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# Mathematics
## Quarter 1 - Module 6:
## Quadratic Function

**AIRs - LM**

**Mathematics 9**
**Alternative Delivery Mode**
**Quarter 1 - Module 6: Quadratic Function**
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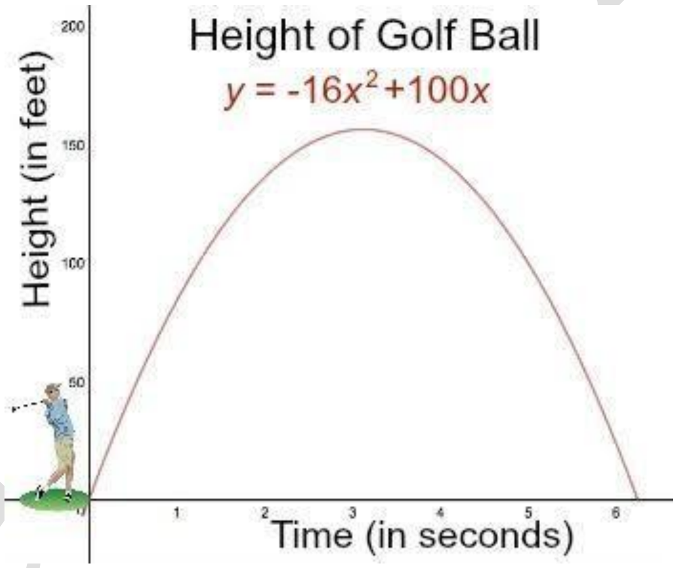
## *Discover*

A **quadratic function** is a function that can be described by the equation of the form $\mathbf{y = ax^2 + bx + c}$ or $\mathbf{f(x) = ax^2 + bx + c}$ where **a**, **b**, and **c** are real numbers and $\mathbf{a \neq 0}$. The highest power of the independent variable x is **2**. Thus, the equation of a quadratic function is of degree 2. The graph of a quadratic function is a smooth curve called **parabola**. But what real-life situations can be modelled by quadratic functions?

**Let's have the following examples!**

Which of the following real-life situations can be modelled by a quadratic function?

**Example 1.** An athlete hitting a golf ball at height $h$ for a given time $t$ define by the function $y = -16x^2 + 100x$.




*Source:https://courses.lumenlearning.com/ivytech-collegealgebra /chapter/finding-x-intercepts-and-y-intercepts/*

> ***Solution:***
> *In the situation, the function is $y = -16x^2 + 100x$ and the degree is 2. The graph also shows a parabola. Thus, hitting a golf ball can be modelled by a quadratic function.*

**Example 2.** The path when a ball is thrown can be modelled by $y = -16x^2 + 64x + 5$, where y (in feet) is the height of the ball x seconds after it is released.

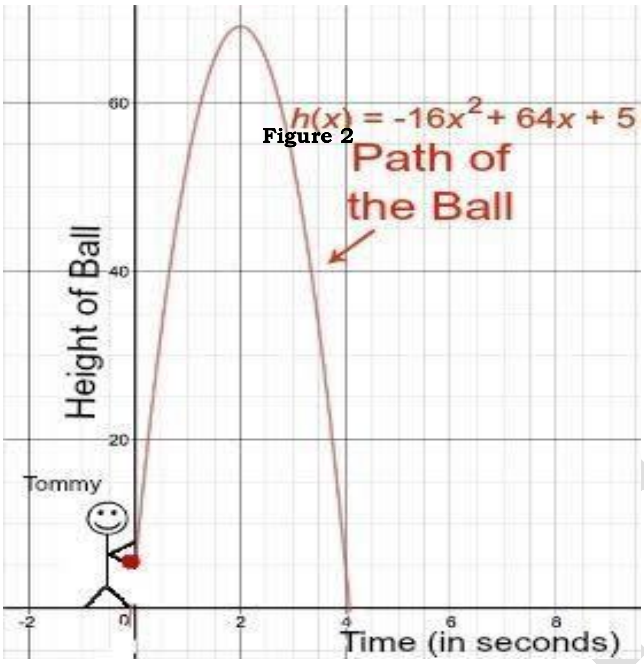



**Figure 2**

***Source:*** *https://study.com/academy/lesson/modeling-the -real-world-with-families-of-functions.html*

> ***Solution:***
> *In the situation, the function is $y = -16x^2 + 64x + 5$ and the degree is 2. The graph also shows a parabola. Thus, throwing a ball can be modelled by a quadratic function.*

**Example 3.** The area $A$ of a rectangular garden whose width is 5m longer than its length.

If we let A = area of the rectangular garden
l = length
w = width

then,

| | |
|---|---|
| $A = lw$ | Area of a rectangle |
| $w = l + 5$ | width is 5 m longer than its length |
| $A = l(l + 5)$ | Express the area in terms of the length |
| $A(l) = l(l + 5)$ | Express the area as a function of length |

In this function, ***l*** is the independent variable and ***A(l)*** read as *"A of l"* is the dependent variable. Hence, if we complete the table of values below,

a. If $l = 1$, substituting the value of $l$ in the rule $A(l) = l(l + 5)$, we have

$$A(l) = l(l + 5)$$
$$A(1) = 1(1 + 5)$$
$$A(1) = 1(6)$$
$$A(1) = 6$$

Using the same procedure to complete the table of values below, the result is:

| ***A(l)*** | 6 | 14 | 24 | 36 | 50 |
|---|---|---|---|---|---|
| ***l*** | 1 | 2 | 3 | 4 | 5 |

which means that the area of the rectangular garden when the length $l$ is 1m is 6m², 14 m² if the length is 2m and so on. So, what is the area $A$ of the rectangular garden if the length $l$ is 7m? 9m? 10m?

In the given situation, the rule of the correspondence is $l(l+5) = l^2 + 5l$ applying distributive property of multiplication. Hence, the function can be written as $A(l) = l^2 + 5l$. Basically, the rule of the correspondence is a polynomial of degree 2 and it is a quadratic function.

A quadratic function just like any other function can be represented using table of values, graphs, and equations. But, how do we know if the given table of values, graphs and equations represent quadratic functions? To answer this, let's have the following examples.

## A. TABLE OF VALUES

The table of values below has columns for x and its corresponding values of y. **To solve for the values of "y", substitute the value of x.**

***1. f(x) = 2x + 1***

1 1 1 1 1 1 → *equal differences in variable "x"*

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| **y** | **-5** | **-3** | **-1** | **1** | **3** | **5** | **7** |

**NOTE**: *To determine the differences in variable x and y, subtract it from right to left.*

2 2 2 2 2 2 → *equal 1st differences in "y".*

**Solution:** To determine the value of "y", substitute the value of "x" in the equation *f(x) = 2x + 1 resulting to:*

| If x = 3, then | If x = 2, then | If x = 1, then | If x = 0, then |
|---|---|---|---|
| y = 2x + 1 | y = 2x + 1 | y = 2x + 1 | y = 2x + 1 |
| y = 2(3) + 1 | y = 2(2) + 1 | y = 2(1) + 1 | y = 2(0) + 1 |
| y = 6 + 1 | y = 4 + 1 | y = 2 + 1 | y = 0 + 1 |
| **y = 7** | **y = 5** | **y = 3** | **y = 1** |

| If x = -1, then | If x = -2, then | If x = –3, then |
|---|---|---|
| y = 2x + 1 | y = 2x + 1 | y = 2x + 1 |
| y = 2(-1) + 1 | y = 2(-2) + 1 | y = 2(-3) + 1 |
| y = -2 + 1 | y = -4 + 1 | y = -6 + 1 |
| **y = -1** | **y = -3** | **y = -5** |

**2. $g(x) = x^2 - 4$**

1 1 1 1 1 1 ⇒ *equal differences in variable "x"*

| X | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| Y | 5 | 0 | -3 | -4 | -3 | 0 | 5 |

**NOTE**: *To determine the differences in variable x and y, subtract it from right to left.*

-5 -3 -1 1 3 5 ⇒ *1st differences in "y"*

2 2 2 2 2 ⇒ *equal 2nd differences in "y".*

**Solution:** To determine the value of "y", substitute the value of "x" in the equation *$g(x) = x^2 - 4$ resulting to:*

If x = 3, then
$y = x^2 - 4$
$y = (3)^2 - 4$
$y = 9 - 4$
**y = 5**

If x = 2, then
$y = x^2 - 4$
$y = (2)^2 - 4$
$y = 4 - 4$
**y = 0**

If x = 1, then
$y = x^2 - 4$
$y = (1)^2 - 4$
$y = 1 - 4$
**y = -3**

If x = 0, then
$y = x^2 - 4$
$y = (0)^2 - 4$
$y = 0 - 4$
**y = -4**

If x = -1, then
$y = x^2 - 4$
$y = (-1)^2 - 4$
$y = 1 - 4$
**y = -3**

If x = -2, then
$y = x^2 - 4$
$y = (-2)^2 - 4$
$y = 4 - 4$
**y = 0**

If x = 3, then
$y = x^2 - 4$
$y = (-3)^2 - 4$
$y = 9 - 4$
**y = 5**

**Remember:**

You have seen that in a linear function, *equal differences in variable "x" produce equal differences in "y".*

However, in a quadratic function, *equal differences in variable "x" produce equal second differences in "y".*

**B. GRAPH**

Using the respective table of values, we have determined the values of "y" in terms of the values of "x".

- From the given function, **y = 2x + 1**, the table yields the *ordered pairs: {(–3, –5), (–2, –3), (–1, –1), (0, 1), (1, 3), (2, 5), (3, 7)}*

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| y | -5 | -3 | -1 | 1 | 3 | 5 | 7 |

- From the function, **$y = x^2 - 4$**, the *ordered pairs are {(–3, 5), (–2, 0), (–1, –3), (0, –4), (1, –3), (2, 0), (3, 5)}*

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| y | 5 | 0 | -3 | -4 | -3 | 0 | 5 |

Now, we can plot the ordered pairs (x, y) using a Cartesian plane and connect the points on the graph.

**$y = 2x + 1$**

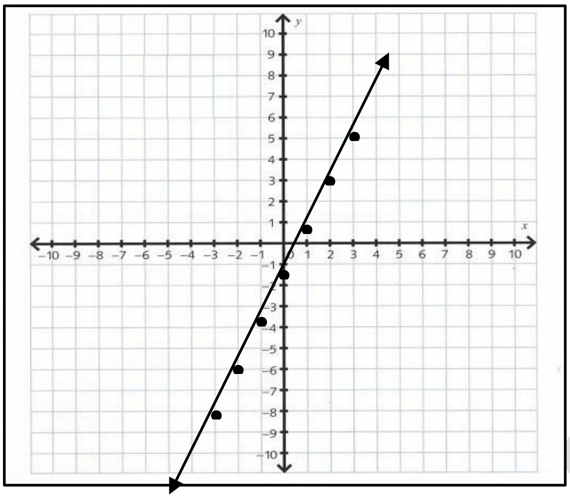


**Figure 1:**
Linear Function

**$y = x^2 - 4$**

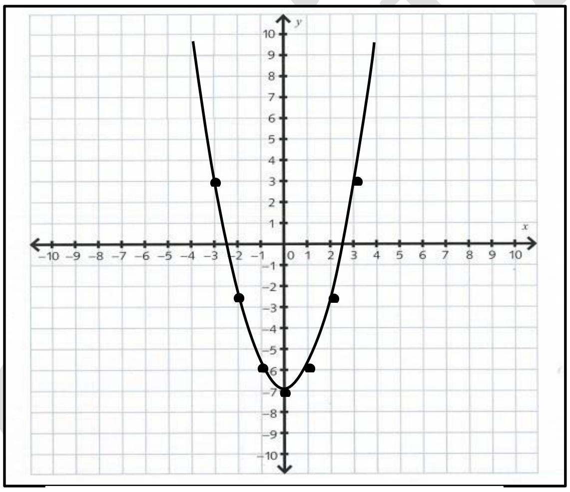


**Figure 2:**
Quadratic Function

**Remember:**
The graph of a linear function is a **straight line** while the graph of a quadratic function is a smooth curve called **parabola.**

**C. EQUATIONS**

Apart from knowing the definition of quadratic function, prior knowledge of products of binomials is also imperative to broaden your knowledge in representing quadratic function by equations.

Products of Binomials

1. *Multiplying monomial to a binomial*
   Examples:

STEPS

a. $3(x^2 + 7) = \underline{\mathbf{3x^2 + 21}}$ -----> (Use Distributive Property of Equality)

1. multiply 3 and 1st term of the binomial

2. multiply 3 and $2^{nd}$ term of the binomial

b. $x(x-4)$ = $\underline{\mathbf{x^2 - 4x}}$ ------> (Use Distributive Property of Equality)

1. multiply x and $1^{st}$ term of the binomial
2. multiply 3 and $2^{nd}$ term of the binomial

*2. Multiplying binomial to a binomial*

Examples:

a. $(x + 2)(x + 7) = x^2 + 7x + 2x + 14$ ------> (Use FACE or FOIL Method)

$= \underline{\mathbf{x^2 + 9x + 14}}$

1. multiply the first terms
2. multiply the outer terms
3. multiply the inner terms
4. multiply the last terms

b. $(x + 7)^2$ = $\underline{\mathbf{x^2 + 14x + 49}}$ -------> (Use Square of a Binomial)

1. square the $1^{st}$ term
2. twice the product of the $1^{st}$ and $2^{nd}$ term
3. square the $2^{nd}$ term

c. $(x - 3)(x + 3)$ = $\underline{\mathbf{x^2 - 9}}$ ------->(Use Sum & Difference of a Binomial)

1. multiply the two $1^{st}$ terms
2. multiply the two $2^{nd}$ terms

## Explore

**Activity 3:**

### Find My Difference!

***Direction:*** Show and tell whether the given table of values below represent a quadratic function or linear function. *(Write your answers in a separate sheet of paper).*

1.

| | | | | | | |
|---|---|---|---|---|---|---|
| **x** | -1 | 0 | 1 | 2 | 3 | 4 |
| **y** | 9 | 4 | 1 | 0 | 1 | 4 |

2.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **x** | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| **y** | 3 | 5 | 7 | 9 | 11 | 13 | 15 |

3.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **x** | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| **y** | -4 | -1 | 0 | -1 | -4 | -9 | -16 |

4.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **x** | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| **y** | 27 | 18 | 11 | 6 | 3 | 2 | 3 |

*The previous activity familiarized you with recognizing whether a given table of values demonstrates quadratic function or not. I'm sure in the next activity, you are now ready to represent table of values using graph. Let's do it!*

## Activity 4: What Do I Model?

***Directions:*** Identify which of the given real-life situations represents a linear function or a quadratic function. Write **QF** if the situation represents a quadratic function and **LF** if the situation represents a linear function.

1. Throwing a ball
2. Distance travelled when jogging
3. Shooting a cannon
4. Diving from a platform
5. Renting a van
6. Richard predicted that the number of mango trees, $\boldsymbol{x}$, planted in a farm could yield $y = -20x^2 + 2800$ mangoes per year.
7. The cost $C$ of milkfish per number of kilogram $n$ at Php 170.00 represented by the function $C(x) = 170n$.
8. A store sells lecture notes, and the monthly revenue $R$ of this store can be modelled by the function $R(x) = 3000 + 500x - 100x^2$, where $x$ is the peso increase over Php 4.00.
9. The maximum height that can be reached by the projectile if the height in meters of a projectile after $t$ seconds is given by $h(t) = 160t - 80t^2$.
10. If 100 m of fencing wire is to be used to enclosed a rectangular lot, then the area of the fenced lot is represented by $A(w) = w(50 - w)$.

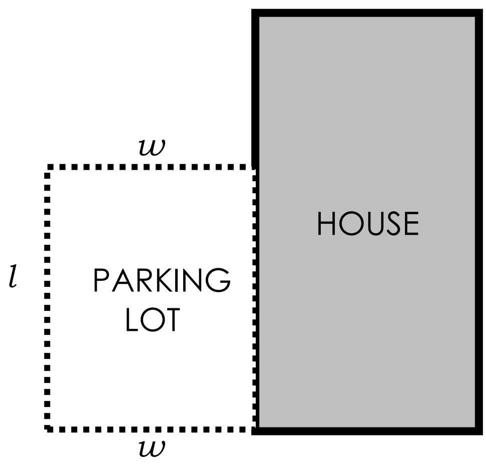
# *Deepen*

## Activity 5

***Directions:*** Read and analyze the problem below and do what is asked.

**Parking Lot Problem**

Mr. Santos wants to enclose the rectangular parking lot beside his house by putting a wire fence on the three sides as shown in figure 1. If the total length of the wire is 80 m, find the dimension of the parking lot that will enclose a maximum area.




**Process Questions:**

1. If we let $w$ be the width and $l$ be the length, what is the equation for the sum of the measures of the three sides of the parking lot?
2. What is the length of the rectangle in terms of the width?
3. Write the quadratic function that models the area of the parking lot.

4. Write the quadratic function to its equivalent quadratic equation then find the values of *a, b,* and *c.*
5. Use the formula $x = \frac{-b}{2a}$ to solve for the width then solve for the length using the obtained value of x.

## Activity 6: Show Me My Graph!

***Directions:*** Do what is asked. Use separate sheet of paper for your answer.

A. Illustrate whether the following table of values below represent a quadratic function or not.

1.

| **x** | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| **y** | 2 | 0 | 0 | 2 | 6 | 12 |

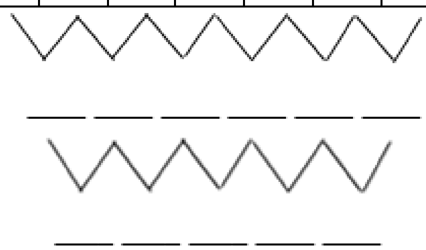
2.

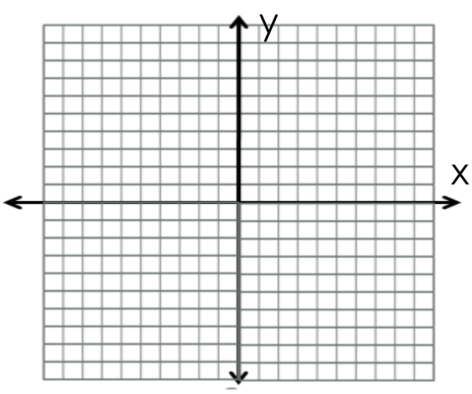
| **x** | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| **y** | -7 | -5 | -3 | -1 | 1 | 3 | 5 |

B. Given the equation y = – $x^2$, find the differences in variable "y" to complete the table of values then plot the points on the Cartesian plane to show it represents a quadratic function.

3.

| **x** | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| **y** | | | | | | | |






*Since you already know the important notes about representing quadratic function, let us go deeper.*