Senior High School

General Mathematics

Module 4:

One-to-one Functions and Inverse Functions

AIRs - LM

GENERAL MATHEMATICS
Module 4: One-to-one Functions and Inverse Functions
Second Edition, 2021

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General Mathematics
Module 4:
One-to-one Functions and Inverse Functions

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

One-to-one functions are special functions which are invertible. When we take its reverse, it is still be a function. And only one-to-one function has its inverse.

In your previous lesson, you have learned about the definition of rational functions, representation of rational function through table of values and graphs, you find the domain and range as well and you solve problems involving rational functions.

This learning material will provide you with information and activities that will help you understand about another type of function which is one-to-one functions and its inverse.

After going through with this learning material, you are expected to:

1. represent real-life situations using one-to-one functions **(M11GM-Id-1),**
2. determine the inverse of one-to-one function **(M11GM-Id-2),**
3. represents an inverse function through its: (a) table of values, and (b) graph (**M11GM-Id-3),**
4. finds the domain and range of an inverse function **(M11GM-id-4);** and
5. solves problems involving inverse functions (**M11GM-le-2).**

Learning Objectives:

1. define one-to-one function
2. define inverse of one-to-one function
3. identify real-life situations using one-to-one functions
4. enumerate the steps in finding the inverse of one-to-one functions
5. find the inverse of one-to-one functions
6. describe inverse functions through table of values and graphs
7. determine the domain and range of an inverse function
8. solve problems involving inverse functions

Before going on, check how much you know about this topic. Answer the pretest on the next page in a separate sheet of paper

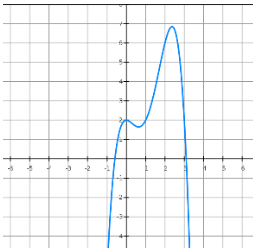
Pretest

Directions: Read each item carefully. Write the letter of the correct answer on a separate sheet of paper.

1. Which of the following relations is a one-to-one function?
   A. $\{(-1,2),(0,1),(1,2),(2,5)\}$ B. $\{(-1,1),(0,0),(1,1),(2,4)\}$
   C. $\{(-2,-8),(-1,-4),(0,0),(1,4)\}$ D. $\{(-1,6),(-2,7),(-3,8),(-2,10)\}$

2. Which of the following does NOT represent one-to-one function?
   A. Sim cards to cellphones
   B. Plate numbers to vehicles
   C. Facebook accounts to passwords
   D. Learner Reference Numbers to students

3. Which graph represents one-to-one function?

A.

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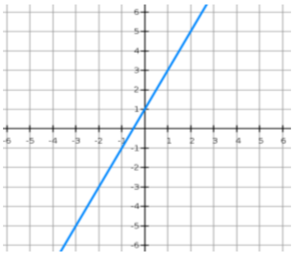
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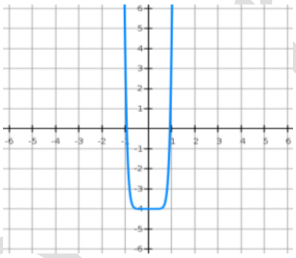
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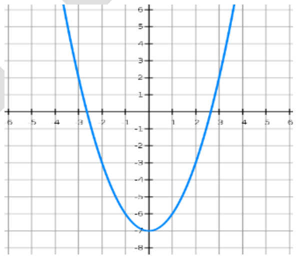


4. Which symbol denotes an inverse of a function?
   A. f B. f^{-1} C. f_{-1} D. $f-1$

5. Which is the correct way of reading this function "$f^{-1}(x) = 2x + 5$"?
   A. f reverse of x is equal to $2x + 5$.
   B. f inverse of x is equal to $2x + 5$.
   C. f raised to -1 of x is equal to $2x + 5$.
   D. The converse of x is equal to $2x + 5$.

6. Which statement is correct about inverse function?
   A. An inverse function is a function which is similar to the original function.
   B. An inverse function is a function created through negating the domain and range of a function.
   C. An inverse function is a function created through interchanging the domain and range of a function.
   D. All of these

7. Which is the correct arrangement of the steps in determining inverse of one-to-one function?
   I. Replace y with $f^{-1}(x)$
   II. Solve for y
   III. Replace $f(x)$ with y
   IV. Interchange x and y
   A. I-II-III-IV B. III-IV-II-I C. II-IV-III-I D. IV-II-III-I

8. Which is the inverse of the function $f(x) = 7x$?
   A. $f^{-1}(x) = \frac{7}{x}$ B. $f^{-1}(x) = \frac{x}{7}$ C. $f^{-1}(x) = -\frac{7}{x}$ D. $f^{-1}(x) = -\frac{x}{7}$

9. Which of the following is the inverse of the solution set $\{(0,1),(2,3),(4,5),(6,7)\}$?

A. $\{(1,0),(3,2),(5,4),(7,6)\}$ B. $\{(0,1),(-2,3),(-4,5),(-6,7)\}$

C. $\{(0,-1),(2,-3),(4,-5),(6,-7)\}$ D. $\{(-1,0),(-3,2),(-5,4),(-7,6)\}$

10. Joana and Jomar are seatmates; their teacher asks them to determine the inverse of the one-to-one function $f(x) = \frac{2x+7}{3x-4}$. Below are their solutions. Who among the two got the correct answer?

Joana's solution

$$\begin{aligned} f(x) &= \frac{2x+7}{3x-4} \\ y &= \frac{2x+7}{3x-4} \\ x &= \frac{2y+7}{3y-4} \\ 3xy - 4x &= 2y + 7 \\ 3xy - 2y &= 4x + 7 \\ y(3x-2) &= 4x + 7 \\ y &= \frac{4x+7}{3x-2} \\ f^{-1}(x) &= \frac{4x+7}{3x-2} \end{aligned}$$

Jomar's solution

$$\begin{aligned} f(x) &= \frac{2x+7}{3x-4} \\ y &= \frac{2x+7}{3x-4} \\ x &= \frac{2y+7}{3y-4} \\ 3xy - 4x &= 2y + 7 \\ 3xy - 2y &= -4x + 7 \\ y(3x-2) &= -4x + 7 \\ y &= \frac{-4x+7}{3x-2} \\ f^{-1}(x) &= \frac{4x+7}{3x-2} \end{aligned}$$

A. Joana B. Jomar C. Both D. None of them

For items 11-13, use the function f(x) = 2x – 5.

11. Which is its table of values?

A.

x	-2	-1	0	1
y	-9	-7	5	-3

B.

x	-2	-1	0	1
y	9	7	5	3

C.

x	-2	-1	0	1
y	-7	-5	-3	-1

D.

x	-2	-1	0	1
y	-9	-7	-5	-3

12. Which is the set of values of its inverse?

A. $f^{-1}(x) = \{(2,9),(1,7),(0,5),(1,3)\}$

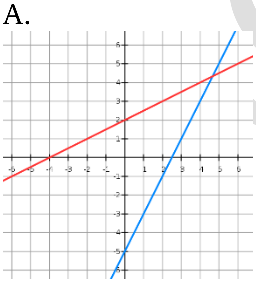
B. $f^{-1}(x) = \{(9,2),(7,1),(5,0),(3,-1)\}$

C. $f^{-1}(x) = \{(-2,-9),(-1,-7),(0,-5),(1,-3)\}$

D. $f^{-1}(x) = \{(-9,-2),(-7,-1),(-5,0),(-3,1)\}$

13. Which graph represents a function and its inverse?

A.



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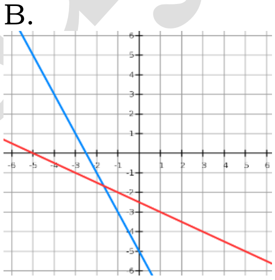
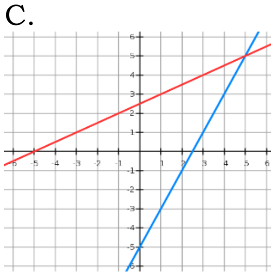
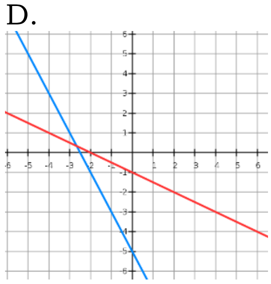


C.



D.



14. What is the domain and range of the inverse function $f^{-1}(x) = 3x + 5$?

A. Domain: All ℝ; Range: All ℝ B. Domain: (0, 3); Range: (0, 5)

C. Domain: (-3, 0); Range: (-5, 0) D. Domain: (0, -3); Range: (0, -5)

15. Liza and Lito are playing number-guessing game. Liza asked Lito to think of a positive number, square the number, multiply the result by 2, and then add 3. If Lito's final answer is 53, what was the original number chosen?

A. 3 B. 4 C. 5 D. 8

Jumpstart

For you to understand the lesson well, do the following activities. Have fun and good luck!

Activity 1: Read and Assess Me!

Directions: Write TRUE if the statement is correct, otherwise write FALSE. Write your answer in a separate sheet of paper.

1. One-to-one function is a special function which each element of the domain corresponds to exactly one element of the range or vice versa.
2. Padlocks to padlock keys is an example of one-to-one function.
3. f^{-1}denotes inverse of a function.
4. Only one-to-one function has its inverse.
5. The inverse of the function can be done through interchanging domain and range of the function.

Activity 2: Analyze, Understand and Tell Me What Happen!

Consider the given one-to-one function $\boldsymbol{f(x) = 2x + 1}$ and its solution in determining its inverse.

Directions: Fill in the correct step in each blank described by the solution on the left side. Use separate sheet of paper for your answer. You can choose from the steps written below:

- Solve for y in terms of x
- Replace $f\ (x)$ with y
- Replace y with $f^{\ -1}(x)$
- Interchange x and y

Solution		**Steps**
$f(x) = 2x + 1$		
$y = 2x + 1$	$\Longrightarrow$	______________________
$x = 2y + 1$	$\Longrightarrow$	______________________
$x - 1 = 2y$		
$\frac{\text{x} - 1}{2} = y$		
$y = \frac{x - 1}{2}$	$\Longrightarrow$	______________________
$f^{\ -1}(x) = \frac{x - 1}{2}$	$\Longrightarrow$	______________________

If you wrote these steps in determining the inverse of one-to-one function orderly: Replace $f(x)$ with y, interchange x and y, solve for y in terms of x, and replace y with $f^{-1}(x)$, then you did a great start. Congratulations!

Activity 3: Complete Me!

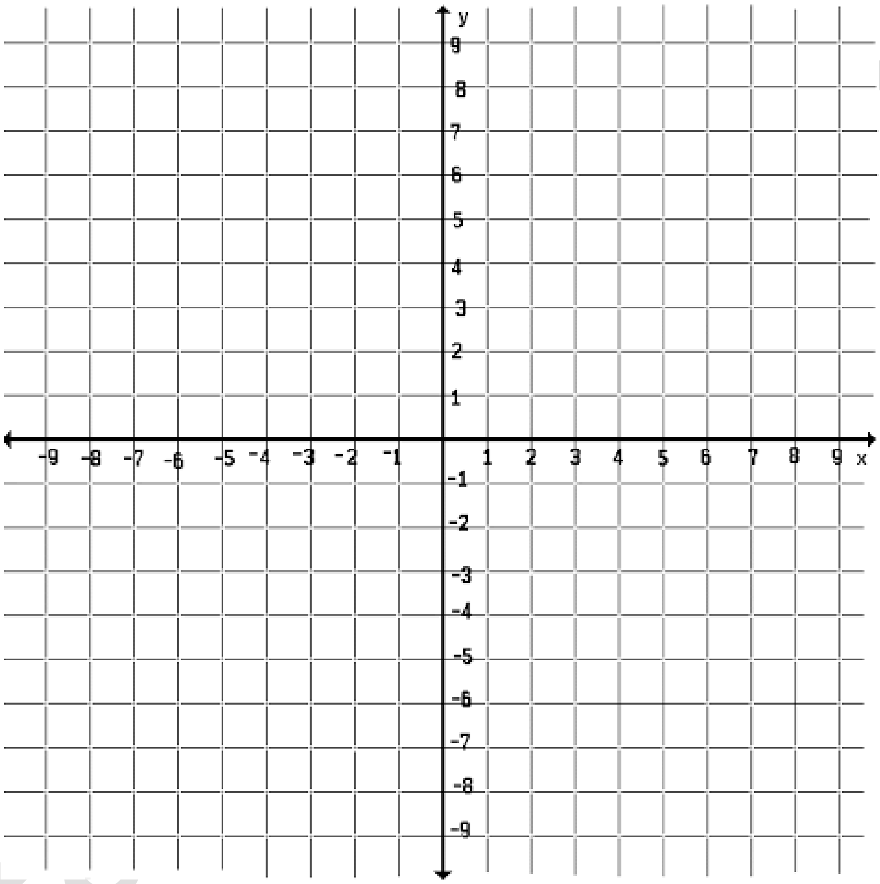
Directions: Complete the table of values below. Plot the points in the Cartesian plane then graph the given functions.

$f(x) = x + 2$

x	1	2	3	4
y				

$f(x) = x - 2$

x	3	4	5	6
y				



Answer the following:

1. Describe the x and y-values ____________________________
2. Describe the graph ______________________________________

Discover

One-to-one Functions

Definition: The function f is **one-to-one** if for any $x_1 = x_2$ in the domain of f, then $f(x_1) = f(x_2)$. That is, the same y-value is never paired with two different x-values.

You can easily understand the definition of one-to-one function by looking at the examples below.

Example 1: Which of the two mapping diagrams is one-to-one function?

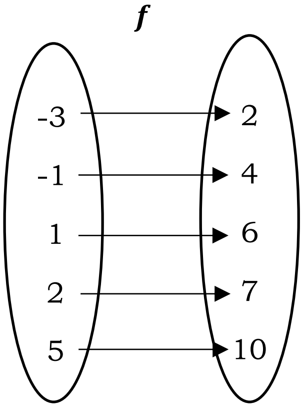


Figure 1

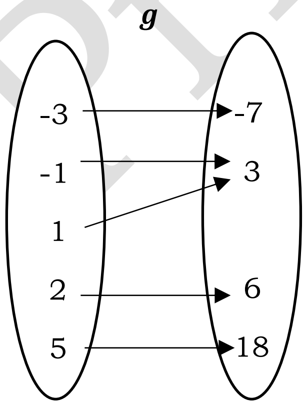


Figure 2

Solution: Function ***f*** in **figure 1** is a **one-to-one function** since NO two inputs have the same output. On the other hand, function ***g*** is NOT one-to-one function since the two inputs, -1 and 1, have the same output of 3.

Example 2: Which of the two table of values is one-to-one function?

A

Input, x	-2	-1	0	2
Output, y	5	6	7	9

B

Input, x	-2	-1	0	2
Output, y	4	1	0	4

Solution: Table of values A shows **one-to-one function** since NO two inputs have the same output. On the other hand, table of values B is NOT one-to-one function since the two inputs, -2 and 2, have the same output which is 4.

Example 3: Which of the two equations is one-to-one function?

$f(x) = 7x + 2$ $\qquad$ $f(x) = x^2 - 2$

Solution: Let x = 2 and -2

$f(x) = 7x + 2$ $\qquad$ $f(x) = x^2 - 2$

$f(x) = 7(2) + 2 = 16$ $\qquad$ $f(x) = (2)^2 - 2 = 2$

$f(x) = 7(-2) + 2 = -12$ $\qquad$ $f(x) = (-2)^2 - 2 = 2$

The function $\boldsymbol{f(x) = 7x + 2}$ is an example of **one-to-one function** since NO two x-values have the same y-value. On the other hand, function $f(x) = x^2 - 2$ is NOT one-to-one function since the two x-values, -2 and 2, have the same y-value of 2.

Example 4: Which of the two graphs below is considered as one-to-one function?

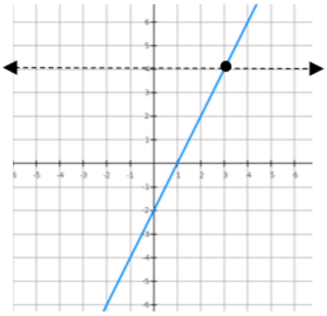


Figure 1

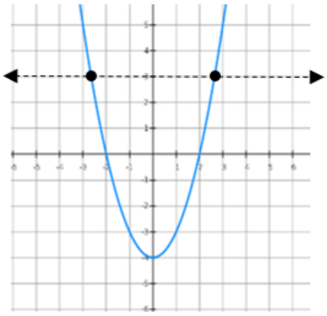


Figure 2

Solution: Figure 1 shows **one-to-one function** since the horizontal line does not intersect the graph at more than one point. On the other hand, figure 2 is NOT one-to-one function because the horizontal line intersects more than one. By the definition of *horizontal line test*, a function is one-to one if each horizontal line does not intersect the graph at more than one point.

Notes to the Teacher
Figure 2 is a graph of quadratic function. It is a function because it satisfies the vertical line test but it is not one-to-one function because it does not satisfy the horizontal line test. For a graph to be considered as one-to-one function, the graph must satisfy both vertical and horizontal line test.

Example 5: Does the pairing of a GSIS members to government employees' GSIS number represents one-to-one function? Why or why not?

Solution: The pairing of GSIS members to government employees' GSIS number represents one-to-one function because each GSIS member is assigned to a unique GSIS number. In general, no two members can have the same GSIS number.

Example 6: Does the pairing of a person to his or her citizenship represents one-to-one function? Why or why not?

Solution: The pairing of a person to his or her citizenship is NOT a one-to-one function because a person can have two citizenships (dual citizen).

Inverse of One-to-one Functions

Definition: Let f be a one-to-one function with domain A and range B. Then its inverse function denoted by $\boldsymbol{f^{-1}}$, has domain B and range A and is defined by $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in B.

The definition of inverse function simply explains that the inverse of the function can be done through interchanging the domain and range of the function.

A function has an inverse if and only if it is one-to-one function. And to determine the inverse of one-to-one function we need to follow the steps below:

1. Replace $f(x)$ with y
2. Interchange x and y variables
3. Solving for y in terms of x
4. Replace y with $f^{-1}(x)$.

Now, let's have examples.

Example 1: Find the inverse of the function $f(x) = 2x + 3$.

Solution:

$f(x) = 2x + 3$	Given
$y = 2x + 3$	Replace $f(x)$ with y
$x = 2y + 3$	Interchange x and y
$x - 3 = 2y$	Apply SPE/Transpose 3 to the left side
$\frac{x-3}{2} = \frac{\cancel{2}y}{\cancel{2}}$	Apply DPE to solve for y
$\frac{x-3}{2} = y$	
$y = \frac{x-3}{2}$	Apply Symmetric Property of Equality
$\boldsymbol{f^{-1}(x) = \frac{x-3}{2}}$	Replace y with $f^{-1}(x)$

∴ The inverse of the function $f(x) = 2x + 3$ is $f^{-1}(x) = \frac{x-3}{2}$

Example 2: Find the inverse of the function $f(x) = x^2$, if exist.

Solution:

$f(x) = x^2$	Given
$y = x^2$	Replace $f(x)$ with y
$x = y^2$	Interchange x and y
$\sqrt{x} = \sqrt{y^2}$	
$\pm\sqrt{x} = y$	Solve for y in terms of x
$\boldsymbol{y = \pm\sqrt{x}}$	

∴ The equation $y = \pm\sqrt{x}$ does not represent one-to-one function because there are some x-values that corresponds to two different y-values. Therefore, $f(x) = x^2$ has no inverse function.

Example 3: Find the inverse of the function $f(x) = \frac{5x-3}{2}$.

Solution: $f(x) = \frac{5x-3}{2}$	Given
$y = \frac{5x-3}{2}$	Replace $f(x)$ with y
$x = \frac{5y-3}{2}$	Interchange x and y
$(2)(x) = \frac{5y-3}{2}(2)$	Multiply both sides by the LCD which is 2
$2x = 5y - 3$	Simplify and apply APE/Transpose -3
$\frac{2x+3}{5} = \frac{\cancel{5}y}{\cancel{5}}$	Apply DPE to solve for y
$\frac{2x+3}{5} = y$	
$y = \frac{2x+3}{5}$	Apply Symmetric Property of Equality
$\boldsymbol{f^{-1}(x) = \frac{2x+3}{5}}$	Replace y with $f^{-1}(x)$

$\therefore$ The inverse of the function $f(x) = \frac{5x-3}{2}$ is $f^{-1}(x) = \frac{2x+3}{5}$.

Example 4: Find the inverse of the function $f(x) = x^3 + 2$.

Solution: $f(x) = x^3 + 2$	Given
$y = x^3 + 2$	Replace $f(x)$ with y
$x = y^3 + 2$	Interchange x and y
$x - 2 = y^3$	Apply SPE/Transpose 2
$\sqrt[3]{x-2} = \sqrt[\cancel{3}]{y^{\cancel{3}}}$	Extract the cube root of both sides to solve for y
$\sqrt[3]{x-2} = y$	
$y = \sqrt[3]{x-2}$	Apply Symmetric Property of Equality
$\boldsymbol{f^{-1}(x) = \sqrt[3]{x-2}}$	Replace y with $f^{-1}(x)$

$\therefore$ The inverse of the function $f(x) = x^3 + 2$ is $f^{-1}(x) = \sqrt[3]{x-2}$.

Example 5: Find the inverse of the function $f(x) = \frac{7-2x}{x+3}$.

Solution: $f(x) = \frac{7-2x}{x+3}$	Given
$y = \frac{7-2x}{x+3}$	Replace $f(x)$ with y
$x = \frac{7-2y}{y+3}$	Interchange x and y
$(y+3)(x) = \left(\frac{7-2y}{\cancel{y+3}}\right)\cancel{(y+3)}$	Multiply both sides by the LCD which is y+3
$xy + 3x = 7 - 2y$	Simplify
$xy + 2y = 7 - 3x$	Combine terms with y on the left sides
$\frac{y\cancel{(x+2)}}{\cancel{x+2}} = \frac{7-3x}{x+2}$	Apply common monomial factoring and DPE to solve for y

$$y = \frac{7 - 3x}{x + 2}$$

$\boldsymbol{f^{-1}(x) = \frac{7 - 3x}{x + 2}}$ Replace y with $f^{-1}(x)$

$\therefore$ The inverse of the function $f(x) = \frac{7-2x}{x+3}$ is $f^{-1}(x) = \frac{7 - 3x}{x + 2}$.

Now, how can we verify whether the two functions are really inverses of one another? Can you still remember the composition of function? To verify that the two functions are inverses to one another, we need to evaluate $f(f^{-1}(x))$ and $f^{-1}(f(x))$. If $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are both equal to x, then we can say that the two functions $f(x)$ and $f^{-1}(x)$ are inverses of one another.

$$f\left(f^{-1}(x)\right) = x = f^{-1}(f(x))$$

Let's prove that the inverse of the function $f(x) = 2x + 1$ is $f^{-1}(x) = \frac{x-1}{2}$.

Given: $f(x) = 2x + 1$; $f^{-1}(x) = \frac{x - 1}{2}$

Prove: $f\left(f^{-1}(x)\right) = x = f^{-1}(f(x))$

Solution:

Perform $\boldsymbol{f(f^{-1}(x))}$

$f(f^{-1}(x)) = 2x + 1$ Write the main or outer function

$f\left(f^{-1}(x)\right) = \cancel{2}\left(\frac{x-1}{\cancel{2}}\right) + 1$ Substitute the full value of $f^{-1}(x)$ to x

$f\left(f^{-1}(x)\right) = x - 1 + 1$ Simplify

$f\left(f^{-1}(x)\right) = x$

Perform $\boldsymbol{f^{-1}(f(x))}$

$f^{-1}\left(f(x)\right) = \frac{x-1}{2}$ Write the main or outer function

$f^{-1}\left(f(x)\right) = \frac{(2x+1)-1}{2}$ Substitute the full value of $f(x)$ to x

$f^{-1}\left(f(x)\right) = \frac{2x+1-1}{2}$ Simplify

$f^{-1}\left(f(x)\right) = \frac{\cancel{2}x}{\cancel{2}}$

$f^{-1}\left(f(x)\right) = x$

Since $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$, then we can say that $f^{-1}(x) = \frac{x - 1}{2}$ is the inverse of $f(x) = 2x + 1$.

The example and solution given displayed the different properties of an inverse of one-to-one function.

Property of an inverse of a one-to-one function

Given a one-to-one function $f(x)$ and its inverse $f^{-1}(x)$, then the following are true:

- The inverse of $f^{-1}(x)$ is $f(x)$.
- *For all x in the domain of f^{-1},* $f\left(f^{-1}(x)\right) = x$.
- *For all x in the domain of f,* $f^{-1}\left(f\ (x)\right) = x$.

Representation of Inverse Function

The inverse of one-to-one function can be represented through table of values and graphs.

A **table of values** can help you understand well the concept of inverse functions. For instance, the following table shows several values of the function. Interchange the rows for x and y of this table to obtain values of the inverse function.

$\boldsymbol{f(x) = x + 4}$

x	**1**	**2**	**3**	**4**
y	**5**	**6**	**7**	**8**

$\boldsymbol{f^{-1}(x) = x - 4}$

x	**5**	**6**	**7**	**8**
y	**1**	**2**	**3**	**4**

A **graph** of an inverse can be obtained by **reflecting the graph** about the line $\boldsymbol{y = x}$. To graph a function and its inverse, all you have to do is graph the function and then switch all x and y values in each point to graph the inverse. Just look at all those values switching places from the $f(x)$ function to its inverse $f^{-1}(x)$ (and back again), reflected over the line $y = x$.

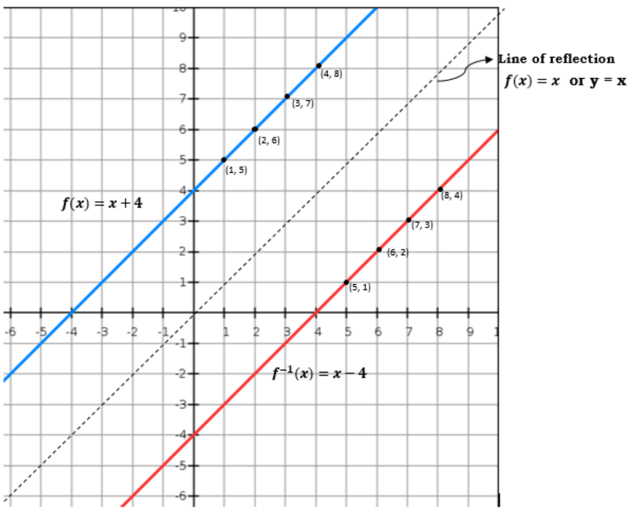


Figure 1. Graph of the function $f(x) = x + 4$ and its inverse $f^{-1}(x) = x - 4$.

To understand more about representation of an inverse function through tables of values and graph, let's consider below examples.

Example 1: Find the inverse function of $f(x) = 2x + 3$ using tables of values and sketch the graph.

Steps:

1. Create a table of values. You may use different values.
   (Your graph will be the same no matter what values you use.)

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**
y						

2. Complete the table. Solve the value of y by substituting the value of x to the equation $y = 2x + 3$.

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**
y	y = 2(-4) +3 y = -5 **-5**	y = 2(-3) +3 y = -3 **-3**	y = 2(-2) +3 y = -1 **-1**	y = 2(-1) +3 y = 1 **1**	y = 2(0) +3 y =3 **3**	y = 2(1) +3 y = 5 **5**

3. To find the inverse of the function, interchange the values of x and y.
   Then, plot the points.

$\boldsymbol{f(x) = 2x + 3}$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**
y	**-5**	**-3**	**-1**	**1**	**3**	**5**

$\boldsymbol{f^{-1}(x) = \dfrac{x-3}{2}}$

x	**-5**	**-3**	**-1**	**1**	**3**	**5**
y	**-4**	**-3**	**-2**	**-1**	**0**	**1**

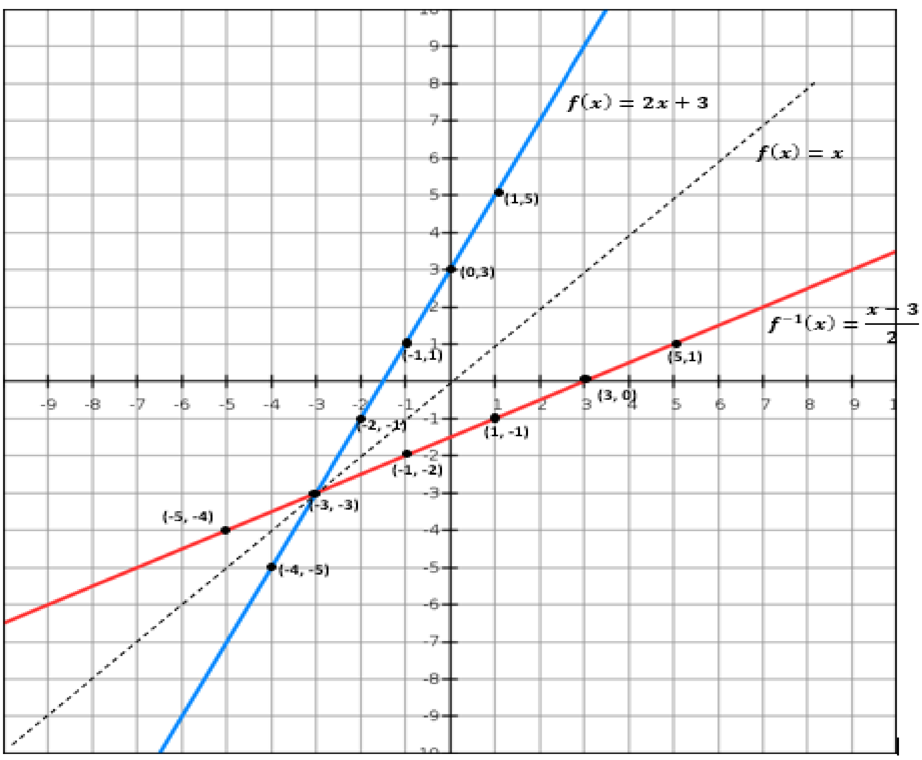


Figure 2. Graph of the function $f(x) = 2x + 3$ and its inverse $f^{-1}(x) = \frac{x-3}{2}$

Observe the graphs of a function and its inverse. Are the graphs symmetrical? Notice that the graphs of the original function and its inverse are reflections of each other along the line $y = x$. When the images are folded along that line, the graphs will coincide.

Example 2: Find the inverse function of $f(x) = \frac{5x-3}{2}$ using tables of values and sketch the graph.

Steps:

1. Create a table of values. You may use different values.

   (Your graph will be the same no matter what values you use.)

x	**-2**	**-1**	**0**	**1**	**2**
y					

2. Complete the table. Solve the value of y by substituting the value of x to the equation $y = \frac{5x-3}{2}$.

x	**-2**	**-1**	**0**	**1**	**2**
y	$y = \frac{5(-2)-3}{2}$ y = -6.5 **-6.5**	$y = \frac{5(-1)-3}{2}$ y = -4 **-4**	$y = \frac{5(0)-3}{2}$ y = -1.5 **-1.5**	$y = \frac{5(1)-3}{2}$ y = 1 **1**	$y = \frac{5(2)-3}{2}$ y =3.5 **3.5**

3. To find the inverse of the function, interchange the values of x and y.

   Then, plot the points.

$$\boldsymbol{f(x) = \frac{5x-3}{2}}$$

x	**-2**	**-1**	**0**	**1**	**2**
y	**-6.5**	**-4**	**-1.5**	**1**	**3.5**

$$\boldsymbol{f^{-1}(x) = \frac{2x+3}{5}}$$

x	**-6.5**	**-4**	**-1.5**	**1**	**3.5**
y	**-2**	**-1**	**0**	**1**	**2**

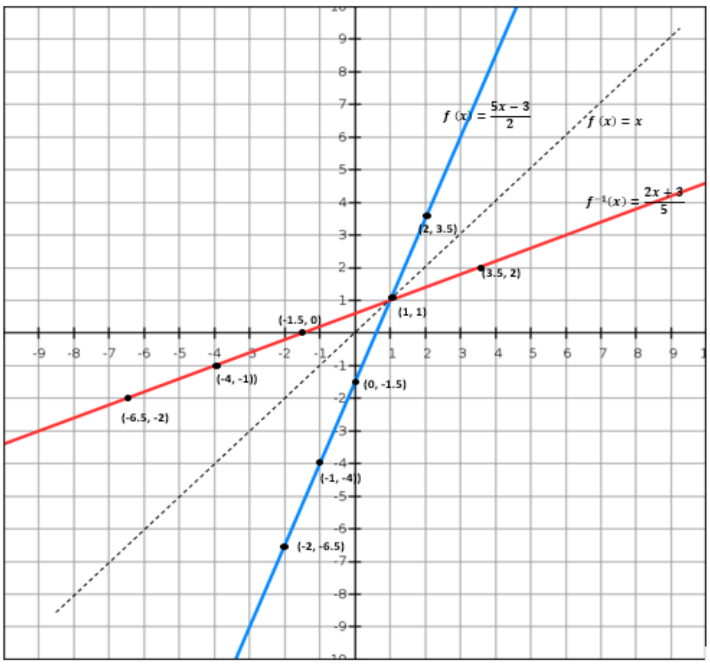


Figure 3. Graph of the function $f(x) = \frac{5x-3}{2}$ and its inverse $f^{-1}(x) = \frac{2x+3}{5}$.

Domain and Range of an Inverse Function

> Suppose we have a function $f(x)$ whose inverse is $f^{-1}(x)$. Then, the **domain of the inverse function** $f^{-1}(x)$ is the range of $f(x)$ and the **range of the inverse function** $f^{-1}(x)$ is the domain of $f(x)$.

We all know that domain is a set of all x-values of the given function and range is the set of all y-values of the given function. To fully understand about the domain and range of inverse function, consider the examples below:

Example 1: If the domain of the function is (1, ∞) and the range of the function is (−∞, -4). Find the domain and range of the inverse function.

Solution:

	$f(x)$	$f^{-1}(x)$
Domain	(1, ∞)	(−∞, -4)
Range	(−∞, -4)	(1, ∞)

Interchange the domain and range of the original function to get the domain and range of the inverse function. Thus, the domain of the inverse function is (−∞, -4) and the range of the inverse function is (1, ∞)**.**

Example 2: Find the domain and range of the function $f(x) = 3x + 5$ and its inverse.

Solution:

Domain: $f(x) = 3x + 5$ Given

$y = 3x + 5$ Replace $f(x)$ with y

> From the equation y = 3x + 5, determine x- values that will not make the y-value undefined. Since, for every value of x we substitute, we can get value of y, ∴ we can say that the set of x-values or the domain is "all real numbers".

$\boldsymbol{x} = (-\infty, \infty)$ **or All** $\mathbb{R}$ Domain of the inverse function

Range: $y = 3x + 5$ Copy the given equation

$y - 5 = 3x$ Solve for x

$3x = y - 5$

$\frac{3x}{3} = \frac{y-5}{3}$

$x = \frac{y-5}{3}$

> From the equation $x = \frac{y-5}{3}$, determine y- values that will not make the x-value undefined. Since, for every value of y we substitute, we can get value of x, ∴ we can say that the set of x-values or the range is "all real numbers".

$\boldsymbol{y} = (-\infty, \infty)$ **or All** $\mathbb{R}$ Range of the inverse function

Therefore, the domain of the function is **(-∞, ∞) or All** $\mathbb{R}$ and the range is **(-∞, ∞) or All** $\mathbb{R}$**.**

Inverse of the function:

$f(x) = 3x + 5$ Given

$y = 3x + 5$ Replace $f(x)$ with y

$x = 3y + 5$ Interchange x and y

$x - 5 = 3y$ Solve for y in terms of x

$\frac{3y}{3} = \frac{x-5}{3}$

$y = \frac{x-5}{3}$

$f^{-1}(x) = \frac{x-5}{3}$ Replace y with $f^{-1}(x)$

Since the domain and range of the function $f(x) = 3x + 5$ are $(-\infty, \infty)$ or All $\mathbb{R}$ and $(-\infty, \infty)$ or All $\mathbb{R}$, respectively. Then, to get the domain and range of the inverse function $f^{-1}(x) = \frac{x-5}{3}$, just interchange the domain and range of original function.

	Domain	Range
$f(x) = 3x + 5$	$(-\infty, \infty)$ or All $\mathbb{R}$	$(-\infty, \infty)$ or All $\mathbb{R}$
$f^{-1}(x) = \dfrac{x-5}{3}$	$(-\infty, \infty)$ or All $\mathbb{R}$	$(-\infty, \infty)$ or All $\mathbb{R}$

To fully understand about the domain and range of the function $f(x) = 3x + 5$ and the domain and range of its inverse $f^{-1}(x) = \frac{x-5}{3}$ look at the graph below.

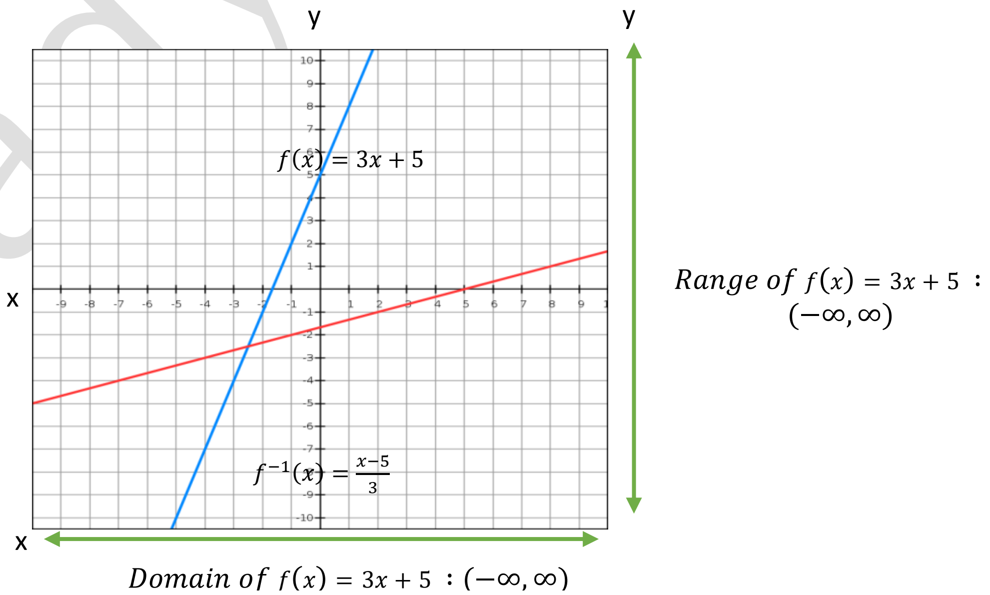


Figure 4. Graph of the function $f(x) = 3x + 5$ and its inverse $f^{-1}(x) = \frac{x-5}{3}$ showing the domain and range of both functions.

Example 3: Find the domain and range of the inverse function $f^{-1}(x) = 2x + 1$.

Solution:

Domain:	$f^{-1}(x) = 2x + 1$	Given
	$y = 2x + 1$	Replace $f^{-1}(x)$ with y

From the equation y = 2x + 1, determine x- values that will not make the y-value undefined. Since, for every value of x we substitute, we can get value of y, ∴ we can say that the set of x-values or the domain is "all real numbers".

	$\boldsymbol{x} = (-\infty, \infty)$ **or All** $\mathbb{R}$	Domain of the inverse function
Range:	$y = 2x + 1$	Copy the given equation
	$y - 1 = 2x$	Solve for x
	$2x = y - 1$	
	$\frac{2x}{2} = \frac{y-1}{2}$	
	$x = \frac{y-1}{2}$	

From the equation $x = \frac{y-1}{2}$, determine y- values that will not make the x-value undefined. Since, for every value of y we substitute, we can get value of x, ∴ we can say that the set of x-values or the range is "all real numbers".

$\boldsymbol{y} = (-\infty, \infty)$ **or All** $\mathbb{R}$ Range of the inverse function

Therefore, the domain of the inverse function is **(-∞, ∞) or All** $\mathbb{R}$ and the range is **(-∞, ∞) or All** $\mathbb{R}$**.**

To fully understand about the domain and range of an inverse function $f^{-1}(x) = 2x + 1$ look at the graph below.

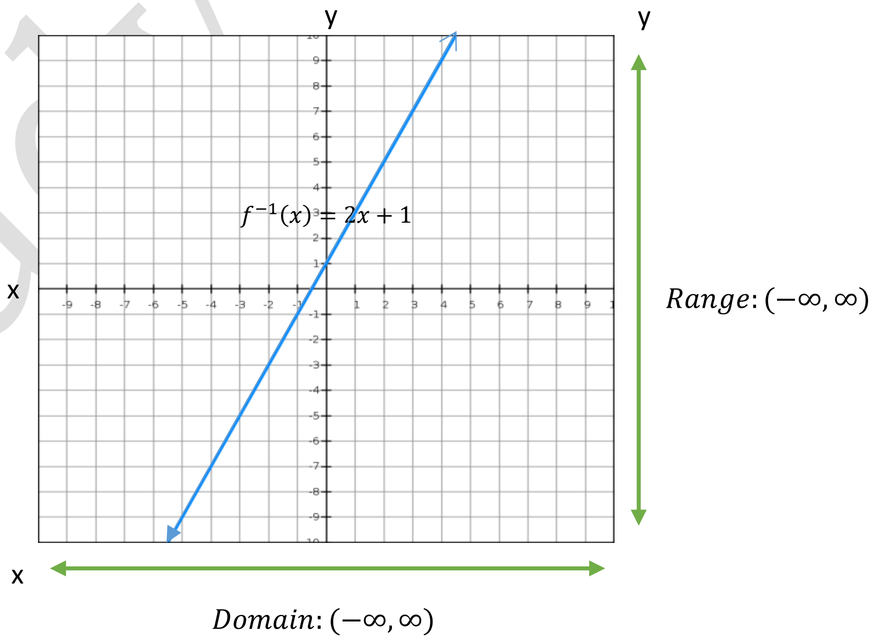


Figure 5. Graph of the inverse function $f^{-1}(x) = 2x + 1$ showing its domain and range

.

Example 4: Find the domain and range of the inverse function $f^{-1}(x) = \frac{2}{x-3}$.

Solution:

Domain:	$f^{-1}(x) = \frac{2}{x-3}$	Given
	$y = \frac{2}{x-3}$	Replace $f^{-1}(x)$ with y

From the equation $y = \frac{2}{x-3}$, determine x- values that will not make the y-value undefined. Substituting the value of x which is 3 to the equation will make the y-value undefined. Therefore, we can say that the set of x-values or the range is "all real numbers except 3".

	x = All $\mathbb{R}$ **except 3**	Domain of the inverse function
Range:	$y = \frac{2}{x-3}$	Copy the given equation
	$xy - 3y = 2$	Solve for x
	$xy = 2 + 3y$	
	$\frac{xy}{y} = \frac{2+3y}{y}$	
	$x = \frac{2+3\mathrm{y}}{\mathrm{y}}$	

From the equation $x = \frac{2+3\mathrm{y}}{\mathrm{y}}$, determine y- values that will not make the x-value undefined. Substituting the value of y which is 0 to the equation will make the x-value undefined. Therefore, we can say that the set of y-values or the range is "all real numbers except 0".

y = All $\mathbb{R}$ **except 0** Range of the inverse function

Therefore, the domain of the inverse function is **All** $\mathbb{R}$ **except 3** and the range if **All** $\mathbb{R}$ **except 0.**

To fully understand about the domain and range of an inverse function $f^{-1}(x) = \frac{2}{x-3}$ look at the graph below.

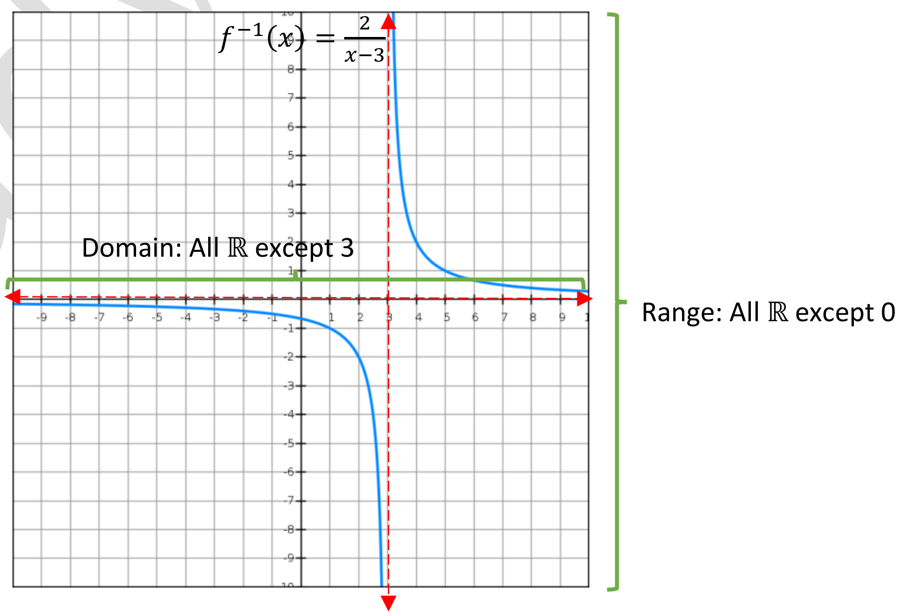


Figure 6. Graph of the inverse function $f^{-1}(x) = \frac{2}{x-3}$ showing its domain and range.

Solving problems involving inverse of one-to-one function

Example 1: The function defined by $f(x) = 2.2x$ converts a weight of x kilogram into $f(x)$ pounds. Find the equivalent weight in kilograms for a 132 lbs. boy.

Given: x - weight in kilogram

$f(x) = 2.2\,x$ - given function

132 lbs. - weight to be converted

Find: equivalent of 132 lbs. in kilograms

Solution:

$f(x) = 2.2x$	Given function
$y = 2.2x$	Replace $f(x)$ with y
$x = 2.2y$	Interchange x and y
$\frac{x}{2.2} = \frac{2.2y}{2.2}$	
$y = \frac{x}{2.2}$	Solve for y in terms of x
$f^{-1}(x) = \frac{x}{2.2}$	Replace $f^{-1}(x)$ with y
$f^{-1}(132) = \frac{132}{2.2}$	Find the equivalent of 132 lbs. in kg.
$\boldsymbol{f^{-1}(132) = 60}$	

Therefore, the equivalent weight of 132 lbs. in kilograms is 60.

Example 2: The formula for converting Celsius to Fahrenheit is given as $F = \frac{9}{5}C + 32$ where C is the temperature in Celsius and F is the temperature in Fahrenheit. Find the formula for converting Fahrenheit to Celsius. If the temperature in the thermometer read 101.3 °F, what is that in °C?

Given: $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$ - given function

101.3 °F - temperature to be converted

Find: equivalent of 101.3 °F to Celsius

Solution:

$F = \frac{9}{5}C + 32$	Given function
$F - 32 = \frac{9}{5}C$	Convert Fahrenheit to Celsius
$(\frac{5}{9})(F - 32) = (\frac{5}{9})(\frac{9}{5}C)$	
$(\frac{5}{9})(F - 32) = C$	
$C = (\frac{5}{9})(F - 32)$	
$C = (\frac{5}{9})(101.3 - 32)$	Substitute the given to the formula derived
$C = (\frac{5}{9})(69.3)$	Solve for °C
$\boldsymbol{C = 38.5}$	

Therefore, the equivalent of 101.3 °F is 38.5 °C.

Example 3: Linda will be celebrating her 18th birthday in October. Her parents are planning for a swimming party for her. The rent to the place is Php 5, 000 plus 150 per guest inclusive of snacks and lunch. How many guests can they invite if their budget is Php 11,000?

Given: x - number of guests
y - total cost
Php 11,000 - budget for the celebration

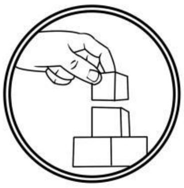
Find: number of guests to be invited for the celebration

Equation: y = 5000 + 150x

Solution:

$$y = 5000 + 150x \quad \text{Copy the equation}$$
$$y - 5000 = 150x \quad \text{Solve for x}$$
$$\frac{y-5000}{150} = \frac{150x}{150}$$
$$x = \frac{y-5000}{150}$$
$$x = \frac{11000-5000}{150} \quad \text{Substitute the total cost/budget to the equation}$$
$$x = \frac{6000}{150}$$
$$\boldsymbol{x = 40}$$

Therefore, they can invite 40 people for the celebration.



Explore

Activity 1. Identify Me!

Directions: Write OTO if the following pairs represents one-to-one function, otherwise write NOTO.

1. Passport to a person.
2. Person to his or her religion.
3. ATM account names to account numbers.
4. Length in inches to its length in centimeters.
5. Degree Fahrenheit to its equivalent degree Celsius measurement.

Activity 2: Choose Me!

Directions: Find the inverse of the following one-to-one functions. Write the letter of the correct answer. Use separate sheet of paper.

1. $f(x) = 4x$

   A. $f^{-1}(x) = \frac{4}{x}$ B. $f^{-1}(x) = \frac{x}{4}$ C. $f^{-1}(x) = -\frac{x}{4}$ D. $f^{-1}(x) = -\frac{4}{x}$

2. $f(x) = 7x - 3$

   A. $f^{-1}(x) = \frac{x+3}{7}$ B. $f^{-1}(x) = \frac{7}{x+3}$ C. $f^{-1}(x) = \frac{x-3}{7}$ D. $f^{-1}(x) = -\frac{x+3}{7}$

3. $f(x) = x^3 + 5$

   A. $f^{-1}(x) = \sqrt[3]{x-5}$ B. $f^{-1}(x) = \sqrt[3]{x+5}$ C. $f^{-1}(x) = -\sqrt[3]{x}$ D. $f^{-1}(x) = \sqrt[3]{x}$

4. $f(x) = \sqrt[3]{x+4}$

   A. $f^{-1}(x) = x^3 + 4$ B. $f^{-1}(x) = x^3 - 4$ C. $f^{-1}(x) = 4 - x^3$ D. $f^{-1}(x) = x^3$

5. $f(x) = \frac{7x+5}{x-2}$

   A. $f^{-1}(x) = \frac{5+2x}{x-7}$ B. $f^{-1}(x) = \frac{7+5x}{x-7}$ C. $f^{-1}(x) = \frac{5-2x}{x+7}$ D. $f^{-1}(x) = \frac{2+7x}{x-5}$

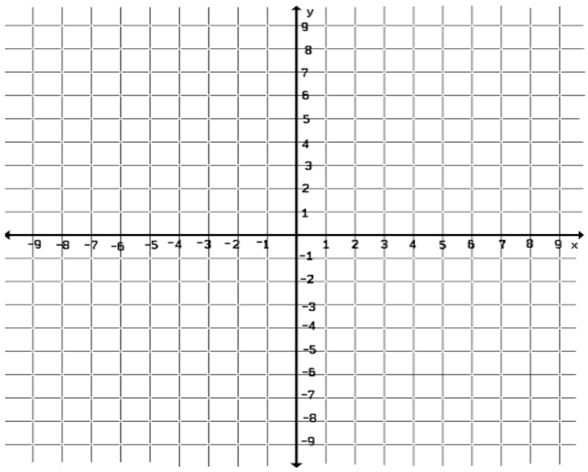
Activity 3. Sketch Me!

Directions: Complete the table of values, sketch the graph and find the domain and range of the inverse function. (You can use Desmos calculator for this activity)

A. $\boldsymbol{f(x) = 2x + 4}$ $\boldsymbol{f^{-1}(x) = \frac{x}{2} - 2}$

x	-2	-1	0	1	2
y					

x					
y					



Domain of the inverse function:

____

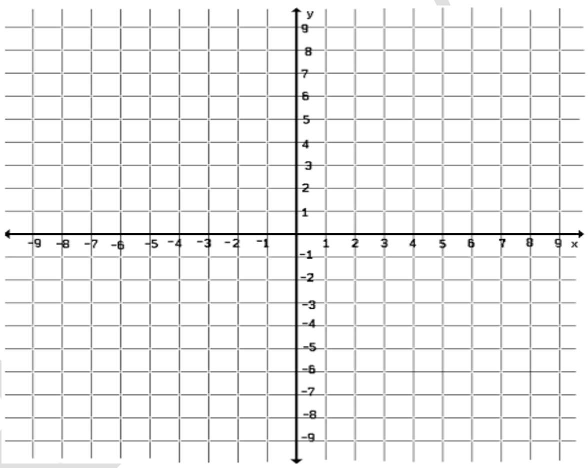
Range of the inverse function:

____

B. $\boldsymbol{f(x) = \frac{2}{x+1}}$ $\boldsymbol{f^{-1}(x) = \frac{2-x}{x}}$

x	-1	0	1	2	3
y					

x					
y					



Domain of the inverse function:

____

Range of the inverse function:

____

Activity 4: Answer Me!

Directions: Solve the given problem in a separate sheet of paper. Show your solution.

The function $y = 150 + 50x$ describes the hourly wage (y) of a math tutor earning a flat fee of Php 150.00 plus Php 50.00 for each student the tutor assists during that hour. If the wage of the Math tutor is Php 650.00 during that hour. How many students did she assists?

Deepen

At this point, you will be assessed how well you have understood the lesson. The scoring rubric on the next page will be used in assessing your answers.

Since you did great in this lesson, your teacher chooses you to be one of the participants for the incoming Mathematics Competition in your Division. Your coach believes in a saying “A healthy body makes a healthy mind” that is why your coach will start your training by asking you to walk along the road in your barangay, pick problems posted and solve it accurately. We all know that a good participant must possess mastery, accuracy and neatness in his work. Begin at the starting point. Good luck! Use separate sheet of paper for your answers.

FINISH

10. The formula for converting Celsius to Fahrenheit is given as $F = \frac{9}{5}C + 32$. After walking along the road in your barangay doing the task, your temperature is 99°F. Do you have any reason to worry? Why or why not?
Answer:

9. Prove that the function and inverse you've created are inverses of one another.
Answer:

6. Represent the function $f(x) = 5x - 2$ through table of values and graphing. Write your answer in a separate sheet.

7. Find the domain and range of the inverse $f^{-1}(x) = 6x - 2$
Answer:

8. Create your own one-to-one function and find its inverse
Answer:

5. Find the inverse of the function $f(x) = x^3 + 4$
Answer:

4. Find the inverse of the function $f(x) = \frac{x+1}{3}$
Answer:

3. Find the inverse of the function $f(x) = 5 - 2x$
Answer:

START

1. Is this one-to-one function or not? {(-2, 1), (-1, 2), (0, 3), (2, 5), (3, 6)} Write OTO if it is one-to-one, otherwise write NOTO.
Answer:

2. Find the inverse of the function $f(x) = 2x$
Answer:

What you need
1. Pen
2. Extra sheet of paper for your solutions

What you have to do
1. Answer the 10 items found in the road game. Show your solution. Begin at the starting point.

ROAD GAME RUBRIC

CRITERIA	4	3	2	1
MASTERY (x2)	10 or 100% of the items in the road game are answered correctly	9 or 90% of the items in the road game are answered correctly	8 or 80 % of the items in the road game are answered correctly	At most 7 or 70% of the items in the road game are answered correctly
ACCURACY (x2)	90% - 100% of the steps and solutions are accurate	Almost all or 80%-89% of the steps and solutions are accurate	Most or 70%-79% of the steps and solutions are accurate	At most 70% of the steps and solutions are accurate
NEATNESS	The solutions are presented in a neat, clear, and organized that is easy to understand	The solutions are presented in a neat, and organized that is usually easy to understand	The solutions are presented in an organized way but may be hard to read at times	The solutions appear sloppy and unorganized. It is hard to know what information goes together

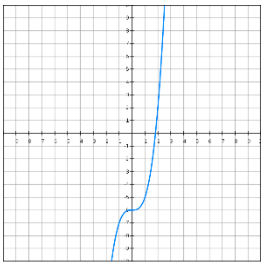
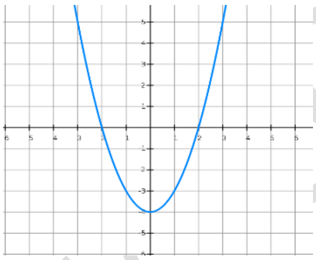
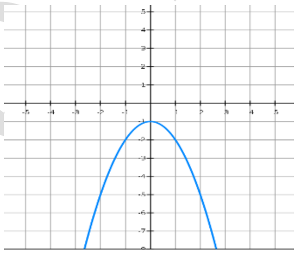
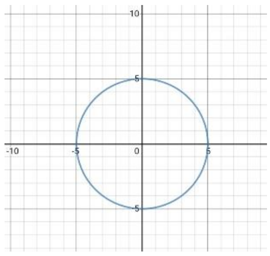
Gauge

Directions: Read each item carefully. Write the letter of the correct answer on a separate sheet of paper.

1. Which of the following relations is a one-to-one function?
   A. {(-6, 2), (-5, 1), (-4, 2), (-3, 3)}
   B. {(0, -2), (1, -1), (1, 0), (3, 1)}
   C. {(-2, -6), (-1, -3), (0, 0), (1, -3)}
   D. {(6, 8), (7, 9), (8, 10), (9, 11)}

2. Which of the following does NOT represent one-to-one function?
   A. Name of school and its school ID
   B. Facebook accounts to passwords
   C. Learner Reference Numbers to students
   D. Mobile phone brands to cellphone models

3. Which graph represents one-to-one function?

A.

B.

C.

D.


4. Which of the following functions does NOT have an inverse?
   A. $f(x) = 3x + 6$
   B. $f(x) = \frac{7x}{2} + 3$
   C. $f(x) = x^2 - 5$
   D. $f(x) = -4x^3$

5. Which statement is correct about inverse function?
   A. An inverse function is a function which is similar to the original function.
   B. An inverse function is a function created through negating the domain and range of a function.
   C. An inverse function is a function created through interchanging the domain and range of a function.
   D. All of these

6. Which is the inverse of the solution set {(-4, 4), (-2, 6), (0, 8), (2, 10)?
   A. {(4, -4), (6, -2), (8, 0), (10, -2)}
   B. {(4, -4), (6, -2), (8, 0), (10, 2)}
   C. {(4, -4), (2, -6), (0, -8), (-2, -10)}
   D. {(-4, 4), (-6, 2), (-8, 0), (-10, -2)}

7. Which is the inverse of the function $f(x) = x^3 - 10$?
   A. $f^{-1}(x) = \sqrt[3]{x+10}$
   B. $f^{-1}(x) = \sqrt{x+10}$
   C. $f^{-1}(x) = -\sqrt[3]{x+10}$
   D. $f^{-1}(x) = \sqrt{x-10}$

8. Which is the inverse of the function $f(x) = 9x + 9$?

A. $f^{-1}(x) = x - 1$ B. $f^{-1}(x) = 9x - 1$

C. $f^{-1}(x) = \frac{x}{9} - 1$ D. $f^{-1}(x) = \frac{x}{9} + 1$

9. Andrea and Andrew are seatmates; their teacher asks them to determine the inverse of the one-to-one function $f(x) = \frac{2x+3}{3x-5}$. Below are their solutions. Who among the two got the correct answer?

Andrea's solution

$$f(x) = \frac{2x+3}{3x-5}$$
$$y = \frac{2x+3}{3x-5}$$
$$x = \frac{2y+3}{3y-5}$$
$$3xy - 5x = 2y + 3$$
$$3xy - 2y = -5x + 3$$
$$y(3x-2) = 5x + 3$$
$$y = \frac{5x+3}{3x-2}$$
$$f^{-1}(x) = \frac{5x+3}{3x-2}$$

Andrew's solution

$$f(x) = \frac{2x+3}{3x-5}$$
$$y = \frac{2x+3}{3x-5}$$
$$x = \frac{2y+3}{3y-5}$$
$$3xy - 5x = 2y + 3$$
$$3xy - 2y = 5x + 3$$
$$y(3x-2) = 5x + 3$$
$$y = \frac{5x+3}{3x-2}$$
$$f^{-1}(x) = \frac{5x+3}{3x-2}$$

A. Andrea B. Andrew C. Both D. None of them

For items 10-12, use the function $f(x) = 3x + 7$

10. Which is its table of values?

A.

x	-2	-1	0	1
y	-1	4	7	10

B.

x	-2	-1	0	1
y	1	-4	-7	3

C.

x	-2	-1	0	1
y	1	4	7	10

D.

x	-2	-1	0	1
y	5	6	7	8

11. Which is the set of values of its inverse?

A. $f^{-1}(x)$= {(5, -2), (6, -1), (7, 0), (8, 1)}

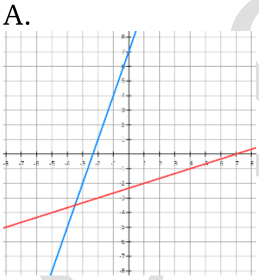
B. $f^{-1}(x)$ ={(-2, 1), (-1, 4), (0, 7), (1, 10)}

C. $f^{-1}(x)$= {(1, -2), (4, -1), (7, 0), (10, 1)}

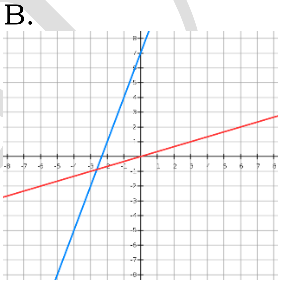
D. $f^{-1}(x)$= {(1, -2), (4, -1), (7, 0), (10, -1)}

12. Which graph represents a function and its inverse?

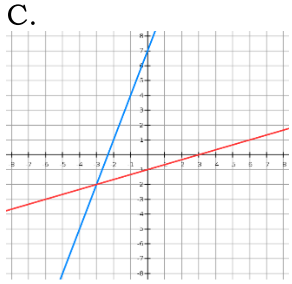
A.



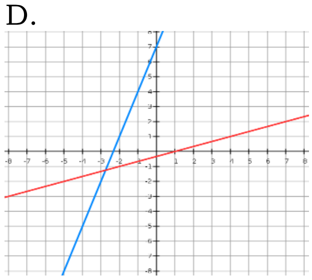
B.



C.



D.



For items 13-14, use the inverse function $f^{-1}(x) = \frac{2}{x-5}$.

13. What is the domain of the inverse?

A. All ℝ except -5 B. All ℝ except 0 C. All ℝ except 2 D. All ℝ except 5

14. What is the range of the inverse?

A. All ℝ except -5 B. All ℝ except 0 C. All ℝ except 2 D. All ℝ except 5

15. The function C described by $C(F) = \frac{5}{9}(F - 32)$ gives the Celsius temperature corresponding to the Fahrenheit temperature (F). Find the Celsius temperature equivalent to 14°F

A. -10 °F B.0 °F C. 5 °F D. 10 °F

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