





# **Mathematics**

Quarter 3 – Week 1 - Module 1: Properties and Measures of Parallelogram



AIRs - LM

SAO LERUNG OF SALL

### **Subject**

Quarter 3 – Week 1 – Module 1: Properties and Measures of Parallelogram First Edition, 2021

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## **Target**

Good day Grade 9 learners! Welcome to the world of Geometry – the mathematics of shapes and sizes.

What does it mean to be opposite? What does it mean to be consecutive? For instance, you are in a rectangular room. If you place your back on one corner of that room and look directly across the room, you would be looking at the opposite corner. If you look to your right, that corner would be consecutive. If you look to your left, that corner would also be consecutive. In this situation, concepts on parallelograms are evident.

This module provides different activities that guide you in determining the conditions that make a quadrilateral a parallelogram. Use these properties to find measures of angles, sides, and other quantities involving parallelograms.

Before you begin, familiarize yourself first with the most essential learning competencies for this module.

### **Most Essential Learning Competencies**

- Determine the conditions that make a quadrilateral a parallelogram **M9GE-IIIa-2**
- Use properties to find measures of angles, sides, and other quantities involving parallelograms **M9GE-IIIb-1**

#### Subtasks:

After going through this module, you are expected to:

- 1. Define what is a parallelogram
- 2. State the conditions that make a quadrilateral parallelogram
- 3. Find the relationships among angles, sides, and diagonals of parallelograms
- 4. Use properties to find the measures of angles, sides, and other quantities involving parallelograms

To start with, let's find out how much you already know about this module. Answer the pre-assessment below using a separate sheet of paper.

### **Pre-Assessment**

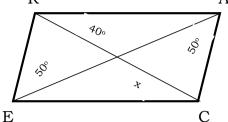
**Directions:** Read each item carefully. Write the letter of the correct answer for each question.

	A. 90°	measure of B. 180º	the meet	C. 270	-	D. 360		
2.	Three angles	s of a qua	drilateral	are 10	0°, 130°	and $50^{\circ}$	. What is	s the
	measure of t	he fourth a	angle?					
	A. 60°	B.70°		C. 80°		D. 90 <sup>o</sup>	)	
3.	Which is <b>NO</b>	<b>T</b> a proper	ty of a par	rallelog	ram?			
	A. Diagonals are congruent.							
	B. Diagonals bisect each other.							
	C. Both pair	s of opposi	te sides a	re para	ıllel.			
	D. Both pair	s of opposi	te angles	are cor	ngruent.			
4.	What can	you say	about a	any tv	vo cons	ecutive	angles i	in a
	parallelograr	n?						
	A. They are both right angles.							
	B. They are always congruent.							
	C. They are	always sup	plementa	ry.				
	D. They are	always con	- iplementa	ary.				
5.	How do you describe any two opposite angles in a parallelogram?							
	A. They are both right angles.							
	B. They are	always con	gruent.					
	C. They are always supplementary.							
	D. They are	always con	- iplementa	ary.				
6.	What condit	ion guaran	tees that	□BES'	Γ is a par	allelogra	m in figu	re 1?
	A. Diagonals	s are congr	uent.	fig	ure 1	В	<del></del>	E
	B. Diagonals	s bisect eac	ch other.			/ ×	$\mathcal{I}$	
	C. Two cons	ecutive sid	es are con	igruent	Ī.		$\sim$ /	
	D. Two cons	ecutive ang	gles are co	ngruei	nt.	T	-	
	How many			_		when a	diagona	al of
	parallelogram is drawn?							
	A. 1	B. 2		C.	_	D. 4		

8.	Which of the following conditions is <b>NOT</b> sufficient to prove that a quadrilateral is a parallelogram?  A. Two pairs of sides are parallel.  B. Two angles are supplementary.  C. Two diagonals bisect each other.  D. Two pairs of opposite sides are congruent.
9.	Which <b>BEST</b> describes the diagonals of a parallelogram?  A. Bisected B. Congruent C. Parallel D. Perpendicular
10	What is the $m \angle A$ if $\square MAKE$ is a parallelogram in figure 2?  A. 60 figure 2  M  120  C. 100  D. 120  K
11	If $\square$ SAVE is a parallelogram and $VM = 5$ , what is the length of $SV$ in igure 3?
	A. 5 B. 8 C. 10 D. 13  Figure 3  S  E  V
12	In figure 4, $\square$ LIKE is a parallelogram. If $m \angle L = x + 40$ and $m \angle I = 2x + 20$ , what is the $m \angle L$ ?
	A. 60 figure 4 B. 70 C. 80 D. 100  Figure 4  F
13	Quadrilateral BUSY is a parallelogram in figure 5. The diagonals $\overline{BS}$ and $\overline{UY}$ intersect at point E. If $YE = 3x - 2$ and $UE = x + 8$ , how long is JY?
	A. 5 B. 13 C. 26 D. 30  figure 5 B Y S

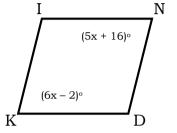
- 14. What is the value of x that will make quadrilateral RACE a parallelogram in figure 6? R
  - A. 40°
  - B. 50°
  - C. 60°
  - D. 70°

figure 6



- 15. What is the measure of  $\angle D$  in parallelogram KIND in figure 7?
  - A. 18°
  - B. 74°
  - C. 106°
  - D. 110°

figure 7



Did you do well in the pretest? Are there items that you were not sure of your answers? No worries! You can go back to those items as you gain new knowledge and skills in this module. But, don't forget to list ideas and concepts in the lessons. Enjoy!

### Lesson

1

## **Properties of Parallelogram**

There are many kinds of quadrilaterals. Some quadrilaterals are parallelograms, some are not. For example, kites and trapezoids are quadrilaterals but they are not parallelograms. Rectangles, squares, and rhombuses are parallelograms. What makes these quadrilaterals parallelograms? To answer this question, perform activity 1.



### **Activity 1: Fantastic Four!**

Materials: Protractor, graphing paper, ruler, pencil, compass

**Directions:** Follow the given procedures below and answer the processing questions.

- 1. Draw each of following quadrilaterals on a graphing paper.
  - a. Parallelogram OBEY
  - b. Rectangle GIVE
  - c. Rhombus THNX
  - d. Square LOVE
- 2. Measure the sides and the angles and record your findings in your own table similar to what is shown below.
- 3. Draw the diagonals and measure the segments formed by the intersecting diagonals. Again, record your findings in the table.

Drawing	In your drawing, following	identify the	Measurement	Are the measuremen ts equal or not equal?
	Opposite sides			
	Opposite angles			
	Consecutive angles			
	Segments formed by intersecting diagonals			

### Processing Questions:

- 1. Based on the table above, what is true about the following?
  - a. Pairs of opposite sides
  - b. Pairs of opposite angles
  - c. Pairs of consecutive angles
  - d. Pairs of segments formed by intersecting diagonals
- 2. What does each diagonal do to a parallelogram?

- 3. Make a conjecture about the two triangles formed when a diagonal of a parallelogram is drawn. Explain your answer.
- 4. Do the findings apply to all kinds of parallelogram? Why?

Your answers to the questions show the conditions that guarantee that a quadrilateral is a parallelogram.



### **Discover**

A parallelogram is a special type of quadrilateral. It is a four-sided polygon with two pairs of opposite sides that are parallel. To write the name of a parallelogram, we use the symbol  $\square$ . When we mark diagrams of quadrilaterals, use matching arrowheads to indicate which sides are parallel. In Figure 1.2, quadrilateral ABCD is a parallelogram where segment AD is parallel to segment CB and segment AB is parallel to segment CD.

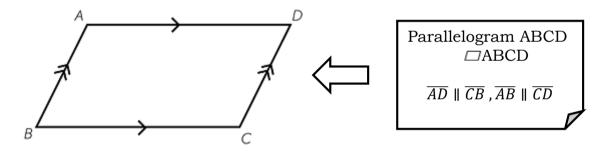


Figure 1.2

In Activity 1, you have just discovered the properties of parallelogram. Now, let's finalize these properties and be ready to use these properties to answer the succeeding activities.

### **Properties of Parallelogram**

1. In a parallelogram, any two opposite sides are congruent.

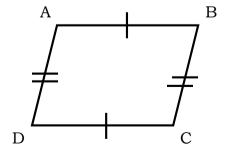


Figure 1.3

In figure 1.3, segment AB is congruent to segment CD, and segment AD is congruent to segment CB.

$$\overline{AB} \cong \overline{CD}$$
 and  $\overline{AD} \cong \overline{CB}$ 

2. In a parallelogram, any two opposite angles are congruent.

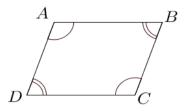


Figure 1.4

In figure 1.4, angle A is congruent to angle C, and angle B is congruent to angle D.

$$\angle A \cong \angle C$$
  
 $\angle B \cong \angle D$ 

3. In a parallelogram, any two consecutive angles are supplementary.

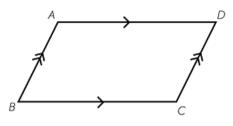
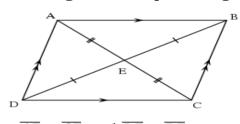


Figure 1.5

In figure 1.5, 
$$m\angle A + m\angle B=180$$
  $m\angle B + m\angle C=180$   $m\angle C + m\angle D=180$   $m\angle D + m\angle A=180$ 

4. The diagonals of a parallelogram bisect each other.



 $\overline{AE} \cong \overline{CE}$  and  $\overline{DE} \cong \overline{BE}$ 

Figure 1.6

In figure 1.6,

The diagonals  $\overline{AC}$  and  $\overline{BD}$  bisect each other.

Segment AE is congruent to segment CE, and segment DE is congruent to segment BE.

5. A diagonal of a parallelogram forms two congruent triangles.

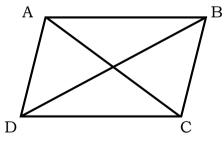


Figure 1.7

In figure 1.7,
Triangle DAB is congruent to triangle DCB.
Triangle ABC is congruent to triangle ADC.

$$\Delta DAB \cong \Delta DCB$$
, and  $\Delta ABC \cong \Delta ADC$ 

Now that you already know the properties of parallelograms, we can already come up with the different conditions that guarantee that a quadrilateral is a parallelogram.

### Conditions that Guarantee that a Quadrilateral is a Parallelogram

- 1. A quadrilateral is a parallelogram if both pairs of opposite sides are congruent.
- 2. A quadrilateral is a parallelogram if both pairs of opposite angles are congruent.
- 3. A quadrilateral is a parallelogram if both pairs of consecutive angles are supplementary.
- 4. A quadrilateral is a parallelogram if the diagonals bisect each other.
- 5. A quadrilateral is a parallelogram if each diagonal divides a parallelogram into two congruent triangles.
- 6. A quadrilateral is a parallelogram if one pair of opposite sides are both parallel and congruent.



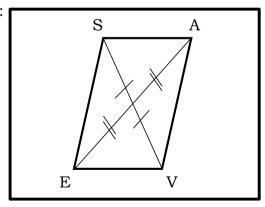
## **Explore**

Let's master and strengthen the basic concepts you have learned from this lesson by answering activity 2. Have fun and enjoy!

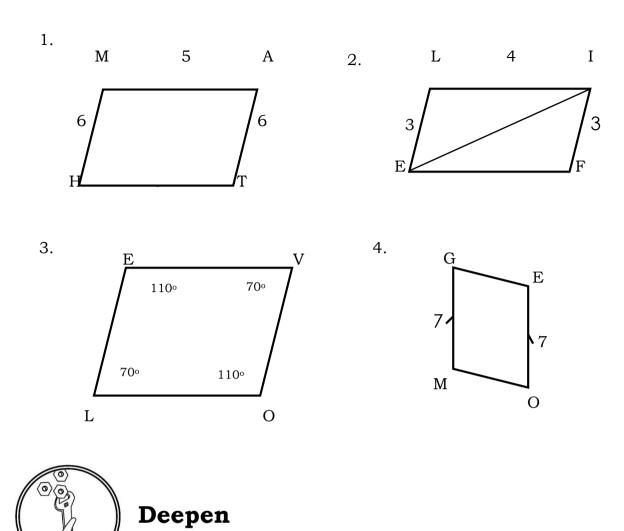
### **Activity 2: State My Condition!**

**Directions:** Study the markings in the parallelograms below then determine the condition that will make each of the given figure a parallelogram. The first item is done for you.

Example:



**Answer:** In the figure,  $\overline{EA}$  and  $\overline{SV}$  are the diagonals of quadrilateral SAVE. Based from the markings,  $\square$ SAVE is a parallelogram because the diagonals bisect each other.



## Activity 3: Is it a Parallelogram?

**Directions:** Read and analyze the situation below then answer the question that follows.

Jomari concluded that if one pair of sides is congruent, and the other pair of sides is parallel then it is sufficient to prove that the given quadrilateral is a parallelogram. Is he correct? Justify your answer.

## Lesson

2

## Find Measures of Angles, Sides, and Other Quantities Involving Parallelograms

Now that you know the properties of parallelogram, let's put this to use by answering the next activity.



## **Jumpstart**

### **Activity 1: TRUE or FALSE**

**Directions:** Read and analyze each statement, then write whether it is **TRUE** or **FALSE**.

- 1. In parallelogram JUMP,  $\overline{JU} \cong \overline{MP}$  and  $\overline{UM} \cong \overline{JP}$ .
- 2. If  $\angle A$  is 70°, then  $\angle Y$  is also 70° in parallelogram RAYS.
- 3. In parallelogram CUTE,  $\overline{CT} \cong \overline{UE}$ .
- 4.  $\overline{ML}$  and  $\overline{IE}$  bisect each other in parallelogram MILE.
- 5. In parallelogram DAYS,  $\overline{AS}$  divides it into two congruent triangles.

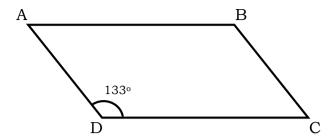
Did you get all the items correctly? Since you have already mastered the properties of parallelograms, you are now ready to apply these properties to uncover missing sides, angles and other quantities of a given parallelogram. Let's get started!



### Discover

If one angle in a parallelogram is a right angle, then all angles are right angles. This means that if we know the properties of parallelograms, we can easily identify missing angles and sides. Let's have some illustrative examples.

**Example 1.** What is the measure of  $\angle A, \angle B, \angle C$  in the parallelogram below?



You know that the opposite angles are congruent and the consecutive angles are supplementary.

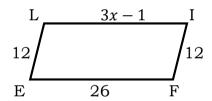
### Solution:

 $\angle B = 133^{o}$  since it is opposite  $\angle D$  and opposite angles are congruent. Since consecutive angles are supplementary,

$$m \angle C + m \angle D = 180$$
  
 $m \angle C + 133 = 180$   
 $m \angle C = 180 - 133$   
 $m \angle C = 47$ 

 $\angle A = 47^{\circ}$  since it is opposite  $\angle C$ .

**Example 2.** What is the value of x in the parallelogram below?

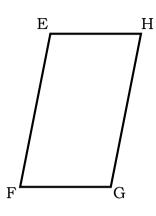


### Solution:

Since  $\overline{LI}$  and  $\overline{FE}$  are opposite sides they are congruent.  $\overline{LI} \cong \overline{FE}$ 

LI = FE	Opposite sides are congruent.	Check:
3x - 1 = 26	Substitution Property	If we substitute
3x - 1 + 1 = 26 + 1	Addition Property of Equality	$x = 9 \text{ in } \overline{LI},$
3x = 27	Division Property of Equality)	
	(Divide both sides by 3	LI = 3x - 1
x = 9	Simplify	LI = 3(9) - 1
		LI = 27 - 1
$\therefore \overline{LI} \cong \overline{FE}$ since they	LI = 26	

**Example 3.** Find the measures of all the angles of parallelogram EFGH.



Given:

$$m \angle E = 6x - 2$$

$$m \angle F = 2x + 6$$

Solution:

Since  $\angle E$  and  $\angle F$  are consecutive angles they are supplementary.

$$m \angle E + m \angle F = 180$$
  
 $6x - 2 + 2x + 6 = 180$   
 $8x + 4 = 180$   
 $8x + 4 - 4 = 180 - 4$   
 $8x = 176$   
 $x = 22$ 

Consecutive angles are supplementary.

Substitute  $m \angle E$  and  $m \angle F$  with their measures

Combine similar terms

Subtract 4 from both sides

Divide both sides by 8

Simplify

To find the measures of the angles, substitute x = 22 in  $m \angle E$  and  $m \angle F$ .

a. 
$$m \angle E = 6x - 2$$
  
 $m \angle E = 6(22) - 2$   
 $m \angle E = 132 - 2$ 

 $m \angle E = 130$ 

c.  $\angle G$  is 130° since it is opposite  $\angle E$  and opposite angles are congruent.

b. 
$$m \angle F = 2x + 6$$
  
 $m \angle F = 2(22) + 6$   
 $m \angle F = 44 + 6$   
 $m \angle F = 50$ 

d.  $\angle H$  is 50° since it is opposite  $\angle F$  and opposite angles are congruent.

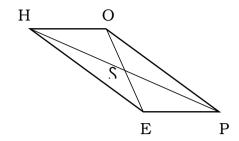
Check:

If we substitute the measures of  $\angle E$  and  $\angle F$ , we have,

$$m \angle E + m \angle F = 180$$
  
 $130^{\circ} + 50^{\circ} = 180^{\circ}$   
 $180^{\circ} = 180^{\circ}$ 

 $\therefore$  Since  $\angle E$  and  $\angle F$  are consecutive angles they are supplementary.

### **Example 4.** What is the length of $\overline{HS}$ and $\overline{PS}$ in $\square$ HOPE below?



Given:

 $\square$  HOPE with diagonals  $\overline{HP}$  and  $\overline{OE}$  intersecting at point S.

$$HS = x + 10$$
$$PS = 2x - 14$$

### Solution:

Since the diagonals bisect each other,  $\overline{HS} \cong \overline{PS}$ .

$$HS = PS$$
 Diagonals bisect each other.  
 $x + 10 = 2x - 14$  Substitution Property  
 $10 + 14 = 2x - x$  Combine similar terms  
 $24 = x$  Simplify

Check:

If we substitute x = 22 in  $\overline{HS}$  and  $\overline{PS}$ , we have HS = x + 10 PS = 2x - 14

HS = 24 + 10 PS = 2(24) - 14HS = 34 PS = 48 - 14

PS = 34

 $\therefore \overline{HS} \cong \overline{PS}$  since diagonals bisect each other.



### **Activity 2: Challenge Time!**

**Directions:** Use the properties of parallelogram to do what is asked in each item. Consider  $\square$ WXYZ in Figure 2.1 below.

**Given:** Quadrilateral WXYZ is a Parallelogram. The diagram is not drawn to scale.

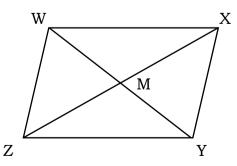


Figure 2.1

1. If WX = 18 and WM = 10, find:

**a.** YZ =\_\_\_\_\_\_

**b.** YM =\_\_\_\_\_

2. If WM = 9 and WZ = 5, find:

**a.** WY =\_\_\_\_

**b.** XY =\_\_\_\_\_\_

3. If  $\overline{XZ} = 14$  and WX = 10, find:

 $\mathbf{a.} \ \ XM =$ 

**b.** YZ =

4. If  $m \angle Z = 42$ , find:

**a.**  $m \angle X =$ \_

**b.**  $m \angle W =$ 

5. If WX = 2x - 1 and YZ = 5x - 10, find:

**a.** x =\_\_\_\_\_

6. If XM = 6x + 1 and ZM = 4x + 31, find:

**a.** x =\_\_\_\_\_

 $m \angle WXY = 3x - 7$ 

 $m \angle XYZ = x + 15$ , find:

**a.** x =\_\_\_\_\_

**b.**  $m \angle WXY =$ \_\_\_\_\_\_

c.  $m \angle XYZ = \underline{\hspace{1cm}}$ 

and 8. If  $m \angle Z = 7x - 3$  and  $m \angle X = 5x + 1$ 

7, find:

**a.** x =\_\_\_\_\_

**b.**  $m \angle W = \_\_\_$ 

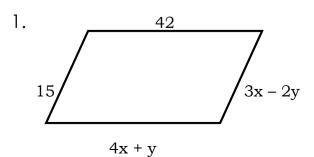
**c.**  $m \angle X =$ \_\_\_\_\_



## Deepen

### Activity 3: X and Y in Parallelogram

**Directions:** Find the values of x and y that will make the given quadrilateral a parallelogram.



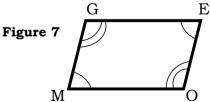


**Directions:** Read each item carefully. Write the letter of the correct answer for each question in a separate sheet of paper.

- 1. Which of the following quadrilaterals is **NOT** a parallelogram?
  - A. Rectangle
- B. Rhombus
- C. Square
- D. Trapezoid
- 2. Which of the following statements **BEST** describes a parallelogram?
  - A. A quadrilateral with one pair of parallel sides.
  - B. A quadrilateral with two pairs of parallel sides.
  - C. A quadrilateral with two pairs of congruent sides.
  - D. Any four-sided figure with opposite sides congruent.
- 3. What reason can be used to prove that  $\square$ LOVE is a parallelogram in Figure 6?
  - A. Opposite sides are parallel.



- B. Opposite sides are congruent.
- C. Opposite angles are congruent.
- D. Opposite sides are both parallel and congruent.
- 4. Which of the following statements is **NOT** a property of parallelogram?
  - A. Diagonals bisect each other.
  - B. Opposite sides are congruent.
  - C. Opposite angles are congruent.
  - D. Opposite sides are not parallel.
- 5. Based on the markings, what reason can be used to prove that  $\square$ GEOM is a parallelogram in Figure 7?  $\mathbf{E}$ 
  - A. Opposite angles are congruent.
  - B. Opposite angles supplementary.
  - C. Consecutive angles are congruent.
  - D. Consecutive angles are supplementary.



- 6. Which of the following statements describes the diagonals of a parallelogram?
  - A. Diagonals are congruent.
  - B. The diagonals bisect each other.
  - C. A diagonal form four congruent triangles.
  - D. The diagonals form two congruent triangles.

- 7. Quadrilateral MNOP is a parallelogram in Figure 8. If MN = 2x 10 and PO = x + 30, how long is  $\overline{MN}$ ?

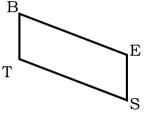
  A. 50
  - A. 50
    B. 60
    C. 70
    D. 80

    Figure 8

    M
    P
- 8. Which of the following statements can be used to prove quadrilateral JUMP is a parallelogram in Figure 9?
  - A.  $\overline{JU} \cong \overline{UM}$  and  $\overline{MP} \cong \overline{PJ}$  Figure 9
  - B.  $\overline{JU} \cong \overline{JP}$  and  $\overline{PM} \cong \overline{UM}$
  - C.  $\overline{JU} \cong \overline{PM}$  and  $\overline{JP} \cong \overline{UM}$
  - D.  $\overline{JM} \cong \overline{PU}$  and  $\overline{JP} \cong \overline{UM}$

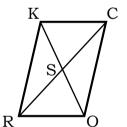
- $P = \frac{1}{2} \sum_{M}^{M}$
- 9. Quadrilateral BEST is a parallelogram in Figure 10. If  $m \angle B = 3x 20$  and  $m \angle S = 2(x+5)$ , what is the value of x?
  - A. 25
  - B. 30
  - C. 35
  - D. 40

Figure 10



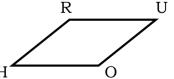
- 10. What is the length of  $\overline{RC}$  if RS = 5x 6 and SC = 3x 1 in  $\square ROCK$  below?
  - A. 6.5
  - B. 12
  - C. 13
  - D. 13.5

Figure 10

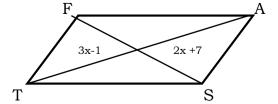


- 11. In figure 11, quadrilateral HOUR is a parallelogram. If  $m \angle RHO = 4(x + x)$ 
  - 5) and  $m \angle HOU = 2(3x + 20)$ , what is  $m \angle RUO$ ?
  - A. 50
  - B. 68
  - C. 100
  - D. 112

Figure 11



- 12. What is the length of  $\overline{AT}$  in parallelogram FAST below?
  - A. 8
  - B. 31
  - C. 46
  - D. 52
- Figure 12



- 13. In Figure 13,  $\square$ JUST is a parallelogram. What is the length of  $\overline{JT}$ ?
  - A. 3
  - B. 11
  - C. 14
  - D. 24





- 14. The sum of two angles of a parallelogram is 110°. What is the measure of each of its angles?
  - A. 55°, 55°, 125°, 125°
- B. 75°, 75°, 165°, 165°
- C. 110°, 110°, 250°, 250°
- D. 110°, 120°, 130°, 140°
- 15. Two adjacent angles of a parallelogram are in the ratio 4:5. What is the measure of each of its angles?
  - A. 90°, 90°, 90°, 90°
- B. 80°, 80°, 100°, 100°
- C. 70°, 90°, 100°, 100°
- D. 60°, 90°, 105°, 105°

Congratulations! You are done with this module.

### References

#### A. Books

Mathematics Grade 9 Learner's Module, First Edition 2014, Reprint 2017

Mathematics Grade 9 Teacher's Guide, First Edition 2014, Reprint 2017

Oronce, Orlando A., Marilyn O. Mendoza, Worktext in Mathematics for Third Year High School, e-math III Geometry, RBS Mathematics Series, First Edition 2007, Rex Bookstore, Inc (RSBI)

### B. Online Resources

https://www.onlinemath4all.com/properties-of-parallelogram.html

https://calcworkshop.com/quadrialterals/properties-parallelograms/

https://www.slideshare.net/mobile/Ysnilsmaili/properties-of-a-parallelogram

https://slideplayer.com/slide/263882

https://www.google.com/imgres?imgurl=https:i2.wp.com/proofsfromthebook.com/

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https://en.m.wikipedia.org/wiki/Quadrilateral