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MATHEMATICS

Quarter 2 - Module 5:
Tangents, Secants, Segments and Sectors of a Circle

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MATHEMATICS 10
Quarter 2 - Module 5: Tangents, Secants, Segments and Sectors of a Circle
Second Edition, 2021

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Printed in the Philippines by: ___________________________

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MATHEMATICS

Quarter 2 - Module 5:
Tangents, Secants, Segments and Sectors of a Circle

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Have a great day! In this module you will be learning about the different geometric relationships involving tangents, secants, segments and sectors of a circle. You are also given the opportunity to design something practical where tangents, secants, segments and sectors of a circle are illustrated and applied. Likewise, it also focuses on the theorems involving secants, tangents and segments and their properties particularly on the angles that they form. This module will also show how the measures of the angles formed by tangents and secants can be determined and other aspects on how to compute for the measures of the angles.

After going through this module, you are expected to attain the following objectives:

Learning Competency:

1. Illustrates secants, tangents, segments and sectors of a circle.(**M10GE-IIe–1)**
2. Proves theorems on secants, tangents and segments.(**M10GE-IIe-f-1)**

Subtasks:

- Recall and identify secants, tangents, segments and sectors of a circle.
- Illustrate and determine the secants, tangents, segments and sectors of a circle.
- Show the relationship between tangents, secants and segments of a circle.

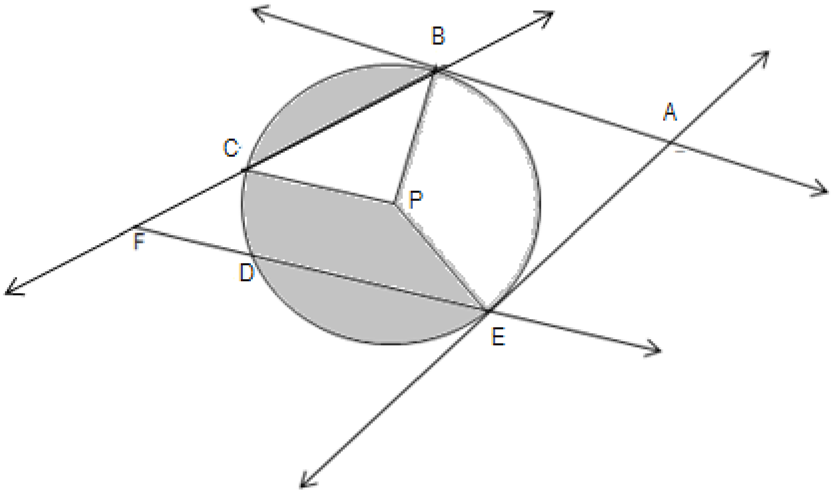
Before going on, find out how much you already know about the topic in this module. Answer the pre-assessment below.

Pre – Assessment

Directions: Read and understand the questions below. Select the best answer to each item then write the letter of the correct answer in a separate sheet.

1. What do you call the line coplanar with the circle and intersects it in exactly two points?
   A. secant B. sector C. segment D. tangent
2. Which of the following terms refers to the region bounded by two radii and the minor arc they determine?
   A. secant segment B. sector of a circle
   C. segment of a circle D. tangent segment
3. Which of the following lines refers to a line that is tangent to two circles in the same plane?
   A. common tangent B. curve
   C. secant D. tangent

For number 4 to 7, refer to the figure below.



4. Which of the following is a tangent line?
   A. $\overleftrightarrow{AB}$ B. $\overline{CB}$ C. $\overrightarrow{FE}$ D. $\overleftrightarrow{BF}$
5. What do you call the region bounded by $\overline{PC}$, $\overline{PE}$ and $\widehat{CE}$?
   A. segment CDE B. sector BPA C. sector CPE D. sector BPC
6. Which of the following segments is an external secant segment?
   A. $\overline{AB}$ B. $\overline{PB}$ C. $\overline{DF}$ D. $\overline{FB}$
7. Which of the following is a secant segment?
   A. $\overline{AB}$ B. $\overline{PB}$ C. $\overline{DF}$ D. $\overline{FB}$
8. If two chords intersect each other inside a circle, then the products of their segments are equal. What theorem corresponds to this statement?
   A. Radius-Tangent B. Intersecting Chord
   C. Perpendicular Chord D. Tangent-Secant
9. What is the process in finding the measure of an angle formed by secants intersecting inside the circle?
   A. twice the sum of the measures of the arc intercepted by the angle and its vertical angle pair
   B. twice the sum of the measures of the arc intercepted by the angle and its intercepted arc
   C. one-half the sum of the measures of the arc intercepted by the angle and its vertical angle pair
   D. one-half the sum of the measures of the arc intercepted by the angle and its intercepted arc
10. How would you find the measure of the angle formed by two secants intersecting outside the circle?
   A. twice the difference of the two intercepted arcs
   B. thrice the difference of the two intercepted arcs
   C. one-half the difference of the two intercepted arcs
   D. one-third the difference of the two intercepted arcs

11. In Figure 1 below, $m\widehat{XY} = 140$ and $m\widehat{MN} = 20$. What is $m\angle XPY$?

A. 60 B. 90 C. 120 D. 180

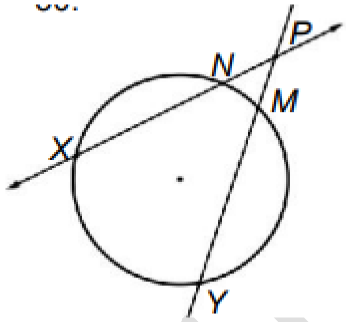


Figure 1

12. In the Figure 2, $\overline{CB}$ and $\overline{CD}$ are tangents to circle A at B and D. If CB = 16, what is CD?

A. 14 B. 16 C. 18 D. 20

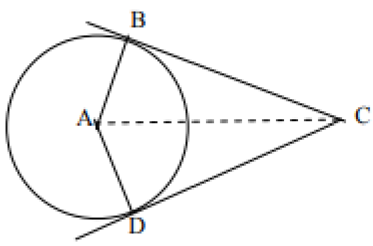


Figure 2

13. In the same figure above, if $m\angle BAC = 40$, what is $m\angle BCA$?

A. 30 B. 40 C. 50 D. 60

14. Which of the following is the length of $\overline{ZK}$ in the figure 3?

A. 2.86 units B. 6 units C. 8 units D. 8.75 units

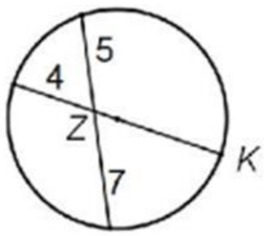
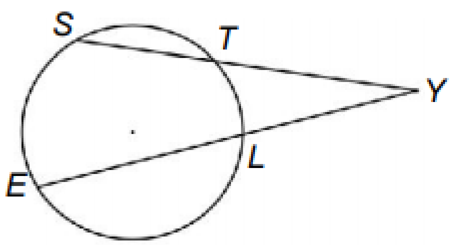


Figure 3

15. In the figure below, $\overline{SY}$ and $\overline{EY}$ are secants. If $SY = 15$ cm, $TY = 6$ cm, and $LY = 8$ cm. What is the length of $\overline{EY}$?

A. 20 cm B. 12 cm
C. 11.25 cm D. 6.75
cm



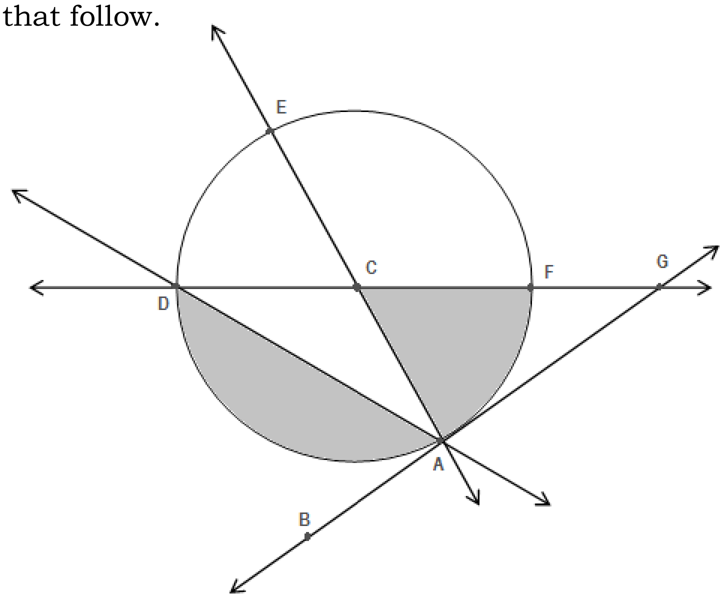
Lesson 1 Tangents, Secants, Segments, and Sectors of a Circle

Jumpstart

Let us start this lesson by assessing your knowledge of the different mathematical concepts previously studied and other mathematical skills learned. These knowledge and skills will help you understand the different geometric relationships involving tangents and secants of a circle, relationships among tangent and secant segments, and segment and sector of a circle. If you find any difficulty in answering the different exercises, seek the assistance of your teacher or peers or refer to the modules you have studied earlier.

For you to understand the lesson well, do the following activity. Good luck!!

Activity 1: Investigate Me!
Direction: In the figure below, C is the center of the circle. Use the figure to answer the following questions that follow.



Questions:
1. Which lines intersect circle C at two points?
2. How about the lines that intersect circle C at exactly one point?
3. Which shaded region bounded by an arc and the segment joining its endpoint?
4. Which shaded region bounded by an arc of the circle and the two radii to the endpoints of the arc?

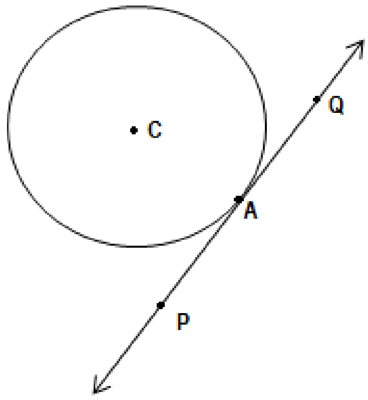
Were you able to determine and identify all the lines, name all the segments, and shaded regions of the circle? I am sure it was! This time, find out the relationships among tangent and secant lines, tangent and secant segments and segment and sector of the circle.

Discover

Tangent Line

A **tangent to a circle** is a line coplanar with the circle and intersects it in one and only one point. The point of intersection of the line and the circle is called the **point of tangency**.

Example: In the figure on the right, $\overleftrightarrow{PQ}$ intersects $\odot C$ at A. $\overleftrightarrow{PQ}$ is a tangent line and A is the point of tangency.



Tangent Segment

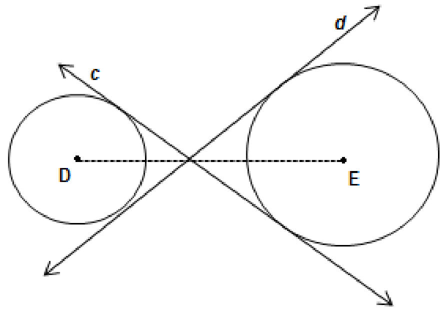
A **tangent segment** is a segment of a tangent line whose endpoints are the point of tangency and any other point on the tangent line.

In the figure above, $\overline{AP}$ and $\overline{AQ}$ are tangent segments.

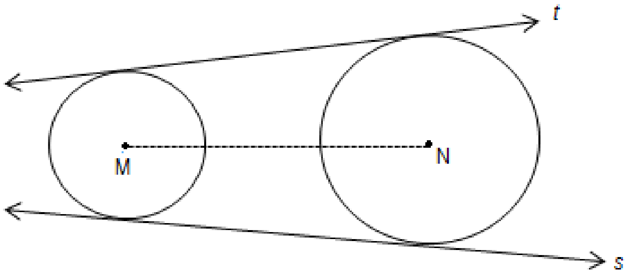
Common Tangent

A **common tangent** is line that is tangent to two circles in the same plane.

Common internal tangents intersect the segment joining the centers of the two circles	**_Common external tangents_** do not intersect the segment joining the centers of the two circles



Lines c and d are
common internal tangents

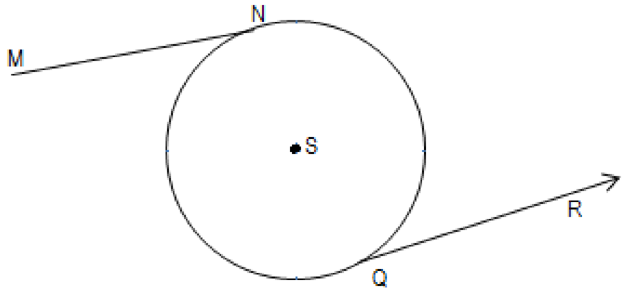


Lines s and t are
common external tangents

Tangent

Segments and rays that are coplanar with the circle and intersect the circle in one and only one point are also said to be **tangent to the circle.**

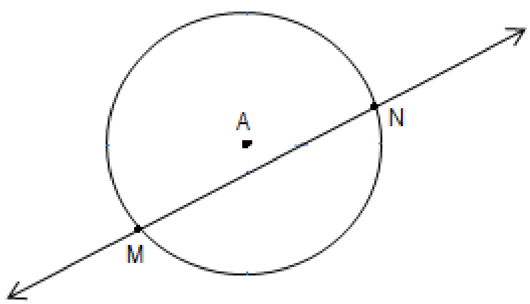
In the figure on the right, $\overline{MN}$ and $\overrightarrow{QR}$ are tangents to ⊙S



Secant Line

A **secant** is a line that intersects a circle at exactly two points. A secant contains a chord of a circle.

In circle A, $\overleftrightarrow{MN}$ is a secant line.



Secant Segment

If a segment intersects a circle in two points, and exactly one of these is an endpoint of the segment, then the segment is called a **secant segment** to the circle.

A secant segment maybe divided into two parts. A secant segment may have an internal and external segment.

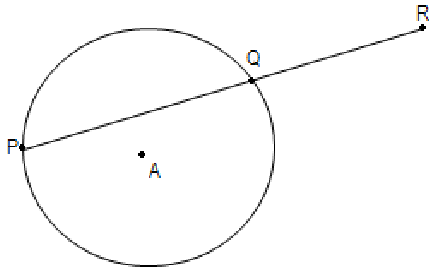
External Secant Segment

An external secant segment is the part of a secant segment that is outside a circle or a segment whose one endpoint is a point on the circle and the other endpoints are in the exterior of the circle.

Internal Secant Segment

An internal secant segment is the part of the secant segment whose endpoints are on the circle and all its points are in the interior of the circle.

In circle A, $\overline{PR}$ is a secant segment.
$\overline{PQ}$ is an internal secant segment
$\overline{QR}$ is an external secant segment

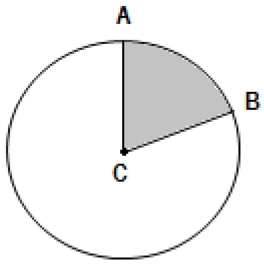


Sector and Segment of a Circle

Sector of a Circle

A sector of a circle is a region in the circle bounded by two radii and the minor arc they determine.

The shaded region of the circle on the figure to the right is an example of a sector.

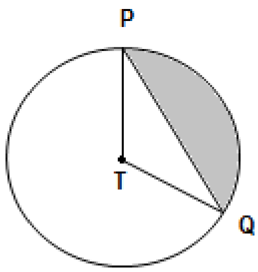


Sector ACB

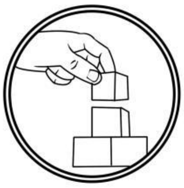
Segment of a Circle

A segment of a circle is a region bounded by an arc and the chord of the arc.

The shaded region in the figure on the right is a **segment of** $\odot$**T**. It is the region bounded by $\widehat{PQ}$ and $\overline{PQ}$



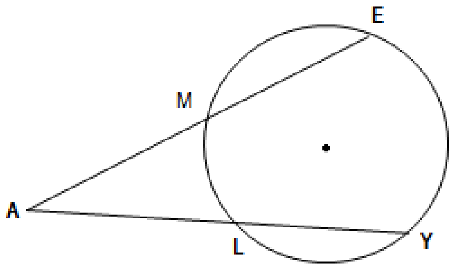
Now that you know the important ideas about the topic, you may now proceed to the next activities.



Explore

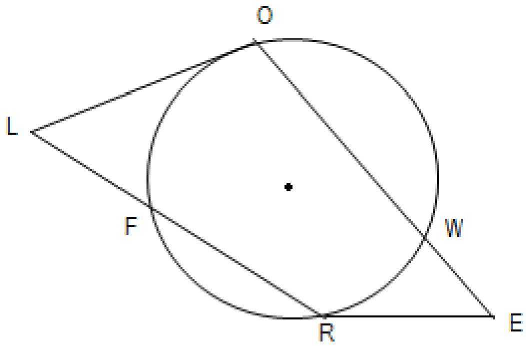
Activity 2: Am I away from you?
Direction: Use the figures below, identify what is being asked.
A.

1. Secant segments:
   ______ and ______
2. External secant segments:
   _____ and _______
3. Internal secant segments:
   _______ and _______



B.

1. Tangent segments:
   ______ and ______
2. Secant segments:
   _____ and _______
3. External secant segments:
   _______ and _______



Here is another activity that lets you apply what you have learned about tangent and secant lines. Tangent and secant segments and segment and sector of a circle.

Deepen

Activity 3: Try to Fit!
Direction: Answer the following. Place your answers in a bond paper.
A. Draw and label a circle that fits the following descriptions.
   1. has center O
   2. has secant line AB
   3. has tangent line CD
B. Draw and label a circle that fits the following descriptions.
   1. has center A
   2. has secant segments MO and QO
   3. has external secant segments NO and PO
   4. has tangent segment RO

Lesson 2	Theorems on Tangents, Secants and Segments of a Circle

Jumpstart

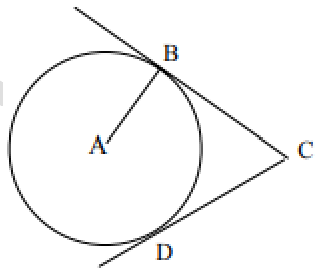
Let us begin this lesson by assessing your knowledge of the different mathematical concepts previously studied and other mathematical skills learned. Furthermore, these knowledge and skills will help you understand the different geometric relationships involving tangents, secants and segments of a circle.

Activity 1: Find Me!

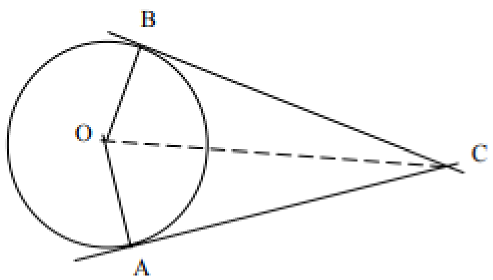
Direction: Solve the following problems completely.

If $\overline{CB}$ and $\overline{CD}$ are tangents to circle A, then

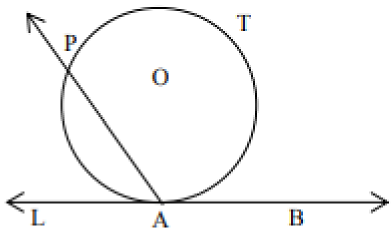
1. $\overline{CB}$ ___ $\overline{CD}$
2. $\overline{CB}$ ___ $\overline{AB}$



3. $\overline{CB}$ and $\overline{CA}$ are tangents to circle O.
   If $m\angle BOA = 160$, then $m\angle C =$ _____.
4. If $m\angle BCO = 22$, what is $m\angle ACO$?



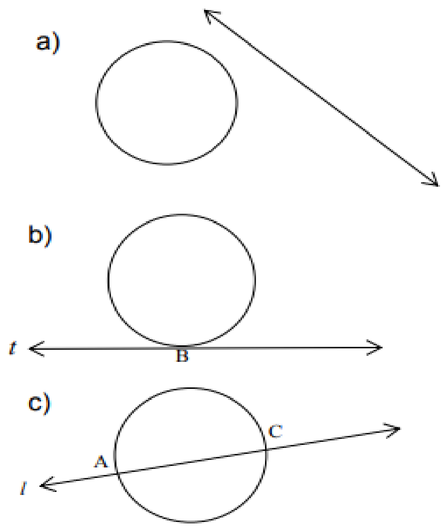
5. In the figure, if $m\widehat{PTA} = 242$, what is $m\ \angle PAL$?



Discover

Below are some important matters that we need to discuss in order for you to understand the theorems regarding secants, tangents and segments of a circle. Read carefully and understand all salient points written in this part of the module.

Tangents and Secants of a Circle



A line on the same plane with a circle may or may not intersect a circle. If ever a line intersects a circle, it could be at one point or at two points.

The figures at the right showed these three instances.

Figure a at the right showed a line that does not intersect the circle.

Figure b showed that line t intersects the circle at only one point.

Figure c showed line l intersecting the circle at two points A and C.

We will focus our study on figures b and c.

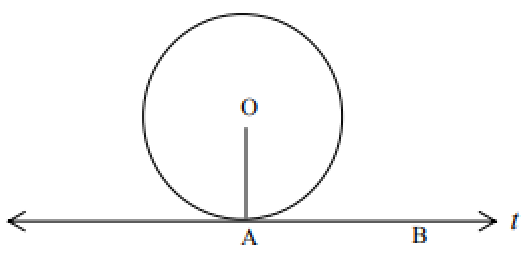
In figure b, line t is called a tangent and point B is called the point of tangency. Therefore, a tangent is a line that intersects a circle at only one point and the point of intersection is called the **point of tangency**.

In figure c, line l intersects the circle at two points A and C. Hence, line l is called a secant. Thus, a **secant** is a line that intersects a circle at two points.

Some properties exist between tangent and circle and they will be discussed here in detail. The first theorem is given below.

THEOREMS ON TANGENT LINE

Radius-Tangent Theorem. If a line is tangent to a circle, then it is perpendicular to the radius at the point of tangency.



Given: line t is tangent to circle O at A.
$\overline{OA}$ is a radius of the circle.

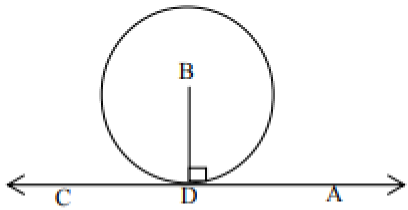
Prove: $t \perp \overline{OA}$

Proof:

Statements	Reasons
1. Let B be another point on line *t*.	1. Line Postulate
2. B is on the exterior of circle O.	2. Definition of a tangent line (A tangent can intersect a circle at only one point)
3. $\overline{OA} < \overline{OB}$	3. The radius is the shortest segment from the center to the circle and B is on the exterior of the circle.
4. $\boldsymbol{t} \perp \overline{OA}$	4. The shortest distance from a point to a line is the perpendicular segment.

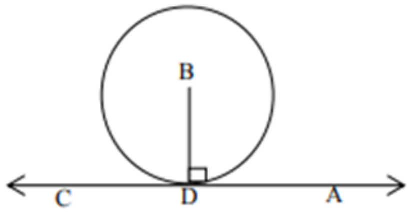
Example:

In the figure, if $\overleftrightarrow{AC}$ is tangent to circle B, then $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ at D.



The converse of the theorem is also true.
Converse: The line drawn perpendicular to the radius of a circle at its end on the circle is tangent to the circle.

Illustration:
If $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ at D, then $\overleftrightarrow{AC}$ is tangent to circle B.

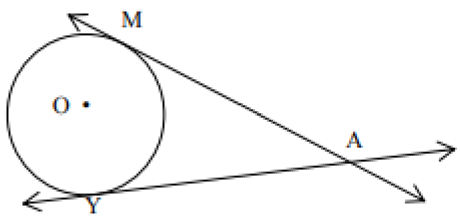


A circle is composed of infinite number of points; thus, it can also have an infinite number of tangents. Tangents of the same circle can intersect each other only outside the circle.

At this point, we will discuss the relationship of tangents that intersect the same circle. As such, those tangents may or may not intersect each other. Our focus here are those tangents that intersect each other outside the circle.

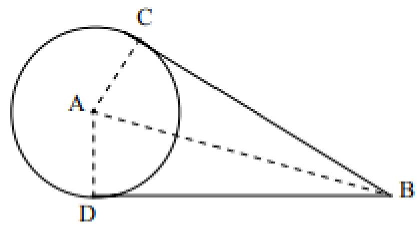
Consider the given figure:

$\overline{AM}$ and $\overline{AY}$ are tangent segments from a common external point A. What relationship exists between $\overline{AM}$ and $\overline{AY}$? The next theorem will tell us about this relationship and other properties related to tangent segments from a common external point.



Theorem: If two tangent segments are drawn to a circle from an external point then

a. the two tangent segments are congruent and
b. the angle between the segments and the line joining the external point and the center of the circle are congruent.



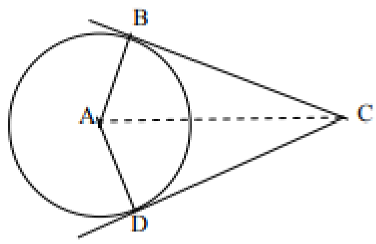
Given: Circle A. $\overline{BC}$ and $\overline{BD}$ are two tangent segments from a common external point B. C and D are the points of tangency.

Prove: a. $\overline{BC} \cong \overline{BD}$
b. $\angle CBA \cong \angle DBA$

Proof:

Statements	Reasons
1. Draw $\overline{AC}$, $\overline{AD}$, $\overline{AB}$.	1. Line determination Postulate
2. $\overline{BC}$ and $\overline{BD}$ are two tangent segments from a common external point B.	2. Given
3. $\overline{AC} \perp \overline{BC}$, $\overline{AD} \perp \overline{BD}$	3. A line tangent to a circle is perpendicular to the radius at the point of tangency.
4. $\angle ACB$ and $\angle ADB$ are right angles.	4. Definition of right angles
5. $\triangle ACB$ and $\triangle ADB$ are right triangles.	5. Definition of right triangles
6. $\overline{AC} \cong \overline{AD}$	6. Radii of the same circle are congruent.
7. $\overline{BC} \cong \overline{BD}$	7. Reflexive property of Congruency
8. $\triangle ACB \cong \triangle ADB$	8. Hypotenuse-Leg Postulate
9. $\overline{BC} \cong \overline{BD}$	9-10. CPCTC (Corresponding parts of congruent triangles are congruent).
10. $\angle CBA \cong \angle DBA$	

Examples:



1. In the figure, $\overline{CB}$ and $\overline{CD}$ are tangents to circle A at B and D.
   a. If CB = 10, what is CD?
   b. If $m\angle BAC = 49$, what is $m\angle BCA$?
   c. If $m\angle BCD = 73$, what is $m\angle BCA$? $m\angle DCA$?

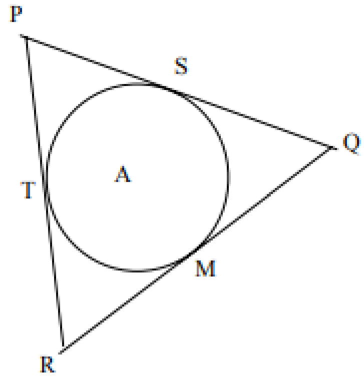
Solution:

a. Since $\overline{CB}$ and $\overline{CD}$ are tangents to the same circle from the same external point, then $\overline{CB} \cong \overline{CD}$, and therefore, CB = CD. Thus, if CB = 10, then CD = 10.

b. $m\angle BAC + m\angle BCA = 90$
$49 + m\angle BCA = 90$
$m\angle BCA = 90 - 49 = 41$

c. $m\angle BCA = \frac{1}{2}(m\angle BCD) = \frac{1}{2}(73) = 36.5$
$\angle BCA \cong \angle DCA$, $m\angle BCA = m\angle DCA = 36.5$

2. $\overline{PQ}$, $\overline{QR}$ and $\overline{PR}$ are tangents to circle A at S, M and T respectively. If PS = 7, QM = 9 and RT = 5, what is the perimeter of $\triangle PQR$?



Solution:

Using the figure and the given information, it is therefore clear that PS = PT, QS = QM and RM = RT.
PQ = PS + SQ QR = QM + MR PR = PT + RT

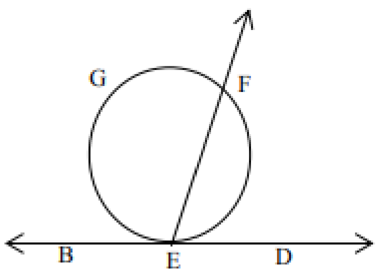
Perimeter of $\triangle PQR$ = PQ + QR + PR
= (PS + SQ) + (QM + MR) + (PT + RT)
= (PS + QM) + (QM + RT) + (PS + RT)
= 2PS + 2QM + 2RT
= 2(PS + QM + RT)
= 2(7 + 9 + 5)
= 2 (21)
= 42

Every time tangents and secants of circles are being studied, they always come with the study of angles formed between them. Coupled with recognizing the angles formed is the knowledge of how to get their measures. The next section will be devoted to studying angles formed by secants and tangents and how we can get their measures.

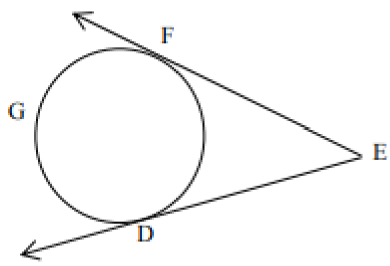
THEOREMS ON ANGLES FORMED BY TANGENTS AND SECANTS

Angles formed by secants and tangents are classified into five categories. Each category is provided with illustration.

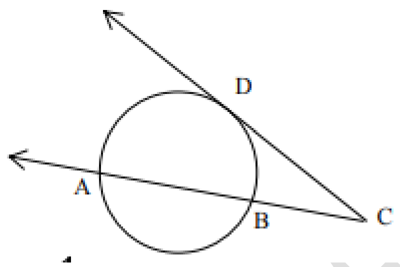
1. Angle formed by secant and tangent intersecting on the circle. In the figure, two angles of this type are formed, $\angle FED$ and $\angle FEB$. Each of these angles intercepts an arc. $\angle FED$ intercepts $\widehat{EF}$ and $\angle FEB$ intercepts $\widehat{EGF}$.



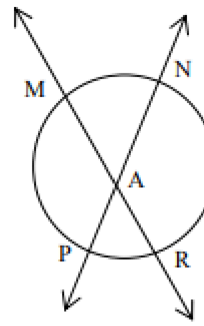
2. Angle formed by two tangents. In the figure, $\angle E$ is formed by two tangents. The angle intercepts the whole circle divided into 2 arcs, minor arc $\widehat{FD}$, and major arc $\widehat{FGD}$.



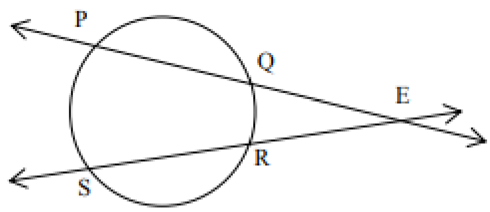
3. Angle formed by a secant and a tangent that intersect at the exterior of the circle. $\angle C$ is an angle formed by a secant and a tangent that intersect outside the circle. $\angle C$ intercepts two arcs, $\widehat{DB}$ and $\widehat{AD}$.



4. Angle formed by two secants that intersect in the interior of the circle. The figure shows four angles formed. $\angle MAN$, $\angle NAR$, $\angle PAR$, and $\angle PAM$. Each of these angle intercepts an arc. $\angle MAN$ intercepts $\widehat{MN}$, $\angle NAR$ intercepts $\widehat{NR}$, $\angle PAR$ intercepts $\widehat{PR}$ and $\angle PAM$ intercepts $\widehat{PM}$.



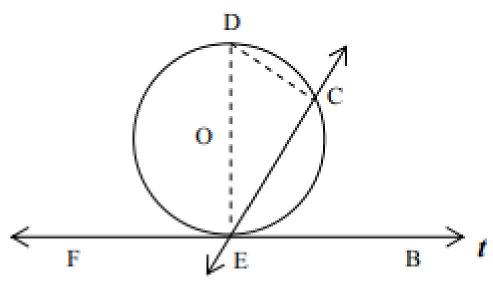
5. Angle formed by two secants intersecting outside the circle. $\angle E$ is an angle formed by two secants intersecting outside the circle. $\angle E$ intercepts two arcs namely, $\widehat{QR}$ and $\widehat{PS}$.



How do we get the measures of angles illustrated in the previous page? To understand the answers to this question, we will work on each theorem proving how to get the measures of each type of angle. It is therefore understood that the previous theorem can be used in the proof of the preceding theorem.

Theorem: The measure of an angle formed by a secant and a tangent that intersect on the circle is one-half its intercepted arc.

Given: Circle O. Secant m and tangent t intersect at E on circle O.
Prove: $m\angle \text{CEB} = \frac{1}{2}\widehat{\text{CE}}$



Proof:

Statements	Reasons
1. Draw diameter $\overline{ED}$. Join DC.	1. Line determination Postulate
2. $\overline{DE} \perp t$	2. Radius-tangent theorem
3. $\angle DCE$ is a right angle	3. Angle inscribed in a semicircle is a right angle.
4. $\angle DEB$ is a right angle	4. Perpendicular lines form right angles
5. $\triangle DCE$ is a right triangle	5. Definition of right triangle
6. $m\angle 1 + m\angle 2 = 90$	6. Acute angles of a right triangle are complementary
7. $m\angle 1 + m\angle BEC = m\angle DEB$	7. Angle addition Postulate
8. $m\angle 1 + m\angle BEC = 90$	8. Definition of complementary angles
9. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle BEC$	9. Transitive Property of Equality
10. $m\angle 1 = m\angle 1$	10. Reflexive Property of Equality
11. $m\angle 2 = m\angle BEC$	11. Subtraction Property of Equality
12. $m\angle 2 = \frac{1}{2} m\widehat{CE}$	12. Inscribed angle Theorem
13. $m\angle BEC = \frac{1}{2} m\widehat{CE}$	13. Substitution

Illustration:

In the given figure, if $m\,\widehat{CE} = 104$, what is the $m\angle BEC$? What is $m\angle CEF$?

Solution:

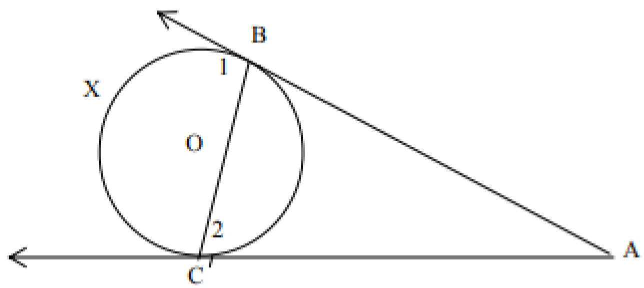
$$m\angle BEC = \frac{1}{2} m\widehat{CE} = \frac{1}{2}(104)$$
$$m\angle BEC = 52$$

$$m\angle CEF = \frac{1}{2} m\widehat{CDE} = \frac{1}{2}(360 - 104) = \frac{1}{2}(256)$$
$$m\angle CEF = 128$$

Theorem: The measure of an angle formed by two tangents from a common external point is equal to one-half the difference of the major arc minus the minor arc.

Given: Circle O. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents

Prove: $m\angle A = \frac{1}{2}\left(\widehat{BXC} - \widehat{BC}\right)$



Proof:

Statements	Reasons
1. Draw chord $\overline{BC}$.	1. Line determination Postulate
2. In $\triangle ABC$, $\angle 1$ is an exterior angle	2. Definition of exterior angle
3. $m\angle 1 = m\angle 2 + m\angle A$	3. Exterior angle theorem
4. $m\angle A = m\angle 1 - m\angle 2$	4. Subtraction Property of Equality
5. $m\angle 1 = \frac{1}{2} m\widehat{BXC}$ $m\angle 2 = \frac{1}{2} m\widehat{BC}$	5. Measure of angle formed by secant and tangent intersecting on the circle is one-half the intercepted arc.
6. $m\angle A = \frac{1}{2} m\widehat{BXC} - \frac{1}{2} m\widehat{BC}$	6. Substitution
7. $m\angle A = \frac{1}{2}\left(m\widehat{BXC} - m\widehat{BC}\right)$	7. Algebraic solution (Common monomial Factor)

Illustration:

Find the $m\angle A$ if $m\widehat{BC} = 162$.

Solution:

Since $m\angle A = \frac{1}{2}\left(m\widehat{BXC} - m\widehat{BC}\right)$, then we have to find first the measure of major arc BXC. To find it, use the whole circle which is 360°.

$$\begin{aligned} m\widehat{BXC} &= 360 - m\widehat{BC} \\ &= 360 - 162 \\ &= 198 \end{aligned}$$

Then we use the theorem to find the measure of $\angle A$,

$$\begin{aligned} m\angle A &= \frac{1}{2}\left(m\widehat{BXC} - m\widehat{BC}\right) \\ &= \frac{1}{2}(198 - 162) \end{aligned} \qquad \begin{aligned} m\angle A &= \frac{1}{2}(36) \\ m\angle A &= 18 \end{aligned}$$

We are now into the third type of angle. Angle formed by secant and tangent intersecting on the exterior of the circle.

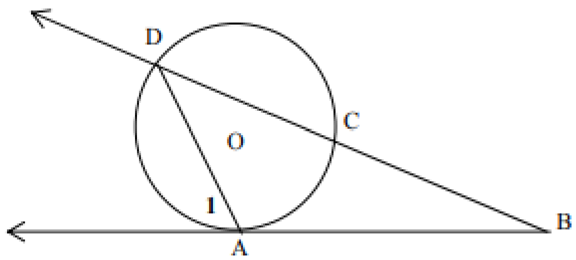
Theorem: The measure of an angle formed by a secant and tangent intersecting on the exterior of the circle is equal to one-half the difference of their intercepted arcs.

Given: $\overrightarrow{BA}$ is a tangent of circle O

$\overrightarrow{BD}$ is a secant of circle O

$\overrightarrow{BA}$ and $\overrightarrow{BD}$ intersect at B

Prove: $m\angle B = \frac{1}{2}\left(\widehat{AD} - \widehat{AC}\right)$



Proof:

Statements	Reasons
1. $\overrightarrow{BA}$ is a tangent of circle O, $\overrightarrow{BD}$ is a secant of circle O	1. Given
2. Draw $\overline{AD}$	2. Line determination Postulate
3. $\angle 1$ is an exterior angle of ΔDAB	3. Definition of exterior angle
4. $m\angle 1 = m\angle B + m\angle ADB$	4. Exterior angle Theorem
5. $m\angle B = m\angle 1 - m\angle ADB$	5. Subtraction Property of Equality
6. $m\angle 1 = \frac{1}{2} m\widehat{AD}$	6. The measure of an angle formed by secant and tangent intersecting on the circle equals one-half its intercepted arc.
7. $m\angle ADB = \frac{1}{2} m\widehat{AC}$	7. Inscribed angle Theorem
8. $m\angle B = \frac{1}{2} m\widehat{AD} - \frac{1}{2} m\widehat{AC}$	8. Substitution
9. $m\angle B = \frac{1}{2}\left(m\widehat{AD} - m\widehat{AC}\right)$	9. Simplifying expression

Illustration:

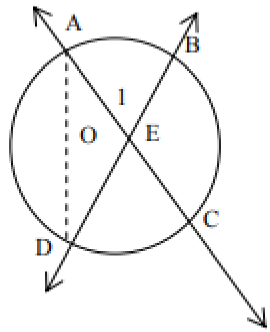
In the figure, if $m\widehat{AD} = 150$ and $m\widehat{AC} = 73$, what is the measure of $m\angle B$?

Solution:

$$m\angle B = \frac{1}{2}\left(m\widehat{AD} - m\widehat{AC}\right)$$
$$= \frac{1}{2}(150 - 73)$$
$$= \frac{1}{2}(77)$$
$$m\angle B = 38.5$$

The next theorem will tell us how angles whose vertex is in the interior of a circle can be derived. Furthermore, this will employ the previous knowledge of vertical angles whether on a circle or just on a plane.

Theorem: The measure of an angle formed by secants intersecting inside the circle equals one-half the sum of the measures of the arc intercepted by the angle and its vertical angle pair.



Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are secants intersecting outside the circle O forming $\angle 1$ with vertical angle pai $\angle CED$. (We will just work on one pair of vertical angles.)

Prove: $m\angle 1 (m\angle AEB) = \frac{1}{2}\left(\widehat{AB} + \widehat{DC}\right)$

Proof:

Statements	Reasons
1. $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are secants intersecting outside the circle O.	1. Given
2. Draw $\overline{AD}$	2. Line determination Postulate
3. $\angle 1$ is an exterior angle of ΔAED	3. Definition of exterior angle
4. $m\angle 1 = m\angle DAC + m\angle ADE$	4. Exterior angle Theorem
5. $m\angle DAC = \frac{1}{2} m\widehat{DC}$ $m\angle ADE = \frac{1}{2} m\widehat{AB}$	5. Inscribed Angle Theorem
6. $m\angle 1 = \frac{1}{2} m\widehat{DC} + \frac{1}{2} m\widehat{AB}$ $m\angle 1 = \frac{1}{2}\left(m\widehat{DC} + m\widehat{AB}\right)$	6. Substitution

Illustration:

Using the figure, find the measure of $\angle 1$ if $m\widehat{AB} = 73$ and $m\widehat{DC} = 90$.

Solution:

Using the formula in the theorem,

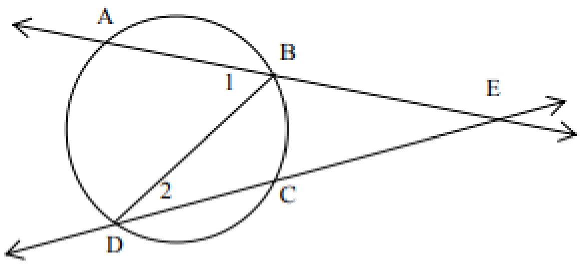
$$m\angle 1 = \frac{1}{2}\left(m\widehat{DC} + m\widehat{AB}\right)$$
$$= \frac{1}{2}(90 + 73)$$
$$= \frac{1}{2}(163)$$
$$m\angle 1 = 81.5$$

Let us discuss how to find the measure of the angle formed by two secants intersecting outside the circle.

Theorem: The measure of the angle formed by two secants intersecting outside the circle is equal to one-half the difference of the two intercepted arcs.

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are secants intersecting outside the circle O forming $\angle BEC$ outside the circle.

Prove: $m\angle BEC = \frac{1}{2}\left(\widehat{AD} - \widehat{BC}\right)$



Proof:

Statements	Reasons
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are secants intersecting outside the circle O forming $\angle$BEC outside the circle.	1. Given
2. Draw $\overline{DB}$	2. Line determination Postulate
3. $\angle 1$ is an exterior angle of ΔDBE	3. Definition of exterior angle of a triangle
4. $m\angle 1 = m\angle 2 + m\angle BEC$	4. Exterior angle Theorem
5. $m\angle BEC = m\angle 1 - m\angle 2$	5. Subtraction Property of Equality
6. $m\angle 1 = \frac{1}{2} m\widehat{AD}$ $m\angle 2 = \frac{1}{2} m\widehat{BC}$	6. Inscribed Angle Theorem
7. $m\angle BEC = \frac{1}{2} m\widehat{AD} - \frac{1}{2} m\widehat{BC}$ $m\angle BEC = \frac{1}{2}\left(m\widehat{AD} - m\widehat{BC}\right)$	7. Substitution

Illustration:

Find the measure of $\angle$BEC if $m\widehat{AD} = 150$ and $m\widehat{BC} = 80$.

Solution:

Again, we apply the theorem using the formula:

$$m\angle BEC = \frac{1}{2}\left(m\widehat{AD} - m\widehat{BC}\right)$$
$$= \frac{1}{2}(150 - 80)$$
$$= \frac{1}{2}(70)$$
$$m\angle BEC = 35$$

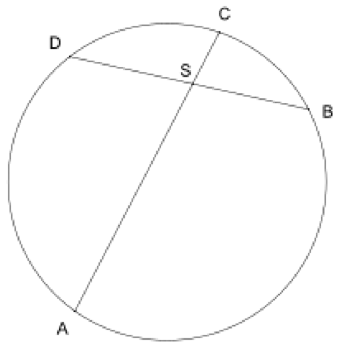
Tangents and Secants Segments

Let us proceed to the theorems on tangent and secant segments of circles with illustrations and examples presented.

Intersecting Chord Theorem: When two chords intersect each other inside a circle, the products of their segments are equal.

In the circle shown at the right, $\overline{AC}$ intersects $\overline{BD}$ at S. AC consists of two segments, namely, AS and SC. Likewise, for BD, we have BS and SD. From the theorem,

$$AS \cdot SC = BS \cdot SD$$



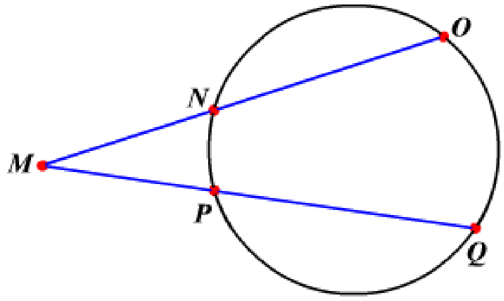
In the illustration above, If AS = 8, SC = 2, BS = 4, what is SD?
Solution:

Using the formula,

$$AS \cdot SC = BS \cdot SD$$
$$8 \cdot 2 = 4 \cdot SD$$
$$16 = 4SD$$
$$SD = 4$$

Intersecting Secants Theorem: If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

In the circle, $\overline{MO}$ and $\overline{MQ}$ are secants that intersects at point M.
So, $MN \cdot MO = MP \cdot MQ$



Given: $MN = 10$, $NO = 17$, $MP = 9$. Find the length of PQ.

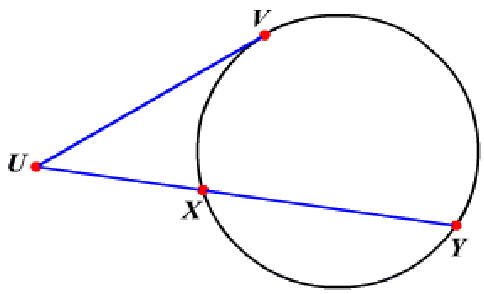
Solution:

$$MO = MN + NO$$
$$MQ = MP + PQ$$
$$MN \cdot MO = MP \cdot MQ$$
$$MN(MN + NO) = MP(MP + PQ)$$
$$10(10 + 17) = 9(9 + PQ)$$
$$270 = 81 + 9PQ$$
$$189 = 9PQ$$
$$21 = PQ$$

Therefore, PQ is equal to 21 units.

Intersecting Secant-Tangent Theorem: If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

In the circle, $\overline{UV}$ is a tangent and $\overline{UY}$ is a secant. They intersect at point U. So, $(UV)^2 = UX \cdot UY$

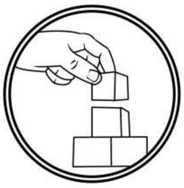


In the circle shown above, if $UX = 8$ and $XY = 10$, find the length of UV.
Solution:

Since $UY = UX + UY$, then $UY = 18$

$$(UV)^2 = UX \cdot UY$$

$$= (8)(18)$$
$$= 144$$
Take the square root of each side,
$$\sqrt{(UV)^2} = \sqrt{144}$$
$$UV = 12$$

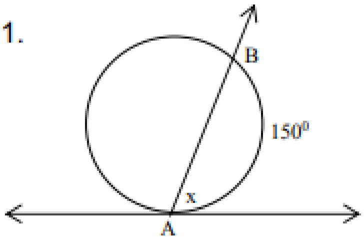


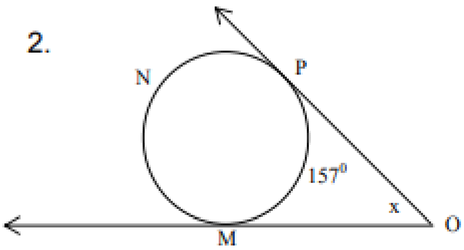
Explore

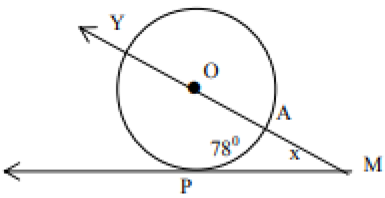
Work on the following enrichment activity for you to apply your understanding on this lesson.

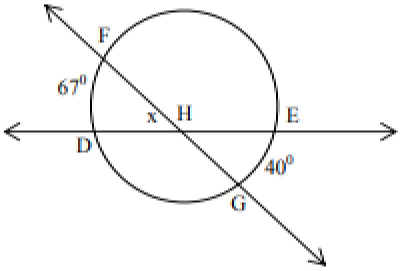
Activity 2: Missing Me!

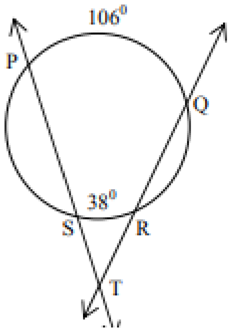
Direction: In each of the given figure, find the measure of the unknown angle (x).

1.


2.


3.


4.


5.


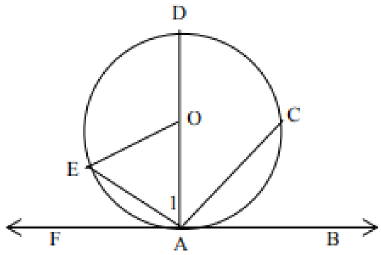
How did you find the activity? What mathematical concepts did you use? Now, here is another activity that deepens your understanding on the concepts and theorems you have studied in this module.

Deepen

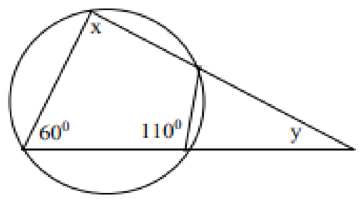
Activity 3: Try Me!

Directions: Answer the following problems completely. Find the missing angles or arcs in the given figure.

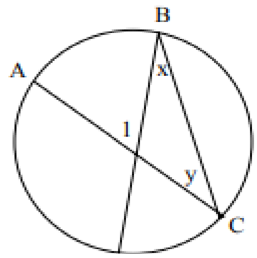
1. If $m\widehat{DE} = 108$ and $m\angle DOC = 85$, find:
   - a. $m\widehat{EA}$
   - b. $m\angle EAF$
   - c. $m\angle DAF$
   - d. $m\angle CAB$
   - e. $m\angle 1$



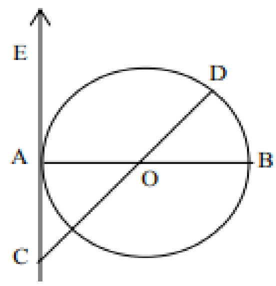
2. Using the given figure, find x and y.



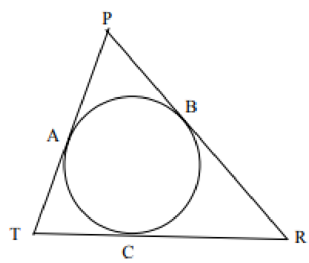
3. If $x = 18$ and $y = 23$, find $m\angle 1$.



4. $\overleftrightarrow{EC}$ is tangent to circle O. $\overline{AB}$ is a diameter. If $m\widehat{DB} = 47$, find $m\widehat{AD}$ and $m\angle ECD$.



5. A polygon is said to be circumscribed about a circle if its sides are tangent to the circle. ΔPRT is circumscribed about circle O. If PT = 10, PR = 13 and RT = 9, find AP, TC and RB.



Gauge

Post-Assessment

Directions: Read and understand the questions below. Select the best answer to each item then write the letter of your choice on your answer sheet.

1. What theorem states that if a line is tangent to a circle, then it is perpendicular to the radius at the point of tangency?
   - A. Radius-Tangent
   - B. Intersecting Chord
   - C. Perpendicular Chord
   - D. Tangent-Secant

2. If two tangent segments are drawn to a circle from an external point, then which of the following statements is true?
   - A. the radius is perpendicular to the chord
   - B. the two tangent segments are congruent
   - C. the two tangents are intersecting
   - D. the products of their segments are equal

3. What is the measure of an angle formed by a secant and a tangent that intersect on the circle?
   - A. one-sixth the measure of its intercepted arc
   - B. one-fifth the measure of its intercepted arc
   - C. one-fourth the measure of its intercepted arc
   - D. one-half the measure of its intercepted arc

4. What is the measure of the angle formed by two secants intersecting outside the circle?
   - A. one-half the difference of the two intercepted arcs
   - B. one-third the difference of the two intercepted arcs
   - C. twice the difference of the two intercepted arcs
   - D. thrice the difference of the two intercepted arcs

5. How do you find the measure of an angle formed by secants intersecting inside the circle?
   - A. twice the sum of the measures of the arc intercepted by the angle and its vertical angle pair
   - B. twice the sum of the measures of the arc intercepted by the angle and its intercepted arc
   - C. one-half the sum of the measures of the arc intercepted by the angle and its vertical angle pair
   - D. one-half the sum of the measures of the arc intercepted by the angle and its intercepted arc

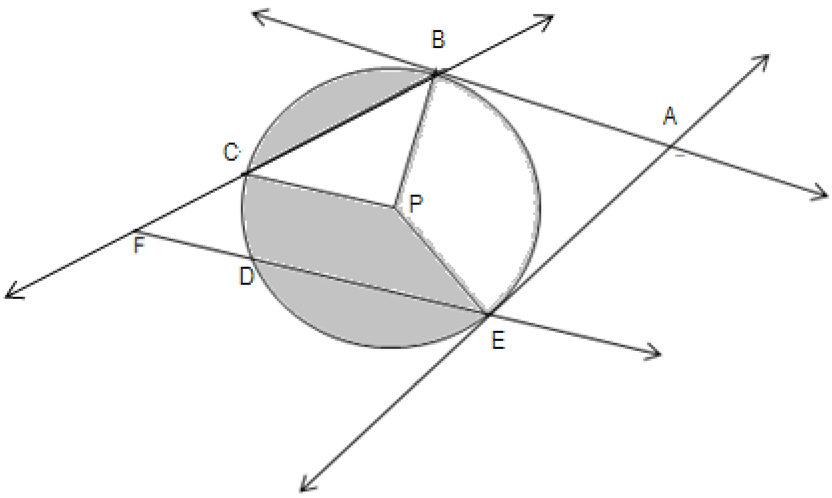
6. What theorem states that "If two chords intersect each other inside a circle, the products of their segments are equal"?
   - A. Radius-Tangent
   - B. Intersecting Chord
   - C. Perpendicular Chord
   - D. Tangent-Secant

7. What do you call the line coplanar with the circle and intersects it in exactly two points?
   A. secant   B. sector   C. segment   D. tangent

8. Which of the following terms refer to the region bounded by an arc and the chord of the arc?
   A. secant segment   B. sector of a circle
   C. segment of a circle   D. tangent segment

9. A secant segment maybe divided into two parts. What do you call the part that is outside the circle?
   A. external secant segment   B. internal secant segment
   C. secant segment   D. tangent segment

For number 10 to 12, refer to the figure below.

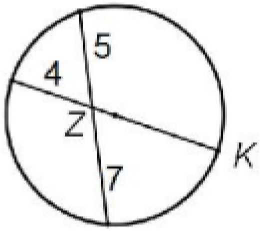


10. Which of the following is a tangent line?
   A. $\overleftrightarrow{AB}$ B. $\overline{CB}$ C. $\overrightarrow{FE}$ D. $\overleftrightarrow{BF}$

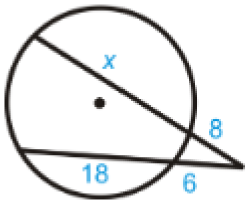
11. Which of the following segments is an external secant segment?
   A. $\overline{AB}$ B. $\overline{PB}$ C. $\overline{DF}$ D. $\overline{FB}$

12. Which of the following is a secant segment?
   A. $\overline{AB}$ B. $\overline{PB}$ C. $\overline{DF}$ D. $\overline{FB}$

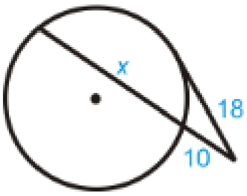
13. What is the length of $\overline{ZK}$ in the figure at the right?
   A. 2.86 units   B. 6 units   C. 8 units   D. 8.75 units



14. Find the value of x in the figure below.
    A. 5 B. 10 C. 15 D. 20



15. What is the value of x in the figure below?
    A. 11.4 B. 12.4 C. 22.4 D. 23.4



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