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MATHEMATICS

Quarter 1 - Module 4: Solving Problems Involving Sequences

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MATHEMATICS 10
Quarter 1 - Module 4: Solving Problems Involving Sequences
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La Union Schools Division
Region I

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MATHEMATICS

Quarter 1 - Module 4:
Solving Problems Involving Sequences

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

During this corona virus pandemic, fake news is putting lives at risk, unreliable information is spreading around the world and the rate of spread is unbelievable. Consider a fake news spread via chain text message. If a person sends the message to five people and the message says that each has to send it to another five people. How many recipients will there be? If this continues how many recipients of the message be after the 6^{th} sending?

The above situation shows an example of a sequence applied in real life. This module focuses on the application of sequence and series and how the concept is utilized in our daily life.

In going over this module, you are expected to:

Learning Competency

- Solves problems involving sequences**(M10AL-If-2)**

Learning Objectives:

1. classify a given sequence whether arithmetic or geometric sequence
2. identify the type of sequence involved in a word problem and the formula to be applied
3. solve problems involving sequences

Before we start the lesson, find out how much you already know about this module by answering the pre – assessment on the subsequent page.

PRE - ASSESSMENT

Directions: Read and answer each statement below carefully. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module. Write your answers in a separate sheet of paper.

1. A car travels 300 m the first minute, 420 m the next minute, 540 m the third minute, and so on in an arithmetic sequence. What' s the total distance the car travels in 5 minutes?

   A.1200 m B.2500 m C.2700 m D.3000 m

2. The sum of the interior angles of a triangle is 180º, of a quadrilateral is 360º and of a pentagon is 540º. Assuming this pattern continues, find the sum of the interior angles of a dodecagon (12 sides).

   A.1,800^0 B.900^0 C.820^0 D.720^0

3. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 12 hours?

   A.20,000 B.32,000 C.40,000 D.42,000

4. An auditorium has 20 seats on the first row, 24 seats on the second row, 28 seats on the third row, and so on and has 30 rows of seats. How many seats are in the theatre?

   A.2340 B.2450 C.2550 D.2620

5. A fruit vendor owes a Cooperative P4200 and agrees to pay it by monthly installment. If she pays P350 the first month, P400 the second month,P450 the third month, and so on, how long will it take her to pay the debt? How much will be her last payment?

   A.6 months, P550 B.7 months, P650
   C.7 months, P700 D.8 months, P600

6. Ten days before Valentine's Day, Pepe decided to give Pilar flowers. On the first day, he sent one red rose, on the second day, two red roses, on the third day,4 red roses, and so on. How many roses did Pilar receive during the tenth day?

   A. 512 B. 511 C.510 D. 509

7. Suppose person X was infected by a virus and it gets to infect three persons in a day. Each of these infected persons gets to infect three other persons. If these cycle goes on, then how many persons will be infected within 5 days including person X?

   A. 120 B.121 C. 130 D.131

8. A snail is crawling straight up a wall. The first hour it climbs 16 inches, the second hour it climbs 12 inches,and each succeeding hour, it climbs only three fourths the distance it climbed the previous hour. How far does the snail climb during the seventh hour?

   A. $\frac{256}{14\,197}$ B.$\frac{256}{727}$ C.$\frac{729}{256}$ D. $\frac{14\,197}{256}$

9. Max bought a car P 600,000. The yearly depreciation of his car is 10% of its value at the start of the year. What is its value after 4 years?

   A. ₱438,000 B. ₱437,400 C. ₱393,660 D. ₱378,000

10. A mine worker discovers an ore sample containing 500 mg radioactive material. It was discovered that the radioactive material has a half-life of 1 day. About how much of the radioactive material will be present after 7 days?

    A. 3.9 mg B.7.8 mg C. 15.6 mg D. 31.2 mg

Now, let us take a look first on the given activity below. This activity will surely a big help to you in understanding the lesson.

Jumpstart

Can you still recall what you have learned in the previous modules about sequences? Let us see how well you can still remember them.

Answer the activity below.

Activity 1: What Am I?

Which animal has a tongue that is one and a half times the length of its body?

Direction**:** Identify whether each of the following sequence whether arithmetic, geometric or neither. Copy the letter under your answer, then write the letter in the answer box where you can find the item number. Good luck!

Sequence	arithmetic	geometric	neither
1.6,18,54,162,...	M	L	S
2.4,10,16,22,...	A	T	E
3.1,1,2,3,5,8,...	V	D	C
4.625,125,25,5,...	R	O	W
5.1/2,1/4,1/6,1/8,...	M	D	E
6.5,8,13,21,34,...	U	C	M
7.-1296,216,-36,6,...	S	E	I
8.8.2,8,7.8,7.6,...	N	S	A
9.11,2,-7,-16,...	H	O	P

Answer Box

3	9	2	6	5	1	7	4	8

Did you enjoy the activity?

Having a thorough understanding of the sequences will help you solve real life problems involving them.

Discover

Do you know that sequences are used to model and solve many mathematical ideas and real-life situations? Sequences are useful in our daily lives as well as in higher mathematics.

Let's remember the lessons in your previous modules about sequence and series. We will focus on arithmetic and geometric sequence and series formula.

A **sequence** is a set of numbers that are arranged in a specific order.

By definition, **arithmetic sequence** whose consecutive terms have a common difference (d) while a **geometric sequence** is a sequence whose consecutive terms have a common ratio(r).

The **nth term** of an arithmetic sequence with a_1 as the first term and d as the common difference is given by the following formula:

$$a_n = a_1 + (n-1)d$$

The **nth term** of a geometric sequence with a1 as the first term and r as the common difference is given by the following formula:

$$a_n = a_1 \cdot r^{n-1}$$

A **series** is the sum of the terms of a sequence.

By definition, an **arithmetic series** is the sum of the terms of an arithmetic sequence while a **geometric series** is the sum of the terms of geometric sequence.

The sum of the first n terms of an arithmetic sequence is given by :

$$S_n = \frac{n(a_1 + a_n)}{2}$$

where a_1 is the first term and an is the nth term of the sequence.

Alternately, the equation may also be written as follows:

$$S_n = \frac{n[2a_1 + (n-1)d]}{2}$$

The sum of the first n terms of a geometric sequence is given by the following formula:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

where a_1 is in the first term and r is the common ratio.

The sum of an infinite geometric sequence is given by:

$$S = \frac{a_1}{1-r}$$

where a_1 is the first term and r is the common ratio.

Now, let us take a look first on some illustrative examples. These examples will surely help you in understanding the lesson.

Example 1: The Prize is Right

To motivate her online class learners, Teacher Mathilda asks a question for a prize of ₱150. If the learner being asked does not answer correctly, the prize money is increased by ₱100 each attempt until someone correctly answers the question. Someone finally got the correct answer on the 10th attempt. How much would be the prize?

Solution:

Let us first illustrate using a table of values.

Attempts	1st	2nd	3rd	4th	5th
Prize	150	250	350	450	550

Continue listing until the 10th attempt. Did you get P1050? That is right!

Now, listing can be a tedious process. There must be a shorter way of solving such type of problem.

a.) Identify the type of sequence the table illustrates.

There is a common difference of P 100.It must be an arithmetic sequence.

b.) Determine what is asked.

The question asks for the prize on the 10th attempt. It is asking for the 10th term.

c.) Decide on what formula to be used.

The formula in finding the nth term of arithmetic sequence is :

$$a_n = a_1 + (n-1)d$$

d.) Substituting in the formula,

$$a_{10} = 150 + (10-1)\ 100$$
$$= 150 + 9(100)$$
$$= 150 + 900$$
$$a_{10} = 1050$$

The prize on the 10th attempt is ₱1,050.

Example 2 : Fake News

Matheo is scrolling his messages. In a minute, he receives a fake news that purports to be true out of rumors regarding coronavirus situation. After a minute, he sends this message to three other accounts through messenger. Every minute,

an account who receives this message forwards it to three other accounts. Assuming this pattern continues where every account receives it only once, how many accounts receive the message on the fifth minute?

Solution:

Let us start by illustrating the situation through a table of values.

No. of Minutes	1st	2nd	3rd
No. of Accounts	1	3	9

a.) What type of sequence is illustrated?

The table shows a geometric sequence since there is a common ratio of 3.

That is r = $\frac{3}{1} = \frac{9}{3} = 3$

b.) What is asked in the problem?

The problem is looking for the number of accounts received the fake news on the 5th minute. Therefore, the problem requires a_5.

c.) What formula will be used?

The formula to be used is $a_n = a_1 \cdot r^{n-1}$ where $a_1 = 1$, $n = 5$ and $r = 3$.

d.) Substituting in the formula,

$a_5 = 1 \bullet 3^{5-1}$

$a_5 = 1 \bullet 3^4$

$a_5 = 1 \bullet 81$

$a_5 = 81$

There are 81 accounts that receive the fake news. If this continues in an hour, imagine how many accounts would receive it?

Example 3 : Smart Ako

For her online class, Jenny wanted to buy a smart phone that costs ₱6,000. She decided to save money and started with ₱500 on the first week, P 550 on the second week , ₱605 on the third week, and so on. If this goes on, how much would Jenny's savings be on the 8th week? How much did she save all in all? Is it enough to buy her the smart phone?

Solution:

Let us start by illustrating Jenny's savings in a table.

1st week	2nd week	3rd week
₱500	₱550	₱605

The table above shows a geometric sequence since it has no common difference, instead a common ratio that is:

$$r = \frac{550}{500} = \frac{605}{550} = \frac{11}{10}$$

The above situation gives us the following data:

a_1 = ₱500 n= 8

a.) Using the nth term of a geometric sequence, $a_n = a_1 \cdot r^{n-1}$, where

a_1 = 500, r = $\frac{11}{10}$ where r ≠ 1 and n=8,then

By substitution; $a_8 = 500\left[\left(\frac{11}{10}\right)^{8-1}\right]$, gives 974.40

Therefore, Jenny's savings on the 8th week is ₱974.40.

b.) To find out the sum of all her savings, let's use the sum of finite geometric series.

$$S_n = \frac{a_1(1-r^n)}{1-r},$$

By substitution,

$$S_8 = \frac{500\left[1-\left(\frac{11}{10}\right)^8\right]}{1-\frac{11}{10}}$$ gives 5717.90

Therefore, Jenny must have saved ₱5,717.90 for the whole 8 weeks and this is not enough to buy her the smartphone.

Now, it's your time to answer on your own. Answer the activity, taking note of the guide questions you had in the previous examples.

Activity 1: Complete Me!

1. **Face Mask is a Must!**

Aling Mathring makes face masks for our front liners. She has 12 face masks left over from last day, and she plans to make 10 more each day . If she decided to finish it for 15 days, how many face masks will she have to make?

Let us start by illustrating the number of Aling Mathring's face mask in a table.

Day 1	Day 2	Day 3	Day 4	Day 5
12	22	32	42	52

a. What type of sequence is illustrated? _____

b. How many face masks will Aling Mathring make on the

6^{th} day? _____

12^{th} day? _____

15^{th} day? _____

Aside from listing method, the number of face masks Aling Mathring will make on the 15^{th} day can also be found using the formula:

$$a_n = a_1 + (n-1)d$$

where $a_1 = 12$, $n = 15$ and $d = 10$

Substituting in the formula, what did you get? Did you arrive at the same answer?

Nice work! Now you are ready to discover more about solving word problems on sequences.

2. **It's Pandemic!**

Suppose patient 1 was infected by a virus and it gets to infect three persons in a day. Each of these infected persons gets to infect three other persons. If these cycle goes on, then how many persons will be infected within 8 days including patient 1?

Solution:

a. Assuming that the number of infected persons continues to increase in the same manner, how many persons were expected to be infected with the virus on the 5^{th} day?

Illustrating using a table of values,

Day 1	Day 2	Day 3	Day 4	Day 5
1	3	9	27	

b. What kind of sequence is illustrated by the situation? _____

Now, continue listing until the 8^{th} day.

c. How many infected persons were expected on the 8^{th} day? _____

Add all the number of infected persons from day 1 to day 8.

d. How many persons in all were infected on the 8^{th} day? _____

To determine the number of infected persons on the 8^{th} day, the formula below will be used:

$$a_n = a_1 \cdot r^{n-1}$$

where $a_1 = 1$, n= 8 and r = 3

To determine how many persons in all were infected on the 8th day, the formula below will be used:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Did you arrive at the same answer?

In solving problems involving sequences and series the following pointers can be of help.

POINTS TO CONSIDER

1.Understand the problem.

2.List or illustrate the given data.

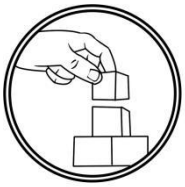
3.Determine the sequence illustrated whether arithmetic or geometric sequence.

4.Identify the formula for each unknown.

5.Solve for what is asked by substituting the needed values in the formula.

NOTE:

There is no best strategy in solving problems on sequences but it is important to stick with the concept.



Explore

At this point you already know some essential ideas about solving problems on sequences. Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 1: Log In !

There are 28 logs in the bottom layer of a pile of logs, 27 in the next layer, 26 in the next, and so on to the top which has one log.

a.) How many logs are there in the 15^{th} layer?

b.) How many logs are there all in the pile?

Activity 2: Double this, Double That!

A research lab is to begin experimentation with a bacterium that doubles every 4 hours. The lab starts with 200 bacteria.

a.) How many bacteria will be present at the end of the 12^{th} hour?

b.) How many bacteria will be present at the end of one day?

Activity 3: Jog for Life!

After a knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 15 minutes each day for the first week. Each week, thereafter, he suggests that you increase that time by 15 minutes per day.

a.) On what week will it be before you are up to jogging 2 hours per day?

b.) How many hours have you spent jogging for those weeks?

Activity 4: Mathsubishi!

Mr. Mathias bought a car for P 500,000. The yearly depreciation of his car is 10% of its value at the start of the year. What is its value after 5 years?

Activity 5: Sky Fall!

During a free- fall, a skydiver jumps 16 feet, 48 feet and 80 feet on the first, second and third fall, respectively. If he continues to jump at this rate, how many feet will he have jumped during the 10^{th} fall?

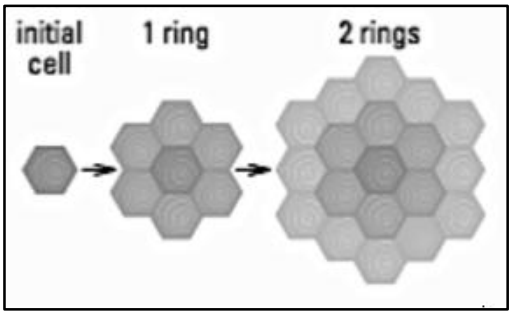
Since you already know the important notes about solving problems involving arithmetic and geometric sequence, let us go deeper.

Deepen

At this point, you are going to apply all the mathematical concepts learned from this module. *(Use separate sheet of paper).*

1. Mr. Mathy Yaga was hired by a prestigious company because of his highly qualified skills. He was offered to choose from the following computation for a 20-day salary. What will be the best choice from the offers?
   a.) ₱250,000.00 for 20 days
   b.) Start at ₱1000.00 for the first day and increase by ₱1000.00 each day for 20 days
   c.) Start at 0.25 and doubling each day for 20 days

2. Domestic bees make their honeycomb by starting with a single hexagonal cell, then forming ring after ring of hexagonal cells around the initial cell, as shown. The number of cells in successive rings form an arithmetic sequence.



https://www.123rf.com/stock-photo/honeycomb.html?sti=nwce2

   a.) Write a rule for the number of cells in the nth ring.
   b.) What is the total number of cells in the honeycomb after the 9th ring is formed? (Hint: Do not forget to count the initial cell.)

Gauge

I. Multiple Choice:

Direction: Choose the letter of the best answer from the given choices. Write your answers in a separate sheet of paper. *(1 point each)*

1.There are 20 rows of seats in a concert hall with 20 seats in the first row, 21 seats in the second row,22 seats in the third row, and so on. In total, how many seats are there in the concert hall?

A. 450 B. 590 C. 650 D. 680

2.A rocket rises 30 feet after 1 second,85 feet after 2 seconds and 140 feet after 3 seconds. If it continues to rise at this rate, how many feet will it rise after 16 seconds?

A. 780 feet B. 830 feet C. 855 feet D. 910 feet

3.Ramilo is the track and field representative of La Union in R1AA. He begins training by running 5 miles during the first week, 6.5 miles during the second week, and 8 miles on the third week. Assume the pattern continues, how far will he run on the 10^{th} week?

A. 18.5 miles B. 20 miles C.21.5 miles D. 23 miles

4.A culture of bacteria doubles every 2 hours. If there are 300 bacteria at the beginning, how many bacteria will there be after 8 hours?

A.4,800 B.1,600 C.600 D.150

5. Lanie has decided to add strength training to her exercise program. Her trainer suggests that she add weight lifting for 5 minutes during her routine for the first week. Each week thereafter, she is to increase the weight lifting time by 2 minutes. If Lanie continues with this increase in weight lifting time, how many minutes will she be devoting to weight lifting in week 10?

A.23 B.25 C.32 D.52

6. Mr. Corpuz suffers from allergies. When allergy season arrives, his doctor recommends that he take 300 mg of his medication the first day, and decrease the dosage by one half each day for one week.
To the *nearest milligram*, what is the amount of medication Mr. Corpuz will take on the 7th day?

A.5 mg B.9 mg C.19 mg D.38 mg

7. The summer Olympics occur every four years. Starting with 2016, in which year will the 12th summer Olympics occur?

A.2056 B.2058 C.2060 D.2068

8. Each hour, a grandfather clock chimes the number of times that corresponds to the time of day. For example, at 3:00, it will chime 3 times. How many times does the clock chime in a day?

A. 24 B. 42 C. 156 D. 165

9. On the first day, 42 liters of water are poured into a tank. Every day thereafter, three times as much water is poured into the tank as was poured on the previous day. On which day are 1,134 liters poured into the tank?

A. 4th B.5th C.8th D.10th

10. A ball is dropped from a height of 16 feet. Each time it drops, it rebounds 80% of the height from which it is falling. Find the total distance traveled in 15 bounces.

A. 77.2 feet B. 82.7 feet C.98.4 feet D. 102.3 feet

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LINKS

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