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Mathematics

Quarter 1 - Module 3
Solving Equations Transformable to Quadratic Equations and Rational Algebraic Equations

AIRs - LM

Mathematics 9
Alternative Delivery Mode
Quarter 1 - Module 3: Solving Equations Transformable to Quadratic Equations and Rational Algebraic Equations
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Mathematics

Quarter 1 - Module 3
Solving Equations Transformable to Quadratic Equations and Rational Algebraic Equations

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

We start this module by gauging your knowledge and understanding of the several mathematical concepts and principles that you have already studied previously as well as to assess your skills in working with mathematical operations. With these, it will equip you better in solving equations transformable into quadratic equations.

As you begin with this lesson, you might be having a big question; "how are these equations be transformed into Quadratic Equations?" To provide you a clearer viewpoint, you have to evaluate a set of activities.

After going through this module, you are expected to attain the following objectives:

Learning Competency

- Solve algebraic equations that can be transformed into quadratic equations (Including Rational Algebraic Equations) **M9AL-Ic-d-1**

Subtasks

1. Enumerate various forms of quadratic equations;
2. Determine the different equations that are transformable to quadratic equations; and
3. Solve equations that are transformable into quadratic equations and rational algebraic equations.

Before doing your main task, try working on the following set of questions. This will gauge the level of your understanding on this module. Work with fun!

Pre-Assessment

Directions: Find out how much you already know about this module. Evaluate the following problems and write your answers on a separate sheet.

1. Which of the following is a quadratic equation written in standard form?
   A. $2x^2 + 4x - 3 = 0$
   B. $x^2 = -x + 5$
   C. $x^2 - 3x = -4$
   D. $x\ (x - 3) = -2$
2. What makes the equation $2t^2 = 200$ quadratic?
   A. 2
   B. t^2
   C. $2t^2$
   D. 200

3. Which of the following is **NOT** considered a quadratic equation when transformed?
   A. $(x\text{-}2)(x+1) = 0$
   B. $x^2 = 4x - 4$
   C. $x^2 = 25$
   D. $5(x - 3) = 5$
4. What is the equivalent of quadratic equation $(2x + 1)(x - 2) = 0$?
   A. $x^2 + 4x + 2 = 0$
   B. $x^2 - 4x - 2 = 0$
   C. $2x^2 - 3x - 2 = 0$
   D. $2x^2 + 3x + 2 = 0$
5. Which of the following is the correct transformation of $\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{2}$ into a quadratic equation?
   A. $x^2 + x + 1 = 0$
   B. $x^2 - 2x - 1 = 0$
   C. $x^2 + 4x - 1 = 0$
   D. $x^2 + 2x - 3 = 0$
6. What is the standard quadratic equation of $(x\text{-}2)(x + 3) = 1$?
   A. $x^2 + 5x + 7 = 0$
   B. $x^2 - 6x - 6 = 0$
   C. $x^2 + x - 7 = 0$
   D. $x^2 + 5x - 6 = 0$
7. Which of the following leads to an equivalent quadratic equation $x^2 + x - 56 = 0$?
   A. $(x + 1)(x - 56)=0$
   B. $(x + 4)(x - 14) = 0$
   C. $(x + 8)(x - 7) = 0$
   D. $(x - 8)(x + 7) = 0$
8. Which of the following are the solutions of the equation $(x - 5)(x + 2) = 8$?
   A. 6 , -3
   B. -6 , -3
   C. 6 , 13
   D. 3 , 10
9. What are the solutions of the equation $2x^2 - 50 = 0$?
   A. 25, -25
   B. 5, -5
   C. 0, 5
   D. 25, -5
10. In the rational algebraic equation $x + \frac{8}{x - 2} = 1 + \frac{4x}{x - 2}$, which of the roots is extraneous?
   A. 2
   B. 4
   C. 5
   D. 8
11. Which of the following is a root of the equation $\frac{x}{2x-1} = \frac{1}{x+4}$?
   A. 4
   B. 2
   C. 1
   D. -1

12. What is the comparison between a true solution with an extraneous solution of a quadratic equation?
    A. They are both solutions that satisfy the given equation.
    B. The extraneous root never satisfy the given quadratic equation.
    C. The true solution is the only root that can satisfy the given quadratic equation.
    D. The extraneous root sometimes satisfy the given quadratic equation as that with the true root.
13. The length of a garden is 4 m longer than its width and the area is 36 m^2? Which of the following equations represents the given situation?
    A. $x^2 - 4x + 36 = 0$
    B. $x^2 + 4x - 36 = 0$
    C. $x^2 + 4x = -36$
    D. $x^2 - 4x - 36 = 0$
14. What is the value of x in the equation $\frac{1}{3x} + \frac{4x}{6} = 1$?
    A. 1, ½
    B. -1, ½
    C. 1, 2
    D. -1, 2
15. Which of the following quadratic equations is equivalent to x(x+5) = 2?
    A. $x^2 + 5x + 2 = 0$
    B. $x^2 + 5x - 2 = 0$
    C. $x^2 - 5x - 2 = 0$
    D. $x^2 - 5x + 2 = 0$

Were you able to answer all the questions? If not, don't worry because the next activity will help you better understand the lesson.

Jumpstart

Activity 1: You Complete Me

In his iconic speech at the Lincoln Memorial for the 1963 March on Washington for Jobs and Freedom, Martin Luther King Jr. urged America to "make real the promises of democracy." More than 200,000 people, black and white came to listen and demanded equal rights for black people.

Find out the title of his speech by solving the quadratic equations below. Choose the right answer from the table and write the appropriate letter on the corresponding number.

TITLE OF THE SPEECH:

I HAVE A _____ _____ _____ _____ _____
1 2 3 4 5

1. $x^2 - 9x + 18 = 0$
2. $3k^2 - 18k - 21 = 0$
3. $x^2 + 14x - 15 = 0$
4. $5k^2 = 60 - 20k$
5. $4b^2 + 8b + 7 = 4$

E (1, -15)	**D** (3, 6)	**I** (-5, 3)	**S** (-2 , 7)
A (2, -6)	**Y** (3, 6)	**R** (7, -1)	**M** $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

Were you able to evaluate the activity with ease? Quadratic Equations are easier to work on when they are written in standard form and with your mastery of the various methods of finding solutions of quadratic

Activity 2:

A. Quadratic or Not Quadratic?

Identify which of the following equations are quadratic and which are not. Put a check mark (✓) if quadratic, otherwise, put a wrong mark (✗).

_____1. $x^2 - 4x + 12 = 0$
_____2. $15 - 3x = 0$
_____3. $2x^2 + 6x = -4$
_____4. $3x\,(x - 2) = -7$
_____5. $x^2 = 100$

B. Perform the indicated operation and express your answer in simplest form. You can have an extra sheet of paper to solve and simplify.

Given	*Your Answer*
1. $\frac{1}{x} + \frac{3x}{2}$	
2. $\frac{4}{y} - \frac{2y-1}{2}$	
3. $\frac{3m}{2} + \frac{m+1}{m}$	
4. $\frac{k+1}{2k} - \frac{k+2}{3k}$	
5. $\frac{x-5}{2x} + \frac{x+1}{x-2}$	

Are you still familiar with those expressions? Recall your learnings in your previous grade level mathematics. Certainly, this is one of the types of expressions you are fond to work with.

Discover

In some instances, there are equations that are transformable into quadratic equations. These equations may be given in different forms. Hence, transforming these into quadratic equations requires varied procedures and processes.

Once the equations are transformed into quadratic equations, then they can be solved using the techniques you have learned in the previous module. These methods of solving quadratic equations may be in the form of extracting square roots, factoring, completing the squares and through quadratic formula, all leading to the solution/s of the transformed equations.

Case 1: Solving Quadratic Equations That Are Not Written In Standard Form

Example 1: Solve $x\,(x-7) = 18$

This is a quadratic equation but it is not yet written in standard form.

To transform the quadratic equation in standard form, follow these steps:

a. *Write the given equation* → $x\,(x-7) = 18$

b. *Use distributive property of multiplication over subtraction* → $x^2 - 7x = 18$

c. *Rewrite quadratic equation in standard form* → $x^2 - 7x - 18 = 0$

Use any of the four methods in finding the solutions of the quadratic equation $x^2 - 7x - 18 = 0$.

In this case we use factoring in finding the roots of the equation.

d. Factor the left side of the equation ⟶ $(x-9)(x+2) = 0$
e. equate each factor to zero ⟶ $x-9=0$ and $x+2=0$
f. solve each resulting equation ⟶ $x = 9$ or $x = -2$

Check whether the obtained values of x make the equation $x\,(x-7) = 18$ true. If the obtained values of x make the equation $x\,(x-7) = 18$ true, then the solutions of the equation are: $x = 9$ or $x = -2$.

Example 2*:* Find the roots of the equation $(x+1)^2 - 9 = 0$

Expand the term $(x+1)^2$: $(x+1)(x+1) = 0$ ⟶ $x^2 + 2x + 1$

Combine like terms: $x^2 + 2x + 1 - 9 = 0$ ⟶ $x^2 + 2x - 8 = 0$
Factor the equation: $x^2 + 2x - 8 = 0$ ⟶ $(x+4)(x-2) = 0$
Equate each factor to zero: $x - 4 = 0$ or $x + 2 = 0$
Check whether the obtained values of x make the equation $(x+1)^2 - 9 = 0$ true.

Alternately, you can also do extracting square roots: $(x+1)^2 - 9 = 0$

That is: $\longrightarrow$ $(x+1)^2 = 9$

Extract square roots of both sides: $\longrightarrow$ $\sqrt{(x+1)^2} = \sqrt{9}$

Simplify the terms: $\longrightarrow$ $x + 1 = \pm 3$

Solve for the values of x: $x = 3 - 1$ *or* 2 and $x = -3 - 1$ *or* -4

Case 2: Solving Rational Algebraic Equations Transformable Into Quadratic Equations

Example 1*:* Solve the rational algebraic equation $\frac{6}{x} + \frac{x-3}{4} = 2$

Solution:

The equation can be transformed into quadratic equation. There are few steps to consider to solve for its solutions.

Find the Least Common Multiple (LCM) of all denominators	4x
Multiply both sides of the equation by the LCM to get rid of the denominator	$4x\left(\frac{6}{x} + \frac{x-3}{4}\right) = 2(4x)$
	$24 + x^2 - 3x = 8x$
Rewrite the resulting equation in standard form	$x^2 - 11x + 24 = 0$
Find the solutions using any of the methods in solving quadratic equation	$(x-3)(x-8) = 0$ x = 3 and x = 8

Example 2*:* Find the solutions of $x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$

Find the Least Common Multiple (LCM) of all denominators	$x - 2$
Multiply both sides of the equation by the LCM to get rid of the denominator	$(x-2)\left(x + \frac{8}{x-2}\right) = (x-2)\left(1 + \frac{4x}{x-2}\right)$
	$x^2 - 2x + 8 = x - 2 + 4x$
Rewrite the resulting equation in standard form	$x^2 - 7x + 10 = 0$
Find the solutions using any of the methods in solving quadratic equation	$(x-5)(x-2) = 0$ x = 5 and x = 2

✓ *Check whether the solutions make the equation true.*

For x = 5

$$x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$$

$$5 + \frac{8}{5-2} = 1 + \frac{4(5)}{5-2}$$

$$5 + \frac{8}{3} \stackrel{?}{=} 1 + \frac{20}{3}$$

$$\frac{15+8}{3} \stackrel{?}{=} \frac{3+20}{3}$$

$$\frac{23}{3} = \frac{23}{3}$$

For x = 2

$$x + \frac{8}{x-2} = 1 + \frac{4x}{x-2}$$

Observe that at x = 2 , the value of $\frac{8}{x-2}$ and $\frac{4x}{x-2}$ are undefined or does not exist.

Why? (Zero denominators)

*Here, x = 2 is an **extraneous root** or **solution** of the equation.*

***An extraneous root or solution** is a solution of an equation derived from the original equation . However, it is **not** a solution of the original equation.*

Explore

Strengthen your understanding and establish mastery in the basic concepts you've learned through the evaluation of these exercises.

Activity 3: Change my view!

Transform each of the following equations into a standard quadratic equation and answer briefly the questions below. You have **20 minutes** to work on this.

1. $x\,(x+4) = 5$
2. $(x+3)^2 = 36$
3. $(t+2)^2 + (t-3)^2 = 9$
4. $\frac{3}{x} + \frac{4}{2x} = x - 1$
5. $\frac{3}{m-2} + \frac{4}{m+5} = 1$

Questions:

1. How did you transform each equation into a quadratic equation? What concepts or principles did you employ?

2. Did you encounter any difficulty transforming each equation into a quadratic equation? Which item/s? Briefly explain why.

Deepen

Activity 4: Trace my roots!

A. Solve for the solution set of the following. Use extra sheet of paper if needed.

1. $3s\,(s-2) = 12s$
2. $(t+1)^2 - 2t - 1 = 9$
3. $\frac{1}{k} - \frac{k}{6} = \frac{2}{3}$
4. $\frac{1}{x} + \frac{x+2}{3} = 2$
5. $\frac{4}{t-3} + \frac{t}{2} = -2$

Activity 5: Set me in your world!

Linda and Lando are frontliners in a public hospital doing a very critical job of collecting swab samples from suspected patients of Covid-19. Working together, they can finish the job in 6 days. If Linda works alone, she will take 5 days less than Lando to complete. How many days required for Linda and Lando to complete their heroic gestures of doing their job alone?

Source: https://www.istockphoto.com/illustrations/covid-testing-kit

Questions:

1. What type of equation can you formulate for this given word problem?
2. Can you set the required mathematical equation for this? State your equation.
3. What mathematical concepts or principles can you employ to evaluate this problem?
4. Is there any extraneous root you got for this situation? Identify
5. How many days do each of these individuals work alone?

You did a very exceptional job of completing these activities.
Keep that intensity resounds as you finally accomplish this last activity.

Gauge

Post - Assessment

Directions: Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which of the following is an example of quadratic equation written in standard form?

   A. $m^2 = -m + 2$

   B. $2x^2 + 5x - 1 = 0$

   C. $t^2 - 5t = -6$

   D. $y(y - 3) = -2$

2. What makes the equation $m^2 = 100$ quadratic?

   A. 2
   B. m
   C. m^2
   D. 100

3. Which of the following is **NOT** considered a quadratic equation?

   A. $(m\text{-}2)(m+1) = 0$

   B. $p^2 = 4p - 4$

   C. $5(t - 3) = 5$

   D. $x^2 = 25$

4. What is the equivalent quadratic equation of the given; $(2w + 1)(w - 2) = 0$?

   A. $2w^2 - 3w - 2 = 0$

   B. $w^2 - 4w - 2 = 0$

   C. $w^2 + 4w + 2 = 0$

   D. $2x^2 + 3x + 2 = 0$

5. Some rational algebraic equations can be transformed into quadratic equations. Which of the following is the correct transformation of $\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{2}$ into a quadratic equation?

   A. $x^2 + x + 1 = 0$

   B. $x^2 - 2x - 1 = 0$

   C. $x^2 + 4x - 1 = 0$

   D. $x^2 + 2x - 3 = 0$

6. What is the correct translation of the quadratic equation (m-2) (m + 3) = 1?

A. $m^2 + 5m + 7 = 0$

B. $m^2 + m - 7 = 0$

C. $m^2 - 6m - 6 = 0$

D. $m^2 + 5m - 6 = 0$

7. Which of the following leads to an equivalent quadratic equation $x^2 + x - 56 = 0$?

A. $(x + 1)(x - 56)$
B. $(x + 4)(x - 14)$
C. $(x - 8)(x + 7)$
D. $(x + 8)(x - 7)$

8. Which of the following are the solutions or roots of the equation $(y - 5)(y + 2) = 8$?

A. 6 , 13
B. -6 , -3
C. 6 , -3
D. 3 , 10

9. What are the solutions of the equation $2w^2 - 50 = 0$?

A. 25, -25
B. 5, -5
C. 0
D. none of these

10. What is the LCM of the equation $\frac{2}{r+1} + \frac{4}{r-5} = 7$?

A. (r -4)
B. (7) r(4)
C. (r+1)(r-5)
D. (r+1)(r-5)(7)

11. Which of the following is a root of the equation $\frac{x}{2x-1} = \frac{1}{x+4}$?

A. 4
B. 2
C. 1
D. -1

12. Which of the following quadratic equations is equivalent to $\frac{2}{x} - \frac{3x}{2} = 7$?

A. $3x^2 - 7x + 4 = 0$
B. $3x^2 + 7x - 4 = 0$
C. $3x^2 + 14x - 4 = 0$
D. $3x^2 - 14x - 4 = 0$

13. Which of following **CANNOT** be transformed into quadratic equation?

A. $x(x - 1) = -5$
B. $3x^2 - 3x = 2x^2 - 3$
C. $\frac{1}{x+3} = \frac{3}{x-2}$
D. $\frac{2}{2x-1} = \frac{x}{2x+1}$

14. Jona and Joseph are doing their math project. Joseph can do the work alone in 5 days more than Jona to complete. If they work together, they can finish the work in 6 days. What mathematical equation best illustrate the problem?

A. $\frac{1}{x} + \frac{1}{x-5} = \frac{1}{6}$

B. $\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$

C. $\frac{1}{x} + \frac{x}{x-5} = 6$

D. $x + x - 5 = 6$

15. Joe's father can paint a room in 3 hours less than Joe. Working together, they can complete the job in 2 hours. How many hours would each require if they work alone?

A. Joe = 6 hrs, Joe's Father = 3 hrs

B. Joe = 8 hrs, Joe's Father = 5 hrs

C. Joe = 7 hrs, Joe's Father = 4 hrs

D. Joe = 10 hrs, Joe's Father = 7 hrs

Congratulations! You finally made it. See you in your next exploration.

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