9

Mathematics

Quarter 3- Week 4 Module 4

Solving Problems Involving Parallelograms, Trapezoids and Kites

AIRs - LM

Mathematics 9
QUARTER 3-WEEK 4 MODULE 4: Solving Problems Involving Parallelograms, Trapezoids and Kites

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Development Team of the Module

Writer: Marites M. Ugay
Editor: SDO La Union, Learning Resource Quality Assurance Team

Lesson 4	Solving Problems Involving Parallelogram, Trapezoids and Kites

In the previous lessons, you have learned about the three types of quadrilaterals: the parallelograms, trapezoids and the kite. You have also learned about each of their properties. It is important to remember the properties since they are useful in solving problems on geometry.

Activity 1: FACT OR BLUFF!

A review on the properties of parallelograms, trapezoids and kites. Read carefully the direction.

DIRECTION: Write **FACT** if the statement identifies the properties of parallelogram, trapezoids and kites. **BLUFF** if it is not a property of a parallelograms, trapezoid, and kites.

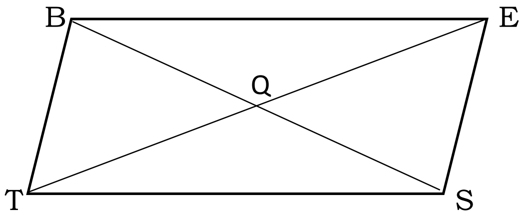
1. In a parallelogram, any two opposite angles are supplementary.
2. In a parallelogram, any two opposite sides are congruent.
3. In a parallelogram, any two consecutive angles are complementary.
4. A diagonal of a parallelogram forms two congruent triangles.
5. The base angles of a an isosceles trapezoid are congruent.
6. The diagonals of an isosceles trapezoid are not equal.
7. If the base angles of a trapezoid are not equal, then the trapezoid is isosceles.
8. If a qudarilateral is a kite, then its diagonals are perpendicular.
9. If exactly one diagonal of a quadrilateral is the perpendicular bisector of the other diagonal, then the quadrilateral is a kite.
10. The area of a kite is half the product of the lengths of its diagonals.

Jumpstart

In the preceding lesson, you have studied some properties of parallelograms, trapezoids and kites.

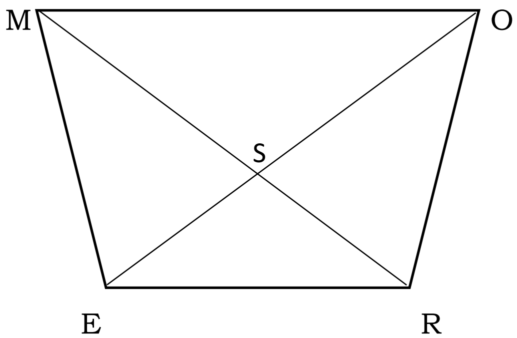
Activity 2: Tests for Parallelograms, Trapezoids and Kites

A. For each of the following, state the condition that supports why BEST is a parallelogram.



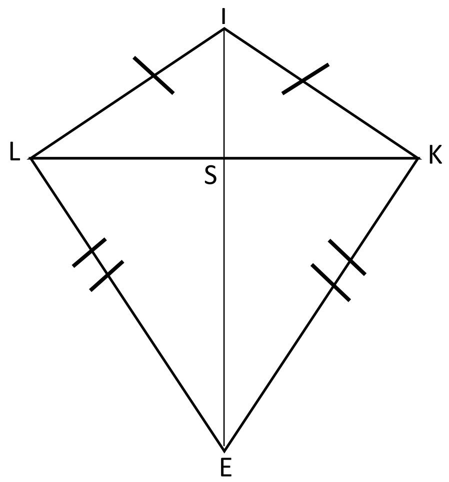
1. $\overline{BQ} \cong \overline{SQ}$, $\overline{TQ} \cong \overline{EQ}$ ______________________________
2. $\overline{BT} // \overline{ES}$, $\overline{BT} \cong \overline{ES}$ ______________________________
3. BE//TS , BT// ES ______________________________

B. MORE is an isosceles trapezoid with bases $\overline{ER}$ and $\overline{MO}$. With this figure and information, find the indicated measures and state the definition or theorem that supports your answer.



4. ME=7.8 cm, find OR=____________, _______________________________________
5. MR= 5.9 cm, find EO=___________, _______________________________________
6. m∠EMO = 78, find m∠ROM=_____,_______________________________________

C. Consider kite LIKE below. Complete the each statement. Give the reason that justifies the statement.



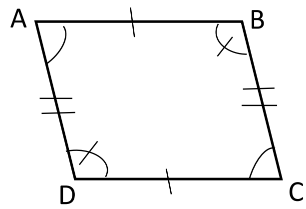
7. If LI=6, what is IK=__________,____
8. If EL=10.5, what is EK=_____,____
9. If LK=7 cm and IE= 13 cm, what is the area? _________,_______________________
10. If the area 96 cm^2 = LK=8 cm, what is IE?___________,_________________________

Discover

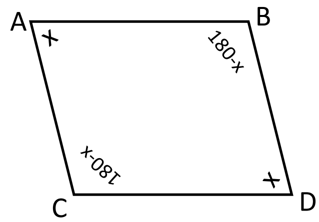
To build your understanding in solving problems on properties of parallelograms, trapezoids and kites, let's discover more of their properties. Let's start.

A. PROPERTIES OF PARALLELOGRAMS
1. The opposite sides of a parallelogram are parallel and congruent.
2. The opposite angles of a parallelogram are congruent.
3. The consecutive angles of a parallelogram are supplementary
4. The diagonals of a parallelogram bisect each other.
5. Each diagonal divides a parallelogram into two congruent triangles.

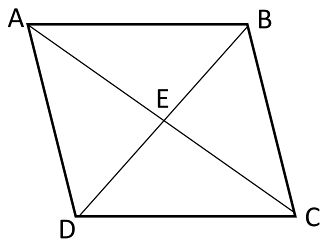
Example 1: ABCD is parallelogram.



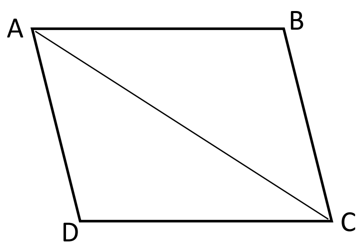
1. The opposite sides are parallel and congruent.
   $\overline{AB} // \overline{DC}$ and $\overline{AD} // \overline{BC}$
   $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$
2. The opposite angles are congruent.
   $\angle A \cong \angle C$ and $\angle B \cong \angle D$
3. The consecutive angles are supplementary.
   $m\angle A + m\angle B = 180$
   $m\angle D + m\angle C = 180$
   $m\angle A + m\angle D = 180$
   $m\angle B + m\angle C = 180$

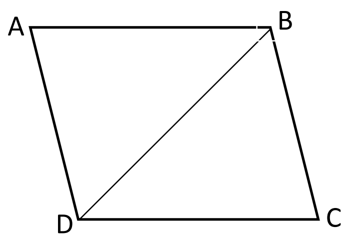


4. The diagonals bisect each other.
   Diagonal $\overline{AC}$ bisects $\overline{BD}$, this means $\overline{BE} \cong \overline{DE}$.
   Diagonal $\overline{BD}$ bisects $\overline{AC}$, this means $\overline{AE} \cong \overline{CE}$.

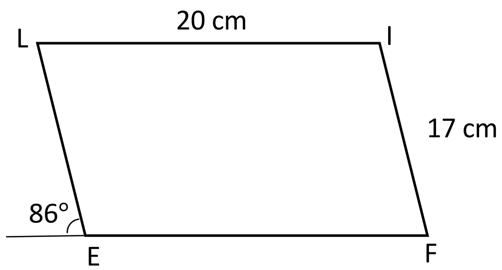


5. Each diagonal bisects the parallelogram into two congruent triangles.
   For diagonal $\overline{AC}$, $\Delta ABC \cong \Delta CDA$
   For diagonal $\overline{BD}$, $\Delta ABD \cong \Delta CDB$



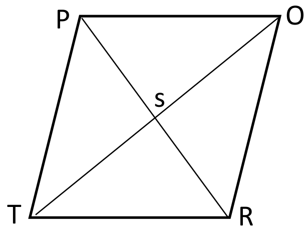


Example 2: Use the properties to solve for the parallelogram LIFE.



Solution: LE=LF= 17 cm and $m\angle LEF$=180-86=94. Also $m\angle I = m\angle LEF$ =94
EF=LI= 20 cm
$m\angle L$ = 180- $m\angle LEF$ =86
$m\angle F$ = $m\angle L$=86

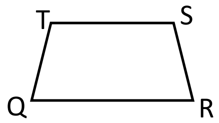
Example 3: ▱PORT is a parallelogram If OT is 6 more than PR, and PS=9, then ST= _________ .



Solution: Since $\overline{PR}$ and $\overline{OT}$ bisect each other, and PS=9, then PR=18. This further implies that OT=PR+6=18+6=24 units and ST=12 units.

B. Definition of Trapezoid

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides.

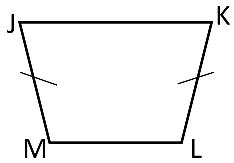


The parallel sides are called the ***bases of the trapezoid.*** The nonparallel sides are called ***legs.*** The angles formed by a base and the legs are called ***base angles***.

Quadrilateral QRST is a trapezoid. $\overline{QR}$ and $\overline{TS}$ are its bases and $\overline{QT}$ and $\overline{RS}$ are its legs. $\angle Q$ and $\angle R$ are its base angles. $\angle T$ and $\angle S$ are also base angles.

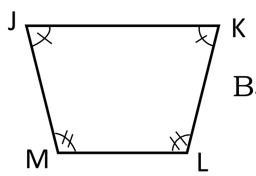
Definition of an Isosceles Trapezoid

An *isosceles trapezoid* is trapezoid with congruent legs.



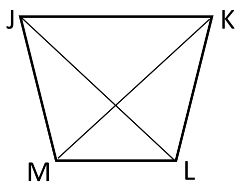
Trapezoid JKLM is an isosceles trapezoid. The consequence of this definition is stated in the next theorems.

Theorem 1: The base angles of an isosceles trapezoid are congruent.



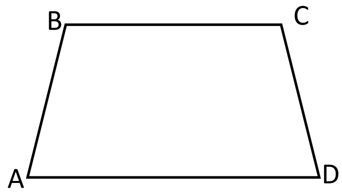
Base angles are $\angle J \cong \angle K$ and $\angle M \cong \angle L$

Theorem 2: The diagonals of an isosceles trapezoid are congruent



Diagonals are $\overline{JL} \cong \overline{KM}$

Example 4: In an isosceles trapezoid ABCD, $m\angle A$ = 3x+40 and $m\angle D$=x+60. Find $m\angle B$.



Solution: By definition, $\angle A \cong \angle D$

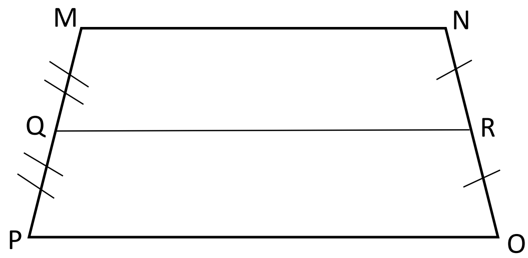
$$m\angle A = m\angle D$$
$$3x+40=x+60$$
$$2x=20$$
$$x=10$$
$$m\angle A = 3x+40$$
$$m\angle A = 3(10) + 40$$
$$m\angle A = 70$$

With AB as transversal to parallel $\overline{BC}$ and $\overline{AD}$, $\angle A$ *and* $\angle B$ are supplementary.

$$m\angle A + m\angle B = 180$$
$$70 + m\angle B = 180$$
$$m\angle B = 180 - 70$$
$$m\angle B = 110$$

Definition of Median of a Trapezoid

The *median* of a trapezoid is the segment joining the midpoints of the legs.



In the trapezoid MNOP, $\overline{QR}$ is the median.

Theorem 3: The median of the trapezoid is parallel to its bases.

Trapezoid MNOP with median $\overline{QR}$.
$\overline{QR} \parallel \overline{MN}$ and $\overline{QR} \parallel \overline{PO}$

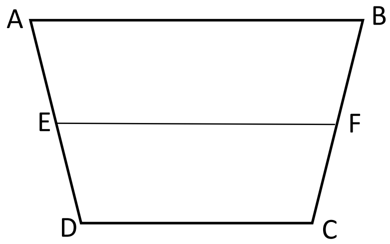
Theorem 4: The median of the trapezoid is half the sum of the lengths of the bases.

Trapezoid MNOP with median $\overline{QR}$.
$QR = \frac{1}{2}(MN + PO)$

Example 5: The length of the bases of a trapezoid are 17 cm and 11 cm. How long is its median?

Solution: By the Theorem 4, the median's length is half the sum of bases.
The sum of the bases is 17 cm+ 11 cm=28 cm.
Thus, the median is 28 cm divided by 2 is equal to 14 cm.

Example 6: ABCD is a trapezoid , with EF as median. If EF=16 and AB=22, find DC.



Solution: $EF=\frac{1}{2}(AB+DC)$

$16=\frac{1}{2}(22+DC)$

$16(2) = 22+DC$

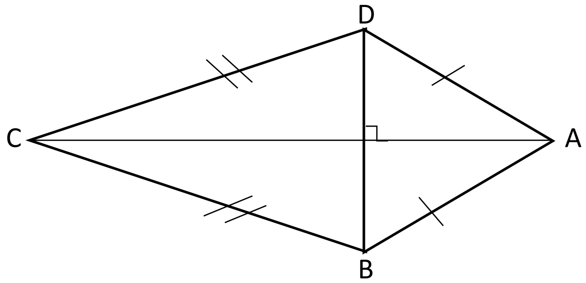
$32=22+DC$

$DC=32-22$

$DC=10$

C. Definition of a Kite

A *kite* is another type of a quadrilateral. It has two distinct pairs of adjacent congruent sides. Quadrilateral ABCD below is a kite.



In kite ABCD, $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$. The two segments $\overline{AC}$ and $\overline{BD}$ are its diagonals.

Some properties of kites are stated in the next theorems.

Theorem 5: If a quadrilateral is a kite, then its diagonals are perpendicular.
$\overline{AC} \perp \overline{DB}$

Theorem 6: The area of a kite is half the product of the length of its diagonals.

Area of kite $ABCD=\frac{1}{2}(\overline{AC}+\overline{BD})$

Example 7: The diagonals of a kite have lengths of 13 cm and 9 cm. Find the area of a kite?

Solution: Area of a Kite= $\frac{1}{2}$ (d_1) (d_2)

$= \frac{1}{2}$ (13 cm) (9cm)

$= \frac{1}{2}$ (117 cm^2)

$= 58.5$ cm^2

Example 8: The area of a kite is 180 cm^2 and the length of the diagonal is 36 cm. How long is the other diagonal?

Solution: Area of a Kite= $\frac{1}{2}$ (d_1) (d_2)

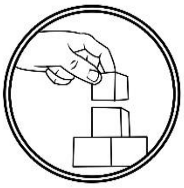
180 cm^2= $\frac{1}{2}$ (36 cm) ((d_2)

2(180 cm^2)= (36 cm) ((d_2)

360 cm^2= (36 cm) ((d_2)

$\frac{360\ cm^2}{36\ cm} = \frac{\cancel{36\ cm}}{\cancel{36 cm}}\ (d_2)$

d_2 = 10 cm

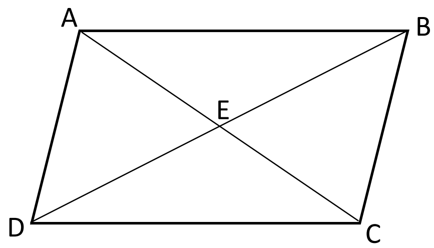


Explore

Now, you are equipped with knowledge on applying the properties of parallelograms, trapezoids, and kites in solving problems, it's about time to find out what you can do. Enjoy and have fun!

Activity 3: YES! YOU CAN DO IT!

Below is parallelogram ABCD. Consider each given information and answer the questions that follow.



1. Given: AB=(3x-5) cm, BC=(2y-7) cm, CD=(x+7) cm and AD=(y+3) cm.

a. What is the value of x?___________

b. How long is AB?______

c. What is the value of y?________

d. How long is AD?_________

e. What is the perimeter of parallelogram ABCD?__________

2. $\angle BAD$ measures $(2a+25)°$ while $\angle BCD$ measures $(3a-15)°$.

a. What is the value of a?_________

b. What is m$\angle BAD$? ____________

c. What is m$\angle CBA$? ____________