

# Mathematics

## Quarter 3 – Week 7 – Module 7: Use Triangle Similarity in Proving Pythagorean Theorem



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## **Mathematics 9**

Quarter 3 – Week 7 – Module 7: Proving Pythagorean Theorem Using Triangle Similarity

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Region I

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## Target

Good day learners!

This module is designed and written to help you apply the theorems to show that triangle is congruent. You will explore more the Pythagorean Theorem. The different theorems on the similarity of triangles are very helpful in this module. The application of the different theorems and Pythagorean Theorem can be seen in many different fields like physics, industry, and business, and various mathematical problems. In the lessons, you are given the opportunity to use your prior knowledge and skills on the similarity of triangles and the Pythagorean Theorem. Activities are also given to process your knowledge and skills acquired, deepen and transfer your understanding. The scope of this module enables you to use it in many different learning situations.

Before we start, let us consider first the learning competencies:

1. Applies the theorems to show that given triangles are similar  
**(M9GE-IIIi-1)**
2. Proves the Pythagorean Theorem **(M9GE-IIIi-2)**

Subtasks:

1. Define and illustrate the different similarity theorems.
2. State and illustrate the Pythagorean Theorem.
3. Solve problems applying the different theorems.

*Before going on, check how much you know about this topic. Answer the pretest on the next page in a separate sheet of paper.*

### Pre- test:

**Choose the letter of the correct answer. Write your answer on a separate sheet of paper.**

1. The lengths of the sides of a triangle are 6cm, 10cm and 13cm. What kind of a triangle is it?  
A. acute      B. obtuse      C. regular      D. right
2.  $\triangle BRY \sim \triangle ANT$ . Which ratio of sides gives the scale factor?  
A.  $\frac{NT}{AN}$       B.  $\frac{AT}{BY}$       C.  $\frac{NT}{RY}$       D.  $\frac{NT}{AT}$

3. What theorem states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs?
  - A. Pythagorean
  - B. Triangle Similarity
  - C. Triangle Proportionality
  - D. Triangle -Angle Bisector
4. Which of the following theorem states that "If a segment bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides"?
  - A. Pythagorean
  - B. Triangle Similarity
  - C. Triangle Proportionality
  - D. Triangle -Angle Bisector
5. Which of the following is a property of right triangles?
  - A. The length of the altitude is the geometric mean between the segments of the hypotenuse.
  - B. The width of the altitude is the geometric mean between the segments of the hypotenuse.
  - C. Each leg is the arithmetic mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.
  - D. All leg is the arithmetic mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.
6. What is the symbol to use to show the relationship of the longest side of a right triangle to the sum of the squares of the shorter side?
  - A. equal to
  - B. greater than
  - C. lesser than
  - D. not equal
7. If the square of the longest side is greater than the sum of the squares of the shorter side, what triangle is it?
  - A. acute
  - B. obtuse
  - C. right
  - D. scalene
8. What kind of triangle is formed if the first side measures 6cm, the second is 8cm and the length is 10cm?
  - A. acute
  - B. obtuse
  - C. right
  - D. scalene
9.  $\triangle WHY$  is a right triangle with  $\angle WHY$  as the right angle.  $HD \perp WY$ . Which of the following segments is a geometric mean?
 

I. HD	II. DY	III. HY	IV. DW	V. HW	VI. WY
A. II, IV, VI	B. I, III, V	C. I only	D. all except VI		
10. The ratio of the volumes of two similar rectangular prisms is 125:64. What is the ratio of their base?
  - A. 25:16
  - B. 25:4
  - C. 4:5
  - D. 5:4
11. What is the perimeter of a 30-60-90 triangles whose shorter leg is 5 inches long?
  - A.  $5\sqrt{3}cm$
  - B.  $15 + 5\sqrt{3}cm$
  - C.  $\frac{7\sqrt{2}cm}{2}$
  - D.  $\frac{7\sqrt{3}cm}{3}$

12. The hypotenuse of an isosceles right triangle measures 7cm. How long is each leg?
- A.  $7\sqrt{2}cm$       B.  $3.5\text{ cm}$       C.  $\frac{7\sqrt{2}cm}{2}$       D.  $\frac{7\sqrt{3}cm}{3}$
13. A map is drawn to the scale of 1 cm: 150m. If the distance between towns A and B measures 8.5 cm on the map, determine the approximate distance between these towns.
- A. 2715 m      B. 2175      C. 1725 m      D. 1275 m
14. The length of the shadow of your one and one half meter height is 2.4 meters at a certain time in the morning. How high is a tree in your backyard if the length of its shadow is 16 meters?
- A. 38.4 m      B. 25.6 m      C. 24 m      D. 10 m
15. The smallest square of grid you made on your original picture is 6cm. if you enlarge the picture on a 15-cm grid, which of the following is NOT true?
- I. The new picture is 250% larger than the original one.  
 II. The new picture is two and a half times larger than the original one.  
 III. The scale factor between the original and the enlarge is 2:5
- A. I only      B. I and II      C. III only      D. I, II and III

# Lesson 1

## Use Triangle Similarity in Proving Pythagorean Theorem

It is easier to understand this module if you know the concepts associated with similarity of triangles particularly the different theorems. Most real-life problems involve concepts of similarities of triangle. If you find difficulty in answering the activities, ask assistance from your teacher.

### Activity 1: Agree or Disagree?

**Directions:** Check the first column if you agree, otherwise disagree if not.

STATEMENTS	AGREE	DISAGREE
1. The hypotenuse of a right triangle is the longest of all its three sides.		
2. In a right triangle, the measure of the hypotenuse is equal to the sum of the measure of its legs.		
3. SSS Similarity Theorem states that two triangles are similar if an angle of one triangle is congruent to an angle of another triangle and the corresponding sides including those angles are in proportion.		
4. The AAA Similarity Postulate states that if the three angles of one triangle are congruent to three angles of another triangle, then the two triangles are similar.		
5. Corresponding sides of similar polygons are proportional		

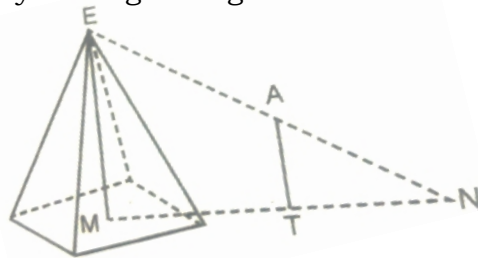
*For you to understand the lesson well, do the following activities.  
Have fun and good luck!*



## Jumpstart

### Activity 2: Fill Me In!

**Directions:** Analyze the given figure and fill in the correct answer to complete the sentences.



1. ME is the unknown \_\_\_\_\_ of the pyramid.
2. MN is the length of the shadow of the \_\_\_\_\_.
3. \_\_\_\_\_ is the height of a vertical post.
4. TN is the length of the \_\_\_\_\_ of the vertical post.
5. Is length ME can be measured by measuring tool? \_\_\_\_\_

### Activity 3: Choozzy!

**Directions:** Choose the correct answer from the box.

C. Pythagoras                      D. hypotenuse  
 E.  $c^2 = a^2 + b^2$

- \_\_\_\_\_ 1. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.
- \_\_\_\_\_ 2. The longest side of a triangle.
- \_\_\_\_\_ 3. The algebraic notation of Pythagorean Theorem.
- \_\_\_\_\_ 4. The triangle that should be used if the square of the hypotenuse is equal to the sum of the two sides.
- \_\_\_\_\_ 5. The Mathematician who discovered Pythagorean Theorem.

How did you find the activity? Can you still recall the properties triangles and understand how the concepts of triangles can be applied to solve geometric problems, study the illustrative examples below.



## Discover

*Applying the theorems to show that given triangles are similar and proving the Pythagorean Theorem is essential in day to day activities. In these lessons, you will realize that your success in writing proofs involving similarity depends upon your skill in making an accurate and appropriate representation of mathematical conditions. Your logical and reasoning skills will be developed as you go through with the different activities.*

How do we create proportionality statements for triangles and how do we show two triangles are similar? Being able to create a proportionality statement and apply the theorems are our goal when dealing with similar triangles. By definition, we know that if two triangles are similar, then the corresponding angles are congruent and their corresponding sides are proportional. Please take a look at the given examples below on how we apply the different theorems.

### Example 1:

A 6-inches-by-5 inches picture is a copy that was reduced from the original one by reducing each of its dimension by 40%. In short, each dimensions of the available copy is 60% of the original one. You would like to enlarge it back to its original size using a copier. What copier settings would you use?

If each dimension of the available picture is 60% of the original one, then we can make the following statements to be able to determine the dimensions of the original picture:

1. The length of 6 inches is 60% of the original length  $L$ . It means that

$$6 = 60\% (L) \text{ or that is } L = \frac{6}{0.6} = 10 \text{ inches}$$

2. The width of 5 inches is 60% of the original width  $W$ . It means that

$$5 = 60\% (W) \text{ or that is } W = \frac{5}{0.6} = 8\frac{1}{3} \text{ inches}$$

To determine the copier settings to use to be able to increase the 6-inches by 5-inches picture back to the 10 inches by  $8\frac{1}{3}$  inches, the following statements should be used:

1. The original length of 10 inches in what percent of 6? It means that

$$10 = \text{rate } R(6) \text{ or that is } R = \frac{10}{6} = \frac{5}{3} \approx 1.67 \approx 167\%$$

2. The original width of  $8\frac{1}{3}$  inches is what percent of 5? It means that

$$8\frac{1}{3} = \text{rate } R(5) \text{ or that is } R = \frac{8\frac{1}{3}}{5} = \frac{\frac{25}{3}}{5} = \frac{25}{3} \cdot \frac{1}{5} = \frac{5}{3} \approx 1.67 \approx 167\%$$

Therefore, the copier should be at 167% the normal size to convert the picture back to its original size.



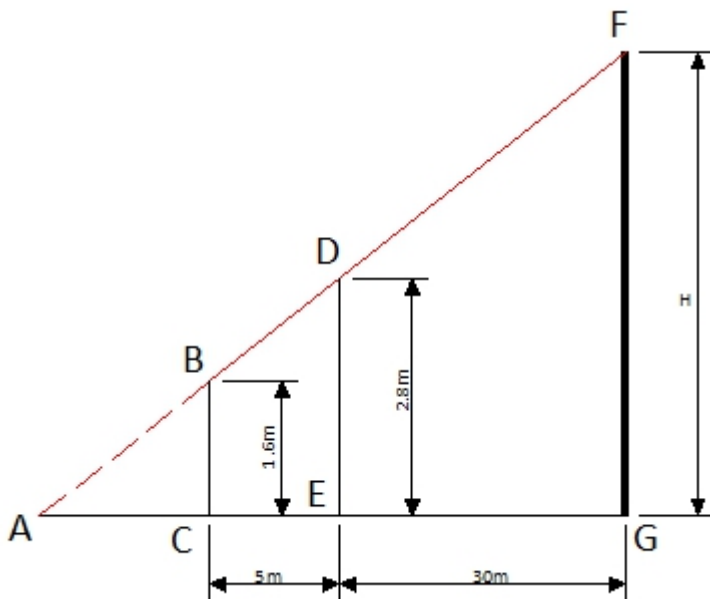
### Example 2

Trisha wants to measure the height of a building but she does not have the tools to do so. She noticed that there is a tree located in front of the building so she decided to use her smartness and the geometry knowledge that she got at school to measure the building height. She measured the distance between the tree and the building and found that it is 30m. She stood in front of the tree and started backing until she could see the top edge of the building from above the tree top. She marked her place and measured it from the tree. It was 5m. Knowing that the tree height is 2.8m and Trisha's eyes height is 1.6m, help Trisha to do the math and calculate the building height.



### Solution:

This problem can be geometrically represented as in the figure below.



First, let us make use of the similarity between the triangles  $\triangle ABC$  and  $\triangle ADE$ .

$$\frac{BC}{DE} = \frac{1.6}{2.8} = \frac{AC}{AE} = \frac{AC}{5+AC} = 2.8 \times AC = 1.6 \times (5 + AC) = 8 + 1.6 \times AC$$

$$(2.8 - 1.6) \times AC = 8 = AC = \frac{8}{1.2} = 6.67$$

We can then use the similarity between triangles  $\triangle ACB$  and  $\triangle AFG$  or between the triangles  $\triangle ADE$  and  $\triangle AFG$ . Let us take the first option.

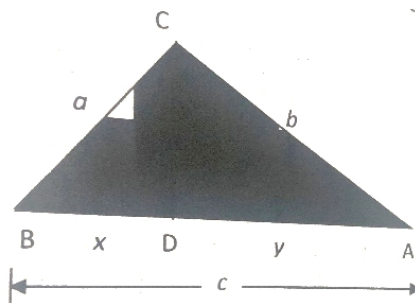
$$\frac{BC}{FG} = \frac{1.6}{H} = \frac{AC}{AG} = \frac{6.67}{6.67+5+30} = 0.16 = H = \frac{1.6}{0.16} = 10m$$

*Consider the next examples to understand how to use the concept of a Pythagorean Theorem in real-life situations.*

The main theorem about right triangles is attributed to the name of a Greek, **Pythagoras** of Samos, born around 570-560 BC. The theorem states that: "In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs." That is the Pythagorean Theorem.

In a right triangle, the longest side is the hypotenuse and the legs are the other two sides.

In algebraic term, it is written as,  $c^2 = a^2 + b^2$ . To satisfy the given equation, let us prove the theorem by letting  $\triangle ABC$  be the right triangle with legs of lengths  $a$  and  $b$  and hypotenuse  $c$ .



Proof: Let  $CD$  be the altitude to the hypotenuse and  $\overline{CD} \perp \overline{BA}$ . Let  $BD = x$  and  $DA = y$ . Since either leg of  $\triangle ABC$  is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg, then,  $\frac{c}{a} = \frac{a}{x}$  and  $\frac{c}{b} = \frac{b}{y}$ .

Hence,  $a^2 = cx$  and  $b^2 = cy$ , by adding the equations we have,

$$a^2 + b^2 = cx + cy$$

$$a^2 + b^2 = c(x + y)$$

Since  $x+y = c$ , we have  $a^2 + b^2 = c(x + y)$

$$a^2 + b^2 = c(c), \text{ therefore } a^2 + b^2 = c^2$$

**Example 1:** Given right  $\triangle ABC$ , if  $a=8$ ,  $b=15$ , find  $c$ .

Solution:

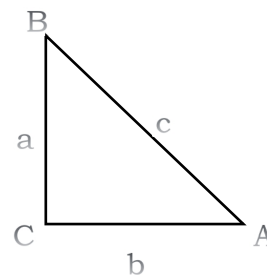
$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{8^2 + 15^2}$$

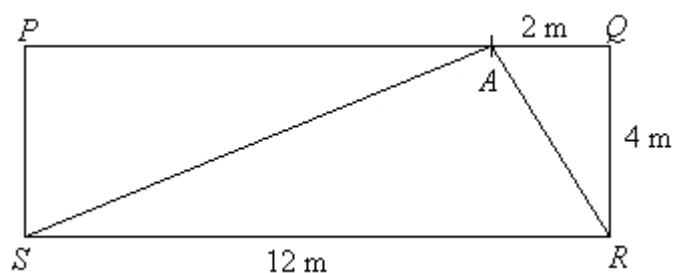
$$c = \sqrt{64 + 225}$$

$$c = \sqrt{289}$$

$$c = 17$$



**Example 2:** The rectangle PQRS represents the floor of a room.



Ivan stands at point A. Calculate the distance of Ivan from

- a) point R, representing the corner of the room
- b) point S, representing the corner of the room

**Solution:**

a)  $AR = \sqrt{2^2 + 4^2} = 4.47 \text{ m}$

Ivan is 4.47 m from the corner R of the room.

b)  $AS = \sqrt{4^2 + 10^2} = 10.77 \text{ m}$

Ivan is 10.77m from the corner S of the room

How did you find the given examples? Did you understand the concepts of Pythagorean Theorem? If not, go back to those parts that you find challenging and study further. If yes, you are now ready to take the challenge!



## Explore

*Here are some enrichment activities for you to explore on to master and strengthen the basic concepts on applying the different theorems and Pythagorean Theorem*

### Activity 4: Solve Me!

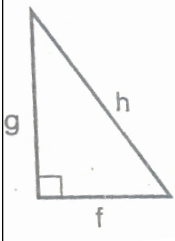
Directions: Read and analyze the given problem. Answer the questions that follow.

Instead of enlarging each dimension of a document by 20%, the dimensions were erroneously enlarged by 30% so that the new dimensions are now 14.3 inches by 10.4 inches.

1. What are the dimensions of the original document?
2. What are the desired enlarged dimensions?

**Activity 5: Complete Me!**

Directions: Use the Pythagorean Theorem to find the unknown side of the given right triangle if two of its sides are given.

Figure	Shorter Leg (f)	Longer leg (g)	Hypotenuse	Solution
	3	_____	5	
	5	12	_____	

*Great job! You have understood the lesson. Are you now ready to summarize?*

Reflect on the activities you have done in this lesson by completing the following sentences. Write your answers on your journal notebook

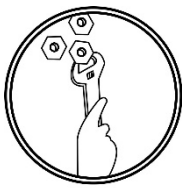
I learned that I \_\_\_\_\_

I was surprised that I \_\_\_\_\_

I noticed that I \_\_\_\_\_

I discovered that I \_\_\_\_\_

I was pleased that I \_\_\_\_\_

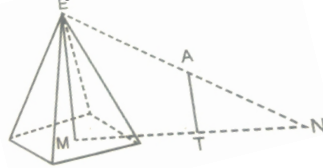


## Deepen

*The following activities will deepen your knowledge, skills and understanding on the concepts on applying theorems and Pythagorean Theorem*

### Activity 6: What If's?

**Directions:** Read and analyze then solve the problems given.



1. The sun shines from the western part of the pyramid and casts a shadow on the opposite side. If  $MN = 80$  ft, and  $AT = 6$  ft, what is the height of the pyramid?
2. If the post was not erected to have its top to be along the line of the line shadow cast by the building such as shown, will you still be able to solve the height of the pyramid? Explain.

### Activity 7: RAS-Now! (Read, Analyze, Solve)

**Directions:** Read and analyze then solve the problems given using Pythagorean Theorem.

1. The size of a TV screen is given by the length of its diagonal. If the dimension of a TV screen is 16 inches by 14 inches, what is the size of the TV screen?
2. A 20-foot ladder is leaning against a vertical wall. If the foot of the ladder is 8 feet from the wall, how high does the ladder reach?

*Congratulations for reaching this far! You are now ready to take the assessment test. Good luck!*



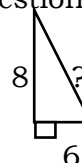
## Gauge

### Post Assessment:

Read each item carefully. Identify the choice that best answers the question.

1. In the given right triangle, what is the measurement of one side?

A. 10                      B. 9                      C. 8                      D. 7

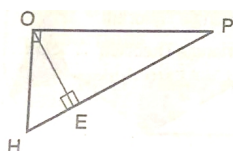


2. Which is the correct representation of Pythagorean Theorem?

A.  $a^2 + b^2 = c^2$                       B.  $a + b = c$                       C.  $a^2 - b^2 = c^2$                       D.  $a + b = c^2$

3. In the figure, there are three similar right triangles by Right Triangle Proportionality Theorem. Name the triangle that is missing in this statement:

$\triangle HOP \sim \underline{\hspace{1cm}} \sim \triangle OEP$



A.  $\triangle HOE$                       B.  $\triangle OEH$                       C.  $\triangle HOP$                       D.  $\triangle HEO$

4. When the altitude is drawn to the hypotenuse of a right triangle, what happens to the length?

A. The length of the altitude is the geometric mean between the segments of the hypotenuse.  
 B. The length of the altitude is the geometric mean after the segments of the hypotenuse.  
 C. The length of the altitude is the geometric mean before the segments of the hypotenuse.  
 D. The length of the altitude is the arithmetic mean between the segments of the hypotenuse.

5. The scale of a drawing is 3 in: 15ft. What is the actual measurements for 4 inches?

A. 20 ft                      B. 21 ft                      C. 22 ft                      D. 23 ft

6. What is the length of a 150 m in a scale drawing having a scale of 1 cm:15m?

A. 8 cm                      B. 9 cm                      C. 10 cm                      D. 11 cm

7. What will happen to the scale of the size of an actual object having 1m=10cm?

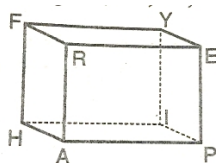
A. enlarges                      B. normalize                      C. preserves                      D. reduces

8. A 20-foot ladder is leaning against a vertical wall. If the foot of the ladder is 8 ft from the wall, how high does the ladder reach?

A.  $8\sqrt{6}$                       B.  $9\sqrt{6}$                       C.  $10\sqrt{6}$                       D.  $9\sqrt{5}$

9. Who is the person behind Pythagorean Theorem?  
 A. Archimedes      B. Einstein      C. Newton      D. Pythagoras

For nos. 10-13



The figure of a rectangular prism shown is not drawn to scale. If  $AH=3$  cm,  $AP=7$  cm, and  $AR = 5$  cm

10. What the measurement of AI?  
 A.  $\sqrt{58}$       B.  $\sqrt{59}$       C.  $\sqrt{60}$       D.  $\sqrt{61}$
11. What is the measurement of AE?  
 A.  $\sqrt{71}$       B.  $\sqrt{72}$       C.  $\sqrt{73}$       D.  $\sqrt{74}$
12. AF measures \_\_\_\_\_.  
 A.  $\sqrt{31}$       B.  $\sqrt{32}$       C.  $\sqrt{33}$       D.  $\sqrt{34}$
13. What is the measurement of AY?  
 A.  $\sqrt{81}$       B.  $\sqrt{82}$       C.  $\sqrt{83}$       D.  $\sqrt{84}$
14. A person has to walk 100 m to go from position X in the north of east direction to the position B and then to the west of Y to reach finally at position Z. The position Z is situated at the north of X and at a distance of 60 m from X. Find the distance between X and Y.  
 A. 65 m      B. 66 m      C. 67 m      D. 68 m
15. If the square of the hypotenuse of an isosceles right triangle is  $128 \text{ cm}^2$ , find the length of each side  
 A. 8 cm      B. 9 cm      C. 10 cm      D. 11 cm

*Great job! You have understood the lesson. Congratulations! An additional activity is given for you to master more the competency.*

## References

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