8

Mathematics

Quarter 3 Week 7 – Module 7

Illustrating an Experimental Probability and a Theoretical Probability

AIRs - LM

Mathematics 8 Quarter 4- Week 7 Module 7: Illustrating an Experimental Probability and a Theoretical Probability
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Target

Probability theory is a field of Mahematics that has an interesting history filled with interesting people. It was created by a mathematician who was either a friend of gamblers or a gambler himself; in order to help gamblers win more often. Every casino uses probabilities to set their house odds so that they will always win profits. Learning about probability and games of chance may convince evryone not to be a gambler since there is no single good bet in a game of chance.

In this lesson, we will look into experimental probability and theoretical probability and you are expected to attain the following:

Learning Competency:

- Illustrates an experimental probability and a theoretical probability. **(M8GE-IVi-1)**

Learning Objectives:

1. Illustrates an experimental probability and a theoretical probability.
2. Identifies a given experiment as experimental probability or theoretical probability.
3. Determines the experimental probability and theoretical probability of an even

LESSON 1	Illustrating an Experimental Probability and a Theoretical Probability

Jumpstart

This activity will enable you to assess your prior knowledge on proving two triangles are congruent deductively.

Let us begin the lesson by accomplishing the activity below

Activity 1. WORDS COME EASY!
Direction: Join Reagan and his friends perform the same activity. Record each outcome of your experiment. Then fill in the blanks using the basic concept of probability to complete the paragraph below.

Reagan and his friends decided to find the number of times two heads {HH} would come up when flipping two coins simultaneously. Everytime they flip the fair coins is an(1)____________. The (2)_____________ that they are looking for is to come up with two heads:{HH}. The (3)_____________ is the set of all possible outcomes: {HH},{HT},{TT}.

These are the results of their experiment. Complete the table.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
Coin 1	H	H	H	T	T	H	T	T	H	T
Coin 2	T	H	T	H	T	H	T	H	H	T
Outcome	HT	HH								
Is it {HH}? YES or NO	NO	YES								

In order to find all the (4)___________, they have to continue flipping the coins at least 30 times.

Questions:

1. Were you able to complete the paragraph with the correct terms?
2. Do the words come easy to you?
3. After 10 trials, Reagan and his friends had 3 HH events. Is the result

of the experiment close to what you have expected? What would have they done to make it closer to what is expected?

4. In your own experiment, how many HH events did you have? Is the result of your experiment close to what is expected? Why?

5. What Reagan and his friends and you had performed uses **Experimental Probability**. In your own understanding, can you define Experimental Probability?

ACTIVITY 2. WHAT IS THE PROBABILITY?

DIRECTION: Look at carefully at the given set, then match Column A with Column B. Your answers will help you understand the concept of probability of an event.

Given: Set E= {2,4,6,8,10,12,14,16,18}

COLUMN A	COLUMN B
The probability of having:	
_________1. a 12	A. $\frac{0}{9}$ or 0
_________2. Odd numbers	B. $\frac{1}{9}$
_________3. Even Numbers	C. $\frac{2}{9}$
_________4. A number divisible by 3	D. $\frac{3}{9}$ *or* $\frac{1}{3}$
_________5. A number divisible by 2	E. $\frac{9}{9}$ or 1

Questions:

1. How many possible outcomes are there?
2. To have an even number, how many favorable outcomes are there?
3. Considering your answers, how do you compute for the probability of an event?
4. What formula can be used?

The concepts you have just learned from the previous activities are helpful in understanding the lesson in illustrating an Experimental Probability and a Theoretical Probability. In your own understanding, can you define Experimental Probability? How do you define Theoretical Probability?

Discover

From your previous lesson, you can predict that if you toss a coin, there are two possible outcomes: a head or a tail. When you are asked about the probability of the coin landing on heads, you would probably answer that the chance is $\frac{1}{2}$ or 50%. Imagine that you toss the coin 20 times, how many times would you expect to land it on heads? You might say $\frac{10}{20}$ $0r$ $\frac{1}{2}$, or 50% of the number of times you toos the coin. So, you would expect it to land on heads 10 times. And this is probability.

Now let us take the two kinds of probability: Theoretical probability and theoretical probability.

EXPERIMENTAL PROBABILITY

Activity 1 shows an Experimental Probability. **It is the probability of an event found by repeating an experiment and observing the outcome.** Each trial in which the event occurs is a success.

Experimental Probability **is what actually happens, instead of what you are expecting to happen.**

Use the formula below to find the experimental probability of an event.

$$P(\text{Event}) = \frac{number\ of\ event}{total\ number\ of\ trials} = \frac{n(E)}{n(S)}$$

Example 1. Tossing a Coin. Use the table below to determine the probability of landing on:

a. heads
b. tails

Outcomes	Frequency
Heads	12
Tails	8
Total	20

Since there are 20 trials, n(S)=20

Solution:

a. Probability of landing on heads

Since the frequency of landing on head is 12, n(head)=12. Thus,

$$P(\text{head}) = \frac{n(head)}{n(S)}$$
$$= \frac{12}{20}\ or\ \frac{3}{5}$$

b. Probability of landing on tailss

Since the frequency of landing on tail is 8, n(tail)=8. Thus,

$$P(\text{head}) = \frac{n(tail)}{n(S)}$$
$$= \frac{8}{20}\ or\ \frac{2}{5}$$

Example 2. A single die is rolled 100 times. The result is shown in the table below.

Event	*1*	*2*	*3*	*4*	*5*	*6*
Frequency	*16*	*18*	*15*	*16*	*18*	*17*

a. How many times the die was rolled? (100 times)
b. How many time the number "1" appeared? (16 times)
c. Find the experimental probability of each event (E):
   a. Getting a three
   b. Getting an odd number
   c. Getting a number greater than 3

Solution:

Since there are 100 rolls/trials in the experiment, n(S)= 100

a. Since the frequency of landing on 3 is 15, n(3)= 15

$P(3) = \frac{n(3)}{n(S)} = \frac{15}{100}$ or $\frac{3}{20}$

b. The odd numbers are 1,3,5, n(odd number) = n(1)+n(3)+n(5)

n(1)= 16; n(3)= 15 and n(5)= 18

n(odd number)= 16+15+18

n(odd number) = 49

$$P(\text{odd number}) = \frac{n(odd\ number)}{n(S)}$$
$$= \frac{49}{100}$$

c. Since there are three numbers greater than 3 in the sample space (4,5,6), n(number greater than three)= n(4)+n(5)+n(6)

n(number greater than 3)= 16+18+17 = 51

$$P(\text{number greater than three}) = \frac{n(number\ greater\ than\ 3)}{n(S)}$$
$$= \frac{51}{100}$$

Example 3. A face-card is drawn from a well-shuffled deck of cards. The table below show the result of the activity.

	Frequency
King	9
Queen	5
Jack	6
Total	20

Find the experimental probability of drawing:

a. a King
b. a Queen
c. a face card

Solution:

There are 20 trials in the activity, thus, n(S)= 20.

a. P(King) = $\frac{n(King)}{n(S)}$

$= \frac{9}{20}$

b. P(queen) = $\frac{n(queen)}{n(S)}$

$= \frac{5}{20}$ or $\frac{1}{4}$

c. P(Face card) = $\frac{n(Face\ card)}{n(S)}$

$= \frac{6}{20}$ or $\frac{3}{10}$

Theoretical Probability

The Theoretical Probability of an event is what you expect to happen, but it is not always what actually happens.

The formula for the Theoretical Probability of an event is :

P(event)=$\frac{Number\ of\ possible\ favorable\ outcomes}{number\ of\ total\ possible\ outcomes}$, in symbols:

P(E) = $\frac{n(E)}{n(S)}$,

where n(E) and n(S) are the number of elements in the event E and the sample space S, respectively.

Example 1. Two coins are tossed. Find the theoretical probability that both coins land tails up (TT).

Solution:

The possible outcomes (S) for tossing two coins are: {HH}, {TH},{HT} and {TT}. Since there are 4 events and only one to get two tails {TT}, the theoretical probability is:

n(S) = 4

n(TT) = 1

P(T) = $\frac{n(TT)}{n(S)} = \frac{1}{4}$

Example 2. A single die is rolled. Find the probability of each event (E):

a. Getting a three

b. Getting an odd number

c. Getting a number greater than 3

Solution:

The sample space is {1,2,3,4,5,6}, so there are six possible outcomes.

n(S)= 6

a. Since there is only one way to obtain 3, n(3)= 1

$P(3) = \frac{n(3)}{n(S)} = \frac{1}{6}$,

b. Since there are three ways to get an odd number (1,3,5), n(odd number) = 3

$$P(\text{odd number}) = \frac{n(odd\ number)}{n(S)}$$
$$= \frac{3}{6}\ or\ \frac{1}{2}$$

c. Since there are three numbers greater than 3 in the sample space (4,5,6), n(number greater than three)= 3

$$P(\text{number greater than three}) = \frac{n(number\ greater\ than\ 3)}{n(S)}$$
$$= \frac{3}{6}\ or\ \frac{1}{2}$$

Example 3. A card is drawn from a well-shuffled deck of cards. Find the probability of drawing:

a. a King (K)
b. a heart (H)
c. a face card (F)

Solution:

An ordinary deck of cards consist of 52 cards, so n(S)= 52. There are 4 Kings, 13 hearts and 12 face cards. If we denote the event of getting a King to be K, the event of getting a heart to be H and the event of getting a face card to be F, then n(K)= 4, n(H)=13 and n(F)= 12.

a. $P(K) = \frac{n(K)}{n(S)}$
$= \frac{4}{52}$ or $\frac{1}{13}$

b. $P(H) = \frac{n(H)}{n(S)}$
$= \frac{13}{52}$ or $\frac{1}{4}$

c. $P(F) = \frac{n(F)}{n(S)}$
$= \frac{12}{52}$ or $\frac{3}{13}$

Example 4. A committee of two is is to be chosen at random from a group of 5 students consisting of 3 boys and 2 girls. What is the theoretical probability that a committee

a. of two boys will be chosen? (B)
b. of two girls will be chosen? (G)
c. consisting of 1 boy and 1 girl will be chosen? (BG)

Solution:

To find the sample space, denote the three boys as B_1, B_2 and B_3 and the 2 girls as G_1 and G_2. Then the sample space is:

S={B_1 B_2, B_1 B_3, B_2 B_3, B_1 G_1, B_1 G_2, B_2 G_1, B_2 G_2, B_3 G_2, B_3 G_1, G_2 G_1}

N(S) = 10

a. If we let B the event of having two boys in the committee, by inspection: n(B)= 3 {B_1 B_2, B_1 B_3, B_2 B_3}

$$P(B) = \frac{n(B)}{n(S)}$$
$$= \frac{3}{10}$$

b. If we let G the event of having two girls in the committee, by inspection: n(G)= 1 {G_1 G_2}

$$P(B) = \frac{n(G)}{n(S)}$$
$$= \frac{1}{10}.$$

If we let BG the event of having 1 boy and 1 girl in the committee, by inspection: n(BG)= 6 { B_1 G_1, B_1 G_2, B_2 G_1, B_2 G_2, B_3 G_1, B_3 G_2}

$$P(B) = \frac{n(BG)}{n(S)}$$
$$= \frac{6}{10} \text{ or } \frac{3}{10}$$

Example 5. A bookshelf contains 10 Science books, 8 Mathematics books, 9 English books and 10 Filipino books. If a book is selected, find the probability that it is:

a. a Science book

b. a Mathematics book

c. a cookbook

d. an English book

Solution:

To find the sample space, count the total outcomes: 10+8+9+10= 37.

n(S) = 37

a. Since there are 10 Science books, n(Science) = 9

$$P(Science) = \frac{n(Science)}{n(S)}$$
$$= \frac{10}{37}$$

b. Since there are 8 Mathematics books, n(Mathematics)=8

$$\text{P(Mathematics)} = \frac{n(Mathematics)}{n(S)}$$
$$= \frac{8}{37}$$

c. Since there is no cookbook in the shelf, n(cookbook)= 0

$$\text{P(cookbook)} = \frac{n(cookbook)}{n(S)}$$
$$= \frac{0}{37} \ or\ 0$$

d. Since there are 9 English books, n(English)= 9

$$\text{P(Science)} = \frac{n(English)}{n(S)}$$
$$= \frac{9}{37}$$

Theoretical Probability

The Theoretical Probability of an event is what you expect to happen, but it is not always what actually happens.

The formula for the Theoretical Probability of an event is :

P(event)=$\frac{Number\ of\ possible\ favorable\ outcomes}{number\ of\ total\ possible\ outcomes}$, in symbols:

P(E) = $\frac{n(E)}{n(S)}$, where n(E) and n(S) are the number of elements in the event **E** and the sample space **S**, respectively.

Example 1. Two coins are tossed. Find the theoretical probability that both coins land tails up (TT).

Solution:

The possible outcomes (S) for tossing two coins are: {HH}, {TH},{HT} and {TT}. Since there are 4 events and only one to get two tails {TT}, the theoretical probability is:

n(S) = 4

n(TT) = 1

P(T) = $\frac{n(TT)}{n(S)} = \frac{1}{4}$

Example 2. A single die is rolled. Find the probability of each event (E):

a. Getting a three

b. Getting an odd number

c. Getting a number greater than 3

Solution:

The sample space is {1,2,3,4,5,6}, so there are six possible outcomes.

n(S)= 6

a. Since there is only one way to obtain 3, n(3)= 1

P(3) = $\frac{n(3)}{n(S)} = \frac{1}{6}$,

b. Since there are three ways to get an odd number (1,3,5), n(odd number) = 3

$$P(\text{odd number}) = \frac{n(odd\ number)}{n(S)}$$
$$= \frac{3}{6}\ or\ \frac{1}{2}$$

c. Since there are three numbers greater than 3 in the sample space (4,5,6), n(number greater than three)= 3

$$P(\text{number greater than three}) = \frac{n(number\ greater\ than\ 3)}{n(S)}$$
$$= \frac{3}{6}\ or\ \frac{1}{2}$$

Example 3. A card is drawn from a well-shuffled deck of cards. Find the probability of drawing:

a. a King (K)
b. a heart (H)
c. a face card (F)

Solution:

An ordinary deck of cards consist of 52 cards, so n(S)= 52. There are 4 Kings, 13 hearts and 12 face cards. If we denote the event of getting a King to be K, the event of getting a heart to be H and the event of getting a face card to be F, then n(K)= 4, n(H)=13 and n(F)= 12.

a. $P(K) = \frac{n(K)}{n(S)}$

$= \frac{4}{52}$ or $\frac{1}{13}$

b. $P(H) = \frac{n(H)}{n(S)}$

$= \frac{13}{52}$ or $\frac{1}{4}$

c. $P(F) = \frac{n(F)}{n(S)}$

$= \frac{12}{52}$ or $\frac{3}{13}$

Example 4. A committee of two is is to be chosen at random from a group of 5 students consisting of 3 boys and 2 girls. What is the theoretical probability that a committee

a. of two boys will be chosen? (B)
b. of two girls will be chosen? (G)
c. consisting of 1 boy and 1 girl will be chosen? (BG)

Solution:

To find the sample space, denote the three boys as B_1, B_2 and B_3 and the 2 girls as G_1 and G_2. Then the sample space is:

$S=\{B_1\ B_2, B_1\ B_3, B_2\ B_3, B_1\ G_1, B_1\ G_2, B_2\ G_1, B_2\ G_2, B_3\ G_2, B_3\ G_1, G_2\ G_1\}$

N(S) = 10

a. If we let B the event of having two boys in the committee, by inspection: n(B)= 3 $\{B_1 B_2, B_1 B_3, B_2 B_3\}$

$$P(B) = \frac{n(B)}{n(S)}$$
$$= \frac{3}{10}$$

b. If we let G the event of having two girls in the committee, by inspection: n(G)= 1 $\{G_1 G_2\}$

$$P(B) = \frac{n(G)}{n(S)}$$
$$= \frac{1}{10}$$

c. If we let BG the event of having 1 boy and 1 girl in the committee, by inspection: n(BG)= 6 $\{ B_1 G_1, B_1 G_2, B_2 G_1, B_2 G_2, B_3 G_1, B_3 G_2\}$

$$P(B) = \frac{n(BG)}{n(S)}$$
$$= \frac{6}{10} \text{ or } \frac{3}{10}$$

Example 5. A bookshelf contains 10 Science books, 8 Mathematics books, 9 English books and 10 Filipino books. If a book is selected, find the probability that it is

a Science book
b. a Mathematics book
c. a cookbook
d. an English book

Solution:

To find the sample space, count the total outcomes: 10+8+9+10= 37.
n(S) = 37

a. Since there are 10 Science books, n(Science) = 9

$$P(Science) = \frac{n(Science)}{n(S)}$$
$$= \frac{10}{37}$$

b. Since there are 8 Mathematics books, n(Mathematics)=8

$$P(Mathematics) = \frac{n(Mathematics)}{n(S)}$$
$$= \frac{8}{37}$$

c. Since there is no cookbook in the shelf, n(cookbook)= 0

$$P(cookbook) = \frac{n(cookbook)}{n(S)}$$

$$= \frac{0}{37} \quad or\ 0$$

d. Since there are 9 English books, n(English)= 9

$$\text{P(Science)} \quad = \frac{n(English)}{n(S)}$$

$$= \frac{9}{37}$$

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3. Experimental or Theoretical?

Direction: Determine whether each problem illustrates an experimental or a theoretical probability. Draw a ☆ if it is experimental, otherwise, draw a ☺.

1. Suppose we toss a coin 100 times and get a head 58 times. Now, we toss a coin at random. What is the probability of getting a head based from the activity?
2. A card is drawn from an ordinary deck of playing cards. Find the probability of drawing a red ace.
3. There were 40 marbles in a jar: 6 red, 10 orange, 7 bue and the remaining are yellow. Find the probability of choosing a blue marble.
4. There are 13 boys and 8 girls in Grade 8- Diamond A class. What is the probability that the teacher will call a boy?
5. A cube is rolled once. What is the probability that the number on the top of the cube is odd?
6. Gianni tossed a coin 30 times and recorded 5 tails and 25 heads. What is the probability of getting head based from the result?
7. A color cube is rolled 100 times. The number 6 comes up 25 times. What is its probability based from the trials done?

8. If three fair coins are tossed randomly 175 times and it is found that **three heads** appeared 21 times, **two heads** appeared 56 times, **one head** appeared 63 times and zero head appeared 35 times.What is the probability of getting (i) three heads, (ii) two heads, (iii) one head, (iv) 0 head.

9. Each letter of the word HONESTY is written in a separate piece of paper and put in a bag. You randomly choose a piece of paper from the bag. Find the probability of choosing letter Y.

10. Suppose a fair coin is randomly tossed for 90 times and it is found that head turns up 35 times and tail 55 times. What is the probability of getting (i) a head and (ii) a tail?

Deepen

Activity 4. Kakasa ka ba sa PROBA?
Direction:

1. A die is thrown randomly four hundred fifty times. The frequencies of outcomes 1, 2, 3, 4, 5 and 6 were noted as given in the following table:

Outcomes	1	2	3	4	5	6
Frequency	70	78	73	75	80	74

Find the theoretical probability and experimental probabilities of the occurrence of the event
(a) 4
(b) a number less than 4
(c) a number greater than 4
(d) a prime number
(e) a number less than 7

2. Use the following results and illustrations of a spinner below:

Color	No. of times it occur
Violet	13
Red	9
Yellow	7
Blue	1

a. How many trials are done in this experiment?
b. What is the theoretical probability that violet will occur?
c. What is the experimental probability that yellow will occur?
d. What is the theoretical probability that blue will occur?
e. What is the experiemtnal probability that red will occur?

Gauge

Directions: Answer the following questions correctly. Write the letter of the correct answer in a separate sheet.

1. Which of the following illustrates a theoretical probability?
   A. what should happen
   B. what will happen
   C. what does happen
   D. what I want to happen
2. Which of the following illustrates an experimental probability?
   A. what should happen
   B. what will happen
   C. what does happen
   D. what I want to happen
3. Which of the following is true about experimental probability?
   A. It is the number of times the event occurs.
   B. It is equal to the number of trials done.
   C. It is the ratio of the number of times the event occurs to the total number of trials.
   D. It is the ratio of the total number of trials to the number of times the event occurs.

For numbers 4 - 6, determine whether the following situations illustrate an experimental probability oe theoretical probability.

4.A balanced die is rolled. The probability of rolling a number that is 3 is $\frac{1}{6}$

What type of probability is illustrated?

A. Experimental Probability B. Theoretical Probability

5. In a society 1000 families with 2 children were selected and the following data was recorded. Find the probability of having 2 boys.

Number of boys in a family	0	1	2
Number of families	333	392	275

A. Experimental Probability B. Theoretical Probability

6. In a 500- ticket draw for an educational prize, Ana's name was written on 50 tickets. What is the probability that she would win?

A. Experimental Probability B. Theoretical Probability

For numbers 7 – 10, refer on the problem below.

A bag has 3 red, 2 blue and 4 yellow marbles. Moana drew marbles from the bag and the result is shown in the chart below.

Color	Number of Marbles
Red	13
Blue	9
Yellow	8

7. What is the theoretical probability of choosing red?

A. $\frac{3}{10}$ B. $\frac{1}{9}$ C. $\frac{1}{6}$ D. $\frac{1}{3}$

8. Based on the chart, what is the experimental probability of getting blue?

A. $\frac{3}{10}$ B. $\frac{1}{9}$ C. $\frac{1}{6}$ D. $\frac{1}{3}$

9. What is the theoretical probability of choosing a yellow marble?

A. $\frac{3}{10}$ B. $\frac{1}{9}$ C. $\frac{4}{9}$ D. $\frac{8}{30}$

10. What is the experimental probability for problem number 9?

A. $\frac{3}{10}$ B. $\frac{1}{9}$ C. $\frac{4}{9}$ D. $\frac{4}{15}$

11. You flip a coin 50 times and get 20 tails. What is the experimental probability of getting tails?

A. $\frac{2}{5}$ B. $\frac{3}{5}$ C. $\frac{1}{2}$ D. $\frac{5}{2}$

12. What is the experimental probability of getting heads?

A. $\frac{2}{5}$ B. $\frac{3}{5}$ C. $\frac{1}{2}$ D. $\frac{5}{2}$

For numbers 13 – 15, refer on the problem below.

Randolf asked a number of people about their age and recorded in the table.

Age Group	Number of People
0 – 20	15
21- 40	24
41- 60	18
60 & over	21

13. Based from the table, what is the probability that the next person will be 40 years old or less?

A. $\frac{9}{39}$ B. $\frac{12}{39}$ C. $\frac{24}{78}$ D. $\frac{39}{78}$

14.What will the probability that it will be over 40 years old?

A. $\frac{9}{39}$ B. $\frac{12}{39}$ C. $\frac{24}{78}$ D.$\frac{39}{78}$

15. What is the probability that the next person will be over 20 years old but not over 40 years old.

A. $\frac{9}{39}$ B. $\frac{12}{39}$ C. $\frac{24}{78}$ D.$\frac{39}{78}$

Great job! You are done with this module.

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