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MATHEMATICS

Quarter 2 - Module 7:
Linear Equations & Inequalities in One Variable

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MATHEMATICS 7
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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

This module will help you understand linear equations and inequalities in one variable. It will also help you solve for the unknown variable applying the different properties of equations and inequalities.

Before we start, let us consider first the learning competencies of this module.

Learning Competencies:

1. Differentiate algebraic expressions, equations and inequalities.

2. Illustrate linear equation and inequality in one variable. **(M7AL-llh-4)**

3. Finds the solution of linear equation or inequality in one variable.

   (M7AL-lli-1)

Learning Objectives:

1. Define equations, inequalities, and solution set of an equation.

2. Identify the different properties of equations and inequalities.

   3. Apply the different properties of equalities solving for the solution/root of the equation or inequality.

Pre – Assessment

Directions: Read carefully each statement below. Select the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following indicates that two expressions are equal?
    A. Equation   B. Inequality
    C. Root of the equation   D. Solution Set

2. Which of the following symbols is used for an equation?
    A. $>$ B. $<$ C. $=$ D. $\neq$

3. Which of the following is ***NOT*** a translation for "="?
    A. equals   B. is   C. is at least   D. results to

4. Which equation represents the sentence "a number added to 30 is 7"?
    A. $a + 7 = 30$ B. $7 – b = 30$ C. $30 + c = 7$ D. $d – 7 = 30$

5. Which equation does ***NOT*** have –3 as its solution?
    A. $x + 5 = 2$ B. $x + 5 = 8$ C. $x + (-3) = - 6$ D. $x + (-2) = - 5$

6. Which equation does ***NOT*** have -4 as its solution?
    A. $5y = - 20$ B. $-x = -4$ C. $-8y = 32$ D. $3y = -12$

7. Which step should you take to solve the equation $x – 11 = 45$?
    A. Add 11 to each side.   B. Add 45 to each side.
    C. Subtract 11 from each side.   D. Subtract 45 from each side

8. Which situation could be modeled by the equation $5x = 12$?
    A. Five increased by 12.   B. A number times 5 is 12.
    C. Twelve is 5 more than a number.   D. A number more than 5 is 12.

9. What equation has – 3 as the solution?
    A. $2x – 3 = - 9$ B. $7 – 2x = 1$ C. $3x + 1 = 10$ D. $-4x + 5 = 7$

10. Which of the following indicates that two expressions are ***NOT*** equal?
    A. Equation   B. Inequality
    C. Root of the equation   D. Solution Set

11. Which of the following is the solution of $x + 12 < 20$?
    A. $x > 8$ B. $x < 8$ C. $x < - 8$ D. $x > - 8$

12. What are the possible values of x in the inequality $2x + 5 > 11$?
    A. $x > – 3$ B. $x > 3$ C. $x < – 4$ D. $x > – 4$

13. What property of inequality is illustrated in $x – 4 > 2$, and 4 is added to both sides of the inequality?
    A. Addition Property of Inequality   B. Division Property of Inequality
    C. Multiplication Property of Inequality D. Subtraction Property of Inequality

14. What is the solution of the inequality $3x + 2 < - 5x – 38$?
    A. $x < - 5$ B. $x > 4$ C. $x < - 4$ D. $x > 5$

15. What is the solution of the statement "three more than x at least -2x"?
    A. $x < - 1$ B. $x \geq - 1$ C. $x \leq 2$ D. $x \geq 2$

Jumpstart

A phrase is defined as a group of words that does not express a complete idea while a sentence is a group of words that brings out a complete idea. The same thing is true in mathematics.

A **word phrase** which can be translated into an **algebraic expression** does not give a complete idea. Below are some algebraic expressions.

$$x + 5, \qquad 12 + y, \qquad 2a - 9, \qquad \frac{c}{2} - 15$$

An ***algebraic expression*** may contain variables that can take on many values. However, when a variable is known to have a specific value, we can substitute this value in the expression. This process is called ***evaluating an algebraic expression.***

Activity 1. Find My Value!

Instructions: Evaluate each expression under Column A if $x = 2$. Match it to its value under Column B and write the corresponding letter within the box provided for each number below. A passage will be revealed if answered correctly.

COLUMN A	COLUMN B
1. $3 + x$	A. - 3
2. $3x - 2$	C. - 1
3. $x - 1$	E. - 5
4. $2x - 9$	F. 1
5. $\frac{1}{2}x + 3$	H. - 2
6. $5x$	I. 4
7. $x - 5$	L. 5
8. $1 - x$	O. 6
9. $-4 + x$	S. 10
10. $3x$	
11. $14 - 5x$	
12. $-x + 1$	
13. $1 - 3x$	

PASSAGE:

1	2	3	4	5	6	7	8	9	10	11	12	13

Activity 2: Me or Not?

Tell whether the given is an algebraic expression or not.

1. a + b
2. 3a + 5 = 20
3. m – 9
4. x – 3 ≤ 5
5. 4x + 7 = 35
6. 2n – 4 > 12

Were you able to identify those algebraic expressions from the table?

Numbers 1 and 3 are examples of algebraic expressions. But items 2, 4, 5 and 6 are not. Numbers 2 and 5 use equal sign. They are known as equations. While numbers 4 and 6 are inequalities because they used the symbols < (less than) and > (greater than).

Discover

This lesson will deal with finding the unknown value of a variable that will make an equation true (or false). You will try to prove if the value/s from a replacement set is/are solution/s to an equation or inequality.

In addition, this lesson will help you think logically via guess and check even if rules for solving equations are not yet introduced.

LESSON A: LINEAR EQUATIONS

Equation is a mathematical sentence indicating that two expressions are equal. The symbol "=" is used to indicate equality.

Examples of equations:

$$x + 7 = 12, \quad 3a + 5 = 20, \quad 4x + 7 = 35, \quad 5x - 4 = 11$$

An equation maybe true or false depending upon the value of the variables used. The values that make an equation true are called the **roots** or **solutions** of the equation. To solve an equation, therefore, means to find its solution. The set of replacements for the variable that yields a true statement for the equation is called its **solution set.**

Example: Given, $x + 5 = 13$, prove that only one of the elements of replacement set {–8, –3, 5, 8, 11} satisfies the equation.

For x = –8:	For x = –3	For x = 5	For x = 8	For x = 11
–8 + 5 = 13	–3 + 5 = 13	5 + 5 = 13	8 + 5 = 13	11 + 5 = 13
–3 $\neq$ 13	2 $\neq$ 13	10 $\neq$ 13	13 = 13	16 $\neq$ 13
Therefore – 8 is not a solution.	Therefore – 3 is not a solution.	Therefore 5 is not a solution.	Therefore 8 is a solution.	Therefore – 8 is not a solution.

Based on the evaluation, only x = 8 satisfied the equation while the rest did not. Therefore, we proved that only one element is the replacement set satisfies the equation.

LINEAR EQUATIONS

A linear equation in one variable is an equation written in the form ax + b = 0, where a and b are real numbers and a ≠ 0. It is also called a first-degree equation since the highest degree of the variable is 1.

To solve a linear equation, use the properties of equality to isolate the variable on one side and the solutions on the other.

The Properties of Equality
To solve equations algebraically, we need to use the various properties of equality

A. *Reflexive Property of Equality*
For each real number a, $a = a$.
Examples: $3 = 3$ $-b = -b$ $x + 2 = x + 2$

B. *Symmetric Property of Equality*
For any real numbers a and b, if $a = b$ then $b = a$.
Examples: If $2 + 3 = 5$, then $5 = 2 + 3$.
                    If $x - 5 = 2$, then $2 = x - 5$.

C. *Transitive Property of Equality*
For any real numbers a, b, and c,
     If $a = b$ and $b = c$, then $a = c$
Examples: If $2 + 3 = 5$ and $5 = 1 + 4$, then $2 + 3 = 1 + 4$.
                    If $x - 1 = y$ and $y = 3$, then $x - 1 = 3$.

D. *Substitution Property of Equality*
For any real numbers a and b: If $a = b$, then a may be replaced by b, or b may be replaced by a, in any mathematical sentence without changing its meaning.
Examples: If $x + y = 5$ and $x = 3$, then $3 + y = 5$.
                    If $6 - b = 2$ and $b = 4$, then $6 - 4 = 2$.

E. *Addition Property of Equality (APE)*
For all real numbers a, b, and c,
     $a = b$ if and only if $a + c = b + c$.
If we add the same number to both sides of the equal sign, then the two sides remain equal.
Example: $10 + 3 = 13$ is true if and only if $10 + 3$ **+ 25** $= 13$**+ 25** is also true (because the same number, 25, was added to both sides of the equation).

F. *Multiplication Property of Equality (MPE)*
For all real numbers a, b, and c, where $c \neq 0$,
     $a = b$ if and only if $ac = bc$.
If we multiply the same number to both sides of the equal sign, then the two sides remain equal.
Example: $3 \cdot 5 = 15$ is true if and only if $(3 \cdot 5) \cdot$ **2** $= 15 \cdot$ **2** is also true   (because the same number, 2, was multiplied to both sides of the equation).

Solving Linear Equations

   To solve linear equations, use the properties of equality to isolate the variable (or *x*) to one side of the equation.

Addition Property of Equality

Example:

   a. b – 12 = 58

     Add 12 to both sides

       b – 12 + 12 = 58 + 12

           b + 0 = 70

              b = 70

   b. c – 15 = – 25

     Add 15 to both sides

       c – 15 + 15 = – 25 + 15

           c + 0 = – 10

              c = – 10

Subtraction Property of Equality

Example:

   a. x + 10 = 32

     Subtract 10 to both sides

       x + 10 – 10 = 32 – 10

           x + 0 = 22

             x = 22

   b. y + 15 = – 35

     Subtract 15 to both sides

     y + 15 – 15 = – 35 – 15

         y + 0 = – 50

           c = – 50

Division Property of Equality

Example:

   a. 5x = 30

     Divide both sides by 5 to isolate x.

$$\frac{5x}{5} = \frac{30}{5}$$

       x = 6

   b. – 8x = 48

     Divide both sides by – 8 to isolate x.

$$\frac{-8x}{-8} = \frac{48}{-8}$$

        x = – 6

Multiplication Property of Equality

Example:

a. $\frac{x}{7} = 2$

Multiply both sides by 7 (the reciprocal of $\frac{1}{7}$) to isolate x.

$(7)\ \frac{x}{7} = 2(7)$

$x = 14$

b. $\frac{x}{-10} = -3$

Multiply both sides by -10 (the reciprocal of $\frac{1}{-10}$) to isolate x.

$(-10)\ \frac{x}{-10} = -3(-10)$

$x = 30$

Point to Remember:

In solving linear equations, it is usually helpful to use the properties of equality to combine all terms involving x on one side of the equation, and all constant terms on the other side.

1. Solve the equation 3x + 2x + 5 = 40

Solution: 3x + 2x + 5 - 5 = 40 – 5

$\underbrace{\qquad}$
5x   + 0 = 35

$\frac{5x}{5} = \frac{35}{5}$

x = 7

2. Solve the equation 3x + 4 – 2x – 7 = 4x + 3

3x – 2x + 4 – 7 = 4x + 3

x   – 3 = 4x + 3

Solution: Combine all terms involving x on the left side and all constant on the right side.

x – 4x = 3 + 3

$\frac{-3x}{-3} = \frac{6}{-3}$

x = – 2

LESSON B: LINEAR INEQUALITIES

> **Inequality** is a mathematical sentence indicating that two expressions are not equal. If two expressions are unequal, then their relationship can any of the following: $>$, $<$, $\geq$, $\leq$.

INEQUALITIES			
Symbols	**Meaning**	**Example**	
$>$	"is greater than"	x + 2 > 5	read as "x + 2 is greater than 5"
$<$	"is less than"	x < 8	read as " x is less than 8"
$\geq$	"is greater than or equal to" "is at least" "is no less than"	2x – 6 ≥18	read as "2x minus 6 is greater than or equal to 18"
$\leq$	"is less than or equal to" "is at most" "do not exceed"	7x ≤ 35	read as "7x is less than or equal to 35"

We can also use similar procedure to find solutions to a mathematical inequality on the following examples below.

Example: Given x – 3 ≤ 5, determine the elements of the replacement set {–8, –3, 5, 8, 11} that satisfies the inequality.

For $x = -8$: –8 – 3 ≤ 5 –8 ≤ 5 Therefore – 8 is a solution.	For $x = -3$ –3 – 3 ≤ 5 – 6 ≤ 5 Therefore – 3 is a solution.	For $x = 5$ 5 – 3 ≤ 5 2 ≤ 5 Therefore 5 is a solution.	For $x = 8$ 8 – 3 ≤ 5 5 ≤ 13 Therefore 8 is a solution.	For $x = 11$ 11 – 3 ≤ 5 8 ≤ 5 Therefore 11 is not a solution.

Based on the evaluation, the inequality was satisfied if x = –8, –3, 5 or 8. The inequality was not satisfied when x = 11. Therefore, there are 4 elements in the replacement set that are solutions to the inequality.

The Properties of Inequality

To solve algebraically, we need to use the various properties of inequality.

1. **Addition Property of Inequality (API)**
   For all real numbers a, b and c:
   (a) if $a < b$, then $a + c < b + c$, and
   (b) if $a > b$, then $a + c > b + c$.

2. **Subtraction Property of Inequality (SPI)**
   For all real numbers a, b and c:
   (a) if $a < b$, then $a - c < b - c$, and
   (b) if $a > b$, then $a - c > b - c$.

3. **Multiplication Property of Inequality**

For all real numbers a, b and c, then all the following are true:

(a) if $c > 0$ and $a < b$, then $ac < bc$;

(b) if $c > 0$ and $a > b$, then $ac > bc$.

Note: the same positive integer can be multiplied to both sides.

(c) if $c < 0$ and $a < b$, then $ac > bc$;

(d) if $c < 0$ and $a > b$, then $ac < bc$.

Note: the direction of the inequality symbol is reversed when $c < 0$ (Negative integer) is multiplied.

Solving Linear Inequalities

To solve linear equations, use the properties of equality to isolate the variable (or x) to one side of the equation.

Note: We can also use similar procedure as solving linear equation to find solutions of linear inequality.

Example: a. $x - 12 > 50$
  Add 12 to both sides
  $x - 12 + 12 > 50 + 12$
  $b > 70$

  b. $-20 < x + 18$
  Subtract 18 to both sides
  $-20 - 18 < x + 18 - 18$
  $-38 < x$ or $x > -38$

  c. $-5x < -45$
  Divide both sides by 5 to isolate x.
  $\dfrac{-5x}{-5} < \dfrac{-45}{-5}$
  $x > 9$ ⟹ The inequality sign is reversed.
  Dividing -5 to both sides of the inequality is the same as multiplying it with $-\dfrac{1}{5}$, which is the reciprocal of -5.

  d. Solve the inequality $\dfrac{x}{6} > 18$
  $(6)\dfrac{x}{6} > 18(6)$ ⟹ Multiply both sides by 6 to isolate x.
  $x > 108$

e. Solve the inequality 5x – 4 $\leq$ 7x – 2

Solution: 5x – 7x $\leq$ – 2 + 4

$$5x – 4 \leq 7x – 2$$

$$5x – 7x \leq – 2 + 4$$

$$\frac{– 2x}{– 2} \leq \frac{2}{– 2}$$

$$x \geq – 1$$

 Explore

Activity 3: **Am I The One?**

A. Verify if the given values of x are solutions to the given equations.
   Write TRUE if it makes the equation true, and FALSE if it does not.

   1. $30 - b = 22$, $b = 8$

   2. $\frac{n}{3} = 4$, $n = 12$

   3. $9 - 5x = 2x + 30$; $x = -3$

   4. $3x = x + 8$, $x = 8$

   5. $\frac{3x}{5} = 3(5)$, $x = 25$

B. Replace x with 5. Write TRUE if it makes the equation true, and FALSE if it does not.

   6. $5 + x = 0$

   7. $-2x = 7 + 3$

   8. $11 = 3x - 4$

   9. $3x + 8 = 2 + 4x$

   10. $6x = 30$

Activity 4: Complete Me!

A. Inequalities help you describe relationships. Insert the correct inequality symbol
   ($<. >. \leq, \geq$) for each of the following situations.

   1. You are free to enter, if your age is less than 5,   a _____ 5

   2. Drink at least 8 glasses of water a day.   g _____ 8

   3. A speed of 40kph or less is legal.   s _____ 40

   4. Your grade in Math is greater than 85   g _____ 85

   5. At most 100 guests attended the party.   g _____ 100

B. Tell whether the given number is a solution of the inequality or not.

   1. $a \leq 40$: 30

   2. $a < 5$: 6

   3. $g > 8$: 15

   4. $x + 4 < 7$: - 1

   5. $2x - 8 > 6$: 5

Deepen

Activity 5:
Solve for the unknown variable. Show your solution.

1. x – 7 = 8

2. x + 6 = -10

3. 3x = 21

4. 5t + 9 = -11

5. 4x – 13 = 3

6. $\frac{n}{7}$ = -10

7. $-6y - 4 = 16$

8. $3x + 4 = 5x - 2$

9. $x - 4 - 4x = 6x + 9 - 8x$

10. $5x - 4(x - 6) = -11$

Activity 6:
Solve for the unknown variable.

1. – 12 + e ≥ 15

2. h – 38 ≤ 80

3. 11h > 121

4. 10a ≤ - 100

5. $\frac{b}{4}$ > -10

6. $\frac{f}{14}$ - 4 < -12

7. $\frac{m}{75}$ ≤ -100

8. 15x – 4 > 11x + 16

9. 5x + 7 < 3 (x + 1)

10. 4 (8 + 6y) > 2(9y – 12)

Gauge

Directions: Read each statement below carefully. Select the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the solution of 28 – x = 20?
   A. 2   B. 4   C. 6   D. 8

2. What must be added to both sides of the equation: x – 12 = 15 to isolate the variable?
   A. - 15   B. - 12   C. 12   D. 15

3. Which property of equality should be used to isolate the variable in the equation 7x = 63?
   A. Addition   B. Division   C. Multiplication   D. Subtraction

4. Which of the following is the solution set of the equation 8x + 12 = 5x – 21?
   A. - 11   B. - 6   C. 3   D. 10

5. What is the solution set ofthe equation 4(x + 2) – 8 = 80?
   A. 8   B. 10   C. 16   D. 20

6. Which of the following can be used as a replacement for b in 6b – 24 = 2b + 48?
   A. 6   B. 12   C. 18   D. 24

7. What is three times a number increased by six is 21?
   A. 5   B. 6   C. 8   D. 12

8. If x = 3,which of the following equation is correct?
   A. 2x – 5 = 5   B. 2x + 5 = 11   C. 3x – 6 = 9   D. 3x + 6 = 12

9. Which is the equivalent mathematical sentence for "four times a number decreased by two is less than twice the number"?
   A. 4x – 2 = 2x   B. 4x – 2 < 2x   C. 4x + 2 = 2x   D. 4x – 2 > 2x

10. What is the solution set of the inequality 3x – 12 < 15?
    A. x > -9   B. x < -9   C. x < 9   D. x > 9

11. Which property of inequality should you use to isolate the variable in the inequality 2x – 5 < 11?
    A. Addition   B. Division   C. Multiplication   D. Subtraction

12. What is the correct translation of the inequality: "Twice a number increased by 15 is at least 7"?
    A. 2x + 15 < 7   B. 2x + 15 > 7   C. 2x + 15 ≤ 7   D. 2x + 15 ≥ 7

13. Which of the following is a linear inequality in one varible?
    A . A = πr^2 B. 4(x – 1) > 5
    C. (x + 3) (x – 6) < 24   D. x + y ≤ 1

14. What is the largest integral value of x that belongs to the solution of 2x+ 12 < 3?
    A. – 6   B. – 5   C. – 4   D. – 3

15. What property of inequality is illustrated in the statement, If $\frac{2}{7}$x ≥ 4, then x ≥ 14?
    A. Addition and Subtraction   B. Multiplication and Division
    C. Multiplication only   D. Division only

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Online Resources

- https://fsw01.bcc.cuny.edu/mathdepartment/Courses/Math/MTH05/05text0916aChap4.pdf

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