Senior High School

General Mathematics
Module 11:
Annuity

AIRs - LM

GENERAL MATHEMATICS
Module 11: Annuity
Second Edition, 2021

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Senior High School

General Mathematics
Module 11:
Annuity

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

In your previous lesson, you learned how to calculate simple and compound interest. You were able to compute the maturity value, future value, and present value in a simple and compound interest environment. You were also able to solve problems involving simple and compound interest.

This module will help you understand another common business practice of payment that is annuity. House rental, life insurance premiums, installment payments, and labor wages are examples of annuities. Annuities take different forms. The mode of payments is basically the basis for the forms of annuities.

After going through this module, you are expected to:

1. illustrates simple and general annuities. **M11GM-IIc-1**
2. distinguishes between simple and general annuities. **M11GM-IIc-2**
3. find the future value and the present value of both simple annuities and general annuities. **M11GM-IIc-d-1**

Learning Objectives:
1. define annuity payment
2. identify the different types of annuity
3. understand the difference between simple annuity and general annuity
4. differentiate ordinary annuity and annuity due
5. calculate the future value and present value of simple annuities
6. calculate the future value and present value of general annuities
7. solve problems involving annuities

Before going on, check how much you know about the topic. Answer the pretest below in a separate sheet of paper

Pretest

Directions: Read each item carefully, and select the correct answer. Write the letter of your choice in separate sheet of paper.

1. What is the common ratio of the following sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$?
   A. $\frac{1}{2}$ B. 2 C. $\frac{1}{4}$ D. 4

2. Consider the geometric sequence: 2, 6, 18, 54, . . . If the last term is 13,122, how many terms are there in the sequence?
   A. 7 B. 8 C. 9 D. 10

3. Refer to item number 2, what is the sum of the geometric sequence?
   A. 9,842 B. 19,682 C. 19,862 D. 20,308

4. Which term refers to the series of equal payments at regular intervals?
   A. Annuity B. Bond C. Interest D. Stocks

5. Which is the periodic payment made at the beginning of each payment interval?
   A. Annuity Certain B. Annuity Due
   C. Contingent Annuity D. Ordinary Annuity

6. Periodic payment made at the end of each payment interval is _____.
   A. Annuity Due B. Future Annuity
   C. Ordinary Annuity D. Present Annuity

7. Which annuity is described when the payment interval is the same as the period of interest?
   A. General annuity B. Simple annuity
   C. Both A and B D. Neither A nor B

8. Rafael has been contributing ₱500 at the end of each quarter for the past 18 quarters to savings plan that earns 10% compounded quarterly. What forms of annuity is the given situation represents?
   A. Simple annuity and annuity due
   B. General annuity and annuity due
   C. Simple annuity and ordinary annuity
   D. General annuity and ordinary annuity

9. Refer to item number 8, what amount will Rafael accumulate if he continues with the plan for another year?
   A. ₱10,211.41 B. ₱11,482.15
   C. ₱12,942.23 D. ₱14,431.43

10. Clyde spends ₱500 per month on cigarettes. If he stops smoking and investing the same amount in a plan paying 10% compounded monthly, how much will he have after 2 years?

A. ₱10,491.22 B. ₱13,223.46
C. ₱13, 491.21 D. ₱14,500.00

For items 11-12, determine if the given situation describes:

I. Simple Annuity
II. General Annuity
III. Ordinary Annuity
IV. Annuity Due

11. Payments are made at the end of the month for a loan that charges 3.5% interest compounded quarterly.

A. I and III B. I and IV C. II and III D. II and IV

12. A deposit of ₱5,000 was made every January, April, July and October to an account that earns 5.6% interest compounded quarterly.

A. I and III B. I and IV C. II and III D. II and IV

13. If ₱2,000 is invested at the end of every year at 8% compounded semiannually, what will be the total value of the periodic investments after 20 years?

A. ₱89,743.12 B. ₱90,392.12
C. ₱90,882.22 D. ₱93,162.27

14. Find the present value of an ordinary annuity of Php1500 payable monthly for 2 years if money is worth 18% compounded monthly.

A. ₱15,019.22 B. ₱22,921.21
C. ₱28,917.92 D. ₱30,045.61

15. How much must be deposited now in a bank paying 13% compounded semiannually if it is desired that a semiannually withdrawal of Php5,200 can be made for 5 years?

A. ₱28,910.22 B. ₱30,920.21
C. ₱33,109.20 D. ₱37,381.92

Jumpstart

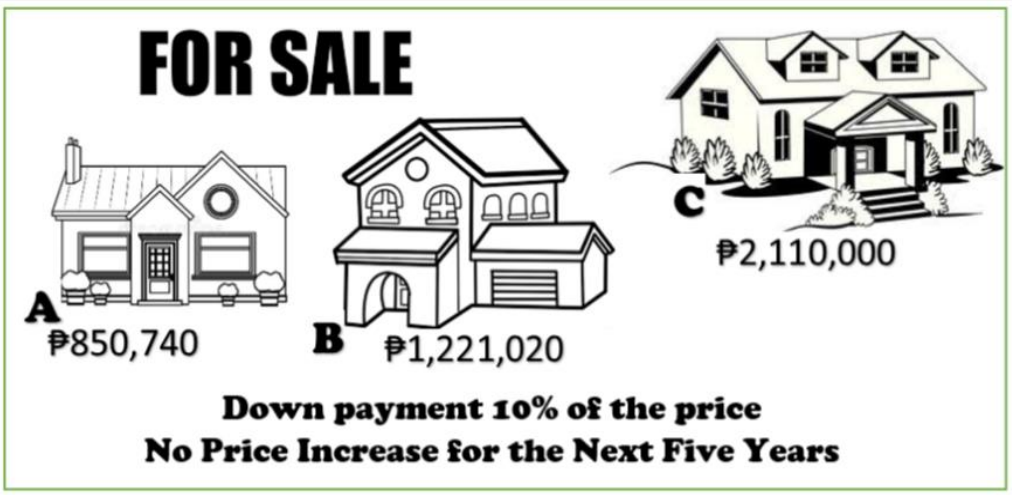
For you to understand the lesson well, do the activity below. Have fun and good luck!

ACTIVITY: Houses For Sale!

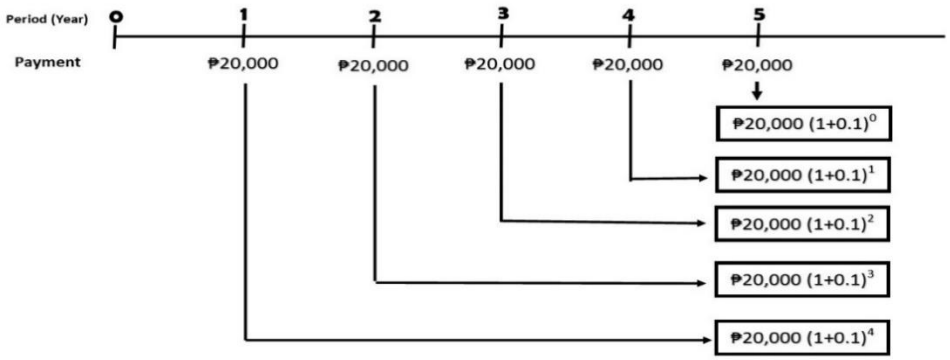
Engr. and Dr. Roxas are planning to have their own home but have limited budget. They went to a bank for some advice as to how they can produce enough amount for the down payment on a house and lot they have chosen

This is the advice of the bank: "If you will invest ₱20,000.00 at the end of each year for 5 years in an account that pays interest compounded annually, you will have the amount for the down payment of the house and lot at the end of 5 years."

Which house did the couple choose?



A. The diagram below will help you answer the questions the follow



1. Calculate the amount to which the first ₱20,000 will grow using the compound interest formula → _____
2. Calculate the amount to which the second ₱20,000 will grow using the compound interest formula → _____
3. Calculate the amount to which the third ₱20,000 will grow using the compound interest formula → _____
4. Calculate the amount to which the fourth ₱20,000 will grow using the compound interest formula → _____
5. Find the total of the separate ₱20,000 investments. Include the fifth or last ₱20,000 in the sum → _____

A. Enter the results from part (A) onto the following table

Period	**Amount in Exponential Form**	**Amount in Pesos**
1	$₱20{,}000(1.1)^4$	₱29,282.00
2		
3		
4		
5		
Total	=	

1. What kind of annuity represents the situation above (Simple Annuity or General Annuity)? _____
2. Determine whether the situation above describes an ordinary annuity or an annuity due. _____
3. Which house did the couple choose? _____
4. If the couple decided to invest ₱10,000 every 6 months instead of ₱20,000 at the end of each year, what kind of annuity it represents? _____
5. If the couple wanted to shoulder the down payment of House A for their son Roy, how much will they give him?_____

Discover

Annuity is a series of equal payments at regular intervals. Literally, annuity means payment made annually. But it can also refer to payment made at different segmental payment period.

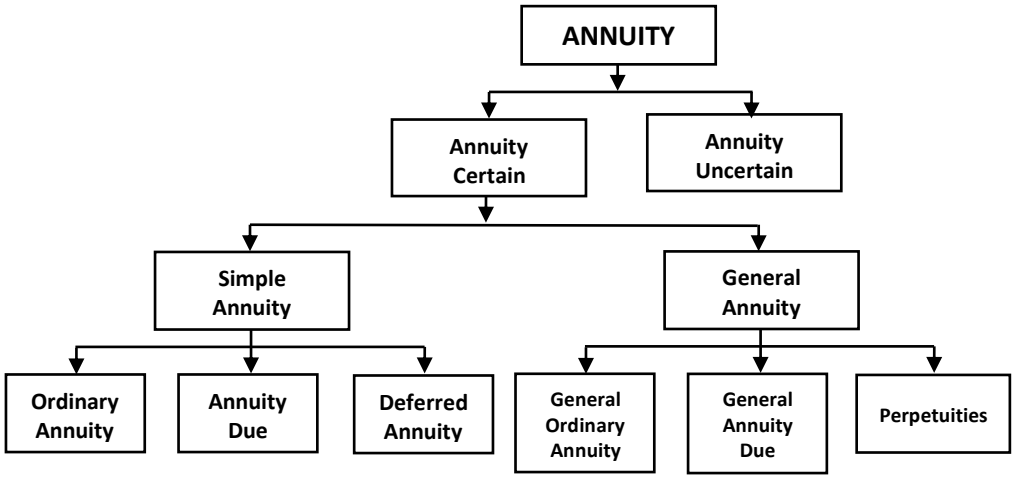
Classification of Annuity based on the Interest Period and Payment Interval

1. **Simple Annuity –** is an annuity where the payment interval coincides with the interest conversion period.
   For example, ₱10,000 invested at the end of every six months for 2 years and that 15% interest is paid compounded semiannually.
2. **General Annuity –** is an annuity where the payment interval does not coincide with the interest conversion period.
   For example, every 3 months, a father deposited ₱15,000 in a trust fund for the son's education for 8 years. The money earns 12% compounded monthly. The payment period is not equal to the interest period.

Classification of Annuity based on Payment Schedule

1. **Annuity Due –** is an annuity that is paid or received at the beginning of the time period.
2. **Ordinary Annuity –** is an annuity that is paid or received at the end of a time period.
3. **Deferred Annuity –** the periodic payment is not made at the beginning nor at the end of each payment interval, but at some later date.

The flowchart below gives the different kinds of annuities



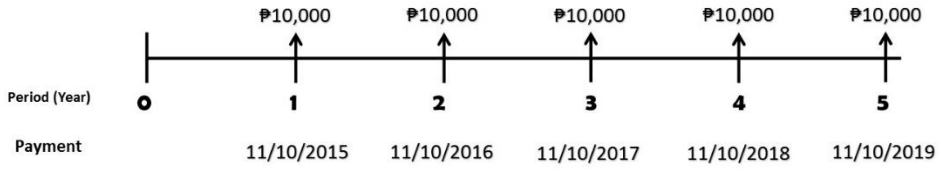
Illustrative Example 1:

Mr. Bingo deposited ₱10,000 on his birthday on November 10, 2015, and had deposited the same amount on the same date every year until his birthday in 2019. The bank credits 3.5% interest compounded annually to Mr. Bingo's account on the same date. Find the future value of Mr. Bingo's annuity.

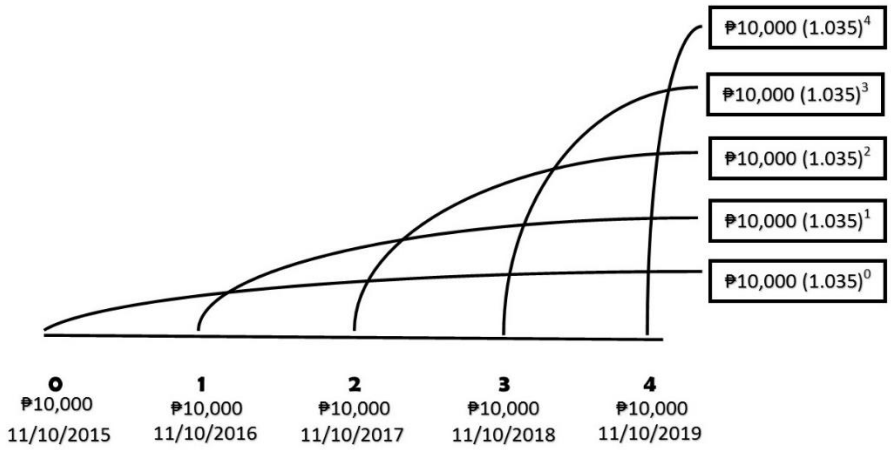
Solution:

This is an example of a simple ordinary annuity. Note that the interest conversion date (annually) and the date of payments (November 10) are the same. And the payments are made at the end of the interest conversion period.

Mr. Bingo's cash flow schedule of payments is shown in the scale below:



The future value of Mr. Bingo's annuity on November 10, 2019 can be illustrated in a time-value scale as follows:



$$FV = ₱10{,}000\,(1.035)^0 + ₱10{,}000\,(1.035)^1 + ₱10{,}000\,(1.035)^2 + ₱10{,}000\,(1.035)^3 + ₱10{,}000\,(1.035)^4$$

$$FV = ₱10{,}000\,(1) + ₱10{,}000\,(1.035) + ₱10{,}000\,(1.071225) + ₱10{,}000\,(1.108717875) + ₱10{,}000\,(1.147523001)$$

$$FV = ₱10{,}000 + ₱10{,}350 + ₱10{,}712.25 + ₱11{,}087.18 + ₱11{,}475.23$$

$$\boldsymbol{FV = ₱53{,}624.66}$$

This amount follows a **geometric series** with the first term $a_1 = ₱10{,}000$, common ratio $r = 1.035$, and the number of terms $n = 5$

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \rightarrow \quad S_5 = \frac{₱10{,}000(1-1.035^5)}{1-1.035}$$

$$S_5 = \frac{₱10{,}000(-0.1876863)}{-0.035}$$

$$S_5 = \frac{-₱1{,}876.863}{-0.035}$$

$$S_5 = ₱53{,}624.66$$

Since most annuities involve relatively small periodic payments over a longer period of time, they are affordable for the average person. Hence, formulas are needed to simplify computations of the future value of annuities.

> The ***future value (FV)*** of an annuity is the total accumulation of the payments and interest earned.
>
> The ***present value (PV)*** of an annuity is the principal that must be invested today to provide regular payments of annuity.

FORMULA **SIMPLE ORDINARY ANNUITY**

Future Value of Simple Ordinary Annuity	**Present Value of Simple Ordinary Annuity**
$FV = P\left[\frac{(1+i)^n - 1}{i}\right]$	$PV = P\left[\frac{1-(1+i)^{-n}}{i}\right]$
where FV = Future Value or Amount P = Periodic Payment i = interest rate per period	where PV = Present Value or Amount P = Periodic Payment i = interest rate per period
where $i = \frac{r}{m} = \frac{r \rightarrow \text{annual rate}}{m \rightarrow \text{number of conversion periods in a year}}$ m = 12→month m = 4→quarter m = 2→semiannual m = 1→year $n = m \cdot t$ = total number of conversion periods $t \rightarrow$ number of years	

Illustrative Example 2: Future Value of Simple Ordinary Annuity

Solve for the future value for ₱75,000 at 2.7% for 3 years compounded monthly.

Solution:

Since the interest is to be compounded monthly

$P = \frac{₱75{,}000}{12} = ₱\,6{,}250 \qquad i = \frac{r}{m} = \frac{0.027}{12} = 0.00225 \quad n = m \cdot t = 12\,(3) = 36$

The amount of annuity is $FV = P\left[\frac{(1+i)^n - 1}{i}\right]$

$FV = ₱6{,}250\left[\frac{(1+0.00225)^{36} - 1}{0.00225}\right] \qquad \boldsymbol{FV = ₱234{,}089.54}$

Illustrative Example 3: Future Value of Simple Ordinary Annuity

Jeffrey and Alfred are college friends. After graduation and being finally able to get a good job, they both plan for retirement as follows:

- Starting at the age of 24, Jeffrey deposited ₱10,000 at the end of each year for 36 years.
- Starting at the age of 42, Alfred deposited ₱20,000 at the end of each year for 18 years.

Who will have the greater amount at retirement if both annuities earn 12% compounded annually? How much is the difference between them?

Solution:

For Jeffrey's plan:

P = ₱10,000

$i = \frac{r}{m} = \frac{12\%}{1} = 0.12$

$n = m \cdot t = 1\,(36) = 36$

$FV = ?$

$FV = P\left[\frac{(1+i)^n - 1}{i}\right]$

$= ₱10{,}000\left[\frac{(1+0.12)^{36} - 1}{0.12}\right]$

$= \boldsymbol{₱4{,}844{,}631.16}$

For Alfred's plan:

P = ₱20,000

$i = \frac{r}{m} = \frac{12\%}{1} = 0.12$

$n = m \cdot t = 1\,(18) = 18$

$FV = ?$

$FV = P\left[\frac{(1+i)^n - 1}{i}\right]$

$= ₱20{,}000\left[\frac{(1+0.12)^{18} - 1}{0.12}\right]$

$= \boldsymbol{₱1{,}114{,}994.30}$

Jeffrey's retirement annuity is greater than Alfred's of ₱3,729,636.86.

NOTE: The example shows the value of time and the advantage of saving early on the amount of money.

Illustrative Example 4: Present Value of Simple Ordinary Annuity

Mr. Luiz wants to buy a simple ordinary annuity of ₱100,000 per year for 5 years. He wants to enter into a contract with Kabayan Company (KC) and deposit a certain amount so that later, the company will pay him ₱100,000 per year for 5 years. Suppose the company offered 2.5% compounded annually. Find the amount Mr. Luiz should deposit to KC.

Solution:

The problem calls for finding the present value of an annuity. It is the sum of the present values of all the 5 payments of the annuity.

Mr. Luiz' present value for each future payment $[P(1+i)^{-n}]$:

$₱100{,}000\ (1.025)^{-1}$ = ₱97,560.98 on the first year

$₱100{,}000\ (1.025)^{-2}$ = ₱95,181.44 on the second year

$₱100{,}000\ (1.025)^{-3}$ = ₱92,859.94 on the third year

$₱100{,}000\ (1.025)^{-4}$ = ₱90,595.06 on the fourth year

$₱100{,}000\ (1.025)^{-5}$ = ₱88,385.43 on the fifth year, for a total of **₱464,582.85**

Using the above formula, the present value of Mr. Luiz' annuity is

$$PV = P\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV = ₱100{,}000\left[\frac{1-(1+0.025)^{-5}}{0.025}\right] \quad = ₱100{,}000\left[\frac{1-(1.025)^{-5}}{0.025}\right]$$

$$PV = ₱100{,}000\left[\frac{0.1161457124}{0.025}\right] \qquad = ₱100{,}000\ (4.645828496)$$

$$\boldsymbol{PV = ₱464{,}582.85}$$

Illustrative Example 5: Future Value and Present Value of Simple Ordinary Annuity

Find the future value and the present value of annuity for ₱500,000 investment at 2% compounded quarterly for 5 years.

Solution:

The interest is to be compounded quarterly. Thus,

$$P = \frac{₱500{,}000}{4} = ₱\ 125{,}000 \qquad i = \frac{r}{m} = \frac{0.02}{4} = 0.005 \qquad n = m \cdot t = 4\ (5) = 20$$

The future value of annuity is $FV = P\left[\frac{(1+i)^n-1}{i}\right]$

$FV = ₱125{,}000\left[\frac{(1+0.005)^{20}-1}{0.005}\right] = ₱125{,}000\left[\frac{0.1048956}{0.005}\right]$

$FV = ₱125{,}000\ (20.97912)$

$\boldsymbol{FV = ₱2,622,390.00}$

The present value of annuity is $PV = P\left[\frac{1-(1+i)^{-n}}{i}\right]$

$PV = ₱125{,}000\left[\frac{1-(1+0.005)^{-20}}{0.005}\right] = ₱125{,}000\left[\frac{1-(1.005)^{-20}}{0.005}\right]$

$PV = ₱125{,}000\left[\frac{0.0949371}{0.005}\right] = ₱125{,}000\ (18.98742)$

$\boldsymbol{PV = ₱2,373,427.50}$

FORMULA **SIMPLE ANNUITY DUE**

Future Value of Simple Annuity Due	**Present Value of Simple Annuity Due**
$\boldsymbol{FV = P(1+i)\left[\frac{(1+i)^n-1}{i}\right]}$	$\boldsymbol{PV = P(1+i)\left[\frac{1-(1+i)^{-n}}{i}\right]}$
where $\boldsymbol{FV}$ = Future Value or Amount $\boldsymbol{P}$ = Periodic Payment $\boldsymbol{i}$ = interest rate per period	where $\boldsymbol{PV}$ = Present Value or Amount $\boldsymbol{P}$ = Periodic Payment $\boldsymbol{i}$ = interest rate per period
where $i = \frac{r}{m} = \frac{r \rightarrow \text{annual rate}}{m \rightarrow \text{number of conversion periods in a year}}$ $\boldsymbol{m}$ = 12→month $\boldsymbol{m}$ = 4→quarter $\boldsymbol{m}$ = 2→semiannual $\boldsymbol{m}$ = 1→year $\boldsymbol{n = m \cdot t}$ = total number of conversion periods $\boldsymbol{t}$ → number of years	

Illustrative Example 6: Future Value of Simple Annuity Due

Angelo's parents saved for his college education by investing ₱15,000 at the beginning of each year in an education plan that earns 6% compounded annually. What is the total amount of investment at the end of 16 years?

Solution:

The interest is to be compounded annually. Thus,

$P = \frac{₱15{,}000}{1} = ₱\,15{,}000 \qquad i = \frac{r}{m} = \frac{0.06}{1} = 0.06 \qquad n = m \cdot t = 1(16) = 16$

The future value of annuity is $FV = P(1+i)\left[\frac{(1+i)^n - 1}{i}\right]$

$$FV = ₱15{,}000(1+0.06)\left[\frac{(1+0.06)^{16}-1}{0.06}\right] \qquad = ₱15{,}000(1.06)\left[\frac{1.5403517}{0.06}\right]$$

$$FV = ₱15{,}900\ (25.672528)$$

$$\boldsymbol{FV = ₱408{,}193.20}$$

Illustrative Example 7: Future Value of Simple Annuity Due

Consider the given annuities:

- **Annuity A**: ₱1,000 deposited at the beginning of each month for 3 years at 12% compounded monthly.
- **Annuity B**: ₱3,000 deposited at the beginning of each quarter for 3 years at 12% compounded quarterly.

Calculate the amount of each annuity. Compare the two annuities.

Solution:

For Annuity A: P = ₱1,000 $\qquad i = \frac{r}{m} = \frac{12\%}{12} = 0.01 \qquad n = m \cdot t = 12\ (3) = 36$

$$FV = P(1+i)\left[\frac{(1+i)^n - 1}{i}\right] \qquad FV = ₱1{,}000(1+0.01)\left[\frac{(1+0.01)^{36}-1}{0.01}\right]$$

$$FV = ₱1{,}000(1.01)\left[\frac{0.4307687836}{0.01}\right] \qquad \underline{\boldsymbol{FV = ₱43{,}507.65}}$$

For Annuity B: P = ₱3,000 $\qquad i = \frac{r}{m} = \frac{12\%}{4} = 0.03 \qquad n = m \cdot t = 4\ (3) = 12$

$$FV = P(1+i)\left[\frac{(1+i)^n - 1}{i}\right] \qquad FV = ₱3{,}000(1+0.03)\left[\frac{(1+0.03)^{12}-1}{0.03}\right]$$

$$FV = ₱3{,}000(1.03)\left[\frac{0.4257608868}{0.03}\right] \qquad \underline{\boldsymbol{FV = ₱43{,}853.37}}$$

The amount in Annuity A is less than the amount in Annuity B.

Illustrative Example 8: Present Value of Simple Annuity Due

Kat borrows money from One Network Bank for house renovation. She is to pay ₱50,000 yearly at the beginning of each year for the period of 10 years at an interest rate of 9% compounded annually. How much did Kat borrow?

Solution:

Given: $P = \frac{₱50{,}000}{1} = ₱\,50{,}000 \qquad i = \frac{9\%}{1} = 0.09 \qquad n = 10$

$$\boldsymbol{PV = P(1+i)\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

$$PV = ₱50{,}000(1+0.09)\left[\frac{1-(1+0.09)^{-10}}{0.09}\right] \qquad = ₱50{,}000(1.09)\left[\frac{0.5775891931}{0.09}\right]$$

$$PV = ₱54{,}500\ (6.417657701)$$

$$\boldsymbol{PV = ₱349{,}762.34}$$

FORMULA **GENERAL ORDINARY ANNUITY**

Future Value of General Ordinary Annuity	**Present Value of General Ordinary Annuity**
$\boldsymbol{FV = P\left[\frac{(1+i)^n - 1}{(1+i)^k - 1}\right]}$	$\boldsymbol{PV = P\left[\frac{1-(1+i)^{-n}}{(1+i)^k - 1}\right]}$

where $\boldsymbol{P}$ = Regular Payment

$\boldsymbol{i}$ = interest rate per compounding period

where $i = \frac{r}{m} = \frac{r \rightarrow \text{annual rate}}{m \rightarrow \text{number of conversion periods in a year}}$

$\boldsymbol{n = m \cdot t}$ = total number of conversion periods

$\boldsymbol{t} \rightarrow$ number of years

$k = \frac{p}{c} = \frac{p \rightarrow \text{number of months in payable interval}}{c \rightarrow \text{number of months in compounding period}}$

Illustrative Example 9: Future Value and Present Value of General Ordinary Annuity

Find the future value and present value of an ordinary annuity of ₱2,000 payable annually for 9 years if the money is 5% compounded quarterly.

Solution:

Given: $P = ₱2{,}000 =$ $i = \frac{5\%}{4} = 0.0125$ $n = 9 \cdot 4 = 36$

$p = 12$ $c = 3$ $k = \frac{p}{c} = \frac{12}{3} = 4$

The future value of annuity is $FV = P\left[\frac{(1+i)^n - 1}{(1+i)^k - 1}\right]$

$FV = ₱2{,}000\left[\frac{(1+0.0125)^{36}-1}{(1+0.0125)^4-1}\right]$ $= ₱2{,}000\left[\frac{(1.0125)^{36}-1}{(1.0125)^4-1}\right]$

$FV = ₱2{,}000\left[\frac{0.5639438187}{0.05094533691}\right]$ $= ₱2{,}000\ (11.0695866)$

$\boldsymbol{FV = ₱22{,}139.17}$

The present value of annuity is $PV = P\left[\frac{1-(1+i)^{-n}}{(1+i)^k - 1}\right]$

$PV = ₱2{,}000\left[\frac{1-(1+0.0125)^{-36}}{(1+0.0125)^4-1}\right]$ $= ₱2{,}000\left[\frac{1-(1.0125)^{-36}}{(1.0125)^4-1}\right]$

$PV = ₱2{,}000\left[\frac{0.3605908422}{0.05094533691}\right]$ $= ₱2{,}000\ (7.077995)$

$\boldsymbol{PV = ₱14{,}155.99}$

Illustrative Example 10: Future Value of General Ordinary Annuity

Twenty-five thousand pesos will be invested in an account at the end of each year at 4% compounded semi-annually. Find the size of the fund at the beginning of the 16th year.

Solution:

Given: $P = ₱25{,}000$ $m = 2$ $t = 15$ $n = 2 \cdot 15 = 30$

$i = \frac{4\%}{2} = 0.02$ $p = 12$ $c = 6$ $k = \frac{12}{6} = 2$

The future value of annuity is $FV = P\left[\frac{(1+i)^n - 1}{(1+i)^k - 1}\right]$

$$FV = ₱25{,}000\left[\frac{(1+0.02)^{30}-1}{(1+0.02)^2-1}\right] \quad = ₱25{,}000\left[\frac{(1.02)^{30}-1}{(1.02)^2-1}\right]$$

$$FV = ₱25{,}000\left[\frac{0.8113615841}{0.0404}\right] \quad = ₱25{,}000\ (20.08320753)$$

$$\boldsymbol{FV = ₱502{,}080.1}$$

Illustrative Example 11: Present Value of General Ordinary Annuity with Down Payment

The latest cell phone sells for ₱10,000 down payment (DP) and 900.00 every end of each quarter for 3 years at a rate of 8% compounded semi-annually. Find the cash equivalent (CE) of the cell phone.

Solution:

Given: $P = ₱900.00$ $m = 2$ $t = 3$ $n = 2 \cdot 3 = 6$

$i = \frac{8\%}{2} = 0.04$ $p = 3$ $c = 6$ $k = \frac{3}{6} = 0.5$

The annuity value is $PV = P\left[\frac{1-(1+i)^{-n}}{(1+i)^k - 1}\right]$

$$PV = ₱900\left[\frac{1-(1+0.04)^{-6}}{(1+0.04)^{0.5}-1}\right] \quad = ₱900\left[\frac{1-(1.04)^{-6}}{(1.04)^{0.5}-1}\right]$$

$$PV = ₱900\left[\frac{0.2096854743}{0.019803903}\right] \quad = ₱900\ (10.58808848)$$

$$\boldsymbol{PV = ₱9{,}529.28}$$

CE = DP + PV = ₱10,000 + ₱9,529.28 = **₱19,529.28**

FORMULA **GENERAL ANNUITY DUE**

Future Value of General Annuity Due	**Present Value of General Annuity Due**
$FV = P\left[\frac{(1+i)^n - 1}{i}\right]\left[\frac{i}{(1+i)^k - 1} + i\right]$	$PV = P\left[\frac{1-(1+i)^{-n}}{i}\right]\left[\frac{i}{(1+i)^k - 1} + i\right]$

where FV = Future Value

PV = Present Value

P = Annuity Payment

i = interest rate per compounding period

where $i = \frac{r}{m} = \frac{r \rightarrow \text{annual rate}}{m \rightarrow \text{number of conversion periods in a year}}$

$n = m \cdot t$ = total number of conversion periods

$t \rightarrow$ number of years

$k = \frac{p}{c} = \frac{p \rightarrow \text{number of months in payable interval}}{c \rightarrow \text{number of months in compounding period}}$

Illustrative Example 12: Present Value of General Annuity Due

Find the present value of an annuity due of ₱10,000 payable quarterly for 10 years if the money is worth 6% compounded semi-annually?

Solution:

Given: $P = ₱10{,}000$ $m = 2$ $t = 10$ $n = 2 \cdot 10 = 20$

$i = \frac{6\%}{2} = 0.03$ $p = 3$ $c = 6$ $k = \frac{3}{6} = 0.5$

$$PV = P\left[\frac{1-(1+i)^{-n}}{i}\right]\left[\frac{i}{(1+i)^k - 1} + i\right]$$

$$PV = ₱10{,}000\left[\frac{1-(1+0.03)^{-20}}{0.03}\right]\left[\frac{0.03}{(1+0.03)^{0.5} - 1} + 0.03\right]$$

$$PV = ₱10{,}000\left[\frac{1-(1.03)^{-20}}{0.03}\right]\left[\frac{0.03}{(1.03)^{0.5} - 1} + 0.03\right]$$

$$PV = ₱10{,}000(14.87747486)(2.044889157)$$

$$\boldsymbol{PV = ₱304{,}227.87}$$

Illustrative Example 13: Future Value of General Annuity Due

Rocky wants to save up to ₱100,000 for his college. He deposits ₱3,500 at the beginning of each month in an account that earns 4% per year compounded semi-annually. Will Rocky have enough money saved at the end of 2 years?

Solution:

Given: $P = ₱3{,}500$ $m = 2$ $t = 2$ $n = 2 \cdot 2 = 4$

$i = \frac{4\%}{2} = 0.02$ $p = 1$ $c = 6$ $k = \frac{1}{6}$

$$FV = P\left[\frac{(1+i)^n - 1}{i}\right]\left[\frac{i}{(1+i)^k - 1} + i\right]$$

$$FV = ₱3{,}500\left[\frac{(1+0.02)^4 - 1}{0.02}\right]\left[\frac{0.02}{(1+0.02)^{1/6} - 1} + 0.02\right]$$

$$FV = ₱3{,}500\left[\frac{(1.02)^4 - 1}{0.02}\right]\left[\frac{0.02}{(1.02)^{1/6} - 1} + 0.02\right]$$

$$FV = ₱3{,}500(4.121608)(6.069807476)$$

$$\boldsymbol{FV = ₱87{,}560.78}$$

Since ₱87,560.78 is less than ₱100,000, Rocky will not have enough money at the end of 2 years.

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Enrichment Activity 1.a:

Directions: Classify the following as to Simple Annuity/General Annuity and as to Ordinary Annuity/Annuity Due using the given situations:

Time of Payment	Length of Annuity	Interest Rate per year	Frequency of Compounding	Simple Annuity /General Annuity	Ordinary Annuity /Annuity Due
1. end of each month	4 years	4%	monthly		
2. end of each quarter	8 years	6%	monthly		
3. every January and July	10 years	10%	Semi-annually		
4. beginning of each year	5 years	8%	annually		
5. end of each month	6 years	$9\frac{1}{2}\%$	quarterly		

Enrichment Activity 1.b:

Directions: Determine ***i*** (interest rate per compounding period and ***n*** (the number of compounding periods) for each annuity above:

1. i = _____ n = _____
2. i = _____ n = _____
3. i = _____ n = _____
4. i = _____ n = _____
5. i = _____ n = _____

Enrichment Activity 2:

Directions: Use the given situation to give the values of the following variables:

The present value of an annuity of ₱5000 every end of 3 months for 10 years when the interest rate is 4% compounded annually is ₱164,631.30.

1. P = _____
2. t = _____
3. m = _____
4. n = _____
5. r = _____
6. i = _____
7. c = _____
8. p = _____
9. k = _____
10. PV = _____

Individual Assessment 1.a: Simple Ordinary Annuity

Directions: Calculate the future value of #1, 2, & 3 and present value of #4 & 5.

	Principal	**Interest Rate**	**Mode of Payment**	**Length of Annuity**	**Future Value**
1.	₱20,000	2%	Annually	3 years	
2.	₱1,500	7%	Monthly	5 years	
3.	₱4,300	3.50%	Quarterly	4 years	
	Principal	**Interest Rate**	**Mode of Payment**	**Length of Annuity**	**Present Value**
4.	₱5,000	3.20%	Semi-annually	10 years	
5.	₱3,800	6%	Bimonthly	2 years	

Individual Assessment 1.b: Simple Annuity Due

Directions: Calculate the future value of #1 & 2 and present value of #3, 4 & 5.

	Principal	Interest Rate	Mode of Payment	Length of Annuity	Future Value
1.	₱1,500	3.2%	Monthly	5 years	
2.	₱3,000	8.2%	Bi-Monthly	3 years	
	Principal	**Interest Rate**	**Mode of Payment**	**Length of Annuity**	**Present Value**
3.	₱4,500	5.0%	Quarterly	5 years	
4.	₱10,000	7.5%	Semi-annually	4 years	
5.	₱8,500	5.5%	Annually	7 years	

Individual Assessment 2:

Directions: Try to analyze and solve these problems

1. In a savings account crediting 3% annual interest rate compounded monthly, ₱1,000 is deposited at the end of each month. What is the value of the account at the end of 18 months?

2. What income will be paid at the end of every month for 10 years if the present value is ₱360,000, and the interest is paid 6% monthly?

3. How much money is accumulated if ₱5,800 is paid at the end of every month for 4 years at 5% compounded quarterly?

4. Find the present value of an annuity ₱10,000 payable at the end of each quarter for 5 years if the rate of interest is 10% compounded monthly.

5. Find the future value given the situation of item number 4.

Deepen

Let's try harder in solving the problems below.

1. Consider the following annuities:

	Annuity X	**Annuity Y**
Monthly investment:	₱2,000	₱4,000
Start:	Now	10 years from now
Time Period:	20 years	10 years
Annual Interest rate:	7%	7%
Compounding period:	Monthly	Monthly

   a. Find the total deposit and the amount of each annuity
   b. Why are the amounts not the same even if the total deposits are the same?

2. Consider the following annuities:

Annuity 1		**Annuity 2**	
Monthly investment:	₱1,000	Quarterly investment:	₱3,000
Time Period:	5 years	Time Period:	5 years
Annual Interest rate:	8%	Annual Interest rate:	8%
Compounding period:	Monthly	Compounding period:	Quarterly

   a. Find the total deposit and the amount of each annuity
   b. Why are the amounts not the same even if the total deposits are the same?

3. As a reward, Teddy receives this offer where he has to choose from one of these payment plans.
   a. A single cash payment of 320,000 to be received immediately
   b. Monthly reward payment of 4,800 for 10 years
      If the money can be invested at 6% per year compounded monthly which offer would you think Teddy should accept?

MATH PROBLEM SOLVING RUBRIC:

Understands the problem (4) Understands enough to solve part of the problem or to get part of the solution (3) Doesn't understand enough to get started or make progress (2)
Applies completely appropriate procedures (3) Applies some appropriate procedures (2) Applies inappropriate procedures (1)
Correct solution (3) Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly (2) No answer or wrong answer based upon an inappropriate plan (1)

Gauge

Directions: Read each item carefully and select the correct answer. Write the letter of your choice on a separate sheet of paper.

1. In a typical loan amortization schedule, the *amount of interest* paid each period _____ .
   A. remains constant with each payment
   B. increases with each payment
   C. decreases with each payment
   D. varies with each payment

2. In a typical loan amortization schedule, the total *amount of money paid* each period _____ .
   A. remains constant with each payment
   B. increases with each payment
   C. decreases with each payment
   D. varies with each payment

3. In which case will an investor receive the most interest?
   A. 10%, compounded continuously.
   B. 10%, compounded annually.
   C. 10%, compounded monthly.
   D. 10%, compounded daily

4. At 12% interest compounded quarterly for 5 years, what is the interest rate and the number of periods that will be computed before a present or future value table can be used?
   A. 12%, 5 periods
   B. 6%, 10 periods
   C. 4%, 15 periods
   D. 3%, 20 periods

5. When comparing an annuity due with an ordinary annuity with the same payment and duration, the annuity due will always have a _____ present value and will always have a _____ future value.
   A. higher; higher
   B. higher; lower
   C. lower; higher
   D. lower; lower

6. You want to buy an ordinary annuity that will pay you ₱4,000 a year for the next 20 years. You expect annual interest rates will be 8 percent over that time period. The maximum price you would be willing to pay for the annuity is closest to
   A. ₱32,000 B. ₱39,272 C. ₱40,000 D. ₱80,000.

7. In 3 years, you are to receive ₱15,000. If the interest rate were to suddenly increase, the present value of that future amount to you would _____.
   A. increase
   B. decrease
   C. remain unchanged
   D. cannot be determined

8. Assume that the interest rate is greater than zero. Which of the following cash-inflow streams should you prefer?

	Year 1	Year 2	Year 3	Year 4
A.	₱4,000	₱3,000	₱2,000	₱1,000
B.	₱1,000	₱2,000	₱3,000	₱4,000
C.	₱2,500	₱2,500	₱2,500	₱2,500

D. Any of the above, since they each sum to ₱10,000

9. Claire spends ₱850 per month on expensive clothes. If she stops buying expensive clothes and investing the same amount in a plan paying 15% compounded monthly, how much will she have after 4 years?

A. ₱51,419.41 B. ₱52,928.11
C. ₱55,444.13 D. ₱60,192.41

10. Find the future value of an investment of ₱1,000 at the end of each year for 3 years if the rate of interest is 20% compounded annually.

A. ₱4,368 B. ₱3,640 C. ₱3,600 D. ₱2,106

11. If college costs are increasing at a 10% annual rate, and if one year of college now costs ₱30,000, how much will one year probably cost 20 years from now?

A. ₱90,000 B. ₱90,932
C. ₱201,825 D. ₱607,450

12. Determine the present value of an annuity if ₱6,800 paid at the end of subsequent calendar quarter for 7 years and the money is worth 6% compounded quarterly

A. ₱162,292.11 B. ₱154,541.67
C. ₱151,892.14 D. ₱145,299.24

13. How much money is accumulated if 3900 is paid at the end of every month for 2 years at 6% compounded quarterly?

A. ₱101,273.12 B. ₱99,155.91
C. ₱98,664.22 D. ₱95,009.41

14. Find the present value of an annuity which pays ₱2700 at the end of every 3 months for 6 years, if the money is worth 18% compounded monthly

A. ₱25,919.12 B. ₱30,018.13 C. ₱38,874.17 D. ₱40,982.64

15. An 8-year annuity has a present value of $1,000. If the interest rate is 5%, the amount of each annuity payment is closest to which of the following?

A. $104.72 B. $109.39 C. $147.36 D. $154.73

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