

MATHEMATICS

Quarter 1 - Module 3:

Determining Geometric Means, n th Term and the Sum of the Terms of a Given Finite or Infinite Geometric Sequence



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MATHEMATICS 10

Quarter 1 - Module 3: Determining Geometric Means, nth Term of a Geometric Sequence, and Sum of the Terms of a Given Finite or Infinite Geometric Sequence
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Region I

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MATHEMATICS

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**Quarter 1 - Module 3:
Determining Geometric Means,
nth Term and Sum of the Terms
of a Given Finite or Infinite
Geometric Sequence**

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



Target

Covid-19 pandemic has greatly affected everyone. It has brought enormous changes in various aspects of human life. However, the exponential increase of cases does not impede the learning growth of a determined student like you.

This module was designed to help you grow and develop your mathematical skills even during this time of crisis. It will help you understand about determining geometric means, n th term of a geometric sequence and the sum of a given finite or infinite geometric sequence. So as you go through this module, do not forget to spread a contagious smile while enjoying to learn!

This module will help you attain the following objectives:

Learning Competency

- Determines geometric means, n th term of a geometric sequence and the sum of a given finite or infinite geometric sequence.

Subtasks:

1. Determines the n th term of a geometric sequence.
2. Finds the geometric means between terms of a geometric sequence.
3. Determines the sum of the first n terms of a given finite and infinite geometric sequence.

Before we start the lesson, find out how much you already know about this module by answering the pre – assessment test.

Pre-Assessment

Directions: Read carefully and analyze each item then select the letter that best corresponds to the question in a separate sheet of paper.

1. What is the next term in the geometric sequence 3, -9, 27?
A.-30 B.-36 C.-81 D.-90
2. What is the common ratio in the geometric sequence 4,-12,36,...?
A.-3 B.-2 C.2 D.3
3. Which geometric sequence has a common ratio of $\frac{1}{2}$?
A.64,16,4,1,... B.8,4,2,... C. $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \dots$ D. $\frac{27}{2}, 54, 216, 864, \dots$
- 4.What is the common ratio for the sequence given by the formula $a_n = 4(3)^{n-2}$?
A.1 B.2 C.3 D.4
5. .What is the 5th term of the geometric sequence 2,6,18,...?
A.45 B.54 C.126 D.162
6. What is the third term of the sequence $a_n=3(2)^{n-1}$?
A.12 B.18 C.24 D.26
7. What is the first term of the geometric sequence having 80 as the 5th term and the common ratio is 2?
A.5 B.10 C.20 D.80
8. What is the 8th term of a geometric sequence where the 4th term is 32 and the common ratio is 2.
A.128 B.256 C.512 D.1024
9. What is the value of k so that k-3,k+2,k+3 form a geometric sequence?
A. $-\frac{1}{5}$ B. $\frac{1}{5}$ C. $-\frac{13}{4}$ D. $\frac{13}{4}$
10. What is the geometric mean between 3 and 27?
A.9 B.10 C.11 D.12
11. What is the sum of the two geometric means between 2 and 54?
A.6 B.18 C.24 D.42
12. Which formula below shows the sum of finite geometric sequence with a common ratio of 1?
A. $S_n = \frac{a_1}{1-r}$ B. $S_n = \frac{a_n}{1-r}$ C. $S_n = a_1$ D. $S_n = na_1$
13. Which of the following will give the sum to infinity of a given geometric sequence?
A. $S_n = \frac{a_1}{1-r}$ B. $S_n = \frac{a_n}{1-r}$ C. $S_n = \frac{a_n}{1+r}$ D. $S_n = \frac{a_n}{1+r}$
14. What is the sum of the first six terms of the geometric sequence with first term 40 and common ratio $\frac{1}{2}$?
A. $\frac{315}{4}$ B. $\frac{315}{2}$ C. $\frac{315}{8}$ D.315
15. What is the sum of the infinite geometric series $\frac{3}{4} - \frac{9}{16} + \frac{27}{64} - \frac{81}{256} + \dots$?
A. $\frac{3}{4}$ B. $\frac{3}{7}$ C. 1 D. 3



Jumpstart

Before we proceed to our lesson, let us have a recall on geometric sequence.

A **geometric sequence** is a sequence where each term after the first term is obtained by multiplying the preceding term by a nonzero constant called the **common ratio**.

The common ratio, r , can be obtained by dividing any term in the sequence by the term that precedes it. Thus, in the geometric sequence 4, 12, 36, ..., the common ratio is 3.

Activity 1: Tell Me!

State whether each of the following sequences is geometric or not.

1. 2, 6, 18, 54, ...
2. 3, -6, 12, -24, ...
3. 16, 8, 4, 2, ...
4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$
5. 2, 0, 0, 0, ...

Let us take a look on the discussions below. This will help you in understanding the lesson.



Discover

Now, you will discover more about geometric sequence.

Activity 2: And This is My Rule!

Problem: What are the first 5 terms of a geometric sequence whose first term is 2 and whose common ratio is 3?

Term	Other Ways to Write the Terms	
	In Factored Form	In Exponential Form
$a_1 = 2$	2	2×3^0
$a_2 = 6$	$2 \cdot 3$	2×3^1
$a_3 = 18$	$2 \cdot 3 \cdot 3$	2×3^2
$a_4 = 54$	$2 \cdot 3 \cdot 3 \cdot 3$	2×3^3
$a_5 = 162$	$2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	2×3^4
...
$a_n =$?

GUIDE QUESTIONS:

1. Look at the two ways of writing the terms. What does 2 represent?
2. For any two consecutive terms, what does 3 represent?
3. What is the relationship between the exponent of 3 and the position of the term?
4. In general, if the first term of geometric sequence is a_1 , and the common ratio is r , what is the n th term of a geometric sequence?

From the activity presented, given the first term a_1 , the common ratio r , the n th term of a geometric sequence is **$a_n = a_1 r^{n-1}$** .

In the next examples, you will learn how to find the n th term of a geometric sequence .

Example 1. What is the 7th term of the geometric sequence 3, 6, 12, 24, ...?

Solution:

$$a_1 = 3, r = 2, n = 7 \quad a_7 = ?$$

$$\text{Since } r = 2, \text{ then } a_n = a_1 r^{n-1}$$

$$a_7 = 3(2)^{7-1}$$

$$a_7 = 3(2)^6$$

$$a_7 = 3(64)$$

$$\mathbf{a_7 = 192}$$

Example 2. What is the 5th term of the geometric sequence 64, 32, 16, ...?

Solution:

$$a_1 = 64, r = \frac{1}{2}, n = 5 \quad a_5 = ?$$

Since $r = \frac{1}{2}$, then $a_n = a_1 r^{n-1}$

$$a_5 = 64\left(\frac{1}{2}\right)^{5-1}$$

$$a_5 = 64\left(\frac{1}{2}\right)^4$$

$$a_5 = 64\left(\frac{1}{16}\right)$$

$$\mathbf{a_5 = 4}$$

Example 3. What is the 6th term of the geometric sequence if the third term is 16 and the common ratio is 2?

Solution:

$$a_3 = 16, r = 2, n = 6, a_1 = ? \quad a_6 = ?$$

Since $a_3 = 16$ and $r = 2$, then

$$16 = a_1(2)^{3-1}$$

$$16 = a_1(2)^2$$

$$\frac{16}{4} = \frac{a_1(4)}{4}$$

$$\mathbf{a_1 = 4}$$

$$a_6 = a_1 r^{n-1}$$

$$a_6 = 4(2)^{6-1}$$

$$a_6 = 4(2)^5$$

$$a_6 = 4(32)$$

$$\mathbf{a_6 = 128}$$

Did you understand the discussion? Are you ready to learn more about geometric sequence?

Geometric Means – terms between any two given terms of a geometric sequence.

Example 4. Insert a geometric mean between 2 and 50.

Solution: 2, __, 50

$$a_n = a_1 r^{n-1}$$

$$50 = 2(r)^{3-1}$$

$$50 = 2(r)^2$$

$$\frac{50}{2} = \frac{2r^2}{2}$$

$$25 = r^2$$

$$\sqrt{25} = \sqrt{r^2}$$

$$\mathbf{5 = r \text{ or } r = 5}$$

Therefore, **the geometric mean is 10** and the geometric sequence is 2, **10**, 50.

Example 5. Insert three geometric means between 3 and 48.

Solution: 3, __ , __ , __ , 48

$$a_n = a_1 r^{n-1}$$

$$48 = 3r^{5-1}$$

$$48 = 3r^4$$

$$\frac{48}{3} = \frac{3r^4}{3}$$

$$16 = r^4$$

$$\sqrt[4]{16} = \sqrt[4]{r^4}$$

$$\mathbf{2 = r \text{ or } r = 2}$$

Therefore, the **geometric means are 6, 12 and 24** and the geometric sequence is 3, 6, 12, 24, 48.

In the next discussion, you will learn how to find the sum of the n terms of a geometric sequence.

Example 6. What is the sum of the first 10 terms of the sequence 2, -2, 2, -2, ...?

$$\text{Solution: } 2 - 2 + 2 - 2 + 2 - 2 + 2 - 2 + 2 - 2 = 0$$

Example 7. What is the sum of the first 11 terms of the sequence 2, -2, 2, -2?

$$\text{Solution: } 2 - 2 + 2 - 2 + 2 - 2 + 2 - 2 + 2 - 2 + 2 = 2$$

If $r = -1$, then

$$S_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ a_1 & \text{if } n \text{ is odd} \end{cases}$$

Example 8. What is the sum of the first 10 terms of $2 + 2 + 2 + 2 + 2 + \dots$?

$$\text{Solution: } 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 10(2) = 20$$

If $r = 1$, then $a_1 + a_1 + a_1 + a_1 + \dots + a_1 =$

Example 9. Consider the geometric sequence 2, 4, 8, 16, 32, ... What is the sum of the first 5 terms?

If $r \neq 1$, then

$$S_n = \frac{a_1 - a_n r}{1 - r}$$

$$\text{Solution: } a_1 = 2, a_n = 32, r = 2$$

$$S_5 = \frac{2 - 32(2)}{1 - 2}$$

$$S_5 = \frac{2 - 64}{1 - 2}$$

$$S_5 = \frac{-62}{-1}$$

$$S_5 = 62$$

Example 10. What is the sum of the first 8 terms of the geometric sequence 3, 6, 12, 24, ...?

Solution: $a_1 = 3$ $r = 2$ $a_8 = ?$ $S_8 = ?$

$$a_8 = a_1 r^{n-1}$$

$$a_8 = 3(2)^{8-1}$$

$$a_8 = 3(2)^7$$

$$a_8 = 3(128)$$

$$\mathbf{a_8 = 384}$$

$$S_8 = \frac{a_1 - a_n r}{1 - r}$$

$$S_8 = \frac{3 - 384(2)}{1 - 2}$$

$$S_8 = \frac{3 - 768}{-1}$$

$$S_8 = \frac{-765}{-1}$$

$$\mathbf{S_8 = 765}$$

Is it possible to get the sum of an infinite number of terms in a geometric sequence?

Infinite Sequence- a function whose domain is the infinite set 1, 2, 3, ...

If $-1 < r < 1$, then

$$\mathbf{S = \frac{a_1}{1-r} -}$$

Example 11. What is the sum to infinity of $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$?

Solution: Since $r = \frac{1}{2}$, then $S = \frac{a_1}{1-r}$

$$S = \frac{\frac{1}{2}}{1 - \frac{1}{2}}$$

$$S = \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$s = 1$$

Example 12. What is the sum to infinity of $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} \dots$?

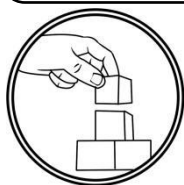
Solution: Since $r = \frac{1}{3}$, then $S = \frac{a_1}{1-r}$

$$S = \frac{\frac{1}{2}}{1 - \frac{1}{3}}$$

$$S = \frac{\frac{1}{2}}{\frac{2}{3}}$$

$$S = \frac{3}{4}$$

Did you enjoy learning about geometric sequence? You are now equipped with the necessary knowledge needed in this session. Keep going!



Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned in this lesson.

Enrichment Activity 1. Can You Geo It?

Use the n th term of a geometric sequence $a_n = a_1 r^{n-1}$ to answer the following questions.

1. What is the 9th term of the geometric sequence 8, 16, 32, ...?
2. Find the 5th term of a geometric sequence where the second term is 108 and the common ratio is 3.
3. In the geometric sequence 4, 12, 36, ... which term is 8748?

Enrichment Activity 2. What do You Mean?

Answer the following questions accurately.

4. What is the geometric mean between 5 and 405?
5. What is the sum of the two geometric means between 3 and 192?

Enrichment Activity 3. Sum All You Can!

Answer the following questions accurately.

6. What is the sum of the first 12 terms of the geometric sequence $-3+3-3+3-3+3+\dots$?
7. What is the sum of the first 10 terms of the geometric sequence $4, 4, 4, 4, \dots$?
8. Find the sum of the terms of a geometric sequence where the first term is 6, the last term is 4,374 and the common ratio is 3.
9. What is the sum of the first 7 terms of the geometric sequence $3, 6, 12, \dots$?
10. What is the sum of the infinite geometric sequence whose first term is -6 and a common ratio of $\frac{1}{4}$?

Well done! Let's dig deeper to the next activities! Do you still have the energy to go on? Come on, let's do it!



Deepen

At this point, you are going to apply the knowledge you have learned on geometric sequence. Try to answer the given problem.

Find the value of x so that $x + 2, 5x + 1, x + 11$ will form a geometric sequence.

Find the sum of the first 10 terms of the given sequence.



Gauge

Post-Assessment

Directions: Read carefully and analyze each item then select the letter that best corresponds to the question in a separate sheet of paper.

1. What is the next term in the geometric sequence $2, -8, 32$?
 A. -16 B. -64 C. -128 D. -256
2. What is the common ratio in the geometric sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$?
 A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. 2

3. Which geometric sequence has a common ratio of $\frac{1}{2}$?
- A. 80,40,20,10,5,... B. 120,80,53, $\frac{1}{3}$,35
 C. $\frac{1}{4}$, $\frac{1}{2}$, 1,2... D.-1,2,-4,8,...
4. What is the common ratio for the sequence given by the formula $a_n = 5(2)^{n-3}$?
- A.2 B.3 C.4 D.5
5. What is the 7th term of the geometric sequence 2, 6, 18, ...?
- A.54 B.162 C.486 D.1458
6. What is the third term of the sequence $a_n = 2(3)^{n-1}$?
- A.15 B.16 C.17 D.18
7. What is the tenth term of a geometric sequence with 32 as the fourth term and 2 as the common ratio?
- A.1024 B.1034 C.2048 D.2058
8. What is the first term of the geometric sequence if the third term is 3 and the sixth term is $\frac{1}{9}$?
- A.3 B.9 C.18 D.27
9. What is the value of k so that $2k + 1$, $3k + 4$, $7k + 6$ form a geometric sequence?
- A.2;-1 B.-2;1 C.2;1 D.-2;-1
10. What is the geometric mean between x and $\frac{1}{x}$?
- A.x B. $\frac{x}{2}$ C.1 D. $\frac{1}{2}$
11. What is the sum of the two geometric means between 3 and 24?
- A.10 B.18 C.21 D.27
12. Which formula will give the sum of an infinite geometric sequence?
- A. $S_n = \frac{a_1}{1-r}$ B. $S_n = \frac{a_1 - r a_n}{1-r}$ C. $S_n = a_1$ D. $S_n = n a_1$
13. Which of the following formula will give the sum of the first 15 terms of the sequence $\frac{3}{5}, -\frac{3}{5}, \frac{3}{5}, \dots$?
- A. $S_n = \frac{a_1}{1-r}$ B. $S_n = \frac{a_n}{1-r}$ C. $S_n = a_1$ D. $S_n = n a_1$
14. What is the sum of the first 10 terms of the sequence 3, 6, 12, 24, ...
- A. 2040 B. 2045 C. 3026 D. 3069
15. What is the sum of the infinite geometric series 8, -4, 2, -1, ...?
- A. $\frac{3}{16}$ B. $\frac{5}{16}$ C. $\frac{16}{3}$ D. $\frac{16}{5}$

References

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LINKS

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