9

Mathematics

Quarter 2 - Module 1:
Variations

AIRs - LM

MATHEMATICS 9
Quarter 2 - Module 1: Variations
Second Edition, 2021

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Printed in the Philippines by: ________________________

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Jumpstart

Let's start this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you understand variations. As you go through this lesson, think of this important question: "How are concepts of variations used in solving real-life problems and in making decisions? If you find any difficulty in answering the exercises, seek assistance of your teacher or refer to the modules you have gone over earlier.

Activity 1: TRANSLATE ME!

Directions: Rewrite the following expressions into verbal statements.

1. 3x
2. 10x + y
3. P = $2l + 2w$
4. A = $\frac{1}{2}bh$
5. Z = 2xy

Activity 2. MAKE ME SIMPLE!

Directions. Express the following ratios in lowest terms:

1. $\frac{25}{50}$
2. 10:15
3. $(-9)\left(\frac{2}{3}\right)$
4. (-100)(-4)
5. $(-3)(5)^2$
6. $(9^2)\ (1^2)$
7. $\frac{-45}{(9)(5)}$
8. $\frac{-20}{-4}$
9. $\frac{(6)(-1^2)}{18}$
10. $\frac{(5)(8)}{50}$

How did you find the activity? I am sure you did not find any difficulty in answering the questions. The next activity will help you fully understand the concepts behind this activity. But before doing the next activity, you have to read thoroughly and understand first some important notes about variations.

Discover

Lesson 1
Direct Variation

There is ***direct variation*** whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:
"y varies directly as x"
"y is directly proportional to x"
"y is proportional to x"

Imply direct variation and are used in many situations. Likewise the statements translate mathematically as $y = kx$. Where k is often referred to as the constant of proportionality or the constant of variation.

For two quantities x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Examples 1:
"The amount that a family (f) gives a charity varies directly as its income (i)" is translated as $f = ki$

Example 2:
"T varies directly as M" is translated as $T = kM$

Example 3:
If y varies directly as x and y = 50 when x = 10, find the constant of variation and the equation of variation.

Solution:

a. Express the statement "y varies directly as x" as y =kx.
b. Solve for k by substituting the given values in the equation.

$y = kx$

$50 = 10k$

$k = \frac{50}{10}$

$k = 5$

Therefore the **constant of variation** is **5**.

c. Form the equation of variation by substituting 5 in the statement, y = kx
So we have **y = 5x**

Example 4:

The table shows that the distance (*d)* varies directly as the time (*t)*. Find the constant of variation and the equation which describes the relation, then graph.

Time (hr)	1	2	3	4	5
Distance(km)	10	20	30	40	50

Solution:

Since the distance d varies directly as the time t, then

$$d = kt$$

Using one of the pairs of values (2, 20) from the table, substitute the values of d and t in $d = kt$ and solve for k.

$d = kt$

$20 = 2k$

$k = \frac{20}{2}$

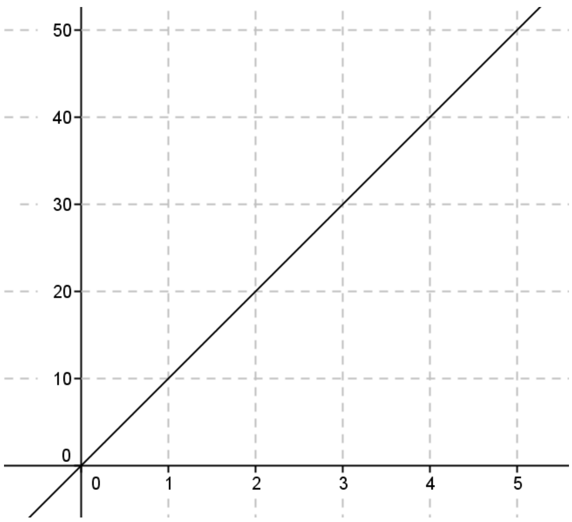
$k = 10$

Therefore, the constant of variation is 10

The equation of the variation is d = 10t

Below is the graph of the relation d = 10t.

distance (d)



time (t)

As the time *(t)* increases, the distance *(d)* being travelled also increases.

Lesson 2
Inverse Variation

This lesson deals with the relation of two quantities where one value increases as the other value decreases and vice versa. Inverse variation concepts can also be used to solve problems in other fields of mathematics.

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or y = $\frac{k}{x}$

Example 1:
"The number of notebooks sold **(N)** varies inversely as the price per notebook **(P)**" is translated as $\mathbf{N = \frac{k}{P}}$

Example 2:
Find the variation constant and the equation of variation if y varies inversely x, and y = 5 when x = 10.

Solution: The relation y varies inversely as x translates to y = $\frac{k}{x}$**,** then substitute the values to find k (constant of variation).

$y = \frac{k}{x}$

$5 = \frac{k}{10}$

k = (5) (10)

k = 50

Therefore, the equation of variation is y = $\frac{50}{x}$

Example 3:
The distance **(d)** from the center of a seesaw varies inversely as the weight of the child **(w)**. Daniel, who weighs 100 lb., sits 10 feet from the fulcrum.

Solution:

$d = \frac{k}{w}$

$10 = \frac{k}{w}$ by Substitution

k = (100) (10)

k = 1000

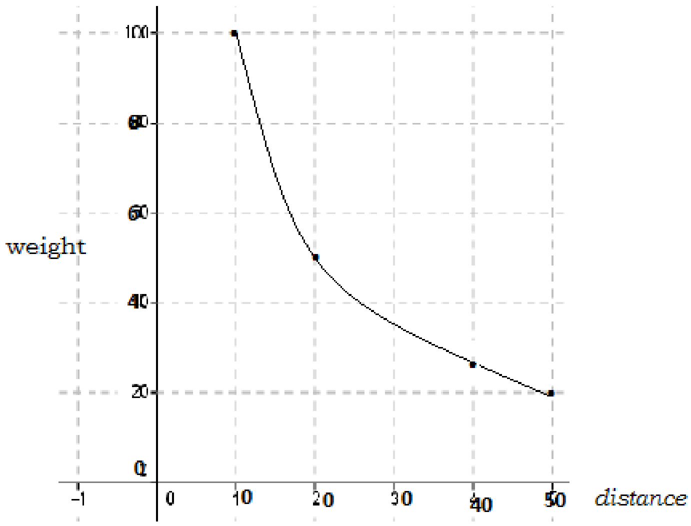
The constant of variation is 1000; therefore, the equation of variation is

$$y = \frac{\mathbf{1000}}{\mathbf{x}}$$

In table form the relation between the distance (d) and the weight of the child is shown

Weight (w)	100	50	40	20
Distance (d)	10	20	25	50

The graph of the relation looks like this:



Lesson 3
Joint Variation

Joint variation is just like direct variation but involves more than one other variable. All the variables are directly proportional, taken one at a time.

The statement "*a varies jointly as b and* c" means a = kbc, or k = $\frac{a}{bc}$ where *k* is the constant of variation

Example 1:

The lateral surface area **(A)** of a cylindrical jar varies jointly as the diameter **(d)** and the height **(h)** of the jar.

Translated as: $A = kdh$

Example 2.

Find an equation of variation where *a* varies jointly as *b* and *c*, and a = 24 when b = 3 and c = 4.

Solution: $a = kbc$

$24 = k(3)(4)$ substitute the set of given data to find k

$k = \frac{24}{12}$ apply the properties of equality

$k = 2$

Therefore, the required equation of variation is: $a = 2bc$

Example 3:

z varies jointly as x and y. If $z = 16$ when $x = 4$ and $y = 6$, find the constant of variation and the equation of the relation.

Solution: $z = kxy$

$16 = k(4)(6)$ Substitute the set of given data to find k

$k = \frac{16}{24}$ Apply the properties of equality

$k = \frac{2}{3}$

The equation of the variation is: $z = \frac{2}{3}xy$

Lesson 4
Combined Variation

Combined variation is another physical relationship among variables. This is the kind of variation that involves both the direct and inverse variations.

Combined variation describes a situation where a variable depends on two (or more) other variables, and varies directly with some of them and varies inversely with others (when the rest of the variables are held constant).. These equations are a little more complicated.

The statement "z varies directly as x and inversely as y" means $z = \frac{kx}{y}$, or $k = \frac{zy}{x}$, where k is the constant of variation.

This relationship among variables will be well illustrated in the following examples:

Examples:

1. Translating statements into mathematical equations using k as the constant of variation.
   a. T varies directly as a and inversely as b.

   $$T = \frac{\text{ka}}{b}$$

   b. Y varies directly as x and inversely as the square of z.

   $$\text{Y} = \frac{kx}{z^2}$$

2. If z varies directly as x and inversely as y , and z = 9 when x = 6 and y = 2, find the constant of variation and the equation of the variation.
   Solution:
   The equation is $z = \frac{kx}{y}$

   $$9 = \frac{6k}{2}$$

   $$\text{k} = \frac{9}{3}$$

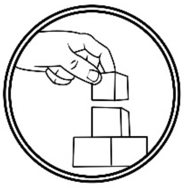
   $$\text{k} = 3$$

   The constant of variation is 3, therefore the equation is $z = \frac{3x}{y}$

3. t varies directly as m and inversely as the square of n. If $t = 16$ when $m = 8$ and $n = 2$, find the constant of variation k
   Solution:
   The equation of the variation: $t = \frac{km}{n^2}$
   To find k, where $t = 16$, $m = 8$ and $n = 2$, substitute the given values

   $$16 = \frac{k(8)}{(2)^2}$$
   $$k = \frac{16(2)^2}{8}$$
   $$k = \frac{(16)(4)}{8}$$
   $$k = \frac{64}{8}$$
   $$k = 8$$

   Therefore, the constant of variation is 8 and the equation is $t = \frac{8m}{n^2}$



Explore

Activity 3: It's Your Turn!

A. Identify whether the following illustrates direct variation **(DV)**, *i*nverse **(IV)**, joint **(JV)** or combined variation **(CV)**. Write only the symbol.

1. y = $\frac{25}{x}$

2.

x	4	8	10	12
y	2	4	5	6

3.

x	*6*	*4*	*3*	*2*	*1*
y	*2*	*3*	*4*	*6*	*12*

4. $A = kbh$

5. $P = \frac{kx^2}{s}$

B. Write an equation for the following statements:
1. The perimeter ***(P)*** of a square varies directly as the distance ***(d).***
2. The weight ***(W)*** of an object is directly proportional to its mass ***(m).***
3. Air pressure ***(P)*** varies inversely as its altitude ***(h)***
4. The volume (***V)*** of a pyramid varies jointly as the base area ***(b)*** and the altitude **(a)**
5. **(U)** varies directly as **(c)** and inversely as **(d)**

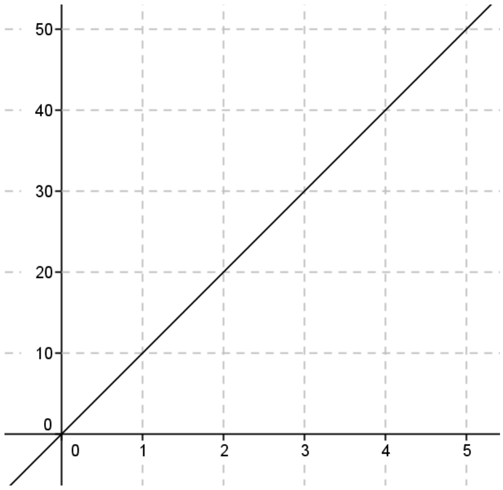
Activity 4

Directions: Find the constant of variation and write the equation representing the relationship between the quantities in each of the following:

1.

x	1	2	3	4
y	3	6	9	12

2. y



x

3. y varies inversely as x and y = 12 when x = 4
4. z varies jointly as x and y if z =27, x = 3 and y = 3
5. "p varies directly as q and the square of r and inversely as s" if p = 40 when q = 5, r = 4 and s = 6.

Now that you have understood the concept and ideas of this topic, let's now deepen your understanding by moving on to the next activity.

Deepen

Activity 5: How Well Do You Understand?

Direction: Find the constant of variation of the following situations below.

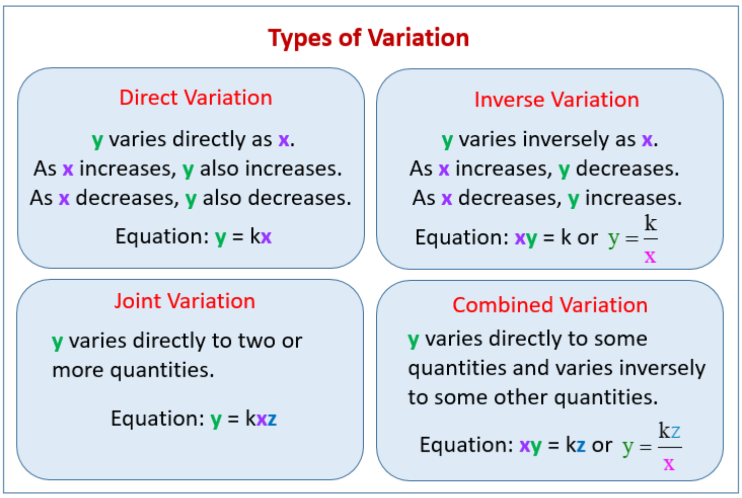
1. The current **I** varies directly as the electromotive force **E** and inversely as the resistance **R**. In a system a current of 20 amperes flows through a resistance of 20 ohms with an electromotive force of 100 volts.
   a. Equation: _______
   b. Current (I) : _____ amperes
   c. Resistance (R): _____ ohms
   d. Electromotive force (E): _____ volts
   e. Find the constant of variation (k)

2. Lory is asking for donations for a charity walk-a-thon. She wants to raise Php100, 000.00 for the charity. Write an inverse variation equation to calculate the amount she should ask each donor for based on the number of donors she plans on asking.

CRITERIA	4	3	2	1
Solution	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.
Mathematical Work and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it easy to understand what was done.	Correct terminology and notation are used, making it easy to understand what was done.	There is little use, or a lot of inappropriate use of terminology and notation.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat, clear, organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.

Generalization

In your notebook, summarize what you have learned from this lesson. Provide one real life example. Then illustrate using table, graph or mathematical equation showing relationship of quantities.



https://www.onlinemathlearning.com/joint-variation.html