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Mathematics

Quarter 2 - Module 7:
Solving Equations Involving
Radical Expressions

AIRs - LM

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Quarter 2 - Module 7: Solving Equations Involving Radical Expressions
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Solving Equations Involving
Radical Expressions

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

This module will help you understand how to solve equations involving radical expressions. Apply your knowledge of simplifying radicals and the fundamental operations on radical expressions to solving radical equations. Why is there a need to know how to solve radical equations? Are these learnings essential in life?

After going through this module, you are expected to

> **Learning Competency**
> Solves equations involving radical expressions. **(M9AL-IIj-1)**
>
> > **Subtasks**
> > 1. Define radical equation
> > 2. Transform and simplify radical expressions.

> *Before going on, answer the pre-assessment on a separate answer sheet. This will gauge your prior knowledge about the content you are about to encounter on this module.*

Pre-Assessment

Directions: Read and analyze carefully the following questions. Choose the letter of the correct answer and write it on a separate answer sheet.

1. Which of the following is a radical equation?
   A. $\sqrt{m} = 2$ B. $\sqrt{121y}$ C. $2x\sqrt{5} + 3x\sqrt{3} + 10$ D. $\sqrt{12} + \sqrt{30} + 7$

2. Which of the following is true about the square of a number?
   A. It is always positive.
   B. It is always negative.
   C. It has positive and negative values.
   D. Its sign depends whether odd or even exponent.

3. Which of the following is True?
   A. If $a = b$ then $a^2 = b^2$.
   B. If $a = \sqrt{b}$ then $a^2 = b^2$.
   C. If $\sqrt{9} = 3$ then $9 = \sqrt{3}$
   D. If $\sqrt{y} = 5$ then $y = \sqrt{25}$

4. What is x in $\sqrt{x} = 9$?
   A. 2   B. 3   C. 18   D. 81

5. What is y in $\sqrt{2y} = 10$?
   A. 12   B. 50   C. 100   D. 120

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6. What is x in $\sqrt{x} - 2 = 10$?

   A. 144   B. 121   C. 100   D. 81

7. Which is the correct value of $\sqrt{5a + 1} + 2 = 6$?

   A. 12   B. 9   C. 6   D. 3

8. Which of the following is the solution of $\sqrt{x + 3} = 8$?

   A. 61   B. 64   C. 90   D. 121

9. Which is the correct value of $\sqrt[3]{x + 1} - 2 = 3$?

   A. 122   B. 123   C. 124   D. 125

10. What is the value of x in the equation $\sqrt{x + 6} = \sqrt{2x - 3}$?

   A. 4   B. 9   C. 16   D. 25

11. Which of the following is the value of x in $\sqrt{2x} - 3 = 5$?

   A. 25   B. 32   C. 64   D. 81

12. Which of the following radical equations will have $x = 4$ as the solution?

   A. $\sqrt{x} + \sqrt{x - 3} = 3$

   B. $\sqrt{x - 3} = 3$

   C. $\sqrt{x - 3} = \sqrt{x} - 3$

   D. $\sqrt{x + 3} + \sqrt{x} = 3$

13. If $\sqrt{y + 2} = 2$, then what is $(y + 2)^2$ equals to?

   A. 2   B. 4   C. 8   D. 16

14. What is the value of y in $\sqrt{y - 1} = \sqrt{2y - 11}$?

   A. 3   B. 7   C. 10   D. 12

15. What is the solution of $5\sqrt{m - 1} = 40$?

   A. 65   B. 199   C. 200   D. 1625

Jumpstart

Activity 1: Simply Me!

Directions: Match the following radicals in column A with their exponential form in column B and the simplified form in column C.

Column A	Column B	Column C
1. $\sqrt{36x^2}$	A. $(9)^{\frac{1}{2}}(y^2)^{\frac{1}{2}} - (16)^{\frac{1}{4}}$	L. $3y-2$
2. $\sqrt[3]{27y^3}$	B. $\dfrac{(4x^2)^{\frac{1}{2}}}{(16y^2)^{\frac{1}{2}}}$	M. $y-3$
3. $\sqrt[4]{256}$	C. $\{(y-3)^4\}^{\frac{1}{4}}$	N. $y-9$
4. $\sqrt{(x-1)^2}$	D. $\{(y-9)^3\}^{\frac{1}{3}}$	O. $\dfrac{x}{2y}$
5. $\sqrt[3]{m^3}$	E. $\{(x-2)^2\}^{\frac{1}{2}}$	P. $x-1$
6. $\sqrt{(x-2)^2}$	F. $(m^3)^{\frac{1}{3}}$	Q. $6x$
7. $\sqrt[3]{(y-9)^3}$	G. $\{(x-1)^2\}^{\frac{1}{2}}$	R. $3y$
8. $\sqrt[4]{(y-3)^4}$	H. $(256)^{\frac{1}{4}}$	S. m
9. $\sqrt{\dfrac{4x^2}{16y^2}}$	I. $(27)^{\frac{1}{3}}(y^3)^{\frac{1}{3}}$	T. $x-2$
10. $\sqrt{9y^2}$ - $\sqrt[4]{16}$	J. $(36)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}$	U. 4

Discover

A **radical equation** is an equation containing a variable in the radicand.

Examples of radical equations:

$$\sqrt{x} = 4$$

$$\sqrt{x+8} = 5$$

$$\sqrt{x-1} = x - 7$$

In solving radical equations, note that if two numbers are equal then their squares are also equal. In symbols; if $a = b$ then $a^2 = b^2.$

> *Power Rule:*
> *If both sides of an equation are raised to the same power, all solutions of the original equation are also solutions of the new equation.*

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For example, if $\sqrt{16} = 4$ are equal, then $(\sqrt{16})^2 = (4)^2$ are equal.

To simplify further: $(\sqrt{16})^2$ is expressed as $\left\{(16)^{\frac{1}{2}}\right\}^2$ by applying the law of radical. So that, if $\left\{(16)^{\frac{1}{2}}\right\}^2 = 16$ then $16 = 16$. The key to solving radical equations is to raise both sides of the equation to the same power.

Solving radical equations means finding the value/s of the variable that would make the radical equation true.

Steps in solving a radical equation:
1. Arrange the terms of the equation such that the term with radical is isolated on the left side of the equation.
2. Square/raise the nth power of both sides of the radical equation.
3. If a radical still remains, repeat steps 1 to 2
4. Combine like terms
5. Solve for the variable
6. Check apparent solution in the original equation.

Note that if the solution would **not** make the equation true, then the solution is an **extraneous solution.**

Example 1: Solve $\sqrt{x} = 4$

$\sqrt{x} = 4$	Arrange the terms of the equation such that the term with radical is isolated on the left side of the equation.
$\left(x^{\frac{1}{2}}\right)^2 = (4)^2$	Square both sides of the radical equation
	Combine like terms
$x = 16$ **The solution is 16.**	Solve for the variable
If $x = 16$ $\sqrt{x} = 4$ $\sqrt{16} \overset{?}{=} 4$ $16 \overset{\sim}{=} 16$	Check apparent solution in the original equation.

Example 2: Solve $\sqrt{x+8} = 5$

$$\left\{(x+8)^{\frac{1}{2}}\right\}^2 = (5)^2 \qquad \text{square both sides}$$
$$(x+8) = 25$$
$$x = 25 - 8 \qquad \text{combine like terms}$$
$$x = 17 \qquad \text{solve for the variable}$$

The solution is 17.

Check: $\sqrt{x+8} = 5$ substitute the value of x in the original

$\sqrt{17+8} = 5$ equation

$\sqrt{25} \overset{?}{=} 5$

$5 = 5$

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Example 3: Solve $\sqrt{x-1} = x - 7$

$$\left\{(x-1)^{\frac{1}{2}}\right\}^2 = (x-7)^2 \qquad \text{square both sides}$$

$$x - 1 = x^2 - 14x + 49$$

$$x^2 - 14x - x + 49 + 1 = 0 \qquad \text{combine like terms}$$

$$x^2 - 15x + 50 = 0 \qquad \text{solve for the variable by factoring}$$

$$(x-10)(x-5) = 0$$

$$x = 10 \ and \ x = 5 \qquad \textbf{The solutions are 10 and 5.}$$

Check:

If x = 10 $\qquad \sqrt{x-1} = x - 7 \qquad$ substitute the value of x in the

$$\sqrt{10-1} = 10 - 7 \qquad \text{original equation}$$

$$\sqrt{9} \overset{?}{=} 3$$

$$3 = 3 \qquad x = 10 \ \textit{is the only solution}$$

If x = 5 $\qquad \sqrt{x-1} = x - 7 \qquad$ substitute the value of x in the

$$\sqrt{5-1} = 5 - 7 \qquad \text{original equation}$$

$$\sqrt{4} \overset{?}{=} -2$$

$$2 \neq -2 \qquad \text{x} = 5 \ \textit{is an extraneous solution.}$$

 Explore

Activity 2: Find my Solutions!

Directions: Solve the following radical equations on a separate sheet of paper.

1. $\sqrt{y} = 10$
2. $\sqrt{2x} = 10$
3. $\sqrt{y-4} = 1$
4. $\sqrt{2x-1} = 5$
5. $\sqrt[4]{2m} = 4$
6. $\sqrt{6y} - 4 = 2$
7. $\sqrt{y+3} + 5 = 12$
8. $\sqrt{6y+5} = \sqrt{2y+10}$
9. $(3\sqrt{3})^2 = \sqrt{x}$
10. $2\sqrt[3]{y+5} = 4\sqrt[3]{2y+10}$

Deepen

Activity 3: Justify your Actions!

Directions: Solve the radical equations. Write your complete solution and indicate the property, definition, or theorem used in your solution.

Radical Equations	Solution	Reason
1. $5\sqrt{5x+2} = 10$		
2. $\sqrt[4]{n+2} = 3$		
3. $\sqrt[3]{3a+9} = \sqrt[3]{6a+15}$		
4. $\sqrt[4]{2m+10} = 4$		

Additional Activity

Directions: Solve the following radical equations and check. Determine whether the obtained value is really a solution or extraneous solution.

1. $x - 4 = \sqrt{2x}$
2. $\sqrt{y-2} = -3 + \sqrt{4y+1}$

Gauge

Post – Assessment

Directions: Read each item carefully and pick out your answer from the choices given. Write the letter of your chosen answer on your answer sheet.

1. Which of the following is a radical equation?

   A. $\sqrt{x+2} = 4$ 　　　B. $\sqrt{m+n}$ 　　　C. $\sqrt{2y} + 3y\sqrt{4}$ 　　　D. $\sqrt[3]{8} + \sqrt[3]{64y} + \sqrt[3]{8}$

2. Which of the following is True?

   A. If $a = b$ then $a^2 = b^2$.　　　　　B. If $a = \sqrt{b}$ then $a^2 = b^2$.

   C. If $\sqrt{81} = 9$ then $(\sqrt{81})^2 = (\sqrt{9})^2$　　　D. If $\sqrt{y} = 5$ then $(\sqrt{y})^2 = \sqrt{25}$

3. What is x in $\sqrt{2x} + 4 = 12$?

   A. 2 　　　　　B. 4 　　　　　C. 18 　　　　　D. 32

4. What is x in $\sqrt{2x-5} = 4$?

   A. 11 　　　　　B. 16 　　　　　C. 20 　　　　　D. 121

5. What is x in $\sqrt[3]{2x+1} = \sqrt[3]{x+8}$?

   A. 6 　　　　　B. 7 　　　　　C. 63 　　　　　D. 65

6. Which is the correct value of $4\sqrt{x-1} = \sqrt{x+4}$?

   A. $\frac{1}{4}$ 　　　　　B. $\frac{2}{3}$ 　　　　　C. $\frac{10}{15}$ 　　　　　D. $\frac{20}{15}$

7. Which of the following is the solution of $\sqrt{x-7} = \sqrt{2}$?

   A. 6 　　　　　B. 7 　　　　　C. 9 　　　　　D. 19

8. Which is the correct value of x in $2\sqrt{3x+2} - 10 = 0$?

   A. $\frac{1}{3}$ 　　　　　B. $\frac{2}{3}$ 　　　　　C. $\frac{13}{3}$ 　　　　　D. $\frac{23}{3}$

9. What is the value of x in the equation $\sqrt[3]{2x-1} = 3$?

   A. 4 　　　　　B. 9 　　　　　C. 14 　　　　　D. 24

10. Which of the following radical equations will have $x = \frac{19}{15}$ as the solution?

    A. $\sqrt{x} = \sqrt{x+3}$ 　　　　　B. $\sqrt{x-1} = \sqrt{x+3}$

    C. $\sqrt{x-1} = \sqrt{x}$ 　　　　　D. $4\sqrt{x-1} = \sqrt{x+3}$

11. Which of the following radical equations will have $m = \frac{21}{4}$ as the solution?

    A. $\sqrt{m^2-5} = 0$ 　　　　　B. $\sqrt{m^2-5} + m = 0$

    C. $\sqrt{m^2} + m - 10 = 0$ 　　　　　D. $\sqrt{m^2-5} + m - 10 = 0$

12. If $\sqrt{b+2} = 4$, what is $(b+2)^2$ equals to?

    A. 16 　　　　　B. 18 　　　　　C. 20 　　　　　D. 256

13. If $x = z + \sqrt{\frac{5}{y}}$, then what is y equals to?

    A. $\frac{5}{x-z}$ 　　　　　B. $\frac{5}{z-x}$ 　　　　　C. $\frac{5}{(z-x)^2}$ 　　　　　D. $\frac{5}{(x-z)^2}$

14. If $x = 3\sqrt{3} + 2$, what is the value of $x^2 + 3x - 2$?

    A. $3 + 21\sqrt{3}$ 　　　B. $5 + 21\sqrt{3}$ 　　　C. $25 + 21\sqrt{3}$ 　　　D. $35 + 21\sqrt{3}$

15. What is the value of y in $\sqrt{y+9} + \sqrt{y-7} = 8$?

    A. 16 　　　　　B. 49 　　　　　C. 64 　　　　　D. 81

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References

Books

      Learners' Manual, K to 12 Grade 9 Mathematics (First Quarter)

      Teacher's Guide, K to 12 Grade 9 Mathematics (First Quarter)

      Mathematics Grade 9 Learner's Module, First Edition 2014, Reprint 2017

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Website:

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