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# MATHEMATICS

**Quarter 1 – Module 6**
**Determining the Relationships Between the Hypothesis**
**and the Conclusion of an If - Then Statement and**
**Transforming a Statement into an Equivalent**
**If – Then Statement**

## AIRs - LM

**MATHEMATICS 8**
**Quarter 2 - Module 6: Determining the Relationships Between the Hypothesis and the Conclusion of an If - Then Statement and Transforming a Statement into an Equivalent If – Then Statement**
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**Conclusion of an If - Then Statement and**
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**Introductory Message**

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

# *Target*

Do you think it is possible to make a valid conclusion without even going through the process of investigation? What would you do if you were asked to make a decision that will affect many people? Many aspects in our life involve decisions. Geometry deals with logical reasoning to prove a certain statement.

This module will help you understand what an if-then statement is. It contains explicit lessons on identifying the hypothesis and the conclusion of an if – then statement, determining its truth values by giving counterexamples and recalling the different definitions, theorems, axioms, properties and concepts of Mathematics you have learned previously; and transforming a given statement into an equivalent if – then statement.  Activities were also given to process your knowledge and skills acquired, deepen and transfer your understanding of the different lessons.

After going through this module, you are expected to attain the following:

**Learning Competencies:**
- Determines the relationship between the hypothesis and the conclusion of an if-then statement. **(M8GE-IIf-1);**
- Transforms a statement into an equivalent if – then statement. **(M8GE-IIf-2)**

**Learning Objectives:**
1. Defines an if - then statement or conditional.
2. Identifies the hypothesis and the conclusion of an if – then statement.
3. Determines whether a conditional is true or false.
4. Rewrites a statement into an equivalent if – then statement.
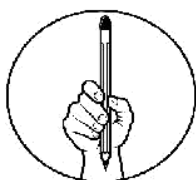
## *Pre-Test*

**Directions:** Read and answer each statement below carefully. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module. Write your answers in a separate sheet of paper.

1. Which of the following is an if-then statement?
   A. If $m\angle B$ = 90⁰ or $m\angle Q$ = 100⁰
   B. If $m\angle B$ = 90⁰, then $m\angle Q$ = 100⁰
   C. $m\angle B$ = 90⁰ when $m\angle Q$ = 100⁰
   D. $m\angle B$ = 100⁰ and $m\angle Q$ = 90⁰

2. What is the hypothesis of the statement, "If you feel unwell, then stay home."?
   A. Stay home.
   B. You feel unwell.
   C. If you feel unwell.
   D. You stay home if you feel unwell.

3. What is the conclusion of the statement in Question No. 3?
   A. Stay home.
   B. You feel unwell.
   C. If you feel unwell.
   D. You stay home if you feel unwell.

4. If <u>you practice the standard health protocols set by IATF</u>, then you can protect yourself from the spread of COVID-19 virus. What do you call the underlined portion in this statement?
   A. argument
   B. conclusion
   C. conditional
   D. hypothesis

5. If you practice the standard health protocols set by IATF, then <u>you can protect yourself from the spread of COVID-19 virus</u>. What do you call the underlined portion in this statement?
   A. argument
   B. conclusion
   C. conditional
   D. hypothesis

6. Which of the following if- then statements is TRUE?
   A. If two angles are adjacent, then they are congruent.
   B. If two angles are complementary, then their sum is 90⁰.
   C. If two angles are congruent, then they are complementary.
   D. If two segments are congruent, then they bisect each other.

*For questions 7-8*, refer to the following statements.
   p: 4=4      r: 4=2      q: 4≠2

7. Which of the following conditionals is false?
   A. $p \rightarrow r$
   B. $p \rightarrow q$
   C. $r \rightarrow q$
   D. $r \rightarrow p$

8. What makes your answer in item number 7 false?
   A. The hypothesis and the conclusion are both true.
   B. The hypothesis is true and the conclusion is false.
   C. The hypothesis and the conclusion are both false.
   D. The hypothesis is false and the conclusion is true.

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9. Which of the following properties determines the relationship between the hypothesis and the conclusion of the statement: If ∠X ≅ ∠Y , ∠Y ≅ ∠Z, then ∠X ≅ ∠Z?
   A. Addition Property
   C. Symmetric Property
   B. Reflexive Property
   D. Transitive Property

10. Given the statement, "If $M$ is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.", which of the following determines the relationship between the hypothesis and the conclusion of the statement?
    A. Definition of Midpoint
    B. Definition of Linear Pair
    C. Definition of Betweenness
    D. Definition of Congruent Segments

11. What is the if-then form of the statement, "Parallel lines never intersect."?
    A. If two lines intersect, then they are parallel.
    B. If two lines intersect, then they are parallel.
    C. If two lines are not parallel, then they intersect.
    D. If two lines are parallel, then they never intersect.

12. Transform the statement "A quadrilateral is a figure with four sides." in the if – then form.
    A. If a figure has four sides, then it is a quadrilateral.
    B. If a figure is a quadrilateral, then it has four sides.
    C. A figure is a quadrilateral if and only if it has four sides.
    D. A figure has four sides if and only if it is a quadrilateral.

13. What is the equivalent if-then form of the statement, "The sum of the measures of complementary angles is 90°"?
    A. If the sum of the measures of complementary angles, then it is 90°.
    B. If the sum of the measures is 90°, then they are complementary angles.
    C. If the sum of the measures of the angles is 90°, then they are complementary.
    D. If two angles are complementary, then the sum of the measures of their angles is 90°.

14. Which of the following is the correct if – then form of the statement, "Two adjacent angles are congruent."?
    A. If two angles are congruent, then they are adjacent.
    B. If two congruent angles are given, then they are adjacent.
    C. If there are two adjacent angles, then they are congruent.
    D. If there are two adjacent angles, then they are complementary.

15. Transform the statement, "An isosceles triangle has two congruent sides."
    A. If a triangle has two congruent sides, then it is an isosceles.
    B. If a triangle is isosceles, then it has two congruent sides.
    C. If a triangle is not isosceles, then it has two congruent sides.
    D. If a triangle is isosceles, then it has no two congruent sides.

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## *Jumpstart*

Let us begin the lesson by accomplishing the activity below.

> This activity will enable you to assess your prior knowledge on if- then statements and transforming a statement into its equivalent if- then statement.

## Activity 1. Match It Up!

**Directions:** Match the expression in Column A with appropriate expression in Column B to make a complete statement.

| COLUMN A | COLUMN B |
|---|---|
| 1. If you study your lessons well<br>2. If people follow health protocols strictly<br>3. If lines do not intersect<br>4. If a figure has four sides<br>5. If Francis listens to his parents' advice | A. then the spread of the virus is prevented.<br>B. then it is a quadrilateral.<br>C. then he'll surely succeed in life.<br>D. then you will get high grades.<br>E. then they are parallel. |

**Questions:**
   a. What have you noticed about the statements you formed?
   b. Take one statement and tell something about it.
   c. What is common to all of the statements?
   d. Which do you think of the statements you formed are true? Which are false? Explain.
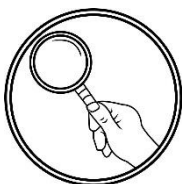
## *Discover*

## Determining the Relationships Between the Hypothesis and the Conclusion of an If - Then Statement

You have just encountered conditional statements or the if – then statements.
An if- then statement, also called as conditional, is a logical statement composed of two clauses: the if-clause and the then-clause.

When a conditional statement is written in if-then form, the "if- clause" contains the hypothesis and the "then-clause" contains the conclusion.

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We can denote a letter for each clause, p for the hypothesis and q for the conclusion. It is denoted by the symbols, $p \rightarrow q$, which is read as "If p, then q" or "p implies q."[1]

**Illustrative Examples:**
Identify the hypothesis and the conclusion in each of the following if – then statements.

1. If two lines intersect to form a right angle, then they are perpendicular.
   Hypothesis (p): Two lines intersect to form a right angle.
   Conclusion (q): They [the lines] are perpendicular.

2. If <u>a number is even</u>, then <u>it is divisible by 2</u>.
         Hypothesis (p)       Conclusion (q)

3. If m∠A=100, then ∠A is obtuse.
   Hypothesis (p): m∠A = 100
   Conclusion (q): ∠A is obtuse.

4. $8y = 40$ implies $y = 5$.
   Hypothesis (p):   8y = 40
   Conclusion (q):      y = 5

5. ∠1 ≅ ∠2 if m∠1 = m∠2.
   Hypothesis (p):  m∠1 = m∠2.
   Conclusion (q):    ∠1 ≅ ∠2

A conditional statement may be true or false.  The statement $p \rightarrow q$ is always true except in the case that p is true and q is false.  Consider the table below:

Table of Truth Values

| p (hypothesis) | q (conclusion) | $p \rightarrow q$ |
|---|---|---|
| True (T) | True (T) | True (T) |
| True (T) | False (F) | False (F) |
| False (F) | True (T) | True (T) |
| False (F) | False (F) | True (T) |

Source: Emmanuel P. Abuzo,et.al.,Mathematics Learner's Module(Pasig City:Department of Education,2013), p. 325

To show that a conditional statement is false, you only need to find one example (called a counterexample) in which the hypothesis is fulfilled and the conclusion is not fulfilled.

**Example 1:** If ∠A is acute, then m∠A= 45. This is false because not all acute angles measure 450.
Counterexample: $m∠A$ can be 500. (Some acute angles do not measure 450.)

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[1]Orlando A. Oronce and Marilyn O. Mendoza, *E-MATH,Worktext in Mathematics*(Sampaloc, Manila:Rex Book Store, Inc., 2015)p. 361

**Example 2:** If two angles are congruent, then they are right angles.
  p: Two angles are congruent. (True)
  q:  They are right angles. (False, not all congruent angles are right angles.
                          Two acute angles can be also congruent.)
Counterexample: The two angles might be both 750.
Therefore, the statement is false, since the hypothesis is true and the conclusion is false.

To show that a conditional statement is true, you must construct a logical argument using reasons, which can be a definition, an axiom, a property, a postulate or a theorem.[2]

**Example 3:** $\angle 1 \cong \angle 2$ if $m\angle 1 = m\angle 2$. This is true because of the Definition of Congruent Angles (If angles have the same measure, then they are congruent.).

**Example 4:** If a rectangle does not have four sides, then a square is not a quadrilateral.
  p: A rectangle does not have four sides. (False)
  q: A square is not a quadrilateral. (False)
Therefore, the statement is true, since the hypothesis and the conclusion are both false.

**Example 5:** If the shape is a triangle, then it is a polygon.
  p: The shape is a triangle. (True)
  q:  The triangle is a polygon. (True)
   Therefore, the statement is true, since the hypothesis and the conclusion are both true.

# Transforming a Statement into an Equivalent If – Then Statement

Sometimes, you encounter conditional statements that are not written in the "if – then" form.

To transform a given statement to an if – then statement, we need to identify first the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

In some cases, a statement does not present the hypothesis and the conclusion right away, like that of the theorems. For cases like these, we need to have a deeper understanding of the statements we are referring to.

**Example 1.** Two angles with the same measure are congruent.
         Hypothesis: Two angles have the same measure.
         Conclusion: They [the two angles] are congruent.

Rewrite the statement by placing "if" before the hypothesis and "then" before the conclusion.

If-then Statement: If two angles have the same measure, then they are congruent.

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[2] Orlando A. Oronce and Marilyn O. Mendoza, *E-MATH,Worktext in Mathematics*(Sampaloc, Manila:Rex Book Store, Inc., 2015)p. 361

**Example 2.** A polygon with four sides is a quadrilateral.
Hypothesis: A polygon has four sides.
Conclusion: It [the polygon] is a quadrilateral.

If-then Statement: If a polygon has four sides, then it is a quadrilateral.

**Example 3.** All congruent segments have the same length.
Hypothesis: Segments are congruent.
Conclusion: They [the segments] have the same length.

If-then Statement: If segments are congruent, then they have the same length.

**Example 4.** Two angles are supplementary if they are a linear pair.
Hypothesis: Two angles are a linear pair.
Conclusion: They [the two angles] are supplementary.

If- then Statement:  If two angles are a linear pair, then they are supplementary.

**Example 5.** Opposite sides of a rectangle are parallel.
Hypothesis: The sides of a rectangle are opposite.
Conclusion: They [sides] are parallel.

If- then Statement:  If sides of a rectangle are opposite, then they are parallel.



## *Explore*

> *Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.*

## Activity 2
### A. Which is ONCE, Which is TWICE!
**Directions:** Underline the hypothesis ONCE and the conclusion TWICE in the following statements.
1. If 4x+1 = 9, then x = 2.
2. If you see lightning, then you hear thunder.
3. If you are a basketball player, then you are at least 5'8" tall.
4. If two lines intersect, then their intersection is a point.
5. If two angles are vertical, then they are congruent.
6. If you practice social distancing, then you can go to the market.
7. If a shape is a square, then it has four equal sides.
8. 5x=10 implies x=2.
9. If a number is divisible by 2 and 3, then it is divisible by 6.
10. If an angle is right, then it measures 900.

**B. True or False!**

**Direction:** Give the truth values of the hypothesis and the conclusion, then write the if-then statement on the space provided for. Determine the truth value of the if-then statement.

|  | Truth Values (True or False) |
|---|---|
| 8 is an odd number. | 12. |
| 9 is a composite number. | 13. |
| 11. $p \rightarrow q$:_____ | 14. |
| $7^2 = 49$ | 16. |
| 49 is a perfect square. | 17. |
| 15. $p \rightarrow q$:_____ | 18. |

**C. Transform Me!**

**Directions:** Rewrite the following statements into their if – then form.

1. A triangle is a polygon with three sides.
2. Two intersecting lines lie in one plane.
3. Equilateral triangles are equiangular.
4. All whole numbers are integers.
5. Good citizens obey rules and regulations.

**D. I Made It...I Transformed It!**

**Directions:** Write 5 declarative statements, then transform into an if-then form.

(2 points each)

| DECLARATIVE STATEMENTS | IF-THEN STATEMENTS |
|---|---|
| 1. | 6. |
| 2. | 7 |
| 3. | 8. |
| 4. | 9. |
| 5. | 10. |

 *Deepen*

## Activity 3

**A. If's and Then's...**

**Directions:** Identify the hypothesis and the conclusion in the following items.

1. If a triangle is equiangular, then the angles are congruent.

Hypothesis:_____

Conclusion:_____

2.  $5y = -20$ implies $y = -4$.
Hypothesis:_____
Conclusion:_____

3.  If $4x - 1 = 11$, then $x = 3$.
Hypothesis:_____
Conclusion:_____

4.  If $RS = TW$, then $TW = RS$.
Hypothesis:_____
Conclusion:_____

5.  If $AB = CD$, $CD = EF$, and $EF = 23$, then $AB = 23$.
Hypothesis:_____
Conclusion:_____

6.  If $2x + y = 70$ and $y = 3x$, then $2x + 3x = 70$.
Hypothesis:_____
Conclusion:_____

7.  If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
Hypothesis:_____
Conclusion:_____

8.  If $\frac{z}{5} = 10$, then $z = 50$.
Hypothesis:_____
Conclusion:_____

9.  If $M$ is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.
Hypothesis:_____
Conclusion:_____

10. If point $E$ lies on the interior of $\angle VGA$, then $m\angle VGE + m\angle AGE = m\angle VGA$
Hypothesis:_____
Conclusion:_____

**B. Which is Which?**
**Directions:** Tell whether each if-then statement is True or False.  If false, give a counterexample. If true, determine what definition, postulate, theorem or concepts will prove that it is true.
1. If m∠B = 37, then ∠B is an acute angle.
2. If a number is greater than 5, then it is greater than 3.
3. If ∠E is an acute angle, then $m\angle E = 69$.
4. If two angles are adjacent, then they are congruent.
5. If $M$ is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

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**Activity 4. You Can Do It!**

**Direction:** Transform the following terms [definitions], theorems, or postulates into equivalent if-then statements. Choose 5 from the following items.

1. Midpoint of a segment
2. All right angles are congruent
3. Triangle Interior Angle Theorem
4. Perpendicular lines
5. Complementary angles
6. Acute Angles
7. Symmetric Property of Equality
8. Transitive Property of Equality
9. Vertical Angles Theorem
10. Supplement Theorem3

## *Gauge*

## Post Test

**Directions:** Read and answer each statement below carefully. Write your answers in a separate sheet of paper.

1. Which of the following is an if-then statement?
    A. Dog barks when cat meows.
    B. A dog barks at a cat's meow.
    C. Either the dog barks or the cat will meow.
    D. If the dog barks, then the cat will meow.

2. What is the hypothesis of the statement, "If two angles are vertical, then they are congruent."?
    A. Two angles are vertical.       B. Two angles are congruent.
    C. Vertical angles are congruent.  D. Congruent angles are vertical.

3. Which of the following represents the conclusion of the statement in Question No. 2?
    A. Two angles are vertical.       B. Two angles are congruent.
    C. Vertical angles are congruent.  D. Congruent angles are vertical.

4. If two angles do not form a linear pair, then they are not supplementary. What do you call the underlined portion in this statement?
    A. argument        B. conditional
    C. conclusion      D. hypothesis

5. If two numbers are odd, then their sum is even. What do you call the underlined portion in this statement?
    A. argument        B. conditional
    C. conclusion      D. hypothesis

6. Which of the following statements is TRUE about an if – then statement?
    A. If both the hypothesis and conclusion are true, then the conditional is false.
    B. If both the hypothesis and conclusion are false, then the conditional is false.

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C. If the hypothesis is false and the conclusion is true, then the conditional is false.

D. If the hypothesis is true and the conclusion is false, then the conditional is false.

7. Which of the following if- then statements is TRUE?
   A. If ∠E is an obtuse angle, then $m∠E= 180$.
   B. If three points are given, then exactly one plane contains them.
   C. If $m∠1 + m∠2 = 180,$ then ∠1 and ∠2 are supplementary.
   D. If two lines intersect, then they are perpendicular.

8. "If two angles are congruent, then they are vertical angles.", which of the following is true about the statement given?
   A. True, by the Definition of Vertical Angles.
   B. True. Both the hypothesis and conclusion are true.
   C. False. Not all congruent angles are vertical angles
   D. False. Both the hypothesis and conclusion are false.

9. Which of the following properties determines the relationship between the hypothesis and the conclusion of the statement: If an angle is a right angle, then its measure is 900.?
   A. Complementary angles are right angles.
   B. Complements of congruent angles are congruent.
   C. All right angles measure 900.
   D. A right angle measures more than 900.

10. "If a number is divisible by 3, then it is odd.", which of the following is true about the statement given?
    A. True. Most odd numbers are divisible by 3.
    B. True. 9 is odd and divisible by 3.
    C. False. 7 is odd but not divisible by 3.
    D. False. Both the hypothesis and conclusion are false.

11. What is the if-then form of the statement, "A negative number is less than 0."?
    A. If a number is negative, then it is less than 0.
    B. If a number is not negative, then it is not less than 0.
    C. If a number is less than 0, then it is a negative number.
    D. If a number is not less than 0, then it is not a negative number.

12. Transform the statement "Two perpendicular lines form four right angles." in the if – then form.
    A. If four right angles are formed, then the two lines are perpendicular.
    B. If the two lines are perpendicular, then they form four right angles.
    C. Two lines are perpendicular if and only if they form four right angles.
    D. Four right angles are formed if and only if there are two perpendicular lines.

13. What is the equivalent if-then form of the statement, "Three noncollinear points determine a plane."?
    A. If the three points are noncollinear, then they determine a plane.
    B. If there is a plane, then there are three noncollinear points.
    C. There is a plane if and only if there are three points.
    D. If there are three points, they determine a plane.

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14. Which of the following is the correct if – then form of the statement, "Two lines intersect at one point."?
    A. If two lines intersect, then there is one point.
    B. If there is one point, then the two lines intersect.
    C. If two lines intersect, then they intersect at one point.
    D. If the lines intersect at one point, then there are two lines.

15. Transform the statement, "All parallelograms are quadrilaterals."
    A. If there is a quadrilateral, then it is a parallelogram.
    B. If a figure is a parallelogram, then it is a quadrilateral.
    C. If the figure is a quadrilateral, then it is a parallelogram.
    D. If the figure is a parallelogram, then it is not a quadrilateral.

*Great job! You are done with this module.*

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