9

Mathematics

Quarter 1 - Module 7: Quadratic Function and its Graph

AIRs - LM

LU_Q1_Mathematics9_Module7

Mathematics 9
Alternative Delivery Mode
Quarter 1 - Module 7: Quadratic Function and its Graph
Second Edition, 2021

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Printed in the Philippines by: ___________________________

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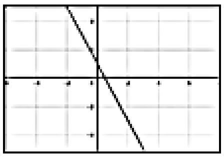
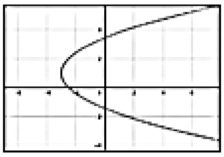
Jumpstart

Let us start this lesson by doing this activity.

Activity 1: Define Me!
Directions: Observe the following graphs. Draw a **FLOWER** in the in your answer sheet if it represents a quadratic function and a **LEAF** if it does not represent a quadratic function.



1. __________



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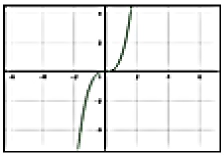

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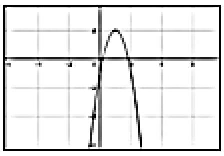

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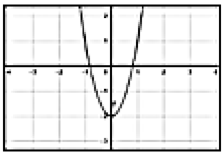

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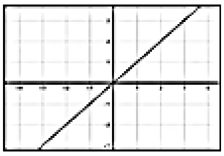


6. __________

Process Question:

1. What kind of graph of a quadratic function?
2. How can you describe the graph of a quadratic function?
3. How can you determine whether the graph is a quadratic or not?

Discover

Transforming Quadratic Function defined by $y = ax^2 + bx + c$ into the form $y = a(x-h)^2 + k$ and vice versa.

It is important to note that the form $y = ax^2 + bx + c$ is called the standard form by some references; others recognized it as the general form and that their standard form is the vertex form which is $y = a(x-h)^2 + k$.

Let us study the following examples in transforming a quadratic function written in the form $y = ax^2 + bx + c$ into the form $y = a(x-h)^2 + k$.

Example 1:

Express $y = 5x^2 - 20x + 10$ in the vertex form $y = a(x-h)^2 + k$ and give the values of h and k.

Solution:

$y = 5x^2 - 20x + 10$
$y = 5(x^2 - 4x) + 10$
$y = 5(x^2 - 4x + 4) + 10 - 5(4)$
$y = 5(x-2)^2 + 10 - 20$
$y = 5(x-2)^2 - 10$

Hence, $y = 5x^2 - 20x + 10$ can be expressed as $y = 5(x-2)^2 - 10$. The values of h and k are 2 and -10, respectively. Take note that h in this case is NOT -2 because the vertex form is always $y = a(x-h)^2 + k$ and NOT $y = a(x+h)^2 + k$.

Example 2:

Transform $y = x^2 + 12x + 32$ into its vertex form and give the values of h and k.

Solution:

$y = x^2 + 12x + 32$
$y = (x^2 + 12x) + 32$
$y = (x^2 + 12x + 36) + 32 - 36$
$y = (x+6)^2 - 4$

Hence, $y = x^2 + 12x + 32$ can be expressed as $y = (x+6)^2 - 4$. The values of h and k are -6 and -4, respectively.

There is another way to rewrite a quadratic function $y = ax^2 + bx + c$ into its vertex form. Let us transform it.

$y = ax^2 + bx + c$

$= (ax^2 + bx) + c$ — Group together the terms containing x.

$= a(x^2 + \frac{b}{a}x) + c$ — Factor out a. Here, a=1.

$= a(x^2 + \frac{b}{a}x \ \frac{b^2}{4a^2}) + c - \frac{b^2}{4a}$ — Complete the enclosed expression to make it into a perfect square trinomial by adding a constant $a\left(\frac{b}{2a}\right)^2$. Subtract the same value from the constant term outside the parentheses to maintain equality.

$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$ — Simplify and express the perfect square trinomial as the square of binomial.

The vertex form is $y = a(x + \frac{b}{2a})^2 + \frac{4ac-b^2}{4a}$. Thus, $\mathbf{h} = \frac{\mathbf{-b}}{\mathbf{2a}}$ and $\mathbf{k} = \frac{\mathbf{4ac-b^2}}{\mathbf{4a}}$. With this formula, we can convert a quadratic function in the form $y = ax^2 + bx + c$ into the vertex form without performing *completing the square*. Let us answer the first example in the previous page using this solution.

Example 3:

Express $y = 3x^2 - 12x + 16$ in the vertex form using the values of h and k.

Solution:

$Let\ a = 3, b = -12, and\ c = 16.$

$h = \frac{-b}{2a} = \frac{-(-12)}{2(3)} = \frac{12}{6} = 2$ and

$$k = \frac{4ac - b^2}{4a} = \frac{4(3)(16) - (-12)^2}{4(3)} = \frac{192 - (144)}{12} = \frac{48}{12} = 4$$

The values of h and k are 2 and 4, respectively. Substituting them to the vertex form $y = a(x - h)^2 + k$, we obtain $y = 3(x - 2)^2 + 4$, which is the same with our answer in Example 1.

The value of k is also equal to $f(h)$ or the function of h. In other words, k is equal to y. Here is a quick hack to obtain the value of k in the example above:

$k = y = 3x^2 - 12x + 16$

Substituting the value of h which is 2 to the variable x,

$k = y = 3(2)^2 - 12(2) + 16$
$k = 3(4) - 24 + 16$
$k = 12 - 24 + 16$
$k = -12 + 16$
$k = 4$

Now, try answering Example 2 using $h = \frac{-b}{2a}$ and $k = \frac{4ac-b^2}{4a}$ *or* $k = f(h)$ and find out if your answer is the same with when using *completing the square*.

Example 4: Transform the vertex form to general form.

Express $y = (x + 6)^2 - 4$ into the form $y = ax^2 + bx + c$.

Solution:

$y = (x + 6)^2 - 4$	
$y = (x^2 + 12x + 36) - 4$	Expand the *square of a binomial*.
$y = x^2 + 12x + 32$	Simplify.

Hence $y = (x + 6)^2 - 4$ can be written as $y = x^2 + 12x + 32$.

Example 5:

Rewrite $f(x) = 2(x+3)^2 - 22$ into the form $y = ax^2 + bx + c$.

Solution:

$f(x) = 2(x+3)^2 - 22$
$f(x) = 2(x^2 + 6x + 9) - 22$
$f(x) = 2x^2 + 12x + 18 - 22$
$f(x) = 2x^2 + 12x - 4$

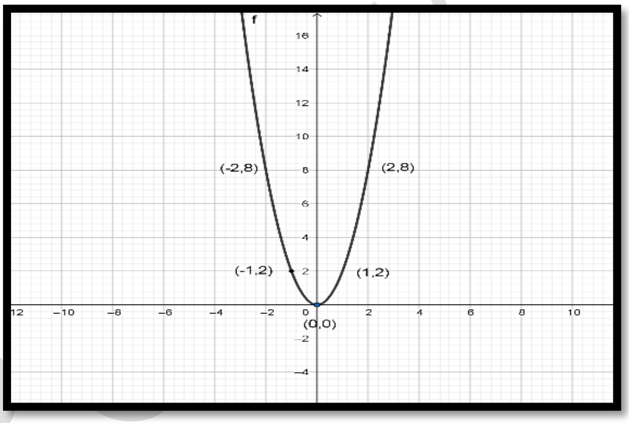
Hence $f(x) = 2(x+3)^2 - 22$ can be written as $f(x) = 2x^2 + 12x - 4$.

Graphs of Quadratic Function

Domain. The domain of the graph of a quadratic function is the set of all real numbers. It is the set of x-values of the graph.
Range. The range of the graph of a quadratic function that opens upward is the set of real numbers from k and up, {y: y ≥ k}. If the graph opens downward, the range is the set of real numbers from k and down, {y: y ≤ k} where k is the y-value of the vertex.

It is time to you to know the significant properties of a graph of quadratic function. Let's start.

Example 1: What is the domain and range of the given function $y = 2x^2$?



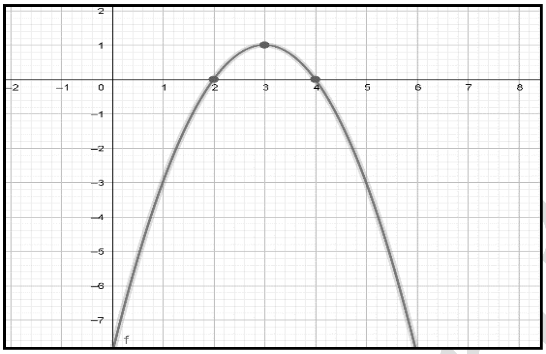
Based from the given graph and its corresponding table of values:
Domain: {-2, -1, 0, 1, 2}
(set of x-values)
Range: {8, 2, 0, 2, 8}
(set of y-values)

x	-2	-1	0	1	2
y	8	2	0	2	8

Example 2: What is the domain and range of the given function y = -(x – 3)²+1?

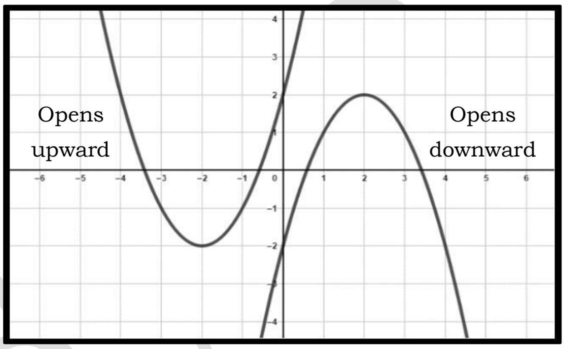
Based from the given graph on the right:

Domain: The domain is the set of all real numbers.
Range: Since, it is downward and the y- values of the vertex (3,1), then the range is {y: y ≤ 1}.

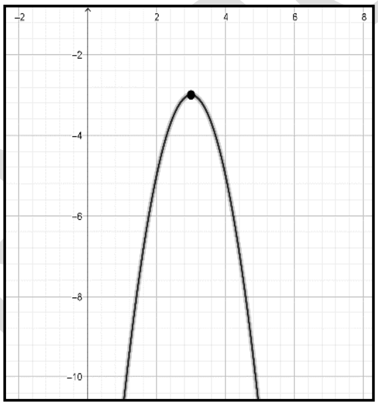


*Quadratic function has a U –shaped graph called a **parabola**. It may open upward or downward depending on the value of a.*

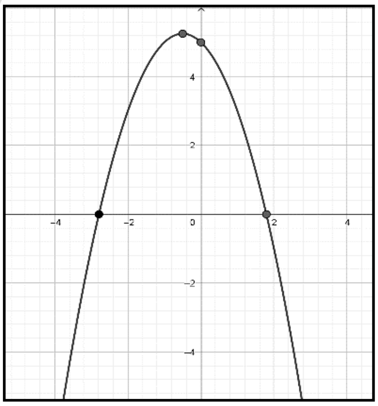
Let's always remember that if a > 0 the graph opens upward and the vertex is the lowest point. On the other hand, if a < 0 the graph opens downward and the vertex is the highest point.



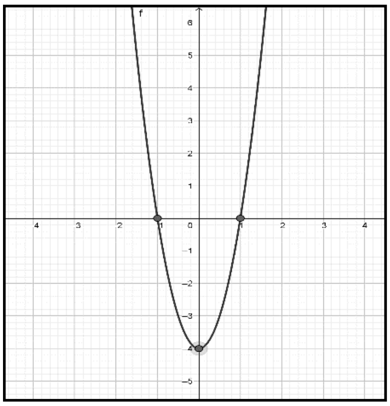
Example 3:



The graph of
f(x) = -x²- x + 5
opens downward
because a = -1.
(a< 0)



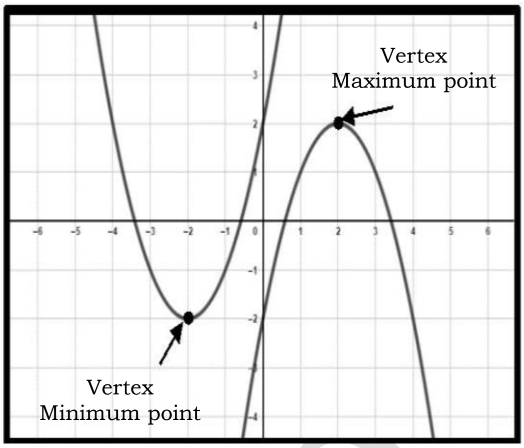
The graph of
f(x) = -2(x – 3)²-3
opens downward
because a = -2.
(a< 0)



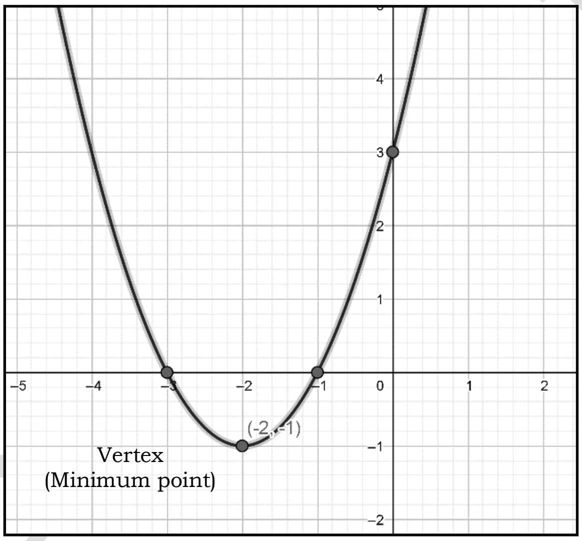
The graph of
f(x) = 4x² – 4
opens upward
because a =4.
(a> 0)

Vertex *is the turning point of a parabola, which can either be the highest or lowest point of the graph. The coordinates of the vertex are (h, k) which can be identified by transforming the quadratic function into standard form or by using the formula* $h = \frac{-b}{2a}$ *and* $k = \frac{4ac-b^2}{4a}$.

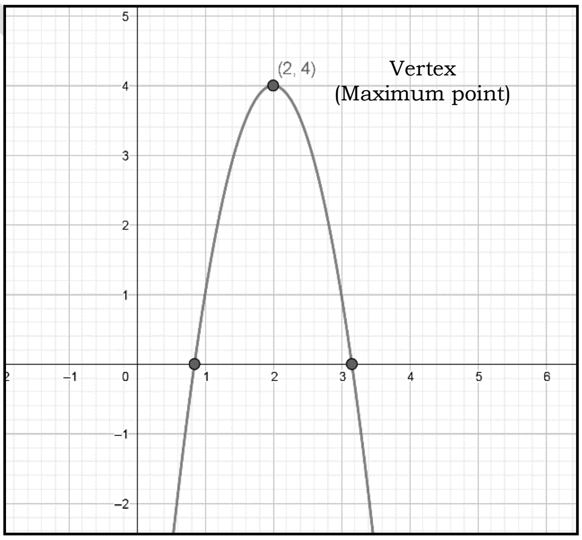
Minimum/Maximum Value *is the value of k. If the parabola opens upward, k is a minimum value. If the parabola opens downward, k is a maximum value.*



Example 4:

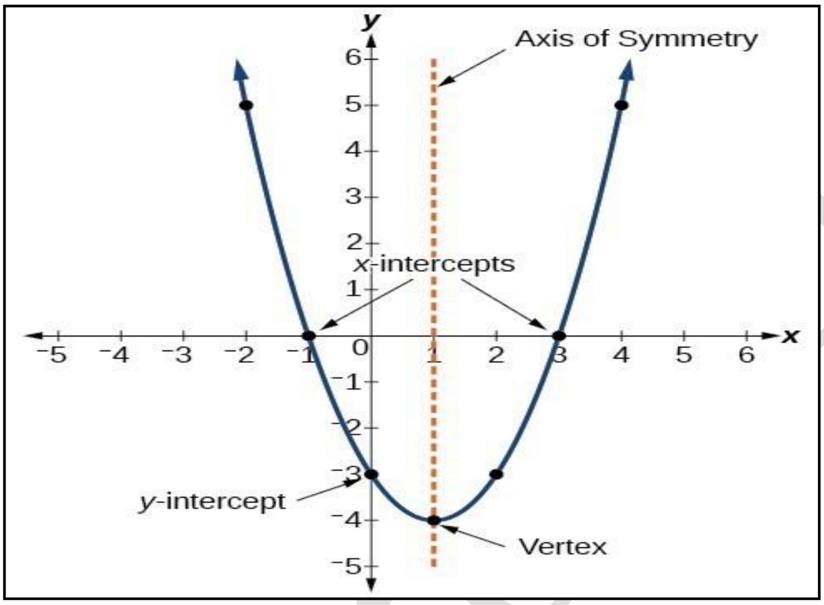


The vertex of the graph $f(x)=x^2+4x+3$ is (-2, -1) and it is the minimum point because the graph opens upward.

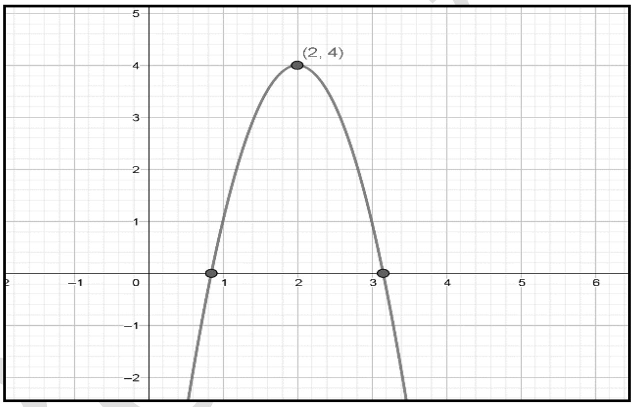


The vertex of the graph $f(x)=-3(x-2)^2+4$ is (2,4) and it is the maximum point because the graphs opens downward.

> ***Axis of symmetry*** *is a line which divides the graph of quadratic function into two parts. It is also the vertical line that passes through the vertex. The equation of the line of the axis of symmetry can be determined by the x-coordinate of the vertex.*



Example 5: What is the axis of symmetry of the equation $f(x)=-3(x-2)^2+4$?



The vertex of the graph $f(x)=-3(x-2)^2+4$ is (2,4).
The x-coordinate of the vertex is $h = 2$.

Therefore, the equation of the line of the axis of symmetry is $x = 2$.

Example 6: What is the axis of symmetry of the equation $f(x)=x^2+4x+3$?

From the given equation: $h=\frac{-b}{2a}=\frac{-4}{2(1)}=\frac{-4}{2}=-2$, therefore, the equation of the axis of symmetry is $x=-2$.

*The **x-intercept/s** is/ are point/ s that lie on the x-axis of the Cartesian Plane. To get x-intercept(s) from the equation, let y = 0 then solve for x.*

Example 7: What is the x-intercept of $y=x^2+4x+3$?

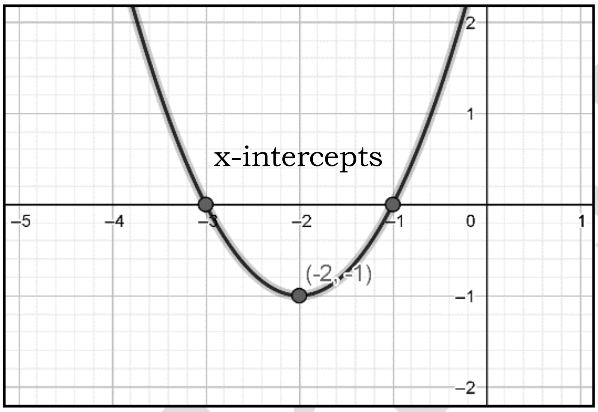
Let y=0, the solve for x.

$0= x^2+4x+3$

$0=(x+3)(x+1)$

$x=-3, \quad x=-1$

The x- intercepts of the function are -3 and -1.



*The **y-intercept** is a point that lie on the y-axis of the Cartesian plane. To get y-intercept(s) from the equation, let x = 0 then solve for y.*

Example 8: What is the y-intercept of $y=x^2+4x+3$?

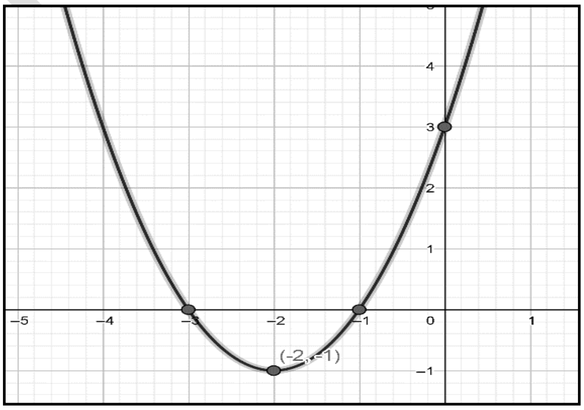
Let x=0, the solve for y.

$y=(0)^2+4(0)+3$

$y=0+0+3$

$y=3$

The y- intercept of the function is 3.

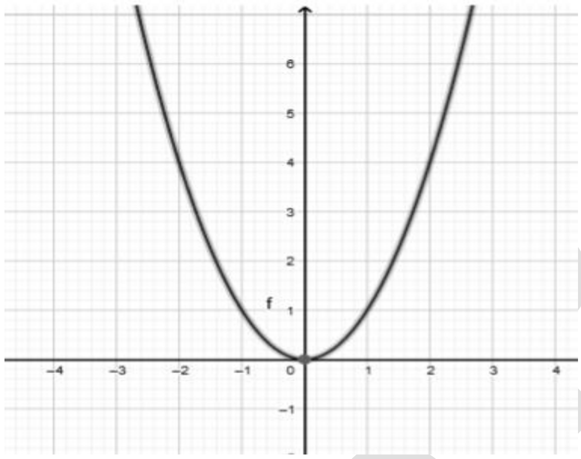


The Effects of Changing the Values of a, h, and k to the Graph of Quadratic Function

Effects of Changing the Value of a

Let us learn about the effects of changing the value of a in the quadratic function **$y = a(x - h)^2 + k$.** Consider the graph of $y = x^2$. The value of $a = 1$ and the vertex is at (0, 0).

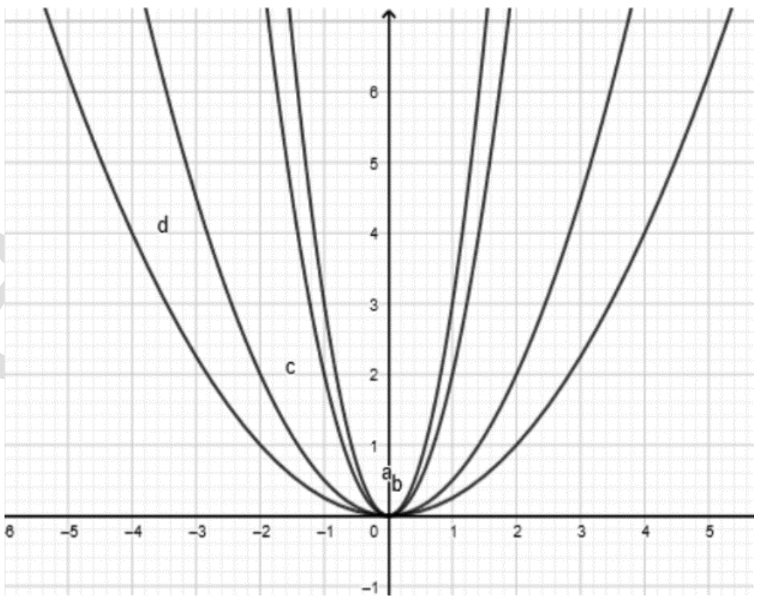
$y = x^2$



What happens to the graph as the value of a changes?

Observe the graphs below.

a. $y = 2x^2$ b. $y = 3x^2$ c. $y = \frac{1}{2}x^2$ d. $y = \frac{1}{4}x^2$



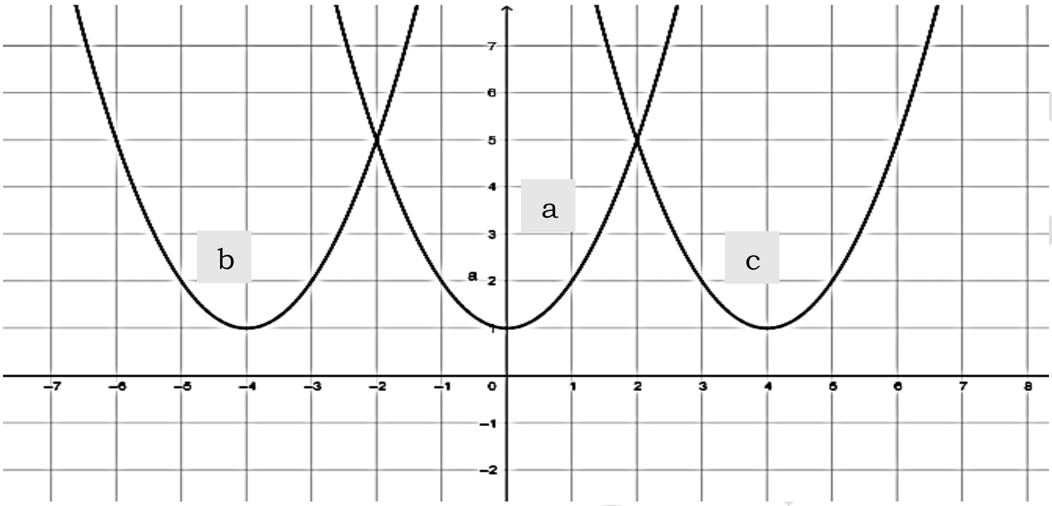
You have noticed that graph with larger values of a have narrower opening. The graphs with smaller values of a have wider openings.

Thus, in the graph of $y = a(x - h)^2 + k$, the larger the $/a/$ is, the narrower the graph.

Effects of Changing the Values of *h* and *k*

Let's see how the quadratic function $\mathbf{y = a(x - h)^2 + k}$ is affected by the changes in *h* and *k*. Observe the graphs of the following quadratic functions.

a. $y = x^2 + 1$ b. $y = (x + 4)^2 + 1$ c. $y = (x - 4)^2 + 1$



The vertex of the quadratic function $y = x^2 + 1$ is at (0, 1). When you add -4 to the value of *h*, the quadratic function becomes $y = (x - 4)^2 + 1$, the graph shifted 4 units to the right. On the other hand, when you add 4 to the value of *h* the quadratic function becomes $y = (x + 4)^2 + 1$, the graph shifted 4 units to the left.

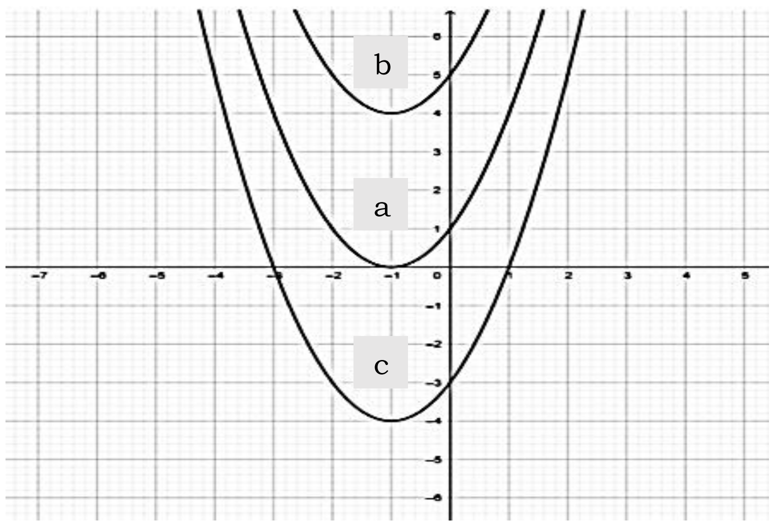
The Shifting of the Parabola and the Value of h

The graph of the quadratic function $y = a(x - h)2 + k$ is shifted c units to the right if $y = [x - (h + c)]2 + k$, and is shifted c units to the left if $y = x - (h - c)]2 + k$.

The value of h corresponds to the horizontal shifting of the graph.

Now, let us observe the following graphs.

a. $y = (x + 1)^2$
b. $y = (x + 1)^2 + 4$
c. $y = (x + 1)^2 - 4$



The vertex of the quadratic function $y = (x + 1)^2$ is at (- 1, 0). When you add -4 to the value of k, the quadratic function becomes $y = (x + 1)^2 - 4$, the graph moves 4 units downward. On the other hand, when you add 4 to the value of h the quadratic function becomes $y = (x + 1)^{2+4}$, the graph shifted 4 units upward.

The Shifting of the Parabola and the Value of ***k***

The graph of the quadratic function $y = a(x - h)^2 + k$ is shifted c units up if $y = (x - h)^2 + (k + c)$, and is shifted c units down if $y = (x - h)^2 + (k - c)$.

The value of *k* corresponds to the vertical shifting of the graph

Look at these examples!

Example 1. Given the quadratic functions below:

a. $y = x^2$ b. $y = 6x^2$ c. $y = (x - 6)^2 + 2$

1. Compare the graphs of the quadratic functions a and b?
.
Answer: The graph of $y = x^2$ is wider than the graph of $y = 6x^2$.
The graph of $y = 6x^2$ is narrower than the graph of $y = x^{2.}$

2. Compare the graphs of quadratic functions a and c?

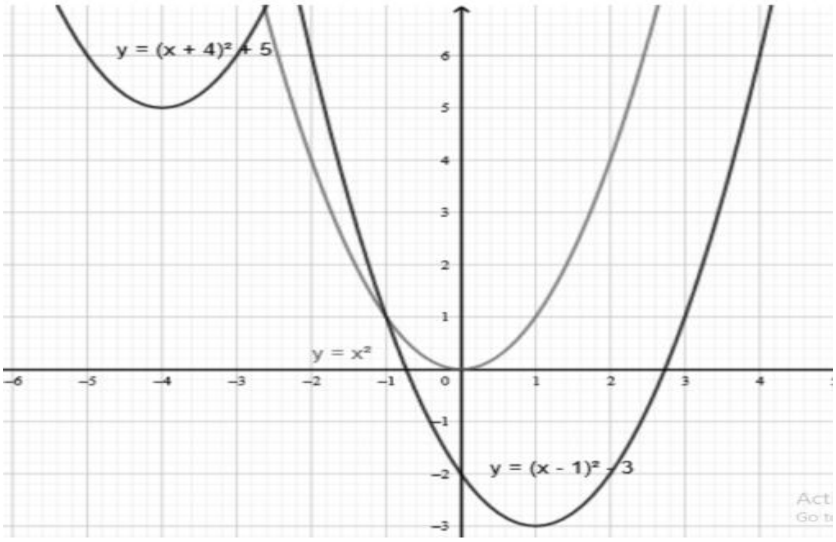
Answer: The graph of $y = (x - 6)^2 + 2$ is shifted 6 units to the right and shifted 2 units down relative to the graph of $y = x^2$.

3. What happens to the graph as the value of *a* increases? decreases?

Answer: As the value of *a* increases, the graph of the quadratic function becomes narrower. On the other hand, as the value of *a* decreases, the graph of the quadratic function becomes wider.

Example 2. Analyze the graphs of the quadratic functions and answer the questions that follow.

a. $y = x^2$
b. b. $y = (x - 1)^2 - 3$
c. c. $y = (x + 4)^2 + 5$

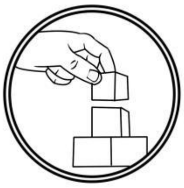


1. What is the effect of the variables h and k on the graph of y = $(x + 4)^2 + 5$ as compared to the graph of y = x^2?

   Answer: The graph of y = $(x + 4)^2 + 5$ is shifted 4 units to the left and shifted 5 units up.

2. How does the value of h and k affect the graph of y = $(x - 1)^2 - 3$ as compared to the graph of y = x^2?

   Answer: The graph of y = $(x - 1)^2 - 3$ is shifted 1 unit to the right and shifted 3 units down



Explore

Activity 2: FIND MY PAIR

Directions: Match the given quadratic function y=ax^2+ bx + c in the first column to its equivalent standard form/vertex form $y = a(x-h)^2 + k$ in the second column. Write the answer on a separate sheet

$y = x^2 - x + \frac{13}{4}$	$y = (x - 2)^2 - 3$
$y = \frac{1}{2}x^2 - 3x + 3$	$y = 2(x - 1)^2 + 2$
$y = -2x^2 + 12x - 17$	$y = -2(x - 3)^2 + 1$
$y = x^2 - 4x + 1$	$y = (x - \frac{1}{2})^2 + 3$
$y = 2x^2 - 4x + 4$	$y = \frac{1}{2}(x - 3)^2 - \frac{3}{2}$

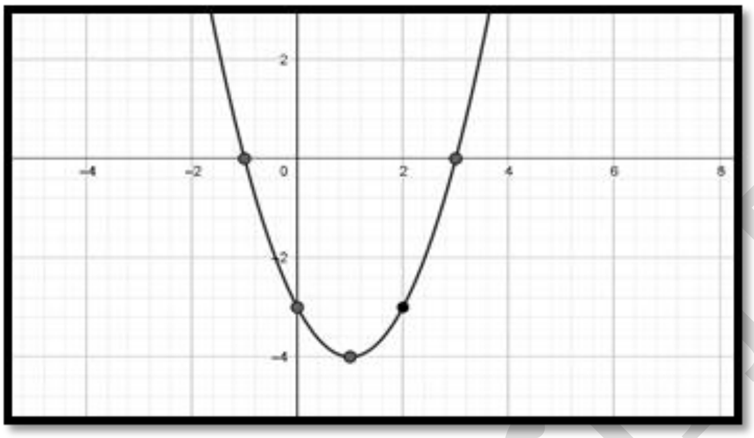
Process Questions

- What mathematical concepts did you used in doing the transformation?
- Explain how the quadratic function in the form y =ax^2+ bx + c into the form $y = a(x-h)^2 + k$

Activity 3: TILL I MEET YOU

Identify the x-intercept/s and y-intercept/s of each function.

1. $y=x^2+5x+6$
   x-intercept/s: ________
   y-intercept/s: ________



2. x-intercept/s: ________
   y-intercept/s: ________

Process Questions:

1. How do you obtain the intercepts of a quadratic function from its graph or its equation?
2. What property of the graph of a quadratic function is used on this activity? Discuss briefly.

Activity 4:

Analyze the following and answer the questions briefly.

1. Given the quadratic function, $\mathbf{y = (x - 1)^2 + 4}$.
   What is the effect of the variables h and k on the graph of $y = (x - h)^2 + k$ as compared to the graph of $y = x^2$?

   ____
   ____
   ____
   ____

2. What is the effect of the variables h and k on the graph of $y = (x + 2)^2 - 2$ as compared to the graph of $y = x^2$?

   ____
   ____
   ____
   ____

Deepen

Activity 5: The Hidden Message

Direction: Write the indicated letter of quadratic function in the form $y = a(x-h)^2 + k$ into the box that corresponds to its equivalent general form y=ax^2+ bx + c.

I	$y=(x-1)^2-4$	**T**	$y=(x-1)^2-16$
S	$y=2(x-\frac{5}{4})^2-\frac{49}{8}$	**F**	$y=(x-3)^2+5$
E	$y=(x-\frac{2}{3})^2+2$	**M**	$y=(x-\frac{1}{2})^2+\frac{3}{2}$
A	$y=3(x+2)^2-\frac{1}{2}$	**U**	$y=-2(x-3)^2+1$
N	$y=(x-0)^2-36$	**H**	$y=2(x+1)^2-2$

Dialog Box

$\square$ $y = x^2 - x + \frac{7}{4}$

$\square$ $y = 3x^2 + 12x + \frac{23}{7}$

$\square$ $y = x^2 - 2x - 15$

$\square$ $y = 2x^2 + 4x$

$\square$ $y = x^2 - 2x - 3$

$\square$ $y = 2x^2 + 5x - 3$

$\square$ $y = x^2 - 6x + 14$

$\square$ $y = -2x^2 + 12x - 17$

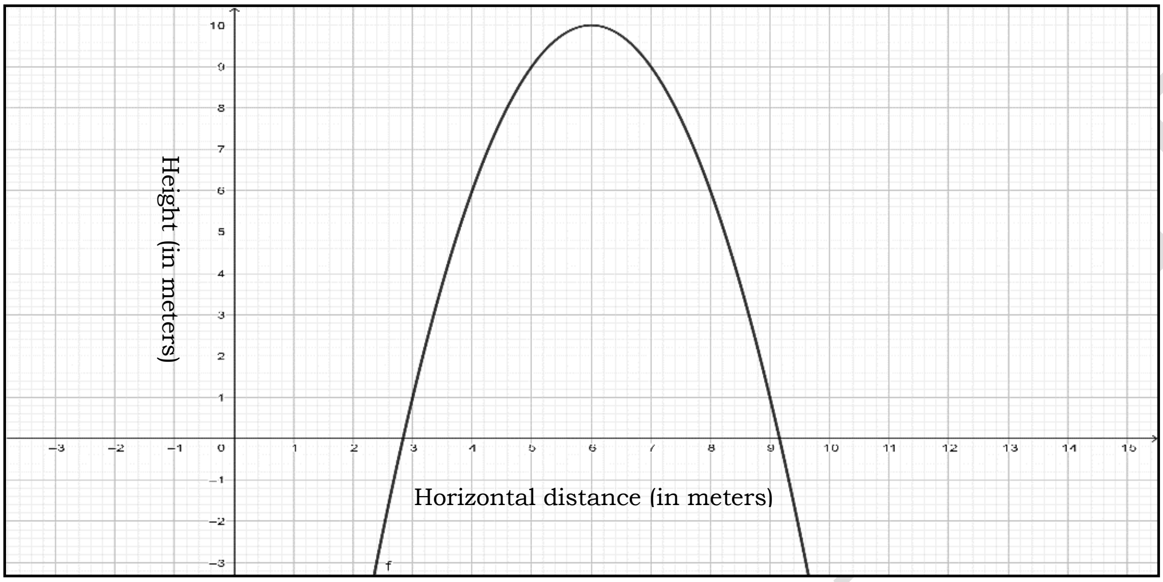
$\square$ $y = x^2 - 36$

Questions:

- How is the square of binomial obtained without using the long method of multiplication?
- Explain how the quadratic function in the form $y = a(x-h)^2 + k$ can be transformed into the form $y = ax^2 + bx + c$.

Activity 6: LET'S ANALYZE!
Direction: Analyze the problem and answer the given questions.

Problem 1: A ball on the playing ground was kicked by Carl Jasper. The parabolic path of the ball is traced by the graph below. Distance is given in meters.



Questions:
1. How would you describe the graph?
2. What is the initial height of the ball?
3. What is the maximum height reached by the ball?
4. Determine the horizontal distance that corresponds to maximum distance.
5. Determine/approximate the height of the ball after it has travelled 2 meters horizontally.
6. How far does the ball travel horizontally before it hits the ground?

Gauge

Post-Assessment

Directions*:* Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which of the following quadratic function is equivalent to $y = -6(x+2)^2 - 6$?
   A. $y = -6x^2 + 24x + 30$
   B. $y = -6x^2 - 24x + 30$
   C. $y = -6x^2 + 24x + 30$
   D. $y = -6x^2 - 24x - 30$