

Mathematics

Quarter 2 – Module 6:

Operations on Radical Expressions



AIRs - LM

Mathematics 9
Quarter 2 - Module 6: Operations on Radical Expressions
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Region I

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Mathematics

Quarter 2 – Module 6: Operations on Radical Expressions

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



Target

Your goal in this module is to construct your understanding on how the laws of radical is applied in performing mathematical operations. Towards the end of this module, you will be encouraged to use your understanding in performing the different operations on radicals and simplifying results.

After going through this module, you are expected to attain the following objectives.

Learning Competency

- Performs operations on radical expressions (M9AL- IIh-1)

Subtasks

1. Describe similar or like radicals.
2. Performs operations on radical expressions.

*Before going on, check how much you know about this topic.
Answer the pre-assessment in a separate sheet of paper.*

Pre – Assessment

Directions: Read and analyze the following questions carefully. Choose the letter of the correct answer and write it on your answer sheets.

1. What is the radical form of the expression $3^{\frac{1}{2}} + 3^{\frac{1}{3}}$?
A. $\sqrt{3} + \sqrt[3]{3}$ B. $\sqrt{3^2} + \sqrt[3]{3}$ C. $\sqrt{3} + \sqrt{3^3}$ D. $\sqrt{3^2} + \sqrt{3^3}$
2. What is the equivalent of $\sqrt[5]{4} + \sqrt[3]{2}$ using exponential notation?
A. $4^{\frac{1}{5}} + 2^{\frac{1}{3}}$ B. $4^5 + 2^3$ C. 6^8 D. $6^{\frac{1}{8}}$
3. Which of the following is an incorrect characteristic of a radical in simplest form?
A. No fraction as radicands.
B. No radicands with variable.
C. No radical appears in the denominator of a fraction.
D. No radicands have perfect square factors other than 1.
4. What is the sum of $7\sqrt{2}$ and $3\sqrt{2}$?
A. $-4\sqrt{2}$ B. $4\sqrt{2}$ C. $10\sqrt{2}$ D. $10\sqrt{4}$
5. What is the simplified form of the radical expression $4\sqrt{45} + \sqrt{5}$?
A. $13\sqrt{10}$ B. $13\sqrt{5}$ C. $4\sqrt{50}$ D. $12\sqrt{5}$
6. What is the sum of $-5\sqrt{2}$, $2\sqrt{3}$, $10\sqrt{2}$, $-14\sqrt{3}$ and $3\sqrt{2}$?
A. $12\sqrt{2} + 8\sqrt{3}$ B. $8\sqrt{2} + 12\sqrt{3}$ C. $12\sqrt{2} - 8\sqrt{3}$ D. $8\sqrt{2} - 12\sqrt{3}$
7. What is the difference of $14\sqrt{3}$ and $20\sqrt{3}$?
A. $6\sqrt{3}$ B. $-6\sqrt{3}$ C. $34\sqrt{3}$ D. -6

8. Evaluate: $\sqrt{18} - 3\sqrt{2}$
 A. $6\sqrt{2}$ B. $2\sqrt{8}$ C. $\sqrt{2}$ D. 0
9. What is the result after performing $2\sqrt{5} + 4\sqrt{5} - 5\sqrt{5}$?
 A. $-\sqrt{5}$ B. $\sqrt{5}$ C. $11\sqrt{5}$ D. $21\sqrt{5}$
10. What is the product of $\sqrt{7}$ and $\sqrt{11}$?
 A. $\sqrt{77}$ B. $\sqrt{17}$ C. 18 D. 77
11. Write the product in simplest form of $\sqrt{3}(\sqrt{6} + \sqrt{7})$?
 A. $2\sqrt{3} + \sqrt{21}$ B. $2\sqrt{6} + 2\sqrt{21}$ C. $3\sqrt{2} + \sqrt{21}$ D. 39
12. Which of the following is the simplified form of $(2 + \sqrt{5})(2 - \sqrt{5})$?
 A. 1 B. $4 - \sqrt{5}$ C. 0 D. -1
13. What is the value of $\frac{\sqrt{7}}{\sqrt{3}}$ when simplified?
 A. $\frac{7}{3}$ B. $\frac{\sqrt{21}}{3}$ C. $\frac{\sqrt{14}}{3}$ D. $\frac{\sqrt{10}}{3}$
14. Write in simplest form: $\sqrt{\frac{3}{4}}$
 A. $\frac{2\sqrt{3}}{4}$ B. $\frac{\sqrt{3}}{4}$ C. $\frac{\sqrt{7}}{2}$ D. $\frac{\sqrt{3}}{2}$
15. What is the result when we simplify $\frac{6-\sqrt{2}}{4-3\sqrt{2}}$?
 A. 5 B. $-2\sqrt{2}$ C. $5 - \sqrt{2}$ D. $-9 - 7\sqrt{2}$



Jumpstart

Activity 1: Find My Index and Radicand

Directions: Identify the index and radicand of the following radicals. Write your answer on your answer sheets.

| Radicals | index | radicand | Radicals | index | radicand |
|---------------------|-------|----------|--------------------|-------|----------|
| 1.) $\sqrt[3]{5}$ | _____ | _____ | 5.) $\sqrt{3}$ | _____ | _____ |
| 2.) $7\sqrt[3]{5}$ | _____ | _____ | 6.) $4\sqrt{7}$ | _____ | _____ |
| 3.) $\sqrt[4]{6}$ | _____ | _____ | 7.) $\sqrt[3]{10}$ | _____ | _____ |
| 4.) $13\sqrt[5]{6}$ | _____ | _____ | 8.) $5\sqrt{21}$ | _____ | _____ |

Process Questions

- Given radicals #1 & #2, what can you say about their indices and radicand?
- Given radicals #3 & #4, what can you say about their indices and radicand?
- Given radicals #5 & #6, what can you say about their indices and radicand?
- Given radicals #7 & #8, what can you say about their indices and radicand?

Activity 2: Translate Me

Directions: Translate or convert radical expressions to exponential expressions and vice versa. Write your answer on your answer sheets.

A. Radical to Exponential

B. Exponential to Radical

1.) $\sqrt[3]{2} = \underline{\hspace{2cm}}$

3.) $5^{\frac{1}{2}} = \underline{\hspace{2cm}}$

2.) $\sqrt{3} = \underline{\hspace{2cm}}$

4.) $7^{\frac{1}{3}} = \underline{\hspace{2cm}}$

You have learned in the previous lessons on how to write expressions with rational exponents to radicals and vice versa and you will need these skills to succeed in the next activities as we apply the different operations in simplifying radicals.



Discover

Operations on Radicals

Adding and subtracting radical expressions works like adding and subtracting expressions involving variables. Just as we need like terms when combining expressions involving variables, we need like radicals in order to combine radical expressions.

Two radical expressions are said to be **like radicals** if they have the same indices and the same radicands.

Examples:

1.) $3\sqrt{5}$ and $8\sqrt{5}$ → like radicals, *they have the same indices and the same radicands.*

2.) $2\sqrt[3]{6}$ and $\sqrt[3]{7}$ → unlike radicals, *they have the same indices but different radicands.*

3.) $4\sqrt{10}$ and $8\sqrt[5]{10}$ → unlike radicals, *they have the same radicands but different indices.*

4.) $5\sqrt[3]{6}$ and $11\sqrt[4]{9}$ → unlike radicals, *they have different radicands and different indices.*

Other Examples:

$$1.) 7\sqrt{11} \text{ and } 3\sqrt{11} \quad \rightarrow \text{ like radicals}$$

$$2.) 5\sqrt{3b}, -2\sqrt{3b} \text{ and } 4\sqrt{3b} \quad \rightarrow \text{ like radicals}$$

There are terms which may not look like they are similar or like radicals, but when some simplifications are done, they are actually like radicals.

Example:

$$\begin{array}{ccc} \sqrt{2} \text{ and } \sqrt{8} & \text{at first look, it seems that they are not like radicals} & \\ \downarrow & \downarrow & \\ \sqrt{4 \cdot 2} & \text{by simplifying radicals} & \\ \downarrow & \downarrow & \\ \sqrt{2} \text{ and } 2\sqrt{2} & \text{are now like radicals} & \end{array}$$

More Examples: *Simplify to make them like radicals*

| | | |
|--|---|---|
| $\begin{array}{l} 1.) \ 2\sqrt{54} \text{ and } 5\sqrt{24} \\ \downarrow \qquad \qquad \downarrow \\ 2\sqrt{9 \cdot 6} \text{ and } 5\sqrt{4 \cdot 6} \\ \downarrow \qquad \qquad \downarrow \\ 6\sqrt{6} \text{ and } 10\sqrt{6} \\ \text{like radicals} \end{array}$ | $\begin{array}{l} 2.) \ \sqrt[3]{16} \text{ and } 7\sqrt[3]{250} \\ \downarrow \qquad \qquad \downarrow \\ \sqrt[3]{8 \cdot 2} \text{ and } 7\sqrt[3]{125 \cdot 2} \\ \downarrow \qquad \qquad \downarrow \\ 2\sqrt[3]{2} \text{ and } 35\sqrt[3]{2} \\ \text{like radicals} \end{array}$ | $\begin{array}{l} 3.) \ \sqrt{x^2y} \text{ and } \sqrt{x^4y^3} \\ \downarrow \qquad \qquad \downarrow \\ x\sqrt{y} \text{ and } x^2y\sqrt{y} \\ \text{like radicals} \end{array}$ |
|--|---|---|

Adding and Subtracting Radical Expressions

To add or subtract radicals, the indices and what is inside the radical (called the radicand) must be exactly the same. If the indices and radicands are the same, then add or subtract the terms in front of each like radical. If the indices or radicands are not the same, then you cannot add or subtract the radicals.

Examples:

$$1.) \ 3\sqrt{5} + 8\sqrt{5} = (3 + 8)\sqrt{5} \quad \text{add 3 \& 8, then copy } \sqrt{5} \\ = 11\sqrt{5}$$

$$2.) \ 8\sqrt{13} - 2\sqrt{13} = (8 - 2)\sqrt{13} \quad \text{subtract 2 to 8, then copy } \sqrt{13} \\ = 6\sqrt{13}$$

$$3.) \ 10\sqrt[3]{7} - 4\sqrt[3]{7} + \sqrt[3]{7} - 2\sqrt[3]{7} = (10 - 4 + 1 - 2)\sqrt[3]{7} \\ = 5\sqrt[3]{7}$$

Other Examples:

$$1.) 2\sqrt{11} + 5\sqrt{11} - 5\sqrt{3} + 8\sqrt{3}$$

combine similar terms or like radicals

$$\underbrace{2\sqrt{11} + 5\sqrt{11}}_{7\sqrt{11}} - \underbrace{5\sqrt{3} - 8\sqrt{3}}_{-3\sqrt{3}}$$

$$7\sqrt{11} + 3\sqrt{3} = 7\sqrt{11} + 3\sqrt{3}$$

this is now the final answer
because you cannot
add/subtract unlike radicals

$$2.) \sqrt[3]{6} - 16\sqrt{2} + 5\sqrt[3]{6} + 10\sqrt{2}$$

rearrange and

$$\sqrt[3]{6} + 5\sqrt[3]{6} - 16\sqrt{2} + 10\sqrt{2}$$

combine similar terms or like radicals

$$\underbrace{\sqrt[3]{6} + 5\sqrt[3]{6}}_{6\sqrt[3]{6}} - \underbrace{16\sqrt{2} - 10\sqrt{2}}_{-6\sqrt{2}}$$

$$6\sqrt[3]{6} - 6\sqrt{2} = 6\sqrt[3]{6} - 6\sqrt{2}$$

this is now the final answer
because you cannot
add/subtract unlike radicals

$$3.) -5\sqrt{2} + 7\sqrt{3} = -5\sqrt{2} + 7\sqrt{3}$$

just copy, cannot add/
subtract unlike radicals

Remember that we can only combine like radicals.

Examples:

$$1.) 2\sqrt{54} + 5\sqrt{24}$$

simplify each radical

$$\begin{array}{c} \downarrow \quad \downarrow \\ 2\sqrt{9 \cdot 6} + 5\sqrt{4 \cdot 6} \\ \downarrow \quad \downarrow \end{array}$$

Add or subtract the radicals.

$$6\sqrt{6} + 10\sqrt{6} = 16\sqrt{6}$$

add 6 & 10, then copy $\sqrt{6}$

$$2.) \sqrt[3]{16} - 7\sqrt[3]{250}$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ \sqrt[3]{8 \cdot 2} - 7\sqrt[3]{125 \cdot 2} \\ \downarrow \quad \downarrow \end{array}$$

$$2\sqrt[3]{2} - 35\sqrt[3]{2} = -33\sqrt[3]{2}$$

$$3.) -2\sqrt{3} + 3\sqrt{27} - \sqrt{12} + 3\sqrt{3}$$

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ -2\sqrt{3} + 3\sqrt{9 \cdot 3} - \sqrt{4 \cdot 3} + 3\sqrt{3} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \end{array}$$

$$-2\sqrt{3} + 9\sqrt{3} - 2\sqrt{3} + 3\sqrt{3}$$

$$= (-2 + 9 - 2 + 3)\sqrt{3} = 8\sqrt{3}$$

$$4.) -\sqrt{45} + 2\sqrt{5} - \sqrt{20} - 2\sqrt{6}$$

simplify each radical

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ -\sqrt{9 \cdot 5} + 2\sqrt{5} - \sqrt{4 \cdot 5} - 2\sqrt{6} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \end{array}$$

$$-3\sqrt{5} + 2\sqrt{5} - 2\sqrt{5} - 2\sqrt{6}$$

combine similar terms or like radicals

$$\underbrace{(-3 + 2 - 2)\sqrt{5}}_{-3\sqrt{5}} - 2\sqrt{6}$$

$$= -3\sqrt{5} - 2\sqrt{6}$$

this is now the final answer because you cannot
add/subtract unlike radicals

$$5.) -3\sqrt{3} + \sqrt{8} - 3\sqrt{3} \rightarrow -3\sqrt{3} + 2\sqrt{2} - 3\sqrt{3} = -6\sqrt{3} + 2\sqrt{2}$$

$$6.) 2x \sqrt[3]{24x} + 5 \sqrt[3]{81x^4} \rightarrow 2x \sqrt[3]{8 \cdot 3x} + 5 \sqrt[3]{27 \cdot 3x^4} \\ = 4x \sqrt[3]{3x} + 15x \sqrt[3]{3x} = 19x \sqrt[3]{3x}$$

$$7.) 3\sqrt{20} - 6\sqrt{125} + 5\sqrt{45} \rightarrow 3\sqrt{4 \cdot 5} - 6\sqrt{25 \cdot 5} + 5\sqrt{9 \cdot 5} \\ = 6\sqrt{5} - 30\sqrt{5} + 15\sqrt{5} = -9\sqrt{5}$$

Multiplying Radical Expressions

You can multiply any two radicals that have the same indices together. If the radicals do not have the same indices, manipulate the equation until they do.

A. Multiply Radicals without Coefficients

$$1.) (\sqrt{18})(\sqrt{2}) \quad \text{Step 1: Make sure that the radicals have the same index}$$

$$\sqrt{(18)(2)} = \sqrt{36} \quad \text{Step 2: Multiply the numbers under the radical signs}$$

$$= \sqrt{36} = 6 \quad \text{Step 3: Simplify the radical expressions.}$$

$$2.) \sqrt{10} \cdot \sqrt{5} = \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$$

$$3.) (\sqrt[3]{3})(\sqrt[3]{9}) = \sqrt[3]{27} = 3$$

$$4.) 5 \cdot \sqrt{6} = 5\sqrt{6}$$

B. Multiply Radicals with Coefficients

$$1.) (3\sqrt{2})(\sqrt{10}) \quad \text{Step 1: Multiply the coefficients.}$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ (3\sqrt{2})(\sqrt{10}) \\ \uparrow \quad \uparrow \end{array} \quad \text{Step 2: Multiply the numbers under the radical signs}$$

$$3\sqrt{20} = 3\sqrt{4 \cdot 5} = 6\sqrt{5} \quad \text{Step 3: Simplify the radical expressions.}$$

$$2. (4\sqrt{3})(3\sqrt{6}) \rightarrow \begin{array}{c} \downarrow \quad \downarrow \\ (4\sqrt{3})(3\sqrt{6}) \\ \uparrow \quad \uparrow \end{array} = 12\sqrt{18} = 12\sqrt{9 \cdot 2} = 36\sqrt{2}$$

C. Multiply Radicals with Different Indices

$$1.) (\sqrt[3]{5})(\sqrt{2})$$

$$\downarrow \quad \downarrow$$

$$5^{\frac{1}{3}} \cdot 2^{\frac{1}{2}}$$

$$\downarrow \quad \downarrow$$

$$5^{\frac{2}{6}} \cdot 2^{\frac{3}{6}}$$

$$\downarrow \quad \downarrow$$

$$(\sqrt[6]{5^2})(\sqrt[6]{2^3})$$

$$\downarrow \quad \downarrow$$

$$(\sqrt[6]{25})(\sqrt[6]{8}) = \sqrt[6]{200}$$

Step 1: Convert radical expression to exponential expression

Step 2: Find the LCD of the exponents $\frac{1}{3}$ and $\frac{1}{2}$ which is 6 and became $\frac{2}{6}$ and $\frac{3}{6}$ respectively (similar fractions)

Step 3: Convert exponential expression to radical expression
Notice that the radicals have the same indices.

Step 4: Simplify the radicand and you can now multiply radicals with the same indices

$$2.) (\sqrt[3]{2})(\sqrt[4]{3}) = 2^{\frac{1}{3}} \cdot 3^{\frac{1}{4}} = 2^{\frac{4}{12}} \cdot 3^{\frac{3}{12}} = (\sqrt[12]{2^4})(\sqrt[12]{3^3}) = (\sqrt[12]{16})(\sqrt[12]{27}) = \sqrt[12]{432}$$

D. Multiplying Radicals with More Than One Term

$$1.) \sqrt{2}(\sqrt{3} + \sqrt{5}) \rightarrow \sqrt{2}(\sqrt{3} + \sqrt{5}) \quad \text{Distributive Property}$$

$$= \underbrace{\sqrt{2} \cdot \sqrt{3}} + \underbrace{\sqrt{2} \cdot \sqrt{5}}$$

$$= \sqrt{6} + \sqrt{10}$$

$$2.) \sqrt{3}(\sqrt{6} - \sqrt{3}) \rightarrow \sqrt{3}(\sqrt{6} - \sqrt{3}) = \sqrt{18} - \sqrt{9}$$

$$= \sqrt{9 \cdot 2} - 3$$

$$= 3\sqrt{2} - 3$$

E. Multiplying Binomials Involving Radicals

$$1.) (2 - \sqrt{7})(5 - \sqrt{3}) \rightarrow (2 - \sqrt{7})(5 - \sqrt{3}) \quad \text{FOIL method}$$

$$= \frac{2 \cdot 5}{\downarrow \text{F}} \quad \frac{2 \cdot -\sqrt{3}}{\downarrow \text{O}} \quad \frac{-\sqrt{7} \cdot 5}{\downarrow \text{I}} \quad \frac{-\sqrt{7} - \sqrt{3}}{\downarrow \text{L}}$$

$$= 10 - 2\sqrt{3} - 5\sqrt{7} + \sqrt{21}$$

$$\begin{array}{lcl}
 2.) (3 - \sqrt{2})(4 + \sqrt{2}) & \rightarrow & \begin{array}{l} \mathbf{F:} 3(4) = 12 \\ \mathbf{O:} 3(\sqrt{2}) = 3\sqrt{2} \\ \mathbf{I:} -\sqrt{2}(4) = -4\sqrt{2} \\ \mathbf{L:} -\sqrt{2}(\sqrt{2}) = -2 \end{array} \\
 & & \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \\ \\ \\ \end{array} = 12 + 3\sqrt{2} - 4\sqrt{2} - 2 \\
 & & \qquad \qquad \qquad = 10 - \sqrt{2}
 \end{array}$$

$$\begin{array}{lcl}
 3.) (4 + \sqrt{5})(3 - \sqrt{7}) & \rightarrow & \begin{array}{l} \mathbf{F:} 4(3) = 12 \\ \mathbf{O:} 4(-\sqrt{7}) = -4\sqrt{7} \\ \mathbf{I:} \sqrt{5}(3) = 3\sqrt{5} \\ \mathbf{L:} \sqrt{5}(-\sqrt{7}) = -\sqrt{35} \end{array} \\
 & & \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} = 12 - 4\sqrt{7} + 3\sqrt{5} - \sqrt{35}
 \end{array}$$

F. Multiplying Radicals with Its Conjugate

A conjugate is a binomial formed by negating the second term of a binomial. An example is, the conjugate of $(x + y)$ is $(x - y)$.

$$\begin{array}{lcl}
 1.) (3 - \sqrt{5})(3 + \sqrt{5}) & \rightarrow & \begin{array}{l} (3 - \sqrt{5})(3 + \sqrt{5}) \\ = 9 - \sqrt{25} \\ = 9 - 5 \\ = 4 \end{array}
 \end{array}$$

$$2.) (\sqrt{6} + 2)(\sqrt{6} - 2) \rightarrow (\sqrt{6} + 2)(\sqrt{6} - 2) = \sqrt{36} - 4 = 6 - 4 = 2$$

$$3.) (\sqrt{2} + \sqrt{7})(\sqrt{2} - \sqrt{7}) \rightarrow (\sqrt{2} + \sqrt{7})(\sqrt{2} - \sqrt{7}) = \sqrt{4} - \sqrt{49} = 2 - 7 = -5$$

$$\begin{array}{lcl}
 4.) (3\sqrt{11} + 5\sqrt{2})(3\sqrt{11} - 5\sqrt{2}) & \rightarrow & (3\sqrt{11} + 5\sqrt{2})(3\sqrt{11} - 5\sqrt{2}) = 9\sqrt{121} - 25\sqrt{4} \\
 & & = 99 - 50 = 49
 \end{array}$$

Dividing Radical Expressions

In the previous activity, you were able to simplify the radicals by rationalizing the denominator. Rationalization is a process where you simplify the expression by making the denominator free from radicals. This skill is necessary in the division of radicals.

A. Dividing Radicals with the Same Indices

$$1.) \sqrt{12} \div \sqrt{3} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

$$2.) \sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

Rationalizing the Denominator

$$3.) \sqrt{\frac{7}{5}} = \frac{\sqrt{7}}{\sqrt{5}} = \frac{\sqrt{7}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{35}}{\sqrt{25}} = \frac{\sqrt{35}}{5}$$

$$4.) \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

B. Dividing Radicals with Different Indices

$$1.) \sqrt[3]{3} \div \sqrt{3} \quad \text{Step 1: Convert radical expression to exponential expression}$$

$$\frac{\sqrt[3]{3}}{\sqrt{3}} = \frac{3^{\frac{1}{3}}}{3^{\frac{1}{2}}}$$

$$= \frac{3^{\frac{2}{6}}}{3^{\frac{3}{6}}} \quad \text{Step 2: Find the LCD of the exponents } \frac{1}{2} \text{ and } \frac{1}{3} \text{ which is 6}$$

and became } \frac{2}{6} \text{ and } \frac{3}{6} \text{ respectively (similar fractions)}

$$= \frac{\sqrt[6]{3^2}}{\sqrt[6]{3^3}} \quad \text{Step 3: Convert exponential expression to radical expression}$$

Notice that the radicals have the same indices.

$$= \frac{\sqrt[6]{3^2}}{\sqrt[6]{3^3}} \cdot \frac{\sqrt[6]{3^3}}{\sqrt[6]{3^3}} \quad \text{Step 4: Rationalize the denominator}$$

$$= \frac{\sqrt[6]{3^5}}{\sqrt[6]{3^6}} = \frac{\sqrt[6]{243}}{3}$$

C. Rationalizing the Denominator Using Its Conjugate

$$1.) \frac{3}{\sqrt{3}-\sqrt{2}} \rightarrow \frac{3}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3(\sqrt{3}+\sqrt{2})}{\sqrt{9}-\sqrt{4}} = \frac{3\sqrt{3}+3\sqrt{2}}{3-2} = \frac{3\sqrt{3}+3\sqrt{2}}{1} = 3\sqrt{3} + 3\sqrt{2}$$

$$2.) \frac{6-\sqrt{3}}{2+\sqrt{7}} \rightarrow \frac{6-\sqrt{3}}{2+\sqrt{7}} \cdot \frac{2-\sqrt{7}}{2-\sqrt{7}} = \frac{12-6\sqrt{7}-2\sqrt{3}+\sqrt{21}}{4-\sqrt{49}} = \frac{12-6\sqrt{7}-2\sqrt{3}+\sqrt{21}}{4-7}$$

$$= \frac{12-6\sqrt{7}-2\sqrt{3}+\sqrt{21}}{-3} \quad \text{or}$$

$$= \frac{-12+6\sqrt{7}+2\sqrt{3}-\sqrt{21}}{3}$$

$$3.) \frac{6-\sqrt{2}}{4-3\sqrt{2}} \rightarrow \frac{6-\sqrt{2}}{4-3\sqrt{2}} \cdot \frac{4+3\sqrt{2}}{4+3\sqrt{2}} = \frac{24+18\sqrt{2}-4\sqrt{2}-3\sqrt{4}}{16-9\sqrt{4}} = \frac{24+14\sqrt{2}-6}{16-18}$$

$$= \frac{18+14\sqrt{2}}{-2}$$

$$= -9 - 7\sqrt{2}$$



Explore

Activity 3: Like or Unlike Radicals

A. Identify whether the following are **Like** radicals or **Unlike** radicals. For like radicals write **LR** otherwise write **UR** on your answer sheets

- 1) $\sqrt{2}$ and $\sqrt{5}$ → _____
- 2) $2\sqrt{7y}$ and $3\sqrt{7y}$ → _____
- 3) $\sqrt{3}$ and $\sqrt[3]{3}$ → _____
- 4) $5\sqrt[6]{3x}$ and $5\sqrt[6]{2x}$ → _____
- 5) $2\sqrt[7]{m^2n^3}$ and $-\sqrt[7]{n^3m^2}$ → _____

B. Simplify to make each pair of radical expression like radicals. An example is provided for you. Write your answer on your answer sheets.

Example $\sqrt{2}$ and $\sqrt{8} = \sqrt{2}$ and $\sqrt{4 \cdot 2} = \sqrt{2}$ and $2\sqrt{2}$

- 6) $\sqrt{2}$ and $\sqrt{18}$ → _____ and _____
- 7) $5\sqrt{3}$ and $\sqrt{27}$ → _____ and _____

$$8) 2\sqrt{28} \text{ and } 5\sqrt{7} \quad \rightarrow \quad \underline{\hspace{2cm}} \text{ and } \underline{\hspace{2cm}}$$

$$9) \sqrt{20a^2} \text{ and } a\sqrt{45} \quad \rightarrow \quad \underline{\hspace{2cm}} \text{ and } \underline{\hspace{2cm}}$$

$$10) 2\sqrt{3a^3} \text{ and } 5a\sqrt{3a} \quad \rightarrow \quad \underline{\hspace{2cm}} \text{ and } \underline{\hspace{2cm}}$$

Activity 4: Adding & Subtracting Radicals

Directions: Add or subtract the following radical expressions. Make sure your answer is in simplest radical form. Write your answer on your answer sheets.

$$1.) 5\sqrt{2} + 3\sqrt{2} \quad = \quad \underline{\hspace{2cm}}$$

$$2.) 4\sqrt[3]{6} - \sqrt[3]{6} \quad = \quad \underline{\hspace{2cm}}$$

$$3.) 10\sqrt{7} + 12\sqrt{7} - 8\sqrt{7} + \sqrt{7} \quad = \quad \underline{\hspace{2cm}}$$

$$4.) \sqrt{12} + \sqrt{27} - \sqrt{3} \quad = \quad \underline{\hspace{2cm}}$$

$$5.) \sqrt{50} + \sqrt{32} - \sqrt{8} \quad = \quad \underline{\hspace{2cm}}$$



Deepen

Activity 5: Multiplying and Dividing Radicals

Performance Task: Simplify each radical expression by multiplying or dividing the radicals. Write your answer on your answer sheets. Please show your *SOLUTIONS*. The rubric below will guide you in the presentation of your solutions.

$$1.) \sqrt{5} \cdot \sqrt{10}$$

$$2.) -3\sqrt{3}(2 + \sqrt{6})$$

$$3.) (7 + \sqrt{12})(7 - \sqrt{12})$$

$$4.) \sqrt{6} \div \sqrt{5}$$

$$5.) \frac{5}{5 + \sqrt{2}}$$

Rubric for the Task

| RATING | 4 Successful | 3 Progressing | 2 Limited | 1 N/A |
|------------------------|---|--|---|---------------|
| Presentation | The presentation shows complete understanding of all important math concepts learned. | The presentation shows partial understanding of all important math concepts learned. | The presentation is incomplete and does not correctly show important math concepts learned. | Not addressed |
| Strategies | The strategies are correct for the task. | The strategies are partially correct for the task. | The strategies are not correct for the task. | Not addressed |
| Computation & Solution | The computation is correct and leads to the correct answer. | The computation is correct but does not lead to the correct answer. | The computation is incorrect and does not relate to the task. | Not addressed |

**Gauge****Post – Assessment**

Directions: Read each item carefully and pick out your answer from the choices given. Write the letter of your chosen answer on your answer sheet.

- What is the exponential form of $2\sqrt{a} + 3\sqrt{b}$?
 A. $2^{\frac{1}{2}}a + 3^{\frac{1}{2}}b$ B. $2a^{\frac{1}{2}} + 3b$ C. $(2a)^{\frac{1}{2}} + (3b)^{\frac{1}{2}}$ D. $(2a + 3b)^{\frac{1}{2}}$
- What is the equivalent of $\sqrt[3]{4} + \sqrt[5]{2}$ using exponential notation?
 A. $4^{\frac{1}{3}} + 2^{\frac{1}{5}}$ B. $4^3 + 2^5$ C. 6^8 D. $6^{\frac{1}{8}}$
- Which of the following is an incorrect characteristic of a radical in simplest form?
 A. No fraction as radicands.
 B. No radicands with variable.
 C. No radical appears in the denominator of a fraction.
 D. No radicands have perfect square factors other than 1.
- What is the sum of $5\sqrt{2}$ and $7\sqrt{2}$?
 A. $-2\sqrt{2}$ B. $2\sqrt{2}$ C. $12\sqrt{2}$ D. $12\sqrt{4}$
- What is the simplified form of the radical expression $3\sqrt{20} + \sqrt{5}$?
 A. $5\sqrt{5}$ B. $6\sqrt{5}$ C. $7\sqrt{5}$ D. $7\sqrt{20}$
- What is the sum of $2\sqrt{3}$, $-5\sqrt{2}$, $10\sqrt{3}$, $14\sqrt{2}$ and $-3\sqrt{2}$?
 A. $34\sqrt{5}$ B. $18\sqrt{5}$ C. $6\sqrt{2} + 12\sqrt{3}$ D. $12\sqrt{2} + 6\sqrt{3}$
- What is the difference of $15\sqrt{3}$ and $17\sqrt{3}$?
 A. $-2\sqrt{3}$ B. $2\sqrt{3}$ C. $32\sqrt{3}$ D. $32\sqrt{6}$

- LU Q2 Mathematics 9 Module6

References

Books

Department of Education (2014), Mathematics Grade 9 Learner's Module, Module 4: Zero Exponents, Negative Integral Exponents, Rational Exponents, and Radicals, First Edition

Department of Education (2014), Mathematics Grade 9 Teacher's Guide, Module 4: Zero Exponents, Negative Integral Exponents, Rational Exponents, and Radicals, First Edition

Websites

"Add & Subtract Radicals", accessed August 10, 2021
<https://mathbitsnotebook.com/Algebra2/Radicals/RDAddSubtract>

"Conjugates & Dividing by Radicals", accessed August 10, 2021
<https://www.purplemath.com/modules/radicals4.htm>

"Multiplication Radicals", accessed August 10, 2021
<https://www.katesmathlessons.com/multiplying-radicals.html>

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