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Mathematics

Quarter 3 – Week 7 – Module 7: Use Triangle Similarity in Proving Pythagorean Theorem

AIRs - LM

Mathematics 9
Quarter 3 – Week 7 – Module 7: Proving Pythagorean Theorem Using Triangle Similarity
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Lesson 1	Use Triangle Similarity in Proving Pythagorean Theorem

It is easier to understand this module if you know the concepts associated with similarity of triangles particularly the different theorems. Most real-life problems involve concepts of similarities of triangle. If you find difficulty in answering the activities, ask assistance from your teacher.

Activity 1: Agree or Disagree?
Directions: Check the first column if you agree, otherwise disagree if not.

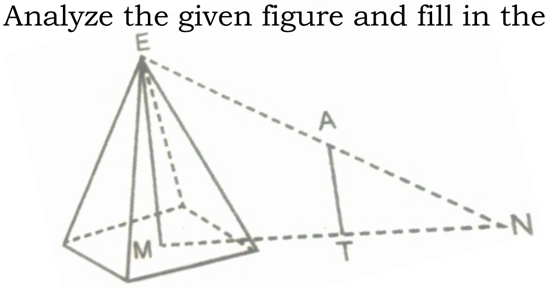
STATEMENTS	AGREE	DISAGREE
1. The hypotenuse of a right triangle is the longest of all its three sides.		
2. In a right triangle, the measure of the hypotenuse is equal to the sum of the measure of its legs.		
3. SSS Similarity Theorem states that two triangles are similar if an angle of one triangle is congruent to an angle of another triangle and the corresponding sides including those angles are in proportion.		
4. The AAA Similarity Postulate states that if the three angles of one triangle are congruent to three angles of another triangle, then the two triangles are similar.		
5. Corresponding sides of similar polygons are proportional		

For you to understand the lesson well, do the following activities.
Have fun and good luck!

Jumpstart

Activity 2: Fill Me In!

Directions: Analyze the given figure and fill in the correct answer to complete the sentences.



1. ME is the unknown _______ of the pyramid.
2. MN is the length of the shadow of the ________.
3. _______ is the height of a vertical post.
4. TN is the length of the ______ of the vertical post.
5. Is length ME can be measured by measuring tool? ______

Activity 3: Choozzy!

Directions: Choose the correct answer from the box.

C. Pythagoras	D. hypotenuse
E. $c^2 = a^2 + b^2$	

_____ 1. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.
_____ 2. The longest side of a triangle.
_____ 3. The algebraic notation of Pythagorean Theorem.
_____ 4. The triangle that should be used if the square of the hypotenuse is equal to the sum of the two sides.
_____ 5. The Mathematician who discovered Pythagorean Theorem.

How did you find the activity? Can you still recall the properties triangles and understand how the concepts of triangles can be applied to solve geometric problems, study the illustrative examples below.

Discover

Applying the theorems to show that given triangles are similar and proving the Pythagorean Theorem is essential in day to day activities. In these lessons, you will realize that your success in writing proofs involving similarity depends upon your skill in making an accurate and appropriate representation of mathematical conditions. Your logical and reasoning skills will be developed as you go through with the different activities.

How do we create proportionality statements for triangles and how do we show two triangles are similar? Being able to create a proportionality statement and apply the theorems are our goal when dealing with similar triangles. By definition, we know that if two triangles are similar, then the corresponding angles are congruent and their corresponding sides are proportional. Please take a look at the given examples below on how we apply the different theorems.

Example 1:

A 6-inches-by-5 inches picture is a copy that was reduced from the original one by reducing each of its dimension by 40%. In short, each dimensions of the available copy is 60% of the original one. You would like to enlarge it back to its original size using a copier. What copier settings would you use?

If each dimension of the available picture is 60% of the original one, then we can make the following statements to be able to determine the dimensions of the original picture:

1. The length of 6 inches is 60% of the original length L. It means that 6 = 60% (L) or that is $L = \frac{6}{0.6} = 10\ inches$
2. The width of 5 inches is 60% of the original width W. It means that 5=60% (W) or that is $W = \frac{5}{0.6} = 8\frac{1}{3}\ inches$

To determine the copier settings to use to be able to increase the 6-inches by 5-inches picture back to the 10 inches by $8\ \frac{1}{3}$ inches, the following statements should be used:

1. The original length of 10 inches in what percent of 6? It means that 10=rate R(6) or that is $R = \frac{10}{6} = \frac{5}{3} \approx 1.67 \approx 167\%$
2. The original width of 8 1/3 inches is what percent of 5? It means that 8 1/3 = rate R (5) or that is $R = \frac{8\frac{1}{3}}{6} = \frac{\frac{25}{3}}{3} = \frac{25}{3}.\frac{1}{5} = \frac{5}{3} \approx 1.67 \approx 167\%$

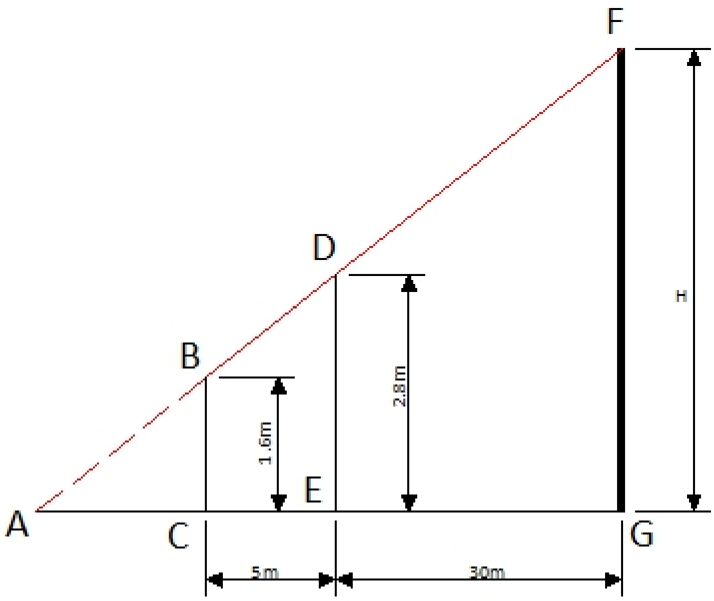
Therefore, the copier should be at 167% the normal size to convert the picture back to its original size.

Example 2

Trisha wants to measure the height of a building but she does not have the tools to do so. She noticed that there is a tree located in front of the building so she decided to use her smartness and the geometry knowledge that she got at school to measure the building height. She measured the distance between the tree and the building and found that it is 30m. She stood in front of the tree and started backing until she could see the top edge of the building from above the tree top. She marked her place and measured it from the tree. It was 5m. Knowing that the tree height is 2.8m and Trisha's eyes height is 1.6m, help Trisha to do the math and calculate the building height.

Solution:

This problem can be geometrically represented as in the figure below.



First, let us make use of the similarity between the triangles ΔABC and ΔADE.

$$\frac{BC}{DE} = \frac{1.6}{2.8} = \frac{AC}{AE} = \frac{AC}{5+AC} = 2.8\ x\ AC = 1.6\ x(5 + AC) = 8 + 1.6\ x\ AC$$

$$(2.8 - 1.6)x\ AC = 8 = AC = \frac{8}{1.2} = 6.67$$

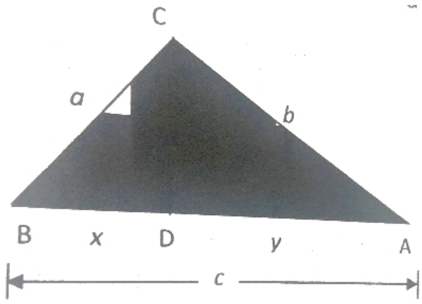
We can then use the similarity between triangles ΔACB and ΔAFG or between the triangles ΔADE and ΔAFG. Let us take the first option.

$$\frac{BC}{FG} = \frac{1.6}{H} = \frac{AC}{AG} = \frac{6.67}{6.67+5+30} = 0.16 = H = \frac{1.6}{0.16} = 10m$$

Consider the next examples to understand how to use the concept of a Pythagorean Theorem in real-life situations.

The main theorem about right triangles is attributed to the name of a Greek, **Pythagoras** of Samos, born around 570-560 BC. The theorem states that: "In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs." That is the Pythagorean Theorem.

In a right triangle, the longest side is the hypotenuse and the legs are the other two sides.
In algebraic term, it is written as, $c^2 = a^2 + b^2$. To satisfy the given equation, let us prove the theorem by letting ΔABC be the right triangle with legs of lengths a and b and hypotenuse c.



Proof: Let CD be the altitude to the hypotenuse and $\overline{CD} \perp \overline{BA}$. Let BD = x and DA = y. Since either leg of ΔABC is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg, then, $\frac{c}{a} = \frac{a}{x}$ *and* $\frac{c}{b} = \frac{b}{y}$.
Hence, $a^2 = cx$ *and* $b^2 = cy$, by adding the equations we have,

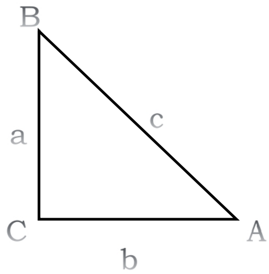
$$a^2 + b^2 = cx + cy$$
$$a^2 + b^2 = c(x + y)$$

Since x+y = c, we have $a^2 + b^2 = c(x + y)$

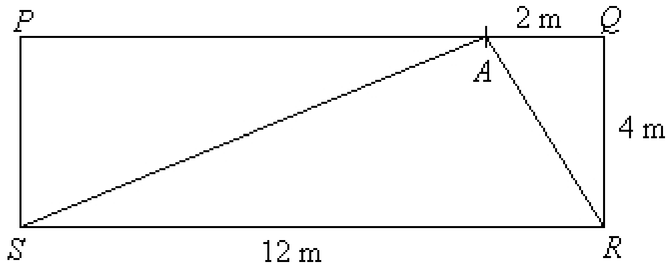
$$a^2 + b^2 = c(c), therefore\ a^2 + b^2 = c^2$$

Example 1: Given right ΔABC, if a=8, b =15, find c.
Solution:

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{8^2 + 15^2}$$
$$c = \sqrt{64 + 225}$$
$$c = \sqrt{289}$$
$$c = 17$$



Example 2: The rectangle PQRS represents the floor of a room.



Ivan stands at point A. Calculate the distance of Ivan from
a) point R, representing the corner of the room

b) point R, representing the corner S of the room

Solution:

a) AR = $\sqrt{2^2 + 4^2}$ = 4.47 m
Ivan is 4.47 m from the corner R of the room.

b) AS = $\sqrt{4^2 + 10^2}$ = 10.77 m
Ivan is 10.77m from the corner S of the room

How did you find the given examples? Did you understand the concepts of Pythagorean Theorem? If not, go back to those parts that you find challenging and study further. If yes, you are now ready to take the challenge!

Explore

Here are some enrichment activities for you to explore on to master and strengthen the basic concepts on applying the different theorems and Pythagorean Theorem

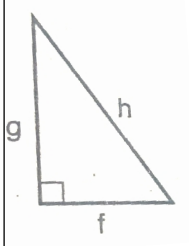
Activity 4: Solve Me!
Directions: Read and analyze the given problem. Answer the questions that follow.

Instead of enlarging each dimension of a document by 20%, the dimensions were erroneously enlarged by 30% so that the new dimensions are now 14.3 inches by 10.4 inches.

1. What are the dimensions of the original document?
2. What arte the desired enlarged dimensions?

Activity 5: Complete Me!

Directions: Use the Pythagorean Theorem to find the unknown side of the given right triangle if two of its sides are given.

Figure	Shorter Leg (f)	Longer leg (g)	Hypotenuse	Solution
	3	______	5	
	5	12	_________	

Great job! You have understood the lesson. Are you now ready to summarize?

Reflect on the activities you have done in this lesson by completing the following sentences. Write your answers on your journal notebook

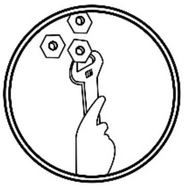
I learned that I ____

I was surprised that I ____

I noticed that I ____

I discovered that I ____

I was pleased that I ____

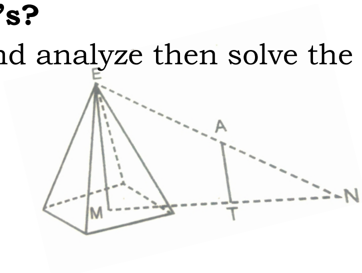


Deepen

The following activities will deepen your knowledge, skills and understanding on the concepts on applying theorems and Pythagorean Theorem

Activity 6: What If's?

Directions: Read and analyze then solve the problems given.



1. The sun shines from the western part of the pyramid and casts a shadow on the opposite side. If MN= 80 ft, and AT= 6ft, what is the height of the pyramid?
2. If the post was not erected to have its top to be along the line of the line shadow cast by the building such as shown, will you still be able to solve the height of the pyramid? Explain.

Activity 7: RAS-Now! (Read, Analyze, Solve)

Directions: Read and analyze then solve the problems given using Pythagorean Theorem.

1. The size of a TV screen is given by the length of its diagonal. If the dimension of a TV screen is 16 inches by 14 inches, what is the size of the TV screen?
2. A 20-foot ladder is leaning against a vertical wall. If the foot of the ladder is 8 feet from the wall, how high does the ladder reach?

Congratulations for reaching this far! You are now ready to take the assessment test. Good luck!