SHS

AIRs - LM in

Statistics and Probability
Module 1:
Random Variable and Probability Distribution

Statistics and Probability
Module 1: Random Variable and Probability Distribution
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Target

Based on your previous lesson in Statistics, you dealt with the concept of probability and were given ideas on how you will come up with a decision based on the presented situation (e.g. drawing of card, rolling a die, picking your name from a raffle and others).

In this part of the module, you will be given more examples and real-life application on how probability will be utilized in dealing with decision-making process.

Decision-making is an important aspect in business, education, insurance, and other real-life situations. Many decisions are made by assigning probabilities to all possible outcomes pertaining to the situation and then evaluating the results.

This module will provide you with information and activities that will help you understand Random Variable and Probability Distribution.

After going through this module, you are expected to:

1. illustrates a random variable (discrete and continuous) (**M11/12SP-IIIa-1**);
2. distinguished between a discrete and a continuous random variable (**M11/12SP-IIIa-2**);
3. finds the possible values of a random variable (**M11/12SP-IIIa-3**);
4. illustrates a probability distribution for a discrete random variable and its properties (**M11/12SP-IIIa-4**); and
5. computes probabilities corresponding to a given random variable (**M11/12SP-IIIa-6**).

Subtasks:

1. define and illustrate random variables;
2. distinguish and give examples of discrete random variable and continuous random variable;
3. find the possible values of discrete random variable;
4. compute sample space of an experiment/ event;
5. illustrate probability distribution for a discrete random variable and its properties; and
6. computes probabilities corresponding to a given random variable.

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

_____1. Which of the following is ***NOT*** a discrete variable?

A. Number of white marbles in the box.
B. Number of students present in the classroom.
C. The weight of a box of soft drinks labeled "8 ounces."
D. The number of customers arrived in the restaurants between 7:00 a. m to 5:00 p.m.

_____2. Which of the following statement describe a continuous random variable?

A. The average distance traveled by a jeep in a week.
B. The number of students present in a Class Anthurium.
C. The number of motorcycles owned by a randomly selected household.
D. The number of girls taller than 5 feet in a random sample of 10 girls.

_____3. Which of the following is a variable whose value is obtained by measuring?

A. Continuous B. Discrete C. Interval D. Nominal

_____4. You decide to collect a bunch of bottles of soft drink and measure the volume of soft drink in each bottle. Let X be the number of milliliter (ml) of soft drink in each bottle. What type of variable is X?

A. X is a constant.
B. X is a place holder.
C. X is a discrete random variable.
D. X is a continuous random variable.

_____5. Which of the following statements describe a discrete random variable?

A. The length of span of a 10-months baby.
B. The average increase in height of a baby each year.
C. The average increase in weight of a baby each year.
D. The number of avocados produced by an avocado tree each year.

_____6. X is the time it takes a chef to cook a specific dish, is a continuous random variable. Why do think the variable X in the statement becomes continuous? The variable X is continuous because it takes….

A. a range of values B. takes a countable value.
C. an integral value. D. a specific numerical value

_____7. If a coin is tossed, what are the possible values of the random variable for the number of heads?

A. 0, 1 B. 0, 1, 2 C. 1, 2, 3 D. 0, 1, 2, 3

_____8. Suppose you tossed two coins. What are the sample spaces for the experiment above?

A. HH, TT C. HH, TH, TT
B. HH, HT, TT D. HH, HT, TT, TH

_____9. If you tossed a die, which of the following is ***NOT*** a possible value of the random variable X representing the number of dots appeared at the top after tossing the die?

A. 1 B. 7 C. 6 D. 3

_____10. The following table shows the probability distribution of a discrete random variable X. Find the value of ***n***.

X	2	4	6
P(X)	0.40	0.15	***n***

A. 0.15 B. 0.25 C. 0.45 D. 0.6

_____11. Which of the following table represents probability distribution?

A.

X	0	1	2	3
P(X)	0.11	0.15	0.42	0.44

C.

X	1	2	3	4
P(X)	0.32	0.28	0.28	0.12

B.

X	1	3	5	7
P(X)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$

D.

X	0	2	4	6
P(X)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

____12. What are the values of P(x) if $P(X) = \frac{X}{3}$, for $X = 0,1$, and 2?

A. $0, 0.33, 0.67$ B. $0, 0.45, 0.55$
C. $0, 1,2$ D. $0.2, 0.3, 0.5$

_____13. The given table represents a probability distribution. What is $P(2) + P(3)$?

X	1	2	3	4
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

Refer to the given problem in answering ***number 14 – 15***

The daily demand for copies of a magazine at a variety store has the probability distribution as follows.

Number of Copies ***X***	*Probability* ***P(X)***
0	0.10
1	0.25
2	0.30
3	0.16
4	0.05
5	0.14

_____14. What is the probability that two or more copies will be demanded on a particular day?

A. 0.30 B. 0.35 C. 0.65 D. 0.75

_____15. What is the probability that the demand will be at least one but not more than four?

A. 0.76 B. 0.71 C. 0.51 D. 0.35

Lesson 1	Random Variable

Jumpstart

Figure 1 shows how human beings are fascinated with the different events that involve probability. Humans are trying to make decisions based on how likely a particular event or experiment will happen. Since probability is all about chance, human-beings are trying to search for the ways to represent these events. Humans created random variable and use the concept to make it easy for an individual to verify the different results or probabilities of a given events or situation.

Figure 1. Different Events that use Probability

For you to understand the lesson well, do the following activities.
Have fun and good luck!

Activity 1: MY SAMPLE, MY SPACE, SAMPLE SPACE?

Directions: For each of the following experiments/events given below, supply the missing part either the possible outcomes (sample space) or the number of sample space.

Experiment	Sample Space	Number of Sample Space n(S)
Example: *Tossing a coin*	*S = {Head, Tail}* *or in symbol* *S = {H, T}*	$n(S) = 2$
1. Rolling a die	$S = \{1, 2, 3, 4, 5, 6\}$	
2. Rolling a die and tossing a coin simultaneously.		$n(S) = 12$
3. Determining the network provider of the cellphone number dialed randomly.	S = {Globe, Smart, Sun, Talk N Text, TM}	
4. Picking two balls inside a container containing 5 red balls and 6 blue balls.		$n(S) = 4$
5. Spinning a roulette containing number from 1 to 5.	$S = \{1,2,3,4,5\}$	
6. Flipping the page of a book with pages 1 to 9.		$n(S) = 9$
7. Answering a true-false type of test.		$n(S) = 2$

Activity 2: TOSSING COINS!

Directions: Perform the experiment below. After performing, try to answer the questions that follows.

If you are going to observe on the characteristics of the coin. One side contains a <u>head</u>, and we will represent that as **H**, while the other side which is the <u>tail</u> or **T**.

Steps:

1. Prepare **3 coins** for the activity.
2. Toss the first coin then the second coin and followed by the last coin.
3. Record the result by writing and indicating whether it is H or T. If the results of your three tosses for example is **heads, tails, heads**, then you will write on the outcome **HTH** on the given table below. (*Note: If the outcome is already repeated, do not write anymore the result.* The outcomes should be unique.)

Tossing the Coins			***Outcomes***
First Tossed	**Second Tossed**	**Third Tossed**	

4. After recording, you will notice that there are **only 8 possible outcomes** and no matter how you will repeat the tossing, the result will always be one of the 8 outcomes you already had.
5. After writing all the possible outcomes, try to fill the given table below:

OUTCOMES	***Number of Heads (H)***	***Number of Tails (T)***

Note*: Count the number of heads or tails in the given outcomes. Example:* ***HHH****, how many heads are there? Correct! There are 3 heads. How about tails? Correct! There are no or 0 tail.*

6. Based on the given table above, what can you say about the number of head? How
about the number of tails? Is there a possibility that the number of heads or tails exceed the highest value or number recorded? Why do you say so?

____
____
____

Discover

The activities on the Jumpstart part are all essential for you to understand the content of this module which on Random Variable. Before proceeding with the discussion, let us have a short recall on the different concepts that we are going to use in tackling the different parts of this module.

In your basic probability class before, you already discussed the following terms: experiments/events, outcomes, sample, and sample spaces.

Experiments are any movement or activity which can be dome repeatedly under similar or comparative condition. The *outcomes* are the result of a given experiment while the *sample space* is the set of all possible outcomes of an experiment.

Let us have an example to illustrate this: ***rolling a die (dice)*** is an example of ***experiment***. Upon rolling the die, it is expected that it will show a dot on the top which are either **1, 2, 3, 4, 5** or **6**, these are what we call the **sample space** of the given experiment. If you are only interested on a given value of dots, let say, **three dots**, then "3" is what we call the **outcome** or **the sample**.

In this module, we are looking for a **number** that will be assigned for the result of an experiment. What you did on Activity 2 is a preparation for the discussion.

If there are 4 coins instead of three coins tossed, what number or value can be assigned for the frequency of heads that will occur? If three cards are drawn from a deck of card, what number can be assigned for the frequency of face cards that will occur? The answer to these questions requires a knowledge of ***random variable***.

Recall also that a ***variable*** is a characteristic or attribute that can assume different values (e.g. in algebra, variables like x can be any number). We are going to use capital letter (X, Y, Z, and others) to denote or represent a variable.

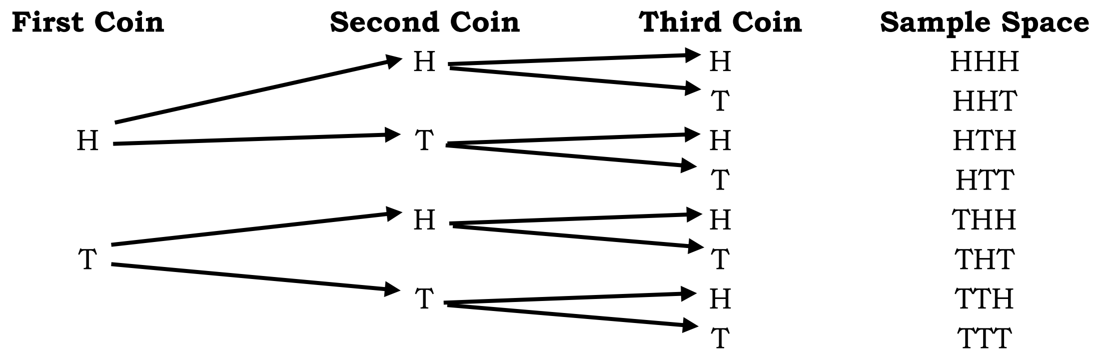
Let us examine the examples below to know more about Random Variable.

Example 1: TOSSING THREE COINS

Suppose you have *three coins*. We want to find the *number of heads* that will occur after tossing the coins.

To determine the number of sample space, we are going to use the basic probability rule. Since there are three coins, and each coin will give us 2 outcomes, then $n(S) = (2)(2)(2) = 2^3 = 8$ possible outcomes or sample space. Let us check!

We are going to use tree diagram to show us the possible outcomes of the said experiment. *Tree diagram* is used in probability to show possibilities of an event.



Sample Space / Possible Outcomes	Number of Heads
HHH	3
HHT	2
HTH	2
HTT	1
THH	2
THT	1
TTH	1
TTT	0

On the given experiment, there are 8 sample spaces and since we are interested to the number of heads (H) in each of the possible outcomes, which in

> A **random variable** is a capacity that connects a real number with every component in the sample space. It is a variable whose qualities are controlled by chance. In this manner, a random variable is a numerical amount that is derived from the results of an arbitrary trial or

this case are 0, 1, 2, and 3. These are what we call the **RANDOM VARIABLE**.

Here is another example to illustrate random variable.

Example 2: BOYS or GIRLS?

An experiment consists of studying the number of boys in families with exactly 2 children. The random variable X may be used to represent the number of boys.

Solution:

Since we are dealing with families with 2 children, and the children can be a boy or a girl, then there are $n(S) = (2)(2) = 2^2 = 4$ sample spaces.

Let **B** denote the boy, while **G** denote the girl.

First Child	**Second Child**	**Sample Space / Possible Outcomes**
B →	B	BB
B →	G	BG
G →	B	GB
G →	G	GG

Sample Space / Possible Outcomes	Random Variable X (number of boys in the family)
BB	2
BG	1

GB	1
GG	0

Based on the sample space, there are 4 possible outcomes. Since we are only interested with the number of boys in the family, then the random variable $X = \{0,1,2\}$. Observed that you cannot have 3 as the value of the random variable since it is impossible to have 3 boys in the family with two children as what the problem given as a condition.

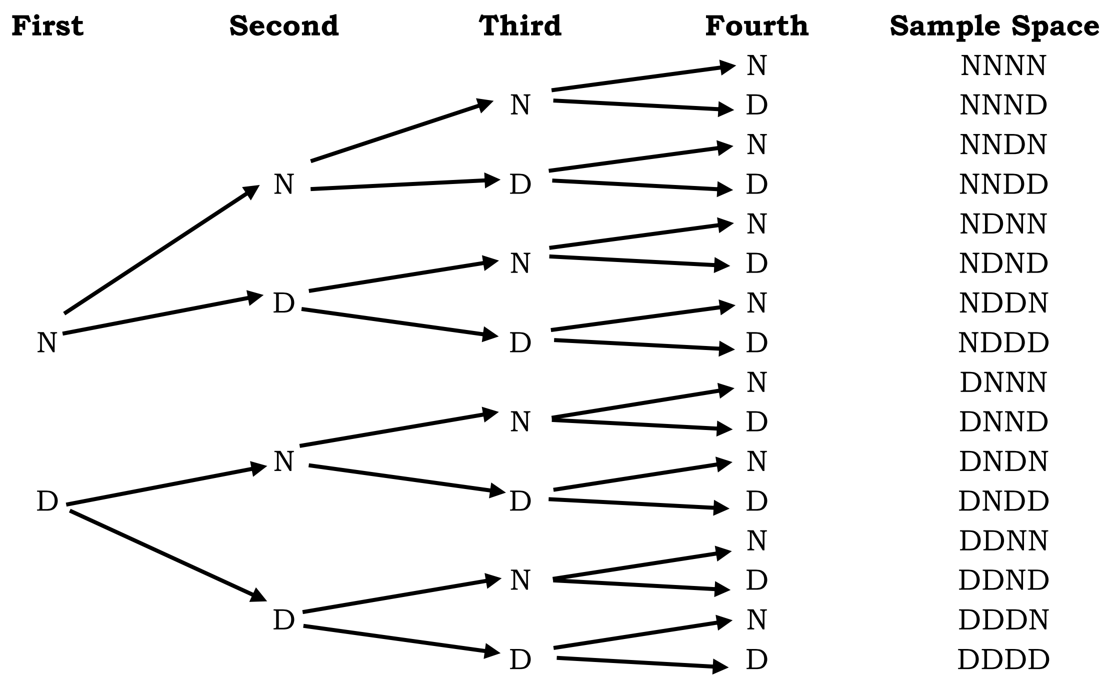
Example 3: DEFECTIVE or NON-DEFECTIVE

Suppose 4 laptops are tested at random. Let **N** represent the non-defective laptops and let **D** represent the defective laptops. If we let **Z** be the random variable for *the number of non-defective laptops*, determine the value of the random variable Z.

Solution:

Based on the given problem, there are 4 laptops that we will be tested at random and each laptop can either be defective or non-defective.

Let us first determine the number of sample space. Since there are 4 laptops and each laptop can be defective or non-defective, then $n(S) = (2)(2)(2)(2) = 2^4 = 16$ sample space / possible outcomes.



Sample Space / Possible Outcomes	**Random Variable Z** (Number of Non-Defective Laptops)
NNNN	4
NNND	3
NNDN	3
NNDD	2
NDNN	3
NDND	2
NDDN	2
NDDD	1
DNNN	3
DNND	2
DNDN	2
DNDD	1
DDNN	2
DDND	1
DDDN	1
DDDD	0

From the table, the values of the random variable Z are 0, 1, 2, 3, and 4. This means that it is possible that out of the 4 laptops you will have, there can be 0, 1, 2, 3, or 4 non-defective. It can also be observed that no number is greater than 4.

TYPES OF RANDOM VARIABLE

The random variable that we had from the three examples are classified as discrete random variable. There are two types of random variables and these are the ***discrete random variable*** and the ***continuous random variable***. In this module we are going to discuss and deal only with the first type, since the second type needs higher mathematics.

Discrete Random Variables are variables that can take on a **finite number** of distinct values. In easier definition, discrete random variable is a set of possible outcomes that is **countable**.

Examples are the number of heads acquired while flipping a coin three times, the number of defective chairs, the number of boys in the family, the number of students present in the online class, and more.

Continuous Random Variable are random variables that take an **infinitely uncountable number** of potential values, regularly **measurable amounts**.

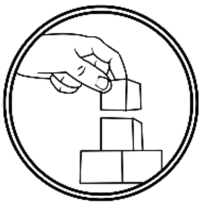
Often, continuous random variables represent measured data, such as height, weights, and temperature.

Let us have examples to illustrate and distinguish discrete from continuous random variable.

Example 3: Am I DISCRETE or CONTINUOUS!

Experiment	Random Variable X	Types of Random Variable
Determine the defective cell phones in the given shipment	The *number of defective* phones.	**Discrete** (Reason: You can count the number of defective phones)
Buying two trays of eggs in the market	The *weight* of eggs in kilograms.	**Continuous** (Reason: Since we are talking about the weight of the eggs, and weight is measurable)
Rolling a pair of dice.	The *sum of the number of dots* on the top faces.	**Discrete** (Reason: Since the number of dots is countable, it takes a finite number: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12)
Vehicular Accidents happened in La Union	The *number of accidents* happened at the intersection	**Discrete** (Reason: Accidents are countable)
Learners will prepare for a quiz in Mathematics	The *time spent* by the learners studying for a quiz in Mathematics	**Continuous** (Reason: Time is a measurable unit)

Just keep in mind that discrete random variables are countable, while continuous random variable are those measured or uncountable.



Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 1: CLASSIFY ME!

***Directions*:** Classify whether the following random variables are **DISCRETE** or **CONTINUOUS**. Write **D** or **C** only. Write your answer on the space provided.

__________1. The number of deaths per year attributed to lung cancer.
__________2. The average amount of electricity consumed per household per month.
__________3. The number of patient arrivals per hour at a medical clinic.
__________4. The number of bushels of mangoes per hectare this year.
__________5. The number of voters favoring a candidate.
__________6. The number of people who are playing LOTTO each day.
__________7. The amount of sugar in a cup of coffee.
__________8. The time needed to finish the test.
__________9. The number of female athletes in R1AA.

__________10. The speed of a car.
__________11. The number of dropouts in a school district for a period of 10 years.
__________12. The amount of paint utilized in a building project.
__________13. The number of siblings in a family of a region.
__________14. The weight of newborns each year in a hospital.
__________15. The number of COVID-19 cases each day.

Activity 2: JUSTIFY ME!

Directions: Given the following experiments, give the random variable X that fits the type of random variable given. The first entry is already answered as your guide.

Experiment	Random Variable X	Type of Random Variable
Example: The car travelling in La Union tourist spots.	The number of La Union tourist spots visited.	**Discrete**
Playing mobile games		Continuous
The learners reading a module.		Discrete
Eating your breakfast every day.		Continuous
Learners going to school.		Discrete
Visiting the social media sites.		Discrete

Activity 3: FINDING MY VALUE!

Directions: Find the possible values of the given random variable of the following experiments below. Write your answer on the space provided.

1. Supposed two coins are tossed, let P be the random variable representing the number of heads that occur. Find the values of the random variable P.

Possible Outcomes	Random Variable P

Answer: The values of the random variable P are ___________________.

2. Inside the box are 2 balls – one white and one yellow. Two balls are picked one at a time with replacement (meaning the ball is replaced once picked). Let X be the random variable representing the number of white balls. Find the values of the random variable X.

Possible Outcomes	Random Variable X

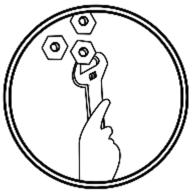
Answer: The values of the random variable X are ___________________.

3. Suppose three cellphones are tested. Let R represent the defective cellphones and N for the non-defective cellphones. Let U be the random variable that represent the defective cellphones. What are the possible values of the random variable R?

Possible Outcomes	Random Variable U

Answer: The values of the random variable U are ___________________.

Great job! You have understood the lesson.
Are you ready to summarize?



Deepen

At this point, you are going to apply what you have learn about the random variables. You are expected to solve problems regarding finding the value of a random variable.

What you need:

A piece of paper/Bond Paper
Ballpen or any writing material

What you must do:

Read the problem below. After reading, prepare a table just like what we did in the different examples previously. You are expected to determine *the number of possible outcomes or sample space (S)*. The *correct values of random variables* and *interpretation of the obtained values (see example 2 and 3 as your reference)*. You will be scored based on the give rubrics found at the end of the module.

Problem:

Five coins are tossed. Let **G** be the random variable representing the number of heads (H) that occur. Find the values of the random variable **G**.

What to find:

1. The number of possible outcomes or sample space (S).
2. The sample spaces (place in the table).
3. The correct values of random variable ***G.***
4. Interpretation or description about the value of the random variable ***G.***

Lesson 2	Probability Distribution and Its Properties

Jumpstart

Activity 1: FACT or BLUFF?

Directions: Determine whether the statement is FACT or BLUFF. If the answer is bluff, you can modify the statement to make it true.

Statement	Fact	Bluff
1. There are 4 outcomes if you tossed two coins.		
2. If you tossed three coins where X be the random variable representing the number of tails that occur. The possible values of the random variable X are 0, 1, and 2.		
3. The sum of $\frac{3}{4}+\frac{1}{2}=\frac{5}{8}$.		
4. The sum of $0.25 + 0.06 + 0.36 + 0.28 = 0.95$		
5. If $P(X) = \frac{X+1}{6}$, then the value of $P(1)$ is $\frac{1}{2}$.		
6. If $P(X) = \frac{3}{x-2}$, then the value of $P(4)$ is $\frac{3}{2}$.		

Activity 2: WHAT'S RANDOM?

Directions: Determine the values of the random variable by answering the given problem below.

Find the values of the random variable **Y** representing the number of green balls when 2 balls are drawn in succession without replacement from a jar containing 4 red balls and 5 green balls.

Solution:

a. Determine the sample space. Let **R** represent the red ball and **G** represent the green ball. The sample size for this experiment is *{RR, RG, GR, GG}*.
b. Count the number of green balls in each outcome in the sample space and assign the number to this outcome.

Possible Outcome	**Random Variable Y** (No. of Green Balls)
RR	
RG	
GR	
GG	

The values of the random variable Y are ____________________.

Discover

Before we have the definition of probability distribution for a discrete random variable and its properties, we need to tackle the problem below to show how to deal with probability distribution. Read and analyze the problem carefully.

Illustrative Example:

In a computer laboratory, the teacher wants to find out if there is a defective keyboard among its computer set. Supposed three keyboards were tested at random, he asked one of his learners to list all the possible outcomes, such that **D** represents the defective keyboard and **N** represents the non-defective. Let **X** be the random variable for the number of defective keyboards. Then, illustrate the probability distribution of the random variable **X**.

Based on the given problem above, observe, analyze, and answer the following questions:

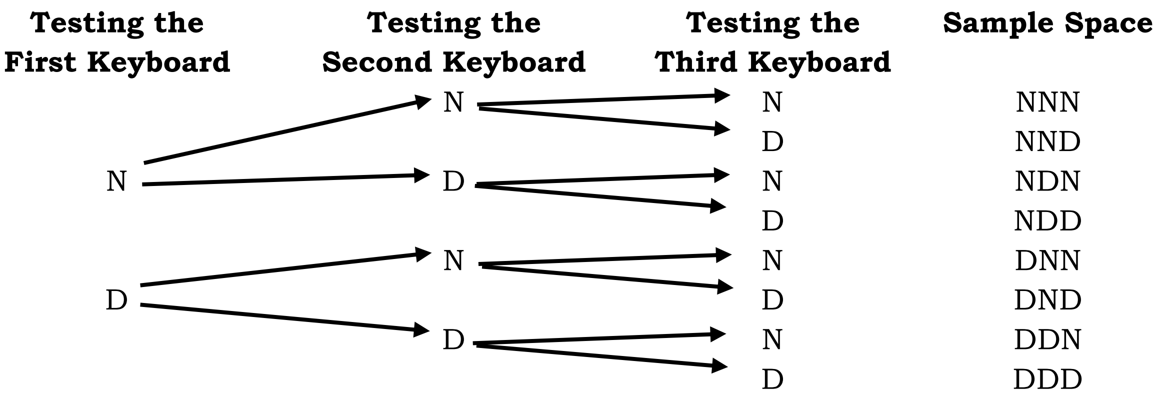
a. List the sample space in the given experiment. How many outcomes are possible?
b. Construct a table showing the number of defective keyboards in each outcome and assign this number to this outcome. What is the value of the random variable X?
c. Illustrate a probability distribution. What is the probability value $P(X)$ to each value of the random variable?
d. What is the sum of the probabilities of all values of the random variable?
e. What do you notice about the probability of each value of the random variable?

Solutions:

To solve the problem, you must consider first the steps in determining the values of the random variable that you had learn from your previous discussion.

a. Let **D** represent the defective keyboard and **N** for non-defective computer.

The number of sample space of the given problem is 8. This is because there are three keyboards subjected to inspection whether defective or non-defective, $n(S) = (2)(2)(2) = 2^3 = 8$ sample space.



The sample space is: $\boldsymbol{S} = \{\boldsymbol{NNN}, \boldsymbol{NND}, \boldsymbol{NDN}, \boldsymbol{NDD}, \boldsymbol{DNN}, \boldsymbol{DND}, \boldsymbol{DDN}, \boldsymbol{DDD}\}$.

b. Count the number of defective keyboards in each outcome in the sample space and assign this number to the outcome. For instance, if you list NND, then the number of defective keyboards is 1.

Possible Outcome	**Random Variable X** (number of defective keyboards)
NNN	0
NND	1
NDN	1
NDD	2
DNN	1
DND	2
DDN	2
DDD	3

There are four possible values of the random variable X representing the number of defective keyboards. The possible values of X are 0, 1, 2, and 3.

c. Each of these numbers corresponds to an event in the sample space **S** of equally likely outcomes for this experiment. Since the value of the random variable X represents the number of defective keyboards.

If each of the outcomes is equally likely to occur, then the probability is:

$$P(E) = \frac{number\ of\ the\ outcomes\ in\ the\ event}{number\ of\ the\ sample\ space}$$

In assigning probability values for P(X) to each value of the random variable, since we know that the number of the sample space is 8, then the probability that:

0 defective keyboard will come out is $\frac{1}{8}$ or $P(0) = \frac{1}{8}$;

1 defective keyboard will come out is $\frac{3}{8}$ or $P(1) = \frac{3}{8}$;

2 defective keyboards will come out is $\frac{3}{8}$ or $P(2) = \frac{3}{8}$;

and

3 defective keyboards will come out is $\frac{1}{8}$ or $P(3) = \frac{1}{8}$.

Random Variable X	Probability P(X)
0	$P(0) = \frac{1}{8}$
1	$P(1) = \frac{3}{8}$
2	$P(2) = \frac{3}{8}$
3	$P(3) = \frac{1}{8}$

Note:
These are the number of random variable X found in the table above. *How many 0, 1, 2, and 3 occurred in the table of random variable X?*

The number 8 is the number of sample space. ($\boldsymbol{n(s) = 8}$)

You can also construct the table in this form:

X	0	1	2	3
P(X)	$\frac{1}{8}$ or 0.125	$\frac{3}{8}$ or 0.375	$\frac{3}{8}$ or 0.375	$\frac{1}{8}$ or 0.125

d. By adding all the probabilities or the values of $P(X)$:

$$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=\frac{1+3+3+1}{8}=\frac{8}{8}=1$$

You can also use the decimals in determining the sum:

$$0.125 + 0.375 + 0.375 + 0.125 = 1$$

If you add all the probabilities, the sum is equal to 1.

e. From the given activity, you can observe that the values of the probability ranges from 0 to 1 only. No values of the probability will be lesser than zero and no values will be greater than 1.

The table given below is what we call the **probability distribution** or also known as the **probability mass function**.

X	0	1	2	3
P(X)	$\frac{1}{8}$ or 0.125	$\frac{3}{8}$ or 0.375	$\frac{3}{8}$ or 0.375	$\frac{1}{8}$ or 0.125

The ***probability distribution of a discrete random variable X*** is a list of the possible values of X and the corresponding probabilities of the values. It specifies the probability associated with each possible value of the random variable. The distribution is generally known as ***Probability Mass function***.'

Properties of Discrete Probability Distribution

1. The probability of each value of the random variable must be between or equal to 0 and 1. In symbol, $0 \leq P(X) \leq 1$.

2. The sum of all the probabilities of all values of the random variable must be equal to 1. In symbol, we write it as $\sum P(X) = 1$.

Example 1:

Determine if the distribution below is a discrete probability distribution:

X	1	5	7	8	9
P(X)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

The distribution must satisfy the two conditions given, *a)* the probability value P(X) must be from 0 to 1 and *b)* the sum of all the values of the probabilities must be equal to 1.

The first condition is met because $\frac{1}{3} = 0.33$ is from 0 to 1. The second condition is not satisfied because, the sum is **NOT** equal to 1.

$$\sum P(X) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3} \; or \; 1.67$$

Hence, this is ***NOT*** a probability distribution.

Example 2:

Determine if the distribution below is a discrete probability distribution:

X	1	3	5	7
P(X)	0.35	0.25	0.28	0.12

Remember, the two conditions that you need to satisfy.

1. Is the value of P(X) between 0 to 1? Based on the table, the values of $P(X)$ are 0.35, 0.25, 0.28, and 0.12, and the values are between 0 and 1. The first condition is SATISFIED.
2. Is the sum of P(X) equal to 1?

$$0.35 + 0.25 + 0.28 + 0.12 = \mathbf{1}$$

Since the sum is 1, then second condition is SATISFIED.

Hence, this is a **PROBABILITY DISTRIBUTION**.

Example 3:

Determine whether the given values can serve as the values of a probability distribution.

a. $P(1) = 0.05,$ $P(2) = 1.01,$ $P(3) = 0.2$
b. $P(4) = \frac{3}{20},$ $P(5) = \frac{7}{20},$ $P(3) = \frac{1}{2}$

Solution:

a. The probability of each value of the random variable **DOES NOT** lie between 0 and 1 because $P(2) = 1.01$. Therefore, this is **not** a probability distribution.
b. The probability of each values of the random variable lies between 0 and 1 because $(\frac{3}{20} = 0.15, \frac{7}{20} = 0.35, \frac{1}{2} = 0.50)$. The sum of its probabilities is equal to 1.

$$\frac{3}{20} + \frac{7}{20} + \frac{1}{2} = \frac{3}{20} + \frac{7}{20} + \frac{10}{20} = \frac{20}{20} = \mathbf{1}$$

Therefore, this is a probability distribution.

> **Note:**
> $\frac{1}{2}$ becomes $\frac{10}{20}$ because we need to make the denominators the same or equal.

Example 4:

Determine whether the given values can serve as the values of a probability distribution of a random variable X.

a. $P(X) = \frac{1}{7}$ for $X = 1, 2, 3, \ldots, 9$

This means that the value of $P(1)$ up to $P(9)$ is $\frac{1}{7}$.

X	1	2	3	4	5	6	7	8	9
P(X)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

The probability value $P(X)$ lies between 0 and 1 because $\frac{1}{7}$ is equal to 0.14. But the sum of its probabilities is not equal to 1.

$$\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{9}{7} \text{ } or \text{ } 1.29$$

Therefore, this is **NOT** a probability distribution.

b. $P(Z) = \frac{12}{25Z}$ for $Z = 1, 2, 3, 4$

Evaluating $P(Z)$, given the value of Z:

$$P(1) = \frac{12}{25(1)} = \frac{12}{25} = \mathbf{0.48}$$

$$P(3) = \frac{12}{25(3)} = \frac{12}{75} = \mathbf{0.16}$$

> **Note:**
> To convert fraction to decimal, divide the numerator by the denominator.

$$P(2) = \frac{12}{25(2)} = \frac{12}{50} = \mathbf{0.24} \quad \Big| \quad P(4) = \frac{12}{25(4)} = \frac{12}{100} = \mathbf{0.12}$$

Illustrating this in a table would give as:

Z	1	2	3	4
P(Z)	0.48	0.24	0.16	0.12

The probability of each value of the random variable lies between 0 and 1 and the sum of its probabilities is equal to 1. Therefore, this is a probability distribution.

Explore

Activity 1: AM I A PROBABILITY DISTRIBUTION?

Directions: Determine whether the following represents a probability distribution or not. Explain your answer.

1.

X	1	5	7	8
P(X)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

2.

X	1	3	5	7
P(X)	0.35	0.25	1.22	0.12

3. $P(1) = 0.42, P(2) = 0.31, P(3) = 0.37$

4. $P(1) = \frac{12}{35}, P(2) = \frac{8}{35}, P(3) = \frac{3}{7}$

5. $\mathrm{P}(X) = \frac{x}{6}$ for $X = 1,2,3$

Deepen

At this point, you are going to apply what you have learn about the random variables. You are expected to solve problems regarding finding the value of a random variable and applying properties of probability distribution.

What you need:

A piece of paper/Bond Paper
Ballpen or any writing material

What you must do:

Read and analyze the problem below. After reading, answer the question that follows. You will be scored based on the give rubrics found at the end of the module.

Problem:

Kate is a boutique shop owner in her town. Due to COVID-19 pandemic, wearing a facemask of a person is required for their safety. Since there are limited stocks available, she decided to start another business by making a face mask. She started selling a face mask from day 1 to day 10. The data she collected is shown in the table below:

Day	Number of Face Mask (X)
1	25
2	20
3	15
4	14
5	15
6	10
7	12
8	10
9	15
10	14

Guide:

X	P(X)
10	$\frac{2}{10}$
12	$\frac{1}{10}$
14	?
15	?
20	?
25	?

This is a sample. Complete this by writing the value of X, then counting how many X (example: 15, there are 3 out of the 10 days) in P(X).

What to find:

Illustrate a probability distribution of a random variable X showing the number of face mask sold per day and its corresponding probabilities. Also, check the two properties of the probability distribution.

Lesson 3	Computing Probability Corresponding to a Given Random Variable

Jumpstart

Activity 1: FIND ME!

Directions: Determine the value of the given random variable using the probability distribution below. Write your answer on any sheet of paper.

Z	2	4	6	8	10	12	14	16
P(Z)	0.07	0.23	0.09	0.06	0.14	0.12	0.28	0.01

Find:

a. $P(8) =$ _____ c. $P(2) + P(10) =$ _____ e. $P(Z \leq 6) =$ _____

b. $P(14) =$ _____ d. $P(4) + P(10) =$ _____ f. $P(8 < Z < 16) =$ _____

Discover

In the previous lesson, you have learned how to illustrate a probability distribution of a discrete random variable. For this lesson you are going to compute for the probability corresponding to a given random variable. Let us start the discussion by presenting a problem.

Illustrative Example 1:

Golden's Bakery is known for its famous Filipino delicacies. Among these foods which is native delicious food called "*kakanin*" is a "leche puto". The bakeshop owner recorded the number of boxes of "leche puto" that were delivered each day. The number of boxes delivered for 10 days is shown below:

Day	Number of Boxes (X)
1	35
2	37
3	50
4	45
5	37
6	45
7	40
8	42
9	45
10	42

Questions:

a. What is the probability that 40 or more boxes will be delivered on a particular day?
b. What is the probability that the number of boxes delivered will be least 37 but not more than 50?
c. What is the probability that at most 40 boxes will delivered on a particular day?
d. Find $P(X \leq 45)$
e. Find $P(40) + P(50)$

To answer the questions above, let us construct first the probability distribution. Let X be the value of the random variable represented by the number of boxes of "leche puto". The probability distribution is shown below.

Number of Boxes X	Probability P(X)
35	$\frac{1}{10}$
37	$\frac{2}{10}$ or $\frac{1}{5}$
40	$\frac{1}{10}$
42	$\frac{2}{10}$ or $\frac{1}{5}$
45	$\frac{3}{10}$
50	$\frac{1}{10}$

Note:
It is helpful if you are going to write the value of the random variable descending or ascending. Do not repeat a value if it is already written.

Note:
The probability P(X) is obtained by counting how many X are there in the sample space (ex. There are 1 "35 boxes" out of the "10 days" recorded.

Solution:

a. The probability that *40 or more* boxes will be sold in a particular day means $P\ (X \geq\ 40)$.

This means that you must add $P(X = 40), P(X = 42), P\ (X =\ 45), and\ P(X =\ 50)$.

$P\ (X \geq\ 40)\ =\ P(40)\ +\ P(42)\ +\ P(45)\ +\ P(50)$ then, substitute its corresponding probability $P(X)$ using the table above:

$$= \frac{1}{10} + \frac{1}{5} + \frac{3}{10} + \frac{1}{10}$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{10}$$

$$= \frac{7}{10} \text{ or } 0.7$$

Note:
$P(40) = \frac{1}{10}$, $P(42) = \frac{1}{5}$,
$P(45) = \frac{3}{10}$, $P(50) = \frac{1}{10}$

Interpretation: There are 0.7 chance that 40 or more boxes will be sold.

b. The probability that the number of boxes delivered will be *at least 37* but *not more than 50* means $P\,(37 \leq X < 50)$.

Hence, the values included are $P(37)$, $P(40)$, $P(42)$ and $P(45)$, then substitute its corresponding probability.

$$P\,(37 \leq X < 50) = P(37) + P(40) + P(42) + P(45)$$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{5} + \frac{3}{10}$$

$$= \frac{2}{10} + \frac{1}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10}\, or\, \frac{4}{5} \text{ or } 0.8$$

Interpretation: There are 0.8 chance that the number of boxes delivered will be at least 37 but not more than 50.

c. The probability that *at most 40* boxes will be delivered in a particular day means $P\,(X \leq 40)$ so the values of X are $P(40)$, $P(37)$, and $P(35)$.

$$P\,(X \leq 40) = P(40) + P(37) + P(35)$$

$$= \frac{1}{10} + \frac{1}{5} + \frac{1}{10}$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{1}{10}$$

$$= \frac{4}{10} \text{ or } \frac{2}{5} \text{ or } 0.40$$

Interpretation: There are 0.40 chance that at most 40 boxes will be delivered.

d. Find $P\,(X \leq 45) = P(45) + P(42) + P(40) + P(37) + P(35)$

$$= \frac{3}{10} + \frac{1}{5} + \frac{1}{10} + \frac{1}{5} + \frac{1}{10}$$

$$= \frac{3}{10} + \frac{2}{10} + \frac{1}{10} + \frac{2}{10} + \frac{1}{10}$$

$$= \frac{9}{10} \text{ or } 0.90$$

Interpretation: There are 0.90 probability that less than or equal to 45 boxes will be delivered.

e. Find $P(40) + P\,(50) = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \text{ or } \frac{1}{5} \text{ or } 0.10$

Interpretation: There are 0.10 chance that 40 and 50 boxes will be delivered.

Illustrative Example 2:

The probabilities that a customer buys 5,6,7, 8, and 9 items in a convenience store has the following probability distribution.

X	5	6	7	8	9
P(X)	0.25	0.12	0.28	0.15	0.20

Questions:

a. What is the probability that a customer will buy at least 6 items?
b. What is the probability that a customer will buy at most 7 items?

Solution:

a. "At least 6 items" means $P(X \geq 6)$, so we are concern with the probabilities of greater than or equal to 6 items. These are $P(6), P(7), P(8)$, and $P(9)$.

$$\begin{aligned} P\,(X \geq 6) &= P(6) + P(7) + P(8) + P(9) \\ &= 0.12 + 0.28 + 0.15 + 0.20 \\ &= 0.75 \end{aligned}$$

Interpretation: There are 0.75 probability that a customer will buy at least 6 items in the convenience store.

b. "At most 7 items" means $P(X \leq 7)$, so we are concern with the probabilities of less than or equal to 7 items. These are $P(5), P(6) and\ P(7)$.

$$\begin{aligned} P\,(X \leq 7) &= P(5) + P(6) + P(7) \\ &= 0.25 + 0.12 + 0.28 \\ &= 0.65 \end{aligned}$$

Interpretation: There are 0.65 probability that a customer will buy at most 7 items in the convenience store.

Explore

Activity 1: FIND MY VALUE!

Directions: Given the problem below, answer the following questions. Show your solution to each question.

Number of Absences

The monthly absences of a learner based on his class adviser's record are presented in the probability distribution below:

X (number of absences)	0	1	2	3	4
P(X) (Probability)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{1}{5}$

Questions:

a. What is the probability that the number of absences is more than 3?
b. What is the probability that the number of absences is at least 2?
c. What is the probability that the number of absences is greater than 1 but less than 4?

Deepen

At this point, you are going to apply what you have learn about the random variables. You are expected to compute probability corresponding to a given random variable.

What you need:

A piece of paper/Bond Paper
Ballpen or any writing material

What you must do:

Read and analyze the problem below. After reading, answer the question that follows. You will be scored based on the give rubric found at the end of the module.

Problem:

"Bayanihan To Heal as One Act"

In a certain barangay, the DSWD conducted a survey among the ages of senior citizens who will receive cash assistance or the Social Amelioration Program (SAP). This program is a cash emergency subsidy program for Filipino families whose lives are greatly affected by the Enhance Community Quarantine (ECQ). It is mandated by the new law, the "Bayanihan To Heal As One Act", which was signed by the President on March 25, 2020. The given data shows the probability distribution among the ages of senior citizens.

Age X	Probability P(X)
60	0.16
61	0.10
62	0.10
63	0.07
65	0.13
67	0.10
68	0.07
70	0.07
72	0.10
73	0.07
80	0.03

Questions:

a. What is the probability that at least 65 years old senior citizen will receive the SAP?
b. What is the probability that at most 70 years old senior citizen will receive the SAP?
c. What is the probability that at least 60 years old but less than 70 years old senior citizen will receive the SAP?
d. What is the probability that more than 60 years old, but less than 67 years old senior citizen will receive the SAP?
e. Give interpretations on your answers obtained from questions ***a*** to ***d***.

Rubric:

Your output in all DEEPEN part will be graded using this rubric.

Criteria	Excellent 10 points	Satisfactory 8 points	Developing 5 points	Beginning 2 points
Accuracy of the Solution	Shows accurate solution.	Shows solution with minimal errors.	Shows solution with plenty of errors.	The solution is all erroneous.
Mathematical Concept	Shows excellent understanding of the concept of solving problems involving random variables.	Shows clear understanding of the concept of solving problems involving random variables.	Shows limited understanding of the concept of solving problems involving random variables.	Did not apply the concept of solving problems involving random variables.

Gauge

Directions: Read carefully each item. Write the letter of the best answer for each test item.

_____1. Which of the following statement describe a continuous random variable?
A. The number of students present in a Class Temperance.
B. The average distance travelled by a tricycle in a month.
C. The number of motorcycles owned by a randomly selected household.
D. The number of girls taller than 5 feet in a random sample of 6 girls.

_____2. Which of the following is an example of discrete variable?
A. Distance travelled between cars.
B. Height of the students in a section Prudence.
C. Number of blue marbles in the box.
D. Weight of potatoes in the basket.

_____3. Which of the following is ***NOT*** a continuous random variable?
A. The height of the airplane's flight
B. The amount of liquid on a container
C. The length of time for the check up in the hospital
D. The number of clients of a certain Insurance Company each day

_____4. Which of the following is discrete random variable?
A. Hipolito weighs 65 kg.
B. Hipolito is 160 cm tall.
C. Hipolito has two brothers.
D. Hipolito ran 100 meters in 10.2 seconds.

_____5. You decided to conduct a survey of families with two children. You are interested in counting the number of girls (out of 2 children) in each family. Is this a random variable?
A. Yes, it is a random variable. B. Maybe.
C. No, it is not a random variable. D. Cannot be determined.

______6. Which of the following statement ***DOES NOT*** describe a continuous random variable?
A. The distance traveled by a truck in an hour.
B. The average height of a coconut tree each day.
C. The number of provinces belong to Region I.
D. The intensity of an earthquake that happens last month.

_____7. Suppose you tossed three coins. What are the sample spaces for the experiment above?
A. HHH, TTT
B. TTT, HHH, TTH, THT, HTH
C. TTT, HHH, HHT, THT, HTH
D. TTT, TTH, THT, HTT, HHT, HTH, THH, HHH

_____8. Suppose three laptops are tested. Let **D** represent the defective laptop and **N** for the non-defective laptop. How many possible outcomes will occur from the experiment?

A. 3 B. 4 C. 8 D. 9

_____9. Based on *number 8*, if we let ***X*** be the random variable representing the number of non-defective laptops. What are the possible values of the random variable?

A. 0, 1 B. 0, 1, 2 C. 1, 2, 3 D. 0, 1, 2, 3

_____10. Which probability distribution represents the given problem?

A.

X	0	1	2
P(X)	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$

B.

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

C.

X	0	1	2	3
P(X)	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

D.

X	0	1	2	3
P(X)	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{35}{8}$	$\frac{1}{8}$

_____11. Which of the following values can serve as the values of a probability distribution?

A. $P(1) = 0.42, P(2) = 0.31, P(3) = 0.37$
B. $P(1) = 9/14, P(2) = 4/14, P(3) = 1/14$
C. $P(1) = 0.08, P(2) = 0.12, P(3) = 1.03$
D. $P(1) = 10/33, P(2) = 12/33, P(3) = 10/33$

_____12. The probabilities that a customer buys 5, 8, 9, 12, and 15 items in a grocery store are 0.06, 0.14, 0.32, 0.28, and 0.20, respectively. Which probability distribution represents the given problem?

A.

X	5	8	9	12	15
P(X)	0.06	0.14	0.2	0.28	0.32

B.

X	5	8	9	12	15
P(X)	0.32	0.28	0.2	0.14	0.06

C.

X	5	8	9	12	15
P(X)	0.06	0.14	0.32	0.28	0.20

D.

X	5	8	9	12	15
P(X)	0.06	0.14	0.2	0.28	0.32

_____13. The given table represents a probability distribution.
What is $P(1) + P(4)$?

X	1	2	3	4
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

For number 14 to 15, refer to the given problem below:

The given table shows the probability distribution of a random variable Z which represents the percentage of male students per section among Grade 11 senior high school students.

Z	8	9	11	14	15	18
P(Z)	8 %	8 %	25%	17%	25%	17%

_____14. Find $P(Z \geq 9)$

A. 92% B.84% C. 59% D.43%

_____15. Find $P(8 < Z < 15)$

A. 84% B.73 % C. 59% D.50%

Great job! You are almost done with this module.

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