SHS

AIRs - LM in

Statistics and Probability
Module 3:
Solving Problems Involving Mean and Variance of Probability Distributions

Statistics and Probability
Module 3: Mean and Variance of Probability Distributions

First Edition, 2021

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Target

In the study of basic probability, you have learned that for a certain experiment, there are various possible outcomes. In tossing a coin, for instance, the possible outcomes are turning up a head or tail. In the experiment of tossing a coin, the number of times the coin turns up a tail is an example of a random variable.

In your previous lesson, you have learned to illustrate the mean and variance of a discrete random variable. Your knowledge in calculating and interpreting the mean and variance of discrete random variables will be very useful in solving problems related to it.

Just like variables from a data set, random variables are described by measures of central tendency (like the mean) and measures of variability (like variance). This module will focus on how to solve problems on these measures for random variables.

After going through this module, you are expected to:

1. Solve problems involving mean and variance of probability distributions **(M11/12SP-IIIb-4)**

Subtasks:

1. Illustrate a probability distribution for a discrete random variable
2. Construct a probability mass function for a discrete random variable
3. Calculate the mean and variance of a discrete random variable.

Before going on, check how much you know about this topic. Answer the pretest in a separate sheet of paper.

Pretest

Directions: Read and analyze each item carefully. Write the letter of the correct answer using a separate sheet of paper.

1. Which of the following is a variable whose possible values are determined by chance?
   A. Dependent Variable B. Independent Variable
   C. Extraneous Variable D. Random Variable

2. What must be the sum of all the probabilities of a random variable?
   A. 0 B. 1 C. 2 D. 3

3. If $P(X) = \frac{X}{6}$, what are the possible values of X for it to be a probability distribution?
   A. 0, 2, 3   B. 1, 2, 3   C. 2, 3, 4   D. 1, 1, 2

4. Which of the following is a discrete random variable?
   A. Height of basketball player
   B. Weight of a newborn baby
   C. Number of books in the library
   D. Length of time in watching TV

5. A researcher surveyed the households in a small town. The random variable X represents the number of college graduates in the households. Does the table illustrate a probability distribution?

X	0	1	2
P(X)	0.25	0.50	0.50

   A. Yes   B. No   C. Maybe   D. Cannot be determined

6. A random variable X has the probability distribution below. What is the expected value or the mean?

X	**P(X)**	**X · P(X)**
1	0.10	
2	0.20	
3	0.45	
4	0.25	

   A. 2.58   B. 2.85   C. 5.82   D. 8.25

7. Mean is the expected value or the weighted average of all the values that the random variable X would assume. What are the steps in computing the mean of a discrete random variable?
   I. Add the results obtained in multiplying the random variable X by the corresponding probability.
   II. Construct the probability distribution for the random variable X.
   III. Multiply the value of the random variable X by the corresponding probability.

   A. III, II, I   B. I, II, III   C. II, III, I   D. III, I, II

8. A random variable X has the following probability distribution. What is the value of c?

X	1	2	3	4	5
P(X)	0.25	0.04	0.37	0.28	***c***

   A. 0.05   B. 0.06   C. 0.50   D. 0.60

9. A random variable X can take only two values, 1 and 2, P (1) = 0.8 and P (2) = 0.2. What is the variance of X?
   A. 0.16 B. 0.26 C. 0.4 D. 0.76

10. Suppose three coins are tossed. Let H be the random variable representing the number of heads that occur. Which of the following is ***NOT*** a possible value of the random variable for the number of heads?
   A. 1 B. C. 2 D. 3 E. 4

11. What is the probability distribution for a box that contains 3 20 peso bills, 2 50 peso bills, 1 100 peso bill and 1 200 peso bill?

A.

X	20	50	100	200
P(X)	3/7	2/7	1/7	1/7

B.

X	1	1	2	3
P(X)	20	50	100	200

C.

X	3	2	1	1
P(X)	20	50	100	200

D.

X	3	2	1	1
P(X)	200	100	50	20

For numbers 11-12, refer to the probability distribution shown below.

X	0	2	4	6	8
P(X)	1/5	1/5	1/5	1/5	1/5

12. What is the mean of the probability distribution?
   A. 1.5 B. 2.0 C. 3.5 D. 4.0

13. What is the variance of the probability distribution?
   A. 4.15 B. 6.35 C. 8.00 D. 7.50

For numbers 14-15, refer to the problem below.

From past experience, a company has found that in carton of transistors, 92% contain no defective transistors, 3% contain one defective transistor, 3% contain two defective transistors, and 2% contain three defective transistors.

1

hat is the mean of the probability distribution?
   A. 0.10 B. 0.15 C. 0.20 D. 0.25

15. What is the variance of the probability distribution?
   A. 0. 3075 B. 0. 2525 C. 0.3500 D. 0.4275

Jumpstart

For you to understand the lesson well, do the following activities. Have fun and good luck!

Activity 1: Toss a Coin!

Suppose four coins are tossed. Let X, the random variable, be the number of heads on all four coins.

Questions:

a. What are the possible values of X?

b. Is the random variable, X, continuous or discrete?
c. Construct a probability distribution for this experiment.

X					
P(X)					

d. What is the mean of the probability distribution?

e. What is the variance of the probability distribution?

Discover

Mean of a Probability Distribution

The **mean of a probability distribution** is the number obtained by multiplying all the possible values of the variables by the respective **probabilities** and adding these products together. It indicates the expected value the corresponding variable would take.

The mean of probability distribution is given by $\boldsymbol{\mu} = \sum \mathbf{X} \cdot \mathbf{P(X)}$**,** where $\mathbf{X}$ is a possible outcome and $\mathbf{P(X)}$ is its probability.

Steps in Finding the Mean of the Probability Distribution:

1. Construct the probability distribution for the random variable X.
2. Multiply the value of the random variable X by the corresponding probability.
3. Add the results obtained in Step 2.

Example 1

A bakery has the following schedule of daily demand for cakes. Find the expected number of cakes demanded per day.

Number of Cakes Demanded in Hundreds, X	Probability P (X)
0	0.02
1	0.07
2	0.09
3	0.12
4	0.20
5	0.20
6	0.18
7	0.10
8	0.01
9	0.01

Solution:

1. Since the probability distribution is given, multiply the value of the random variable X by the corresponding probability.

Number of Cakes Demanded in Hundreds, X	Probability P (X)	X • P (X)
0	0.02	0
1	0.07	0.07
2	0.09	0.18
3	0.12	0.36
4	0.20	0.80
5	0.20	1.00
6	0.18	1.08
7	0.10	0.70
8	0.01	0.08
9	0.01	0.09

2. Add the results obtained in Step 1.

Number of Cakes Demanded in Hundreds, X	Probability P (X)	X • P (X)
0	0.02	0
1	0.07	0.07
2	0.09	0.18
3	0.12	0.36
4	0.20	0.80
5	0.20	1.00
6	0.18	1.08
7	0.10	0.70
8	0.01	0.08
9	0.01	0.09
$\mu = \sum X \bullet P(X) = 4.36$		

The expected/mean number of cakes demanded per day is 4.36 or 436 (in hundreds).

Example 2

A researcher conducted a study to investigate how a newborn baby's crying after midnight affects the sleep of the baby's mother. The researcher randomly selected 50 new mothers and asked how many times they were awakened by their newborn baby's crying after midnight per week. Two mothers were awake zero times, 11 mothers were awake one time, 23 mothers were awake two times, nine mothers were awake three times, four mothers were awakened four times, and one mother was awake five times. Find the expected value of the number of times a newborn baby's crying wakes its mother after midnight per week.

Solution:

1. Construct the probability distribution for the random variable X.
   Let the random variable X = the number of times a mother is awakened by her newborn's crying after midnight per week
   Let P(X) = the probability of each x value

X	P (X)
0	$\frac{2}{50}$
1	$\frac{11}{50}$
2	$\frac{23}{50}$
3	$\frac{9}{50}$
4	$\frac{4}{50}$
5	$\frac{1}{50}$

2. Multiply the value of the random variable X by the corresponding probability.

X	P (X)	X • P (X)
0	$\frac{2}{50}$	$(0)(\frac{2}{50}) = 0$
1	$\frac{11}{50}$	$(1)(\frac{11}{50}) = \frac{11}{50}$
2	$\frac{23}{50}$	$(2)(\frac{23}{50}) = \frac{46}{50}$
3	$\frac{9}{50}$	$(3)(\frac{9}{50}) = \frac{27}{50}$
4	$\frac{4}{50}$	$(4)(\frac{4}{50}) = \frac{16}{50}$
5	$\frac{1}{50}$	$(5)(\frac{1}{50}) = \frac{5}{50}$

3. Add the results obtained in Step 2.

X	P (X)	X • P (X)
0	$\frac{2}{50}$	0
1	$\frac{11}{50}$	$\frac{11}{50}$
2	$\frac{23}{50}$	$\frac{46}{50}$
3	$\frac{9}{50}$	$\frac{27}{50}$
4	$\frac{4}{50}$	$\frac{16}{50}$
5	$\frac{1}{50}$	$\frac{5}{50}$
$\mu = \sum X \bullet P(X) = \frac{105}{50} = 2.1$		

Therefore, we expect a newborn to wake its mother after midnight 2.1 times per week, on the average.

Variance of Probability Distribution

Variance is a measure of how spread the data are, so instead of multiplying each probability with a single data point, it is multiplied by the sequence of the distance of each data point from the mean.

The variance of a probability distribution is given by $\sigma^2 = \sum (X - \mu)^2 \cdot P(X)$.

Steps in Finding the Variance of a Probability Distribution

1. Find the mean of the probability distribution.
2. Subtract the mean from each value of the random variable X.
3. Square the results obtained in Step 2.
4. Multiply the results obtained in Step 3 by the corresponding probability.
5. Get the sum of the results obtained in Step 4.

Example 1

The probabilities that a surgeon operates on 3, 4, 5, 6 or 7 patients in any day are 0.15, 0.10, 0.20, 0.25, and 0.30. Compute the variance of the probability distribution by following the steps.

Solution:

1. Find the mean of the probability distribution.

Number of Patients, X	Probability $P(X)$	X • $P(X)$
3	0.5	0.45
4	0.10	0.40
5	0.20	1.00
6	0.25	1.50
7	0.30	2.10
$\mu = \sum X \bullet P(X) = 5.45$		

2. Subtract the mean from each value of the random variable X.

X	P (X)	X • P (X)	X-μ
3	0.5	0.45	3 – 5.45 = -2.45
4	0.10	0.40	4 – 5.45 = -1.45
5	0.20	1.00	5 – 5.45 = -0.45
6	0.25	1.50	6 – 5.45 = 0.55
7	0.30	2.10	7 – 5.45 = 1.55

3. Square the results obtained in Step 2.

X	P (X)	X • P (X)	X-μ	$(X-\mu)^2$
3	0.5	0.45	-2.45	$(-2.45)^2$ = 6.003
4	0.10	0.40	-1.45	$(-1.45)^2$ = 2.103
5	0.20	1.00	-0.45	$(-0.45)^2$ = 0.203
6	0.25	1.50	0.55	$(0.55)^2$ = 0.303
7	0.30	2.10	1.55	$(1.55)^2$ = 2.403

4. Multiply the results obtained in Step 3 by the corresponding probability.

X	P (X)	X • P (X)	X-μ	$(X-\mu)^2$	$(X-\mu)^2$• P (X)
3	0.5	0.45	-2.45	6.003	(6.003)(0.5)=3.002
4	0.10	0.40	-1.45	2.103	(2.103)(0.10)=0.210
5	0.20	1.00	-0.45	0.203	(0.203)(0.20)=0.041
6	0.25	1.50	0.55	0.303	(0.303)(0.25)=0.076
7	0.30	2.10	1.55	2.403	(2.403)(0.30)=0.721

5. Get the sum of the results obtained in Step 4.

X	P (X)	X • P (X)	X-μ	$(X-\mu)^2$	$(X-\mu)^2$• P (X)
3	0.5	0.45	-2.45	6.003	3.002
4	0.10	0.40	-1.45	2.103	0.210
5	0.20	1.00	-0.45	0.203	0.041
6	0.25	1.50	0.55	0.303	0.076
7	0.30	2.10	1.55	2.403	0.721
$\sigma^2 = \sum (X-\mu)^2 \cdot P(X) = 4.05$					

The variance is 4.05.

Example 2.

The probabilities that a patient will have 0, 1, 2, or 3 medical tests performed on entering a hospital are 6/15, 5/15, 3/15, and 1/15 respectively. Compute the variance for the probability distribution.

Solution:

1. Find the mean of the probability distribution.

X	P(X)	X • P(X)
0	$\frac{6}{15}$	0
1	$\frac{5}{15}$	$\frac{5}{15}$
2	$\frac{3}{15}$	$\frac{6}{15}$
3	$\frac{1}{15}$	$\frac{3}{15}$
$\mu = \sum X \bullet P(X) = \frac{14}{15} = 0.93$		

2. Subtract the mean from each value of the random variable X.

X	P(X)	X • P(X)	X-μ
0	$\frac{6}{15}$	0	0-0.93 = -0.93
1	$\frac{5}{15}$	$\frac{5}{15}$	1-0.93 = 0.07
2	$\frac{3}{15}$	$\frac{6}{15}$	2-0.93 = 1.07
3	$\frac{1}{15}$	$\frac{3}{15}$	3-0.93 = 2.07

3. Square the results obtained in Step 2.

X	P(X)	X • P(X)	X-μ	$(X-\mu)^2$
0	$\frac{6}{15}$	0	-0.93	$(-0.93)^2 = 0.8649$
1	$\frac{5}{15}$	$\frac{5}{15}$	0.07	$(0.07)^2 = 0.0049$
2	$\frac{3}{15}$	$\frac{6}{15}$	1.07	$(1.07)^2 = 1.1449$
3	$\frac{1}{15}$	$\frac{3}{15}$	2.07	$(2.07)^2 = 4.2849$

4. Multiply the results obtained in Step 3 by the corresponding probability.

X	P(X)	X • P(X)	X-μ	$(X-\mu)^2$	$(X-\mu)^2 \bullet P(X)$
0	$\frac{6}{15}$	0	-0.93	0.8649	$(0.8649)\left(\frac{6}{15}\right) = 0.346$
1	$\frac{5}{15}$	$\frac{5}{15}$	0.07	0.0049	$(0.0049)\left(\frac{5}{15}\right) = 0.0016$
2	$\frac{3}{15}$	$\frac{6}{15}$	1.07	1.1449	$(1.1449)\left(\frac{3}{15}\right) = 0.229$
3	$\frac{1}{15}$	$\frac{3}{15}$	2.07	4.2849	$(4.2849)\left(\frac{1}{15}\right) = 0.286$

5. Get the sum of the results obtained in Step 4.

X	P(X)	X • P(X)	X-μ	$(X-\mu)^2$	$(X-\mu)^2$• P (X)
0	$\frac{6}{15}$	0	-0.93	0.8649	0.346
1	$\frac{5}{15}$	$\frac{5}{15}$	0.07	0.0049	0.0016
2	$\frac{3}{15}$	$\frac{6}{15}$	1.07	1.1449	0.229
3	$\frac{1}{15}$	$\frac{3}{15}$	2.07	4.2849	0.286
$\sigma^2 = \sum (X-\mu)^2 \cdot P(X) = 0.8626$					

The variance is 0.8626.

Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 1: Test Your Knowledge

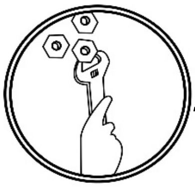
Directions: Compute the mean and variance of the following problems by following the step by step procedure as shown in the examples.

1. The number of X days in the rainy months that a construction worker cannot work because of the weather has the following probability distribution.

X	P(X)
6	0.03
7	0.08
8	0.15
9	0.20
10	0.19
11	0.16
12	0.10
13	0.07
14	0.02

2. Mark enters a local branch bank at 4:30 pm every payday, at which time there are always two tellers on duty. The number X of customers in the bank who are either at a teller window or are waiting in a single line for the next available teller has the following probability distribution.

X	P(X)
0	0.135
1	0.192
2	0.284
3	0.230
4	0.103
5	0.051
6	0.005



Deepen

Directions: Analyze and solve the following problems carefully.

Problem 1

The Vice-President of ABC Bank feels that each savings account customer has, on average, four credit cards. The following distribution represents the number of credit cards people own.

X	0	1	2	3	4
P(X)	0.18	0.44	0.27	0.08	0.03

a. Find the mean and variance of the probability distribution. Fill the table with the correct values.

X	P(X)	X • P(X)	X-μ	$(X-\mu)^2$	$(X-\mu)^2$• P (X)
0	0.18				
1	0.44				
2	0.27				
3	0.08				
4	0.03				

b. Is the Vice-President correct? Justify your answer.

Problem 2.

A die is loaded in such a way that the probabilities of getting 1, 2, 3, 4, 5 and 6 are 1/2, 1/6, 1/12, 1/12, 1/12 and 1/12.

a. Construct the probability distribution for the data.

b. Find the mean and the variance of the probability distribution.

Gauge

Directions: Read each item carefully and write the correct answer in a separate sheet of paper.

1. Which of the following is a discrete random variable?
   A. Average weight of female athletes.
   B. Average amount of electricity consumed.
   C. Number of suspected cases of COVID-19.
   D. Amount of paint used in repainting a building.

2. Which of the following represents a probability distribution?

   A.

X	1	3	5	7
P(X)	0.35	0.31	0.22	0.12

   B.

X	1	2	3	4
P(X)	1/6	3/6	2/6	1/6

   C.

X	0	1	2	3
P(X)	0.38	0.36	0.30	0.12

   D.

X	2	4	6	8
P(X)	1/3	4/3	2/3	1/3

3. Which of the following is the expected value of the variable x?
   A. Mean B. Median C. Mode D. Variance

4. If P(X) = $\frac{X}{6}$, what are the possible values of X for it to be a probability distribution?
   A. 0, 2, 3 B. 1, 2, 3 C. 2, 3, 4 D. 1, 1, 2

5. A random variable X has the following probability distribution. What is the value of c?

X	1	2	3	4	5
P(X)	0.25	***c***	0.37	0.28	0.06

A. 0.01 B. 0.02 C. 0.03 D. 0.04

6. If two coins are tossed, which is ***NOT*** a possible value of the random variable for the number of heads?
A. 0 B. 1 C. 2 D. 3

7. A researcher surveyed the households in a small town. The random variable X represents the number of college graduates in the households. Does the table illustrate a probability distribution?

$\boldsymbol{x}$	0	1	2
$\boldsymbol{P(x)}$	0.25	0.50	0.50

A. Yes B. No C. Maybe D. Cannot be determined

8. A coin is tossed thrice. Let the variable X represent the numbers of heads. What will be the values of variable X?

A. 0, 1, 2, 3 B. 1, 2, 3, 4 C. 5, 6, 7, 8 D. 1, 3, 5, 7

9. Mean is the expected value or the weighted average of all the values that the random variable X would assume. What are the steps in computing the mean of a discrete random variable?
I. Add the results obtained in multiplying the random variable X by the corresponding probability.
II. Construct the probability distribution for the random variable X.
III. Multiply the value of the random variable X by the corresponding probability.

A. III, II, I B. I, II, III C. II, III, I D. III, I, II

10. In a recent little league softball game, each player went to bat 4 times. The number of hits made by each player is described by the following probability distribution. What is the mean of the probability distribution?

Number of hits, x	0	1	2	3	4
Probability, P(X)	0.10	0.20	0.30	0.25	0.15

A. 1.00 B. 1.75 C. 2.00 D. 2.15

For numbers 11-13, refer to the problem below.

Suppose three coins are tossed. Let X be the random variable representing the number of tails that occur.

11. What are the possible values of the random variable X?
A. 0, 2, 4, 6 B. 1, 5, 7, 9 C. 0, 1, 2, 3 D. 5, 6, 7, 8

12. What is the probability distribution for the number of tails?

A.

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

B.

X	1	3	5	7
P(X)	2	4	6	8

C

X	1	3	5	7
P(X)	3/8	2/8	1/8	1/8

D.

X	0	1	2	3
P(X)	1	3	3	1

13. What is the mean of the probability distribution?
A. 1.00 B. 1.50 C. 2.00 D. 2.50

For numbers 14-15, refer to the probability distribution shown below.

X	0	2	4	6	8
P(X)	1/5	1/5	1/5	1/5	1/5

14. What is the mean of the probability distribution?
A. 1.5 B. 2.0 C. 3.5 D. 4.0

15. What is the variance of the probability distribution?
A. 4.15 B. 6.35 C. 8.00 D. 7.50

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