

Mathematics

Quarter 3- Week 7

Module 5: Proving Statements on Triangle Congruence



AIRs - LM

GOVERNMENT PROPERTY
NOT FOR SALE

Mathematics 8
Quarter 3- Week 7 Module 5: Proving Statements on Triangle
Congruence
First Edition, 2021

COPYRIGHT 2021

La Union Schools
Division Region I

All rights reserved. No part of this module may be reproduced in any form without written permission from the copyright owners.

Development Team of the Module

Writer: Aldrin A. Galvez

Editor: SDO La Union, Learning Resource Quality Assurance Team

Layout Artist: Ernesto F. Ramos Jr., *P II*

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent

Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent

German E. Flora, PhD, *CID Chief*

Virgilio C. Boado, PhD, *EPS in Charge of LRMS*

Erlinda M. Dela Peña, EdD, *EPS in Charge of Mathematics*

Michael Jason D. Morales, *PDO II*

Claire P. Toluyen, *Librarian II*



Target

Have you ever thought how triangles make a triangular window design, a doorway, basketball ring board with triangular design, seaplane wings more stable?

In this module, you will learn how to apply your knowledge on triangle congruent to prove that the triangles are congruent. As you go over, you will enhance your skills in writing proofs which guarantees that the given triangles are congruent.

In this lesson you will learn to:

1. prove statements on triangle congruence. **(M8GE-IIIh-1)**

At the end of this module, you are expected to:

1. recall the concept of SAS, ASA and SSS congruence postulates;
2. recall and apply the different geometrical properties and theorems; and
3. proves statements on triangle congruence.

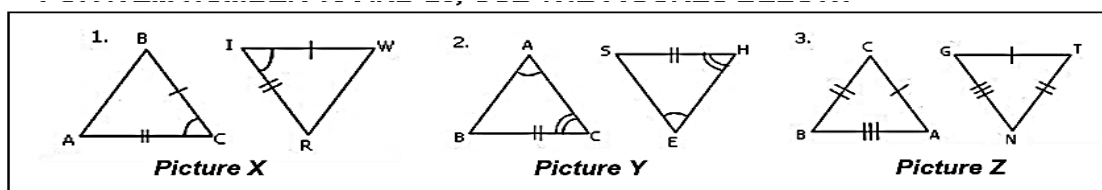
Let us find out how much you already know about this module. Answer the pre-assessment in a separate sheet of paper.

PRE-ASSESSMENT

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. It is a logical argument in which each statement you make is backed up by a statement that is accepted as true.
A. Conclusion B. Postulate C. Proof D. Theorem
2. What is the first step in proving triangle congruence?
A. Choose the correct theorem to prove congruency
B. Set up a two - column proof
C. Identify the known information
D. Write down the given

For item number 3 and 4, use the figure below.



3. Which of the following illustrates the SAS Postulate?
 - A. Picture X
 - B. Picture Y
 - C. Picture Z
 - D. None of the above
4. What congruence postulate does picture Y illustrates?
 - A. AAS
 - B. ASA
 - C. SAS
 - D. SSS
5. In the statement, "If the two triangles are congruent to the three corresponding sides of another triangle, then the two triangles are congruent", what congruence postulate is being described?
 - A. AAS
 - B. ASA
 - C. SASA
 - D. SSS

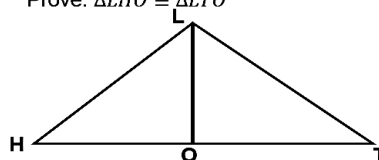
For item number 6 and 7, use the two-column proof at the right.

6. What concept/definition must be used to prove the statement number 2?
 - A. definition of angle bisector
 - B. definition of midpoint
 - C. definition of segment bisector
 - D. vertical angles theorem (VAT)
7. Which of the following statements is best to support the reflexive property in statement number 4?
 - A. $\overline{HT} \cong \overline{HT}$
 - B. $\overline{LO} \cong \overline{LO}$
 - C. $\overline{HL} \cong \overline{TL}$
 - D. $\overline{OH} \cong \overline{OT}$

Given: O is the midpoint of \overline{HT}

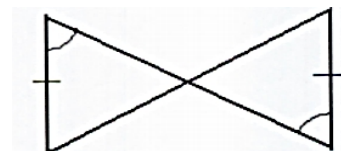
$$\overline{LH} \cong \overline{LT}$$

Prove: $\triangle LHO \cong \triangle LTO$



STATEMENTS	REASONS
1. O is the midpoint of \overline{HT}	Given
2. $\overline{HO} \cong \overline{TO}$	
3. $\overline{LH} \cong \overline{LT}$	Given
4.	Reflexive Property
5. $\triangle LHO \cong \triangle LTO$	SSS Congruence

8. Mr. Alden knows that in $\triangle ALD$ and $\triangle TEL$, $AL = TE$, $AD = TL$, $LD = EL$. Which postulate or theorem can he use to prove that the triangles are congruent?
 - A. ASA
 - B. AAS
 - C. SAS
 - D. SSS
9. Mr. Jeremie knows that $AB = XY$ and $AC = XZ$. What other information must he know to prove that $\triangle ABC \cong \triangle XYZ$ by SAS postulate?
 - A. $\angle B \cong \angle Y$
 - B. $\angle C \cong \angle Z$
 - C. $\angle A \cong \angle X$
 - D. $\angle C \cong \angle X$
10. In the figure at the right, can you use the SAS Postulate, the AAS theorem or both to prove that the triangles are congruent?
 - A. either SAS or AAS
 - B. SAS only
 - C. AAS only
 - D. neither



For item number 11 and 12, use the two-column proof at the right.

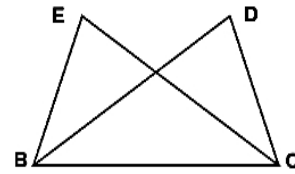
11. Which of the following statements make the proof incorrect?

A. $\overline{BE} \cong \overline{CD}$ B. $\overline{EC} \cong \overline{DB}$
 C. $\overline{BC} \cong \overline{BC}$ D. None

12. Are the statements and reasons on the proof enough to prove that $\triangle EBC \cong \triangle DCB$ by SSS Congruence?

A. Yes B. No C. Maybe
 D. the given information are not enough

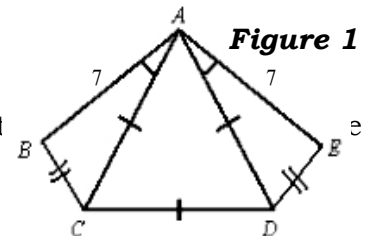
Given: $\overline{BE} \cong \overline{CD}$
 $\overline{EC} \cong \overline{DB}$
 Prove: $\triangle EBC \cong \triangle DCB$



STATEMENTS	REASONS
1. $\overline{BE} \cong \overline{CD}$	Given
2. $\overline{EC} \cong \overline{DB}$	Given
3. $\overline{BC} \cong \overline{BC}$	Reflexive Property
4. $\triangle EBC \cong \triangle DCB$	SSS Congruence

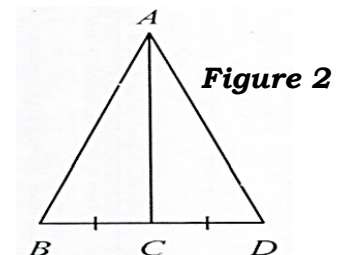
13. In figure 1, are $\triangle ABC$ and $\triangle AED$ congruent? Justify your answer.

A. Yes, by SAS only
 B. Yes, by SSS only
 C. Yes, by either SAS or SSS
 D. No, the information are not enough to conclude they are congruent.



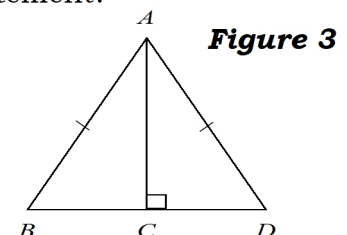
14. In figure 2, what other information do you need to prove that the triangles are congruent using the SAS congruence postulate?

A. $\overline{AC} \perp \overline{BD}$ C. $\angle CBA \cong \angle CDA$
 B. $\overline{AC} \cong \overline{BD}$ D. $\angle BAC \cong \angle DAC$



15. In figure 3, are the information enough to conclude that the two triangles are congruent? If so, what is the correct congruence statement?

A. Yes, $\triangle ABC \cong \triangle ACD$
 B. Yes, $\triangle ACB \cong \triangle ACD$
 C. Yes, $\triangle ABC \cong \triangle ADC$
 D. No, the triangles cannot be proven congruent



Lesson 1

Proving Statements on Triangle Congruence

Previously, you studied about proving two triangles are congruent. Likewise, you have honed the knowledge and skills of illustrating the SAS, ASA and SSS congruence postulates as well as completing the two-column proof.

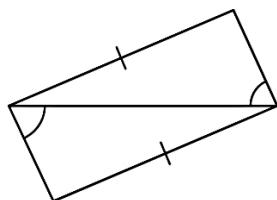
In this module, you will be considering important properties to prove statements on triangle congruence. In this lesson, you will recall the concept of SAS, ASA, SSS congruence postulates in proving statements on triangle congruence.



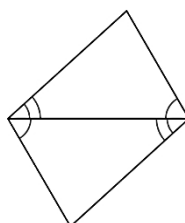
Jumpstart

Activity 1: Who Am I?

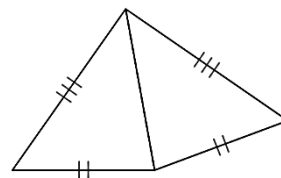
Directions: Identify if the two triangles are congruent. If they are, state how they are made congruent. Write your answer on the space below each figure.



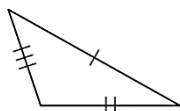
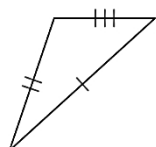
1.



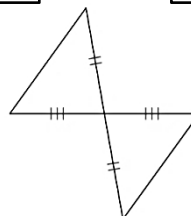
2.



3.



4.

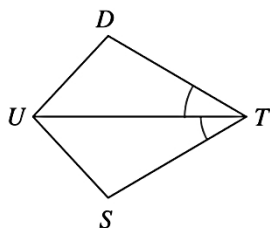


5.

Activity 2: Complete Me!

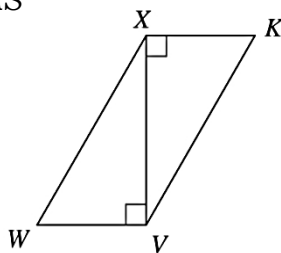
Directions: State what additional information is required to know that the triangles are congruent for the given reason. Write your answer on the space below each figure.

ASA



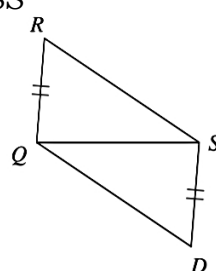
1.

SAS



2.

SSS



3.

Were you able to answer the given activity correctly? In the next activity you will enhance and deepen your understanding on the concept of triangle congruence.



Discover

At this point, you are going to take everything you have learned about triangle congruence and apply it to our lesson. This activity will enable you to add information to complete the proving process on triangle congruence.

Consider and observe the figures below:

Be safe during Earthquake!

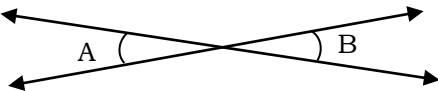
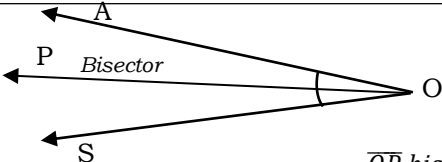
To prevent a doorway from collapsing after an earthquake, you can reinforce it. Explain why doorway with diagonal brace is more stable while the one without the brace can collapse.



Guide Questions:

1. Why do you think that a doorway with diagonal brace is stable?
2. Why is it that the doorway without diagonal brace is unstable?
3. What geometrical figure can be formed in the doorway with a diagonal brace?
4. What congruence postulate proves/guarantees that the triangles are rigid?
Explain your answer.

The above activity illustrates how to prove statements on triangle congruence. In doing so, some helpful and useful properties will guide you on how to successfully come up with the correct reasons or statements in proving.

DEFINITIONS	IF THEN FORM
1. Segment Addition Postulate	If points P, Q, and R are collinear and Q is between points P and R, then $PQ + QR = PR$
2. Definition of Midpoint	<i>The middle point of a line segment</i> If A is the midpoint of \overline{BC} , then $AB = AC$
3. Definition of Segment Bisector	<i>Divides a line segment into two equal parts</i> If \overline{AB} , bisects \overline{PQ} at B, then $\overline{PB} \cong \overline{QB}$,
4. Definition of congruent segments	<i>Line Segments that are equal in length</i> If $\overline{AB} \cong \overline{CD}$, then $AB \cong CD$
PROPERTIES OF EQUALITY	
1. Reflexive Property of Equality	If a, then $a = a$
2. Symmetric Property	If $a = b$, then $b = a$
3. Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$
VERTICAL ANGLES THEOREM (VAT) Angles that are opposite to each other and formed by two intersecting lines are congruent.	 <p>Since $\angle A$ and $\angle B$ are vertical angles. Therefore, $\angle A \cong \angle B$</p>
DEFINITION OF ANGLE BISECTOR It is a line or ray that divide an angle into two congruent angles. Bisect means to divide into two equal parts	 <p>\overline{OP} bisects $\angle AOS$ Therefore, $\angle AOP \cong \angle POS$</p>

Steps in Proving Statements in Triangle Congruence

1. Label your diagram with all given information.
2. Analyze your diagram to see what else you can find to be congruent. You will be including this information in your proof.
3. Decide what method you are going to use to prove the triangles congruent.
(Example: SSS Postulate, SAS Postulate)
4. Start your proof. List your given information as need is step number 1 with

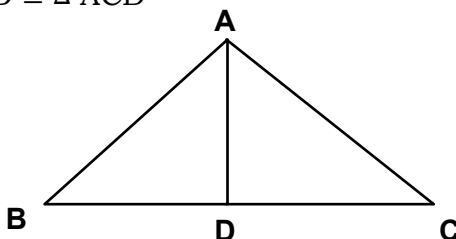
- “given” as your reason. (Remember some given information may be used later in the proof. Only list the given information that you need to start the proof.
- Continue your proof by listing your congruent statement with their reasons.
 - finally, show your triangles are congruent using a congruence statements and a reason.

Consider the following examples:

Example 1:

Given: D is the midpoint of \overline{BC} , $\overline{AB} \cong \overline{AC}$.

Prove: $\triangle ABD \cong \triangle ACD$



Proof:

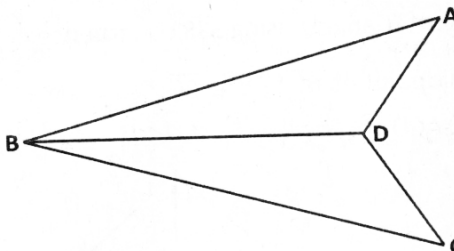
Statements	Reasons
1. D is the midpoint of \overline{BC}	1. Given
2. $\overline{DB} \cong \overline{DC}$	2. Definition of midpoint
3. $\overline{AB} \cong \overline{AC}$	3. Given
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\triangle ABD \cong \triangle ACD$	5. SSS Postulate

Example 2:

Given: $\overline{AB} \cong \overline{CB}$

$\overline{AD} \cong \overline{CD}$

Prove: $\triangle ABD \cong \triangle CBD$



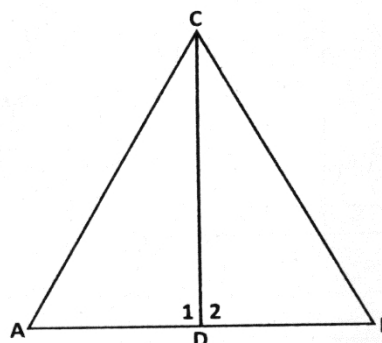
Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$	1. Given
2. $\overline{AD} \cong \overline{CD}$	2. Given
3. $\overline{DB} \cong \overline{DB}$	3. Reflexive Property
4. $\triangle ABD \cong \triangle CBD$	4. SSS Postulate

Example 3:

Given: D is the midpoint of \overline{AB}

$\angle 1 \cong \angle 2$

Prove: $\triangle ADC \cong \triangle BDC$

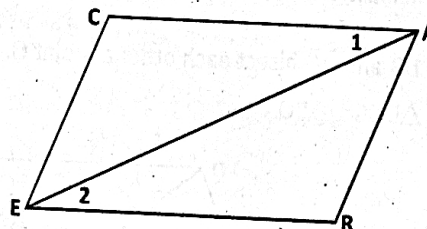


Statements	Reasons
1. D is the midpoint of \overline{AB}	1. Given
2. $\overline{AD} \cong \overline{BD}$	2. Definition of midpoint
3. $\angle 1 \cong \angle 2$	3. Given
4. $\overline{CD} \cong \overline{CD}$	4. Reflexive Property
5. $\triangle ABD \cong \triangle CBD$	5. SAS Postulate

Example 4:

Given: $\overline{CA} \cong \overline{RE}$, $\angle 1 \cong \angle 2$

Prove: $\triangle CAE \cong \triangle REA$

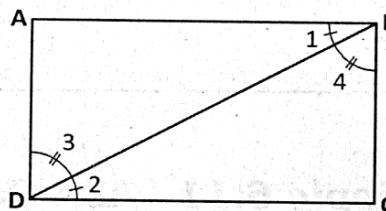


Statements	Reasons
1. $\overline{CA} \cong \overline{RE}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\overline{AE} \cong \overline{EA}$	3. Reflexive Property
4. $\triangle CAE \cong \triangle REA$	4. SAS Postulate

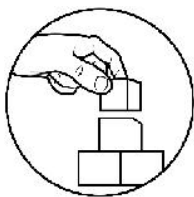
Example 5:

Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\triangle ABD \cong \triangle CDB$



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overline{BD} \cong \overline{BD}$	2. Reflexive Property
3. $\angle 3 \cong \angle 4$	3. Given
4. $\triangle ABD \cong \triangle CDB$	4. ASA Postulate



Explore

Activity 3: Fill Me In!

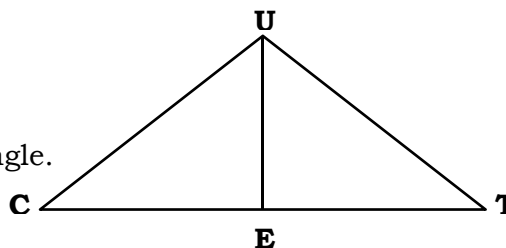
To enhance your skills in writing a two-column proof, let us answer this activity. Fill in the missing part of the two-column proof with the correct answer by choosing the answer from the box. Write your answers on a separate sheet of paper.

Problem 1:

Given: $\triangle CUT$ is an isosceles triangle.

E is the midpoint of \overline{CT}

Prove: $\triangle CUE \cong \triangle TUE$



Statements	Reasons
1. $\triangle CUT$ is an isosceles triangle.	1. Given
2.	2. Definition of isosceles triangle
3. E is the midpoint of \overline{CT}	3.
4.	4. Definition of midpoint
5.	5.
6. $\triangle CUE \cong \triangle TUE$	6.

Choices:

SSS Postulate

$\overline{UE} \cong \overline{UE}$

$\overline{CU} \cong \overline{UT}$

$\overline{EC} \cong \overline{ET}$

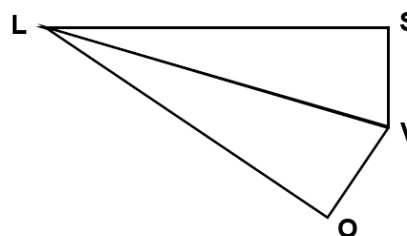
Reflexive property

Given

Problem 2:

Given: $\overline{SL} \cong \overline{OL}$, $\overline{SV} \cong \overline{OV}$

Prove: $\triangle SLV \cong \triangle OLV$



Statements	Reasons
1. $\overline{SL} \cong \overline{OL}$	1. Given
2. $\overline{SV} \cong \overline{OV}$	2.
3.	3.
4. $\triangle SLV \cong \triangle OLV$	4.

Choices:

Given

$\overline{LV} \cong \overline{LV}$

Reflexive Property

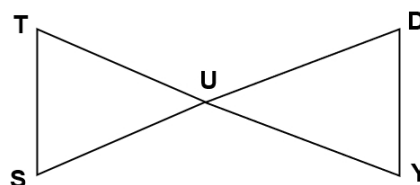
SSS Postulate

Activity 4: You Complete Me!

To improve your skills in writing a two-column proof, let us answer this activity. Fill in the missing part of the two-column proof with the correct answer. Write your answers on a separate sheet of paper.

Problem 1:

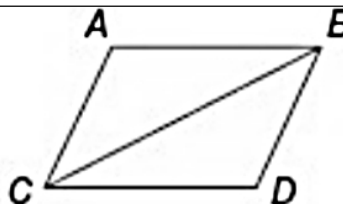
Given: \overline{TY} and \overline{SD} bisect each other at U.
Prove: $\triangle SUT \cong \triangle DUY$



STATEMENTS	REASONS
1. \overline{TY} and \overline{SD} bisect each other at U.	1. Given
2. $\angle SUT \cong \angle YUD$	2.
3. $\overline{TU} \cong \overline{YU}$	3.
4. $\overline{SU} \cong \overline{DU}$	4.
5. $\triangle SUT \cong \triangle DUY$	5.

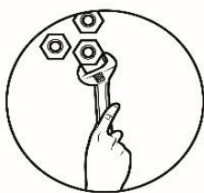
Problem 2:

Given: \overline{CB} bisects $\angle ABD$ and $\angle ACD$
Prove: $\triangle ABC \cong \triangle DCB$



STATEMENTS	REASONS
1. \overline{CB} bisects $\angle ABD$ and $\angle ACD$	1.
2. $\angle ABC \cong \angle DBC$	2. Definition of Angle Bisector
3.	3.
4.	4. Definition of Angle Bisector
5. $\triangle SUT \cong \triangle DUY$	5.

Now that you know the important ideas about the topic, let us go deeper by moving on to the next section.



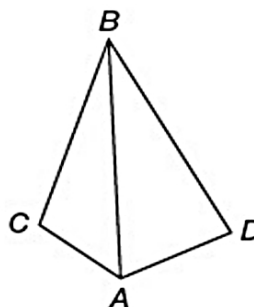
Deepen

Activity 5: Prove Me Right!

Problem 1:

Given: \overline{AB} bisects $\angle CBD$ and $\angle CAD$.

Prove: $\triangle ABC \cong \triangle ABD$



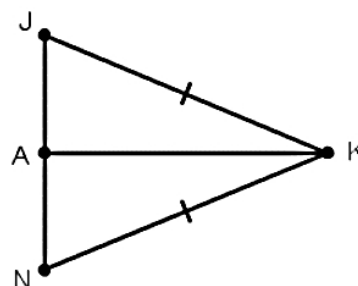
STATEMENTS	REASONS
1.	1.
2.	2.
3.	3.
4.	4.

Problem 2:

Given: $\overline{JK} \cong \overline{NK}$

A is the midpoint of \overline{JN}

Prove: $\triangle JAK \cong \triangle NAK$



STATEMENTS	REASONS
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.



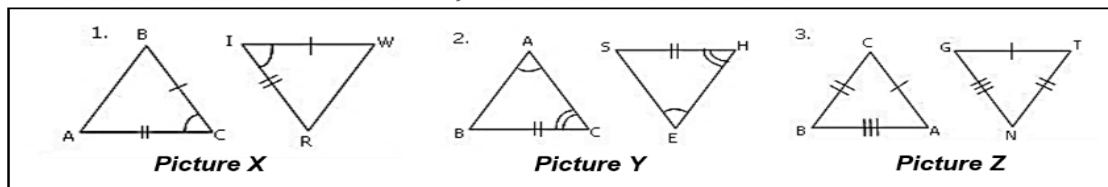
Gauge

Assessment: Post-Test

Direction: Find out how much have you learned from the lesson. Choose the letter which you think best answer to the question. Write your answer in a separate sheet of paper.

- It is a logical argument in which each statement you make is backed up by a statement that is accepted as true.
A. Conclusion B. Postulate C. Proof D. Theorem
- What is the first step in proving triangle congruence?
A. Choose the correct theorem to prove congruency
B. Set up a two - column proof
C. Identify the known information
D. Write down the given

For item number 3 and 4, use the figure below.



- Which of the following illustrates the SSS Postulate?
A. Picture X B. Picture Y C. Picture Z D. None of the above
- What congruence postulate does picture Y illustrates?
A. AAS B. ASA C. SAS D. SSS
- In the statement, "If two sides and the included angle of one triangle are congruent to the corresponding sides and the included angle of another triangle then the two triangles are congruent, what congruence postulate is being described?
A. AAS B. ASA C. SAS D. SSS

For item number 6 and 7, use the two-column proof at the right.

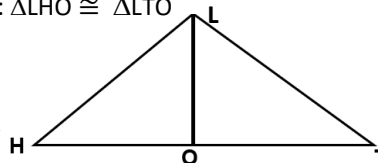
6. What concept/definition must be used to prove the statement number 2?

- A. definition of angle bisector
B. definition of midpoint
C. definition of segment bisector
D. vertical angles theorem (VAT)

Given: O is the midpoint of HT

$$LH \cong LT$$

Prove: $\triangle LHO \cong \triangle LTO$



7. Which of the following statements is best to support the reflexive property in statement number 4?

- A. $\overline{HT} \cong \overline{HT}$
B. $\overline{LO} \cong \overline{LO}$
C. $\overline{HL} \cong \overline{TL}$
D. $\overline{OH} \cong \overline{OT}$

STATEMENTS	REASONS
1. O is the midpoint of \overline{HT}	Given
2. $\overline{HO} \cong \overline{TO}$	
3. $\overline{LH} \cong \overline{LT}$	Given
4.	Reflexive Property
5. $\triangle LHO \cong \triangle LTO$	SSS Congruence

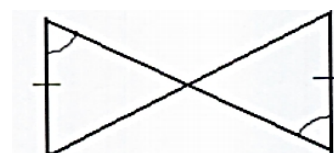
8. Mr. Alden knows that in $\triangle ALD$ and $\triangle TEL$, $AL = TE$, $AD = TL$, $LD = EL$. Which postulate or theorem can he use to prove that the triangles are congruent?

- A. ASA
B. AAS
C. SAS
D. SSS
9. Mr. Jeremie knows that $AB = XY$ and $AC = XZ$. What other information must he know to prove that $\triangle ABC \cong \triangle XYZ$ by SAS postulate?

- A. $\angle B \cong \angle Y$
B. $\angle C \cong \angle Z$
C. $\angle A \cong \angle X$
D. $\angle C \cong \angle X$

10. In the figure at the right, can you use the SAS Postulate, the AAS theorem or both to prove that the triangles are congruent?

- A. either SSS or AAS
B. SAS only
C. AAS only
D. neither



For item number 11 and 12, use the two-column proof at the right.

11. Which of the following statements make the proof incorrect?

- A. $\overline{BE} \cong \overline{CD}$
B. $\overline{EC} \cong \overline{DB}$
C. $\overline{BC} \cong \overline{BC}$
D. None

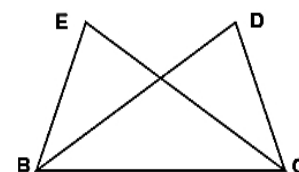
12. Are the statements and reasons on the proof enough to prove that $\triangle EBC \cong \triangle DCB$ by SSS Congruence?

- A. Yes
B. No
C. Maybe
D. the given information are not enough

Given: $\overline{BE} \cong \overline{CD}$

$\overline{EC} \cong \overline{DB}$

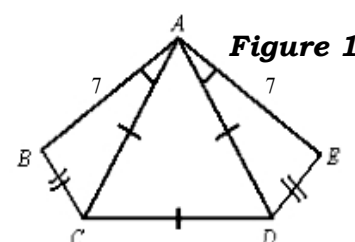
Prove: $\triangle EBC \cong \triangle DCB$



STATEMENTS	REASONS
1. $\overline{BE} \cong \overline{CD}$	Given
2. $\overline{EC} \cong \overline{DB}$	Given
3. $\overline{BC} \cong \overline{BC}$	Reflexive Property
4. $\triangle EBC \cong \triangle DCB$	SSS Congruence

13. In figure 1, are $\triangle ABC$ and $\triangle AED$ congruent? Justify your answer.

- A. Yes, by SAS only
B. Yes, by SSS only
C. Yes, by either SAS or SSS
D. No, the information are not enough to conclude that the two triangles are congruent



14. In figure 2, what other information do you need to prove that the triangles are congruent using the SAS congruence postulate?

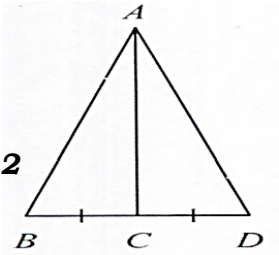
A. $\overline{AC} \perp \overline{BD}$

C. $\angle CBA \cong \angle CDA$

B. $\overline{AC} \cong \overline{BD}$

D. $\angle BAC \cong \angle DAC$

Figure 2



15. In figure 3, are the information enough to conclude that the two triangles are congruent? If so, what is the correct congruence statement?

A. Yes, $\triangle ABC \cong \triangle ACD$

B. Yes, $\triangle ACB \cong \triangle ACD$

C. Yes, $\triangle ABC \cong \triangle CAD$

D. No, the triangles cannot be proven congruent

References

Printed Materials:

Exploring Math 8 Textbook by Orlando A. Oronce and Marilyn O. Mendoza, 2018

Mathematics Grade 8 Learner's Module First Edition, 2014 Reprint 2017

Website:

https://www.wyzant.com/resources/lessons/math/geometry/triangles/congruent_sss_sas[https://](https://www.onlinemathlearning.com/two-column-proofs.html)

<https://www.onlinemathlearning.com/two-column-proofs.html>