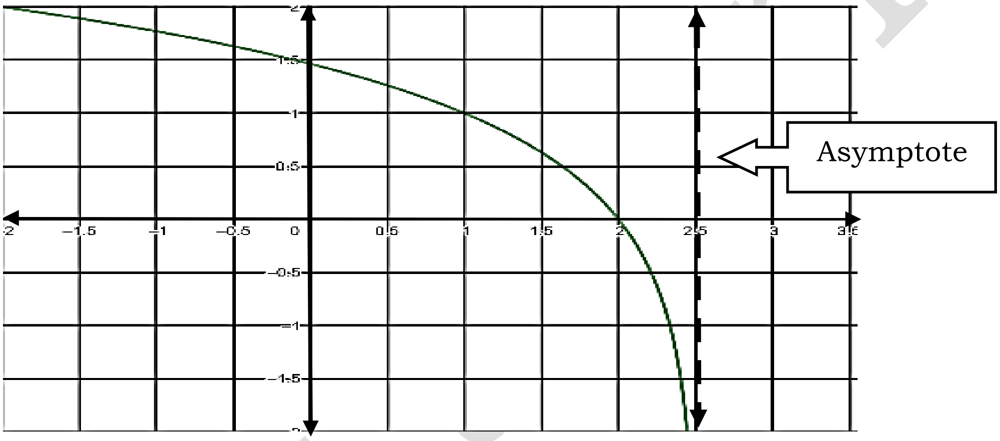
$5 = 2x$ By Addition Property of Equality

$\frac{5}{2} = \frac{2x}{2}$ Divide both sides by 2

$\frac{5}{2} = x$

Notice that if you substitute $x = \frac{5}{2}$ to $f(x) = \log_3(5-2x)$, it will make the function equal to $\log 0$ which is undefined. Therefore, the vertical asymptote is $x = \frac{5}{2}$.

Now, let us take a look at the graph of $\boldsymbol{f(x)} = \mathbf{log_3(5-2}\boldsymbol{x}\mathbf{)}$ below.



By graphing, we could see and verify that the function has a vertical asymptote at x = 2.5 or 5/2.

Example 2: What is the vertical asymptote of $\boldsymbol{f}(\mathbf{x}) = \mathbf{log_5(x^2-9)}$?

Solution: $x^2 - 9 = 0$ Copy the argument and set it equal to zero.

$x^2 = 9$ By Addition Property of Equality

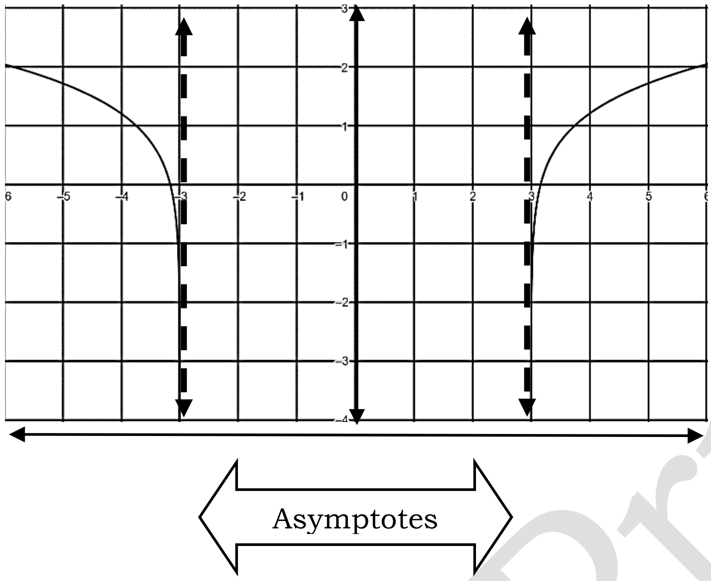
$\sqrt{x^2} = \sqrt{9}$ Extract the square root of both sides.

$x = \pm 3$

Therefore, the function has a vertical asymptote at $x = \pm 3$.

Ready Print

Now, let us take a look at the graph of $f(x) = \log_5(x^2 - 9)$ below.



Looking into its graph, we could see and verify that the function has vertical asymptotes at x = 3 and x = -3.

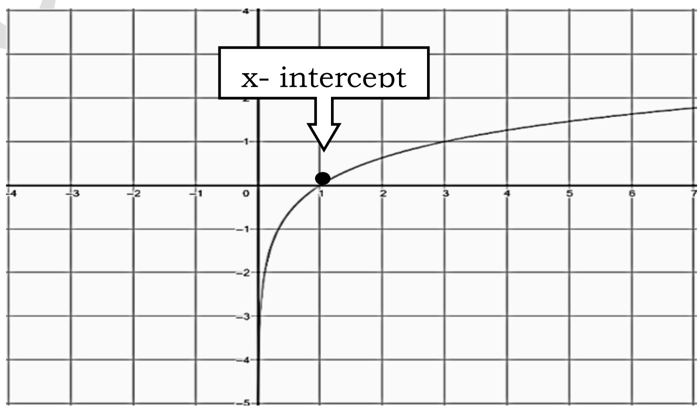
INTERCEPTS and ZEROES OF LOGARITHMIC FUNCTIONS

Intercepts are the points at which a graph crosses either the x – axis or y – axis. In finding for the x – intercept, you let y = 0. On the other, you let x = 0 if we wish to look for the y – intercept.

What are zeroes of a function?

The zero of a function is the point (x, y) on which the graph of the function intersects with the x – axis. Determining the zeros of a logarithmic function is just the same with finding its x – intercept wherein the function is set equal to 0.

Example 1: Consider the graph of $f(x) = log_3\, x$.



By looking at the graph, you could see that the function intersects the x – axis at (1, 0). Thus, the x – intercept is (1, 0). On the other hand, since the graph does not intersect the y – axis, then it has no y – intercept.

Ready to Print

Example 2: Find the intercepts and zeros of $\boldsymbol{f(x) = \log_2(x-1)}$.

Solution: For **x - intercept,** you let $f(x) = 0$.

$0 = \log_2(x-1)$ Change to exponential form ($y = \log_b x \rightarrow b^y = x$)

$2^0 = x - 1$ By Addition Property of Equality

$2^0 + 1 = x$ Simplify

$1 + 1 = x$

$2 = x$ Therefore, (2, 0) is the x – intercept.

Also, the function has only one zero and that is 2.

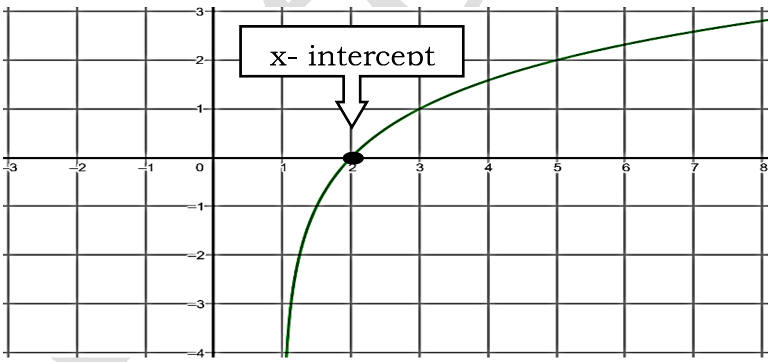
For **y – intercept**, you let x = 0.

$y = \log_2(0-1)$

$y = \log_2(-1)$

Notice that if you plugged in x = 0 to the equation, you get an argument equal to -1. Since the argument of a logarithmic function can only take positive values, this means that the function has no y – intercept.

Now, looking at its graph below, we could verify that the x – intercept is (2, 0), and that the function has no y – intercept.



Example 3: Find the intercepts of $\boldsymbol{f(x) = -\log_{\frac{2}{3}}(x-2) + 3}$.

Solution: For **x- intercept**, you let $f(x) = 0$.

$0 = -\log_{\frac{2}{3}}(x-2) + 3$ By Subtraction Property of Equality

$-3 = -\log_{\frac{2}{3}}(x-2)$ Divide both sides by -1

$3 = \log_{\frac{2}{3}}(x-2)$ Change to exponential form ($y = \log_b x \rightarrow b^y = x$)

$\left(\frac{2}{3}\right)^3 = x - 2$

$\frac{8}{27} = x - 2$

$\frac{8}{27} + 2 = x$

$\frac{62}{27} = x$ Therefore, the x – intercept is ($\frac{62}{27}$, 0).

Also, the function has only one zero and that is $\frac{62}{27}$.

Ready to Print

For **y – intercept**, you let x = 0.

$$y = -\log_{\frac{2}{3}}(0-2)\ +3$$

$$y = -\log_{\frac{2}{3}}(-2)\ +3$$

Notice that, if you plugged in x = 0 to the equation, you get a negative argument. Therefore, there is no y – intercept.

Example 4: Find the intercepts of $\boldsymbol{f(x) = 2 + log_2(x+1)}$.

Solution: For **x- intercept**, you let $f(x) = 0$.

$0 = 2 + log_2(x+1)$ By Subtraction Property of Equality

$-2 = \log_2(x+1)$ Change to exponential form ($y = \log_b x \rightarrow \ b^y = x$)

$2^{-2} = x+1$ Subtract 1 on both sides

$2^{-2} - 1 = x$

$\frac{1}{2^2} - 1 = x$

$\frac{1}{4} - 1 = x$

$-\frac{3}{4} = x$ Therefore, the x – intercept is $(-\frac{3}{4}, 0)$.

Also, the function has only one zero and that is $-\frac{3}{4}$.

For **y – intercept**, you let x = 0.

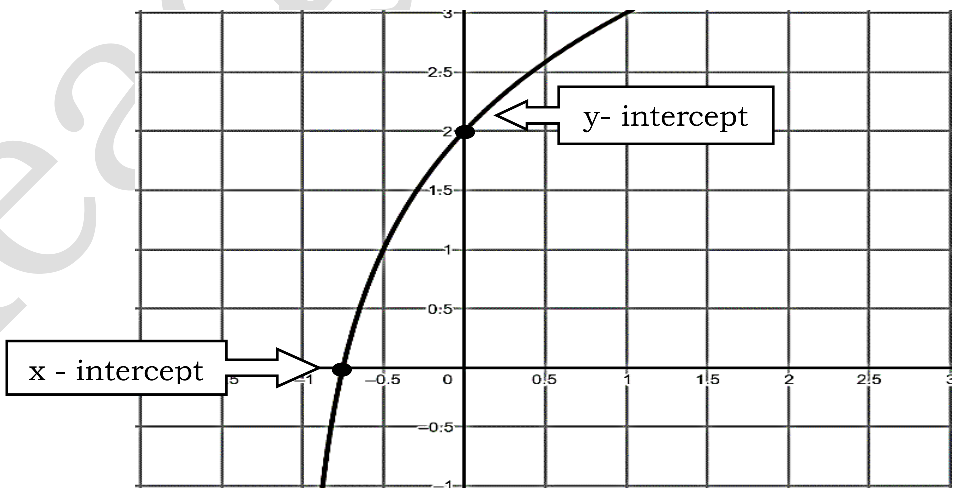
$$y = 2 + log_2(0+1)$$

$$y = 2 + log_2(1)$$

$$y = 2 + 0$$

$y = 2$ Therefore, the y – intercept is $(0, 2)$.

Now, let's look at the graph of $\boldsymbol{f(x) = 2 + log_2(x+1)}$ shown below.



Looking at the graph, we could see that the function crosses the x – axis at point (-3/4, 0) which we call as the x – intercept. Also, we could see that the function crosses the y – axis at point (0, 2) which we call as the y – intercept.

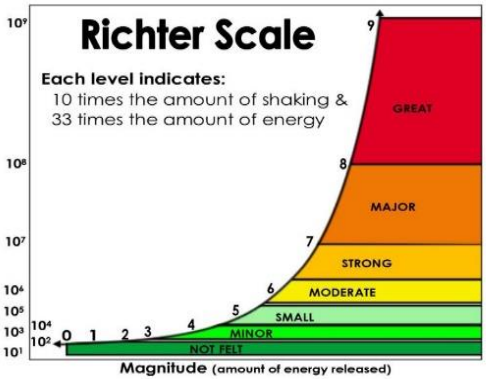
Ready to Print

PROBLEMS INVOLVING LOGARITHM

The most common real-life applications of logarithmic functions are Richter Scale, Sound Intensity, pH level and compound interest.

A. Richter Scale

It is the quantitative measure of an earthquake's magnitude (size) devised in 1935 by American seismologists Charles Francis Richter. (https://www.britannica.com>science).



Logarithm is used to convert the energy released by an earthquake (E) in joules to Richter Scale.

The magnitude R of an earthquake is:

$$R = \frac{2}{3}\log\frac{E}{10^{4.4}}$$

where E (in joules) is the energy released by the earthquake. The quantity $10^{4.4}$ *joules is the energy released by a very small reference earthquake (DepEd Learner's Material in General Mathematics, 2016).*

Sample Problems

1. Suppose an earthquake with an epicenter at City of San Fernando, La Union released approximately 10^9 joules of energy.

a. What is its magnitude in Richter Scale?

b. How much more energy does this earthquake release than the reference earthquake?

Solution:

a. Since E = 10^9 then $= \frac{2}{3}\log 10^{9-4.4}$ $R = \frac{2}{3}\log\frac{10^9}{10^{4.4}}$ $= \frac{2}{3}\log 10^{4.6}$ but $\log 10^{4.6} = 4.6$ because t $\log 10^x = x$. Thus, $R = \frac{2}{3}(4.6) \approx 3.07$. Therefore, the earthquake's magnitude is 3.07 in Richter Scale.

b. The earthquake releases $\frac{10^9}{10^{4.4}} = 10^{4.6} \approx 39810.72$ times more energy than the reference earthquake.

2. “The Big One” which is the most feared earthquake that could hit Metro Manila would release $10^{15.2}$ joules of energy. What is its magnitude in Richter Scale?

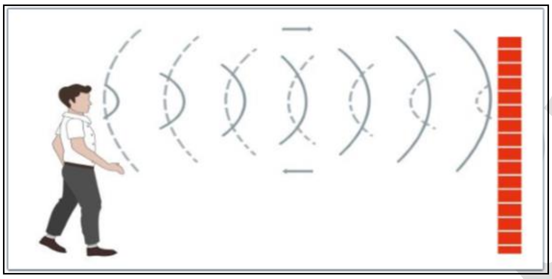
Solution: Since E = $10^{15.2}$ then $R = \frac{2}{3}\log\frac{10^{15.2}}{10^{4.4}} = \frac{2}{3}\log 10^{15.2-4.4} = \frac{2}{3}\log 10^{10,8}$

but $\log 10^{10.8} = 4.6$ because $\log 10^{x} = x$ thus, $R = \frac{2}{3}(10.8) = 7.2$

Therefore “The Big One’s” magnitude is 7.2 in Richter Scale.

B. Sound Intensity

It is also known as acoustic intensity, is the sound power per unit area (hyperphysics.phy-astr.gsu.edu>hbase).



If the unit of sound is given in $watts/m^2$, you can convert this to decibel (dB) using logarithmic function.

In acoustics, the decibel level of a sound is $D = 10\log\frac{l}{10^{-12}}$ where l is the sound intensity in $watts/m^2$. The quantity $10^{-12}\, watts/m^2$ is the least audible sound a human can hear (DepEd Learner’s Material in General Mathematics, 2016).

Sample Problems

1. The decibel level of sound in a public market of a town under Modified General Community Quarantine (MGCQ) is $10^{-9}\, watt/m^2$.

a. What is the corresponding sound intensity of the public market in decibel?
b. How much more intense is this sound than the least audible sound a human can hear?

Solution: a. Since $l = 10^{-9}$ then $D = 10\log\frac{10^{-9}}{10^{-12}} = 10\log 10^{-9-(-12)} = 10\log 10^{3}$

but $\log 10^{3} = 3$ because $\log 10^{x} = x$ thus, $D = 10(3) = 30$

Therefore, the sound intensity of the public market is in 30 dB

b. The sound is $\frac{10^{-9}}{10^{-12}} = 10^{3} = 1000$ times more intense than the least audible sound a human can hear.

2. The sound of fireworks has an intensity $10^2\ watt/m^2$. What is the corresponding sound intensity of the fireworks in decibel?

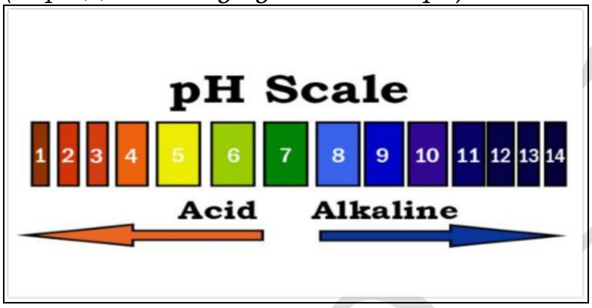
Solution: Since $I = 100 = 10^2$ then $D = 10\log\frac{10^2}{10^{-12}} = 10\log 10^{2-(-12)} = 10\log 10^{14}$

but $\log 10^{14} = 14$ because $\log 10^x = x$ thus, $D = 10(14) = 140$

Therefore, the sound intensity of the fireworks is in 140 dB.

C. pH Level

It is the measure of how acidic/basic water is. The range goes from 0 to 14 with 7 being neutral. *(https://wwwusgs.gov>science>ph)*



For easier understanding of how acidic or basic a water solution is, it is advisable to convert the hydrogen ions in moles per liter to pH level using logarithmic function.

The pH level of water-based solution is defined by $pH = -\log[H^+]$ *where* $[H^+]$ *is the concentration of hydrogen ions in moles per liter. Solutions with pH of 7 are defined as neutral; those with pH of less than 7 are acidic, and those with pH of more than 7 are basic (DepEd Learner's Material in General Mathematics, 2016).*

Sample Problems

1. Pure water has a hydrogen ion concentration of 0.0000001 or 10^{-7} moles per liter. Find its pH level.

Solution: Since $[H^+] = 10^{-7}$ then $pH = -\log[10^{-7}]$

but $\log[10^{-7}] = -7$ because $\log 10^x = x$ thus, $pH = -(-7) = 7$

Therefore, the pH level of pure water is 7 which is neutral.

2. Find the pH level of milk with a hydrogen ion concentration of 0.0001 per liter.

Solution: Since $[H^+] = 0.0001 = 10^{-4}$ then $pH = -\log[10^{-4}]$

but $\log[10^{-4}] = -4$ because $\log 10^x = x$ thus, $pH = -(-4) = 4$

Therefore, the pH level of the milk is 4.

D. Compound Interest

This type of interest is calculated on the initial principal, which also includes all the accumulated interest from previous periods on deposits or loans (www.investopedia.com). This means that the previous interest will become a part of the principal, thus will also earn interests.

Logarithmic function is needed to determine how long will you invest a Principal in order to accumulate a specific amount if invested at a given rate which is compounded annually, semi-annually, quarterly, monthly or continuously.

$$A = P\left[1 + \frac{r}{n}\right]^{nt}$$

The aforementioned formula is used to compute the total amount (A) in a compound interest.

But if the interest rate is compounded annually then the value of n=1 and the formula can be simplified as:

$$A = P[1 + r]^{t}$$

where:	*A*	=	*total Amount*
	P	=	*Principal/Amount invested*
	r	=	*rate of interest*
	t	=	*no. of years you are investing the Principal*

Sample Problems

1. How long would you keep Php 4 000 in a bank that pays 4.5% interest compounded annually in order to be worth Php 6 000?

Solution: Substituting the values of A = 4 000, P = 6 000 and r = 4.5% = 0.045 in the formula, we have

$$6000 = 4000(1 + 0.045)^{t} \quad \text{then} \quad 6000 = 4000(1.045)^{t}$$

$$\frac{6000}{4000} = \frac{4000(1.045)^{t}}{4000}$$

$$1.5 = (1.045)^{t}$$

$$\log 1.5 = \log(1.045)^{t}$$

Using the power of logarithm rule, we have

$$\log(1.5) = t(\log 1.045)$$

Solving for t, we have $\dfrac{\log 1.5}{\log 1.045} = t$

$t = 9.21 years$

Explore

Activity 1: MY TABLE OF VALUES!

Directions: Make a table of values of the following logarithmic functions. Use a separate sheet of paper for your answers.

- $\boldsymbol{f(x) = \log_{\frac{1}{2}} x}$

X					
f(x) or y	-2	-1	0	1	2

- $\boldsymbol{f(x) = \log_3 (x-2)}$

X					
f(x) or y	-2	-1	0	1	2

Activity 2: IDENTIFY MY DOMAIN AND RANGE

Directions: Find the domain and range of the given logarithmic functions. You can use table of values, graphing, or using the equation to find the domain and range. Use a separate sheet of paper for the solution.

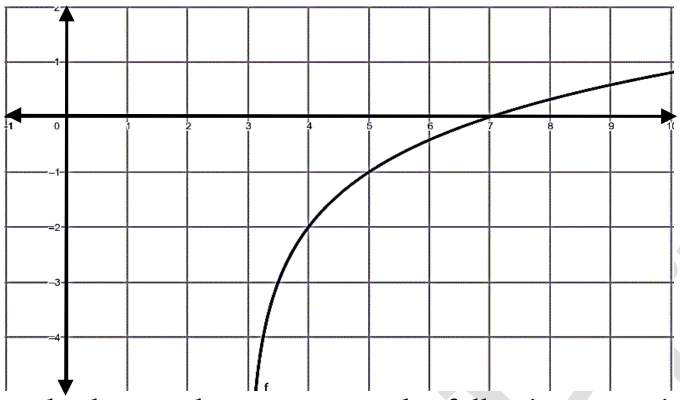
	Logarithmic Function	**Domain**	**Range**
Ex.	$f(x) = 2\log_3(4x+2)$	$\boldsymbol{D = \{x \mid x > -\frac{1}{2}\}}$	$\boldsymbol{R = \{y \mid y \in \mathbb{R}\}}$
1.	$g(x) = \log_5(3x-4)$		
2.	$r(x) = \log_2(x+3)$		
3.	$p(x) = -\frac{1}{2}\log_5(-x+4)$		
4.	$f(x) = -6\log_3 4x$		
5.	$v(x) = \log(5+2x)$		

Ready to Print

Activity 3: DOES IT MATCH?

Directions: Read and answer the following items. Use a separate sheet of paper for your solutions.

Consider the graph of $\boldsymbol{f(x) = \log_2(4x - 12) - 4}$.



A. Based from the graph shown above, answer the following questions.

1. What is the vertical asymptote? ________________________________
2. What is the x – intercept? y – intercept? __________________________
3. What is the zero of the function? ______________________________

B. Using the same function $\boldsymbol{f(x) = \log_2(4x - 12) - 4}$. Verify your answers in items 1, 2, and 3 on part A through solving. Show your solutions on a separate sheet of paper.

4. What is the vertical asymptote? ________________________________
5. What is the x – intercept? y – intercept? __________________________
6. What is the zero of the function? ______________________________

Activity 4: Complete Me!

Directions: Complete the solution of the given problems by writing your answers on the line provided for.

1. A normal conversation is about $10^{-6} watts/m^2$. How many decibel is that?

Solution: Since $I = 10^{-6}$ then $D =$ ______________ $= 10\log 10^{-6-(-12)} = 10\log$____

but $\log 10^6 =$ ______ thus, $D =$ _______

Therefore, the sound intensity of the public market in is ____ dB.

2. The hydrogen ion of fruit juice is 0.001 moles per Liter. What is its pH level?

Solution: Since $\left[H^+\right] =$ ____ then $pH = -\log[\_\_\_]$

but $\log\left[10^{-3}\right] =$ _____ thus, $pH = -(-3) =$ _____

Therefore, the pH level of the fruit juice is ________.

Ready to Print

Deepen

Direction: Analyze the given situation then answer the questions that follows. Show your solution and enclose your answers on your answer sheet.

A senior citizen wanted to triple his retirement benefit amounting to

Php1 000 000 by keeping it in a bank that pays 7.5% interest compounded annually.

1. How long should he keep his money in that bank in order to achieve his goal?

2. If you are the senior citizen, will you also keep your retirement benefit for that long? Justify your answer.

Rubrics for Scoring the Output

Criteria	Excellent 10 points	Satisfactory 8 points	Developing 5 points	Beginning 2 points
Accuracy of the Solution	Shows accurate solution.	Shows solution with minimal errors.	Shows solution with plenty of errors.	The solution is all erroneous.
Mathematical Concept	Shows excellent understanding of the concept of solving problems involving random variables.	Shows clear understanding of the concept of solving problems involving random variables.	Shows limited understanding of the concept of solving problems involving random variables.	Did not apply the concept of solving problems involving random variables.

Ready to Print

Gauge

Directions: Read carefully each item. Write the letter of the best answer for each test item.

___1. What is the domain of the given logarithmic function, $f(x) = \log_2(2x - 3) + 4$?

A. $D = \{x|x > -\frac{3}{2}\}$ B. $D = \{x|x > -\frac{2}{3}\}$

C. $D = \{x|x > \frac{3}{2}\}$ D. $D = \{x|x > \frac{2}{3}\}$

_____2. Which of the following logarithmic function has this domain $D = \{x|x > 2\}$?

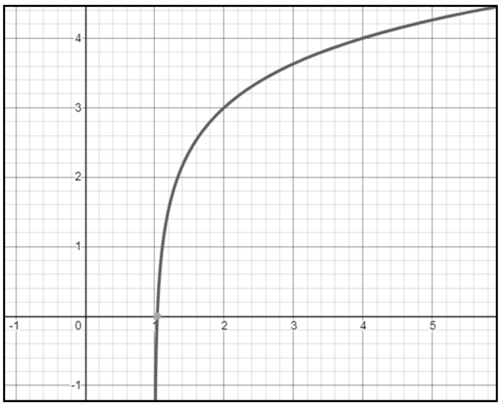
A. $f(x) = \log_2(10 - 5x) - 9$ B. $f(x) = \log_4(x + 10) - 8$

C. $f(x) = -2\log_3(x + 2)$ D. $f(x) = 5\log_6(x - 2)$

_____3. Which is the range of the logarithmic function $f(x) = 2\log_2 x$?

A. $R = \{y|y > 2\}$
B. $R = \{y|y < 2\}$
C. $R = \{y|y > 0\}$
D. $R = \{y|y \in \mathbb{R}\}$

_____4. Given the graph on the right, what is the range?



E. $R = \{y|y > 1\}$
F. $R = \{y|y < 1\}$
G. $R = \{y|y > 0\}$
H. $R = \{y|y \in \mathbb{R}\}$

_____5. Given the same graph, what is the domain?

A. $D = \{x|x > 0\}$
B. $D = \{x|x > 1\}$
C. $D = \{x|x > -1\}$
D. $D = \{x|x \in \mathbb{R}\}$

_____6. What is the vertical asymptote of $\boldsymbol{f(x) = \log_2 \frac{2x+8}{x-3}}$?

A. x = -3, x = 4 B. x = 3, x = -4
C. x = -3, x = -4 D. x = 4

For numbers 7 – 10, consider the function $\boldsymbol{f(x) = log_2(2x + 8) - 1.}$

_____7. What is the vertical asymptote of the function?

A. x = 3 B. x = 4
C. x = –3 D. x = –4

_____8. What is the x - intercept of the function?

A. (0, –3) B. (3, 0)
C. (–3, 0) D. (0, 3)

Ready to Print

_____9. What is the y - intercept of the function?

A. (0, –2)   B. (2, 0)
C. (–2, 0)   D. (0, 2)

_____10. What is the zero of the function?

A. –3   B. 2
C. 0   D. 3

____11. What is the value of $-3(\log 10^3)$?

A.-3   B. -6   C. -9   D. -12

____12. In a monitoring station of PHIVOLCS-DOST, they recorded a tremor of Mt. Bulusan which is $10^{10}\ joules$. What is the approximate magnitude of the earthquake in Richter Scale?

A. 3.5   B. 3.6   C. 3.7   D. 3.8

For numbers 13 - 15, use the given situation.

Mother is preparing the pre-ordered "Ube Pandesal" for delivery. The level of sound in our kitchen is approximately $10^{-3} watts/m^2$.

_____13. What is the corresponding sound intensity of our kitchen in decibel?

A. 30   B. 60   C. 90   D. 120

_____14. If the sound intensity of a quiet room is 20 dB, a normal conversation is 60 dB and a busy street traffic is 70 dB, how do you describe my mother's behavior while working in our kitchen?

A. She is very quiet while working.
B. She is relaxed in preparing the orders.
C. She still has a lot of time to prepare the orders.
D. She is in a hurry in preparing the orders.

_____15. Due to this pandemic (COVID - 19), Carlos is more conscious of his food intake because he is acidic. His doctor advised him to refrain from eating highly acidic foods. His caring mother prepares him a fruit juice that contains 0.000001 moles of hydrogen ion per Liter. What is the pH level of the fruit juice prepared by his mother?

A. 6   B. 7   C. 8   D. 9

Ready to Print

References

Printed Materials

Debbie Marie B. Verzosa, et. al., *General Mathematics Learner's Material* (Pasig City: Lexicon Press Inc., 2016), 99-101.

Luis Allan B. Melosantos, et. al., *Math Connections in the Digital Age: General Mathematics Grade 11* (Quezon City, Philippines: Sibs Publishing House Inc., 2016), 65-72.

Murray R. Piegel and Robert E. Moyer, *College Algebra - Fourth Edition* (U. S. A. McGraw-Hill Education, 2014), 57-63.

Orlando A. Oronce, *General Mathematics* (Manila, Philippines: Rex Book Store Inc., 2016), 68-75.

Rosita P. Abubo, et al., *General Mathematics* (Malabon City, Philippines: Jimczyville Publications, 2016), 68-75.

Websites

"Analysis of Logarithmic Graphs," CK-12, last accessed July 25, 2020, https://www.ck12.org/c/calculus/analysis-of-logarithmic-graphs/lesson/Graphing-Logarithmic-Functions-ALG-II/.

"Domain and range of logarithmic functions," OnlineMath4All.com, last accessed July 24, 2020, https://www.onlinemath4all.com/domain-and-range-of-logarithmic-functions.html.

"Finding Zeroes of Functions," Study.com, last accessed July 25, 2020, https://study.com/academy/lesson/finding-zeroes-of-functions.html.

"Graphing Logarithmic Functions," Elizabeth Stapel, last accessed July 25, 2020, https://www.purplemath.com/modules/graphlog2.htm.

"Horizontal and Vertical Asymptotes," CK-12, last accessed July 25, 2020, https://www.ck12.org/algebra/horizontal-and-vertical-asymptotes/lesson/Horizontal-and-Vertical-Asymptotes-MAT-ALY/.

"Hyperphysics," C.R. Nave, last accessed July 26, 2020, http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html.

"pH and Water," U.S. Geological Survey, last accessed July 26, 2020, https://wwwusgs.gov>science>ph.

"Properties of Logarithm," Paul Dawkins, last accessed August 3, 2020, https://tutorial.mathlamar.edu>alg.

Ready to Print

"Quiz: Logarithmic functions," Houghton Mifflin Harcourt, last accessed July 23, 2020, https://www.cliffsnotes.com/study-guides/algebra/algebra-ii/exponential-and-logarithmic-functions/quiz-logarithmic-functions.

"Vertical Asymptotes and Domain of Logarithmic Functions," Mathispower4u, last accessed July 25, 2020, https://www.youtube.com/watch?v=Z9zLGlvWcBQ&t=69s.

Ready to Print

Ready to Print

For inquiries or feedback, please write or call:

Department of Education – SDO La Union
Curriculum Implementation Division
Learning Resource Management Section
Flores St. Catbangen, San Fernando City La Union 2500
Telephone: (072) 607 - 8127
Telefax: (072) 205 - 0046
Email Address:
launion@deped.gov.ph
lrm.launion@deped.gov.ph