10

Mathematics

Quarter 3- Week 2 Module 2: Combinations

AIRs - LM

Mathematics 10
Quarter 3 –Week 2- Module 2: Combinations

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Target

This module was designed and written with you in mind. This will help you illustrate the combination of objects. The scope of this module will be used in many different learning situations. The language used recognizes the diverse vocabulary level of learners. The lessons are arranged to follow the standard sequence of the course but how the learner read and answer this module is dependent on his/her ability.

After going through this module, the learner is expected to be able to demonstrate knowledge and skills related to combinations which will further help in forming conclusions and in making decisions.

Learning Competency:

Illustrates the combination of objects **(M10SP - IIIc– 1)**

Subtasks:

- Defines the combination of objects
- Gives the combination of n objects taken r at a time
- Illustrates real life situations involving the combination of objects

Pre-Assessment:

The questions below are intended to check your prior knowledge, skills, and understanding on the illustration of combinations. Choose the letter of your choice from the given options. Take note of the items that you are not able to answer correctly and find the correct answer as you go through this module. Write your answers on a separate sheet of paper.

1. Which of the following refers to the number of ways of selecting objects from a set when the order or arrangement is not important?
   A. Circular permutation B. Combination
   C. Fundamental Counting Principle D. Permutation

2. Which of the following is the formula used to determine the number of combinations of n objects taken r at a time?
   A. C(n,r) = $\frac{n!}{(n-r)!}$ B. C(n,r) = $\frac{n!}{r!}$
   C. C(n,r) = $\frac{n!}{r!(n-r)!}$ D. C(n,r) = $\frac{n}{r(n-r)}$

3. Which of the following does NOT illustrate a combination?
   A. taking 5 modules from the pile
   B. drawing 6 cards from a deck of cards
   C. entering the PIN of ATM card
   D. choosing 4 persons to clean the yard

4. Which of the following is a combination?
   A. announcing the 5 finalists
   B. opening a combination lock
   C. assembling a jigsaw puzzle
   D. arranging 5 persons in a row

5. Which of the following most likely describe *choosing a subset of a set*?
   A. combination B. factorial C. order D. permutation

6. Which of the following shows the combination of letters ABC taken two at a time?
   A. AB, AC, BC, BA, CB,CA B.AB,BC,AC
   C. ABC,BAC,CBA D.A,B,C

7. Given S = { apple, orange, melon },how many possible selections of 2 fruits from the set are there?
   A. 2 B. 3 C. 4 D. 5

8. Which of the following is the same as C(5,3)?
   A.5!/3! B.5!/2! C.5!/2!3! D.5!

9. What is the expression in taking the number of ways can a basketball team of 5 players be chosen from 9 players?
   A. 9C5 B. 5C9 C. 5!9! D. (9-5)!

10. If the number of objects is represented by n taken 4 at a time and the combination is 5, then which of the following equation is to be used?
   A. C(4,n) = 5 B. C(n,4) = 5 C. C(5,n) = 4 D. C(n,5) = 5

11. How many combinations are there if the number of objects is the same as the value of r?
   A. 1 B. 2 C. 3 D. 4

12. What equation best represents the combination of 5 objects taken 2 at a time?
   A. C(x, 2) = 5 B. C(2, 5) = x C. C(5, 2) = x D. C(5, x) = 2

13.If C(n,r) =20,which of the following are possible values of n and r?

A. n=5,r=2 B.n=6, r=3 C.n=7,r=5 D.n=5,r=3

14. There are 11 volunteers to clean the room. If the teacher was able to form 462 possible ways of grouping them, which of the following did the teacher possibly do?

A. Choose 4 out of the 11 volunteers.
B. Choose 6 out of the 11 volunteers.
C. Choose 8 out of the 11 volunteers.
D. Choose 7 out of the 11 volunteers.

15. If w=C(5,2), x =C(6,5) , y =(7,2) and z=(6,4),which of the following is the correct order when arranged from least to greatest?

A.w,x,y,z B.x,w,z,y C.y,z,w,x D.w,y,x,z

Jumpstart

In the previous module, you learned about permutations and the Fundamental Counting Principle. This knowledge and skill will help you understand combinations, which will further help you in forming conclusions and in making decisions. To be able to do this, perform each activity that follows.

Activity 1: REMEMBER ME?

Answer the following questions.

1. Matheo has 10 t-shirts,5 pairs of pants and 3 pairs of shoes,in how many possible ways can he dress up himself for the day?
2. In how many ways can 7 people arrange themselves in a row for picture taking?
3. A dress shop owner has 8 new different dresses that she wants to display in a window. If the display window has 5 identical mannequins, in how many ways can she dress them up?
4. In how many ways can 8 people be seated around a circular table?
5. How many distinguishable permutations of the letters of the word PASS?

Activity 2: WATCH YOUR ORDER!

Study the situation given in each item below then identify if order or arrangement is important. Write **OI** if order or arrangement is important and **ONI** if order is not important.

1. Opening a combination lock
2. Choosing leaders from the different groups
3. Selecting 6 questions from 15 questions in a test
4. Awarding the top 5 winners in a contest
5. Designing seats to VIPs for dinner
6. Pressing the PIN of ATM card to withdraw money
7. Selecting 5 persons to represent the organization
8. Choosing 3 teachers to distribute the modules
9. Forming triangles from 7 distinct points in which no 3 points are collinear.
10. Appointing 4 learners to lead the class

Questions:

1.In which tasks/activities above is order important?

2.In which tasks/activities is order NOT important?

Discover

Situations where *order is important* are **permutations**, while situations where *order is not important* are **combinations**. Permutation is an ordered combination. Items 2,3,7,8,9 and 10 in Activity 2 are situations where order is not important. They are examples of combinations.

Let's try to analyze the situation below.

Suppose you were assigned to be the representative of your group for a certain task. You have to choose 5 of your classmates to be your members. If you choose Ara, Bea, Carlo, Dario and Ellen, does it make any difference if you choose instead Bea, Carlo, Ara, Ellen and Dario? Of course not, because the list refers to the same persons. Each selection that you could possibly make is **combination**.

Let's discover something by performing this simple activity.

S= 4 kinds of flowers : rose, santan, daisy, gumamela

1. Select 1 flower at a time.

   Selection: 1. rose or 1. gumamela
   2. gumamela 2. Rose
   3. daisy 3.santan
   4.santan 4. daisy

   You can see that there are 4 ways in selecting 1 flower at a time.

2. Select 2 flowers at a time.

   Selection: 1. rose, santan or santan, rose (1 combination only)
   2. rose, daisy
   3. rose, gumamela
   4. santan, daisy
   5. santan, gumamela
   6. daisy, gumamela

   You can see that there are 6 ways in selecting 2 flowers at a time

3.Select 3 flowers at a time.

   Selection:

   1. rose, santan, daisy
   2. rose, santan,gumamela
   3. daisy, rose, gumamela
   4. daisy, santan, gumamela

   You can see that there are 4 ways in selecting 3 flowers at a time.

4.Select 4 flowers at a time.

   Selection: rose, gumamela, santan, daisy

There is only 1 way of selecting 4 flowers at a time.

Take note of this!

If there is a set S with n elements, and if r is a nonnegative integer less than or equal to n, then each subset of S containing r distinct elements is called a **combination** of S.

The number of combinations of n objects taken r at a time is denoted by C(n,r) or nCr .

The above examples can be illustrated as:

1. C(4,1) = 4
2. C(4,2) = 6
3. C(4,3) = 4
4. C(4,4) = 1

Activity 3: WATCH MY PATTERN!

Study and complete the table below then answer the questions that follow.

Number of Objects(n)	Number of Objects Taken at a Time(r)	Number of Possible Selections	Written as C(n,r)= no. of possible selections
2	1	2	C(2,1)=2
2	2		
3	1		
3	2		
3	3		
4	2		
4	3		
5	2		
5	3		
5	4		

Questions:

1. Can you find any pattern in the results?
2. Can you think of other ways of finding these answers?

Activity 4: Let's Explore MATH!

Suppose you are to take the combinations of the 3 letters of the word **MATH.** We can see that taking MAH is the same as taking MHA, AHM, AMH/HMA/HAM. Hence, all these arrangements are considered as 1 combination only.

How can we find the number of combinations more systematically? Let's study this: If order of the letters is important, then we have the following possibilities in taking 3 letters from the word MATH.

MAT	MAH	ATH	THM
MTA	MHA	AHT	TMH
ATM	AHM	THA	HMT
AMT	AMH	TAH	HTM
TMA	HAM	HAT	MHT
TAM	HMA	HTA	MTH

We can clearly see that in each column of arrangements or permutation, it is considered only as 1 combination. So, the number of different orders of 4 letters taken 3 at a time is given by

P (4,3) = $\frac{4!}{(4-3)!}$ = 24 , there are 24 possibilities, thus the actual number of combinations is C(4,3) = $\frac{24}{6}$ = 4. There are only 4 possibilities for combination or for taking the combination it could be done as $\frac{P(4,3)}{6}$ or $\frac{P(4,3)}{3!}$ because for every 6 arrangements there is only 1 combination as you can see in the list of possibilities above. Remember that there are 4 objects (M,A,T,H) , so n=4, and they are selected 3 at a time so, r=3.

Therefore, C(n,r) = $\frac{P(n,r)}{r!}$ =$\frac{n!}{r!(n-r)!}$.
The combination of n objects taken r at a time is :

C(n,r) = $\frac{n!}{r!(n-r)!}$, where n≥r≥0 .

Try to work on this:

Give the combination in each of the following given the number of objects (n) and the number of objects taken at a time (r). The first number is done for you.

1. n = 5, r = 3
   C(n,r) = $\frac{n!}{r!(n-r)!}$
   C(5,3) = $\frac{5!}{3!(5-3)!}$ =$\frac{5.4.3.2.1}{3.2.1(2.1)}$ = 10
2. n = 4, r = 2
3. n = 6, r = 3
4. n = 5, r = 2
5. n = 5, r = 4

If you already know how to illustrate combination of objects, then you have already understood the concepts of combinations

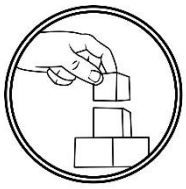
Activity 5: Reveal My Message!

Write the letter or symbol of the given situation on combination listed below into the box that corresponds to its value. Write the hidden message on the blank below the given situation on combination.

5	35	28	20	10	3	56	9	6	7	15

* n = 5, r = 2	**T**. n= 8, r = 6	**I.** n=3, r = 1
U. n = 7, r = 6	* n = 9, r = 1	**N.** n = 6, r = 2
M. n=5, r =4	**A**. n = 7, r = 4	**S.** n = 8, r = 5
H. n = 6, r = 3	**F**. n= 6, r = 1	

Hidden message: _____



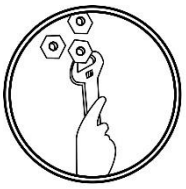
Explore

Activity 6: STRETCH IT OUT!

Find the unknown in each item.

1. C(7,4)=___
2. C(8,r)= 56
3. C(n,8)=45
4. C(8,8)=___
5. C(n,4)=5
6. C(10,r)=252
7. C(n,4)= 70
8. C(6,r)=1
9. C(n,2)=10
10. C(11,r)=330

How did you find the activity? Was it easy? I guess so. Go on to the next activity to apply your knowledge of combinations in real life situations.



Deepen

In this activity you are going to think more deeply. You are now required to apply your knowledge and skills learned from the previous activities. Let us see how well you can work on real life situations involving combinations.

Activity 7: TRANSLATE ME

Here are real life situations. How many selections can be made out of the given situations? Illustrate your answer using the notation **C(n,r)**.**Do not solve.** An example has been done for you.

Example: Selections of 3 from your 10 favorite songs
Answer: C(10,3)

1. Selections of 8 books from a list of 40
2. Selections of 6 people to test the anti Covid-19 drug out of 13 volunteers
3. Number of committees formed from 11 members of the organization if the committee has to be composed of 4
4. Number of handshakes if there are 15 people and they have to shake hands with each other
5. Selections of 3 toys from a box with 12 toys on it

Activity 8: COMBINATIONS FOR REAL

Illustrate the combinations asked in each situation using the formula then solve.

1. In how many ways can 5 ice cream flavors be selected from 10 possible choices?
2. Mathsagana High School is planning to conduct their Mathematics Festival. All Math classes will send 5 representatives to compete. How many different groups of students can be chosen from a class of 16 students?
3. A photographer is taking pictures of a bride and a groom and 6 guests. If he takes photographs of 3 people in a group, how many different groups can he photograph?
4. Five vials of Covid 19 vaccines will be pilot tested to a group of 12 individuals. In how many ways can the 5 recipients of the vaccines be selected?
5. In how many ways can a 6/42 lotto player choose a single bet? If each bet costs Php 20, how much will he pay if he so decides to bet on all of the choices

Congratulations! You are almost done with this module. To check your understanding of combinations, answer the post-test below.

Gauge

Directions: Read and analyze each item carefully then write the letter of the correct answer.

1. It refers to the number of ways of selecting from a set when the order or arrangement is not important.
   A. Circular permutation B. Combination
   C. Fundamental Counting Principle D. Permutation

2. Which of the following could be the equation for the number of combinations of n objects taken r at a time?
   A. C(n,r) = $\frac{n!}{(n-r)!}$ B. C(n,r) = $\frac{n!}{r!}$
   C. C(n,r) = $\frac{n!}{r!(n-r)!}$ D. C(n,r) = $\frac{n}{r(n-r)}$

3. Which of the following is NOT a combination?
   A. taking 5 modules from the pile
   B. drawing 6 cards from a deck of cards
   C. entering the PIN of ATM card
   D. choosing 4 persons to clean the yard

4. Which of the following is a combination?
   A. Selecting 9 members to be the committee on foods
   B. arranging 5 persons to have their seats in a row
   C. assembling a jigsaw puzzle
   D. opening a combination lock

5. Which of the following most likely describe *choosing a subset of a set*?
   A. order B. factorial C. combination D. permutation

6. Which of the following shows the combination of letters JOY taken two at a time?
   A. JO, JY, OY, OJ, YJ,YO B.JO,OY,JY
   C. JOY,OYJ,YOJ D.JOY

7. Given S = { chico, guyabano, mango },select 2 fruits from the set then determine the number of possibilities in selecting the chosen fruits.
   A. 2 B. 3 C. 4 D. 5

8. What must be the value of r in C(n,r) ?

A. $n \geq r \geq 0$ B. $n \geq 0 \geq r$ C. $n = r \geq 0$ D. $n = r = 0$.

9. What is the expression in taking the number of ways can a basketball team of 5 players be chosen from 9 players?

A. 9C5 B. 5C9 C. 5!9! D. (9-5)!

10. Which of the following is/are true?

i. C(n,n) =n ii. C(n,n-1)=n iii. C(n,n) = 1

A.i only B.i and iii C.ii only D.ii and iii

11. If C(n,4)=126,what is n?

A. 11 B. 10 C. 9 D. 7

12. What equation best represents the combination of 7 objects taken 4 at a time?

A. C(x, 7) = 4 B. C(4, 7) = x C. C(7, 4) = x D. C(7, x) = 4

13.If C(n,r) =21,which of the following are possible values of n and r?

A. n=5,r=2 B.n=6, r=3 C.n=7,r=5 D.n=5,r=3

14. There are 10 volunteers to clean the room. If the teacher was able to form 210 possible ways of grouping them, which of the following did the teacher possibly do?

A. Choose 4 out of the 10 volunteers.
B. Choose 6 out of the 10 volunteers.
C. Choose 8 out of the 10 volunteers.
D. Choose 7 out of the 10 volunteers.

15. If w=C(6,2), x =C(6,5) , y =(7,4) and z=(5,4),which of the following is the correct order when arranged from least to greatest?

A.w,x,y,z B.x,w,z,y C.y,z,w,x D.z,x,w,y

References

Books:

Mathematics Grade 10 Learner's Module, First Edition 2015, pp. 301-309.
Mathematics Grade 10 Teacher's Guide, First Edition 2015, pp. 259 -264.
Advanced Algebra: Trigonometry and Statistics, Dilao, et. Al., .

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