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# Mathematics

## Quarter 2 – Module 2:
## Solving Problems Involving Variation

**AIRs - LM**

**MATHEMATICS 9**
**Quarter 2 - Module 2: Solving Problems Involving Variation**
**Second Edition, 2021**



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# Mathematics

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## Solving Problems Involving Variation

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# Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

# *Target*

Your goal in this module is to see the application of variations in real-life situation. There are many ways to apply your understanding in variations with the application of mathematics principles and concepts. In this module, you will learn how to solve word problems involving Variations. You will discover the significance of having mastery on the various methods of solving problems involving variations as applied in real life situations.

After going through this module, you are expected to:

**Learning Competency**

- Solve problems involving variation. **(M9AL-IIb-c-1)**

**Subtasks**

1. Enumerate the different ways of solving problems involving variations.
2. Translate a verbal phrase into mathematical phrase.
3. Apply the steps in solving word problems involving variations.

*Let's find out how much you already know about this module.*
*Answer the pre-assessment in a separate sheet of paper.*

## Pre-Assessment

*Directions:* Find out how much you already know about this module. Choose the letter of the correct answer and write it on a separate sheet of paper.

1. If 36 men can do a piece of work in 25 days, in how many days will 15 men do it?
   A. 40
   B. 50
   C. 60
   D. 70
2. If y varies inversely as x, and y = 8 when x = 2, what is the constant of variation?
   A. 14
   B. 15
   C. 16
   D. 17

3. If y varies directly as x and y = 45 when x = 5, what is the value of y when x=13?
    A. 115
    B. 116
    C. 117
    D. 118
4. If y varies directly as x and inversely as the square of z, and when x = 32, y = 6 and z =4. What is the value of x when y = 10 and z = 3?
    A. 15
    B. 20
    C. 25
    D. 30
5. Shaun eats six egg whites a day with a protein content of 21.6 grams. How much is his protein intake, if he consumes 9 egg whites a day?
    A. 30.4 grams
    B. 31.4 grams
    C. 32.4 grams
    D. 33.4 grams
6. At a science exhibition, students learnt that it takes 5 hours for a block of ice to melt at 32° C. How many time taken to melt a similar-sized block of ice at 20° C?
    A. 7 hours
    B. 8 hours
    C. 9 hours
    D. 10 hours
7. If y varies inversely as x and y=12 when x=8, what is the value of x when y=4?
    A. 22
    B. 23
    C. 24
    D. 25
8. Daryl hired a tile specialist to do up the walls in his home. The specialist used 24 tiles of the same size to cover a wall of 156 square feet. How many similar-sized tiles will be required to fill a wall measuring an area of 338 square feet?
    A. 46 tiles
    B. 48 tiles
    C. 50 tiles
    D. 52 tiles
9. It took 500 workers to build a suspension bridge in 400 days. How many days will 800 workers take to complete the same task?
    A. 200 days
    B. 250 days
    C. 300 days
    D. 350 days

10. If 3 men can do a portion of a job in 8 days, how many men can do the same job in 6 days?
    A. 4
    B. 5
    C. 6
    D. 7
11. In a business, if Allan can earn Php 7500.00 in 2.5 years. At the same rate, how much his earning for 4 years?
    A. Php11 000.00
    B. Php12 000.00
    C. Php13 000.00
    D. Php14 000.00
12. If 5 men can paint a house in 18 hours, how many men will be able to paint it in 10 hours?
    A. 7
    B. 8
    C. 9
    D. 10
13. On a map, 180 miles are represented by 4 inches. How many miles are represented by 6 inches?
    A. 240 miles
    B. 250 miles
    C. 260 miles
    D. 270 miles
14. Time varies inversely as speed if the distance is constant. A trip takes 4 hours at 80 km/h. How long does it take at 64 km/h?
    A. 5 hours
    B. 6 hours
    C. 7 hours
    D. 8 hours
15. The bending of a beam varies directly as its mass. A beam is bent 20mm by a mass of 40 kg. How much will the beam bend with a mass of 100 kg?
    A. 20 mm
    B. 30 mm
    C. 40 mm
    D. 50 mm

# *Jumpstart*

## Activity1. Find My Constant and My Equation!

*Directions:* Find the constant of variation and write the equation representing the relationship between the quantities in each of the following.

1. y varies directly as x and y = 81 when x = 3
2. m varies directly as n and m = 120 when n = 5
3. y varies inversely as x and y = 18 when x = 2/3
4. p varies jointly as q and square of r and p = 240 when q = 6 and r = 2
5. w varies directly as x and inversely as z and w = 18 when x = 9 and z = 3

## Activity 2: Find My Value!

*Directions:* Solve for the indicated variable in each of the following.

1. If y varies inversely as x and y = 36 when x = 5, find y when x = 6.
2. If z varies jointly as x and y and z = 240 when x = 8 and y = 2, find z when x = 7 and y = 6.
3. If x varies directly as the square of y and inversely as z and x =12 when y =3 and z = 6, find x when y =9 and z =6.
4. If y varies directly as x and y = -18 when x = 9, find y when x = 7.
5. If m varies inversely as n and m = 8 when n = 3, find m when n = 12.

# *Discover*

## Solving Problems Involving Variation

We will now discuss the concept of variations and its applications to real life situations.

### Direct Variation

There is direct variation whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"
"y is directly proportional to x" and
"y is proportional to x"

may be translated mathematically as y = kx, where k is the constant of variation. For two quantities x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Example 1:
The amount of paint **P** needed to paint the walls of a room varies directly as the area **A** of the wall. If 2 gallons of paint is needed to paint a 40 sq. meter wall, how many gallons of paint are needed to paint a wall with an area of 100 sq. meters?

*Solution:*

| Solve first for k. | Solve for the number of gallons of paint |
|---|---|
| P = kA | P = kA |
| 2 = k(40) | P = $\frac{1}{20}$(100) |
| k = 2/40 | P = 5 gallons of paint |
| k = $\frac{1}{20}$ | |

**Inverse Variation**

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or y=$\frac{k}{x}$. The statement, "y varies inversely to x", translate to y = $\frac{k}{x}$, where k is the constant of variation.

Example 2:
Cathy and Anthony are figuring out a way to balance themselves on a seesaw. Cathy who weighs 15 kilograms sits 2 meters from the fulcrum. Anthony who weighs 20 kilograms tried sitting at different distances from the fulcrum in order to balance the weight of Cathy. If you were Anthony, how far from the fulcrum should you sit?

To balance the weight of Cathy, Anthony has to sit at a distance closer to the fulcrum.

The relation shows that the distance d varies inversely as the weight w and can be transformed into a mathematical equation as

$$d = \frac{k}{w}$$

We can now solve for distance from the fulcrum where Anthony has to sit.

Solution:

Solve first for k.

$d = \frac{k}{w}$

$15 = \frac{k}{2}$

$k = 15(2)$

$k = 30$

Solve for the distance of Anthony from the fulcrum.

$d = \frac{k}{w}$

$d = \frac{30}{20}$

$d = 1.5$ m

Hence, Anthony has to sit 1.5 meters from the fulcrum.

**Joint Variation**

Joint variation describes a situation where one variable depends on two (or more) other variables, and varies directly as each of them when the others are held constant. We say z varies jointly as x and y if $z = kxy$ for k is the constant of variation.

Example 3:
The mass of a rectangular sheet of wood varies jointly as the length and the width. When the length is 20 cm and the width is 10 cm, the mass is 400 grams. Find the mass when the length is 25 cm and the width is 20 cm.

Solution:

Solve first for constant k.

$m = klw$

$400 = k\,(20)(10)$

$400 = k(200)$

$k = \frac{400}{200}$

$k = 2$

Solve for the mass.

$m = klw$

$m = 2(25)(20)$

$m = 1000$ grams

**Combined Variation**

Combined variation describes a situation where a variable depends on two (or more) other variables and varies directly with some of them and varies inversely with others (when the rest of the variables are held constant).
The statement "z varies directly as x and inversely as y" means $z = \frac{kx}{y}$ , or $k = \frac{zy}{x}$ , where k is the constant of variation.

Example 4:
The force of attraction, F of a body varies directly as its mass m and inversely as the square of the distance d from the body. Wen m = 8 kilograms and d = 5 meters, F = 100 Newtons. Find F when m = 12 kilograms and d = 5 meters.

Solution:

Solve for the constant k.

$F = \frac{km}{d^2}$

$k = \frac{Fd^2}{m}$

$k = \frac{100(5)^2}{8}$

$k = \frac{2500}{8}$

$k = 312.5$

Solve for the force of attraction F.

$F = \frac{km}{d^2}$

$F = \frac{(312.5)(12)}{5^2}$

$F = \frac{3750}{25}$

F = 150 Newtons

## *Explore*

### Activity 3: How Well Do You Understand?

*Directions:* Solve the following problems.

1. A mailman can sort out 738 letters in 6 hours. If the number of sorted letters varies directly as the number of working hours, how many letters can be sorted out in 9 hours?
2. The number of days needed in repairing a house varies inversely as the number of men working. It takes 15 days for 2 men to repair the house. How many men are needed to complete the job in 6 days?
3. The area of triangle varies jointly as the base and the height. A triangle with a base of 8 cm and a height of 9 cm has an area of 36 square centimeters. Find the area when the base is 10 cm and the height is 7 cm.
4. The current ***I*** varies directly as the electromotive force ***E*** and inversely as the resistance R. If in a system a current of 20 amperes flows through a resistance of 20 ohms with an electromotive force of 100 volts, find the current that 150 volts will send through the system.

# *Deepen*

## Activity 4: Think Deeper!

*Directions:* Solve the following problems.

1. The amount of gasoline used by a car varies jointly as the distance travelled and the square root of the speed. Suppose a car used 25 liters on a 100 kilometer trip at 100 km/hr. About how many liters will it use on a 192 kilometer trip at 64 km/hr?

2. The area ***A*** of a triangle varies jointly as the base ***b*** and the altitude ***h*** of the triangle. If A = 65 $cm^2$ when b = 10 cm and h = 13 cm, find the area of a triangle whose base is 8 cm and whose altitude is 11 cm.

3. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If a wire 30 meters long, and 0.75 mm in diameter has a resistance of 25 ohms, find the length of a wire of the same material whose resistance and diameter are 30 ohms and 1.25 mm, respectively.

4. The weight ***W*** of a cylindrical metal varies jointly as its length ***l*** and the square of the diameter ***d*** of its base.
    a. If $W$ = 6 kg when $l$ = 6 cm and $d$ = 3 cm, find the equation of variations.
    b. Find $l$ when $W$ = 10 kg and $d$ = 2 cm.
    c. Find $W$ when $d$ = 6 c and $l$ = 1.4 cm

# *Gauge*

## Post - Assessment

*Directions:* Read and analyze the following questions carefully. Choose the letter of the correct answer. Write it on your answer sheet.

1. If car covers 102 km in 6.8 litres of petrol, how much distance will it cover in 24.2 litres of petrol?
    A. 303 km
    B. 323 km
    C. 333 km
    D. 363 km

2. If 3 persons can weave 168 shawls in 1 4 days, how many shawls will be woven by 8 persons days?
    A. 153
    B. 160
    C. 175
    D. 178
3. If 32 men can reap a field in 15 days, in how many days can 20 men reap the same field?
    A. 21 days
    B. 22 days
    C. 23 days
    D. 24 days
4. The number of kilograms of water in a person's body varies directly as the person's mass. A person with a mass of 100 kg contains 70 kg of water. How many kilograms of water are in a person with a mass of 50 kg?
    A. 30 kg
    B. 30.5 kg
    C. 35 kg
    D. 35.5 kg
5. The volume $V$ of a gas varies inversely with the pressure $P$. When the volume is 75 inches cubed, the pressure is 30 pounds per square inch. What is the volume when the pressure is 25 pounds per square inch?
    A. 60 cubic inches
    B. 70 cubic inches
    C. 80 cubic inches
    D. 90 cubic inches
6. The volume of a pyramid varies jointly as its height and the area of its base. A pyramid with a height of 12 feet and a base with area of 23 square feet have a volume of 92 cubic feet. What is the volume of a pyramid with a height of 17 feet and a base with an area of 27 square feet?
    A. 153 cubic feet
    B. 163 cubic feet
    C. 173 cubic feet
    D. 183 cubic feet
7. The number of minutes needed to solve an exercise set of variation problems varies directly as the number of problems and inversely as the number of people working on the solutions. It takes 4 people and 36 minutes to solve 18 problems. How many minutes will it take 6 people to solve 42 problems?
    A. 52 minutes
    B. minutes
    C. minutes
    D. 58 minutes

8. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. A wire with a length of 200 inches and a diameter of one-quarter of an inch has a resistance of 20 ohms. What is the electrical resistance in a 500 inch wire with the same diameter?
    A. 40 ohms
    B. 50 ohms
    C. 60 ohms
    D. 70 ohms
9. The number of girls varies directly as the number of boys and inversely as the number of teachers. When there were 50 girls, there were 20 teachers and 10 boys. How many boys were there when there were 10 girls and 100 teachers?
    A. 10 boys
    B. 15 boys
    C. 20 boys
    D. 25 boys
10. Strawberries varies jointly as plums and tomatoes. If 500 strawberries went with 4 plums and 25 tomatoes, how many plums would go with 40 strawberries and 2 tomatoes?
    A. 2 plums
    B. 4 plums
    C. 6 plums
    D. 8 plums
11. Reds varies directly as yellows and inversely as greens squared. If 100 reds and 40 yellows went with 10 greens, how may reds went with 20 yellows and only 5 greens?
    A. 150 reds
    B. 200 reds
    C. 250 reds
    D. 300 reds
12. Horses varies directly as goats and inversely as pigs. When the barnyard contained 3 horses there were 8 pigs and only 4 goats. How many goats went with 12 pigs and 10 horses?
    A. 14 goats
    B. 16 goats
    C. 18 goats
    D. 20 goats
13. Cheers varies jointly as the number of fans and the square of the jubilation factor. When there were 100 fans and jubilation factor was 4 there were 1000 cheers. How many cheers were there when there were only 10 fans whose jubilation factor was 20?
    A. 1500 cheers
    B. 2500 cheers
    C. 3500 cheers
    D. 4500 cheers

14. Roger works at a flower shop and he notices that the more flowers people buy, the more chocolates are sold too. If Roger sells 45 chocolates when 90 flowers are sold, how many chocolates does he sell if 100 flowers are sold?
    A. 40
    B. 50
    C. 60
    D. 70
15. The volume V of gas varies inversely to the pressure P. The volume of a gas is 200 $cm^3$ under pressure of 32 $kg/cm^2$. What will be its volume under pressure of 40 $kg/cm^2$ ?
    A. 130 $cm^3$
    B. 140 $cm^3$
    C. 150 $cm^3$
    D. 160$cm^3$

*Great job! You made it. Congratulations!*

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