8

Mathematics

Quarter 3 - Week 1 to 2

Module 1: Axiomatic Structure of a Mathematical System

AIRs - LM

Mathematics 8
Quarter 3- Week 1 to 2 Module 1:
Axiomatic Structure of a Mathematical System
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Target

This module will help you understand the key concepts of Mathematical System and the need for an axiomatic structure in Geometry. Moreove, you' ll find out how these mathematics concepts are used in solving real-life problems. In all lessons, you are given the opportunity to use your prior knowledge and skills in linear inequalities in one variable. Activities are also given to process your knowledge and skills acquired, deepen and transfer your understanding of the different lessons. The scope of this modules enables you to use it in many different learning situations. The lesson are arranged to follow the standard sequence of the course. But in order in which you read them can be changed to corresponds with the textbooks you are using.

This module contains the following lessons:

Lesson 1: Mathematical System

Lesson 2: Defined terms, Undefined terms, Postulates and Theorems

After going through this module, you are expected to

1. describes a mathematical system; and
2. illustrates the need for an axiomatic structure of a mathematical system in general, and in Geometry in particular.

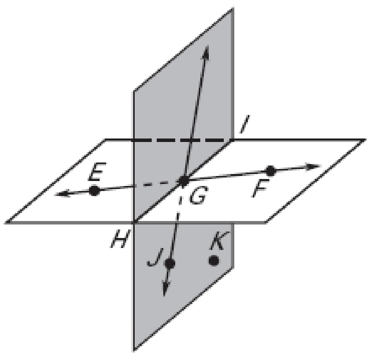
Pre - test

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What mathematical statement is accepted without proof?
   A. paragraph B. phrase C. postulate D. theorem

2. It has no dimension, no width, no length and no thickness and is represented by a dot. What undefined term is being described in the statement?
   A. angle B. line C. plane D. point

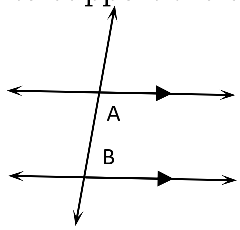
3. What is being illustrated by the points E, G. and F?
   A. They are collinear.
   B. They determine a plane.
   C. They are non-coplanar points.
   D. They are contained in only plane JKH.



4. What is being illustrated by the line that contains F and H?
   A. It intersects plane EGI but not plane JKH.
   B. It intersects plane JKH but not plane EGI.
   C. It does not intersect either plane.
   D. It intersects both planes.

5. If 3 points lie in a plane, what must be true about the three points?
   A. They must be collinear.
   B. They must be coplanar.
   C. None of the points lie on the same line.
   D. None of the points lie in any other plane.

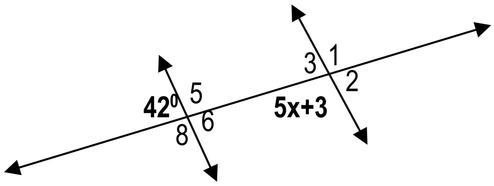
6. Based on the diagram, which theorem would you use to support the statement $m\angle A + m\angle B = 180^{\circ}$
   A. Alternate interior angles theorem
   B. Alternate exterior angles theorem
   C. Same side interior angles theorem
   D. Parallel lines theorem



7. What is the measure of $\overline{AB}$ if D is between A and B, AD = 7 and DB = 12.
   A. 5 B. 12 C. 10 D. 19

8. If $\angle CAT$ and $\angle EAT$ form a linear pair and $\angle EAT$ is acute, then $\angle CAT$ is
   A. acute B. obtuse C. right D. straight

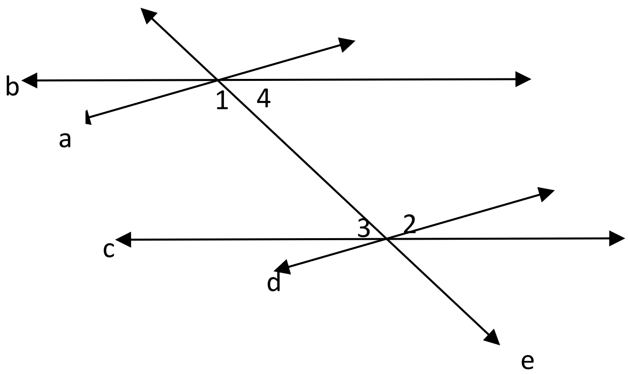
For numbers 9 – 10:



9. What is the value of x?
   A. 27° B. 30° C. 40° D. 50°

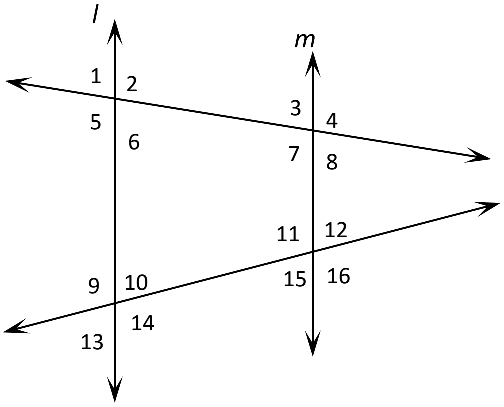
10. What is the measure of $\angle 8$?
   A. 45° B. 90° C. 138° D. 180°

11. In the figure below, a| | d with e as the transversal. What must be true about angle 3 and angle 4 if b| |c?



A. angle 3 is greater than angle 4
B. angle 3 is congruent to angle 4
C. angle 3 is a supplement of angle 4
D. angle 3 is a complement of angle 4

For numbers 12 – 15:



12. What special angle pair is being identified by $\angle 9$ and $\angle 11$?
    A. Alternate interior angles
    B. Corresponding Angles
    C. Same Side Exterior Angles
    D. Vertical Angles

13. Given $l \mid\mid m$ and $m\angle 15 = 82°$ find $m\angle 13$.
    A. $78°$ B. $82°$ C. $98°$ D. $108°$

14. Given $l \mid\mid m$ and $m\angle 6 = 45°$ and $m\angle 3 = (4x+9)°$ find the value of x.
    A. 5 B. 9 C. 36 D. 42

15. Given $m\angle 12 = 75°$ and $m\angle 10 = (15x-15)°$, find the value of x so that $l \mid\mid m$
    A. 3 B. 4 C. 5 D. 6

Lesson 1

Describing Mathematical System

Jumpstart

Activity 1. TRUE or FALSE

Directions: Write TRUE if the statement is correct. Otherwise, write FALSE.

______ 1. A good definition names the term being defined.

______ 2. The characteristics of a good definition places the term into set or category.

______ 3. A good definition distinguishes itself from the terms in that category.

______ 4. A good definition is reversible.

______ 5. Points, lines, and planes are the three defined terms.

______ 6. A lemma is a helping postulate.

______ 7. The midpoint which bisects the line into two congruent parts is a postulate.

______ 8. A postulate is a statement that needs to be proven.

______ 9. A theorem is a statement that is assumed to be true.

______ 10. A corollary is a theorem that follows from another theorem as a “by - product”.

Discover

There are some undefined terms (primitive terms) in an axiomatic scheme and a set of statements about the undefined terms, called axioms or postulates. By proving new statements, called theorems, one obtains a mathematical theory using only the axioms (postulates), the logic system, and previous theorems. In the method, definitions are made in order to be more concise.

A distinction was made between axioms and postulates by most early Greeks. There is proof that Euclid made the distinction that an axiom (standard notion) is an assertion common to all sciences and that a postulate is an assumption unique to the science being studied in particular. Today, no distinction is made between the two in modern times; an axiom or postulate is an implied assertion.

Usually an axiomatic system does not stand alone, but other systems are also assumed to hold. For example, we will assume:

1. the real number system,
2. some set theory,
3. Aristotelian logic system, and
4. the English language.

Too much familiarity with the subject matter of the method is one of the drawbacks of dealing with a deductive system. We need to be cautious of what we believe to be true, because when writing a proof, stating something is obvious. We need to take extreme care that, beyond the framework being analyzed, we do not make an additional inference.

A common mistake in the writing of evidence in geometry is to base the evidence on an image. Either by not covering all possibilities or by representing our implicit prejudice as to what is right, an image can be misleading. It is crucially important in a proof to use only the axioms and the theorems which have been derived from them and not depend on any preconceived idea or picture.

In developing the proof, pictures should only be used as an intuitive aid, but each step in the proof should rely only on the axioms and theorems without reliance on any picture.

As they are useful in improving conceptual comprehension, diagrams can be used as an aid, but caution must be taken to ensure that the diagrams do not lead to confusion.

Typically, at the beginning of the construction of an axiomatic system, not all the axioms are given; this helps one to prove very general theorems that hold for many axiomatic systems. An abstract algebra example is: group theory → ring theory → field theory. A second example is that a parallel postulate is often not added early in Euclidean geometry studies, so both Euclidean and hyperbolic geometry theorems developed will hold for (called a neutral geometry).

To avoid circular definitions, some terms are left undefined, and the axioms are specified to provide the undefined terms with properties. There are two types of ambiguous terms: terms implying objects, called components, and terms implying connections between objects, called relationships. Examples of undefined terms (primitive terms) in geometry are point, line, plane, on, and between. For these undefined terms, on and between would indicate some undefined relationship between undefined objects such as point and line. An example would be: A point is on a line. Early geometers tried to define these terms:

point

Pythagoreans, “a monad having position"
Plato, “the beginning of a line"
Euclid, “that which has no part"

line

Proclus, “magnitude in one dimension", “flux of a point"
Euclid, “breadthless length"

If there is no assertion such that both the statement and its negation are axioms or theorems of the axiomatic system, an axiomatic system is consistent. Since in an axiomatic system, contradictory axioms or theorems are typically not needed, we will consider consistency as a necessary condition for an axiomatic system. An axiomatic system that does not have the property of consistency has no mathematical value and is generally not of interest.

If we can attach significance to the ambiguous terms of the axiomatic system that translate the axioms into true statements about the concepts assigned, a model of an axiomatic system is obtained. Relevant models and abstract models are used for two types of models. A model is concrete if objects and relations adapted from the real world are the definitions assigned to the unknown concepts. A model is abstract if the meanings assigned to the undefined terms are objects and relations adapted from another axiomatic development.

An axiomatic system is a system composed of the following:

• Undefined terms
• Definitions or Defined terms
• Axioms or Postulates
• Theorems

Undefined terms are terms that are left undefined in the system. Instead of providing a definition for them, we resort to a description, illustration, or demonstration.

Examples:

• Point - represented by a dot, it has no length, width, or thickness.
• Line - represented by a straight line with two arrowheads, it has no thickness but its length extends and goes on forever in both directions.

Definitions or **defined terms** on the other hand are terms defined from the undefined terms in the system.

Examples:

• Angle - A figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle.
• Parallel lines - Lines in a plane which do not meet.

Postulates are statements that are considered true without proof or validation.

Examples:

• Postulate: Through any two points there is exactly one line.
• Postulate: If two lines intersect, then they intersect at exactly one point.

Theorems are statements proved to be true using postulates, definitions, other established theorems, and logic.

Example:

- Vertical Angles Theorem: Vertical angles are equal in measure. •

Consistency

An axiomatic system is said to be consistent if there are no axiom or theorem that contradict each other. So if the following statement is an axiom or a theorem:

"There exist two lines that are parallel."

Then its negation should not be an axiom or a theorem:

"No two lines are parallel."

An axiomatic system should be consistent for it to be logically valid. Otherwise, the axiomatic system and its statements are all flawed.

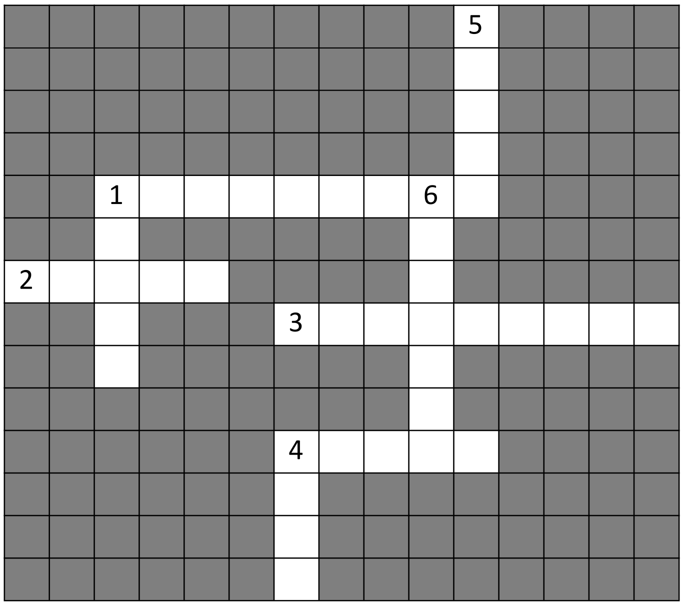
Independency

A postulate is said to be independent if it cannot be proven true using other axioms in the system. An axiomatic system is said to be independent if all of its axioms are independent.

The word "axiom" can be used interchangeably with "postulate." If a statement is an axiom, it is regarded as true whether it makes sense or not.

Explore

Activity 1: **CROSS WORD PUZZLE!**
Directions: Fill in the cross word puzzle.



Across
1 – Statement assumed to be true
2 – Statement accepted as true as basis for argument or inference
3 – Consequence of another theorem
4 – Helping theorem
Down
1 – Dot
4 – Consists of infinite points
5 – Flat surface has length and width
6 – Statement that can be proved to be true

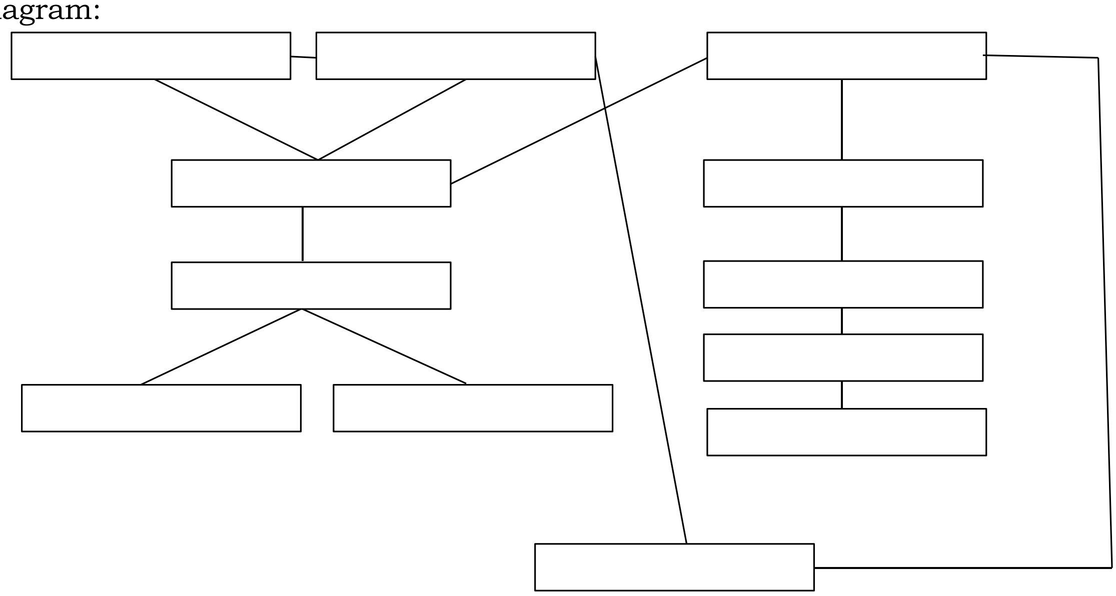
Deepen

Activity 1: **WHERE DO I BELONG?**

Directions: Complete the diagram by writing the correct mathematical terms listed in the box.

Axiom	Axiomatic system	Corollary	Defined terms	
Lemma	Line	Plane	Point	Postulates
Theorem	The Five Postulates	The Three Undefined Terms		

Diagram:



Lesson 2 Defined terms, Undefined terms, Postulates and Theorems

Jumpstart

Have you seen a ring light used by a tiktoker? How does it look like? Have you notice its stand? If it has three legs, it is called tripod.

The legs of the tripod touch a table or a floor at three points. The legs suggest lines, and the table/floor surface suggests a plane.

Activity Point, Line or Plane
Directions: Determine whether the following is a representations of a **point**, a **line** or a **plane**.
1. TV screen
2. a mole in a face
3. hair strand
4. the corner of sheet of paper
5. a pebble on the sand
6. ceiling
7. edge of a notebook
8. tip of a ballpen
9. star in the sky
10. chord of a charger

Discover

Geometry relies on terms such as point, line, and plane for a common understanding. Since these terms cannot be described using other known words mathematically, they are called **undefined terms**.

A **point** has no dimension. It is represented by a small dot.

● A

Figure 1 *point A.*

A **line** has one dimension. It extends without end in two directions. It is represented by a line with two arrowheads.

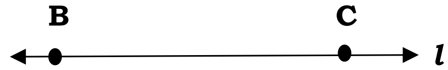


Figure 2 *line **l** or line **BC.***

A **plane** has two dimensions. It is represented by a shape that looks like a floor or wall. You have to imagine that it extends without end.

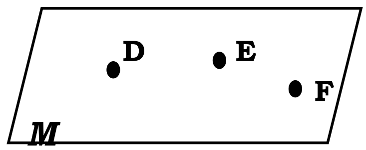


Figure 3 *plane **M** or plane **DEF***

You need two points to describe a line, and you need three points to describe a plane, because the geometry in this book follows the two postulates given below. are statements that are accepted without further justification.

A. Postulate and Theorems

A **postulate** is a statement that is assumed true without proof. A **theorem** is a true statement that can be proven.

Listed below are six postulates and the theorems that can be proven from these postulates.

- ***Postulate 1****:* A line contains at least two points.
- ***Postulate 2:*** A plane contains at least three non-collinear points.
- ***Postulate 3****:* Through any two points, there is exactly one line.
- ***Postulate 4****:* Through any three non-collinear points, there is exactly one plane.
- ***Postulate 5****:* If two points lie in a plane, then the line joining them lies in that plane.
- ***Postulate 6****:* If two planes intersect, then their intersection is a line.

- ***Theorem 1:*** If two lines intersect, then they intersect in exactly one point.
- ***Theorem 2****:* If a point lies outside a line, then exactly one plane contains both the line and the point.
- ***Theorem 3****:* If two lines intersect, then exactly one plane contains both lines

Example 1: The following postulate or theorem are used to justify the statement made in figure 1.

(a) Through any three non-collinear points, there is exactly one plane *(Postulate 4).*

(b) Through any two points, there is exactly one line *(Postulate 3).*

(c) If two points lie in a plane, then the line joining them lies in that plane *(Postulate 5).*

(d) If two planes intersect, then their intersection is a line *(Postulate 6).*

(e) A line contains at least two points (*Postulate 1*).
(f) If two lines intersect, then exactly one plane contains both lines *(Theorem 3)*.
(g) If a point lies outside a line, then exactly one plane contains both the line and the
point *(Theorem 2)*.
(h) If two lines intersect, then they intersect in exactly one point *(Theorem 1)*.

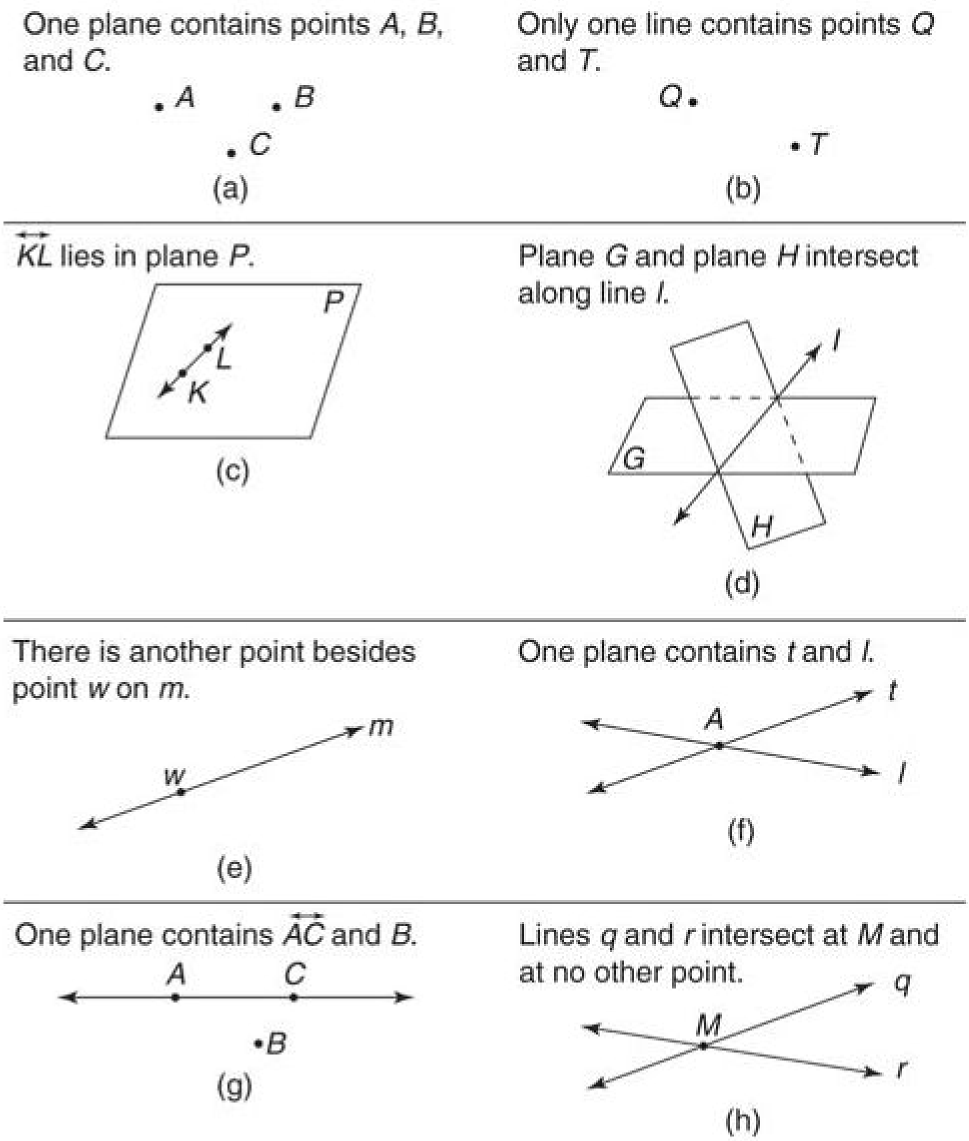


Figure 1 *Illustrations of Postulates 1–6 and Theorems 1–3.*

B. Segments, Midpoints and Rays

The concept of lines is straightforward, but much of geometry is concerned with portions of lines. Some of those portions are so special that they have their own names and symbols.

a. Line segment

A **line segment** is a connected piece of a line. It has two endpoints and is named by its endpoints. Sometimes, the symbol – written on top of two letters is used to denote the segment. This is line segment *CD* (Figure 1).

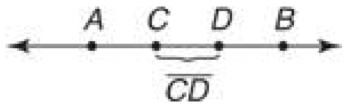


Figure 1 *Line segment.*

It is written *CD* (Technically, *CD* refers to the points *C* and *D* and all the points between them, and *CD* without the ▬ refers to the distance from *C* to *D*.)

Note that *CD* is a piece of $\overleftrightarrow{AB}$.

***Postulate 7* (Ruler Postulate):** Each point on a line can be paired with exactly one real number called its **coordinate**. The distance between two points is the positive difference of their coordinates (Figure 2).

Figure 2 *Distance between two points.*

If $a > b$, then $AB = a - b$.

Example 1**:** In Figure 3, find the length of *QU*.

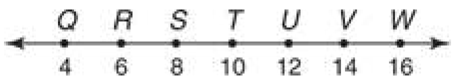


Figure 3 *Length of a line segment.*

$QU = 12 – 4$

$QU = 8$ (The length of line segment QU is 8)

Postulate 8 *(Segment Addition Postulate):* If *B* lies between *A* and *C* on a line, then $AB + BC = AC$ (Figure 4).

Figure 4 *Addition of lengths of line segments.*

Example 2: In Figure 5, *A* lies between *C* and *T*. Find *CT* if $CA = 5$ and $AT = 8$.

Figure 5 *Addition of lengths of line segments.*

Because *A* lies between *C* and *T*, Postulate 8 tells you

$$\boldsymbol{CA} + \boldsymbol{AT} = \boldsymbol{CT}$$
$$5 + 8 = 13$$
$$\boldsymbol{CT} = 13$$

b. Midpoint

A **midpoint** of a line segment is the halfway point, or the point equidistant from the endpoints (Figure 6).

Figure 6 Midpoint of a line segment.

R is the midpoint of *QS* because *QR* = *RS* or because *QR* = ½ *QS* or *RS* = ½ *QS*
Example 3: In Figure 7, find the midpoint of *KR* .

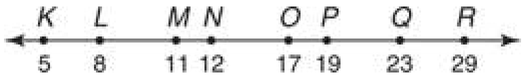


Figure 7 Midpoint of a line segment.

$$\boldsymbol{KR} = 29 - 5$$
$$\boldsymbol{KR} = 24$$

The midpoint of *KR* would be ½(24), or 12 spaces from either *K* or *R*. Because the coordinate of *K* is 5, and it is smaller than the coordinate of R (which is 29), to get the coordinate of the midpoint you could either add 12 to 5 or subtract 12 from 29. In either case, you determine that the coordinate of the midpoint is 17. That means that point *O* is the midpoint of *KR* because *KO* = *OR*.

Another way to get the coordinate of the midpoint would be to find the average of the endpoint coordinates. To find the average of two numbers, you find their sum and divide by two. (5 + 29) ÷ 2 = 17. The coordinate of the midpoint is 17, so the midpoint is point *O*.

Theorem 4*:* A line segment has exactly one midpoint.

c. Ray

A **ray** is also a piece of a line, except that it has only one endpoint and continues forever in one direction. It could be thought of as a half-line with an endpoint. It is named by the letter of its endpoint and any other point on the ray. The symbol → written on top of the two letters is used to denote that ray. This is ray *AB* (Figure 8).

Figure 8 Ray *AB*.

It is written as $\overrightarrow{AB}$
This is ray *CD* (Figure 9).

Figure 9 Ray *CD*

It is written as $\overrightarrow{CD}$ or $\overleftarrow{DC}$
Note that the non-arrow part of the ray symbol is over the endpoint.

C. Angle Pairs Created with a Transversal

A **transversal** is any line that intersects two or more lines in the same plane but at different points. In Figure 1, line *t* is a transversal

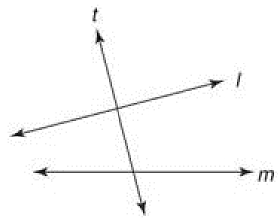


Figure 1 *A transversal intersecting two lines in the same plane.*

A transversal that intersects two lines forms eight angles; certain pairs of these angles are given special names. They are as follows:

- **Corresponding angles** are the angles that appear to be in the same relative position in each group of four angles. In Figure, ∠1 and ∠5 are corresponding angles. Other pairs of corresponding angles in figure are: ∠4 and ∠8, ∠2 and ∠6, and ∠3 and ∠7.

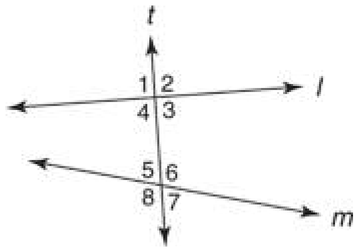


Figure 2 *A transversal intersecting two lines and forming various pairs of corresponding angles, alternate interior angles, alternate exterior angles, consecutive interior angles, and consecutive exterior angles.*

- **Alternate interior angles** are angles within the lines being intersected, on opposite sides of the transversal, and are not adjacent. In Figure 2, ∠4 and ∠6 are alternate interior angles. Also, ∠3 and ∠5 are alternate interior angles.
- **Alternate exterior angles** are angles outside the lines being intersected, on opposite sides of the transversal, and are not adjacent. In Figure 2, ∠1 and ∠7 are alternate exterior angles. Also, ∠2 and ∠8 are alternate exterior angles.
- **Consecutive interior angles** (same-side interior angles) are interior angles on the same side of the transversal. In Figure 2, ∠4 and ∠5 are consecutive interior angles. Also, ∠3 and ∠6 are consecutive interior angles.
- **Consecutive exterior angles** (same-side exterior angles) are exterior angles on the same side of the transversal. In Figure 2, ∠1 and ∠8 are consecutive exterior angles. Also, ∠2 and ∠7 are consecutive exterior angles.

The Parallel Postulate

Postulate 11 (Parallel Postulate): If two parallel lines are cut by a transversal, then the corresponding angles are equal (Figure 1).

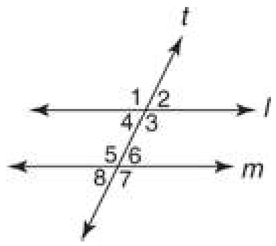


Figure 1 *Corresponding angles are equal when two parallel lines are cut by a transversal.*

This postulate says that if l // m, then

- $m\angle 1 = m\angle 5$
- $m\angle 2 = m\angle 6$
- $m\angle 3 = m\angle 7$
- $m\angle 4 = m\angle 8$

D. Consequences of the Parallel Postulate

Postulate 11 can be used to derive additional theorems regarding parallel lines cut by a transversal. Because $m\angle 1 + m\angle 2 = 180°$ and $m\angle 5 + m\angle 6 = 180°$ (because adjacent angles whose non-common sides lie on a line are supplementary), and because $m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 4$, $m\angle 5 = m\angle 7$, and $m\angle 6 = m\angle 8$ (because vertical angles are equal), all of the following theorems can be proven as a consequence of *Postulate 11.*

Theorem 13 (Alternate Interior Angles Theorem)*:* If two parallel lines are cut by a transversal, then alternate interior angles are equal.

Theorem 14 (Alternate Exterior Angles Theorem)*:* If two parallel lines are cut by a transversal, then alternate exterior angles are equal.

Theorem 15 (Same Side Interior Angles Theorem)*:* If two parallel lines are cut by a transversal, then same side interior angles are supplementary.

Theorem 16 (Same Side Exterior Angles Theorem)*:* If two parallel lines are cut by a transversal, then same side exterior angles are supplementary.

The above postulate and theorems can be condensed to the following theorems:

Theorem 17*:* If two parallel lines are cut by a transversal, then every pair of angles formed are either equal or supplementary.

Theorem 18*:* If a transversal is perpendicular to one of two parallel lines, then it is also perpendicular to the other line.

Based on *Postulate 11* and the theorems that follow it, all of the following conditions would be true if $l \parallel m$ (Figure 1).

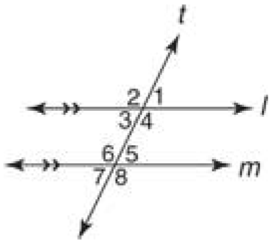


Figure 1 *Two parallel lines cut by a transversal.*
In figures, single or double arrows on a pair of lines indicate that the lines are parallel.

Based on *Postulate 11:*

- $m\angle 1 = m\angle 5$, $m\angle 4 = m\angle 8$, $m\angle 2 = m\angle 6$, $m\angle 3 = m\angle 7$

Based on *Theorem 13:*

- $m\angle 3 = m\angle 5$, $m\angle 4 = m\angle 6$,

Based on *Theorem 14:*

- $m\angle 1 = m\angle 7$, $m\angle 2 = m\angle 8$

Based on *Theorem 15:*

- $\angle 3$ and $\angle 6$ are supplementary, $\angle 4$ and $\angle 5$ are supplementary

Based on *Theorem 16:*

- $\angle 1$ and $\angle 8$ are supplementary, $\angle 2$ and $\angle 7$ are supplementary

Based on *Theorem 18:*

- If $t \perp l$, then $t \perp m$

E. Testing for Parallel Lines

Postulate 11 and Theorems 13 through 18 tell you that *if* two lines are parallel, *then* certain other statements are also true. It is often useful to show that two lines are in fact parallel. For this purpose, you need theorems in the following form: *If* (certain statements are true) *then* (two lines are parallel). It is important to realize that the **converse** of a theorem (the statement obtained by switching the *if* and *then* parts) is not always true. In this case, however, the converse of postulate 11 turns out to be true. We state the converse of Postulate 11 as Postulate 12 and use it to prove that the converses of Theorems 13 through 18 are also theorems.

Postulate 12*:* If two lines and a transversal form equal corresponding angles, then the lines are parallel.
In Figure 1, if $m\angle 1 = m\angle 2$, then $l \parallel m$. (Any pair of equal corresponding angles would make $l \parallel m$.)

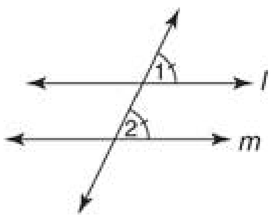


Figure 1 *A transversal cuts two lines to form equal corresponding angles.*

This postulate allows you to prove that all the converses of the previous theorems are also true.
Theorem 19: If two lines and a transversal form equal alternate interior angles, then the lines are parallel.
Theorem 20: If two lines and a transversal form equal alternate exterior angles, then the lines are parallel.
Theorem 21: If two lines and a transversal form consecutive interior angles that are supplementary, then the lines are parallel.
Theorem 22: If two lines and a transversal form consecutive exterior angles that are supplementary, then the lines are parallel.
Theorem 23: In a plane, if two lines are parallel to a third line, the two lines are parallel to each other.
Theorem 24: In a plane, if two lines are perpendicular to the same line, then the two lines are parallel.

Based on *Postulate 12* and the theorems that follow it, any of following conditions would allow you to prove that a // b. (Figure 2).

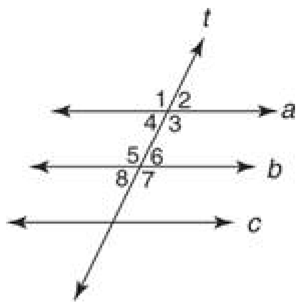


Figure 2 *What conditions on these numbered angles would guarantee that lines* a *and* b *are parallel?*

Postulate 12:

- $m\angle 1 = m\angle 5$, $m\angle 2 = m\angle 6$, $m\angle 3 = m\angle 7$, $m\angle 4 = m\angle 8$

Use *Theorem 19:*

- $m\angle 4 = m\angle 6$, $m\angle 3 = m\angle 5$

Use *Theorem 20:*

- $m\angle 1 = m\angle 7$; $m\angle 2 = m\angle 8$

Use *Theorem 21:*

- $\angle 4$ and $\angle 5$ are supplementary ; $\angle 3$ and $\angle 6$ are supplementary

Use *Theorem 22:*

- $\angle 1$ and $\angle 8$ are supplementary ; $\angle 2$ and $\angle 7$ are supplementary

Use *Theorem 23:*

- a // c and b // c

Use *Theorem 24:*

- $a \perp t$ and $b \perp t$

Example 1: Using Figure 3, identify the given angle pairs as alternate interior, alternate exterior, consecutive interior, consecutive exterior, corresponding, or none of these: ∠1 and ∠7, ∠2 and ∠8, ∠3 and ∠4, ∠4 and ∠8, ∠3 and ∠8, ∠3, and ∠2, ∠5 and ∠7.

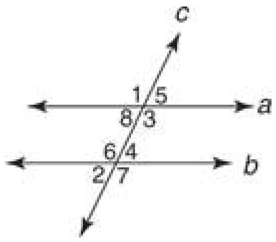


Figure 3 *Find the angle pairs that are alternate interior, alternate exterior, consecutive interior, consecutive exterior, and corresponding.*

∠1 and ∠7 are alternate exterior angles.
∠2 and ∠8 are corresponding angles.
∠3 and ∠4 are consecutive interior angles.
∠4 and ∠8 are alternate interior angles.
∠3 and ∠2 are none of these.
∠5 and ∠7 are consecutive exterior angles.

Example 2: For each of the figures in Figure 4, determine which postulate or theorem you would use to prove *l* // *m.*

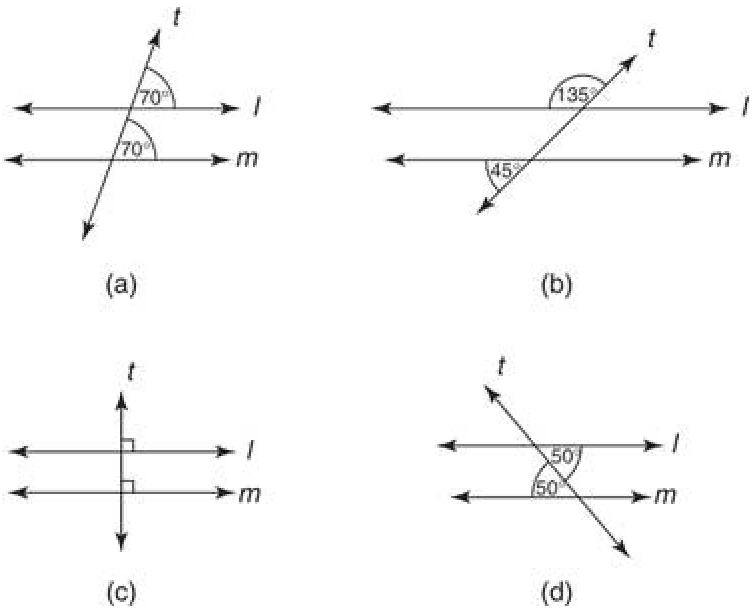


Figure 4 *Conditions guaranteeing that lines l and m are parallel.*

Figure 4 (a): If two lines and a transversal form equal corresponding angles, then the lines are parallel *(Postulate 12).*
Figure 4 (b): If two lines and a transversal form consecutive exterior angles that are supplementary, then the lines are parallel *(Theorem 22).*
Figure 4 (c): In a plane, if two lines are perpendicular to the same line, the two lines are parallel *(Theorem 24).*
Figure 4 (d): If two lines and a transversal form equal alternate interior angles, then the lines are parallel *(Theorem 19).*

Example 3: In Figure 5, $a \parallel b$ and $m\angle 1 = 117°$. Find the measure of each of the numbered angles.

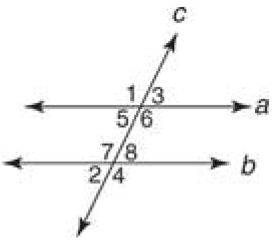


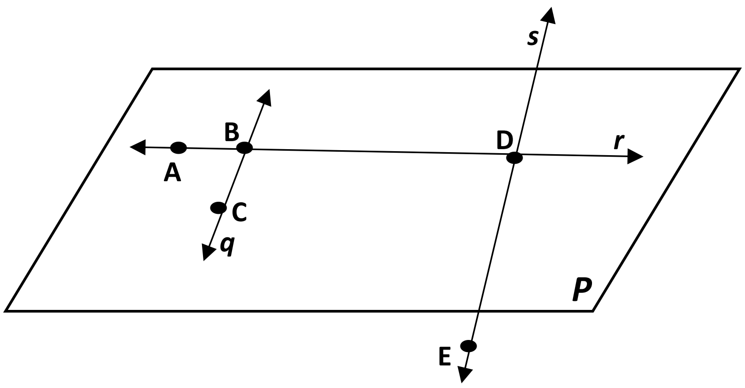
Figure 5 *When lines* a *and* b *are parallel, knowing one angle makes it possible to determine all the others pictured here.*

m $\angle 2 = 63°$, $m\angle 3 = 63°$, $m\angle 4 = 117°$, $m\angle 5 = 63°$, $m\angle 6 = 117°$, $m\angle 7 = 117°$, and $m\angle 8 = 63°$

Explore

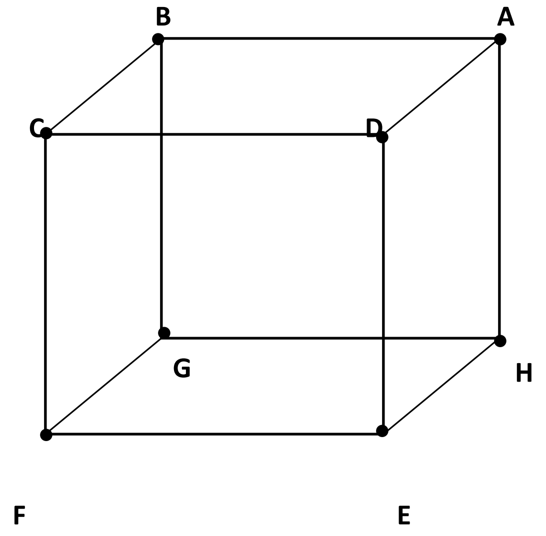
Activity 1. Defined and Undefined Terms

A. Use the figure to name each of the following.



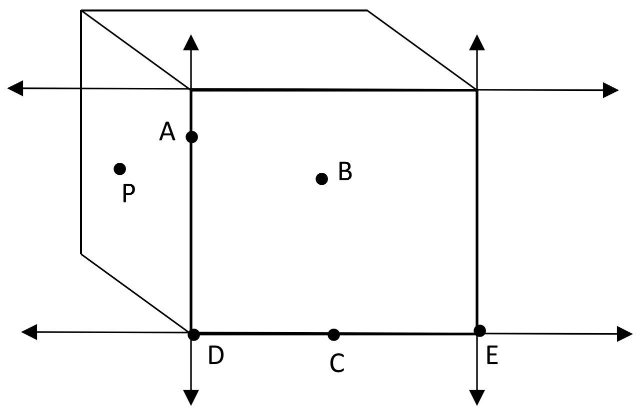
1. a line containing point **A**
2. a plane containing points **C** and **D**

B. Name the line segments and plane represented by the front of the cube in several different ways.



3. Line segments:
4. Plane:

Activity 2. Postulates
Directions: Explain how the figure below illustrates that each statement is true, then state the postulate or theorem that can be used to show each statement is true.



1. If two planes intersect, then their intersection is a line.
2. If two lines intersect, then their intersection is exactly one point.
3. A line contains at least two points.
4. If two points lie in a plane, then the line joining them lies in that plane.
5. Through any two points, there is exactly one line.

Activity 3. Theorems

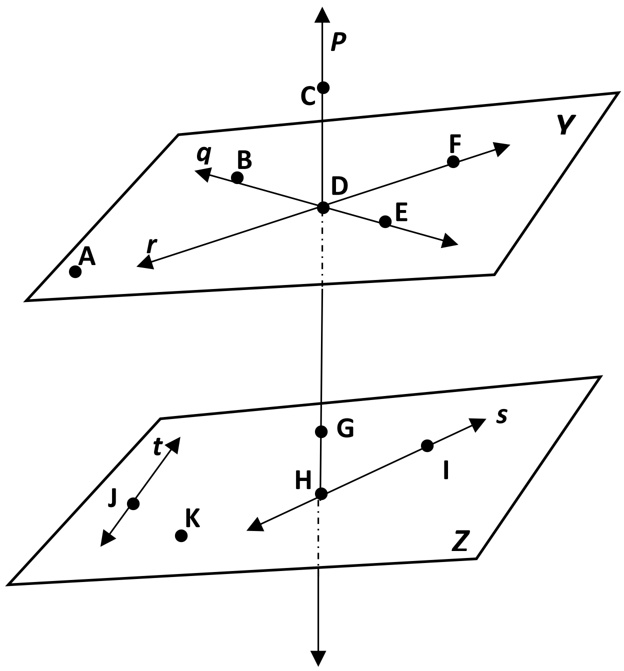
Directions: Match the statements from column A to its corresponding theorems in column B.

Column A	Column B
______1. If two angles form a linear pair, then they are supplementary.	a. Alternate Interior Angles Theorem
______2. If two angles are supplements of the same angle, then they are congruent.	b. Vertical Angles Theorem
______3. If two angles are complements of the same angle, then they are congruent.	c. Perpendicular Transversal Theorem
______4. All right angles are congruent.	d. Linear Pair Theorem
______5. Vertical angles are equal in measure.	e. Parallel Lines Theorem
______6. If two parallel lines are intersected by a transversal, then alternate interior angles are equal in measure.	f. Alternate Exterior Angles Theorem
______7. If two parallel lines are intersected by a transversal, then alternate exterior angles are equal in measure.	g. Right Angle Congruence Theorem
______8. If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary.	h. Same-side Interior Angles Theorem
______9. If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one.	i. Congruent complements theorem
______10. In a coordinate plane, two non - vertical lines are parallel if and only if they are the same slope.	j. Congruent supplements theorem

Deepen

Activity 1. Defined and Undefined Terms

Directions: Refer to the figure on the right. Answer the following questions.

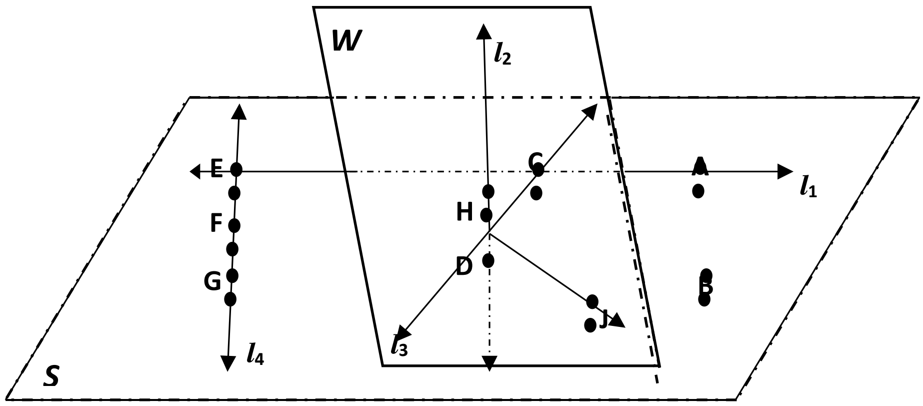


1. What are the lines in plane ***Z*** only?
2. How many planes are labeled in the figure?
3. What plane is contained in the lines ***q*** and ***r***?
4. What is the intersection of lines ***q*** and ***r***?
5. What point that is not coplanar with points **A**, **B**, and **D**?
6. Are points **E**, **J**, **K**, and **H** coplanar? Explain.

7. What are the points not contained
   in a line shown?
8. What is another name for line ***r***?
9. Does line ***t*** intersect line ***s***? Explain.
10. Does line ***q*** intersect plane ***Y*** at a single point?

Activity 2. Postulates

Directions: In the figure below, line segment CD and line segmet CE lie in plane S and line segment DH and line segment DJ lie in plane W . Given the postulate, make a statement in the figure presented.

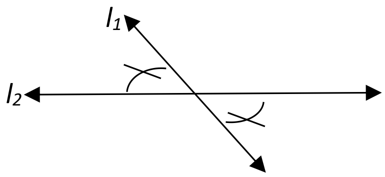


1. Postulate 1
2. Postulate 2
3. Postulate 3
4. Postulate 4
5. Postulate 5
6. Postulate 6

Activity 3. Theorems

Directions: Study each illustration below, then identify the theorem/s is/are shown.

1.



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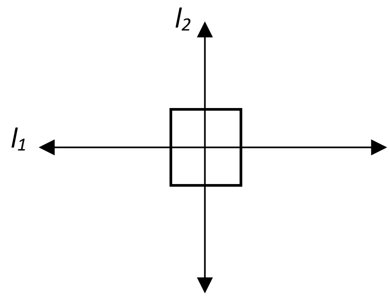

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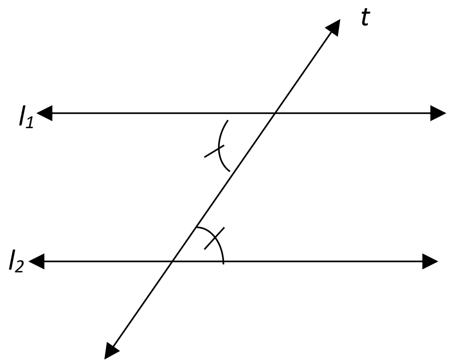

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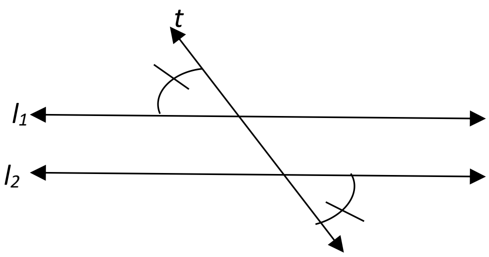

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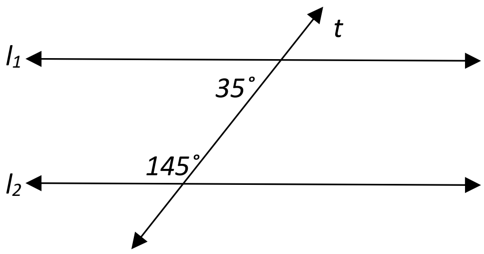

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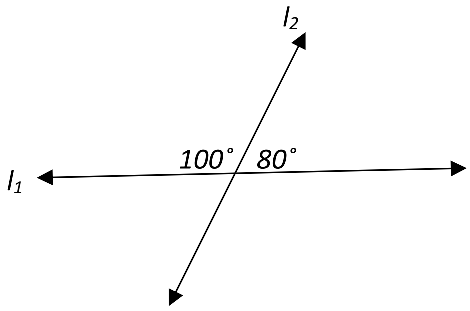

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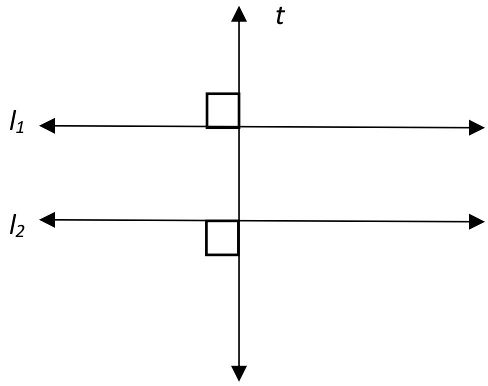

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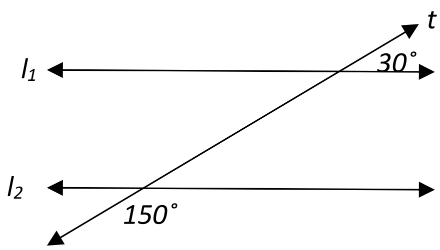
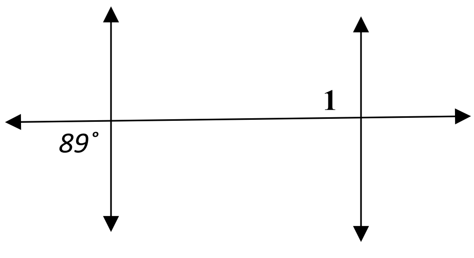


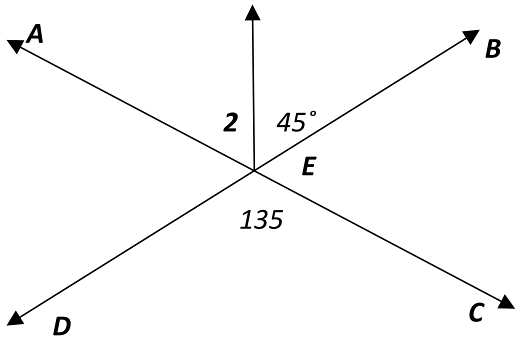
Activity 4. Theorems

Directions: Solve the following.

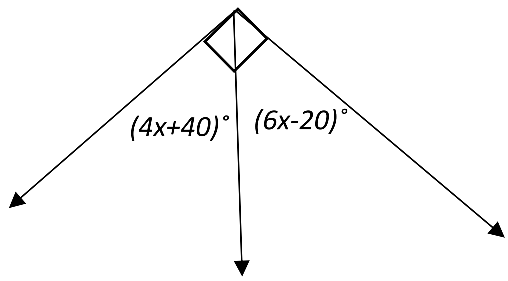
1. There are 2 parallel lines in the figure. What is the value of angle 1? State the theorem/s being used.



2. Both AEC and DEB are straight lines. What is the value of 2? What theorem/s is/are being used?



3. What is the value of x in the figure below? What theorem is being used?



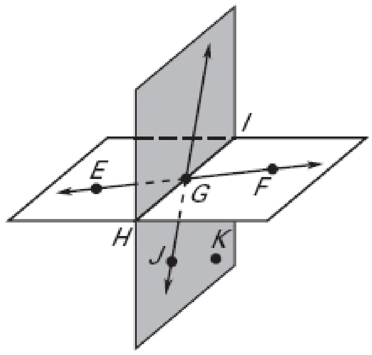
Gauge

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What mathematical statement that is true and needs to be proven?
   A. paragraph B. phrase C. postulate D. theorem

2. It has no thickness but its length extends and goes on forever in both directions, what undefined term is being described?
   A. angle B. line C. plane D. point

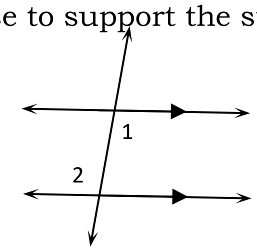
3. What is being illustrated by the points J,G. and K?
   A. They are collinear.
   B. They determine a plane.
   C. They are noncoplanar points.
   D. They are contained in only plane JKH.



4. What is being illustrated by the line that contains H and I?
   A. It intersects plane EGI but not plane JKH.
   B. It intersects plane JKH but not plane EGI.
   C. It does not intersect either plane.
   D. It intersects both planes.

5. If 3 points lie in a line, what must be true about the three points?
   A. They must be collinear.
   B. They must be coplanar.
   C. None of the points lie on the same line.
   D. None of the points lie in any other plane.

6. Based on the diagram, which theorem would you use to support the statement angle 1 is congruent to angle 2?



A. Alternate interior angles theorem
B. Alternate exterior angles theorem
C. Same side interior angles theorem
D. Parallel lines theorem

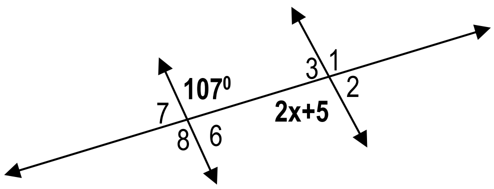
7. What is the measure of $\overline{AB}$ if C is between A and B, AC=10 and DB=14.

A. 5 B. 7 C. 12 D. 24

8. If $\angle CAT$ and $\angle EAT$ form a linear pair and $\angle EAT$ is a right angle, then what angle is $\angle CAT$?

A. acute B. obtuse C. right D. straight

For numbers 9 – 10:



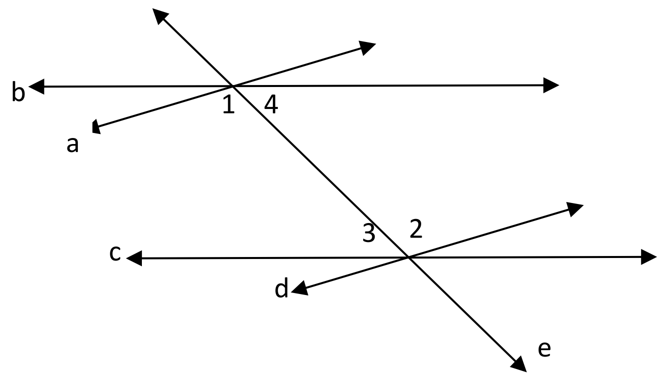
9. What is the value of x?

A. 51° B. 56° C. 102° D. 107°

10. What is the measure of $\angle 3$?

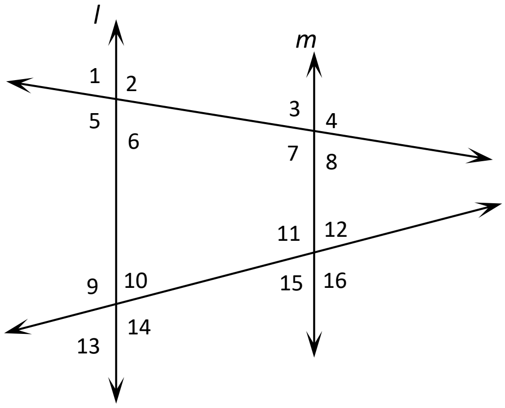
A. 51° B. 73° C. 83° D. 107°

11. In the figure below, b| | c with e as the transversal. What must be true about angle 1 and angle 2 if a| |d?



A. angle 1 is greater than angle 2
B. angle 1 is congruent to angle 2
C. angle 1 is a supplement of angle 2
D. angle 1 is a complement of angle 2

For numbers 12 – 15:



12. What special angle pair is being identified by angle 2 and angle 5?
    A. Alternate interior angles
    B. Corresponding Angles
    C. Same Side Exterior Angles
    D. Vertical Angles

13. Given $l \parallel m$ and $m\angle 9 = 96^\circ$ find $m\angle 11$?
    A. 48° B. 84° C. 96° D. 180°

14. Given $l \parallel m$ and $m\angle 1 = 84^\circ$ and $m\angle 8 = (2x - 4)^\circ$ find the value of x.
    A. 40 B. 42 C. 44 D. 84

15. Given $m\angle 6 = 80^\circ$ and $m\angle 7 = (x - 20)^\circ$, find the value of x so that $l \parallel m$
    A. 60 B. 100 C. 120 D. 180

Great job! You are done with this module.

References

A. Books

Abuzo, E. P., et.al. (2013). Mathematics – Grade 8 Learner's Module First Edition, 2013. Published by the Department of Education. Printed in the Philippines by Book Media Press, Inc.

Oronce, O. A. and Mendoza M. O. (2018). Exploring Math 8. Metro Manila. Rex Books Store

B. Online Resources

faculty.pingry.org › vmcgrath. Chapter 1 Points, Lines, Planes and Angles

https://www.google.com/url?sa=t&source=web&rct=j&url=http://www.wahibluedevils.org/images/courses_programs/Math/geometry_eoc_exam_practice_test1.pdf&ved=2ahUKEwiBhI2avvztAhXZLqYKHb3xD_cQFjAEegQIDxAB&usg=AOvVaw2g2ZlivMwsDOa2WNYU7cUK

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rexinteractive.com › Supplemental Math High School Grade 8 3rd Q.indd - Rex Interactive

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