Senior High School

General Mathematics

Module 16:
Valid Arguments and Fallacies

AIRs - LM

GENERAL MATHEMATICS
Module 16: Valid Arguments and Fallacies
Second Edition, 2021

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Printed in the Philippines by: _________________________

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Senior High School

General Mathematics
Module 16:
Valid Arguments and Fallacies

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

Based on your previous lessons in logic, you are expected to have rich understanding of propositions, the difference between simple and compound propositions, the different symbols used, the different operations involving propositions and lastly, you were able to determine the truth value of propositions.

At the end of this module, you are expected to distinguish and to determine whether an argument is valid or not using the different rules. And apply also those learnings you had in determining the validity and falsity of real-life arguments.

This module will provide you with information and activities that will help you understand tautologies and fallacies, validity of categorical syllogisms and more.

After going through this module, you are expected to:

1. illustrate different types of tautologies and fallacies **(M11GM-IIi-1)**,
2. determine the validity of categorical syllogisms **(M11GM-IIi-2**); and
3. establish the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies **(M11GM-IIi-3).**

Learning Objectives:

1. define arguments, tautology, and fallacy
2. convert arguments into its equivalent symbolic form and vice versa
3. apply the validity condition and truth condition in determining validity falsity, and soundness of arguments
4. identify the types of tautologies and fallacies of a given argument
5. construct own valid arguments.

Pretest

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

_____1. Which of the following is the correct symbol for the given compound propositions, *"If classes are suspended, then typhoon signals are declared."*?

P - Classes are suspended. **Q** – Typhoon signals are declared.

A. $P \leftrightarrow Q$ B. $P \rightarrow Q$ C. $Q \rightarrow P$ D. $P \wedge Q$

_____2. Which of the following sentences is an example of a compound proposition?

A. I can sing.
B. I can play the guitar.
C. Today is not my birthday.
D. Mindanao is an island in the Philippines.

_____3. Which of the following is a proposition?

A. February has 30 days.
B. Welcome to the Philippines!
C. What is the capital of Canada?
D. What is the domain of the function?

_____4. Given the following statements:

P: *I am beautiful.* **Q**: *I am a Senior High School student.*
R: *I am good in Mathematics.*

Which of the following is the correct translation of the given operation, $(\boldsymbol{P} \wedge \boldsymbol{R}) \rightarrow \boldsymbol{Q}$?

A. If I am beautiful and good in Mathematics, then I am a Senior High School student.
B. I am beautiful or good in Mathematics if an only is I am a Senior High School student.
C. I am beautiful but not good in Mathematics, then I am not a Senior High School student.
D. If I am not beautiful and not good in Mathematics, then I am not a Senior High School student.

_____5. Determine the truth value of the given table,

P	**Q**	**P ∧ Q**
T	T	
T	F	
F	T	
F	F	

A.

P ∧ Q
T
T
F
F

B.

P ∧ Q
T
T
T
T

C.

P ∧ Q
T
F
F
F

D.

P ∧ Q
F
F
F
F

______6. Let $\boldsymbol{P}$: We should be honest., $\boldsymbol{Q}$: We should be dedicated., $\boldsymbol{R}$: We should be overconfident. Which of the following is the best representation of the statement, *'We should be honest or dedicated but not overconfident.'?*

A. $\sim P \vee \sim Q \vee R$ B. $P \wedge \sim Q \wedge R$
C. $P \vee Q \wedge R$ D. $P \vee Q \wedge \sim R$

_____7. What do you call a valid argument that satisfies the validity condition; that is, the conclusion is true whenever the premises are all true?

A. Contradiction B. Fallacy
C. Tautology D. Contrapositive

_____8. Let ***P***: If Tina bowls, then Saurabh hits a century. and ***Q***: If Raju bowls, then Tina gets out on first ball.

Now, if ***P*** is true and ***Q*** is false then which of the following can be **TRUE**?

A. Raju did not bowl.

B. Tina bowled and Saurabh got out.

C. Tina bowled and Saurabh hits a century.

D. Raju bowled, and Tina got out on first ball

_____9. Which of the following standard form of argument is the translation of the given arguments given below?

*"If Tomas was absent **(A)**, then he missed **(M)** the review.*
Tomas was absent.
Therefore, Tomas missed the review.

A.	B.	C.	D.
$A \rightarrow M$	$A \rightarrow M$	$A \rightarrow M$	$M \rightarrow A$
A	M	$\sim M$	A
$\therefore M$	$\therefore M$	$\therefore \sim M$	$\therefore \sim M$

_____10. What is the correct translation of the given propositional form given below?

$$\frac{p}{\therefore p \vee q}$$

Let ***p*** – Antonio Luna is a scientist.
q – Jose Rizal is a scientist.

A. Antonio Luna is a scientist.
Therefore, Antonio Luna and Jose Rizal are scientist.

B. Antonio Luna is not a scientist.
Therefore, Antonio Luna and Jose Rizal are not a scientist.

C. Jose Rizal is a scientist.
Therefore, Antonio Luna and Jose Rizal are scientist.

D. Antonio Luna is a scientist.
Therefore, Antonio Luna or Jose Rizal is a scientist.

_____11. Which of the following is the symbol for the following argument?

*The fish is fresh (**P**) or I will not order it. **(Q)***
The fish is fresh.
Therefore, I will order it.

A. $((P \vee Q) \rightarrow P) \rightarrow Q$

B. $((P \vee Q) \rightarrow P) \rightarrow \sim Q$

C. $((P \vee Q) \rightarrow \sim P) \rightarrow Q$

D. $((P \vee \sim Q) \rightarrow P) \rightarrow Q$

_____12. Given the following statements, determine what type of fallacy was committed.

If it rains, then the road is wet.
It did not rain.
Therefore, the road is not wet.

A. Affirming the disjunct

B. Fallacy of the converse

C. Fallacy of the inverse

D. Fallacy of the consequent

_____13. Which of the following arguments is considered as a tautology?

A. Her new boyfriend drives an old car. He must be poor. She should break up with him.

B. I see dark clouds on the horizon. Dark clouds mean rain. It's going to rain here today.

C. All humans are mortal. Khloe is human. Khloe is mortal.

D. The house looks old. Therefore, the house is over 100 years old.

For numbers 14 and 15, refer to the given categorical syllogism below:

All attendees are senior citizens.
Mrs. dela Cruz is an attendee.
Therefore, Mrs. dela Cruz is a senior citizen.

_____14. Which among the statements is/are the premise/s?
A. All attendees are senior citizens.
B. Mrs. dela Cruz is an attendee.
C. Both A and B are correct.
D. Neither A nor B is correct

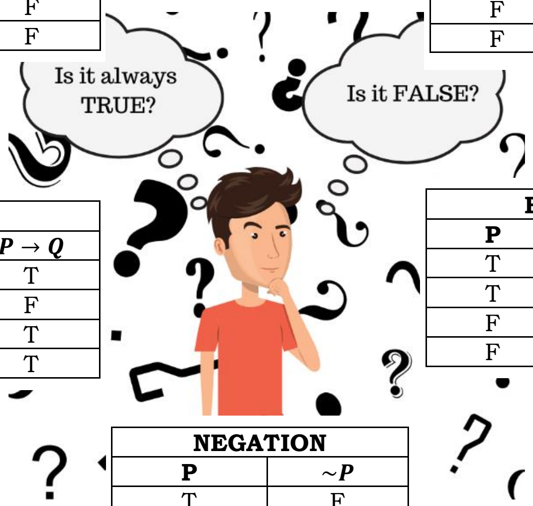
_____15. Which among the statements is the conclusion?
A. All attendees are senior citizens.
B. Mrs. dela Cruz is an attendee.
C. Mrs. dela Cruz is a senior citizen.
D. None of the above.

Jumpstart

Figure 1 shows that human-beings always think and analyze whether an argument they received, heard, or read is valid and sound. It can also be interpreted from the figure that individual must practice a questioning mind in order not to be easily deceived in an argument.

AND		
P	**Q**	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

OR		
P	**Q**	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F



CONDITIONAL		
P	**Q**	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

BICONDITIONAL		
P	**Q**	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

NEGATION	
P	$\sim P$
T	F
F	T

Figure 1. Logic is always searching arguments' validity and falsity

Activity 1: RECALL IT PLEASE!

***Directions*:** Fill in the given table for the different operations of their corresponding truth table value. Write your answer on a separate sheet of paper.

Conjunction of P and Q		
P	Q	$P \wedge Q$

Disjunction of P or Q		
P	Q	$P \vee Q$

Negation	
P	~P

Conditional Statement		
P	Q	$P \rightarrow Q$

Biconditional Statement		
P	Q	$P \leftrightarrow Q$

Activity 2: COMPLETE ME!

Directions: Complete the truth table of the given propositions below. Write your answer on a separate sheet of paper.

- $(\boldsymbol{P} \rightarrow \boldsymbol{Q}) \wedge \boldsymbol{P} \rightarrow \boldsymbol{Q}$

P	**Q**	**P**	$\rightarrow$	**Q**	$(\boldsymbol{P} \rightarrow \boldsymbol{Q})$	$\wedge$	**P**	$[(\boldsymbol{P} \rightarrow \boldsymbol{Q}) \wedge \boldsymbol{P}]$	$\rightarrow$	**Q**
1	*2*	*1*	*3*	*2*	*3*	*4*	*1*	*4*	*5*	*2*

The numbers below the table are the patterns you need to follow for you to get the value of the last operation which is the highest number. (Note: Numbers that are repeated mean that the column shares the same answer or value.)

Question:

Describe what you obtained in the circled column of the truth table.

____.

Activity 3: AM I TRUE?

Directions: Determine whether the given statements below are ALWAYS TRUE, TRUE or NOT TRUE. Write **AT** if you think it is always true, if true, write **T**, otherwise **NT**. Write your answer on a separate sheet of paper.

____1. If you are poor, then you cannot help to solve the problem of the country.
____2. Complexion is a measurement of beauty in the Philippines.
____3. If you are good in Mathematics, then you are poor in English.
____4. If I study hard, then I can have a better future.
____5. If you respect the elders, then they will also show respect to you.

____6. If you plant trees, then you are helping the world.
____7. If you know how to read well, then it can help you succeed.
____8. Being literate in technologies nowadays makes you difficult to be fooled.
____9. If you lack sleeps, then it is hard for you to think clearly and sound.
____10. Jumping during the New Year's Eve makes you taller.

Discover

Recall that from the previous module, you dealt with simple and compound propositions. Compound propositions are combination of two or more simple propositions being connected by connectors (*e.g. and, or, if…then…, if and only if, but, not*) which later becomes the operations.

In real-life, we are given set of information and our goal is to *logically* understand from the given some new information.

Let us have an example, we know that only a small portion of the earth's water is freshwater (mostly is saltwater). *We know that if there is a limited supply of freshwater, then we should conserve water.* Combining these information needs us to conclude that *we should conserve water.*

From the given example above about freshwater, the set/statement that gives us information are '*If there is a limited supply of freshwater, then we should conserve water.*' as the <u>premise</u>, while the derived new information *'We should conserve water'* is referred to as the <u>conclusion</u>. The ***premises*** together with the ***conclusion*** form what we call an ***argument***.

DEFINITION OF ARGUMENT

An **argument** is a compound proposition of the form

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q.$$

> ***Remember:***
> $\wedge$ stands for 'and'
> $\rightarrow$ stands for the 'if…then…'

The propositions $p_1, p_2, \cdots, p_n$ are the **premises** of the argument, and q is the **conclusion**.

Arguments can be written in **proposition form**,

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q.$$

or in column or **standard form** or also the form of **categorical syllogism:**

$$\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ \underline{\;p_n\;} \\ \therefore q \end{array}$$

The premises of an argument are intended to act as reasons to establish the validity or acceptability of the conclusions.

The *categorical syllogism* has two parts, the **premises** *(major and minor premise)***,** and the **conclusion** as what will be observed in the succeeding examples.

Illustrative Example 1:

Explain why the following set of propositions is an argument.

If General Antonio Luna is a national hero, then he died at the hands of the Americans in 1899.
General Antonio Luna is a national hero.
Therefore, General Antonio Luna died at the hands of the Americans in 1899.

Solution:

The set of propositions is an argument because it has premises and conclusion.

Its **premises** are the propositions '*If General Antonio Luna is a national hero, then he died at the hands of the Americans in 1899.*' and '*General Antonio Luna is a national hero.*'

The ***conclusion***, which is signaled by the word 'therefore' (*or in symbol*, ∴), is the proposition '*General Antonio Luna died at the hands of the Americans in 1899.*'

Illustrative Example 2:

Write the following argument presented in *propositional form* and in *standard form.*

If there is limited freshwater supply, then we should conserve water.
There is limited freshwater supply.
Therefore, we should conserve water.

Solution: Let us convert the statements into symbol:

$\boldsymbol{p_1}$: There is limited freshwater supply.

$\boldsymbol{p_2}$: We should conserve water

> ***Remember:***
> ***p*** stands for <u>premise</u>.
> Therefore (∴) means for the conclusion

The <u>premises</u> of the argument are:

$(p_1 \rightarrow p_2)$: *If* there is limited freshwater supply, *then* we should conserve water.

p_1: There is a limited fresh water supply.

> ***Recall:***
> In $(p_1 \rightarrow p_2)$, the symbol $(\rightarrow)$ means *If.., then..*

and its <u>conclusion</u> is

p_2: We should conserve water.

In symbols, we can write the whole argument in *propositional form;*

$$[(\boldsymbol{p_1} \rightarrow \boldsymbol{p_2}) \wedge \boldsymbol{p_1}] \rightarrow \boldsymbol{p_2}$$

> ***Note:***
> In propositional form, the symbol $(\wedge)$ indicates the connection between the two premises.

and in *standard form*;

$$\begin{array}{l} \boldsymbol{p_1} \rightarrow \boldsymbol{p_2} \\ \underline{\boldsymbol{p_1} \qquad\quad} \\ \therefore \boldsymbol{p_2} \end{array}$$

> **Note:**
> $\wedge$ is removed in the standard form

Illustrative Example 3:

Consider the argument *A* and *A'* given below. Notice that the two arguments have *equivalent logical forms*, even if they are different in *content*.

A		**A'**	
$p \rightarrow q$	If my alarm sounds, then I will wake up.	$p \rightarrow q$	If my alarm sounds, then I will wake up.
p	My alarm sounded.	q	I woke up.
$\therefore q$	Therefore, I woke up.	$\therefore p$	Therefore, my alarm sounded.

To determine the validity of the given arguments, you need to analyze these arguments separately by answering the following question.

> Validity Condition:
>
> Is it ***logically impossible*** for the PREMISES to be ***TRUE*** and the CONCLUSION ***FALSE***?
>
> If the answer is ***YES***, we say that the argument satisfies the validity condition.
>
> ***The argument is valid.***

Solution:

Argument A

Can $\boldsymbol{p \rightarrow q}$ and $\boldsymbol{p}$ both be true and $\boldsymbol{q}$ be false?

Suppose that the premises $p \rightarrow q$ and p are true. The truth table below show that both p and $p \rightarrow q$ are true only in the first row. In this row, q is also true.

Observe that, only in the first row, $\boldsymbol{p \rightarrow q}$ and $\boldsymbol{p}$ are both true (T). $\boldsymbol{q}$ is also true (T) on the same row.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

****Remember***: This is the TRUTH TABLE of a Conditional Proposition ($\rightarrow$).

Hence, **YES**, it is logically impossible for the premises to be true and the conclusion to be false. We can say that A satisfies the validity condition, and so it is valid.

Argument A'

Can $\boldsymbol{p \rightarrow q}$ and $\boldsymbol{q}$ both be true and $\boldsymbol{p}$ be false?

Suppose that the premises $p \rightarrow q$ and q are true. The truth table below show that both q and $p \rightarrow q$ are true in the first and third row. However, in the third row, the conclusion p is false, even if the premises ($p \rightarrow q$ and q) are true.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Observe that, in first and third row, $\boldsymbol{p \rightarrow q}$ and $\boldsymbol{q}$ are both true (T). $\boldsymbol{p}$ is true (T) on the first row. However, $\boldsymbol{p}$ is also false (F) on the third row.

Argument A' does not satisfy the validity condition. In practical terms, *it is possible that I woke up, but my alarm did not sound. I had woken up due to a bad dream, for example.*

In summary, we have the following:

$p \rightarrow q$	If my alarm sounds, then I will wake up.	False $\rightarrow$ True
q	I woke up.	True
$\therefore p$	Therefore, my alarm sounded.	$\therefore$ False

Based on the different examples above, we are now ready to define what is a **valid argument**.

DEFINITION OF VALID ARGUMENT

A *valid argument* satisfies the validity condition; that is, the conclusion **q** is true whenever the premises $p_1 \wedge p_2 \wedge \cdots \wedge p_n$ are ***all true***.

The conditional $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q$ is called a ***tautology***.

In simpler definition, a compound statements which are true for any combination of truth values of the variables in the statement is called ***tautology***.

Recall that in *Activity 2: Complete Me! (on page 5)*, the circled column showed all **TRUE** (**T**). Therefore, the argument of the form $[(P \rightarrow Q) \wedge P] \rightarrow Q$ is a tautology.

$$[(\boldsymbol{P} \rightarrow \boldsymbol{Q}) \wedge \boldsymbol{P}] \rightarrow \boldsymbol{Q}$$

P	**Q**	**P**	**→**	**Q**	**(P→Q)**	**∧**	**P**	**[(P → Q) ∧ P]**	**→**	**Q**
T	T	T	T	T	T	T	T	T	**T**	T
T	F	T	F	F	F	F	T	F	**T**	F
F	T	F	T	T	T	F	F	F	**T**	T
F	F	F	T	F	T	F	F	F	**T**	F
1	*2*	*1*	*3*	*2*	*3*	*4*	*1*	*4*	*5*	*2*

You are now ready to know the *different types of tautologies and fallacies*. Do not worry, you will be guided throughout the discussion.

LIST OF TAUTOLOGIES or also known as the RULES OF INFERENCES

Let p, q, and r be propositions.

Name	**Tautology**	
	Propositional Form	**Standard Form**
Rule of Simplification	$(p \wedge q) \rightarrow p$	$\underline{p \wedge q}$ $\therefore p$
Rule of Addition	$p \rightarrow (p \vee q)$	$\underline{p}$ $\therefore p \vee q$
Rule of Conjunction	$(p) \wedge (q) \rightarrow (p \wedge q)$	p $\underline{q}$ $\therefore p \wedge q$
Modus Ponens	$((p \rightarrow q) \wedge p) \rightarrow q$	$p \rightarrow q$ $\underline{p}$ $\therefore q$
Modus Tollens	$\big((p \rightarrow q) \wedge (\sim q)\big) \rightarrow \sim p$	$p \rightarrow q$ $\underline{\sim q}$ $\therefore \sim p$
Law of Syllogism	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	$p \rightarrow q$ $\underline{q \rightarrow r}$ $\therefore p \rightarrow r$
Rule of Disjunctive Syllogism	$((p \vee q) \wedge (\sim p)) \rightarrow q$	$p \vee q$ $\underline{\sim p}$ $\therefore q$
Rule of Contradiction	$((\sim p) \rightarrow \emptyset) \rightarrow p$	$\underline{(\sim p) \rightarrow \emptyset}$ $\therefore p$
Rule of Proof by Cases	$((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$	$p \rightarrow r$ $\underline{q \rightarrow r}$ $\therefore (p \vee q) \rightarrow r$

The list of tautologies or rules of references only tells us that arguments are valid only in terms of its form and not on its content. The validity of the content will be discussed after tautologies and fallacies.

Illustrative Example 1:

Here is an example of an argument:

Premise 1: If one loves algebra, then he loves mathematics.
Premise 2: Mike loves algebra.
Conclusion: Therefore, Mike loves mathematics.

Representing each simple statement with a letter:

p: One loves algebra.
q: One loves mathematics.

Writing the two premises and the conclusion in symbolic form, we have:

Premise 1:	$p \rightarrow q$	*If* one loves algebra, *then* he loves mathematics.
Premise 2:	$\underline{p}$	Mikes loves algebra.
Conclusion:	$\therefore q$	Therefore, Mike loves mathematics.

Is the argument valid? If yes, what type of tautology is the argument?

The given argument is **valid** because of **Modus Ponens**.

But why *Modus Ponens*? Look at the table of the *List of Tautologies*, look the standard form of the Modus Ponens and try to compare it with the example. If you already see the pattern or similarity, then you are right!

Illustrative Example 2:

Determine whether the following argument is valid and identify what type of tautology or rule of inference it is.

Premise 1: If Antonio and Jose are friends, then they are Facebook friends.
Premise 2: Antonio and Jose are not Facebook friends.
Conclusion: Therefore, they are not friends.

Solution: Let

***p**:* Antonio and Jose are friends.
***q**:* Antonio and Jose are Facebook friends.

Writing the two premises and the conclusion in symbolic form, we have:

Premise 1:	$p \rightarrow q$	*If* Antonio and Jose are friends, *then* they are Facebook friends.
Premise 2:	$\underline{\sim q}$	Antonio and Jose are **not** Facebook friends.
Conclusion:	$\therefore \sim p$	Therefore, they are not friends.

Then, the given argument is of the form

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

Recall:

~ is the symbol for **NOT**.

Hence, the *argument is valid* because of **Modus Tollens** as rule of inference.

Why Modus Tollens? Is the argument valid? Why? Of course, if two people are really friends, they should be friend also in social media.

Illustrative Example 3:

Determine which rule of inference is the basis of each argument below:

(a) Antonio Luna *and* Jose Rizal like Nelly Boustead.
Therefore, Antonio Luna likes Nelly Boustead.

(b) Antonio Luna is a scientist.
Therefore, *either* Antonio Luna *or* Jose Rizal is a scientist.

(c) *If* the Spaniards imprison Antonio Luna, *then* he will repent and not join the revolution.
If Antonio Luna regrets *not* joining the revolution, *then* he will go to Belgium to study the art of war.
Therefore, *if* the Spaniards imprison Antonio Luna, *then* he will go to Belgium to study the art of war.

Solution:

(a) Let

p: Antonio Luna likes Nelly Boustead.
q: Jose Rizal likes Nelly Boustead.

Then, the given argument is of the form

$$\frac{p \wedge q}{\therefore p}$$

Recall:
$\wedge$ is the symbol for **AND**.

By the ***Rule of Simplification***, the arguments are ***valid***.

(b) Let

p: Antonio Luna is a scientist.
q: Jose is a scientist.

The argument in standard form is

$$\frac{p}{\therefore p \vee q}$$

Recall:
$\vee$ is the symbol for **OR**.

which is valid by the ***Rule of Addition***.

(c) Let

p: The Spaniards imprison Antonio Luna.
q: Antonio Luna regrets not joining the revolution.
r: Antonio Luna goes to Belgium to study the art of war.

In standard form, we have

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

Recall:
$\rightarrow$ is the symbol for **IF…, THEN…**?

Thus, the argument is valid by ***Law of Syllogism***.

From the previous examples we deal with those arguments which are *valid*. How about those arguments which are ***invalid***? These will be the focus of the next part of this lesson which is on **FALLACY**.

DEFINITION OF FALLACY

An argument

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q,$$

which is not valid is called a **fallacy**.

In a ***fallacy***, it is possible for the premises $p_1, p_2, \cdots, p_n$ to be true, while the conclusion q is false.

Equivalently, for this case, the conditional

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q.$$

is ***not*** a tautology.

The following table lists some common fallacies in logic.

Table of Fallacies

Let p, q, and r be propositions.

Name	**FALLACY**	
	Propositional Form	**Standard Form**
Fallacy of the Converse	$\big((p \rightarrow q) \wedge q\big) \rightarrow p$	$p \rightarrow q$ q $\therefore p$
Fallacy of the Inverse	$\big((p \rightarrow q) \wedge (\sim p)\big) \rightarrow (\sim q)$	$p \rightarrow q$ $\sim p$ $\therefore \sim q$
Affirming the Disjunct	$\big((p \vee q) \wedge p\big) \rightarrow (\sim q)$	$p \vee q$ p $\therefore \sim q$
Fallacy of the Consequent	$(p \rightarrow q) \rightarrow (q \rightarrow p)$	$p \rightarrow q$ $\therefore q \rightarrow p$
Denying a Conjunct	$(\sim(p \wedge q) \wedge (\sim p)) \rightarrow q$	$\sim(p \wedge q)$ $\sim p$ $\therefore q$
Improper Transposition	$(p \rightarrow q) \rightarrow ((\sim p) \rightarrow (\sim q))$	$p \rightarrow q$ $\therefore (\sim p) \rightarrow (\sim q)$

Illustrative Example 1:

Determine whether the given is a valid argument or a fallacy.

(a) *Either* Alvin sings *or* dances with Nina.
Alvin sings with Nina.
Therefore, Alvin did *not* dance with Nina.

(b) *Either* Alvin sings *or* dances with Nina.
Alvin did *not* dance with Nina.
Therefore, Alvin sang with Nina.

(c) It is *not* true that Alvin sings *and* dances with Nina.
Alvin did *not* sing with Nina.
Therefore, Alvin dances with Nina.

Solution: Let

$\boldsymbol{p}$: Alvin sings with Nina.
$\boldsymbol{q}$: Alvin dances with Nina.

(a) The given argument is of the form:

$$\begin{array}{l} p \vee q \\ \underline{p \qquad\quad} \\ \therefore \sim q \end{array}$$

> ***Recall***:
> ∨ is the symbol for **OR**

Thus, the argument is the fallacy of **Affirming the Disjunct**.

(b) In symbols, the given argument has of the form

$$\begin{array}{l} p \vee q \\ \underline{\sim q \qquad\quad} \\ \therefore\ p \end{array}$$

If you are going to look into the <u>Table of Fallacies</u>, there is no such form. What you can do is to transform it. By the *commutative law*, which states that $\big((p \wedge q) \Leftrightarrow (q \wedge p)\big)$, you will have

$$\begin{array}{l} q \vee p \\ \underline{\sim q \qquad\quad} \\ \therefore\ p \end{array}$$

> ***Observe***:
> ***p*** and ***q*** just interchange.
> That is commutative law!

Thus, by the **Rule of Disjunctive Syllogism**, the argument is **valid**.

(c) Transforming the argument in symbols yields the following

$$\begin{array}{l} \sim(p \wedge q) \\ \underline{\sim p \qquad\quad} \\ \therefore\ q \end{array}$$

> ***Recall***:
> ∧ is the symbol for **AND**

This is the fallacy of **Denying a Conjunct**.

Illustrative Example 2:

Determine whether the given is a valid argument or a fallacy.

(a) If today is Wednesday, then I have math class.
Today is not Wednesday.
Therefore, today I do not have math class.

(b) If today is Wednesday, then I have math class.
I have math class.
Therefore, today is Wednesday.

Solution: Let

p: Today is Wednesday.
q: I have math class.

(a) The given argument is of the form:

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim p \qquad} \\ \therefore \sim q \end{array}$$

Thus, the argument is the **Fallacy of the Inverse**.

(b) Transforming the argument in symbols yields the following

$$\begin{array}{l} p \rightarrow q \\ \underline{q \qquad} \\ \therefore p \end{array}$$

This is the **Fallacy of the Converse**.

For you to easily identify the type of fallacy or tautology a given argument, you need to always practice writing it to its **equivalent symbols** like how the examples are presented.

And now you already have an idea about the valid argument which is **tautology** and the not valid which is the **fallacy**. We are now ready to know the difference between **valid argument** and a **sound argument**.

Illustrative Example 1:

Let us go back to our previous example to show their validity and soundness.

$p \rightarrow q$	If General Antonio Luna is a national hero, then he died at the hands of the Americans in 1899.
$\underline{p}$	General Antonio Luna is a national hero.
$\therefore q$	Therefore, General Antonio Luna died at the hands of the Americans in 1899.

You already know that this argument is *valid* because of *Modus Ponens*. The argument satisfies the *validity condition.* **BUT IS THE ARGUMENT SOUND?**

To determine if the argument is **SOUND**, we are going to ask the question:

> *Truth Condition*
> Are the premises of the argument all generally **TRUE**?
> If the answer is **YES**, the argument is said to satisfy the truth condition.

Let us try with the example:

We know that General Luna was killed by fellow Filipinos at the height of Filipino-American War. Then, the premise $p \rightarrow q$ is **FALSE** because General Luna is a national hero (p is true), but he did not die at the hands of the American (q is false).

Hence, the argument, though it is **valid**, does *not satisfy the truth condition.* We say that the argument is **not sound**.

For an argument to be considered **sound** it must satisfy **both** the **validity condition** and the **truth condition**.

DEFINITION OF SOUND ARGUMENT and BAD ARGUMENT

A **sound argument** is a <u>valid argument</u> which also *satisfies* the <u>truth condition</u>.

An argument which *does not satisfy* either the <u>validity condition</u> or the <u>truth condition</u> is called a **bad argument**.

Illustrative Example 2:

Determine whether each of the following is **valid**, and if each is **sound**.

(a) If I was born poor, then I cannot serve my country.
I was born poor.
Therefore, I cannot serve my country.

(b) If I study every day, then I will develop a good work ethic.
I study every day.
Therefore, I will develop a good work ethic.

(c) If you study hard, then you refine your communication skills and build up your confidence.
If you refine your communication skills and build up your confidence, then your job opportunities increase.
Hence, if you study hard, your job opportunities increase.

(d) If I have dark skin, then I am not beautiful.
I have dark skin.
Therefore, I am not beautiful.

Solution:

(a) Let

p: I was born poor.

q: I serve my country.

Then, the given argument is of the form

$$\begin{array}{l} p \rightarrow \sim q \\ \underline{p \qquad\quad} \\ \therefore\ \sim q \end{array}$$

> ***Note:***
> The given form of Modus Ponens is different from what is given on the list of tautologies, but if you remove the negation (~), it will now be the same form as what it is in the list.

The argument is **valid** by **Modus Ponens**. *But is the argument sound?*

Note that being poor does not prevent one from serving one's country (Think of someone who is poor but manage to serve the country). Hence, the given argument is a **bad argument** or **not sound**.

(b) Let

p: I study every day.

q: I will develop a good work ethic.

The argument in standard form is

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad\quad} \\ \therefore\ q \end{array}$$

which is **valid** by **Modus Ponens**. *But is the argument sound?*

It is accepted as true that if I or you study every day, then I or you will develop a good work ethic. Now, is the statement 'I study every day' true? You should know! If you do, then this argument satisfies both the validity and truth conditions, and you can rightfully claim that you will develop work ethic. Hence, the argument is ***sound argument***.

(c) Let

p: You study hard.
q: You refine your communication skills and build up your confidence
r: Your job opportunities increase

The argument in standard form is

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r \quad} \\ \therefore\ p \rightarrow r \end{array}$$

which is **valid** by **Law of Syllogism**. *But is the argument sound?*

If you are going to look into it, the reason why you are studying is for you to develop certain skills like communication skills and as you develop certain skills, you also build your confidence to conquer everything. And if you have communication skills and confidence then, there is a greater chance of you to be hired in any job you want. Then, it is okay to conclude that studying hard will increase your job opportunities. Is that a good claim? If yes, then the argument is **valid** (because of Law of Syllogism) as well as **sound** (because the premises can be both generally true).

(d) Let

p: I have dark skin.

q: I am beautiful.

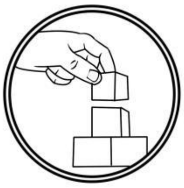
Then, the given argument is of the form

$$\begin{array}{l} p \rightarrow \sim q \\ \underline{p \qquad\quad} \\ \therefore\ \sim q \end{array}$$

> ***Note:***
> The given form of Modus Ponens is different from what is given on the list of tautologies, but if you remove the negation (~), it will now be the same form as what it is in the list.

The argument is **valid** by **Modus Ponens**. *But is the argument sound?*

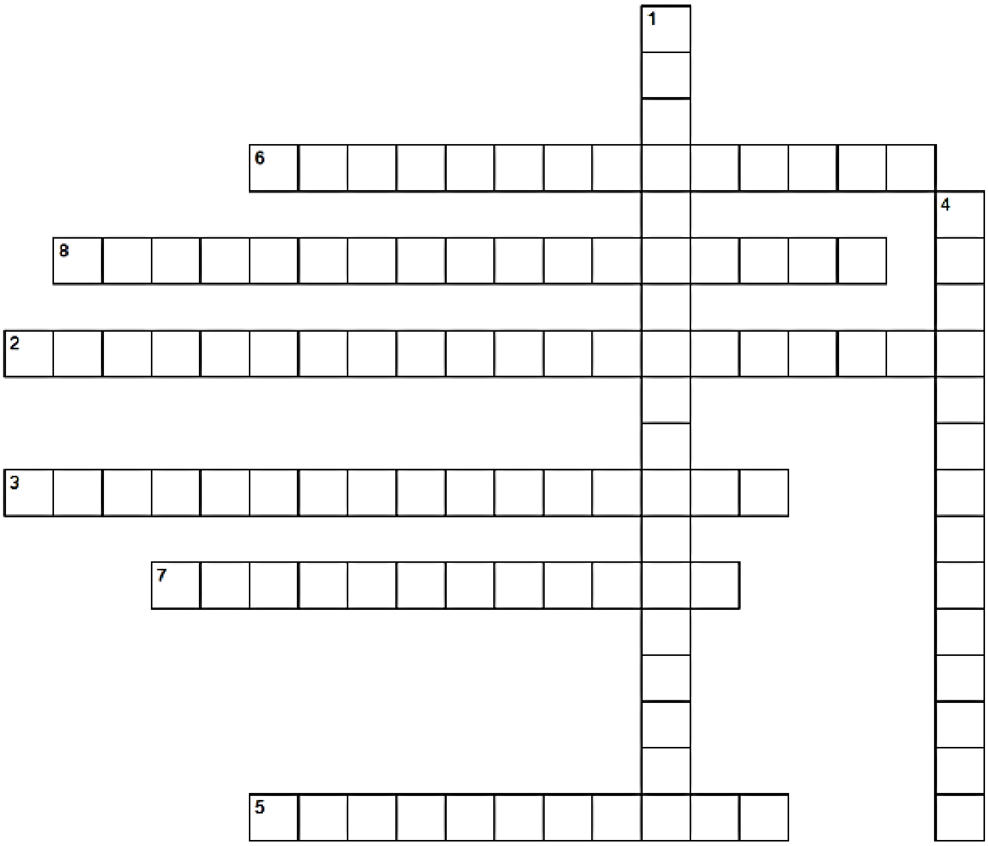
Note that having dark skin <u>does not</u> mean that you are ugly. Complexion is not criteria of being beautiful or any other. Everyone is beautiful no matter what her color is, race, or anything. Hence, the given argument is a **bad argument** or **not sound**.



Explore

Activity 1: IDENTIFY ME!

Directions: Determine the different tautologies and fallacies as illustrated below. Write your answer on another sheet of paper. You may not copy the puzzle.



ACROSS:

2
$$\begin{array}{l} p \to q \\ \underline{q \qquad\quad} \\ \therefore\ p \end{array}$$

3 $(\sim(p \wedge q) \wedge (\sim p)) \to q$

5
$$\begin{array}{l} p \to q \\ \underline{p \qquad\quad} \\ \therefore\ q \end{array}$$

6 $((p \to q) \wedge (q \to r)) \to (p \to r)$

7 $\big((p \to q) \wedge (\sim q)\big) \to (\sim p)$

8 $(p \wedge q) \to (p \wedge q)$

DOWN:

1
$$\begin{array}{l} p \to q \\ \underline{\sim p \qquad\quad} \\ \therefore \sim q \end{array}$$

4
$$\begin{array}{l} \underline{\ p \qquad\quad} \\ \therefore\ p \vee q \end{array}$$

Activity 2: CONVERT AND IDENTIFY WHAT I AM!

Directions: Convert the following arguments into *standard form* and *identify the type of tautology or fallacy* it illustrates. Use the letters indicated on each argument. Write your answer on a clean sheet of paper.

	Arguments	**Standard Form**	**What type of Tautology/ Fallacy?**
1.	If Nicanor (N) is a famous author, then he knows how to write (W). But Nicanor is not a famous author. Hence, Nicanor does not know how to write.		
2.	If Liway(L) is a famous author, then she knows how to write(W). Moreover, Liway knows how to write. So, she is a famous author.		
3.	If the solution (S) turns blue litmus paper red, then the solution contains acid(A). The solution turns blue litmus paper red. So, the solution contains acid.		
4.	If Zeus is human(H), then Zeus is mortal(M). Zeus is not mortal. Therefore, Zeus is not human.		
5.	If you study hard(H), then you refine your communication skills and build up your confidence(C). If you refine your communication skills, build up your confidence, then your job(J) opportunities increase. Therefore, if you study hard, then your job opportunities increase.		

Activity 3: AM I BAD? TELL ME WHY?

Directions: Given the following arguments, discuss or tell WHY each are considered as bad argument. Write your answer on a clean sheet of paper.

1. If I have curly hair and dark skin, then I am not beautiful. I have curly hair and dark skin. Therefore, I am not beautiful.
2. If I drink energy drink every day, then I will be good in basketball. I drink energy drink every day. Hence, I will be good in basketball.

Activity 4: WHO IS GIVING A VALID ARGUMENT?

Directions: Based on the situation presented below. Determine who is stating a valid argument. Write your answer on a clean sheet of paper.

Two of your classmates, Juan and Pedro, are arguing. Each one claims that his argument is valid and the argument of the other is not. They call your attention to judge and determine who is giving a valid argument. Presented in the table are the arguments of your classmates. Fill out the table below to guide you in deciding. After doing so, write your decision together with your reasons on the space provided below.

	Argument	Argument in Symbols (**Standard Form**)	What Law of Tautologies or Fallacy?	Validity/ Falsity (**Valid** or **Invalid**)
Juan	If my allowance will be increased, then I will have more money to spend. I have more money to spend. Therefore, my allowance was increased.			
Pedro	My parents will increase my allowance or get me a new mobile phone. They did not increase my allowance. Therefore, they got me a new phone.			
Decision:				
Reason:				

Deepen

At this point, you are going to apply what you have learn about the different types of tautologies and fallacies and the process of checking arguments whether it is valid, invalid, sound, or bad arguments. You are expected to **create your own arguments** which illustrates tautology and/or fallacy. You must **add creativity on your work** by placing it on a colored paper or place it in a card or any art materials available at your home.

What you need:

A piece of paper/Bond Paper
Ballpen or any writing material
Any art materials

What you must do:

1. You need to create two (2) original arguments. Choose at least one type of tautologies and one type of fallacies (*e.g. modus tollens, modus ponens, fallacy of an inverse, fallacy of converse and so on.*)

2. From each type, construct an argument which is **OBSERVED NOWADAYS**. It should be timely and relevant. Getting from news are highly recommended.

   Example of possible topics:
   - *Education*
   - *Technology*
   - *Mental Health*
   - *Politics*
   - *COVID-19*
   - *Module*

3. Write the arguments and convert it into its equivalent forms both <u>propositional form</u> and <u>standard form</u>.

4. Validate by explain your arguments using the *Validity Condition* and *Truth Condition* which were discussed in this lesson.

Rubrics for Scoring the Output

Criteria	Poor (1 point)	Fair (3 points)	Good (5 points)
Construction (Grammar, Punctuation marks)	The student writes the argument with more than 5 grammatical and punctuation errors.	The student writes the arguments with at least 3 grammatical and punctuation errors.	The student writes the arguments with no grammatical and punctuation errors.
Conversion (Process of Converting the arguments into different forms)	The student needs help in converting the arguments into different forms.	The student shows a good conversion of the arguments into different forms	The student shows a commendable conversion of the arguments into different forms.
Validation and Explanation	The student explains and validates the arguments with help.	The student explains and validates the arguments with minor error.	The student explains and validates well the arguments.
Timeliness/Relevance	The arguments created are not timely at all.	The arguments created are timely but not relevant at this time.	The arguments created are timely and relevant.
Creativity of the Work	The learners did not make his/her work creative. The work shows no balance and artistry in placing of elements like color.	The learners used art materials available at their home, but the artwork lacks in balance and artistry in placing of elements like color.	The learner used art materials available at their home and the artwork shows balance and artistry in placing of elements like.

Gauge

Directions: Read carefully each item. Write the letter of the best answer for each test item.

_____1. What do you call a valid argument that satisfies the validity condition; that is, the conclusion is true whenever the premises are all true?

A. Contradiction B. Fallacy
C. Tautology D. Contrapositive

_____2. Which of the following standard form of argument is the translation of the given arguments given below?

If Tomas was absent (A), then he missed (M) the review.
Tomas was absent.
Therefore, Tomas missed the review.

A. $A \rightarrow M$
$\underline{A}$
$\therefore M$

B. $A \rightarrow M$
$\underline{M}$
$\therefore M$

C. $A \rightarrow M$
$\underline{\sim M}$
$\therefore \sim M$

D. $M \rightarrow A$
$\underline{A}$
$\therefore \sim M$

_____3. Based on your answer on number 2, what kind of valid argument is the given argument?

A. Modus Ponens
B. Modus Tollens
C. Rule of Contradiction
D. Law of Syllogism

_____4. What is the correct translation of the given propositional form given below?

$$\frac{p}{\therefore p \vee q}$$

Let ***p*** – *Antonio Luna is a scientist.*
q – *Jose Rizal is a scientist.*

A. Antonio Luna is a scientist.
Therefore, Antonio Luna and Jose Rizal are scientist.
B. Antonio is not a scientist.
Therefore, Antonio Luna and Jose Rizal are not a scientist.
C. Jose Rizal is a scientist.
Therefore, Antonio Luna and Jose Rizal are scientist.
D. Antonio Luna is a scientist.
Therefore, Antonio Luna or Jose Rizal is a scientist.

_____5. Which of the following types of fallacies satisfies the standard form of arguments below?

$$\begin{array}{l} \sim(p \wedge q) \\ \underline{\sim p \qquad\quad} \\ \therefore q \end{array}$$

A. Fallacy of the Inverse
B. Affirming the Disjunct
C. Fallacy of the Converse
D. Denying the Conjunct

_____6. Convert the given categorical syllogism below into its propositional form.

If it is sunny, then Ben will go biking.
It is not sunny.
Therefore, Ben did not go biking.

Let ***p***: It is sunny.
q: Ben will go biking.

A. $\big((p \to q) \wedge q\big) \to p$
B. $\big((p \to q) \wedge (\sim p)\big) \to (\sim q)$
C. $\big((p \vee q) \wedge p\big) \to (\sim q)$
D. $(p \to q) \to (q \to p)$

_____7. Determine whether the given argument below is valid and sound.

'If the solution turns blue litmus paper into red, then the solution contains acid. The solution turns blue litmus paper to red. So, the solution contains acid.'

A. Valid and sound argument
B. Invalid but sound argument
C. Valid but not sound argument
D. Invalid and not sound argument

_____8. Which of the following fallacy was illustrated by the given argument below?

'Either Alvin sings or dances with Nina.
Alvin sang with Nina
Therefore, Alvin did not dance with Nina.'

Guide:
p: Alvin sings with Nina.
q: Alvin dances with Nina.

A. Fallacy of Converse
B. Affirming the Disjunct
C. Fallacy of the Inverse
D. Denying the Conjunct

_____9. Which of the following is the correct categorical syllogism or argument of the given propositional form below?

$$((p \vee q) \wedge (\sim p)) \rightarrow q$$

Let ***p***: I will eat an apple.
q: I will drink tea.

A. I will eat an apple and I will drink tea.
I will not eat an apple.
Therefore, I will drink tea.

B. If I will eat an apple, then I will drink tea.
I will eat an apple.
Therefore, I will not drink tea.

C. I will eat an apple, or I will drink tea.
I will not eat an apple.
Therefore, I will drink tea.

D. I will not eat an apple and I will not drink tea.
I will not eat an apple.
Therefore, I will drink tea.

_____10. What do you call an argument that satisfies the validity condition and the truth condition?

A. Bad Argument
B. Contrapositive
C. Sound Argument
D. Tautology

_____11. Which of the following argument is valid and sound?

A. If he is a thief, he is a lawbreaker.
He is a thief.
Hence, he us a lawbreaker.

B. If I am not good in math, then I am not intelligent.
I am not good in math.
Then, I am not intelligent.

C. If I have dark skin, then I am not beautiful.
I have dark skin.
Therefore, I am not beautiful.

D. If I drink Gilas power energy drink every day, then I will be good in basketball.
I drink Gilas power energy every day.
Hence, I will be good in basketball.

_____12. Determine whether the given argument below is valid and sound.

'If overeating causes disease, then it is not healthy. Overeating does not cause disease. So, overeating is healthy.'

A. Valid and sound argument
B. Invalid but sound argument
C. Valid but not sound argument
D. Invalid and not sound argument

Guide:
p: Overeating causes disease.
q: It is not healthy.

_____13. Which of the following standard form of argument is the translation of the given arguments given below?

If Tina is happy (T), then she passed (P) the examination.
Tina was not happy.
Therefore, Tina did not pass the examination.

A. $$\begin{array}{l} T \rightarrow P \\ \underline{T\qquad\quad} \\ \therefore P \end{array}$$

B. $$\begin{array}{l} T \rightarrow P \\ \underline{P\qquad\quad} \\ \therefore P \end{array}$$

C. $$\begin{array}{l} T \rightarrow P \\ \underline{\sim T\qquad} \\ \therefore \sim P \end{array}$$

D. $$\begin{array}{l} T \rightarrow P \\ \underline{T\qquad\quad} \\ \therefore \sim P \end{array}$$

_____14. Which of the following tautology was illustrated by the given argument below?

'If Alvin sings, then Nina dances.
If Nina dances, then Elsa claps.
Therefore, If Alvin sings, then Elsa claps.'

Guide:
p: Alvin sings **r**: Elsa claps
q: Nina dances.

A. Law of Syllogism
B. Modus Tollens
C. Rule of Contradiction
D. Rule of Simplification

_____15. Convert the given categorical syllogism below into its propositional form.

If I pass the final exam, I will graduate.
I did not graduate.
Therefore, I did not pass the final exam.

Let ***p***: I pass the final exam.
q: I will graduate.

A. $((p \rightarrow q) \wedge q) \rightarrow p$
B. $((p \vee q) \wedge p) \rightarrow (\sim q)$
C. $((p \rightarrow q) \wedge (\sim q)) \rightarrow (\sim p)$
D. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

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