10

MATHEMATICS

Quarter 1 - Module 2: Arithmetic Sequence VS. Geometric Sequence

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MATHEMATICS 10
Quarter 1 - Module 2: Arithmetic Sequence VS. Geometric Sequence
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Development Team of the Module

Author: Rustico R. Diaz
Editor: SDO La Union, Learning Resource Quality Assurance Team
Illustrators: Ernesto F. Ramos, Jr.
Christian R. Bumatay
Content Reviewer: Rhonavi Q. Masangkay
Language Reviewer: Concesa M. Juntilano
Design and Layout: Mark Jesus M. Mulato

Management Team:

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Assistant Schools Division Superintendent
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Erlinda M. Dela Peña, Ph.D, *EPS in Charge of Mathematics*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

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Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

10

MATHEMATICS

Quarter 1 - Module 2:
Arithmetic Sequence VS. Geometric Sequence

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DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Sequence plays an important role in our daily lives. Not only in nature, it is often used as part of our daily routine such as creating designs, analyzing data, computation of interest portion of monthly payments and many more.

This module will illustrate another type of sequence which is of equal importance with the previous learned type of sequence.

This module will help you attain the following objectives:

- Illustrates a geometric sequence. **(M10AL-Id-1)**
- Differentiate a geometric sequence from an arithmetic sequence. **(M10AL-Id-2)**

After going through this module, you are expected to:

1. Recall concepts on arithmetic sequence;
2. Define and illustrates a geometric sequence; and
3. Differentiates a geometric sequence from an arithmetic sequence.

Before you start doing the activities in this lesson, let's find out how much you already know about this module by answering the pre-test on the next page.

PRE-ASSESSMENT

Directions: Read each mathematical statement carefully. Write the letter of your choice in a separate sheet of paper.

1. In the arithmetic sequence, common difference is obtained by subtracting successive terms from right to left while in geometric sequence common ratio is obtained by _____ successive terms from right to left. Which of the following will complete the statement?
   A. adding B. dividing C. multiplying D. subtracting
2. Which of the following illustrates a geometric sequence?
   A. 2, 5, 10, 20, . . . B. 1, 2, 3, 5, . . .
   C. -2, -4, -6, -8, . . . D. -2, -6, -18, -54, . . .
3. If common difference is for arithmetic sequence, what is for the geometric sequence?
   A. common ratio B. common term C. first term D. last term
4. If the formula in solving for the general term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, what is the formula for the general term of a geometric sequence?
   A. $a_n = a_1 r^{n-1}$ B. $a_n = a_1 r^n$ C. $S_n = a_1 r^{n-1}$ D. $S_n = a_1 r a_n$
5. What is the common ratio of the geometric sequence 8, 4, 2, 1, $\frac{1}{2}$, . . . ?
   A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{5}$
6. What is the fifth term in the geometric sequence 3, 9, 27, 81, . . .?
   A. 240 B. 241 C. 242 D. 243
7. Which of the following illustrates a geometric sequence?
   A. 1, 1, 2, 3, 5, . . . B. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots$
   C. 10, 15, 20, 25, . . . D. 5, 1, $\frac{1}{5}$, . . .
8. What is the common ratio of the geometric sequence 4, -16, 64, -256, . . . ?
   A. 2 B. 0 C. -2 D. -4
9. What type of sequence is 3, 6, 9, 12, . . .?
   A. Arithmetic B. Fibonacci C. Geometric D. Harmonic
10. Given the rule: $a_n = 7(½)^{n-1}$. Which is the common ratio?
    A. a_n B. 7 C. $\frac{1}{2}$ D. n - 1
11. Given the rule: $a_n = 7(½)^{n-1}$. Which is the first term?
    A. a_n B. 7 C. $\frac{1}{2}$ D. n – 1
12. Given $a_1 = 4$ and r = -3, what are the first four terms of the geometric sequence?
    A. 4, 8, 16, 32 B. 4, -4, 4, -4
    C. 4, -8, 16, -32 D. 4, -12, 36, -108
13. What are the first four terms of the geometric sequence given the rule: $a_n = 4(5)^{n-1}$?
    A. 20, 100, 500, 2500 B. 5, 20, 80, 320
    C. 4, 20, 100, 500 D. 4, 9, 14, 19
14. What is the explicit rule for the geometric sequence: 2, -12, 72, . . . ?
    A. $a_n = 72(-6)^{n-1}$ B. $a_n = 2(-6)^{n-1}$
    C. $a_n = 2(6)^{n-1}$ D. $a_n = 72(-6)^{n-1}$
15. What is the explicit formula for the geometric sequence 3, -12, 48, -192, . . .?
    A. $a_n = 4(3)^{n-1}$ B. $a_n = 3(4)^{n-1}$
    C. $a_n = -4(3)^{n-1}$ D. $a_n = 3(-4)^{n-1}$

In this module, the lesson starts with assessing your prior knowledge on the different concepts you've learned involving arithmetic sequence.

Jumpstart

Activity 1: Correct Practice Makes Perfect!

Complete the table below involving arithmetic sequence and series.

Arithmetic Sequence	d	a_{10}	S_{10}
1. 3,10, 17, 24, 31, . . .			
2. 11, 6, 1, -4, -9, . . .			
3. -15, -7, 1, 9, 17, . . .			
4. a_1= 3 and a_3 = 19			
5. a_1= -2 and a_{10} = -65			

Discover

In the previous lesson, we learned that in arithmetic sequence**,** the difference between one term and the next is a constant. In other words, we just add the same value each time ... infinitely.

In a **Geometric Sequence** each term is found by **multiplying** the previous term by a **constant ratio**. The common ration can be determined by dividing any term in the sequence by the term that precedes it.

$$\mathbf{r} = \frac{a_{n+1}}{a_n}$$

The nth term of the geometric sequence can be obtained using the formula

$$a_{n=}\, a_1 r^{n-1}$$

where:

a_n = nth term
a_1 = first term
r = common ratio
n = number of term

Example 1. Find the next term or the 5th term in the geometric sequence 3, -9, 27, -81, ...

Step 1. Find the common ratio by dividing any term by the preceding term.

3	-9	27	-81
$\frac{-9}{3} = -3$		$\frac{27}{-9} = -3$	$\frac{-81}{27} = -3$

r = $\frac{-9}{3}$ = -3

Step 2. The next term to -81 is the 5th term so to solve for the fifth term

$a_5 = a_1 \bullet r^{n-1}$

$= 3 \bullet -3^{5-1}$

a_5 = 243

Therefore, the next term or 5th term in the geometric sequence is 243.

Example 2. Given a_1 = -4 and r = -4, write the first four terms of the geometric sequence.

Solution:

a_1 = -4

$a_2 = a_1 \bullet$ r = -4 $\bullet$ -4 = 16

$a_3 = a_2 \bullet$ r = 16 $\bullet$ -4 = -64

$a_4 = a_3 \bullet$ r = -64 $\bullet$ -4 = 256

Therefore, the first four terms of the geometric sequence are -4, 16, -64 and 256.

After learning the two types of sequences, arithmetic and geometric, the real challenge is to determine which sequence shows such. The following examples will help you unlock the problem as to whether it is an arithmetic or geometric.

Example 1. Is the sequence 5, 15, 45, 135, . . . an arithmetic or geometric?

Solution:

To identify the type of sequence, you have to compute both the common difference (d) and common ratio (r).

d = 15 – 5 = 10 r = $\frac{15}{5}$ = 3

d = 45 – 15 = 30 r = $\frac{45}{15}$ = 3

d = 135 – 45 = 90 r = $\frac{135}{45}$ = 3

Since *r* shows the same result, then the sequence 5, 15, 45, 135, . . . is geometric.

Example 3. Is the sequence -8, -17, -26, -35, . . . an arithmetic or geometric?

Solution:

To identify the type of sequence, you have to compute both the common difference (d) and common ratio (r).

d = -17 – (-8) = -17 + 8 = -9 $\qquad$ r = $\frac{-17}{-8} = \frac{17}{8}$

d = -26 – (-17) = -26 + 17 = -9 $\qquad$ r = $\frac{-26}{-17} = \frac{26}{17}$

d = -35 – (-26) = -35 + 26 = -9 $\qquad$ r = $\frac{-35}{-26} = \frac{35}{26}$

Since *d* shows the same result, then the sequence -8, -17, -26, -35, . . . is arithmetic.

Example 4. Find the missing term to make the sequence an arithmetic.
-5, -12, ____, -26, . . .?

Solution:

Since the problem states that it is an arithmetic sequence, then we have to compute the value of the common difference first.

d = -12 – (-5) = -12 + 5 = -7

Now that you know the value of *d*, you can now find the missing term by adding -7 to its preceding term -12.

$a_3 = a_2$ + d = -12 + (-7) = -19.

Therefore, the missing term in the sequence is -19.

Example 4. What value/s of *m* will make the sequence m – 2, m + 4, 3m, . . . geometric?

Since the sequence is geometric, then we will apply the common ratio formula to find the value of m.

r = $\frac{m+4}{m-2} = \frac{3m}{m+4}$

Apply cross-multiplication to $\frac{m+4}{m-2} = \frac{3m}{m+4}$

3m(m-2) = (m+4)(m+4)
$3m^2 – 6m = m^2 + 8m + 16$
$3m^2 – 6m - m^2 – 8m – 16 = 0$
$3m^2 - m^2 – 6m – 8m – 16 = 0$
$2m^2 – 14m – 16 = 0$

$\frac{2m2 - 14m - 16 = 0}{2}$

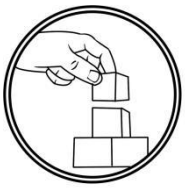
$m^2 - 7m - 8 = 0$

$(m - 8)(m + 1) = 0$

$m - 8 = 0$; $m + 1 = 0$

$m_1 = 8$; $m_2 = -1$

The values of m that will make the sequence geometric are -1 and 8.



Explore

Activity 2: You Make Me Feel Complete!

Complete the table by identifying the common ratio and the next four terms of the given geometric sequence.

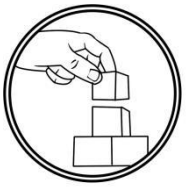
Geometric Sequence	r	Next Four Terms
1. 3, 6, 12, 24, 48, . . .		
2. 5, 15, 45, . . .		
3. 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, . . .		
4. 10, -100, 1 000, . . .		
5. 100, 20, 4, $\frac{4}{5}$, . . .		

Deepen

Activity 3: Who Am I!

Determine whether each sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference, if geometric, find the common ratio.

Sequence	Arithmetic/Geometric /Neither	value of r/d
1. 4, 9, 14, 19, . . .		
2. 10, 2, $\frac{2}{5}$, $\frac{2}{25}$, . . .		
3. -8, -15, -22, -29, . . .		
4. 0.3, 0.9, 1.5, 2.1, . . .		
5. -2, 8, -32, 128, . . .		



Explore

Activity 4: Where is My Princess!

Find the first three terms of the given sequence and state whether it is arithmetic or geometric.

1. a_n = 2n + 5
2. a_n = 3^{n-1}
3. a_n = $2(3)^{n-1}$
4. a_n = 4 - 5n
5. a_n = n^2 + 1

Gauge

POST ASSESSMENT

Directions*:* Read each mathematical statement carefully. Write the letter of your choice in a separate sheet of paper.

1. Which basic operation is needed in identifying the common ratio of the given geometric sequence?
   A. addition B. division C. multiplication D. subtraction
2. Which of the following illustrates an arithmetic sequence?
   A. 2, 5, 10, 20, . . . B. 1, 2, 3, 5, . . .
   C. -2, -4, -6, -8, . . . D. -2, -6, -18, -54, . . .
3. If *d* is for arithmetic sequence, what is for the geometric sequence?
   A. m B. n C. r D. s
4. Which of the following is needed in identifying the explicit formula of geometric sequence?
   A. a_1 B. d C. r D. Both A and C
5. What is the common ratio of the geometric sequence 100, 20, 4, $\frac{4}{5}$, . . . ?
   A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{5}$
6. What is the fifth term in the geometric sequence 1, -5, 25, -125, . . .?
   A. -625 B. -526 C. 526 D. 625
7. Which of the following illustrates a geometric sequence?
   A. 1, 1, 2, 3, 5, . . . B. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$, . . .
   C. -10, -5, $\frac{-5}{2}$, . . . D. -24, -12, 0, . . .

8. What is the common ratio of the geometric sequence -4, -16, -64, -256, . . . ?
   A. 4 B. 0 C. -2 D. -4
9. To properly identify whether the sequence is arithmetic or geometric, which of the following will you consider?
   A. find the explicit formula
   B. solve for the common difference
   C. solve for the common ratio
   D. Both B and C
10. Given the rule: $a_n = 2(½)^{n-1}$. Which is the common ratio?
   A. a_n B. 7 C. $\frac{1}{2}$ D. n - 1
11. Given the rule: $a_n = 2(½)^{n-1}$. Which is the first term?
   A. a_n B. 2 C. $\frac{1}{2}$ D. n – 1
12. Given a_1 = 10 and r = -4, what are the first four terms of the geometric sequence?
   A. 10, -40, 160, -640 B. 10, -10, 10, -10
   C. 10, -40, 80, -320 D. 4, -12, 36, -108
13. What are the first four terms of the geometric sequence given the rule: $a_n = -20\left(\frac{1}{10}\right)^{n-1}$?
   A. -20, -2, $\frac{-1}{5}$, $\frac{-1}{50}$ B. 5, 20, 80, 320
   C. 20, 2, $\frac{1}{5}$, $\frac{1}{50}$ D. 4, 9, 14, 19
14. What is the explicit rule for the geometric sequence: $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \ldots$?
   A. $a_n = \frac{-1}{2}\left(\frac{-2}{3}\right)^{n-1}$ B. $a_n = \frac{1}{2}\left(\frac{2}{3}\right)^{n-1}$
   C. $a_n = \frac{1}{2}\left(\frac{1}{3}\right)^{n-1}$ D. $a_n = \frac{-1}{2}\left(\frac{-1}{3}\right)^{n-1}$
15. What value of m will make the sequence -4, m, -36, . . . geometric?
   A. -21 B. -12 C. 12 D. Both B and C

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For inquiries or feedback, please write or call:

Department of Education – SDO La Union
Curriculum Implementation Division
Learning Resource Management Section
Flores St. Catbangen, San Fernando City La Union 2500
Telephone: (072) 607 - 8127
Telefax: (072) 205 - 0046
Email Address:
launion@deped.gov.ph
lrm.launion@deped.gov.ph