10

Mathematics

Quarter 3- Week 1 Module 1:
Permutations

AIRs - LM

Mathematics 10
Quarter 3 – Week 1 Module 1- Permutations
Lesson 1: Illustrating permutations of objects
Lesson 2: Solving problems involving permutations

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Target

This module will discuss in detail about permutations and its applications in real life. This module will also provide you with opportunities to identify situations that describe permutations and differentiate them from those that do not. In addition, you are given the opportunity to formulate and solve problems on permutations and apply the knowledge to formulating conclusions and making decisions.

After going through this module, you are expected to attain the following objectives:

Learning Competencies:

1. Illustrates the permutations of objects (**M10SP-IIIa-1)**
2. Solves problems involving permutations (**M10SP-IIIb-1)**

Subtasks:

- Recall the fundamental counting principle
- Define and illustrate permutations of objects
- Solve problems involving permutations of objects

Pre-Assessment

Directions: Read and understand the questions below. Select the best answer to each item then write your choice on your answer sheet.

1. Which of the following situations or activities involve permutation?
   A. matching shirts and pants
   B. enumerating the subsets of a set
   C. assigning telephone numbers to subscribers
   D. forming a committee from the members of a club

2. Which of the following refers to two different arrangements of objects where some of them are identical?
   A. circular combinations
   B. circular permutations
   C. unique combinations
   D. distinguishable permutations

3. What term refers to the arrangement of objects in a circle?
   A. circular combinations
   B. circular permutations
   C. unique combinations
   D. distinguishable permutations

4. Which of the following is the formula for finding the permutation of r objects from the set in order given n distinct objects?

A. $P\,(r, n) = \frac{r!}{(r-n)!}$

B. $P\,(n, r) = \frac{n!}{(n-r)!}$

C. $P\,(r, n) = \frac{n!}{(r-n)!}$

D. $P\,(n, r) = \frac{r!}{(n-r)!}$

5. What do you call the product of positive integer n and all positive integers less than it?

A. powers of n

B. n- factors

C. multiples of n

D. n factorial

6. How many ways can 8 people be seated around a circular table?

A. 360 B. 720 C. 1440 D. 5040

7. How many different 4-digit numbers can be formed from the digits 1, 3, 5, 6, 8, and 9 if no repetition of digits is allowed?

A. 120 B. 240 C. 360 D. 840

8. Find the number of distinguishable permutations of the letters of the word SUMS.

A. 4 B. 12 C. 24 D. 36

9. Find the value of $P(8, 4)$.

A. 1680 B. 6720 C. 7400 D. 8520

10. Solve for the value of r if $P(9, r) = 15120$.

A. 2 B. 4 C. 5 D. 6

11. If $P(n, 4) = 5040$, find the value of n.

A. 8 B. 9 C. 10 D. 12

12. Find the number of distinguishable permutations of the letters in the word EDUCATED.

A. 5040 B. 10 080 C. 20 160 D. 40 320

13. In a Brain Collision Quiz Bee with 12 contestants, how many ways can the organizer arrange the first three winners?

A. 1320 B. 1716 C. 360 D. 120

14. How many ways can a code be formed from the digits 0 to 9 if a lock passcode must contain 5 different digits?

A. 604 800 B. 151 200 C. 30 240 D. 15 120

15. Given $x = P(n, n)$ and $y = P(n, n-1)$, what can you conclude about x and y?

A. $x = y$ B. $x = -y$ C. $x > y$ D. $x < y$

Jumpstart

Let us begin this lesson by assessing your knowledge of the basic counting technique called the Fundamental Counting Principle. This knowledge and skill will help you understand permutations of objects.

Activity 1: Show Me the Way!

Directions: Solve the following problems completely.

1. Rachel invited Althea to her birthday party. Althea has **4 new blouses (stripes, with ruffles, long-sleeved, and sleeveless)** and **3 skirts (red, pink, and black**) in her closet reserved for such occasions.

   a. Assuming that any skirt can be paired with any blouse, in how many ways can Althea select her outfit? List the possibilities. **(one example is the pair between stripe blouse and red skirt)**

   b. How many blouse-and-skirt pairs are possible?

2. Suppose you secured your computer using a passcode lock. Later, you realized that you forgot the four-digit code. You only remembered that the code contains the digits 1, 3, 4, and 7.

   a. List all the possible codes out of the given digits. **(example is 1347)**

   b. How many possible codes are there?

How did you determine the different possibilities asked for in the two situations? What method did you use? What did you feel when you are listing the answers?

Discover

We recall that **fundamental counting principle states that if there are p ways to do one thing, and q ways to do another thing, then there are $p\ x\ q$ ways to do both things** that you have learned in Grade 8 Mathematics. The counting principle can be extended to situations where you have more than 2 choices. For instance, if there are p ways to do one thing, q ways to a second thing, and r ways to do a third thing, then there are $p\ x\ q\ x\ r$ ways to do all three things. This is the **Multiplication Principle of the Fundamental Counting Principle.**

In the first problem in Jumpstart, since there are 4 blouses and 3 skirts, we use the Multiplication Principle, so therefore, the number of pairs is $\mathbf{4\ x\ 3} =$

12. Likewise, for Problem 2, there are $\mathbf{4 \; x \; 3 \; x \; 2 \; x \; 1 = 24}$ possible codes since it is a 4-digit code and no digit is repeated.

From the activity you have done, you recalled the Fundamental Counting Principle which is essential tool in understanding about arrangement, or permutations.

Below are some important matters that we need to discuss in order for you to understand permutations. Read carefully and understand all salient points written in this part of the module.

How do we find the permutations of objects?

Suppose we have 6 different potted plants and we wish to arrange 4 of them in a row. In how many ways can this be done?

We can determine the number of ways these plants can be arranged in a row if we arrange only 4 of them at a time. Each possible arrangement is called a **permutation.** One type of problem involves placing objects in order. We arrange letters into words and digits into numbers, line up for photographs, decorate rooms, and more. An ordering of objects is called a **permutation**. The permutation of 6 potted plants taken 4 at a time is denoted by $P(6,4)$, ${}_6P_4$, $P_{6,4}$, or P_4^6.

Similarly, if there are n objects which will be arranged r at a time, it will be denoted by $P(n,r)$.

Another way to write this is $\mathbf{{}_nP_r}$, a notation commonly seen on computers and calculators.

To calculate $P(n,r)$, we begin by finding $n!$,the number of ways to line up all n objects. $\boldsymbol{n}!$ **(n-factorial)** means the product of a positive integer n and all positive integers less that it. In this learning material, we will use $\boldsymbol{P(n,r)}$ as our notation for permutation.

Formula for Permutations of n distinct Objects

Given n distinct objects, the number of ways to select r objects from the set in order is:

$$\boldsymbol{P(n,r) = \frac{n!}{(n-r)!}} \textbf{ where } \boldsymbol{n \geq r}$$

The permutation of n objects taken all at a time is :

$$\boldsymbol{P(n,n) = n!}$$

Examples: Evaluate the following:

1. $P(10,5)$

   Substitute $n = 10$ and $r = 5$ into the formula,

   $P(10,5) = \frac{10!}{(10-5)!}$

   $P(10,5) = \frac{10!}{5!}$

   $= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

   $P(10,5) = 30240$

2. $P(6,6)$

   $n = 6$ and $r = 6$

Using $\boldsymbol{P(n,n) = n!}$,
$P(6,6) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

For convenience, we define $0! = 1$
Thus, in example 2, $P(6,6) = 6!$
Also, $P(8,8) = 8! = 40320$
$P(4,4) = 4! = 24$

3. In a school club, there are 5 possible choices for the president, a secretary, a treasurer, and an auditor. Assuming that each of them is qualified for any of these positions, in how many ways can the 4 officers be elected?

Solution:
Substitute $\boldsymbol{n = 5}$ and $r = 4$ into the formula,
$P(5,4) = \frac{5!}{(5-4)!}$

$P(10,5) = \frac{5!}{1!}$

$P(5,4) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways

Notice that, in the previous examples, the objects to be arranged are all distinct. Suppose some of the objects to be arranged are not distinct, that is, some are identical.

Finding the Number of Permutations of n Non-Distinct Objects

Distinguishable permutations are different arrangements of objects where some of them are identical or alike.

The number of **distinguishable permutations**, P**,** of $\boldsymbol{n}$ objects where p objects are alike, q objects are alike, r objects are alike, and so on, is

$$\boldsymbol{P = \frac{n!}{p!\,q!\,r!\ldots}}$$

Examples:
1. Find the number of permutations of the letters of the word STATISTICS.

Solution:
There are 10 letters of the word. Assuming that the letters are distinct, there are $P(10,10) = 10!$ permutations.
However, we have to take into considerations that the 3 S's are alike, the 3 T's are alike, and the 2 I's are also alike. The permutations of the 3 S's is $P(3,3) = 3!$. The permutations of the 3 T's is $P(3,3) = 3!$ The permutations of the I's is $P(2,2) = 2!$.
So we must divide $10!$ by $3!\,3!\,2!$ in order to eliminate the duplicates. Thus,

$$\boldsymbol{P = \frac{10!}{3!\,3!\,2!}}$$

$$\boldsymbol{P = 50\,400 \text{ permutations}}$$

2. Find the number of rearrangements of the letters in the word DISTINCT.

Solution:

There are 8 letters. Both I and T are repeated 2 times.

$$\boldsymbol{P} = \frac{\mathbf{8!}}{\mathbf{2!\,2!}}$$

$$\boldsymbol{P} = \mathbf{10\,080 \;\; arrangements}$$

3. Find the number of distinguishable permutations of the given letters "ABCCDDD".

Solution:

There are 7 letters. Letters C and D are repeated 2 and 3 times respectively.

$$\boldsymbol{P} = \frac{\mathbf{7!}}{\mathbf{2!\,3!}}$$

$$\boldsymbol{P} = \mathbf{420 \;\; permutations}$$

Let us now consider arrangement of objects in a circle, which we call **circular permutations. The circular permutations of $\boldsymbol{n}$ objects is $(\boldsymbol{n}-\mathbf{1})!$.**

Examples:

1. In how many ways can 3 people be seated around a circular table?

Solution:

The number of ways will be $(\mathbf{3}-\mathbf{1})! = \mathbf{2}! = \mathbf{2}$**.**

2. In how many ways can 3 men and 3 women be seated at around table such that no two men sit together?

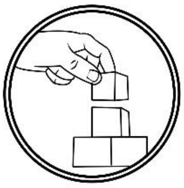
Solution:

Since we don't want the men to be seated together, the only way to do this is to make the men and women sit alternately. We'll first seat the 3 women, on alternate seats, which can be done in (3 – 1)! or 2 ways, as shown below. (We're ignoring the other 3 seats for now.)

That is, if each woman shifts by a seat in any direction, the seating arrangement remains exactly the same. That is why we have only 2 arrangements.

Now that we've done this, the 3 men can be seated in the remaining seats in 3! or 6 ways. Note that we haven't used the formula for circular arrangements now. This is because after the women are seated, shifting the each of the men by 2 seats will give a different arrangement. After fixing the position of the women (same as 'numbering' the seats), the arrangement on the remaining seats is equivalent to a linear arrangement.

Therefore, the total number of ways in this case will be **2! x 3! or 12.**



Explore

Work on the following enrichment activities for you to apply your understanding on this lesson.

Activity 2: Warm That Mind Up!

Direction: Solve for the unknown in each item.

1. $P(6,6) =$ _____
2. $P(7,r) = 840$
3. $P(n,3) = 60$
4. $P(n,3) = 504$
5. $P(10,5) =$ _____
6. $P(8,r) = 6720$
7. $P(8,3) =$ _____
8. $P(n,4) = 3024$
9. $P(12,r) = 1320$
10. $P(13,r) = 156$

How did you find the activity? What mathematical concepts or principles did you apply to solve each permutation?

Now, here is another activity that lets you apply what you have learned about the concept of permutations to solve real-life problems.

Activity 3: Mission Possible!

Directions: Answer each permutation problem completely.

1. In how many different ways can 5 bicycles be parked if there are 7 available parking spaces?

2. How many distinguishable permutations are possible with all the letters of the word ELLIPSES?

3. In a certain general assembly, three major prizes are at stake. In how many ways can the first, second, and third prizes be drawn from a box containing 120 names?

4. How many ways can the letters of the word PHOENIX be arranged?

5. A shopping mall has a straight row of 5 flagpoles at its main entrance plaza. It has 3 identical green flags and 2 identical yellow flags. How many distinct arrangements of flags on the flagpoles are possible?

6. In how many different ways can 5 people occupy the 5 seats in a front row of a mini-theater?

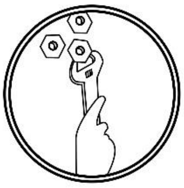
7. Find the number of ways that a family of 6 can be seated around a circular table with 6 chairs.

8. How many 4-digit numbers can be formed from the digits 1, 3, 5, 6, 8, and 9 if no repetition is allowed?

9. If there are 10 people and only 6 chairs are available, in how many ways can they be seated?

10. Find the number of distinguishable permutations of the digits of the number 348 838.

Excellent! You were able to apply the concepts you have learned. Answer the next activity to deepen your understanding in the concepts you have studied in this module.



Deepen

Activity 4: Decisions from Permutations!

Directions: Answer the following problems completely.

1. A license plate begins with three letters. If the possible letters are A, B, C, D and E, how many different permutations of these letters can be made if no letter is used more than once?

2. Find the number of different permutations of the letters of the word MISSISSIPPI.

3. In how many ways can four couples be seated at a round table if the men and women want to sit alternately?

4. Three couples want to have their pictures taken. In how many ways can they arrange themselves in a row if

   a. couples must stay together?

   b. they may stand anywhere?

5. How many ways can you order 2 blue marbles, 4 red marbles and 5 green marbles? Marbles of the same color look identical

Gauge

Directions: Read and understand the questions below. Select the best answer to each item then write your choice on your answer sheet.

1. Which of the following situations or activities involve permutation?
   A. matching shirts and pants
   B. assigning telephone numbers to subscribers
   C. forming a committee from the members of a club
   D. forming different triangles out of 5 points on a plane, no three of which are collinear

2. What do you call the two different arrangements of objects where some of them are identical?
   A. circular combinations
   B. circular permutations
   C. unique combinations
   D. distinguishable permutations

3. Which of the following refers to the arrangement of objects in a circle?
   A. circular combinations
   B. circular permutations
   C. unique combinations
   D. distinguishable permutations

4. Which of the following is the formula for finding the number of ways to select r objects from the set in order given n distinct objects?
   B. $P\,(n,r) = \frac{n!}{(n-r)!}$ B. $P\,(r,n) = \frac{r!}{(r-n)!}$
   C. $P\,(n,r) = \frac{r!}{(n-r)!}$ D. $P\,(r,n) = \frac{n!}{(r-n)!}$

5. How many different 4-digit even numbers can be formed from the digits 1, 3, 5, 6, 8, and 9 if no repetition of digits is allowed?
   A. 120 B. 420 C. 840 D. 1680

6. Find the number of distinguishable permutations of the letters of the word PASS.
   A. 144 B. 36 C. 12 D. 4

7. What do you call the product of positive integer n and all positive integers less than it?
   A. powers of n B. n- factors
   C. multiples of n D. n factorial

8. How many ways can 8 people be seated around a circular table if two of them insist on sitting beside each other?
A. 360 B. 720 C. 1440 D. 5040

9. What is the value of $P(8,5)$?
A. 56 B. 336 C. 1400 D. 6720

10. If $P(9,r) = 3024$, solve for the value of r.
A. 2 B. 4 C. 5 D. 6

11. Find the number of rearrangements of the letters in the word DISTINCT.
A. 5040 B. 10 080 C. 20 160 D. 40 320

12. In a town fiesta singing competition with 12 contestants, how many ways can the organizer arrange the first three singers?
A. 132 B. 990 C. 1320 D. 1716

13. If a combination lock must contain 5 different digits, how many ways can a code be formed from the digits 0 to 9?
A. 15 120 B. 30 240 C. 151 200 D. 604 800

14. If $P(n,4) = 5040$, find the value of n.
A. 12 B. 10 C. 9 D. 8

15. Given $x = P(n,n)$ and $y = P(n,n-1)$, what can you conclude about x and y?
A. $x > y$ B. $x < y$ C. $x = y$ D. $x = -y$

References

Books:

Mathematics Grade 10 Learner's Module

New Century Mathematics 10 Math book for Grade 10

Links:

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