9

Mathematics

Quarter 3 – Week 6 -Module 6: Illustrating and Proving the Conditions for Similarity of Triangles

AIRs - LM

Mathematics 9
Quarter 3 – Week 6 -Module 6: Illustrating and Proving the Conditions for Similarity of Triangles
First Edition, 2021

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Target

This lesson is intended to help learners reason about geometry and in particular, triangle similar theorem. Similar triangles have the same shape, but not necessarily the same size. Corresponding angles are equal and corresponding sides are in the same ratio.

After going through this module, you are expected to attain the following objectives:

Learning Competency:

Illustrates similarity of figures.

Proves the conditions for similarity of triangles.

1.1 SAS Similarity Theorem
1.2 SSS Similarity Theorem
1.3 AA Similarity Theorem
1.4 Right Triangle Similarity Theorem
1.5 Special Right Triangle Theorems

Subtasks:

- describe a proportion
- illustrate similarity of polygons
- prove the conditions for
  - º similarity of triangles
    - 1.1 SAS Similarity Theorem
    - 1.2 SSS Similarity Theorem
    - 1.3 AA Similarity Theorem
  - º similarity of right triangles
    - 1.4 Right Triangle Similarity Theorem
    - 1.5 Special Right Triangle Theorem
      - 1.5.1. Pythagorean Theorem
      - 1.5.2. 45-45-90 Right Triangle Theorem
      - 1.5.3.30-60-90 Right Triangle Theorem

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper.

Pre-test

Directions: Choose the letter of your choice and write your answer in a separate sheet of paper.

1. If m:n=3:2, what is the correct order of the steps in determining m^2-n^2:m^2-$2n^2$?

   I. m=3k; n=2k   III $\frac{(3k)^2-(2k)^2}{(3k)^2-2(2k)^2}$

   II. m^2-n^2:m^2-$2n^{2=}$5:1   IV. $\frac{m}{3}=\frac{n}{2}=k$

   A. I,IV,III,II   B. IV,I,III,II   C. I,IV,II,III   D.I,III,II,I

2. The ratio of the volumes of two similar rectangular prisms is 125:64. What is the ratio of their base areas?

   A. 25:16   B. 25:4   C. 4:5   D. 5:4

3. The lengths of the sides of a triangle are 6cm, 10cm, and 13 cm. What kind of triangle is it?

   A. Acute   B. Obtuse   C. Regular   D. Right

4. What is the perimeter of a 30-60-90 triangle whose shorter leg is 5 inches long?

   A. $5\sqrt{3}cm$ B.$15+5\sqrt{3}cm$ C.$15+\sqrt{3}cm$ D. $10+5\sqrt{3}cm$

5. The ratio of the sides of the original triangle to its enlarged version is 1:3.The enlarged triangle is expected to have

   A. sides that are thrice as long as the original
   B. an area that is thrice as large as the original
   C. sides that are one-third the lengths of the original
   D. angles that are thrice the measurement of the original

6. A map is drawn to the scale of 1 cm: 150m. If the distance between towns A and B measures 8.5 cm on the map, determine the appropriate distance between these towns.

   A. 2175 m   B. 1725   C.1275cm   D. 1270m

7. The length of the shadow of your one-and-a-half meter high height is 2.4 meters at a certain time in the morning. How high is a tree in your backyard if the length of its shadow is 16 meters?

   A. 25.6 m   B. 10 m   C. 9.5 m   D. 6.7 m

8. The smallest square of the grid you made on your original picture is 6cm. If you enlarge the picture on a 15-cm grid, which of the following is not true?

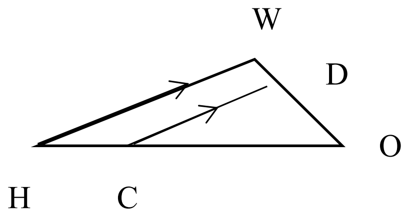
   I. The new picture is 250% larger than the original one.
   II. The new picture is two and a half time larger than the original one.
   III. The scale factor between the original and the enlarged picture 2:5.

   A. I only   B. I and II   C. III only   D. I,II and III

9. A document is 80% only of the size of the original document. If you were tasked to convert this document back to its original size, what copier enlargement settings will you use?

   A. 125%`   B. 120%   C. 110%   D. 100%

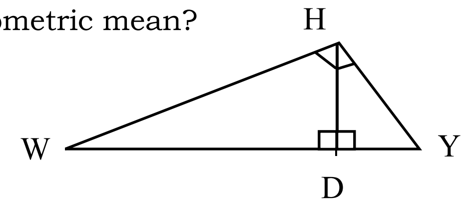
10. ΔCOD ~ ΔHOW Because $\overline{CD}$ | | $\overline{HW}$, which of the following is not true?



A. $\frac{OD}{DW} = \frac{OC}{CH} = \frac{CD}{HW}$ B . $\frac{OD}{OW} = \frac{OC}{OH} = \frac{CD}{HW}$

C. $\frac{DW}{OW} = \frac{CH}{OH} = \frac{HW-CD}{HW}$. D. $\frac{OD}{DW} = \frac{OC}{CH} = \frac{CD}{HW-CD}$

11. ΔWHY is a right triangle with ∠WHY as the right angle. $\overline{HD} \perp \overline{WY}$. Which of the following segments is a geometric mean?



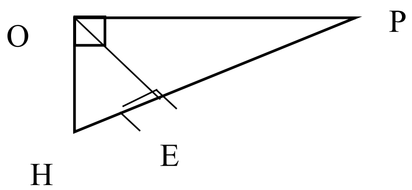
I. HD IV. DW

II. DY V. HW

III. HY VI. WY

A. II, IV, VI B. I, III, V C. I only D. All except VI

12. In the figure, there are three similar right triangle by Right Triangle Proportionality Theorem. Name the triangle that is missing in this statement: △HOP ~ _________ ΔOEP.

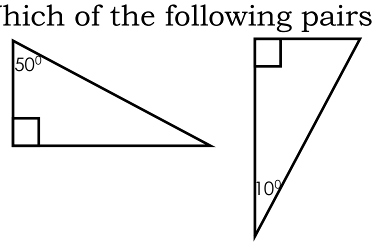


A. △HOE B. △OEH

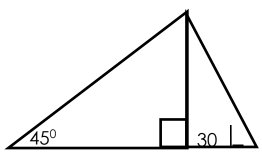
C. △HOP D. △HEO

13. Which of the following pairs of triangles cannot be proved similar?

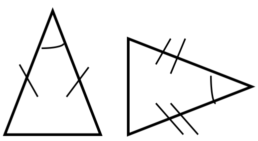
A.



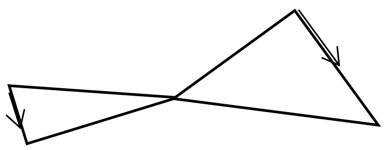
B.



C



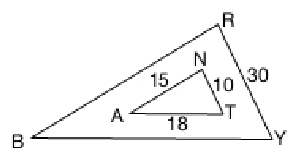
D



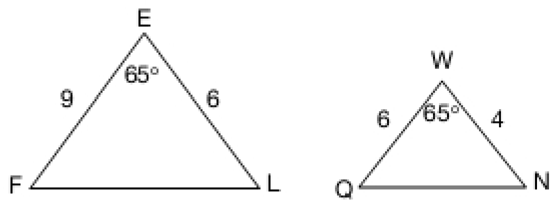
14. ΔBRY :ΔANT .Which ratio of sides gives the scale factor?

A. $\frac{NT}{AN}$ B. $\frac{NT}{RY}$

C. $\frac{AT}{BY}$ D. $\frac{NT}{AT}$



15. What similarity concept justifies that △FEL ~ △QWN?
A. Right Triangle Proportionality Theorem
B. Triangle Proportionality Theorem
C. SSS Similarity Theorem
D. SAS Similarity Theorem



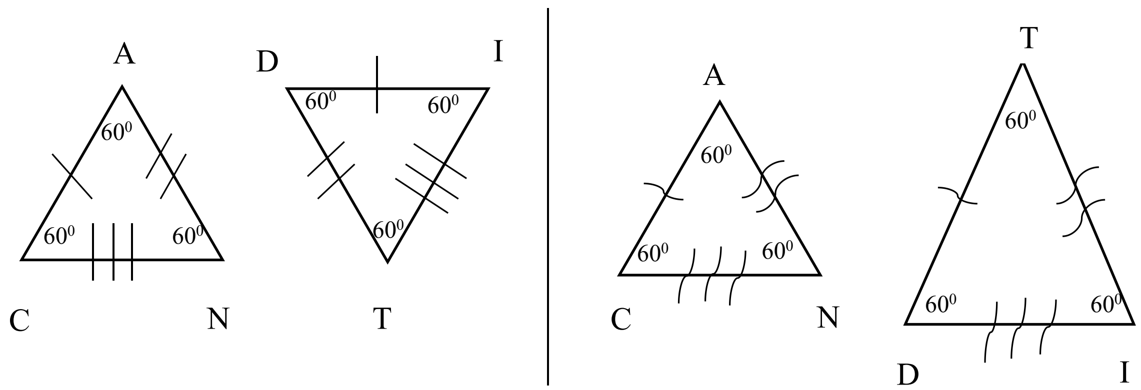
Lesson 6	**Illustrating and Proving the Condition for Similarity of Triangles**

Jumpstart

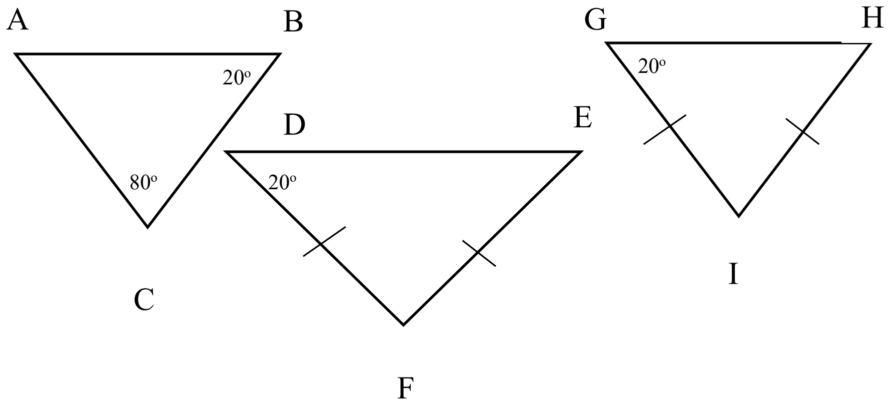
How do we create proportionality statements for triangles? How do we show triangles are similar? By definition, two triangles are similar if their corresponding angles are congruent and their corresponding sides are congruent. The symbol for similarity is ~.

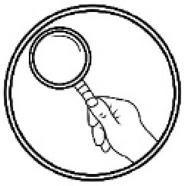
Activity 1: Resemblant

Which among the two figures are similar?
Which among the two figures are congruent?



Activity 2: Are these triangles mathematically similar?





Discover

Similar triangles have the same shape but not necessarily the same size. When triangles are similar, they have many of the same properties and characteristics. Triangle similarity theorems specify the conditions under which two triangles are similar, and they deal with the sides and angles of each triangle. Once a specific combination of angles and sides satisfy the theorems, then they are considered to be similar.

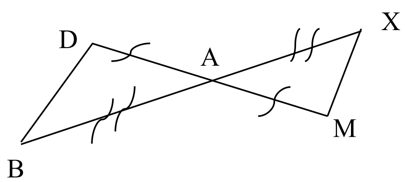
Triangle Similarity Illustration and Proof

SAS Similarity Theorem

Two triangles are similar if two of the sides of two triangles are proportional and the included angle or the angle between the sides is the same.

Example: If two sides of the triangles are 2 and 3 inches and those of another triangle are 4 and 6 inches, the sides are proportional, but the triangles may not be similar because the third sides could be any length. If the included angle is the same, then all the three sides of the triangles are proportional and the triangles are similar.

Given the figure, prove that $\triangle DAB \sim \triangle MAX$

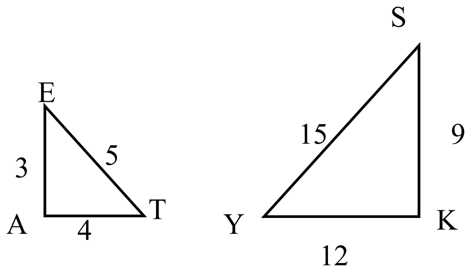


	Hints:	Statements	Reasons
1.	Write in a proportion the ratios of two corresponding proportional sides.	$\frac{DA}{MA} = \frac{XA}{BA}$	Corresponding sides are congruent
2.	Describe included angles of the proportional sides	$\angle A \cong \angle A$	Vertical angles are congruent
3.	Conclusion based on the simplified ratios	$\triangle$ DAB ~ $\triangle$ MAX	SAS Theorem

SSS Similarity Theorem

If all three sides of two triangles are the same, the triangles are not only similar, they are congruent or identical. For similar triangles, the three sides of two triangles only have to be proportional.

Example:
If one triangle has 3, 5 and 6 inches and a second triangle has sides of 9,15 and 18 inches, smaller triangle. The sides are in proportion to each other, and the triangles are similar.
Prove that $\triangle$EAT~ $\triangle$SKY.



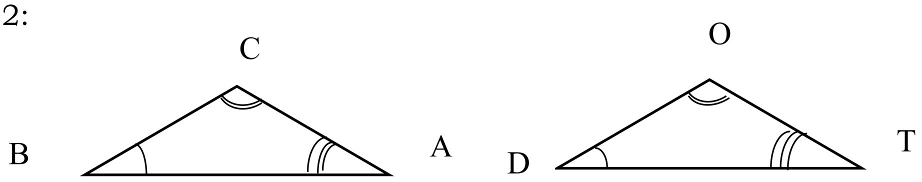
	Hints	Statements	Reasons
1.	Do all corresponding sides have uniform proportionality? Verify by substituting the lengths of the sides. Simplify afterwards	$\frac{3}{9} = \frac{4}{12} = \frac{5}{15}$; $\frac{1}{3} = \frac{1}{3} = \frac{1}{3} = \frac{1}{3}$	By Computation
2.	What is the conclusion based on the simplified ratios?	$\triangle$ EAT ~ $\triangle$ SKY.	SSS Similarity Theorem

AA Similarity Theorem

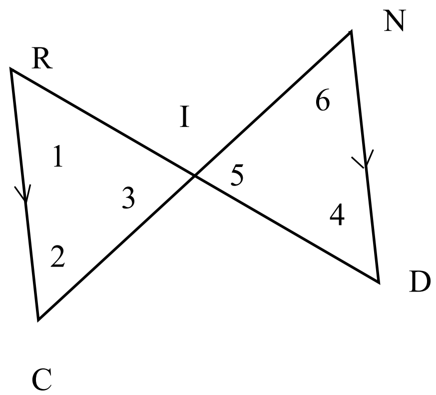
If two of the angles of two triangles are the same, the triangles are similar.
Remember: The sum of the angles of a triangle is 180°
Example 1:

If two of the angles are known, the third can be found by subtracting the two known angles from 180. If the three angles of the two triangles are the same, the triangles have the same shape and similar

Example 2:



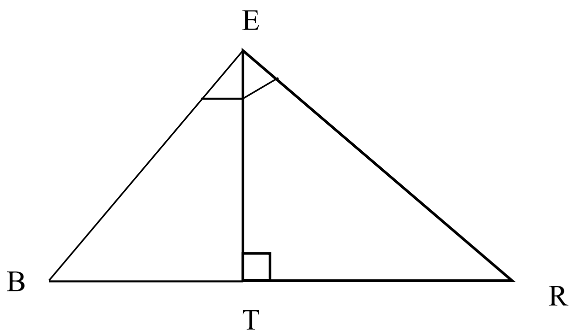
If: ∠B≅∠D; ∠C≅∠O;   Then: △BCA~ △DOT



Proof:
The figure shows that $\overline{RC}$ of △RIC and $\overline{DN}$ of △DIN are parallel. It follows that (∠1 & ∠4 and ∠2 & ∠6) determine by these parallel lines and their transversals ($\overline{DR}$ and $\overline{CN}$) are congruent. That is ∠1 & ∠4 and ∠2 & ∠6: By the vertical angle theorem, ∠3 &∠ 5. Since all their corresponding angles are congruent, then △ RIC ~ △DIN

Right Triangle Similarity Theorem (RTST)

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



Given
∠BER is a right triangle with ∠BER as the right angle and $\overline{MR}$ as the hypotenuse. $\overline{ET}$ is an altitude to the hypotenuse of △ BER

Prove
$\triangle BER \cong \triangle ETR \cong \triangle BTE$

Proof

Statements	Reasons
1.1 △BER is a right triangle with ∠BER as right angle and BR as the hypotenuse.	1. Given
1.2 $\overline{ET}$ is an altitude to the hypotenuse	

of $\triangle BER$	
2. $\overline{ET} \perp \overline{BR}$	2. Definition of altitude
3. $\angle$BTE and $\angle$ETR are rigth angles	3. Definition of perpendicular lines
4. $\boldsymbol{\angle BTE \cong \angle ETR \cong \angle BER}$	4. Definition of right angles
5. $\boldsymbol{\angle TBE \cong \angle EBR; \angle TRE \cong \angle ERB}$	5. Reflexive property
6. $\triangle BTE \sim \triangle BER; \triangle BER \sim \triangle ETR$	6. AA Similarity Theorem
7.$\boldsymbol{\triangle BER \cong \triangle ETR \cong \triangle BTE}$	7. Transitive property

Special properties of Right Triangles

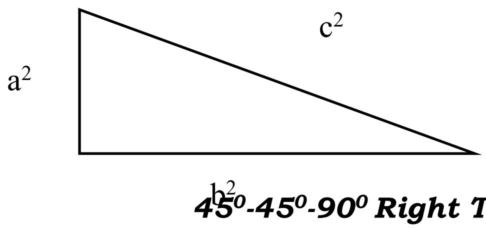
When the altitude is drawn to the hypotenuse of a right triangle,

1. The length of the altitude is the geometric mean between the segments of the hypotenuse; and;
2. Each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Similar triangles fit into each other, can have parallel sides and scale from one to the other. Determining whether two triangles are similar using the triangle similarity theorems is important to solve geometrical problems.

Pythagorean Theorem

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs. To illustrate:



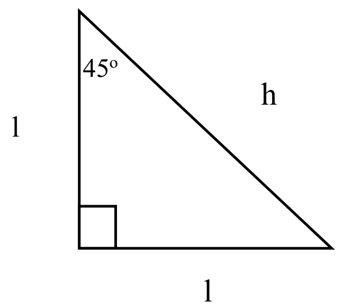
Pythagorean Formula

$c^2 = a^2 + b^2$

45^0-45^0-90^0 Right Triangle Theorem

In a 45^0-45^0-90^0 right triangle:

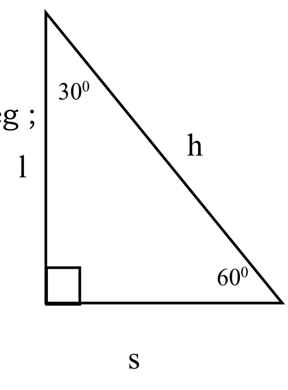
- each leg is $\frac{\sqrt{2}}{2}$ times the hypotenuse; and
- the hypotenuse is $\sqrt{2}$ times each leg l



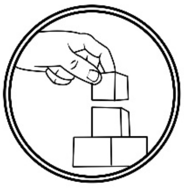
30^0-60^0-90^0 Right Triangle Theorem

In a 30^0-60^0-90^0 right triangle:

- the shorter leg is $\frac{1}{2}$ the hypotenuse h or $\frac{\sqrt{3}}{3}$ times the longer leg ;



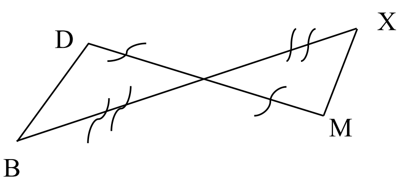
• the longer leg l is $\sqrt{3}$ times the shorter legs; and
• the hypotenuse h is twice the shorter leg



Explore

Work on the following enrichment activities for you to apply your understanding on this lesson.

Activity 3. Identify Me!
State the similarity type on each /pair of triangles.

1.


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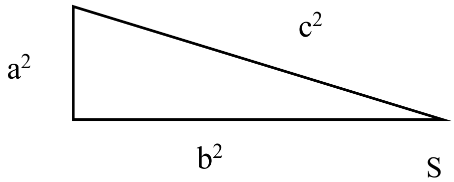
2.


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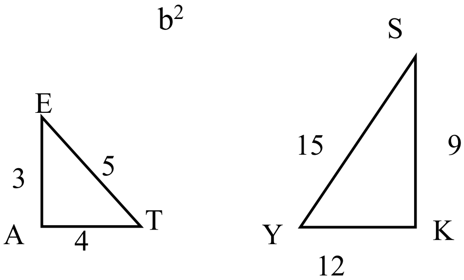
3.


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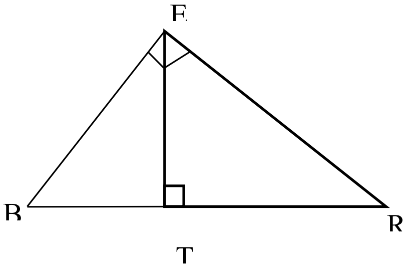

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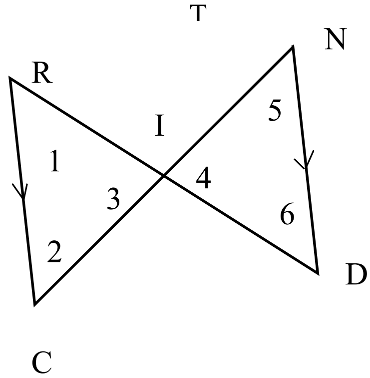
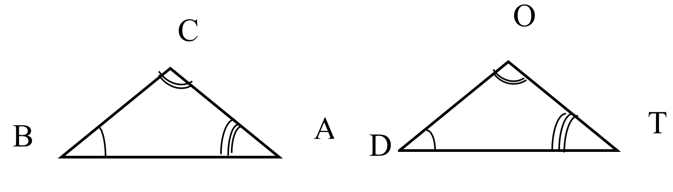
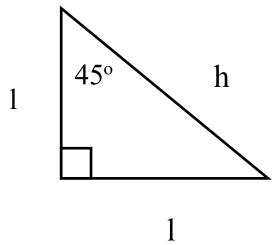
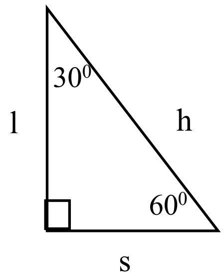
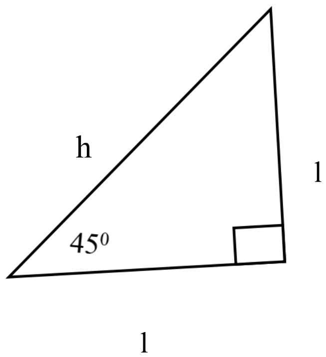
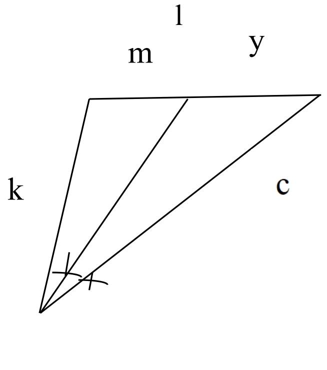
5.


6. 

7. 

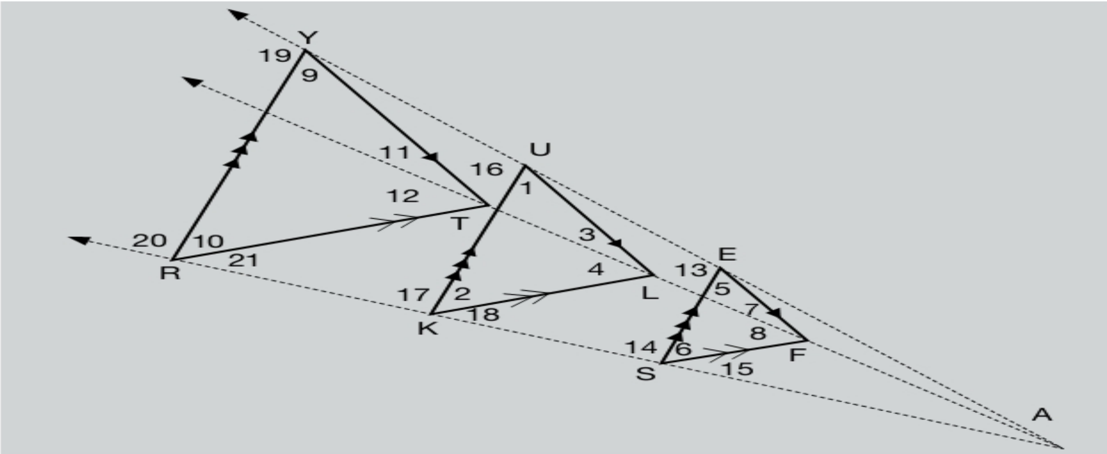
8. 

9. 

10. 

$$\frac{m}{y} = \frac{k}{c}$$

Activity 4. Dilation: Reducing or Enlarging Triangles



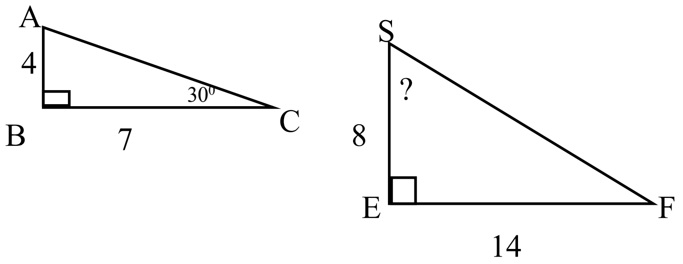
Triangles KUL and RYT are similar images of the original triangle SEF through dilation, the extending of rays that begin at a common endpoint A. The point A is called the center of dilation. Give justifications to the statements of its proof.

	Statements	Reasons
1.	• $\overline{RY}$II$\overline{KU}$II$\overline{SE}$ • $\overline{YT}$II$\overline{UL}$II$\overline{EF}$ • $\overline{RT}$II$\overline{KL}$II$\overline{SF}$	___________
2.	• $\angle 20 \cong \angle 17 \cong \angle 14$ *and* $\angle 19 \cong \angle 16 \cong \angle 13$ • $\angle TYU \cong \angle LUE \cong \angle FEA$ • $\angle 21 \cong \angle 18 \cong \angle 15 and \angle 12 \cong \angle 4 \cong \angle 8$	___________ angles are congruent.
3. 1	• $m\angle 20 + m\angle 21 + m\angle 10 = 180$ • $m\angle 17 + m\angle 18 + m\angle 2 = 180$ • $m\angle 14 + m\angle 15 + m\angle 6 = 180$	___________ on a Straight Line Theorem
3. 2	• $m\angle 19 + m\angle TYU + m\angle 9 = 180$ • $m\angle 16 + m\angle LUE + m\angle 1 = 180$ • $m\angle 13 + m\angle FEA + m\angle 5 = 180$	
4.	• $m\angle 20 + m\angle 21 + m\angle 2 = 180$ • $m\angle 20 + m\angle 21 + m\angle 6 = 180$	Substitution
	• $m\angle 19 + m\angle TYU + m\angle 1 = 180$ • $m\angle 19 + m\angle TYU + m\angle 5 = 180$	
5.	• $m\angle 20 + m\angle 21 + m\angle 10 = m\angle 20 + m\angle 21 + m\angle 2 = m\angle 20 + m\angle 21 + m\angle 6$	___________ Property of Equality
	• $m\angle 19 + m\angle TYU + m\angle 9 = m\angle 19 + m\angle TYU + m\angle 1 + m\angle 19 + m\angle TYU + m\angle 5$	
6	• $m\angle 10 = m\angle 2 = m\angle 6$	___________ Property of Equality
	• $m\angle 9 = m\angle 1 = m\angle 5$	
	• $\triangle RYT \sim \triangle KUL \sim \triangle SEF$	___________ Similarity

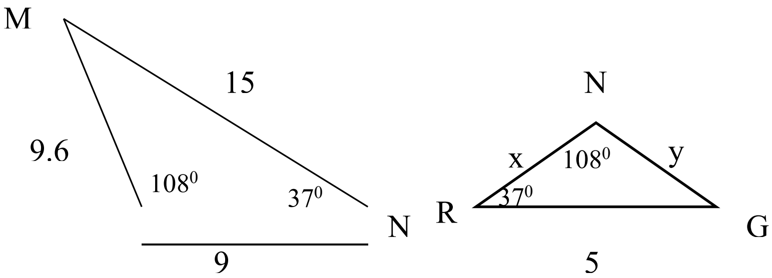
		Theorem

Deepen

Activity 5. Determine the measurement of the angle S using similarity theorem.



Activity 6. Determine the measurement of the sides x and y.



Gauge

Directions: Encircle the letter of the correct answer.

1. A flagpole cast a shadow of 25 ft at the same time the shadow of a person 6 feet tall is 2 ft long. How tall is the flagpole?
   A. 150ft B. 100ft C. 84 ft D.76 ft
2. The lengths of the sides of a triangle are 4cm, 5cm, and 6 cm. What kind of triangle is it?
   A. Acute B. Obtuse C. Regular D. Right
3. The ratio of the volumes of two similar rectangular prisms is 125:512. What

is the ratio of their base areas?

A. 25:64 B. 25:41 C. 5:8 D. 5:4

4. What is the perimeter of a 30^0-60^0-90^0 right triangle whose shorter leg is 5 inches long?

A. $5\sqrt{3}$cm B.$15+5\sqrt{3}$ cm C. $15+\sqrt{3}$ cm D. $10+5\sqrt{3}$ cm

5. The ratio of the sides of the original triangle to its enlarged version is 3:9.The enlarged triangle is expected to have

A. sides that are thrice as long as the original
B. an area that is thrice as large as the original
C. sides that are one-third the lengths of the original
D. angles that are thrice the measurement of the original

6. A map is drawn to the scale of 1 cm: 100m. If the distance between towns A and B measures 6.5 cm on the map, determine the appropriate distance between these towns.

A. 750 m B. 700 m C.650 m D. 550 m

7. The length of the shadow of your three meter high height is 4.8 meters at a certain time in the morning. How high is a tree in your backyard if the length of its shadow is 16 meters?

A. 25.6 m B. 10 m C. 8.4m D. 5.6 m

8. The smallest square of the grid you made on your original picture is 6cm. If you enlarge the picture on a 15-cm grid, which of the following is not true?

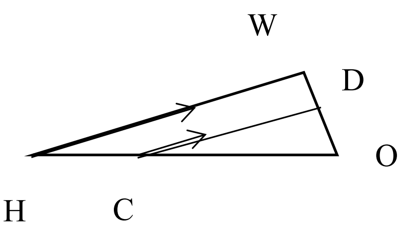
I. The new picture is 250% larger than the original one.
II. The new picture is two and a half time larger than the original one.
III. The scale factor between the original and the enlarged picture 2:5.

A. I only B. I and II C. III only D. I,II and III

9. A document is 75% only of the size of the original document. If you were tasked to convert this document back to its original size, what copier enlargement settings will you use?

A. 200% B.150% C. 133% D. 125%`

10. △COD ~ △HOW Because CD || HW, which of the following is not true?



A. $\frac{OD}{DW}=\frac{OC}{CH}=\frac{CD}{HW}$ B .$\frac{DW}{OW}=\frac{CH}{OH}=\frac{HW-CD}{HW}$

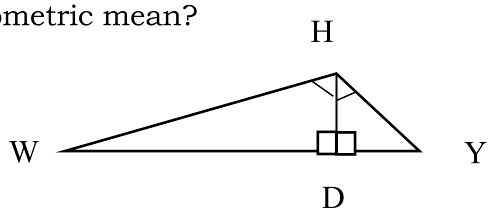
C.$\frac{OD}{OW}=\frac{OC}{OH}=\frac{CD}{HW}$ D. $\frac{OD}{DW}=\frac{OC}{CH}=\frac{CD}{HW-CD}$

11. ΔWHY is a right triangle with ∠WHY as the right angle. HD ⊥ WY. Which of the following segments is a geometric mean?

I. HD IV. DW

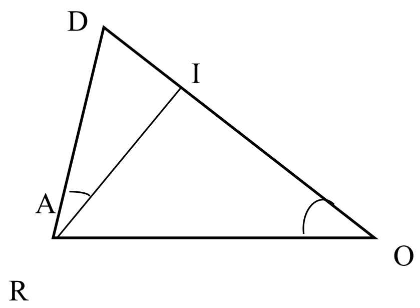
II. DY V. HW

III. HY VI. WY



A. II, IV, VI B. I, III, V C. I only D. All except VI

12. Name the similar triangles in the figure.



A. $\triangle$ADI~$\triangle$ODR B. $\triangle$ADI~$\triangle$OIA

C. $\triangle$AID~$\triangle$OIA D. $\triangle$AIO~$\triangle$ODR

13. A right and equilateral triangles are_________ similar?

A. always B. either C. never D. sometimes

14. Two equilateral triangles are ___________similar?

A. always B. either C. never D. so: etimes

15. In 30^0-60^0-90^0 right triangle, how can you find the shorter leg?

A. Double the long leg

B. Twice the hypotenuse

C. Divide the long leg by the $\sqrt{3}$

D. Double the length of the longer leg

References

https://www.varsitytutors.com/hotmath/hotmath_help/topics/similar-triangles

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Using Similar Triangle retrieved from onlinemathlearning.com