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Mathematics

Quarter 1 - Module 8:
Finding the Equation and Solving Problems Involving Quadratic Function

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Quarter 1 - Module 8: Finding the Equation and Solving Problems Involving Quadratic Function
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Jumpstart

Activity 1. Where do you belong?

Directions: Copy the table below in your answer sheet and write the letter of the given equation in the table.

a. $y = x2 - 1$	e. $y = (2x+ 3)\ (x- 1)$	i. $3x + x^2 = y$
b. $y = x$	f. $y = x^3 + 1$	j. $2x(x - 3) - y = 0$
c. $2x^2 - 2x + 1 = y$	g. $2^2 + x = y$	
d. $3x-1 + y = 0$	h. $y = 3x + 2x$	

Quadratic Function	Not Linear nor Quadratic	Linear Function

Discover

Finding the Equation of a Quadratic Function

We can determine the equation of a quadratic function using the three methods: (a) table of values (b) graph and (c) zeros. Let us find out how to use these methods as we go on with this module.

A. Using the table of values

Assuming you are given table of values that represents a parabola, you can create system of three equations. Create the equations by substituting the ordered pair for each point into the general form of the quadratic equation, $ax^2 + bx + c$. Simplify each equation, then use the method of your choice to solve the system of equations for *a*, *b* and *c*. Finally substitute the values you found for *a*, *b*, and *c* into general equation to generate the equation for your parabola.

Example 1. Given table of values, determine the quadratic function.

$$f(x) = ax^2 + bx + c$$

x	0	-1	2	3	4
y	c	a+b+c	4a+2b+c	9a+3b+c	16a +4b+c

First difference	a+b	3a+b	5a+b	7a+b
Second difference	2a	2a	2a	

1. Equate 2a with the second difference in the table of values to find a

   2a = 4 ⟶ a = 2
2. Equate a+b with the first difference in the table of values and use the value a to find b.

   a + b =-3 ⟶ 2+b = -3 ⟶ b = -5
3. To find c, locate the ordered pair where x equals zero.

   c=-12

The quadratic function is $f(x)=2x^2-5x-12$.

Example 2. Given the following table of values, determine the quadratic function.

x	1	2	3
y	5	11	19

You are given three points along a parabola, you can find the quadratic equation that represents that parabola by creating a system of three equations. The following are the steps to follow:

1. Substitute the first pair of values into the general form of the quadratic equation: Solve for *a*.	$f(x) = ax^2 + bx + c$ $5 = a(1)^2 + b(1) + c$ simplifies to $a = -b - c + 5$
2. Substitute the second ordered pair and the value of *a* into the general equation. Solve for *b*	$11 = (-b - c + 5)(2)^2 + b(2) + c$ simplifies to $b = -1.5c + 4.5$.

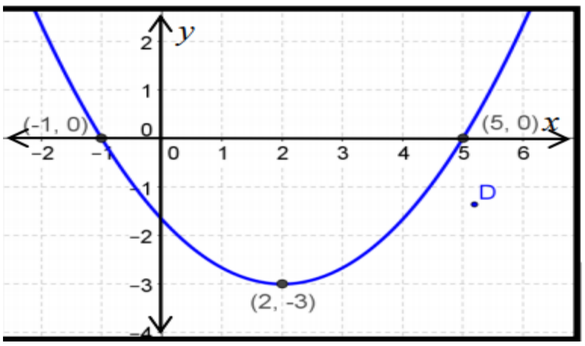
3. Substitute the third ordered pair and the values of *a* and *b* into the general equation. Solve for *c*.	19 = -(-1.5c + 4.5) - c + 5 + (-1.5c + 4.5)(3) + c simplifies to c = 1.
4. Substitute any ordered pair and the value of *c* into the general equation. Solve for *a*.	For instance, you can substitute (1, 5) into the equation to yield $5 = a(1)^2 + b(1) + 1$, which simplifies to a = -b +4
5. Substitute another ordered pair and the values of *a* and c into the general equation. Solve for *b*.	For example, $11 = (-b + 4)(2)^2 + b(2) + 1$ simplifies to b = 3.
6. Substitute the last ordered pair and the values of *b* and *c* into the general equation. Solve for *a*.	The last ordered pair is (3, 19), which yields the equation: $19 = a(3)^2 + 3(3) + 1$. This simplifies to a = 1.
7. Substitute the values of *a*, *b* and *c* into the general quadratic equation.	The equation that describes the graph with points (1, 5), (2, 11) and (3, 19) is $x^2 + 3x + 1$.

B. Graph

The graph of a quadratic function $y = ax^2 + bx + c$ is called **parabola.** Parabola is the locus of points in the plane that are equidistant from both the directrix and the focus. Parabola is like an umbrella, it could open upward or downward depending on the value of quadratic term ax^2. Usually $-ax^2$ creates a parabola that opens downward and ax^2 creates a parabola that opens upward. The vertex of the parabola is the point where the graph attains its minimum point if opens upward and maximum point if opens downward.

Study the illustrative examples presented below.

Example 1. Find the equation of the quadratic function determined from the graph below.



Solution:

The vertex of the graph of the quadratic function is (2,-3). The graph passes the point (5,0). By replacing x and y with 5 and 0, respectively, and h and k with 2 and -3 respectively, we have

$$y = a(x-h)^2 + k$$
$$0 = a(5-2)^2 + (-3)$$
$$0 = a(3)^2 - 3$$
$$3 = 9a$$
$$a = \frac{1}{3}$$

Thus, the quadratic equation is $y = \frac{1}{3}(x-2)^2 - 3$ or $y = \frac{1}{3}x^2 - \frac{4}{3}x - \frac{5}{3}$

Example 2. Express $y = 3x^2 - 4x + 1$ in the form $y = a(x - h)^2 + k$ form and give the values of h and k.

Solution:

$y = 3x^2 - 4x + 1$ $\quad$ $y = (3x^2 - 4x) + 1$

$y = 3(x^2 - 4x) + 1$ $\longrightarrow$ Group together the terms containing x

$y = 3\left(x^2 - \frac{4}{3}x\right) + 1$ $\longrightarrow$ Factor out a. Here a=3

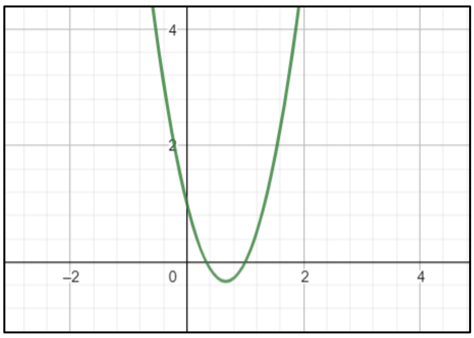
$y = \left[x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2\right] + 1 - \left(\frac{2}{3}\right)^2$ $\longrightarrow$ Compete the expression in parenthesis to make it a perfect square trinomial by adding the constant. $3(4/3/3)^2=3(2/3)^2=3(4/9)=4/3$ and subtracting the same value from the constant term.

$y = 3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + 1 - \frac{4}{9}$ $\longrightarrow$ Simplify and express the perfect square trinomial as the square of a binomial

$y = 3\left(x - \frac{2}{3}\right)^2 - \frac{1}{3}$ $\longrightarrow$ Hence, $y=3x^2-4x+1$ can be expressed as $y=3(x - \frac{2}{3})^2 - \frac{1}{3}$

In this case, h = $\frac{2}{3}$ and k = $\frac{1}{3}$.

We can graph the function as shown below.



C. Zeros

If r_1 and r_2 are the zeros of a quadratic function, then $f(x) = a\,(x- r_1)\,(x - r_2)$ where a is a nonzero constant that can be determined from other point on the graph. Also, you can use the sum and product of the zeros to find the equation of the quadratic function.

Example 1: Find an equation of a quadratic function whose zeros are -3 and 2.

Solution: Since the zeros are $r_1 = -3$ and $r_2 = 2$, then

$f(x) = a(\,x- r_1\,)\,(\,x - r_2\,)$

$f(x) = a[x - (-3)](\,x - 2)\,]$

$f(x) = a(x + 3)(\,x - 2\,)$

$f(x) = a(x^2 + x - 6)$ where a is any nonzero constant.

Example 2: Find an equation of a quadratic function with zeros $\frac{3 \pm \sqrt{23}}{3}$.

Solution: A quadratic expression with irrational roots cannot be written as a product of linear factors with rational coefficients. In this case, we can use another method. Since the zeros are $\frac{3 \pm \sqrt{23}}{3}$ then,

$$x = \frac{3 \pm \sqrt{23}}{3} \longrightarrow 3x=3\pm\sqrt{2} \longrightarrow 3x-3=\pm\sqrt{2}$$

Square both sides of the equation and simplify $\longrightarrow$ $9x^2-18x+9=2$

$9x^2-18x+7=0$

Thus, the equation of a quadratic function is $f(x)=9x^2-18x+7$

Solving Problems Involving Quadratic Equations

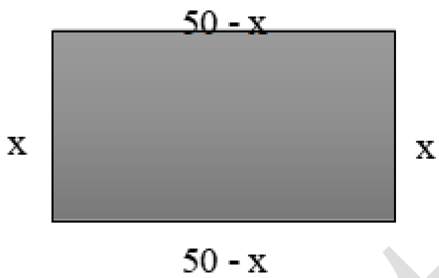
To solve for problems involving quadratic equation, concepts associated with quadratic functions particularly the maximum point and the minimum point of the graph. Most real-life problems involve concepts of maxima-minima. Consider the examples below:

Example 1. A rectangular garden will be enclosed by 100 m of fencing materials. Find the greatest possible area that the materials can enclose?

Representation: The problem asks for the maximum area that can be enclosed using 100m of fencing materials.

Let x = length of one side of the rectangular garden, then

$\frac{100-2x}{2}$ *or* $50 - x =$ the length of the other side adjust to it



Equation:
Express the area (A) as function of x,

A = x (50 – x)
A = -x² + 50 x

Solution:

Here a = -1, thus the graph opens downward and the vertex gives the maximum value at a given value of x.

To find the vertex, you can choose between completing the square or using the vertex formula. By completing the square,

$$A = -x^2 + 50x$$
$$= -(x^2 - 50x)$$
$$= -(x^2 - 50x + 625) + 625$$
$$= -(x - 25)^2 + 625$$

The vertex is at (25, 625). It means that the maximum area that can be enclosed from 100m of fencing materials is 625m².

The length of one side is x = 25m
The length of the other side 50 – x, or 25m

It indicates that the largest area for 100m fencing materials can be given by a square of side 25m.

Example 2: $f(x) = 2x^2 + 10x - 7$

$-2(x^2 - 5x) - 7$	⟶	Factor the coefficient
$-2\left(x^2 - 5x + \frac{25}{4}\right) + \frac{25}{4} - 7$	⟶	$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$; add and subtract
$-2(x - \frac{5}{2})^2 + \frac{11}{2}2$	⟶	Factor $\left(x^2 - 5x + \frac{25}{4}\right)$

The vertex of the graph of this function is at $(\frac{5}{2}, \frac{11}{2})$. The value of k or the y- coordinate of the vertex gives the maximum or minimum value depending on the opening of the graph.

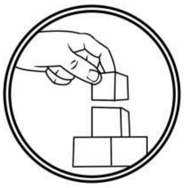
Therefore, the equation has a maximum value of $\frac{11}{2}$ at x = $\frac{5}{2}$.

REMEMBER:

The following are the steps to solve problems involving quadratic functions particularly maximum or minimum value:

1. Rewrite the unknowns using a single variable.
2. Write the equation of quadratic function in a general form.

3. Choose the appropriate method for the problem
    a. Complete the square and find (h, k)
    b. Use the vertex formula: V
    $V = (\frac{-b}{2a}, \frac{4ac-b^2}{4a})$
4. The value of k gives the maximum value or minimum value of the function at the given value of h.



Explore

Activity 3

***Directions**: The table of values below describes a quadratic function. Find the equation of the quadratic function by following the given procedure. Write your answer on your answer sheet.*

x	-3	-2	-1	0	1	2	3
y	24	16	10	6	4	4	6

Steps	Solutions
A. Substitute 3 ordered pairs (x,y) in $y = ax^2 + bx + c$	
B. What are the three equations you came up with?	
C. Solve for the values of a, b and c.	
D. Write the equation of the quadratic function $y = ax^2 + bx + c$.	

Activity 4: Step by Step!

***Directions:** Transform the given quadratic functions into the form $y = a(x - h)^2 + k$ by following the steps below. Write your answer on your answer sheet.*

1. $y = x^2 - 4x - 10$
2. $y = 3x^2 - 4x + 1$

Steps	Task
1. Group the terms containing x	
2. Factor out a	
3. Complete the expression in parenthesis to make it a perfect square trinomial.	
4. Express the perfect square trinomial as square of a binomial	
5. Give the value of h	
6. Give the value of k	

Deepen

Activity 5: Hit the Mark!

Directions: Analyze and solve the problem carefully. Write your answer on your answer sheet.

A company of cellular phones can sell 200 units per month at Php2,000.00 each. They found out that they can sell 50 more cellphone units every month for each Php100.00 decrease in price.

a. How much is the sales amount if cellphone units are priced at Php2000.00?
b. How much would be their sales if they sell each cellphone unit at Php 1600.00?
c. What is the equation for the revenue function?
d. What price per cell phone unit gives them the maximum monthly sales?
e. How much is the maximum sale?

Great job! You have understood the lesson. Are you now ready to summarize?

Gauge

Post-Assessment

***Directions**:* Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which of the following equations represents a quadratic function?
   A. $9y^2 + 3 = x$
   B. $y = 2x^2 – 7x + 1$
   C. $y = 3x - 8^2$
   D. $y = 3x – 3$
2. What is the quadratic function that this table of values represents?

x	-2	-1	0	1	2
y	5	0	-3	-4	-3

A. $y = 3x^2 -2x-2$
B. $y = 2x^2 -2x-3$
C. $y = -2x^2 +3x-2$
D. $y = -3x^2 +2x$