Senior High School

General Mathematics

Module 15:

Propositions, Logical Operators, and Truth Values

AIRs - LM

GENERAL MATHEMATICS
Module 15: Propositions, Logical Operators, and Truth Values
Second Edition, 2021

Development Team of the Module

Author: Meryl Mae R. Nelmida
Editor: SDO La Union, Learning Resource Quality Assurance Team
Content Reviewer: Catherine F. Carbonell
Language Reviewer: Lovely G. Tadije
Illustrator: Ernesto F. Ramos Jr.
Design and Layout: Antoniette G. Padua

Management Team:

Atty. Donato D. Balderas Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent
German E. Flora, PhD, *CID Chief*
Virgilio C. Boado, PhD, *EPS in Charge of LRMS*
Erlinda M. Dela Pena, Ed.D., *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

Printed in the Philippines by: ________________________

Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

Senior High School

General Mathematics

Module 15:

Propositions, Logical Operators, and Truth Values

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

In your previous lessons in General Mathematics, you were able to study about functions and their graphs and basic business mathematics. In this module, you will be introduced to another key content in General Mathematics which is logic.

One area of mathematics that has its roots deep in philosophy is the study of logic. According to Price, Rath, Leschensky (1992), logic is the study of formal reasoning based upon statements or propositions. Logic evolved out of a need to fully understand the details associated with the study of mathematics. At the most fundamental level, Wheeler (1995) emphasized that mathematics is a language and it is a language of choice and must be communicated with great precision. The idea of logic was a major achievement of Aristotle. In his effort to produce correct laws of mathematical reasoning, Aristotle was able to codify and systemize these laws into a separate field of study.

This learning material will provide you with information and activities that will deepen your understanding about logic specifically on propositions, logical operators, and truth values.

After going through this module, you are expected to:

1. illustrate and symbolize propositions **(M11GM-IIg-2)**;
2. distinguish between simple and compound propositions **(M11GM-IIg-3)**;
3. perform the different types of operations on propositions **(M11GM-IIg-4)**;
4. determine the truth values of propositions **(M11GM-IIh-1)**; and
5. illustrate the different forms of conditional propositions **(M11GM-IIh-2)**.

Learning Objectives:

1. define a proposition
2. define simple and compound propositions
3. define conditional, converse, inverse, and contrapositive statements and their corresponding logical notations
4. enumerate factors for a statement to be a proposition
5. determine simple propositions from compound propositions
6. describe the rules, logical connectives, symbols, and truth table of the different operations on propositions
7. apply the rules of the different operations on propositions
8. derive converse, inverse, and contrapositive statements from conditional statements and vice versa.

Before going on, check how much you know about this topic. Answer the pretest on the next page in a separate sheet of paper.

Pretest

Directions: Read each item carefully. Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

For nos. 1 – 2, refer to the following:

I. 2.5 is an integer
II. 2 is even and prime number.
III. If an integer is even, then its square is also even.
IV. 2 is a square number

1. Which of the statements above illustrate a simple proposition?
   A. I and II B. II and III C. III and IV D. I and IV

2. Which of the statements above illustrate a compound proposition?
   A. I and II B. II and III C. III and IV D. I and IV

For nos. 3-5, express the statements in symbols. Refer to the following propositions:

p: I will go to the mall.
q: I will sleep at the cinema.
r: I will eat at the gym.

3. I will not eat at the gym or sleep at the cinema.
   A. $\sim r \vee q$ B. $r \vee \sim q$ C. $\sim r \vee \sim q$ D. $q \vee r$

4. If I will not go to the mall, then I will eat at the gym.
   A. $\sim p \rightarrow r$ B. $r \vee \sim p$ C. $\sim p \rightarrow \sim r$ D. $r \rightarrow p$

5.I will not sleep at the cinema if and only if I will not go to the mall and I will sleep at the gym.
   A. $q \leftrightarrow (\sim p \wedge \sim q)$ B. $\sim q \leftrightarrow (\sim p \wedge \sim q)$ C. $\sim q \rightarrow (\sim p \wedge \sim q)$ D. $q \rightarrow (\sim p \wedge \sim q)$

6. Using the propositions p, q, and r above, what is the translation of $p \vee \sim r$ in words?
   A. I will go to the mall or eat at the gym.
   B. I will go to the mall and eat at the gym.
   C. I will go to the mall or not eat at the gym.
   D. I will go to the mall and not eat at the gym.

For numbers 7 - 9, translate the following symbols to English sentences; refer to the following given:

p: Logic is fun. q: Logic is boring.

7. $p \wedge \sim q$

   A. Logic is fun or boring.
   B. Logic is fun and not boring.
   C. If logic is fun, then it is boring.
   D. Logic is fun if and only if it is boring.

8. $p \rightarrow q$

A. Logic is fun or boring.
B. Logic is fun and not boring.
C. If logic is fun, then it is boring.
D. Logic is fun if and only if it is boring.

9. $p \wedge q$

A. Logic is fun and boring.
B. Logic is fun or not boring.
C. If logic is fun, then it is boring.
D. Logic is fun if and only if it is boring.

For numbers 10 – 12, consider the given statement $p \vee [\sim (p \wedge q)]$. What is the truth value of the statement if:

10. p and q are true?

A. True B. False C. Cannot be determined

11. p and q are false?

A. True B. False C. Cannot be determined

12. p is true and q is false?

A. True B. False C. Cannot be determined

For numbers 13 – 15, let p be" I am an achiever". and q be "I get my reward.", identify the form of the conditional propositions being illustrated.

13. If I get my reward, then I am an achiever.

A. Conditional B. Contrapositive C. Converse D. Inverse

14. If I am not an achiever, then I will not get my reward.

A. Conditional B. Contrapositive C. Converse D. Inverse

15. If I am an achiever, then I will get my reward.

A. Conditional B. Contrapositive C. Converse D. Inverse

Jumpstart

Activity: Intelligence Test

The following short IQ test consists of 4 short questions which test your intelligence, and the results will tell you, whether you are truly a manager/leader or a child. The questions like: "How do you put a giraffe into a refrigerator?" are easy — the answers may be not: The questions are NOT that difficult. Get a piece of paper and answer the following questions consecutively.

How do you put a giraffe into a refrigerator?
How do you put an elephant into a refrigerator?
The Lion King is hosting an animal conference. All the animals attend except one. Which animal did not attend?
There is a river you must cross but it is used by crocodiles, and you do not have a boat. How do you manage it?

Let’s check if your answers are correct. Turn to the next page ☺

1. How do you put a giraffe into a refrigerator?

Correct Answer: Open the refrigerator, put in the giraffe, and close the door. This question tests whether you tend to do simple things in an overly complicated way.

2. How do you put an elephant into a refrigerator?

Did you say, Open the refrigerator, put in the elephant, and close the refrigerator? Wrong Answer.

Correct Answer: Open the refrigerator, take out the giraffe, put in the elephant and close the door. This tests your ability to think through the repercussions of your previous actions.

3. The Lion King is hosting an animal conference. All the animals attend except one. Which animal did not attend?

Correct Answer: The Elephant. The elephant is in the refrigerator. You just put him in there. This tests your memory.

Okay, even if you did not answer the first three questions correctly, you still have one more chance to show your true abilities.

4. There is a river you must cross but it is used by crocodiles, and you do not have a boat. How do you manage it?

Correct Answer: You jump into the river and swim across. Have you not been listening? All the crocodiles are attending the Animal Meeting. This tests whether you learn quickly from your mistakes.

Did you have fun answering the NOT-SO-DIFFICULT questions above? I bet you are laughing right now. The questions above prompted you to show off your reasoning skills. But did you really expect for the correct answers to really be the answers to those questions?

Did you know that there is a study of methods and principles used to distinguish correct reasoning from incorrect reasoning? That is exactly what we are going to learn in this module – LOGIC. Let us introduce ourselves with the basics of logic – propositions, logical operators, and truth values. ☺

Discover

Propositions

A proposition is the basic building block of logic. It is defined as a declarative sentence that is either True or False, but not both. The Truth Value of a proposition is True (denoted as T) if it is a true statement, and False (denoted as F) if it is a false statement.

Definition: A **proposition** is a declarative sentence that is either true or false, but not both. If a proposition is true, then its truth value is true which is denoted by T; otherwise, its truth value is false which is denoted by F.

Look at the following sentences.

1. The sun rises in the East and sets in the West.
2. 1 + 1 = 2
3. 'b' is a vowel.

All of the above sentences are propositions, where the truth value of the first two is true and the third one is false.

Some sentences that do not have a truth value or may have more than one truth value are not propositions. Look at the following sentences.

1. What time is it?
2. Go out and play.
3. x + 1 = 2.

The above sentences are not propositioning as the first two do not have a truth value and are not declarative sentences, and the third one may be true or false.

Propositions are usually denoted by small letters. For example, the proposition

p: The sun rises in the East and sets in the West.

may be read as

p is the proposition "The sun rises in the East and sets in the West."

Try to determine whether each of the following statements is a proposition or not. If it is a proposition, give its truth value.

p: Luzon is an island in the Philippines.

q: Find a number which divides your grade level.

r: My friend will get a perfect score in his performance task.

s: Welcome to Module Legends!

t: 4 + 6 =10

u: What is the inverse of the function?

v: I am lying.

p_1: It is not the case that $\sqrt{\mathbf{2}}$ is a rational number.

p_2: Either Math is fun and interesting, or it is boring.

p_3: If you are a Grade 11 student, then you are 16 years old.

p_4: If the quadrilateral has four congruent sides and angles, then the quadrilateral is a square and if the quadrilateral is a square, then the quadrilateral has four congruent sides and angles.

Recall that for a statement to be proposition, it must be a declarative sentence and it should have a truth value of either true or false, but not both true and false at the same time.

Proposition	Declarative Sentence	Truth Value	Proposition or Not
p	✔	T	Proposition
q	✖ (imperative)	-	Not a proposition
r	✔	Either true or false (its truth value will only be known after the performance task grading)	Proposition
s	✖ (exclamatory)	-	Not a proposition
t	✔ (A mathematical sentence that can be read as “The sum of four and sic is ten.” When translated in English)	T	Proposition
u	✖ (interrogative)	-	Not a proposition
v	✔	Neither true or false	Not a proposition
p_1	✔	F	Proposition
p_2	✔	T	Proposition
p_3	✔	F	Proposition
p_4	✔	T	Proposition

Simple and Compound Propositions

Propositions can be simple or compound. A statement that conveys one thought with no connectives is a simple proposition. A proposition is simple if it cannot be broken down any further into other component propositions; while a proposition is compound if it can be broken down into simpler propositions. Compound propositions are formed using logical connectors or combination of these logical connectors. Some logical connectors are *not, and, or,* and, *if…, then…* Logical connectors involving propositions p and q may be expressed as follows:

not p
p and q
p or q
If p, then q.

Referring to the propositions in the previous example, determine whether it is a simple or compound proposition. If it is a compound proposition, identify the simple components.

The propositions p, r, and t are simple propositions. On the other hand, the following are compound propositions:

p_1: It is not the case that $\sqrt{2}$ is a rational number.
p_2: Either Math is fun and interesting, or it is boring.
p_3: If you are a Grade 11 student, then you are 16 years old.
p_4: If the quadrilateral has four congruent sides and angles, then the quadrilateral is a square and if the quadrilateral is a square, then the quadrilateral has four congruent sides and angles.

Furthermore, we can determine the simple propositions that make up the compound propositions

Proposition	Simple Component/s
p_1	- $\sqrt{2}$ is a rational number.
p_2	- Math is fun. - Math is interesting. - Math is boring.
p_3	- You are a Grade 11 student. - You are 16 years old.
p_4	- The quadrilateral has four congruent sides. - The quadrilateral is a square.

The Truth Table

Mathematics normally uses a two-valued logic: every statement is either true or false. You use truth tables to determine how the truth or falsity of a complicated statement depends on the truth or falsity of its components.

Definition: Given a proposition, its **truth table** shows all its possible truth values.

Since a proposition has two possible truth values, a proposition *p* would have the following truth table.

p
T
F

Truth values can also be used to display various combinations of the truth values of two propositions p and q. The rows of the table will correspond to each truth value combination of p and q., so there will be $2^2 = 4$ rows. The truth values for propositions p and q are as follows.

p	q
T	T
T	F
F	T
F	F

Similarly, suppose p, q and r are propositions, then the truth table involving the given propositions has $2^3 = 8$ rows, as shown below.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

In general, a truth table involving *n* propositions has 2^n rows; Thus, given propositions p, q, r, and s, its truth table will contain 2^4 rows as shown below.

p	q	r	s
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

Operations of Propositions/Logical Operators

There are five (5) logical operators for propositions: negation, conjunction, disjunction, conditional, and biconditional.

The **negation** of a proposition p is denoted by

~p (read as "not p")

and is defined through its truth table

p	~p
True (T)	False (F)
False (F)	True (T)

Negation is the result of reversing the truth value of a given proposition. If a proposition is true, its negation is false; and if a proposition is false, its negation is true. Given the proposition *p:* "This book is interesting." *~p* can be translated to either of the following:

- This book is not interesting.
- This book is uninteresting.
- It is not the case that this book is interesting.

The **conjunction** of propositions p and q is denoted by

p ^ q (read as "p and q")

and is defined through its truth table

P	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

The propositions p and q are called conjuncts. Conjunction is the result of combining two other propositions called conjuncts. The conjunction of two statements is true only if both conjuncts are true.

Given the following propositions,

p: This book is interesting.

q: I am staying at home.

p ∧ q: This book is interesting and I am staying at home.

The **disjunction** of propositions p and q is denoted by

$p \vee q$ (read as "p or q")

and is defined through its truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The propositions p and q are called disjuncts. Disjunction is the result of combining two other propositions called disjuncts. The disjunction of two statements is false only if both disjuncts are false.

Given the following propositions,

p: This book is interesting.

q: I am staying at home.

$p \vee q$: This book is interesting or I am staying at home.

The **conditional/implication** of propositions p and q is denoted by

$p \longrightarrow q$ (read as "p implies q" or "If p, then q.")

and is defined through its truth table

P	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication is the result of combining a hypothesis (antecedent) to a conclusion (consequent). The implication is true in all cases, except when the antecedent is true and the consequent is false.

Given the following propositions,

p: This book is interesting.

q: I am staying at home.

$p \longrightarrow q$: If this book is interesting, then I am staying at home.

The **biconditional/equivalence** of propositions p and q is denoted by

$p \leftrightarrow q$ (read as "p if and only if q")

and is defined through its truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional or equivalence is the result of combining propositions in the form "…if and only if…" The equivalence is true if both propositions are true or both are false.

Given the following propositions,

p: This book is interesting.

q: I am staying at home.

$p \leftrightarrow q$: This book is interesting if and only if I am staying at home.

Let us combine the different logical operators. Look at how the following statements were translated using the logical connectives.

p or not q.	$\mathbf{p \vee (\sim q)}$
If p then q	$\mathbf{p \rightarrow q}$
If p and r, then q.	$\mathbf{(p \wedge r) \rightarrow q}$
p if and only if q.	$\mathbf{p \leftrightarrow q}$
p if and only if q and r.	$\mathbf{p \leftrightarrow (q \wedge r)}$

Next, look at how the propositions are translated in logical notations.

Let: p = "The pancake is hot."

q = "The kakanin is cold."

r = "The fried chicken will be delivered.

The pancake is hot and the fried chicken will not be delivered.	$p \wedge (\sim r)$
If the kakanin is cold, then the fried chicken will be delivered.	$q \rightarrow r$
If the fried chicken won't be delivered, then the pancake is hot and the kakanin is not cold.	$(\sim r) \rightarrow (p \wedge \sim q)$

This time, look at how the following logical notations are translated in compound propositions.

$(\sim p) \leftrightarrow (q \vee r)$	The pancake is not hot if and only if the kakanin is cold or the fried chicken will be delivered.
$\sim p \wedge q \rightarrow \sim r$	If the pancake is not hot and the kakanin is cold, then the fried chicken will not be delivered.
$\sim p \vee q \wedge r$	The pancake is not hot or the kakainin is cold and the fried chicken will be delivered.

Here is a summary table on the different logical operators.

<table>
<tr><th>Logical Operator and its Rule</th><th>Logical Connective</th><th>Symbol</th><th>Truth Table</th></tr>
<tr><td>Negation

If a proposition is true, its negation is false; and if a proposition is false, its negation is true.</td><td>NOT

It is not the case that</td><td>~
¬</td><td><table><tr><td>p</td><td>~p</td></tr><tr><td>True (T)</td><td>False (F)</td></tr><tr><td>False (F)</td><td>True (T)</td></tr></table></td></tr>
<tr><td>Conjunction

The conjunction of two statements is true only if both conjuncts are true.</td><td>AND

BUT</td><td>∧</td><td><table><tr><td>p</td><td>q</td><td>p ^ q</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table></td></tr>
<tr><td>Disjunction

The disjunction of two statements is false only if both disjuncts are false.</td><td>OR</td><td>∨</td><td><table><tr><td>p</td><td>q</td><td>p ∨ q</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>T</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table></td></tr>
<tr><td>Conditional/Implication

The implication is true in all cases, except when the antecedent is true and the consequent is false.</td><td>IMPLIES

IF ___,
THEN ___.</td><td>→</td><td><table><tr><td>p</td><td>q</td><td>p→q</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table></td></tr>
<tr><td>Biconditional/Equivalence

The equivalence is true if both propositions are true or both are false.</td><td>IF AND ONLY IF</td><td>↔</td><td><table><tr><td>Equivalence</td><td>q</td><td>p↔q</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table></td></tr>
</table>

Truth Tables of Compound Propositions

Now that you already know the different operation on propositions, let us combine what you have learned in the initial parts of this module to construct truth tables for compound propositions.

1. Let p and q be propositions. Construct the truth table for the compound proposition $p \rightarrow \sim q$.

Note that there are two propositions, p and q, involved in the compound proposition. Thus, the truth table will contain 4 rows, the first two columns of which are

p	q
T	T
T	F
F	T
F	F

Using the truth table for the definition of negation, we add one more column to indicate the truth value of $\sim q$.

p	q	$\sim q$
T	T	F
T	F	T
F	T	F
F	F	T

In the final column, we add the truth value of $p \rightarrow \sim q$ which is the implication of p and $\sim q$ as hypothesis and conclusion, respectively.

p	q	$\sim q$	$p \rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

2. Let p and q be propositions. Construct the truth table for the compound proposition $(p \rightarrow q) \wedge (q \rightarrow p)$.

Note that there are two propositions, p and q, involved in the compound proposition. Thus, the truth table will contain 4 rows, the first two columns of which are

p	q
T	T
T	F
F	T
F	F

Using the truth table for the definition of implication, we add two more columns to indicate the truth value of $(p \rightarrow q)$ and $(q \rightarrow p)$.

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Note: In getting the truth values of $q \rightarrow p$, look at the column for q first, then p after. In obtaining the truth values of compound propositions, start with the first proposition seen from the compound proposition.

In the final column, we add the truth value of $(p \rightarrow q) \wedge (q \rightarrow p)$. which is a conjunction involving $p \rightarrow q$ and $q \rightarrow p$ as conjuncts.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

3. Consider the compound proposition $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$. Construct its truth table.

We first consider the truth table pertaining to $(p \rightarrow r) \wedge (q \rightarrow r)$.

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Next, we consider (p ∨ q) ⟶ r. For this, we add first the truth value for p ∨ q and (p ∨ q) ⟶ r, which will be appended to the table above.

p	q	r	p ⟶ r	q ⟶ r	(p ⟶ r) ^ (q ⟶ r)	p ∨ q	(p ∨ q) ⟶ r
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Lastly, we consider the truth value for the compound proposition

[(p ⟶ r) ^ (q ⟶ r)] ⟶ [(p ∨ q) ⟶ r]

p	q	r	p ⟶ r	q ⟶ r	(p ⟶ r) ^ (q ⟶ r)	p ∨ q	(p ∨ q) ⟶ r	[(p ⟶ r) ^ (q ⟶ r)] ⟶ [(p ∨ q) ⟶ r]
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	T
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

Conditional Propositions

Conditional propositions can be converted into various forms. Suppose p and q are propositions, from the conditional proposition **p ⟶ q**, we derive three other conditional propositions namely its

Converse: q ⟶ p

Inverse: ~p ⟶ ~q

Contrapositive: ~q ⟶ ~p

Example 1. Given the propositions
p: A number is even.
q: It is divisible by 2
Look at how the conditional propositions are constructed.

Conditional	If a number is even, then it is divisible by 2.
Converse	If it is divisible by 2, then the number is even.
Inverse	If a number is not even, then it is not divisible by 2
Contrapositive	If it is not divisible by 2, then the number is not even

Example 2. Given the contrapositive statement, “If it is not consumed in large volumes, then chocolate cannot be harmful to one’s health.” Determine the other forms of conditional proposition.

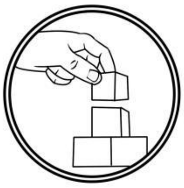
Since contrapositive statements are denoted by $\sim q \rightarrow \sim p$, the following will be the two simple propositions in the conditional statement.

p: Chocolate is harmful to one’s health.

q: It is consumed in large volumes.

The other forms of conditional statement are as follows:

Conditional	If chocolate can be harmful to one’s health, then it is consumed in large volumes.
Converse	If it is consumed in large volumes, then chocolate can be harmful to one’s health.
Inverse	If chocolate cannot be harmful to one’s health, then it is not consumed in large volumes.



Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 1: Break it Down, Pro!

Directions: Determine whether the following statements are propositions. If the proposition is a compound proposition, identify the simple components and the logical connectors used.

1. Define a logarithmic function.
2. Anne Curtis has over 15 million followers on Instagram.
3. If Popoy’s score is less than 50 percent, then Popoy will fail the exam.
4. Where is the party?
5. Either it is sunny in Metro Manila or its streets are flooded.
6. If a, b, and c denote the lengths of the legs and the hypotenuse of a right triangle, then $a^2 + b^2 = c^2$.
7. Dinner is served with coffee or tea.
8. Natasha’s average is at least 96 and she is getting an excellence award for the first semester.
9. -7 is not a negative number.
10. If Bryan receives a scholarship, then he will go to college.

Activity 2: Express Me!

Directions: Express the following in logical notations or statements as the case may be.

1. Let p, q, and r be the propositions.
   - p: Pam has a stomachache.
   - q: Pam misses the exam.
   - r: Pam receives a passing grade in Science.

   a. Pam does not have a stomachache, but she misses the exam.
   b. If Pam has a stomachache, then she misses the exam and does not receive a passing grade in Science.
   c. Either Pam has a stomachache and misses the exam, or she does not miss the exam and she receives a passing grade in Science.
   d. $q \longrightarrow (\sim r)$
   e. $p \longrightarrow (q \wedge \sim r)$
   f. $\sim q \leftrightarrow r$

2. Let p, q, and r be the propositions.
   - p: The classroom is clean.
   - q: The corridor is dirty.
   - r: The students are busy.

a. The classroom is clean but the corridor is dirty.
b. If the classroom is clean, then the corridor is dirty or the students are busy.
c. If the classroom is not clean, then the students are busy if and only if the corridor is dirty.
d. $p \longrightarrow (q \wedge \sim r)$
e. $(\sim q \wedge \sim r) \longrightarrow \sim p$

Activity 3: We Deserve the Truth!

A. Fill in the correct truth values for each row to make the truth table for each compound proposition accurate.

$p \vee [\sim (p \wedge q)]$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee [\sim (p \wedge q)]$
T	**T**			
T	**F**			
F	**T**			
F	**F**			

$[\sim(p \wedge q)] \rightarrow [(\sim p) \vee (\sim q)]$

p	q	~p	~q	p ^ q	~ (p ^ q)	(~p) ∨ (~q)	[~ (p ^ q)] ⟶ [(~p) ∨ (~q)]
T	**T**						
T	**F**						
F	**T**						
F	**F**						

B. Construct the truth table for the following compound propositions.

1. $(\sim p) \wedge (q \wedge \sim r)$
2. $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
3. $[(p \rightarrow q) \wedge (q \rightarrow p)] \rightarrow (p \vee q)$

Activity 4: Complete Me!

Directions: Complete the table by writing the different forms of conditional propositions.

Conditional	Converse	Inverse	Contrapositive
If the clothes are neatly stacked and pressed, then the house helper arrived today.			
	If Paul studied alone, then he will get the highest score in the class.		
		If it did not flood yesterday, then the streets are dry today.	
			If Mariah did not hit the highest whistle note, then the audience will not give her a standing ovation.

Deepen

Proposition Posing

In this activity, create a compound proposition involving propositions p, q, and r in logical notation, then construct its truth table.

Problem Posing Rubric					
	1	**2**	**3**	**4**	**Score**
The Degree of the Proposition "What is the level of difficulty of the proposition?" x 3	It is an exercise type proposition which can be solved very easily.	It is an exercise type proposition which can be solved easily.	It is a normal proposition which can be solved easily.	It is a normal proposition which is difficult to solve.	
Originality of the Proposition "Is this an original proposition?" x 2	It is not an original proposition.	It is partially an original proposition.	It is almost an original proposition.	It is an original proposition.	
Solution "Is the solution properly presented?" x 5	The data and information provided in the problem is not sufficient for any solution.	The problem is too complicated to be solved although the data is sufficient.	The problem has a solution but the data is mistaken and missing.	The problem has a solution since all the data and information of the problem is complete and proper.	
				Total	

Gauge

Directions: Read each item carefully. Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

For nos. 1 – 2, refer to the following:

I. 2.5 is an integer
II. 2 is even and prime number.
III. If an integer is even, then its square is also even.
IV. 2 is a square number

1. Which of the statements above illustrate a simple proposition?
   A. I and II B. II and III C. III and IV D. I and IV
2. Which of the statements above illustrate a compound proposition?
   A. I and II B. II and III C. III and IV D. I and IV

For nos. 3-5, express the statements in symbols. Refer to the following propositions:

p: I will go to the mall.
q: I will sleep at the cinema.
r: I will eat at the gym.

3. I will not eat at the gym or sleep at the cinema.
   A. $\sim r \vee q$ B. $r \vee \sim q$ C. $\sim r \vee \sim q$ D. $q \vee r$
4. If I will not go to the mall, then I will eat at the gym.
   A. $\sim p \rightarrow r$ B. $r \vee \sim p$ C. $\sim p \rightarrow \sim r$ D. $r \rightarrow p$
5. I will not sleep at the cinema if and only if I will not go to the mall and I will sleep at the gym.
   A. $q \leftrightarrow (\sim p \wedge \sim q)$ B. $\sim q \leftrightarrow (\sim p \wedge \sim q)$ C. $\sim q \rightarrow (\sim p \wedge \sim q)$ D. $q \rightarrow (\sim p \wedge \sim q)$
6. Using the propositions p, q, and r above, what is the translation of $p \vee \sim r$ in words?
   A. I will go to the mall or eat at the gym.
   B. I will go to the mall and eat at the gym.
   C. I will go to the mall or not eat at the gym.
   D. I will go to the mall and not eat at the gym.

For numbers 7 - 9, translate the following symbols to English sentences; refer to the following given:

p: Logic is fun. q: Logic is boring.

7. $p \wedge \sim q$

   A. Logic is fun or boring.
   B. Logic is fun and not boring.
   C. If logic is fun, then it is boring.
   D. Logic is fun if and only if it is boring.

8. $p \rightarrow q$

A. Logic is fun or boring.
B. Logic is fun and not boring.
C. If logic is fun, then it is boring.
D. Logic is fun if and only if it is boring.

9. $p \wedge q$

A. Logic is fun and boring.
B. Logic is fun or not boring.
C. If logic is fun, then it is boring.
D. Logic is fun if and only if it is boring.

For numbers 10 – 12, consider the given statement $p \vee [\sim (p \wedge q)]$*. What is the truth value of the statement if:*

10. p and q are true?

A. True B. False C. Cannot be determined

11. p and q are false?

A. True B. False C. Cannot be determined

12. p is true and q is false?

A. True B. False C. Cannot be determined

For numbers 13 – 15, let p be" I am an achiever". and q be "I get my reward.", identify the form of the conditional propositions being illustrated.

13. If I get my reward, then I am an achiever.

A. Conditional B. Contrapositive C. Converse D. Inverse

14. If I am not an achiever, then I will not get my reward.

A. Conditional B. Contrapositive C. Converse D. Inverse

15. If I am an achiever, then I will get my reward.

A. Conditional B. Contrapositive C. Converse D. Inverse

References

Printed Materials:

Department of Education. (2016). Unit X: Logic, General Mathematics Learner's Material (pp. 240-269). Pasig City, Philippines.

Website:

Ikenega, Bruce. (n.d.) Truth Tables, Tautologies, and Logical Equivalences. Retrieved October 21, 2020 from http://sites.millersville.edu/bikenaga/math-proof/truth-tables/truth-tables.html

Propositional Logic. (September 2, 2019). Retrieved October 20, 2020 from https://www.stat.berkeley.edu/~stark/SticiGui/Text/logic.htm

Geeks for Geeks. (February 14, 2018). Introduction to Propositional Logic. Retrieved October 21, 2020 from https://www.geeksforgeeks.org/proposition-logic/

Conditional Statements. (January 21, 2020). Retrieved October 20, 2020 from https://calcworkshop.com/reasoning-proof/conditional-statement/

Propositional Logic. (n.d.). Retrieved October 20, 2020 from http://intrologic.stanford.edu/textbook/chapter_02.html

For inquiries or feedback, please write or call:

Department of Education – SDO La Union
Curriculum Implementation Division
Learning Resource Management Section
Flores St. Catbangen, San Fernando City La Union 2500
Telephone: (072) 607 - 8127
Telefax: (072) 205 - 0046
Email Address:
launion@deped.gov.ph
lrm.launion@deped.gov.ph