7

Mathematics

Quarter 3: Week 5 - Module 5
Deriving the Relationship of Interior and Exterior of a Convex Polygon

AIRs - LM

Mathematics Grade 7

Quarter 3: Week 5 - Module 5: **Deriving the Relationship of Interior and**

Exterior of a Convex Polygon

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Region I

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Target

You were acquainted to polygon early on. You have probably been taught basic polygons like the triangle, square, rectangle and parallelogram. You began to familiarize yourself with these polygons since everywhere you direct your eyes on, you are sure to see objects that take these polygons.

Because you see these objects quite often, there is no doubt that knowledge of these different polygons and the segments and angles that make them up is basic and essential. More complex applications such as architecture and urban planning are founded on concepts of basic polygons.

In this module, you shall extend your knowledge of polygons as you study and solve problems concerning their angles. You shall also review the relationships of angles and solve problems involving the angles of convex polygon

After going through this module, you are expected to:

Learning Competency:

derives inductively the relationship of exterior and interior angles of a convex polygon **(M7GEIIIf-1)**

Subtasks:

1) Determine linear pair of angles;
2) Solve the supplement of an angle;
3) Determine the sum of the interior and exterior angles of a convex polygon;
4) Determine the number of sides of a convex polygon; and
5) Solve interior and exterior angles of a convex polygon

> Before we start the lesson, find out how much you already know about this module by answering the pre – assessment on the subsequent page.

Jumpstart

This activity will enable you to assess your prior knowledge in the relationship of interior and exterior angles of a convex polygon.

PRE – ASSESSMENT

Directions: Read and answer each statement below carefully. Write your answers in a separate sheet of paper.

1. What is the relationship of two adjacent angles with a sum of 180º?

   A. Congruent   B. Linear Pair

   C. Supplementary   D. Vertical

2. What is the relationship of any two angles with the same measurement?

   A. Complementary   B. Congruent

   C. Linear Pair   D. Supplementary

3. What is the relationship of adjacent interior and exterior angles of any convex polygon?

   A. Complementary   B. Congruent

   C. Linear Pair   D. Vertical

4. What convex polygon has congruent sides and congruent angles?

   A. Equiangular   B. Equilateral

   C. Regular   D. Similar

5. What is the sum of exterior angles of any convex polygon?

   A. 90º   B. 180º   C. 360º   D. 540º

   For items 6 to 10, refer to the following representations:

   S = Sum of interior angle   *n= number of sides*

   a = interior angle   *x = exterior angle*

6. What is the formula in getting the sum of interior angles of any convex polygon?

   A. $S = \frac{360}{n}$ B. $S = 180 - x$ C. S = 180 - a   D. $S = 180(n - 2)$

7. What is the formula in getting the measurement of an interior angle?

    A. $a = \dfrac{360}{n}$ B. $a = \dfrac{180}{x}$ C. $a = 180 - x$ D. $a = 180(n - 2)$

8. What is the formula in getting the measurement of an exterior angle?

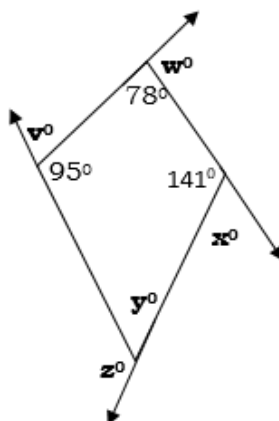   A. $x = \dfrac{360}{a}$ B. $x = 180n$ C. $x = 180 - a$ D. $x = 180(n - 2)$

9. What is the formula in getting each measurement of the exterior angle of any regular polygon?

    A. $x = \dfrac{360}{n}$ B. $x = 180n$ C. $x = 180 - a$ D. $x = 180(n - 2)$

10. What is the formula in getting the number of sides of any convex polygon?

    A. $n = 180 - a$ B. $n = \dfrac{360}{x}$ C. $n = S - 360$ D. $n = \dfrac{S}{180} + 2$

For items 11 – 15, refer to the figure below. (Figure is not drawn to scale)

w^0

78^0

v^0

95^0 141^0

x^0

y^0

z^0

Based from the figure, what is the value of:

11. v = _____ ?   A. 39   B. 85   C. 92   D. 141

12. w = _____?   A. 39   B. 46   C. 102   D. 134

13. x = _____ ?   A. 39   B. 85   C. 92   D. 141

14. y = _____?   A. 46   B. 92   C. 102   D. 134

15. z = _____?   A. 46   B. 85   C. 102   D. 134

Jumpstart

This activity will enable you to assess your prior knowledge in the relationship of angles and sum of interior angles of a convex polygon.

Activity 1: Get Straight!
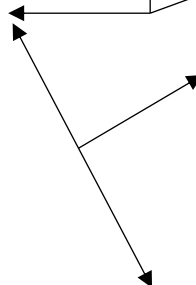**Direction:** Determine which of the following illustrations show linear pair of angles.

A.

B.

C.
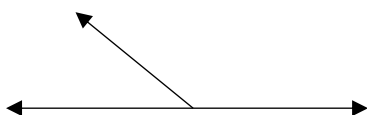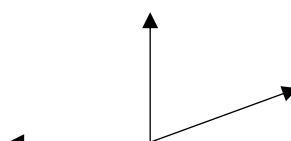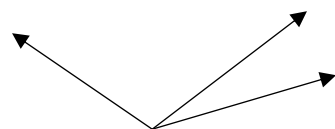
D.

Activity 2: What Supplements Me?

Direction: Determine the supplement of the following measures of angles.

1) 29° → _____   3) 85° → _____   5) 133° → _____
2) 73° → _____   4) 107° → _____

Activity 3: Sum of the Interior Angles of a Convex Polygon

Direction: Complete the table by following the pattern. Write the formula in getting the sum of interior angles of a convex polygon in the last row.

| Name of Polygon	Number of Sides	Sum of the Interior Angles
Triangle	3	1(180) = **180**
Quadrilateral	4	2 (180) = **360**
Pentagon	5	3(180) = **540**
Hexagon	6	
Heptagon	7	
Octagon	8	
Nonagon	9	
Decagon	10	
n – gon	n	

Discover

Discovering the Relationship of Interior and Exterior Angles

1. Draw a 4-sided polygon and name the vertices like the one below.
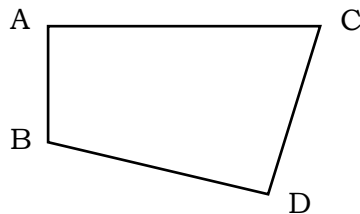
2. From one vertex, extend the point going outside to form a segment and name the angles formed inside and outside the polygon using numbers like the one below.
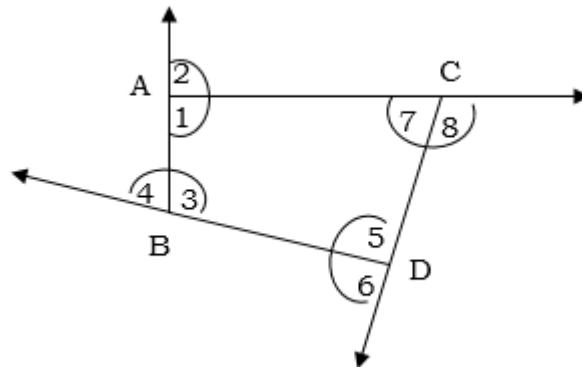
3. Using the figure in step 2, we name all the exterior and interior angles.

Interior Angles				Exterior Angles			
∠1	∠3	∠5	∠7	∠2	∠4	∠6	∠8

4. Using the figure in step 2, what is the relationship of the following angles: ∠1 & ∠2 ? ∠3 & ∠4 ? ∠5 & ∠6 ? ∠7 & ∠8 ?

5. Applying your answer in no. 4, give the measurements of the angles if:

Interior	Exterior
∠1 = 90	∠2 = _____
∠3 = 100	∠4 = _____
∠5 = _____	∠6 = 89
∠7 = _____	∠8 = 101

6. To check if your answer is correct, get the sum of quadrilateral using the formula you get in Activity 3: Sum of the Interior Angles of a Convex Polygon, compare it to the sum of interior angles of the polygon ABDC. Did you get the same sum?

7. What is the sum of the exterior angles?

Relationship of Interior and Exterior angle of a Convex Polygon

The adjacent interior and exterior angles of a convex polygon are linear pair of angles. We can solve angles of convex polygons using the following formula:

Let: a = interior angle   x = exterior angle

1) If the interior angle (*a*) is given, the **exterior angle** (*x*) adjacent to it can be solved using:
$$x = 180 - a$$

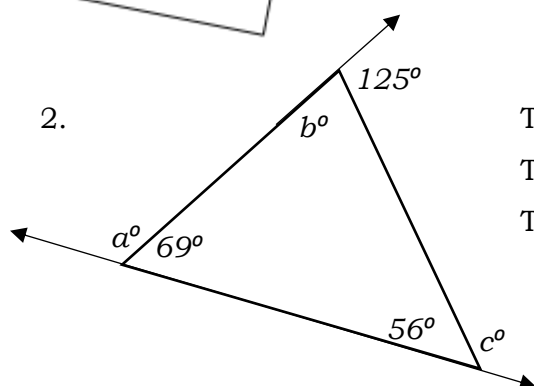2) If the exterior angle (*x*) is given, the **interior angle** (*a*) adjacent to it can be solved using:
$$a = 180 - x$$

Examples: (figures are not drawn to scale)
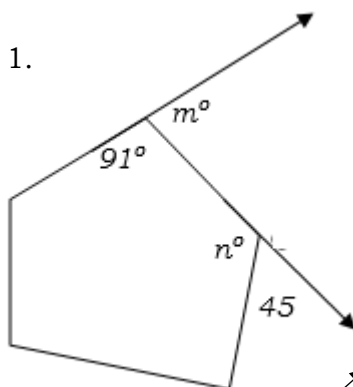
1.

To get ∠m: ∠m = 180 – 91 = **89⁰**
To get ∠n: ∠n = 180 – 45 = **135⁰**

2.

To get ∠a: ∠a = 180 – 69 = **111⁰**
To get ∠b: ∠b = 180 – 125 = **55⁰**
To get ∠c: ∠c = 180 – 56 = **124⁰**

> ### Sum of the Interior and Exterior angle of a Convex Polygon
>
> Interior and exterior angles can also be solved with the help of the sum of interior and exterior angles of it.
>
> *Let:*   *S = Sum of interior angle*   *n= number of sides*
>
>   *a = interior angle*   *x = exterior angle*
>
> 1) If the number of sides (*n*) of a convex polygon is given, we can solve the sum of interior angles using the formula:
>
> $$S = 180 (n - 2)$$
>
> 2) The sum of the exterior angles of any convex polygon is always **360⁰**.

Examples:

1) The polygon is a triangle. Applying the formula of the sum of the interior angles, we will get:

S = 180 (n – 2)

= 180 (3 – 2)

= 180 (1)

= 180

We subtract the given measurements of the angles to the sum to get the remaining angle.
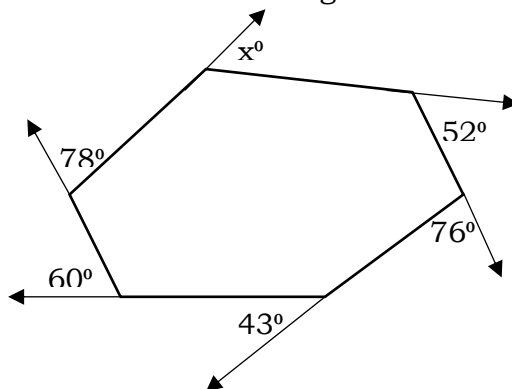
a = 180 – (43+97)

   = 180 – 140

   = 40
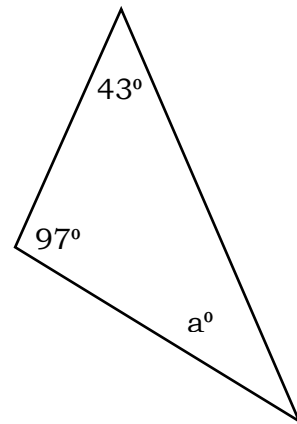
Therefore, ***a = 40⁰***

2) The polygon is a hexagon. Knowing that the sum of the exterior angles of any convex polygon is 360⁰, all we have to do is to add the angles and subtract it from 360.

x = 360 – (78+60+43+76+52)

= 360 – 309
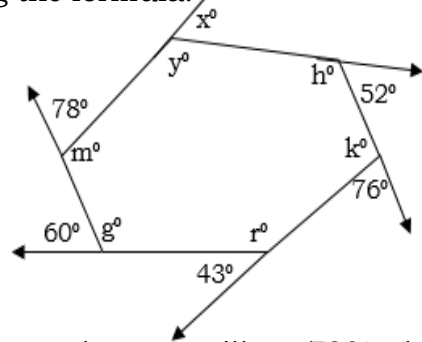
= 51

Therefore, ***x = 51⁰***

3) We already know, how to get the value of x (refer to example 2). Now, we solve the interior angles of the polygon by applying the formula:

$a = 180 - x$

$\angle m = 180 - 78 = \mathbf{102}$ $\angle k = 180 - 76 = \mathbf{104}$

$\angle g = 180 - 60 = \mathbf{120}$ $\angle h = 180 - 52 = \mathbf{128}$

$\angle r = 180 - 43 = \mathbf{137}$ $\angle y = 180 - 51 = \mathbf{129}$

Notice that when you add all measures of the interior angles, you will get 720° which can be solve also using the formula S = 180 (n-2).

Regular Polygon and Number of Sides of a Convex Polygon

A ***regular polygon*** is a polygon which is equilateral and equiangular. All sides and angles are congruent with each other. If the convex polygon is regular, we can solve the measurement of the each angles in the interior and exterior of the polygon using the formula:

Let: *S = Sum of interior angle*   *n= number of sides*

          a = interior angle   *x = exterior angle*

1) If the number of sides (*n*) of a regular convex polygon is given, we can solve the of measurement of each interior angle using the formula:

$$a = \frac{180(n-2)}{n}$$

2) If the number of sides (*n*) of a regular convex polygon is given, we can solve the measurement of each exterior angle using the formula: $$x = \frac{360}{n}$$

3) If any of the interior *(a)* or exterior (*x*) of a regular polygon is given, we can solve the number of sides using the formula:

$$n = \frac{360}{180-a} \quad or \quad n = \frac{360}{x}$$

4) If the sum of the interior angles (*S)* of a convex polygon is given, we can solve the number of sides using the formula:

$$n = \frac{S}{180} + 2$$

Examples:

Let's solve what is asked in each item.

1. Find the measurement of each interior angle of a regular quadrilateral.

    To solve:

    Step 1: Identify the given. It is quadrilateral so, $n = 4$.

    Step 2: Know what is ask. Interior angle (a)

    Step 3: Determine what formula is suited to find what is asked.

    $$a = \frac{180(n-2)}{n}$$

    Step 4: Solve.

    $$a = \frac{180(n-2)}{n} = \frac{180(4-2)}{4} = \frac{180(2)}{4} = \frac{360}{4} = 90$$

    Therefore the measurement of each angle of a regular quadrilateral is 90º.

2. Find the measurement of each exterior angle of a regular decagon.

    To solve:

    Step 1: Identify the given. It is a decagon so, $n = 10$.

    Step 2: Know what is ask. Exterior angle (x)

    Step 3: Determine what formula is suited to find what is asked.

    $$x = \frac{360}{n}$$

    Step 4: Solve.

    $$x = \frac{360}{n} = \frac{360}{10} = 36$$

Therefore the measurement of each exterior angle of a regular decagon is 36º.

3. Find the number of sides of a regular polygon which has an interior angle measures 156º.

    To solve:

    Step 1: Identify the given. Interior angle, $a = 156$.

    Step 2: Know what is ask. Number of sides (n)

    Step 3: Determine what formula is suited to find what is asked.

    $$n = \frac{360}{180 - a}$$

    Step 4: Solve.

    $$n = \frac{360}{180 - a} = \frac{360}{180 - 156} = \frac{360}{24} = 15$$

    Therefore the regular polygon is 15 - sided.

4. Find the number of sides of a regular polygon which has an exterior angle measures 60º.

　　To solve:

　　Step 1: Identify the given. Exterior angle, $x = 60$.

　　Step 2: Know what is ask. Number of sides (n)

　　Step 3: Determine what formula is suited to find what is asked.

$$n = \frac{360}{x}$$

　　Step 4: Solve.

$$n = \frac{360}{x} = \frac{360}{60} = 6$$

　　Therefore the regular polygon is 6 – sided, which is a hexagon.

5. Find the number of sides of a regular polygon which sum of interior angles is 3060º

　　To solve:

　　Step 1: Identify the given. Sum of the interior angles, $S = 3060$.

　　Step 2: Know what is ask. Number of sides (n)

　　Step 3: Determine what formula is suited to find what is asked.

$$n = \frac{S}{180} + 2$$

　　Step 4: Solve.

$$n = \frac{S}{180} + 2 = \frac{3060}{180} + 2 = 17 + 2 = 19$$

　　Therefore the regular polygon is 19– sided.

Explore

Directions: Solve the missing angles.(Figures are not drawn to scale)

1.

55

63 x^0 y^0

$\angle x = $ _____

$\angle y = $ _____

2.

57 x^0 116^0

43 y^0 z^0

$\angle x = $ _____

$\angle y = $ _____

$\angle z = $ _____

3.

75^0

y^0 z^0

116^0

97^0

112^0 110^0 x^0

$\angle x = $ _____

$\angle y = $ _____

$\angle z = $ _____
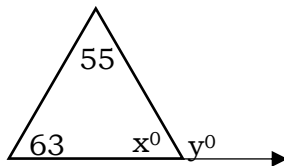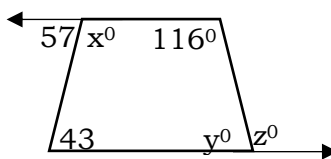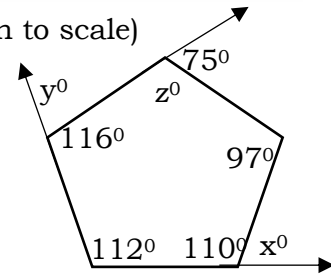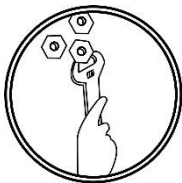
Deepen

Since you already know the important notes about the relationship of angles of a convex polygon, let us widen and go deeper to our understanding.

A. Complete the table by solving the number of sides, sum of the angles or measure of each interior of a regular convex polygon.

Number of sides (n)	Sum of the angles (S)	Measurement of each interior angle (a)
1)	**2880 ^{0}**	2)
6	3)	4)
5)	6)	**144^0**
15	7)	8)
9)	**5040**	10)

B. Complete the table by solving the measure of each exterior angle, number of sides or measure of each interior angle of a regular convex polygon.

Number of sides (n)	Measurement of each interior angle(a)	Measurement of each exterior angle(x)
8	1)	2)
3)	**60⁰**	4)
5)	6)	**90⁰**
7)	8)	**12**
9)	**144**	10)
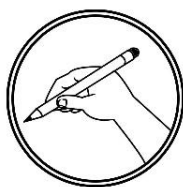
Gauge

Directions: Read and answer each statement below carefully. Write your answers in a separate sheet of paper. (Figures are not drawn to scale)

1. What is/are the characteristic/s of angles that are linear pair?

   A. Adjacent and Congruent   B. Adjacent and Supplementary

   C. Congruent   D. Vertical

2. What is the relationship of adjacent interior and exterior angles of any convex polygon?

   A. Complementary   B. Congruent

   C. Linear Pair   D. Vertical

3. Which of the following formula can be used to solve the measurement of the adjacent interior angle of an exterior angle of a convex polygon?

   A. $a = \frac{360}{x}$ B. $a = \frac{360}{n}$ C. $a = 180 - x$ D. $a = 180(n-2)$

4. What is the formula can be used to solve each interior angle of a regular polygon?

   A. $a = \frac{360}{n}$ B. $a = \frac{180\ (n-2)}{n}$ C. $a = 180 - n$ D. $a = 180(n-2)$

5. What is the formula can be used to solve the measurement of an exterior angle of any convex polygon?

   A. $x = \frac{360}{n}$ B. $x = 180n$ C. $x = 180 - a$ D. $x = 180(n-2)$

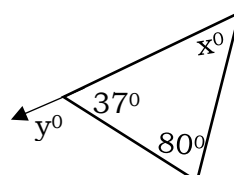6. What is the formula in solving the sum of interior angles of a convex polygon?

   A. $S = \frac{360}{n}$ B. $S = 180n$ C. $S = 180 - a$ D. $S = 180(n-2)$

7. What is the sum of exterior angles of any convex polygon?

   A. 90º   B. 180º   C. 360   D. 540

8. What is the value of x and y in the figure?

   A. x = 53, y = 117   B. x = 63, y = 143
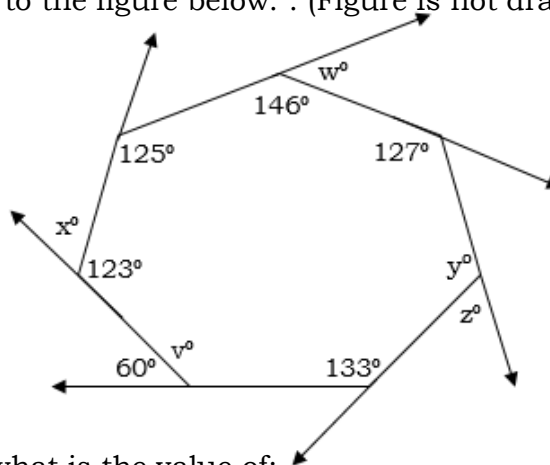
   C. x = 117, y = 53   D. x = 143, y = 63

   (figure: triangle with angles x^0, 37^0, 80^0 and exterior y^0)

9. What is measurement of each interior (a) and exterior (x) angle of a regular nonagon?

   A. a = 36, x = 144   B. a = 40, x = 140

   C. a = 140, x = 40   D. a = 144, x = 36

10. How many sides does a regular polygon has if the sum of the interior angles is 4140º

   A. 11   B. 13   C. 23   D. 25

For items 11 – 15, refer to the figure below. . (Figure is not drawn to scale)

Based from the figure, what is the value of:

11. v = _____?   A. 30   B. 60   C. 90   D. 120

12. w = _____?   A. 34   B. 44   C. 124   D. 146

13. x = _____ ?   A. 57   B. 67   C. 123   D. 143

14. y = _____?   A. 44   B. 54   C. 126   D. 136

15. z = _____?   A. 44   B. 54   C. 126   D. 136

Great job! You are done with this module.

References

Books

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