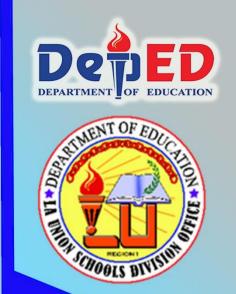
SHS

AIRs - LM in

Statistics and Probability Quarter 3: Week 5 - Module 5

Understanding Z-scores,







Statistics and Probability

Grade 11 Quarter 3: Week 5 - Module 5 First Edition, 2021

Copyright © 2021 La Union Schools Division Region I

All rights reserved. No part of this module may be reproduced in any form without written permission from the copyright owners.

Development Team of the Module

Author: Carla E. Collado, TIII

Catherine F. Carbonell, MT1

Editor: SDO La Union, Learning Resource Quality Assurance Team

Illustrator: Ernesto F. Ramos Jr., PII

Management Team:

ATTY. Donato D. Balderas, Jr. Schools Division Superintendent

Vivian Luz S. Pagatpatan, PhD Assistant Schools Division Superintendent

German E. Flora, PhD, CID Chief

Virgilio C. Boado, PhD, EPS in Charge of LRMS

Erlinda M. Dela Peña, EdD., EPS in Charge of Mathematics

Michael Jason D. Morales, PDO II

Claire P. Toluyen, Librarian II



Conversion is one of the most common terms we encounter in Mathematics subject. It is the process of changing or causing something to change from one form to another.

In your previous lesson, you already learned the definition of normal random variable and standard normal variable. This time you will be asked to convert normal random variable to standard normal variable and vice versa. In addition, you will learn how to compute probabilities and percentile of the random variables.

This module shall present you the information and activities that will help you comprehend conversion of normal random variable to standard normal variable, and vice versa, and to compute probabilities, and percentiles using the standard normal table.

After going through this module, you are expected to:

- 1. convert a normal random variable to a standard normal variable and vice versa (M11/12SP-IIIc-4), and
- 2. compute probabilities and percentiles using the standard normal table (M11/12SP-IIIc-d1).

Subtasks:

- 1. differentiate normal random variable and standard normal variable.
- 2. transform normal random variable to standard normal vice versa;
- 3. locate z-value from the standard normal table; and
- 4. analyze the correct probability based on the given situations.

Let's assess first how much you know about this topic. Answer the pretest below in a separate sheet of paper.

Pretest

Directions: Choose the letter of the correct answer. Write your answer in a separate sheet of paper.

- 1. What is being represented by z in the equation $z = \frac{x \overline{\mu}}{\sigma}$?
 - A. Normal Random Variable
- B. Standard Normal Variable
- C. Mean of the scores
- D. Standard Deviation
- 2. Which symbol represents population mean?
 - Α. σ

- B. \bar{X}
- C. µ

- D. z
- 3. Pedro, a garbage collector decided to sort soda cans to earn extra income. Let x = the total number of soda cans that he will collect for each day. What type of variable is x?
 - A. Concrete

- B. Continuous Random
- C. Discrete Random

- D. Not a Random Variable
- 4. The government suggested the implementation of National ID. Maria, and Juan were tasked to conduct census to take the total of legible people who can avail the said ID in San Juan, La Union. Let x = the total number of people that will be reflected in their census. What type of variable is x?
 - A. Constant

B. Continuous Discrete Random

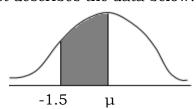
C. Discrete

- D. Not a Random Variable
- 5. What table is used to know the percentage under the curve at any particular point?
 - A. s-table

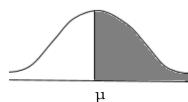
B. t-table

C. v-table

- D. z-table
- 6. Which best describes the data below?



- A. All data is $-1.5 > \mu$.
- B. All data is $-1.5 < \mu$.
- C. All data is -1.5 or lower.
- D. All data is -1.5 or higher.
- 7. Which best describes the data below?



- A. The scores are lower than the mean.
- B. The scores are higher than the mean.
- C. The scores could be equal or lower than the mean.
- D. The scores could be equal or higher than the mean.

- 8. Refer to the figure in item 7, what property of normal curve is being presented?
 - A. The total area under the curve is 1.
 - B. The curve is symmetric at the center.
 - C. Each division reflects 50% probability.
 - D. The mean is in the middle part of a normal curve.
- 9. Which is true among the following?
 - A. Converting standard normal variable to normal random variable has the formula $z = \frac{X + \mu}{\sigma}$.
 - B. Converting standard normal variable to normal random variable has the formula $z = \frac{X - \mu}{\sigma}$.
 - C. Converting standard normal variable to normal random variable has the formula $X = \mu + \sigma z$.
 - D. Converting standard normal variable to normal random variable has the formula $X = \overline{\mu} - \sigma z$.
- 10. A standard normal distribution has which of the following properties?
 - A. The mean is equal to the standard deviation.
 - B. The mean and the standard deviation both equal 1.
 - C. The mean is equal to the standard deviation.
 - D. The mean is equal to 0 and the standard deviation is equal to 1.
- 11. What is z when X = 45, $\mu = 67$, and $\sigma = 8$?
 - A. -2.75
- B. -0.55
- C. 0.55
- D. 2.75
- 12. Let z be a normal random variable with a mean of 0 and a standard deviation of 1. Determine $P(z \le 1.4)$.
 - A. 0

B. 0.0808

C. 0. 4192

- D. 0.9192
- 13. Which is the upper 10% of the normal curve?
 - A. z = 0.1554 B. z = 1.28
- C. z = 1.54 D. z = 2.43
- 14. Which is the correct probability notation for the area between z = -2 and z = -1.5?
 - A. P(-2 < z < -1.5)

B. P(-1.5 < z < -2)

C. P(z < -1.5 + -2)

- D. P(-2 1.5 < z)
- 15. What is the area between z = -2 and z = -1.5?
 - A. 0.0440 B. 0.4332
- C. 0.4772
- D. 0.5



Jumpstart

For you to use your prior knowledge to better understand this topic, do the activity below. Hope you enjoy it. Good luck!

Activity 1: Fact or Bluff!

Directions: Read each item carefully. Write FACT if the statement is correct otherwise write BLUFF.

- 1. A standard normal random variable is a normally distributed random variable with mean equal to 0 and standard deviation equal to 1.
- 2. The normal random variable is denoted by letter Z
- 3. The probability value corresponds to the area of the normal curve.
- 4. A percentile is a measure of relative standing.
- 5. The formula to be used in converting standard normal variable to normal random variable is $z = \frac{X-\mu}{\sigma}$
- 6. A probability is a descriptive measure of the relationship of a measurement to the rest of the data.
- 7. The z-scores to the left of the mean are negative values.
- 8. In locating area at the body of the table, if the exact area is not available, we take the nearest area.
- 9. Standard normal table is also known as table of areas under the normal curve or z-table.
- 10. In converting normal random variable to standard normal variable the formula to be used is $x = \mu \sigma z$



The **standard normal distribution** has the special case where the mean (μ) , equals 0, and the standard deviation (σ) equals 1. The normally distributed random variable under this standard normal distribution is known as **standard normal variable** and it is always denoted by letter Z. We use the standard score, also known as z-scores or z-values (z) to represent raw scores(X) may be composed of large values that cannot be accommodated at the baseline of the normal curve. Thus, conversion from normal random variable(X) to standard normal variable(z) is highly encouraged. In addition, if someone wants to determine the exact or raw scores in the standard normal distribution, conversion from standard normal variable to normal random variable will be done.

Below are the formulae to be used in converting normal random variable(X) to standard normal variable (z) also known as the z-scores.

z-score for population

$$z = \frac{X - \mu}{\sigma}$$

where: z = z-score or z-value

X = raw score or the given measurement

 μ = population mean

 σ = population standard deviation

z-score for sample

$$z = \frac{X - \bar{X}}{S}$$

where: z = z-score or z-value

X = raw score or the given measurement

 \bar{X} = sample mean

s = sample standard deviation

On the other hand, the formula to be used in converting standard normal variable(z) to normal random variable(x) is

$$X = \mu + \sigma z$$

where: X = raw score

z = z-score or z-value

 μ = population mean

 σ = population standard deviation

Example 1: Given the mean μ = 50 and the standard deviation, σ = 5 of a population of Statistics and Probability scores. Find the z-value that corresponds to a score X = 65.

$$\mu = 50$$

$$\sigma$$
 = 5

Find: the z-value that corresponds to a score x = 65.

Solution:
$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{65 - 50}{5}$$

$$\mathbf{Z} = \frac{15}{5}$$

z = 3



-3 -2 -1 0 1 2 3 35 40 45 50 55 60 65

Answer: The z-value that corresponds to a score X = 65 is 3 in a population distribution.

Interpretation: Since the score X = 65 corresponds to z = 3, We can say that, with respect to the mean, the score of 65 is above average.

Example 2: The mean μ = 20, and the standard deviation σ = 10 of a set of summative scores of a Grade 11 class, Find the z-value that corresponds to a score X = 32.

$$\mu = 20$$

$$\sigma = 10$$

Find: the z-value that corresponds to a score X = 32.

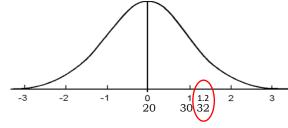
Solution:
$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{32 - 20}{10}$$

$$z = \frac{12}{10}$$

$$z = 1.2$$

Illustration:



Answer: The z-value that corresponds to a score x = 32 is 1.2 in a population distribution.

Example 3. A pretest was given to Tourism 11 class with a mean μ = 30, and the standard deviation σ = 10. If a student's z-score is -1, then what is the score of the students before the conversion?

Given: z = -1

$$\mu = 30$$

$$\sigma = 10$$

Find: the score(X) of the student before the conversion

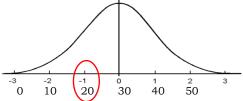
Solution: $X = \mu + \sigma z$

$$X = 30 + 10(-1)$$

$$X = 30 + (-10)$$

$$X = 20$$

Illustration:



Answer: The raw score that corresponds to a z = -1 is 20 in a population distribution. It is below average.

Example 4: A farmer recorded that this past 2020, he only gained a mean μ = 20, and the standard deviation σ = 5 sacks of rice in each hectare due to the typhoon. If he assumed that he had less sacks of rice equivalent to -1.6, how many sacks will it be?

Given: z = -1.6

$$\mu = 20$$

$$\sigma$$
 = 5

Find: number of sacks of rice

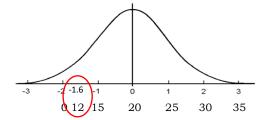
Solution: $X = \mu + \sigma z$

$$X = 20 + 5(-1.6)$$

$$X = 20 + (-8)$$

$$X = 12$$

Illustration:



Answer: There will only be 12 sacks of rice.

COMPUTING THE PROBABILITIES USING THE STANDARD NORMAL TABLE

The z-values are matched with specific areas under the normal curve using the standard normal table. The area under the curve is the **probability**, this probability gives the desired percentage for x.

The following notations for a random variable are used in various solutions concerning the normal curve.

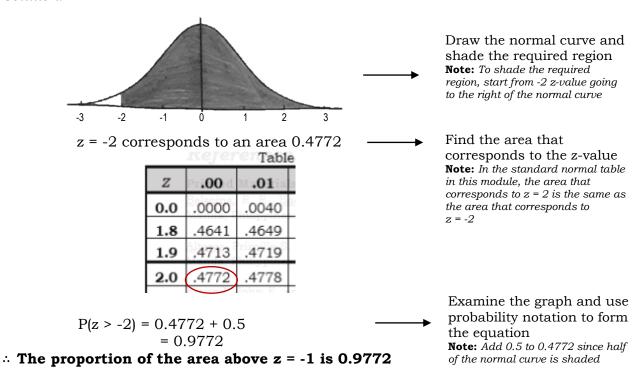
P(a < z < b) denotes the probability that the z-score is between a and b.

P(z > a) denotes the probability that the z-score is greater than a.

P(z < a) denotes the probability that the z-score is less than a.

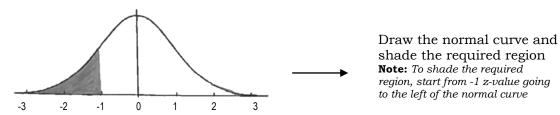
Example 1: Find the proportion of the area above -2.

Solution:



Example 2: Find the area to the left of z = -1

Solution:



z = -1 corn	responds to a	an area 0.34	413 —
--------------	---------------	--------------	-------

		1401
Z	.00	.01
0.0	.0000	.0040
0.8	.2881	.2910
0.9	.3159	.3186
1.0	.3413	.3438

$$P(z < -1) = 0.5 - 0.3413$$
$$= 0.1587$$

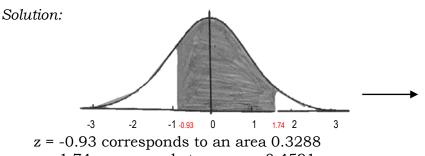
Find the area that corresponds to the z-value **Note:** *In the standard normal table* in this module, the area that corresponds to z = 1 is the same as the area that corresponds to z = -1

Examine the graph and use probability notation to form the equation

Note: Subtract 0.3413 from 0.5

\therefore The proportion of the area above z = -1 is 0.1587

Example 3: Find the area between z = -0.93 and z = 1.74



Draw the normal curve and shade the required region **Note:** To shade the required region, start from -0.93 to 1.74 zvalue

z = 1.74 corresponds to an area 0.4591

0 0 2288	Z	.03
0.9 (.3200)	0.9	.3288

Find the area that corresponds to the z-value **Note:** *In the standard normal table* in this module, the area that corresponds to z = 0.93 is the same as the area that corresponds to z = -0.93

Examine the graph and use probability notation to form the equation

Note: Add 0.3288 and 0.4591

 \therefore The area between z = -0.93 and z = 1.74 is 0.07879

Example 4: Ms. Santos, a Statistics and Probability teacher of San Juan Senior High School-Stand Alone wants to know the probability of the scores of her students of getting lower than 80 with a μ = 75, and σ =10. We know that the probability P(x < 80) = z-value

Solution:

$$z = \frac{X - \mu}{\sigma} = \frac{80 - 75}{10} = 0.5$$

→ Convert to z-score form

0.5 corresponds to z = 0.1915

$$P(x < 80) = 0.5 + 0.1915$$

→ Locate 0.5 in z-table

$$P(x < 80) = 0.5 + 0.19$$

→ Use the probability notation to form

1 - (0.5 + 0.3944)P(x<80) = 0.6915

the equation

: The probability that the students will get lower than 80 is 0.6915 or 69.15%.

COMPUTING PERCENTILE USING THE STANDARD NORMAL TABLE

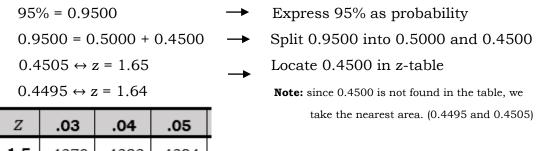
A **percentile** is a measure of relative standing. It is a descriptive measure of the relationship of a measurement to the rest of the data. For example, you got 78 in a test in Statistics and Probability and you want to know how's your standing in comparison with your classmates. If your teacher tells you that you scored at the 89th percentile, then it means that 89% of the grades were lower than yours and 11% lies above or higher.

Four important things to remember:

- 1. A probability value corresponds to an area under the normal curve
- 2. The numbers in the body of the table of areas under the normal curve or the z-table, also known as the standard normal table are areas or probability.
- 3. z-scores to the left of the mean are negative values.
- 4. In locating area at the body of the table, if the exact area is not available, we take the nearest area.

Example 1: Find the 95^{th} percentile(P_{95}) of a normal curve.

Solution:



Z	.03	.04	.05
1.5	.4370	.4382	.4394
1.6	.4484	4495	4505
1.7	.4582	.4591	.4599

$$\frac{1.65 + 1.64}{2}$$
 = 1.645 Find the average of the two-values

 \therefore The 95th percentile is z = 1.645

Example 2: Find the upper 20% of the normal curve.

Solution:

0.3000	$\leftrightarrow z =$	0.84
--------	-----------------------	------

Z	.03	.04	.05
0.8	.2967	.2995)	.3023
0.9	.3288	.3264	.3289

Locate 0.3000 in z-table

Note: since 0.3000 is not found in the table, we take the nearest area. 0.2995

\therefore The upper 20% is above z = 0.84

Example 3: The results of the National Achievement Test in Mathematics are normally distributed with μ = 75 and σ = 10. What is the percentile rank of the score 88?

Solution:

: The percentile rank of the score 88 in a test is 90.32%



Explore

Enrichment Activity 1: Convert Me!

Directions: Read carefully each item. Write the letter of the best answer for each test item.

- 1. What is z when x = 16, $\mu = 40$, and $\sigma = 10$?
 - A. -2.4
- B. -1.2
- C 1 2
- D. 2.4
- 2. What is z when x = 55, μ = 25, and σ = 20?
 - A. -2.4
- B. -1.25
- C. 1.5
- D. 2.4
- 3. What is z when x = 100, μ = 112, and σ = 8?
 - A. -1.5
- B. -1.2
- C. 1.2
- D. 1.5
- 4. What is x when z = 0.5, $\mu = 40$, and $\sigma = 10$?
 - A. 30
- B. 35
- C. 40
- D. 45
- 5. What is x when z = 1.15, $\mu = 200$, and $\sigma = 120$?
 - A. 318
- B. 328
- C. 338
- D. 328

Assessment 1

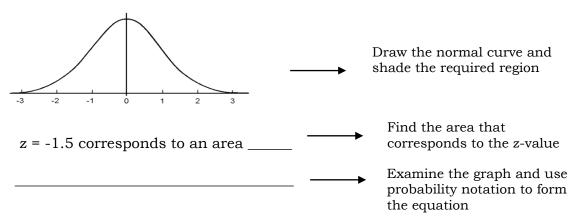
Directions: Read, analyze and solve the problem correctly. Show your complete solution with illustration.

Ms. Perez, a call center agent computed a μ = 20, and the standard deviation σ = 5 of the number of clients she assists in half of her working hours. Find the z-value that corresponds to a score X = 12.

Enrichment Activity 2: Complete Me!

Find the area to the left of z = -1.5

Solution:



 \therefore The proportion of the area to the left of z = -1.5 is _____.

Assessment 2

Directions: Read, analyze and solve the problem correctly. Show your complete solution with illustration.

An agriculturist wants to find out the probability of growing trees more than 50 years. Given that μ = 30, and σ = 20.



At this point, you are going to apply what you have learn about converting normal random variable to standard normal variable and vice versa, compute probabilities and percentile using the standard normal table or z-table.

What you need:

Extra sheet of paper

Pencil

What you have to do:

Read and analyze the problem below. After reading, answer the question that follows. You will be scored based on the rubric below.

Problem:

The weights of the Grade 11 students are known to be normally distributed with a mean of 54 kg and a standard deviation of 8 kg.

- 1. Find the percentage of Grade 11 students with weights between 45 kg and 66 kg.
- 2. Find the 60th percentile for the weight of Grade 11 students known to be normally distributed with a mean of 54 kg and a standard deviation of 8 kg.

Scoring Rubric

Criteria	4 points	3 points	2 points	1 point
	Shows	Shows	Shows	The solution
Accuracy of	accurate solution with		solution with	is all
the Solution	solution.	minimal	plenty of	erroneous.
		errors.	errors.	
	Shows	Shows clear	Shows limited	Did not apply
	excellent understandin		understanding	the concept of
	understanding of the concept		of the concept	solving
Mathematical	of the concept	of solving	of solving	problems
Concept	of solving problems		problems	involving
Concept	problems	involving	involving	random
	involving	random	random	variables.
	random	variables.	variables.	
	variables.			



Gauge

Directions: Choose the letter of the correct answer. Write your answer in a separate sheet of paper.

1. What is being represented by z in the equation $z = \frac{x - \overline{\mu}}{\sigma}$?

B. Normal Random Variable

B. Standard Normal Variable

C. Mean of the scores

D. Standard Deviation

2. Which symbol represents population mean?

Β. σ

 $\mathbf{R}^{\hat{\lambda}}$

C. 1

D. z

3. Pedro, a garbage collector decided to sort soda cans to earn extra income. Let x = the total number of soda cans that he will collect for each day. What type of variable is x?

A. Concrete

B. Continuous Random

C. Discrete Random

D. Not a Random Variable

4. The government suggested the implementation of National ID. Maria, and Juan were tasked to conduct census to take the total of legible people who can avail the said ID in San Juan, La Union. Let x = the total number of people that will be reflected in their census. What type of variable is x?

A. Constant

B. Continuous Discrete Random

C. Discrete

D. Not a Random Variable

5. What table is used to know the percentage under the curve at any particular point?

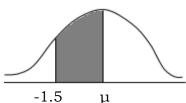
B. s-table

B. t-table

C. v-table

D. z-table

6. Which best describes the data below?



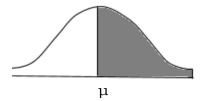
A. All data is $-1.5 \ge \mu$.

B. All data is $-1.5 \le \mu$.

C. All data is -1.5 or lower.

D. All data is -1.5 or higher.

7. Which best describes the data below?



- A. The scores are lower than the mean.
- B. The scores are higher than the mean.
- C. The scores could be equal or lower than the mean.
- D. The scores could be equal or higher than the mean.
- 8. Refer to the figure in item 7, what property of normal curve is being presented?
 - A. The total area under the curve is 1.
 - B. The curve is symmetric at the center.
 - C. Each division reflects 50% probability.
 - D. The mean is in the middle part of a normal curve.
- 9. Which is true among the following?
 - A. Converting standard normal variable to normal random variable has the formula $z = \frac{X + \mu}{\sigma}$.
 - B. Converting standard normal variable to normal random variable has the formula $z = \frac{X - \mu}{\sigma}$.
 - C. Converting standard normal variable to normal random variable has the formula $X = \mu + \sigma z$.
 - D. Converting standard normal variable to normal random variable has the formula $X = \overline{\mu} - \sigma z$.
- 10. A standard normal distribution has which of the following properties?
 - A. The mean is equal to the standard deviation.
 - B. The mean and the standard deviation both equal 1.
 - C. The mean is equal to the standard deviation.
 - D. The mean is equal to 0 and the standard deviation is equal to 1.
- 11. What is z when X = 45, $\mu = 67$, and $\sigma = 8$?

C. 0.55

D. 2.75

12. Let z be a normal random variable with a mean of 0 and a standard deviation of 1. Determine $P(z \le 1.4)$.

B. 0.0808

C. 0. 4192

D. 0.9192

13. Which is the upper 10% of the normal curve?

A.
$$z = 0.1554$$

B.
$$z = 1.28$$

$$C_{7} = 1.54$$

C.
$$z = 1.54$$
 D. $z = 2.43$

14. Which is the correct probability notation for the area between z = -2 and z = -1.5?

A.
$$P(-2 < z < -1.5)$$

B.
$$P(-1.5 < z < -2)$$

C.
$$P(z < -1.5 + -2)$$

D.
$$P(-2 - 1.5 < z)$$

15. What is the area between z = -2 and z = -1.5?

References

Printed Materials:

Belencina, R.R., Baccay E.S. & Mateo, E.B. (2016). Statistics and Probability. Sampaloc, Manila, Philippines: Rex Book Store, Inc.

Websites:

Z-table (Right of Curve or Left) - Statistics How To

Normal Distribution Practice | Statistics - Quizizz

https://www.slideshare.net/AileenLositano1/converting-normal-to-standard-

normal-distribution-and-vice-versa-ppt

z-table - Bing images

How to do Normal Distributions Calculations | Laerd Statistics

3.3.3 - Probabilities for Normal Random Variables (Z-scores) | STAT 500 (psu.edu)

Table I: Table of Areas under the Normal Curve

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0150	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1253	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3288	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3557	.3559	.3621
1.1	.3642	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
	For values of z above 3.09, use 0.4999 Joomla SEF URLs by Artio									