10

Mathematics

Quarter 3- Week 5 Module 4
Union and Intersection of Events

AIRs - LM

Mathematics 10
Quarter 3 – Week 5- Module 4
Union and Intersection of Events

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Target

This module will assess your knowledge and skills of the different mathematics concepts related to events as well as concepts of sets and its operations you previously studied. These knowledge and skills are important in understanding the union and intersection of events as well as the complement of an event.

In this module, you will learn to:

- illustrate events, and union and intersection of events. **M10SP-IIIf-1**

At the end of this module, you are expected to:

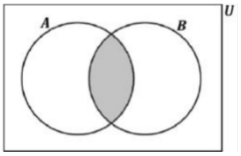
1. identify and distinguish simple events from compound events.
2. illustrate and differentiate union and intersection of events.
3. apply the concepts of sets and Venn diagram in determining the union, intersection and complements of an event.

Let us find out how much you already know about this module. Answer the pre-assessment in a separate sheet of paper.

PRE-ASSESSMENT

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper. Take note of the items that you were not able to answer correctly and find the right answer as you go through this module.

1. Which of the following is NOT a simple event?
   A. tossing a head with a coin B. rolling an even number on fair die
   C.rolling a 3 on a fair die D. getting a king of heart from a deck of cards
2. A fair of die numbered 1-6 is rolled once, which of the following is a compound event?
   A. rolling a 2 B. rolling a number greater than 5
   C.rolling a 4 D. rolling an odd number
3. It is defined as the event containing all outcomes that are in A or in B or in both.
   A. complement B. empty C. intersection D. union
4. This image/symbol "∩" represents _____.
   A. intersection, "and" B. union, "and"
   C. intersection, "or" D. union, "or"
5. Which of the following represents the shaded region?



A. A union B B. A intersection B

C.$A \subset B$ D. $B \subset A$

6. If A = {1, 3, 5, 7, 9} and B = {2, 3, 5, 7}, what is $A \cap B$?
   A. {3, 5, 7} B. {2, 3, 5, 7, 9}
   C.{2, 3, 5, 7) D. {1, 2, 3, 5, 7, 9}

For items number 7 – 10, refer to the situation below.

In the experiment of rolling a single die, event E is "the number rolled is even" and event T is "the number rolled is greater than two".

7. Find the $E \cap T$.
   A. {2, 4} B. {4, 6} C. {2, 4, 6} D. {2, 3, 4, 5, 6}
8. What is $E \cup T$?
   A. {2, 4} B. {4, 6} C. {2, 4, 6} D. {2, 3, 4, 5, 6}
9. Find the complement of T(T').
   A. {1, 2} B. {2, 3} C. { 3, 4} D. {4, 5}
10. What is the complement of E(E')?
    A. {2, 4, 6} B. {1, 3, 5} C. {1, 2, 3} D. {4, 5, 6}
11. Let C = {2, 5, 7, 10} and B = {x|x is an odd whole number less than 10}. Find $C \cup B$.
    A. {1, 3, 5, 7, 9, 10} B. {1, 2, 3, 5, 7, 9, 10}
    C.{2, 4, 5, 6, 7, 8, 10} D. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
12. Given: The Universal Set(U) = {the set of counting numbers from 1 to 10} and event A = (1, 3, 6, 7, 9}. What is the complement of event A (A')?
    A. {2, 4, 6, 8, 10} B. {1, 3, 5, 7, 9}
    C.{2, 4, 5, 8, 10} D. {1, 3, 5, 8, 10}
13. In the game of snakes and ladders, a fair die is thrown. If event E_1 represents all the events of getting a natural number less than 4, event E_2 consists of all the events of getting an even number and E_3 denotes all the events of getting an odd number. Find $E_1 \cup E_2$? (E_1 = { 1, 2, 3} ; E_2 = {2, 4, 6} ; E_3 = {1, 3, 5})
    A. {1, 2, 2, 3, 4, 6} B. {1, 2, 3, 5}
    C. {1, 2, 3, 4, 6} D. {1, 2, 3, 4, 5, 6}
14. Refer to item #14. What is $E_1 \cap E_3$?
    A. {1. 2} B. {1, 3} C. {1, 4} D. {1, 5}
15. There are 500 students in a school, 220 like science subject, 180 like math and 40 like both science and math. Find the number of students who like math only but not science.
    A. 40 B. 140 C. 180 D. 220

Lesson 1 Union and Intersection of Events

In the lower grade level, you studied about the basic concepts on sets and its operations and Venn Diagram. Likewise, you have the knowledge and skills of identifying universal sets, union sets, intersection sets and complement sets. In this lesson, you will need to apply the basic concepts you have learned about the operation of sets.

Jumpstart

At this point, you are going to recall the operation of sets. The activities provided will enable you to identify simple and compound events, union and intersection events and complement of an event.

Activity 1: Do You Remember

In this activity, you are going to recall the operation of sets.

1. Situation A: Esteph likes bananas, mangoes and star-apples. Rey likes oranges, pineapples and mangoes.
   a. What fruit/s do they both like?
   b. What kind of fruits do they have in all?

2. Situation B: The school trains 5 students for the Math Challenge. The students are; Jovic, Vincent, Lou, Eric and Bernadette. For the actual competition, the school sends only the top three. Suppose Jovic, Lou and Eric were selected for the competition, who were not selected?

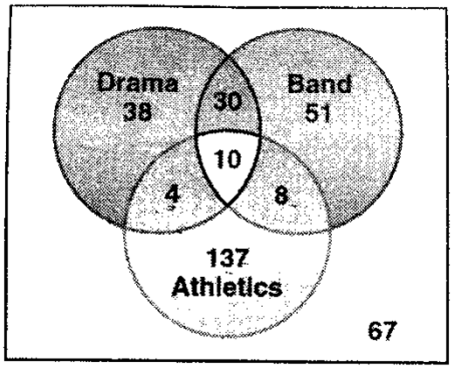
Activity 2: Simple and Compound

Given the following events, determine if simple or compound event.

1. Getting a sum of 10 in a roll of 2 dice.
2. Getting a sum of 2 in a roll of 2 dice.
3. Getting a hand of 8 cards consisting of 3 aces and 5 face cards.
4. Getting at least 2 heads in a toss of 4 coins.
5. Getting the pair (HT) when two coins are tossed simultaneously.

Activity 3: Intersection and Union of Events

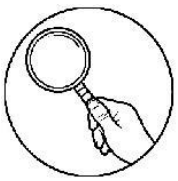
The Extracurricular activities in which the Senior class at San Jose National High School participated are shown in the Venn diagram below.



Extra-Curricular Activities Participated by Senior Students

Questions:

1. How many students are in the senior class?
2. How many students participate in athletics?
3. How many students participate in drama?
4. How many students participate in band?
5. How many students participate in band or drama?
6. How many students participate in band and athletics?
7. How many students did not participate in drama?



Discover

The activities above show how to determine if an event is simple or compound. It also shows how to determine and differentiate the intersection and union of two or more events and identifying complement of an event using Venn diagram. This section will discuss this further.

In the first activity, situation A, illustrates union and intersection.

a. Both Esteph and Rey like mangoes. If A is the set of Esteph's favorite fruits {banana, mango, star-apple} and B is the set of Rey's favorite fruits {orange, pineapple, mango}, then mango represents the

intersection of sets A and B denoted by **A ∩ B**. "Mango" is the only element of A ∩ B. The word **'and'** suggests intersection.

A ∩ B = {mango}

b. If Esteph and Rey combine their fruits, the fruits they have in all is {banana, mango, star-apple, orange, pineapple}. By combining the fruits, you form a union of the fruits belonging to either Esteph or Rey or both, written **A ∪ B**. The word **'or'** suggests union.

A ∪ B = {banana, mango, star-apple, orange, pineapple}

Union and **intersection** are called set operations because they replace two sets with a third set.

Example: The students who love swimming S = {Bert, Fred, Sam, Butch} and those who love volleyball V = {Luis, Joey, Sam, Butch}.

a. Find S ∩ V.
b. Find S ∪ V.

Solution:

a. S ∩ V = {Sam, Butch}
b. S ∪ V = {Bert, Fred, Sam, Butch, Luis, Joey}

Situation B, illustrates **complement of a set,** this is another operation on a set. Let U(Universal Set) = { Jovic, Vincent, Lou, Eric and Bernadette}. Suppose A = {Jovic, Lou, Eric} as the selected students for the competition, then the set of students who were not selected is usually referred to as the complement of set A, denoted as **A'**.

A' = {Vincent, Bernadette}

The **complement of a set A,** written **A'**, is the set of elements in the universal set that are not in set A.

Example: Find the complement of each set.

If universal set U = { 1, 3, 5, 7, 9, 11, 13, 15, 17}

1. C = { 5, 7, 9, 11}
2. D = { 13, 15, 17}

Solution:

1. C' = { 1, 3, 13, 15, 17}
2. D' = { 1, 3, 5, 7, 9, 11}

In activity 2, you were ask to determine if the given event is simple or compound. In statistics, ***event*** is a collection of one or more outcomes in an experiment. If the event has only 1 outcome, the event is called a ***simple event***. A simple event is usually denoted by E_1, E_2, E_3, E_4, and so forth. Any other capital letter could be used as well.

On the other hand, if the event has at least 1 outcome, it is called a ***compound event***. A compound event is usually denoted by A, B, C, and so forth or A_1, A_2, A_3, and so forth.

EVENT EXAMPLES

1. If a single face is considered when a die is rolled, then it will be simple event. For example suppose getting 5 or 6 or 3 or 2 etc… on the die when it is thrown, is called as simple event. If the event is any even number on the die, then the event is consist of points {2, 4, 6}, which is known as compound event. That compound event is consisting of three simple events i.e., {2}, {4} and {6}.

2. Suppose two dice are rolled simultaneously, then the pair (1, 1) will be the simple event. This is so, because it is a single outcome in the sample space. If event consists of the sum of two dice is "5" than it consists of four outcomes i.e., (1, 4), (2, 3), (3, 2), (4, 1) and this is considered to be a compound event.

3. Suppose two coins are tossed simultaneously, then the pair (HT) will be the simple event. If condition is defined that an event should consist of at least one head then there are three outcomes. These outcomes are (HH), (HT) and (TH) and this is said to be as compound event. That compound event consists of three simple events i.e., {HH}, {HT} and {TH}.

In activity 3, it is often useful to use a Venn Diagram to visualize the outcomes of events. You can explore the use of a Venn Diagram to determine the union of events, the intersection of events and the complement of an event. To understand more about union and intersection of events, let's differentiate first union of events, intersection of events and complement of an event.

-Union of Events-

Suppose you roll a die. Then the possible outcome is S = {1,2,3,4,5,6}.
Consider the following two events A = {1,3,5} and B = {1,3,6}. What is A ∪ B?

A ∪ B is defined as an event that consists of the basic outcomes that are either in A or B.

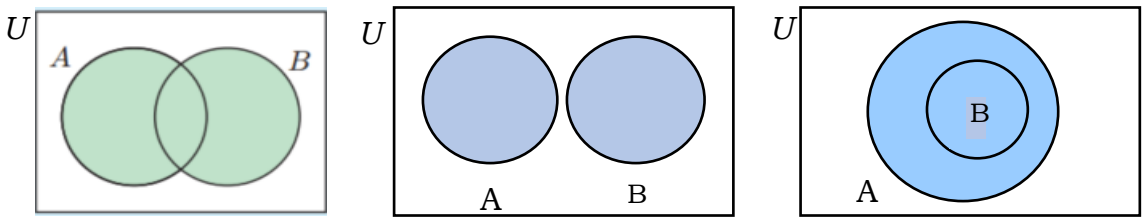
A ∪ B reads **"A union B" (A or B)**.

Answer: **A ∪ B = {1, 3, 5, 6}**

Venn Diagram

Venn Diagram - a diagram that uses circles to represent sets/events, in which the relations between the sets are indicated by the arrangement of the circles.

Venn Diagram of Union of Events A or B



A ∪ B all the shaded area

-Intersection of Events-

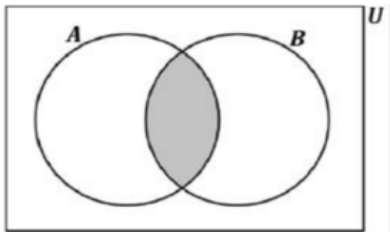
Suppose you roll a die. Then the possible outcome is S = {1,2,3,4,5,6}.
Consider the following two events A = {1,3,5} and B = {1,3,6}. What is **A∩B**?

A∩B is defined as an event that consists of the basic outcomes that are common to both A and B.

A∩B reads "**A intersection B**" (**A and B**).

Answer: **A∩B** = {1, 3}

Venn Diagram of Intersection of Events A and B



A∩B is the shaded region

Complement of an Event

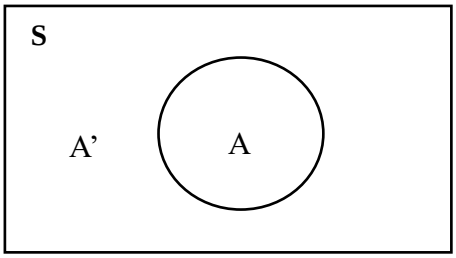
Let A be an event in the sample space S. The set of basic outcomes of a random experiment belonging to S but not to A is called the complement of A and is denoted by A'.

Examples:

In tossing a coin, when the event is head, the complement is tail.
In a week, when the event is (Monday, Wednesday), the complement is (Tuesday, Thursday, Friday, Saturday, Sunday)

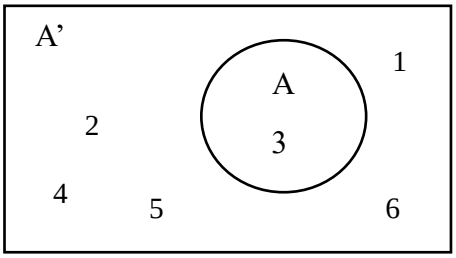
In a deck of 52 cards, when the event is (hearts), the complement is (Spades, Clubs, Diamonds)

Venn Diagram for the Complement of Event A



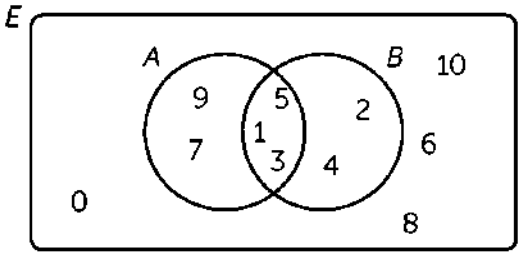
Illustrative Example: In rolling a die, the event is to "get a 3" or A = {3}. What is the complement of event A?

Solution:



The complement of the event, A' = {1, 2, 4, 5, 6}

Another illustrative Example: Use the Venn diagram below to answer the following.



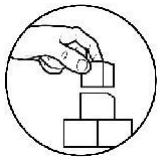
Questions:

a. What is A∪B / A or B?
b. What is A∩B / A and B?
c. What is the complement of A(A')?
d. What is the complement of B(B')?
e. What is A' ∪ B'?

f. What is A' ∩ B'?

Solution:

a. A∪B = {1, 2, 3, 4, 5, 7, 9}
b. A∩B = {1, 3, 5}
c. A' = {0, 2, 4, 6, 8, 10}
d. B' = {0, 6, 7, 8, 9, 10}
e. A' ∪ B' = {0, 2, 4, 6, 7, 8, 9, 10}
f. A' ∩ B' = {0, 6, 8, 10}



Explore

Let's have some practice about the concepts of simple and compound events, union, intersection and compliments of events that you have learned.

Activity 1: Dealing with "ONE" or "MORE THAN ONE"

In this activity, you are going to identify if the given probability is ***simple event*** or ***compound event***.

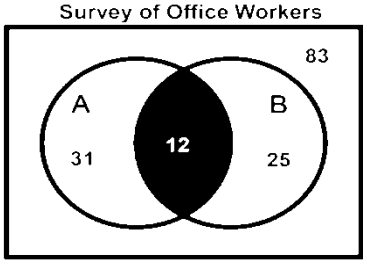
1. The event of rolling a 5 on a die. _____
2. The event of rolling an odd number less than 5 on a die. _____
3. The event of drawing a red king from a deck of cards. _____
4. The event of drawing the queen of hearts from a deck of cards. _____
5. The event of rolling an even number on a die, then tossing a tail on a coin. _____

Activity 2: Interaction of Sets in a Venn Diagram

In this activity, you are going to find the union, intersection and complement of events. Set A = workers that drink coffee and set B = workers that drink soda, then the rest do not drink coffee nor soda.

Answer the following correctly.

1. How many workers are there?
2. How many workers drink coffee?
3. How many workers drink soda?
4. How many workers drink coffee or soda (A∪B)?
5. How many workers drink coffee and soda (A∩B)?
6. How many workers do not drink coffee (A')?
7. How many workers do not drink soda (B')?



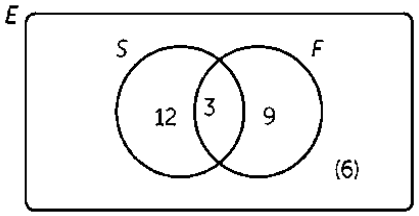
Now that you know the important concepts about this topic, let's go deeper by moving to the next session.

Deepen

Union and Intersection in Real Life

Directions: Read and analyze the problems then answer the question/s. Write your solution in your answer sheet.

1. Refer to the Venn Diagram below. Twenty-four people go on holidays. If 15 go swimming(S), 12 go fishing(F) and 6 do neither(N),
   a. How many people will do swimming?
   b. How many people will do fishing?
   c. What is S ∪ F?
   d. What is S ∩ F?
   e. What is S'?



2. A special deck of 16 cards has 4 that are blue, 4 yellow, 4 green, and 4 red. The four cards of each color are numbered from one to four. A single card is drawn at random.

   Event B: blue card = {*b1, b2, b3, b4*}
   Event R: red card = {*r1, r2, r3, r4*}
   Event N: the number on the card is at most two = {*b1, b2, y1, y2, g1, g2, r1, r2*}

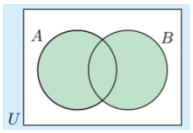
   Answer the following correctly:
   a. B ∪ R =
   b. B ∩ R =
   c. B ∩ N =
   d. R ∪ N =
   e. B' =

3. Make a statement in simple word/s that describes the complement of each event (without simply inserting the word 'not').
    a. In the roll of a die: 'five or more."
    b. In a roll of a die: "an even number."
    c. In two tosses of a coin: "at least one heads."
    d. In the random selection of a college student: "Not a freshmen."

Gauge

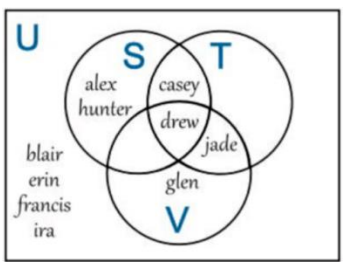
Directions: Find out how much have you learned from the lesson. Choose the letter of the correct answer to the question. Write your answer in a separate sheet of paper.

1. Which of the following illustrates a simple event?
    A. getting an even number less than 5 when rolling a die.
    B. drawing a red ace from a deck of cards.
    C. drawing the ace of hearts from a deck of cards.
    D. tossing three coins and getting at least 2 heads.
2. It is defined as an event that consists of the basic outcomes that are common to both events A and B.
    A. intersection B. union C. complement D. event
3. Refer to the figure at the right. What statement does the shaded region represent?
    A. A or B B. A and B C. A D. B



4. This image/symbol "∪" represents _____.
    A. intersection, "and" B. union, "and"
    C. Intersection, "or" D. union, "or"
5. Let A = {1, 3, 4} and B = {x|x is an even whole number less than 9}. Find A∩B?
    A. { 8 } B. { 6 } C. { 4 } D. { 2 }
6. What event is denoted by A^c?
    A. complement B. intersection C. subset D. union

7. The Universal Set U = { -4, -3, -2, -1, 0, 1, 2, 3, 4,) and A = { 0 }. What is the complement of A?
    A. {-4, -3, -2, -1, 0, 1, 2, 3} B.. {-3, -2, -1, 1, 2, 3}
    C{-4, -3, -2, -1, 1, 2, 3, 4} D. {-4, -3, -2, -1, 1, 2, 3}



For questions 8 to 12. Refer to the Venn Diagram above.

8. What is the set of $S \cap T$?

A. {casey, drew, jade, glen}
B. {alex, casey, drew, hunter}
C. {casey, drew}
D. {drew, jade}

9. What is the set of $V \cup T$?

A. {casey, drew, jade, glen}
B. {alex, casey, drew, hunter}
C. {casey, drew}
D. {drew, jade}

10. What is S ∪ T?

A. {alex, hunter, casey, glen, jade}
B. {alex, casey, hunter, drew, glen}
C. {alex, hunter, casey, drew, jade}
D. {blair, erin, francis, ira}

11. What is (S ∪ V ∪ T)'?

A. {alex, hunter, casey, glen, jade}
B. {alex, casey, hunter, drew, glen}
C. {alex, hunter, casey, drew, jade}
D. {blair, erin, francis, ira}

12. What is S ∩ V ∩ T?

A. {casey}
B. {jade}
C. {glen}
D. {drew}

Given: S = {1, 2, 3, 4, 5} and event A = {1, 3, 5}. What is the complement of A (A')?

A. A' = {1, 2, 3}
B. A' ={4, 5}
C. A' = {2, 4}
D. A' = { }

13. Of the customers who bought items at a store on a particular day, 53 of them bought clocks and 47 of them bought watering cans. What is the largest possible number of people that bought either a clock or a watering can on that day?

A. 47
B. 53
C. 100
D. 106

14. In a school there are 25 teachers who teach math or science. Of these, 15 teach math and 6 teach both math and science. How many teach science?

A. 6
B. 15
C. 16
D. 25

15. In a college, there are 20 students who enrolled for commerce only,90 students enrolled for mathematics only, 30 students enrolled for commerce and mathematics both and 60 students enrolled for others. Find total number of students enrolled either for commerce or mathematics.

A. 50
B. 120
C. 140
D. 200

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e-Math Grade 10 Revised Edition 2015

Website:

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