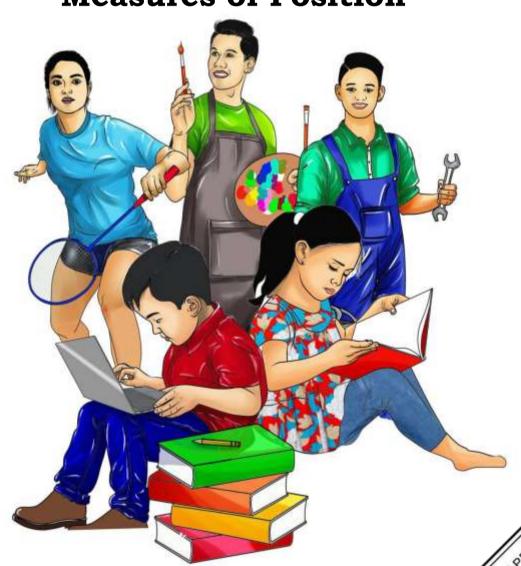




Mathematics

Quarter 4 – Week 4 to 5 Module 4 Solving Problems Involving Measures of Position



AIRs - LM

SONO LEGICAL POR SERVER LOS SERVE

Mathematics 10

Quarter 4- Week 4-5 Module 4: Solving Problems Involving Measures of Position First Edition, 2021

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This module covers the key concepts on measures of position. You are given practical tasks to utilize your prior knowledge and skills in learning measures of position. You are also expected to investigate and analyze thoroughly the mathematical relationships in various situations, formulate real – life problems and solve them using a variety of strategies.

After going through this module, you are expected to attain the following objectives:

Learning Competency:

Solves problems involving measures of position. (M10SP-IVd-e-1)

Subtasks:

- 1. understand and analyze problems involving measures of position;
- 2. identify the formula to be applied; and
- 3. solve problems involving measures of position.

Before going on, find out how much you already know about the topic in this module. Answer the pre- assessment below.

Pre - Assessment

A. measures of central tendency

belongs to?

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following statistical measures does quartile, decile and percentile

B. measures of position

C. measures of d	lispersion	D. measures of v	ariability
	tla in solving for the B. $Q_3 + Q_1$	ne interquartile range? $C. Q_3 - Q_1$	D. Q ₃ – Q ₂
_			. What is the product of
the lower and the mi	-		
A. 129	B. 4004		D. 8008
4. Which of the follow ungrouped data?	wing is the formul	a in finding the positi	on of deciles for
10	+ 1) B.	P of $D_k = \frac{k}{4}(n+1)$	
C. P of $D_k = \frac{k}{100} (n)$	n+1) D.	$P of D_k = k(n+1)$	
5. Which of the follow	wing is equivalent	to 50 th percentile?	
		C. 1st quartile	D. 2 nd quartile
6. In a set of 15 dat	a values, what is	the classification of t	he 8th data value in the
data set?	,		
A. D ₃	B. D ₅	C. D ₈	D. D ₉
7. What value does t			
A. 40	B. 40.5	C. 42	D. 42.5
8. What is the positi	on of the 50 th perc	entile of an ordered d	ata set of 17 values?
A. 10th	B. 9th	C. 8th	D. 7th
9. Find the upper qu	artile Q_3 of the fol	lowing set of data.	
), 14, 9, 6, 1 , 12 1	_	
A. 6	B. 10	C. 14	D. 16
10. The following ar	re scores in a stat	tistics test: 2, 3, 5, 6	, 8, 10, 12, 15, 18, 20.
Find the value corre	sponding to the 25	5 Th percentile.	
A. 3	В. 5	C. 6	D. 8
11. The scores of Ma	iss Philippines ca	ndidates from seven j	udges were recorded as
follows: 8.45, 9.20, scores?	8.56, 9.13, 8.67, 8	3.85, and 9.17. What	is the P_{35} of the judges'
A. 8.65	B. 7.65	C. 6.65	D. 5.65
	is a veterinarian	one morning, she	asked her secretary to
•		•	g are service times in
			7, 52, and 17. Find the
value of the 6th decil		, , , , ,	,
A. 25.6	B. 26.6	C. 36.6	D. 46.6

13. Albert has an assignment to ask at random 10 students in their school about their ages. The data are given in the table below.

Name	Age	Name	Age
Ana	10	Tony	11
Ira	13	Lito	14
Susan	14	Christian	13
Antonette	13	Michael	15
Gladys	15	Dennis	12

How many students belong to Q_2 in terms of their ages? $Q_3 = 9$

A. 3

B. 5

C. 6

D. 9

For numbers 14 & 15

The following distribution represents the data on the scores of 40 Grade 10 students in a math test.

Scores	f
53 – 56	1
49 – 52	3
45 – 48	3
41 – 44	4
37 – 40	2
33 – 36	4
29 – 32	4
25 – 28	8
21 – 24	6
17 – 20	2
13 - 16	3

14. Find the value of P_{36} of the students so	core.
---	-------

A. 26.2

B. 27.2

C. 28.2

D. 29.2

15. How many percent of the scores is less than the cumulative frequency of 35?

A. 36%

B. 46%

C. 54%

D. 64%

Module 4

Solving Problems Involving Measures of Position

Statistical data is arranged either in ascending or descending order which can be divided into various parts. These may be quartiles, deciles or percentiles. All these values can be determined in the same way as the median. The only difference is in their location.

In this lesson, you are expected to demonstrate understanding of the key concepts of measures of position for grouped and ungrouped data in solving problems involving measures of position. You are also expected to solve problems correctly involving measures of position through appropriate and accurate presentation and to justify the usefulness of quantities in dealing with real – life situations. If you find any difficulty in answering the different exercises, seek the assistance of your teacher or peers or refer to the modules you have studied earlier.



Activity 1: Watch This!

1. You are the fourth tallest student in a group of 10. If you are the 4th tallest student, therefore 6 students are shorter than you.

It also means that 60% of the students are shorter than you. If you are the 8th tallest student in a group of 10, how many percent of the students are shorter than you? _____

2. A group of students obtained the following scores in their Math quiz	
8, 2, 5, 4, 8, 5, 1, 7, 3, 9, 6	
Arrange the scores in ascending order	
The least value is	
The greatest value is	
The third data value is	
The middle value in the data is	
The 9th data value is	

Activity 2: Match Me!

Match column A with column B. Write the letters only

- 1. Position of Pk
- 2. Position of Qk
- 3. Position of Dk
- 4. Pk
- 5. Q_k
- 6. D_k
- 7. PR of Ungrouped
- 8. PR of Grouped
- 9. Interquartile Range
- 10. Median

- a. $\frac{k}{4}(n+1)$ b. $\frac{k}{10}(n+1)$ c. $\frac{k}{100}(n+1)$
- d. $Q_3 Q_1$
- e. $LB + \left[\frac{kN}{4} cf_b\right]i$
- f. $LB + \left[\frac{kN}{10} cf_b f_{D_k}\right] i$
- g. $LB + \left[\frac{kN}{100} cf_b\right]i$
- h. $\frac{100}{N} \left[\frac{(P-LB)f_P}{i} + cf_P \right]$ i. $\frac{number\ of\ values\ below\ X+0.5}{total\ number\ of\ values}\ x\ 100\%$
- j. Q_2

Activity 3. Complete Me!

The table below shows the frequency distribution of the mathematical test scores of students. Complete the given table.

Mathematical Test Scores of 100 Students

Scores	Frequency (f)	Lower Boundaries (LB)	Less than Cumulative Frequency (<cf)< th=""></cf)<>
95 - 99	5		
90 - 94	11		
85 - 89	17		
80 - 84	25		
75 - 79	20		
70 - 74	12		
65 - 69	7		
60 - 64	3		

Were you able to answer the different activities? I am sure it was! This time, to help you understand the concepts of measures of position and be able to solve problems involving measures of position, read the discussion below.



There are times when we want to know the position of a value relative to the other observations in a data set. For instance, you took a 100 – item test. You might want to know how your score of 90 compares to the scores of the others.

The measures of positions that divide a distribution into four, ten, and hundred equal parts is called **quantiles**. Such measures of position are **quartiles**, **deciles and percentiles**.

Quantiles can be applied when:

- 1. dealing with large amount of data, which includes the timely results for standardized tests in school, etc.
- 2. trying to discover the smallest as well as the largest values in a given distribution.
- 3. examining financial fields for academic as well as statistical studies.

Quantiles are very useful because they help the government to find how the income in a country is distributed, how much of the total income is earned by low wage earning groups and by high wage earning groups.(If both groups earn the same proportion of the income, then there is income equality)

Let's remember the lessons in your previous modules about measures of position for grouped and ungrouped data.

Ungrouped Data

Quartiles are scores points which divide the distribution into four equal parts. The twenty – five percent (25%) of the distribution fall below the first quartile or lower quartile denoted by Q_1 , fifty percent (50%) fall below the second quartile or middle quartile denoted by Q_2 , and seventy – five percent (75%) fall below the third quartile or upper quartile denoted by Q_3 . The difference between Q_3 and Q_1 is the interquartile range.

Position of
$$Q_k = \frac{k}{4}(n+1)$$

Deciles are the nine score points which divide the distribution into ten equal parts. They are deciles and denoted by D_1 , D_2 , D_3 , ..., D_9

Position of
$$D_k = \frac{k}{10}(n+1)$$

Percentiles are the ninety – nine score points which divide a distribution into one hundred equal parts, so that each part represents the data set. It is used to characterize values according to the percentage below them. Percentiles are denoted by P_1 , P_2 , P_3 , ..., P_{99} .

Position of
$$P_k = \frac{k}{100}(n+1)$$

Grouped Data

Quartiles

In computing the quartiles of grouped data, the following formula is used:

$$Q_k = LB + \left[\frac{\frac{kN}{4} - cf_b}{f_{Q_k}} \right] i$$

where:

LB = lower boundary of the Q_k class

N = total frequency

 cf_b = cumulative frequency of the class before the Q_k class

 f_{Qk} = frequency of the Q_k class

i = size of the class interval

k = nth quartile, where n = 1, 2, and 3

Deciles

The kth decile denoted by D_k is computed as follows:

$$D_k = LB + \left[\frac{\frac{kN}{10} - cf_b}{f_{D_k}} \right] i$$

where:

LB = lower boundary of the D_k class

N = total frequency

 cf_b = cumulative frequency of the class before the D_k class

 f_{Dk} = frequency of the D_k class

i = size of the class interval

k = nth decile, where n = 1, 2, 3, 4, 5, 6, 7, 8, and 9

Percentiles

The kth percentile denoted by P_k is computed as follows:

$$P_k = LB + \left[\frac{kN}{100} - cf_b \right] i$$

where:

LB = lower boundary of the P_k class

N = total frequency

 cf_b = cumulative frequency of the class before the P_k class

 f_{Pk} = frequency of the P_k class

i = size of the class interval

k = nth percentile, where n = 1, 2, 3, ..., 97, 98, and 99

Percentile Rank

Percentile ranks are particularly useful in relating individual scores to their positions in the entire group. A percentile rank is typically defined as the proportion of scores in a distribution that a specific score is greater than or equal to.

Ungrouped Data

The percentile corresponding to a given value X is computed by using the formula:

$$PR = \frac{\text{number of values below } X + 0.5}{\text{total number of values}} \times 100\%$$

Grouped Data

$$PR = \frac{100}{N} \left[\frac{(P - LB)f_P}{i} + cf_P \right]$$

where:

PR = percentile rank, the answer will be a percentage

cf_P = cumulative frequency of all the values below the critical value

P = raw score or value fro which one wants to find a percentile rank

LB = lower boundary of the kth percentile class

N = total frequency

i = size of the class interval

Illustrative examples

Problem 1. The owner of a Sari - Sari Store recorded the number of customers who came into his store each hour in a day. The results were 12, 7, 9, 6, 14, 2, 5, 6, 12, 7 and 8. Find the lower quartile Q_I and the upper quartile Q_3 of the data set.

Solution:

Arrange the data from the lowest to the highest

Using the formula,

$$Q_1 = \frac{1}{4}(n+1) = \frac{1}{4}(11+1) = \frac{1}{4}(12) = 3$$
 \rightarrow the 3rd data: 6

$$Q_3 = \frac{3}{4}(n+1) = \frac{3}{4}(11+1) = \frac{3}{4}(12) = 9$$
 \rightarrow the 9th data : 12

Problem 2. The numbers of days taken by each employee to finish a job were 33, 40, 28, 24, 41, 36, 43, 30, 47, 38 and 43. Find the value of the 7th decile.

Solution:

Arrange the data from the lowest to the highest

Using the formula,

 $D_7 = \frac{7}{10}(n+1) = \frac{7}{10}(11+1) = \frac{7}{10}(12) = 8.4$, since the result is a decimal number, interpolation is needed.

Interpolate the value to obtain the D₇

$$43 - 41 = 2$$
 Subtract the 8th data from the 9th data 2 (0.4) = 0.8 multiply the result by the decimal part add the result to the 8th data or smaller value

Therefore, the value of the 7th decile is 41.8

Problem 3. Mrs. Labonete gave a test to her students in Statistics. The students finished the test in 35 minutes. This time is the 2.5th decile of the allotted time. What does this mean?

Solution:

This means that 25% of the allotted time the students finished the test. A low decile considered good, because it means the students finished the test in a short period of time.

Problem 4. Find the percentile rank of a test score of 49 in the data set 12, 28, 35, 42, 47, 49, 50

Solution:

Arrange the data in order from lowest to highest. Then substitute in the formula

Percentile =
$$\frac{number\ of\ values\ below\ X+0.5}{total\ number\ of\ values}\ x\ 100\%, \qquad X = 49$$
$$= \frac{5+0.5}{7}\ x\ 100\%$$
$$= \frac{5.5}{7}\ x\ 100\%$$
$$= 78.57\%$$

Problem 5. The ages of 115 residents along Ma. Martha St. in Congress Park Subdivision is given below. Find the middle quartile (Q_2) , 3rd decile (D_3) , 80^{th} percentile (P_{80}) , and find how many percent of their ages are less than the cumulative frequency of 30.

Agas	(f)	Lower	Less than	
Ages	(1)	Boundaries (LB)	Cumulative	

			Frequency (<cf)< th=""><th></th></cf)<>	
63 - 69	3	62.5	115	
56 - 62	11	55.5	112	
49 - 55	18	48.5	101	P ₈₀ class
42 - 48	21	41.5	83	
35 - 41	26	34.5	62	Q ₂ class
28 - 34	15	27.5	36	D₃ class
21 - 27	12	20.5	21	
14 - 20	7	13.5	9	
7 - 13	2	6.5	2	
	N = 115			

Solution:

Solving for Q₂

Q₂ class:
$$\frac{2N}{4} = \frac{2(115)}{4} = 57.5$$

LB = 34.5
N = 115
cf_b = 36
f_{Q2} = 26
i = 7
Q₂ = LB + $\left[\frac{2N}{4} - cf_{b}\right]i$
= 34.5 + $\left[\frac{57.5 - 36}{26}\right]7$
= 34.5 + 5.79
= 40.29

Solving for D₃

D₃ class:
$$\frac{3N}{10} = \frac{3(115)}{10} = 34.5$$

LB = 27.5
N = 115
cf_b = 21
f_{D3} = 15
i = 7

$$D_3 = LB + \left[\frac{\frac{3N}{10} - cf_b}{f_{D_3}}\right]i$$

$$= 27.5 + \left[\frac{34.5 - 21}{15}\right]7$$

$$= 27.5 + 6.3$$

$$= 33.8$$

Solving for P₈₀

$$P_{80}$$
 class: $\frac{80N}{100} = \frac{80(115)}{100} = 92$
 $LB = 48.5$
 $N = 115$
 $cf_b = 83$
 $f_{P80} = 18$

$$P_{80} = LB + \left[\frac{\frac{80N}{100} - cf_b}{f_{P_{80}}}\right]i$$

$$= 48.5 + \left[\frac{92 - 83}{18}\right]7$$

$$= 48.5 + 3.5$$

$$= 52$$

Finding PR

In the table, age 30 is within 28 - 34

LB = 27.5
P = 30
N = 115

$$f_p = 15$$

 $cf_p = 21$
 $i = 7$

$$PR = \frac{100}{N} \left[\frac{(P-LB)f_P}{i} + cf_P \right]$$

$$= \frac{100}{115} \left[\frac{(30-27.5)15}{7} + 21 \right]$$

$$= \frac{100}{115} [26.36]$$

$$= 22.92 \sim 23\%$$

Now that you know the important ideas about the topic, use the mathematical ideas you learned from the activity and from the discussion. Answer the problems in the following activities in different ways when possible.



Activity 4: Do you have a sharp memory?

The following are scores of ten students in their 40 – item test quiz.

34 23 15 27 36 21 20 13 33 25

- 1. What are the scores of the students which are less than or equal to 25% of the data?
- 2. What are the scores of the students which are less than or equal to 65% of the data?
- 3. What are the scores of the students which are less than or equal to 8% of the data?

Activity 5. That's My Place

Daily Allowance of 60 Students

5						
Class Interval	f	<cf< td=""></cf<>				
81 – 90	7	60				
71 – 80	10	53				
61 – 70	15	43				
51 – 60	4	28				
41 - 50	12	24				
31 - 40	6	12				
21 - 30	3	6				
11 - 20	2	3				
1 - 10	1	1				

D_6	P ₁₅	P ₃₅	D_8	D_4	P ₇₀	Q_1	Q_2	D_8	Q_3
							•		

Given the frequency distribution, compute for each quantile and match it with the letter code of its corresponding value to complete the phrase above.

Q_1	T. 43
Q_2	I. 61.83
Q_3	N. 72.5
P_{15}	Y. 35.5
P ₃₅	L. 48
P ₇₀	A. 69.83
D_6	M. 65.83
D_4	C. 60.5
D_8	O. 75.5
	R 3



Activity 6: Am I a Scholar?

Dennis and Christine scored 32 and 23, respectively, in the National Career Assessment Examination (NCAE). The determining factor for a college scholarship is that a student's score should be in the top 10% of the scores of his/her graduating class. The students in the graduating class obtained the following scores in the NCAE.

NCAE Scores	f	LB	<cf< th=""></cf<>
39 – 41	6		
36 – 38	7		
33 – 35	9		
30 – 32	13		
27 – 29	22		
24 – 26	10		
21 – 23	9		
18 – 20	7		
15 – 17	8		
12 – 14	4		
9 – 11	2		
6 – 8	1		
3 – 5	1		

- 1. Complete the table by filling in the values of LB (lower boundaries) and <cf (less than cumulative frequency). Explain how you arrived at your answers.
- 2. Find the 3^{rd} quartile (Q₃), 72^{nd} percentile (P₇₂), and the 8^{th} decile of the set of data.
- 3. What is the percentile rank of Dennis and Christine?
- 4. Based on their percentile and percentile ranks, will Dennis and Christine receive a scholarship? Explain your answer.

Activity 7: 1 - 4 - 3 List

In this activity, you will be asked to complete the 1-4-3 chart. Write down what is being asked regarding the problems involving measures of position.

1 – 4 – 3 LIST		
One thing I really love about this topic		
1.		
Four important reasons why I love this topic		
1.		
2.		
3.		
4.		
Three things I still need to understand about this topic		
1.		
2.		
3.		

Your goal in this section is to take a closer look at some aspects of the topic. You are going to think deeper and test further your understanding on solving problems involving measures of position. Work on the last activity.



Assessment: Answer Me, PLEASE!

- A. Directions: Select the letter of the correct answer. Write your answer on a separate sheet of paper.
- 1. When a distribution is divided into four equal parts, what do you call each score point that describes the distribution?

A. decile

B. percentile

C. quantile

D. quartile

2. The number of games won by a basketball team each year from year 1990 to 2000 are 15, 20, 25, 20, 45, 35, 50, 30, 35, 45 and 35. Find the interquartile range of the data set.

A. 65

B. 35

C. 25

D. 20

3. To find the position of Q₁ in the distribution, which of the following formulas is to be used?

A. P of $Q_1 = \frac{1}{4} (n + 1)$ B. P of $Q_1 = \frac{1}{2} (n + 1)$ C. P of $Q_1 = \frac{3}{4} (n+1)$ D. P of $Q_1 = (n + 1)$

4. Which of the following is the formula in finding deciles of grouped data?

A. $D_k = LB + \begin{bmatrix} \frac{N}{10} - cf_b \\ f_{D_k} \end{bmatrix} i$

 $C. D_k = LB + \left\lceil \frac{kN}{10} - cf_b \right\rceil i$

B. $D_k = LB + \begin{bmatrix} \frac{kN}{100} - cf_b \\ f_{D_k} \end{bmatrix} i$ D. $D_k = LB + \begin{bmatrix} \frac{kN}{4} - cf_b \\ f_{D_k} \end{bmatrix} i$

5. Which of the following is equivalent to 5th decile?

A. 5th percentile

B. 50th percentile

C. 1st quartile

D. 3rd quartle

6. The lower quartile of a data set is the 8th data value. How many data values are there in the data set?

A. 31

B. 32

C. 30

D. 34

7. Which of the following values is to be used to determine the 5th decile (D₅) class if N = 100?

A. 25

B. 50

C. 75

D. 100

8. What is the position of the 50th percentile of an ordered data set of 21 values?

A. 10th

B. 11th

C. 12th

D. 13th

9. Find the upper quartile Q_3 of the following set of data.

19, 12, 16, 0, 14, 9, 6, 1, 12, 13, 10, 19, 7, 5, 8

C. 14

D. 16

10. The following are scores in a statistics test: 2, 3, 5, 6, 8, 10, 12, 12, 18, 20. Find the value corresponding to the 75^{Th} percentile.

A. 18

B. 12

C. 10

D. 8

11. The scores of Miss Philippines candidates from seven judges were recorded as follows: 8.45, 9.20, 8.56, 9.13, 8.67, 8.85, and 9.17. What is the P_{60} of the judges' scores?

A. 9.07

B. 8.07

C. 7.07

D. 6.07

12. Mrs. Marasigan is a veterinarian. One morning, she asked her secretary to record the service time for 15 customers. The following are service times in minutes: 20, 35, 55, 28, 46, 32, 25, 56, 55, 28, 37, 60, 47, 52, and 17. Find the value of the 2^{nd} decile (D_2)

A. 25.6

B. 26.6

C. 36.6

D. 46.6

13. Albert has an assignment to ask at random 10 students in their school about their ages. The data are given in the table below.

Name	Age	Name	Age
Ana	10	Tony	11
Ira	13	Lito	14
Susan	14	Christian	13
Antonette	13	Michael	15
Gladys	15	Dennis	12

How many students belong to Q₃ in terms of their ages?

A. 3

B. 5

C. 6

D. 9

For numbers 14 and 15

The following distribution represents the data on the scores of 40 Grade 10 students in a math test.

Scores	f
53 – 56	1
49 – 52	3
45 – 48	3
41 – 44	4
37 – 40	2
33 – 36	4
29 – 32	4
25 – 28	8
21 – 24	6
17 – 20	2
13 - 16	3

14. Calculate the 85th percentile of the Math scores of 40 students.

A. 25.85

B. 35.85

C. 45.83

D. 55.83

15. How many percent of the scores are greater than the cumulative frequency of 35?

A. 36%

B. 46%

C. 54%

D. 64%

References

Books:

Mathematics Grade 10 Learner's Module, First Edition 2015,
Mathematics Grade 10 Teacher's Guide, First Edition
MSA Statistics and Probability, 2010 Revised Edition
Advanced Algebra, Trigonometry and Statistics, Reprinted 2005
Mathematics for Grade 10: A Spiral Approach, Explanation, Examples, Exercises, copyright 2017