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DepED DEPARTMENT OF EDUCATION

Mathematics

Quarter 1- Module 4:
Solving Problems Involving Quadratic Equations and Rational Algebraic Equations

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Quarter 1 - Module 4: **Solving Problems Involving Quadratic Equations and Rational Algebraic Equations**
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In this module, the lesson starts with assessing your prior knowledge of the diverse mathematics principles and concepts studied previously and enhancing your skills in performing mathematical operations. All these skills and knowledge may help you in applying the solutions and processes to real-life problems involving Quadratic Equations and Rational Algebraic Equations.

Jumpstart

Were you able to apply correctly the different methods, mathematical concepts or principles in finding the solutions of the quadratic equations? Why is there a need to perform and master the different methods and perform such mathematical tasks? You will find this out as you go through this module.

Activity 1: Where's My Match

Directions: This activity will enable you to translate each verbal sentence into a mathematical equation and vice versa. Represent each of the following algebraically by finding its match on the box below.

Let **w** represent the width of a rectangle. The length is 7 more than the width.

1. Four times the length is 49
2. One-fourth of the length equals 49
3. 7 cm less than twice the width is 49
4. 7 times the sum of the width and 4 cm is 49
5. Twice the width plus twice the length is 49
6. The product of the width and the length is equal to 49

$4w-2=49$

$4w+2=49$

$w(w+7)=49$

$2w-7=49$

$4w+7=49$

$\frac{w+4}{2}=49$

$7(w+4)=49$

$\frac{w+7}{4}=49$

$\frac{2w+4}{2}=4$

$4(w+7)=49$

$2w+2(w+7)=49$

Process Questions:

a. How do you classify the mathematical equations?
b. How will you describe a quadratic equation? A rational equation?
c. What are the common terms used to represent the "=" sign?
d. Use the phrase "is equal" to in your own sentence.

Activity 2: Translate Me!

Directions: Use variable x to represent the unknown quantity, write an equation from the given information.

1. The product of two consecutive integers is 33.
2. If a number is added to its reciprocal the sum is 25/12.
3. The length of a rectangle is 3cm more than twice the width. The area is 152 square cm.
4. Jeffrey is 5 years older than Fe. Find their present ages if the product of Jeffrey's age 4 years from now and Fe's age 5 years ago is 51.
5. The sum of twice a number and seven is 15.

Were you able to represent each situation by an equation? If YES, then you are ready to perform the next activity.

Discover

Quadratic equations refer to **equations** with at least one squared variable, with the most standard form being $ax^2 + bx + c = 0$.

A **rational equation** is a type of **equation** where it involves at least one **rational** expression, a fancy name for a fraction. Rational expressions typically contain a variable in the denominator. For this reason, we will take care to ensure that the denominator is not 0 by making note of restrictions and checking our solutions. The best approach to address this type of equation is to eliminate all the denominators using the idea of LCD (least common denominator).

To systematically solve a given problem, some steps are to be considered.

Step 1: **READ** the problem. To solve a verbal problem, read the problem carefully and explore what the problem is about. Identify the given, know what is asked, and choose a variable to represent the unknown.

Step 2: **PLAN** the solution. Create an equation from the statement in the problem. Choose the method that best applies to the problem.

a. Extracting the square root
b. Factoring
c. Completing square method
d. Quadratic formula

Step 3: **SOLVE** the problem. Remember the rules and properties in simplifying.
Step 4: **EXAMINE** the solution. Use the solution of the equation to write a statement that settles the problem. Check that the conclusion agrees with the problem situation, otherwise you have to rework the problem.

Let us try to solve word problems applying the concepts of quadratic and rational algebraic equations. This example will help us integrate problem solving in real life situations.

Example 1: The sum of two numbers is 5 and their product is −84. Find these two numbers.

Solution 1:

Step 1: Let x = be the number

Remember that if the sum and the product of the roots of a quadratic equation are given, the roots can be determined. This can be done by inspection or by using the equation $x^2 + \frac{-b}{a}x + \frac{c}{a} = 0$ where $\frac{-b}{a}$ is the sum of the roots and $\frac{c}{a}$ is the product.

Step 2: Equation: $x^2 - 5x - 84 = 0$

Step 3: Solution: Let us try using the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{5 \pm \sqrt{25+336}}{2} = \frac{5 \pm \sqrt{361}}{2} = \frac{5 \pm 19}{2} =$$

$$x_1 = \frac{24}{2} = 12$$

$$x_2 = \frac{-14}{2} = -7$$

Step 4: Since the given are sum and product, we can use the formula

$x_1 + x_2 =$ *sum of roots*	$x_1 \cdot x_2 =$ *product of roots*
When x= 12	x= -7
$12 + (-7) \stackrel{?}{=} 5$	$12 \cdot (-7) \stackrel{?}{=} -84$
✓	✓
5=5	-84 = -84

The solutions are **x=12** and **= -7**

Solution 2:

Another method of finding the roots is to use the equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$. Let $-\frac{b}{a} = 5$ and $\frac{c}{a} = -84$. Then substitute these values in the equation.

$$x^2 + \frac{-b}{a}x + \frac{c}{a} = 0 \longrightarrow x^2 - (5)x + (-84) = 0$$

$$\longrightarrow x^2 - 5x - 84 = 0$$

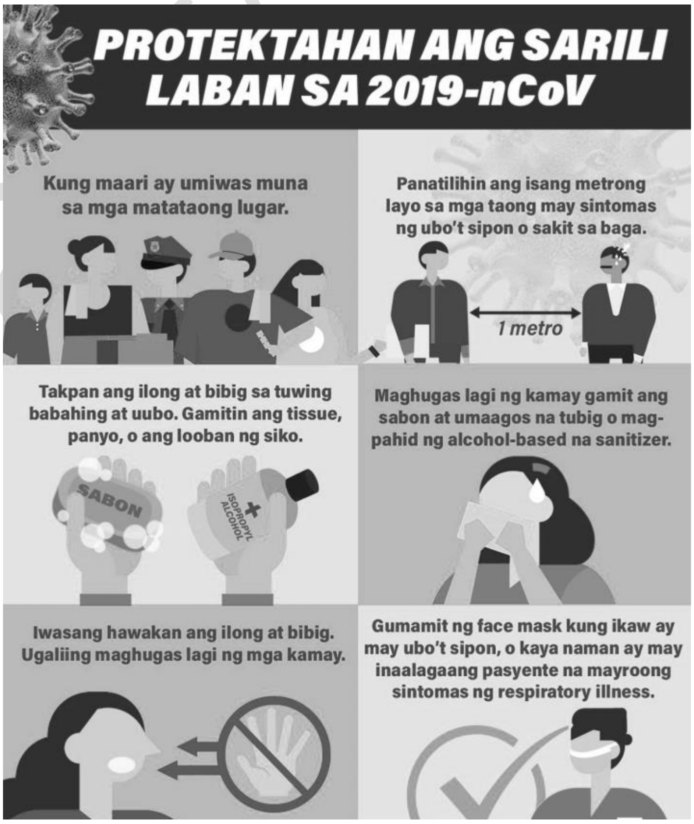
Solve the resulting equation $x^2 - 12x + 27 = 0$ using any of the methods of solving quadratic equation. Try factoring.

$$x^2 - 5x - 84 = 0$$
$$(x + 7)(x - 12) = 0$$

$x + 7 = 0$	$x - 12 = 0$
$x = -7$	$x = 12$

With the obtained roots of the quadratic equation, the numbers are -7 and 12, respectively.

Example 2. The Local Government of La Union wants to place a new rectangular billboard to inform and give awareness to the residents on how to protect themselves from the spread of COVID19. Suppose the length of the billboard to be placed is 6m longer than its width and the area is112 m^2. What will be the length and the width of the billboard?



Source: "ProtectiveMeasures7", Supreme Court of the Philippines, accessed September 24,2020, https://sc.judiciary.gov.ph/protectivemeasures7/.

Step 1: Let x = represent the width of the billboard in meters

x + 6 = represent the length.

Since the area of the billboard is 112 m^2, then (x)(x+6) = 112.

Step 2: The equation (x)(x+4) = 96 is a quadratic equation that can be written in the form $ax^2 + bx + c = 0$.

$$(x)(x+6) = 112 \quad \longrightarrow \quad x^2 + 6x = 112$$
$$\longrightarrow \quad x^2 + 6x - 112 = 0$$

Step 3: Solve the resulting equation (by factoring)

$$x^2 + 6x - 112 = 0$$
$$(x - 8)(x + 14) = 0$$

$x - 8 = 0$	$x + 14 = 0$
$x = 8$	$x = -14$

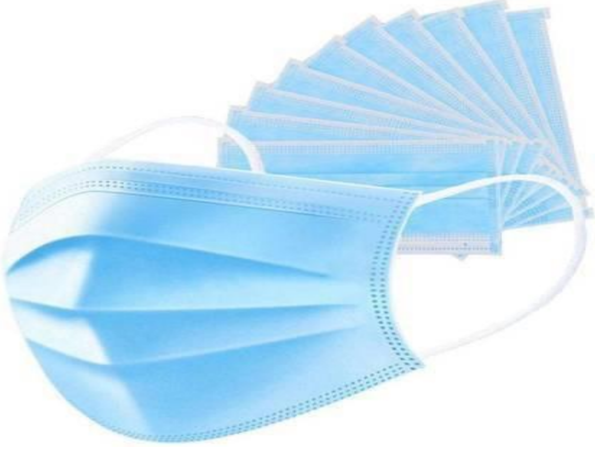
Step 4: Substitute x = 8

$$x^2 + 6x - 96 = 0$$
$$8^2 + 4(8) - 96 = 0$$
$$64 + 32 - 96 = 0$$
$$\checkmark$$
$$0 = 0$$

The equation $x^2 + 6x - 112 = 0$ has two solutions: x = 8 or x = -14.

However, we only consider the positive value of x since the situation involves measure of length. Hence, the width of the billboard is 8m and its length is 14m.

Example 3. Kelly is a workaholic mom and she finds it so timely to sell face mask around the city. She charges 20 pesos per face mask for 60 face masks sold. If Kelly decreases her price for each face mask, 5 more face masks would be sold. What is the price that will maximize her revenue?



Source: "Disposablefacemask", accessed September 24,2020, https://www.flipkart.com/celwark-kn95-n95-kn95-v-5-layer-reusable-mask-respirator-valve-disposal-use-throw-good-health/p/itm5490e3d5b37f0

Step 1: Let n denotes the decrease in price

20-n is the maximize price

Revenue R = price × no. of face masks sold

Step 2:

$$R = (20 - n)(60 + 5n)$$
$$R = 1200 - 60n + 100n - 5n^2$$
$$R = -5n^2 + 40n + 1200$$

Step 3: Now we have to maximize R.

Above is a quadratic equation in n with coefficient of n^2, its maximum value will be

$$-\frac{b}{2a} = -\frac{40}{2(-5)}$$
$$-\frac{b}{2a} = 4$$

Thus, 4 pesos is the decrease in price. At n = 4,

$$\begin{aligned} R &= -5n^2 + 40n + 1200 \\ &= -5(4)^2 + 40(4) + 1200 \\ &= -5(16) + 160 + 1200 \\ &= -80 + 160 + 1200 \\ R &= 1280 \end{aligned}$$

Step 4: $R = -5n^2 + 40n + 1200$

$$\begin{aligned} 1280 &= -5(4)^2 + 40(4) + 1200 \\ 1280 &= -5(16) + 160 + 1200 \\ 1280 &= -80 + 1360 \\ 1280 &= 1280 \end{aligned}$$

Since 20 -n = maximize price, 20 – 4 = 16

Therefore, the price that will maximize her revenue is 16 pesos

Example 4. The sum of a number and its reciprocal is $\frac{29}{10}$. Find the number.

Step 1: Translating the words in the problem:

Let x = the number, then $\frac{1}{x}$ = it's reciprocal

The sum of a number and its reciprocal is $\frac{29}{10}$

$$x + \frac{1}{x} = \frac{29}{10}$$

Step 2: Thus, the equation is $x + \frac{1}{x} = \frac{29}{10}$

Step 3 : LCD = 10x

$10x\left[x + \frac{1}{x} = \frac{29}{10}\right]10x$ multiply both sides by the LCD

$10x\,(x) + 10x\,(\frac{1}{x}) = 10x\,(\frac{29}{10})$ by distributive property

$10x^2 + 10x\,(\frac{1}{x}) = 10x\,(\frac{29}{10})$

$10x^2 + 10 = 29x$ write equation in standard form

$10x^2 - 29x + 10 = 0$ the factoring method works

$(5x - 2)(2x - 5) = 0$

$5x - 2 = 0$ or $2x - 5 = 0$

$x = \frac{2}{5}$ or $x = \frac{5}{2}$

Notice, that the two solutions are reciprocals of each other.

Step 4: Check that both solutions satisfy the statement of the problem.

$$x + \frac{1}{x} = \frac{29}{10}$$

	$x = \frac{2}{5}$ or	$x = \frac{5}{2}$
LCD = 10	$\frac{2}{5} + 1 \bullet \frac{5}{2} = \frac{29}{10}$	$\frac{5}{2} + 1 \bullet \frac{2}{5} = \frac{29}{10}$
	$10\left[\frac{2}{5} + \frac{5}{2} = \frac{29}{10}\right]10$	$10\left[\frac{5}{2} + \frac{2}{5} = \frac{29}{10}\right]10$
	$4 + 25 \stackrel{?}{=} 29$	$4 + 25 \stackrel{?}{=} 29$
	$29 \stackrel{\checkmark}{=} 29$	$29 \stackrel{\checkmark}{=} 29$

Example 5. In driving to Manila, a distance of 540 km, a motorist discovered that he could make the trip via the expressway in 2 hours less time by increasing his speed by 9 km per hour. What was his original speed?

Step 1: Let x = the original speed

	Distance	**Rate**	**Time**
Ordinary road	540 km	x	$\frac{540}{x}$
Expressway	540 km	x+9	$\frac{540}{x+9}$

Step 2: Equation: $\frac{540}{x} - 2 = \frac{540}{x+9}$

Step 3: LCD = x(x+9)

$540(x+9)-2(x)(x+9) = 540x$
$540x + 4860 - 2x^2 - 18x = 540x$
$2x^2 + 18x - 4860 = 0$
$x^2 + 9x - 2430 = 0$
$(x + 54)(x - 45) = 0$
$x = -54$ or $x = 45$

Here, we only consider the positive value since there is no negative distance.

Step 4: Substitute 45 in the equation

$\frac{540}{x} - 2 = \frac{540}{x+9}$
$\frac{540}{45} - 2 = \frac{540}{45+9}$
$12 - 2 = \frac{540}{54}$

Therefore, the original speed is 45 km per hour. $10 \overset{\checkmark}{=} 10$

Example 6: Bobby can clean the yard alone in four hours. It takes Peter five hours to clean the same yard alone. If they work together, how long will it take them to plow the field?

Step 1: Let t = be the amount of time spent for work

	Work Rate	Time	Work Done
Bobby	$\frac{1}{4}$	t	$\frac{1}{4}t$
Peter	$\frac{1}{5}$	t	$\frac{1}{5}t$

Step 2: Equation

$\underbrace{\frac{1}{4}t}_{\text{Part of work done by Bobby}} + \underbrace{\frac{1}{5}t}_{\text{Part of work done by Peter}} = \underbrace{1}_{\text{Work done Together}}$

Step 3: LCD = 20

$\frac{1}{4}t + \frac{1}{5}t = 1$

$20(\frac{t}{4} + \frac{t}{5} = 1)$ multiply both sides by the LCD

$20\,(\frac{t}{4}) + 20\,(\frac{t}{5}) = 20(1)$ by distributive property

$$5t + 4t = 20 \qquad \text{combine similar terms}$$

$$9t = 20 \qquad \text{simplify}$$

$$t=\frac{20}{9} \text{ or } 2\frac{2}{9} \text{ hours}$$

Step 4: Substitute $\frac{20}{9}$ in the equation

$$\frac{1}{4}t + \frac{1}{5}t = 1$$

$$\frac{1}{\cancel{4}}\left(\frac{\overset{5}{\cancel{20}}}{9}\right)+\frac{1}{\cancel{5}}\left(\frac{\overset{4}{\cancel{20}}}{9}\right) = 1$$

$$\frac{5}{9}+\frac{4}{9} = 1$$

$$\frac{9}{9} \stackrel{?}{=} 1$$

$$\checkmark$$
$$1 = 1$$

The next activity will enhance your skills and enrich the concepts and principles of quadratic equations. You will be tasked to write Quadratic equations that would represent real life situations. You will also compare and discuss problems involving quadratic equations that will give you the opportunity to assess your skills and examine errors if there are any.

Explore

Now that you know the important ideas about quadratic and rational algebraic equations and their application in real-life, let's go deeper and have some enrichment activities for you to work on to master the basic concepts you have learned from this lesson.

Activity 3: Find Those Missing!

Directions: Solve each problem algebraically on a separate sheet of paper and identify what method did you use.

1. The sum of the number and its reciprocal is 13/6. Find the number.
2. The sum of two numbers is 20, and their product is 96. Find the two numbers.
3. One number is 5 more than 3 times a second number. If the reciprocal is -2, what are the numbers?

4. The length of a rectangle is three more than twice its width, and its area is 90 square meters. Find its dimensions.
5. The width of a rectangle in one - third its length. If the area of the rectangle is 20 square inches, what are the dimensions of the rectangle?

In this section, the discussion was about solving problems involving quadratic and rational algebraic equations in reals-life. You were given the opportunities to see the real-life application of quadratic equations. What new insights do you have from this lesson? How would you connect this in your daily life? How will you use this in making decisions? Your understanding of this lesson and other previously learned concepts and principles will facilitate your understanding of the succeeding lessons.

Deepen

Performance Task

***Directions**:* Make a comic strip using a situation in real life where the concepts of quadratic or rational algebraic equation is applied. Formulate and solve problems out of these situations.

Rubric for Real-Life Situation Involving Quadratic/Rational Equation

RATING	4	3	2	1
COMIC STRIP WITH EQUATIONS FORMULATED AND SOLVED	The situation is clearly drawn, similar to a real-life situation and the use of quadratic/ rational equation and other mathematical concepts are properly illustrated.	The situation is clearly drawn but the use of quadratic/ rational equation and other mathematics concepts are not properly illustrated.	The situation is not so clearly drawn, and the use of quadratic/ rational equation is not illustrated.	The situation is not clearly drawn, and the use of quadratic/ rational equation is not illustrated.