Senior High School

General Mathematics

Module 6:

Problem Solving Involving Exponential Functions, Equations, and Inequalities

AIRs - LM

General Mathematics
Module 6: Problem Solving Involving Exponential Functions, Equations, and Inequalities
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General Mathematics
Module 6:
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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Based on your lessons during Junior High School Mathematics, exponents are defined as the *number placed on the upper right of a certain number, letter/variable or an expression indicating how many times it will be multiplied by itself.* (e.g. $4^{3}, 2x^{5}, (x+3)^{2},\ x^{2}+y^{6}$).

In the previous modules, you dealt with **exponential functions.** These are functions that can be written as $\boldsymbol{f(x) = b^{x}}$ or $\boldsymbol{y = b^{x}}$, where **b** is the ***base*** ($b \neq 1$ and $b > 0$), and **x** is the ***exponent*** (any real number). Notice that the exponent is not a number but a variable. It will be a number if it will be substituted with a certain value/number. Let say, you have $f(x) = 2^{x}$ and you will take 3 as the value of x, then the exponential function will become $f(3) = 2^{3}$.

This module will provide you with information and activities that will help you understand more about Exponential Functions.

After going through this module, you are expected to:

1. represent an exponential function through its: (a) table of values, (b) graph, and (c) equation **(M11GM-If-2),**
2. find the domain and range of an exponential function **(M11GM-If-3),**
3. determine the intercepts, zeroes, and asymptotes of an exponential function **(M11GM-If-4**); and
4. solve problems involving exponential functions, equations, and inequalities **(M11GM-Ig-2).**

Learning objectives:

1. identify the equation of an exponential function
2. evaluate an exponential function
3. describe the graph of an exponential function
4. determine the domain and range of an exponential function
5. identify the intercepts, zeroes, and asymptotes of exponential functions
6. show step-by-step process on solving exponential functions

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper.

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

_____1. Given the function, $\boldsymbol{f(x) = 5^x + 1}$, find the value of $\boldsymbol{f(2)}$.

A. $f(2) = 6$ B. $f(2) = 26$

C. $f(2) = -6$ D. $f(2) = -26$

_____2. Identify the correct table of values for the exponential function, $\boldsymbol{f(x) = 3^x}$.

A.

x	-1	0	1	2
f(x)	$\frac{1}{3}$	1	27	243

B.

x	-1	0	1	2
f(x)	$\frac{1}{3}$	3	27	243

C.

x	-1	0	1	2
f(x)	$\frac{1}{3}$	0	3	9

D.

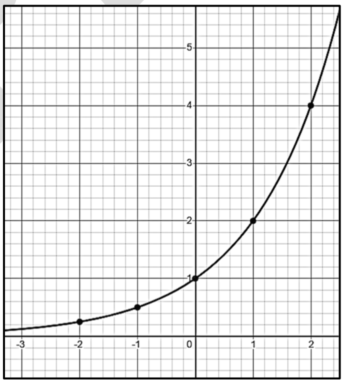
x	-1	0	1	2
f(x)	$\frac{1}{3}$	1	3	9

_____3. Which of the following is an exponential function?

A. $f(x) = 2x - x^2$ B. $x(t) = 2x^2 - x - 3$

C. $g(x) = 2^3 + 5^x$ D. $v(t) = \left(\frac{1}{3} \cdot x^3\right) + 12$

_____4. Given the graph on the right, identify the equation that best fits it.



A. $f(x) = 2^x$

B. $f(x) = 2^{x+1}$

C. $f(x) = 2^x + 1$

D. $f(x) = 2^{x+1} + 1$

_____5. Which is TRUE about the domain of an exponential function? The domain is….

A. any positive number.

B. restricted only to negative numbers.

C. the set of all real numbers.

D. the set of negative numbers.

_____6. Which of the following is NOT a characteristic of $f(x) = b^x$ ($b > 0$ and $b \neq 1$)?

A. The domain is (-∞, ∞).

B. The x-axis is a horizontal asymptote

C. The range is (-∞, ∞).

D. The y-intercept is (0,1).

_____7. Which of the following is TRUE about the range of an exponential function? The range of….

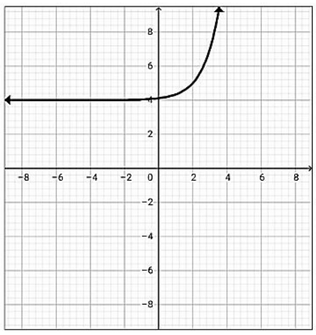
A. an exponential function is a real number.

B. an exponential function is the same as its domain.

C. the exponential function is always a positive number.

D. the exponential function could either be positive or negative.

_____8. What is the horizontal asymptote of the given graph on the right?



A. y = 0 B. y = 1

C. y = 2 D. y = 4

_____9. Which of the following is TRUE about the intercepts of the function $f(x) = b^x$, where $b > 0$ and $b \neq 1$? The y-intercept….

A. 1 and the x-intercept is 0.

B. and x-intercept are the same.

C. is 1 and there is no x-intercept.

D. is 0 and there is no x-intercept.

_____10. What is the zero of the exponential function $f(x) = 2^x$?

A. 0 B. 1 C. 2 D. no zero

_____11. What is the x-intercept of the function $g(x) = 3^{x+1} - 9$?

A. (1,0) B. (2,0) C. (3,0) D. (-1, 0)

_____12. What is the y-intercept of the exponential function $p(x) = 5^x - 2$?

A. (0,0) B. (0,-1) C. (0,1) D. (0, 2)

_____13. You buy a new computer for Php 48,000 for your online class. The computer decreases by 50% annually. When will the computer have a value of Php 24,000?

A. 1 year B. 2 years C. 3 years D. 4 years

_____14. A certain isotope has a half-life of 3.5 minutes. At present time (t = 0), there were y_0 grams of that isotope, but only 25 % of this amount remains after some time. How much time has passed?

A. 5 minutes B. 7 minutes
C. 6 minutes D. 8 minutes

_____15. Cobalt-60 is a substance used in manufacturing. It has a half-life of 5.3 years. If an old sample of 16 grams has now decayed to 2 grams, how much time has passed?

A. 15.9 years B. 16.9 years
B. 17.9 years D. 18.9 years

Jumpstart

For you to understand the lesson well, do the following activities.
Have fun and good luck!

Activity 1: HOW IS IT EXPONENTIALLY DONE?

Directions: Read and understand the given situation below. After reading, answer the questions based on the given situation. Choose the letter of the correct answer.

Some organizations need to spread accurate information to as many people in the shortest time possible. One way to do this is by cellphone texting or sending SMS.

The ABCXYZ Club has members of 500 families who jog every Sunday. When it rains, everyone wants to know if the jogging activity is cancelled. As a Club President, you make a decision and then send a text message to two families. Each of the family who received the decision sends a text message to two other families, and so on.

_____1. At the start of the process, only the Club president knows whether the jogging is cancelled or not. After the Club President texted the decision, there are two new families get the decision. In the next stage of texting, how many new families read the decision?

A. Four B. Five C. Six D. Eight

_____2. How will you describe the number of families reading the message grow as the texting continues? *As the number/ stage of texting increases the number of families that reads the message…*

A. increases rapidly. B. decreases rapidly.
C. increases slowly. D. decreases slowly.

_____3. Based on the process presented on the situation above, which of the following table of value represent the number of new families who read the decision.

A.

Stage of Texting	0	1	2	3	4	5	6	7	8	9	10
Families Informed	1	2	4	6	10	20	40	80	160	320	640

B.

Stage of Texting	0	1	2	3	4	5	6	7	8	9	10
Families Informed	1	2	4	8	16	24	32	64	128	256	512

C.

Stage of Texting	0	1	2	3	4	5	6	7	8	9	10
Families Informed	1	2	4	8	16	32	64	128	256	512	1024

D.

Stage of Texting	0	1	2	3	4	5	6	7	8	9	10
Families Informed	0	2	4	8	16	32	64	128	256	512	1024

_____4. How many stages of texting will be needed to inform the 500 families about the decision? *To inform all 500 families about the decision, the stage of texting should be ….*

A. Eight B. Seven C. Nine D. Six

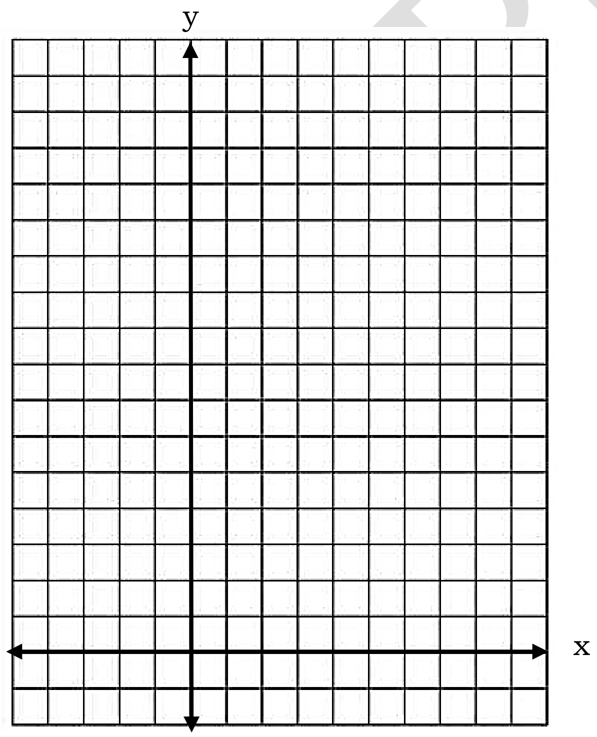
_____5. Which of the following equation best described the given situation above?

A. $f(x) = 2x$ B. $f(x) = x^2$

C. $f(x) = 2^x$ D. $f(x) = \frac{2}{x}$

For Question 6, Based on the situation, plot the values in the Rectangular Coordinate/Cartesian plane.

Stage of Texting (x)	0	1	2	3	4	5	6	7	8	9	10
Families Informed (y)											



Discover

Representation of Exponential Function in Table of Values, Graph and Equation

Recall your previous lessons in different functions, you observed that the functions can be represented in various ways. It can be through its **(a) table of values**, **(b) graph**, and **(c) equation**.

Since you started dealing with Exponential functions in the previous modules, how did you visualize or represent this kind of function? Let us have its equation, table, and graph.

EQUATION

Recall that an exponential function can be written as

$$\boldsymbol{f(x) = b^x}$$

*where **b>0**, **b ≠ 1**, and **x** is any real number.*

***Note**: reads as greater than (>), not equal to (≠)*

In the equation $\boldsymbol{f(x) = b^x}$, **b** is a constant/number called the **base** and **x** is the variable called the **exponent**.

Here are some examples of exponential functions.

$f(x) = \mathbf{3}^x$

Base is 3

$f(x) = \mathbf{10}^x$

Base is 10

$h(x) = \mathbf{2}^{x+1}$

Base is 2

The following are **<u>not</u>** exponential function

$F(x) = \boldsymbol{x}^2$

Variable is the base and not the exponent.

$G(x) = \mathbf{1}^x$

The base of an exponential function must be a positive number other than 1

$H(x) = \boldsymbol{x}^x$

Both the base and the exponent are variables.

$R(x) = (-\mathbf{2})^{2x-1}$

The base of an exponential function must **positive** and not negative.

The two restrictions on ***b*** in the definition are important. First, the definition does **not** include b=1 because 1^x has a value of 1 for all values of x. Why? What is the answer of 1^2? 1^3? 1^{100}? Yes, the answer will always be 1 even what exponent you place in it.

The definition also requires ***b*** to be positive so that the function can be defined for all real numbers x.

TABLE OF VALUES

Exponential functions can be represented also using **table of values**. But before we proceed in filling up the table of values of exponential function, let us try evaluating the following:

Illustrative Example 1:

- $\boldsymbol{f(x) = 2^x}$

$\boldsymbol{x = -3}$	$\boldsymbol{x = -2}$	$\boldsymbol{x = -1}$	$\boldsymbol{x = 0}$	$\boldsymbol{x = 1}$	$\boldsymbol{x = 2}$	$\boldsymbol{x = 3}$
$f(-3) = 2^{-3}$ $f(-3) = \frac{1}{2^3}$	$f(-2) = 2^{-2}$ $f(-2) = \frac{1}{2^2}$	$f(-1) = 2^{-1}$ $f(-1) = \frac{1}{2^1}$	$f(0) = 2^0$	$f(1) = 2^1$	$f(2) = 2^2$	$f(3) = 2^3$
$f(-3) = \boldsymbol{\frac{1}{8}}$	$f(-2) = \boldsymbol{\frac{1}{4}}$	$f(-1) = \boldsymbol{\frac{1}{2}}$	$f(0) = \mathbf{1}$	$f(1) = \mathbf{2}$	$f(2) = \mathbf{4}$	$f(3) = \mathbf{8}$

Note: **Negative Exponent Rule**: $a^{-n} = \frac{1}{a^n}$

This rule means that if a base contains a negative exponent the base together with its exponent moved to the denominator and the negative exponent becomes positive.

Zero-Exponent Rule: $a^0 = 1$

This says that any number/expression raised to the zero power is equal to 1.

Summary:

x	-3	-2	-1	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Any observations about the table of values of $\boldsymbol{f(x) = 2^x}$?

- Observed that as the value of **x** increases there is also an increase in the value of **f(x)** or **y**.
- Another is that, noticed that there is **no** negative value of **f(x)**, even there is a negative value in **x**.

Illustrative Example 2:

- $\boldsymbol{f(x) = 2^{-x}}$

$\boldsymbol{x=-3}$	$\boldsymbol{x=-2}$	$\boldsymbol{x=-1}$	$\boldsymbol{x=0}$	$\boldsymbol{x=1}$	$\boldsymbol{x=2}$	$\boldsymbol{x=3}$
$f(-3) = 2^{-(-3)}$ $f(-3) = 2^3$	$f(-2) = 2^{-(-2)}$ $f(-2) = 2^2$	$f(-1) = 2^{-(-1)}$ $f(-1) = 2^1$	$f(0) = 2^{-(0)}$	$f(1) = 2^{-(1)}$ $f(1) = 2^{-1}$ $f(1) = \frac{1}{2^1}$	$f(2) = 2^{-(2)}$ $f(2) = 2^{-2}$ $f(2) = \frac{1}{2^2}$	$f(3) = 2^{-(3)}$ $f(3) = 2^{-3}$ $f(3) = \frac{1}{2^3}$
$f(-3) = \mathbf{8}$	$f(-2) = \mathbf{4}$	$f(-1) = \mathbf{2}$	$f(0) = \mathbf{1}$	$f(1) = \boldsymbol{\frac{1}{2}}$	$f(2) = \boldsymbol{\frac{1}{4}}$	$f(3) = \boldsymbol{\frac{1}{8}}$

Note: $a^{-n} = \frac{1}{a^n}$ $\qquad a^0 = 1$

Summary:

x	-3	-2	-1	0	1	2	3
f(x)	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Any observations about the table of values of $\boldsymbol{f(x) = 2^{-x}}$?

- Observed that as the value of **x** increases there is a corresponding decrease in the value of **f(x)** or **y**.
- Another is that, noticed that there is no negative value of f(x), even there is a negative value in x.

GRAPH

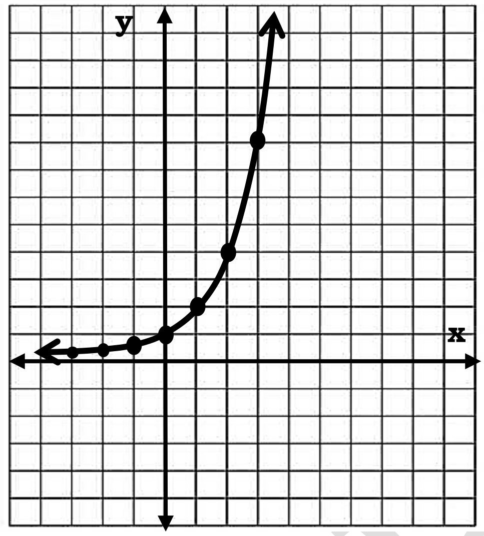
Graph is the best way to visualize a function. In this part, you are going to discover how the graph of an exponential function looks like. But before proceeding, make sure you still remember your ordered pair and how to plot and connect it in your Cartesian / Rectangular Coordinate Plane.

Illustrative Example 1:

- Let us graph the function $\boldsymbol{f(x) = 2^x}$
- First, complete the table of values.

x	-3	-2	-1	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
(x, f(x)) or (x, y)	$(-3, \frac{1}{8})$	$(-2, \frac{1}{4})$	$(-1, \frac{1}{2})$	(0,1)	(1,2)	(2,4)	(3,8)

- Plot the ordered pairs in Cartesian plane.
- Connect the points using a SMOOTH CURVE.



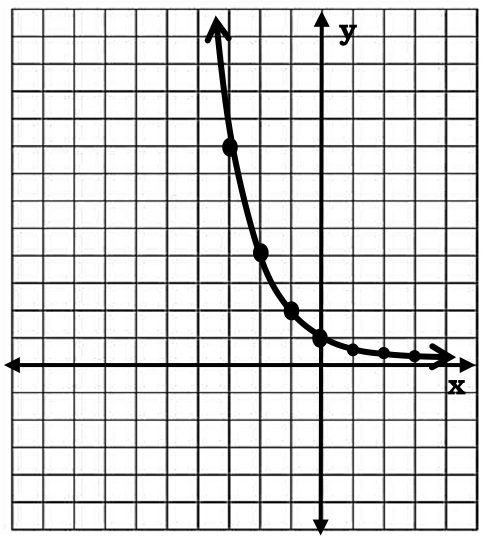
Observed that the graph increases from left to right. It passes through the y-axis at point (0,1). The graph is asymptotic to x-axis.

Illustrative Example 2:

- Let us graph the function $f(x) = 2^{-x}$
- First, complete the table of values.

x	-3	-2	-1	0	1	2	3
f(x)	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
(x, f(x))	$(-3, 8)$	$(-2, 4)$	$(-1, 2)$	(0,1)	$(1, \frac{1}{2})$	$(2, \frac{1}{4})$	$(3, \frac{1}{8})$

- Plot the ordered pairs in Cartesian plane.
- Connect the points using a SMOOTH CURVE.



Observed that the graph decreases from left to right. It passes through the y-axis at point (0,1). The graph is asymptotic to the x-axis.

What did you notice about the graph of $\boldsymbol{f(x) = 2^{-x}}$ and $\boldsymbol{f(x) = 2^{x}}$? Yes, the two graphs are the same, but they are just facing differently. The graph of $\boldsymbol{f(x) = 2^{x}}$ **increases** from left to right while the graph of $\boldsymbol{f(x) = 2^{-x}}$ **decreases** from right to left.

Domain and Range of an Exponential Function

The ***domain*** of a function refers to the set of input values for the independent variable, these are simply the x-values of the given function. The domain of an exponential function is all real numbers, that is, x can be any real number.

The ***range*** of a function is the set of output values for the dependent variable (y-values). The range of an exponential function $f(x) = b^x$ is a set of real numbers above and below the horizontal asymptote (a line that a curve approaches arbitrarily).

Example 1: Find the domain and range of $f(x) = 3^x$.

Applying the previous lesson, let us first represent the given exponential function through its table and graph.

x	-4	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81

Solutions:

Using the function $f(x) = 3^x$, substitute the values of x:

x = -4
$f(-4) = 3^{-4}$
$f(-4) = \frac{1}{3^4}$
$f(-4) = \frac{1}{81}$ or y = $\frac{1}{81}$

x = -3
$f(-3) = 3^{-3}$
$f(-3) = \frac{1}{3^3}$
$f(-3) = \frac{1}{27}$ or y = $\frac{1}{27}$

x = -2
$f(-2) = 3^{-2}$
$f(-2) = \frac{1}{3^2}$
$f(-2) = \frac{1}{9}$ or y = $\frac{1}{9}$

x = -1
$f(-1) = 3^{-1}$
$f(-1) = \frac{1}{3^1}$ or y = $\frac{1}{3}$

x = 0
$f(0) = 3^0$
$f(0) = 1$ or y = 1

x = 1
$f(0) = 3^1$
$f(0) = 3\ or$ y = 3

x = 2
$f(2) = 3^2$
$f(2) = 9$ or y = 9

x = 3
$f(3) = 3^3$
$f(3) = 27$ or y = 27

x = 4
$f(4) = 3^4$
$f(4) = 81$ or y = 81

To identify the domain and range of the exponential function, look into the value of x and y on the given table above. What do you notice about the value of x?

Based on the table, the values of x are **all real numbers**, meaning, the x-variable on the equation $f(x) = 3^x$ can take any values of x, infinitely. It can take 100, -100, and others. Therefore, the **domain** is the **set of all real numbers** or in interval notation (symbol), it could be written as $(-\infty, \infty)$.

On the other hand, take a look on the value of y in the table. What do you notice? Is there any value of y that is negative?

On the table above, there are no negative values of y even though there are x-values that are negative. This may bring us to a conclusion that the **range** of the given exponential function is the **set of all positive real numbers** or in interval notation (symbol), it could be written as $(\mathbf{0}, \infty)$

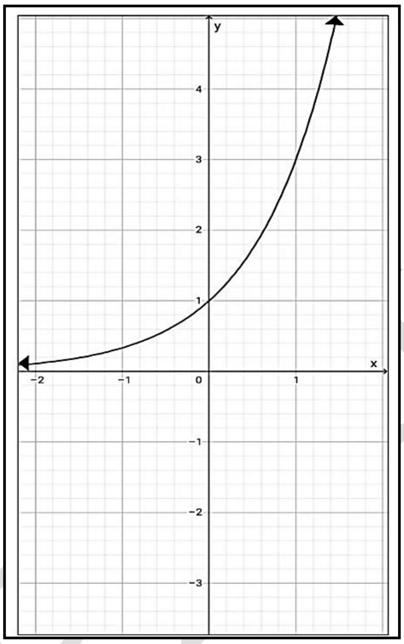


Figure 1. Graph of $f(x) = 3^x$

You can also visualize the domain and range of the given function based on its graph which is on figure 3. The **domain** is the **horizontal line**, which is also the x-axis, observe that the graph will hit all the values of x as it extends to the left or to the right. On the other hand, the **range** is the **vertical line**, which is the y-axis, you can say that the range is **only restricted** on the values which are **above $y = 0$**, or all **positive real numbers**.

From the given graph, you can also observe that the behavior of the given exponential function is increasing.

Example 2: Sketch the graph of $f(x) = 3^{x-2} + 4$,then find the domain and range.

Since, the table of values is not present, observe the graphs of the given function. What can you say about the domain or the values of x? Is there any restriction as you move to the left, or right? If there is none, then you can say that the **domain** is still the **set of all real numbers**.

How about the y-values or the range? Is the graph restricted? Yes, it is restricted, but on what part? Yes, the y-values are restricted on the values above $y = 4$. This means that the range is the set of all positive numbers greater than 4 or in interval notation $(\mathbf{4}, \infty)$.

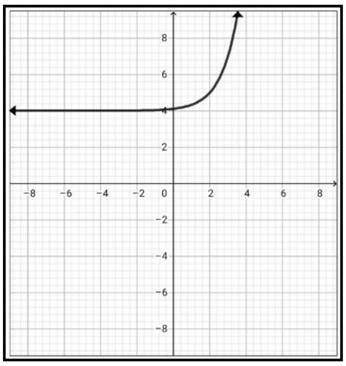


Figure 2. Graph of $f(x) = 3^{x-2} + 4$

Intercepts, Zeroes, and Asymptotes of an Exponential Function

Exponential function is a function with the form $f(x) = b^x$, where $b > 0$ and $b \neq 1$. It can be observed that exponential functions possess different characteristics which can be observed through its graph and can be obtained using the equation.

The following properties are the **intercepts (x and y-intercepts)**, **the zeroes**, and **the asymptotes**. As you continue reading this part of the module, you will differentiate the three and you will be able to determine where they can be located in the given graph of the given exponential function or how can they be obtained using the equation.

INTERCEPTS OF EXPONENTIAL FUNCTIONS

The **intercept of a function** refers to the **points** at which it crosses either the **x or y axis**. The x-intercept of a function is the point (x, y) such that the value of $y = 0$. This implies that the graph crosses the x-axis. Similarly, the y-intercept of a function is the point (x, y) such that the value of $x = 0$. This also implies that the graph crosses the y-axis.

Let us take the following examples to illustrate the x-intercept and the y-intercepts.

Example 1: Sketch the graph and determine the intercepts of $f(x) = 2^x$.

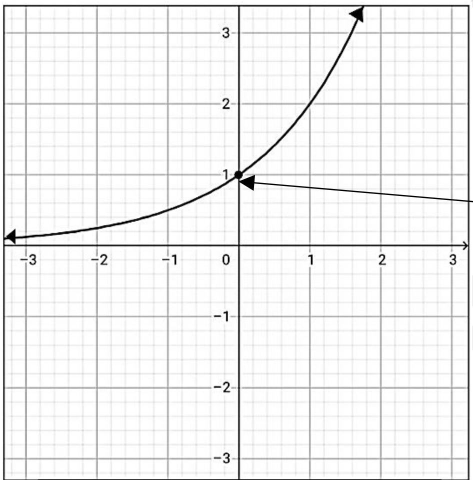


Figure 3. Graph of $f(x) = 2^x$

As you can see in the given graph, it crosses the y-axis at (0,1). Therefore, the y-intercept is (0,1) or $y = 1$.

y-intercept (0,1) or $y = 1$

Notice that the function is in the form $f(x) = b^x$, and the graph did not cross the x-axis. Therefore, there is no x-intercept in this example.

Example 2: Determine the intercepts of then given function $f(x) = 8^x - 2$.

Solution:

For x-intercept: Let y = 0

$f(x) = 8^x - 2$

$y = 8^x - 2$

$0 = 8^x - 2$

$2 = 8^x$

$2^1 = 2^{3x}$

$1 = 3x$

$\boldsymbol{x = \frac{1}{3} \text{ or } 0.333}$

For y-intercept: Let x = 0

$f(x) = 8^x - 2$

$f(o) = 8^0 - 2$

$f(o) = 1 - 2$

$f(o) = -1$

$\boldsymbol{y = -1}$

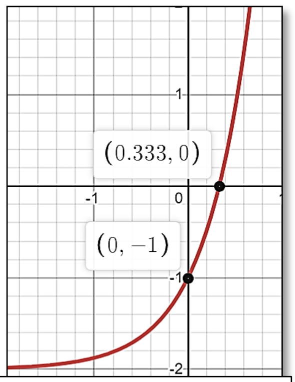


Figure 4. Graph of $f(x) = 8^x - 2$

ZEROES OF EXPONENTIAL FUNCTIONS

The **zeroes of an exponential function** is the value of x that gives $y = 0$. To find zeros, set y = 0 then solve for x.

Observed that the *process of getting the zeroes of the function* is **the same** in *getting the x-intercept* which is correct because the zeroes is the value of x that will make the function equal to 0. Note that, not every exponential function has zeroes.

Example 1: Find the zeroes of the exponential function $g(x) = 3^{4x-8} - 1$.
Solution:

Let y = 0

$$g(x) = 3^{4x-8} - 1$$
$$y = 3^{4x-8} - 1$$
$$0 = 3^{4x-8} - 1$$
$$1 = 3^{4x-8}$$
$$3^0 = 3^{4x-8}$$
$$0 = 4x - 8$$
$$4x = 8$$
$$\frac{4x}{4} = \frac{8}{4}$$
$$\boldsymbol{x = 2}$$

To check:

$$g(x) = 3^{4x-8} - 1$$
$$g(2) = 3^{4(2)-8} - 1$$
$$g(2) = 3^{8-8} - 1$$
$$g(2) = 3^0 - 1$$
$$g(2) = 1 - 1$$
$$g(2) = 0$$

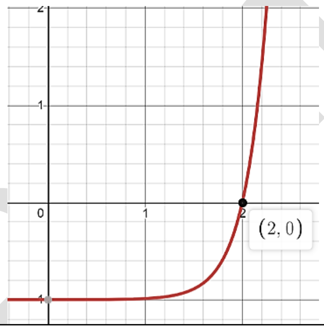


Figure 5. Graph of $g(x) = 3^{4x-8} - 1$

ASYMPTOTES OF AN EXPONENTIAL FUNCTIONS

Asymptote refers to the line that the graph gets closer and closer to, it could be horizontal or vertical. In the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, the horizontal asymptote is the line y = 0 (or the x-axis). There is no vertical asymptote.

Example 1: Determine the horizontal asymptote of the given graph.

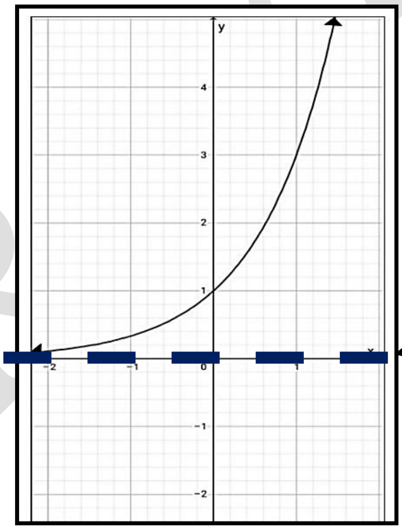
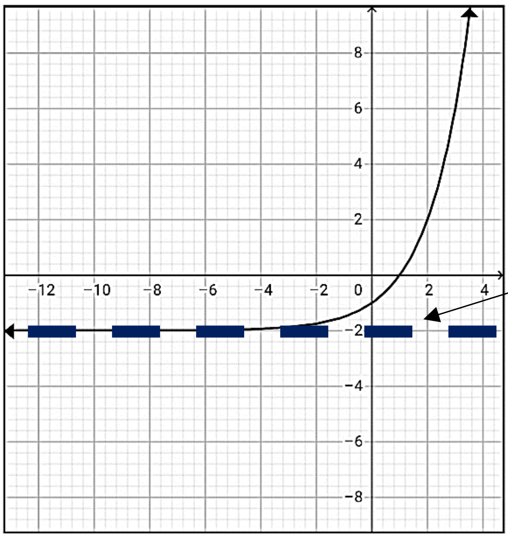


Figure 6. Graph of the Exponential Function

The dotted line shows the horizontal asymptote of the given function which is the x-axis or y = 0. As you can see, the graph gets closer and closer to the x-axis.

horizontal asymptote – x-axis or y = 0

Example 2: Sketch the graph of the function $g(x) = 2^x - 2$. Determine its horizontal asymptote.



horizontal asymptote $y = -2$

Figure 7. Graph of $g(x) = 2^x - 2$

The graph of the function $g(x) = 2^x - 2$ gets closer and closer to y = -2. Therefore, the horizontal asymptote is **y = -2**.

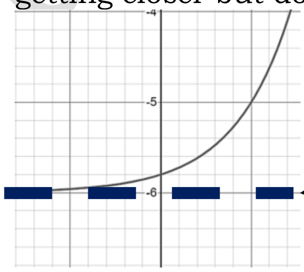
Example 3: What is the y –intercept and the horizontal asymptote of $g(x) = 5^{x+1} - 6$?

Solution:

To solve for the y-intercept, let x = 0

$$g(x) = 5^{x+1} - 6$$
$$g(0) = 5^{0+1} - 6$$
$$g(0) = -1$$

To get the horizontal asymptote, you may plot the given exponential function and look for the value of y that it may getting closer but does not intersect.



horizontal asymptote $y = -6$

Hence, the y-intercept is **-1** or **(0, -1)** and the horizontal asymptote is $\boldsymbol{y = -6}$.

Problems Involving Exponential Function and Equation

Exponential function has a variety of real-world phenomena where it can be modeled to solve certain problems especially when a percent of a quantity changes by overtime stays constant. In this part, we will discover how to solve a certain situation that increases as a function of time.

❖ **Exponential Models and Population Growth**

Suppose a quantity y doubles every T unit of time. If y_o is the initial amount, then the quantity y after t units of time is given by

$$y = y_o(2)^{\frac{t}{T}}$$

Note that that **2** in the formula $\boldsymbol{y = y_o(2)^{\frac{t}{T}}}$ refers to the word "**doubles**" in the problem/situation. If the problem stated that a certain population or substance is "**triples**" then the formula becomes $\boldsymbol{y = y_o(3)^{\frac{t}{T}}}$.

In the examples below, the problem is about half-life. **Half-life** refers to the time it takes the radioactive substance to decay into half of its original amount or quantity.

Example 1. A certain isotope with a half-life of 20 hours is present at time t=0 with 32 grams. How much time will have elapsed when only 1 gram remain?

Solution: The amount of the isotope after t hours $32\left(\frac{1}{2}\right)^{t/20}$

$32\left(\frac{1}{2}\right)^{t/20} = 1$ Exponential Equation

$\frac{32\left(\frac{1}{2}\right)^{t/20}}{32} = \frac{1}{32}$ Divide both side by 32

$\left(\frac{1}{2}\right)^{t/20} = \left(\frac{1}{2}\right)^{5}$ Rewrite as $\frac{1}{32}$ as base $\left(\frac{1}{2}\right)^{5}$

$\frac{t}{20} = 5$ Solve for t

$\boldsymbol{t = 100}$ **Final Answer**

Therefore, 100 hours will elapse to have only 1 gram.

Example 2: Cobalt-60 is a substance used in manufacturing. It has a half-life of 5.3 years. If an old sample of 32 grams has now decayed to 2 grams, how much time has passed?

Solution: The amount of the substance after t years $32\left(\frac{1}{2}\right)^{t/5.3}$

$32\left(\frac{1}{2}\right)^{t/5.3} = 2$ Exponential equation

$\frac{32\left(\frac{1}{2}\right)^{t/5.3}}{32} = \frac{2}{32}$ Divide both sides by 32

$\left(\frac{1}{2}\right)^{t/5.3} = \left(\frac{1}{16}\right)$ Simplify $\frac{2}{32}$ into $\frac{1}{16}$

$\left(\frac{1}{2}\right)^{t/5.3} = \left(\frac{1}{2}\right)^{4}$ Rewrite $\frac{1}{16}$ as the same with $\frac{1}{2}$

$\frac{t}{5.3} = 4$ Solve for t

$\boldsymbol{t = 21.2}$ Thus, 21.2 years have passed since $t = 0$

❖ **Exponential Growth**

Consider the following:

$Y = A(1+r)^t$ where $A = initial\ amount$

$r = rate\ (decimal\ form)$

$t\ =\ time$

Example 3: Sherry started to have chain of online business that operated 100 stores S nationwide since 2018. If the rate of increase is 9% annually, how many online stores does her business operate in 2027?

Given: A = 100 | r = 9% --- 0.09 | t = 9 ---- 2027 – 2018 = 9 years

Solution: $Y = A(1+r)^t$

$S = 100(1+0.09)^9$

$S = 100(1.09)^9$

$S = 100(2.17189)$

$\boldsymbol{S = 217.19\ \approx 217}$ **Thus, there will be 217 online stores in year 2027.**

Example 4: There is a small town in Region I that has a population of 750 in year 2015 and it increases at an annual rate of 12%. Find the number of population *(P)* in the said town in year 2021.

Given: A = 750 | r = 12% --- 0.12 | t = 9 ---- 2015 – 2021 = 6 years

Solution: $Y = A(1+r)^t$

$P = 750(1+0.12)^6$

$P = 750(1.12)^6$

$P = 750(1.97382)$

$\boldsymbol{P = 1,480.37\ \approx 1480}$

Thus, a population of 1480 is present in year 2021.

❖ **Exponential Decay**

Consider the following:

$Y = A(1-r)^t$ where $A = initial\ amount$

$r = rate\ (decimal\ form)$

$t\ =\ time$

Example 5: A radioactive substance decays at a rate of 2.5 % per hour. What percent of the substance is left after 8 hours?

Given: A=100 (original amount) | r = 2.5% --- 0.025 | t = 8

Solution: $Y = A(1-r)^t$

$C = 100(1-0.025)^8$

$C = 100(0.975)^5$

$C = 100(0.8811)$

$\boldsymbol{C = 88.11}$ **Therefore, 88.11% of the substance is left after 8 hours.**

Example 6: You buy a new model of smartphone *(C)* for Php 52,000.00. Its value depreciates 7% every 6 months. How much is the smartphone worth after 5 years?

Given:

A=52,000	r = 7% --- 0.07	t = 10

Note: (6 months = 1 period)
Since 1 year = 2 periods
2 years = 4 periods
3 years = 6 periods
4 years = 8 periods
5 year = 10 periods

Solution: $Y = A(1-r)^t$

$C = 52000(1-0.07)^{10}$

$C = 52000(0.93)^{10}$

$C = 52{,}000(0.48398)$

$\boldsymbol{C = 25{,}167.08}$ **Therefore, after 5 years the smartphone will worth Php 25, 167.08**

❖ **Compound Interest**

Consider the following:

$A = P\left(1+\frac{r}{n}\right)^{nt}$ where $A = amount\ after\ t\ years$

$P = principal\ amount\ invested$

$r = interest\ rate$

$n = no.of\ compounding\ periods\ each\ year$

$t = no.of\ year$

Example 7: Jano puts Php 5,000.00 in a savings account that has 5% annual interest compounded monthly. At this rate, how much money will be in the account after 10 years?

Given: A = ? P = 5,000 r = 5% --- 0.05 n = 12 t = 10

Solution: $A = P\left(1+\frac{r}{n}\right)^{rt}$

$A = 5000\left(1+\frac{0.05}{12}\right)^{(12)(10)}$

$A = 5000(1.00417)^{(120)}$

$A = 5000(1.6477)$

$\boldsymbol{A = 8{,}235.5}$ **Thus, Jano will have Php 8,235.50 in his account after 10 years**

Note: n = 12 (monthly)
n = 4 (quarterly/every 3 months)

Example 8: Mica wants to have a Php 30,000 to buy a brand-new laptop after 3 years. She invested in a bitcoin trading with an average of 9% rate interest each year compounded quarterly. How much should she deposit in her bitcoin account?

Given: A = 30,000 P = ? r = 9% --- 0.09 n = 4 t = 3

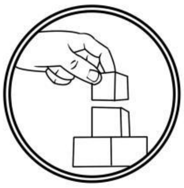
Solution: $A = P\left(1+\frac{r}{n}\right)^{rt}$

$$30000 = P\left(1+\frac{0.09}{4}\right)^{(4)(3)}$$

$$30000 = P(1.0225)^{(12)}$$

$$30000 = P(1.30605) \quad \text{Divide both side by 1.30605}$$

$\boldsymbol{P = 22,970.02}$ **Thus, Mica initially deposited Php 22,970.02 in her bitcoin account.**



Explore

Activity 1: IDENTIFY ME!

Directions: Identify whether the given equation is an Exponential Function or NOT. Write EF if it is an exponential function, otherwise NOT.

_____1. $f(x) = 3^x$ _____3. $f(x) = 6x - 5$ _____5. $f(x) = (3x-5)^5$

_____2. $f(x) = (-0.9)^{5x}$ _____4. $f(x) = \left(\frac{3}{4}\right)^{-x}$

Activity 2: FILL ME OUT!

Directions: Given the exponential function, fill out the missing part of the table of values. Some parts of the table are already fill out for your guidance. Always simplify your answer.

$$\boldsymbol{f(x) = 2^x + 1}$$

$\boldsymbol{x = -3}$	$\boldsymbol{x = -2}$	$\boldsymbol{x = -1}$	$\boldsymbol{x = 0}$	$\boldsymbol{x = 1}$	$\boldsymbol{x = 2}$	$\boldsymbol{x = 3}$
		$f(x) = 2^x + 1$ $f(-1) = 2^{-1} + 1$ $f(-1) = \frac{1}{2^1} + 1$ $f(-1) = \frac{1+2}{2}$				$f(x) = 2^x + 1$ $f(3) = 2^3 + 1$ $f(3) = 8 + 1$
		$f(-1) = \boldsymbol{\frac{3}{2}}$				$f(3) = \boldsymbol{9}$

x	-3	-2	-1	0	1	2	3
f(x)			$\mathbf{\frac{3}{2}}$				**9**

Activity 3: Find MY DOMAIN and MY RANGE!

Directions: The figure on the below shows the graph of the different exponential functions. Study the figure, then identify the domain and range of each function. Use a separate sheet of paper for your answers.

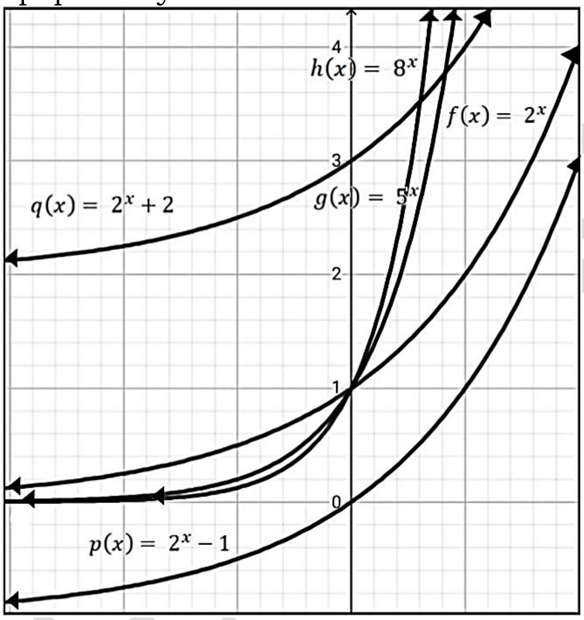


Figure 8. Graph of the Exponential Functions

1. $h(x) = 8^x$ Domain:________________ Range:________________
2. $q(x) = 2^x + 2$ Domain:________________ Range:________________

Activity 4: Fill the Missing Link!

Directions: Complete the table below by writing the horizontal asymptote and the y-intercept of each exponential function. Use a separate sheet of paper for your answers.

Exponential Functions	***Horizontal Asymptote***	***y-intercept***
$f(x) = 5^{x-1}$	y = 0	**(3)**
$g(x) = 10^x + 3$	**(1)**	(0, 4)
$h(x) = 7^x - 9$	**(2)**	(0, -8)

Activity 5: Fix Your Problem!

Directions: Read and analyze the following problem and write your solution and final answer in a separate sheet of paper.

1. The half-life of Silver-42 is 42 seconds. There were y_0 grams at first but only 6.25% of this amount remains after some time. How much time has passed?

2. Find the number of populations of 450 animals that increases at an annual rate of 3% after 5 years.

3. Supposed the cost of a smart tablet is worth Php 7,500. Its value depreciates 6% every 4 months. How much is the tablet cost after 3 years?

Deepen

At this point, create your own design of a brochure about your own online investment and a certain radioactive substance. The content of your brochures should contain the information as indicated in the following given.

The scoring rubric is also shown in the next page in assessing your outputs.

What you need:

Construction paper/Oslo Paper, marker, any coloring materials

What you need to do:

1. Using the materials given, design your own brochure that market your two outputs. You can insert a picture in it and a brief introduction. Indicated also in your brochure are the following information.

   * Brochure 1: Online Investment
   Capital = Php 2,000.
   Expected increase after 3 years if its interest rate is:
   a.) 2% compounded monthly
   b.) 3.5 % compounded quarterly
   c.) 7% compounded annually

   * Brochure 2: Radioactive Substance
   A radioactive substance with a half-life of 32 hours at t = 0. Find the number of time elapsed when:
   a.) $\frac{1}{64}$of this amount remains after $t = 0$
   b.) Only 17 grams remain, if there are 2176 grams at t = 0
   c) Only 23 grams remain, if there are 736 grams at t = 0

2. Make it creative and resourceful. You must be able to answer the given above to be able to put that important information in your brochure.

3. Write your solution for the given problem in your brochure.

Rubrics for Scoring the Output

	Excellent **(5 points)**	**Good** **(4 points)**	**Satisfactory** **(3 points)**	**Needs Improvement** **(1-2 points)**
Content	The brochure portrays correct content and organization of ideas.	The brochure portrays correct content and some parts have organization of ideas.	The brochure portrays some parts that have correct content	The brochure does not demonstrate clear content with wrong solution.
Creativity /Design	The brochure shows creativity and resourcefulness	The brochure shows creativity and with lack of resourcefulness	Only resourcefulness/creativity is shown in the brochure	The brochure shows lack of creativity and lack of resourcefulness
Final Output	Final output looks professional with understandable content.	Final output looks decent and with understandable content.	Final output required more revisions and the content/concept were not clearly demonstrated	Final output looks unrefined and the content is not clearly stated.

Gauge

Directions: Read carefully each item. Write the letter of the best answer for each test item.

_____1. Which of the following is an exponential function?

A. $f(x) = 2 + 3x^2$ B. $x(t) = x^2 + 2x + 1$

C. $g(x) = 1^3 - 5^x$ D. $v(t) = (2 \cdot x^2) - 1$

_____2. Identify the correct table of values for the exponential function, $\boldsymbol{f(x) = 3^{2x+1}}$.

A.

x	-1	0	1	2
f(x)	$\frac{1}{3}$	1	27	243

B.

x	-1	0	1	2
f(x)	$\frac{1}{3}$	3	27	243

C.

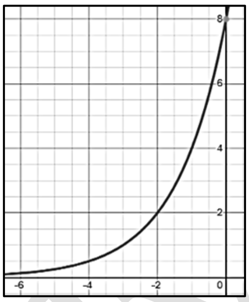
x	-1	0	1	2
f(x)	$\frac{1}{3}$	1	9	27

D.

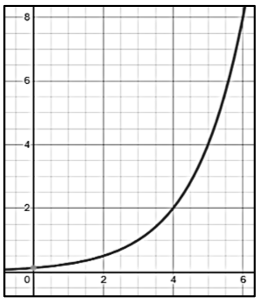
x	-1	0	1	2
f(x)	$\frac{1}{3}$	0	27	81

_____3. Which of the following is the correct graph of the function, $\boldsymbol{f(x) = 2^{x-3}}$?

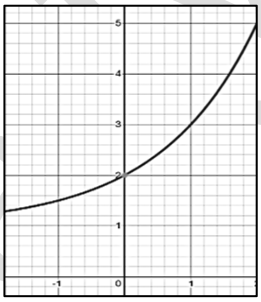
A.



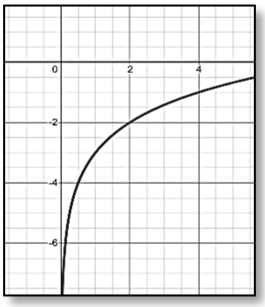
B.



C.



D.



_____4. Which of the following exponential function has this table of values?

x	-3	-2	-1	0	1	2	3
f(x)	$\frac{126}{125}$	$\frac{26}{25}$	$\frac{6}{5}$	2	6	26	126

A. $f(x) = 5^{x+1}$ B. $f(x) = 5^x + 1$

C. $f(x) = 5^{x+1} + 1$ D. $f(x) = 5^{x-1}$

_____5. Given the function, $\boldsymbol{f(x) = 2(3)^{x+1}}$, find the value of $\boldsymbol{f(-1)}$.

A. $f(-1) = 1$ B. $f(-1) = 0$

C. $f(-1) = -1$ D. $f(-1) = 2$

_____6. Which of the following is correct about the domain of an exponential function? The domain is…

A. all positive numbers.
B. any real number.
C. all the set of negative number.
D. is the set of numbers including zero.

_____7. What is the x-intercept of the function $g(x) = 5^{x+1} - 25$?

A. (1,0) B. (2,0) C. (3,0) D. (-1, 0)

_____8. What is the y-intercept of $p(x) = 4^{x+1} + 1$?

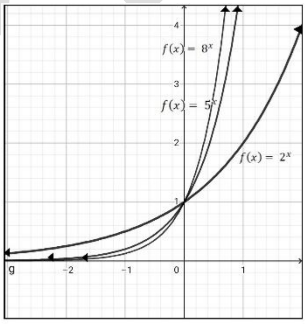
A. (0,0) B. (0,3) C. (0,4) D. (0, 5)

_____9. What is the y-intercept of the function $g(x) = 7^x$?

A. (0,0) B. (0,1) C. (0,3) D. (0, 7)

_____10. What is the horizontal asymptote of the exponential functions in the given graph on the right side?

A. y = 0 B. y = 1
C. y = 2 D. y = -1



_____11. The half-life of a certain bacteria is 2.45 minutes. At present there were y_0 grams of that bacteria, but only $\frac{1}{128}$ of this amount remains after some time. How much time has passed?

A. 17 minutes B. 17.15 minutes
C. 17.6 minutes D. 20 minutes

_____12. A given 1600 grams of isotope has a half-life of 32 hours. How much time it will decay to have only 25 grams remain?

A. 24 hours B. 172 hours
C. 56 hours D. 192 hours

_____13. A radioactive substance decays at a rate of 4.5 % per hour. What percent of the substance is left after 12 hours?

A. 57.549 % B. 58.001%
C. 58.112% D. 60.55%

_____14. Anton purchased a new house for his family worth Php 172,000.00. Its value increases 2% annually. How much is house worth after 6 years?

A. Php 139,699.05 B. Php 193,699.94
C. Php 159,699.05 D. Php 139,799.05

_____15. You decided to deposit Php 3,000.00 in a savings account for your college tuition fee that has 6.5% annual interest compounded quarterly. At this rate, how much money will be in your account after 5 years?

A. Php 4,131.259 B. Php 4,010.00
C. Php 4,141.259 D. Php 4,000.00

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