9

Mathematics

Quarter 1 - Module 5:
Quadratic Inequalities

AIRs - LM

LU_Q1_Mathematics9_Module5

Government Property
NOT FOR SALE

Mathematics 9
Quarter 1 - Module 5: Quadratic Inequalities
Alternative Delivery Mode
Second Edition, 2021

Copyright © 2021
La Union Schools Division
Region I

All rights reserved. No part of this module may be reproduced in any form without written permission from the copyright owners.

Development Team of the Module

Authors: Teresa A. Villanueva, Edel Jayne E. Villanueva, and Jon-Jon V. Orpilla
Editor: SDO La Union, Learning Resource Quality Assurance Team
Content Reviewers: Philip R. Navarette and Jocelyn G. Lopez
Language Reviewers: Teresa A. Villanueva and Cleofe M. Lacbao
Illustrator: Ernesto F. Ramos Jr. and Christian Bautista
Design and Layout: Dana Kate J. Pulido

Management Team:

Atty. Donato D. Balderas Jr.
Schools Division Superintendent
Vivian Luz S. Pagatpatan, Ph D
Assistant Schools Division Superintendent
German E. Flora, Ph D, *CID Chief*
Virgilio C. Boado, Ph D, *EPS in Charge of LRMS*
Erlinda M. Dela Peňa, Ph D, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

Printed in the Philippines by: ___________________________

Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

Ready to Print

13. A garment store sells about 40 t-shirts per week at a price of ₱ 100 each. For each ₱10 decrease in price, the sales lady found out that 5 more t-shirts per week were sold. What price produces the maximum revenue?
   A. ₱65
   B. ₱80
   C. ₱90
   D. ₱100
14. From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height S(t) of the ball above the building after t seconds is given by the function $S(t) = 80t - 16t^2$. What is the maximum height will the object reach from the top of the building?
   A. 75 ft
   B. 100 ft
   C. 88 ft
   D. 116 ft
15. A ball is tosses upward from the ground. Its height in feet above the ground after t seconds is given by the function $g(t) = -16t^2 + 22t$. Find the maximum height that the ball will reach.
   A. 7.56 ft
   B. 8.14 ft
   C. 6.62 ft
   D. 5.98 ft

Jumpstart

Activity 1: Which are Not Quadratic Equations?

Directions: Use the mathematical sentences below to answer the questions that follow.

$x^2 + 9x + 20 = 0$	$2t^2 < 21 - 9t$	$r^2 + 10r \leq -16$	$3w^2 + 12w \geq 0$
$2s^2 + 7s + 5 > 0$	$15 - 6n^2 = 10$	$4x^2 - 25 = 0$	$m^2 = 6m - 7$

Ready to Print

A. Which of the given mathematical sentences are quadratic equations?

Quadratic Equation	Not Quadratic Equation

B. How do you describe quadratic equation?

____

____

C. How would you describe those mathematical sentences which are not quadratic equations?

____

____

D. How are they different from those mathematical sentences which are not quadratic equations?

____

____

Discover

Quadratic inequalities in one variable are inequalities that contain a polynomial of degree 2 and can be written in any of the following forms:

$$ax^2 + bx + c > 0$$
$$ax^2 + bx + c < 0$$
$$ax^2 + bx + c \geq 0$$
$$ax^2 + bx + c \leq 0$$
$$\textit{where } a, b \textit{ and } c \textit{ are real number and } a \neq 0$$

A quadratic inequality uses an inequality symbol instead of an equal sign.

Symbol	***Words***	***Example***
$>$	greater than	$x^2 + 3x > 2$
$<$	less than	$7x^2 < 28$
$\geq$	greater than or equal to	$5 \geq x^2 - x$
$\leq$	less than or equal to	$2y^2 + 1 \leq 7y$

Ready to Print

In Activity1, those mathematical sentences which are not quadratic equations are considered quadratic inequalities.

Other Examples:

1.) $x^2 - x - 6 < 0$ → quadratic inequality
2.) $t^2 + 4t \leq 10$ → quadratic inequality
3.) $x^3 + 4 \geq 3x^2 + x$ → not quadratic inequality (degree 3)
4.) $8 = x^2 - 6x$ → not quadratic inequality (equal sign)

Quadratic inequalities illustrated in real life;

The city government is planning to construct a new children's playground. They plan to fence a rectangular ground using one of the walls of a building. The length of the new playground is 15 m longer than its width and its area is greater than the old playground with 2,200 m².

.

We can represent the situation into quadratic inequality as;

$$Area\ of\ rectangle = length\ x\ width$$

$$length = width + 15m \qquad \text{or} \qquad l = w + 15$$

$$\boldsymbol{w(w + 15) \geq 2,200} \quad \text{or} \quad \boldsymbol{w^2 + 15w \geq 2,200}$$

Solving Quadratic Inequalities in One Variable

Solving quadratic inequalities means finding the roots of its corresponding equality. The points corresponding to the roots of the equality, when plotted on the number line, separate the line into two or three intervals. An interval is a part of the solution of the inequality if a number in that interval makes the inequality true.

Illustrative Example 1:

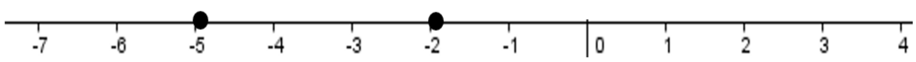
Let us now solve the quadratic inequality $x^2 + 7x + 10 > 0$.

Solution:

a. Solve the corresponding equation of $x^2 + 7x + 10 > 0$, which is $x^2 + 7x + 10 = 0$ by factoring.

$(x + 2)(x + 5) = 0$ *factors of the quadratic equation*
$x = -2$ and $x = -5$ *get the resulting equation*

b. Plot the points corresponding to -2 and -5 on the number line.



c. Determine the three intervals

$x < -5 \qquad -5 < x < -2 \qquad x > -2$

Ready to Print

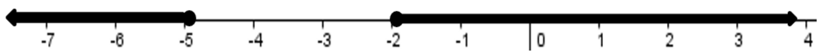
d. Determine the solution set using the three-point test.

For $x < -5$	For $-5 < x < -2$	For $x > -2$
Choose a value of x which is less than -5 Let $x = -6$	Choose a value of x which is less than -2 but greater than -5 Let $x = -4$	Choose a value of x which is greater than -2 Let $x = -1$
$x^2 + 7x + 10 > 0$ $(-6)^2 + 7(-6) + 10 > 0$ $36 - 42 + 10 > 0$ $4 > 0$ True	$x^2 + 7x + 10 > 0$ $(-4)^2 + 7(-4) + 10 > 0$ $16 - 28 + 10 > 0$ $-2 > 0$ False	$x^2 + 7x + 10 > 0$ $(-1)^2 + 7(-1) + 10 > 0$ $1 - 7 + 10 > 0$ $4 > 0$ True

e. Test whether the points $x = -2$ and $x = -5$ satisfy the inequality

$x^2 + 7x + 10 > 0$ $(-2)^2 + 7(-2) + 10 > 0$ $4 - 14 + 10 > 0$ $0 > 0$ False	$x^2 + 7x + 10 > 0$ $(-5)^2 + 7(-5) + 10 > 0$ $25 - 35 + 10 > 0$ $0 > 0$ False

f. Therefore, the quadratic inequality is true for any value of x in the interval $x < -5$ or solution set $x > -2$ and these intervals exclude -2 and -5.



g. The solution set of the inequality is $\{x : \boldsymbol{x} < \boldsymbol{-5}\ or\ \boldsymbol{x} > \boldsymbol{-2}\}$.

Note that solid circle is used when the roots satisfy the quadratic inequality. If otherwise, use a hollow circle.

Solving Quadratic Inequalities in Two Variables

To determine the solution set of quadratic inequalities in two variables, use the graphical method. First, write the corresponding equation of the given quadratic inequality and then show the graph of the resulting parabola. From the graph, identify the shaded region which will represent the solution of the quadratic inequality.

Example: Find the solution set of $y \le x^2 + 3x + 2$

Solution:

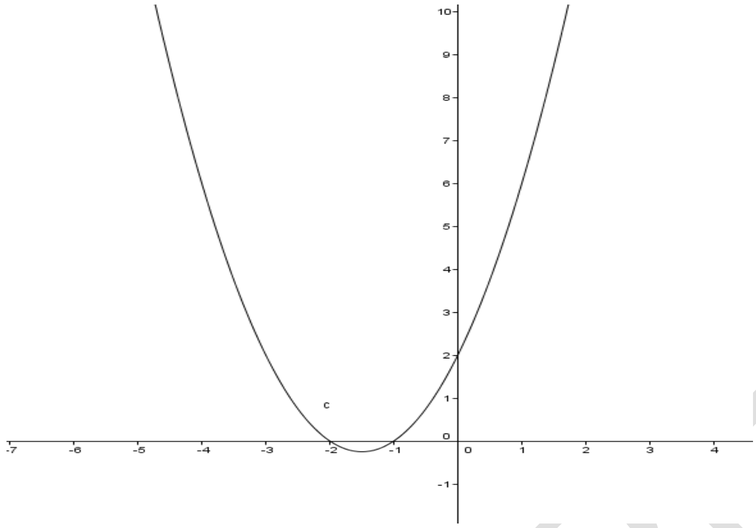
1. Write the quadratic inequality to its corresponding quadratic equation.
   $y = x^2 + 3x + 2$

2. Construct table of values.

x	-4	-3	-2	-1	0	1
y	6	2	0	0	2	6

Ready to Print

3. Use the table of points, plot the points and show the graph of the quadratic equation.



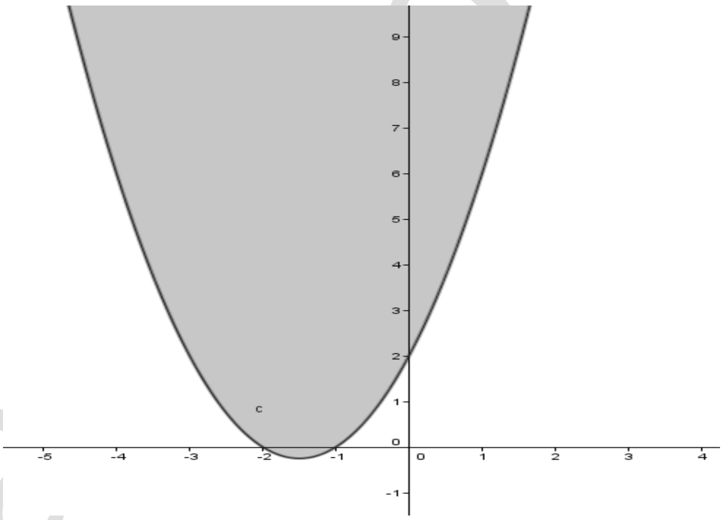
4. Test the points to determine the boundary and the shaded region. Choose at least 2 points along the parabola, within the parabola and outside the parabola.

Use points (-3,2) & (-2,0) along the parabola	**For (-3, 2),** $\boldsymbol{x = -3 \quad y = 2}$ $y \leq x^2 + 3x + 2$ $2 \leq (-3)^2 + 3(-3) + 2$ $2 \leq 9 - 9 + 2$ $2 \leq 2$ True **For (-2, 0),** $\boldsymbol{x = -2 \quad y = 0}$ $y \leq x^2 + 3x + 2$ $0 \leq (-2)^2 + 3(-2) + 2$ $0 \leq 4 - 6 + 2$ $0 \leq 0$ True *Therefore (-3,2) and (-2,0) are part of the solution set. Since the points are along the parabola, use solid line for the parabola.*
Use points (-1,2) and (-2,1) which are inside the parabola	**For (-1, 2),** $\boldsymbol{x = -1 \quad y = 2}$ $y \leq x^2 + 3x + 2$ $2 \leq (-1)^2 + 3(-1) + 2$ $2 \leq 1 - 3 + 2$ $2 \leq 0$ False **For (-2, 1),** $\boldsymbol{x = -2 \quad y = 1}$ $y \leq x^2 + 3x + 2$ $1 \leq (-2)^2 + 3(-2) + 2$ $1 \leq 4 - 6 + 2$ $1 \leq 0$ False

Ready to Print

	These points in this region do not satisfy the inequality. Therefore, this region is not part of the solution set of the inequality.
Use points (-3,1) and (1,2) which are outside the parabola	**For (-3, 1),** $\boldsymbol{x = -3 \quad y = 1}$ $y \leq x^2 + 3x + 2$ $1 \leq (-3)^2 + 3(-3) + 2$ $1 \leq 9 - 9 + 2$ $1 \leq 2$ True **For (1, 2),** $\boldsymbol{x = 1 \quad y = 2}$ $2 \leq (1)^2 + 3(1) + 2$ $1 \leq 1 + 3 + 2$ $1 \leq 6$ True *These points in this region satisfy the inequality. Therefore, this region is part of the solution set of the inequality.*

5. The solution set of the inequality is the set of points on the shaded region of the graph. To check, choose any point on the shaded region. If it satisfies the inequality, then it is a solution.



Broken line is used in the graph of the parabola when the points along the parabola do not satisfy the quadratic inequality. If otherwise, solid line is used.

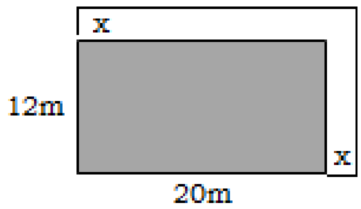
Solving Problems Quadratic Inequalities

Now that you know how to solve quadratic inequalities, you can proceed to applying the concept in real – life problems. There are a lot of situations that can be represented by quadratic inequalities. Consider the situation below.

Mr. Ramon Magsaysay wants to expand his lot for his garden. The lot measures 12m by 20m and he wants to expand the size by adding an equal distance

Ready to Print

to two of its side as shown below. However, the new lot with expansion should not exceed 345 m², what ranges of distance in meters can Mr. Magsaysay add in his lot?



Let us analyze.

The phrases "*longer than*", "*smaller than*" , "*should not exceed*", "*less than*", "*more than*", "*range of distance*", "*range of cost*" and the like are indicators of inequalities.

In solving problems involving quadratic inequalities, we need to read through the entire problem. Highlight the important information and key words that we need to solve the problem. And then identify our variables.

Solution:

Let x be the additional constant width to be added to each side;

$(x + 12)\,m$ will be the width of the expanded rectangle

$(x + 20)\,m$ will be the length

If the area should not exceed $345m^2$, then the inequality that will represent the situation will be: $(x + 12)(x + 20) \leq 345$

Solving for the possible value of x,

$$(x + 12)(x + 20) \leq 345$$
$$x^2 + 32x + 240 \leq 345$$
$$x^2 + 32x + 240 - 345 \leq 0$$
$$x^2 + 32x - 105 \leq 0$$

Then, solve for the roots of the equivalent quadratic equation,

$$x^2 + 32x - 105 \leq 0$$
$$x^2 + 32x - 105 = 0$$
$$(x + 35)(x - 3) = 0$$
$$x + 35 = 0 \quad \text{and} \quad x - 3 = 0$$
$$x = -35 \qquad\qquad x = 3$$

Since the value of x refers to the length of expansion, it cannot be negative. Thus the range of distance that Mr. Ramon Magsaysay can add on both sides is $0m \leq x \leq 3m$ in order not to exceed $345m^2$.

Ready to Print

Another Example:

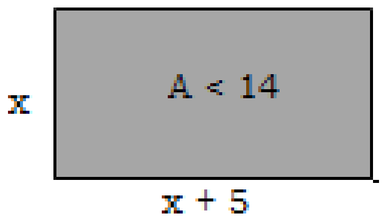
Mang Carding wants to build a rectangular garden enclosed with a fence. He wants the length of the garden to be 5 meters longer than the width. If the area of the garden must be less than 14 square meters, what are the possible dimensions of Mang Carding's garden?

Let the following;

x = the width of the garden

x + 5 = length of the garden

less than 14 → area of the garden



Since Area of a rectangles = lw,
We can substitute

$A = lw$

$(x + 5)(x) < 14$

$x^2 + 5x - 14 < 0$

Then, solve for the roots of the equivalent quadratic equation,

$x^2 + 5x - 14 < 0$

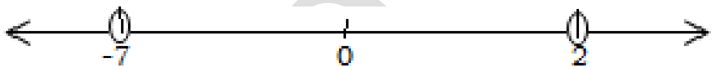
$x^2 + 5x - 14 = 0$

$(x + 7)(x - 2) = 0$

$x + 7 = 0$ and $x - 2 = 0$

$x = -7$ $\quad$ $x = 2$

Plot the points



Test the points

For $x < -7$	For $-7 < x < 2$	For $x < 2$
Choose a value of x which is less than -7 Let $x = -8$	Choose a value of x which is less than 2 but greater than -7 Let $x = 0$	Choose a value of x which greater than 2 Let $x = 3$
$x^2 + 5x < 14$ $(-8)^2 + 5(-8) < 14$ $64 - 40 < 14$ $24 < 14$ *False*	$x^2 + 5x < 14$ $(0)^2 + 5(0) < 14$ $0 + 0 < 14$ $0 < 14$ *True*	$x^2 + 5x < 14$ $(3)^2 + 5(3) < 14$ $9 + 15 < 14$ $24 < 14$ *False*

Thus, the solution is $-7 < x < 2$

Since we do not use negative numbers, because there are no such negative measurements. We will be using numbers that are greater than 0 but less than 2.

If we let the width(x) = 1

$(x + 5)(x) < 14$

$(1 + 5)(1) < 14$

$(6)(1) < 14$

$6 < 14$, TRUE

Therefore, 6 is the length

If we let the width(x) = 1.5

$(x + 5)(x) < 14$

$(1.5 + 5)(1.5) < 14$

$(6.5)(1.5) < 14$

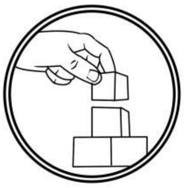
$9.75 < 14$, TRUE

Therefore, 6.5 is the length

We can conclude that the possible dimensions of Mang Carding's garden will be 1m by 6m or 1.5m by 6.5m.

Now that you have learned how to solve real – life problems that involves quadratic inequalities, you can proceed to the next activities

Ready to Print



Explore

Activity 2: Quadratic Inequality or Not

Direction: Write *quadratic inequality* if the mathematical sentence illustrates quadratic inequality, otherwise write *not quadratic inequality*.

1.) $x^2 - 6x - 16 \leq 0$ → ____________________
2.) $x^2 + 4 > 0$ → ____________________
3.) $x^2 - 3x + 2 = 0$ → ____________________
4.) $6x^2 - 7x + < 0$ → ____________________
5.) $2(x^2 + 1) \geq 5x$ → ____________________
6.) $y^2 - 4y > -3$ → ____________________
7.) $x^3 - x > 12$ → ____________________
8.) $2x^2 < 9x + 5$ → ____________________
9.) $-x^2 + 4 < 0$ → ____________________
10.) $-x^2 + 3x - 2 = 0$ → ____________________

Activity 3: Fill Me In

Direction: Fill in the table with what is being asked.

Quadratic Inequality	$x^2 + x - 6 > 0$		
Quadratic Equation	____________________		
Roots of corresponding equation	$x =$ _____	$x =$ _____	
Intervals	$x <$ _____	_____ $< x <$ _____	$x >$ _____
Assigned Value for x	$x =$ _____	$x =$ _____	$x =$ _____
Test the three points	$x^2 + x - 6 > 0$ $(\_\_\_)^2 + (\_\_\_\_) - 6 > 0$ $(\_\_\_\_) + (\_\_\_\_) - 6 > 0$ _____ > 0	$x^2 + x - 6 > 0$ $(\_\_\_)^2 + (\_\_\_\_) - 6 > 0$ $(\_\_\_\_) + (\_\_\_\_) - 6 > 0$ _____ > 0	$x^2 + x - 6 > 0$ $(\_\_\_)^2 + (\_\_\_\_) - 6 > 0$ $(\_\_\_\_) + (\_\_\_\_) - 6 > 0$ _____ > 0
True or False	____________	____________	____________
Solution set	____________________________		

Ready to Print

Activity 4: Am I a Solution?

Direction: Determine whether the following points are solutions of the inequality $y > x^2 + 5x + 6$. Write ***S*** if solution and ***NS*** if otherwise.

1. A (-1, 2)
2. B (-2, 1)
3. C (-5, 2)
4. D (-4, 2)
5. E. (3, 3)
6. F (1, 12)
7. G (0, 1)
8. H (-2, -2)
9. I (-2 5)
10. J (-3, 4)

Deepen

Activity 5: Ready, Solution Set, Go!

Direction: On a separate answer sheet, solve the following quadratic inequalities.

1. $y^2 + y - 2 < 0$
2. $x^2 + 2x - 8 > 0$
3. $s^2 - 9 < 0$
4. $t^2 + 5t + 6 > 0$
5. $x^2 + 2x - 10 < 0$

Activity 6: Graph My Solution!

Direction: On a separate answer sheet, find the solution set of each of the following quadratic inequalities then graph.

1. $y < x^2 - 2x - 8$
2. $y > x^2 - 3x$
4. $y \leq x^2 + 2x$
5. $y > x^2 - 6x + 5$
5. $y > x^2 - 2x - 3$

Activity 7: Let Me Solve It!

Direction: Read and analyze the following problems. Solve by applying the concepts of quadratic inequalities. Write your answer on a separate sheet of paper.

1. The area of a rectangle is 20 square inches. The length is 4 more than three times the width. Find the length and the width of the rectangle.

2. A rectangular box is completely filled with dice. Each die has a volume of 1 cm^3. The length of the box is 3 cm greater than ots width and its height is 5 cm. suppose the box holds at most 140 dice. What are the possible dimensions of the box?

Ready to Print