9

# Mathematics

## Quarter 4 - Week 2 - Module 2: The Trigonometric Ratios of Special Angles

**AIRs - LM**

Mathematics 9
Quarter 4-Week 2- Module 2: The Trigonometric Ratios of Special Angles
First Edition, 2021



**Development Team of the Module**

**Writer:** Cleofe M. Lacbao
**Editor:** SDO La Union, Learning Resource Quality Assurance Team
**Layout Artist:** Ernesto F. Ramos Jr., *P II*

**Management Team:**

Atty. Donato D. Balderas, Jr.
*Schools Division Superintendent*

Vivian Luz S. Pagatpatan, PhD
*Assistant Schools Division Superintendent*

German E. Flora, PhD, *CID Chief*

Virgilio C. Boado, PhD, *EPS in Charge of LRMS*

Erlinda M. Dela Peña, EdD, *EPS in Charge of Mathematics*

Michael Jason D. Morales, *PDO II*

Claire P. Toluyen, *Librarian II*

# *Target*

Trigonometry is discovered through the special right triangles and the Pythagorean Theorem. Sine, cosine, tangent, secant, cosecant, and cotangent are all functions of angles and the result is the ratio of the sides of a right triangle. Only our special right triangles generate sine, cosine, tangent values that can be found without using scientific calculator. This module aims to demonstrate your understanding of finding the trigonometric ratios of special angles, the 45°-45°-90° and 30°-60°-90° right triangles.

After going through this module, you are expected to:
Finds the trigonometric ratios of special angles. **(M9GE-IVB-C1)**

**Subtask:**

- Determine trigonometric ratios involving special angles
- Compute the numerical values of trigonometric expressions involving special angles.

*Before you start doing the activities in this lesson, find out how much you already know about this module. Answer the pretest on the next page in a separate sheet of paper. Write the letter that corresponds to the best answer.*

**Pre-Assessment Test**

Directions: Choose the letter of the correct answer. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. Given the **c** as the hypotenuse, **a** and **b** as legs of a right triangle. which of the following is correct?
   A. $a = \sqrt{c^2 + b^2}$ B. $a^2 = c^2 + b^2$
   C. $c^2 = a^2 - b^2$ D. $c^2 = a^2 + b^2$

2. What is the other name of a 45°-45°-90° right triangle?
   A. isosceles right triangle B. scalene right triangle
   C. obtuse right triangle D. equiangular triangle

3. Which of the following is true in a 45°-45°-90° right triangle?
   A. legs are congruent B. hypotenuse is equal to a leg times $\sqrt{2}$
   C. **a** only D. both **a** & **b**

4. In a 45°-45°-90° triangle, the length of each of the congruent sides is $4\sqrt{3}$ cm. How long is its hypotnuse?
   A. $4\sqrt{3}$ cm B. $8\sqrt{3}$ cm
   C. $4\sqrt{5}$ cm D. $4\sqrt{6}$ cm

5. In a 30°-60°-90° right triangle, what is the relation between the shorter leg (**a**) and the hypotenuse (**c**)?
   A. c= ½ a B. c = 2a
   C. $c = a\sqrt{3}$ D. $c = a\sqrt{2}$

6. In a 30°-60°-90° right triangle, what is the relation between the longer leg (**b**) and the shorter leg (**a**)?
   A. b = ½ a B. $b = a\sqrt{3}$
   C. $b = a\sqrt{2}$ D. b = 2a

7. In a 30°-60°-90° right triangle, what is the value of the shorter leg (**a**) in terms of the hypotenuse (**c**)?.
   A. a = ½ c B. $a = c\sqrt{3}$
   C. $a = \frac{c\sqrt{2}}{2}$ D. $a = c\sqrt{2}$

8. With respect to the given right triangle, what is the ratio of the opposite side to the hypotenuse?
   A. cosecant B. cosine
   C. sine D. tangent

9. It is described as the ratio of the opposite side to the adjacent side of a right triangle?
   A. cosecant B. cosine
   C. sine D. tangent

10. What is the correct trigonometric ratio to be used in solving x in figure 1?

A. cosecant B. cosine

C. sine D. tangent

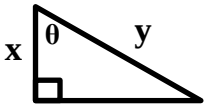


Figure 1

11. Which of the following ratios describe a tangent?

A. $\frac{\text{adjacent side}}{\text{opposite side}}$ B. $\frac{\text{opposite side}}{\text{adjacent side}}$

C. $\frac{\text{opposite side}}{\text{hypotenuse}}$ D. $\frac{\text{adjacent side}}{\text{hypotenuse}}$

12. In figure 1, suppose $\theta = 60^{\circ}$, what is the value of y if x = 6 inches?

A. 3 inches B. $6\sqrt{2}$ inches

C. $6\sqrt{3}$ inches D. 12 inches

13. Solve for x in the figure 2.

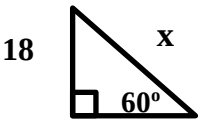


Figure 2

A. $6\sqrt{3}$ B. $9\sqrt{3}$

C. $12\sqrt{3}$ D. $18\sqrt{3}$

14. In figure 3, y = 10, which ratio is used to solve x?

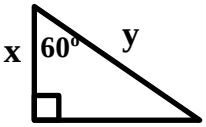


Figure 3

A. $\cos 60^{o} = \frac{x}{y}$ B. $\tan 60^{o} = \frac{x}{y}$

C. $\sin 60^{o} = \frac{x}{y}$ D. $\csc 60^{o} = \frac{y}{x}$

15. The length of the shorter leg of a 30°-60°-90° triangle is 15 cm, find the length of the longer leg.

A. 30 cm B. $5\sqrt{3}$ cm

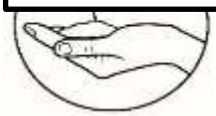
C. $\sqrt{2}$ cm D. $15\sqrt{3}$ cm

*How was your performance in the pre-assessment? Were you able to answer all the questions? Did you find difficulties in answering them? Are there questions familiar to you? Keep yourself on track as we learned new concepts in this module.*

# Module 2

# The Trigonometric Ratios of Special Angles

In this lesson you will use the concepts you have learned in the previous lessons to evaluate the trigonometric ratios of special angles. There are two triangles, the isosceles and equilateral triangles that are frequently used in mathematics to generate exact values for the trigonometric ratios. Let us consider the succeeding activities to develop mastery of the topic.

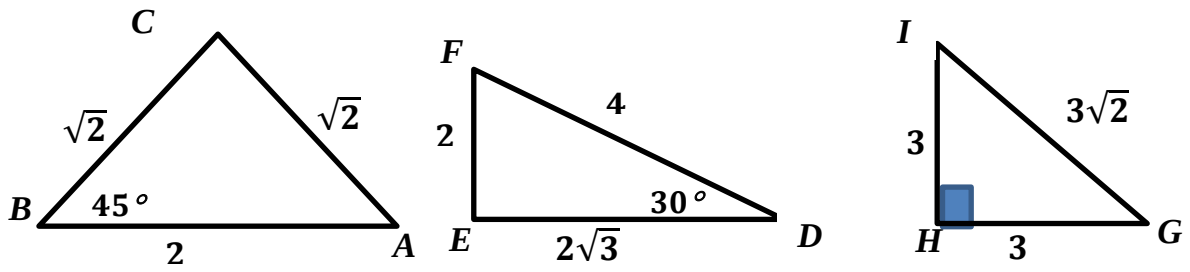


## *Jumpstart*

**Activity 1: Let's Get Familiar**
**Directions:**

1) a) Draw a right triangle that has one angle measuring 30°. Label the sides using lengths $\sqrt{3}$, 2, and 1.
   b) Identify the adjacent and opposite sides relative to the 30° angle.
   c) Redraw the triangle and identify the adjacent and opposite sides relative to the 60° angle.
2) a) Draw a right triangle that has one angle measuring 45°. Label the sides using the lengths 1, 1, and 2.
   b) Identify the adjacent and opposite sides relative to one of the 45° angles.

**Activity 2: Compare Us**
**Directions:** Do the following activities and answer the questions that follow. Use a protractor to find the measures of the angles of each triangle.




1. ∠A = _____
2. ∠C = _____
3. ∠E = _____
4. ∠F = _____
5. ∠G = _____
6. ∠I = _____

*Questions*

a. What have you noticed about the lengths of sides of each triangle?
b. What have you observed about the measures of the angles of each triangle?
c. What do you call these triangles?
d. What the mathematical concepts that you have learned from the activity?

*In this activity, you have learned about some special angles. To evaluate the trigonometric ratios of these special acute angles, we can use geometric methods. These special acute angles are 30°, 45°, and 60°.*

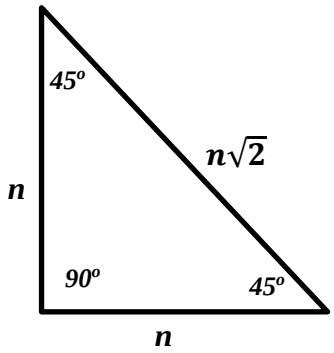
## *Discover*

In trigonometry, 30°, 45°, 60° and 90° are called as special angles and they always lie in the first quadrant. These special angles 30°, 45° and 60° are frequently seen in applications and we can use geometry to determine the trigonometric ratios of these angles. Let us see, how to determine trigonometric ratios of these special angles using geometry.

**45°-45°-90° Right Triangle Theorem**

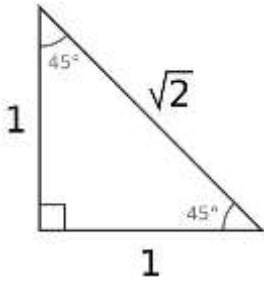
In a 45°-45°-90° Right Triangle,

- the legs are congruent;
- the length of the hypotenuse is $\sqrt{2}$ times the length of the leg
- hypotenuse = $\sqrt{2}$ leg

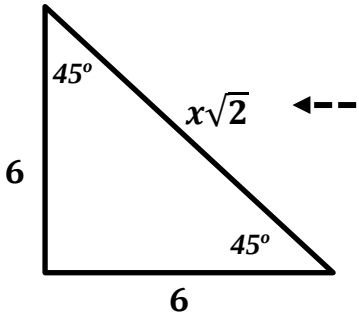



**An Isosceles Right Triangle**

The ratio of the sides is
n : n: $n\sqrt{2}$



**Example 1**. Find the length of the indicated side.




Using the ratio

n: n : n$\sqrt{2}$

6 : 6 : 6 $\sqrt{2}$, then x=6

$6\sqrt{2}$ is the hypotenuse

Now let us apply the six trigonometric ratios for the 45° angle.

$\sin 45° = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{6}{6} = 1$

$\sec 45° = \frac{6\sqrt{2}}{6} = \sqrt{2}$

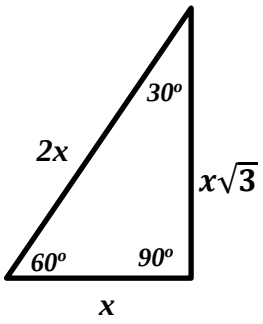
$\csc 45° = \frac{6\sqrt{2}}{6} = \sqrt{2}$

$\cot 45° = \frac{6}{6} = 1$

**30°-60°-90° Right Triangle Theorem**

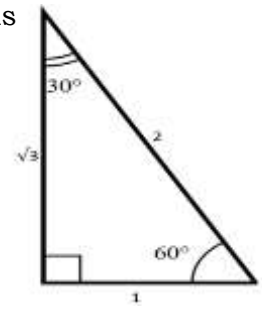
In a 30°-60°-90° Right Triangle,

- the length of the hypotenuse is twice the length of the shorter leg
- the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg
- hypotenuse = 2 shorter leg
- longer leg = $\sqrt{3}$ shorter leg

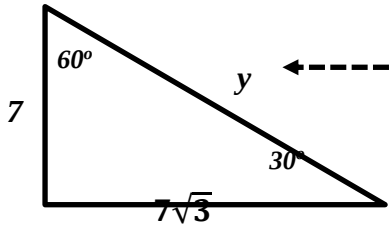



The ratio of the sides is

n: 2n: n$\sqrt{3}$




**Example 2**. Find the length of the indicated side.




Using the ratio n : 2n: n $\sqrt{3}$

**n** = shorter leg = 7

**2n** = hypotenuse = 2 (7)

**y** = 14

**n** $\sqrt{3}$ = longer leg = $7\sqrt{3}$

Now let us apply the six trigonometric ratios for the 30° angle.

$\sin 30° = \frac{7}{14} = \frac{1}{2}$ $\quad\quad$ $\sec 30° = \frac{14}{7\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos 30° = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$ $\quad\quad$ $\csc 30° = \frac{14}{7} = 2$

$\tan 30° = \frac{7}{7\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad\quad$ $\cot 30° = \frac{7\sqrt{3}}{7} = \sqrt{3}$

Let us also solve for the 60° angle.

$\sin 60° = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$ $\quad\quad$ $\sec 60° = \frac{14}{7} = 2$

$\cos 60° = \frac{7}{14} = \frac{1}{2}$ $\quad\quad$ $\csc 60° = \frac{14}{7\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan 60° = \frac{7\sqrt{3}}{7} = \sqrt{3}$ $\quad\quad$ $\cot 60° = \frac{7}{7\sqrt{3}} = \frac{\sqrt{3}}{3}$

Now we are ready to evaluate the exact values of the following expressions base from the values that we have derived;

**Example 3.** sec 60° + cot 45°

Since sec 60° = 2 ; cot 45° = 1

Then: sec 60° + cot 45° = 2 + 1 = **3**

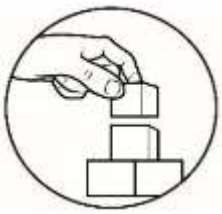
**Example 4**. sin 45° (tan 45°- cos 60°).

Since sin 45° = $\frac{\sqrt{2}}{2}$ ; tan 45°= 1 and cos 60°= $\frac{1}{2}$

Then: sin 45° (tan 45°- cos 60°)= $\frac{\sqrt{2}}{2}$ ( 1- $\frac{1}{2}$ ) = $\frac{\sqrt{2}}{2}(\frac{1}{2}) = \frac{\sqrt{2}}{4}$

**Example 5**. Determine the value of angle x when tan x = 1.

Referring to the trigonometric ratios above, tan 45° =1. Therefore, x = 45°



## *Explore*

After deepening your understanding on the trigonometric ratios of special angles, let us put those skills in higher level through the different activities.

**Activity 3: Complete the table**

Using the concepts you have learned on special angles, complete the table to summarize the trigonometric values of 30°,45° and 60° angles.

| θ | sin | cos | tan | csc | sec | cot |
|---|---|---|---|---|---|---|
| 30° | | | | | | |
| 45° | | | | | | |
| 60° | | | | | | |

## *Deepen*

**Activity 4: My Missing Part**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 2 | $\sqrt{3}$ | $\frac{3\sqrt{3}}{2}$ | tan 30° | sin 60° |

Find the missing value that will complete each mathematical statement below.

1. $\tan 60^0$(_____) = 1
2. $(\sin 45°)^2$ = _____
3. $3 \csc 60^0 - \cot 30° =$ ___
4. $\sin 30° - \cos 60° =$ _____
5. $\cos 30° +$ _____ = $\sqrt{3}$
6. $\tan 45^0$= _____
7. $2 \sin 30^0 + 3 \cos 60° - 3 \tan 45° =$ ____
8. $\cot 30^0 + \sin 60° =$ _____

## *Gauge*

**Post Assessment:**

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. In a 30°-60°-90° right triangle, suppose the hypotenuse is $10\sqrt{3}$, how long is the longer leg (b)?
   A. 15 B. $5\sqrt{3}$
   C. $5\sqrt{2}$ inches D. 25

2. In a 45°-45°-90° right triangle, suppose the hypotenuse is 10, how long are the legs (b)?
   A. 5 inches B. $5\sqrt{3}$ inches
   C. $5\sqrt{2}$ inches D. 20 inches

3. In a $30^0 - 60^0 - 90^0$, which side is the shortest?
   A. the hypotenuse
   B. the side opposite the $60^0$ angle
   C. the side opposite the $30^0$ angle
   D. the included side of the acute angles

4. In a 45°-45°-90° special right triangle what is the relation of the hypotenuse (c) to its shorter side (a)?
   A. c = 2a B. $c = a\sqrt{2}$
   C. $c = 2\sqrt{a}$ D. $2\sqrt{3}$

5. In a 30°-60°-90° special right triangle what is the relation of the hypotenuse (c) to its shortest side (a)?
   A. c = 2a B. $c = 2\sqrt{a}$
   C. $c = a\sqrt{2}$ D. $c = 2\sqrt{3}$

For numbers 6 to 8, give the exact value of the trigonometric ratios of the special angles.

6. $\sin 60^\circ$
   A. $\frac{1}{2}$ B. $\frac{\sqrt{2}}{2}$
   C. $\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{3}$

7. $\cos 45^\circ$

A. $\frac{1}{2}$ B. $\frac{\sqrt{2}}{2}$

C. $\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{3}$

8. tan 30°

A. $\frac{1}{2}$ B. $\frac{\sqrt{2}}{2}$

C. $\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{3}$

9. What if the value of x in figure 1?

A. $6\sqrt{3}$ B. $6\sqrt{2}$

C. 6 D. 3

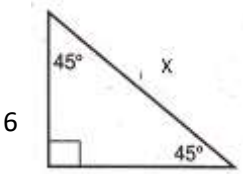


Figure 1

10. What is the value of 2(sin30°) – tan 45°?

A. 2 B. 1

C. 0 D. -1

11. What is the exact value of cos $30^0$ + sin $60^0$?

A. 0 B. 1

C. $\sqrt{2}$ D. $\sqrt{3}$

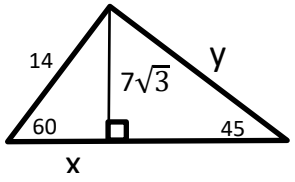


Figure 2

12. What is the estimated value for y in figure 2?

A. 7 B. 9

C. $7\sqrt{5}$ D. $7\sqrt{6}$

13. Suppose figure 3 is a 30-60-90 rt triangle, if $\boldsymbol{z} = 6\sqrt{3}$ what is **y**?

A. $3\sqrt{3}$ B. $4\sqrt{3}$

C. 6 D. 9

14. In figure 3, when x = 6 , z = 12 and $y = 6\sqrt{3}$. What type triangle is illustrated?

A. 30-60-90 B. 45-45-90

C. 60-60-60 D. none of these

15. What must be the measure of y in figure 3 when x = 6 and z = 12?

A. $6\sqrt{2}$ B. $12\sqrt{2}$

C. $6\sqrt{2}$ D. $12\sqrt{2}$

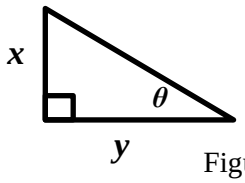


Figure 3

# References


**Printed Materials:**

Bryant, Merden L., Leonides E Bulalayao, Melvin M. Callanta, Jerry D Cruz Richard F. De Vera, Gilda T. Garcia, Sonia E Javier, Roselle A. Lazaro, Bernadeth J. Mesterio Rommel Hero A Saladino, *Mathematics Grade 9 Learner's Module* First Edition 2014, Reprinted 2016, Sunshine Interlinks Publishing House, Inc. pages 447-455.

Rivera, Gemma E., Albert Flores, *Advanced Algebra & Trigonometry*, BEC 2004, Philippine Copyright 2006 by Victorious Publishing Inc. pages 319-356.

**Links:**

https://www.ck12.org/book/ck-12-trigonometry-second-edition/section/1.0/

https://www.google.com/search?q=30+60+90+triangle+example+problems&tbm=isch&ved=2ahUKEwjM35OyuJDuAhWXzYsBHWALA4cQ2-cCegQIABAA&oq=30+90+triangle+example+problems&gs_lcp=CgNpbWcQARgA

https://www.onlinemathlearning.com/30-60-90-right-triangle.html