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# AIRs - LM in

## Statistics and Probability
## Module 7:
## Central Limit Theorem

**Statistics and Probability**
Module 7: Central Limit Theorem
First Edition, 2021



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# *Target*

One of the fundamental theorems of probability is the Central Limit Theorem. The **Central Limit Theorem** tells us about the shape of the distribution of the sample mean $\bar{x}$ when the sample size $n$ is sufficiently large.

In your previous lesson, you are done defining the sampling distribution of sample mean for normal population when the variance is known and unknown.

This learning material will provide you with information and activities that will help you understand and illustrate the Central Limit Theorem and it will help you solve problems involving sampling distributions of the sample mean.

After going through this learning material, you are expected to attain the following objectives:

1. illustrate the Central Limit Theorem **(M11/12SPIIIe-2);**
2. define the sampling distribution of the sample mean using the Central Limit
   Theorem **(M11/12SPIIIe-3);** and
3. solve problems involving sampling distributions of the sample mean.
   **(M11SP-IIIe-f1)**

*Subtasks:*

1. define central limit theorem.
2. describe sampling distribution of the sample mean.
3. construct the sampling distribution of the sample mean.
4. calculate problems on sampling distribution of the sample mean.

*Before going on, check how much you know about this topic. Answer the pretest on the next page in a separate sheet of paper.*

***Pretest***

**Directions**: Read and analyze each item carefully. Write the letter of the correct answer using a separate sheet of paper.

1. Which of the following is correct about the sampling distribution of the sample mean using the Central Limit Theorem?
   A. The mean of the sampling distribution of the means is not equal to the mean of the population.
   B. As the sample size *n* increases, the sampling distribution of the means approaches a normal distribution.
   C. The variance of the sampling distribution of the means is equal to the variance of the population multiplied by the sample size *n.*
   D. The standard deviation of the sampling distribution of the means is equal to the standard deviation of the population multiplied by the square root of the sample size *n.*

2. Which of the following statements is correct if a certain population has a mean of 15.4, a standard deviation of 5.6 and the random sample size is 5?
   A. The sample mean is equal to 15.4.
   B. The sample mean is less than 15.4.
   C. The standard deviation of sample is equal to 15.6.
   D. The standard deviation of sample is more than 15.4.

3. Which of the following statements best describes a z-value?
   A. The z-value leads to the area under the curve found in the normal curve table.
   B. The z-value leads to the area under the curve found in the normal distribution table, which is a probability.
   C. The z-value leads to the area under the curve found in the normal curve table, which is a percentage, and that percentage gives the desired probability for X.
   D. The z-value leads to the are under the curve found in the normal curve table, which is a probability, and that percentage gives the desired percentage for X.

4. Which of the following refers to the standard deviation of a sampling distribution?
   A. It is the sum of squares.
   B. It is the standard error of the mean.
   C. It only applies only to population data.
   D. It can be larger than the standard deviation for the population.

5. Which of the following properties describes the sampling distribution of the sample mean?
   A. The mean of the sampling distribution of the means is equal to the mean of the population.
   B. As the sample size *n* decreases, the sampling distribution of the means approaches a normal distribution.
   C. The variance of the sampling distribution of the means is equal to the variance of the population multiplied by the sample size *n.*
   D. The standard deviation of the sample is equal to the standard deviation of the population multiplied by the square root of the sample size *n.*

6. Who is the French-born Mathematician who proved the first version of the Central Limit Theorem?
   A. Abraham de Moivre
   B. Aristotle
   C. Aleksander Lyapunov
   D. Rene Descartes

7. Which of the following describes the central limit theorem?
   A. The mean of the sampling distribution of the means is not equal to the mean of the population.
   B. As the sample size *n* decreases, the sampling distribution of the means approaches a normal distribution.
   C. The variance of the sampling distribution of the means is equal to the variance of the population multiplied by the sample size *n.*
   D. The standard deviation of the sampling distribution of the means is equal to the standard deviation of the population divided by the square root of the sample size *n.*

8. Which of the following statements is correct about population mean and sample mean?
   A. The mean of the sample is less than the population mean.
   B. The mean of the samples is greater than the population mean.
   C. The means of the samples drawn from a population are always equal to the population mean.
   D. The means of the samples drawn from a population may be equal, greater than or less than the population mean.

9. Which of the following refers to the frequency distribution using the means computed from all possible random samples of a specific size take from the population?
   A. Population Distribution
   B. Frequency Distribution Table
   C. Sampling Distribution of the Sample Means
   D. Sampling Distribution of the Population Mean

10. If the mean of the sampling distribution is 9.5, what is the mean of the population?
    A. 9.5
    B. 19
    C. 28.5
    D. 95

11. What formula is use when computing for the probability $\bar{x}$ that will take on a value within a given range in the sampling distribution of the sample mean?
    A. $z = \frac{\bar{x}-\mu}{\sigma}$
    B. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$
    C. $z = \frac{\bar{x}-\sigma}{\mu}$
    D. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{N}}}$

12. What is the standard deviation of sampling distribution if standard deviation of population is 35 and sample size is 9?
    A. 11.67
    B. 12.67
    C. 13.67
    D. 14.67

13. What is the variance of the sampling distribution of the means of size 5 drawn from a population with a variance of 10?
    A. 1.41
    B. 2
    C. 4.47
    D. 10

For numbers 14-15, use the given problem below.

| |
|---|
| *The average time it takes for high school students to complete certain examination is 46.2 minutes. The standard deviation is 8 minutes. There will be 50 random samples to be selected. Assume that the variable is normally distributed.* |

14. What is the value of the population mean in the given problem?
    A. 0    B. 8    C. 46.2    D. 50

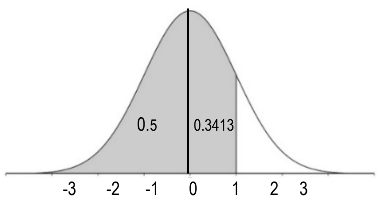
15. If 50 random selected high school students take the exam, what is the probability that the mean time it takes the group to complete the test will be less than 43 minutes?
    A. 0.23%    B. 2.3%    C. 3%    D. 23%

## *Jumpstart*

*For better understanding of the given topic, do the following activities. Have fun and good luck!*

**Activity 1: Fill Me!**

**Directions:** Complete the table below by writing the missing information in the given table.

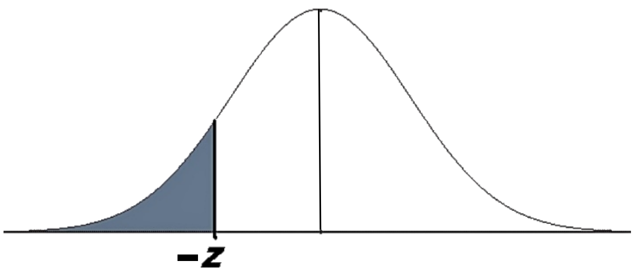
| **Conditions** | **Illustration** | **Area** |
|---|---|---|
| Less than z = 1 |  | The required area is equal to 0.5 + 0.3413 = 0.8413. |
| Above z = -1 | | |

| | | |
|---|---|---|
| Between z = 0.98 and z = 2.58 | | |
| Less than z = 1.5 | | |

**Activity 2: Choose Me!**

**Directions:** Read and understand the given question, then write the letter of the correct answer in a separate sheet of paper.

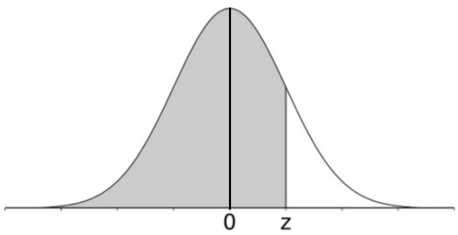
1. What is the required area as depicted by the shaded region of the given figure below?




A. "At least z" B. "Less than -z"
C. "Greater than z" D. "To the right of -z"

2. What is the correct step in getting for the area between z =-1 and z=1?
A. Add the area of z = -1 and z = 1.
B. Get the difference between the areas of z = 2 and z=1.
C. Multiply the area of z =1 to itself.
D. Subtract the area of z = -1 and z= 1.

3. What is the area of the shaded region in the given figure below if z = 1?




A. 0.1587 B. 0.3413 C. 0.5000 D. 0.8413

4. Which of the following is the formula to be used in solving for z-value applying Central Limit Theorem?
   A. $z = \frac{\bar{x}-\mu}{\sigma}$ B. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ C. $z = \frac{\bar{x}-\sigma}{\mu}$ D. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{N}}}$

5. Which of the following is the symbol for sample size?
   A. n B. N C. $\bar{x}$ D. σ

## *Discover*

**Central Limit Theorem**

The **Central Limit Theorem** states that "as the sample size becomes bigger, the sampling distribution of the sample mean can be approximated by a normal probability distribution". The sampling distribution of the sample means taken with replacement from a population $N$ with a population mean $\mu$ and variance $\sigma$ will approach a normal distribution according to the **Central Limit Theorem**. A **sampling distribution of sample means** is a frequency distribution using the means computed from all possible random samples of a specific size taken from a population. It is also called as probability distribution.

- As the sample size increases, the sampling distribution of the sample means approaches the normal distribution.
- The mean of the sampling distribution of means is equal to the mean of the population.

$$\boldsymbol{\mu_{\bar{x}} = \mu}$$

- The variance of the sampling distribution of the sample means is equal to the variance of the population divided by the sample size *n.*

$$\boldsymbol{\sigma^2{}_{\bar{x}} = \frac{\sigma^2}{n}}$$

- The standard deviation of the sampling distribution of the sample means is equal to the standard deviation of the population is divided by the square root of the sample size *n.*

$$\boldsymbol{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

An essential component of the Central Limit Theorem, the average of your samples is equal to the population mean**.** The Central Limit Theorem is important in Statistics because it allows us to safely assume that the sampling distribution of the mean will be normal in most cases.

The French-born mathematician Abraham de Moivre (16667-1754) proved the first version of the Central Limit Theorem. He used the normal distribution to approximate the distribution of the number of heads that will result when a fair coin is tossed a large number of times. But it was Russian mathematician and physicist, Aleksander Lyapunov (1857-1918) who gave the first rigorous proof of the general Central Limit Theorem.

Central limit theorem is applicable for a sufficiently large sample sizes (n ≥ 30). In Central Limit Theorem, when computing for z-value, you are going to use the formula: $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

where $\bar{x}$ = sample mean
$\mu$ = population mean
$\sigma$ = = population standard deviation
$n$ = sample size

**Example 1:**

The record of weights of the male population follows the normal distribution. Its mean and standard deviations are 70 kg and 15 kg respectively. If a researcher considers the records of 50 males, then what would be the mean and standard deviation of the chosen sample?

**Step 1:** Identify the given in the problem.

$\boldsymbol{\mu_{\bar{x}}}$ = ?
$\mu$ = 70
$\sigma$ = 15
$n$ = 50

**Step 2:** Solve the given problem using the given formula for the mean and standard deviation of the chosen sample.

Since, $\boldsymbol{\mu_{\bar{x}}} = \boldsymbol{\mu}$ and value of μ = 70. Therefore, $\boldsymbol{\mu_{\bar{x}}}$ **=70.**

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$\sigma_{\bar{x}} = \frac{15}{\sqrt{50}}$

$\sigma_{\bar{x}} = 2.12$

Therefore, the mean of the chosen sample is 70 kg and its standard deviation is 2.12 kg.

**Example 2:**

During this time of pandemic, a certain group of welfare recipients receives cash benefits of ₱1,100.00 per week with a standard deviation of ₱200. If a random sample of 25 is taken, what is the probability that their mean benefit is greater than ₱ 1,200.00 per week?

**Step 1:** Identify the given in the problem.

$\bar{x}$ = 1,200
$\mu$ = 1, 100
$\sigma$ = 200
$n$ = 25

**Step 2:** Substitute the value in the given formula.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

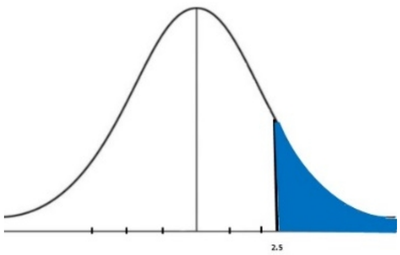
$$z = \frac{1,200 - 1,100}{\frac{200}{\sqrt{25}}}$$

$$z = \frac{100}{\frac{200}{5}}$$

$$z = \frac{100}{40}$$

$z = 2.5$

**Step 3:** Sketch the area under the normal curve then find the probability.




Based on the Table of Areas Under the Standard Normal Curve, $z$ =2.5 corresponds to 0.4938. We shall find P ($\bar{x} > 1200$) by getting the area under the normal curve.

P ($\bar{x}$ >1200) = P(z>2.5)
= 0.5000 – 0.4938
= 0.0062

Therefore, the probability that their mean benefit is greater than ₱ 1,200.00 per week is 0.0062 or 0.62%.

**Example 3:**

There are 64 *pawikan* hatchlings in a marine sanctuary in Batangas which can creep their way to the sea from the shore at an average speed of 0.025 meter per second with a standard deviation of 0.012 meter per second. Assuming that the variable is normally distributed and 16 *pawikan* hatchlings are chosen at random, what is the probability that they have an average speed of less than 0.03 meter per second?

**Step 1:** Identify the given in the problem.

$\bar{x}$ = 0.03
μ = 0.025
$\sigma$ = 0.012
$n$ = 16

**Step 2:** Substitute the value in the given formula.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

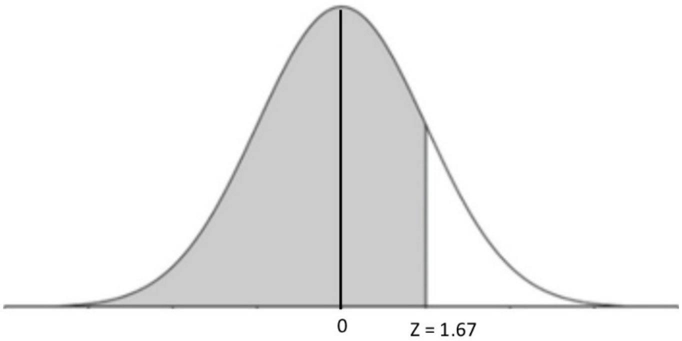
$$z = \frac{0.03 - 0.025}{\frac{0.012}{\sqrt{16}}}$$

$$z = \frac{0.005}{\frac{0.012}{4}}$$

$$z = \frac{0.005}{0.003}$$

$$z = 1.67$$

**Step 3:** Sketch the area under the normal curve then find the probability.




Based on the Table of Areas Under the Standard Normal Curve, $z = 1.67$ corresponds to 0.4525. We shall find $P(\bar{x} < 0.03)$ by getting the area under the normal curve.

$$\begin{aligned} P(\bar{x} < 0.03) &= P(z < 1.67) \\ &= 0.5000 + 0.4525 \\ &= 0.9525 \end{aligned}$$

Therefore, the probability that they have an average speed of less than 0.03 meter per second is 0.9525 or 95.25%.

# *Explore*

*Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.*

**Enrichment Activity 1: Complete Me!**

**Directions:** Complete the following statements about Central Limit Theorem and Sampling Distribution of the Sample means.

1. A sampling distribution of sample means is a frequency distribution using the means computed from all possible random samples of specific size taken from a _________.
2. The Central Limit Theorem states that if random samples of size *n* are drawn from a population, then as *n* becomes larger, the sampling distribution of the mean approaches the normal distribution, regardless of the shape of the _______________.
3. The mean of the population is equal to the mean of the __________.
4. The Russian Mathematician who gave the first rigorous proof of the general Central Limit Theorem is ____________.
5. If a population has a mean of 25.6, then the mean of the sampling distribution of the sample means is ________.

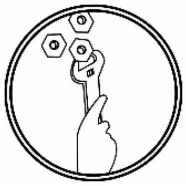
**Assessment 1: Fix the Problem!**

**Directions:** Read and analyze the given problems below. Write your solutions and final answer on a separate sheet of paper.

1. ILAW Manufacturing company produces bulbs that last a mean of 900 hours with a standard deviation of 110 hours. What is the probability that the mean lifetime of a random sample of 15 of these bulbs is less than 850 hours?

2. A school principal claims that grade 11 students have a mean grade of 86 with a standard deviation of 4. Suppose that the distribution is approximately normal. What is the probability that a randomly selected grade will be greater than 82 but less than 90?

*Great job! You have understood the lesson. Are you now ready to summarize?*



## *Deepen*

At this point, you will be applying the concepts of Central Limit Theorem. Specifically, in solving problems involving sampling distribution of the sample means. The instructions for the given activity are provided below. The scoring rubric below will be used in assessing your output.

As a Senior High School Student, how much time do you spent in studying and answering the activities in your modules? Do you spend 25 hours in a week or more than that? Suppose that the average number of hours spent by senior high school students in your school for their modular classes in a week is 25 hours with a standard deviation of 4 hours. Assuming that the study is true and the data is normally distributed. What is the probability that a random sample of 12 senior high school students spends more than 24 hours?

**Rubric for Scoring the Output**

| **Categories** | **Excellent**<br>**5** | **Fair**<br>**3** | **Poor**<br>**1** | **Score** |
|---|---|---|---|---|
| Content | Appropriate content is used in the given activity. Student clearly understands the mathematical concepts. | Appropriate content is used in the given activity. Student understands most of the mathematical concepts. | Appropriate content is not observed. Student does not demonstrate understanding of the mathematical concepts. | |
| Applies appropriate procedures | All procedures are appropriate for the problem. | Applies mostly appropriate procedures for the problem. | Applies inappropriate procedures for the problem. | |
| Solution and Answer | Correct solution and information about the problem. Gives correct and complete answer. | Computation and answer may give contain minor flaws. | Computation in incorrect, attempts an answer. | |

# Gauge

**Directions:** Read carefully each item. Use a separate sheet for your answers. Write only the letter of the best answer for each test item.

1. Which of the following is correct about the sampling distribution of the sample mean using the Central Limit Theorem?
   A. The mean of the sampling distribution of the means is not equal to the mean of the population.
   B. As the sample size *n* increases, the sampling distribution of the means approaches a normal distribution.
   C. The variance of the sampling distribution of the means is equal to the variance of the population multiplied by the sample size *n.*
   D. The standard deviation of the sampling distribution of the means is equal to the standard deviation of the population multiplied by the square root of the sample size *n.*

2. Which of the following refers to the standard deviation of a sampling distribution?
   A. It is the sum of squares.
   B. It is the standard error of the mean.
   C. It only applies only to population data.
   D. It can be larger than the standard deviation for the population.

3. Who is the French-born Mathematician who proved the first version of the Central Limit Theorem?
   A. Abraham de Moivre B. Aristotle
   C. Aleksander Lyapunov D. Rene Descartes

4. Which of the following properties describes the sampling distribution of the sample mean?
   A. The mean of the sampling distribution of the means is equal to the mean of the population.
   B. As the sample size *n* decreases, the sampling distribution of the means approaches a normal distribution.
   C. The variance of the sampling distribution of the means is equal to the variance of the population multiplied by the sample size *n.*
   D. The standard deviation of the sampling distribution of the means is equal to the standard deviation of the population multiplied by the square root of the sample size *n.*

5. Which of the following statements best illustrates a z-value?
   A. The z-value leads to the area under the curve found in the normal curve table.
   B. The z-value leads to the area under the curve found in the normal distribution table, which is a probability.
   C. The z-value leads to the area under the curve found in the normal curve table, which is a percentage, and that percentage gives the desired probability for X.
   D. The z-value leads to the area under the curve found in the normal curve table, which is a probability, and that percentage gives the desired percentage for X.

6. Which of the following describes the central limit theorem?
   A. The mean of the sampling distribution of the means is not equal to the mean of the population.
   B. As the sample size $n$ decreases, the sampling distribution of the means approaches a normal distribution.
   C. The variance of the sampling distribution of the means is equal to the variance of the population multiplied by the sample size $n$.
   D. The standard deviation of the sampling distribution of the means is equal to the standard deviation of the population divided by the square root of the sample size $n$.

7. Which of the following statements is correct about mean of the samples?
   A. The mean of the sample is less than the population mean.
   B. The mean of the samples is greater than the population mean.
   C. The means of the samples drawn from a population are always equal to the population mean.
   D. The means of the samples drawn from a population may be equal, greater than or less than the population mean.

8. Which of the following refers to the frequency distribution using the means computed from all possible random samples of a specific size take from the population?
   A. Population Distribution
   B. Frequency Distribution Table
   C. Sampling Distribution of the Sample Means
   D. Sampling Distribution of the Population Mean

9. Which of the following is used when computing for the probability $\bar{x}$ that will take on a value within a given range in the sampling distribution of the sample mean?
   A. $z = \frac{\bar{x}-\mu}{\sigma}$ B. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ C. $z = \frac{\bar{x}-\sigma}{\mu}$ D. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{N}}}$
10. What is the mean of sampling distribution if mean of population is 25?
   A. 25 B. 5 C. 30 D. 35
11. What is the standard deviation of sampling distribution if standard deviation of population is 35 and sample size is 9?
   A. 11.67 B. 12.67 C. 13.67 D. 14.67
12. If a population has a mean of 6.5, what is the mean of the sampling distribution of its means?
   A. 5.6 B. 6.5 C. 6.7 D. 65

For numbers 13-15, use the problem below.

*The average time it takes for high school students to complete certain examination is 46.2 minutes. The standard deviation is 8 minutes. There will be 50 random samples to be selected. Assume that the variable is normally distributed.*

13. What is the population mean in the given problem?
   A. 0 B. 8 C. 46.2 D. 50
14. What is the value of $\boldsymbol{n}$ in the given problem?
   A. 0 B. 8 C. 46.2 D. 50
15. If 50 random selected high school students take the exam, what is the probability that the mean time it takes the group to complete the test will be less than 43 minutes?
   A. 0.23% B. 2.3% C. 3% D. 23%

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