

MATHEMATICS

Quarter 4 – Week 8 - Module 6 Solving Problems Involving Oblique Triangles



AIRs - LM

GOVERNMENT PROPERTY
NOT FOR SALE

Mathematics 9

Quarter 2 – Week 8 Module 6: Solving Problems Involving Oblique Triangles
First Edition, 2021

Copyright © 2021
La Union Schools Division
Region I

All rights reserved. No part of this module may be reproduced in any form without written permission from the copyright owners.

Development Team of the Module

Author: Teresa A. Villanueva, MT 1

Editor: SDO La Union, Learning Resource Quality Assurance Team

Illustrator: Ernesto F. Ramos, Jr., P II

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent

Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent

German E. Flora, PhD, *CID Chief*

Virgilio C. Boado, PhD, *EPS in Charge of LRMS*

Erlinda M. dela Peña, EdD, *EPS in Charge of Mathematics*

Michael Jason D. Morales, *PDO II*

Claire P. Toluyen, *Librarian II*



Target

Triangles you encountered are not always right triangles. When a triangle has one right angle, it is said to be a Right Triangle. But what if it does not have any right angle? What do you call that triangle? Thus, an oblique triangle does not contain any right angle. In this module, you will be asked to solve problems on oblique triangles.

After going through this module, you are expected to:

Most Essential Learning Competency:

Solves problems involving oblique triangles. **(M9GE-IVh-j-1)**

Subtasks:

1. Recall the law of sines and law of cosines
2. Apply the law of sines and law of cosines in solving problems involving oblique triangles.

Before going on, check how much you know about this topic.
Answer the pre-assessment on a separate sheet of paper.

Pre-Assessment

Directions. Choose the letter that you think best answers the question. Write your answer on a separate sheet of paper.

- Which of the following best describes oblique triangles?
 - Triangles with either three acute angles, or an obtuse angle and two acute angles.
 - Triangles with two congruent angles.
 - Triangle with three congruent angles.
 - Triangles with no congruent sides.
- Which of the following is the correct formula for law of sines?
 - $\frac{\sin A}{a} = \frac{\sin B}{b}$
 - $a \sin A = b \sin B$
 - $\frac{\sin A}{b} = \frac{\sin B}{a}$
 - $b \sin B = c \sin C$

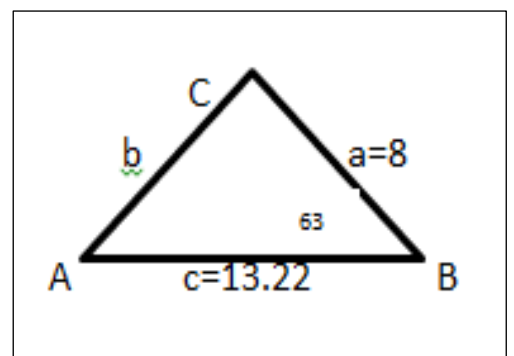
Refer to this problem to answer items number 3 – 6

Given an oblique triangle ABC, $m\angle A = 42^\circ$, $m\angle C = 70^\circ$ and $a = 6$

- What is the law/equation to be used to solve for b?
 - $\frac{\sin A}{a} = \frac{\sin B}{b}$
 - $a \sin A = b \sin B$
 - $\frac{\sin A}{b} = \frac{\sin B}{a}$
 - $a \sin B = b \sin A$
- What is the exact measure of angle B?
 - 68°
 - 70°
 - 72°
 - 74°
- What is the measure of c?
 - 5.5
 - 6.3
 - 7.4
 - 8.4
- What is the measure of b?
 - 8.2
 - 8.3
 - 8.4
 - 8.7

Refer to the figure at the right to answer items number 7-12

- What case is being illustrated in the figure?
 - ASA case
 - ASS case
 - SAS case
 - SSS case
- What is the equation to be used to solve for **b**?
 - $a^2 = b^2 + c^2 - 2bc (\cos A)$
 - $b^2 = a^2 + c^2 - 2ac (\cos A)$
 - $b^2 = a^2 + c^2 - 2ac (\cos B)$
 - $c^2 = a^2 + b^2 - 2ab (\cos C)$



9. What is the measure of b ?
A. 10.8 B. 11.94 C. 13 D. 14
10. What is the equation to be used to solve for C ?
A. $a^2 = b^2 + c^2 - 2bc (\cos A)$
B. $b^2 = a^2 + c^2 - 2ac (\cos A)$
C. $b^2 = a^2 + c^2 - 2ac (\cos B)$
D. $c^2 = a^2 + b^2 - 2ab (\cos C)$
11. What is the measure of C ?
A. 80 B. 82 C. 93 D. 94
12. What is the measure of A ?
A. 30° B. 32° C. 37° D. 44°
13. What is the total measure of all the angles in a triangle?
A. 90 B. 100 C. 180 D. 360
14. Given $\triangle LEX$, if $m\angle L = 71.51^\circ$, $m\angle X = 59.05^\circ$, what is $m\angle E$?
A. 49.44° B. 59.44° C. 69.44° D. 79.44°
15. From DMTC SNHS, Frexyl walks 15 m N to reach the basketball court of San Isidro and 4m E to be at home which makes an angle measuring to 138° with the school. How far is Frexyl's house to the school?
A. 8m B. 10m C. 12m D. 25m

Were you able to answer all the questions correctly? If not, don't worry because the next activity will help you better understand the lesson.

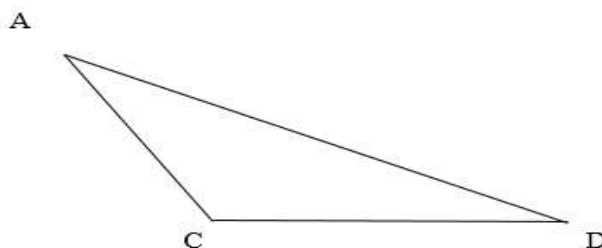


Jumpstart

Activity 1: Know Me First!

Directions: Label the parts of the oblique triangle ACD. Such that $A = 46.57^\circ$,

$D = 28.96^\circ$, $C = 104.47^\circ$, $a = 15$, $c = 20$, and $d = 10$



Activity 2: Complete Me!

Direction: Fill in the missing terms in the given law.

A. Law of Sines

In any $\triangle ABC$,

$$1. \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

B. Law of Cosines

In any $\triangle ABC$,

$$2. a^2 = b^2 + c^2 - 2bc (\cos \text{ ___ })$$

$$3. b^2 = \text{ ___ } + c^2 - 2ac (\cos B)$$

$$4. c^2 = a^2 + b^2 - 2\text{ ___ } (\cos C)$$

How did you find the activity? Were you able to recall the laws of Sines and Cosines? You will now apply these 2 laws in solving problems on oblique triangles



Discover

Oblique Triangle ABC can be solved using three tools: the sum of angles, the Law of Sines and the Law Cosines.

Consider the illustrative example #1 below

From DMTC SNHS, Frexyl walks 15 m North to reach the basketball court of San Isidro and 4 m East to be at home which makes an angle measuring 138° with the school. How far is Frexyl's house from the school?



In solving word problem, follow the IDEA method.

Step 1. **I**dentify the given

Step 2. **D**esign a solution

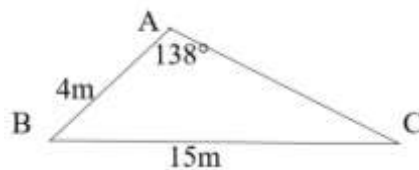
Step 3. **E**valuate the solution

Step 4. **A**ssess or check the answer

Here is how you are going to solve the problem.

Step 1. **I**dentify the given

To solve the given problem, sketch the triangle and label the parts with the given measures.



FYI: The longest side is opposite the largest angle, and the shortest side is opposite the smallest angle

Given: $a = 15$ $\angle A = 138^\circ$

$c = 4$

Unknown: b , B and C

Step 2. **D**esign a solution

Based from the illustration, the measures of 2 sides and non-included angle of the triangle is given. This illustrates SSA case. The law of Sines may be applied to solve for the unknown in the problem.

Law of Sines

$$\text{In } \triangle ABC, \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Step 3. **E**valuate the solution

Since the problem is asking for the distance of Frexyl's house from the school which is represented by b , we can use the formula $\frac{\sin B}{b} = \frac{\sin C}{c}$.

But the measure of angle C is still unknown. Therefore, solve for the measure of angle C first. We can use the formula $\frac{\sin A}{a} = \frac{\sin C}{c}$ since measures of side a and $\angle A$ are given.

$$\frac{\sin 138^\circ}{15} = \frac{\sin C}{4}$$

find the value of $\sin 138^\circ$

$$\sin 138^\circ = 0.6691$$

$$\sin C = \frac{4(\sin 138^\circ)}{15}$$

express the unknown in terms of the other

$$\sin C = \frac{4(0.6691)}{15}$$

substitute the value of $\sin 138^\circ$

$$\sin C = 0.1784$$

simplify

$$C = \sin^{-1} 0.1784$$

solve for C, take inverse sine 0.1784 since measure of the

$$\angle C = 10.16^\circ$$

angle is the unknown

Now that you solved for the measure of angle C and measure of angle A is 138° , you may use the sum of angles to solve for the measure of angle B.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$m\angle A = 138^\circ \quad m\angle C = 10.16^\circ$$

$$138^\circ + \angle B + 10.16^\circ = 180^\circ$$

$$\angle B = 180^\circ - 148.16^\circ$$

$$\angle B = 31.84^\circ$$

You are now ready to solve for b . Use the formula $\frac{\sin B}{b} = \frac{\sin C}{c}$ since measures of side c , angle B and angle C are already determined.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 31.84^\circ}{b} = \frac{\sin 10.16^\circ}{4}$$

find the value of $\sin 31.84^\circ$ and $\sin 10.16^\circ$

$$b = \frac{4(\sin 31.84^\circ)}{\sin 10.16^\circ}$$

express the unknown in terms of the other

$$b = \frac{4(\sin 31.84^\circ)}{\sin 10.16^\circ}$$

substitute the value of $\sin 138^\circ$

$$\sin 31.84 = 0.5275 \text{ and } \sin 10.16^\circ = 0.1764$$

$$b = \frac{4(0.5275)}{0.1764}$$

simplify

$$b = 11.96 \approx 12$$

solve for b

Therefore, Frexyl's house is approximately 12m away from the school.

Step 4. **A**ssess or Check the answer.

$$a = 15$$

$$\angle A = 138^\circ$$

$$b = 12$$

$$\angle B = 31.84^\circ$$

The longest side is opposite the largest angle, and the shortest side is opposite the smallest angle

$$c = 4 \quad \angle C = 10.16^\circ$$

Illustrative Example #2

Find the distance between the two trees such that the trees are 8 cm and 13.22 cm away from the signage iron stand. The angle formed by these two trees with the signage stand is 63.39° .



Step 1. Sketch the triangle and label the parts. Identify the given.

$$\text{Given: } b = 13.22 \quad \angle A = 63.39^\circ \\ c = 8$$

Unknown: a , the distance between the two trees

Step 2. Based from the illustration, the measures of 2 sides and an included angle of the triangle are given. This illustrates an SAS case. The law of Cosines may be applied to solve for the unknown in the problem.

<p>Law of Cosines In any $\triangle ABC$,</p> $a^2 = b^2 + c^2 - 2bc (\cos A)$ $b^2 = a^2 + c^2 - 2ac (\cos B)$ $c^2 = a^2 + b^2 - 2ab (\cos C)$

Step 3. The distance between the two trees, represented by a , is the unknown. Since the measures of the 2 sides (b and c) and the included angle A are given, use the equation $a^2 = b^2 + c^2 - 2bc (\cos A)$.

$$a^2 = (13.22)^2 + 8^2 - 2(13.22)(8) (0.4479)$$

$\cos 63.39 = 0.4479$

$$a^2 = 174.77 + 64 - (211.52) (0.4479)$$

$$a^2 = 238.77 - 94.74$$

$$a^2 = 144.03$$

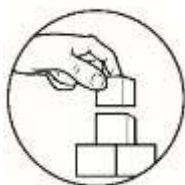
$$a = \sqrt{144.03}$$

$$a = 12$$

Therefore, the distance between the two trees is 12cm.

Step 4. Assess and check the answers

$$b = 13.22 \quad \angle A = 63.39^\circ \\ c = 8 \\ a = 12$$



Explore

Here is an enrichment activity for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3. Three Sided

Directions. Read and analyze the problem carefully. Answer correctly what is being asked in the problem.

Two barangay roads intersect at 100° . Jan's pet house is 8m from the intersection. Lester's pet shop is on the other road and is 12m from the intersection. How far is Jan's pet house from that of Lester's pet shop? Express answer in two decimal places.



Deepen

Now that you have understood the concept and ideas of this topic, let's now deepen your understanding by moving on to the next activity.

Activity 4. Love Tri-Angle

Directions. Read and analyze the problem carefully. Answer correctly what is being asked in the problem.

A triangular piece of land measures 35m, 40m and 50m. What are the measures of the angles between the sides? Express answers to the nearest degree.

Great Job. Having answered all the activities, I am pretty sure that you can answer correctly all the items in the Post-test. Good luck!



Gauge

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

- Which of the following equations illustrate the law of Sines?
 - $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
 - $\frac{\sin a}{A} = \frac{\sin c}{C}$
 - $\frac{\sin a}{A} = \frac{\sin b}{B}$
 - $\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{c}$
- Which of the following equations illustrate the law of Cosines?
 - $a^2 = b^2 + c^2 - 2bc (\cos A)$
 - $a^2 = b^2 + c^2 - 2ac (\cos A)$
 - $b^2 = a^2 + c^2 - 2ab (\cos B)$
 - $c^2 = a^2 + b^2 - 2bc (\cos C)$
- Which of the following is the correct equation for sin B?
 - $\frac{\sin B}{b} = \frac{\sin C}{c}$
 - $\frac{\sin B}{b} = \frac{\sin c}{C}$
 - $\frac{\sin a}{A} = \frac{\sin b}{B}$
 - $\frac{\sin C}{c} = \frac{\sin b}{B}$
- Which of the following is the correct formula cos A?
 - $a^2 = b^2 + c^2 - 2bc (\cos A)$
 - $a^2 = b^2 + c^2 - 2ac (\cos A)$
 - $b^2 = a^2 + c^2 - 2ab (\cos A)$
 - $c^2 = a^2 + b^2 - 2bc (\cos A)$
- A triangular piece of land measures 15m and 25m and the included angle is 50° . What is the measure of the third side?

A. 19m B. 20m C. 24m D. 30m
- Refer to item number 5, what is the correct equation used to solve for the third side? Assume that the third side is represented by c .
 - $c^2 = a^2 + b^2 - 2ac (\cos C)$
 - $c^2 = a^2 + b^2 - 2ab (\cos C)$
 - $\frac{\sin C}{c} = \frac{\sin B}{b}$
 - $\frac{\sin c}{C} = \frac{\sin A}{a}$

Refer to this problem to answer items number 7- 10.

Mrs. Monis has a triangular-shaped backyard. Two of its sides measure 8 m and 10 m, and the angle between these sides is 54° .

- What is the correct equation to solve for the third side? Assume that the third side is represented by c .

A. $c^2 = a^2 + b^2 - 2ab (\cos C)$

B. $c^2 = a^2 + b^2 - 2ab (\cos A)$

C. $\frac{\sin C}{c} = \frac{\sin B}{b}$

D. $\frac{\sin c}{c} = \frac{\sin A}{a}$

8. What is the measure of the third side?

A. 8.36m

B. 8.63m

C. 69.95m

D. 95.69m

9. What is the correct ratio for h ?

A. $h = \frac{\sin 54^\circ}{8}$

B. $\sin 54^\circ = \frac{h}{8}$

C. $\cos 54^\circ = \frac{h}{8}$

D. $\cos 54^\circ = \frac{8}{h}$

10. What is the area of the backyard if two of its sides measure 8 m and 10 m, and the angle between these sides is 54° ?

A. 36.23 m^2

B. 36.32 m^2

C. 32.63 m^2

D. 32.36 m^2

Refer to this problem to answer items number 11- 15.

A triangular field has sides 500m, 450m and 720m. Find the measure of the angles between the sides of the triangular field. Round off your answers to the nearest degree.

11. What is the measure of the angle between the sides of 500m and 720m?

A. 37°

B. 38°

C. 39°

D. 40°

12. What is the correct equation used to solve for measure of the angle between the sides of 500m and 720m? Represent the angle by C.

A. $450^2 = 500^2 + 720^2 - 2(500)(720) (\cos C)$

B. $450^2 = 500^2 + 720^2 - 2(500)(720) (\cos C)$

C. $500^2 = 450^2 + 720^2 - 2(450)(720) (\cos C)$

D. $720^2 = 450^2 + 500^2 - 2(450)(500) (\cos C)$

13. What is the measure of the angle between the sides of 500m and 450m?

A. 38°

B. 40°

C. 82°

D. 120°

14. What is the correct equation used to solve for measure of the angle between the sides of 500m and 400m? Represent the angle by A.

A. $450^2 = 500^2 + 720^2 - 2(500)(720) (\cos A)$

B. $450^2 = 500^2 + 720^2 - 2(500)(720) (\cos A)$

C. $500^2 = 450^2 + 720^2 - 2(450)(720) (\cos A)$

D. $720^2 = 450^2 + 500^2 - 2(450)(500) (\cos A)$

15. What is the measure of the angle between the sides of 450m and 720m?

A. 38°

B. 40°

C. 82°

D. 120°

References

Bryant, Merden L., et al. 2014. Mathematics Learner's Material 9. Pasig City

Nivera, Gladys C. and Lapinid Minie Rose C. 2013. *Grade 9 Mathematics Patterns and Practicalities* .Makatu City, Don Bosco Press.