8

Mathematics

Quarter 1 - Module 1:

Factoring Polynomials

AIRs - LM

MATH 8
Quarter 1 - Module 1: Factoring Polynomials

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Development Team of the Module

Author: Virginia G. Acas
Editor: SDO La Union, Learning Resource Quality Assurance Team
Content Reviewer: Joel C. Garcia
Language Reviewer: Lorna M. Tampos
Illustrator: Ernesto F. Ramos Jr.
Design and Layout: Jerome P. Medriano

Management Team:

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Schools Division Superintendent
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Assistant Schools Division Superintendent
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Belen C. Aquino, PhD, *EPS in Charge of English*
Michael Jason D. Morales, *PDO II*
Claire P. Toluyen, *Librarian II*

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Department of Education – SDO La Union
Office Address: Flores St. Catbangen, San Fernando City, La Union
Telefax: 072 – 205 – 0046
Email Address: launion@deped.gov.ph

8

Mathematics

Quarter 1- Module 1:

Factoring Polynomials

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

The purpose of this module is to familiarize ourselves with the different techniques of factoring polynomials and they are used in solving real-life problems. We will reverse the operation of multiplication and show how to find the factors of a known product, which we call the process as factoring.

After going through this module, you are expected to attain the following:

Learning Competencies:

1. Factors completely different types of polynomials (polynomials with common monomial factor, difference of two squares, sum and difference of two cubes, perfect square trinomials, and general trinomials(M8AL-Ia-b-l);
2. Solves problems involving factors of polynomials (M8ALIb-2)

Learning Objectives:

1. Identifies the Greatest Common Monomial of two or more polynomials.
2. Determines if a trinomial is a perfect square or not;
3. Determines if a trinomial is factorable or not;
4. Factors general trinomials by AC Method or by Grouping Method
5. solves problems involving factors of polynomials

Before you start studying this module, check how much you already know about the topic by answering the Pre-Test.

Pre-Test:

DIRECTION. Read the following statements correctly. Write the letter that corresponds to the correct answer on the space provided for.

____1. Which of the following is the Greatest Common Factor of 21abc and $14a^2bc^2$?

A. 7abc B. 14abc C. 21abc D. 21 a^2bc^2

____2. What are the factors of $4x^3y-2x^2y^2+2xy^3$?

A. $xy(2x^2-xy+y^2)$ B. $2xy(2x^2+xy-y^2)$ C. $2xy(2x^2-xy+y^2)$ D. $2xy(2xy+xy-y)$

____3. Find the factors of $4x^2-9$.

A. (2x+3)(2x+3) B. (2x-3)(2x-3) C. (2x-3)(2x+3) D. 2x(2x-3)

____4. If (3x+4) is one of the factors of $9x^2-16$, what is the other factor?

A. 3x+4 B. 3x-4 C. 4x+3 D. 4x-3

____5. Factor y^3-125 completely.

A. $(y^2 -25)(y+5)$ B. $(y-5)(y^2 -25)$

C. $(y-5)(y^2 -5y+25)$ D. $(y-5)(y^2 +5y+25)$

____6. Which of the following are the factors of $125m^3+343n^3$?

A. $(5m+7n)(5m^2+35n+49n^2)$ B. $(5m+7n)(5m^2-35n+49n^2)$

C. $(5m-7n)(5m^2+35n+49n^2)$ D. $(5m-7n)(5m^2-35n+49n^2)$

____7. Which is of the following polynomials is NOT a perfect square trinomial?

A. $9x^2-24x+64$ B. $9x^2+48x+64$ C. $9x^2-48x+64$ D. $4x^2-12x+9$

____8. Which of the following are the factors of the perfect square trinomial $b^2+12b+36$?

A. $(b+6)^2$ B. $(b-6)^2$ C. (b-6)(b+6) D. $(b\pm6)^2$

____9. What factoring technique is applied to $x^2-4x+3= (x-3)(x-1)$?

A. Factoring Difference of Two Squares

B. Factoring Perfect Square Trinomials

C. Factoring Sum or Difference of Two Cubes

D. Factoring General Trinomials of the Form x^2+Bx+C

____10. Which of the following are the factors of $m^4-3nm^2-18n^2$?

A. $(m^2-3n)(m^2-6n)$ B. $(m^2-3n)(m^2+6n)$

C. $(m^2+3n)(m^2-6n)$ D. $(m^2+3n)(m^2+6n)$

____11. What is the missing term in the statement $2x^2$-_____+3= (2x-1)(x-3)?

A. -x B. -5x C. -6x D. -7x

_____12. Find the factors of $6x^2-13xy+6y^2$?

A. (3x-2y)(2x-3y) B. (3x+2y)(2x-3y)

C. (3x-2y)(2x+3y) D. (3x+2y)(2x+3y)

_____13. A rectangular garden has a length of (3x-1)m. If the area is ($3x^2+2x-1$) sq. m., what is the width of the garden?

A. (x+1)m B. (x-1)m C. (3x+1)m D. (3x-1)m

_____14. The area of the square is five times its perimeter. What is the measure of the side of the square?

A. 4 units B. 5 units C. 10 units D. 20 units

_____15. The volume of an open box is represented by $3x^3+3x^2-36x$ cubic centimeters. What are the dimensions of the box if the factors of the volume are the dimensions themselves?

A. 3x (x+4)(x+3) B. 3x (x+4)(x-3) C. 3x (x-4)(x+3) D. 3x (x-4)(x-3)

Lesson 1 Factoring Polynomials

Jumpstart

Concepts such as special products, factors, prime factorization have been discussed and have been used in many instances in your previous Math classes. Let us reactivate what you previously learned by answering the activity below.

Activity 1: AM I SPECIAL OR NOT?

Directions: Determine which of the following are special products. Indicate whether the product is a square of a binomial, sum and difference of two terms, cube of a binomial, square of a trinomial or neither of them.

1. $(5x-2)(2x-5)$ ______________________
2. $2(3+x)$ ______________________
3. $(p-q)(p+q)$ ______________________
4. $(2xy-9)(2xy+9)$ ______________________
5. $(2a+b)(2a+b)$ ______________________
6. $(8x+ab)(8x+ab)$ ______________________
7. $(5x^2)(5xy-x^2+6y)$ ______________________
8. $(2x-y)(4x^2+2xy+y^2)$ ______________________
9. $(x-3)(x+5)$ ______________________
10. $8a^3+24a^2b+2ab^2-8b^3$ ______________________

Activity 2: FACTOR ME!

Directions: Identify the prime factors of the following polynomials. Number 1 is done for you.

Polynomial	Prime Factors
1. $18y$	$2\cdot3\cdot3\cdot y$
2. $25x$	
3. $100y^2$	
4. $64a^3b^2$	
5. $343xy^3$	

Activity 3: WHICH IS COMMON?

Directions: Encircle the Greatest Common Factor (GCF) of each set of polynomials from the expressions at the right. Number 1 is done for you.

Polynomials	Greatest Common Factor (GCF)
1. 8,16,24	2 4 8 16
2. 15a, 12b	a b 3 5 ab
3. x^2, $4x^3$, $8x^4$	x x^2 $4x$ $4x^2$
4. $7y$, $21y^2$, 49	7 $7y$ $7y^2$
5. a^2b^2, $6ab$, $5a^2b$	ab a^2b a^2b^2

Discover

The above activities will help you understand further the lessons in this module, that is, how to factor polynomials completely.

Factoring is the reverse process of multiplication. To factor polynomials means to express the polynomial as a product of other polynomials, which are usually with integral exponents.[i]

We say that a polynomial is ***completely factored*** if all its factors are *prime* or if it is expressed as the product of two or more *irreducible polynomials*. A *polynomial is said to be prime* if its only factors are 1 and itself. At the same time, a *polynomial is irreducible* if it cannot be expressed as a product of two polynomials of lower degree and if the coefficients have no common factor other that 1.[ii]

There are different techniques of factoring, depending on the given polynomial, which will be discussed in this module.

A. FACTORING POLYNOMIALS WITH COMMON MONOMIAL FACTOR

One way of factoring a polynomial is determine the Greatest Common Monomial Factor or the Greatest Common Factor (GCF).

Consider the following illustrative examples:

Example 1. Factor $4x^2-10$ completely.

Solution: To factor the polynomial, $4x^2-10$

FIRST STEP: Determine the prime factors of each term in the polynomial.

Prime factors of $4x^2$ are: (2)(2)(x)

Prime factors of -10 are: (2) (-5)

SECOND STEP: Determine the common factors in the prime factors of the terms.

$4x^2$	=	$\boxed{2}$	2	x	x
-10	=	$\boxed{2}$	-5		

2 is the only common factor. Therefore, the GCF is 2.

THIRD STEP: Write the GCF as the first factor, and the remaining factors as the other factor of the polynomial.

That is, 2(2x-5).

Therefore, if you factor $4x^2-10$ completely, it would be $4x^2-10 = 2(2x-5)$.

Example 2. Factor the polynomial completely.

$25x^4y-15x^3y^2+10x^2y^3-20xy^4$

Solution: Factor each term of the polynomials.

$25x^4y$	=	$\boxed{5}$	5	x	$\boxed{x}$	x	x	$\boxed{y}$	
$-15x^3y^2$	=	$\boxed{5}$	-3	x	$\boxed{x}$	x	y	$\boxed{y}$	
$10x^2y^3$	=	$\boxed{5}$	2	x	$\boxed{x}$	y	y	$\boxed{y}$	
$-20xy^4$	=	$\boxed{5}$	2	-2	$\boxed{x}$	y	y	$\boxed{y}$	y

Since all terms have the common factor of 5, x and y, the GCF is (5) (x) (y) or 5xy. We can use the distributive property to factor the given polynomial.

Therefore, $25x^4y-15x^3y^2+10x^2y^3-20xy^4= 5xy\ (5x^3-3x^2y+2xy^2-4y^3)$

Example 3. Factor the polynomial $-6m^3n^2+ 12mn^2-9mn$ completely.

Solution: Find the GCF of all the terms in the polynomial.

Since the leading coefficient -6 is negative, we factor out with a negative coefficient.

$-6m^3n^2$	=	-3	2	m	m	m	n	n
$12mn^2$	=	-3	-2	2	m	n	n	
$9mn$	=	-3	-3	m	m			

-3 and m are common in all the factors of the terms in the polynomial. Therefore the GCF is (-3)(m) or -3m.

Therefore, the complete factorization of $-6m^3n^2+12mn^2-9mn$ is:

$-3m(2m^2n^2-4mn^2-3m^2)$

Express each term as the product of the GCF and the other factors of the term.

B. FACTORING THE DIFFERENCE OF TWO SQUARES

The difference of the squares of two terms is equal to the product of the sum and difference of two terms.

In symbols, $\mathbf{x^2-y^2 = (x+y)\ (x-y)}$

Remember that **the difference of two squares are two perfect squares separated by a minus sign.**

If the binomial you wan to factor matches the pattern x^2-y^2, then the factors are x+y and x-y.

Here are some examples: Original Polynomial Written as a Difference of Two Squares	Factored Form
1. $x^2-25 = x^2 - 5^2$ $\uparrow \quad \uparrow$ $x^2 - y^2$	$(x+5)\ (x-5)$ $\uparrow \quad \uparrow$ $(x+y)\ (x-y)$
2. $4a^2-100 = (2a)^2 - 10^2$ $\uparrow \quad \uparrow$ $x^2 - y^2$	$(2a+10)\ (2a-10)$ $\uparrow \quad \uparrow$ $(x+y)\ (x-y)$
3. $121a^2- 25b^4 = (11a)^2 - (5b^2)^2$ $\uparrow \quad \uparrow$ $x^2 - y^2$	$(11a+5b^2)\ (11a-5b^2)$ $\uparrow \quad \uparrow$ $(x+y)\ (x-y)$

Note: For variable to be a perfect square, it must be raised to an even power. The perfect squares less than 300 are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, and 289.[iii]

If the terms of a binomials have a common factor, first factor out the common factor, then continue factoring.

Examples:

4. $18x^3 - 8x = 2x(9x^2 - 4)$	Factor out 2x, the GCF of $18x^3$ and -8x.
$= 2x(3x)^2 - 2^2$	Express the second factor as the difference of two squares.
$= 2x(3x+2)(3x-2)$	The factors are the sum and difference of the squared terms.
5. $100x^5 - 36x^3 = x^3(100x^2-36)$	Factor out x^3, the GCF of $100x^5$ and $-36x^3$.
$= x^3(10x)^2-(6)^2$	Express the second factor as the difference of two squares.
$= x^3(10x+6)(10x-6)$	The factors are the sum and difference of the squared terms.

C. FACTORING SUM AND DIFFERENCE OF TWO CUBES

Sum of Two Cubes

Let x and y be real numbers, variables or algebraic expressions.

Factoring Sum of Two Cubes: $\mathbf{x^3 + y^3 = (x+y)(x^2-xy+y^2)}$

That is, the factors of the sum of two cubes are two polynomials:

a) the first factor is a binomial (the **sum** of the cube root of the first and last term); and

b) the second factor is a trinomial (the square of the first term in the binomial, **minus** the product of the first and second terms of the binomial, **plus** the square of the last term of the binomial).

Example 1. Factor a^3+64 completely.

Solution: 64 can be written as $(4)^3$, thus, $a^3+64 = a^3+4^3$

First Factor = cube root of first term + cube root of the second term

$$= \sqrt[3]{a^3} \quad + \quad \sqrt[3]{4^3}$$

$$\uparrow \qquad\qquad \uparrow$$

$$\mathbf{= \quad a \quad + \quad 4}$$

Second Factor =(Square of the first term) – (product of the first and second terms) + (square of the last term)

$$= (a)^2 - (a)(4) \quad + \quad (b)^2$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$\mathbf{= a^2 \quad -4a \quad + \quad b^2}$$

Therefore, $a^3+64 = (a+4)\ (a^2-4a+b^2)$

Example 2. Factor $8b^3+343c^3$ completely.

Solution: $8b^3+343c^3 = (2b)^3 + (7c)^3$

$$\sqrt[3]{(2b)^3} + \sqrt[3]{(7c)^3}$$

$$\uparrow \qquad \uparrow$$

First Factor $= 2b + 7c$

$$[(2b)^2 - \underbrace{(2b)\,(7c)} + (7c)^2$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

Second Factor = $\mathbf{4b^2 \quad -14bc \quad +49c^2}$

Therefore the factors of $\mathbf{8b^3+343c^3}$ are $\mathbf{(2b+7c)\ (4b^2-14bc+49c^2)}$.

Difference of Two Cubes
Let x and y be real numbers, variables, or algebraic expressions.
Factoring Difference of Two Cubes: $\mathbf{x^3 - y^3 = (x+y)\ (x^2-xy+y^2)}$

Just like the factors of the sum of two cubes, the factors of the difference of two cubes are also two polynomials:

a) the first factor is a binomial (the **difference** of the cube root of the first and last term); and

b) the second factor is a trinomial (the square of the first term in the binomial, **plus** the product of the first and second terms of the binomial, **plus** the square of the last term of the binomial).

Example 3. Factor a^3-64 completely.

Solution: 64 can be written as $(4)^3$, thus, $a^3-64= a^3-4^3$

First Factor = cube root of first term - cube root of the second term

$$= \sqrt[3]{a^3} \qquad - \qquad \sqrt[3]{4^3}$$

$$\uparrow \qquad\qquad \uparrow$$

$$\mathbf{= \quad a \quad - \quad 4}$$

Second Factor =(Square of the first term) – (product of the first and second terms) + (square of the last term)

$$= (a)^2 + (a)(4) + (b)^2$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$\mathbf{= a^2 \quad +4a \quad + \quad b^2}$$

Therefore, $a^3-64 = (a-4)\ (a^2+4a+b^2)$

Example 4. Factor $8b^3-343c^3$ completely.

Solution: $8b^3-343c^3 = (2b)^3 \;-\; (7c)^3$

$$\sqrt[3]{(2b)^3} \;-\; \sqrt[3]{(7c)^3}$$

$$\uparrow \qquad \uparrow$$

First Factor $= 2b \quad - \quad 7c$

$$[(2b)^2 + \underbrace{(2b)\,(7c)} + (7c)^2$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

Second Factor = $\mathbf{4b^2 \quad +14bc \quad +49c^2}$

Therefore the factors of $\mathbf{8b^3-343c^3}$ are $\mathbf{(2b-7c)\,(4b^2+14bc+49c^2)}$.

D. Factoring Perfect Square Trinomials

A perfect square trinomial is the product if a binomial is squared.

$(a+b)^2= (a+b)\,(a+b) = a^2+2ab+b^2$

$(a-b)^2= (a-b)\,(a-b) = a^2-2ab+b^2$

$a^2+2ab-b^2$ and $a^2-2ab+b^2$ are examples of perfect square trinomials.

How can you determine if the trinomial is a perfect square trinomial or not? Here is how:

1. The first and the last terms must be both perfect squares; and,
2. The middle term must be equal to twice the product of the square roots of the first and last terms, regardless of the sign.

Let us look at these illustrative examples.

a. Is $x^2+10x+25$ a perfect square trinomial?

Solution: To check if the trinomial is a perfect square trinomial, it must satisfy the given conditions.

The first and last terms are x^2 and 25 respectively. Both are perfect squares. Condition number 1 is satisfied.

The square root of x^2 is x and the square root of 25 is 5. The product of x and 5 is 5x, and twice the product is 10x. Condition number 2 is also satisfied.

Therefore, $x^2+10x+25$ is a perfect square trinomial.

b. Is $36m^2 – 54m +81$ a perfect square trinomial?

Solution:

Condition 1: $36m^2$ and 81 are both perfect squares.

Condition 2: $\sqrt{36m^2} = 6m$

$\sqrt{81} \;= 9$

$(6m)\,(9)(2) = 108m$

The middle term is -54m. It does not satisfy Condition number 2. Therefore, $36m^2 – 54m +81$ is not a perfect square trinomial.

How do we factor perfect square trinomials then?

To factor perfect square trinomials, follow the following steps:

1. Extract the square root of the first term.
2. Extract the square root of the second term.
3. Write these two terms as binomial raised to the second power, following the sign of the middle term in the binomial.

Illustrative examples:

1. Factor $4x^2+16xy+16y^2$ completely.

Step 1. Extract the square root of $4x^2$.

$\sqrt{4x^2} = 2x$

Step 2. Extract the square root of $16y^2$.

$\sqrt{16y^2} = 4y$

Step 3. Write the square roots as binomial raised to the second power, following the sign of 16xy.

Therefore, the factors of $4x^2+16xy+16y^2$ are $(2x+4y)(2x+4y)$ or $(2x+4y)^2$.

2. Factor $81a^2-18a+1$ completely.

Step 1. Extract the square root of $81a^2$.

$\sqrt{81a^2} = 9a$

Step 2. Extract the square root of 1.

$\sqrt{1} = 1$

Step 3. Write the square roots as binomial raised to the second power, following the sign of 16xy.

Therefore, the factors of $81a^2-18a+1$ are $(9a+1)(9a+1)$ or $(9a+1)^2$.

3. Factor $4x^2+16xy+16y^2$ completely.

Step 1. Extract the square root of $4x^2$.

$\sqrt{4x^2} = 2x$

Step 2. Extract the square root of $16y^2$.

$\sqrt{16y^2} = 4y$

Step 3. Write the square roots as binomial raised to the second power, following the sign of 16xy.

Therefore, the factors of $4x^2+16xy+16y^2$ are $(2x+4y)(2x+4y)$ or $(2x+4y)^2$.

E. FACTORING the GENERAL TRINOMIAL, ax^2+bx+c

A general trinomial, also called a quadratic trinomial, is a second-degree polynomial containing three terms. It is in the form, $\mathbf{ax^2 + bx + c}$, where a, b and c are nonzero integral exponents.[iv]

If a trinomial of this form where $a \neq 0$, and a, b, and c are constant, and the value of b^2-4ac is a perfect square, then the factors are two binomials. For example, x^2+5x+6 is factorable since $b^2-4ac = 5^2 - 4(1)(6) = 1$, and 1 is a perfect square.[v]

Examples of general trinomials are:

$x^2+13x+30$	$n^2-11n+10$	$3y^2+y-4$
b^2+6b+5	$s^2-9s+20$	$x^2-13x-140$

In factoring a general trinomial, we take the two cases.

CASE 1. Factoring the general trinomial, $ax^2 + bx + c$, where $a=1$. If the constant *c* (last term) is not a perfect square, the trinomial cannot be factored into a square of a binomial. It may, however, be possible to factor it into a product of two different binomials.[vi]

Consider the following examples in factoring a trinomial, ax^2+bx+c, where $a=1$:

Example 1: Factor $x^2+11x+18$ completely.

STEPS:

1. Write two pairs of parentheses.

   (   ) (   )

2. The x^2 in the trinomial is the product of the first terms of the binomials.

   (x   ) (x   )

3. Since the coefficient of x is 11, which is positive, list the pairs of positive factors of 18.

18	
1	18
2	9
3	6

4. Find the factors that have the sum of 11.

18		Sum
1	18	19
2	**9**	**11** ←**The sum is 11.**
3	6	9

5. Complete the parentheses.

   $(x + 2)(x + 9)$ or $(x + 9)(x + 2)$

Therefore, $x^2+11x+18 = (x + 2)(x + 9)$ or $x^2+11x+18 = (x + 9)(x + 2)$.[vii]

Example 2: Factor $x^2-10x+16$ completely.

STEPS:

1. Write two pairs of parentheses.

   (   ) (   )

2. The x^2 in the trinomial is the product of the first terms of the binomials.

   (x   ) (x   )

3. Since the last term is positive and coefficient of x is -10, which is negative, list the pairs of negative factors of 16.

16	
-1	-16
-2	-8
-4	-4

4. Find the factors that have the sum of -10

16		Sum
-1	-16	-17
-2	**-8**	**-10** ←**The sum is -10.**
-4	-4	-8

5. Complete the parentheses.

(x - 2) (x - 8) or (x - 8) (x - 2)

Therefore, x^2-10x+16 = (x - 2) (x -8) or x^2-10x+16 = (x -8)(x -2).

Example 3: Factor x^2-8x -9 completely.

STEPS:

1. Write two pairs of parentheses.

( ) ( )

2. The x^2 in the trinomial is the product of the first terms of the binomials. Since the last term is -9, which is negative, the second terms of the binomial factors are positive and negative.

(x +) (x -)

3. Since the coefficient of x is -8, which is negative, list the pairs of factors of -9 whose sum is -8.

-9	
-1	9
-3	3
1	-9
3	-3

4. Find the factors that have the sum of -9.

-9		Sum	
-1	9	8	
-3	3	0	
1	**-9**	**-8**	←**The sum is -8.**
3	-3	0	

5. Complete the parentheses.

(x + 1) (x - 9) or (x - 9) (x + 1)

Therefore, x^2-8x-9 = (x + 1) (x -9) or x^2-8x-9 = (x - 9)(x + 1).

Example 4: Factor x^2+7x - 30 completely.

STEPS:

1. Write two pairs of parentheses. ( ) ( )
2. The x^2 in the trinomial is the product of the first terms of the binomials. Since the last term is -30, which is negative, the second terms of the binomial factors are positive and negative.

(x +) (x -)

3. Since the coefficient of x is 7, which is positive, list the pairs of factors of -30 whose sum is 7.

-30	
-1	30
-2	15
-3	10
-5	6
-6	5
-10	3
-15	2
-30	1

4. Find the factors that have the sum of 7.

-30		Sum
-1	30	29
-2	15	13
-3	**10**	**7**
-5	6	1
-6	5	-1
-10	3	-7
-15	2	-13
-30	1	-29

5. Complete the parentheses.

(x + 10) (x - 3) or (x - 3) (x + 10)

Therefore, $x^2+7x-30$ = (x + 10)(x -3) or $x^2+7x-30$ = (x - 3)(x + 10).

CASE 2. Factoring the general trinomial, $ax^2 + bx + c$, where $a \neq 1$.

The process of factoring these trinomials is similar to Case 1, but it involves the Try, Check and Revise method. [viii]

Let us proceed to the following examples:

Example 5: Factor $2x^2+3x-2$ completely.

STEPS:

1. Find two factors such that the product of the first two terms is $2x^2$. The trinomial has a negative constant term, so one factor will be negative and the other will be positive, The possibilities are:

$2x^2+3x-2 = (2x-\_\_\_)(x+\_\_\_)$
$2x^2+3x-2 = (2x+\_\_\_)(x-\_\_\_)$
$2x^2+3x-2 = (x+\_\_\_)(2x-\_\_\_)$
$2x^2+3x-2 = (x-\_\_\_)(2x+\_\_\_)$

2. The product of the last two terms in each factor must be -2.

-2		Product
-1	2	-2
-2	1	-2

3. Write the factors of -2 with the factors of $2x^2$. Determine the middle term of each product using the FOIL Method.

Factors of 4	Possible Factors of $2x^2+3x-2$	Sum of the Outer terms and the Inner terms
-1, 2	(2x-1)(x+2)	(4x) + (-x) = 3x
-2, 1	(2x+1)(x-2)	(-4x) + (x) = -3x
2, -1	(2x+2)(x-1)	(-2x) + (2x) = 0
-2, 1	(2x-2)(x+1)	(2x) + (-2x) = 0

The required middle term is 3x.

Therefore, $2x^2+3x-2$ = (2x -1)(x +2) or $2x^2+3x-2$ = (x +2)(2x -1).

Example 6: Factor $8x^2+17x+2$ completely.

STEPS:

1. Find two factors such that the product of the first two terms is $8x^2$. The trinomial has a positive constant term and the middle term is positive, so the factors will be both positive. The possible factors are:

$$8x^2+17x+2 = (8x+\_\_\_)(x+\_\_\_)$$
$$8x^2+17x+2 = (4x+\_\_\_)(2x+\_\_\_)$$
$$8x^2+17x+2 = (2x+\_\_\_)(4x+\_\_\_)$$
$$8x^2+17x+2 = (x+\_\_\_)(8x+\_\_\_)$$

2. The product of the last two terms in each factor must be 2. Since the middle term is positive, both factors are positive.

2		Product
1	2	2
2	1	2

3. Write the factors of 2 with the factors of $8x^2$. Determine the middle term of each product using the FOIL Method.

Factors of	Possible Factors of $2x^2+3x-2$	Sum of the Outer terms and the Inner terms
1, 2	$(8x+1)(x+2)$	$(16x)+(x) = 17x$
2, 1	$(8x+2)(x+1)$	$(8x)+(2x) = 10x$
1,2	$(4x+1)(2x+2)$	$(8x)+(2x) = 10x$
2, 1	$(4x+2)(2x+1)$	$(4x)+(4x) = 8x$

The required middle term is 17x.

Therefore, $8x^2+17x+2 = (8x+1)(x+2)$ or $8x^2+17x+2 = (x+2)(8x+1)$

POINTS TO REMEMBER IN FACTORING the TRINOMIAL, ax^2+bx+c, where $a \neq 1$:

1. If the terms of a trinomial do not have common factor, then the terms of a binomial factor cannot have a common factor.
2. If the constant term of a trinomial is:
   a. positive, the constant terms of the binomials have the same signs, as the coefficient of x in the trinomial.
   b. negative, the constant terms of the binomials have opposite signs.

***Source*:** Orlando A. Oronce, Marilyn O. Mendoza, *Exploring Math Textbook,* (Sampaloc:Rex Book store, Inc., 2018) p.46

Another way of factoring the trinomial, **ax^2+bx+c, where $a \neq 1$** is by using the AC Method or by grouping method.[ix]

Example 7. Factor $6x^2-7x-20$ by AC Method or by grouping method.
Solution:

Step 1. Find ac.

In the polynomial, $6x^2-7x-20$, $a=6$, $b=-7$ and $c=-20$.

$ac = (6)(-20) = -120$

Step 2. Find the factors of ac whose sum is b.

-120		Sum
-1	120	119
-2	60	58
-3	40	37
-4	30	26
-5	24	19
-6	20	14
-8	15	8
-10	12	2
-12	10	-2
-15	**8**	**-7**
-20	6	-14
-24	5	-19
-30	4	-26
-40	3	-37
-60	2	-58
-120	1	-119

The required factors are $-15x$ and $8x$.

Step 3. Express the middle term of $6x^2-7x-20$ as the sum of the factors in Step 2.

$$\begin{aligned} 6x^2-7x-20 &= 6x^2+(-15x+8x)-20 \\ &= 6x^2-15x+8x-20 \end{aligned}$$

Step 4. Factor the resulting polynomial by grouping.

$6x^2-7x-20 =$	$6x^2-15x+8x-20$	
$=$	$(6x^2-15x)+(8x-20)$	Group the terms.
$=$	$3x(2x-5) + 4(2x-5)$	Factor each group using the GCF
$=$	$(2x-5)(3x+4)$	Factor out the common binomial $(2x-5)$

Thus, $6x^2-7x-20=(2x-5)(3x+4)$.

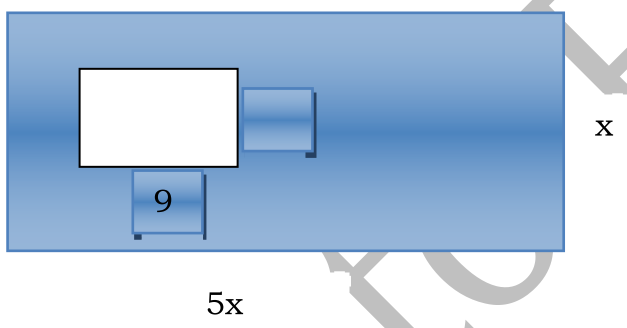
Lesson 2 Solving Problems Involving Factors of Polynomials

Some problems encountered in Algebra need to be expressed in terms of polynomials before they can be solved. Use of appropriate formulas and accurate manipulation of expressions in terms of special products and factoring techniques are essential for finding correct solutions.[x]

Consider the following examples of problems involving factors of polynomials.

Illustrative Examples:

1. Given the diagram below, find the area of the shaded region of the larger rectangle.



Solution:

Find the area of the larger rectangle:

Area = length (l) x width (w)
= 5x (x) square units
= $5x^2$ square units

Find the area of the smaller rectangle:

Area = lw
= 9(5) square units
= 45 square units

To find the area of the shaded region:

Subtract the area of the smaller rectangle from the area of the larger rectangle.

That is, $5x^2 - 45$.

Factor $5x^2 - 45$

$5x^2 - 45 = 5(x^2 - 9)$	Factor out the GCF of $5x^2$ and 45.
$5(x^2 - 9) = 5(x+3)(x-3)$	Factor completely by difference of two squares.

Therefore, the area of the shaded region is 5(x+3)(x-3) square units.

2. If the area of a square is $4x^2-4x+1$ square units, what is the measure of
its side?
Solution:
We know that the formula for the area of a square is the square of one side, that is, $A= s^2$.
It is given in the problem that the area of the square is $4x^2-4x+1$ sq. units. Therefore, we can find the measure of its side by factoring:
$A= s^2 \rightarrow s^2 = 4x^2-4x+1$
The factors of $4x^2-4x+1$ is $(2x-1)(2x-1)$ or $(2x-1)^2$.

We extract both sides of $s^2 = (2x-1)^2$,

$$\sqrt{s2} = \sqrt{(2x-1)2}$$
$$s = 2x-1$$

Therefore, the measure of the side of the square is (2x-1) units.

3. The product of two consecutive positive numbers is 110. What are the numbers?
Solution:
Let x - be the first number
$x + 1$ - be the second number
Equation:
$x(x+1) = 110$

$x(x+1) = 110$	
$x^2+x = 110$	By distributive property
$x^2+x -110 = 110 -110$	Place all terms in the left side of the equation.
$x^2+x -110 = 0$	By Subtraction Property of Equality
$(x-10)(x+11) = 0$	Factor the trinomial, $x^2+x -110$.
$(x-10) = 0$; $(x+11) = 0$	By Zero Property of Equality
$x=10$; $x = -11$	

The problem asked for consecutive positive numbers, so reject -11.

Therefore, the two positive numbers are 10 and 11.

4. A rectangle has sides of (x+6) units and (x+2) units. What value(s) of x gives and area of 17 sq. units. (Remember that there is no such thing as negative length.)
Solution:
We know that the area of a rectangle is the product of the length and the width, that is, A=lw.
Given: length = (x+6) units
width = (x+2) units
area = 21 sq. units
Equation: $(x+6)(x+2)= 21$
$x^2+8x +12= 21$ By FOIL Method
$x^2+8x +12 -21= 21-21$

$x^2+8x-9=0$ By Subtraction Property of Equality
$(x+9)(x-1)=0$ Factoring General Trinomial
$x+9=0$; $x-1=0$
$x=-9$; $x=1$ By Zero Property of Equality

There is no such thing as negative length, so **x=1** to have an area of 21 sq. units.

Explore

Activity 1. PERFECT OR NOT!

DIRECTIONS. Determine whether the trinomial is a perfect square or not. If it is, write PST, otherwise write NPST.

1. $x^2-12x+36$
2. $x^2-9x-10$
3. $x^2-13x+22$
4. $x^2-6x-27$
5. $81t^2-90t+25$
6. $k^2+9k+14$
7. $x^2-18xy+81y^2$
8. $6m^2+33m+15$
9. $4x^2+28x+49$
10. $x^2-x+\frac{1}{4}$

Activity 2. I AM FACTORABLE!

DIRECTIONS. Determine whether the general trinomial is factorable or not. Write **F** if factorable and **NF** if it is not.

1. $x^2-12x+25$
2. x^2-x-56
3. $x^2-3x+10$
4. $x^2-6x-27$
5. $2t^2+3t-2$
6. $k^2+9k+14$
7. x^2-9y+8
8. $6m^2+33m+15$
9. $4x^2+28x+49$
10. x^2-x+-6

Activity 3. COMPLETE ME!

Complete the table with the correct polynomial. Number 1 is done for you.

Polynomial	Greatest Common Monomial Factor (GCF)	Quotient of Polynomial and GCF	Factored Form
$12a-8$	4	$3a-2$	$4(3a-2)$
x^2y-xy^2	(1)	$x-y$	(2)
$12abc+8b^2c+20bc$	$4bc$	(3)	(4)
(5)	$3m^2$	(6)	$3m^2(2m-3)$

Activity 4. PAIR MO KO NYAN!

DIRECTIONS. Form 10 problems on difference of two squares by pairing two squared quantities, then find their factors. (Hint: You can create expressions that may require the use of the GCF.)

	x^2y^3	$16s^2$	25	$81m^4$	k^2 $4m^6$
$24p^2$	$100n^8$	w^6d^{18}	k^6u^{12}	$36h^{10}$	9
	$20a^4$	$25a^2b^3$	144	$121c^4$	
$49x^2y^8$	1	$(x+3)$	$\frac{16}{64}$	$36z^4x^{16}$	$121h^{18}$
	169	$225v^{22}$	$(x-7)^2$	$30o^4p^6$	$196d^{18}$

Source: Emmanuel P. Abuzo, et. al., *Mathematics Leaner's Module* (Department of Education, 2013) p. 34

Deepen

Activity 5. DECODE THE MESSAGE

DIRECTION. Factor the following polynomials correctly. Then decode the hidden message by shading the cells containing your answer. The cells not shaded will reveal the hidden message. (Show your solution in separate sheet of paper.)

1. $25x^2-5x$
2. $121x^2-44x+11$
3. $3x^2-12$
4. $169x^2-121y^2$
5. $24x^3+3$
6. $27-64x^3$
7. $x^2-3x-28$
8. $25x^2-10x+1$
9. $x^2+8x+16$
10. $x^2-11x+10$
11. $3x^2+5x+2$
12. $2x^2+4xy+x+2y$

$(x-10)(x-1)$	$5x(5x+1)$	$(11x+13y)(11x-13y)$	$(3x+2)(x+1)$
Life on earth	Blessed are	those who	is a test.
$3(x^2-4)$	$(13x+11y)(13x-11y)$	$(x+2)(3x+1)$	$3(2x+1)(4x^2-2x+1)$
endure	The good news	when they	is that
$(2x+1)(x+2y)$	$(2x+y)(x+1)$	$5x(5x-1)$	$(x+7)(x+4)$
God wants you	are tested	to pass	they will
$3(x+2)(x-2)$	$(5x-1)(5x+1)$	$3(2x-1)(4x^2+2x+1)$	$(5x-1)(5x-1)$
The tests of life	receive	the Crown of life	Life on earth
$(x-4)(x-4)$	$(3-4x)(9+12x+16x^{2)}$	$(5x+1)(5x+1)$	$11(11x^2-4x+1)$
is a trust.	We never really	that God has	own anything
$(x+4)(x+4)$	$(x+2y)(x+2y$	$(13x+11y)(13x+11y)$	$(x-7)(x+4)$
during our brief	Promised to those	who love Him	stay on earth.

Hidden Message:____
____.

James 1:12

RUBRICS:

CRITERIA	5	4	3	2	1
Accuracy of Answers	Has shaded all correct answers without erasures.	Shaded all correct answers but with few erasures and shaded few incorrect answers.	Shaded some correct answers and some incorrect answers.	Shaded fewer correct answers than incorrect answers.	Shaded no correct answer.
Hidden Message	Writes the correct hidden message.	Writes the correct hidden message with few unnecessary words/expressions.	Writes the correct hidden message with some unnecessary words/expressions.	Writes the correct hidden message with more that 50% words/expressions inserted.	No hidden message written.
Solution	With complete and 100% correct solution.	With solution but with few incorrect answers.	With solution but with some incorrect answers.	With solution but more than 50% incorrect answers.	No solution at all.

Activity 6. SOLVE ME!

DIRECTION: Read the following problems carefully, then solve them completely. Use the variable x to represent the unknown in the problem.

1. The difference between the squares of two consecutive positive integers is 29. Find the integers.

Representation:

Equation:

Solution:

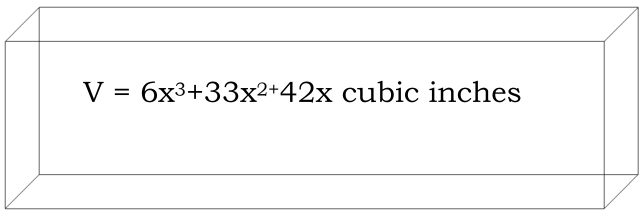
2. A rectangle has sides of $(5x+2)$ units and $(x+2)$ units. What value(s) of x gives and area of 7 sq. units.

Representation:

Equation:

Solution:

3. The volume of an open box is represented by $6x^3+33x^{2+}42x$ cubic inches. What are the dimensions of the box if the factors of the volume are the dimensions themselves?



Equation:

Solution:

4. The area of a square is $16x^2+72x+81$ square units. What is the measure of its one side?

Area = $16x^2+72x+81$ square units

Equation:

Solution:

Gauge

DIRECTION. Read the following statements correctly. Write the letter that corresponds to the correct answer on the space provided for.

____1. What are the factors of $6a^4+ 3a^3b$?

A. $3a(2a^3+a^2b)$ B. $3a^2(2a^2+ab)$ C. $3a^3(2a+b)$ D. $3ab(2a^4+a^2)$

____2. What are the factors of $12c^2d^2-24cd^2+8cd^3$?

A. $cd(12cd-24d+8d^2)$ B. $4cd^2(3c-6+2d)$ C. $4cd^2(3c+6-2d)$ D. $-4cd^2(3c-6+2d)$

____3. Find the factors of $125-20x^2$.

A. $(5+10x)(25+2x)$ B. $(25-10x)(5-2x)$ C. $5(5-2x)(5+2x)$ D. $5(25-4x^2)$

____4. If $(9x+2)$ is one of the factors of $81x^2-4$, what is the other factor?

A. $9x+2$ B. $9x-2$ C. $2x+9$ D. $2x-9$

____5. Your sister asks you to factor $8b^3+ 27c^3$ completely. What would be your answer?

A. $(b^2 -3c) (c+3)$ B. $(b-3c) (b^2 -9c^2)$

C. $(2b+3c)(b^2 -3bc+9c^2)$ D. $(2b-3c) (b^2 +3bc+9c^2)$

____6. Which of the following mathematical statements is correct?

A. $25m^3+ 343n^3=(5m+7n)(5m^2+35n+49n^2)$

B. $25m^3+ 343n^3= (5m+7n)(5m^2-35n+49n^2)$

C. $25m^3+ 343n^3= (5m-7n)(5m^2+35n+49n^2)$

D. $25m^3$+ $343n^3$= $(5m-7n)(5m^2-35n+49n^2)$

____7. Which is of the following polynomials <u>DOES NOT</u> belong to the group?

A. $m^2+12m+36$ B. $9n^2+30nd+25d^2$ C. $9x^2-48x+64$ D. $25r^2+40rn-16n^2$

____8. Which of the following gives a product of $4x^2-12x+9$?

A. $(2x+3)^2$ B. $(2x-3)^2$ C. $(2x-3)(2x+3)$ D. $(2x\pm3)^2$

____9. What factoring technique is applied to $x^2-11x+18= (x-9)(x-2)$?

A. Factoring Difference of Two Squares

B. Factoring Perfect Square Trinomials

C. Factoring Sum or Difference of Two Cubes

D. Factoring General Trinomials of the Form x^2+Bx+C

____10. Which of the following are the factors of $h^4-3mh^2-18m^2$?

A. $(h^2-3m)(h^2-6m)$ B. $(h^2-3m)(h^2+6m)$

C. $(h^2+3m)(h^2-6m)$ D. $(h^2+3m)(h^2+6m)$

____11. What is the missing term in the statement $2d^2$-_____+3= $(2d-1)(d-3)$?

A. -d B. -5d C. -6d D.-7d

_____12. Which of the following gives the correct factors of $6x^2-5xy-6y^2$?

A. $(3x-2y)(2x-3y)$ B. $(3x+2y)(2x-3y)$

C. $(3x-2y)(2x+3y)$ D. $(3x+2y)(2x+3y)$

_____13. Find two numbers whose product is -10 and whose sum is 3. Which of the following is the correct mathematical statement to solve the problem?

A. $x^2+3x-10 = (x-2)(x+5)$ B. $x^2-3x+10 = (x-2)(x-5)$

C. $x^2+3x-10 = (x+2)(x-5)$ D. $x^2+3x-10 = (x+2)(x+5)$

_____14. The area of the square is five times its perimeter. What is the measure of the side of the square?

A. 20 units B. 10 units C. 5 units D. 4 units

_____15. The volume of an open box is represented by $4x^3+16x^2-48x$ cubic centimeters. What are the dimensions of the box if the factors of the volume are the dimensions themselves?

A. $4x(x+6)(x+2)$ B. $4x(x-6)(x+2)$ C. $4x(x+6)(x-2)$ D. $4x(x-6)(x-2)$

Great job! You are done with this module.

References

A. Books

Abuzo, E. P., et.al. (2013). Mathematics – Grade 8 Learner's Module First Edition, 2013. Published by the Department of Education. Printed in the Philippines by Book Media Press, Inc.

Oronce, O. A. and Mendoza M. O. (2018). Exploring Math 8. Metro Manila. Rex Books Store

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[i] Feljone G. Ragma, et.al., *Food for the Brain Mathematics* (Bulacan:St.Andrew Publishing House,2016), p. 23

[ii]Maxima J. Acelajado, *Intermediate Algebra* (Makati City, Diwa Scholastic Press Inc, 2008) p. 11

[iii] Orlando A. Oronce, Marilyn O. Mendoza, *Exploring Math Textbook,* (Sampaloc: Rex Book store, Inc., 2018) p.13

[iv] Feljone G. Ragma, et.al., *Food for the Brain Mathematics* (Bulacan:St.Andrew Publishing House,2016), p. 23

[v] Virginia G. Acas, "A Workbook in Special Products and Factors and Rational Algebraic Expressions and Equations"(master's thesis, Osias Educational Foundation, 2013) p.16

[vi] Orlando A. Oronce, Marilyn O. Mendoza, *Exploring Math Textbook,* (Sampaloc: Rex Book store, Inc., 2018) p.35

[vii] Feljone G. Ragma, et.al., *Food for the Brain Mathematics* (Bulacan:St.Andrew Publishing House,2016), p. 28

[viii] Orlando A. Oronce, Marilyn O. Mendoza, *Exploring Math Textbook,* (Sampaloc: Rex Book store, Inc., 2018) p.45

[ix] Orlando A. Oronce, Marilyn O. Mendoza, *Exploring Math Textbook,* (Sampaloc: Rex Book store, Inc., 2018) pp. 50-51

[x] Virginia G. Acas, "A Workbook in Special Products and Factors and Rational Algebraic Expressions and Equations"(master's thesis, Osias Educational Foundation, 2013)p.20