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MATHEMATICS

Quarter 4 – Week 8 - Module 6
Solving Problems Involving Oblique Triangles

AIRs - LM

Mathematics 9
Quarter 2 – Week 8 Module 6: Solving Problems Involving Oblique Triangles
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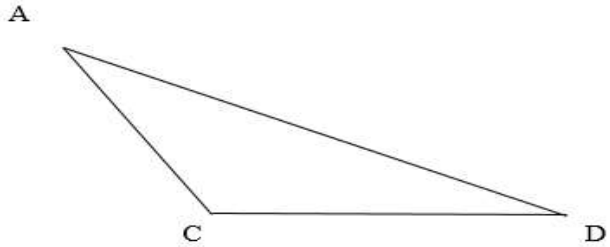
Jumpstart

Activity 1: Know Me First!

Directions: Label the parts of the oblique triangle ACD. Such that $A = 46.57°$,

$D = 28.96°$, $C = 104.47°$, $a = 15$, $c = 20$, and $d = 10$



Activity 2: Complete Me!

Direction: Fill in the missing terms in the given law.

A. Law of Sines

In any ΔABC,

1. $\frac{\sin A}{a} = \frac{}{b} = \frac{\sin C}{c}$

B. Law of Cosines

In any ΔABC,

2. $a^2 = b^2 + c^2 - 2bc\,(\cos \_\_)$
3. $b^2 = \_\_ + c^2 - 2ac\,(\cos B)$
4. $c^2 = a^2 + b^2 - 2\_\_\,(\cos C)$

How did you find the activity? Were you able to recall the laws of Sines and Cosines? You will now apply these 2 laws in solving problems on oblique triangles

Discover

Oblique Triangle ABC can be solved using three tools: the sum of angles, the Law of Sines and the Law Cosines.

Consider the illustrative example #1 below

From DMTCSNHS, Frexyl walks 15 m North to reach the basketball court of San Isidro and 4 m East to be at home which makes an angle measuring 138° with the school. How far is Frexyl's house from the school?

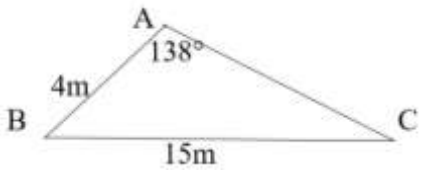
In solving word problem, follow the IDEA method.

Step 1. ***I***dentify the given
Step 2. ***D***esign a solution
Step 3. ***E***valuate the solution
Step 4. ***A***ssess or check the answer

Here is how you are going to solve the problem.

Step 1. ***I***dentify the given

To solve the given problem, sketch the triangle and label the parts with the given measures.



FYI: The longest side is opposite the largest angle, and the shortest side is opposite the smallest angle

Given: $a = 15$ $\angle A = 138°$
$c = 4$
Unknown: $b,\ B\ and\ C$

Step 2. ***D***esign a solution

Based from the illustration, the measures of 2 sides and non-included angle of the triangle is given. This illustrates SSA case. The law of Sines may be applied to solve for the unknown in the problem.

Law of Sines
$In\ \Delta ABC,\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Step 3. ***E***valuate the solution

Since the problem is asking for the distance of Frexyl's house from the school which is represented by *b*, we can use the formula $\frac{\sin B}{b} = \frac{\sin C}{c}$. But the measure of angle C is still unknown. Therefore, solve for the measure of angle C first. We can use the formula $\frac{\sin A}{a} = \frac{\sin C}{c}$ since measures of side a and $\angle$A are given.

$\sin 138° = 0.6691$

$\frac{\sin 138°}{15} = \frac{\sin C}{4}$ *find the value of sin 138°*

$\sin C = \frac{4(sin138°)}{15}$ *express the unknown in terms of the other*

$\sin C = \frac{4(0.6691)}{15}$ *substitute the value of sin 138°*

$\sin C = 0.1784$ *simplify*

$C = \sin^{-1} 0.1784$ *solve for C, take inverse sine* 0.1784 *since measure of the*

$\angle C = 10.16°$ *angle is the unknown*

Now that you solved for the measure of angle C and measure of angle A is 138°, you may use the sum of angles to solve for the measure of angle B.

$m\angle A = 138°$ $m\angle C = 10.16°$

$\angle A + \angle B + \angle C = 180°$

$138° + \angle B + 10.16° = 180°$

$\angle B = 180° - 148.16°$

$\angle B = 31.84°$

You are now ready to solve for *b*. Use the formula $\frac{\sin B}{b} = \frac{\sin C}{c}$ since measures of side c, angle B and angle C are already determined.

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 31.84°}{b} = \frac{\sin 10.16°}{4}$ *find the value of* $\sin 31.84°$ *and* $\sin 10.16°$

$b = \frac{4(sin31.84°)}{\sin 10.16°}$ *express the unknown in terms of the other*

$b = \frac{4(sin31.84°)}{\sin 10.16°}$ *substitute the value of sin 138°*

$sin\ 31.84 = 0.5275$ *and* $\sin 10.16° = 0.1764$

$b = \frac{4(0.5275)}{0.1764}$ *simplify*

$b = 11.96 \approx 12$ *solve for b*

Therefore, Frexyl's house is approximately 12m away from the school.

Step 4. ***A***ssess or Check the answer.

$a = 15$ $\angle A = 138°$

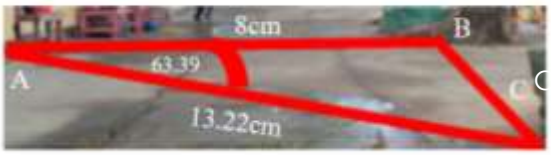
$b = 12$ $\angle B = 31.84°$

The longest side is opposite the largest angle, and the shortest side is opposite the smallest angle

$c = 4$ $\angle C = 10.16°$

Illustrative Example #2

Find the distance between the two trees such that the trees are 8 cm and 13.22 cm away from the signage iron stand. The angle formed by these two trees with the signage stand is 63.39°.



Step 1. Sketch the triangle and label the parts. Identify the given.

Given: $b = 13.22$ $\angle A = 63.39°$

$c = 8$

Unknown: a , the distance between the two trees

Step 2. Based from the illustration, the measures of 2 sides and an included angle of the triangle are given. This illustrates an SAS case. The law of Cosines may be applied to solve for the unknown in the problem.

Law of Cosines

In any ΔABC,

$a^2 = b^2 + c^2 - 2bc\ (\cos A)$

$b^2 = a^2 + c^2 - 2ac\ (\cos B)$

$c^2 = a^2 + b^2 - 2ab\ (\cos C$

Step 3. The distance between the two trees, represented by ***a***, is the unknown. Since the measures of the 2 sides (b *and* c) and the included angle A are given, use the equation $a^2 = b^2 + c^2 - 2bc\ (\cos A)$.

$a^2 = (13.22)^2 + 8^2 - 2(13.22)(8)\ (0.4479)$

$\cos 63.39 = 0.4479$

$a^2 = 174.77 + 64 - (211.52)\ (0.4479)$

$a^2 = 238.77 - 94.74$

$a^2 = 144.03$

$a = \sqrt{144.03}$

$a = 12$

Therefore, the distance between the two trees is 12cm.

Step 4. Assess and check the answers

$b = 13.22$ $\angle A = 63.39°$

$c = 8$

$a = 12$

Explore

Here is an enrichment activity for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3. Three Sided

Directions. Read and analyze the problem carefully. Answer correctly what is being asked in the problem.

Two barangay roads intersect at 100°. Jan's pet house is 8m from the intersection. Lester's pet shop is on the other road and is 12m from the intersection. How far is Jan's pet house from that of Lester's pet shop? Express answer in two decimal places.

Deepen

Now that you have understood the concept and ideas of this topic, let's now deepen your understanding by moving on to the next activity.

Activity 4. Love Tri-Angle

Directions. Read and analyze the problem carefully. Answer correctly what is being asked in the problem.

A triangular piece of land measures 35m, 40m and 50m. What are the measures of the angles between the sides? Express answers to the nearest degree.

Great Job. Having answered all the activities, I am pretty sure that you can answer correctly all the items in the Post-test. Good luck!