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# Mathematics

## Quarter 2 – Module 2:
## Solving Problems Involving Variation

**AIRs - LM**

**MATHEMATICS 9**
**Quarter 2 - Module 2: Solving Problems Involving Variation**
**Second Edition, 2021**

## *Jumpstart*

## Activity1. Find My Constant and My Equation!
*Directions:* Find the constant of variation and write the equation representing the relationship between the quantities in each of the following.

1. y varies directly as x and y = 81 when x = 3
2. m varies directly as n and m = 120 when n = 5
3. y varies inversely as x and y = 18 when x = 2/3
4. p varies jointly as q and square of r and p = 240 when q = 6 and r = 2
5. w varies directly as x and inversely as z and w = 18 when x = 9 and z = 3


## Activity 2: Find My Value!
*Directions:* Solve for the indicated variable in each of the following.

1. If y varies inversely as x and y = 36 when x = 5, find y when x = 6.
2. If z varies jointly as x and y and z = 240 when x = 8 and y = 2, find z when x = 7 and y = 6.
3. If x varies directly as the square of y and inversely as z and x =12 when y =3 and z = 6, find x when y =9 and z =6.
4. If y varies directly as x and y = -18 when x = 9, find y when x = 7.
5. If m varies inversely as n and m = 8 when n = 3, find m when n = 12.


## *Discover*

## Solving Problems Involving Variation
We will now discuss the concept of variations and its applications to real life situations.


**Direct Variation**

There is direct variation whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"
"y is directly proportional to x" and
"y is proportional to x"

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may be translated mathematically as y = kx, where k is the constant of variation. For two quantities x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Example 1:
The amount of paint **P** needed to paint the walls of a room varies directly as the area **A** of the wall. If 2 gallons of paint is needed to paint a 40 sq. meter wall, how many gallons of paint are needed to paint a wall with an area of 100 sq. meters?

*Solution:*

Solve first for k.

P = kA

2 = k(40)

k = 2/40

$k = \frac{1}{20}$

Solve for the number of gallons of paint

P = kA

$P = \frac{1}{20}(100)$

P = 5 gallons of paint

## Inverse Variation

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or $y=\frac{k}{x}$. The statement, "y varies inversely to x", translate to $y = \frac{k}{x}$, where k is the constant of variation.

Example 2:
Cathy and Anthony are figuring out a way to balance themselves on a seesaw. Cathy who weighs 15 kilograms sits 2 meters from the fulcrum. Anthony who weighs 20 kilograms tried sitting at different distances from the fulcrum in order to balance the weight of Cathy. If you were Anthony, how far from the fulcrum should you sit?

To balance the weight of Cathy, Anthony has to sit at a distance closer to the fulcrum.

The relation shows that the distance d varies inversely as the weight w and can be transformed into a mathematical equation as

$$d = \frac{k}{w}$$

We can now solve for distance from the fulcrum where Anthony has to sit.

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Solution:
Solve first for k.          Solve for the distance of Anthony from the fulcrum.

$d = \frac{k}{w}$                    $d = \frac{k}{w}$

$15 = \frac{k}{2}$                   $d = \frac{30}{20}$

k = 15(2)                  d = 1.5 m

k = 30

Hence, Anthony has to sit 1.5 meters from the fulcrum.


## Joint Variation

Joint variation describes a situation where one variable depends on two (or more) other variables, and varies directly as each of them when the others are held constant. We say z varies jointly as x and y if z = kxy for k is the constant of variation.

Example 3:
The mass of a rectangular sheet of wood varies jointly as the length and the width. When the length is 20 cm and the width is 10 cm, the mass is 400 grams. Find the mass when the length is 25 cm and the width is 20 cm.

Solution:

Solve first for constant k.            Solve for the mass.

m = klw                              m = klw

400 = k (20)(10)                     m = 2(25)(20)

400 = k(200)                         m = 1000 grams

$k = \frac{400}{200}$

k = 2


## Combined Variation

Combined variation describes a situation where a variable depends on two (or more) other variables and varies directly with some of them and varies inversely with others (when the rest of the variables are held constant).
The statement "z varies directly as x and inversely as y" means $z = \frac{kx}{y}$ , or $k = \frac{zy}{x}$ , where k is the constant of variation.

Example 4:
The force of attraction, F of a body varies directly as its mass m and inversely as the square of the distance d from the body. Wen m = 8 kilograms and d = 5 meters, F = 100 Newtons. Find F when m = 12 kilograms and d = 5 meters.

Solution:

Solve for the constant k.          Solve for the force of attraction F.

$$F = \frac{km}{d^2}$$          $$F = \frac{km}{d^2}$$

$$k = \frac{Fd^2}{m}$$          $$F = \frac{(312.5)(12)}{5^2}$$

$$k = \frac{100(5)^2}{8}$$          $$F = \frac{3750}{25}$$

$$k = \frac{2500}{8}$$          F = 150 Newtons

k = 312.5

## *Explore*

## Activity 3: How Well Do You Understand?

*Directions:* Solve the following problems.

1. A mailman can sort out 738 letters in 6 hours. If the number of sorted letters varies directly as the number of working hours, how many letters can be sorted out in 9 hours?
2. The number of days needed in repairing a house varies inversely as the number of men working. It takes 15 days for 2 men to repair the house. How many men are needed to complete the job in 6 days?
3. The area of triangle varies jointly as the base and the height. A triangle with a base of 8 cm and a height of 9 cm has an area of 36 square centimeters. Find the area when the base is 10 cm and the height is 7 cm.
4. The current $I$ varies directly as the electromotive force $E$ and inversely as the resistance R. If in a system a current of 20 amperes flows through a resistance of 20 ohms with an electromotive force of 100 volts, find the current that 150 volts will send through the system.

# *Deepen*

## Activity 4: Think Deeper!

*Directions:* Solve the following problems.

1. The amount of gasoline used by a car varies jointly as the distance travelled and the square root of the speed. Suppose a car used 25 liters on a 100 kilometer trip at 100 km/hr. About how many liters will it use on a 192 kilometer trip at 64 km/hr?

2. The area **A** of a triangle varies jointly as the base **b** and the altitude **h** of the triangle. If A = 65 cm² when b = 10 cm and h = 13 cm, find the area of a triangle whose base is 8 cm and whose altitude is 11 cm.

3. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If a wire 30 meters long, and 0.75 mm in diameter has a resistance of 25 ohms, find the length of a wire of the same material whose resistance and diameter are 30 ohms and 1.25 mm, respectively.

4. The weight **W** of a cylindrical metal varies jointly as its length **l** and the square of the diameter **d** of its base.
   a. If $W$ = 6 kg when $l$ = 6 cm and $d$ = 3 cm, find the equation of variations.
   b. Find $l$ when $W$ = 10 kg and $d$ = 2 cm.
   c. Find $W$ when $d$ = 6 c and $l$ = 1.4 cm

# *Gauge*

## Post - Assessment

*Directions:* Read and analyze the following questions carefully. Choose the letter of the correct answer. Write it on your answer sheet.

1. If car covers 102 km in 6.8 litres of petrol, how much distance will it cover in 24.2 litres of petrol?
   A. 303 km
   B. 323 km
   C. 333 km
   D. 363 km