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LA UNION SCHOOLS DIVISION OFFICE
REGION I

Mathematics

Quarter 3- Week 8
Module 6: Applications of Triangle Congruence

AIRs - LM

Mathematics 8
Quarter 3- Week 8 Module 6: Applications of Triangle Congruence
First Edition, 2021

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Target

Have you ever wondered how bridges and buildings are designed? What factors are being considered in the construction of bridges and buildings?

Designing structures requires the knowledge of triangle congruence, its properties, and principles.

This module includes triangle congruence and its application to construct perpendicular lines and angle bisector.

Before we start, let us consider first this learning competency.

1. Applies triangle congruence to construct perpendicular lines and angle bisectors. **(M8GE-IIIi-j-1)**

After going through this module, you are expected to:

1. Identify the different triangle congruence;
2. Illustrate the different triangle congruence;
3. Define perpendicular lines and angle bisector;
4. Apply triangle congruence to construct perpendicular lines and angle bisectors; and
5. Solve problems involving perpendicular lines, angle bisectors and isosceles triangle.

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper.

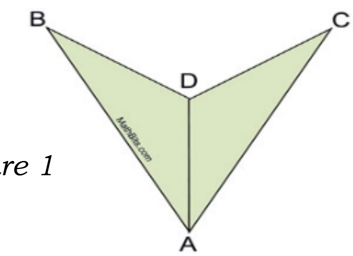
Pre-Assessment

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Two triangles are compared to see if they are congruent. Which of the following conditions will always mean that the triangles are congruent?
   A. both triangles contain a right angle
   B. three pairs of sides are the same
   C. two pairs of angles are congruent
   D. two pairs of sides are congruent
2. Given $\Delta ABC \cong \Delta DEF$, which statement is true?
   A. $\angle A \cong \angle E$ B. $\angle B \cong \angle F$ C. $BC \cong EF$ D. $AC \cong DE$

3. Which congruence postulate can be used to prove $\Delta BDA \cong \Delta CDA$?
   A. ASA B. AAS
   C. SAS D. SSS

figure 1



4. A pair of congruent triangles is shown at the right as marked. Which statement must be TRUE?

   A. $\angle DAC \cong \angle ACB$ B. $\angle DCB \cong \angle DAB$
   C. $CD \cong AB$ D. $AD \cong AB$

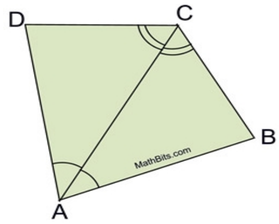


figure 2

5. If two sides of a triangle are congruent, what can be said about the angles opposite these sides?

   A. complementary B. congruent
   C. right angles D. supplementary

6. When are two intersecting lines or segments said to be perpendicular?

   A. they form acute angles B. they form obtuse angles
   C. they form right angles D. cannot be determined

7. Which of the properties described below DOES NOT apply to the perpendicular bisector of a segment?

   A. Every point on the perpendicular bisector is the same distance from both endpoints of the segment.
   B. It must be longer than the original segment.
   C. It is perpendicular to (makes a 90^0 angle with) the original segment.
   D. It divides the original segment into two equal pieces.

8. ΔABC is an isosceles triangle. BD is a perpendicular bisector of AC. Which statement can NOT always be proven?

   A. $\angle ADB \cong \angle CDB$ B. $\Delta ABD \cong \Delta CBD$ C. $AD \cong CD$ D. $BD \cong DC$

For items 9-11, BD is a perpendicular bisector of AC**.**
Use figure 3 at the right.

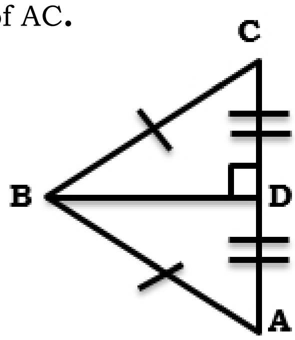


figure 3

9. What is the value of x?

   A. 1 B. 2
   C. 3 D. 4

10. What is the length of CD?

   A. 6 B. 8
   C. 10 D. 12

11. How long is AC?

   A. 12 B. 16 C. 20 D. 24

12. Which triangle has two equal sides and two equal angles?

   A. equilateral B. isosceles C. right D. scalene

13. Which of the following is an incorrect statement?

   A. All the perpendicular bisector of an isosceles triangle are equal.
   B. If the perpendicular bisector from one vertex of a triangle bisects the base of the triangle, then the triangle is isosceles.
   C. If the bisector of the vertical angle of the triangle bisects the base of the triangle, then the triangle is isosceles.
   D. Sides opposite to equal angles are equal in a triangle.

For items 14 and 15, $\overrightarrow{QS}$ bisects $\angle RQP$. Use figure 4 at the right.

14. What is m$\angle SQP$?

   A. 29^0 B. 58^0
   C. 68^0 D. 87^0

15. How large is $\angle RQP$?

   A. 29^0 B. 58^0
   C. 68^0 D. 87^0

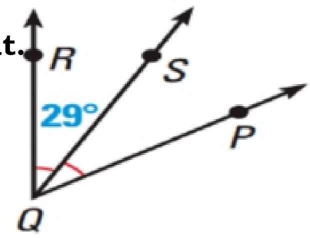


figure 4

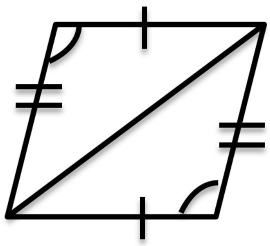
Lesson 1 | Applications of Triangle Congruence

Let's start this module by assessing your knowledge of the different mathematics concepts previously studied. These knowledge will help you understand on how construct perpendicular lines and angle bisector using triangle congruence. If you find difficulty in answering the activities, seek the assistance of your teacher.

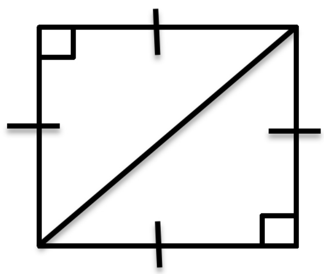
Activity 1: State My Congruence

Directions: State the triangle congruence postulate to show that the pairs of triangles are congruent.

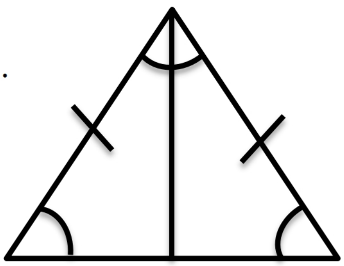
________1.



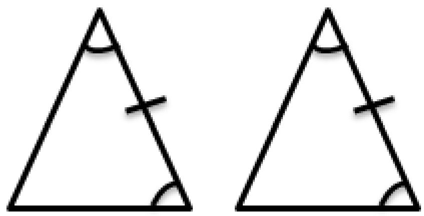
________2.



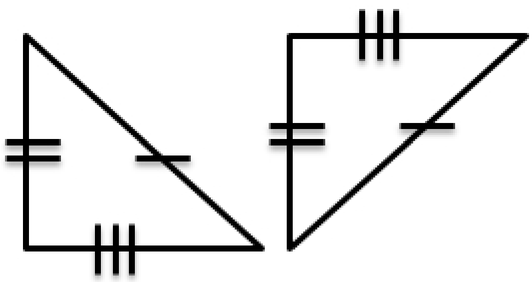
________3.



________4.



________5.



Were you able to identify the triangle congruence postulate used to show the two triangles congruent? In the next activity, you will indicate the additional corresponding congruent parts of the two triangles. You are done with this activity in your previous lesson so I am sure you can do it!

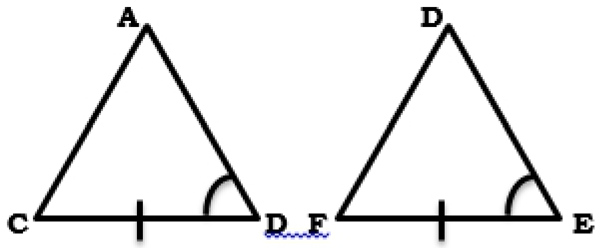
Jumpstart

For you to understand the lesson well, do the following activities. Have fun and good luck!

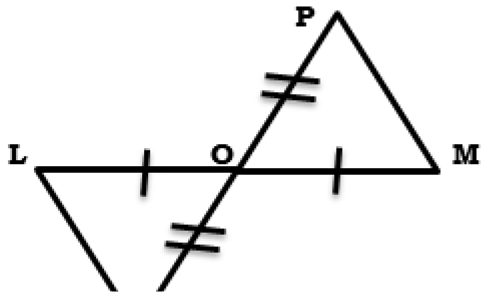
Activity 2: Mark Me

Directions: Corresponding congruent parts are marked. Indicate the additional corresponding parts needed to make the triangles congruent by using the specified congruence postulates.

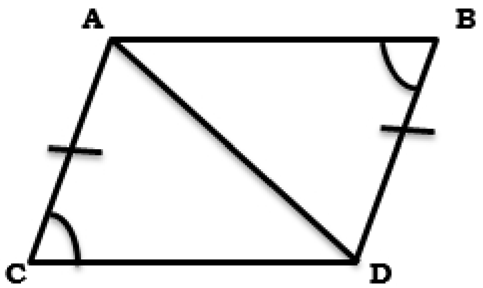
1. a. ASA _______
   b. SAS _______

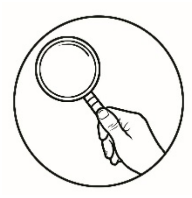


2. a. SAS ______
   b. SSS ______



3. a. SAS ______
   b. ASA ______



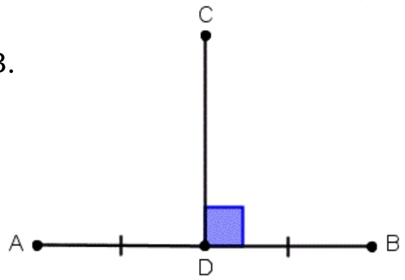


Discover

As you well know by now, being able to deduce key information from a limited set of facts is the basis of Geometry. An important type of segment, ray, or line that can help us prove congruence is called an angle bisector. Understanding what angle bisectors are and how they affect triangle relationships is crucial as we continue our study of geometry. Let's investigate different types of bisectors and the theorems that accompany them.

Perpendicular Bisector

Two lines are perpendicular when they intersect and meet at right angle.
A bisector is a line, ray or segment that cuts another segment or angle into two equal parts.
Perpendicular bisector is a line, ray or segment that bisects a segment at a right angle.
Segment CD is the perpendicular bisector to segment AB.



We derive two important theorems
from the characteristics of perpendicular bisectors.

Perpendicular Bisector Theorem

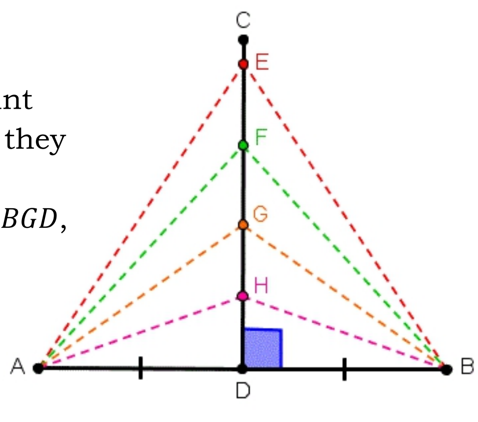
If a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Converse of Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

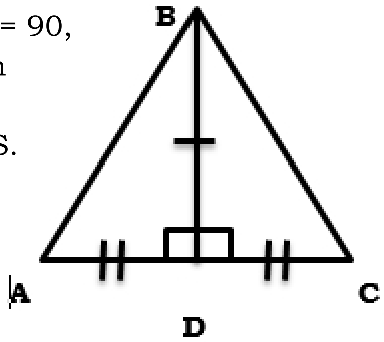
These theorems essentially just show that there exist a locus of points (which form the perpendicular bisector) that are equidistant from the endpoints of a given segment which meet at the midpoint of the segment at a right angle. An illustration of this concept is shown below.

Points E, F, G, and H (along with an infinite amount of points) are equidistant from A and B. Together, they form the perpendicular bisector of segment AB. Therefore,$\Delta AED \cong \Delta BED$,$\Delta AFD \cong \Delta BFD$, $\Delta AGD \cong \Delta BGD$, and $\Delta AHD \cong \Delta BHD$ by SSS Congruence Postulate.



Recognizing Perpendicular Bisectors

In the triangle at the right, BD is the perpendicular bisector of AC. Therefore AD≅CD. Also, m∠ADB = 90 and m∠CDB = 90, so ∠ADB ≅ ∠CDB. You also know that BC is a side of both triangles, and is clearly congruent to itself (this is called the reflexive property). The triangles are congruent by SAS.



Angle Bisectors

Now, we will study a geometric concept that will help us prove congruence between two angles. Any segment, ray, or line that divides an angle into two congruent angles is called an angle bisector.

We will use the following angle bisector theorems to derive important information from relatively simple geometric figures.

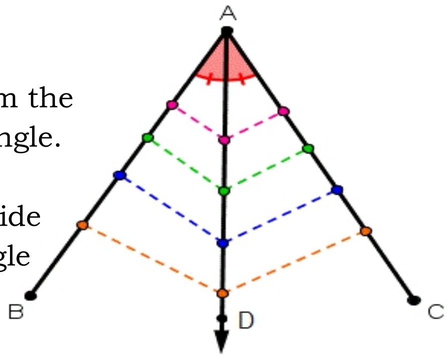
Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the sides of the angle.

Converse of Angle Bisector Theorem

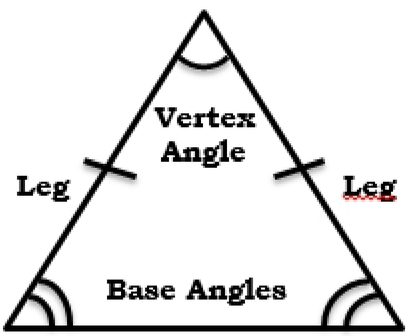
If a point in the interior of an angle is equidistant from the sides of the angle, then it lies on the bisector of the angle.

The points along ray AD are equidistant from either side of the angle. Together, they form a line that is the angle bisector.



Isosceles Triangle

A triangle is isosceles if two sides are congruent. The congruent sides are its legs; the third side is the base; the angles opposite the congruent sides are the base angles; and the angle included by the legs is the vertex angle.



Isosceles Triangle Theorem

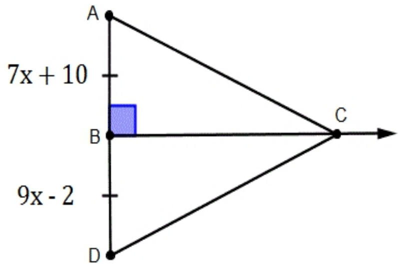
If two sides of a triangle are congruent, then the angle opposite these sides are congruent.

Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

Theorem
The bisector of the vertex of an isosceles triangle is perpendicular to the base at its midpoint.

Thus, the angle bisector of an isosceles triangle to the vertex is also the perpendicular bisector.
Let's work on some exercises that will allow us to put what we've learned about perpendicular bisectors and angle bisectors to practice.
Exercise 1:
BC is the perpendicular bisector of AD. Find the value of x.



Solution:

The most important fact to notice is that BC is the perpendicular bisector of AD. Although it is just one statement, we can derive much information about it. The fact that it is a perpendicular bisector implies that segment DB is equal to segment AB since it passes through the midpoint of segment AD. Therefore, we have

$\overline{DB} = \overline{AB}$

$9x - 2 = 7x + 10$ Subtracting 7x from both sides of the equation yields

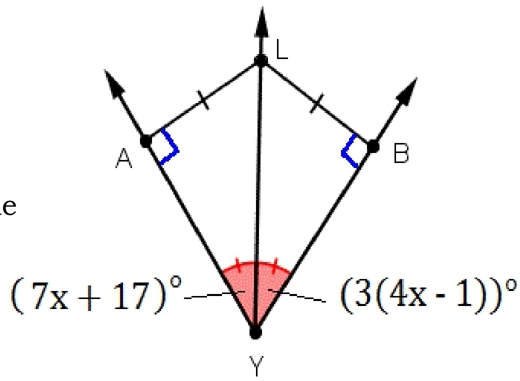
$2x - 2 = 10$ Adding both sides of the equation by 2

$2x = 12$ Dividing both sides of the equation by 2

$x = 6$
So, we have x = 6.

Exercise 2:

Find the value of x.



Solution:
The illustration shows that points A and B are equidistant from point L. By the converse of the Angle Bisector Theorem, we know that L must lie on the angle bisector of ∠AYB. This means that ∠AYL= ∠BYL, so we can solve for x as shown below:

$m\angle AYL = m\angle BYL$

$7x + 17 = 3(4x - 1)$ Distributive Property

$7x + 17 = 12x - 3$ Subtracting 12x both sides of the equation

$-5x + 17 = -3$ Subtracting 17 both sides of the equation

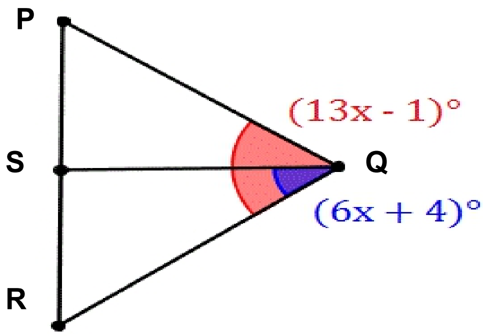
$-5x = -20$ Dividing -5 both sides of the equation

$x = 4$

So, our answer is x = 4.

Exercise 3:

QS is the angle bisector of ∠PQR. ΔPQR is an isosceles triangle. Find the value of x.



From the information we've been given, we know that ∠PQS is congruent to ∠SQR because QS bisects the whole angle, ∠PQR. We have been given the measure of the whole angle and the measure of ∠SQR, which is half of the entire angle (since the angle has been bisected). Therefore, we have

$$\frac{1}{2}m\angle PQR = m\angle SQR$$
$$\frac{1}{2}(13x - 1) = 6x + 4$$
$$\frac{13}{2}x - \frac{1}{2} = 6x + 4$$
$$\frac{1}{2}x - \frac{1}{2} = 4$$
$$\frac{1}{2}x = \frac{9}{2}$$
$$x = 9$$

Thus, we get x = 9.

Now that you have learned about the applications of triangle congruence, so you can proceed to the next activities.

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 3: Fill Me Up

Directions: Fill the blanks to complete each statement. Choose the answer at the box below.

1. The bisector of a line segment divides the line segment in two _____ parts.
2. The angle bisector of an angle divide the angle in two _____ angles.
3. If AB = 6cm and BC = 6cm, then the ΔABC is _____.
4. The angle made by perpendicular bisector of a segment is equal to _____.
5. In ΔXYZ, WY is an angle bisector of $\angle XYZ$. $\angle XYW$ and _____ are congruent.

unequal	scalene	similar	equal	90^0
equal	isosceles	45^0	$\angle XYZ$	$\angle ZYW$

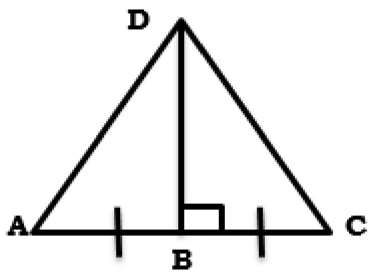
Activity 4: Right or Wrong

Directions: Shade **right** if the given value or measure is correct and **wrong** if it is not correct.

1. Line BD bisects $\angle ABC$, m$\angle ABD = 4x$, and m$\angle DBC = x + 36$.
   A. m$\angle ABC = 48^0$ ○ right ○ wrong
   B. m$\angle ABC = 96^0$ ○ right ○ wrong
   C. m$\angle DBC = 48^0$ ○ right ○ wrong

2.

A. x = 1 ○ right ○ wrong
B. AD = 4 ○ right ○ wrong
C. DC = 8 ○ right ○ wrong

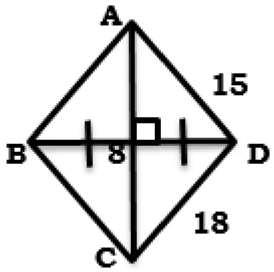


Deepen

Activity 5: Practice and Problem Solving

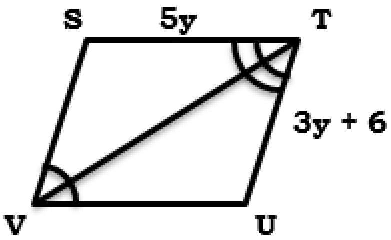
Directions: Answer the following questions.

Use the figure at the right for items 1-4

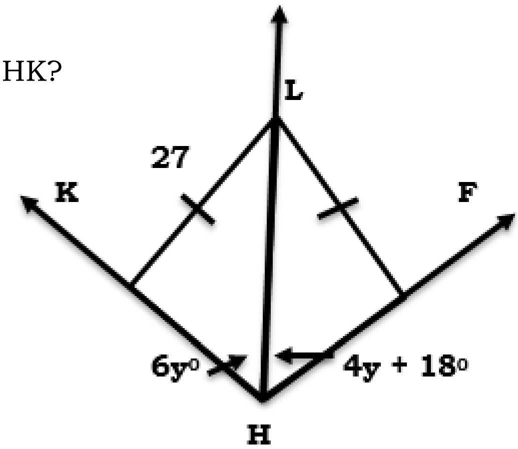


1. From the information given in the figure, how is AC related to BD?
2. Find AB.
3. Find BC.
4. Find ED.

5. On a piece of paper, mark a point H for home and point S for school. Describe the set of points equidistant from H and S.
6. Find y, ST and TU.



Use the figure at the right for items 7-10.



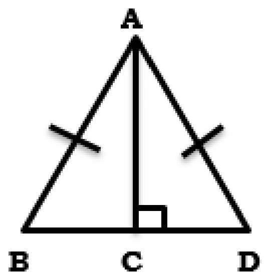
7. According to the diagram, how far is L from HK?
8. How is HL related to $\angle KHF$?
9. Find the value of y.
10. Find m$\angle KHL$ and m$\angle FHL$.

Gauge

Assessment

Directions: Read each item carefully. Write the letter of your choice on a separate sheet of paper.

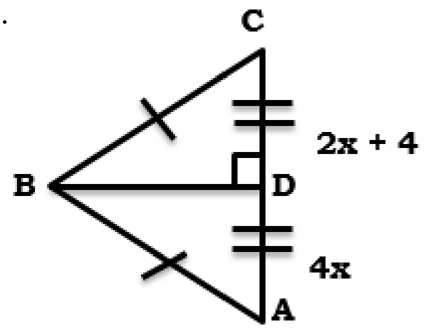
1. What segment will divide the angle of a triangle into two equal parts?
   A. angle bisector B. angle divider
   C. perpendicular bisector D. perpendicular divider
2. Which pair of lines formed a right angle?
   A. concurrent lines B. parallel lines
   C. perpendicular lines D. skew lines
3. Where can the bisector of the angles of an obtuse triangle intersect?
   i. inside the triangle ii. on the triangle iii. outside the triangle
   A. I only B. III only C. I or III only D. I, II or III
4. What will happen to the side of a triangle perpendicular to the bisector drawn from one the angle of the triangle?
   A. The side will be divided into two equal parts.
   B. The side will be divided into two unequal parts.
   C. The side will be divided into three equal parts.
   D. The side will be divided into three unequal parts.
5. Is there enough information to conclude that the two triangles are congruent? If so, what is a correct congruence statement?
   A. Yes, $\Delta ABC \cong \Delta ACD$.
   B. Yes, $\Delta ACB \cong \Delta ACD$.
   C. Yes, $\Delta CAB \cong \Delta DAC$.
   D. No, the triangles cannot be proven congruent.



6. $\Delta ABC \cong \Delta DEF$. Which side is congruent to side BC?
   A. EF B. DE C. DF D. none of the choices
7. If all pairs of sides of the two triangles are congruent, are the triangle congruent?
   A. always congruent B. sometimes congruent
   C. never congruent D. cannot be determined
8. In ΔABC, $\angle A = \angle B$. Which statement is correct?
   A. AC ≠ BC B. AC = BC C. AB = AC D. AB = BC
9. Which triangle has two equal sides?
   A. equilateral triangle B. isosceles triangle
   C. obtuse-angled triangle D. scalene triangle

For items 7-9, BD is a perpendicular bisector of AC.
Use the figure at the right.

10. What is the measure of $\angle BDA$?
    A. 45^0 B. 90^0
    C. 135^0 D. 180^0



11. What is the value of x?

A. 1　　B. 2
C. 3　　D. 4

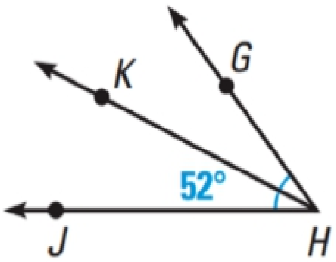
12. What is the length of CD?

A. 6　　B. 8　　C. 10　　D. 12

13. How long is AC?

A. 12　　B. 16　　C. 20　　D. 24

For items 14 and 15, HK bisects $\angle JHG$. Use the figure below.



14. What is the measure of $\angle KHG$?

A. 52^0　　B. 62^0　　C. 104^0　　D. 124^0

15. How large is $\angle JHG$?

A. 52^0　　B. 62^0　　C. 104^0　　D. 124^0

Congratulations! You are done with this module.

References

Mathematics-Grade 8 Learner's Module First Edition 2013,ISBN: 978-971-9990-70-3, pages 357, 364-366
https://quizizz.com/admin/quiz/5808aa38b1205a28377ac9f5/perpendicular-angle-bisectors
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https://mathbitsnotebook.com/Geometry/CongruentTriangles/CTproofsMC.html
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