SHS

AIRs - LM in

Statistics and Probability
Quarter 4: Week 4- Module 12
Test-Statistic Value
(Population Mean)

Statistics and Probability
Grade 11 Quarter 4: Week 4 - Module 12: Test-Statistic Value(Population Mean)
First Edition, 2021

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Target

In the previous lesson, statistical validity of the tests was insured by the Central Limit Theorem, with essentially no assumptions on the distribution of the population. When sample sizes are small, as is often the case in practice, the Central Limit Theorem does not apply.

This module presents the appropriate test statistic in testing a hypothesis and making conclusion about the population mean based on the test-statistic value and the rejection region.

After going through this module, you are expected to:

1. computes for the test-statistic value (population mean) **(M11/12SP-IVd-1)**
2. draws conclusion about the population mean based on the test-statistic value and the rejection region. **(M11/12SP-IVd-2)**

Subtasks:

1. define test statistic
2. identify the formula to be used in finding the test statistic value concerning the mean of the population.
3. recall some terms frequently encountered in doing hypothesis testing.
4. understand the underlying procedures in hypothesis testing

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the critical value/s of α= 0.05 in a two tailed test?
   A.+ 1.94 B. + 1.95 C. +1.96 D. +1.97

2. What is the critical value of 0.01 level of significance when the alternative hypothesis indicates $\neq$?
   A. 2.375 B. 2.575 C. 2.675 D. 2.775

3. The average test score of entire school is 78 with standard deviation 10. A teacher took a random sample of 10 students and scored 75? What is the parameter mean?
   A. 8 B. 10 C. 75 D.78

4. The average test score for entire school is 78. The standard deviation of a random sample of 12 students is 10 with an average test score of 80. What is the sample mean?
   A. 10 B. 12 C. 78 D. 80

5. The average test score for entire school is 85 with a standard deviation of 8. A random sample of 10 students scored above 87. Find the statistic value using $z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}$
   A. 0.5906 B. 0.6906 C. 0.7906 D. 0.8906

For items 6-8, use the problem below.

A company claims that the label on a can of pineapple slices states that the mean carbohydrates content per serving of canned pineapple is over 50 grams. It may be assumed that the standard deviation of the carbohydrate content σ is 4 grams. A random sample of forty servings has a mean carbohydrate content of 52.3 grams. Use α= 0.05.

6. What is the tabulated value of Z?
   A. 1.92 B. 1.93 C. 1.95 D. 1.96

7. What is the value of computed Z?
   A. 3.6 B. 3.61 C. 3.63 D. 3.64

8. Will you accept the claim of the company?
   A. No, because the Z_t is greater than Z_c
   B. No, because the Z_t is less than Z_c.
   C. Yes, because the Z_t is greater than Z_c.
   D. Yes, because the Z_t is less than Z_c

9. The average test score of entire school is 78. The random sample of 32 with standard deviation 9. The average test score of the sample is 84. Find the z-value.
   A. 3.771 B. 3.778 C. 3.877 D. 3. 678

10. What is the test statistic when the population mean is 0.610, the sample mean 0.587, standard deviation is 0.123 and the number of samples is 40?

A. -1.18 B. -1.19 C. -1.20 D. -1.21

11. The null hypothesis H_0: $\mu = 12$ will be rejected in favor of the alternative hypothesis H_a: $\mu > 12$ at the $\alpha = 0.05$ level, what would be the value of the *z* test statistic?

A. Greater than 1.65 B. Greater than 1.68
C. Greater than 1.96 D. Greater than 2.0

12. A test is conducted with the null hypothesis H_0: $\mu = 10$ vs. the alternative hypothesis H_a: $\mu < 10$ at the $\alpha = 0.05$ level. The test statistic is $z = -1.75$. What would be your conclusion?

A. Fail to reject the null hypothesis B. Reject the null hypothesis
C. Only possible at the $\alpha = 0.01$ level D. Accept the null hypothesis

For items 13-15, use the problem below.

The average score in the entrance examination in Mathematics at SRNHS is 80 with a standard deviation of 10. A random sample of 40 students taken from this year's examinees and it was found the mean score of 84. The level of significance is 0.05.

13. Which of these are the critical values?

A. $\pm$ 1.93 B. $\pm$1.94 C. $\pm$ 1.95 D. $\pm$ 1.96

14. Using the Z- test formula, what is the computed test-statistic?

A. 2.53 B. 2.54 C. 2.55 D. 2.56

15. Which is the appropriate conclusion about the hypothesized and the sample mean?

A. The hypothesized mean is greater than the sample mean.
B. There is no relationship between the hypothesized and the sample mean.
C. There is significant difference between the hypothesized mean and the sample mean.
D. There is no significant difference between the hypothesized mean and the sample mean.

Jumpstart

For you to understand the lesson well, do the following activities.
Have fun and good luck!

Activity 1. Recalling It!

Directions. Using the z-table, find the critical value/s for each.

1. α= 0.01 two-tailed test
2. α= 0.10 left -tailed test
3. α= 0.005 right-tailed test
4. α= 0.02 right-tailed test
5. α= 0.01 left-tailed test

Activity 2. Accept or Reject!

Directions: Complete the table below. Write accept or reject the hypothesis given the level of significance, Ztab, Zcom and the rejection region.

Level of significance	**Ztabulated (Z_t)**	**Zcomputed (Z_c)**	**Rejection region**	**Accept/ Reject**
0.05	±1.96	± 2.53	Two-tailed	
0.05	± 1.65	± 2.53	Two -tailed	
0.01	-2.33	-1.67	Left-tailed	
0.01	2.33	1.67	Right-tailed	
0.05	-1.708	-1.85	Left-tailed	

Activity 3. Bowling League

Directions: Read the situation and answer the following questions.

Myla runs a large bowling league. She suspects that the league average score is greater than 150 per game. She takes a random sample of 36 game scores from the league data. The scores in the sample have a mean of 156 and a standard deviation of 30.

Fernanda wants to use these sample data to conduct a test-statistic on the mean. Assume that all conditions for inference have been met.

a. What is the sample mean (x)?

b. What is the population mean (μ)?

c. What is the sample standard deviation (s)?

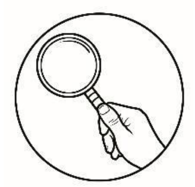
d. What is the sample score (n)?

e. Which of the two formulae will be used?

1. $z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$ 2. $t = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$

f. Calculate the test-statistic for Myla's test.

You may round your answer to two decimal places.



Discover

A **test statistic** is used in a hypothesis test when you are deciding to support or reject the null hypothesis. The test statistic takes your data from an experiment or survey and compares your results to the results you would expect from the null hypothesis.

In large sample test concerning the population mean, the test statistic to be used is the **z.**

The **Z -test statistic** is use when the sample size is greater than 30(n ≥30), or when the population is normally distributed and α is known. The formula and the steps below will be used and followed to solve problems concerning the mean of the population.

Formula for *z*-test statistic:

$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where: x = mean score of the sample

μ = population mean

σ = population standard deviation

n = sample size

Steps in hypothesis testing using the Z-test statistic.

1. State the hypotheses
2. Identify the level of significance
3. Determine the critical values and rejection region

4. State the decision rule
5. Compute the test statistic
6. Make a decision.

Let's consider the examples illustrating the steps in hypothesis testing using the z-test statistic.

Example 1. A manufacturer claims that the average lifetime of his lightbulbs is 3 years or 36 months. The standard deviation is 8 months. Fifty bulbs are selected, and the average lifetime is found to be 32 months. Should the manufacturer's statement be rejected at α = 0.01?

Solution : Step 1. State the hypotheses

$H_0 : \mu = 36$ months

$H_a : \mu \neq 36$ months

Step 2. Level of significance α = 0.01

Thus, $\frac{0.01}{2} = 0.005$

Step 3. Determine the critical values and rejection region.

Since α = 0.01, and it is two-tailed, the critical values are

$Z_t = \underline{+}2.57$

Step 4. Compute the test-statistic

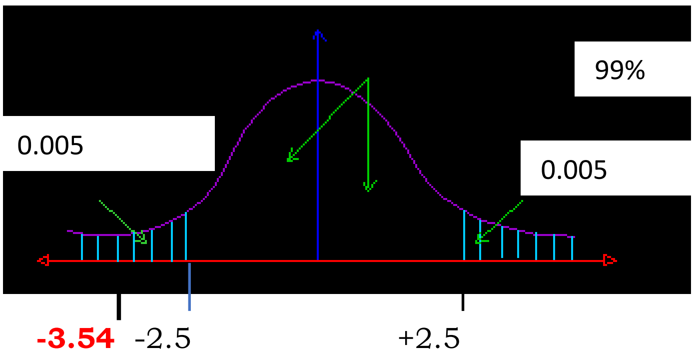
Given: X = 32, μ = 36 σ = 8 n= 50

Solution

$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{32-36}{\frac{8}{\sqrt{50}}} = \mathbf{-3.54}$$

Step 5. Decision rule: Reject H_o if the test statistic is lesser than critical value

Illustrating our solution in the normal curve, we have



Step 6.

Conclusion: Since -3.54 < -2.5, which falls in the rejection region in the left tail. **we reject H_0**.. Therefore, the average lifetime of lightbulbs of the manufacturer is not 6 months.

Example 2.

A test on car braking reaction times for men between 18 and 30 years old have produced a mean and standard deviation of 0.610 sec and 0.123 sec. respectively. When 40 male drivers of this age group were randomly selected and tested for their breaking reaction times, a mean of 0.587 second came out. At the α= 0.10 level of significance, test claim of the driving instructor that his graduates had faster reaction times.

Solution: The claim of instructor means that his graduates have a mean breaking reactions time of less than 0.610 sec.

Step 1. $H_o : \mu = 0.610$ sec

$H_a : \mu < 0.610$ sec

Step 2. α= 0.10

Step 3. Since α= 0.10 and the test is left-tailed, $Z_t = -1.28$

Step 4. Test statistics

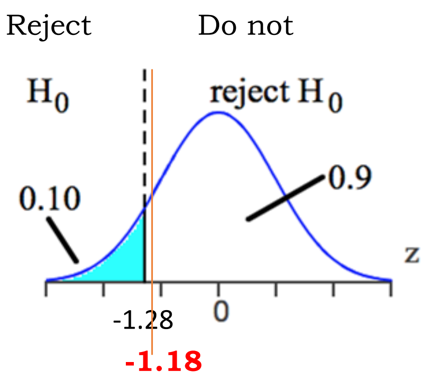
Given: X = 0.587, μ = 0.610 σ = 0.123 n= 40

Solution:

$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{0.587 - 0.610}{\frac{0.123}{\sqrt{40}}} = \mathbf{-1.18}$$

Step 5. Decision Rule: Reject H_o if $Z_c > -1.28$.

Illustrating our solution in the normal curve, we have



Step 6.

Conclusion: Since the test statistic falls within the noncritical region,

do not reject H_o. There is not enough evidence to support the instructor's claim.

Example 3. A researcher reports that the average salary of College Deans is more than P 63,000. A sample of 35 College Deans has a mean salary of P 65,700. At α= 0.01, test the claim that the College Deans earn more than P63,000 a month. The standard deviation is P 5,250.

Step1. State the hypothesis and the alternative hypothesis.

$H_o: \mu \leq$ P 63,000

$H_a: \mu >$ P 63,000

Step 2. The level of significance: α= 0.01

Step 3. The Z critical value is 2.326 (it is a one-tailed test, since it does mention about the direction of the distribution.

Step 4. Compute the Z-test value using the formula,

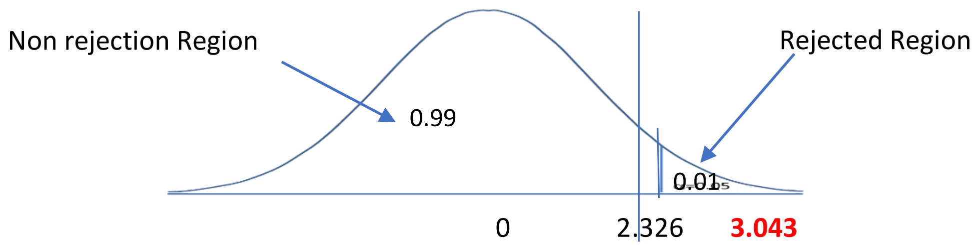
Given: X = P 65,700, μ = P 63,000 σ = P 5,250 n= 35

Solution:

$$Z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{65{,}700 - 63{,}000}{\frac{5250}{\sqrt{35}}} = 3.043$$

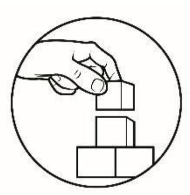
Step 5. Decision rule: Reject the null hypothesis at α = 0.01,

since 3.043 > 2.326.



Step 6.

Conclusion: Since we **reject the null hypothesis**, we can conclude that there is enough evidence to support the claim that the monthly salary of College Deans is more than P 63,000.



Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 4: Test me!
Directions: Complete the solution in each of the following problem.

1. The average score in the entrance examination in Mathematics at Sto. Rosario National High School is 80 with a standard deviation of 10. A random of 40 students was taken from this year's examinees and it was found to have a mean score of 84.

a. Is there a significant difference between the known mean and the sample mean? Test at $\alpha = 0.05$

Solution :

Step 1. $H_o : \mu = 80$: There is no significant difference between the hypothesized and the sample mean.

$H_a : \mu \neq 80$: ______________________

Step 2. Level of significance, $\alpha = 0.05$,

Step 3. two tailed test, find the critical value, Zt = _______

Step 4. Compute the test-statistic value:

Zc = _________________

Step 5. Decision Rule : _________________________

Step 6. Conclusion: ____________________________

b. Does this indicate that this year's batch is better in mathematics than the previous batches?

Solution

Step 1. Ho : μ This year's batch is as good as the previous batches in mathematics.

Ha : μ ______________________________

Step 2. Level of significance, α = _______________

Step 3. Find the critical value, Zt = _______________

Step 4. Compute the test statistic: Zc= __________________

Step 5. Decision Rule ______________________

Step 6. Conclusion: ________________________

2. A diet clinic states that there is an average loss of 24 pounds for those who stay on the program for 20 weeks. The standard deviation is 5 pounds. The clinic tries a new diet reducing salt intake to see whether that strategy will produce a greater weight loss. A group of 40 volunteers loses an average of 16.3 pounds each over 20 weeks. Should the clinic change the new diet? Use $\alpha = 0.05$

Solution:

Step 1: $H_0 : \mu$ ______________________________________

$H_a : \mu$ ______________________________________

Step 2: α = ______________

Step 3: Zt = ______________

Step 4: Zc________________

Step 5; Decision Rule:_____________

Step 6. Conclusion: _______________________

3. The manufacturer of a certain brand of auto batteries claims that the mean life of these batteries is 45 months. A consumer protection agency that wants to check this claim took a random sample of 36 such batteries and found that the mean life for this sample is 43.75 months with the standard deviation of 4 months. Using the 0.025 significance level, would you conclude that the mean life of these batteries is less than 45 months?

Step 1. State the null hypothesis and alternative hypothesis:

(Ho) μ _____

Ha: μ_____

Step 2. What is the level of significance ? ______

Step 3. What is the critical value? Zt = ______

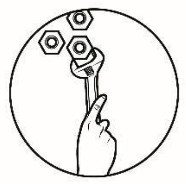
Step 4. Compute the test – statistic value: Zc= ________

Step 5. What is your decision rule? _______________________________

Step 6. What would be your conclusion based on the test-statistic value?

____

Great job! You have understood the lesson.
Are you ready to summarize?



Deepen

Activity 5: Repulse!

Answer the following questions.

1. A paint manufacturing company claims that the mean drying time for its paints is 45 minutes. A random sample of 35 gallons of paints selected from the production line of this company showed that the mean drying time for this sample is 50 minutes with standard deviation of 3 minutes. Assume that the drying time for these paints have a normal distribution, using the 1% significance level.

2. A researcher claims that the monthly load consumed of Grade 11 for their online learning class is more than P 3,000. In a sample of 35 randomly selected students, the mean monthly load consumed was P2,700 with the standard deviation of P 600. Is there sufficient evidence at 0.01 level of significance that the average monthly load consumed is more than P 3,000?

3. Average college cost of tuition fee for all private institutions last year was P 36,400. A random sample of costs this year for 45 institutions of higher learning indicated that the sample mean was P 37,900 and a sample standard deviation was P 5,600. At the 0.10 level of significance, is there sufficient evidence to conclude that the cost has increased?

Rubric for Problem Solving

4	3	2	1
Follow the steps to come up with a correct solution and draw at the correct conclusion	Follow the steps to come up with a solution but a part of the solution led to incorrect conclusion	Follow the steps but came up with an entirely wrong solution and led to incorrect conclusion	Attempt to solve but does not follow the steps that led to a wrong solution and incorrect conclusion

Gauge

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the critical value/s of α= 0.05 in a two tailed test?

   A. ± 1.95 B. ± 1.96 C. ±1.97 D. +1.98

2. What is the critical value of 0.01 level of significance when the alternative hypothesis indicates ≠?

   A. 2.575 B. 2.675 C. 2.775 D. 2.875

3. The average test score of entire school is 78 with standard deviation 10. A teacher took a random sample of 10 students and scored 75? What is the parameter mean?

   A. 8 B. 10 C. 75 D.78

4. The average test score for entire school is 78. The standard deviation of a random sample of 12 students is 10 with an average test score of 80. What is the sample mean?

   A. 10 B. 12 C. 78 D. 80

5. The average test score for entire school is 85 with a standard deviation of 8. A random sample of 10 students scored above 87. Find the statistic value using $z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}$.

   A. 0.6906 B. 0.7906 C. 0.8906 D. 0.9906

For items 6-8, use the problem below.

A company claims that the label on a can of pineapple slices states that the mean carbohydrates content per serving of canned pineapple is over 50 grams. It may be assumed that the standard deviation of the carbohydrate content σ is 4 grams. A random sample of forty servings has a mean carbohydrate content of 52.3 grams. Use α= 0.05.

6. What is the tabulated value of Z?

   A. 1.93 B. 1.94 C. 1.96 D. 1.97

7. What is the value of computed Z ?

   A.3.62 B. 3.63 C. 3.64 D. 3.65

8. Will you accept the claim of the company?

   A. No, because the Zt is less than Zc

   B. No, because the Zt is greater than Zc.

   C. Yes, because the Zt is greater than Zc.

   D. Yes , because the Zt is less than Zc

9. The average test score of entire school is 78. The random sample of 32 with standard deviation 9. The average test score of the sample is 84. Find the z-value.

A. 3.771 B. 3.778 C. 3.877 D. 3. 678

10. What is the test statistic when the population mean is 0.610, the sample mean 0.587, standard deviation is 0.123 and the number of samples is 40?

A.-1.17 B. -1.18 C. -1.19 D. -1.20

11. The null hypothesis H_0: $\mu = 12$ will be rejected in favor of the alternative hypothesis H_a: $\mu > 12$ at the $\alpha = 0.05$ level, what would be the value of the z test statistic?

A. Greater than 1.93 B. Greater than 1.94

C. Greater than 1.96 D. Greater than 2.0

12. A test is conducted with the null hypothesis H_0: $\mu = 10$ vs. the alternative hypothesis H_a: $\mu < 10$ at the $\alpha = 0.05$ level. The test statistic is $z = -1.75$. What would be your conclusion?

A. Accept the null hypothesis B. Reject the null hypothesis

C. Only possible at the $\alpha = 0.01$ level D. Undecided

For items 13-15, use the problem below.

The average score in the entrance examination in Mathematics at SRNHS is 80 with a standard deviation of 10. A random sample of 40 students taken from this year's examinees and it was found the mean score of 84. The level of significance is 0.05.

13. Which of these are the critical values?

A. $\pm$ 1.96 B. $\pm$1.97 C. $\pm$ 1.98 D. $\pm$ 1.99

14. Using the Z- test formula, what is the computed test-statistic?

A. 2.50 B. 2.51 C. 2.52 D. 2.53

15. Which is the appropriate conclusion about the hypothesized and the sample mean?

A. The hypothesized mean is greater than the sample mean.

B. There is no relationship between the hypothesized and the sample mean.

C. There is significant difference between the hypothesized mean and the sample mean.

D. There is no significant difference between the hypothesized mean and the sample mean.

References

Printed Materials:

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DIWA Textbook, Mathematics for the New Millennium

Introduction to Business Statistics by Winston S. Sirug

Math Connections in the Digital Age Statistics and Probability by Luis

MSA Statistics and Probability by Merle S. Alferez

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