9

Mathematics

Quarter 3: Week 3 - Module 3:

The Midline Theorem, Theorems on Trapezoids and Theorems on Kites

AIRs - LM

Mathematics 9
Quarter 3: Week 3 - Module 3: The Midline Theorem, Theorems on Trapezoids and Theorems on Kites

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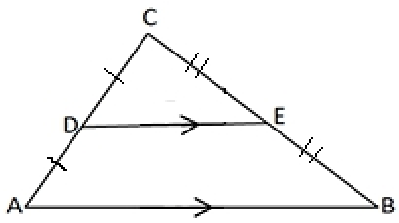
Lesson 1	The Midline Theorem, Theorems on Trapezoids and Theorems on Kites

Jumpstart

Let us start this lesson by doing the Activity 1 and 2. These statements will serve as your guide as you go through the lesson.

Activity 1. Am I Right or Wrong?
Directions: Use the figure at the below, write **R** if the statement is correct and **W** if incorrect.

______1. D is the midpoint of $\overline{AC}$
______2. E is the midpoint of $\overline{CB}$
______3. $\overline{DE} \parallel \overline{AB}$
______4. $\overline{DE}$ is the mid-segment
______5. DE =AB



Activity 2. True or False
Directions: Write T if your guess on the statement is true; otherwise, write F.

______1. The median of the trapezoid is parallel to the bases and its length is equal to half the sum of the lengths of the bases.

______2. The legs of an isosceles trapezoid are parallel and congruent.

______3. The bases of an isosceles trapezoid are congruent.

______4. The segment that joins the midpoints of the legs of a trapezoid is called the median.

______5. The area of a kite is half the product of the lengths of its diagonals.

Discover

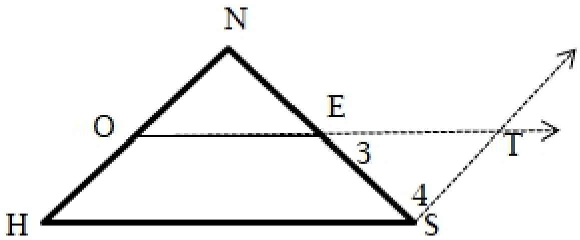
The Midline Theorem

Theorem 5. The segment that joins the midpoint of two sides of a triangle is parallel to the third side and half as long.

Let us prove this theorem by two-column proof.

Given: ΔHNS, O is the midpoint of $\overline{HN}$
E is the midpoint of $\overline{NS}$

Prove: $\overline{OE} \parallel \overline{HS}$, OE = $\frac{1}{2}$ HS

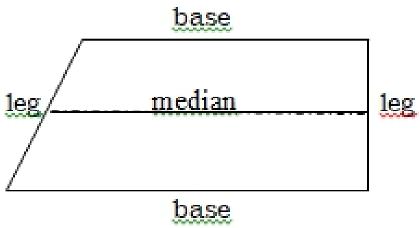


Proof:

Statements	Reasons
1. In ΔHNS, O is the midpoint of $\overline{HN}$ E is the midpoint of $\overline{NS}$	1. Given
2. In a ray opposite $\overrightarrow{EO}$, there is a point T such that OE = ET	2. In a ray, point at a given distance from the endpoint of a ray.
3. $\overline{EN} \cong \overline{ES}$	3. Definition of Midpoint
4. $\angle 2 \cong \angle 3$	4. Vertical Angle Theorem
5. ΔONE ≅ ΔTSE	5. SAS Congruence Postulate
6. $\angle 1 \cong \angle 4$	6. Corresponding parts of congruent triangles are congruent
7. $\overline{HN} \parallel \overline{ST}$	7. If Alternate Interior Angles are Congruent, then the lines are parallel.
8. $\overline{OH} \cong \overline{ON}$	8. Definition of Midpoint
9. $\overline{QN} \cong \overline{TS}$	9. CPCTC (SN 5)
10. $\overline{OS} \cong \overline{ST}$	10. Transitive Property
11. Quadrilateral HOTS is a parallelogram.	11. Definition of parallelogram.
12. $\overline{OE} \parallel \overline{HS}$	12. $\overline{OE}$ is on the side of $\overline{OT}$ of HOTS
13. OE + ET = OT	13. Segment Addition Postulate (SAP
14. OE + OE = OT	14. Substitution (SN 2)
15. 2OE = OT	15. Addition
16. $\overline{HS} \parallel \overline{OT}$	16. Parallelogram Property
17. 2OE = HS	17. Substitution
18. OE = $\frac{1}{2}$ HS (The segment joining the midpoints of two sides of a triangle is half as long as the third side.)	18. Substitution (SN 14 and 15)

You've just completed the proof of The Midline Theorem. Let us continue with the next theorem.
Given

But before that, let us recall the different parts of a trapezoid. All trapezoids have two main parts: bases and legs. The opposite sides of a trapezoid that are parallel to each other are the bases and the remaining sides are called the legs of the trapezoid. The segment that connects the midpoints of the legs of trapezoid is called the mid-segment or the median.



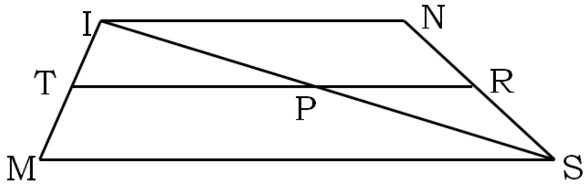
The Mid-segment Theorem of Trapezoid

Theorem 6. The median of a trapezoid is parallel to each base and its length is one-half the sum of the lengths of the bases.

$$\text{Mid-segment} = \frac{1}{2}(\text{base}_1 + \text{base}_2)$$

Let us now prove the theorem using trapezoid MINS.

Given: Trapezoid MINS with median $\overline{TR}$
Prove: $\overline{TR} \parallel \overline{IN}$, $\overline{TR} \parallel \overline{MS}$
$TR = \frac{1}{2}(MS + IN)$



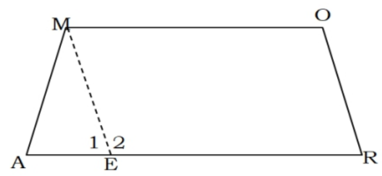
Proof:

Statements	Reasons
1. Trapezoid MINS with median $\overline{TR}$	1. Given
2. Draw $\overline{IS}$, with P as it's midpoint.	2. Line Postulate
3. $TP = \frac{1}{2}MS$ and $\overline{TP} \parallel \overline{MS}$	3. Theorem 5 (Midline theorem), on Δ IMS
4. $PR = \frac{1}{2}IN$ and $\overline{PR} \parallel \overline{IN}$	4. Theorem 5 (Midline Theorem) on Δ INS
5. $\overline{MS} \parallel \overline{IN}$	5. Definition of trapezoid
6. $\overline{TP} \parallel \overline{IN}$	6. Definition of parallel, $\overline{TP} \parallel \overline{MS}$ and $\overline{MS} \parallel \overline{IN}$
7. $\overline{TP}$ And $\overline{PR}$ are both parallel to $\overline{TP} \parallel \overline{IN}$. Thus, T, P and R are collinear.	7. $\overline{TP}$ and $\overline{PR}$ are either parallel or the same line (definition of parallel). Since they contain a common point P, then $\overline{TP}$ and $\overline{PR}$ are contained in the same line.
8. TR = TP + PR	8. Segment Addition Postulate
9. $TR = \frac{1}{2}MS + \frac{1}{2}IN$	9. Substitution
10. $TR = \frac{1}{2}(MS + IN)$	10. Distributive Property of Equality

We've just proven Theorem 6. Now what if the legs of the trapezoid become congruent? What must be true about its base angles and its diagonals? To answer this, we will study the different theorems on isosceles trapezoid.

Theorem 7. The base angles of an isosceles trapezoid are congruent.

Given: Isosceles Trapezoid AMOR
$\overline{MO} \parallel \overline{AR}$
Prove: ∠A ≅ ∠R, ∠AMO ≅ ∠O



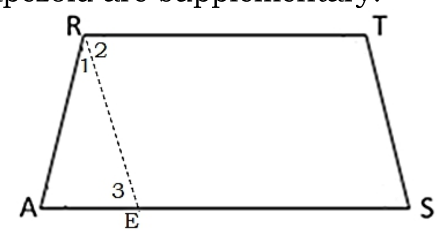
Proof:

Statements	Reasons
1. Isosceles Trapezoid AMOR	1. Given
2. $\overline{AM} \cong \overline{OR}$; $\overline{MO} \parallel \overline{AR}$	2. Definition of Isosceles Trapezoid
3. From M, draw $\overline{ME} \parallel \overline{AR}$ were E lies on $\overline{AR}$.	3. Parallel Postulate
4. MORE is a parallelogram	4. Definition of a parallelogram
5. $\overline{ME} \cong \overline{OR}$	5. Parallelogram Property 1
6. $\overline{OR} \cong \overline{ME}$	6. Symmetric Property
7. $\overline{AM} \cong \overline{ME}$	7. Transitive Property (SN 2 and 6)
8. ΔAME is an isosceles triangle.	8. Definition of an Isosceles Triangle
9. ∠1 ≅ ∠A	9. Base angles of an isosceles triangle are congruent.
10. ∠1 ≅ ∠R	10. Corresponding angles are congruent.
11. ∠R ≅ ∠A	11. Substitution
12. ∠A ≅ ∠R	12. Symmetric Property
13. ∠A and ∠AMO are supplementary angles. ∠O and ∠R are supplementary angles.	13. Same side interior angles are supplementary
14. ∠AMO ≅ ∠O	14. Supplements of congruent angles are congruent.

We have just proven Theorem 7. Let us now proceed to the next theorem.

Theorem 8. Opposite angles of an isosceles trapezoid are supplementary.

Given: Isosceles Trapezoid ARTS.
Prove: ∠ ART and ∠S are supplementary.
∠A and ∠T are supplementary.



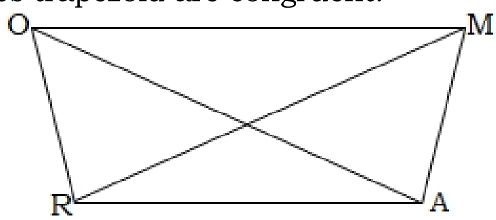
Proof:

Statements	Reasons
1. Isosceles Trapezoid ARTS	1. Given
2. $\overline{AR} \cong \overline{TS}$; $\overline{RT} \cong \overline{AS}$	2. Definition of Isosceles Trapezoid

3. From R, draw $\overline{RE}$ // $\overline{TS}$ where E lies on $\overline{AS}$.	3. Parallel Postulate
4. REST is a parallelogram.	4. Definition of parallelogram
5. $\overline{TS} \cong \overline{RE}$	5. Parallelogram Property 1
6. $\overline{AR} \cong \overline{RE}$	6. Transitive Property
7. ΔARE is an isosceles triangle	7. Definition of isosceles triangle
8. ∠3 ≅ ∠A	8. Isosceles Triangle Theorem
9. m∠1 + m∠3 + m∠A	9. Interior Angle Sum Theorem on Triangle
10. ∠3 ≅ ∠2	10. Alternate Interior Angles are Congruent
11. ∠A ≅ ∠S	11. Theorem 7
12. m∠1 + m∠2 + m∠S	12. Substitution (SN 9, 10 & 11)
13. ∠1 + ∠2 ≅ ∠ART	13. Angle Addition Postulate
14. m∠ART + m∠S	14. Substitution
15. m∠S + m∠T	15. Same Side Interior Angles are Supplementary
16. m∠A + m∠T	16. Substitution
17. ∠ ART and ∠S are supplementary. ∠A and ∠T are supplementary.	17. Definition of Supplementary Angles

Theorem 9. The diagonals of an isosceles trapezoid are congruent.

Given: Isosceles Trapezoid ROMA
Prove: $\overline{RM} \cong \overline{AO}$



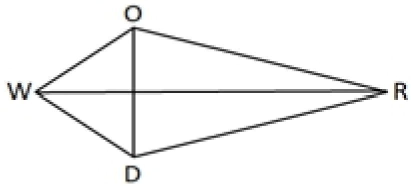
Proof:

Statements	Reasons
1. Isosceles Trapezoid ROMA	1. Given
2. $\overline{OR} \cong \overline{MA}$	2. Definition of Isosceles Trapezoid
3. ∠ROM ≅ ∠AMO	3. Theorem 7
4. $\overline{OM} \cong \overline{MO}$	4. Reflexive Property
5. ΔROM ≅ ΔAMO	5. SAS Congruence Postulate
6. $\overline{RM} \cong \overline{AO}$	6. CPCTC

We've just proven the different theorems concerning trapezoids. Now, you will prove another set of theorems, this time concerning kites.

Kites are quadrilaterals with two pairs of congruent and adjacent sides. Note that rhombus is a special kind of kite where all adjacent sides are equal. There are two theorems related to kite.

Theorem 10. In a kite, the perpendicular bisector of at least one diagonal is the other diagonal.



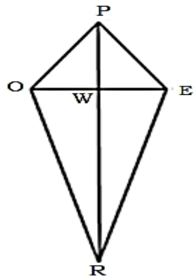
Given: Kite WORD with diagonals $\overline{WR}$ and $\overline{OD}$

Prove: $\overline{WR}$ is the perpendicular bisector of $\overline{OD}$.

Proof:

Statements	Reasons
1. Kite WORD with diagonals $\overline{WR}$ and $\overline{OD}$	1. Given
2. $\overline{WO} \cong \overline{WD}$; $\overline{OR} \cong \overline{DR}$	2. Definition of kite
3. WO = WD; OR = DR	3. Definition of Congruent segments
4. $\overline{WR} \perp \overline{OD}$	4. If a line contains two points each of which is equidistant from the endpoints of a segment, then the line is the perpendicular bisector of the segment.

Theorem 11. The area of a kite is half the product of the lengths of its diagonals.



Given: Kite ROPE

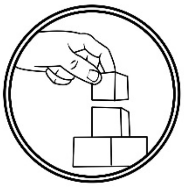
Prove: Area of kite ROPE = $\frac{1}{2}(OE)(PR)$

Proof:

Statements	Reasons
1. Kite ROPE	1. Given
2. $\overline{PR} \perp \overline{OE}$	2. The diagonals of a kite are perpendicular to each other.
3. Area of kite ROPE = Area of ΔOPE + Area of ΔORE	3. Area Addition Postulate
4. Area of ΔOPE = $\frac{1}{2}(OE)(PW)$ Area of ΔORE = $\frac{1}{2}(OE)(WR)$	4. Area Formula for Triangles
5. Area of kite ROPE = $\frac{1}{2}(OE)(PW) + \frac{1}{2}(OE)(WR)$	5. Substitution
6. Area of kite ROPE = $\frac{1}{2}(OE)(PW + WR)$	6. Distributive Property of Equality
7. PW + WR = PR	7. Segment Addition Postulate
8. Area of kite ROPE = $\frac{1}{2}(OE)(PR)$	8. Substitution

It's amazing that the area of a kite has been derived from the formula in finding for the area of a triangle.

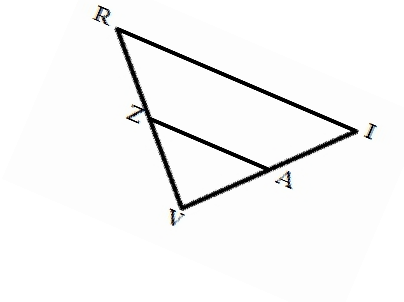
Now that you have proven the midline theorem and the different theorems on trapezoid and kites, you are now ready to apply it in doing the next activity



Explore

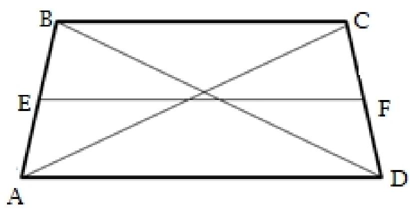
Work on the following enrichment activities for you to apply your understanding on this lesson.

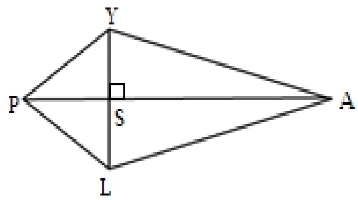
Activity 3: Fill Me!
Directions: Use the Triangle Mid-segment Theorem to name parts of ΔRIV. Z and A are the midpoints of $\overline{RV}$ and $\overline{IV}$, respectively.



1. ________ is a mid-segment of ΔRIV.
2. ________ is a segment parallel to $\overline{AZ}$.
3. $\overline{RZ}$ is a segment that has the same length as _______.
4. _______ is half as long as $\overline{RI}$.
5. _______ is twice as long as $\overline{AZ}$.

Activity 4: Complete Me!
Directions: Use the figures below to complete the sentences that follow.





1. In trapezoid ABCD, _________ is the midline.
2. $\overline{AD}$ is parallel to _________.
3. $\overline{BD}$ is congruent to ________.
4. ∠BAD is the supplement of ________.
5. In kite PLAY, ________ is the perpendicular bisector of $\overline{LY}$.

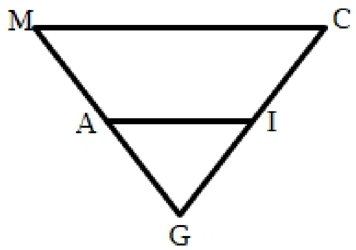
Did you apply the knowledge and concepts of Midline Theorem and the different theorems on trapezoid and kites? If you did then you are now ready for the next activity.

Answer the next activity to deepen more your understanding.

Deepen

Activity 5: Go For It!
Directions: Consider each given information and answer the questions that follow.

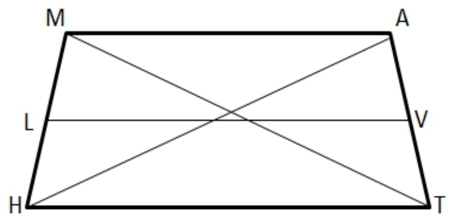
In ΔMCG, A and I are the midpoints of $\overline{MG}$ and $\overline{GC}$, respectively.



1. If AI = 10, what is MC?
2. If MC = 32, what is AI?
3. If MG = 12, what is AG?

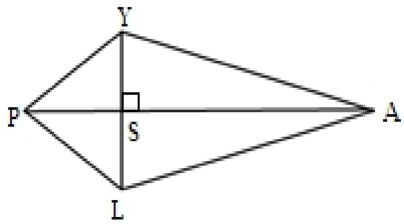
Activity 6: Show More What You've Got!
Directions: Consider the figure that follows and answer the given questions.

A. Quadrilateral MATH is an isosceles trapezoid with bases $\overline{MA}$ and $\overline{HT}$, $\overline{LV}$ is the median.



1. If MA = 10 and HT = 14, what is LV?
2. If MT = 25, what is AH?
3. If m∠MHT = 65, what is m∠ATH?

B. Quadrilateral PLAY is a kite.



1. If m∠SPL =30, what is m∠SLP?
2. If PA=12cm and LY=6cm, what is the area of kite PLAY?

The activities you did above clearly reflect your deeper understanding of the lessons taught to you in this module. Now you are ready to put your knowledge and skills to practice and be able to answer the questions you've instilled in your mind from the very beginning of this module.