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Mathematics

Quarter 2 - Module 3: Chords, Arcs, Central and Inscribed Angles

AIRs - LM

MATHEMATICS 10
Quarter 2 - Module 3: Chords, Arcs, Central and Inscribed Angles
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Mathematics
Quarter 2 - Module 3:
Chords, Arcs, Central and Inscribed Angles

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

This module will assess your knowledge on the different mathematical concepts previously studied and your skills in performing mathematical operations that will help you understand **circles**.

As you go through this module, you will be performing activities to establish the relations among chords, arcs, central and inscribed angles.

If you find any difficulty in answering the exercises, seek assistance to your teacher. Let's start and be amused with the learnings that you will get along the way!

Learning Competency:
- Derives inductively the relations among chords, arcs, central and inscribed angles (**M10GE-IIc-1)**

After going through this module, you are expected to:
- Recall and name the parts of a circle.
- Identify central and inscribed angles and their corresponding intercepted arcs.
- Describe the relationships among chords, arcs, central and inscribed angles.
- Determine the measure of a central angle and its intercepted arc and vice versa
- Solve for the arc measure given the central angle
- Find the measure of an inscribed angle and its intercepted arc and vice versa

> *Begin this module by checking how much you know about the topic to be discussed by answering the pre-test.*

Pre-assessment

Read and analyze each item carefully then select the letter that best corresponds to the question. Write your answer in a separate sheet of paper.
1. What do you call a segment whose endpoints are points on the circle?
   A. arc   B. center
   C. chord   D. radius
2. What do you call a segment from the center to any point on the circle?
   A. center   B. chord
   C. diameter   D. radius
3. What is an angle whose vertex is on the circle and whose sides contain chords of the circle?
   A. central angle   B. circumscribed Angle
   C. inscribed Angle   D. intercepted Angle
4. The degree measure of a minor arc is ____ the measure of the central angle which intercepts the arc.
   A. half   B. equal

C. thrice   D. twice

5. What is the total measure of the central angles of a circle with no common interior points?

     A. 90^0 B. 180^0

     C. 360^0 D. 480^0

6. In circle I at the right, what is the measure of $\overset{\frown}{BG}$?

     A. 25°   B. 45°

     C. 50°   D. 65°

7. If an inscribed angle of a circle intercepts a semicircle, then the angle is _____.

     A. acute   B. obtuse

     C. right   D. straight

8. In circle **N**, which of the following angles does not intercept $\overset{\frown}{AE}$?

     A. ∠ALE   B. ∠ACE

     C. ∠CAL   D. ∠ANE

*For numbers 9 – 10, refer to circle **A**.*

9. Which of the following arc is a major arc?

     A. $\overset{\frown}{BED}$ B. $\overset{\frown}{BC}$

     C. $\overset{\frown}{BEC}$ D. $\overset{\frown}{CDE}$

10. Which of the following arc is a semicircle?

     A. $\overset{\frown}{BED}$ B. $\overset{\frown}{BC}$

     C. $\overset{\frown}{BEC}$ D. $\overset{\frown}{CDE}$

11. In circle **A**, what is the measure of ∠SAY if $\overset{\frown}{DSY}$ is a semicircle and $m\angle SAD=70$?

     A. 20^0 B. 70^0

     C. 110^0 D. 150^0

12. In circle **O**, what is the measure of $\overset{\frown}{PM}$

     A. 120^0 B. 180^0

     C. 60^0 D. 30^0

13. How about the measure of $\overset{\frown}{LMQ}$ in circle **O**?

     A. 60^0 B. 120^0

     C. 120^0 D. 240^0

14. Find the measure of $\angle APF$ in circle **P**.

     A. 50^0 B. 60^0

     C. 70^0 D. 160^0

15. Again from circle **P**, what is the measure of $\angle FPD$?

     A. 20^0 B. 90^0

     C. 110^0 D. 180^0

How did you find the pre-test? Were you able to answer all the items?
Are there questions familiar to you?
What are the terms mentioned in the test would you like to know more about?...

Jumpstart

Before exploring this module, let us have a quick tour on circle and its parts! This has been discussed in your grade 7 mathematics class and it is a pre-requisite of the lesson we will be going through.

A **circle** is a closed curved where all points are equidistant from a fixed point called center.
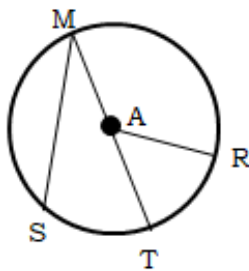
A – center	$\overline{SM}, \overline{MT}$ - chords
$\overline{AM}, \ \overline{AR}, \ \overline{AT}$ – radii	$\overset{\frown}{MR}$-minor arc
$\overline{MT}$ – diameter	$\overset{\frown}{MTS}$-major arc
$\overset{\frown}{MRT}$ and $\overset{\frown}{MST}$ – semicircles	
∠RAM & ∠RAT – central angles	
∠SMT – inscribed angle	

Radius – the line segment from the center to any point on the circle.

Chord – a line segment joining any two points on a circle.

Diameter is a chord that contains the center.

Arc – subset of a circle or part of a circle

➢ **Major arc** – an arc which is more that a semi-circle
➢ **Minor arc** – an arc which is less than a semi-circle

Central Angle – an angle formed by two radii or an angle whose vertex is the center of a circle.

Inscribed Angle – an angle whose vertex is a point on the circle and whose sides are the chords.

Activity 1. Name Me!

*In the figure below, identify what part of circle **I** are the following:*
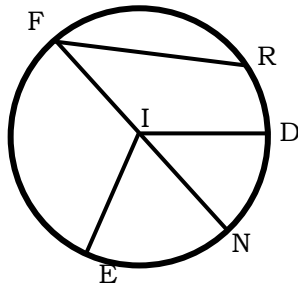
1. $\overline{FR}$ = _____
2. $\overline{IE}$ = _____
3. $\overline{ID}$ = _____
4. I = _____
5. $\overline{FN}$ = _____

6. $\overparen{RDN}$ = _____
7. $\overparen{FEN}$ = _____
8. $\overparen{FDE}$ = _____
9. $\angle RFN$ = _____
10. $\angle EID$ = _____

Activity 2: Find Me!

Using the proper notations, name the indicated parts of circle E.

_____1. a semicircle
_____2. a major arc
_____3. a minor arc
_____4. a central angle
_____5. an inscribed angle
_____6. a radius
_____7. a diameter
_____8. a chord
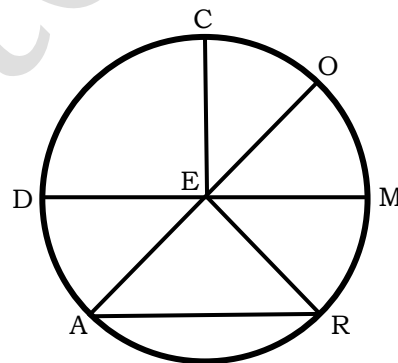
Questions:

1. How did you identify the parts of the circle?
2. How do you describe the radius, diameter, and a chord? How about the semicircle, major arc and minor arc? Inscribed angle and central angle?

Discover

How did you find the preceding activities? I am sure you have successfully identified and described the terms related to circle. This time, I want you to seat back in your most comfortable angle as you will learn about the relations among chords, arcs, central angles and inscribed angles... Let the discussion begin...

Central Angle and Arcs

Central Angle

Step 1. Construct a circle.

Example: Circle **C**.
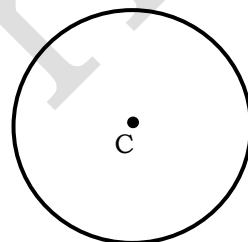
Step 2. Construct two non-collinear radii.

Example: $\overline{CA}$ and $\overline{CB}$.
$\angle$**ACB** is a **central angle**.
$\overset{\frown}{AB}$ is the intercepted arc of $\angle$**ACB.**
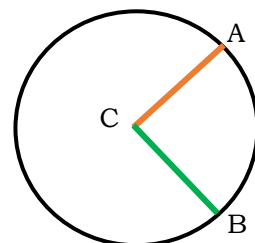
Note: A central angle of a circle is an angle formed by two segments whose vertex is the center of the circle. Each segment intersects the circle at a point, dividing it into **arcs.**

In the figure at the right, $\angle$**ACB** is a central angle. Its sides divide ⊙C into arcs. One arc is the curve containing points A & B. The other arc is the curve containing points A, D and B.
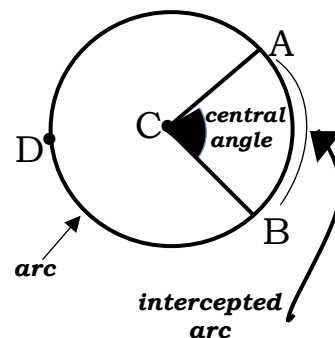
Sum of Central Angles

The sum of the measures of the central angles of a circle with no common interior points is 360⁰.

In the figure, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360$.

(**Note:** *All measures of angles and arcs are in degrees*).
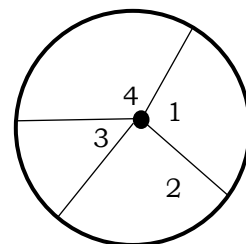
Arcs of a Circle

An **arc** with the symbol ⌢, is a part of the circumference of a circle. A **semicircle** is an arc with a measure equal to one-half the circumference of a circle that is named by using the two endpoints and another point on the arc.

Example: The curve from point M to point T is an arc of ⊙A and is named as $\overarc{MT}$. The other arcs of ⊙A are $\overarc{TH}$, $\overarc{HM}$, $\overarc{THM}$, $\overarc{HMT}$.

$\overarc{MTH}$ and $\overarc{HM}$ are semicircles.

A **minor arc** is an arc of a circle that measures less than a semicircle which is named by using the two endpoints of the arc.

Examples: $\overarc{GO}$, $\overarc{OM}$, $\overarc{MG}$

A **major arc** is an arc of a circle that measures greater than a semicircle and is named using the two endpoints and another point on the arc.

Examples: $\overarc{GOM}$, $\overarc{OMG}$, $\overarc{MGO}$

Degree Measure of an Arc

1. The degree measure of a minor arc is the measure of the central angle which intercepts the arc.

   Example: ∠RAP is a central angle and $\overarc{PR}$ is its intercepted arc. If m∠RAP = 115, then m$\overarc{PR}$ = 115.

2. The degree measure of a major arc is 360^0 minus The measure of the minor arc with the same endpoints.

   Example: If m$\overarc{PR}$ = 115, then m$\overarc{RTP}$ = 360 – m$\overarc{PR}$. Hence, m$\overarc{RTP}$ = 360 – 115 m$\overarc{RTP}$ = 245.
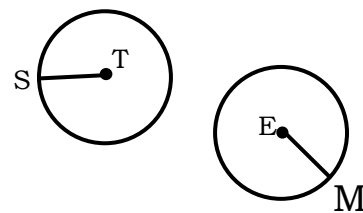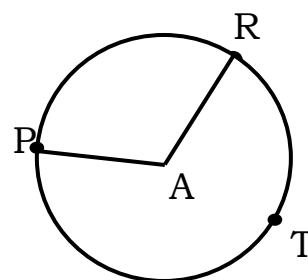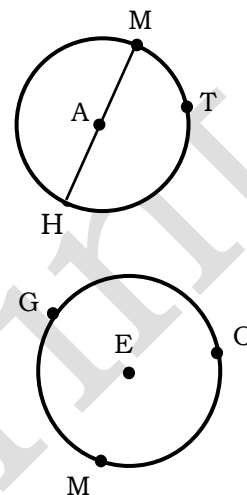
3. The degree measure of a **semicircle** is 180^0.

Congruent Circles and Congruent Arcs

Congruent Circles are circles with congruent radii.
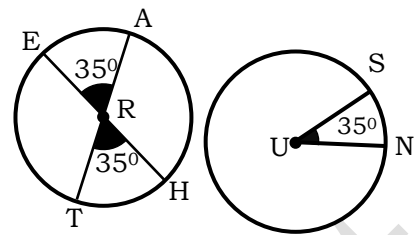
Example: ST is a radius of ⊙T.

EM is a radius of ⊙E.

If $\overline{ST} \cong \overline{EM}$, then ⊙T ≅ ⊙E

Congruent Arcs are arcs of the same circle or of congruent circles with equal measures.

Example: In ⊙R, $\overset{\frown}{EA} \cong \overset{\frown}{TH}$.

If ⊙R ≅ ⊙U then $\overset{\frown}{EA} \cong \overset{\frown}{SN}$

and $\overset{\frown}{TH} \cong \overset{\frown}{SN}$.

Inscribed Angle

Step 1. Construct a circle.

   Example: Circle K.

Step 2. Construct a chord.

   Example: Chord DE.

Step 3. Construct another chord from an endpoint of the <u>first</u> chord to any point on the circle, DF

∠EDF is called an **INSCRIBED ANGLE**.

$\overset{\frown}{EF}$ is the intercepted arc of ∠EDF.
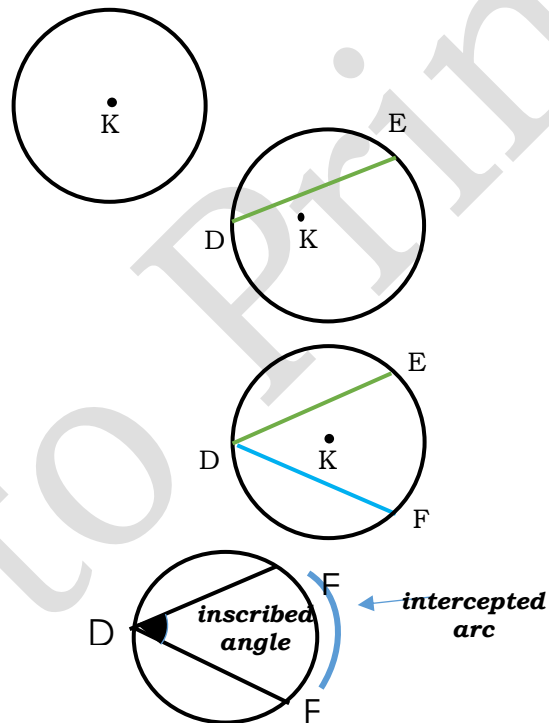
The measure of the inscribed angle is HALF the measure of the intercepted arc.
The measure of the intercepted arc is TWICE the measure of the inscribed angle.

Example:

If $\overset{\frown}{EF}$ = 80⁰, then the inscribed angle ∠EDF is half of 80⁰. ∠EDF = 40⁰.

If ∠EDF is 40⁰, then the intercepted arc is twice as 40⁰. $\overset{\frown}{EF}$ = 80⁰.

> *Now that you have learned about the relations among chords, arcs, central angles and inscribed angles...*
> *Check the clock and show what your brains have got!...*

Explore

> *Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.*

Activity 3: **Fill Me In!**

Directions: Complete the table below by determining whether the given angle is a central angle or inscribed angle, then identify the arc they intercept.
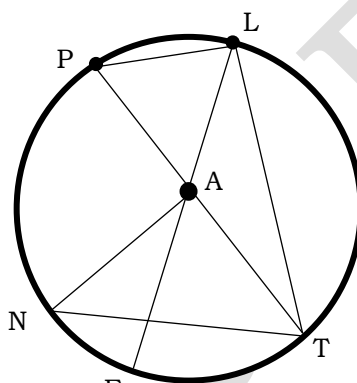

	Angle	Central ∠ or Inscribed ∠	Intercepted Arc
1	∠LAP		
2	∠ELT		
3	∠TAN		
4	∠LAN		
5	∠PLE		
6	∠PAN		
7	∠NTL		
8	∠NAE		
9	∠TLP		
10	∠PTL		
11	∠PTN		
12	∠EAT		
13	∠LPT		
14	∠PAE		
15	∠LAT		

Questions:
1. Which angle/s intercept the same arc?
2. Which angle/s intercept a semicircle?
3. Which arcs are congruent?

Activity 4: Find Me!

In the figure, HT and $\overline{RE}$ are diameters of circle A. Use the figure and the given information to answer the following.
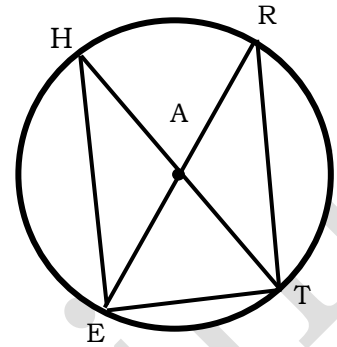
1. Which central angles are congruent? Why?

2. If m∠RAT = 118, find:

    a. m∠EAT

    b. m∠HAE

    c. m∠THE

    d. m∠REH

    e. m∠THE

    f. m∠RTE

3. Is RH ≅ TE? How about HE and RT? Justify your answer.

4. Which minor arcs are congruent? Explain your answer.

5. If m∠HER = 36, find:

    a. $m\overset{\frown}{HR}$

    b. $m\overset{\frown}{HE}$

    c. $m\overset{\frown}{HRT}$

    d. $m\overset{\frown}{ETH}$

    e. $m\overset{\frown}{RTH}$

6. Which arcs are semicircles? Why?

> *Well done!*
> *Let's dig deeper by moving on to the next activities!*

![Deepen logo] ***Deepen***

Activity 5: **Identify Me and Find my Measure!**

State whether the following arcs is a minor arc, a major arc, or a semicircle. Then, find the measure of each.

A. *Given circle **E** at the right, m∠REI = 38, m∠QEU = 24 and ∠AEQ is a right angle.*

1. $\overset{\frown}{AR}$

2. $\overset{\frown}{CQ}$

3. $\overset{\frown}{RIQ}$

4. $\overset{\frown}{CQU}$

5. $\overset{\frown}{ARI}$

6. $\overset{\frown}{ARU}$

7. $\overset{\frown}{QCI}$

8. $\overset{\frown}{RAQ}$

9. $\overset{\frown}{AQU}$

10. $\overset{\frown}{CUA}$
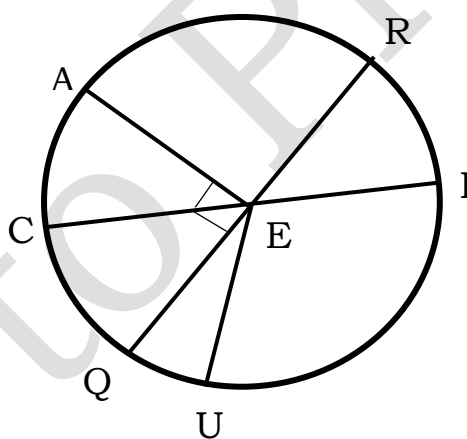
B. *Given circle S at the right, m∠RCE = 60, m∠AER = 30 and m∠CEA = 40*

11. $\overset{\frown}{CAR}$

12. $\overset{\frown}{CE}$

13. $\overset{\frown}{RE}$

14. $\overset{\frown}{ARC}$

15. $\overset{\frown}{REA}$
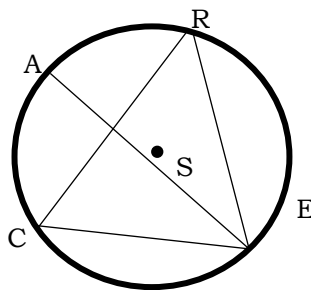
Activity 6: Matchmaker!

Match column **A** to column **B**. Write the letter of your answer on the space provided before each number. Use the given figures as your reference and then answer the question that follows.

COLUMN A

_____1. Find $m\angle TMH$
_____2. Find $m\angle TAH$
_____3. Find $m \overarc{TMH}$
_____4. Find $m \overarc{TMH} + m \overarc{TH}$

For items 1-4

_____5. Find $m \overarc{RD}$
_____6. Find $m\angle IRD$
_____7. Find $m\angle RID$
_____8. Find $m \overarc{IR}$

For items 5-8

_____9. Find $m \overarc{OE}$
_____10. Find $m\angle EDZ$

For items 9-10
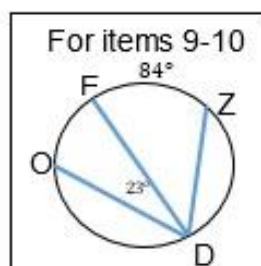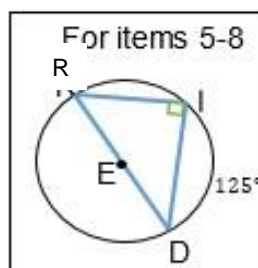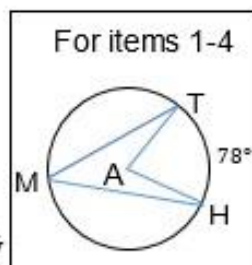
COLUMN B

a. 180°
b. 78°
c. 39°
d. 42°
e. 282°
f. 46°
g. 90°
h. 62.5°
i. 360°
j. 55°
k. 60°

11 – 15. What is the relationship between the central angle and its intercepted arc? How about the inscribed angle and its intercepted arc?

> *Well done! You made it this far…*
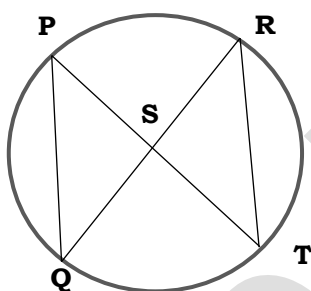> *Your final task is the assessment part!*
> *Good Luck!*

Assessment

Let us determine how much you have learned from this module. Read and understand each item carefully, then choose the letter of your answer and write it on your answer sheet.
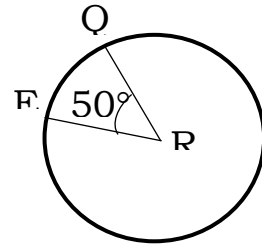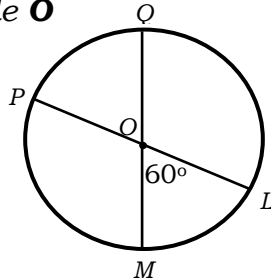
For items 1-5, Use the figure below

1. Which of the following is an inscribed angle?

    A. ∠RST   B. ∠PQR   C. ∠QSP   D. ∠QST

2. Which of the following is a central angle?

    A. ∠RST   B. ∠PQR   C. ∠QRT   D. ∠STR

3. What is the intercepted arc of ∠PQR?

    A. $\overarc{RT}$ B. $\overarc{QT}$ C. $\overarc{PQ}$ D. $\overarc{PR}$

4. In ⊙S, $\overline{QR}$ is a diameter. What is m $\overarc{RTQ}$?

    A. 90   B. 45   C. 180   D. 100

5. Which of the following is **NOT** a central angle?

    A. ∠RST   B. ∠QST   C. ∠RSQ   D. ∠QRT

6. Which statement is true?

    A. The inscribed angle is congruent to the intercepted arc.

    B. The measure of a central angle is twice the measure of its intercepted arc.

    C. The measure of a central angle is one half the measure of its intercepted arc.

    D. The measure of a central angle is equal to the measure of its intercepted arc.

7. From circle **A**, which of the following arc is a minor arc?

    A. $\overarc{BED}$ B. $\overarc{BEC}$

    C. $\overarc{BE}$ D. $\overarc{CDE}$
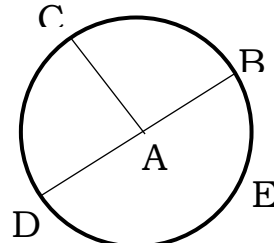
8. Which of the following is the measure of $\overset{\frown}{OE}$?
    A. 65^0 B. 45^0
    C. 50^0 D. 25^0

For items 9-10, refer to circle **O**

9. What is the measure of $\overset{\frown}{PML}$?
    A. 60^0 B. 120^0 C. 180^0 D. 240^0
10. What is the measure of $\overset{\frown}{PQ}$?
    A. 60^0 B. 120^0 C. 180^0 D. 240^0

For items 11 to 15, refer to the figure below where $\overset{\frown}{AC}$ measures 135^0.
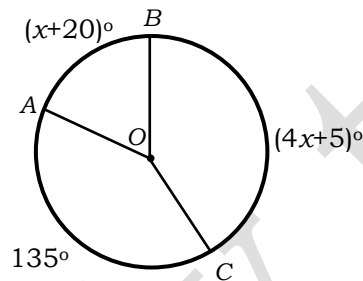
11. What will be the value of x?
    A. 40   B. 60   C. 70   D. 120
12. What is the measure of $\overset{\frown}{AB}$?
    A. 40^0 B. 60^0 C. 80^0 D. 140^0
13. What is the measure of $\overset{\frown}{BC}$?
    A. 300^0 B. 285^0 C. 140^0 D. 165^0
14. What is the measure of $\overset{\frown}{ABC}$?
    A. 135^0 B. 180^0 C. 225^0 D. 360^0
15. What is the measure of $\angle AOC$?
    A. 135^0 B. 180^0 C. 225^0 D. 360^0

Congratulations!
You have completed all the activities of this module!

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