



Mathematics

Quarter 3- Week 8 - Module 8

Solving Problems Involving Sides and Angles of a Polygon



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Mathematics 5

Quarter 1- Week 1 Module A: Using Divisibility Rules for 2, 5 and 10 to find the Common Factors

Mathematics 7

Quarter 3: Week 8 - Module 8: **Solving Problems Involving Sides and Angles of a Polygon**

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Target

This module was designed and written with you in mind. It is here to help you master your skills in solving problems involving sides and angles of a polygon. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course.

After going through this module, you are expected to:

Learning Competency:

solve problems involving sides and angles of a polygon (**M7GE-IIIj-1**)

Subtasks:

- calculate the interior or exterior angle of a polygon
- find the sum of interior or exterior angles of a polygon
- solve the number of sides of a regular polygon given its: (a) interior angle; (b) exterior angle; and (c) sum of interior angles

Before going on, check how much you know about this topic.

PRE-TEST

Directions: Read and understand the questions below. Select the best answer to each item then write your choice on your answer sheet.

1. What is the measure of each interior angle of a regular pentagon?
A. 90° B. 108° C. 120° D. 135°
2. What is the measure of each exterior angle of a regular hexagon?
A. 45° B. 60° C. 72° D. 90°
3. What is the sum of the interior angles of a regular heptagon?
A. 360° B. 540° C. 720° D. 900°
4. What is the sum of the exterior angles of a regular octagon?
A. 45° B. 90° C. 270° D. 360°
5. How many sides does a regular nonagon have?
A. 5 B. 7 C. 9 D. 11
6. The sum of the measures of the interior angles of a polygon is 1980 degrees. How many sides does the polygon have?
A. 11 B. 12 C. 13 D. 14

7. The sum of the interior angles of a polygon is 900° . What is the measure of each interior angle in nearest hundredths?
A. 128.57° B. 128.68° C. 128.84° D. 182.92°
8. In a triangle, the second angle is twice the first angle and the third angle is 12 degrees less than 5 times the first angle. What is the measure of the largest angle?
A. 108° B. 120° C. 132° D. 144°
9. The sum of the interior angles of a polygon is 1080° . What is the measure of each exterior angle?
A. 45° B. 90° C. 270° D. 360°
10. Each exterior angle of a regular polygon measures 24° . How many sides does it have?
A. 10 B. 15 C. 20 D. 25
11. ABCDEFGH is a regular octagon. What is the measure of $\angle BAD$?
A. 25° B. 35° C. 45° D. 55°
12. How many sides does a regular polygon have if the interior angle of a regular polygon is 150 degrees?
A. 6 B. 8 C. 10 D. 12
13. Each interior angle of a regular polygon measures 140° . What is the sum of all its interior angles?
A. 900° B. 1080° C. 1260° D. 1440°
14. If the number of sides of a polygon is increased by 5, by how much would the sum of the interior angles of a polygon increase?
A. 720° B. 900° C. 1080° D. 1260°
15. If the number of sides of a regular polygon is increased from 12 to 16, by how much would the measure of each of its exterior angles decrease?
A. 7.5° B. 7.75° C. 8.25° D. 8.5°

Lesson

Problems Involving Sides and Angles of a Polygon

A polygon is a closed shape that has three or more sides. Each side is a line segment. The line segments form the sides of the polygon. All polygons in this lesson are assumed to be convex polygons. A *convex polygon* has all angles less than 180° .



Jumpstart

In the previous module, you were able to construct triangles, squares, rectangles, regular pentagons, and regular hexagons. Let us begin this lesson by recalling the different concepts learned in constructing a polygon in terms of the measure of each interior and exterior angle, the sum of the exterior angles, and sum of the interior angles.

The knowledge and mathematical skill mentioned will help you to solve problems involving sides and angles of a polygon.

Activity 1: Complete Me!

Directions: Complete the table below.

| Polygon | Number of Sides | Measure of each Interior Angle | Measure of each Exterior Angle | Sum of the Interior Angles | Sum of the Exterior Angles |
|------------------|-----------------|--------------------------------|--------------------------------|----------------------------|----------------------------|
| Triangle | 3 | 60° | 120° | 180° | 360° |
| Square | | | | | |
| Rectangle | | | | | |
| Regular Pentagon | | | | | |
| Regular Hexagon | | | | | |

You did it! Congratulations!

What conclusions can you make as the number of sides of the polygon increases with regards to:

- (a) the measure of each interior angle?
- (b) the measure of each exterior angle?
- (c) the sum of the interior angles? and
- (d) the sum of the exterior angles?



Discover

Below are some important matters that we need to discuss in order for you to understand how to solve problems involving sides and angles of a polygon. Read carefully and understand all salient points written in this part of the module.

PROBLEMS INVOLVING SIDES AND ANGLES OF A POLYGON

Triangle Sum Theorem

The sum of the measures of the interior angles in any triangle will always be 180° .

Example 1

The measures of the angles of a triangle are in the ratio 2:3:4. What is the measure of the largest angle?

Solution:

Let the measures of the angles be equal to $2x$, $3x$, and $4x$.

Using the Triangle Sum Theorem, we have:

$$2x + 3x + 4x = 180^\circ$$

$$9x = 180^\circ$$

$$x = 20^\circ$$

Therefore, the measure of the largest angle is $4x = 4(20^\circ) = 80^\circ$.

Example 2

In a triangle, the second angle is twice the first angle and the third angle is 12 degrees less than 5 times the first angle. What is the measure of the second angle?

Solution:

Let x be the first angle, $2x$ be the second angle, and $5x - 12^\circ$ be the third angle.

Using the Triangle Sum Theorem, we have:

$$x + 2x + 5x - 12^\circ = 180^\circ$$

$$8x - 12^\circ = 180^\circ$$

$$8x = 192^\circ$$

$$x = 24^\circ$$

Therefore, the measure of the second angle is $2x = 2(24^\circ) = 48^\circ$.

Quadrilateral Sum Theorem

The sum of the measures of the interior angles of a convex quadrilateral is 360° .

Example 3

Find the measure of each interior angle in quadrilateral PURE if the measure of each consecutive angle is a consecutive multiple of x .

Solution:

Let x = measure of $\angle P$

$2x$ = measure of $\angle U$

$3x$ = measure of $\angle R$

$4x$ = measure of $\angle E$

Using the Quadrilateral Sum Theorem, we have:

$$\begin{aligned}x + 2x + 3x + 4x &= 360^\circ \\10x &= 360^\circ \\x &= 36^\circ\end{aligned}$$

Use the value of x to find the measure of each angle.

$$\begin{aligned}\angle P &= 36^\circ \\ \angle U &= 2(36^\circ) = 72^\circ \\ \angle R &= 3(36^\circ) = 108^\circ \\ \angle U &= 4(36^\circ) = 144^\circ\end{aligned}$$

Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex polygon with n sides is $180^\circ(n - 2)$.

Example 4

Find the sum of the measures of the interior angles of a regular octagon.

Solution:

An octagon has 8 sides. Use the Polygon Interior Angles Theorem and substitute 8 for n .

$$\begin{aligned}180^\circ(n - 2) &= 180^\circ(8 - 2) \\ &= 180^\circ(6) \\ &= 1080^\circ\end{aligned}$$

Therefore, the sum of the measures of the interior angles of a regular octagon is 1080° .

Example 5

Three angles of a pentagon measures 120° , 130° , and 150° . Find the other two angles if they are in the ratio 2:3.

Solution:

A pentagon has five sides.

$$180^\circ(5 - 2) = 180^\circ(3) = 540^\circ$$

The sum of the measures of the three angles is:

$$120^\circ + 130^\circ + 150^\circ = 400^\circ$$

Thus, the sum of the measures of the other angles is:

$$540^\circ - 400^\circ = 140^\circ$$

If x represents the measure of an angle, then:

$$\begin{aligned}2x + 3x &= 140^\circ \\ 5x &= 140^\circ \\ x &= 28^\circ\end{aligned}$$

$$2x = 2(28^\circ) = 56^\circ$$

$$3x = 3(28^\circ) = 84^\circ$$

The measures of the other two angles are 56° and 84° , respectively.

Example 6

The sum of the measures of the interior angles of a polygon is 2160 degrees. How many sides does the polygon have?

Solution:

The sum of the measures of the interior angles is:

$$180^\circ(n - 2) = 2160^\circ$$

Solve for n:

$$\frac{180^\circ(n-2)}{180^\circ} = \frac{2160^\circ}{180^\circ}$$

$$n - 2 = 12$$

$$n = 12 + 2$$

$$n = 14$$

Therefore, the polygon has 14 sides.

Corollary 1

The measure of each interior angle of a regular n -gon is $\frac{180^\circ(n-2)}{n}$.

Example 7

Find the measure of each interior angle of a regular decagon.

Solution:

A decagon has 10 sides. Use the corollary and substitute 10 for n .

$$\frac{180^\circ(n-2)}{n} = \frac{180^\circ(10-2)}{10}$$

$$= \frac{180^\circ(8)}{10}$$

$$= \frac{1440^\circ}{10}$$

$$= 144^\circ$$

Therefore, the measure of each interior angle of a regular decagon is 144° .

Corollary 2

The measure of each exterior angle of a regular n -gon is $180^\circ - \frac{180^\circ(n-2)}{n}$.

Example 8

The sum of the interior angles of a polygon is 1260° . What is the measure of each exterior angle?

Solution:

The sum of the interior angles is:

$$180^\circ(n - 2) = 1260^\circ$$

Solve for n:

$$\frac{180^\circ(n-2)}{180^\circ} = \frac{1260^\circ}{180^\circ}$$

$$n - 2 = 7$$

$$n = 7 + 2$$

$$n = 9$$

The measure of each interior angle is:

$$\frac{1260^\circ}{9} = 140^\circ$$

Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles, one at each vertex, of any convex polygon is 360° .

Thus, the measure of each exterior angle is $180^\circ - 140^\circ = 40^\circ$.

Example 9

The measures of the exterior angles of a convex quadrilateral, one at each vertex, are $6x$, $4x - 10$, $4x + 10$, and $2x + 40$. Find the measure of each exterior angle.

Solution:

$$6x + 4x - 10 + 4x + 10 + 2x + 40 = 360$$

$$16x + 40 = 360$$

$$16x = 320$$

$$x = 20$$

$$6x = 6(20) = 120$$

$$4x - 10 = 4(20) - 10 = 70$$

$$4x + 10 = 4(20) + 10 = 90$$

$$2x + 40 = 2(20) + 40 = 80$$

The measures of the exterior angles are 70° , 80° , 90° and 120° .

The number of sides (n) of a regular n -gon given the measure of each exterior angle (E) can be obtained by the formula below:

$$n = \frac{360^\circ}{E}$$

Example 10

Each exterior angle of a regular polygon measures 10° . How many sides does it have?

Solution:

Substitute $E = 10^\circ$ into the formula:

$$n = \frac{360^\circ}{E} = \frac{360^\circ}{10^\circ} = 36$$

Thus, the polygon has 36 sides.



Explore

Work on the following enrichment activities for you to apply your understanding on this lesson.

Activity 2: Make Me Complete!

Directions: Complete the table for regular polygon.

| Polygon | Heptagon | Octagon | Nonagon | Decagon | Undecagon | Dodecagon |
|--|----------|---------|---------|---------|-----------|-----------|
| Number of sides | | | | | | |
| Sum of the measures of interior angles | | | | | | |
| Sum of the measures of exterior angles | | | | | | |
| Measure of each interior angle | | | | | | |
| Measure of each exterior angle | | | | | | |

How did you find the activity? What mathematical concepts did you use?

Now, here is another activity that lets you apply what you have learned about solving problems involving sides and angles of a polygon.

7. The sum of the interior angles of a polygon is 2340° . What is the measure of each interior angle?
A. 156° B. 160° C. 162° D. 170°
8. The sum of two angles of a triangle is three times the third angle. What is the sum of the first two angles?
A. 45° B. 90° C. 135° D. 180°
9. The sum of the interior angles of a polygon is 1800° . What is the measure of each exterior angle?
A. 10° B. 20° C. 30° D. 40°
10. Each exterior angle of a regular polygon measures 9° . How many sides does it have?
A. 30 B. 40 C. 50 D. 60
11. ABCDEF is a regular hexagon. What is the measure of $\angle BAD$?
A. 15° B. 30° C. 45° D. 60°
12. How many sides does a regular polygon have if the interior angle of a regular polygon is 162 degrees?
A. 20 B. 22 C. 24 D. 26
13. Each interior angle of a regular polygon measures 120° . What is the sum of all its interior angles?
A. 720° B. 900° C. 1080° D. 1260°
14. If the number of sides of a polygon is increased by 7, by how much would the sum of the interior angles of a polygon increase?
A. 720° B. 900° C. 1080° D. 1260°
15. If the number of sides of a regular polygon is increased from 16 to 20, by how much would the measure of each of its exterior angles decrease?
A. 4° B. 4.25° C. 4.5° D. 4.75°

References

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Links:

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