Senior High School

General Mathematics

Module 3:

Problem Solving Involving Rational Functions, Equations, and Inequalities

AIRs - LM

GENERAL MATHEMATICS
Module 3: Problem Solving Involving Rational Functions, Equations, and Inequalities
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General Mathematics
Module 3:
Problem Solving Involving Rational Functions, Equations, and Inequalities

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

A rational function is a very useful type of function in mathematics. It is a function whose numerator and denominator are both polynomials.

Rational formulas can be useful tools for representing real-life situations and for finding answers to real-life problems. Equations representing direct, inverse, and joint variation are examples of rational formulas that can model many real-life situations.

From the previous lesson, you have learned how to distinguish the similarities and differences among rational functions, rational equations, and rational inequalities, and finding the domain and range of a rational function.

In this module, you need to apply the previous concepts to determine and solve for the intercepts, zeroes, and asymptotes of rational functions and to solve problems involving rational functions, equations, and inequalities as well as to apply them in dealing with daily life problems which you may encounter.

After learning the lesson, you are expected to:

a. determine the (a) intercepts; (b) zeroes; and (c) asymptotes of rational functions **(M11GM-Ic-1);** and
b. solve problems involving rational functions, equations, and inequalities **(M11GM-Ic-3).**

Learning objectives:

1. define intercepts, zeroes, and asymptotes of rational functions
2. determine the (a) intercepts; (b) zeroes; and (c) asymptotes of rational functions
3. solve problems involving rational functions, equations, and inequalities in real-life situations

Pretest

Directions: Read and analyze each item carefully. Write the letter of the best answer using a separate sheet of paper.

1. Which of the following is a line (or a curve) that the graph gets close to but does not touch?
   A. Asymptote B. Intercept C. Line D. Zero

2. Which of the following should be determined when adding and subtracting rational expressions with different denominators?
   A. Composite factors B. Prime factors
   C. Least common denominator D. Greatest common factor

3. What value is obtained when solving rational equations but makes the equation false or makes an expression undefined?
   A. Factor B. Variable
   C. Solution set D. Extraneous solution

4. Which of the following are x or y coordinates of the points at which a graph crosses the x-axis or y-axis, respectively?
   A. Asymptote B. Intercepts C. Parabola D. Roots

For items 5 and 6, use the rational function, $f(x) = \frac{4x-4}{2x+1}$

5. What is the vertical asymptote for the rational function?
   A. $x = -1/2$ B. $x = -2$ C. $x = 1/2$ D. $x = 2$

6. What will be the horizontal asymptote of the rational function?
   A. 1 B. 2 C. 3 D. 4

7. To solve the value of x in $\frac{x+1}{2x-1} = \frac{3}{4x-1}$, what should be multiplied to both sides of this equation?
   A. $(2x-1)\,(4x-1)$ B. $(2x-1)\,(4x+1)$
   C. $(4x-1)\,(2x+1)$ D. $(4x+1)\,(2x+1)$

8. What will be the horizontal asymptote if the degree (highest power) of the numerator is larger than the degree of the denominator?
   A. The horizontal asymptote is 1.
   B. The horizontal asymptote is -1.
   C. There is no horizontal asymptote.
   D. The horizontal asymptote is at $y = 0$.

9. Which of the following is an extraneous solution to $\frac{n}{n+2} + \frac{1}{n+4} = \frac{2}{n^2+6n+8}$?
   A. -5 B. -2 C. 0 D. none

10. What is the first step in solving rational inequality?
    A. Reverse the inequality sign.
    B. Solve the inequality immediately.
    C. Rewrite the inequality so that zero is on one side.
    D. Rewrite the inequality so that zero is on both sides.

11. Which of the following is NOT a rational function?
    A. $f(x) = \frac{\sqrt{x}+1}{x}$
    B. $f(x) = \frac{9x^2+6x+1}{3x+1}$
    C. $f(x) = \frac{x-5}{x+2}$
    D. $f(x) = \frac{x-5}{x^2-25}$

12. What is the vertical asymptote of the function $g(x) = \frac{3}{x-2}$?
    A. $x = 1$
    B. $x = 2$
    C. $x = 3$
    D. $x = 4$

13. Which of the following functions has an oblique or slant asymptote?
    A. $f(x) = \frac{2x-1}{x+3}$
    B. $f(x) = \frac{2}{x-1}$
    C. $f(x) = \frac{x^2-x-2}{x-1}$
    D. $f(x) = \frac{1}{x^2}$

14. What is the x-intercept of $f(x) = \frac{x-5}{x-2}$?
    A. 2
    B. 3
    C. 4
    D. 5

15. How long would it take two painters to paint a car together if one painter can paint the car in six hours and the second painter in four hours?
    A. 1.4 hours
    B. 2.4 hours
    C. 3.4 hours
    D. 4.4 hours

Jumpstart

Activity 1

Direction: Fill in the blanks to make each statement true. Choose your answers on the box below. Write your answers in a separate paper.

horizontal asymptote	numerator	domain
vertical asymptote	denominator	extraneous solution
larger	asymptote	intercepts
zero	least common denominator	

1. ______________ is an apparent solution that does not solve its equation.
2. A ______________ is a line or (a curve) that the graph of a fucntion gets close to but does not touch.
3. If the degree of the ______________ is smaller than the degree of the ______________, then the horizontal asymptote is at $y = 0$.
4. A ______________ is a horizontal line that a part of the graph approaches (as x increases or decreases without bound) but never touches.
5. The first step in solving rational equations is to eliminate denominators by multiplying each term of the equation by the _______________.
6. If the degree (highest power) of the numerator is _______________ than the degree of the denominator, then there is no horizontal asymptote.
7. The first step in solving inequality is to rewrite it with ______________ on one side.
8. A ______________ is a vertical straight line toward which a function approaches closer and closer, but never reaches (or touches).
9. ______________ are x or y coordinates of the points at which a graph crosses the x-axis or y-axis, respectively.
10. The ______________ of a rational function is the set of all real numbers, excluding the zeroes of the denominator.

Discover

Intercepts of Rational Functions

Intercepts are x or y coordinates of the points at which a graph crosses the x-axis or y-axis, respectively.

Note: Not all rational functions have both x and y intercepts. If the rational function f has no real solution, then it does not have intercepts.

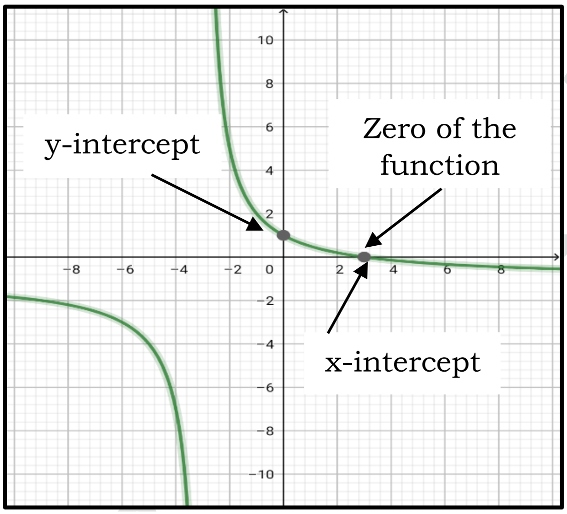

Figure 1. x and y-intercepts using Graph Sketch

Rule:

1. To find the y-intercept, substitute 0 for x and solve for y or *f(x).*
2. To find the x-intercept, substitute 0 for y and solve for x.

Example 1: Find the intercepts of the following rational functions:

a. $f(x) = \frac{x+8}{x-2}$ b. $f(x) = \frac{x^2-5x+6}{x^2-2x+3}$ c. $f(x) = \frac{x^2+9}{x^2-3}$

Solutions:

a. $f(x) = \frac{x+8}{x-2}$

For y-intercept

$f(0) = \frac{0+8}{0-2}$ Substitute 0 for x

$= \frac{8}{-2}$ Simplify

$= -4$

Therefore, the y-intercept is -4.

For x-intercept

$$0 = \frac{x+8}{x-2}$$ Substitute 0 for y or *f(x)*

$$0 = x + 8$$ Multiply both sides by $x - 2$

$$\boldsymbol{x = -8}$$ Simplify

Therefore, the x-intercept is -8.

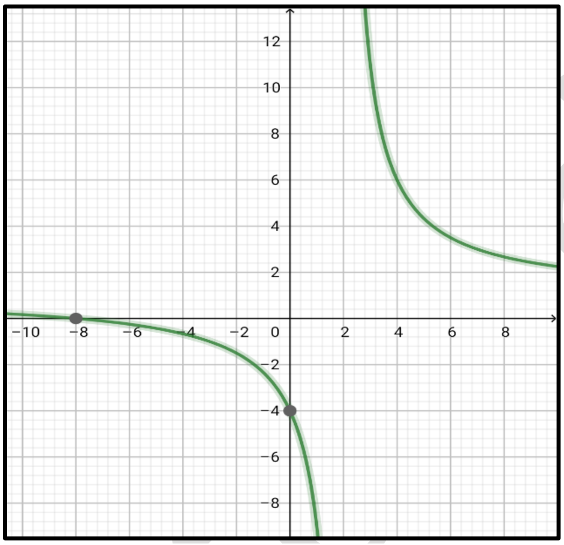

Figure 2. Intercepts of $f(x) = \frac{x+8}{x-2}$ using Graph Sketch

b. $f(x) = \frac{x^2-5x+6}{x^2-2x+3}$

For y-intercept

$$f(0) = \frac{(0)^2 - 5(0) + 6}{(0)^2 - 2(0) + 3}$$ Substitute 0 for x

$$= \frac{6}{3}$$

$$= 2$$

Therefore, the y-intercept is 2.

For x-intercept:

$$0 = \frac{x^2-5x+6}{x^2-2x+3}$$ Substitute 0 for y or *f(x)*

$$0 = x^2 - 5x + 6$$ Multiply both sides by $x^2 - 2x + 3$

$$0 = (x-2)(x-3)$$ Factor

$$x = 2 \quad or \quad x = 3$$

Therefore, the x-intercepts are 2 and 3.

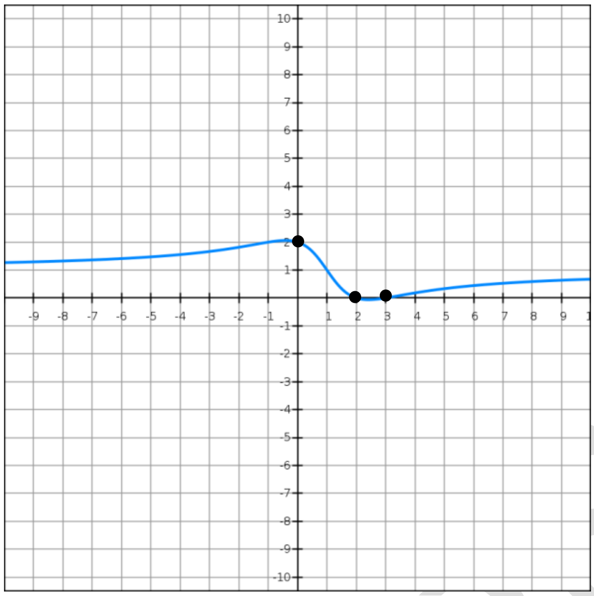

Figure 3. Intercepts of $f(x) = \frac{x^2-5x+6}{x^2-2x+3}$ using Graph Sketch

c. $f(x) = \frac{x^2+9}{x^2-3}$

For y-intercept

$$f(0) = \frac{(0)^2+9}{(0)^2-3} \qquad \text{Substitute 0 for x}$$

$$= \frac{9}{-3} \qquad \text{Simplify}$$

$$= -3$$

Therefore, the y-intercept is -3.

For x-intercept

$$0 = \frac{x^2+9}{x^2-3} \qquad \text{Substitute 0 for y or } f(x)$$

$$0 = x^2+9 \qquad \text{Multiply both sides by } x^2-3$$

$$-9 = x^2$$

There is no real solution for $x^2 + 9 = 0$. Hence, there is no x-intercept.

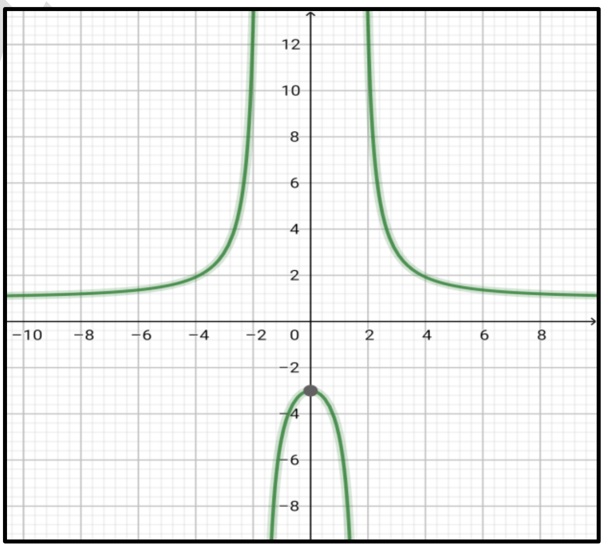

Figure 4. Intercepts of $f(x) = \frac{x^2+9}{x^2-3}$ using Graph Sketch

Zeroes of Rational Functions

In general, when dealing with rational functions, whatever value of x that will make the numerator zero without simultaneously making the denominator equal to zero will be a zero of the said rational function.

Note: To find the zeroes of a rational function, equate the function to 0 or solve for the x-intercept of the function by equating the numerator to 0.

Example 1: Find the zeroes of the rational function $f(x) = \frac{x-2}{x+6}$

Solution:

$$f(x) = \frac{x-2}{x+6}$$

$x - 2 = 0$ Equate the numerator to 0

$\boldsymbol{x = 2}$ Solve for x

Thus, the zero of the rational function $f(x) = \frac{x-2}{x+6}$ is 2.

Example 2. Find the zeroes of the rational function $f(x) = \frac{x-2}{x^2-4}$

Solution:

$f(x) = \frac{x-2}{x^2-4}$ Simplify by factoring the denominator

$f(x) = \frac{\overset{1}{\cancel{x-2}}}{\cancel{(x-2)}(x+2)}$ Remove common factors

$0 = \frac{1}{x+2}$ Equate the numerator to 0

$\boldsymbol{0 \neq 1}$ False statement

So, there is **no zero** of the function which means that no point on the graph touches the x-axis. Thus, 1 is a false solution. Such false solutions are often called *extraneous solutions.*

An **extraneous solution** is an apparent solution that does not solve its equation.

Example 3: Find the zeroes of the rational function $f(x) = \frac{x^2+x-2}{x^2-4}$

Solution:

$$f(x) = \frac{x^2+x-2}{x^2-4}$$

$f(x) = \frac{(x+2)(x-1)}{(x+2)(x-2)}$ Simplify by factoring both the numerator and denominator

$f(x) = \frac{x-1}{x-2}$ Remove the common factors

$x - 1 = 0$ Equate the numerator to 0

$\boldsymbol{x = 1}$ Solve for x

Thus, the zero of the rational function $f(x) = \frac{x^2+x-2}{x^2-4}$ is 1.

Asymptotes of Rational Functions

The graphs of rational functions can be recognized by the fact that they often break into two or more parts. These parts go out of the coordinate system along an imaginary straight line called an asymptote. Asymptotes are lines that the graph approaches but does not touch.

Finding Asymptotes

1. Vertical Asymptotes

A *vertical asymptote* is a vertical straight line toward which a function approaches closer and closer, but never reaches (or touches). Vertical asymptote corresponds to the undefined locations of rational functions. To find the vertical asymptotes, simplify first the factored rational expression. Set any remaining factors of the denominator equal to zero. A vertical asymptote will occur at each of these x-values.

2. Horizontal Asymptotes

A *horizontal asymptote* is a horizontal line that a part of the graph approaches (as x increases or decreases without bound) but never touches. The rule for finding a horizontal asymptote depends on the largest power in the numerator and denominator.

1. If the degree (highest power) of the numerator is larger than the degree of the denominator, then there is no horizontal asymptote.

2. If the degree of the numerator is smaller than the degree of the denominator, then the horizontal asymptote is at $y = 0$ (the x–axis).

3. If the degree of the numerator is equal to the degree of the denominator, then you must compare the coefficients in front of the terms with the highest power. The horizontal asymptote is the coefficient of the highest power of the numerator divided by the coefficient of the highest power of the denominator.

3. Oblique or Slant Asymptotes

Oblique asymptotes occurs when the numerator of $f(x)$ has a degree that is one higher than the degree of the denominator. If you have this situation, simply divide the numerator by the denominator by either using long division or synthetic division.

Example 1 Find the asymptote/s of each rational function and sketch the graph

a. $f(x) = \frac{1}{x+1}$

b. $f(x) = \frac{4x}{2x+1}$

c. $f(x) = \frac{x^2}{x-1}$

d. $f(x) = \frac{x^2-4x+3}{x^2-5x+4}$

e. $f(x) = \frac{x^2-6x-1}{x+3}$

Solutions:

a. $f(x) = \frac{1}{x+1}$

To find the vertical asymptote, set the denominator equal to zero and solve for x.

$$x + 1 = 0$$
$$\boldsymbol{x = -1}$$

Therefore, the *vertical asymptote* is $x = -1$.

Since, the numerator is less than the degree of the denominator, then the *horizontal asymptote* is $y = 0$.

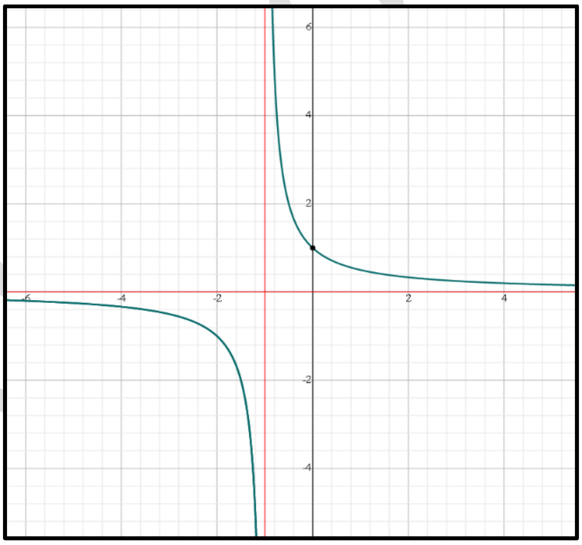

Figure 5. Vertical Asymptote of $f(x) = \frac{1}{x+1}$ using Graph Sketch

b. $f(x) = \frac{4x}{2x+1}$

To find the vertical asymptote, set the denominator equal to zero and solve for x.

$$2x + 1 = 0$$
$$2x = -1$$
$$\boldsymbol{x = -\frac{1}{2}}$$

The vertical asymptote is $x = -\frac{1}{2}$.

The degree of the numerator is equal to the degree of the denominator. The leading coefficient of the numerator is 4 and the leading coefficient of the denominator is 2. So, the horizontal asymptote is $y = \frac{4}{2}$ or $y = 2$.

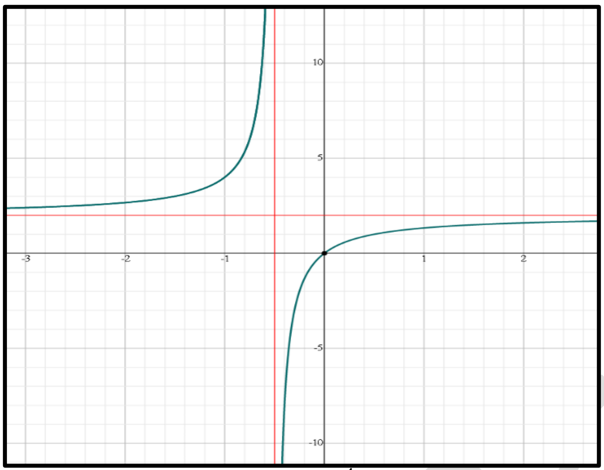

Figure 6. Asymptotes of $f(x) = \frac{4x}{2x+1}$ using Graph Sketch

c. $f(x) = \frac{x^2}{x-1}$

Set the denominator equal to zero and solve for x to get the vertical asymptote.

$$x - 1 = 0$$
$$\boldsymbol{x = 1}$$

The vertical asymptote is $x = 1$.

The degree of the numerator is greater than the degree of the denominator. Hence, there is no horizontal asymptote.

Since the polynomial in the numerator is a degree higher than the denominator, we know we have a slant/oblique asymptote. To find it, we must divide the numerator by the denominator using long division,

$$\begin{array}{r} x+1 \\ x-1\,\overline{)\,x^2} \\ \underline{x^2-x} \\ x \\ \underline{x-1} \\ 1 \end{array}$$

The slant/oblique asymptote: $y = x + 1$.

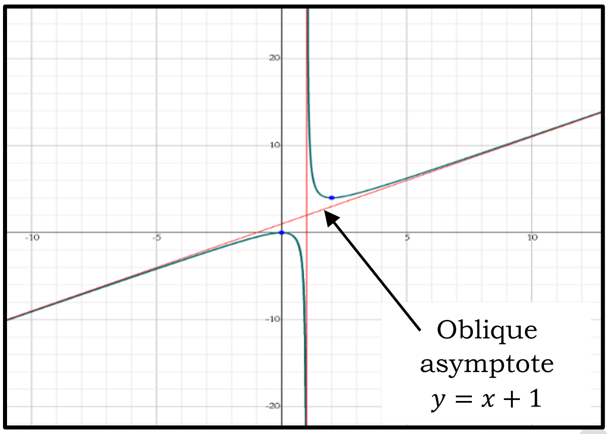

Figure 7. Asymptotes of $f(x) = \frac{x^2}{x-1}$ using Graph Sketch

d. $f(x) = \frac{x^2-4x+3}{x^2-5x+4}$

To find the vertical asymptote, reduce the function by factoring the numerator and the denominator.

$$f(x) = \frac{\cancel{(x-1)}(x-3)}{\cancel{(x-1)}(x-4)} = \frac{x-3}{x-4}$$

The vertical asymptote is $x = 4$.

The degree of the numerator is equal to the degree of the denominator. The leading coefficient of both the numerator and denominator is 1. So, the horizontal asymptote is $y = 1$.

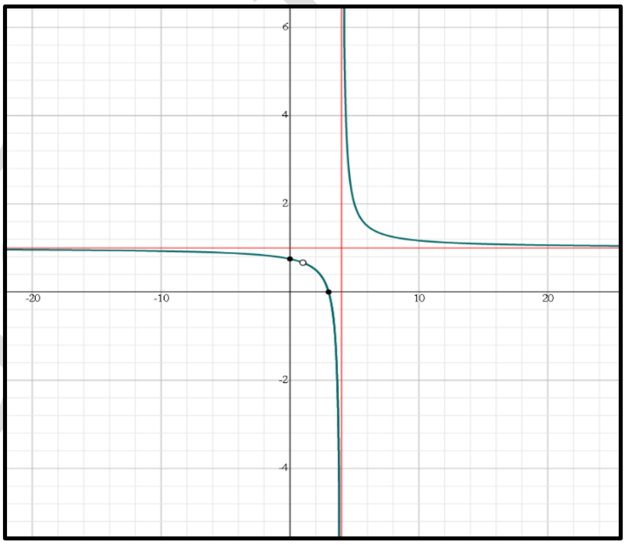

Figure 8. Asymptotes of $f(x) = \frac{x^2-4x+3}{x^2-5x+4}$ using Graph Sketch

e. $f(x) = \frac{x^2-6x-1}{x+3}$

To find the vertical asymptote, set the denominator equal to zero and solve for x.

$$x + 3 = 0$$
$$\boldsymbol{x = -3}$$

Since the polynomial in the numerator is a degree higher than the denominator, we know we have a slant/oblique asymptote. To find it, we must divide the numerator by the denominator using long division,

$$
\begin{array}{r l}
 & \quad x - 9 \\
x + 3 & \overline{)\,x^2 - 6x - 1} \\
 & \underline{x^2 + 3x} \\
 & \qquad -9x - 1 \\
 & \qquad \underline{-9x - 27} \\
 & \qquad\qquad 26
\end{array}
$$

The slant/oblique asymptote: $y = x - 9$

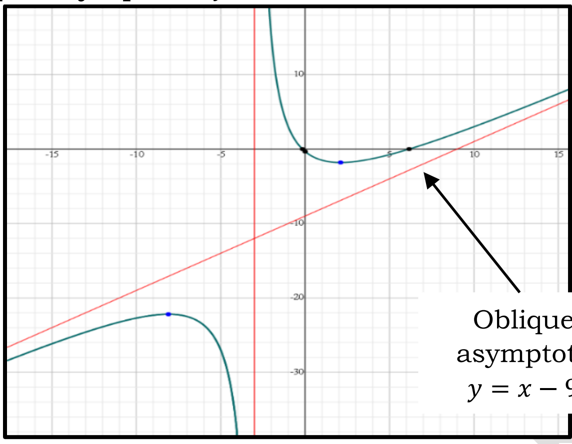

Figure 9. Asymptotes of a Rational Function $f(x) = \frac{x^2-6x-1}{x+3}$ using Graph Sketch

Solving Problems on Rational Functions, Equations, and Inequalities

Solving rational functions, equations, and inequalities is necessary in solving word problems. One of the basic applications of rational functions as real-life models are problems like number, work, distance, mixture, and other related problems.

Example 1: The numerator of a fraction is 4 less than the denominator. If both the numerator and the denominator of the fraction are increased by 2, the resulting fraction is equal to $\frac{3}{5}$. Find the original fraction.

Solution:

Step 1: Make representations.

Let x be the denominator and (x – 4) be the numerator. Thus, the fraction is $\frac{x-4}{x} = \frac{3}{5}$

Step 2: Set up the working equation where both numerator and denominator are increased by 2

$$\frac{\mathrm{x} - 4 + \mathbf{2}}{\mathrm{x} + \mathbf{2}} = \frac{3}{5}$$

$$\frac{\mathrm{x} - 2}{\mathrm{x} + 2} = \frac{3}{5}$$

Step 3: Find the least common denominator.

The LCD of all the denominators is $5(x + 2)$.

Step 4: Multiply both sides of the equation by the LCD then simplify the result.

$$5(x+2)\left[\frac{x-2}{x+2}\right]=\left[\frac{3}{5}\right](5)(x+2)$$

$$5(x-2)=3(x+2)$$

$$5x-10=3x+6$$

$$2x=16$$

Step 5: Solve the resulting equation.

$$2x=16$$

$$x=8$$

Step 6: Check the obtained values by substituting each to the original equation.

$$\frac{x-2}{x+2}=\frac{3}{5}$$

$$\frac{8-2}{8+2}=\frac{3}{5}$$

$$\frac{6}{10}=\frac{3}{5}$$

$$\frac{3}{5}=\frac{3}{5}$$

Step 7: Finalize the answer.

Thus, the original fraction is $\frac{8-4}{8}=\frac{4}{8}$.

Example 2: Working together, it takes Sam, Jenna, and Francis two hours to paint one room. When Sam works alone, he can paint one room in 6 hours. When Jenna works alone, she can paint one room in 4 hours. Determine how long would it take Francis to paint one room on his own.

Solution:

Step 1: Make representations.

Sam can paint one room in 6 hours, $\frac{1}{6}$

Jenna can paint one room in 4 hours, $\frac{1}{4}$

We do not know yet how much Francis can paint in one hour so we can denote it as, $\frac{1}{x}$

Together the three can paint the room in 2 hours, $\frac{1}{2}$

Step 2: Set up the working equation.

$$\frac{1}{6}+\frac{1}{4}+\frac{1}{x}=\frac{1}{2}$$

Step 3: Find the least common denominator.

The LCD of all the denominators is 12x.

Step 4: Multiply both sides of the equation by the LCD then simplify the result.

$$(12x)\frac{1}{6}+\frac{1}{4}+\frac{1}{x}=\frac{1}{2}(12x)$$
$$2x+3x+12=6x$$
$$\boldsymbol{x=12}$$

Therefore, Francis can paint the room in 12 hours.

Example 3: A box with a square base is to have a volume of 8 cubic meters. Let x be the length of the side of the square base and h be the height of the box. What are the possible measurements of a side of the square base if the height should be longer than a side of the square base?

Solution:

The volume of a rectangular box is the product of its width, length, and height. Since the base of the box is square, its width and length are equal.

Let x = length of the side of the box

Let h = height of the box

Using the formula $v = l \times w \times h$ to relate x *and* h.

$$v = x^2h$$

Since, $v = 8 \rightarrow 8 = x^2h$

Express h in terms of x $h = \frac{8}{x^2}$

Since the height is greater than the width, $h > x$ and our inequality is $\frac{8}{x^2} > x$.

a. Solve the inequality by rewriting it with zero on one side.

$$\frac{8}{x^2} - x > 0$$
$$\frac{8-x^3}{x^2} > 0$$
$$\frac{(2-x)(x^2+2x+4)}{x^2} > 0$$

b. By factoring the numerator to find the critical values, we get x =2 and x =0 from the denominator. Plot on a number line and use hollow circles since these values are not part of the solution.

c. Construct a table of signs

Interval	$x < 0$	$0 < x < 2$	$x > 2$
Test point	$x = -1$	$x = 1$	$x = 3$
$2 - x$	+	+	-
$x^2 + 2x + 4$	+	+	+
x^2	+	+	+
$\frac{(2-x)(x^2+2x+4)}{x^2}$	+	+	-

d. Since the rational expression is positive in the interval $0 < x < 2$, this is the solution set of the inequality. We reject the interval $x < 0$ even if the expression is positive here since we only consider positive values of x.

Therefore, the height of the box should be between 0 to 2 meters.

Explore

Activity 1: Fill Me Out!

Directions: For each of the following functions, determine the zeroes, x-intercept, y-intercept, vertical asymptote, and horizontal asymptote by filling out the table below. Use a separate sheet of paper for your answers.

Function	Zeroes	x-intercept	y-intercept	Horizontal Asymptote	Vertical Asymptote
1. $f(x) = \frac{4}{x}$					
2. $f(x) = \frac{5}{x-6}$					
3. $f(x) = \frac{x+1}{x-3}$					
4. $f(x) = \frac{x+2}{x^2-4}$					
5. $f(x) = \frac{2x^2-5x-12}{x^2-16}$					

Activity 2. Solve Me!

Direction: Analyze and solve the following problems carefully. Use a separate sheet of paper for your answers.

1. A manufacturer is producing special candles. The average cost of producing x candles is given by $C(x) = \frac{500}{x+10}$. How many candles should be produced for the average cost per candle to be at most ₱20?

2. Leana can sanitize their house in three hours. Her younger sister Leila can do the same job in five hours. If they will work together, how long will it take for them to finish the job?

3. A dressmaker ordered several meters of red cloth from a vendor, but the vendor only had 4 meters of red cloth in stock. The vendor bought the remaining lengths of red cloth from a wholesaler for P1,120.00. He then sold those lengths of red cloth to the dressmaker along with the original 4 meters of cloth for a total of P1,600.00. If the vendor's price per meter is at least P10.00 more than the wholesaler's price per meter, how many additional meters of red cloth did the vendor purchase from the wholesaler?

Deepen

At this point, you are going to apply your knowledge about rational functions, equations, and inequalities. You are expected to solve problems on rational functions, equations, and inequalities. The scoring rubric will be used in assessing your output.

Directions: Read and analyze the problem below. After reading, answer the questions that follows. Use a separate sheet of paper for your answers.

1. Barangay Dos allocated a budget amounting to P150, 000.00 to provide relief goods for each family in the barangay due to Covid-19 pandemic situation.

   a. Write an equation representing the relationship of the allotted amount per family versus the total number of families.

   b. How much will be the amount of each relief packs if there are 200 families in the barangay?

2. A sponsor wants to supplement the budget allotted for each family by providing an additional P1, 500.00.

   a. If $g(x)$ represents this new amount allotted for each family, construct a function representing the family.

   b. What will be the amount of each relief packs?

Rubrics for Scoring

Criteria	Excellent (5 points)	Satisfactory (3 points)	Developing (2 points)
Representation	Shows a complete understanding of the concept of rational functions, equations, and inequalities.	Shows a partial understanding of the concept of rational functions, equations, and inequalities	Shows limited understanding of the concept of rational functions, equations, and inequalities
Computation and Solution	Computation is correct and leads to the correct answer	Computation is correct but does not lead to the correct answer	Computation is incorrect and does not relate to the task.

Gauge

Directions: Read and analyze each item carefully. Write the letter of the best answer using a separate sheet of paper.

1. Which of the following is the set of x-values that will NOT make the denominator equal to zero?
   A. Domain B. Intercept C. Range D. Zero

For items 2 and 3, use the rational function $f(x) = \frac{2x-1}{x+3}$.

2. What is the x-intercept of the rational function?
   A. (1/2, 0) B. (-1/2, 0) C. (3, 0) D. (-3, 0)

3. What is the vertical asymptote of the rational function?
   A. $x = 1/2$ B. $x = -3$ C. $y = 1/2$ D. $y = -3$

4. Which of the following functions has $x = 0$ as a vertical asymptote?
   A. $f(x) = \frac{x^2-5x+6}{(x+1)(x+4)}$
   B. $f(x) = \frac{x^2+5x-14}{x^2-4x}$
   C. $f(x) = \frac{3x+9}{x^2-9}$
   D. $f(x) = \frac{x(x+4)(2x+3)}{x^2+4}$

5. How many elements are there in the solution set of a rational inequality?
   A. No element
   B. One element
   C. Two elements
   D. Infinite elements

6. What is the x-intercept of $f(x) = \frac{x-1}{x}$?
   A. -1
   B. 0
   C. 1
   D. All real numbers

7. What is the vertical asymptotes of the function $f(x) = \frac{2}{x^2-1}$?
   A. $x = 1$ only B. $x = -1$ only C. $x = \pm 1$ D. $x = 2$

8. What is the x-intercept of the function $f(x) = \frac{x^2-2x+1}{x^2-1}$?
   A. (0, 1) B. (1, 0) C. (-1, 0) D. none

9. What is the horizontal asymptote of the function: $g(x) = \frac{x^2+1}{x-2}$?
   A. $y = -1$
   B. $y = \frac{1}{2}$
   C. $y = 2$
   D. No horizontal asymptote

10. What is the x-intercept of the function?

A. 0 B. 1 C. 2 D.

11. What is the y-intercept of the function?

A. -1 B. -2 C. -3 D. -4

12. What is/are the zero/es of the function?

A. 1 B. 2 C. 3 D. 4

13. It takes Lance 12 minutes to stack 80 chairs. Working together, it takes Lance and Peter 6 minutes to stack 60 chairs. How long will it take for Peter to stack 50 chairs by himself?

A. 15 minutes B. 20 minutes
C. 25 minutes D. 30 minutes

14. Bobby can paint a fence in 5 hours, and working with Jenny, the two of them painted a fence in 2 hours. How long would it take Jenny to paint the fence alone?

A. 1 hour 20 minutes B. 2 hours 20 minutes
C. 3 hours 20 minutes D. 4 hours 20 minutes

For items 12-14, use the rational function $f(x) = \frac{x-2}{x^2+3x+2}$

15. Linda cuts a rectangular cloth with a perimeter of 150 meters. Which of the following functions represents the width *(y)* of the cloth as a function of the length *(x)*?

A. $y = \frac{150}{x}$ B. $y = \frac{x}{150}$ C. $y = \frac{150}{x+1}$ D. $y = 75 - x$

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