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MATHEMATICS

Quarter 2 - Module 2:
Solving Problems on Polynomial Functions

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MATHEMATICS 10
Quarter 2 - Module 2: Solving Problems on Polynomial Functions
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MATHEMATICS

Quarter 2 - Module 2:
Solving Problems on Polynomial Function

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Problem-Solving always plays a vital role in the success of every individual. It gives us a mechanism for identifying things, figuring out why they are broken and determining a course of action to fix them.

In this module, you will realize the importance of having proper skills in solving problems, choosing appropriate tools or skills to solve real-life problems involving polynomial functions and learn to appreciate the importance of polynomial functions in the real-world. Most problems involving polynomials can be solved by factoring, by evaluating and by modelling. However, there is no best strategy in solving problems but it is important to stick with the concept. In this module, it is suggested that you use calculator in your computation.

The most essential learning competency (MELC) for this module is:

- Solves problems involving polynomial functions. **(M10AL-IIb-2)**

After going through this module, you are expected to:

1. identify what technique to be used in solving problems involving polynomial functions
2. Solve problems involving polynomial functions.

Before you start doing the activities in this lesson, find out how much you already know about this module. Answer the pre-test in a separate sheet of paper.

PRE-ASSESSMENT

Directions: Read each problem carefully. Solve each applying what you have learned in your previous lessons. Write the letter of your choice in a separate sheet of paper.

1. A furniture dealer determines that its profit, **P**, in thousands of pesos, can be modeled by the function $P(x) = 0.0012x^3 + 2x - 25$, where **x** represents the number of furniture sold. What is the profit when x = 50?
   A. Php 200, 000 B. Php 225, 000
   C. Php 250, 000 D. Php 275, 000

2. The profit, **P** (in millions of pesos), for a flash drive manufacturer can be modeled by $P = n^3 + 6n^2 - 7n$ where **n** (in millions) is the number of flash drive produced. How many flash drives was produced if the profit reached 60 million pesos?

A. 3, 000, 000 B. 4, 000, 000
C. 5, 000, 000 D. 6, 000, 000

3. During a 20-year period, the amount (in millions of pesos) of school supplies(**S**) sold can be modeled by $S(t) = -20t^3 + 25t^2 - 280t + 3320$, where **t** is in years. Find the amount of school supplies sold in 5 years.

A. Php 35, 000, 000 B. Php 40, 000, 000
C. Php 45, 000, 000 D. Php 50, 000, 000

4. The weight, **w** (in kilograms), of a certain fish can be modeled by $w = 0.00304x^3$, where **x** is the length of the fish in inches. What is the weight of a 12-inch fish?

A. 5.00 kg B. 5.25 kg C. 5.50 kg D. 5.75 kg

5. The volume, **V** (in cubic feet), of a rectangular hydraulic block is modeled by the polynomial function $V(w) = 6w^3 - 19w^2 - 52w$ where **w** is the width (in feet) of the hydraulic block. What is the width of the hydraulic block if its volume is 15 cubic feet?

A. 4 feet B. 5 feet C. 6 feet D. 7 feet

6. The estimated number of electric vehicles, **V** (in thousands), in the Philippines can be modeled by the polynomial function:
$V(y) = 0.15128y^3 - 3.28234y^2 + 23.7565y - 2.041$ where **y** represents the number of years after the year 2005. Find the estimated number of electric vehicles at the end of the year 2007. Round up to the nearest thousands.

A. 32, 000 B. 33, 000
C. 34, 000 D. 35, 000

7. In this school year, 50 high school students participated in one-week Human vs Zombie game. The number of zombies(**Z)**, after days(**d**) of playing can be modeled by the function $Z(d) = 5d^3 - 3d^2 + 15$. How many days did it take for the number of zombies to reach 565?

A. 4 B. 5 C.6 D. 7

8. Faye is going to throw a rock from the top of a trail overlooking the ocean. When she throws the rock upward from 160 ft above the ocean, the function $h(t) = -16t^2 + 48t + 160$ models the height(**h)**, of the rock above the ocean as a function of time(**t**). Find the height of the rock at t = 1.5 seconds.

A. 148 ft B. 164 ft C. 180 ft D. 196 ft

For Item number 9 – 11, refer to the problem below.

On a glass factory, paperweights are created by pouring molten glass into molds. Each mold is a rectangular prism with a height 3 cm greater than the length of each side of its square base. Each mold holds 112 cubic centimeters of glass.

9. If **x** represents the length of each side of the square base, which of the following mathematical model satisfy the condition of the above problem?
   A. (x + 3) (x + 3) (x) = 112 B. (x + 3) (x) = 112
   C. (x + 3) (x) (x) = 112 D. 112 (x + 3) = (x) (x)

10. What is the height of the glass mold?
   A. 4 cm B. 5cm C. 6 cm D. 7 cm

11. What are the dimensions of the glass molds? (Follow V = LWH)
   A. 4 cm by 5 cm by 8 cm B. 5 cm by 5 cm by 8 cm
   C. 4 cm by 4 cm by 7 cm D. 5 cm by 8 cm by 8 cm

For Item number 12 – 15, refer to the problem below.

A construction company was hired to build a swimming pool for public use. The client wants the depth of the pool to be 2 meters less than the width and the length is 10 meters more than the width. The client also specified that the pool must have a water capacity of 595 cubic meters.

12. If **w** represents the measurement of the width, which of the following mathematical model satisfy the condition of the above problem?

   A. (w - 2) (w + 2) (w) + 10 = 595

   B. (w - 2) (w + 10) (w) = 595

   C. (w - 2) (w - 10) (w) = 595

   D. (w - 10) (w + 2) (w) = (595

13. How deep is the pool?
   A. 3 meters B. 4 meters C. 5 meters D. 6 meters

14. How long is the pool?
   A. 15 meters B. 17 meters C. 19 meters D. 21 meters

15. What are the dimensions of the swimming pool? (Follow V = LWH)
   A. 17 m by 5 m by 5 m B. 12 m by 5 m by 8 m
   C. 12 m by 2 m by 7 m D. 17 m by 7 m by 5 m

Were you surprised that polynomial functions can be applied in real – life problems and also have practical uses? What do you need to solve these problems? Enjoy learning as you proceed to the next section.

Jumpstart

To solve problems involving polynomials you need to recall the concepts learned in your previous modules. Your prior knowledge on the different concepts of polynomial functions will help you in solving problems related to real-life.

As you go through this lesson, always bear in mind that learning problem solving skills will build a mentally-ready individual like you in facing different challenges in life.

Let's Recall:

A polynomial function is a function of the form

$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + \ldots + + a_1 x + a_0 \quad a_n \neq 0$

where ***n*** is a **nonnegative integer**, $a_0, a_1, a_2, \ldots, a_n$ are real numbers called **numerical coefficients**, $\mathbf{a_n x^n}$ is the **leading term**, $\mathbf{a_n}$ is the **leading coefficient** and $\mathbf{a_0}$ is the **constant term**.

The term of a polynomial function may be written in any order. However, if they are written in decreasing powers/exponents of x, we say the polynomial function is in **standard form**.

The degree(n) of P(x) is the highest exponent or the highest sum of exponent of the variables of the leading term of the polynomial function.

Example 1:

$f(x) = 12x - 3x^3 - 8x^2 + x^4 + 16$ → is an example of polynomial function since all exponents are positive integers and $a_n \neq 0$

degree(n) = 4

$f(x) = x^4 - 3x^3 - 8x^2 + 12x + 16$ → is the polynomial function written in **standard form** since the exponent of x is written in decreasing power.

constant term($\mathbf{a_0}$) = 16

Leading term ($\mathbf{a_n x^n}$) = x^4

- In the leading term = x^4, the leading coefficient($\mathbf{a_n}$) is 1

Polynomial Function in Factored Form

- Another way to write polynomial function is the **Factored Form.** We can do this by factoring the polynomial part of the function using factoring polynomials, dividing polynomials or using synthetic division.
- The most effective and easiest way to factor polynomials with degree 3 or higher is using **synthetic division.**

Example 2:

What is the Factor Form of $f(x) = 12x - 3x^3 - 8x^2 + x^4 + 16$?

$f(x) = 12x - 3x^3 - 8x^2 + x^4 + 16$ Given Polynomial Function

$f(x) = x^4 - 3x^3 - 8x^2 + 12x + 16$ Standard Form

Applying synthetic division to determine its factors;

2	1	–3	–8	12	16
		2	–2	–20	–16
	1	–1	–10	–8	0

Since the remainder is 0, x = 2 is a solution of f(x).

To get the factor $x = 2$

$x - 2 = 2 - 2$ Use Addition Proper of Equality or Transposition

$x - 2 = 0$ Simplify

Therefore: (x – 2) is a factor of f(x)

–2	1	–1	–10	–8	
		–2	6		8
	1	–3	–4	0	

Since the remainder is 0, x = –2 is a solution of f(x).

To get the factor $x = -2$

$x + 2 = -2 + 2$ Use Addition Proper of Equality or Transposition

$x + 2 = 0$ Simplify

Therefore: (x + 2) is a factor of f(x)

4	1	–3	–4	0
		4	4	
	1	1	0	

Since the remainder is 0, x = 4 is a solution of f(x).

To get the factor x = 4

x + 4 = 4 – 4 — Use Addition Proper of Equality or Transposition

x + 4 = 0 — Simplify

Therefore: (x + 4) is a factor of f(x)

-1	1	1
		–1
	1	0

When the remaining quotient coefficient of the Synthetic Division is 1 and the remainder is 0, that means **the given polynomial function is now factored completely**

Since the remainder is 0, x = -1 is a solution of f(x).

To get the factor x = –1

x + 1 = –1 + 1 — Use Addition Proper of Equality or Transposition

x + 1 = 0 — Simplify

Therefore: (x + 2) is a factor of f(x)

Rewriting the given polynomial in its factored form:

f(x) = (x – 2) (x +2) (x – 4) (x +1)

The Zeros of Polynomial Function

The **zero of a polynomial function** f(x) is the value of the variable x which makes the polynomial function equal to zero or f(x)=0. Further, they are the x intercepts of the graph of the function. Hence, they can be computed by equating the function to zero. In the above example, the zeros of the function can be determined by:

f(x) = (x – 2) (x +2) (x – 4) (x +1)

Replace f(x) = 0; 0 = (x – 2) (x +2) (x – 4) (x +1)

By the zero-product property:

x – 2 = 0	x + 2 = 0	x – 4 = 0	x +1 = 0
x = 2	x = –1	x = 4	x = –1

Therefore, the zeros of the function are 2, –2, 4 and –1.

Finding Values of Polynomial Functions

Synthetic division, hand-in-hand with the Remainder Theorem can be used as a convenient way to find values of polynomial functions. The Remainder Theorem states that when the polynomial function f(x) is divided by (x – c), then the remainder is f (c).

For example, if the polynomial $f(x)= 2x^3 – 8x^2 +19x – 12$ is divided by x – 3, the remainder is f (3).

By synthetic division,

3	2	–8	19	–12
		6	–6	39
	2	–2	13	**27**

By the Remainder Theorem

$f(3) = 2(3)^3 – 8(3)^2 + 19(3) –12$

$= 2(27) – 8(9) + 57 – 12$

$= 54 – 72 + 57 – 12$

$f(3) =$ **27**

The remainder is 27. Therefore, f (3) = 27.

Activity 1: A Call for Recall

Answer each of the problems below by applying the previously learned skills in solving problems.

A. Determine the zeros of the following polynomial functions by Synthetic Division.

1. $f(x)= x^3 + 8x^2 +19x +12$
2. $f(x)= x^4 – x^3 – 11x^2 + 9x + 18$
3. $f(x)=x^4 – 7x^2 + 6x$

B. Find the value of the polynomial function using Synthetic Division or by Remainder Theorem.

4. $f(x)= x^3 – 4x^2 + x + 6,$ $x = 2$
5. $f(x)= 2x^3 – 9x – 5x^2 – 8,$ $x = 4$
6. $f(x)= 8x + x^3 – 5x^2 + 2,$ $x = 2$

The next activity will let you see the connection of the above concepts in the real-life situations. Study the following illustrative examples.

Discover

A. Problem Solving by Evaluating Polynomials

Example 1:

Suppose a windmill generates power based on the speed of the wind. This is represented by the polynomial function $P(s) = \frac{s^4}{1000}$, where s represents the speed of the wind in kilometers per hour. If the wind speed is 15 kilometers per hour, how many units of power, P(s), can the windmill generate?

Procedure:

Step 1	$P(s) = \frac{s^4}{1000}$, s = 15kph	Rewrite the given.
Step 2	Find P (15)	Identify what is asked on the problem. We have P (15) since it is asking on how many units of power can the wind mill generate if the windspeed is 15 kph.
Step 3	$P(15) = \frac{(15)^4}{1000}$	Replace all the variable s with 15, since s = 15 based on the given.
Step 4	$P(15) = \frac{50625}{1000}$	Simplify the equation. Remember that $(15)^4$ = (15((15) (15) (15) = 50625.
Step 5	P (15) = 50.625	By dividing 50625 by 1000, we arrive on the result 50.625.

Answer: The windmill can generate 50.625 units of power if the wind speed is 15 kilometers per hour.

Example 2:

The number of citizen (N) of Barangay San Isidro in t years is predicted to increase and can modeled by the function $N(t) = 7t^4 – 4t^3 + 150t + 17500$. After 3 years, how many citizens will be residing now at Barangay San Isidro?

Procedure:

Step 1	$N(t) = 7t^4 – 4t^3 + 150t + 17500$, $t = 3$ years	Rewrite the given.
Step 2	Find N (3)	Identify what is asked on the problem. We have N (3) since it is asking on how many citizens will be residing at Barangay San Isidro after 3 years.
Step 3	$N (3) = 7(3)^4 – 4(3)^3 + 150(3) + 17500$	Replace all the variable t with 3, since t = 3 based on the given.
Step 4	$N (3) = 7(81) – 4(27) + 150(3) + 17500$	Simplify the equation. Remember that $(3)^4 = (3)(3)(3)(3) = 81$ and $(3)^3 = (3)(3)(3) = 27$. This is to make multiplication easier.
Step 5	$N (3) = 567 – 108 + 450 + 17500$	Simplify each term. This is by multiplying 7 by 81 which is equal to 567, 4(27) = 108, and 150(3) = 450.
Step 6	N (3) = 18, 409	Add all values to get the final result. Thus, 567 – 108 + 450 + 17 500 is equal to 18 409.

Answer: There will be 18, 409 citizens residing at Barangay San Isidro after 3 years.

Activity 2: Substitution Time!

1. The number of eggs, **E(x)**, in a female butterfly is a function of her abdominal width, **x**, in millimeters, modeled by $E(x) = 14x^3 – 17x^2 – 16x + 34$. How many eggs can the butterfly carry if the width of her abdomen is 4 millimeters?

Step 1		Rewrite the given.
Step 2	Find E (4)	Identify what is asked on the problem. We have E(4) since it is asking on how many eggs can the butterfly carry if the width is 4 millimeters.
Step 3	E (4) = _____ – $17(4)^2$ – 16(4) +34	Replace all the variable x with 4, since x = 4 based on the given.
Step 4	E4) = 896-_____ – _____ + 34	Simplify the equation.
Step 5	E (4) = _____	Simplify to get the final result.

2. The polynomial function, $w(d) = 0.0071d^3 – 0.09d^2 + 0.48d$, models the weight of the ideal round-cut diamond where w is the diamond's weight (in carats) and d is the diameter (in millimeters). Based on the model, find the weight of the diamond with a diameter of 20 millimeters.

Step 1	
Step 2	
Step 3	
Step 4	
Step 5	w (20) = 30.4 carats

3. A drugstore that sells a certain brand of Vitamin C tablets predicts that their profit P can be modeled by the polynomial function $P(a) = –50a^3 + 2400a^2 – 2000$, where a is the amount spent on advertising (in thousands of pesos). What is its profit if a = 16?

Step 1	
Step 2	
Step 3	

Step 4	
Step5	P(a)= 407,600

B. Problem Solving by Factoring

The previous activity tells us how to solve problems involving polynomial functions through evaluation. That is, solving for the value of the dependent variable (example of this is P(x), V(x) given the value of the independent variable. Now, we will solve the problems using factoring, but the difference is the problems below are more on solving for the value of the independent variable given the value of the dependent variable. Here, we are going to discuss and solve problems together step by step using the lessons we have discussed previously. The problems presented below are things that we could relate to real-life situations.

Example 1:

A sculptor uses ice blocks to carve the wings of a dragon. The volume, V (in cubic centimeters), of a block of ice can be modeled by the $V(t) = t^3 + 8t^2 – 83t$, where t represents the thickness of the block of ice in centimeters. How thick is the block of ice with a volume of 630 cm^3?

Procedure:

Step 1	$V(t) = t^3 + 8t^2 – 83t$, $V(t) = 630\ cm^3$	First let us rewrite the given.
Step 2	Find t when V(t) = 630.	Then we identify what is asked on the problem.
Step 3	$630 = t^3 + 8t^2 – 83t$	We replace V(t) with 630 since it is given that V(t) = 630.
Step 4	$0 = t^3 + 8t^2 – 83t – 630$	We add –630 to both sides so that the other side is equal to zero.
Step 5	$0 = (t + 7)(t + 10)(t – 9)$	We now then solve the equation using factoring. You could also use synthetic division on solving for the value of t.
Step 6	$t + 7 = 0$ $t + 10 = 0$ $t –9 = 0$	We equate each factor by zero using the zero-product property and to solve for the value of t.

Step 7	t = –7, t = –10, t = 9	By solving each linear equation, we got t = –7, t = –10 and t = 9, but we are going to reject – 7 and –10 since there is no negative measurement for thickness. Thus, we got t = 9.

Answer: The block of ice should be 9 centimeters thick to have a volume of 630 cubic centimeters.

Example 2:

You own a succulent plant that propagates rapidly. You noticed that it propagates every week and by recording the number of seedlings, you came up with a polynomial function that tells you how many seedlings are there. The polynomial function $N(w) = 3w^3 + 10w^2 – 53w – 20$ represents the number of seedlings after **w** weeks. How many weeks are needed for the succulents to propagate up to 120 seedlings?

Procedure:

Step 1	$N(w) = 3w^3 + 10w^2 – 53w – 20$ $N(w) = 120$	First let us rewrite the given.
Step 2	Find w when N(w) = 120.	Then we identify what is asked on the problem.
Step 3	$120 = 3w^3 + 10w^2 – 53w – 20$	We replace N(w) with 120 since it is given that N(w)= 120.
Step 4	$0 = 3w^3 + 10w^2 – 53w – 20 – 120$ $0 = 3w^3 + 10w^2 – 53w – 140$	We add –120 to both sides so that the other side is equal to zero, then simplify the equation by combining similar terms.
Step 5	$0 = (w – 4)(3w + 7)(w + 5)$	We now then solve the equation using factoring. You could also use synthetic division on solving for the value of w.
Step 6	$w – 4 = 0$ $3w + 7 = 0$ $w + 5 = 0$	We equate each factor by zero using the zero-product property and to solve for the value of t.

Step 7	w = 4 w = –7/3 w = –5	By solving each linear equation, we got w = 4, w = –7/3 and w = –5, but we are going to reject –7/3 and –5 since there is no negative number of weeks Thus, we got w = 4.

Answer: Four weeks are needed for the succulents to propagate up to 120 seedlings.

Activity 3: Complete Me: Factoring is the Key

1. Suppose a windmill generates power based on the speed of the wind. This is represented by the polynomial function (s) = $s^2 - 5s$, where s represents the speed of the wind in kilometers per hour. If the windmill generated 50 units of power, what is the speed of the wind?

Procedure:

Step 1		First let us rewrite the given.
Step 2	Find s when P(s) = 50.	Then we identify what is asked on the problem.
Step 3	$50 = s^2 - 5s$	We replace P(s) with 50 since it is given than P(s) = 50.
Step 4	$0 = s^2 - 5s - 50$	We add –50 to both sides so that the other side is equal to zero.
Step 5	0 = (_____) (_____)	We now then solve the equation using factoring. You could use also quadratic equation on solving for the value of s.
Step 6		We equate each factor by zero by zero–product property and to solve for the value of s.
Step 7	s =_____ and s = _____	By solving each linear equation,

Answer: The speed of the wind should be _____kph to generate 50 units of power.

2. Alice was told that the volume of a rectangular block is modeled by the function $V(x) = 14x^3 - 57x^2 - 419x - 3$, where **x** is the length in meters of a rectangular block. How long is a rectangular block if it has a volume of 165 m^3?

Step 1	
Step 2	
Step 3	
Step 4	
Step 5	
Step 6	
Step 7	

Answer: The rectangular block must be 8 m long to have a volume of 165 m^3

3. A drugstore that sells a certain brand of Vitamin **C** tablets predicts that their profit, **P** (in millions of pesos), can be modeled by the polynomial function $P(a) = a^3 + 2a^2 - 5a - 1$, where a is the amount spent on advertising (in thousands of pesos). How much did they spent on advertising if they profited 5 million pesos?

Step 1	
Step 2	
Step 3	
Step 4	
Step 5	
Step 6	
Step 7	

Answer: The drug store must spend _____million pesos on advertisement to have a profit of 5 million pesos.

Were you able to apply all the necessary concepts in factoring polynomial function? The next activity will let you see another way of solving problems involving polynomial functions.

C. Problem Solving by Modelling Polynomials

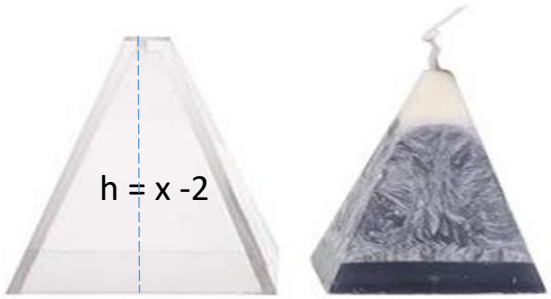
Example 1.

You are designing candle-making kits. Each kit contains 25 in^3 of wax and a mold for making a pyramid-shape candle with a square base. You want the height of the candle to be 2 inches less than the edge of the base. What should the dimensions of your candle mold be?

Step 1: Understand the problem by reading it many times if necessary.

Step 2: Devise a plan by translating it into your own language, illustrate the given problem if possible and put appropriate labeling as shown below and identify the appropriate theorem. Draw and illustration if needed to.

Let x be the side of the square base of the pyramid



- the height of the candle to be 2 inches less than the edge of the base

Volume of Square pyramid($25in^3$) = $\frac{1}{3}$ (Area of the square base)(height of the pyramid)

Step 3: Carry out the plan by solving the problem.

$25 = \frac{1}{3}x^2(x - 2)$ Substitution

$25 = \frac{1}{3}(x^3 - 2x^2)$ Distributive Property

$3\left[25 = \frac{1}{3}(x^3 - 2x^2)\right]3$ To eliminate the denominator multiply both sides of the equation by 3.

$75 = x^3 - 2x^2$

$0 = x^3 - 2x^2 - 75$ Transposition

Find real factors of $0 = x^3 - 2x^2 - 75$ using synthetic division.

Trial 1: x + 5

x = -5

-5	1	−2	0	−75
		−5	35	−175
	1	−7	35	−250

x + 5 is not a factor

Trial 2: x – 5
x = 5

5	1	–2	0	–75
		5	15	75
	1	3	15	0

x – 5 is a factor

Rewriting the remaining coefficient as polynomial
$x^2 + 3x + 15 = 0$ is not factorable and has no real roots.

The only real solution to the equation is 5.

Step 4: Look Back (check and interpret).

The side of the square base
x = 5 inches
The height of the pyramid candle molder
h = x – 2
h = 5 – 2
h = 3 inches

Therefore; The side of the square base is 5 inches and the height of the pyramid is 3 inches.

Example 2:

Mitzi is planning to make an ice sculpture for her parents. She has a rectangular block of ice, with a dimension 3ft x 4ft x 5ft, that she wants to reduce in size by shaving off the same amount from the length, width, and height. She wants to reduce the volume of the ice block into 24 cubic feet. What are the dimensions of the new ice block?

Step 1: Understand the problem by reading it many times if necessary. Identify what are given and asked.

Given: 3ft x 4ft x 5ft block of ice

Volume of reduced block is $24ft^3$

Required: What are the dimensions of the new ice block?

Step 2: Devise a Plan

Representation: x = amount of ice to be shaved off

(3 – x), (4 – x), (5 – x) = dimensions shaved off

Equating to the volume of reduced block:

(3 – x) (4 – x) (5 – x) = 24

Step 3: Carrying Out the Plan

Simplify the polynomial, first multiply (3 – x) by (4 – x) which results to $12 - 7x + x^2$. (You may also multiply the other terms first since multiplication is commutative.)

$$(12 - 7x + x^2)(5 - x) = 24$$

Next is multiply $(12 - 7x + x^2)$ by $(5 - x)$

$$60 - 47x + 12x^2 - x^3 = 24$$

Add -24 to both sides of the equation so that the other side will be equal to zero.

$$36 - 47x + 12x^2 - x^3 = 0$$

Arrange the polynomial in standard form so that it would be easier to factor later on.

$$-x^3 + 12x^2 - 47x + 36 = 0$$

Multiply both sides of the equation by –1 so that the leading term is positive and easier way when factoring the polynomial.

$$x^3 - 12x^2 + 47x - 36 = 0$$

Factor the polynomial. (Synthetic division may also be used in finding the factor of the polynomial).

$$(x - 1)(x^2 - 11x + 36) = 0$$

Set each factor equal to zero by zero product property.

$$(x - 1) = 0, \quad (x^2 - 11x + 36) = 0$$

Since the trinomial factor is no longer factorable, quadratic formula was used.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 - 11x + 36 = 0$; a = 1, b = –11, c = 36

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(36)}}{2(1)}$$

$$x = \frac{11 \pm \sqrt{121 - 144}}{2}$$

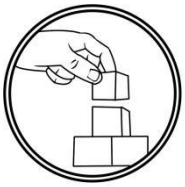
$$x = \frac{11 \pm \sqrt{-23}}{2}$$

$$x = 1, \; x = \frac{11 \pm \sqrt{-23}}{2}$$

By substituting x = 1 on (3 – x), (4 – x), and (5 – x), the new dimension of the ice block is 2ft by 3ft by 4 ft.

Step 4: Look Back (check and interpret)

2ft x 3 ft x 4 ft = 24 ft³



Explore

Activity 4:

Read the following problem carefully then solve.
(Show your solution in a separate sheet of paper.)

1. The dimensions of a cage are:

   height = 3x – 4 feet,

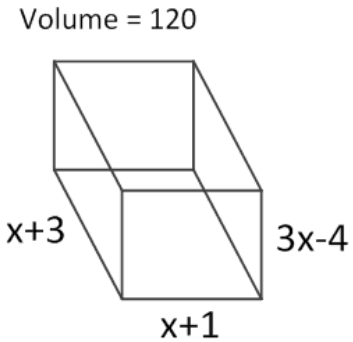
   length = x + 3, feet, and

   width = x + 1 feet.

   If the volume of the cage is 120 cubic feet, what is the value of x?

Rubric for Problem Solving

4	3	2	1
Used an appropriate strategy to come up with a correct solution and arrived at a correct answer	Used an appropriate strategy to come up with a solution, but a part of the solution led to an incorrect answer	Used an appropriate strategy but came up with an entirely wrong solution that led to an incorrect answer	Attempted to solve the problem but used an inappropriate strategy that led to a wrong solution



Deepen

Activity 5:

Read the following problems carefully then solve.

1. The member of a group of packaging designer of a gift shop are looking for a precise procedure to make an open rectangular box with a volume of 560 cubic inches from 24-inch by 18-inch rectangular piece of material. The main problem is how to identify the side of identical squares to be cut from the four corners of the rectangular sheet so that the box can be made. What will you do to meet the specification needed for the box? Show a mathematical solution. The figure below can help solve the problem.

Rubric for Problem Solving

4	3	2	1
Used an appropriate strategy to come up with a correct solution and arrived at a correct answer	Used an appropriate strategy to come up with a solution, but a part of the solution led to an incorrect answer	Used an appropriate strategy but came up with an entirely wrong solution that led to an incorrect answer	Attempted to solve the problem but used an inappropriate strategy that led to a wrong solution

Gauge

POST ASSESSMENT

Directions*:* Answer each of the given problems applying the techniques learned from this module. Write the letter of your choice in a separate sheet of paper.

1. The weight, **w** (in kilograms), of a certain crab can be modeled by $w = 0.00304x^3$, where **x** is the length of the arm span of the crab in inches. What is the weight of a crab with an arm span of 12 inch?

   A. 5.00 kg B. 5.25 kg C. 5.50 kg D. 5.75 kg

2. The profit, **P** (in millions of pesos), for a flash drive manufacturer can be modeled by $P = n^3 + 2n^2 - 23n$ where **n** (in millions) is the number of flash drive produced. How many flash drives was produced if the profit reached 60 million pesos?

   A. 3, 000, 000 B. 5, 000, 000
   C. 4, 000, 000 D. 6, 000, 000

3. An object moves along the horizontal in a straight line according to the function $d = 4t - 2t^2$, where **d** is the distance (in meters) and **t** is the time in seconds. How long will it take the object to travel 70 meters?

   A. 5 sec B. 6sec C. 7 sec 8. 9 sec

4. A sofa set dealer determines that its profit, **P**, in thousands of pesos, can be modeled by the function $P(x) = 0.0012x^3 + 2x - 25$, where **x** represents the number of sofa set sold. What is the profit when x = 20?

   A. Php 24, 600 B. Php 25, 700
   C. Php 26, 300 D. Php 27, 900

5. Edd is going to throw a rock from the top of a trail overlooking the ocean. When he throws the rock upward from 160 ft above the ocean, the function $h(t) = -16t^2 + 48t + 160$ models the height, **h**, of the rock above the ocean as a function of time, **t**. Find the height of the rock at t = 2 seconds.

   A. 148 ft B. 164 ft C. 180 ft D. 192 ft

6. For 12 years that a grocery store has been open. Its annual revenue (in million pesos) can be modeled by the function $R = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$ when **t** is the number of years the store is open. In which year was the revenue 1.5 million?

   A. 4^{th} and 5^{th} B. 3^{rd} and 6^{th} C. 3^{rd} and 9^{th} D. 11^{th} and 12^{th}

7. Suppose you have 150 cubic inches of clay with which to make a sculpture shaped as a rectangular prism. You want the height and width to be the same and the length is 1 inch more than the width. What should the length of the prism be?

   A.5 in B.6 in C.7 in D.8 in

8. You are designing a rectangular swimming pool that is to be set into the ground. The width of the pool is 5 feet more than the depth, and the length is 35 feet more than the depth. The pool holds 2,000 cubic feet of water. What are the dimensions of the pool?

A. 5,20,20 B. 4,10,50 C. 4,20,25 D. 5,10,40

For Item number 9 – 11, refer to the problem below.

On a glass factory, paperweights are created by pouring molten glass into molds. Each mold is a rectangular prism with a height 5 cm greater than the length of each side of its square base. Each mold holds 1008 cubic centimeters of glass.

9. If **x** represents the length of each side of the square base, which of the following mathematical model satisfy the condition of the above problem?

A. (x + 5) (x) = 1008 B. (x + 5) (x + 5) (x) = 1008

C. (x + 5) (x) (x) = 1008 D. 1008 (x + 5) = (x) (x)

10. What is the height of the glass mold?

A. 5 cm B. 7 cm C. 12 cm D. 16 cm

11. What are the dimensions of the glass molds? (Follow V = LWH)

A. 7 cm by 5 cm by 12 cm B. 7 cm by 7 cm by 12 cm
B. 7 cm by 12 cm by 12 cm D. 7 cm by 8 cm by 8 cm

For Item number 12 – 15, refer to the problem below.

A construction company was hired to build a swimming pool for an attraction and high lights in a wedding venue. The client wants the width of the pool to be 2 meters less than the depth and the length is 10 meters more than the depth. The client also specified that the pool must have a water capacity of 595 cubic meters.

12. If d represents the measurement of the depth, which of the following mathematical model satisfy the condition of the above problem?

A. (d – 2) (d + 2) (d) + 10 = 595 B. (d – 10) (d + 2) (d) = 595
C. (d – 2) (d – 10) (d) = 595 D. (d – 2) (d + 10) (d) = 595

13. How deep is the pool?

A. 1 meters B. 3 meters C. 5 meters D. 7 meters

14. How long is the pool?

A. 15 meters B. 17 meters C. 19 meters D. 21 meters

15. What are the dimensions of the swimming pool? (Follow V = LWH)

A. 17 m by 5 m by 7 m B. 12 m by 5 m by 7 m
C. 12 m by 2 m by 7 m D. 17 m by 8 m by 5 m

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