8

Mathematics

Quarter 3 Week 6 – Module 4: Proving Two Triangles to be Congruent

AIRs - LM

Mathematics 8 Quarter 3- Week 6 Module 4
Proving Two Triangles to be Congruent
First Edition, 2021

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Target

Have you ever wondered how bridges and buildings are designed? What factors are considered in the construction of bridges and buildings? Designing them requires the knowledge of triangle conruence, its properties and principles.

In your previously studied lesson, you are done with the different postulates, theorems, definitions and concepts that will help you a lot in understanding the lessons in this module.

After going through this module, you are expected to attain the following:

Learning Competency:

- Proves two triangles are congruent. **(M8GE-IIIg-1)**

Learning Objectives:

1. Proves two triangles are congruent by applying the SSS, SAS, ASA Congruence Postulates and AAS/SAA Congruence Theorem
2. Uses the two – column proof in proving two triangles are congruent deductively.

LESSON 1

Proving Two Triangles to be Congruent

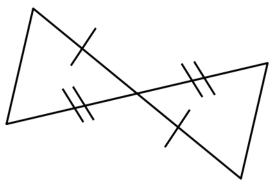
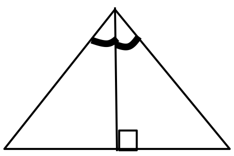
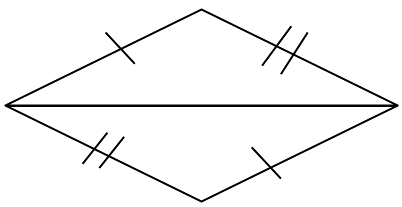
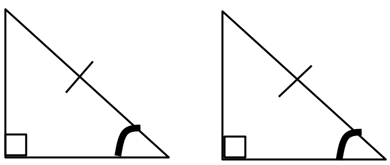
Jumpstart

This activity will enable you to assess your prior knowledge on proving two triangles are congruent deductively.

Let us begin the lesson by accomplishing the activity below.

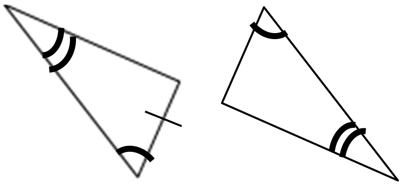
Activity 1. Can You Match Us!

A. Directions: Match the given figures, together with their markings in Column A with appropriate postulate or theorems that they illustrate in Column B. Write the letter of your correct answer only.

	COLUMN A	COLUMN B
	1. 	A. AAS Congruence Theorem
	2. 	B. SAS Congruence Postulate
	3. 	C. SSS Congruence Postulate
	4. 	D. ASA Congruence Postulate

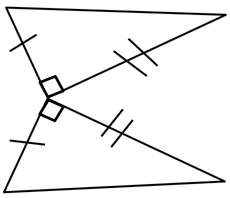
B. Directions: If enough information is/are given, state the postulate ot theorem that proves the congruence of each pair of triangles; otherwise write no congruence.

5.



Answer:________

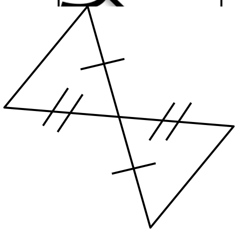
6.



Answer:________

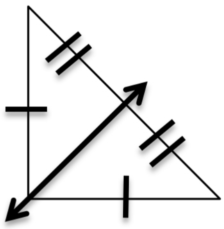
7.

8.



Answer:________

9.



Answer:________

Have you answered the activities correctly? Do you need all the six corresponding parts of two triangles to prove that they are congruent?

The concepts you have just learned from the previous activities are helpful in understanding the lesson in proving two triangles are congruent.

Discover

To prove that two triangles are congruent, you need to show that all the corresponding parts of two triangles are congruent. However, you **do not need all of the corresponding sides and corresponding angles to be congruent to prove that the two triangles are congruent.**

Let us find out how we apply the different postulates and theorems on triangle congruence to prove deductively that two triangles are congruent.

A. SSS Congruence Postulate (Side-side-side Postulate)

"If three sides of one triangle are congruent to the corresponding sides of another triangle, then the two triangles are congruent."

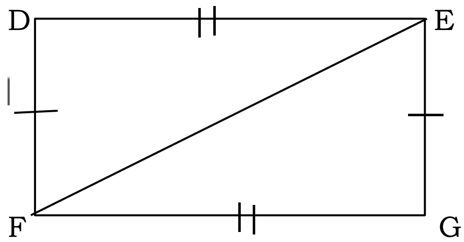
Study the following examples:

Illustrative Example1. Write a two-column proof.

Given: $\overline{DE} \cong \overline{GF}$

$\overline{DF} \cong \overline{GE}$

Prove: $\Delta DEF \cong \Delta GFE$



Proof:

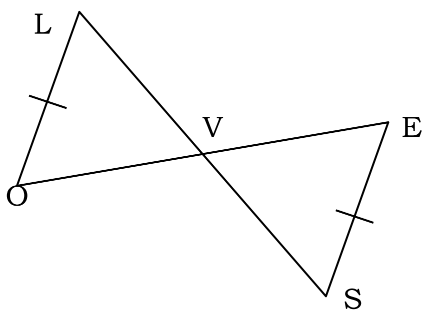
Statements	Reasons
1. $\overline{DE} \cong \overline{GF}$ (S)	Given
2. $\overline{DF} \cong \overline{GE}$ (S)	Given
3. $\overline{EF} \cong \overline{EF}$ (S)	Reflexive Property
4. $\therefore \Delta DEF \cong \Delta GFE$	SSS Congruence Postulate

From the diagram, we know that $\overline{DE} \cong \overline{GF}$ and $\overline{DF} \cong \overline{GE}$. By Reflexive Property, $\overline{EF} \cong \overline{EF}$. Thus, enough information is given.

Because corresponding sides are congruent, we can use the SSS Congruence Postulate to prove that $\Delta DEF \cong \Delta GFE$.

Illustrative Example 2

Write a two-column proof.



Given: $\overline{LS}$ *and* $\overline{OE}$ bisect each other at V

$\overline{LO} \cong \overline{SE}$

Prove: $\Delta LVO \cong \Delta SVE$

Proof:

Statements	Reasons
1. $\overline{LS}$ *and* $\overline{OE}$ bisect each other at V	Given
2. $\overline{LV} \cong \overline{SV}$ (S)	Definition of Segment Bisector
3. $\overline{OV} \cong \overline{EV}$ (S)	Definition of Segment Bisector
4. $\overline{LO} \cong \overline{SE}$ (S)	Given
4. $\therefore \Delta LVO \cong \Delta SVE$	SSS Congruence Postulate

The corresponding three sides of the two triangles, therefore, by SSS Congruence Postulate, $\Delta LVO \cong \Delta SVE$.

B. SAS Congruence Postulate (Side-Angle-Side Postulate)

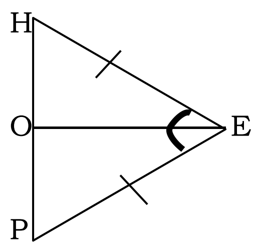
If two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent.

Illustrative example 3. Write a two-column proof.

Given: $\overline{HE} \cong \overline{PE}$

$\angle$HEO $\cong \angle$PEO

Prove: $\Delta HEO \cong \Delta PEO$



Proof:

Statements	Reasons
1. $\overline{HE} \cong \overline{PE}$ (S)	Given
2. $\angle$HEO $\cong \angle$PEO (A)	Given
3. $\overline{EO} \cong \overline{EO}$ (S)	Reflexive Property
4. $\therefore \Delta HEO \cong \Delta PEO$	SAS Congruence Postulate

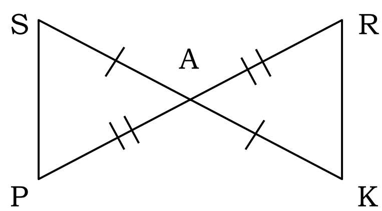
From the diagram, it was given that $\overline{HE} \cong \overline{PE}$ (side) and $\angle$HEO $\cong \angle$PEO (angle). By Reflexive Property, $\overline{EO} \cong \overline{EO}$ (side). Because corresponding two sides and the included angle of ΔHEO *and* ΔPEO are congruent, we can use the SAS Congruence Postulate to prove that $\Delta HEO \cong \Delta PEO$.

Illustrative example 4. Write a two-column proof.

Given: $\overline{SA} \cong \overline{KA}$

$\overline{PA} \cong \overline{RA}$

Prove: $\Delta SPA \cong \Delta KRA$



Proof:

Statements	Reasons
1. $\overline{SA} \cong \overline{KA}$ (S)	Given
2. ∠SAP ≅ ∠KAR (A)	Vertical angles are congruent (Vertical Angles Theorem)
3. $\overline{PA} \cong \overline{RA}$ (S)	Given
4. $\therefore \Delta SPA \cong \Delta KRA$	SAS Congruence Postulate

From the diagram, it is given that $\overline{SA} \cong \overline{KA}$ (side) and $\overline{PA} \cong \overline{RA}$ (side). We can conclude that ∠SAP ≅ ∠KAR by Vertical Angles Theorem (Vertical angles are congruent).

∠SAP is the included angle of $\overline{SA}$ *and* $\overline{PA}$.

∠KAR is the included angle of $\overline{KA}$ *and* $\overline{RA}$.

Because corresponding two sides and the included angle of ΔSPA *and* ΔKRA are congruent, we can use the SAS Congruence Postulate to prove that $\Delta SPA \cong \Delta KRA$.

C. ASA Congruence Postulate (Angle-Side-Angle Postulate)

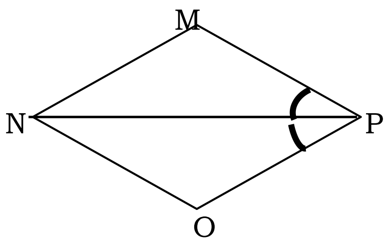
If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the two triangles are congruent.

Illustrative example 5. Write the complete proof.

Given: $\overline{NP}$ bisects ∠MNO

∠MPN ≅ ∠OPN

Prove: $\Delta NMP \cong \Delta NOP$



Proof:

Statements	Reasons
1. $\overline{NP}$ bisects ∠MNO	Given
2. ∠MNP ≅ ∠ONP (A)	Definition of Angle Bisector
3. $\overline{NP} \cong \overline{NP}$ (S)	Reflexive Property
3. ∠MPN ≅ ∠OPN (A)	Given
4. $\therefore \Delta NMP \cong \Delta NOP$	ASA Congruence Postulate

From the diagram, it is given that $\overline{NP}$ bisects ∠MNO. We can conclude that ∠MNP ≅ ∠ONP by the Definition of Angle Bisector.

$\overline{NP}$ is the included side of ∠MNP and ∠NPM.

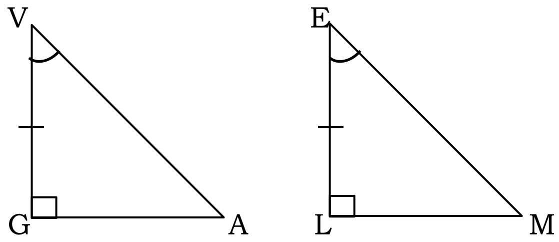
$\overline{NP}$ is the included side of ∠ONP and ∠NPO.

Since corresponding two angles and the included side of ΔNMP and ΔNOP are congruent, we can use the ASA Congruence Postulate to prove that $\Delta NMP \cong \Delta NOP$.

Illustrative example 6. Write a complete proof for the following.
Given: ΔVGA and ΔELM are right triangles.

$\angle V \cong \angle E$
$\overline{VG} \cong \overline{EL}$

Prove: $\Delta VGA \cong \Delta ELM$



Proof:

Statements	Reasons
1. ΔVGA and ΔELM are right triangles.	Given
2. $\angle G$ and $\angle L$ are right angles	Definition of Right Triangle
3. $\angle G \cong \angle L$ (A)	Right Angles Theorem (All right angles are congruent.)
3. $\overline{VG} \cong \overline{EL}$ (S)	Given
3. $\angle V \cong \angle E$ (A)	Given
4. $\therefore \Delta VGA \cong \Delta ELM$	ASA Congruence Postulate

From the diagram, it is given that ΔVGA and ΔELM are right triangles. We can conclude that $\angle$ $\angle G$ and $\angle L$ are right angles by the Definition of Right Triangle.

$\overline{VG}$ is the included side of $\angle AVG$ and $\angle VGA$.
$\overline{EL}$ is the included side of $\angle MEL$ and $\angle ELM$.

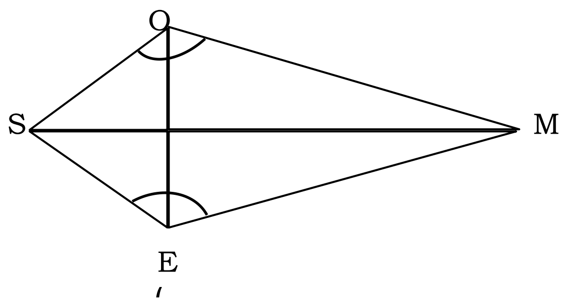
Since the two corresponding angles and the included side of ΔVGA and ΔELM are congruent, we can use the ASA Congruence Postulate to prove that $\Delta VGA \cong \Delta ELM$.

D. AAS or SAA Congruence Theorem

Another method of proving congruent triangles is the AAS or SAA Congruence Theorem. It states that, ***"If two angles and a non-included side of one triangle are congruent to the corresponding parts of another thriangle, then the two traingles are congruent."***

Illustrative example 7. Write a two-column proof.

Given: $\overline{SM}$ bisects $\angle OME$
$\angle SOM \cong \angle SEM$
Prove: $\Delta MOS \cong \Delta MES$



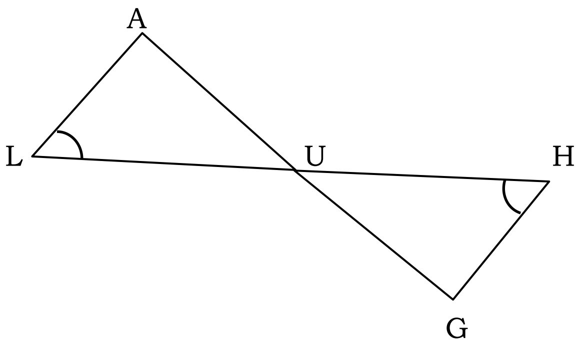
Proof:

Statements	Reasons
1. $\overline{SM}$ bisects ∠OME	Given
2. ∠SMO ≅ ∠SME (A)	Definition of Angle Bisector
3. ∠SOM ≅ ∠SEM (A)	Given
3. $\overline{SM}$ ≅ $\overline{SM}$ (S)	Reflexive Property
4. ∴ $\Delta MOS \cong \Delta MES$	SAA or AAS Congruence Postulate

Two corresponding angles and a non-included sides of the two triangles were proven to be congruent. Thus, we can say that the two triangles are congruent by the AAS/SAA Congruence Theorem.

Illustrative example 8. Write a two-column proof.

Given: U is the midpoint of $\overline{AG}$
∠L ≅ ∠H
Prove: $\Delta LAU \cong \Delta HGU$



Proof:

Statements	Reasons
U is the midpoint of $\overline{AG}$	Given
2. $\overline{UA} \cong \overline{UG}$ (S)	Definition of Midpoint
3. ∠L ≅ ∠H (A)	Given
3. ∠LUA ≅ ∠HUG (A)	Vertical Angles Theorem
4. ∴ $\Delta LAU \cong \Delta HGU$	SAA or AAS Congruence Postulate

There are three corresponding parts that were proven to be congruent: (1) $\overline{UA} \cong \overline{UG}$, by the Definition of Midpoint; (2) ∠L ≅ ∠H , it was a given information; and (3) ∠LUA ≅ ∠HUG, from the diagram, these are vertical angles, and we know that vertical angles are congruent (Vertical Angles Theorem). There is enough information, thus, we can say that $\Delta LAU \cong \Delta HGU$ by the AAS or SAA Congruence Theorem.

There is no ASS or SSA Postulate or Theorem (except for right triangles).

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

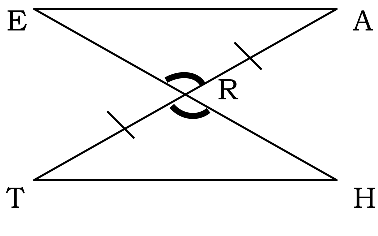
Activity 2. Can You Tell Me?

A. Directions: Identify the conclusion from Column B and the reason of your conclusion from Column C of the following given statements in Column A. Write the letter of your answer on the space provided for.

	COLUMN A		COLUMN B	COLUMN C
1.	$\angle AES$ and $\angle AEV$ are right angles.	8.	A. $\angle 1 \cong \angle 2$	A. Vertical Angles Theorem
2.	X is the midpoint of $\overline{EG}$	9.	B. $\overline{EC} \cong \overline{EC}$	B. Right Angles Theorem
3.	$\angle 1$ and $\angle 2$ are vertical angles	10.	C. $\overline{PB} \cong \overline{QB}$	C. Definition of Angle Bisector
4.	$\overline{EC} \cong \overline{EC}$	11.	D. $\angle BAD \cong \angle DAC$	D. Reflexive Property
5.	$\overline{AB}$ bisects $\overline{PQ}$ at B	12.	E. $\angle AES \cong \angle AEV$	E. Definition of Midpoint
6.	$\overline{AD}$ bisects $\angle BAC$	13.	F. b=a	F. Definition of Segment Bisector
7.	a=b	14.	G. $\overline{EX} \cong \overline{GX}$	G. Symmetric Property

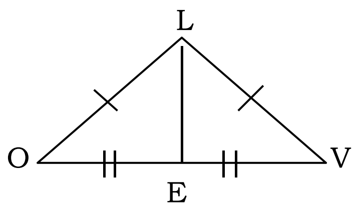
B. Directions: Indicate the additional information required to prove that the two triangles are congruent applying the specified congruence postulate ot theorem.

1. ASA Congruence Postulate



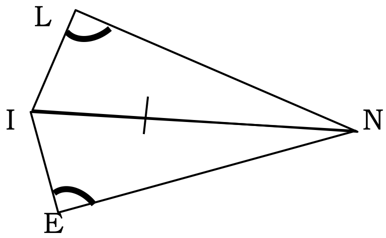
Answer:_________

2. SSS Congruence Postulate



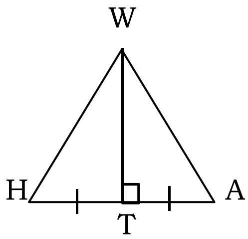
Answer:_________

3. AAS Congruence Theorem



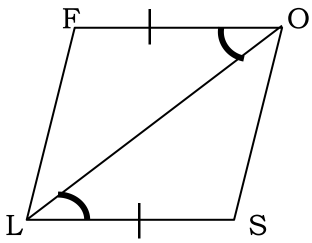
Answer:_________

4. SAS Congruence Postulate

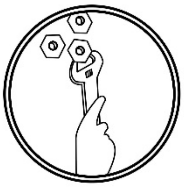


Answer:_________

5. SAA Congruence Theorem



Answer:________



Deepen

Activity 3. Fill – in – the- Missing…

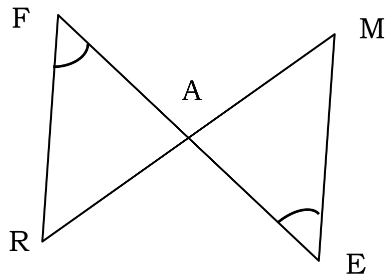
Directions: Fill - in the missing statements and reasons to prove that the two triangles are congruent.

A. Given: A is the midpoint of $\overline{FE}$

$\overline{FE}$ intersects $\overline{RM}$ at point A

∠RFA ≅ ∠MEA

Prove: $\Delta FAR \cong \Delta EAM$



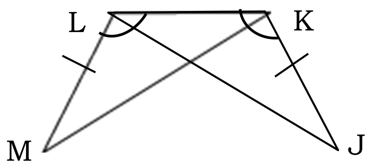
Proof:

Statements	Reasons
1.A is the midpoint of $\overline{FE}$	1.__________________
2.________________________	Definition of Midpoint
3.________________________	Given
∠FAR ≅ ∠EAM	4.__________________
$\Delta FAR \cong \Delta EAM$	5.__________________

B. Given: ∠KLM ≅ ∠LKJ

$\overline{KJ} \cong \overline{LM}$

Prove: $\Delta MLK \cong \Delta JKL$



Proof:

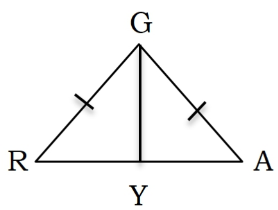
Statements	Reasons
1. _________________	Given
$\overline{KJ} \cong \overline{LM}$	2.______________
3.__________________	Reflexive Property
4. _________________	5.____________________

C. Given: $\overline{GY}$ bisects $\overline{RA}$

$\overline{RG} \cong \overline{AG}$

Prove: $\Delta RGY \cong \Delta AGY$

Proof:



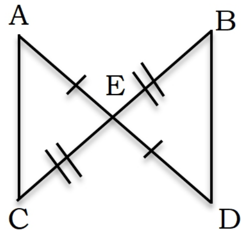
Statements	Reasons
$\overline{GY}$ bisects $\overline{RA}$	Given
$\overline{RY} \cong \overline{AY}$	1.______________
2.__________________	Given
$\overline{GY} \cong \overline{GY}$	3.____________________
4.__________________	SSS Postulate

Gauge

I. DIRECTIONS: For each pair of triangles, tell which postulate or theorem (SSS,SAS,ASA,AAS/SAA), if any, can be used to prove that the triangles are congruent.

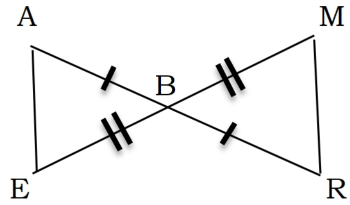
1. $\Delta ACE \cong DBE$

Answer:________



2. $\Delta ABE \cong RBM$

Answer:_______

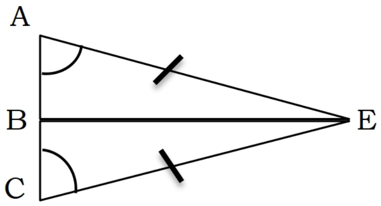


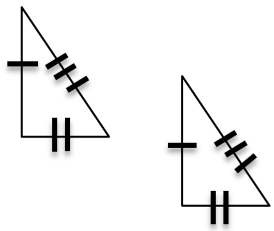
3. $\Delta ABE \cong CBE$

Given:

$\overline{BE}$ bisects ∠AEC

Answer:________



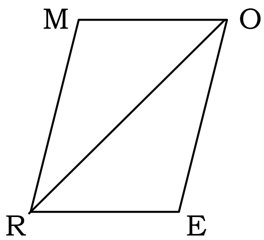


II. DIRECTIONS: Fill in the blanks to complete the proof. Write the letter of your answer from the choices given.

A. Given: $\overline{MO} \cong \overline{ER}$

$\overline{MR} \cong \overline{EO}$

Prove: $\Delta MOR \cong ERO$



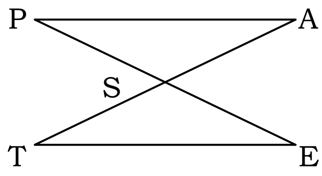
Proof:

Statement	Reason
$\overline{MO} \cong \overline{ER}$	1.___________
2.___________	3.___________
$\overline{RO} \cong \overline{RO}$	4.___________
$\Delta MOR \cong ERO$	5.___________

A. Given
B. Symmetric Property
C. Reflexive Property
D. SSS Postulate
E. SAS Postulate
F. $\overline{MR} \cong \overline{EO}$

B. Given: $\angle A \cong \angle T$
S is the midpoint of $\overline{TA}$

Prove: $\Delta PSA \cong \Delta EST$



Proof:

Statement	Reason
$\angle A \cong \angle T$	1.
2.	Given
3.____________	Definition of Midpoint
4.	Vertical angles are congruent.
$\Delta PSA \cong \Delta EST$	5._____________

III. DIRECTIONS: Do the following:

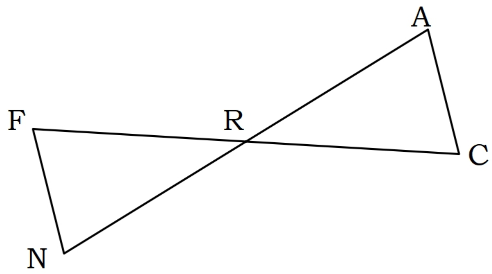
A. Show the given information in the diagram (using tick marks to show congruent sides and arcs to show congruent angles)

B. Show any other congruent parts you notice (from vertical angles, sides shared in common)

C. Prove completely that $\Delta RFN \cong \Delta RCA$ using two-column proof.

D. Give the postulate or theorem that proves the triangles congruent (SSS, SAS, ASA, AAS)

1. Given: $\overline{FR} \cong \overline{RC}$
$\overline{NR} \cong \overline{AR}$
Prove: $\Delta RFN \cong \Delta RCA$



Proof:

Statement	Reason

Great job! You are done with this module.