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LA UNION SCHOOLS DIVISION OFFICE

Mathematics

Quarter 4: Week 3 - Module 3
Proving Properties of Parallel Lines Cut by a Transversal

AIRs- LM

Mathematics 8
Quarter 4- Week 3 Module 3: Proving Properties of Parallel Lines Cut by a Transversal
First Edition, 2021

La Union Schools Division
Region I

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Target

This module was designed and written with you in mind. It is here to help you master on how to prove properties of parallel lines cut by a transversal. The scope of this module permits it to be used in many different learning situations. The language and numeric used recognize the diverse vocabulary and numeracy level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to attain the following objectives:

Learning Competency:

- Proves properties of parallel lines cut by a transversal.
  (M8GE-IVd-1)

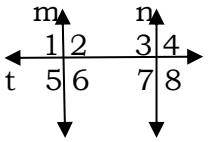
Subtasks:

- Define and illustrate parallel lines.
- Define transversal line.
- Illustrates properties of parallel lines cut by a transversal.
- Use properties to find measures of angles and other quantities involving parallel lines cut by a transversal.

Pre-Assessment

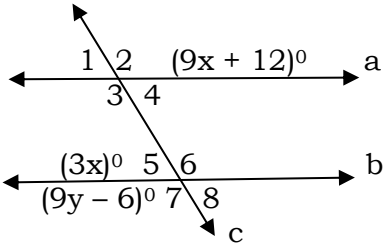
Find out how much you already know about this module. Choose the letter of the correct answer. Write your answers on a separate sheet of paper.

1. What do you call the line that intersects two or more coplanar lines at different points?
   A. Parallel lines B. Transversal
   C. Perpendicular lines D. Line segment
2. What do you call the angels that lie between the parallel lines on the same side of the transversal?
   A. Alternate Exterior Angles B. Alternate Interior Angles
   C. Same-Side Interior Angles D. Corresponding Angles
3. The following pairs of angles are formed when a transversal intersects two or more lines in the same plane. EXCEPT one. Which one is not?
   A. Alternate Exterior Angles B. Alternate Interior Angles
   C. Right angles D. Corresponding Angles
4. "If two parallel lines are cut by a transversal, then the two pairs of angles are supplementary". What theorem does the statement denote?
   A. Same-Side Interior Angles Theorem B. Corresponding Angles Postulate
   C. Alternate Interior Angles Theorem D. Alternate Exterior Angles Theorem
5. Lines *m* and *n* are parallel cut by transversal *t* which is also perpendicular to *m* and *n*. Which of following statements is NOT true?



   A. ∠1 and ∠6 are congruent.
   B. ∠2 and ∠3 are supplementary.
   C. ∠3 and ∠5 are congruent angles.
   D. ∠1 and ∠4 form a linear pair.

For item no. **6-13.** Refer to the figure at the right.



6. Identify the transversal line.
   A.line a B. line b C. line c D. line a & line c

7. What is the value of x?
   A.12 B. 14 C. 45 D. 90

8. Find: m∠ 3
   A. 42 B. 90 C. 138 D. 180

9. What is the value of y?
   A. 16 B. 90 C. 138 D. 180

10. For them to be called corresponding angles, what angle must be paired to ∠ 1?
    A. ∠2 B. ∠3 C. ∠4 D. ∠5

11. What angle must be paired to ∠ 7 to call them "alternate exterior angles"?
    A. ∠2 B. ∠3 C. ∠4 D. ∠5

12. What theorem satisfies the relationship between ∠1 & ∠5?
    A. Alternate Exterior Angle Theorem B. Alternate Interior Angle Theorem
    C. Same-side Interior Angle Theorem D. Corresponding Angle Postulate

13. What theorem satisfies the relationship between ∠4 & ∠5?
    A. Alternate Exterior Angle Theorem B. Alternate Interior Angle Theorem
    C. Same-side Interior Angle Theorem D. Corresponding Angle Postulate

14. You are tasked to sketch a plan of a parking lot of a mall. Which of the following should you include in the plan in order to maximize the use of the area?
    A. landscaping designs B. use of parallel lines
    C. entrance art design D. use of different shapes]

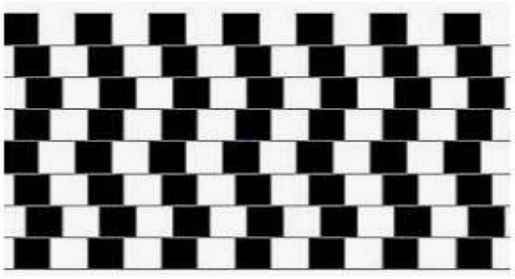
15. Michael is repairing a wooden clothes stands with damaged legs. Which action should he consider?
    A. Check if the clothes stand is high enough for the lengthy garments.
    B. Check if the legs of the clothes stands are parallel to one another.
    C. Check if the distance between legs is greater than the length of the base.
    D. Check if the length of the base is the same as the length of the legs.

Lesson 1 Proves Properties of Parallel Lines Cut by a Transversal

Start this module by assessing your knowledge and skills in determining the properties of parallel lines. These knowledge and skills may help you in proving properties of parallel lines cut by a transversal and achieve the targets for this module.

Review

Activity: Optical Illusion

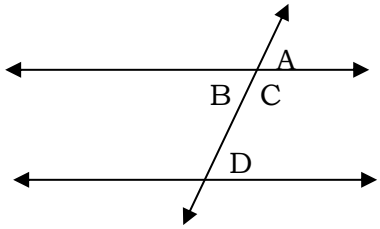


a. Can you see straight lines in the picture above?
b. Do these lines meet/intersect?
c. Are these lines parallel? Why?

Jumpstart

Activity: My Pair and I, Who Are We?

Use the diagram of a parallel lines cut by a transversal below to answer and complete the mathematical sentence.



1. ∠A and ∠D are called _____ angles.
2. ∠B and ∠D are called _____ angles.
3. ∠C and ∠D are called _____ angles.

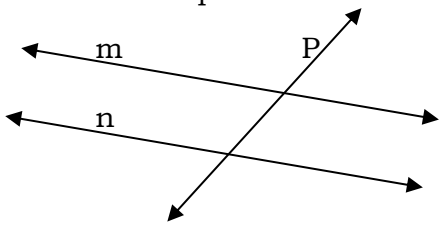
Guide Questions:

1. Were you able to determine the given pairs of angles correctly?
2. Is there any confusion in naming the pair of angles given?

3. Did you find difficulty in the conduct of the activity? What did you do to overcome this difficulty?

Discover

A transversal is a line that intersects two or more coplanar lines at different points. When a transversal intersects two or more lines in the same plane, a series of angles are formed. Certain pairs of angles are given specific "names" based upon their locations in relation to the lines. These specific names may be used whether the lines involved are parallel or not parallel.

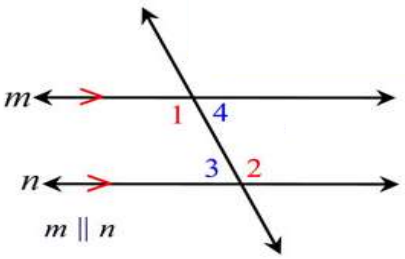


In the figure, $\overleftrightarrow{P}$ is the transversal. Lines m and n are parallel. You can write this as a ∥ b.

Alternate Interior Angles:

The word "alternate" means "alternating sides" of the transversal. This name clearly describes the "location" of these angles. Alternate interior angles are "*interior*" (between the parallel lines), and they "*alternate*" sides of the transversal. Notice that they are not adjacent angles (next to one another sharing a vertex).
When the lines are parallel, the alternate interior angles are equal in measure.

$m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

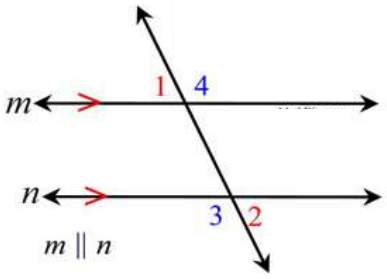


∠1 and ∠2 are alternate interior angles
∠3 and ∠4 are alternate interior angles

Alternate Exterior Angles:

The word "alternate" means "alternating sides" of the transversal. The name clearly describes the "location" of these angles. Alternate exterior angles are "*exterior*" (outside the parallel lines), and they "*alternate*" sides of the transversal. Notice that, like the alternate interior angles, these angles are not adjacent.
When the lines are parallel, the alternate exterior angles are equal in measure.

and $m\angle 3 = m\angle 4$



∠1 and ∠2 are alternate exterior angles
∠3 and ∠4 are alternate exterior angles

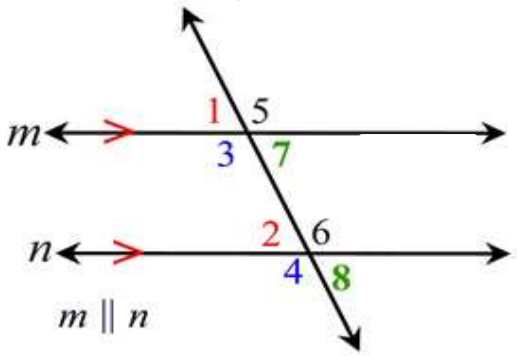
Corresponding Angles:

The name does not clearly describe the "location" of these angles. The angles are on the same side of the transversal, one interior and one exterior, but not adjacent. The angles lie on the same side of the transversal in "corresponding" positions. If you copy one of the corresponding angles and you translate it along the transversal, it will coincide with the other corresponding angle. For example, slide ∠1 down the transversal and it will coincide with ∠2.

When the lines are parallel, the corresponding angles are equal in measure.

$m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

$m\angle 5 = m\angle 6$ and $m\angle 7 = m\angle 8$



∠1 and∠2 are corresponding angles
∠3 and∠4 are corresponding angles
∠5 and ∠6 are corresponding angles
∠7 and∠8 are corresponding angles

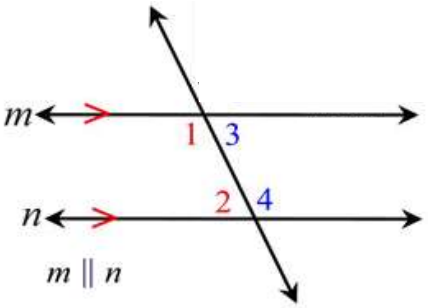
Interior Angles on the Same Side of the Transversal/Consecutive Interior Angles:

The name is a description of the "location" of the these angles. When the lines are parallel, the measures are supplementary. These angles are located exactly as their name describes. They are "*interior*" (between the parallel lines), and they are on the *same side of the transversal*.

When the lines are parallel, the interior angles on the same side of the transversal are supplementary.

$m\angle 1 + m\angle 2 = 180$

$m\angle 3 + m\angle 4 = 180$

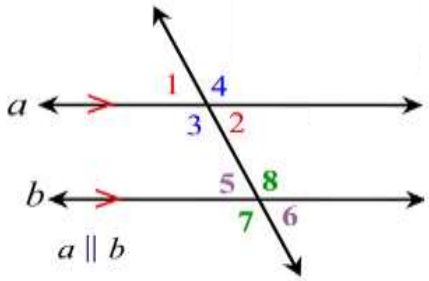


∠1 and ∠2 are interior angles on the same side of transversal
∠3 and ∠4 are interior angles on the same side of transversal

In addition to the 4 pairs of named angles that are used when working with parallel lines (listed above), there are also some pairs of *"old friends"* that are also working in parallel lines.

Vertical Angles:

When straight lines intersect, vertical angles appear. Vertical angles are always equal in measure, whether the lines are parallel or not.



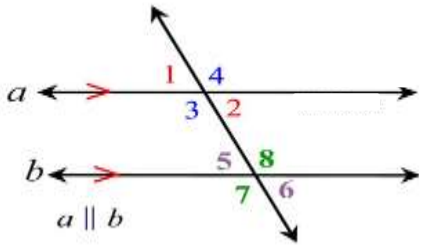
There are 4 sets of vertical angles in this diagram.

∠1 and ∠2
∠3 and ∠4
∠5 and ∠6
∠7 and ∠8

Remember: the lines need not be parallel to have vertical angles of equal measure.

Linear Pair Angles:

A linear pair are two adjacent angles forming a straight line. Angles forming a linear pair are always supplementary.



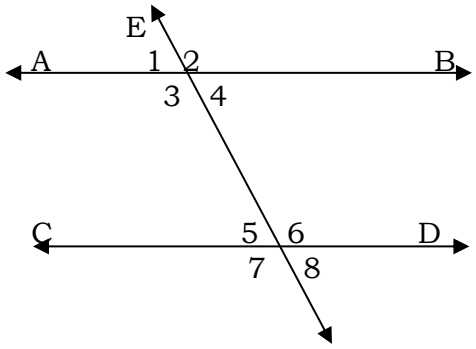
Since a straight angle contains 180°, the two angles forming a linear pair also contain 180° when their measures are added (making them supplementary).

$m\angle 1 + m\angle 4 = 180$
$m\angle 1 + m\angle 3 = 180$
$m\angle 2 + m\angle 4 = 180$
$m\angle 2 + m\angle 3 = 180$
$m\angle 5 + m\angle 8 = 180$
$m\angle 5 + m\angle 7 = 180$
$m\angle 6 + m\angle 8 = 180$
$m\angle 6 + m\angle 7 = 180$

To help you understands better, try to explore the following illustrative examples below on proving properties of parallel lines cut by a transversal.

Illustrative Example: Refer to the figure below, A ∥ B cut by a transversal C.

A. List all the pairs of angles formed according to their classification.



Solution:

Corresponding Angles	Alternate Exterior Angles	Alternate Interior Angles	Same-Side Interior Angles	Vertical Angles	Linear Pair Angles
∠1 and ∠5	∠1 and ∠8	∠3 and ∠6	∠3 and ∠5	∠1 and ∠4	∠1 + ∠3
∠3 and ∠7	∠2 and ∠7	∠4 and ∠5	∠4 and ∠6	∠2 and ∠3	∠2 + ∠4
∠2 and ∠6				∠5 and ∠8	∠3 + ∠4
∠4 and ∠8				∠8 and ∠7	∠1 + ∠2
					∠5 + ∠6
					∠6+ ∠8
					∠8 + ∠7
					∠7 + ∠5

B. a. if m∠2 = 120, find m∠1.

Answer: $m\angle 1 = 180^0 - 120^0 = 60^0$ since ∠1 and ∠2 formed a linear pair

b. if $m\angle 4 = (5x - 2)^0$ and $m\angle 5 = 53^0$), find x.

Answer: Since the angles are alternate interior angles, they are congruent.

$(5x - 2) = 53$

$5x = (53 + 2)$

$5x = 55$

$x = 11^0$

To check: $(5x - 2) = 53$

$(5(11) - 2) = 53$

$55 - 2 = 53$

$53^0 = 53^0$

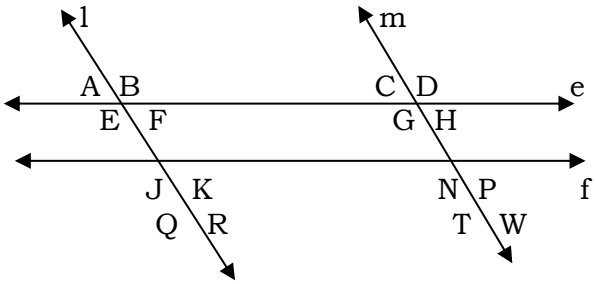
C. Name all the angles that would be congruent to ∠1.

Answer: ∠4, ∠5 and ∠8

Explore

Activity 1: Find My Twins

Use the diagram below, then answer what is being asked.



1. Assume lines l and m are parallel, and lines e and f are parallel. List all the angles that would be congruent to ∠K.
2. Determine which of the following angle relationships would prove lines e ∥ f. For each of the problems, explain why or why not.
   a. ∠A ≅ ∠J b. ∠A ≅ ∠C c. ∠D ≅ ∠T
   d.∠G ≅ ∠P e. ∠N ≅ ∠W
3. a. If m∠F = 87^0 and m∠R = $(4x + 3)^0$. What is the value of x?
   b. If m∠D = $(5x – 5)^0$; m∠H = $(6y – 4y)^0$ and m∠P = $(4x + 10)^0$. Find x and y.

What I Have Learned

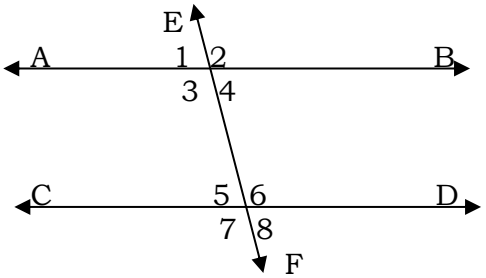
Journal Writing: Reflect on the activities you have done in this lesson by completing the following sentences. Write your answers on your journal notebook.

I learned that I _____
I was surprised that I _____
I noticed that I _____
I discovered that I _____
I was pleased that I _____

Deepen

Activity: Time To Solve Me

Refer to the diagram below for the following exercises.

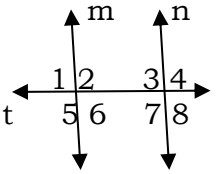


1. If the m∠3 = 4x + 18 and m∠5 = 6x + 2, find the measure of x that proves AB ‖ CD. Explain why your solution is valid.
2. If the m∠1 = 3x + 5 and m∠8 = 4x – 22, Find the measure of x that proves AB ‖ CD. Explain why your solution is valid.
3. EF intersects AB and CD . ∠4 and ∠6 are supplements. Prove AB ‖ CD.

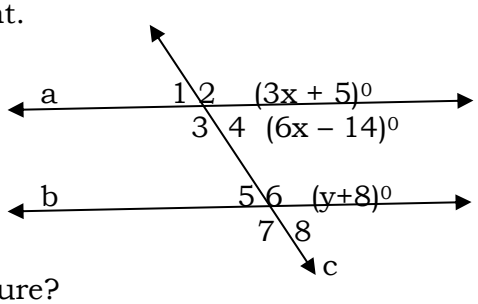
Gauge

Directions: Read each item very carefully. Choose the letter of the correct answer. Write your answers on a separate sheet of paper.

1. What do you call the line that intersects two or more coplanar lines at different points?
   A. Parallel lines B. Transversal
   C. Perpendicular lines D. Line segment
2. The relationship between two angles formed by parallel lines cut by a transversal is supplementary. Which pair of angles is this?
   A. Alternate Exterior Angles B. Alternate Interior Angles
   C. Same-Side Interior Angles D. Corresponding Angles
3. The following pairs of angles are formed when a transversal intersects two or more lines in the same plane EXCEPT what?
   A. Alternate Exterior Angles B. Alternate Interior Angles
   C. Right angles D. Corresponding Angles
4. "If two parallel lines are cut by a transversal, then the two pairs of angles are supplementary". What theorem does the statement denote?
   A. Same-Side Interior Angles Theorem B. Corresponding Angles Postulate
   C. Alternate Interior Angles Theorem D. Alternate Exterior Angles Theorem
5. Lines *m* and *n* are parallel cut by transversal *t* which is also perpendicular to *m* and *n*. What judgment can you make on the following statements which is NOT true?
   A. ∠1 and ∠6 are congruent.
   B. ∠2 and ∠3 are supplementary.
   C. ∠3 and ∠5 are congruent angles.
   D. ∠1 and ∠4 form a linear pair.



For item no. **6-13.** Refer to the figure at the right.



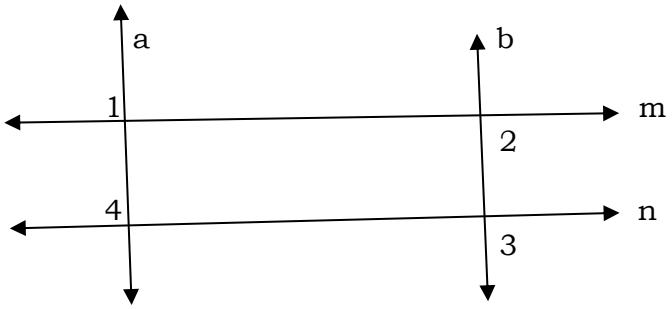
6. How many transversal line we have in the figure?
   A.0 B. 1 C. 2 D. 3
7. What is the value of x?
   A.8 B. 21 C. 48 D. 90
8. Find: m∠ 3
   A. 68 B. 90 C. 128 D. 180
9. What is the value of y?
   A. 30 B. 60 C. 90 D. 180
10. To call them corresponding angles, what angle must be paired to ∠8?
   A. ∠2 B. ∠3 C. ∠4 D. ∠5
11. What angle must be paired to ∠6 to call them "alternate interior angles"?
   A. ∠2 B. ∠3 C. ∠4 D. ∠5
12. What theorem satisfies the relationship between ∠1 & ∠8?
   A. Alternate Exterior Angle Theorem
   B. Alternate Interior Angle Theorem
   C. Same-side Interior Angle Theorem
   D. Corresponding Angle Postulate
13. What are the angles that are congruent to ∠4?
   A. ∠1; ∠2 and ∠3
   B. ∠1; ∠5 and ∠8
   C. ∠1 and ∠ 5
   D. ∠ 8 only
14. You are tasked to sketch a plan of a parking lot of a mall. Which of the following should you include in the plan in order to maximize the use of the area?
   A. landscaping designs
   B. use of parallel lines
   C. entrance art design
   D. use of different shapes
15. Michael is repairing a wooden clothes stands with damaged legs. Which action should he consider?
   A. Check if the clothes stand is high enough for the lengthy garments.
   B. Check if the legs of the clothes stands are parallel to one another.
   C. Check if the distance between legs is greater than the length of the base.
   D. Check if the length of the base is the same as the length of the legs.

Additional Activities

Refer to the figure below.

Given: Line **a** is parallel to Line **b**, Line **m** is parallel to line **n**.

If m∠1 = 4x + 45, m∠2 = 6x + 25, m∠3 = 3y + 50 and m∠4 = 6y + 5, find x and y.



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