9

Mathematics 9

Quarter 3-Week 2 – Module 2:
Theorems on the Different Kinds of Parallelogram

AIRs - LM

Mathematics 9
Quarter 3- Week 2 – Module 2: Theorems on the Different Kinds of Parallelogram

First Edition, 2021

Target

This module was designed and written to help you understand the proof of theorems on the different kinds of a parallelogram (rectangle, rhombus, square). As you go through this lesson, keep on asking yourself, "How do the relationships among rectangle, rhombus, and square help us find solutions to our real-life problems?

In going over this module, you are expected to:

Learning Competency

- proves theorems on the different kinds of a parallelogram (rectangle, rhombus, square). **M9GE-IIIc-1**

Learning Objectives:

1. Identify the relationships among rectangle, rhombus, and square.
2. Fill in the missing data in the two-column proof to prove theorems on the different kinds of a parallelogram (rectangle, rhombus, square).
3. Apply the theorems to the different kinds of a parallelogram (rectangle, rhombus, square).

> Before we start the lesson, find out how much you already know about this module by answering the pre – assessment.

PRE - ASSESSMENT

Directions: Read and answer each statement below carefully. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module. Write your answers on a separate sheet of paper.

1. Which of the following theorems is **NOT** true?

    A. If a parallelogram has one right angle, then it has four right angles, therefore the parallelogram is a rectangle
    B. Diagonals of a rectangle form perpendicular
    C. Diagonals of a square are congruent
   D. Diagonals of a rhombus form perpendicular

2. Which of the following statements could be **FALSE**?
   A. Opposite sides of a rectangle are congruent .
   B. The diagonals of a square are perpendicular and bisect each other.
   C. Square has four right angles.
   D. The diagonals of a rhombus are congruent and perpendicular to each other.

3. Which of the following quadrilaterals have congruent diagonals?
   A. Parallelogram   B. Rectangle
   C. Rhombus   D. Trapezoid

4. Which of the following kinds of parallelograms has congruent diagonals and form perpendicular when bisecting each other?
   A. Rectangle   B. Rhombus
   C. Square   D. Trapezoid

5. The following statements hold about a rhombus, **EXCEPT**...
   A. it has four congruent sides   B. diagonals bisect angles
   C. diagonals are perpendicular   D. congruent diagonals

Use figure 1 for numbers 6 – 8.
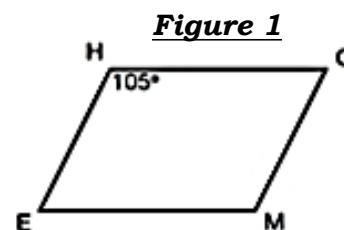
6. HOME is a rhombus, if a diagonal HM is drawn, what is the measure of $m\angle MHO$ when $m\angle H$ is 105?
   A. 37.5   B. 52.5
   C. 75   D. 105

Figure 1

7. HOME is a rhombus, if $m\angle H = 105$ what is the measure of $m\angle HEM$?
   A. 37.5   B. 52.5   C. 75   D. 105

8. In rhombus HOME, if $m\angle OEM = 40$ what is the measure of $m\angle HMO$?
   A. 40   B. 50   C. 60   D. 90

Use figure 2 for numbers 9 – 12.
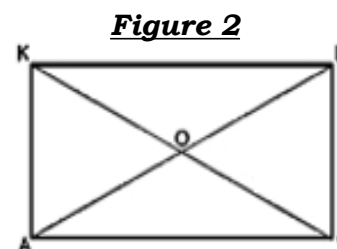
9. What condition will **NOT** make parallelogram KAYE a rectangle?
   A. $\overline{KA} \cong \overline{EY}$ B. $\overline{KY} \perp \overline{AE}$
   C. $\angle A$ is a right angle   D. $\overline{KY}$ bisects $\overline{AE}$

Figure 2

10. If the diagonal of a rectangle measures 14 cm, what is the measure of the other diagonal?
   A. Twice as long as 14 cm   B. They have the same measure
   C. Half as long as 14 cm   D. They are not congruent

11. KAYE is a rectangle. if diagonal KY = 2x + 6 and diagonal AE = 10, what is x?
   A. 1   B. 2   C. 3   D. 4

12. Quadrilateral KAYE is rectangle, $m\angle KYA = 6x + 4$ and $m\angle YKE = 2x + 16$. What is the measure of $m\angle KYA$?
   A. 16   B. 18   C. 20   D. 22

Figure 3
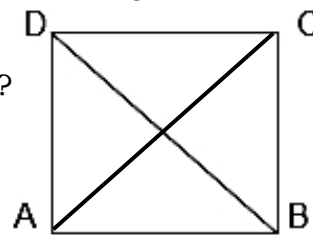
13. ABCD is a square with diagonals AC and BD.
    If AC = 10x – 10 and BD = x + 71. How long is each diagonal?

    A. 9   B. 80   C. 90   D. 100

14. ABCD is a square. If $m \angle ABD$ = 3(x + 10), what is x?
    A. 1   B. 3   C. 5   D. 7

15. If DC = 4m, what is the length of each diagonal?
    A. $2\sqrt{2}$ B. $3\sqrt{2}$ C. $4\sqrt{2}$ D. $5\sqrt{2}$

> You've just answered the pre – assessment of this module. This time, let us test your prior knowledge about the properties of a parallelogram before proceeding to the main topic of this module by answering the given activities below. Goodluck!
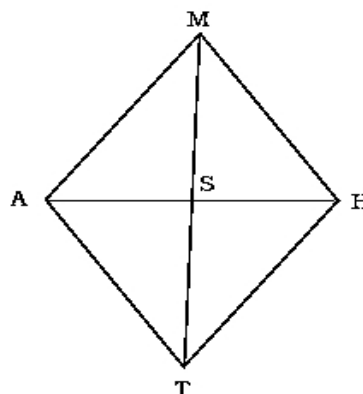
Jumpstart

Activity 1: YES, YOU CAN!

Refer to the given figure at the right and answer the following using properties of a parallelogram.

GIVEN: ▱ MATH is a parallelogram

1. $\overline{MA} \cong$ _____
2. $\overline{MS} \cong$ _____
3. $\Delta THM \cong$ _____
4. $\angle ATH \cong$ _____
5. If $m \angle MHT = 100,$ then $m \angle MAT =$ _____
6. If $m \angle AMH = 70,$ then $m \angle MHT =$ _____
7. If $MS = 7,$ then $MT =$ _____
8. If $AH = 14,$ then $SH =$ _____
9. If $MT = 21,$ then $ST =$ _____
10. If $AT = 34,$ then $MH =$ _____

Activity 2: FIND MY VALUE!

Answer the following by applying the different properties of a parallelogram.

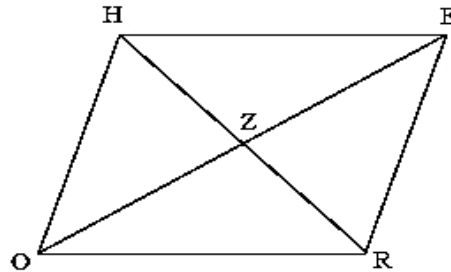1. Given: HE = 2x

   OR = x + 7

   Find: a) x

   b) HE

2. Given: $m \angle HER = 5y - 3$

   $m \angle ROH = 2y + 9$

   Find: a) y

   b) $m \angle ROH$

KEY POINTS

Properties of Parallelogram

1. Any two opposite sides are congruent
2. Any two opposite angles are congruent
3. Any two consecutive angles are supplementary
4. The diagonals of a parallelogram bisect each other
5. A diagonal of a parallelogram divides the parallelogram into two congruent triangles

You are doing great! Now, you are ready to learn more about theorems on the different kinds of a parallelogram from a deeper perspective.
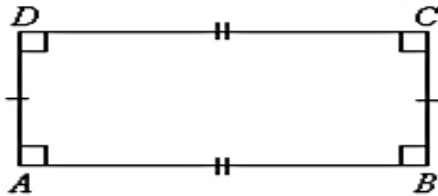
Before you proceed to the different proofs of the theorems on parallelogram, you first study the definition and properties of rectangle, rhombus and square and its relationships to one another.

Definition

Rectangle

Example:
$m\angle DAB = m\angle ABC$
$mDC = mAB$
$If\ AD = 7cm, then\ BC = 7cm$

Rhombus

Example:
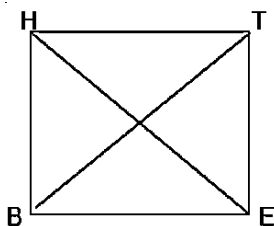$m\angle UCE = m\angle ETU$
$mCU = mTE$
$m\angle CUE = m\angle TUE$

Square
- **Rectangle + Rhombus = Square**

Example:
1. $m\angle HBE = 90$, then what is $m\angle BET = $?
    Solution: $m\angle BET = 90$ since all angles of a square measures 90^0
2. HE = BT
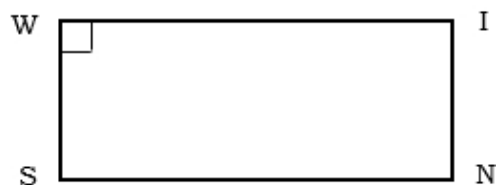   Solution: Diagonals of a square are equal

Theorems on Rectangle

Theorem 1: If a parallelogram has a right angle, then it has four right angles and the parallelogram is a rectangle.

Given: ▭ WINS is a parallelogram

       $\angle W$ is a right angle

Prove: $\angle I, \angle N$ and $\angle S$ are right angles

PROOF:

Statements	Reasons
1. WINS is a parallelogram with $\angle W$ is a right angle	1. Given
2. $\angle W = 90$	2. Definition of right angle
3. $\angle W \cong N$ & $\angle I \cong \angle S$	3. In a parallelogram, opposite angles are congruent
4. $m\angle W = m\angle N$ $m\angle I = m\angle S$	4. Definition of congruent angles
5. $m\angle N = 90$	5. Substitution (SN 2 & 4)
6. $m\angle W + m\angle I = 180$	6. Consecutive angles are supplementary
7. $90 + m\angle I = 180$	7. Substitution (SN 2 & 6)
8. $90 = 90$	8. Reflexive Property
9. $m\angle I = 90$	9. Subtraction Property (SN 7 & 8)
10. $m\angle S = 90$	10. Substitution (SN 4 and 9)
11. $\angle I, \angle N$ and$\angle S$ are right angles.	11. If the measure of an angle is 90, then it is a right angle.
12. WINS is a rectangle.	12. Definition of rectangle.

Theorem 2: The diagonals of a rectangle are congruent.

Given: Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

Prove: $\overline{BD} \cong \overline{AC}$
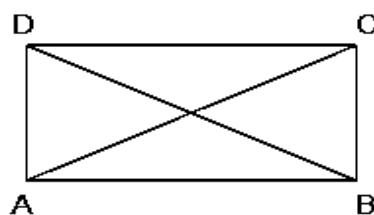
PROOF:

Statements	Reasons
1. Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$	1. Given
2. $\overline{AD} \cong \overline{BC}$	2. Opposite sides of a parallelogram are congruent (Remember, a rectangle is a parallelogram)
3. ∠DAB and ∠CBA are right angles	3. A rectangle is a parallelogram with four right angles
4. ∠DAB ≅ ∠CBA	4. Any two right angles are congruent
5. $\overline{AB} \cong \overline{AB}$	5. Reflexive Property of Congruence
6. Δ DAB ≅ Δ CBA	6. SAS Congruence
7. $\overline{BD} \cong \overline{AC}$	7. Corresponding Parts of Congruent Triangles are Congruent

Theorems on Rhombus

Theorem 3: The diagonals of a rhombus are perpendicular.

Given: Rhombus ROSE

Prove: $\overline{RS} \perp \overline{OE}$
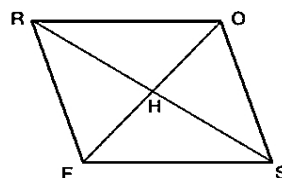
PROOF:

Statements	Reasons
1. Rhombus ROSE	1. Given
2. $\overline{OS} \cong \overline{RO}$	2. Definition of rhombus
3. $\overline{RS}$ and $\overline{EO}$ bisect each other	3. The diagonals of a parallelogram bisect each other.
4. H is the midpoint of RS	4. $\overline{EO}$ bisects $\overline{RS}$ at H
5. $\overline{RH} \cong \overline{HS}$	5. Definition of midpoint
6. $\overline{OH} \cong \overline{OH}$	6. Reflexive Property
7. Δ RHO ≅ Δ SHO	7. SSS Congruence Postulate
8. ∠RHO ≅ ∠SHO	8. CPCTC
9. ∠RHO and ∠SHO are right angles	9. ∠ RHO and ∠ SHO form a linear pair and are congruent.
10. $\overline{RS} \perp \overline{OE}$	10. Perpendicular lines meet to form right angles.

Theorem 4: Each diagonal of a rhombus bisects opposite angles.

Given: Rhombus VWXY

Prove: ∠1 ≅ ∠2

  ∠3 ≅ ∠4
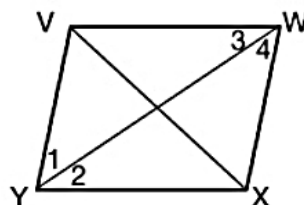
PROOF:

Statements	Reasons
1. Rhombus VWXY	1. Given
2. $\overline{YV} \cong \overline{VW} \cong \overline{WX} \cong \overline{XY}$	2. Definition of rhombus
3. $\overline{WY} \cong \overline{YW}$	3. Reflexive Property
4. $\Delta YVW \cong \Delta WXY$	4. SSS Congruence Postulate
5. ∠1 ≅ ∠2	5. CPCTC

Remember:

  The theorems true to a rectangle and the theorems true to a rhombus are both **true to a square**.

  Were you able to follow and understand the discussion of the proofs presented? Let's continue exploring!
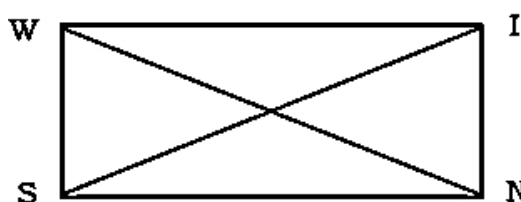
Explore

  Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3: COMPLETE ME!

  Complete the proof of the following theorem. Choose your answer on the word bank below.

Given: Rectangle WINS with

  diagonals $\overline{WN}$ and $\overline{SI}$

Prove: $\overline{WN} \cong \overline{SI}$

Statements	Reasons
1. Rectangle WINS with diagonals $\overline{WN}$ and $\overline{SI}$	1.
2. $\overline{WS} \cong \overline{IN}$	2.
3. ∠WSN and ∠INS are right angles	3.
4.	4. Any two right angles are congruent
5. $\overline{SN} \cong \overline{SN}$	5.
6.	6. SAS Congruence
7. $\overline{WN} \cong \overline{IS}$	7.

Word Bank

Given

CPCTC

Theorem 1
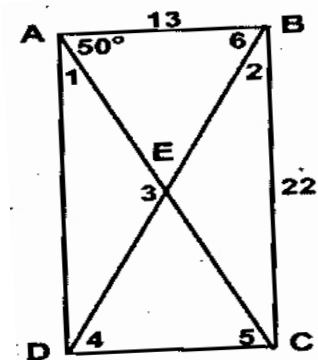
∠WSN ≅ ∠INS

Δ WSN ≅ Δ INS

Reflexive Property

Opposite sides of a parallelogram are congruent

Now that you have learned how to complete the proof, you can now proceed to the next activity.
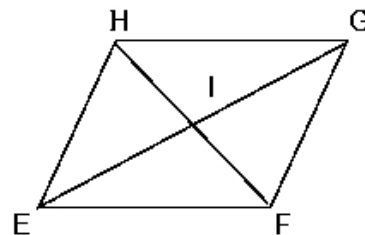
Activity 4: Find My Measure!

A. Given: *ABCD is a rectangle.*

1. What is $m \angle 1$? _____
2. What is $m \angle E$? _____
3. What is $m \angle 5$? _____
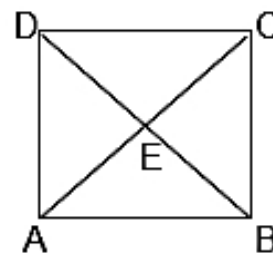4. What is the Perimeter of ABCD? _____

B. Given: *Rhombus EFGH.*

5. If $m \angle HGE = 34$. Find $m \angle FGE$.

6. If $m \angle GHE = 110$. Find $m \angle GHF$.

7. If $m \angle HGF = 80$. Find $m \angle FHG$.

8. If $m \angle IGF = 12$ and $m \angle IGH = 3x$, what is x?

C. Given: *Square ABCD.*

9. If ABCD is a square, then what is $m \angle ACB$?

10. If ABCD is a square, then what is $m \angle DEC$?

How was the activity? Did you enjoy applying your knowledge on the different theorems presented? Now let's go deeper!

Deepen

At this point, you are going to apply the mathematical concepts learned from this module.

BETH is a rhombus with diagonals BT and HE intersect each other at M.
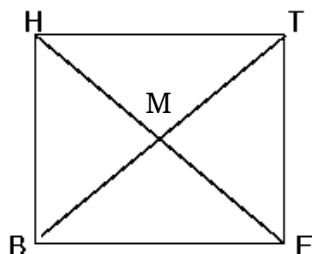
1. If HM = x + 15 and HE = 40, what is x?

2. If $m\angle HBT = 5x - 5$ and $m\angle EBT = 2x + 25$.
    a. What is x?
    b. What is the measure of $m\angle HBT$?

Gauge

I. Multiple Choice:

Directions: Choose the letter of the best answer from the given choices. Write your answers in a separate sheet of paper. *(1 point each)*

1. Which of the following theorems is **NOT** true?

    A. Diagonals of a rhombus are congruent
    B. Diagonals of a square are congruent
    C. Diagonals of a rhombus form perpendicular
    D. Diagonals of a rectangle are congruent

2. The following statements hold about a square, **EXCEPT**...

    A. it has four right angles
    B. diagonals form perpendicular.
    C. diagonals are congruent
    D. Each diagonal does not bisect opposite angles.

3. Which of the following statements could be **FALSE**?
    A. The diagonals of a rectangle are congruent.
    B. The diagonals of a square are perpendicular and bisect each other.
    C. The diagonals of a square are congruent
    D. The diagonals of a rhombus are congruent and perpendicular to each other.

4. Which of the following kinds of parallelograms has four right angles and diagonals are perpendicular when bisect each other?
    A. Rectangle   B. Rhombus
    C. Square   D. Trapezoid

5. The following statements hold about a rhombus, **EXCEPT**...

    A. opposite sides are congruent   B. diagonals divide opposite angles equally
    C. diagonals form perpendicular   D. diagonals are congruent

Use figure 1 for numbers 6 – 8.
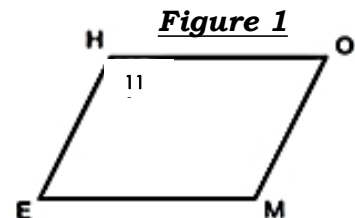
6. HOME is a rhombus, if a diagonal HM is drawn,

    what is the measure of ***m∠MHO*** when *m∠H* is 110?

    A. 35   B. 55

    C. 70   D. 110

Figure 1

7. HOME is a rhombus, if a diagonal EO is drawn, what is the measure of ∠OEM?

    A. 35   B. 55   C. 70   D. 110

8. In rhombus HOME, what is the measure of $m\angle HMO$ if $m\angle OEM = 30$

    A. 30   B. 40   C. 60   D. 120

Figure 2

***Use figure 2 for numbers 9 – 12.***
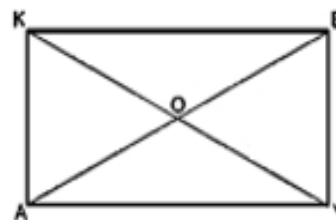
9. What condition will **NOT** make parallelogram
    KAYE a rectangle?

    A. $\overline{KE} \cong \overline{AY}$ B. ∠K is a right angle

    C. $\overline{KY} \perp \overline{AE}$ D. $\overline{KY} \text{ and } \overline{AE}$ intersect at point O

10. If the diagonal of a rectangle measures 21 cm, what is the measure of the other
diagonal?

    A. Twice as long as 21 cm   B. The have the same measure

    C. Half as long as 21 cm   D. They are not congruent

11. KAYE is a rectangle. if diagonal KY = 2x + 4 and diagonal AE = 10, what is x?

    A. 1   B. 2   C. 3   D. 4

12. Quadrilateral KAYE is rectangle, $m\angle KAE = 6x + 2$ and $m\angle YEA = 2x + 4$. What is
the measure of $m\angle KAE$?

    A. 2   B. 5   C. 8   D. 14

Use figure 3 for numbers 13 – 15.

Figure 3
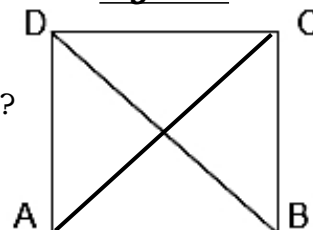
13. ABCD is a square with diagonals AC and BD.
    If AC = 10x + 10 and BD = x + 73. How long is each diagonal?
    A. 7   B. 80   C. 90   D. 100

14. ABCD is a square. If $m\angle ABD$ = 3(x + 8), what is x?
    A. 1   B. 3   C. 5   D. 7

15. If DC = 7m, what is the length of each diagonal?
    A. $4\sqrt{2}$ B. $5\sqrt{2}$ C. $6\sqrt{2}$ D. $7\sqrt{2}$

References

BOOKS
- MATHEMATICS 9 Learners Material (Module 2: Quadratic Functions)

LINKS
- https://www.depedk12.com/2018/02/grade-9-learners-module-and-teachers.html
- https://images.app.goo.gl/1qanhjY63iTTppPf8
- https://images.app.goo.gl/Q9gspHGSmD4Cxgto8
- https://images.app.goo.gl/hV9ziFXR6oHdym399
- https://images.app.goo.gl/hdUDQ4mvkyYke5kK6
- https://images.app.goo.gl/AuGeR1carAkBMeMN6
- https://images.app.goo.gl/sNUTvUJAvPviE8Bj8
- https://images.app.goo.gl/kttcp3eU4iwtCVPJA
- https://en.wikipedia.org/wiki/Rectangle
- https://www.google.com/search?bih=657&biw=1366&hl=en&ei=g9_zX8OyJpXZhwOQgaaQBg&q=what+is+rectangle&oq=what+is+rectangle&gs_lcp=CgZwc3ktYWIQAzIFCAAQyQMyAggAMgIIADICCAAyAggAMgIIADICCAAyAggAMgIIADICCAA6BwgAEEcQsAM6BwgAELADEEM6CAgAEMkDEJECOgUIABCRAjoFCAAQsQM6CAgAELEDEIMBOgcIABDJAxAKOgQIABAKOgcIABDJAxANOgQIABANUNxOWMp3YNV6aANwAngAgAGBAYgB8gaSAQQxMC4xmAEAoAEBqgEHZ3dzLXdpesgBAQ&sclient=psy-ab&ved=0ahUKEwiDgLmq74PuAhWV7GEKHZCACWIQ4dUDCA0&uact=5
- https://www.google.com/search?bih=657&biw=1366&hl=en&ei=YN_zX-ySDInM-QaJlo-oDA&q=what+is+rhombus&oq=what+is+rhombus&gs_lcp=CgZwc3ktYWIQAzIICAAQyQMQkQIyAggAMgIIADICCAAyAggAMgIIADICCAA6BwgAEEcQsAM6BwgAELADEEM6BQgAEJECOgUIABCxAzoICAAQsQMQgwE6BwgAEMkDEA06BAgAEA1Q4VdYr3Bg2XFoAnACeACAAAV2IAe4EkgEBOJgBAKABAaoBB2d3cy13aXrIAQrAAQE&sclient=psy-ab&ved=0ahUKEwjswsaZ74PuAhUJZt4KHQnLA8UQ4dUDCA0&uact=5
- https://www.google.com/search?bih=657&biw=1366&hl=en&ei=cN_zX-K-CIf8wAOLgLCwBw&q=what+is+square&oq=what+is+square&gs_lcp=CgZwc3ktYWIQAzIFCAAQyQMyBAgAEEMyBAgAEEMyBAgAEEMyAggAMgIIADICCAAyAggAMgIIADICCAA6BwgAEEcQsAM6BwgAELADEEM6BwgAEMkDEEM6BQgAELEDUO5vWPiFAWDIjQFoAXACeACAAAVWIAccDkgEBNpgBAKABAaoBB2d3cy13aXrIAQrAAQE&sclient=psy-ab&ved=0ahUKEwjitpOh74PuAhUHPnAKHQsADHYQ4dUDCA0&uact=5