10

Mathematics

Quarter 3- Week 3-4 Module 3
Problems Involving Permutation and Combination

AIRs - LM

Mathematics 10
Quarter 3 – Week 3-4 Module 3
Problems Involving Permutation and Combination

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Target

Permutations and combinations are some of the counting techniques that you have studied and are actually using when asked about the number of ways you can arrange and select objects from a given set. The difference lies on whether the order or the arrangement is considered or not. A simple example where you can apply this is when you try to count the ways the top students in your class be identified. You will be using permutation if you are to determine the first, second, third and so on, in rank since they are being arranged in a definite order. But when you just want to identify the number of ways 10 students be selected without the need to rank them, then we are dealing with a combination problem this time.

This module will guide you in determining activities or tasks that involve permutations and combinations. After identifying such, you also need to recall the concepts and skills on the different counting techniques you have learned from the previous modules and apply the appropriate formulas to be used in solving the word problems on permutations and combinations.

In this module, you will learn to:

1. differentiate permutation from combination of objects taken *r* at a time. **(M10SP-IIIc-2)**
2. solve problems involving permutations and combinations. **(M10SP-IIId-e-1)**

At the end of this module, you are expected to:

1. recall concepts on the different counting techniques;
2. identify examples that illustrate permutations and combinations; and
3. solve word problems that involve permutations and combinations.

Let's start! Answer the pre-assessment in a separate sheet of paper and find out how much you already know about the topics included in this module.

PRE-ASSESSMENT

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. What is term used to refer to the arrangement of a set in a definite order?
   A. combination B. factorial C. permutation D. probability
2. Which of the following is used in solving for combination of n objects taken r at a time?
   A. (n-1)! B. $\frac{n!}{a!b!c!\ldots}$ C. $\frac{n!}{(n-r)!}$ D. $\frac{n!}{(n-r)!r!}$
3. Which of the following tasks involve permutation?
   A. picking fruits in a basket

   B. matching blouses and skirts
   C. assigning seating arrangement for a pictorial
   D. forming a dance group from a group of aspiring applicants
4. Which of the following tasks involve combination?
   A. matching blouses and skirts
   B. seating around a table
   C. arranging books in a shelf
   D. entering a password in a social media account
5. Which of the following best illustrate the act of selecting chocolates to eat from a jar of sweets?
   A. combination B. factorial C. permutation D. probability
6. Which of the following best illustrate the act of forming five-digit numbers from the digits 0, 2, 5, 6, 8 and 9?
   A. combination B. factorial C. permutation D. probability
7. Which of the following formulas will be used to determine the number of ways a committee of 4 members can be formed from a group of 10 persons?
   A. (n-1)! B. $\frac{n!}{a!b!c!\ldots}$ C. $\frac{n!}{(n-r)!}$ D. $\frac{n!}{(n-r)!r!}$
8. How many 2-digit numbers can be formed from the digits 3, 5, 7, and 9 if repetition is not allowed?
   A. P(9,2) B. C(9,2) C. P(4,2) D. P(4,2)
9. In a group of 7 boys and 5 girls, how many ways can they be arranged in a line if they may stand anywhere?
   A. C(12,12) B. P(12,12) C. P(7,1) • P(5,1) D. P(7,7) • P(5,5)
10. There are 4 different Science books, 3 different Math books, and 5 different English books. In how many ways can the books be arranged in a shelf if the books of the same subjects must be kept together?
    A. C(4,4) • C(3,3) • C(5,5) B. C(4,4) • C(3,3) • C(5,5) • C(3,3)
    C. P(4,4) • P(3,3) • P(5,5) D. P(4,4) • P(3,3) • P(5,5) • P(3,3)
11. How many different quadrilaterals can be formed from 8 non-collinear points on a plane?
    A. P(8,5) B. C(8,5) C. P(8,4) D. C(8,4)

12. How many ways can 7 people seat around a table if two of them insist on sitting beside each other?
A. 60 B. 120 C. 240 D. 720
13. How many ways can the letters of the word MATHEMATICS be arranged?
A. 4 989 600 B. 6 652 800 C. 9 974 200 D. 39 916 800
14. In a 20-item assessment test, how any ways can you select 12 questions to answer?
A. 240 B. 40 320 C. 125 970 D. 479 001 600
15. A company has 11 software developers and 5 system architects. In how many ways can a project team be identified if it is composed of 4 software developers and 2 system architects?
A. 340 B. 462 C. 3 300 D. 8 008

Lesson 1 Permutation vs. Combination

Permutations and combinations are some of the counting techniques that you have learned which are helpful in determining the number of ways a certain task is to be done, such as arranging and selecting objects from a given set. For you to have a deeper understanding on the topic, you are encouraged to go through the different activities in this lesson.

Jumpstart

Recall the topic about permutation and combination that were discussed in the previous modules, and then answer the activities that follow.

Activity 1: Note that Notation

Match the given permutation and combination notations in column A with the corresponding factorial notations in column B. Write the letter of your answer.

A	B
1. P(4,2)	A. $\frac{6!}{(6-6)!6!}$

2. C(5,3)
3. P(6,6)
4. C(4,2)
5. P(5,3)

B. $\frac{6!}{(6-6)!}$

C. $\frac{4!}{(4-2)!}$

D. $\frac{5!}{(5-3)!}$

E. $\frac{5!}{(5-3)!3!}$

F. $\frac{4!}{(4-2)!2!}$

Activity 2: Weigh and Decide

Tell whether the given situation is an example of permutation or combination. Write **P** for permutation, and **C** of combination.

1. arranging books in a shelf
2. forming a five-person committee from a group of 10 persons
3. choosing doughnuts to eat from a box of flavored doughnuts
4. opening your Facebook account using your password
5. sitting arrangement around a circular table

Discover

Determining whether a given situation involves permutation or combination is quite a tricky task. It requires in depth knowledge and understanding on the taken previously.

Let us start with a simple recall on the basic definition of permutation and combination.

Permutation is a counting technique that determines the number of ways a set of object can be arranged or ordered. It is denoted by ${}_nP_r$ or P(n,r), where in n is the total number of objects in a set and r is the number of objects selected to be arranged. There are also other types of permutations such as circular permutation (arranging objects around a circle) and distinguishable permutation (arranging distinct objects). Below are the different formulas used in permutation.

n objects taken r at a time	${}_nP_r = P(n,r) = \frac{n!}{(n-r)!}$
circular permutation	$(n-1)!$
distinct/distinguishable permutation	$\frac{n!}{a!\,b!\,c!\,\ldots}$

On the other hand, combination is another counting technique that determines the number of ways a set of object can be selected. It is denoted by ${}_nC_r$ or C(n,r), where n is the total number of objects in a set and r is the number of selected objects. The formula for combination is

$${}_nC_r = C(n,r) = \frac{n!}{(n-r)!r!}$$

How to tell the difference between permutation and combination? The answer is ordering. When the situation cares about the order of the elements, then it involves permutation. If not, it's combination.

The following are examples of permutations:

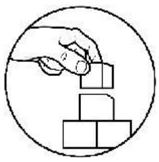
- seating around a table
- arranging books in a shelf
- entering a password in a social media account

Notice that in the given examples, the order of the objects matter. Let's say in opening a social media account, it cannot be opened unless the correct arrangement of numbers/letters is entered correctly.

The next examples are situations that involve combination.

- picking fruits in a basket
- matching blouses and skirts
- forming a dance group from a group of aspiring applicants

In the given tasks above, they only require selection and forming group. In these cases, the order does not matter.



Explore

Activity 3: What's in the box?

Identify which of the situations below are examples of permutation and combination. Copy the table in your answer sheet, then rewrite each phrases in their corresponding columns.

Permutation	Combination

- selecting the first 5 basketball players to play from a team of 12 members
- forming an identification number consisting of 5 distinct letters and numbers
- arranging letters of the word MATHEMATICS
- drawing 3 balls in a bag containing 10 colored balls
- identifying the total number of handshakes made by 8 persons
- identifying the number of possible seating arrangement in a classroom
- choosing 8 questions to answer from a given 10-item summative test
- forming 3-digit numbers from the digits 2,3,5 and 7
- arranging six precious stones in a bracelet
- counting the number of polygons that can be formed from five non-collinear points

Now that you have learned to differentiate situations that involve permutation and combination, let's see how much you have understood the topic by answering the next activity.

Deepen

Activity 4: Pen Works

On your answer sheet, discuss the following questions on permutation and combination briefly but substantially.

1. Describe the difference between permutation and combination. Discuss the important characteristic that a counting task/situation has to have in order to classify it as a permutation or combination.

2. Enumerate real life situations that involve permutation and combination. Give two examples each.

3. Discuss briefly when to use formulas for circular permutation and distinguishable permutation.

4. Given a group of ten runners joining race, construct a counting problem situation that will make it an example that illustrates permutation.

5. Given a list of Korean drama series, construct a counting problem situation that will make it an example that illustrates combination.

Lesson 2	Word Problems Involving Permutation and Combination

In this lesson, you will be able to apply the different counting techniques in solving worded problems that involve permutation and combination. It is important that you have mastered the skills in determining whether a given task is an example of permutation or combination, and be able to establish appropriate solutions to get the accurate answer for each problem to be solved.

Jumpstart

This will serve as a review on evaluating factorials, permutation and combination notations and situational examples.

Activity 1: Fact-orial!

Evaluate the given factorials and notations given.

1. $8!$
2. $\frac{6!}{3!}$
3. $(9\text{-}5)!$
4. $\frac{(6-1)!}{2}$
5. $\frac{(7-2)!}{4!}$
6. $\frac{8!}{3!4!}$
7. $P(4,2)$
8. $C(4,2)$
9. $P(6,6)$
10. $C(6,6)$

Activity 2: Choose Wisely

Choose the letter of the notation that will generate the correct answer for each question below.

Given: There are 6 men and 5 women in a senior citizens' club.

1. How many ways can 5 persons be selected to form a small group?

A. P(11,5)   B. C(11,5)

2. How many ways can 5 persons be arranged in a row of seats?
   A. P(11,5)   B. C(11,5)
3. How many ways can they be arranged in a line if they can sit anywhere?
   A. P(11,11)   B. C(11,11)
4. How many ways can they be arranged in a line if men are grouped together and women are also grouped together?
   A. $P(6,6) \cdot P(5,5) \cdot P(2,2)$ B. $C(6,6) \cdot C(5,5) \cdot C(2,2)$
5. How many ways can a group of 3 men and 3 women be formed?
   A. $C(6,3) \cdot C(5,3)$ B. $C(6,3) \cdot C(5,3)$

Discover

This section will guide you in solving real-life counting problems that involve permutation and combination. Your acquired skill in differentiating situations that illustrate permutation and combination is salient in applying proper formula to solve a particular problem.

Word Problems Involving Permutation and Combination

The following are some worded problems that involve permutation and combination. Remember that when solving, the following should be identified: given, question, and formula to be used. Determining whether a counting problem is an example of permutation or combination is very crucial. Therefore, make sure to analyze each problem carefully.

Examples

1. How many possible 5-digit passwords can be formed given the following situations:

   Given:

   There are 10 digits from 0 to 9 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. These digits can be used to form the password.

   Find:

   Number of 5-digit passwords that can be formed

A. repetition of the digits is allowed

Solution:

Using fundamental counting principle,

$$\begin{bmatrix} no.of \\ choices\ for \\ the\ first \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ second \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ third \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ fourth \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ fifth \\ digit \end{bmatrix} = \begin{bmatrix} no.of \\ possible \\ passwords \end{bmatrix}$$

10 x 10 x 10 x 10 x 10 = 1 000 000

Answer:

There are 1 000 000 possible 5-digit passwords that can be formed.

B. repetition of the digits is not allowed.

Solution no. 1:

Using fundamental counting principle,

$$\begin{bmatrix} no.of \\ choices\ for \\ the\ first \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ second \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ third \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ fourth \\ digit \end{bmatrix} x \begin{bmatrix} no.of \\ choices\ for \\ the\ fifth \\ digit \end{bmatrix} = \begin{bmatrix} no.of \\ possible \\ passwords \end{bmatrix}$$

10 x 9 x 8 x 7 x 6 = 30 240

Solution no. 2:

Using the formula on permutation, n=10 and r=5.

$$P(n,r) = P(10,5) = \frac{n!}{(n-r)!} = \frac{10!}{(10-5)!} = \frac{10!}{5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}} = \mathbf{30\ 240}$$

Answer:

There are 30 240 possible 5-digit passwords that can be formed.

2. In how many ways can 8 children be arranged in a carousel if 3 of them want to sit next to each other?

Given:

8 children in a carousel

3 will sit next to each other

Find:

number of arrangement in the carousel

Solution:

Since the arrangement is on a carousel, circular permutation will be applied.

The 3 children sitting next to each other is considered 1 element, (6-1)!=5!=120.

But, there are also 3!=6 ways to arrange the 3 children.

Thus, the total number of ways of arranging them is

$$\begin{aligned} (6-1)! \cdot 3! &= 5! \cdot 3! \\ &= 120 \cdot 6 \\ &= 720 \end{aligned}$$

Answer:

There are 720 ways to arrange the 8 children in a carousel given that 3 of them sit next to each other.

3. In a container, there are 6 black pens, 4 blue pens and 3 red pens. In how many ways can you select 6 pens given the situations below?

Given:

6 black pens, 4 blue pens and 3 red pens

Find:

Number of ways 6 pens can be selected

A. 2 pens of each color

Solution:

This involves the product of three combinations, one for each type of item.

C(6,2)	two of the 6 black pens will be selected
C(4,2)	two of the 4 blue pens will be selected
C(3,2)	two of the 3 black pens will be selected

Therefore,

$$
\begin{aligned}
C(6,2) \cdot C(4,2) \cdot C(3,2) &= \frac{6!}{(6-2)!2!} \; x \; \frac{4!}{(4-2)!2!} \; x \; \frac{3!}{(3-2)!2!} \\
&= \frac{6!}{4!2!} \; x \; \frac{4!}{2!2!} \; x \; \frac{3!}{1!2!} \\
&= \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!} \cdot 2} \; x \; \frac{4 \cdot 3 \cdot \cancel{2!}}{2 \cdot \cancel{2!}} \; x \; \frac{3 \cdot \cancel{2!}}{1 \cdot \cancel{2!}} \\
&= \frac{30}{2} \; x \; \frac{12}{2} \; x \; \frac{3}{1} \\
&= 15 \; x \; 6 \; x \; 3 \\
&= 270
\end{aligned}
$$

Answer:

There are 270 ways to select 6 pens consisting of 2 pens of each color.

B. 4 of the pens are black

Solution:

This involves the sum of the products of combinations. Note that each set should consist of 6 pens.

C(6,4) · C(4,2)	4 of the 6 black pens and 2 of the 4 blue pens will be selected
C(6,4) · C(3,2)	4 of the 6 black pens and 2 of the 3 red pens will be selected
C(6,4)·C(4,1)·C(3,1)	4 of the 6 black pens, 1 of the blue pens, and 1 of the red pens will be selected

Therefore,

$$[C(6,4) \cdot C(4,2)] + [C(6,4) \cdot C(3,2)] + [C(6,4) \cdot C(4,1) \cdot C(3,1)]$$
$$(15 \cdot 6) \quad + \quad (15 \cdot 3) \quad + \quad (15 \cdot 4 \cdot 3)$$
$$90 \quad + \quad 45 \quad + \quad 180$$
$$315$$

Answer:

There are 315 ways to select 6 pens consisting of 4 black pens.

4. How many polygons can be formed from 5 non-collinear points on a plane?

Given:

n=5 5 non collinear points on a plane

Find:

number of polygons

Solution:

The polygons that can be made from 5 non-collinear points are triangle, quadrilateral, and pentagon. This involves the sum of three combinations.

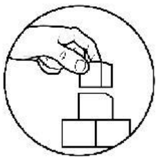
C(5,3) triangles that can be formed
C(5,4) quadrilaterals that can be formed
C(5,5) pentagon that can be formed

Therefore,

$$C(5,3) + C(5,4) + C(5,5) = \frac{5!}{(5-3)!3!} + \frac{5!}{(5-4)!4!} + \frac{5!}{(5-5)!5!}$$
$$= \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!}$$
$$= \frac{5 \cdot 4 \cdot \cancel{3!}}{2 \cdot \cancel{3!}} + \frac{5 \cdot \cancel{4!}}{1 \cdot \cancel{4!}} + \frac{\cancel{5!}}{1 \cdot \cancel{5!}}$$
$$= 10 + 5 + 1$$
$$= 16$$

Answer:

There are 16 polygons that can be formed from the 5 non-collinear points on a plane.



Explore

Let's try to apply the things that you have learned on solving permutation and combination problems.

Activity 3: Match Up!

Match the word problems on permutation and combination in column A with its corresponding answers in column B. Write the letter of your answer

A

1. In how many ways can 7 books be arranged in a shelf?
2. How many ways can you select 3 books to read from a collection of 7 books?
3. How many ways can you select 3 books from a collection of 7 books and then arrange them in s shelf?
4. There are 8 bicycle riders, how many ways can they be arranged in a line?
5. How many ways can the first, second, and third finishers be identified from the 8 bicycle riders in a race?
6. In a group of seven junior high school students and seven senior high school students, how many ways can a committee of 6 persons be identified?
7. In a group of seven junior high school students and seven senior high school students, how many ways can a committee with 3 juniors and 4 seniors be formed?
8. In a group of seven junior high school students and seven senior high school students, how many ways can they be arranged in a line if juniors stand beside each other?
9. During a Noche Buena dinner, there are 4 meat dishes, 2 vegetable dishes, fruit salad, ice cream and cake for desserts, cola and wine for the drinks, and plain rice in the buffet table. How many ways can you select your meal?
10. How many ways can you arrange the letters of the word LEARNERS?

B

A. 336

B. 1 225

C. 48

D. 5 040

E. 56

F. 40 320

G. 35

H. 3 003

I. 210

J. 203 212 800

K. 25 401 600

L. 20 160

M. 10 080

Now that you have practiced solving word problems on permutations and combinations, let’s have more challenging problems on to the next section.

Deepen

Activity 4: Wrap Around

Solve the following word problems on permutation and combination substantially. Show your complete solutions.

1. Given the digits 1, 2, 5, 6, 7 and 9, how many four-digit even numbers can be formed if:
   a. repetition of the digits is allowed
   b. repetition of the digits is not allowed
2. There are a group of 6 engineers and 3 architects. Solve for the following.
   a. How many ways can they be seated at a round table if all the architects are seated together?
   b. How many ways can they be arranged in a line if both ends should be occupied by engineers?
   c. How many ways can a project team be formed if it should consist of 2 engineers and 1 architect?
3. A bowl contains 4 red marbles, 5 yellow marbles, and 6 blue marbles. In how many ways can 4 balls be selected given each condition?
   a. all are yellow
   b. 1 red, 3 blue
   c. exactly 2 red
4. How many triangles and pentagons can be formed by joining the vertices of a heptagon?
5. In a shelf, there are 5 identical Algebra books, 3 identical Geometry books, and 3 identical Statistics books. In how many different ways can you arrange these books in a row?

Gauge

Directions: Find out how much have you learned from the lesson. Choose the letter of the correct answer to the question. Write your answer in a separate sheet of paper.

1. What is term used to refer to the selection made from a set without regard to their order?
   A. combination B. factorial C. permutation D. probability
2. Which of the following is used in solving for permutation of n objects taken r at a time?
   A. (n-1)! B. $\frac{n!}{a!b!c!\ldots}$ C. $\frac{n!}{(n-r)!}$ D. $\frac{n!}{(n-r)!r!}$
3. Which of the following tasks involve permutation?
   A. selecting shoes to wear
   B. electing a president, vice president, secretary and treasurer
   C. forming a set of board of judges
   D. identifying the number of lines that can be formed from 4 non-collinear points
4. Which of the following tasks involve combination?
   A. identifying first 5 placers in a marathon
   B. arranging horses in a carousel
   C. playing 3 songs in a karaoke
   D. selecting food to eat in a buffet restaurant
5. Which of the following best illustrate the act of falling in line during a flag raising ceremony?
   A. combination B. factorial C. permutation D. probability
6. Which of the following best illustrate the act of identifying the first ten top students in a class?
   A. combination B. factorial C. permutation D. probability
7. Which of the following formulas will be used to determine the number of ways 4 persons from a group of 10 be arranged in a row?
   A. (n-1)! B. $\frac{n!}{a!b!c!\ldots}$ C. $\frac{n!}{(n-r)!}$ D. $\frac{n!}{(n-r)!r!}$
8. How many 3-digit numbers can be formed from the digits 3, 5, 7, and 9 if repetition is not allowed?
   A. P(9,3) B. P(4,3) C. C(9,3) D. C(4,3)
9. In a group of 7 boys and 5 girls, how many ways can they be arranged in a line if seven of them will be seated in a row?
   A. C(12,12) B. P(12,12) C. C(12,7) D. P(12,7)

10. There are 4 different Science books, 3 different Math books, and 5 different English books. In how many ways can the books be arranged in a shelf if English books are kept together?

A. P(12,5) B. P(8,5) C. P(8,5) • P(5,5) D. P(8,8) • P(5,5)

11. How many different hexagons can be formed from 8 non-collinear points on a plane?

A. C(8,6) B. C(8,7) C. P(8,6) D. P(8,7)

12. How many ways can 7 people seat around a table if two of them resist on sitting beside each other?

A. 240 B. 480 C. 720 D. 5 040

13. How many ways can the letters of the word ASSESSMENT be arranged?

A. 720 B. 75 600 C. 362 880 D. 3 628 800

14. In a fruit basket consisting of 5 apples, 4 oranges, and 3 mangoes, how many ways can you select four fruits if there should be 2 apples and one orange and mango?

A. 12 B. 60 C. 120 D. 495

15. You have a collection of 12 toys. In how many ways can you select 8 toys then arrange 5 of them on a display cabinet?

A. 495 B. 6 720 C. 7 215 D. 3 326 400

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