9

Mathematics

Quarter 4 - Week 2 - Module 2:
The Trigonometric Ratios of Special Angles

AIRs - LM

Mathematics 9
Quarter 4-Week 2- Module 2: The Trigonometric Ratios of Special Angles
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Module 2 — The Trigonometric Ratios of Special Angles

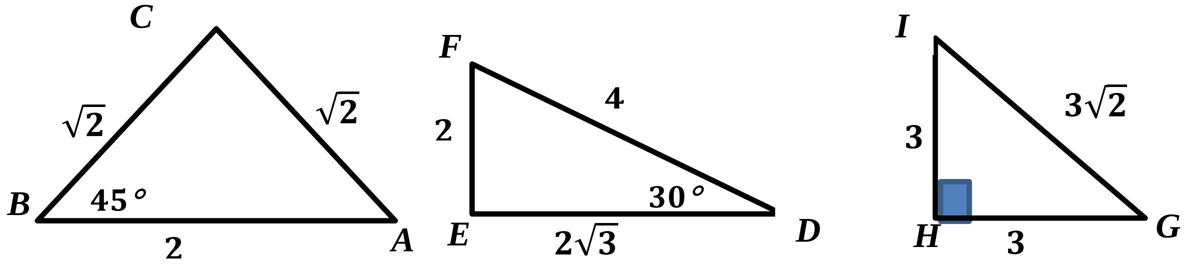
In this lesson you will use the concepts you have learned in the previous lessons to evaluate the trigonometric ratios of special angles. There are two triangles, the isosceles and equilateral triangles that are frequently used in mathematics to generate exact values for the trigonometric ratios. Let us consider the succeeding activities to develop mastery of the topic.

Jumpstart

Activity 1: Let's Get Familiar
Directions:

1) a) Draw a right triangle that has one angle measuring 30°. Label the sides using lengths $\sqrt{3}$, 2, and 1.
   b) Identify the adjacent and opposite sides relative to the 30° angle.
   c) Redraw the triangle and identify the adjacent and opposite sides relative to the 60° angle.
2) a) Draw a right triangle that has one angle measuring 45°. Label the sides using the lengths 1, 1, and 2.
   b) Identify the adjacent and opposite sides relative to one of the 45° angles.

Activity 2: Compare Us
Directions: Do the following activities and answer the questions that follow. Use a protractor to find the measures of the angles of each triangle.



1. $\angle A =$ ________
2. $\angle C =$ ________
3. $\angle E =$ ________
4. $\angle F =$ ________
5. $\angle G =$ ________
6. $\angle I =$ ________

Questions

a. What have you noticed about the lengths of sides of each triangle?
b. What have you observed about the measures of the angles of each triangle?
c. What do you call these triangles?
d. What the mathematical concepts that you have learned from the activity?

In this activity, you have learned about some special angles. To evaluate the trigonometric ratios of these special acute angles, we can use geometric methods. These special acute angles are 30°, 45°, and 60°.

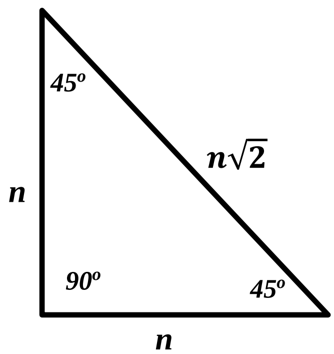
Discover

In trigonometry, 30°, 45°, 60° and 90° are called as special angles and they always lie in the first quadrant. These special angles 30°, 45° and 60° are frequently seen in applications and we can use geometry to determine the trigonometric ratios of these angles. Let us see, how to determine trigonometric ratios of these special angles using geometry.

45°-45°-90° Right Triangle Theorem

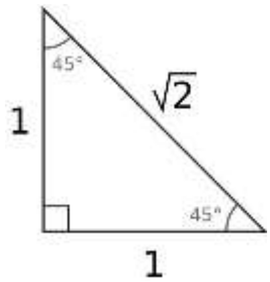
In a 45°-45°-90° Right Triangle,

- the legs are congruent;
- the length of the hypotenuse is $\sqrt{2}$ times the length of the leg
- hypotenuse = $\sqrt{2}$ leg

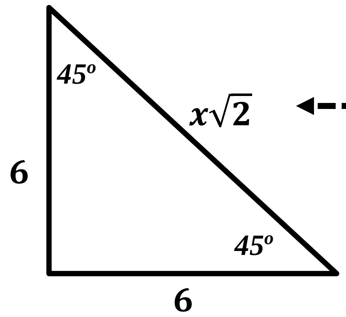


An Isosceles Right Triangle

The ratio of the sides is
n : n: $n\sqrt{2}$



Example 1. Find the length of the indicated side.



Using the ratio

n: n : n$\sqrt{2}$

6 : 6 : 6 $\sqrt{2}$, then x=6

$6\sqrt{2}$ is the hypotenuse

Now let us apply the six trigonometric ratios for the 45° angle.

$\sin 45° = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\qquad$ $\sec 45° = \frac{6\sqrt{2}}{6} = \sqrt{2}$

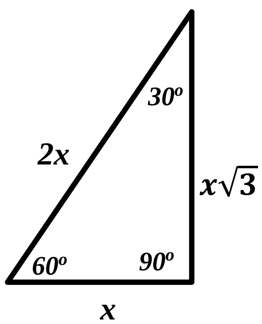
$\cos 45° = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\qquad$ $\csc 45° = \frac{6\sqrt{2}}{6} = \sqrt{2}$

$\tan 45° = \frac{6}{6} = 1$ $\qquad$ $\cot 45° = \frac{6}{6} = 1$

30°-60°-90° Right Triangle Theorem

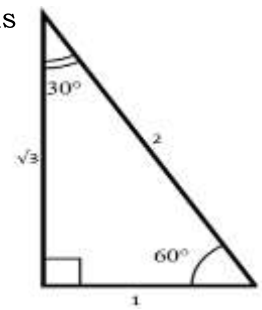
In a 30°-60°-90° Right Triangle,

- the length of the hypotenuse is twice the length of the shorter leg
- the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg
- hypotenuse = 2 shorter leg
- longer leg = $\sqrt{3}$ shorter leg

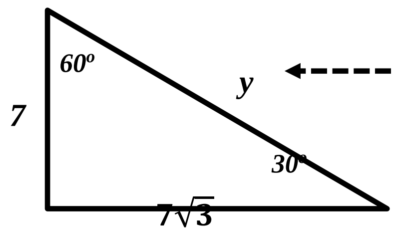


The ratio of the sides is

n: 2n: n$\sqrt{3}$



Example 2. Find the length of the indicated side.



Using the ratio n : 2n: n $\sqrt{3}$

n = shorter leg = 7

2n = hypotenuse = 2 (7)

y = 14

n $\sqrt{3}$ = longer leg = $7\sqrt{3}$

Now let us apply the six trigonometric ratios for the 30° angle.

$\sin 30° = \frac{7}{14} = \frac{1}{2}$ $\sec 30° = \frac{14}{7\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos 30° = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$ $\csc 30° = \frac{14}{7} = 2$

$\tan 30° = \frac{7}{7\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\cot 30° = \frac{7\sqrt{3}}{7} = \sqrt{3}$

Let us also solve for the 60° angle.

$\sin 60° = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$ $\sec 60° = \frac{14}{7} = 2$

$\cos 60° = \frac{7}{14} = \frac{1}{2}$ $\csc 60° = \frac{14}{7\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan 60° = \frac{7\sqrt{3}}{7} = \sqrt{3}$ $\cot 60° = \frac{7}{7\sqrt{3}} = \frac{\sqrt{3}}{3}$

Now we are ready to evaluate the exact values of the following expressions base from the values that we have derived;

Example 3. sec 60° + cot 45°

Since sec 60° = 2 ; cot 45° = 1

Then: sec 60° + cot 45° = 2 + 1 = **3**

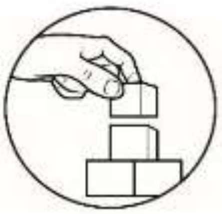
Example 4. sin 45° (tan 45°- cos 60°).

Since $\sin 45° = \frac{\sqrt{2}}{2}$; tan 45°= 1 and $\cos 60° = \frac{1}{2}$

Then: $\sin 45° (\tan 45° - \cos 60°) = \frac{\sqrt{2}}{2} \left(1 - \frac{1}{2}\right) = \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4}$

Example 5. Determine the value of angle x when tan x = 1.

Referring to the trigonometric ratios above, tan 45° =1. Therefore, x = 45°



Explore

After deepening your understanding on the trigonometric ratios of special angles, let us put those skills in higher level through the different activities.

Activity 3: Complete the table

Using the concepts you have learned on special angles, complete the table to summarize the trigonometric values of 30°,45° and 60° angles.

θ	sin	cos	tan	csc	sec	cot
30°						
45°						
60°						

Deepen

Activity 4: My Missing Part

0	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	$\sqrt{3}$	$\frac{3\sqrt{3}}{2}$	tan 30°	sin 60°

Find the missing value that will complete each mathematical statement below.

1. $\tan 60^0(\_\_\_\_\_\_\_\_\_\_\_\_) = 1$
2. $(\sin 45°)^2 =$ ______________
3. $3 \csc 60^0 - \cot 30° =$ ___
4. $\sin 30° - \cos 60° =$ ______
5. $\cos 30° +$ ________________________ $= \sqrt{3}$
6. $\tan 45^0 =$ ______________________________
7. $2 \sin 30^0 + 3 \cos 60° - 3 \tan 45° =$ ____
8. $\cot 30^0 + \sin 60° =$ ____________________