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Mathematics

Quarter 2 – Module 4:
Simplifying Expressions with
Rational Exponents

AIRs - LM

MATHEMATICS 9
Quarter 2 - Module 4: Simplifying Expressions with Rational Exponents
Second Edition, 2021

Printed in the Philippines by: _____

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Mathematics

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Simplifying Expressions with
Rational Exponents

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

We are now on the lesson which is about rational exponents. These are exponents which are rational numbers. A rational number is any number that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. In other words, *rational exponents* are *fractional exponents.*

After reading and studying this lesson, you will be able to simplify expressions with rational exponents. Let's start with a pre-assessment to check your prior knowledge about this lesson.

After going through this module, you are expected to attain the following objectives:

Learning Competencies
- Simplifies expressions with rational exponents. **(M9AL-IIe-1)**
- Write expressions with rational exponents as radicals and vice versa. **(M9AL-IIf-1)**

Subtasks
1. Simplify expressions with $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$.
2. Use the properties of exponents to simplify expressions with rational exponents.
3. Write expressions with rational exponents as radicals and vice versa.

> *Let's find out how much you already know about this module.*
> *Answer the pre-assessment in a separate sheet of paper.*

Pre-Assessment
Directions: Find out how much you already know about this module. Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which of the following has an exponent in the form of $\frac{m}{n}$ where m and n are integers and $n \neq 0$?
   A. exponent
   B. rational
   C. radical exponent
   D. rational exponent

2. Which of the following is the simplified form of $\dfrac{2^{2\frac{1}{2}}}{2^{\frac{1}{3}}}$?

   A. $2^{\frac{1}{6}}$ B. $2^{\frac{2}{6}}$ C. $2^{\frac{5}{6}}$ D. $2^{\frac{2}{5}}$

3. Which of the following is equivalent to $(x^{\frac{3}{7}})(x^{\frac{5}{7}})$?

   A. $x^{\frac{15}{7}}$ B. $x^{\frac{15}{49}}$ C. $x^{\frac{8}{7}}$ D. $x^{\frac{8}{14}}$

4. Which of the following illustrates expressions with rational exponents?

   A. $(x^{1/2}y^{1/2}z^{1/2})^2$ B. $(-x^2y^3z)^{1/2}$ C. $(x^2y^5z^{10})^2$ D. $(-2x^3y^4z)^{-2}$

5. What is the simplified form of $(x^6)^{1/3}$?

   A. $\dfrac{1}{x^2}$ B. $\dfrac{1}{x^{-2}}$ C. x^{-2} D. x^2

6. Which of the following is **TRUE**?

   A. $5^{1/2} + 5^{1/3} = 5^3$ B. $\dfrac{2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{\frac{2}{9}}$ C. $(3^{1/3})^2 = 3^{\frac{2}{3}}$ D. $4^{\frac{2}{3}} = \dfrac{1}{4^{\frac{2}{3}}}$

7. Which of the following is equivalent to $(\frac{25}{49})^{1/2}$?

   A. $\dfrac{5}{7}$ B. $\dfrac{7}{5}$ C. $\dfrac{8}{5}$ D. $\dfrac{9}{5}$

8. What is the equivalent of $[(\frac{1}{3})(\frac{1}{27})]^{1/4}$?

   A. $\dfrac{1}{6}$ B. $\dfrac{1}{5}$ C. $\dfrac{1}{4}$ D. $\dfrac{1}{3}$

9. Which of the following is an expression in the form $\sqrt[n]{a^m}$ where n is a positive integer and a is an element of the real number system?

   A. radical   B. radical equation   C. radical exponent   D. rationalization

10. What is the index in the expression $\sqrt[n]{a^m}$?

    A. a B. m C. n D $\sqrt{}$

11. What is the radicand in the expression $\sqrt[n]{a^m}$?

    A. a^m B. m C. n D. $\sqrt{}$

12. What is the exponent of the radicand in the expression $\sqrt[n]{a^m}$?

    A. a B. m C. n D. $\sqrt{}$

13. Which of the following is the radical form of $x^{\frac{4}{5}}$?

    A. $\sqrt[5]{x^4}$ B. $\sqrt[4]{x^5}$ C. $4\sqrt{x^5}$ D. $5\sqrt{x^4}$

14. What is the radical form of $((a^2b^3c^5)^{1/3})^{1/2}$?

    A. $\sqrt{\sqrt[3]{a^2b^3c^5}}$ B. $\sqrt[3]{\sqrt{a^2b^3c^5}}$ C. $\sqrt[3]{(a^2b^3c^5)^2}$ D. $\sqrt{(a^2b^3c^5)^3}$

15. What is the simplified form of $\left(m^{\frac{1}{3}} n^{\frac{1}{2}} p^{\frac{1}{4}}\right)^{12}$?

    A. $m^4n^6p^3$ B. $m^{\frac{1}{4}} n^{\frac{1}{6}} p^{\frac{1}{3}}$ C. $m^{\frac{3}{4}} n^{\frac{2}{6}} p^{\frac{4}{3}}$ D. $m^{\frac{12}{3}} n^{\frac{12}{2}} p^{\frac{12}{4}}$

Jumpstart

Activity 1: Follow Me!

Directions: Fill in the missing parts of the solution in simplifying expressions with rational exponents. The first one is done for you.

1. $(m^{\frac{2}{3}})(m^{\frac{4}{3}})$ = m $^{2/3\,+\,4/3}$ = m $^{6/3}$ = m^2

2. $(k^{\frac{1}{4}})(k^{\frac{2}{3}})$ = k $^{?/12\,+\,?/12}$ = k $^{?/12}$

3. $\dfrac{a^{\frac{5}{7}}}{a^{\frac{3}{2}}}$ = a$^{10/?\,-\,21/?}$ = a $^{-11/?}$ = $\dfrac{1}{a^{\frac{11}{14}}}$

4. $(r^{12}s^9)^{1/3}$ = r $^{?/3}$ s$^{?/3}$ = r^4 s^3

Process Questions:
  a. Based on the activity, how do you simplify expressions involving rational exponents?
  b. What are the necessary skills in simplifying expressions with rational exponents?

Activity 2. Find My Partner!

Directions: Write each rational exponent to radical form and radical form to exponential form. Choose the letter of the correct answer and write in a separate sheet of paper.

Exponential Form	Radical Form
1. $16^{\frac{1}{2}}$	a. $\sqrt[3]{x^2}$
2. $x^{\frac{2}{3}}$	b. $\sqrt{16}$
3. $x^{\frac{1}{3}}$	c. $3\sqrt{x^3}$
4. $3x^{\frac{3}{2}}$	d. $\sqrt{(3x)^3}$
5. $(3x)^{\frac{3}{2}}$	e. $\sqrt[3]{x}$

Radical Form	Exponential Form
6. $\sqrt[3]{27^2}$	f. $16^{\frac{3}{4}}$
7. $\sqrt[4]{16^3}$	g. $27^{\frac{2}{3}}$
8. $\sqrt[3]{\sqrt{25}}$	h. $25^{\frac{1}{6}}$
9. $\dfrac{1}{\sqrt{25}}$	i. $8x^{\frac{3}{2}}$
10. $8\sqrt{x^3}$	j. $25^{\frac{-1}{2}}$

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Discover

The previous activities enabled you to realize that the laws of exponents for integral exponents may be used in simplifying expressions with rational exponents.

Use the Properties of Exponents to simplify expressions with rational exponents.

The same properties of exponents that we have already used also apply to rational exponents. We will list the Properties of Exponents here to have them for reference as we simplify expressions.

Properties of Exponents

If a and b are real numbers and m and n are rational numbers, then

1. Product Property $(a^m)(a^n) = a^{m+n}$

2. Power Property $(a^m)^n = a^{mn}$

3. Product to a Power $(ab)^n = a^m b^n$

4. Quotient Property $\dfrac{a^m}{a^n} = a^{m-n} , a \neq 0$

5. Zero Exponent Definition $a^0 = 1 ; a \neq 0$

6. Quotient to a Power Property $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m} ; b \neq 0$

7. Negative Exponent Property $a^{-n} = \dfrac{1}{a^n}; a \neq 0$

We will apply these properties in the next examples.

Examples:

1. Simplify: $\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}}\right)$
   Solution:

   $\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}}\right) = x^{\frac{1+1}{3}}$ The bases are the same, so we add the exponents.

   $= x^{\frac{2}{3}}$ Add the fractions

   Therefore, $\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}}\right) = x^{\frac{2}{3}}$

   ➢ *The Power Property tells us that when we multiply the same base, we add the exponents.*

2. Simplify: $(x^{\frac{1}{4}})^8$

   Solution:

   $(x^{\frac{1}{4}})^8 = x^{(\frac{1}{4})(8)}$ To raise a power to a power, we multiply the exponents.

   $\quad\quad = x^{\frac{8}{4}}$

   $\quad\quad = x^2$ Simplify

Therefore, $(x^{\frac{1}{4}})^8 = x^2$

   ➢ *The Power Property tells us that when we raise a power to a power, we multiply the exponents.*

3. Simplify: $27^{\frac{2}{3}}$

   Solution:

   $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}$ Express 27 to exponential form

   $\quad\quad = 3^{(\frac{2}{3})(3)}$ Definition of laws of exponent

   $\quad\quad = 3^2$ Simplify

   $\quad\quad = 9$

Therefore, $27^{\frac{2}{3}} = 9$

   ➢ *The Power Property tells us that when we raise a power to a power, we multiply the exponents.*

4. Simplify: $\dfrac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}}$

   Solution:

   $\dfrac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}} = x^{\frac{1-5}{3}}$ To divide with the same base, we subtract the exponents.

   $\quad\quad = x^{\frac{-4}{3}}$

   $\quad\quad = \dfrac{1}{a^{\frac{4}{3}}}$ Simplify

Therefore, $\dfrac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}} = \dfrac{1}{a^{\frac{4}{3}}}$

   ➢ *The Quotient Property tells us that when we divide with the same base, we subtract the exponents.*

Transforming Rational Exponents to Radical Form and Vice versa

We define $x^{\frac{1}{n}}$ as the nth root of x. We use the symbol $\sqrt[n]{x}$ to mean $x^{\frac{1}{n}}$ where n is the **index** of the radical, x is the **radicand** and $\sqrt[n]{x}$ is itself the **radical**. If the index is not indicated, then it is understood to be 2.

The symbol $\sqrt[n]{a^m}$ is called **radical**. A **radical expression** or a **radical** is an expression containing the symbol $\sqrt{}$ called **radical sign**. In the symbol $\sqrt[n]{a^m}$, n is called the **index** or **order** which indicates the degree of the radical such as square root $\sqrt{}$, cube root $\sqrt[3]{}$, and fourth root $\sqrt[4]{}$, a^m is called the **radicand** which is a number or expression inside the radical symbol and m is the power or exponent of the radicand.

If $\frac{m}{n}$ is a rational number and **a** is a positive real number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ provided that $\sqrt[n]{a^m}$ is a real number. The form $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$ is called the principal nth root of a^m. Through this, we can write expressions with rational exponents as radicals.

Examples:

A. Write as a radical expression.

       1. $x^{\frac{1}{2}}$ 2. $x^{\frac{1}{3}}$ 3. $x^{\frac{3}{2}}$

We want to write each expression in the form $\left(\sqrt[n]{a}\right)$

1. $x^{\frac{1}{2}} = \sqrt{x}$	The denominator of the rational exponent is 2, so the index of the radical is 2. We do not show the index when it is 2.
2. $x^{\frac{1}{3}} = \sqrt[3]{x}$	The denominator of the exponent is 3, so the index is 3.
3. $x^{\frac{2}{3}} = \sqrt[3]{x^2}$	The denominator of the exponent is 3, so the index is 3. The numerator 2 becomes the exponent.

B. Write as a rational exponent.

       1. $\sqrt{y^3}$ 2. $\left(\sqrt[3]{2x}\right)^4$ 3. $\sqrt{\left(\frac{3a}{4b}\right)^3}$

We want to use $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ to write each radical in the form $a^{\frac{m}{n}}$.

1. $\sqrt{y^3} = y^{\frac{3}{2}}$	The numerator of the exponent is the exponent *3*. The denominator of the exponent is the index of the radical *2*.
2. $\left(\sqrt[3]{2x}\right)^4 = (2x)^{\frac{4}{3}}$	The numerator of the exponent is the exponent *4*. The denominator of the exponent is the index of the radical *3*.
3. $\sqrt{\left(\frac{3a}{4b}\right)^3} = \left(\frac{3a}{4b}\right)^{\frac{3}{2}}$	The numerator of the exponent is the exponent *3*. The denominator of the exponent is the index of the radical *2*.

Explore

Activity 3: Fill-Me-In!

Directions: Simplify the following expressions with rational exponents by filling in the boxes with solutions. Number 1 is done for you.

1. $(4^2)^{\frac{1}{2}}$ ⟹ $4^{(\frac{1}{2})(2)}$ ⟹ $4^{\frac{2}{2}}$ ⟹ 4 ⟹

2. $(x^6)^{\frac{-1}{2}}$ ⟹ [ ] ⟹ [ ] ⟹ $\dfrac{1}{x^3}$ ⟹

3. $(n^4)^{\frac{3}{2}}$ ⟹ [ ] ⟹ [ ] ⟹ n^6 ⟹

4. $(p^{\frac{3}{2}})^{-2}$ ⟹ [ ] ⟹ [ ] ⟹ $\dfrac{1}{p^3}$ ⟹

5. $(x^{12}y^{10})^{\frac{-1}{2}}$ ⟹ [ ] ⟹ [ ] ⟹ $\dfrac{1}{x^6 y^5}$ ⟹

Activity 4. Transformers I!

Directions: Transform the given radical form into exponential form and exponential form into radical form. Assume that all the letters represent positive real umbers.

Radical Form	Exponential Form
1. $\sqrt{6}$	
2.	$26^{\frac{1}{4}}$
3. $\sqrt[4]{x}$	
4.	$9^{\frac{1}{3}}$
5.	$x^{\frac{3}{5}}$
6. $\sqrt[5]{a^3}$	
7. $\sqrt[4]{(xy)^3}$	
8.	$(5a^3b^2)^{\frac{2}{3}}$
9. $\sqrt{3m^2}$	
10.	$(4r^2s^3)^{\frac{2}{5}}$

Deepen

Activity 5: Make Me Simple!

Directions: Using your knowledge of rational expressions, simplify the following,

Given	Final Answer
1. $c^{\frac{1}{4}} c^{\frac{5}{8}}$	$c^{\frac{7}{8}}$
2. $(p^{12})^{\frac{3}{4}}$	
3. $\dfrac{r^{\frac{4}{5}}}{r^{\frac{9}{5}}}$	
4. $y^{\frac{1}{2}} y^{\frac{3}{4}}$	
5. $(x^{16} y^{20} z^{8})^{\frac{1}{4}}$	

Activity 6. Transform Me!.

A. Express the following in radical form.

1. $x^{\frac{4}{5}}$
2. $(5x^2 y)^{\frac{2}{5}}$
3. $(a+b)^{\frac{1}{3}}$
4. $(4xy)^{\frac{3}{4}}$
5. $(2x - 5y)^{\frac{1}{3}}$

B. Change the following radicals to exponential form.

1. $\sqrt[3]{1000}$
2. $\sqrt[5]{32^3}$
3. $\sqrt{16xy^3}$
4. $\sqrt[3]{(a+b)^2}$
5. $\sqrt[4]{\sqrt[5]{x}}$

Post - Assessment

Directions: Read and analyze the following questions carefully. Choose the letter of the correct answer. Write it on your answer sheet.

1. Which of the following illustrates expressions with rational exponents?
    A. $(m^{1/2}n^{1/2}o^{1/2})^4$ B. $(-m^3y^6z)^{1/2}$ C. $(m^6n^8p^{18})^2$ D. $(-5m^{10}n^4p6)^{-2}$

2. Which of the following is equivalent to $\left(3^{\frac{1}{3}}\right)^2$?
    A. $3^{\frac{2}{3}}$ B. $3^{\frac{7}{3}}$ C. $3^{\frac{3}{2}}$ D. $3^{\frac{3}{3}}$

3. Which of the following is **TRUE**?
    A. $2^{1/2} + 5^{2/3} = 5^3$ B. $\frac{3^{\frac{1}{2}}}{3^{\frac{1}{3}}} = 3^{\frac{1}{9}}$ C. $(3^{4/3})^2 = 3^{\frac{8}{3}}$ D. $5^{\frac{2}{3}} = \frac{1}{5^{\frac{2}{3}}}$

4. Which of the following is the equivalent of $(\frac{81}{64})^{1/2}$?
    A. $\frac{5}{7}$ B. $\frac{7}{5}$ C. $\frac{8}{5}$ D. $\frac{9}{8}$

5. What is the equivalent of $(\frac{1}{256})^{1/4}$?
    A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$

6. Which of the following is the radical form of $((7)^{2/3})^{1/2}$?
    A. $\sqrt{\sqrt[3]{49}}$ B. $\sqrt[3]{\sqrt{49}}$ C. $\sqrt[3]{(7)^2}$ D. $\sqrt{(7)^3}$

7. Write $5\sqrt{y}$ with rational exponent.
    A. $(5y)^{1/2}$ B. $(5y)^2$ C. $5y^2$ D. $5y^{1/2}$

8.   What is the equivalent form of $\left(\frac{s^{\frac{1}{4}}}{t^{\frac{1}{8}}}\right)^{24}$?
    A. $\frac{s^{20}}{t^{12}}$ B. $\frac{s^6}{t^3}$ C. $\frac{s^2}{t}$ D. $\frac{s^{28}}{t^{32}}$

9. Which of the following is equivalent to $\frac{y^{\frac{7}{10}}}{y^{\frac{2}{10}}}$?
    A. y^5 B. y^2 C. $y^{\frac{9}{10}}$ D. $y^{\frac{1}{2}}$

10. What is the simplified form of $(x^{16} y^0 z^8)^{\frac{1}{4}}$?
    A. $x^{20}y^4 z^{12}$ B. $x^4y^1z^2$ C. $x^4y\ z^2$ D. x^4z^2

11. What is the value of $32^{\frac{3}{5}}$ when simplified?
    A. 8   B. 12   C. 14   D. 16

12. Which of the following expressions with rational exponent is equivalent to $9\sqrt{m^3}$?
    A. $(9m)^{1/2}$ B. $(9m)^2$ C. $9m^3$ D. $9m^{3/2}$

13. Which of the expressions below is equal to the expression $\sqrt{(25 \llbracket a^\wedge 4\ b \rrbracket ^\wedge 5)}$ when written using a rational exponent?
    A. $5a^2b^2\sqrt{b}$ B. $(25a^4b^5)^{\frac{1}{2}}$ C. $25a^4b^5$ D. $25(a^4b^5)^{\frac{1}{2}}$

14. Which of the following is the radical form of $x^{\frac{5}{4}}$?
    A. $\sqrt[5]{x^4}$ B. $\sqrt[4]{x^5}$ C. $4\sqrt{x^5}$ D. $5\sqrt{x^4}$

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15. What is the exponential form of $7\sqrt{a} + 12\sqrt{b}$?

   A. $7^{\frac{1}{2}}a + 12^{\frac{1}{2}}b$ B. $(7a)^{\frac{1}{2}} + (12b)^{\frac{1}{2}}$ C. $7a^{\frac{1}{2}} + 12b^{\frac{1}{2}}$ D. $(7a + 12b)^{\frac{1}{2}}$

   Great job! You made it. Congratulations!

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