7

Mathematics

Quarter 4: Week 6 - Module 6

Illustrating Measures of Variability of a Statistical Data

AIRs - LM

Mathematics Grade 7
Quarter 4: Week 6 - Module 6: **Illustrating Measures of Variability of a Statistical Data**

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Target

In making decisions and choosing what's best to fit, we often compare our choices to each other. Sometimes we compare performance of a group of persons to another group. In most situations, the measures of central tendency may not be enough as the basis of comparison. However, if we wish to know how distributed a set of data is over the values they assume, and compare it with another set of data, we would have to use the *measures of variability*.

Measures of variability may be used in interpreting data or results obtained from researches done in a plethora of fields including psychology, economics, medicine, education, and many more. In this lesson you will learn about the most basic measures of variability such as range, mean deviation, variance, and standard deviation.

After going through this module, you are expected to:

- illustrates the measures of variability (range, mean deviation, variance, standard deviation) of a statistical data (**M7SP-IVf-1)**

Learning Objectives:

1) Perform series of operations;
2) Solve the absolute value of a number;
3) Solve the mean deviation, variance and standard deviation of a set of scores; and
4) Compare the data by using the measures of variability

Before we start the lesson, find out how much you already know about this module by answering the pre – assessment on the subsequent page.

PRE – ASSESSMENT

This activity will enable you to assess your prior knowledge about the measures of variability of a statistical data.

Directions: Read and answer each statement below carefully. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module. Write your answers in a separate sheet of paper.

1. Which of the following refers to how "spread out" a group of scores or data is?
   A. Central Tendency B. Median
   C. Normal Distribution D. Variability
2. Which of the following is ***NOT*** used to measure dispersion of data set?
   A. Mean B. Mean Deviation
   C. Range D. Standard Deviation
3. What statistical measure of variability gives the difference of highest value and lowest value?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
4. What statistical measure of variability gives average of absolute differences between each value in a data set and the mean?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
5. What statistical measure of variability gives the average of the squared differences of the mean and each value in a data set?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
6. What statistical measure of variability tells the average distance of the values in a data set are from the mean?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
7. Which of the following statements is ***NOT*** true about range of a statistical data?
   A. Simplest measure of dispersion
   B. Gives an equal weight to each observation
   C. Ignores the way in which data are distributed
   D. Gives the difference between the largest and the smallest

8. Which of the following statements is true about mean deviation of a statistical data?
   A. It is the square root of variance
   B. It is the square of standard deviation
   C. Gives an equal weight to each observation
   D. Ignores the way in which data are distributed
9. Which of the following statements is true about variance of a statistical data?
   A. Simplest measure of dispersion
   B. It is the square of standard deviation
   C. Ignores the way in which data are distributed
   D. Gives the difference between the largest and the smallest
10. Which of the following statements is true about standard deviation of a statistical data?
   A. Simplest measure of dispersion
   B. It is the square root of variance
   C. Ignores the way in which data are distributed
   D. Gives the difference between the largest and the smallest
11. If the lowest value is 35, and the range of the data is 7, what is the highest value?

   A. 5 B. 28 C. 42 D. 245

12. If the variance of a set of values is 4, what is the standard deviation?

   A. 2 B. 4 C. 8 D. 16

13. If the standard deviation of a set of values is 4, what is the variance?

   A. 2 B. 4 C. 8 D. 16

For numbers 12 – 15, refer to the table below:

The table shows the scores of Vener and Ronnel in their Mathematics Quiz

Learners	Scores	Mean	Standard deviation
Vener	94, 94, 92, 94, 96	94	1.4
Ronnel	91, 97, 92, 95,95	94	2.4

14. What is the range of the scores of the two learners?
   A. Vener: 94; Ronnel: 94 B. Vener: 2; Ronnel: 3
   C. Vener: 1.4; Ronnel: 2.4 D. Vener: 4; Ronnel: 6
15. Which of the following statements is true about the learners' scores?
   A. Ronnel's score in their Math test is better than Vener.
   B. Vener's score in their Math test is more consistent than Ronnel
   C. Ronnel's score in their Math test is more consistent than Vener
   D. Vener and Ronnel's score in their Math test has the same consistency

Jumpstart

Activity 1: You're Absolutely Right!

Directions: Give the absolute value of the following data.

1. /3 – 7/=_____
2. /5 – 9 / = _____
3. /6 – 12/=_____
4. /23 – 25/=_____
5. /29 – 35/=_____
6. /5 – 4/=_____
7. /8 – 2/=_____
8. /10 – 7/=_____
9. /33 – 33/=_____
10. /49 – 42/=_____

Activity 2: What do you Mean?

Directions: Solve the absolute mean of the following data.

1. 2, 3, 4, 4, 5, 6: $\bar{X}$ = _____
2. 89, 91, 95, 97, 97, 98: $\bar{X}$ = _____
3. 1, 1, 3, 3, 3, 5, 5: $\bar{X}$ = _____
4. 10, 12, 13, 15, 15, 19, 21: $\bar{X}$ = _____
5. 18, 25, 25, 26, 26, 30, 33, 35, 36, 36: $\bar{X}$ = _____

Discover

Measures of Variability

- The measure of variability or dispersion shows how the data is spread or scattered around the mean.

  There are four basic measure of variability. These are, *range, average deviation, variance* and *standard deviation.*

The **range (*R*)** is the difference between the largest value and the lowest value in a data set.

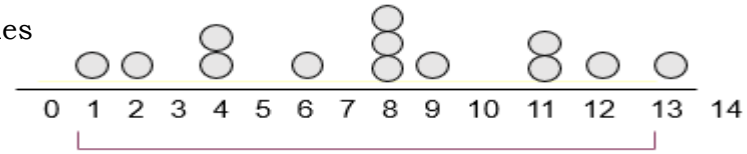
The simplest and easiest measure of variability to understand is the range. The **range** (*R*) gives a general idea on how spread out the data is, but it ignores the way in which data are distributed. This only tells how far the largest value is from the lowest value in a data set.

Range of ungrouped data can be solved using the formula:

Range = Highest Value – Lowest Value

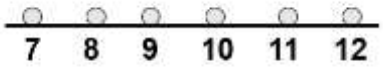
Illustrative examples

1:



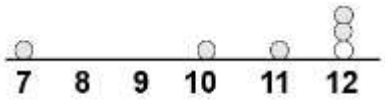
The highest value of this data is 13 and the lowest is 1, therefore the range is 12. This means that distance of the highest and lowest value is 12.

2.



Range = 12 - 7 = 5

3.



Range = 12 - 7 = 5

4. **1**,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,**5**

Range = 5 – 1 = **4**

5. **1**,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,**120**

Range = 120 – 1 = **119**

Range ignores the way in which data are distributed. It is only sensitive to outliers

The ***mean deviation*** (or average deviation) is the average of absolute differences between each value in a data set and the mean.

It gives an equal weight to each observation, and generally more sensitive than the range since a change in any value will affect it.

The formula in solving the mean deviation of ungrouped data is:

Mean Deviation (MD) = $\frac{\sum/X - \bar{X}/}{n}$

The ***variance*** and ***standard deviation*** are most common measures of variability

The **variance** is the average of the squared differences of the mean and each value in a data set. It is denoted by the symbol $\boldsymbol{\sigma^2}$ for a population and $\boldsymbol{S^2}$ for a sample. We can get the variance by getting the square of the *standard deviation*

The formula in solving the variance of ungrouped data is:

$$S^2 = \frac{\sum(X - \overline{X})^2}{n-1}$$

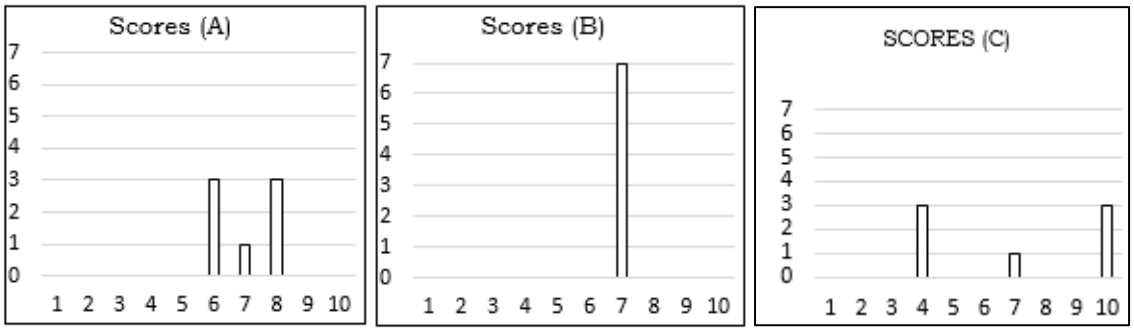
The **standard deviation** tells how far the values in a data set are from the mean. It is denoted by $\boldsymbol{\sigma}$ for a population and $\boldsymbol{S}$ for a sample. The lower the value of the standard deviation the more consistent the values are. We can get the standard deviation by getting the square root of the *variance*

The formula in solving the standard deviation of ungrouped data is:

$$S = \sqrt{\frac{\sum(X - \overline{X})^2}{n-1}}$$

Examples:

The figures below illustrate four sets of scores organized in histograms. Find the mean deviation, variance and standard deviation.



The scores in each set are as follows:

Scores (A) : 6, 6, 6, 7, 8, 8, 8

Scores (B) : 7, 7, 7, 7, 7, 7, 7

Scores (C) : 4, 4, 4, 7, 10, 10, 10

Organize the set of scores in a table, then solve the mean deviation, variance and standard deviation.

Group A:

SCORES	$X - \bar{X}$	$\lvert X - \bar{X} \rvert$	$(X - \bar{X})^2$
6	6 – 7 = -1	$\lvert 6 - 7 \rvert = 1$	$(-1)^2 = 1$
6	6 – 7 = -1	$\lvert 6 - 7 \rvert = 1$	$(-1)^2 = 1$
6	6 – 7 = -1	$\lvert 6 - 7 \rvert = 1$	$(-1)^2 = 1$
7	7 – 7 = 0	$\lvert 7 - 7 \rvert = 0$	$0^2 = 0$
8	8 – 7 = 1	$\lvert 8 - 7 \rvert = 1$	$1^2 = 1$
8	8 – 7 = 1	$\lvert 8 - 7 \rvert = 1$	$1^2 = 1$
8	8 – 7 = 1	$\lvert 8 - 7 \rvert = 1$	$1^2 = 1$
$\sum X = 49$		$\sum \lvert X - \bar{X} \rvert = 6$	$\sum (X - \bar{X})^2 = 6$
n = 7			
$\bar{X} = \frac{49}{7} = 7$			

MD = $\frac{\sum |X - \bar{X}|}{n} = \frac{6}{7} = 0.857$; This means that, in the average, the distances of each value from the mean is **0. 857**

$$S^2 = \frac{\sum (X - \overline{X})^2}{n - 1} = \frac{6}{7 - 1} = \frac{6}{6} = 1$$

This means that, the average squared differences of the mean and each value is **1.**

$$S = \sqrt{\frac{\sum (X - \overline{X})^2}{n - 1}} = \sqrt{\frac{6}{7 - 1}} = \sqrt{\frac{6}{6}} = \sqrt{1} = 1$$

This means that, the average distances of scores from the mean **1.**

Group B:

SCORES	$\boldsymbol{X - \bar{X}}$	$\boldsymbol{/X - \bar{X}/}$	$\boldsymbol{(X - \overline{X})^2}$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
$\boldsymbol{\sum X = 49}$		$\boldsymbol{\sum /X - \bar{X}/ = 0}$	$\boldsymbol{\sum (X - \overline{X})^2 = 0}$
n = 7			
$\boldsymbol{\bar{X} = \frac{49}{7} = 7}$			

MD = $\frac{\sum /X - \bar{X}/}{n} = \frac{0}{7} = 0$; This means that, in the average, the distances of each value from the mean is **0**

$S^2 = \frac{\sum (X - \overline{X})^2}{n - 1} = \frac{0}{7 - 1} = \frac{0}{6} = 0$

This means that, the average squared differences of the mean and each value is **0.**

$S = \sqrt{\frac{\sum (X - \overline{X})^2}{n - 1}} = \sqrt{\frac{0}{7 - 1}} = \sqrt{\frac{0}{6}} = \sqrt{0} = 0$

This means that, the average distances of scores from the mean **0.**

Observe that the values in B are all the same so the standard deviation, which is zero, shows that data are not spread

Group C:

SCORES	$\boldsymbol{X - \bar{X}}$	$\boldsymbol{/X - \bar{X}/}$	$\boldsymbol{(X - \overline{X})^2}$
4	4 – 7 = -3	/4 – 7/ = 3	$(-3)^2 = 9$
4	4 – 7 = -3	/4 – 7/ = 3	$(-3)^2 = 9$
4	4 – 7 = -3	/4 – 7/ = 3	$(-3)^2 = 9$
7	7 – 7 = 0	/7 – 7/ = 0	$0^2 = 0$
10	10 – 7 = 3	/10 – 7/ = 3	$3^2 = 9$
10	10 – 7 = 3	/10 – 7/ = 3	$3^2 = 9$
10	10 – 7 = 3	/10 – 7/ = 3	$3^2 = 9$
$\boldsymbol{\sum X = 49}$		$\boldsymbol{\sum /X - \bar{X}/ = 18}$	$\boldsymbol{\sum (X - \overline{X})^2 = 54}$
n = 7			
$\boldsymbol{\bar{X} = \frac{49}{7} = 7}$			

MD = $\frac{\sum /X - \bar{X}/}{n} = \frac{18}{7} = 2.571$; This means that, in the average, the distances of each value from the mean is **2.571**

$S^2 = \frac{\sum(X - \overline{X})^2}{n-1} = \frac{54}{7-1} = \frac{54}{6} = 9$

This means that, the average squared differences of the mean and each value is **9**

$S = \sqrt{\frac{\sum(X - \overline{X})^2}{n-1}} = \sqrt{\frac{54}{7-1}} = \sqrt{\frac{54}{6}} = \sqrt{9} = 3$

This means that, the average distances of scores from the mean **3**

In summary, the ***standard deviation*** of the groups are:

A: 1; B: 0; C: 3;

Observe that the lower the value of the standard deviation, the more consistent the scores.

Therefore, Group B has the most consistent scores among the three and the least consistent is group C.

Comparing the results and the table of values, you can observe that the **standard deviation gives more accurate value** on the distances of values from the mean.

Observe also that the value of standard deviation is the square root of variance and the variance is the squared value of standard deviation.

Example:

1. If the value of variance is 9, the standard deviation is 3.
2. If the value of standard deviation is 5, the variance 25.

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from these lessons.

A. Solve the missing value of the following data.

1. Highest score = 15; Lowest score =2; Range = _____
2. Highest score = 100; Lowest score=_____ ; Range = 52
3. Highest score = _____ ; Lowest score =24; Range = 12
4. Highest score = _____; Lowest score =2; Range =15
5. Highest score = 28; Lowest score =14; Range = _____

B. Compare the two set of data by completing the data in the table.

GROUP A		
SCORES	$X - \bar{X}$	$/X - \bar{X}/$
4	4 – 7 = -3	3
5	5 – 7 = -2	2
6	6 – 7 = -1	1
6	6 – 7 = _____	
8	8 – 7 =_____	
8	8 – 7 = _____	
9	9 – 7 =_____	
10	10 – 7 =_____	
$\sum X = 56$		$\sum /X - \bar{X}/ =$ ____
n = 8		
$\bar{X} = \frac{56}{8} = 7$		

GROUP B		
SCORES	$X - \bar{X}$	$/X - \bar{X}/$
6	6 – 7 = -1	1
6	6 – 7 = -1	1
6	6 – 7 = -1	1
6	6 – 7 = _____	
8	8 – 7 =_____	
8	8 – 7 = _____	
8	8 – 7 =_____	
8	8 – 7 =_____	
$\sum X = 56$		$\sum /X - \bar{X}/ =$ ____
n = 8		
$\bar{X} = \frac{56}{8} = 7$		

$MD = \frac{\sum /X - \bar{X}/}{n} = \frac{\square}{\square} =$ _____	$MD = \frac{\sum /X - \bar{X}/}{n} = \frac{\square}{\square} =$ _____

Conclusion:

Therefore, Group _____, has less spread scores than Group _____.

Deepen

Since you already know the important notes about the measures of variability of a statistical data, let us widen and go deeper to our understanding.

Fid the variance and standard deviation of the scores in each of the groups shown below then compare the results.

Group A: 20, 21, 25, 26, 26, 27, 30

Group B: 11, 15, 20, 26, 30, 33, 40

GROUP A		
SCORES	$X - \bar{X}$	$(X - \bar{X})^2$
20		
21		
25		
26		
26		
27		
30		
$\sum X =$ ____		$\sum (X - \bar{X})^2 =$ ____
n = ____		
$\bar{X} =$ ____		

GROUP B		
SCORES	$X - \bar{X}$	$(X - \bar{X})^2$
11		
15		
20		
26		
30		
33		
40		
$\sum X =$ ____		$\sum (X - \bar{X})^2 =$ ____
n = ____		
$\bar{X} =$ ____		

$$S^2 = \frac{\sum (X - \bar{X})^2}{n-1} = \frac{\square}{\square} = \_\_\_\_\_$$

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}} = \sqrt{\square} = \_\_\_\_\_$$

$$S^2 = \frac{\sum (X - \bar{X})^2}{n-1} = \frac{\square}{\square} = \_\_\_\_\_$$

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}} = \sqrt{\square} = \_\_\_\_\_$$

Conclusion:

The scores in Group _____ are more spread out than the scores in Group _____.

Gauge

Assessment

Directions: Read and answer each statement below carefully. Write your answers in a separate sheet of paper.

1. Which of the following measures of variability is the most reliable?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
2. Which of the following measures dispersion of a data set?
   A. Mean B. Mean Deviation C. Median D. Mode
3. Which measure of variability is the simplest to use?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
4. What statistical measure of variability gives average of absolute differences between each value in a data set and the mean?
   A. Mean Deviation B. Range C. Standard Deviation D. Variance
5. Which of the following statements is true about range of a statistical data?
   A. Sensitive only to outlier values
   B. Most reliable measure of dispersion
   C. Gives an equal weight to all observation
   D. Gives the difference between the largest and the smallest
6. Which of the following statements is true about standard deviation of a statistical data?
   A. The simplest measure of variability
   B. The least reliable measure of variability
   C. The lower the value the more consistent the data
   D. The higher the value the more consistent the data
7. If the lowest value is 15, and the range of the data is 3, what is the highest value?
   A. 5 B. 12 C. 18 D. 45
8. If the variance of a set of values is 1, what is the standard deviation?
   A. 0 B. 1 C. 2 D. 3
9. If the standard deviation of a set of values is 1.2, what is the variance?
   A. 0 B. 1.2 C. 1.21 D. 1.44

For numbers 10 – 12, refer to the table below:

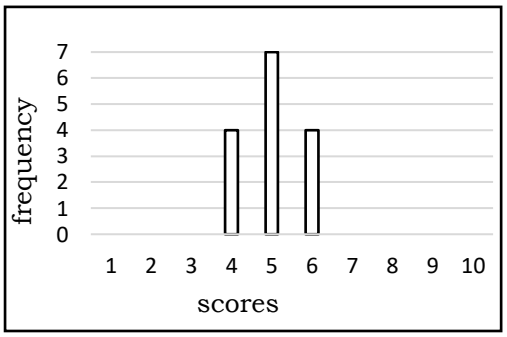
The table shows the results of Rizalyn's scores in her quizzes.

Subjects	Mean	Standard Deviation
Math	56	1.3
Science	56	2.5
English	56	0.6

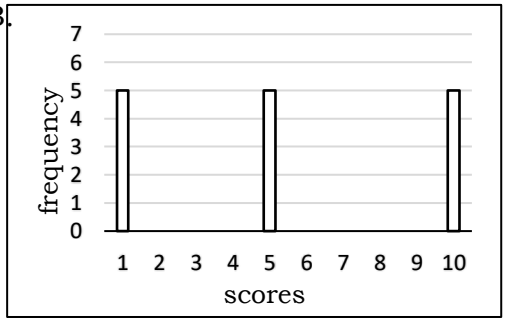
10. Which of the following statements is true about the Rizalyn's scores in her subject areas?
    A. She has more consistent scores in Math than English
    B. She has more consistent scores in Science than Math
    C. She has more consistent score in Science than English
    D. She has more consistent score in Math than Science
11. What subject has most consistent scores?
    A. English B. Math C. Science D. None
12. What subject has the least consistent scores?
    A. English B. Math C. Science D. None

For numbers 13 – 15, refer to the graphs below.

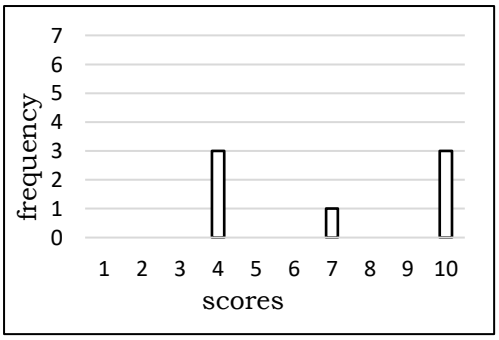
A.



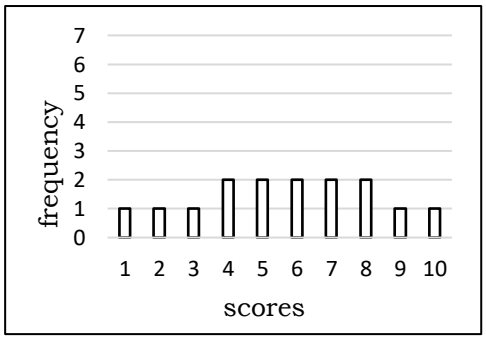
B.



C.



D.



13. What graph shows a range of 6?
14. What graph has the most consistent data?
15. What graph has the least consistent data?

Great job! You are done with this module.

References

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