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MATHEMATICS

Quarter 2 - Module 6:
Solving Problems on Circles

AIRs - LM

MATHEMATICS 10
Quarter 2 - Module 6: Solving Problems on Circles
Second Edition, 2021

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MATHEMATICS

Quarter 2 - Module 6:
Solving Problems on Circles

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Problem-Solving always plays a vital role in the success of every individual. It gives us a mechanism for identifying things, figuring out why they are broken and determining a course of action to fix them.

In this module, you will realize the importance of having proper skills in solving problems, choosing appropriate tools or skills to solve real-life problems involving circles and learn to appreciate the importance of circles in the real-world.

The most essential learning competency (MELC) for this module will be:

- Solves problems on circles **(M10GE-IIf-2)**

After going through this module, you are expected to:

1. Recall theorems involving tangents, secants, segments and sectors of a circle;
2. Identify theorems used in solving problems involving circles;
3. Solve problems on circles using different theorems involving tangents, secants, segments and sectors of a circle; and
4. Appreciate the importance of circles in real-life.

Before you start doing the activities in this lesson, find out how much you already know about this module. Answer the pre-test on the next page in a separate sheet of paper.

PRE-ASSESSMENT

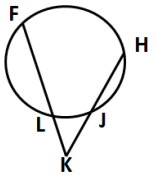
Directions: Read each mathematical statement carefully. Write the letter of your choice in a separate sheet of paper.

1. The gear of a grandfather clock has a radius of 3 in. To the nearest tenth of an inch, what distance does the gear cover when it rotates through an angle of 88°?
   A. 4.3 in. B. 4.6 in. C. 4.9 in. D. 5.2 in.

2. In the company logo shown, m$\widehat{FH}$ = 108°, and m$\widehat{LJ}$ = 12°. What is m∠FKH?

   A. 48^0 B. 45^0

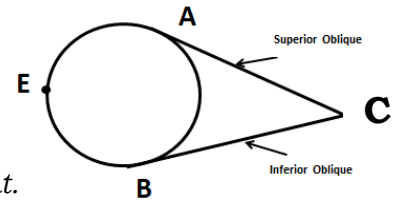
   C. 42^0 D. 39^0



3. Two of the six muscles that control eye movement are attached to the eyeball and intersect behind the eye. If m$\widehat{AEB}$ = 225°, what is m∠ACB?

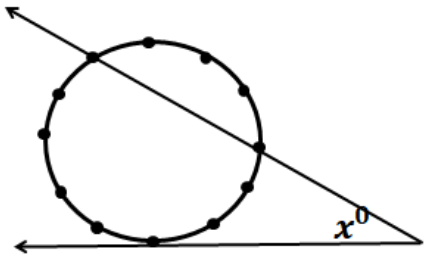
   A. 48^0 B. 45^0

   C. 42^0 D. 39^0

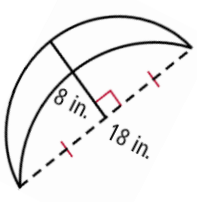


For numbers 4-8 use the illustration at the right.

4. How many degrees do two consecutive dots formed?
   A. 30^0 B. 45^0
   C. 75^0 D. 90^0
5. How many degrees will the bigger arc formed by the angle x?
   A. 60^0 B. 90^0
   C. 120^0 D. 150^0
6. How many degrees will the smaller arc formed by the angle x?
   A. 60^0 B. 90^0
   C. 120^0 D. 150^0



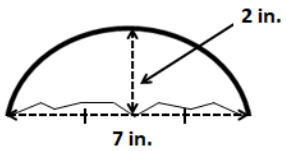
7. What formula will solve the value of x?
   A. $x = \frac{1}{2}(bigger\ arc - smaller\ arc)$ B. $x = \frac{1}{2}(bigger\ arc + smaller\ arc)$
   C. $x = \frac{1}{2}(smaller\ arc - bigger\ arc)$ D. $x = \frac{1}{2}(smaller\ arc + bigger\ arc)$
8. Find the value of x.
   A. 30^0 B. 45^0 C. 60^0 D. 90^0
9. The art department is contracted to construct a wooden moon for a play. One of the artists creates a sketch of what it needs to look like by drawing a chord and its perpendicular bisector. Find the diameter of the circle used to draw the outer edge of the moon.
   A. $18\frac{1}{8}$ in. B. $18\frac{1}{2}$ in.
   C. $16\frac{1}{2}$ in. D. $16\frac{1}{4}$ in.



10. Find the diameter of the illustration at the right.

A. $8\frac{1}{8}$ in. B. $8\frac{1}{2}$ in.

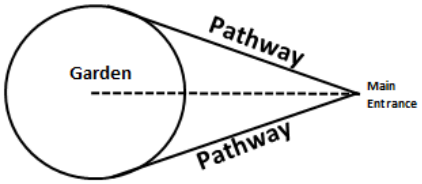
C. $6\frac{1}{2}$ in. D. $6\frac{1}{4}$ in.



11. From the main entrance of a park, there are two pathways where visitors can walk along going to the circular garden. The pathways are both tangent to the garden whose center is 40 m away from the main entrance. If the area of the garden is about 706.5 m^2, how long is each pathway?

A. $5\sqrt{55}$ m. B. $6\sqrt{55}$ m.

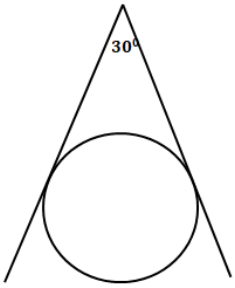
C. $7\sqrt{55}$ m. D. $8\sqrt{55}$ m.



12. Karen has a necklace with a circular pendant hanging from a chain around her neck. The chain is tangent to the pendant. If the chain is extended as shown in the diagram on the right, it forms an angle of 30° below the pendant. What is the measure of the arc at the bottom of the pendant?

A. 60^0 B. 90^0

C. 120^0 D. 150^0



13. Mang Mario cut a circular board with a diameter of 80 cm. Then, he divided the board into 20 congruent sectors. What is the area of each sector?

A. 80π cm^2 B. 320 cm^2 C. 800π cm^2 D. 1 600π cm^2

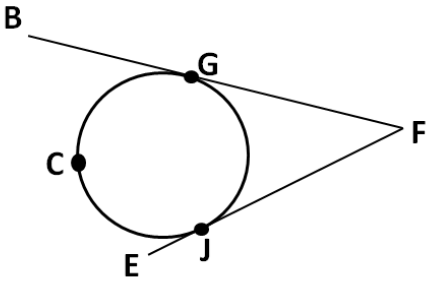
14. Mary designed a pendant. It is a regular octagon set in a circle. Suppose the opposite vertices are connected by line segments and meet at the center of the circle. What is the measure of each angle formed at the center?

A. 22.5^0 B. 45^0 C. 67.5^0 D. 135^0

15. In the figure at the right, $\overleftrightarrow{FB}$ and $\overleftrightarrow{FE}$ are two tangents intersecting outside the circle at point F. $\widehat{GCJ}$ and $\widehat{GJ}$ are the two intercepted arcs of $\angle$GFJ. If $\widehat{GCJ}$ measures 200^0 and $\widehat{GJ}$ measures 160^0, what is the measure of $\angle$GFJ?

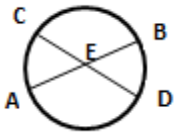
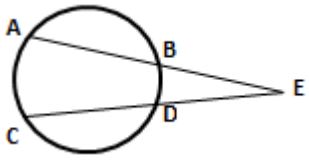
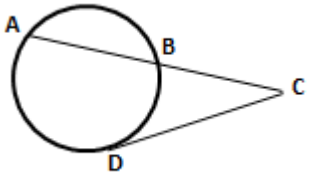
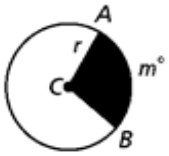
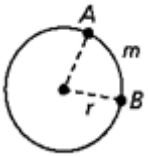
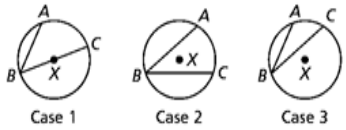
A. 30^0 B. 25^0

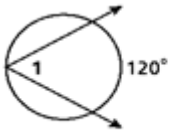
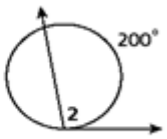
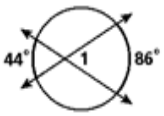
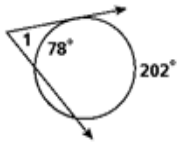
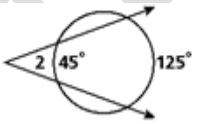
C. 20^0 D. 15^0



Let's Recall:

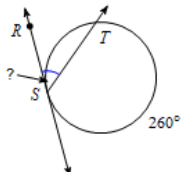
Here are some of the theorems you've learned involving tangents, secants segments and sectors of a circle

Chord-Chord Product Theorem		
THEOREM	ILLUSTRATION	FORMULA
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.		AE • EB = CE • ED
Secant-Secant Product Theorem		
If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.		AE • BE = CE • DE
Secant-Tangent Product Theorem		
If a secant and a tangent intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.		AC • BC = CD^2
Sector of a Circle		
A **sector of a circle** is a region bounded by two radii of the circle and their intercepted arc.	Sector ACB 	A = $\pi r^2 \left(\frac{m^0}{360^0}\right)$
Arc Length		
Arc Length is the distance along an arc measured in linear units.		L = $2\pi r\left(\frac{m^0}{360^0}\right)$
Inscribed Angle Theorem		
The measure of an inscribed angle is half the measure of its intercepted arc.		m $\angle ABC = \frac{1}{2}$ m$\overset{\frown}{AC}$

Vertex of the Angle	Measure of an Angle	Illustrations
On a Circle	Half the measure of its intercepted arc	 $m\angle 1 = 60°$  $m\angle 2 = 100°$
Inside a Circle	Half the sum of the measure of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(44° + 86°)$ $= 65°$
Outside a Circle	Half the difference of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(202° - 78°)$ $= 62°$  $m\angle 2 = \frac{1}{2}(125° - 45°)$ $= 40°$

Activity 1A:

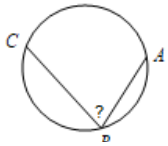
Identify the theorems applicable in the following illustrations then find the measure of the arc or angle indicated. Assume that lines which appear tangent are tangent.

1. 

150°

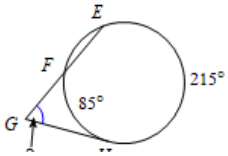
Theorem: _____

Angle/Arc: _____

2. 

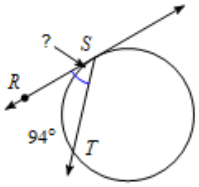
Theorem: _____

Angle/Arc: _____

3. 

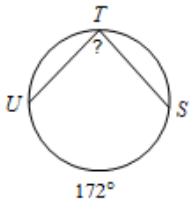
Theorem: _____

Angle/Arc: _____

4. 

Theorem: _____

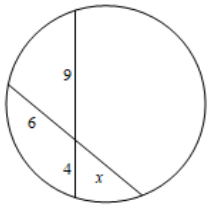
Angle/Arc: _____

5. 

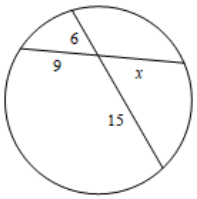
Theorem: _____

Angle/Arc: _____

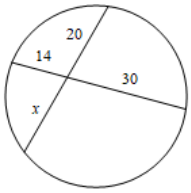
Activity 1B:
Identify the theorems applicable in the following illustrations then find the value of x. Assume that lines which appear tangent are tangent.

6. 

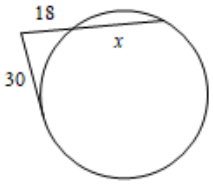
Theorem: _____
Value of x: _____

7. 

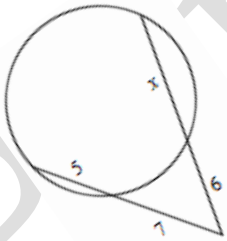
Theorem: _____
Value of x: _____

8. 

Theorem: _____
Value of x: _____

9. 

Theorem: _____
Value of x: _____

10. 

Theorem: _____
Value of x: _____

Well done. Now let's proceed to our module Solving Problems on Circles.

Jumpstart

CIRCLES IN REAL-LIFE

Circles are present in real life, both in the natural world and in manmade creations. Manicouagan Reservoir in Canada is a ring-shaped lake that formed in the remains of a crater. Mushrooms with domed caps have circular bases. Ferris wheels take the circle to vertical heights at amusement parks and carnivals. Many household items, including cups, candles, and doorknobs have circles in their designs.

One application of circles in science is in the design of particle separators. The Large Hadron Collider in Europe is a tunnel in the shape of a circle. This shape helps force the particles to move. NASA uses pi — the ratio of the circumference to the diameter — in several applications. This includes calculation trajectories, determining the size of distant planets, and measuring craters.

The invention of the wheel remains one of the most important inventions of all time. This circle made it possible for people to move and move things greater distances at faster speeds. Circles are still evident in transportation where they appear in vehicle tires, roundabouts in roads, engine crankshafts, and road designs. GPS also relies on circles when determining distance. It identifies points and calculates the distance between the satellite and the point using a circle theory.

Discover

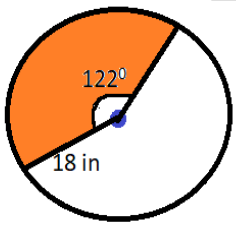
Now let's discover the following real-life examples involving circles for us to learn different problem-solving skills using George Polya's Four-Step Procedure in Problem Solving such as:

- Understand the problem
- Devise a plan (translate)
- Carry out the plan (solve)
- Look back (check and interpret)

Example 1.

A windshield wiper blade is 18 inches long. To the nearest square inch, what is the area covered by the blade as it rotates through an angle of 122°?

Step 1: Understand the problem by reading it many times if necessary.
Step 2: Devise a plan by translating it into your own language, illustrate the given problem if possible and put appropriate labeling as shown below and identify the appropriate theorem.



Step 3: Carry out the plan by solving the problem.

$Area_{sector\ of\ blade} = \left(\frac{m^0}{360^0}\right)\pi r^2$ *formula of area of the sector*

$= \left(\frac{122^{0\not}}{360^{0\not}}\right)(3.1416)(18)^2$ *substitute the value of m, π and r*

$= \left(\frac{122^{\not 0}}{360^{\not 0}}\right)(3.1416)(324)$ *simplify* $(18)^2 = 324$

$= \frac{(122)(3.1416)(324)}{360}$

$Area_{sector\ of\ blade} = 345\ in^2$ *simplify*

Step 4: Look Back (check and interpret)

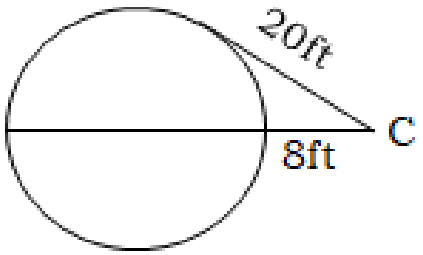
The area covered by the blade is 345 in^2.

Example 2. You are standing at point C, about 8 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is about 20 feet. Estimate the radius of the tank.

Step 1: Understand the problem

Step 2: Devise a plan

The problem describes the Secant-Tangent Product Theorem as shown in the illustration at the right.



Step 3: Carry out the plan by solving the problem.

First: Solve for $\overline{CD}$

$\overline{CE} \bullet \overline{CD} = \overline{BC}^2$

$(8)(\overline{CD}) = 20^2$

$(8)(\overline{CD}) = 400$

$\frac{(8)(\overline{CD})}{8} = \frac{400}{8}$

$\overline{CD} = 50$

Second: Solve for $\overline{DE}$ that will represent the diameter of the aquarium tank

$\overline{DE} = \overline{CD} - \overline{CE}$

$= 50 - 8$

$\overline{DE} = 42$

Third: Find the radius using the formula radius = $\frac{Diameter}{2}$

radius = $\frac{42}{2}$

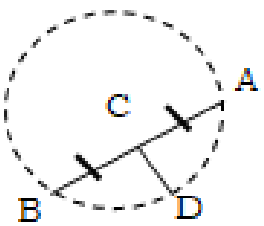
radius = 21 feet

Step 4: Look Back (check and interpret)

The estimated radius of the tank is 21 feet.

For the next two examples it will only show a brief discussion on the solution but if you find it difficult to follow don't hesitate to refer on the detailed solution above.

Example 3. Given the illustration of a circular plate with $\overline{AB}$ = 8 in. and $\overline{CD}$ = 2 in, what is the diameter of the plate?



Solution: The problem applies the Chord-Chord Product Theorem and we will let x as the missing value of the diameter of the plate.

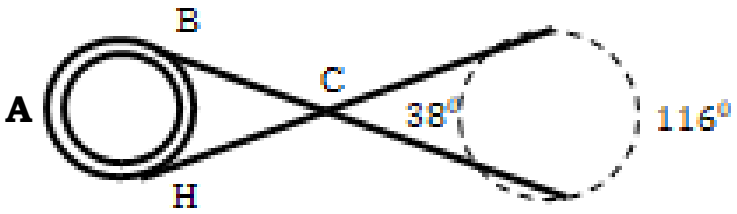
$(2)(x) = (4)(4)$ Diameter = 2 + x

$2x = 16$ = 2 + 8

$\frac{2x}{2} = \frac{16}{2}$ Diameter = 10 inches

$x = 8$

Example 4. Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown. Note that the yarn appears to intersect itself at C, but in reality it does not. Use the information from the illustration to find $m\widehat{BH}$.



Solution:

Step 1. Find $\angle C$

$\angle FCE = \frac{1}{2}(\widehat{EF} - \widehat{DG})$

$= \frac{1}{2}(116^0 - 38^0)$

$= \frac{1}{2}(78^0)$

$\angle C = 39^0$

Step 2. Let $x = m\widehat{BH}$

Using the Vertical Angle Theorem

$\angle BCH = \frac{1}{2}(\widehat{BAH} - \widehat{BH})$

$39^0 = \frac{1}{2}[(360^0 - x) - x]$

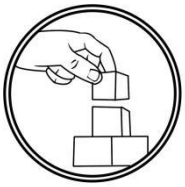
$78^0 = [(360^0 - x) - x]$

$78^0 = 360^0 - 2x$

$2x = 360^0 - 78^0$

$2x = 282^0$

$x = m\widehat{BH} = 141^0$

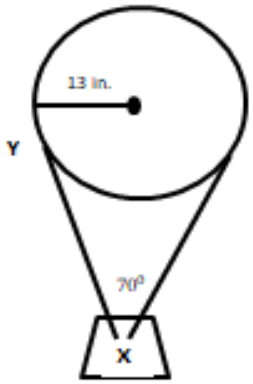


Explore

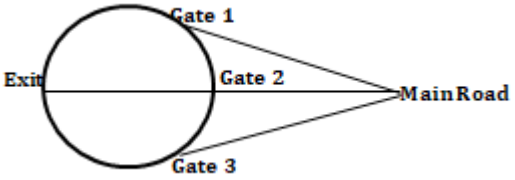
Activity 2:

Read the following problems carefully then solve.
(Show your solution in a separate sheet of paper)

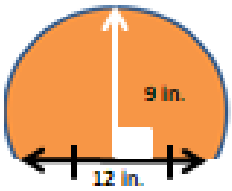
1. A company builds metal stands for bicycle wheels. A new design calls for a V-shaped stand that will hold wheels with a 13 in. radius. The sides of the stand form a 70° angle. To the nearest tenth of an inch, what should be the length XY of a side so that it is tangent to the wheel?



2. The figure below shows a sketch of a circular children's park and the different pathways from the main road. If the distance from the main road to Gate 2 is 70 m and the length of the pathway from Gate 2 to the Exit is 50 m, about how far from the main road is Gate 1?



3. Jheyrus is serving cake at a party. If the dimensions of the remaining cake are shown below, what was the original diameter of the cake?



Rubric for Problem Solving

4	3	2	1
used an appropriate strategy to come up with a correct solution and arrived at a correct answer	used an appropriate strategy to come up with a solution, but a part of the solution led to an incorrect answer	used an appropriate strategy but came up with an entirely wrong solution that led to an incorrect answer	attempted to solve the problem but used an inappropriate strategy that led to a wrong solution

Deepen

Activity 3:

Read the following problems carefully then solve.

1. Two straight pipes intersect under a circular garden and intercept non-adjacent arcs that measure 38^0 and 40^0. What is the measure of the obtuse vertical angles formed where the two pipes intersect?

2. A park maintenance person stands 18 m from a circular monument. If you assume her lines of sight form tangents to the monument and make an angle of 40^0, what is the measure of the arc of the monument that her lines of sight intersect?

3. Cardo is standing 16 feet from a giant Acacia tree and Lia is standing next to the tree. The distance between Cardo and Lia is 27 feet. Find the diameter of the tree.

Rubric for Problem Solving

4	3	2	1
used an appropriate strategy to come up with a correct solution and arrived at a correct answer	used an appropriate strategy to come up with a solution, but a part of the solution led to an incorrect answer	used an appropriate strategy but came up with an entirely wrong solution that led to an incorrect answer	attempted to solve the problem but used an inappropriate strategy that led to a wrong solution

Gauge

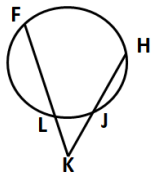
POST ASSESSMENT

Directions*:* Read each mathematical statement carefully. Write the letter of your choice in a separate sheet of paper.

1. The gear of a grandfather clock has a diameter of 10 in. To the nearest tenth of an inch, what distance does the gear cover when it rotates through an angle of 60°?

A. 4.3 in. B. 4.6 in. C. 4.9 in. D. 5.2 in.

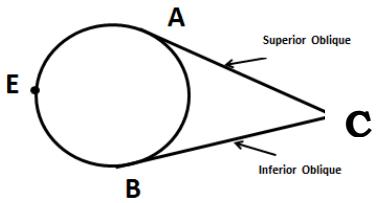
2. In the company logo shown, m$\widehat{FH}$ = 115°, and m$\widehat{LJ}$ = 13°. What is m$\angle FKH$?



A. 48^0 B. 50^0

C. 51^0 D. 52^0

3. Two of the six muscles that control eye movement are attached to the eyeball and intersect behind the eye. If m$\angle ACB$ = 35°, what is m$\widehat{AEB}$?



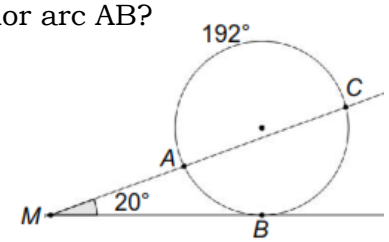
A. 200^0 B. 205^0

C. 215^0 D. 220^0

4. A dart board has a diameter of 40 cm and is divided into 20 congruent sectors. What is the area of one of the sectors?

A. 20π cm^2 B. 40π cm^2 C. 80π cm^2 D. 800π cm^2

5. Mr. Cruz wanted to plant three different colors of roses on the outer rim of a circular garden. He stretched two strings from a point external to the circle to see how the circular rim can be divided into three portions as shown in the figure at the right. What is the measure of minor arc AB?



A. 64^0 B. 104^0

C. 168^0 D. 192^0

6. A windshield wiper blade is 12 inches long. To the nearest square inch, what is the area covered by the blade as it rotates through an angle of 120°?

A. 149 in^2 B. 151 in^2 C. 153 in^2 D. 155 in^2

7. Two straight pipes intersect under a circular garden and intercept non-adjacent arcs that measure 34^0 and 48^0. What is the measure of the acute vertical angles formed where the two pipes intersect?

A. 41^0 B. 42^0 C. 43^0 D. 44^0

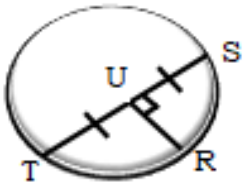
8. A park maintenance person stands 18 m from a circular monument. If you assume her lines of sight form tangents to the monument and make an angle of 33^0, what is the measure of the arc of the monument that her lines of sight intersect?

A. 141^0 B. 143^0 C. 145^0 D. 147^0

9. Charmaine has a circular garden that she separates into three equal parts. If the radius of the garden is 9 m, what is the length of the arc of each part?

A. 6π m B. 7π m C. 8π m D. 9π m

10. Given the illustration with $\overline{ST}$ = 12.2 m, and $\overline{UR}$ = 3.9 m. What was the diameter of the circle to the nearest hundredth?

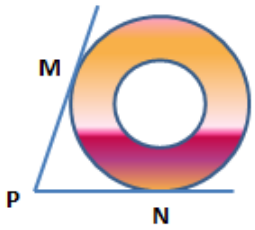


A. 13.44 m B. 13.54 m

C. 13.64 m D. 13.74 m

For numbers 11-12, refer on the given situation.

A manufacturing company is creating a plastic stand for DVDs. They want to make the stand with m$\widehat{MN}$ = 102^0



11. What is the measure of the bigger $\widehat{MN}$?

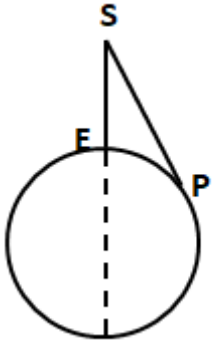
A. 250^0 B. 254^0

C. 258^0 D. 262^0

12. What should be the measure of ∠MPN?

A. 78^0 B. 82^0 C. 86^0 D. 90^0

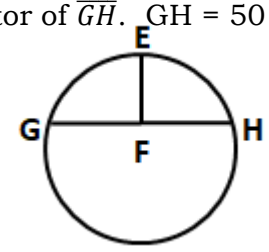
13. A weather satellite S orbits Earth at a distance SE of 6 000 mi. Given that the diameter of the Earth is approximately 8 000 mi, what is the distance from the satellite to P to the nearest mile?



A. 9 100 mi B. 9 150 mi

C. 9 165 mi D. 2120 mi

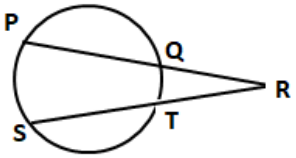
14. In the illustration, $\overline{EF}$ is the perpendicular bisector of $\overline{GH}$. GH = 50 ft, and EF = 20 ft. What is the diameter of the circle?



A. 51 ft B. 52 ft

C. 51.25 ft D. 52.25 ft

15. Using the illustration at the right with $\widehat{PS}$ = 60^0 and $\widehat{QT}$ = 14^0, what is m∠PRS?



A. 20 B. 21

C. 22 D. 23

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