9

Mathematics

Quarter 3-Week 8 – Module 8: Solving Problems Involving Triangle Similarity and Right Triangles

AIRs - LM

Mathematics 9
Quarter 3 Week 8- Module 8: Solving Problems Involving Triangle Similarity and Right Triangles
First Edition, 2020

Development Team of the Module

Writer: Leian Gretel D. Nerida
Editor: SDO La Union, Learning Resource Quality Assurance Team
Illustrator: Ernesto F. Ramos, Jr., *P II*

Management Team:

Atty. Donato D. Balderas, Jr.
Schools Division Superintendent

Vivian Luz S. Pagatpatan, PhD
Assistant Schools Division Superintendent

German E. Flora, PhD, *CID Chief*
Virgilio C. Boado, PhD, *EPS in Charge of LRMS*
Erlinda M. dela Peña, EdD, *EPS in Charge of English*
Michael Jason D. Morales, P*DO II*
Claire P. Toluyen, *Librarian II*

Target

Are you looking forward to the idea of being able to measure tall heights and far distances without directly measuring them? Are you wondering how you can draw a replica of an object that is enlarged or reduced proportionately and accurately to a desired size? It is a guarantee that with focus and determination, you will be able to answer this question: How useful are the concepts of similarity of objects measurement-related problems?

In this lesson you will learn to:

1. Solve problems involving similarity and right triangles (**M9AL-IIIj-i-1**)

At the end of this module, you are expected to

1. recall the concepts and principles on triangle similarity
2. illustrate a given problem
3. apply the concept of similarity and right triangles in solving related problems; and
4. appreciate the uses of triangle similarity and right triangles in real-life situations.

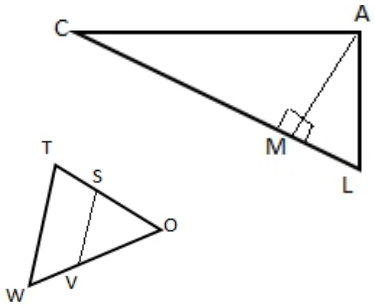
Before going on, check how much you know about this topic.
Answer the pretest in a separate sheet of paper.

Pre-Assessment

Directions. Find out how much you already know about this module. Choose the letter that you think best answers the question. Please answer all items. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module. Write your answer on a separate sheet of paper.

1. What theorem states that "If a line segment is parallel to one side of a triangle and intersects the other two sides at points except the vertex, then it divides the two sides into segments of proportional lengths?
   A. The Basic Proportionality Theorem
   B. The Pythagorean Theorem
   C. The Right Triangle Proportional Theorem
   D. The Triangle Angle Bisector Theorem

2. Which of the following statements is true about the figure?



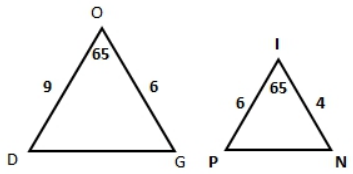
I. ΔCAL ~ $\Delta CMA \sim \Delta AML$
II. AM is the geometric mean of CM and LM
III. AL is the geometric mean of LM and CL
IV. AC is the geometric mean of CM and CL
V. AM is the altitude to hypotenuse CL

A. I only C. I and V only
C. I, III, IV only D. II and III only

3. In triangle TWO, which sides is parallel to $\overline{SV}$?

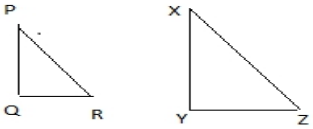
A. $\overline{SO}$ B. $\overline{TW}$ C. $\overline{ST}$ D. $\overline{VO}$

4. Which similarity concept justifies that ΔDOG ~ ΔPIN?



A. Right Triangle Proportionality Theorem
B. Triangle Proportionality Theorem
C. SSS Similarity Theorem
D. SAS Similarity Theorem

5. ΔPQR and ΔXYZ are similar as shown in the figure. Then,



A. ∠R ≅ ∠Z B. ∠P ≅ ∠Z
C. ∠Y > ∠Q D. ∠P ≅ ∠Q

6. What theorem states that the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.

A. The 30°-60°-90° Triangle Theorem
B. The Pythagorean Theorem
C. The Isosceles Right Triangle Theorem
D. The Triangle Proportional Theorem

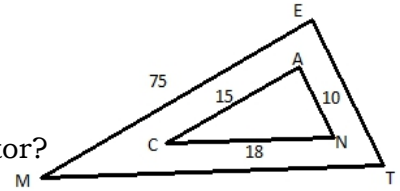
7. What theorem will you use to find the diagonal of a 15 cm by 12 cm rectangle?

A. Pythagorean Theorem
B. Triangle Proportional Theorem
C. Right Triangle Proportionality Theorem
D. Triangle Angle Bisector Theorem

8. What theorem will you use to find the length of the hypotenuse of a 45°-45°-90°

A. The 30°-60°-90° Triangle Theorem
B. The Triangle Proportionality Theorem
C. The Isosceles Right Triangle Theorem
D. The Triangle Similarity Theorem

9. ΔMET ~ ΔCAN. Which ratio of sides gives the scale factor?



A. $\frac{CA}{ME}$ B. $\frac{CN}{ME}$ C. $\frac{CA}{CN}$ D. $\frac{AN}{ME}$

10. If a man walks 4 km east and then 5 km south, how far is he from the point where he started?

A. 7 km B. 6.4 km C. 3 km D. 5.5 km

11. ΔMNO and ΔABC are similar, if the measure of ∠N is 50^0, what is the measure of ∠B?

A. 50^0 B. 60^0 C. 70^0 D. 90^0

12. What is the length of the hypotenuse of a right triangle if the legs are 6 and 8?
A. 10 B. 14 C. 28 D. 48

13. The length of the shadow of your 1.6 meter height is 2.8 meters at a certain time in the afternoon. How high is an electrical post in your backyard if the length of its shadow is 20 meters?
A. 7.14 m B. 10.5 m C. 11.43 m D. 13.21 m

14. A tree 24 feet tall casts a shadow 12 feet long. If Brad's shadow is 3ft, how tall is he?
A. 6 ft. B. 8 ft. C. 9 ft. D. 12 ft.

15. The area of a square is 100 square inches. What is the perimeter of the square in inches?
A. 40 inches B. 80 inches C. 400 inches D. 625 inches

Were you able to answer all the questions? If not, don't worry because the next activity will help you better understand the lesson.

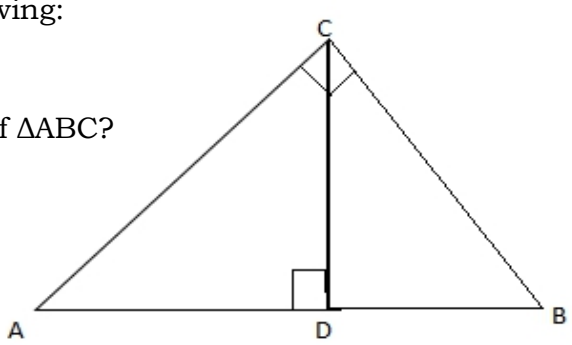
Solving Problems Involving Triangle Similarity and Right Triangles

Let's start this module by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills will help you understand solving problems involving similarity and right triangles. As you go through this lesson, think of this important question: "How are concepts of similarity and right triangles used in solving real-life problems and in making decisions? If you find any difficulty in answering the exercises, seek assistance of your teacher or refer to the modules you have gone over earlier.

Review

Activity 1. Let Me RECALL YOU!

A. Use the figure to answer each of the following:



1. Name the right angle of ΔABC
2. What is the altitude to the hypotenuse of ΔABC?
3. Name the hypotenuse of ΔABC
4. Two segments of the hypotenuse are $\overline{AD}$ and ____.
5. The hypotenuse of ΔBCD if $\overline{CD} \perp \overline{AB}$ is ____.
6. Name the right angle of ΔADC
7. Name the hypotenuse of right ΔBCD

B. True or False. Write T if the statement is true, otherwise write F.

1. In Pythagorean Theorem, the square of the hypotenuse is equal to the sum of the square of the legs.
2. Triangles with the same perimeter are similar.
3. In an isosceles right triangle, the length of the hypotenuse is equal to $\sqrt{2}$ times the length of a leg.
4. In a 30°-60°-90° Triangle Theorem, the length of the shorter leg is twice the length of the hypotenuse and the length of the longer leg is $\sqrt{3}$ times the length of the other leg.
5. If the length of the diagonal of a square is 10 units then the length of each side is 5 units

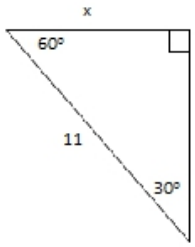
Were you able to recall the concepts on triangle similarity? I'm sure you were good at it. Now let us test your memory further.

Jumpstart

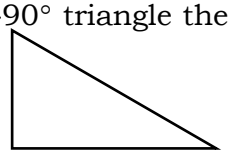
Activity 2: Illustrate Me!

A. Directions: Illustrate the problem completely. On a separate sheet of paper, copy each figure drawn below the problem then label using the given information.

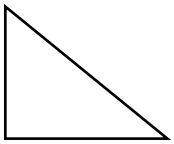
Example: In a 30°-60°-90° triangle, the length of the hypotenuse is 11 and the shorter leg is x.



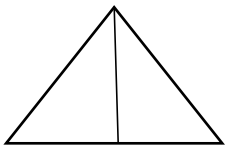
1. In a 30°-60°-90° triangle the length of the hypotenuse is 14 and the longer leg is $7\sqrt{3}$.



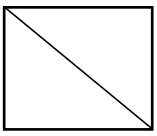
2. In a 45° - 45° - 90° triangle, the length of the hypotenuse is 16.



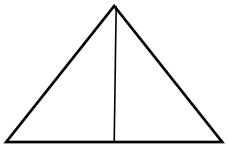
3. The length of the altitude of an equilateral triangle is x and the length of a side is 6.



4. The length of the diagonal of a square is 12 and the length of a side is $6\sqrt{2}$.



5. ΔBAC is a right triangle, $\angle C$ is right angle, $\overline{CD} \perp \overline{AB}$ AD = 14, DB = 6



B. Read the problem, and answer the questions below.

Maria is reading a book for her English class. Last night, she read 84 pages in 2 hours. Tonight she plans to read for 5 hours. If she reads at the same rate tonight as she read last night, how many pages will she read tonight?

1. What are the two related quantities that are being compared?
2. What information is given in the problem that relates the two quantities?
3. Does it make more sense to use this information in pages per hour or in hours per page?
4. Write the information from the problem as a ratio.
5. Use this ratio to set up a proportion that you could use to solve for p, the number of pages she reads in 5 hours.
6. Solve your proportion for p.

How did you find the activity? I am sure you did not find any difficulty in illustrating the problem. The next activity will help you fully understand the concepts behind this activity. But before doing the next activity, you have to read thoroughly and understand first some important notes on how to solve problems involving triangle similarity and right triangles.

Discover

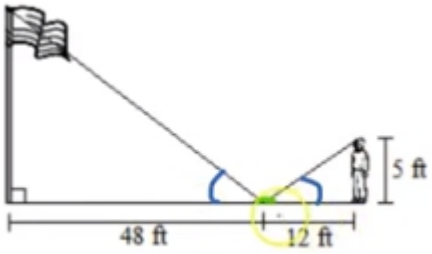
The above activity illustrates how the concept of triangle similarity can be used in solving real-life situations. In doing so, some helpful tips will guide on how to successfully come up with the correct solution.

1. Read the problem thoroughly.
2. Illustrate the problem.
3. Set up the equation.
4. Solve the equation.
5. Write a therefore statement.

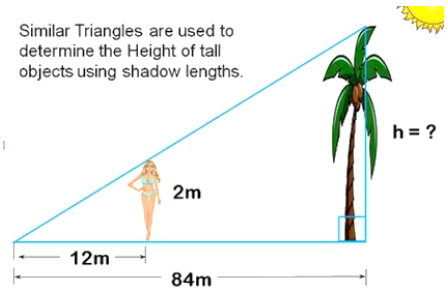
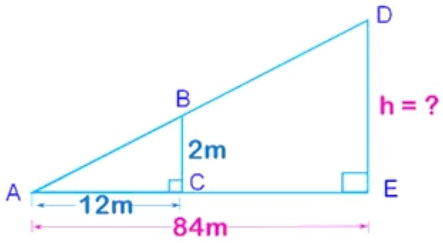
Consider the following examples:

Problem 1. *Keian wanted to measure the height of his school's flagpole. She placed a mirror on the ground 48 feet from the flagpole, then walked backwards until she was able to see the top of the pole in the mirror. His eyes were 5 feet above the ground, and she was 12 feet from the mirror. Using similar triangles, find the height in ft. of the flagpole.*

Solution:

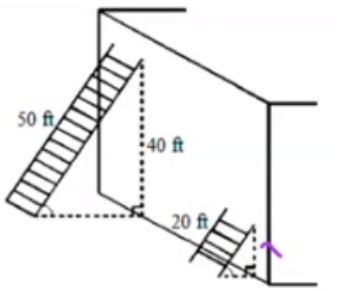
STEPS	SOLUTION
1. Read the problem carefully.	
2. Illustrate the problem. (Draw and label the triangle)	 https://www.youtube.com/watch?v=YCmaFkDdn4c
3. Set up the equation.	Establish the best proportion $\frac{x}{5} = \frac{48}{12}$
4. Solve the equation.	Setting up the ratios and then cross multiply. $\frac{x}{5} = \frac{48}{12}$ $(x)(12) = (48)(5)$ $12x = 240$ $\frac{12x}{12} = \frac{240}{12}$ $x = 20$
5. Write a therefore statement.	➢ Therefore, the height of the flagpole is 20 feet.

Problem 2. *The 2-meter tall lady makes a 12-meter long shadow, and the palm tree makes an 84m long shadow. What is the height in meter of the tree?*

STEPS	SOLUTION
1. Read the problem carefully. 2. Illustrate the problem. (Draw and label the triangle)	 Pairs of similar triangles are being formed. 
3. Set up the equation.	Establish the best proportion $\frac{h}{2} = \frac{84}{12}$
4. Solve the equation.	Setting up the ratios and then cross multiply. $\frac{h}{2} = \frac{84}{12}$ $(h)(12) = (84)(2)$ $12h = 168$ $\frac{12h}{12} = \frac{168}{12}$ $h = 14$
5. Write a therefore statement.	➢ Therefore, the height of the tree is 14 meters.

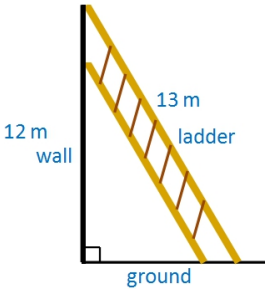
Problem 3. *Two ladders are leaning against a wall as shown, making the same angle with the ground. The longer ladder reaches 40 feet up the wall. How far up the wall does the short ladder reach?*

Solution:

STEPS	**SOLUTION**
1. Read the problem carefully. 2. Illustrate the problem. (Draw and label the triangle)	 https://www.youtube.com/watch?v=YCmaFkDdn4c
3. Set up the equation.	Establish the best proportion $\frac{x}{40} = \frac{20}{50}$
4. Solve the equation.	Setting up the ratios and then cross multiply. $\frac{x}{40} = \frac{20}{50}$ $(x)(50) = (20)(40)$ $50x = 800$ $\frac{50x}{50} = \frac{800}{50}$ $x = 16$
5. Write a therefore statement.	Therefore, the short ladder reach the wall up to 16 feet.

Problem 4. *A ladder 13 m long is placed on the ground in such a way that it touches the top of a vertical wall 12 m high. Find the distance of the foot of the ladder from the bottom of the wall.*

Solution:

STEPS	**SOLUTION**
1. Read the problem carefully. 2. Illustrate the problem. (Draw and label the triangle)	 https://www.youtube.com/watch?v=YCmaFkDdn4c
3. Set up the equation.	Applying the Pythagorean Formula: $a^2 + b^2 = c^2$ $x^2 + 12^2 = 13^2$
4. Solve the equation.	$x^2 + 12^2 = 13^2$ by SPE $x^2 = 13^2 - 12^2$ $x^2 = (13 + 12)(13 - 12)$ $x^2 = (25)(1)$ $x^2 = 25$ $x = \sqrt{25}$ $x = 5$
5. Write a therefore statement.	Therefore, the distance of the foot of the ladder from the bottom of the wall is 5m.

Explore

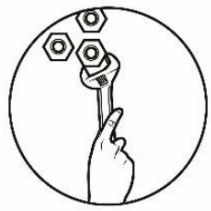
Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3: Show Me More!

A man who is 6 feet tall casts a shadow that is 8 feet long. At the same time, a tree's shadow is 22 feet long. How tall is the tree?

Follow the steps in answering the problem.

a. Illustrate the problem.
b. Set up the equation.
c. Solve the equation.
d. Write a therefore statement.



Deepen

Activity 4: Performance Task

Directions: Make a comic strip using a situation in real life where the concept of triangle similarity or on right triangles is applied. Formulate and solve problems out of these situations.

Remember:

Similar triangles are different only in size. The corresponding angles still have the same measure.

The Pythagorean Theorem states that the square of the hypotenuse is equal to the sum of the square of the two legs. Thus, $c^2 = a^2 + b^2$, where a and b represent the legs of the right triangle and c represents the hypotenuse.

In solving word problems involving triangle similarity and right triangles, always remember the following tips to come up with the correct solution.

1. *Read the problem thoroughly.*
2. *Illustrate the problem.*
3. *Set up the equation.*
4. *Solve the equation.*

Gauge

Assessment:

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. During a workout, Kylie has run 3.2 miles on the treadmill in 25 minutes. She plans to run 1.8 more miles (for a total of 5 miles). How much longer will he need to run assuming she keeps the same pace?

A. 13.05 B. 14.06 C. 15. 01 D. 16.25

2. A pine tree stands 30 ft from Joe's house and needs to be cut down. In the yard, further out next to the tree stands a 20 ft tall flag pole. At 3:00 pm the flagpole casts a 25 ft long shadow and the tree casts a 35 ft long. If the tree falls towards the house, will it hit the house?

A. Yes, because the distance of the pine tree from Joe's house is 2 ft longer than the height of the tree.
B. Yes, because the shadow of the tree casts 35 ft long.
C. No, because the height of the tree is only 28 ft.
D. Can't determine because the time is only at 3:00 pm.

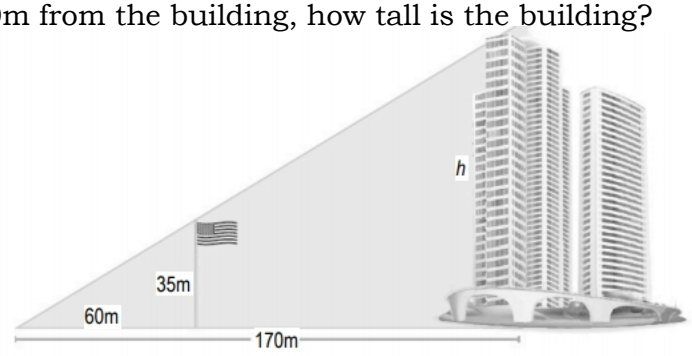
3. Nick is standing 3 ft from a 10 feet tall street light. Nick's shadow is 1 foot long and touches the end of the street light's shadow. How tall is Nick?

A. 2.5 ft tall B. 3 ft tall C. 4 ft tall D. 5 ft tall

4. Eishel, who is 1.82 meters tall, wants to find the height of a tree in her backyard. From the tree's base, she walks 12.20 meters along the tree's shadow to a position where the end of her shadow exactly overlaps the end of the tree's shadow. She is now 6.10 meters from the end of the shadows. How tall is the tree?

A. 4.56 meters B. 5.46 meters C. 6.32 meters D. 7.21 meters

5. In the diagram below, a large flagpole stands outside of an office building. Marquis realizes that when he looks up from the ground, 60m away from the flagpole, that the top of the flagpole and the top of the building line up. If the flagpole is 35m tall, and Marquis is 170m from the building, how tall is the building?



https://www.rcsdk12.org/cms/lib/NY01001156/Centricity/Domain/6881/6.5%20Similar%20Triangle%20Applications.pdf

A. 64.17m B. 84.21m C. 102.3m D. 121.6m

6. A 35-foot ladder is leaning against the side of a building and is positioned such that the base of the ladder is 21 feet from the base of the building. How far above the ground is the point where the ladder touches the building?

A. 18 ft. B. 23 ft. C. 28 ft D. 36 ft

7. The main mast of a fishing boat is supported by a sturdy rope that extends from the top of the mast to the deck. If the mast is 20 feet tall and the rope attaches to the deck 15 away from the base of the mast, how long is the rope?

A. 20 ft B. 25 ft C. 31ft D. 36 ft

8. If a man walks 6 km west and then 8 km north, how far is he from the point where he started?

A. 8 km B. 10 km C. 12 km D. 14 km

9. ΔDRO and ΔSKY are similar, if the measure of ∠0 is 50^0, what is the measure of ∠Y?

A. 50^0 B. 60^0 C. 70^0 D. 90^0

10. What is the length of the hypotenuse of a right triangle if the legs are 3 and 4?

A. 2 B. 5 C. 7 D. 12

11. Anna is standing near a tree on a sunny day. She figures that she can calculate the height of the tree if she measures her shadow and the tree's shadow, and then uses a proportion. How tall is the tree?

A. 7.14 m B. 10.5 m C. 15 m D. 28 m

12. A tree 24 feet tall casts a shadow 12 feet long. Brad is 6 feet tall. How long is Brad's shadow?

A. 3 ft. B. 6 ft. C. 9 ft. D. 12 ft.

13. The area of a square is 64 square inches. What is the perimeter of the square in inches?

A. 32 inches B. 40 inches C. 128 inches D. 256 inches

14. A carpet measures 7 feet long and has a diagonal measurement of $\sqrt{74}$ feet. Find the width of the carpet.

A. 5 ft B. 8 ft C. 11 ft D. 13 ft

15. The 2m tall lady makes a 12m long shadow, and the palm tree makes an 84m long shadow. How tall is the palm tree?

A. 14m B. 15m C. 16m D.17

Rubric for Real-Life Situation Involving Triangle Similarity

RATING	4	3	2	1
COMIC STRIP AND PROBLEMS FORMULATED AND SOLVED	The situation is clearly drawn, similar to a real life situation and the use of triangle similarity and other mathematical concepts are properly illustrated.	The situation is clearly drawn but the use of triangle similarity and other mathematical concepts are not properly illustrated.	The situation is not so clearly drawn and the use of triangle similarity concept is not illustrated.	The situation is not clearly drawn and the use of triangle similarity concept is not illustrated.
RATING	4	3	2	1
COMIC STRIP AND PROBLEMS FORMULATED AND SOLVED	The situation is clearly drawn, similar to a real life situation and the use of triangle similarity and other mathematical concepts are properly illustrated.	The situation is clearly drawn but the use of triangle similarity and other mathematical concepts are not properly illustrated.	The situation is not so clearly drawn and the use of triangle similarity concept is not illustrated.	The situation is not clearly drawn and the use of triangle similarity concept is not illustrated.
RATING	4	3	2	1
COMIC STRIP AND PROBLEMS FORMULATED AND SOLVED	The situation is clearly drawn, similar to a real life situation and the use of triangle similarity and other mathematical concepts are properly illustrated.	The situation is clearly drawn but the use of triangle similarity and other mathematical concepts are not properly illustrated.	The situation is not so clearly drawn and the use of triangle similarity concept is not illustrated.	The situation is not clearly drawn and the use of triangle similarity concept is not illustrated.

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Bryant, Merden L. et.al *Mathematics Grade 9 Learner's Module*, First Edition 2014, Reprint 2017

Pantoja, Sarah Pamela N. and San Juan, Sjiela Ann D. *Interactive Mathematics 9* Copyright 2015 by Innovative Educational Materials, INc.

Website:

https://www.texasgateway.org/resource/estimating-and-finding-solutions-problems-involving-similarity-and-rates

https://www.math10.com/en/geometry/similar-triangles/similar-triangles-theory-and-problems.html

http://passyworldofmathematics.com/similar-triangles-applications/

https://www.rcsdk12.org/cms/lib/NY01001156/Centricity/Domain/6881/6.5%20Similar%20Triangle%20Applications.pdf

https://www.youtube.com/watch?v=YCmaFkDdn4c