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Mathematics

Quarter 3 – Week 5 - Module 5: Proportion

AIRs - LM

Mathematics 9
Quarter 3- Week 5 - Module 5: Proportion
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Target

This module was designed to help learners gain understanding about proportion and apply its properties in solving problems.

After going through the module, you are expected to:

1. describe a proportion (M9GE-IIIf-1); and
2. apply the fundamental theorems of proportionality to solve problems involving proportions. (M9GE-IIIf-2)

   Subtasks :
   1. Recall concepts on ratios and proportions.
   2. Solve problems involving proportions.
   3. Apply the fundamental rule and properties of proportion in solving problems.

Before going on, let's check how much you know about the lesson. Answer the pre-assessment below.

Pre-Assessment

Directions: Read and analyze each item carefully. Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. It is a statement that two ratios are equal.
   A. Fraction B. Proportion C. Ratio D. Similarity

2. What is used to compare two or more quantities?
   A. Fraction B. Proportion C. Ratio D. Similarity

3. Which are equivalent ratios?
   A. 4 : 8 and 1 : 2
   B. 3 : 7 and 4 : 5
   C. 5 : 3 and 11 : 6
   D. 8 : 6 and 6 : 8

4. If $\frac{m}{n} = \frac{o}{p}$ then, $\frac{m}{o} = \frac{n}{p}$ determine the property of proportion that describes the example.
   A. Cross-multiplication Property B. Alternation Property
   C. Inverse Property D. Addition Property

5. Which of the following ratios is proportion to 3: 5?
   A. 9 : 15 B. 12 : 20 C. 6 : 18 D. 9 : 12

6. What is the missing term in the proportion 6: 4 = 12: **x**?
   A. 4 B. 6 C. 7 D. 8

7. If a: b = 1: 3, find 2a – 5b: 4a + b.
   A. $\frac{13}{7}$ B. $\frac{7}{13}$ C. $\frac{-7}{13}$ D. $\frac{-13}{7}$

8. Solve the proportion: $\frac{3}{8} = \frac{15}{x}$.
   A. 40 B. 30 C. 20 D. 10

9. If $\frac{x}{y} = \frac{2}{3}$, what can you conclude?
   A. 3x = 2y B. x + y = 5 C. 2x = 3y D. xy = 6

10. Errol can run 1 km in 12 minutes. How many minutes would it take to run for 4 km?
    A. 36 B. 48 C. 54 D. 60

11. The measures of the acute angles of a right triangle are in the ratio of 2:3. What is the measure of the larger angle?
    A. 18 B. 36 C. 54 D. 72

12. Solve the proportion: $\frac{6}{9} = \frac{b-3}{6}$.
    A. 4 B. 7 C. 12 D. 16

13. A room is 24 feet wide. A scale model of a room is 1 inch = 6 feet. What is the width of the room in the model?
    A. 4 inches B. 5 inches C. 6 inches D. 7 inches

14. The length and the width of the rectangle are in the ratio of 3: 1. If the width is 7 cm, what is the length of the rectangle?
    A. 14 cm B. 16 cm C. 18 cm D. 21 cm

15. Two complementary angles are in the ratio of 1: 2. Find the ratio of the larger angle.
    A. 15 B. 30 C. 45 D. 60

Lesson 1	Proportion

Jumpstart

For you to understand the lesson well, let's first recall concepts on ratio.

Let us consider the following problem:

> *If there are 40 students and 2 wall fans in a classroom. What is the student-wall fan ratio?*
>
> The student-wall fan ratio is 40: 2 or 20: 1.

A ***ratio*** is a comparison of two or more quantities. It can be written in the variety of forms or ways: a to b, a: b, or $\frac{a}{b}$.

Given two numbers ***a*** and ***b***, ***b*** ≠0, a ratio is ***a*** divided by ***b***. Ratios, like fraction can be written in lowest terms.

Now, try the following activities:

ACTIVITY 1. Make Me Simple!

Express the following ratio of the first quantity to the second quantity in simplest form.

1. 4 boys : 8 girls
2. 16 dm : 48 dm
3. 5m : 300 cm
4. 400 g : 600 g
5. 5 days : 1 week

ACTIVITY 2. Find my Ratio!

Give the ratio of the following statements.

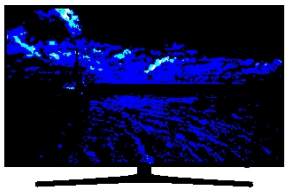
1. The measure of one angle of an equilateral triangle to the sum its interior angles.
2. The side of the square is 4 cm to its perimeter.
3. The sum of the interior angle of a triangle to the sum of the interior angle of a quadrilateral.

Discover

Now, let us discover concepts and skills involving proportion.

Let us consider a problem involving two ratios.

Suppose the length and the width of a flat-screen television screen are 48 inches and 32 inches, respectively. If the image for the advertisement of the television has a length of 12 inches, how wide should the image be?



This situation forms two equivalent ratios expressed as proportion.

A **proportion** states that two ratios are equal.

In the given problem above, the ratio of the dimensions of the television is 48: 32, while the ratio of the dimension of the advertisement is 12: x. Since the new length, 12 inches is one-fourth of 48, the width of the image should also be one-fourth of the width of the original. Hence, the width of the image must be $\frac{1}{4}$ (32) = 8 inches.

By using **proportion**, we will be able to solve the missing dimension of the image.

If the ratios 48: 32 and 12: 8 are expressed in lowest terms, they will be both 3: 2; therefore, 48: 32 and 12: 8 are equal ratios or simply, ***proportional.***

PROPORTION AND ITS PARTS

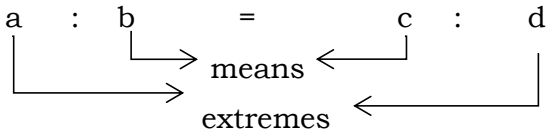
A **proportion** is a statement of equality between two ratios.

In symbols; $\frac{a}{b} = \frac{c}{d}$ or a : b = c : d, where b ≠ 0, d ≠ 0.

Each number in a proportion is called *term.*

$$\frac{\boldsymbol{a}\ (\boldsymbol{first\ term})}{\boldsymbol{b}\ (\boldsymbol{second\ term})} = \frac{\boldsymbol{c}\ (\boldsymbol{third\ term})}{\boldsymbol{d}\ (\boldsymbol{fourth\ term})}$$

The second and third terms are called the *means* and the first and fourth terms are called the *extremes* of the proportion.



FUNDAMENTAL RULE OF PROPORTION

If a : b = c : d, then $\frac{a}{b} = \frac{c}{d}$ provided that b≠0, d≠0.

PROPERTIES OF PROPORTION

1. Cross-Multiplication Property If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$; $b \neq 0, d \neq 0$

2. Alternation Property If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$; $b \neq 0, c \neq 0, d \neq 0$

3. Inverse Property If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{b}$; $b \neq 0, c \neq 0, d \neq 0$

4. Addition Property If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$; $b \neq , d \neq 0$

5. Subtraction Property If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$; $b \neq , d \neq 0$

6. Numerator-Denominator Sum Property If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$, then $\frac{a+C+e\ldots}{b+d+f} = \frac{a}{b} = \frac{c}{d} \ldots;$

Applying the fundamental rule of proportion and the properties of proportion, study the given examples.

Example 1.

Find the value of x in the proportion $\frac{4}{x} = \frac{6}{9}$.

Solution:

a. Using the Cross-Multiplication Property

$$\frac{4}{x} = \frac{6}{9} \longrightarrow 6(x) = 4(9) \longrightarrow \mathbf{x = 6}$$

b. Using the Alternation Property

$$\frac{4}{x} = \frac{6}{9} \longrightarrow \frac{4}{6} = \frac{x}{9} \longrightarrow 6(x) = 4(9) \longrightarrow \mathbf{x = 6}$$

c. Using the Inverse Property

$$\frac{4}{x} = \frac{6}{9} \longrightarrow \frac{x}{4} = \frac{9}{6} \longrightarrow 6(x) = 4(9) \longrightarrow \mathbf{x = 6}$$

d. Using Addition Property

$$\frac{4}{x} = \frac{6}{9} \longrightarrow \frac{4+x}{x} = \frac{6+9}{9}$$

$9(4 + x) = x (6 + 9)$

$36 + 9x = 6x + 9x$

$-6x = -36$

$\mathbf{x = 6}$

e. Using Subtraction Property

$$\frac{4}{x} = \frac{6}{9} \longrightarrow \frac{4-x}{x} = \frac{6-9}{9}$$

$9(4 - x) = x (6 - 9)$

$36 - 9x = 6x - 9x$

$-6x = -36$

$\mathbf{x = 6}$

Example 2.

If a: b = 4: 3, find 3a – 2b: 3a + b.

Solution:

$\frac{a}{b} = \frac{4}{3} \longrightarrow a = \frac{4b}{3}$

Using a $= \frac{4b}{3}$

$$\frac{3a-2b}{3a+b} = \frac{3(\frac{4b}{3})-2b}{3(\frac{4b}{3})+b} = \frac{4b-2b}{4b+b} = \frac{2b}{5b} = \frac{2}{5}$$

Therefore,

3a – 2b: 3a + b = 2: 5

Example 3.

If a and b represent two non-zero numbers, find the ratio a: b if $2a^2 + ab - 3b^2 = 0$.

Solution:

$2a^2 + ab – 3b^2 = 0$

$(2a + 3b)(a – b) = 0 \longrightarrow$ Factor $2a^2 + ab – 3b^2$

$2a + 3b = 0 \longrightarrow$ Equate each factor to 0

$a – b = 0$

For each factor: $2a + 3b = 0$ $\qquad$ $a – b = 0$

$2a = -3b$ $\qquad$ $a = b$

$\frac{2a}{2b} = \frac{-3b}{2b}$ $\qquad$ $\frac{a}{b} = \frac{b}{b}$

$\frac{a}{b} = \frac{-3}{2}$ $\qquad$ $\frac{a}{b} = \frac{1}{1}$

Hence, $\frac{a}{b} = \frac{-3}{2} = \frac{1}{1}$ or a:b = -3:2 or 1:1

Example 4.

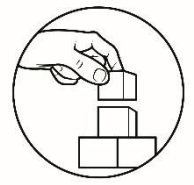
If $\frac{p}{2} = \frac{q}{3} = \frac{r}{4} = \frac{5p-6q-7r}{x}$. Find x.

Solution:

Let $\frac{p}{2} = \frac{q}{3} = \frac{r}{4} = \frac{5p-6q-7r}{x}$ = k. Then,

p= 2k, q = 3k, r = 4k, and $5p - 6q - 7r$ = kx.

5(2k)- 6(3k) -7(4k) = kx
-36k = kx
x = -36



Explore

ACTIVITY 3. Find my X!

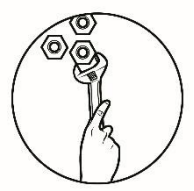
Find the value of x in the proportion.

1. 2 : x = 15 : 30
2. x : 3 = 18 : 2
3. 16 : x = 4 : 8
4. $\frac{5}{6} = \frac{x}{12}$
5. $\frac{12}{x} = \frac{x}{3}$

ACTIVITY 4: Solve Me!

Solve the given problems.

1. Find $\frac{y}{x}$ = 5y – 2x : 10 = 3y – x : 7
2. Solve for the ratio x: y if $x^2 + 3xy - 10y^2 = 0$
3. Solve for the ratio x: y if $x^2 + 3xy - 10y^2 = 0$



Deepen

Apply the concepts and skills learned in the lesson by doing the following activities.

ACTIVITY 5

Using the fundamental rule and the properties proportion, determine what is asked in the following situations:

1. The measures of the three angles of a triangle are in the ratio of 1:2: 3. Find their measures.
2. Two complementary angles are in the ratio of 2: 3. Find the measure of each angle.
3. The angles of a triangle are in the ratio of 2: 3: 5. Find the measure of the smallest angle.

ACTIVITY 6

Solve the following real-life problems involving proportion.

1. The ratio of the female and male in a certain barangay is 7: 5. If there are 3,192 females, what is the total population of the school?
2. A ladder measures 9 feet long leans against a building 7 feet above the ground. At what height would a 15 feet ladder touch the building form the same angle with the ground?

Gauge

Multiple Choice: Choose the letter of the correct answer. Write the chosen letter on a separate answer sheet.

1. What do you call a statement that two ratios are equal?
   A. Fraction B. Proportion C. Ratio D. Similarity

2. It is used to compare two or more quantities.
   A. Fraction B. Proportion C. Ratio D. Similarity

3. In the proportion, a: b = c: d, the second and the third terms are called the ____
   A. Denominator B. Extremes C. Means D. Numerator

4. Which property of proportion states that the product of the means is equal to the product of the extremes?
   A. Cross-multiplication Property
   B. Alternation Property
   C. Inverse Property
   D. Addition Property
5. If $\frac{m}{n} = \frac{o}{p}$ then, $\frac{m}{o} = \frac{n}{p}$, determine the property of proportion that describes the illustration.
   A. Cross-multiplication Property
   B. Alternation Property
   C. Inverse Property
   D. Addition Property
6. Which of the following ratios is proportion to 3: 4?
   A. 9:15 B. 12:20 C. 6:18 D. 9:12
7. What is the missing term in the proportion 6: 8 = 3: x?
   A. 4 B. 5 C. 6 D. 7
8. Express in the simplest form the ratio AB: CD, if AB = 20 m and CD = 40 cm.
   A. 1:5 B. 1:2 C. 2:1 D. 5:1
9. If a:b = 1:3, find 2a – 5b:4a + b.
   A. $\frac{13}{7}$ B. $\frac{7}{13}$ C $\frac{-7}{13}$ D. $\frac{-13}{7}$
10. What is the ratio of the measure of one angle of a rectangle to the sum of the angles of the square?
   A. 1:4 B. 13 C.2:3 D. 4:1
11. What is the simplest form of the ratio $6x^2$ to $18x^3$? Assume that no zero denominator.
   A. 1:6x B. 1:3x C. x:6 D. x:3
12. A pole casts a shadow of 27 feet long. A person standing nearby casts a shadow of 8 feet long. If a person is 6 feet tall. How tall is the pole?
   A. 13. 5 feet B. 20. 25 feet C. 24 feet D. 48 feet
13. Find the missing term in the proportion $\frac{4}{8} = \frac{2}{x}$.
   A. 2 B. 4 C. 6 D. 8
14. Joel is making a model of a car. The real car is 8 feet long and 4 feet wide. Joel's model will be 12 cm long. Which proportion could be used to solve for the width x?
   A. 8:4 = 12:x B. 8:4 = x:12 C. 8:12 = x:4 D. 12:8 = 4:x
15. It takes 6 hours for 2 persons to paint a room. How many hours will it take for 8 persons to do the same task?
   A. 1 B. 1.5 C. 2 D. 3

References

Diaz, Z. B., et.al. (2014). *Next Century Mathematics*. Quezon City. Phoenix Publishing House Inc.

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