9

Mathematics

Quarter 4 - Module 5
Illustrates Laws of Sine and Cosines

AIRs - LM

Mathematics 9
Quarter 2: Week 6-7, Module 5: Illustrates Law of Sines and Cosines
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Target

Good day!

This module will assess your knowledge on the different mathematical concepts and skills in performing mathematical operations that will help you understand **laws of sines and cosines**.

Before we start, let us first consider the learning competency.

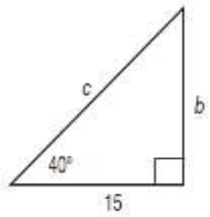
1. Illustrates laws of sines and cosines **(M9GE-IVe-1)**

After going through this module, you are expected to:

1. illustrates the law of sine and law of cosine
2. use the concept of the sum of the measures of the interior angles is 180° in finding the missing one angle.
3. apply the formula of sine and cosine in finding the missing part of an oblique triangle.

Pre – Test:

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper



1. What is the measure of the third angle opposite to 15?

   A. 40° B. 45°

   C. 50° D. 55°

2. Find the value of side *c* to the nearest unit.

   A. 20 B. 22 C. 23 D. 24

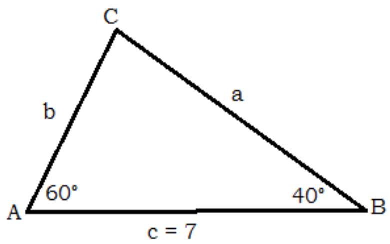
3. Find the value of side *b* to the nearest unit.

   A. 9 B. 13 C. 18 D. 21

4. ΔXYZ is a non – right triangle. If XY measures 20 cm, XZ measures 15 cm and ∠Z measures 35° then what is the measure of ∠Y?

   A. 25.84° B. 24.85° C. 25.48° D. 24.58°

5. Using the sine law, which of the following formula ***DOES NOT*** applicable in finding the triangle's side?



   A. $\frac{\sin A}{a} = \frac{\sin B}{b}$ B. $\frac{\sin B}{b} = \frac{\sin C}{c}$ C. $\frac{\sin A}{a} = \frac{\sin C}{c}$ D. $a^2 + b^2 = c^2$

6. Oblique triangles can also be solved using the Law of Sines and Cosines. This law states the following:

   I. Three sides known

   II. Two sides and the included angle are known

   III. Two angles and one side (SAA Case & ASA Case)

   IV. Two sides and an angle opposite one of these sides (SSA Case)

   Which of the following conditions describes the use of the law of sine?

   A. I & II B. II & III C. III & IV D. I, III & IV

7. Given the conditions in item 6, which of the following conditions describes the use of the law of cosine?

   A. I & II B. II & III C. III & IV D. I, III & IV

8. Which of the following should be used to find the measure of side b using the law of cosine?

A. $b^2 = a^2 + c^2 - 2ac(\cos B)$ B. $b^2 = a^2 + c^2 - 2ac(\cos C)$

C. $b^2 = a^2 + c^2 - 2ac(\cos A)$ D. $b^2 = a^2 + b^2 - 2ab(\cos B)$

9. Which of the following is the formula used in the law of sine?

A. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ B. $a^2 = b^2 + c^2 - 2bc(\cos A)$

C. $b^2 = a^2 + c^2 - 2ac(\cos B)$ D. $c^2 = a^2 + b^2 - 2ab(\cos C)$

10. Which of the following is ***NOT*** the formula used in the law of cosine?

A. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ B. $a^2 = b^2 + c^2 - 2bc(\cos A)$

C. $b^2 = a^2 + c^2 - 2ac(\cos B)$ D. $c^2 = a^2 + b^2 - 2ab(\cos C)$

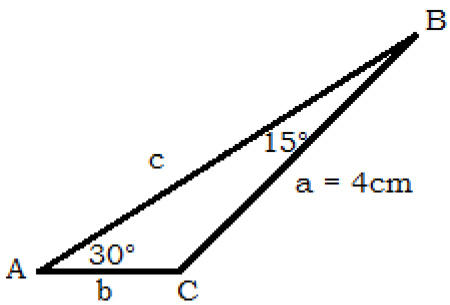
11. Which one will ***NOT*** give you information about law of sine?

A. SAA Case B. SSA Case C. SSS Case D. ASA Case

12. Which one will give you information about law of cosine?

A. SAA Case B. SSA Case C. SSS Case D. ASA Case

For item 13, refer to the triangle below.



13. Find the measure of side b?

A. 2 cm B. 3 cm C. 4 cm D. 5cm

14. Solve the triangle: a = 31, b = 36, c = 42. What is the measure of angle B?

A. 77° B. 67° C. 57° D. 47°

15. Solve the triangle: a = 31, b = 36, c = 42. What is the measure of angle C?

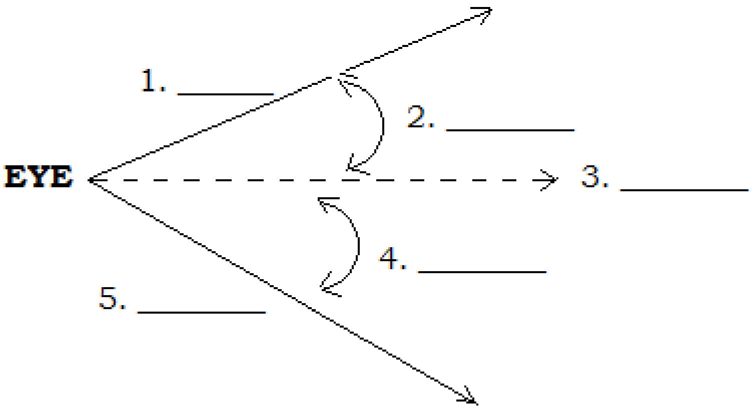
A. 77° B. 67° C. 57° D. 47°

Let us recall on what you had learned in angle of elevation and depression.

REVIEW ACTIVITY

A: Identify the parts of the figure below the angle of elevation and depression. Choose your answers on the box below.

Angle of elevation	Line of sight above the observer
Angle of depression	Line of sight below the observer
Horizontal line of sight	



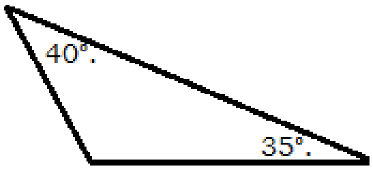
Jumpstart

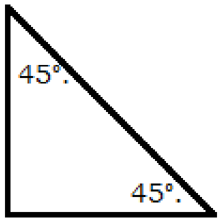
This activity will enable you to assess your prior knowledge on illustrating law of sines and cosines

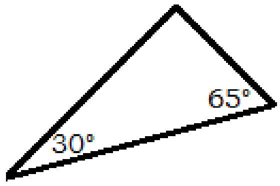
The triangles we see around us are not all right triangles. These triangles are called **oblique triangles.**

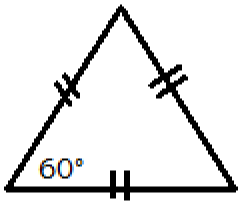
An **oblique triangle** is a triangle that does not contain any right angle.
Oblique triangles may be classified into two --- acute and obtuse.
An **acute angle** is a triangle whose angles are all less than 90°.
An **obtuse angle** is a triangle whose one of the angles is more than 90°.

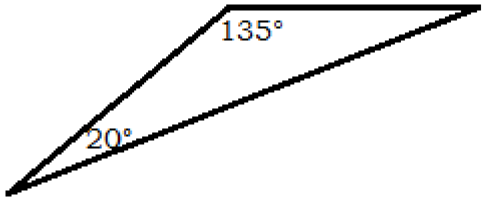
Activity 1: Missing Me!
Directions: Find the missing angle/s and identify whether the triangle is ***acute***, ***obtuse***, or ***neither***. Write your answers on a separate sheet of paper.

1. 

2. 

3. 

4. 

5. 

Discover

To solve problems involving radicals, you must answer the question asked. Well-labeled diagrams and pictures will help you understand the problem.

Lesson 1.1 Illustrating Law of Sines and Its Application

Start this module's lesson by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. This knowledge and skills will illustrate the laws of sines and cosines. If you find any difficulty in answering the activities, seek the assistance of your teacher or refer to the modules you have gone over earlier.

The Law of Sines is easy to follow and very useful in solving oblique triangles when you know the following information:

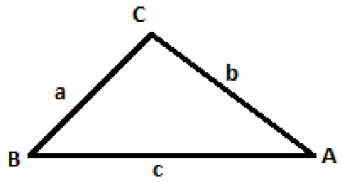
- Two angles and one side (SAA Case & ASA Case)
- Two sides and an angle opposite one of these sides (SSA Case)

The Law of Sines is described by the relation,

Law of Sines

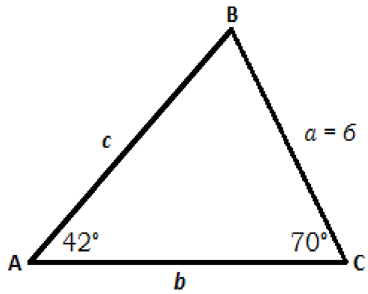
In any ΔABC,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Example 1: SAA Case

Find the missing parts of ΔABC on the right.



Given: two angles and one side

∠ A = 42°

∠ C = 70°

a = 6

Solutions:

Since, side b and ∠ B are unknown, we can use the formula

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$\frac{\sin A}{a} = \frac{\sin C}{c}$	*Formula to use to solve for c*
$\frac{\sin 42°}{6} = \frac{\sin 70°}{c}$	*Substitute the given values*
$c \sin 42° = 6 \sin 70°$	*Cross multiply*
$0.6691\ c = 6\ (0.9397)$	*Compute for the values of sin 42° and sin 70° using a scientific calculator*
$0.6691\ c = 5.6382$	*Simplify the resulting equation*
$c = \frac{5.6382}{0.6691}$	*Solve for c*
$c = 8.43$	

To solve for *b*, the formula to be used is $\frac{\sin A}{a} = \frac{\sin B}{b}$. Notice that ∠ B is unknown. You have learned in Mathematics that in any triangle, the sum of the measures of the three angles is 180°. Using this concept, ∠ A + ∠B + ∠C = 180°, we have

42°+∠B + 70° = 180°

112° + ∠B = 180°

∠B = 180° - 112°

∠ B = 68°

Thus, we can now solve for side *b*.

Following the steps used earlier in solving for *c*, we now have

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\qquad$ *0.6691 b = 5.5632*

$\frac{\sin 42°}{6} = \frac{\sin 68°}{b}$

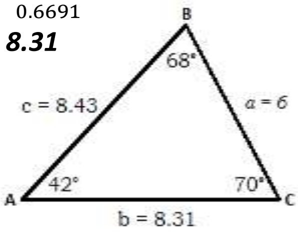
b sin 42° = 6 sin 68°

0.6691 b = 6(0.9272)

$b = \frac{5.5632}{0.6691}$

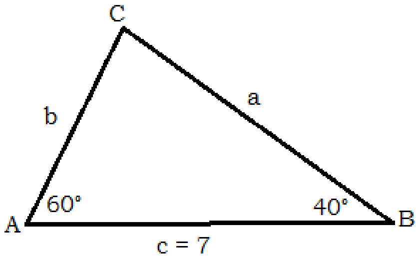
b = 8.31

Thus, the triangle with the measures of its parts is shown at the right.



Example 2: ASA Case

Determine the measure of the missing parts of ΔABC on the right.



Given: two angles and one side

∠ A = 60°

∠ B = 40°

c = 7

Solutions:

Since the measures of the two angles of the triangle are known, the measure of the third angle can be determined using the concept that the sum of the angles of the triangle is 180°.

∠ A + ∠B + ∠ C = 180°

60° + 40° + ∠ C = 180°

100° + ∠ C = 180°

∠ C = 180° - 100°

∠ C = 80°

To solve for side a, we can use the formula $\frac{\sin A}{a} = \frac{\sin C}{c}$.

$\frac{\sin A}{a} = \frac{\sin C}{c}$	*Formula to use to solve for c*
$\frac{\sin 60°}{a} = \frac{\sin 80°}{c}$	*Substitute the given values*
a sin 80° = 7 sin 60°	*Cross multiply*
0.9848 a = 7(0.8660)	*Compute for the values of sin 80° and sin 60° using a scientific calculator*
0.9848 a = 6.062	*Simplify the resulting equation*
$a = \frac{6.062}{0.9848}$	*Solve for a*
a = 6.16	

To solve for side b, use the formula $\frac{\sin B}{b} = \frac{\sin C}{c}$ and follow the steps earlier.

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 40°}{b} = \frac{\sin 80°}{7}$

b sin 80° = 7 sin 40°

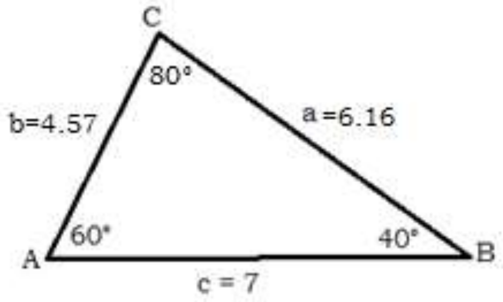
0.9848 b = 7(0.6428)

0.9848 b = 4.4996

$b = \frac{4.4996}{0.9848}$

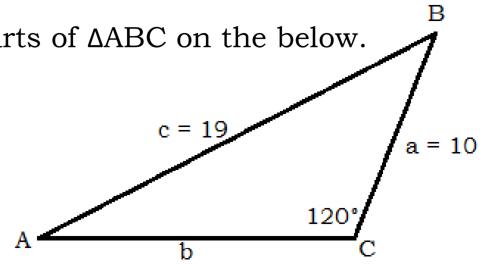
b = 4.57

Shown below is the triangle with its complete parts.



Example 3: SSA Case

Solve for the missing parts of ΔABC on the below.



Given: two sides and an angle opposite of these sides

a = 10

c = 19

∠ *A = 120°*

Solutions:

∠C is an obtuse angle and *c* > *a*, thus there is exactly one solution.

Since *a, c* and ∠C are known, we can use the formula formula $\frac{sin\ A}{a} = \frac{\sin C}{c}$.

$\frac{sin\ A}{a} = \frac{\sin C}{c}$	*Formula to use to solve for A*
$\frac{sin\ A}{10} = \frac{\sin 80°}{19}$	*Substitute the given values*
19 (sin A) = 10 (sin 120°)	*Cross multiply*
19 (sin A) = 10(0.8660)	*Compute for the values of sin 120° and sin 60° using a scientific calculator*
19 (sin A) = 8.66	*Simplify the resulting equation*
$\sin A = \frac{8.66}{19} = 0.4558$	*Solve for A*
A = 27.12°	

Using the concept that the sum of the angles of a triangle is 180°, we have

∠A + ∠ B + ∠C= 180° ∠ B = 180° - 147.12°

27.12° +∠ B + 120° = 180° ∠ **B = 32.88°**

∠B + 147.12° **=** 180°

Following the steps used earlier in solving for *c*, we can now solve for *b*.

$\frac{sin\ B}{b} = \frac{sin\ C}{c}$ *b(0.8660) = 19(0.5429)*

$\frac{\sin 32.88°}{b} = \frac{\sin 120°}{19}$

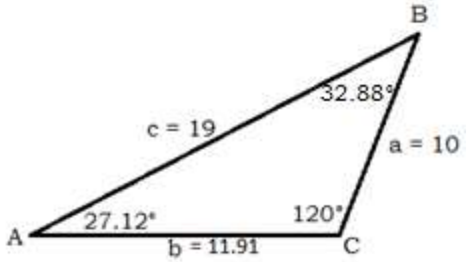
b sin 120° = 19 sin 32.88°

0.9848 b = 7(0.6428)

0.8660 b = 10.3151

$b = \frac{10.3151}{0.8660} = \mathbf{11.91}$

Shown below is the triangle with its complete parts.



Lesson 1.2 Illustrating Law of Cosines and Its Application

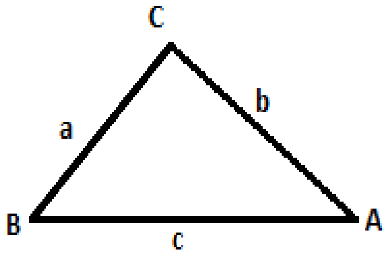
Oblique triangles can also be solved using the Law of Cosines. This law states the following:

The square length of one side is equal to the sum of the other two sides minus the product of twice the two sides and the cosine of the angle between them.

Law of Cosines

In any ΔABC,

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$b^2 = a^2 + c^2 - 2ac(\cos B)$
$c^2 = a^2 + b^2 - 2ab(\cos C)$



The Law of Cosines can be used in the following situations:

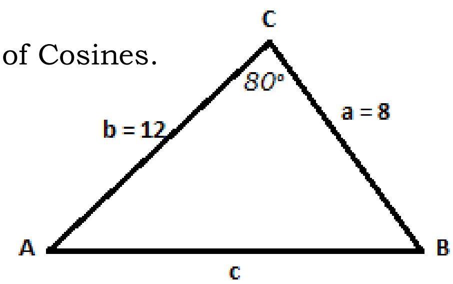
- Two sides and the included angle are known
- Three sides known

Example 1:

Let us use ΔABC to illustrate the Law of Cosines.

Given: two sides and the included angle

∠ C = 80°



a = 8
b = 12

Solutions:

To solve for c,

$c^2 = a^2 + b^2 - 2ab(\cos C)$
$c^2 = 8^2 + 2^2 - 2(8)(2)(\cos 80°)$
$c^2 = 64 + 144 - 192(0.1736)$
$c^2 = 208 - 33.3312$
$c^2 = 174.6688$
$\boldsymbol{c^2 = 13.22}$

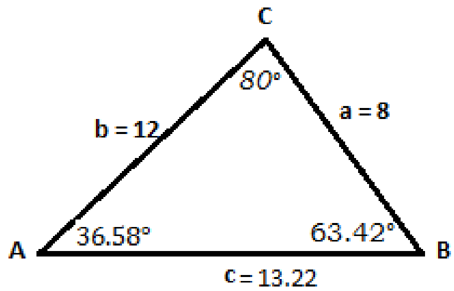
To determine the measure of ∠A,

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$8^2 = 12^2 + 13.22^2 - 2(12)(13.22)(\cos A)$
$64 = 144 + 174.7684 - (37.22)cos\ A$
$64 = 318.7684 - (317.28)(cos\ A)$
$(317.28)(cos\ A) = 318.7684 - 64$
$(317.28)(cos\ A) = 254{,}7684$
$cosA = \frac{2547684}{317.28} = 0.8030$
$\boldsymbol{A = 36.58°}$

Since the measure of ∠C is given, and the measure of ∠ A is now known, the measure of ∠B can be computed using the equatior∠ A +∠ B +∠C = 180°.

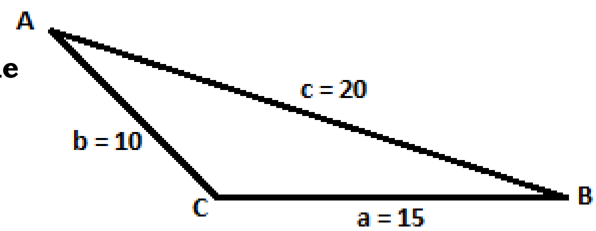
∠A + ∠B + ∠C = 180°
36. 58° +∠ B + 80° = 180°
116.58 + ∠B = 180°
∠ B = 180° - 116.58°
∠ **B = 63.42°**

The triangle with its complete parts is shown below.



Example 2:

Determine the missing parts of ΔABC.



Given: two sides and the included angle

a = 15
b = 10
c = 20

Solutions:

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$b^2 = a^2 + c^2 - 2ac(\cos B)$
$c^2 = a^2 + b^2 - 2ab(\cos C)$

Let us solve for the measure of ∠A.

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$15^2 = 10^2 + 20^2 - 2(10)(20)(cosA)$

$225 = 100 + 400 - 400(cosA)$

$-275 = -400(cosA)$

$\frac{-275}{-400} = \frac{-400(cosA)}{-400}$

$0.6875 = cosA$

$\boldsymbol{A = 46.57°}$

Using the formula $b^2 = a^2 + c^2 - 2ac(\cos B)$ and following the steps used above, let's find the measure of ∠B.

$b^2 = a^2 + c^2 - 2ac(\cos B)$

$10^2 = 15^2 + 20^2 - 2(15)(20)(cosB)$

$100 = 225 + 400 - 600(cosB)$

$-525 = -600(cosB)$

$\frac{-525}{-600} = \frac{-600(cosB)}{-600}$

$0.8750 = cosB$

$\boldsymbol{B = 28.96°}$

Since two angles are already known, substitute their values in the equation ∠A + ∠B + ∠C = 180° to solve for ∠C.

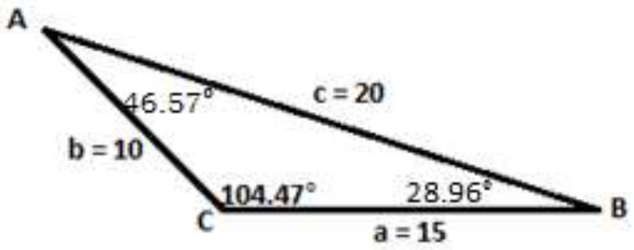
46.57° + 28.96 + ∠C = 180°

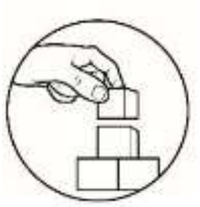
75.53° + ∠C = 180°

∠C = 180° - 75.53°

∠C = 104.47°

Below is the triangle with its complete parts.



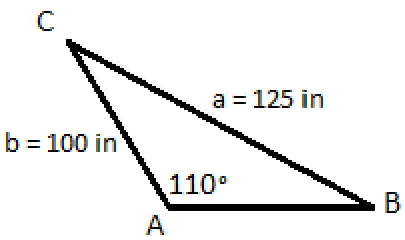


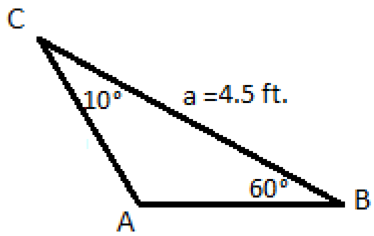
Explore

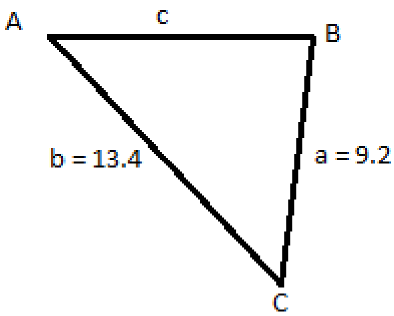
Here are some enrichment activities to master and strengthen the basic concepts you have learned from this lesson.

Activity 2: Practice makes perfect!
Directions: Find all the missing parts of each oblique triangle.

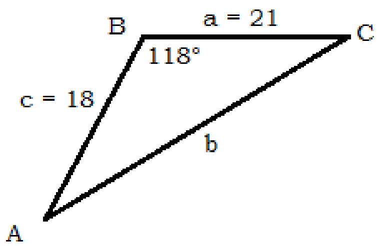
A. Use the Law of Sines to find the all the missing parts of ΔABC.

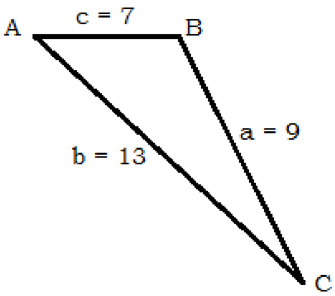
1. 

2. 

3. 

B. Use the Law of Cosines to find the all the missing parts of ΔABC.

4. 

5. 

In the previous activity, you were able to apply your understanding of the illustrating law of sine and cosine.

Let us put that understanding to the test by answering the next activity.

Deepen

Activity 3: Illustrate then Solve!

Directions: Illustrate the oblique triangle then find its missing parts using the Law of Sine and Law of Cosine.

1. Given: $a = 3$; $b = 2$; $\angle A = 50°$
   Find: $\angle B$, $\angle C$ and side c

2. Given: $b = 4$; $c = 6$; $\angle B = 20°$
   Find: $\angle A$, side a and $\angle C$

3. Given: $a = 29$; $b = 39$; $\angle C = 49°$
   Find: $\angle A$, $\angle B$ and side c

4. Given: $a = 3$; $b = 7$; $c = 9$
   Find: $\angle A$, $\angle B$ and $\angle C$

In this activity, the discussion was about your understanding of illustrating the law of sines and cosines. Let us see what you had learned on this module by filling – out the next activity.

Activity 4: Synthesis Journal

Directions: Fill in the table by answering the given questions. Use a separate sheet of paper for your answer.

Synthesis Journal		
What interest me.	**What I learned.**	**How can the knowledge of the law of sine and cosine help us in solving oblique triangles?**

Gauge

Post – Test

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper

1. Which of the following is the formula used in the law of sine?

   A. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ B. $a^2 = b^2 + c^2 - 2bc(\cos A)$

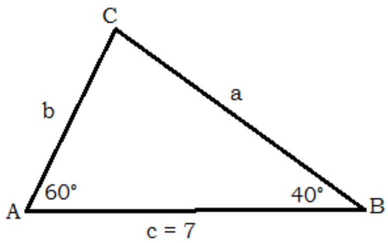
   C. $b^2 = a^2 + c^2 - 2ac(\cos B)$ D. $c^2 = a^2 + b^2 - 2ab(\cos C)$

2. Which of the following is ***NOT*** the formula used in the law of cosine?

   A. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ B. $a^2 = b^2 + c^2 - 2bc(\cos A)$

   C. $b^2 = a^2 + c^2 - 2ac(\cos B)$ D. $c^2 = a^2 + b^2 - 2ab(\cos C)$

3. Using the sine law, which of the following formula should be used in finding the measure of side a in the given figure below?



   A. $\frac{\sin A}{a} = \frac{\sin B}{b}$ B. $\frac{\sin B}{b} = \frac{\sin C}{c}$ C. $\frac{\sin A}{a} = \frac{\sin C}{c}$ D. $a^2 + b^2 = c^2$

4. Using the law of cosine, which of the following should be used to find the measure of side c?

   A. $c^2 = a^2 + b^2 - 2ab(\cos C)$ B. $c^2 = a^2 + b^2 - 2ab(\cos A)$

   C. $c^2 = a^2 + b^2 - 2ab(\cos B)$ D. $c^2 = a^2 + b^2 - 2bc(\cos C)$

5. Oblique triangles can also be solved using the Law of Sines and Cosines. This law states the following:

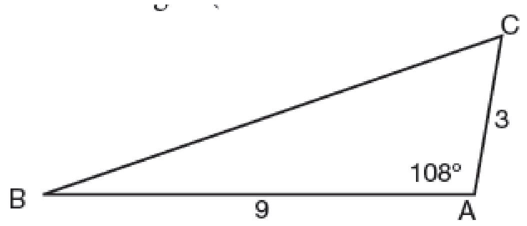
I. Three sides known
II. Two sides and the included angle are known
III. Two angles and one side (SAA Case & ASA Case)
IV. Two sides and an angle opposite one of these sides (SSA Case)
Which of the following conditions that describes the use of law of sine?
A. I & II B. II & III C. III & IV D. I, III & IV

6. Given the conditions in item 5, which of the following conditions that describes the use of law of cosine?
A. I & II B. II & III C. III & IV D. I, III & IV

7. Which one will ***NOT*** give you information about law of sine?
A. ASA Case B. SAA Case C. SSA Case D. SSS Case

8. Which one will give you information about law of cosine?
A. ASA Case B. SAA Case C. SSA Case D. SSS Case

For items 9 – 10, refer to the ΔABC below.

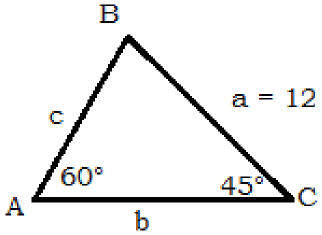


9. Find side a.
A. 10.27 B. 10.33 C. 13.33 D. 15

10. Find angle B.
A. 12° B. 14° C. 16° D. 18°

11. Find angle C.
A. 45° B. 56° C. 63° D. 81°

For items 12 – 14, refer to the ΔABC below.



12. Find angle B.
    A. 25° B. 50° C. 75° D. 90°

13. Find side b.
    A. 11.18 B. 12.28 C. 13.38 D. 14.48

14. Find side c.
    A. 9.80 B. 10.12 C. 11.15 D. 12.29

15. Solve the triangle: a = 31, b = 36, c = 42. What is the measure of angle A?
    A. 76° B. 66° C. 56° D. 46°

References
A. Books:

Mathematics Grade 9 Learner's Module, First Edition 2014, Reprint 2017
Mathematics Grade 9 Teacher's Guide, First Edition 2014, Reprint 2017
Advanced Algebra, Trigonometry and Statistics, Textbook for Fourth Year, Reprinted 2005