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DEPARTMENT OF EDUCATION

MATHEMATICS

Quarter 2 - Module 4:
Proving Theorems Related to Chords, Arcs, Central Angles and Inscribed Angles

AIRs - LM

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MATHEMATICS 10
Quarter 2 - Module 4: Proving Theorems Related to Chords, Arcs, Central Angles and Inscribed Angles
Second Edition, 2021

Development Team of the Module

Author: Concesa M. Juntilano
Editor: SDO La Union, Learning Resource Quality Assurance Team
Illustrators: Ernesto F. Ramos, Jr.
Christian R. Bumatay
Content Reviewer: Richie C. Noveloso
Language Reviewer: Evangeline G. Dimalanta
Design and Layout: Mark Jesus M. Mulato

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent
German E. Flora, PhD, *CID Chief*
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Erlinda M. Dela Peña, PhD, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

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Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

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MATHEMATICS

Quarter 2 - Module 4:

Proving Theorems Related to Chords, Arcs, Central Angles and Inscribed Angles

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

This module was designed and written to help you understand the proof of theorems related to chords, arcs, central angles and inscribed angles. As you go through this lesson, keep on asking yourself the question: “How do the relationships among chords, arcs, central angles and inscribed angles help you find solutions to your real-life problems”?

In going over this module, you are expected to:

Learning Competency

Proves theorems related to chords, arcs, central angles and inscribed angles. (**M10GE-IIc-d-1)**

Objectives:

1. Identifies the relationships among chords, arcs, central angles and inscribed angles.
2. Illustrates how to prove theorems related to chords, arcs, central angles and inscribed angles.
3. Applies the theorems on chords, arcs, central angles and inscribed angles in solving the measurements of chords, arcs and angles.

Before you start the lesson, find out how much you already know about this module by answering the pre – assessment.

Jumpstart

PRE - ASSESSMENT

Directions: Read and answer each statement below. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module. Write your answers on a separate sheet of paper.

1. What kind of angle is resulted when opposite angles of a quadrilateral is inscribed in a circle?

   A. right B. obtuse C. complementary D. supplementary

2. What do you call circles with equal radii?

   A. Unit B. Point C. Congruent D. Circumference

3. In two congruent circles, if $\overset{\frown}{PO} \cong \overset{\frown}{SI}$, what can be concluded about $\overline{PO}$ and $\overline{SI}$?

   A. $\overline{PO} \cong \overline{SI}$ B. PO = ½ SI C. PO = SI D. $\overline{PO} \perp \overline{SI}$

4. What angle is formed when an inscribed angle of a circle intercepts a semicircle?

   A. acute B. right C. obtuse D. straight

5. What is the relationship of the measure of the central angle to the measure of its intercepted arc?

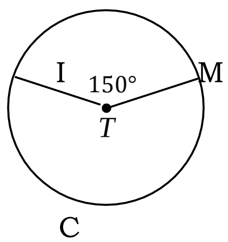
   A.half B.equal C.twice D.thrice

6. What is the relationship of the measure of an inscribed angle to the measure of its intercepted arc?

   A.half B.equal C.twice D.thrice

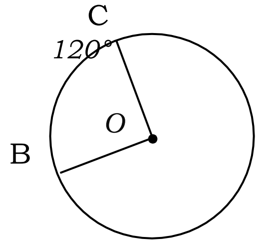
7. In the figure on the right, what is $m\overset{\frown}{IM}$ if $m\angle ITM = 150$?

   A.75 B.150 C.300 D.360



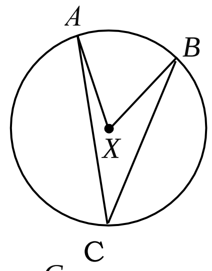
8. In $\odot O$ on the right, what is $m\angle BOC$ if $m\overset{\frown}{BC} = 120$?

   A.60 B.120 C.240 D.300



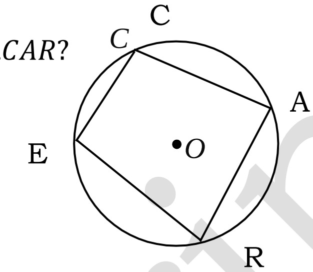
9.Given$\odot X$, if $m\overset{\frown}{AB}$ is 60, what is the measure of $\angle ACB$?

A.30 B.60 C.120 D.200



10. In the figure, m$\angle CER$ =75, m$\angle ARE$ =95 , what is $m\angle CAR$?

A.75 B.85 C.95 D.105



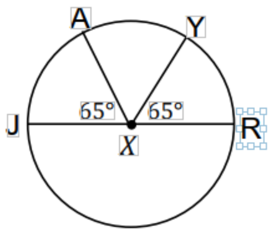
11. In the figure, $\overline{\text{JR}}$ is a diameter of $\odot X$. If $\angle JXA \cong \angle YXR$ which of the following relationship is true?

A. $\angle JXA \cong \angle AXY$

B. $\angle AXY \cong \angle YXR$

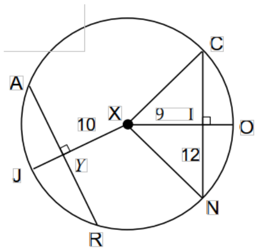
C. $\overset{\frown}{\text{JA}} \cong \overset{\frown}{\text{AY}}$

D. $\overset{\frown}{\text{JA}} \cong \overset{\frown}{\text{YR}}$



For numbers 12-15,use the figure and the given information.

In $\odot X$ on the right, $\overline{\text{XY}}$ = 10 cm, $\overline{\text{IX}}$ = 9 cm and $\overline{\text{IN}}$ = 12 cm.



12. What is the measure of $\overline{\text{XN}}$?

A.12 cm B.13 cm C.14 cm D. 15 cm

13. What is the measure of $\overline{\text{CI}}$?

A.10 cm B.11 cm C.12 cm D.13 cm

14. What is the measure of $\overline{\text{JY}}$?

A. 5 cm B. 6 cm C. 7 cm D. 8 cm

15. What is the measure of $\overline{\text{IO}}$?

A. 5 cm B. 6 cm C. 7 cm D. 8 cm

Now, let us take a look on the discussions below. This will help you in understanding the lesson.

Discover

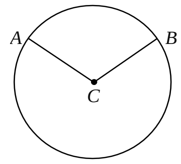
Before you proceed to the different proofs of the theorems on Central Angles and Inscribed Angles, you first study the relationships of chords, arcs and angles.

The degree measure of an arc is defined in terms of its central angle.

Definition

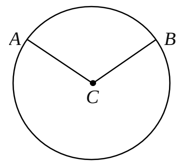
The degree measure of minor arc is equal.
to the degree measure of central angle

$$m\overset{\frown}{AB} = m\angle ACB$$



The degree measure of major arc is equal to 360 minus the degree measure of central angle

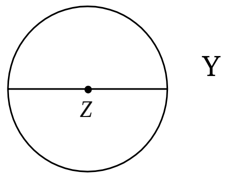
$$m\overset{\frown}{ADB} = 360 - m\angle ACB$$



D

The degree measure of semicircle is equal to 180.

$$m\overset{\frown}{XZY} = 180$$



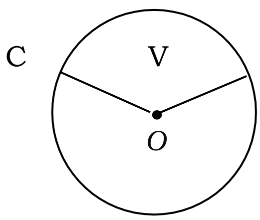
The Central Angle-Intercepted Arc Postulate

The measure of a central angle of a circle is equal to the measure of its intercepted arc.

Example 1.

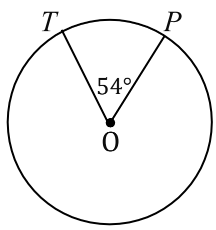
Given: $\odot O$ with $m\angle COV = 120$

Find: $m\overset{\frown}{CV}$



Given: $\odot O$ with $m\angle TOP = 54$

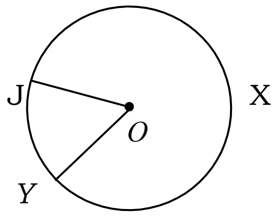
Find: $m\overset{\frown}{TP}$



Answer: $m\overset{\frown}{CV} = 120$ Answer: $m\overset{\frown}{TP} = 54$

Given: $\odot O$ with $m\angle JOY = 55$

Find: $m\overset{\frown}{JXY}$



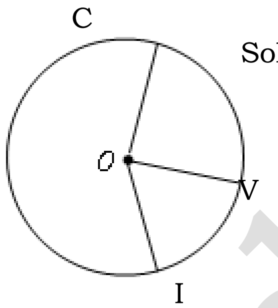
Answer: $m\overset{\frown}{JXY} = 360-55 = 305$

Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the two arcs.

Example 2:

Given $\odot O$, find $m\overset{\frown}{CVI}$ if $m\overset{\frown}{CV}=75$ and $m\overset{\frown}{VI}=40$

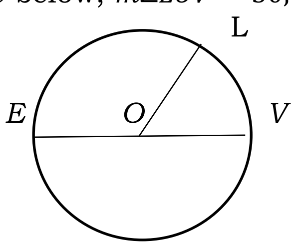


Solution: $m\overset{\frown}{CVI} = m\overset{\frown}{CV} + m\overset{\frown}{VI}$
$=75+40$
$=115$

A diameter divides a circle into two semicircles.

Example 3. In the figure below, $m\angle LOV = 50$, find

a. $m\overset{\frown}{LV}$
b. $m\overset{\frown}{EV}$
c. $m\overset{\frown}{EL}$
d. $m\angle EOL$



Solutions:

a. $m\overset{\frown}{LV} = 50$, since a central angle and its intercepted arc have equal measures
b. $m\overset{\frown}{EV} = 180$ since $\overset{\frown}{EV}$ is a semicircle
c. By the Arc Addition Postulate, we have
$m\overset{\frown}{EL} = 180-50$
$m\overset{\frown}{EL} = 130$

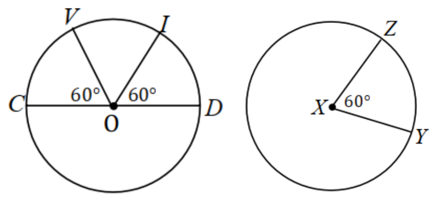
d. m$\angle EOL$ = 130 since a central angle has a measure equal to the measure of its intercepted arc.

Theorems on Central Angles, Arcs, and Chords

1. *In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.*

In ⊙O below, $\angle COV \cong \angle DOI$. Since the two central angles are congruent, the minor arcs they intercept are also congruent. Hence, $\overparen{CV} \cong \overparen{DI}$.

If $\odot O \cong \odot X$ and $\angle COV \cong \angle DOI \cong \angle ZXY$, then $\overparen{CV} \cong \overparen{DI} \cong \overparen{ZY}$.



Proof of the Theorem

Part 1: Given are two congruent circles and a central angle from each circle which are congruent. The two-column proof below shows that their corresponding intercepted arcs are congruent.

Given: $\odot O \cong \odot X$

$\angle COV \cong \angle YXZ$

Prove: $\overparen{CV} \cong \overparen{YZ}$

Proof:

Statements	Reason
1. $\odot O \cong \odot X$ $\angle COV \cong \angle YXZ$	1.Given
2. In $\odot O$, $m\angle COV = m\overparen{CV}$. In $\odot X$, $m\angle YXZ = m\overparen{YZ}$.	2.The degree measure of a minor arc is the measure of the central angle which intercepts the arc.
3. $m\angle COV = m\angle YXZ$	3.From 1, definition of congruent angles
4. $m\overparen{CV} = m\overparen{YZ}$	4.From 2 &3, substitution
5. $\overparen{CV} \cong \overparen{YZ}$	5.From 4, definition of congruent arcs

Part 2. Given are two congruent circles and intercepted arcs from each circle which are congruent. The two-column proof shows that their corresponding angles are congruent.

Given: $\odot O \cong \odot X$

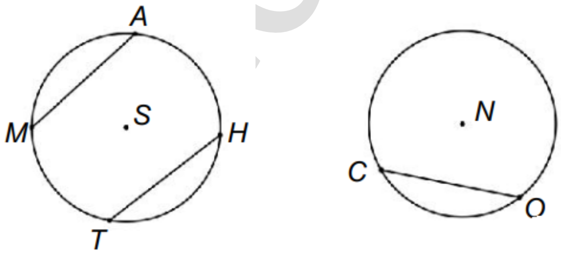
$\overset{\frown}{CV} \cong \overset{\frown}{YZ}$

Prove: $\angle COV \cong \angle YXZ$

Statements	Reason
1. $\odot O \cong \odot X$ $\overset{\frown}{CV} \cong \overset{\frown}{YZ}$	1.Given
2. In $\odot O$, $m\overset{\frown}{CV} = m\angle COV$. In $\odot X$, $m\overset{\frown}{YZ} = m\angle YXZ$.	2.The degree measure of a minor arc is the measure of the central angle which intercepts the arc.
3. $m\overset{\frown}{CV} = m\overset{\frown}{YZ}$	3.From 1, definition of congruent angles
4. $m\angle COV = m\angle YXZ$	4.From 2 & 3, substitution
5. $\angle COV \cong \angle YXZ$	5.From 4, definition of congruent angles

2.In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

In $\odot S$ below, $\overline{MA} \cong \overline{TH}$. Since the two are chords are congruent, then and $\overset{\frown}{MA} \cong \overset{\frown}{TH}$ If $\odot S \cong \odot N$ and $\overline{MA} \cong \overline{TH} \cong \overline{CO}$, then $\overset{\frown}{MA} \cong \overset{\frown}{TH} \cong \overset{\frown}{CO}$.



Proof of the Theorem:

Part 1.Given two congruent circles $\odot S \cong \odot N$ and two congruent corresponding chords $\overline{AM}$ and $\overline{OC}$.

Given: $\odot S \cong \odot N$ and

$\overline{AM} \cong \overline{OC}$

Prove: $\overset{\frown}{AM} \cong \overset{\frown}{OC}$

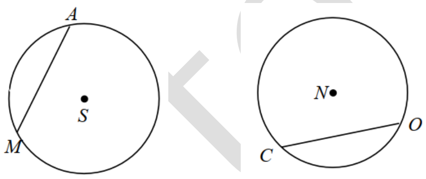
Statements	Reason
1. $\odot S \cong \odot N$ $\overline{AM} \cong \overline{OC}$	1.Given
2. $\overline{SA} \cong \overline{SM} \cong \overline{NC} \cong \overline{NO}$	2.Radii of the same circle or of congruent circles are congruent
3. $\Delta ASM \cong \Delta ONC$	3.SSS Postulate
4. $\angle ASM \cong \angle ONC$	4.Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
5. $\overset{\frown}{AM} \cong \overset{\frown}{OC}$	5.From the previous theorem, “In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.

Part 2. Given two congruent circles $\odot S \cong \odot N$ and two congruent corresponding chords $\overline{AB}$ and $\overline{OE}$.

Given: $\odot S \cong \odot N$ and

$\overline{AM} \cong \overline{OC}$

Prove: $\overline{AM} \cong \overline{OC}$

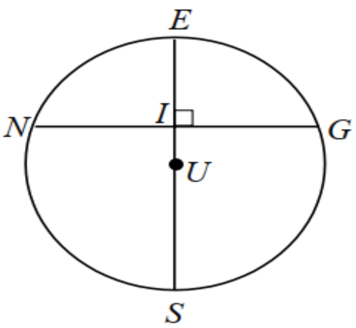


Proof:

Statements	Reason
1. $\odot S \cong \odot N$ $\overline{AM} \cong \overline{OC}$	1.Given
2. $m\overset{\frown}{AM} = m\overset{\frown}{OC}$	2.Definition of congruent arcs
3. $\angle ASM$ $and \angle ONC$ are central angles	3.Definition of central angles
4. $m\angle MSA = m\overset{\frown}{MA}$ $m\angle ONC = m\overset{\frown}{OC}$	4.The degree measure of a minor arc is the measure of the central angle which intercepts the arc.
5. $m\angle MSA = m\angle ONC$	5.From 2,4, substitution
6. $\overline{SA} \cong \overline{SM} \cong \overline{NC} \cong \overline{NO}$	6.Radii of the same circle or of congruent circles are congruent
7. $\triangle MSA \cong \triangle ONC$	7.SAS Postulate
8. $\overline{AM} \cong \overline{OC}$	8.Corresponding Parts of Congruent Triangles are Congruent (CPCTC)

3.In a circle, a diameter bisects a chord and an arc with the same endpoints if and only if it is perpendicular to the chord.

In $\odot U$ below, $\overline{ES}$ is a diameter and $\overline{GN}$ is a chord. If $\overline{ES} \perp \overline{GN}$, then $\overline{GI} \cong \overline{IN}$ and $\widehat{GE} \cong \widehat{EN}$.



Given: $\overline{ES}$ is a diameter of $\odot U$ and perpendicular to chord $\overline{GN}$ at I.

Prove:

1. $\overline{NI} \cong \overline{GI}$
2. $\widehat{EN} \cong \widehat{EG}$
3. $\widehat{NS} \cong \widehat{GS}$

Proof of Part 1: Show that $\overline{ES}$ bisects $\overline{GN}$ and the minor arc $\widehat{GN}$.

Statements	**Reason**
1. $\odot U$ with diameter ES and chord $\overline{GN}$; $\overline{ES} \perp \overline{GN}$	1.Given
2. $\angle GIU$ and $\angle NIU$ are right angles	2.Definition of perpendicular lines
3. $\angle GIU \cong \angle NIU$	3.Right angles are congruent
4. $\overline{UG} \cong \overline{UN}$	4.Radii of the same circle are congruent
5. $\overline{UI} \cong \overline{UI}$	5.Reflexive/Identity Property
6.$\triangle GIU \cong \Delta NIU$	6.Hyl Theorem
7. $\overline{GI} \cong \overline{NI}$	7.Corresponding parts of congruent triangles are congruent (CPCTC)
8. $\overline{ES}$ bisects $\overline{GN}$	8.Definition of segment bisector
9. $\angle GUI \cong \angle NUI$	9.From 6, CPCTC
10.$\angle GUI$ and $\angle GUE$ are the same angles $\angle NUI$ and $\angle NUE$ are the same angles	10.E,I,U are collinear
11.$m\angle GUE = m\angle NUE$	11.From 9,10, definition of congruent angles
12.$m\widehat{EG} = m\angle GUE$ $m\widehat{EN} = m\angle NUE$	12.Degree measure of an arc

13. $m\overparen{EN} = m\overparen{EG}$	13.From 11, 12, substitution
14. $m\angle GUS = m\angle NUS$	14.From 11, definition of supplementary angles that are supplementary to congruent angles are congruent
15. $m\overparen{GS} = m\angle GUS$ $m\overparen{NS} = m\angle NUS$	15.Degree measure of an arc
16. $m\overparen{NS} = m\overparen{GS}$	16.From14, 15, substitution
17. $\overline{ES}$ bisects $\overparen{GN}$	17.Definition of arc bisector

Proof of Part 2:

Given: $\overline{ES}$ is a diameter of $\odot U$; $\overline{ES}$ bisects $\overline{GN}$ at I and the minor arc $\overparen{GN}$

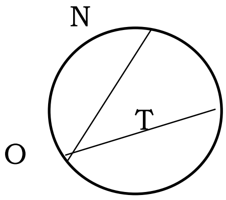
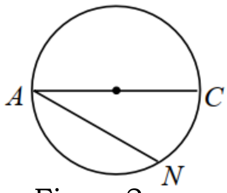
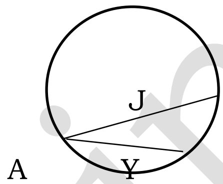
Statements	**Reason**
1. $\odot U$ with diameter $\overline{\mathrm{ES}}$, $\overline{\mathrm{ES}}$ bisects $\overline{\mathrm{GN}}$ at I and the minor arc $\overparen{\mathrm{GN}}$	1.Given
2. $\overline{\mathrm{GI}} \cong \overline{\mathrm{NI}}$ $\overparen{\mathrm{GE}} \cong \overparen{\mathrm{NE}}$	2.Definition of bisector
3. $\overline{\mathrm{UI}} \cong \overline{\mathrm{UI}}$	3.Reflexive/Identity property
4. $\overline{UG} \cong \overline{UN}$	4.Radii of the same circle are congruent
5. $\Delta GIU \cong \Delta NIU$	5.SSS Postulate
6. $\angle \mathrm{UIG} \cong \angle \mathrm{UIN}$	6.CPCTC
7. $\angle \mathrm{UIG}$ *and* $\angle \mathrm{UIN}$ are right angles	7.Angles which form a linear pair and are congruent are right angles
8. $\overline{\mathrm{IU}} \perp \mathrm{GN}$	8.Definition of perpendicular lines
9. $\overline{\mathrm{ES}} \perp \overline{\mathrm{GN}}$	9.IU is on ES

Through this discussion, you have identified how to prove theorems on Central Angles, Arcs and Chords. To enrich your knowledge on the relationships of inscribed angles and its intercepted arcs, please study the discussion presented below.

Inscribed Angles and Intercepted Arcs

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

Examples:

 Figure 1	 Figure2	 Figure 3

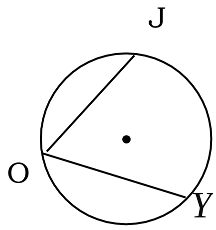
In figure 1**,** $\angle NOT$ is an inscribed angle and its intercepted arc is $\overset{\frown}{NT}$. The center of the circle is in the interior of the angle.

In figure 2, $\angle CAN$ is an inscribed angle and its intercepted arc is $\overset{\frown}{CN}$. One side of the angles is the diameter of the circle.

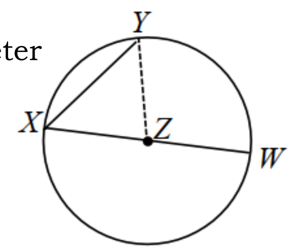
In figure 3, $\angle JAY$ is an inscribed angle and its intercepted arc is $\overset{\frown}{JY}$. The center of the circle is in the exterior of the angle.

Theorems on Inscribed Angles

1. *If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.*



Example: $\angle JOY$ in the figure is an inscribed angle with $\overset{\frown}{JY}$ as its intercepted arc.
If m$\overset{\frown}{JY}$ = 150, then
m$\angle JOY$= ½ (150)
m$\angle JOY$= 75



Proof of the Theorem:
Given: $\angle WXY$ inscribed in $\odot Z$ and $\overline{WX}$ is a diameter
Prove: $\angle WXY = \frac{1}{2} m\overset{\frown}{WY}$
Draw $\overline{YZ}$ and let m$\angle WXY$ = x

Proof:

Statements	Reason
1. $\angle WXY$ is inscribed in $\odot Z$ and $\overline{WX}$ is a diameter	1.Given
2. $\overline{XZ} \cong \overline{WZ}$	2.Radii of a circle are congruent
3. $\triangle XYZ$ is an isosceles triangle	3.Definition of isosceles triangle
4. $\angle WXY \cong \angle XYZ$	4.The base angles of an isosceles triangle are congruent
5. $m\angle WXY = m\angle XYZ$	5.The measures of congruent angles are equal
6. $m\angle WXY = x$	6.Transitive Property
7. $m\angle WZY = 2x$	7.The measure of an exterior angles of a triangle is equal to the sum of the measures of its remote interior angles
8. $m\angle WZY = m\overset{\frown}{WY}$	8.The measure of a central angle is equal to the measure of its intercepted arc
9. $m\overset{\frown}{WY} = 2x$	9.Transitive Property
10. $m\overset{\frown}{WY} = 2(m\angle WXY)$	10Substitution
11. $m\angle XYZ = \frac{1}{2}m\overset{\frown}{WY}$	11.Multiplication Property of Equality

2. *If two inscribed angles of a circle (or in congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.*

Example 1: In figure 1 below, $\angle PIE$ *and* $\angle PCE$ intercept PE. Since $\angle \overset{\frown}{PIE}$ *and* $\angle PCE$ intercept the same arc, then the two angles are congruent.

Figure 1

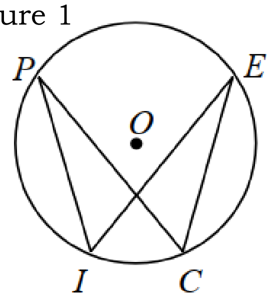
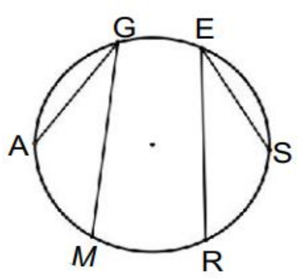


Figure 2



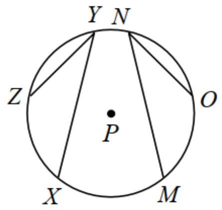
Example 2: In figure2 , $\angle AGM$ *and* $\angle SER$ intercept $\overset{\frown}{AM}$ and $\overset{\frown}{SR}$, respectively. If $\overset{\frown}{AM} \cong \overset{\frown}{SR}$, then $\angle AGM \cong \angle SER$.

Proof of theTheorem:

Given: In $\odot P$, $\overset{\frown}{XZ}$ and $\overset{\frown}{MO}$ are the intercepted arcs of $\angle XYZ$ *and* $\angle MNO$, respectively.

$\overset{\frown}{XZ} \cong \overset{\frown}{MO}$

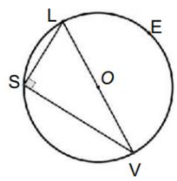
Prove: $\angle XYZ \cong \angle MNO$



Statements	Reason
1. $\overset{\frown}{XZ} \cong \overset{\frown}{MO}$	1.Given
2. $m\overset{\frown}{XZ} \cong m\overset{\frown}{MO}$	2.Congruent arcs have equal measures
3. $m\angle XYZ \cong \frac{1}{2} m\overset{\frown}{XZ}$ and $m\angle MNO \cong \frac{1}{2} m\overset{\frown}{MO}$	3.The measure of an inscribed angle is one-half the measure of its intercepted arc
4. $m\angle XYZ \cong \frac{1}{2} m\overset{\frown}{XZ}$	4.Substitution
5. $m\angle XYZ = m\angle MNO$	5.Transitive Property
6. $\angle XYZ = \angle MNO$	6.Angles with equal measures are congruent

3. *If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.*

Example: In the figure on the right, $\angle LSV$ intercepts $\overset{\frown}{LEV}$. If $\overset{\frown}{LEV}$ is a semicircle, then $\angle LSV$ is a right angle.



Given: In $\odot O$, $\angle LSV$ intercepts a semicircle LEV.

Prove: $\angle LSV$ is a right angle.

Proof:

Statements	Reason
1. $\angle LSV$ intercepts semicircle $\overparen{LEV}$	1.Given
2. $m\overparen{LEV} = 180$	2.The degree measure of a semicircle is 180.
3. $m\angle LSV \cong \frac{1}{2} m\overparen{LEV}$	3.The measure of an inscribed angle is one-half the measure of its intercepted arc
4. $m\angle LSV = \frac{1}{2}(180)$ or $m\angle LSV = 90$	4.Substitution
5. $\angle LSV$ is a right angle	5.Definition of right angle

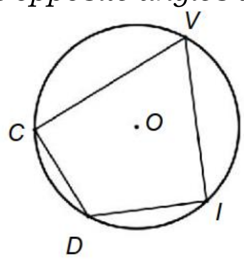
4. *If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.*

Example:

Quadrilateral CVID is inscribed in $\odot O$.

$m\angle DCV + m\angle DIV = 180$

$m\angle CVI + m\angle CDI = 180$

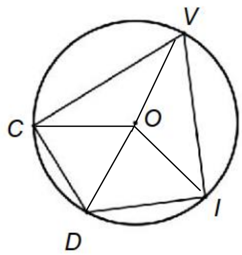


Proof of the Theorem:

Given: Quadrilateral CVID is inscribed in $\odot O$

Prove: 1. $\angle C$ and $\angle I$ are supplementary

2. $\angle V$ and $\angle D$ are supplementary



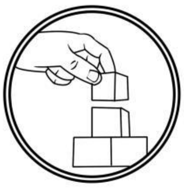
To prove: Draw $\overline{CO}$, $\overline{VO}$, $\overline{IO}$ and $\overline{DO}$

Statements	Reason
1. $m\angle COV + m\angle VOI + m\angle IOD + m\angle DOC = 360$	1.The sum of the measures of the central angles of a circle is 360.
2. $m\angle COV = m\overparen{CV}$, and $m\angle VOI = m\overparen{VI}$ $m\angle IOD = m\overparen{ID}$, and $m\angle DOC = m\overparen{DC}$	2.The measures of a central angle is equal to the measure of its intercepted arc.
3. $m\overparen{CV} + m\overparen{VI} + m\overparen{ID} + m\overparen{DC} = 360$	3.Substitution
4. $m\overparen{DIV} + m\overparen{DCV} = 360$	4.Arc Addition Postulate
5. $m\angle DCV = \frac{1}{2} m\overparen{DIV}$ and $m\angle DIV = \frac{1}{2} m\overparen{DCV}$	5.The measures of an inscribed angle is one-half the measure of its intercepted arc.

6. $m\angle DCV + m\angle DIV = \frac{1}{2} m\overparen{DIV} + \frac{1}{2} m\overparen{DCV}$	6.By Addition

7. $m\angle DCV + m\angle DIV = \frac{1}{2}(360)$ *or* $m\angle DCV + m\angle DIV = 180$	7.Substitution
8. $\angle C$ *and* $\angle I$ *are supplementary*	8.Definition of supplementary angles
9. $m\angle C + m\angle V + m\angle I + m\angle D = 360$	9.The sum of the measures of the angles of a quadrilateral is 360.
10. $m\angle V + m\angle D + 180 = 360$	10.Substitution
11. $m\angle V + m\angle D = 180$	11.Addition Property
12. $\angle V$*and* $\angle D$ *are supplementary*	12.Definition of supplementary Angles

Were you able to follow and understand the discussion of the proofs presented? Let's continue exploring!



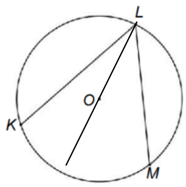
Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned in this lesson.

Enrichment Activity 1: You Complete ME!

Complete the proof of the following theorem. Choose your answer on the word bank below.

1. If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.
   Given: $\angle KLM$ *is inscribed in* $\odot O$
   Prove: $m\angle KLM = \frac{1}{2}m\widehat{KM}$
   To prove: Draw diameter LN



Statements	Reason
1. $m\angle KLN = \frac{1}{2}m\angle MLN = \frac{1}{2}m\widehat{MN}$	
2. $m\angle KLN = m\angle MLN = \frac{1}{2}m\widehat{KN} + \frac{1}{2}m\widehat{MN}$	
3. $m\angle KLN + m\angle MLN = m\angle KLM$	
4. $m\widehat{KN} + m\widehat{MN} = m\widehat{KM}$	
5. $m\angle KLM = \frac{1}{2}m\widehat{KM}$	

Word Bank

Angle addition Postulate

Substitution

Arc Addition Postulate

The measure of an inscribed angle is one-half the measure of its intercepted arc.

Addition Property

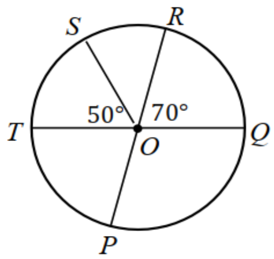
Now that you have learned to how complete the proof, you can proceed to the next activity.

Enrichment Activity 2: Guess My Degree!

$\overline{TQ}$ and $\overline{PR}$ are diameters of $\odot O$. Find the measure of the following.

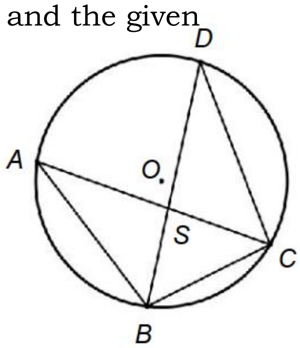
1. $m\angle SOR$ _____
2. $m\overset{\frown}{ST}$ _____
3. $m\angle TOP$ _____
4. $m\overset{\frown}{PQ}$ _____
5. $m\overset{\frown}{SRQ}$ _____

6.$m\angle PQR$ _____
7.$m\overset{\frown}{SR}$ _____
8.$m\angle TOQ$ _____
9.$m\overset{\frown}{TP}$ _____
10. $m\overset{\frown}{STP}$ _____



Enrichment Activity 3: Half, Equal or Twice As?

In $\odot O$, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{BD}$ and $\overline{AC}$ are chords. Use the figure and the given information to answer the following questions.



1.If $m\angle ABD$ = 130, what is $\overset{\frown}{AD}$?

2. If $m\angle BAC$ = 60, what is $\overset{\frown}{BC}$?

3.If $\overset{\frown}{BC}$ = 35, what is $m\angle BDC$?

4. If $m\angle BAC$= *6x+2 and* $m\angle BDC$= *4x+12, find*

a.the value of x c. $m\angle BDC$

b. $m\angle BAC$ *d.*$m\overset{\frown}{BC}$

How was the activity? Did you enjoy applying your knowledge on the different theorems presented? Now let's go deeper!

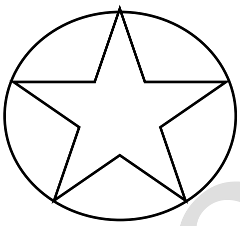
Deepen

At this point, you are going to apply the mathematical concepts learned from this module.

Activity 4: Take me To Your Real World

There are circular gardens having paths in the shape of an inscribed regular star like the one shown below. Answer the following questions.

Hint: one revolution = 360°



a.Determine the measure of an arc intercepted by an inscribed angle formed by the star in the garden.

b.What is the measure of an inscribed angle in a garden with a five-pointed star? Explain.

Gauge

Post Assessment

Direction: Choose the letter of the best answer from the given choices. Write your answers in a separate sheet of paper. *(1 point each)*

1. Which statement is true?
   A. If a quadrilateral is inscribed in a circle, then its consecutive angles are supplementary.
   B. The measure of a central angle of a circle is half the measure of its intercepted arc.

C. The measure of an inscribed angle is one-half the measure of its intercepted arc.

D. An angle inscribed in a semicircle is an acute angle.

2. Quadrilateral *MNOP* is inscribed in a circle. Which of the following is true about the angle measures of the quadrilateral?

I. $m\angle M + m\angle O = 180$

II. $m\angle N + m\angle P = 180$

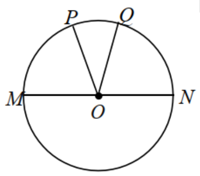
III. $m\angle M + m\angle O = 90$

A. I and II B. I and III C. II and III D.I,II and III

3. What kind of angle is the inscribed angle that intercepts a semicircle?

A. acute B. right C.obtuse D.straight

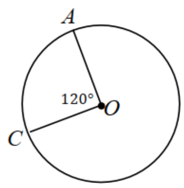
4. In the figure, $\overline{MN}$ is a diameter of $\odot O$. If $\widehat{MP} \cong \widehat{ON}$, which statement is true?



A. $\angle MOP \cong \angle QON$ B. $\angle POQ \cong \angle QON$ C. $\overline{PQ} \cong \overline{QN}$ D. $\widehat{MP} \cong \widehat{PQ}$

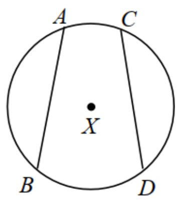
5. If $m\angle AOC = 120$, then what is $m\widehat{AC}$?

A.60 B.120 C.180 D.240



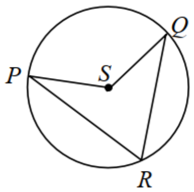
6.In $\odot X$, $\overline{AB} \cong \overline{CD}$, if $m\widehat{AB} = 130$, then what is $m\widehat{CD}$?

A.65 B.130 C.150 D.260



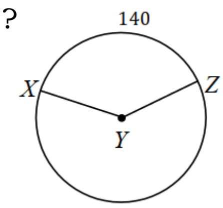
7.In $\odot S$ on the right, what is $\angle PRQ$ if $\angle PSQ = 160$?

A.80° B. 100° C. 160° D. 320°



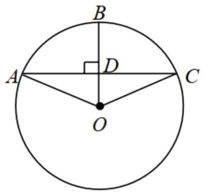
8. In the figure on the right, $m\widehat{XZ} = 140$, what is $\angle XYZ$?

A.70 B.140 C.280 D.300



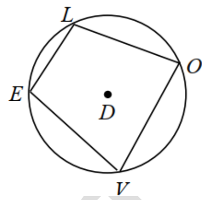
9. In $\odot O$, if OA=5cm, AD = 4 cm, what is OD?

A.2 cm B.3 cm C.4 cm D.5 cm



10. Quadrilateral LOVE is inscribed in $\odot D$. If m$\angle OLE$=80 and m$\angle LEV$=95, find m$\angle LOV$.

A.80 B.85 C.95 D.100



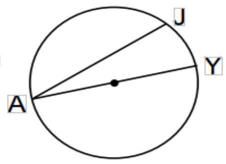
11. Solve for x if $\overparen{JY}$ = 40 and $\angle JAY = (2x - 4)°$

A.9

B.10

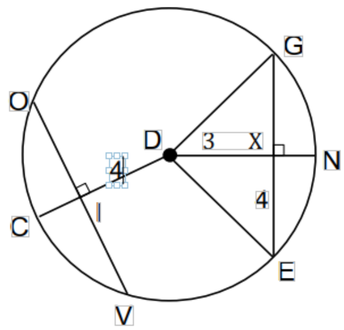
C.11

D.12



For numbers 12-15, use the figure and the given information.

In $\odot D$ on the right, $\overline{DX}$ = 3 cm, $\overline{XE}$ = 4 cm and $\overline{DI}$ = 3 cm



12. What is the measure of $\overline{DE}$?

A.3 cm B.4 cm C.5 cm D. 6 cm

13. What is the measure of $\overline{GX}$?

A.3 cm B.4 cm C.5 cm D.6 cm

14. What is the measure of $\overline{CI}$?

A. 1 cm B. 2 cm C. 3 cm D. 4 cm

15. What is the measure of $\overline{XN}$?

A. 2 cm B. 3 cm C. 4 cm D. 5 cm

References

BOOKS

Callanta,Melvin M et al .Mathematics Grade 10 Learner's Module.Rex Bookstore First Edition 2015

Oronce,Orlando M. and Mendoza,Marilyn.E-Math III (Geometry).Quezon City. REX Publishing Inc.,2007

LINKS

http://www.khanacademy.org/geometry/math/geometry/hs-geo-circles/hs/geo-inscribed-angles/v/inscribed-angles-exercise-example

http://www.mathisfun.com/geometry/circle-theorem.html

For inquiries or feedback, please write or call:

Department of Education – SDO La Union
Curriculum Implementation Division
Learning Resource Management Section
Flores St. Catbangen, San Fernando City La Union 2500
Telephone: (072) 607 - 8127
Telefax: (072) 205 - 0046
Email Address:
launion@deped.gov.ph
lrm.launion@deped.gov.ph