8

Mathematics

Quarter 4 - Week 1 Module 1

Theorems on Triangle Inequalities

AIRs - LM

Mathematics 8
Quarter 4- Week 1 Module 1: Theorems on Triangle Inequalities
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Development Team of the Module

Writer: John Michael L. Ordinario

Editor: SDO La Union, Learning Resource Quality Assurance Team

Layout Artist: Ernesto F. Ramos Jr., *P II*

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent
German E. Flora, PhD, *CID Chief*
Virgilio C. Boado, PhD, *EPS in Charge of LRMS*
Erlinda M. Dela Peña, EdD, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian I*

Target

This module will help you understand the key concepts of Triangle Inequality Theorems. Moreover, you' ll find out how these mathematical concepts are used in solving real-life problems. In all the lessons, you are given the opportunity to use your prior knowledge and skills in linear inequalities in one variable. Activities are also given to process your knowledge and skills acquired, deepen and transfer your understanding of the different lessons. The scope of this module enables you to use it in many different learning situations. The lessons are arranged to follow the standard sequence of the course. But in order in which you read them can be changed to corresponds with the textbooks you are using.

After going through this module, you are expected to

1. Illustrates Theorems on Triangle Inequalities; and
2. Applies Theorems on Triangles Inequalities.

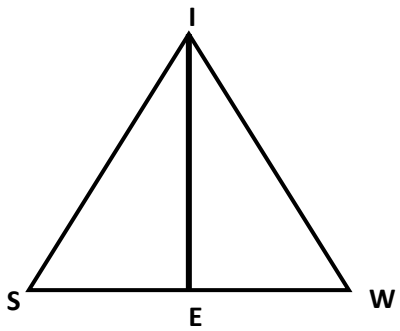
Pre - test

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. The measure of an exterior angle of a triangle is always _____. What phrase should be in the blank?

A. greater than its adjacent interior angle.
B. less than its adjacent interior angle.
C. greater than either remote interior angle.
D. less than either remote interior angle.

2. In the given figure, Kyle has proved that IS > IW. Which of the following statements is NOT part of his proof?

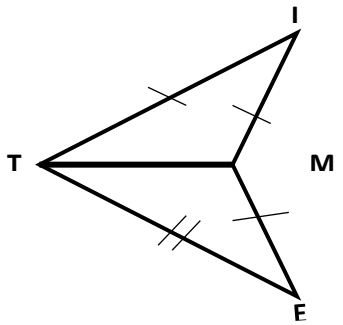


A. ES ≅ EW B. EI ≅ EI C. ∠WEI + ∠SEI = 180 D. ∠W < ∠S

3. What theorem should Kyle use to justify his proved statement in no. 2?

A. Converse of Hinge Theorem
B. Hinge Theorem
C. Triangle Inequality Theorem 1 (Ss◊Aa)
D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

4. Chloe studies the triangles in the figure carefully. Which should be her final conclusion?



A. TM ≅ TM B. IM ≅ EM C. ET > IT D. ∠EMT > ∠ITM

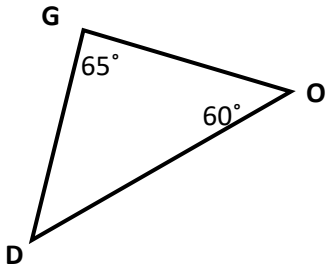
5. Which theorem justifies Chloe's conclusion in no. 4?

A. Converse of Hinge Theorem
B. Hinge Theorem
C. Triangle Inequality Theorem 1 (Ss◊Aa)
D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

6. Will you be able to conclude that EM > EF if one of the following statements is not established: AE ≅ AE, AF ≅ AM, m∠MAE > m∠FAE?

A. Yes, I will.
B. No, I won't.
C. It is impossible to decide.
D. It depends on which statement is left out.

7. Which side of ΔGOD is the shortest?

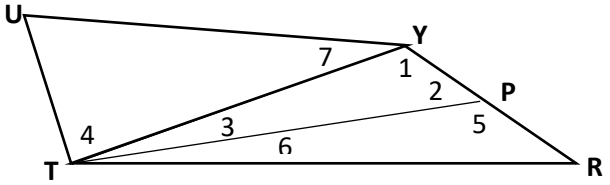


A. GO B. DG C. DO D. GD

8. In ΔTRU, TR = 8 cm, RU = 9 cm, and TU = 10 cm. List the angles in order from least to greatest measure.

A. m∠T, m∠R, m∠U
B. m∠U, m∠T, m∠R
C. m∠R, m∠T, m∠U
D. m∠U, m∠R, m∠T

9. Which of the following angles is an exterior angle of ΔTYP?

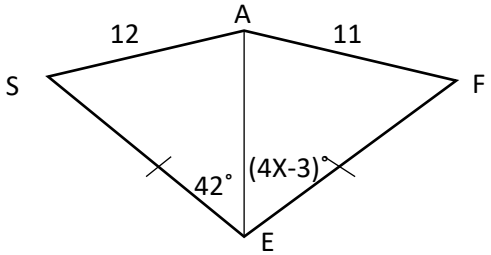


A. ∠4 B. ∠5 C. ∠6 D. ∠7

10. Each of Xylie, Marie, Angel and Chloe was given an 18-inch piece of stick. They were instructed to create a triangle. Each cut the stick in their own chosen lengths as follows: Xylie — 6 in, 6 in, 6 in; Marie — 4 in, 5 in, 9 in; Angel —7 in, 5 in, 6 in; and Chloe — 3 in, 7 in, 5 in. Who among them was not able to make a triangle?

A. Xylie B. Marie C. Angel D. Chloe

11. What are the possible values for x in the figure?



A. x < 11.25 B. x ≤ 11.25 C. x > 11.25 D. x ≥ 11.25

12. What considerations should you emphasize in your design presentation so that the balikbayan would award you the contract to build the houses?

I. Kinds of materials to use considering the climate in the area
II. Height of floor-to-ceiling corner rooms and its occupants
III. Extra budget needed for top-of-the-line furnishings
IV. Architectural design that matches the available funds
V. Length of time it takes to finish the project

A. I, II, and IV B. I, II, IV, and V C. I, IV, and V D. I, II, III, IV, V

13. Hikers Oliver and Ruel who have uniform hiking speed walk in opposite directions- Oliver, eastward whereas Ruel, westward. After walking three kilometers each, both of them take left turns at different angles- Oliver at an angle of 30˚ and Ruel at 40˚. Both continue hiking and cover another four kilometers each before taking a rest. Which of the hikers is farther from their point of origin?
    A. Ruel
    B. Oliver
    C. It cannot be determined.
    D. Ruel is as far as Oliver from the rendezvous

14. A balikbayan chose you to be one of the contractors to design an A-frame house maximizing the size of two square lots with dimensions 18 ft and 24 ft on each side. Which of the following is affected by the dimensions of the lot if the owner would like to spend the same amount of money on the roofs?
    I. The width of the base of the house frames
    II. Design of the windows
    III. The height of the houses
    IV. The roof angles

    A. I and IV B. III and IV C. II, III and IV D. I, II, III, and IV

15. Which of the following theorems justifies your response in item no. 15?
    I. Triangle Inequality Theorem 1
    II. Triangle Inequality Theorem 2
    III. Triangle Inequality Theorem 3
    IV. Hinge Theorem
    V. Converse of Hinge Theorem
    A. I, II, and III B. IV only C. IV and V D. V only

Lesson 1

Illustrating Theorems on Triangle Inequalities (Exterior Angle Inequality Theorem, Triangle Inequality Theorem, Hinge Theorem

Jumpstart

Have you ever wondered how artists utilize triangles in their artworks? Have you ever asked yourself how contractors, architects and engineers make use of triangular features in their designs? What mathematical concepts justify all triangular intricacies of their designs? The answers to these queries are unveiled in this module.

Find out the answer to this essential question: "How can you justify inequalities in triangle?"

Activity 1: Scalene, Isosceles or Equilateral

Directions: Determine by encircling the word "yes" or "no" if the following sides will make a triangle and if yes, then classify by sides. Write Scalene, Isosceles or Equilateral on the blank.

1. 8, 9, 10 yes or no _____
2. 1, 1, 2 yes or no _____
3. 6, 6, 10 yes or no _____
4. 3, 5, 7 yes or no _____
5. 4, 4, 4 yes or no _____

Discover

Triangles are one of the most common geometrical shapes found around us. They consist of three sides containing three angles within. So, let's explore some geometrical theorems related to inequalities in triangles.

Exterior Angle Inequality Theorem

An **exterior angle of a triangle** is the angle formed between any side of the triangle and an external extension of the adjacent side. There are six external angles formed in a triangle, two at each of the vertexes.

Exterior Angle Inequality Theorem

The measure of an exterior angle of a triangle is greater than the measure of either remote interior angles.

For a triangle

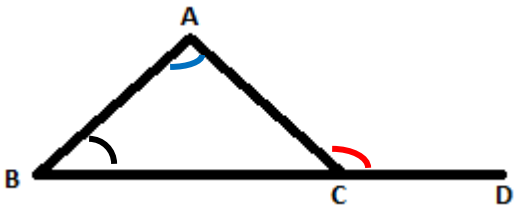
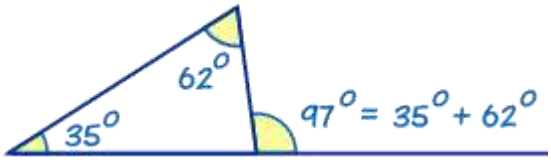


Figure 2.

$m\angle ACD = m\angle CAB + m\angle CBA$
$m\angle ACD > m\angle CAB$
$m\angle ACD > m\angle CBA.$

Example:

Figure 3.



The exterior angle is 35° + 62° = 97° and 97° > 35°

Interior angles of a triangle add to 180°, and angles c + d also add to 180°:

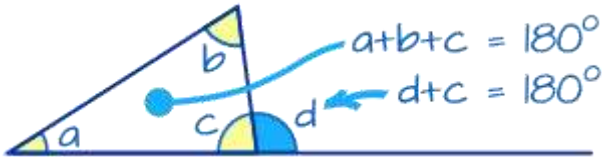


Figure 4.

The interior angles of a triangle add to 180°: a + b + c = 180°
Angles **c** and **d** make a straight angle, which is 180°: d + c = 180°
So **d + c** equals **a + b + c** : d + c = a + b + c
Subtract **c** from both sides: d = a + b

Angle-Side Relationship

In a triangle, the angle opposite a longer side will be greater than the angle opposite a shorter side. Let's look at the angle-side relationship in a triangle that has three unequal sides, with AC being the smallest and BC being the longest.
In triangle ABC there are three inequalities that exist:

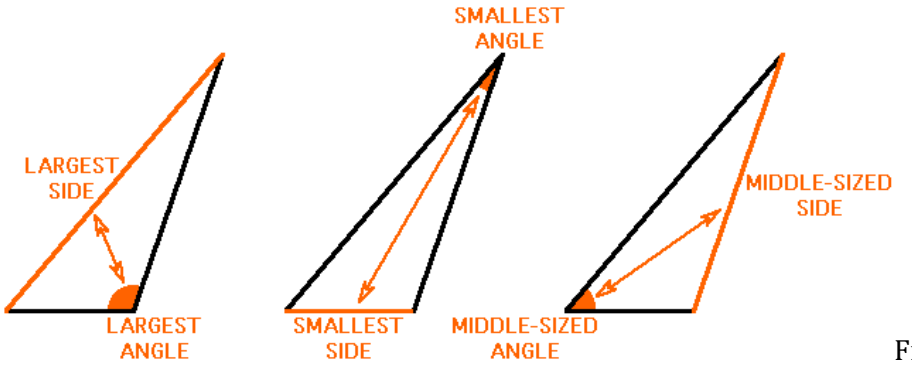


Figure 5.

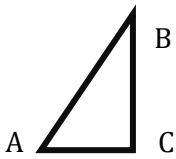


Figure 6.

largest side AB	Largest angle C
smallest side AC	smallest angle B
middle-sized side	middle-sized angle

∠A > ∠B
∠A > ∠C
∠C > ∠B

Triangle Inequality Theorem 3

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Let's try to draw an imaginary triangle with sides:

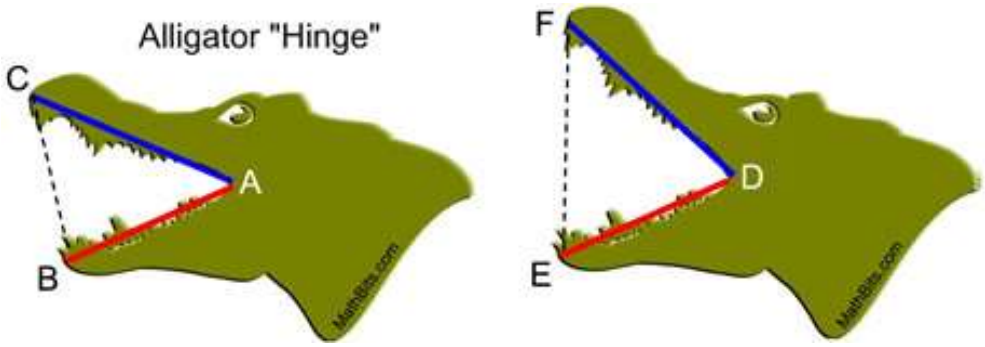
a. 3cm, 4cm, and 10cm.
10 + 3 > 4 10 + 4 > 3 3+ 4 $\not>$ 10,
13 > 4 14 > 3 7 $\not>$ 10,
One statement not true, hence, not a triangle.

b. 6cm, 8cm, 11 cm
6 + 8 > 11 6 + 11 > 8 8 + 11 > 6
14 > 11 17 > 8 19 > 6
All the statements correct, hence, it is a triangle.

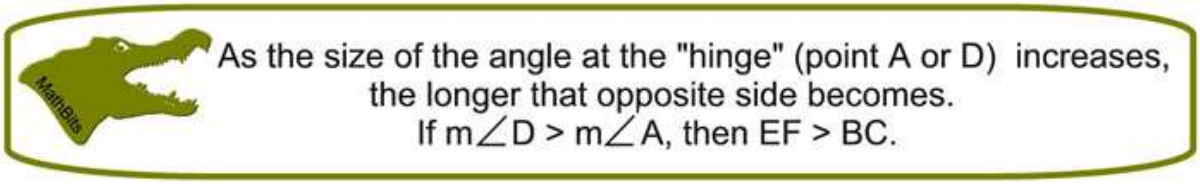
Hinge Theorem or SAS Inequality Theorem

If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Consider the alligator jaws below.



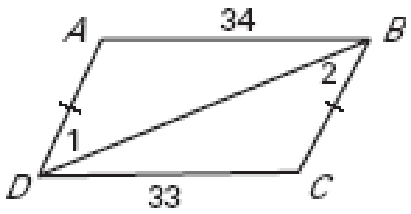
The sides described in this theorem are the jaw lengths of the alligator with the "hinge" being at the corner of the alligator's mouth (point A or D). While the jaw lengths of the alligator will not change, the jaw "hinge" does allow the alligator to open, or close, its mouth with varying angular degrees (at point A or D).



Converse of Hinge Theorem or SSS Inequality Theorem

If two sides of one triangle are congruent to two sides of another triangle, but the third side of one triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

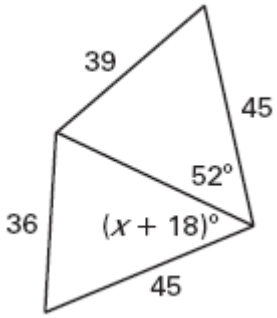
Example 1. Given that $\overline{AD} \cong \overline{BC}$, how does $\angle 1$ compare to $\angle 2$?



Solution:

You are given that $\overline{AD} \cong \overline{BC}$ and $\overline{BD} \cong B\overline{D}$ you know that by the Reflexive property. Because 34 > 33, $\overline{AB} > \overline{CD}$. So, two sides of ΔADB are congruent to two sides of ΔCBD and the third side in ΔADB is longer. By the Converse of the Hinge Theorem, $m\angle 1 > m\angle 2$.

Example 2. Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of x.



Solution:

Since the side (39 units) opposite the angle measuring 52˚ is longer than the side (36), then 52˚ is larger than (x + 18)˚. So we can have the following inequality,

52˚ > (x + 18)˚ or (x + 18)˚ < 52˚.

We can solve now the possible values of x.

(x + 18)˚ < 52˚

x + 18 < 52

x < 52 -18

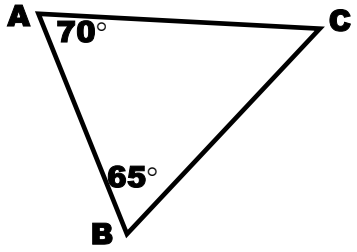
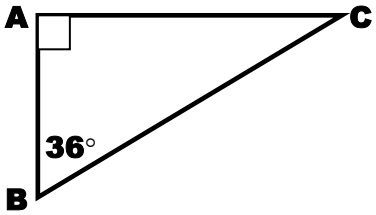
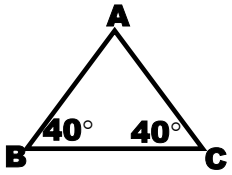
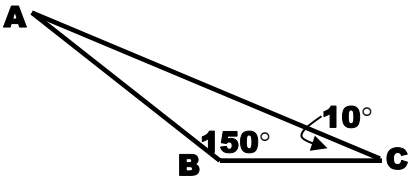
x < 34.

Therefore the solution is x < 34.

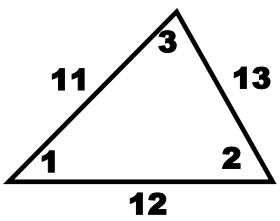
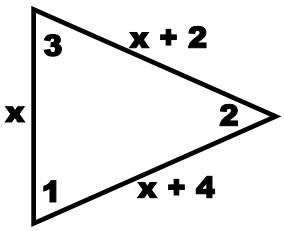
Explore

Activity 2: Triangle Inequality

A. Name the longest segment in each of the following triangles.

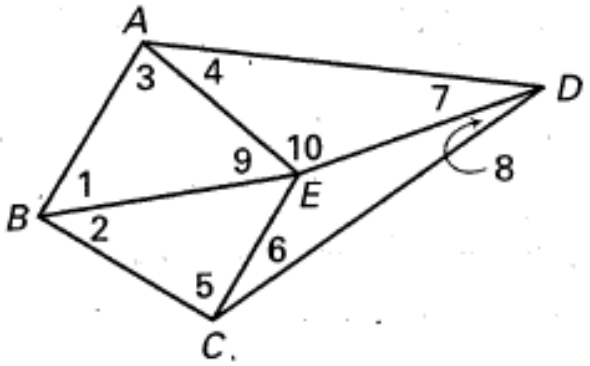
1. _____	
2. _____	
3. _____	
4. _____	

B. Name the largest angle in each of the following.

5. _____		6. _____	

Activity 3 Pair Me?

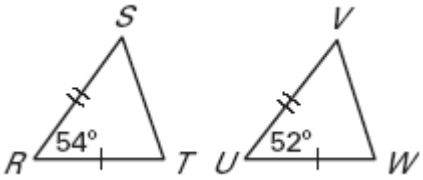
A. Match the conclusion on the right with the given information. Use the diagram below.



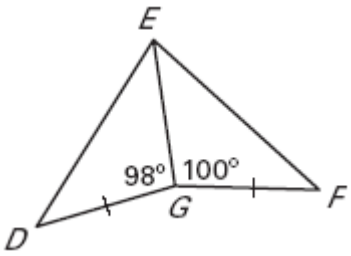
_____1. AB = BC, m∠1 > m∠2	A. m∠7 > m∠8
_____2. AE > EC, AD = CD	B. AD > AB
_____3. m∠9 < m∠10, BE = ED	C. m∠3 + m∠4 = m∠5 + m∠6
_____4. AB = BC, AD = CD	D. AE > EC

B. Fill in the blanks. Complete with <, > or =.

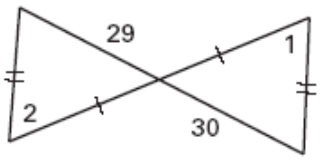
1. *ST* _____ *VW*



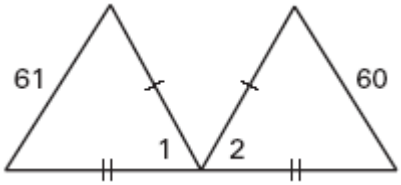
2. DE _____ *EF*



3. *m*∠ 1 _____ *m*∠ 2



4. *m*∠ 1 *m*∠ 2



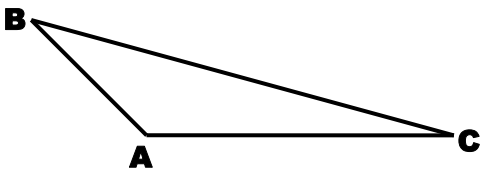
Deepen

Activity 4. Am I a triangle?

A. Complete the table by filling out what us being asked for every column.

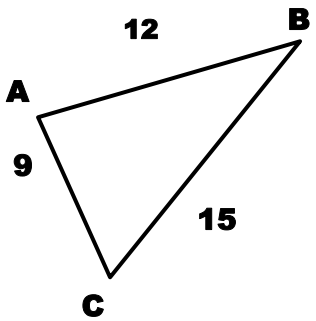
Sides			**Could a triangle be made: Yes or No?**	**Classify by Sides (Scalene, Isosceles, Equilateral)**	**Sum of shorter sides**	**>, <, or =**	**Larger side**
2	2	2					
2	2	3					
2	3	4					
2	3	5					
2	3	6					
3	4	5					
3	3	6					
3	3	3					
3	3	4					
3	5	6					

B. Determine the longest and smallest sides of the given triangle.
$\angle A = 120°$, $\angle B = 40°$, $\angle C = 20°$



Longest side:
Smallest side:

C. List the angles from smallest to largest.



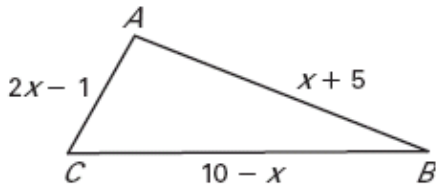
Angles:_____, _____, _____

D. List the sides from smallest to longest based on the given measures of the angles of a triangle.

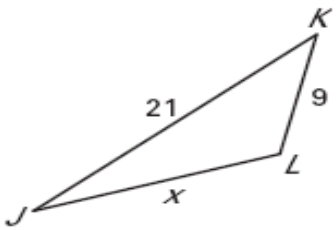
1. Sides _____, _____, _____	$m\angle A = (5x + 2)^\circ$, $m\angle B = (6x - 10)^\circ$, and $m\angle C = (x + 20)^\circ$.
2. Sides: _____, _____, _____	$m\angle A = (x + 16)^\circ$, $m\angle B = (x)^\circ$, and $m\angle C = (x + 29)^\circ$.

Activity 5. Find me!

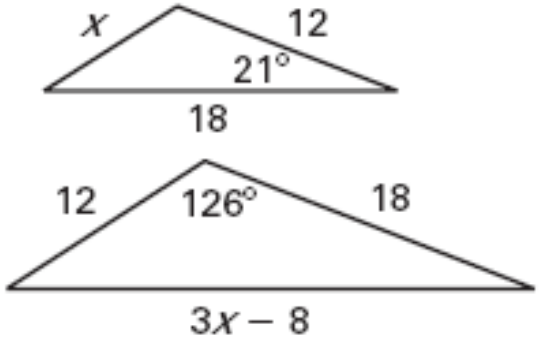
A. Solve the inequality $AB + AC > BC$ for x



B. Find all possible values of x.



C. Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of x.



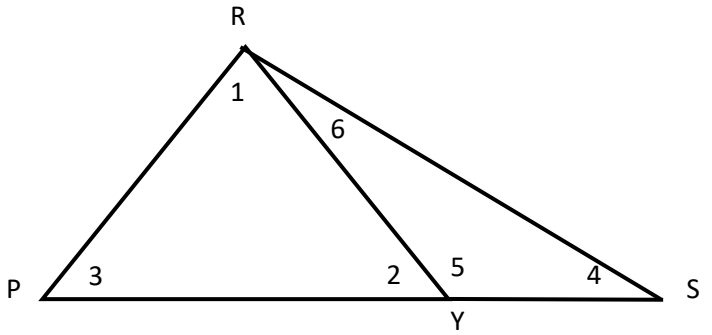
D. Problem Solving: You are asked to fence in a triangular playground. Two sides of the playground have lengths of 100 feet and 200 feet. What is the maximum total length of fence you could possibly need?

Gauge

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following is not an inequality theorem for one triangle?
   A. Hinge Theorem
   B. Triangle Inequality Theorem 1
   C. Triangle Inequality Theorem 2
   D. Triangle Inequality Theorem 3

2. Which of the following angles is an exterior angle of ΔRPY?



   A. ∠2 B. ∠3 C. ∠4 D. ∠5

3. Study the figure in no. 2. Notice that m∠5 > m∠3 and m∠5 > m∠1. Which theorem justifies these observation?

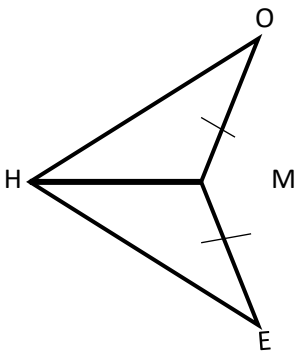
   A. Exterior Angle Inequality Theorem
   B. Triangle Inequality Theorem 1 (Ss◊Aa)
   C. Triangle Inequality Theorem 2 (Aa◊Ss)
   D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

4. Chris forms triangles by bending a 16-inch wire. Which of the following sets of wire lengths successfully form a triangle?
   I. 4 in, 5 in, 6 in
   II. 4 in, 5 in, 7 in
   III. 4 in, 4 in, 8 in
   IV. 3 in, 4 in, 9 in

   A. I, II B. III, IV C. II, IV D. I, III

5. From the inequalities in the triangles shown, Jarold concluded that ∠OHM > ∠EHM. Which theorem on inequalities in triangle justifies his answer?

A. Converse of Hinge Theorem
B. Hinge Theorem
C. Triangle Inequality Theorem 1 (Ss◊Aa)
D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

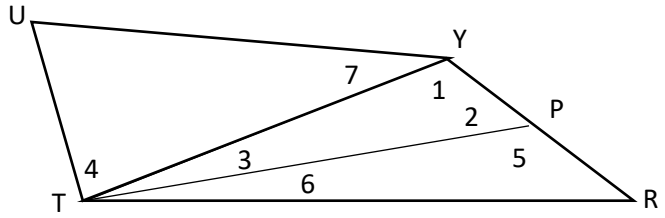


6. In ΔGUD, GU = DU and GD > DU. Which of the following statements may NOT be true?
   A. GU < GD − DU B. m∠U > m∠D C. m∠U > m∠G D. m∠D = m∠G

7. In ΔTRY, if TR = 3, RY = 5, and TY = 2, which statement is true?
   A. m∠R > m∠Y B. m∠Y > m∠T C. m∠R > m∠T D. m∠T > m∠R

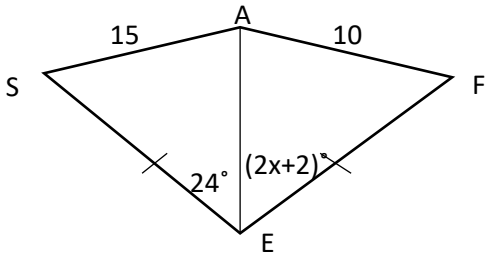
8. Which theorem of inequality in triangles helps you in determining who is farther from the rendezvous?
   A. Converse of Hinge Theorem
   B. Hinge Theorem
   C. Triangle Inequality Theorem 1 (Ss◊Aa)
   D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

9. Which of the following angles is an exterior angle of ΔTPR?



   A. ∠2 B. ∠3 C. ∠4 D. ∠5

10. What are the possible values for x in the figure?



   A. x < 11 B. x ≤ 11 C. x > 11 D. x ≥ 11

11. What considerations should you emphasize in your design presentation so that the balikbayan would award you the contract to build the houses?
    I. Kinds of materials to use considering the climate in the area
    II. Height of floor-to-ceiling corner rooms and its occupants
    III. Extra budget needed for top-of-the-line furnishings
    IV. Architectural design that matches the available funds
    V. Length of time it takes to finish the project
    A. I, II, and IV B. I, II, IV, and V C. I, IV, and V D. I, II, III, IV, V

12. Why is it not practical to design a house using A-Frame style in the Philippines?
    I. A roof also serving as wall contributes to more heat in the house.
    II. Placement of the windows and doors requires careful thinking.
    III. Some rooms of the house would have unsafe low ceiling.
    IV. An A-Frame design is an unusually artful design.
    A. I and III B. I, II, and III C. II and IV D. I, II, III, IV

13. If the owner would like the same height for both houses, which of the following is true?
    I. Roof costs for the larger lot is higher than that of the smaller lot.
    II. The roof of the smaller house is steeper than the larger house.
    A. I only B. neither I nor II C. II only D. I and II

14. Which theorem justifies the then-statement in no. 11?
    A. Exterior Angle Inequality Theorem
    B. Triangle Inequality Theorem 1 (Ss◊Aa)
    C. Triangle Inequality Theorem 2 (Aa◊Ss)
    D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

15. The chairs of a swing ride are farthest from the base of the swing tower when the swing ride is at full speed. What conclusion can you make about the angles of the swings at different speeds?
    A. The angles of the swings remain constant whether the speed is low or full.
    B. The angles of the swings are smaller at full speed than at low speed.
    C. The angles of the swings are larger at full speed than at low speed.
    D. The angles of the swings are larger at low speed than at full speed.

Great job! You are done with this module.

References

A. Books

Abuzo, E. P., et.al. (2013). Mathematics – Grade 8 Learner's Module First Edition, 2013. Published by the Department of Education. Printed in the Philippines by Book Media Press, Inc.

Oronce, O. A. and Mendoza M. O. (2018). Exploring Math 8. Metro Manila. Rex Books Store

B. Online Resources

https://www.onlinemathlearning.com/triangle-inequality.html

https://www.cliffsnotes.com/study-guides/geometry/triangles/the-triangle-inequality-theorem

https://www.mathsisfun.com/geometry/triangle-inequality-theorem.html

https://mathbitsnotebook.com/Geometry/SegmentsAnglesTriangles/SATHinge.html