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Mathematics

Quarter 1 - Module 2:

The Nature & Sum and Product of Roots of Quadratic Equations

AIRs - LM

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Quarter 1-Module 2:The Nature & Sum and Product of Roots of Quadratic Equations
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In the previous lesson, any quadratic equation will have two roots (even though one may be a repeated roots or the roots may not even be real). In this module you will be considering some further properties of these two roots. So, study the discussions on this lesson and answer activities assigned to you to practice skills.

To be able to answer the following activities, you will need to understand the relationship between the coefficients and the roots of a quadratic equation. You will also know on how to determine the nature of the roots of a quadratic equation. If you have some difficulties along the way, you may seek help from your teacher and refer your answers to them.

Jumpstart

Activity 1: MY A, B, C?

Directions: Write the following quadratic equation in standard form and identify the values of a, b, and c. Answer the question that follows.

$$ax^2 + bx + c = 0$$

1. $x^2 + 5x = 2$ ______________________ a= ___ b= ___ c= ___
2. $3x^2 = x - 1$ ______________________ a= ___ b= ___ c= ___
3. $x^2 = 2x - 10$ ______________________ a= ___ b= ___ c= ___
4. $x + 2x^2 - 1 = 0$ ______________________ a= ___ b= ___ c= ___
5. $x(x + 2) = 7$ ______________________ a= ___ b= ___ c= ___

Process Question:

1. Aside from your answer, do you think there is another way of writing each quadratic equation in another way? If YES, show your answer and determine the values of a, b, and c.

Were you able to answer the given activity correctly? In the next activity you will enhance your mathematical skills in finding the roots of the quadratic equation.

Activity 2. Relate Me to My Roots!

Directions: Given the following quadratic equation, complete the table below, then answer the following questions. The first one is done for you.

Quadratic Equation	Coefficients			Roots		Sum of Roots	Product of Roots
	a	b	c	r_1	r_2	$r_1 + r_2$	$(r_1)(r_2)$
$x^2 + 4x - 12 = 0$	1	4	-12	- 6	2	- 4	-12
$x^2 + 7x + 12 = 0$							
$3x^2 + 3x = 6$							

Process Questions:

1. What do you observe about the sum and product of the roots of each quadratic equation in relation to the values of the coefficients a, b, and c?
2. Can you solve for the quadratic equation given its roots? How will you do it?
3. How about if the sum and product of the roots are given? Can you determine the quadratic equation?

Discover

NATURE OF THE ROOTS OF A QUADRATIC EQUATION

The value of the expression $b^2 - 4ac$ is called the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$. This value can be used to describe the nature of the roots of a quadratic equation. It can be zero, positive and perfect square, positive but not perfect square or negative.

1. When $b^2 - 4ac$ is equal to zero, then the roots are real numbers and are equal.

   Example: Describe the roots of $x^2 - 4x + 4 = 0$.

   The values of a, b, and c in the equation are the following:

   a = 1 b = -4 c = 4

   Substitute these values of a, b and c in the expression $b^2 - 4ac$.

   $$b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$$

   Since the value of $b^2 - 4ac$ is zero, we can say that the roots of the quadratic equation $x^2 - 4x + 4 = 0$ are real numbers and equal.

2. When $b^2 - 4ac$ is greater than zero and a perfect square, then the roots are rational numbers but are not equal.

   Example: Determine the nature of the roots of $x^2 + 7x + 10 = 0$.

   In the equation, the values of a, b, and c are 1, 7, and 10, respectively. Use these values to evaluate $b^2 - 4ac$.

$$\begin{aligned} b^2 - 4ac &= (7)^2 - 4(1)(10) \\ &= 49 - 40 \\ &= 9 \end{aligned}$$

   Since the value of $b^2 - 4ac$ is greater than zero and a perfect square, then the roots of the quadratic equation $x^2 + 7x + 10 = 0$ are rational numbers and not equal.

3. When $b^2 - 4ac$ is greater than zero but not a perfect square, then the roots are irrational numbers and are not equal.

   Example: Describe the roots of $x^2 + 6x + 3 = 0$

   Evaluate the expression $b^2 - 4ac$ using the values a, b, and c.

   In the equation, the values of a, b, and c are 1, 6, and 3, respectively.

$$\begin{aligned} b^2 - 4ac &= (6)^2 - 4(1)(3) \\ &= 36 - 12 \\ &= 24 \end{aligned}$$

   Since the value of $b^2 - 4ac$ is greater than zero but not a perfect square, then the roots of the quadratic equation $x^2 + 6x + 3 = 0$ are irrational numbers and not equal.

4. When $b^2 - 4ac$ is less than zero, then the equation has no real roots.

   Example: Determine the nature of the roots of $x^2 + 2x + 5 = 0$

   In the equation, the values of a, b, and c are 1, 2, and 5, respectively. Use these values to evaluate $b^2 - 4ac$.

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(1)(5) \\ &= 4 - 20 \\ &= -16 \end{aligned}$$

   Since the value of $b^2 - 4ac$ is less than zero, then the quadratic equation $x^2 + 2x + 5 = 0$ has no real roots.

SUM AND PRODUCT OF ROOTS OF A QUADRATIC EQUATION

We will now discuss on how the sum and product of roots of the quadratic equation $ax^2 + bx + c = 0$ can be determined using the coefficients a, b, and c.

We have seen that the $\mathbf{b^2 - 4ac}$ is the radicand of the quadratic formula, called the *discriminant*, can tell us the type of roots of a quadratic equation. The quadratic formula can also give us information about the relationship between the roots and the coefficient of the second term and the constant of the equation itself. Consider the following:

Given a quadratic equation: $ax^2 + bx + c = 0$. By the quadratic formulas, the two roots can be represented as:

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of the Roots, $\mathbf{r_1 + r_2}$:

$$r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$r_1 + r_2 = \frac{-b}{a}$$

Product of the Roots, $\mathbf{x_1 \bullet x_2}$:

$$r_1 \bullet r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \bullet \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} - b- \sqrt{b^2 - 4ac} - (b^2 - 4ac)}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$r_1 + r_2 = \frac{c}{a}$$

The ***sum of the roots*** of a quadratic equation is equal to the inverse of the coefficient of the second term, divided by the leading coefficient.

$$(r_1 + r_2) = -\frac{b}{a}$$

The ***product of the roots*** of a quadratic equation is equal to the constant term, divided by the leading coefficient.

$$r_1 \bullet r_2 = \frac{c}{a}$$

Example 1:

Find the sum and product of roots of the quadratic equation $x^2 - 5x + 6 = 0$.

Given the equation $x^2 - 5x + 6 = 0$, we get $a = 1$, $b = -5$ and $c = 6$.

Sum of the roots $= \frac{-b}{a} \longrightarrow \frac{-b}{a} = \frac{-(-5)}{1} = 5$

Product of the roots $= \frac{c}{a} \longrightarrow \frac{c}{a} = \frac{6}{1} = 6$

The roots of the equation $x^2 - 5x + 6 = 0$ are 3 and 2 (using factoring or any method). To check, find the sum and product of these roots.

Let $r_1 = 3$ and $r_2 = 2$

$$r_1 + r_2 = 3 + 2 = 5$$

$$r_1 \bullet r_2 = (3)(2) = 6$$

Therefore, the sum and product of roots of $x^2 - 5x + 6 = 0$ are 5 and 6, respectively.

Example 2:

Given the values $a = 1$, $b = 4$, and $c = -21$. What is the quadratic equation? Solve for the sum and product of roots.

Since $a = 1$, $b = 4$, and $c = -21$ thus the equation is $x^2 + 4x - 21 = 0$.

Sum of the roots $= \frac{-b}{a} \longrightarrow \frac{-4}{1} = -4$

Product of the roots $= \frac{c}{a} \longrightarrow \frac{-21}{1} = -21$

By inspection, the two numbers that give a sum of -4 and a product of -21 are -7 and 3.

Let $r_1 = -7$ and $r_2 = 3$

$$r_1 + r_2 = -7 + 3 = -4$$

$$r_1 \bullet r_2 = (-7)(3) = -21$$

Therefore, the quadratic equation is $x^2 + 4x - 21 = 0$ and its sum and product of roots of are -4 and -21, respectively.

Substitute in the form:

x^2 - (sum of roots) x + (product of roots) = 0

$x^2 - (-4)x + (-21) = 0$

Equation: $x^2 + 4x - 21 = 0$

Example 3:

Write a quadratic equation whose roots are 2 and 5.

Solution: $r_1 + r_2 = 2 + 5 = 7$

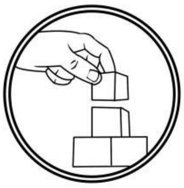
$(r_1)(r_2) = (2)(5) = 10$

Substitute in the form:

x^2 – (sum of roots) x + (product of roots) = 0

$x^2 - (7)x + (10) = 0$

Equation: $x^2 - 7x + 10 = 0$



Explore

Activity 3: State My Nature!

Directions: Determine the nature of the roots of the quadratic equations using the discriminant. Answer the questions that follow.

1. $x^2 + 6x + 9 = 0$ discriminant ________ nature of the roots________
2. $x^2 + 9x + 20 = 0$ discriminant ________ nature of the roots________
3. $2x^2 - 10x + 8 = 0$ discriminant ________ nature of the roots________
4. $3x^2 - 5x = -4$ discriminant ________ nature of the roots________
5. $x^2 - 400 = 0$ discriminant ________ nature of the roots________

Process Questions:

1. When do you say that the roots of a quadratic equation are real or not real?
2. When do you say that the roots of a quadratic equation are rational or irrational?
3. When do you say that the roots of a quadratic equation are equal or not equal?

Activity 4: This is My Sum, and This is My Product. Who Am I?

Directions: Using the values of a, b, and c of each of the following quadratic equation solve for the sum and product of roots. Check your answer by using the roots of the quadratic equation.

Quadratic Equation	Sum of the Roots	Product of the Roots	Roots
$x^2 + 4x + 3 = 0$		3	
$6x^2 + 12x - 18 = 0$	-2		$x = 1 \quad x = -3$
	-4	-21	$x = 3 \quad x = -7$
$2x^2 + 3x - 2 = 0$			$x = \frac{1}{2} \quad x = -2$
$x^2 - 9x + 14 = 0$		14	$x = 7 \quad x = 2$

Process Questions:

1. How did you find answering or finding the sum and product of the roots of a quadratic equation?
2. How did you get the roots of the quadratic equation?
3. How did you obtain the quadratic equation given its roots?

Now that you know the important ideas about the topic, let's go deeper by moving on to the next section.

Deepen

Activity 5: Complete Me!

Direction: Complete the table below. Given the roots, find the sum and product of roots and write a quadratic equation in the form $ax^2 + bx + c = 0$.

Roots	Sum of the Roots	Product of the Roots	Quadratic Equation
6 and 3			
-3 and -7			
-6 and 10			
4 and 7			
$\frac{5}{6}$ and $\frac{1}{6}$			

Process Questions:

1. What did you realize in solving for the sum and product of the roots of a quadratic equation?
2. How to find the quadratic equation using its roots?
3. Which is easier, finding the roots given quadratic equation or finding the quadratic equation given its roots? Justify your answer.

Activity 6: Table Making Time!

Direction: Study the situation below and answer the questions that follow.

Philip Navarette wants to make a table which has an area of 12m². The length of the table has to be 1 meter longer than its width.

Process Questions:

1. If the width of the table is p, what is its length?
2. Form a quadratic equation that represents the situation.
3. Compute for the dimensions of the table and determine whether the dimensions of the table are rational. Explain.

In the previous activities, you were able to understand the nature, sum, and product of roots of quadratic equations.

Gauge

Post-Assessment

Directions: Find out how much have you learned from the lesson. Choose the letter of the correct answer. Write it on a separate sheet of paper.

1. The roots of a quadratic equation are -5 and 3. Which of the following quadratic equations has these roots?
   A. $x^2 - 8x + 15 = 0$
   B. $x^2 + 8x + 15 = 0$
   C. $x^2 - 2x - 15 = 0$
   D. $x^2 + 2x - 15 = 0$
2. What is the sum of the roots of the quadratic equation $x^2 - 9 = 0$?
   A. -9
   B. 3
   C. 0
   D. 9