Senior High School

General Mathematics

Module 7:
Logarithmic Functions, Equations, and Inequalities

AIRs - LM

General Mathematics
Module 7: Logarithmic Functions, Equations, and Inequalities
Second Edition, 2021

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SHS

General Mathematics

Module 7:

Logarithmic Functions, Equations, and Inequalities

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

Target

"Logarithm" is a word made up by Scottish Mathematician John Napier (1550-1617), from the Greek word *logos* meaning "proportion, ratio or word" and *arithmos* meaning "number", which when combined together makes "ratio-number". In its simplest form, logarithms.

This learning material will provide you with information and activities that will deepen your understanding of logarithms.

After going through this module, you are expected to:

1. represent real-life situations using logarithmic functions **(M11GM-Ih-1)**,
2. distinguish logarithmic function, logarithmic equation, and logarithmic inequality **(M11GM-Ih-2)**; and
3. solve logarithmic equations and inequalities **(M11GM-Ih-i-1).**

Learning Objectives:

1. define logarithmic function
2. identify real-life situations using logarithmic functions
3. distinguish logarithmic function, logarithmic equation, and logarithmic inequality
4. determine the properties of logarithm
5. solve logarithmic equations and inequalities

John Napier, (born 1550, Merchiston Castle, near Edinburgh, Scot.—died April 4, 1617, Merchiston Castle), Scottish mathematician and theological writer who originated the concept of logarithms as a mathematical device to aid in calculations.

His contributions to this powerful mathematical invention are contained in two treatises: Mirifici Logarithmorum Canonis Descriptio (Description of the Marvelous Canon of Logarithms), which was published in 1614, and Mirifici Logarithmorum Canonis Constructio (Construction of the Marvelous Canon of Logarithms), which was published two years after his death. In the former, he outlined the steps that had led to his invention.

https://www.britannica.com/biography/John-Napier

Before going on, check how much you know about this topic. Answer the pretest on the next page in a separate sheet of paper.

Pretest

I. Multiple Choices

Directions: Read and understand the following questions carefully. Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which of the following functions is written in the form $y = log_b x$, where b > 0 and b ≠ 1?
   A. Exponential B. Linear C. Logarithmic D. Rational
2. Which of the following is a logarithmic function?
   A. $log_3(2x-1) = 2$ B. $x + 2 = log_{0.25}x$
   C. $log_3(2x+1) > log_3(x+2)$ D. $f(x) = log_3\text{x}$
3. Which of the following is a logarithmic inequality?
   A. $log_3(2x-1) = 2$ B. $x + 2 = log_{0.25}x$
   C. $log_3(2x+1) > log_3(x+2)$ D. $f(x) = log_3\text{x}$
4. Which of the following is a logarithmic equation?
   A. $log_3(2x-1) > log_3 x + 2$ B. $h(x) = log_{0.25}x$
   C. $y = 2log_4 x$ D. $2x + y = log_3 x$
5. What is the value of x in the equation $log_x(32)=\frac{5}{2}$?
   A. 1 B. 2 C. 3 D. 4
6. What is the value of x in the equation $log_x(27)=\frac{3}{2}$?
   A. 4 B. 6 C. 9 D. 11
7. What is the value of x in the equation $log_{\frac{1}{2}}(2x-1) = -3$?
   A. $\frac{1}{2}$ B. $\frac{3}{2}$ C. $\frac{7}{2}$ D. $\frac{9}{2}$
8. What is the value of $\ln(6x-5) = 3$?
   A. 2.326 B. 1.623 C. 1.702 D. 1.815
9. What is the value of x in $log_2(\text{x} + 1) + log_2(\text{ (x} - 1) = 3$?
   A. 1 B. 2 C. 3 D. 4
10. What is the value of x in $log3(2x - 1) > log3(x + 2)$?
   A. $(3, +\infty)$ B. $(-3, -\infty)$ C. $(3, -\infty)$ D. $(-3, +\infty)$

II. Identification

Directions: Below are situations/problems depicting representations of functions in real-life situations. Write **R** if it is a representation of a rational function, **I** if inverse function, **E** if exponential function, and **L** if it represents logarithmic function.

11. A scientist starts with 100 bacteria in an experiment. After 5 days, she discovers that the population has grown to 350. Find out the population after 15 days.

12. A speedboat can travel 32 miles per hour in still water. It travels 150 miles upstream against the current then returns to the starting location. The total time of the trip is 10 hours. What is the speed of the current?

13. An earthquake is measured with a wave amplitude 392 times as great as the standard wave. What is the magnitude of this earthquake using the Richter scale?

14. Bill can finish a report in 2 hours. Maria can finish the same report in 4 hours. How long will it take them to finish the report if they work together?

15. Engineers have determined that the maximum force in tons that a bridge can carry is related to the distance in meters between it supports by the following function: t(d) = (12.5/d). How far should the supports be if the bridge is to support 6.5 tons?

Jumpstart

For you to understand the lesson well, do the following activities.

Directions: Accomplish the FRAYER models on the next page. Choose from the given choices below. Use a separate sheet of paper.

Characteristics

- It is an inverse of exponential function, and any exponential function can be expressed in logarithmic form.
- Equations in which one or both sides can be logarithms
- Inequalities in which one or both sides can be logarithms

Definitions

- It is an exponent that indicates the power to which a base number is raised to produce a given number.
- It is an equation involving logarithms.
- It is an inequality involving logarithms.

Examples and Non-examples

- $log_8 64 = 2$
- $log_7(x+2) \geq log_7(6x-3)$
- $5 + \ln 2x = 4$
- $log_{25} 625 = 2$
- $log_3(2x-1) > log_3 x + 2$
- $h(x) = log_{0.25} x$
- y = $log_3 x$
- $log_6 x = 3$
- $y = 2log_5 x$

A. FRAYER Model on Logarithmic function

Definition	Characteristics
Logarithmic Function	
Examples	Non-examples

B. FRAYER Model on Logarithmic equation

Definition	Characteristics
Logarithmic Equation	
Examples	Non-examples

C. FRAYER Model on Logarithmic inequality

Definition	Characteristics
Logarithmic Inequality	
Examples	Non-examples

Discover

Representation of Logarithmic Function to Real-life Situation

A **logarithm** is defined as the exponent that indicates the power to which a base number is raised to produce a given number. The logarithm of a with base b is denoted by $log_b a$ and is defined as $c = log_b a$ if and only if $a = b^c$. In the example shown below, 3 is the exponent to which the base 2 must be raised to produce the answer, which is 8, or $2^3 = 8$.

answer

$\boxed{log_2 8 = 3} \leftarrow$ exponent

base

Note that in both the logarithmic and exponential forms, b is the base. In the exponential form, c is an exponent; this implies that the logarithm is actually an exponent. Hence, logarithmic and exponential functions are inverses.

exponent

Exponential form $\qquad$ Logarithmic form

$$\boldsymbol{b^c = a} \quad \longrightarrow \quad \boldsymbol{c = \log_b a}$$

base

In dealing with logarithms, it is important to note the following.

1. In both the logarithmic and exponential forms, b is the base. In the exponential form, c is an exponent; this implies that the logarithm is actually, an exponent. Hence, logarithmic and exponential functions are inverses.
2. In the logarithmic form $log_b x$, x cannot be negative.
3. The value of $log_b x$ can be negative.

Logarithms with a base of **10** are called **common logarithms.** When the base is not indicated, base 10 is implied. For example, $\log x$ is the same as $log_{10} x$. Logarithms with a base of ***e*** are called **natural logarithms**. Natural logarithms are denoted by ln. **ln x** is the same as $\boldsymbol{log_e x}$.

A **logarithmic function** expresses a relationship between two variables (such as x and y) and can be represented by a table of values or a graph. The logarithmic function is the function $y = log_b x$ where b is any number such that b > 0, b ≠ 1, and x > 0.

Let's examine the table of values for the function $y = log_2 x$ below.

x	y
1/4	$log_2 1/4 = -2$
½	$log_2 1/2 = -1$
1	$log_2 1 = 0$
2	$log_2 2 = 1$
3	$log_2 3 \approx 1.585$
4	$log_2 4 = 2$

The most common applications of logarithmic functions to real-life situations are to measure the decibel level of sounds through this equation D = 10 log $\frac{I}{10^{-12}}$ where I is the sound intensity; to determine the magnitude of an earthquake through the equation R = $\frac{2}{3}$ log$\frac{E}{10^{4.4}}$ where E is the energy released by an earthquake; to know the pH level of water-based solution which is defined by pH = -log [H^+] where H^+ is the concentration of hydrogen ions in moles per liter; and to solve the compound interest problems using the formula A = P$[1 + r]^t$ where P is the principal amount, r is the rate and t is the time.

The table below allows you to better understand and distinguish logarithmic function, logarithmic equation, and logarithmic inequality.

	Logarithmic Function	**Logarithmic Equation**	**Logarithmic Inequality**
Definition	It is a function involving logarithms.	It is an equation involving logarithms.	It is an inequality involving logarithms.
Examples	$g(x) = log_3x$ $h(x) = log_{0.25}x$ $y = 2log_5x$ $f(x) = log_4x$	$\log_2 8 = 3$ $2 = log_3x$ $log_3(2x - 1) = 2$ $log4x = -\log(3 + 5)$	$\ln x^2 > (\ln x)^2$ $log_3(2x - 1) > log_3x + 2$ $log_{\frac{1}{2}}(5x - 1) \geq 0$ $log_5(3x - 1) < 1$

Rewriting logarithmic equation to its exponential form and vice versa, and knowing the different properties of logarithms are very helpful in finding the solutions to logarithmic equations and inequalities' problems.

In its simplest form, a logarithm answers the question: "What exponent [c] do we need (for one number [b] to become another number [a])?"

Take for example

$$\log_3 81$$

To get the logarithm, we answer the question "What exponent do we need for 3 to become 81?" Now, we rewrite it in exponential form.

$$3^? = 81$$

$$3^4 = 81$$

The logarithmic form of $3^4 = 81$ is $\log_3 81 = 4$ or we can say that the logarithm of 81 to the base 3 is 4.

Example 1: Rewrite the following logarithmic equations to their exponential form and vice versa.

1. $log_4 64$=3
   Answer: $4^3 = \mathbf{64}$
2. $log_7 1 = 0$
   Answer: $7^0 = \mathbf{1}$
3. $log_5 125 = 3$
   Answer: $5^3 = \mathbf{125}$
4. $log_2 \frac{1}{8} = \mathbf{-3}$
   Answer: $2^{-3} = \mathbf{\frac{1}{8}}$
5. $log_{\frac{1}{3}} 3 = -1$
   Answer: $\frac{1}{3}^{-1} = \mathbf{3}$
6. $3^4 = 81$
   Answer: $log_3 81 = \mathbf{4}$
7. $4^1 = 4$
   Answer: $log_4 4 = \mathbf{1}$
8. $6^0 = 1$
   Answer: $log_6 1 = \mathbf{0}$
9. $\left(\frac{1}{2}\right)^{-2} = 4$
   Answer: $log_{\frac{1}{2}} 4 = \mathbf{-2}$
10. $10^{-2} = 0.01$
    Answer: $\log 0.01 = \mathbf{-2}$

Properties of Logarithms

Before you solve logarithmic equation and logarithmic inequality, you need to know first the different properties that you will use in finding their solutions. You have already been exposed to certain properties of logarithms that follow directly from the definition. Recall that y = $log_b x$ is equivalent to $b^y = x$ for x > 0, b > 0 and b ≠ 1.

The following properties follow directly from the definition.

Property 1: $\boldsymbol{log_b 1 = 0}$

Example: a. $log_5(1) = \mathbf{0}$ b. $log_7(1) = \mathbf{0}$ c. $log_8(1) = \mathbf{0}$

Property 2: $\boldsymbol{log_b b = 1}$

Example: a. $log_5(5)$=**1** b. $log_7(7)$=**1** c. $log_8(8)$=**1**

Property 3: Product Property of Logarithms

Let b, M and N be positive real numbers where $b \neq 1$. Then

$$\boldsymbol{log_b MN = log_b M + log_b N}$$

This property implies that the **logarithm of a product of two numbers** is the **sum** of the **logarithms of the numbers**.

Example: Use the product property of logarithms to expand the following:

a. $log_2(4 \cdot 8)$ b. $log_3(9x)$

Solutions:

a. $log_2(4 \cdot 8) = log_2(4) + log_2(8)$
$= log_2(2^2) + log_2(2^3)$
$= 2 + 3$
$=$ **5**

b. $log_3(9x) = log_3(9) + log_3(x)$
$= log_3(3^2) + log_3(x)$
$= \mathbf{2 + log_3(x)}$

Property 4: Quotient Property for Logarithms

Let b, M and N be positive real numbers where b ≠ 1. Then

$$\boldsymbol{log_b \frac{M}{N} = log_b M - log_b N}$$

This property means that the **logarithm of a quotient** is the **difference** of the **logarithm of the numerator** and **the logarithm of the denominator**.

Example: Expand the following expressions using the quotient rule of logarithms.

a. $\log\left(\frac{2x^2+6x}{3x+9}\right)$

b. $log_8\left(\frac{\sqrt{x}}{y^3}\right)$

Solutions:

a. $\log\left(\frac{2x^2+6x}{3x+9}\right) = log\left(\frac{2x(x+3)}{3(x+3)}\right)$
$= log\left(\frac{2x}{3}\right)$
$= \log(2x) - \log(3)$
$= \mathbf{\log(2) + \log(x) - \log(3)}$

b. $log_8\left(\frac{\sqrt{x}}{y^3}\right) = log_8\left(\frac{x^{\frac{1}{2}}}{y^3}\right)$
$= log_8\left(x^{\frac{1}{2}}\right) - log_8(y^3)$
$= \boldsymbol{\frac{1}{2} log_8(x) - 3log_8(y)}$

Property 5: Power Property of Logarithms

Let b and M be positive real numbers where b ≠ 1. Let p be any real number. Then,

$$\boldsymbol{1log_b M^p = plog_b M}$$

That is, **the logarithm of a power of a number** is the **exponent times the logarithm of the number.**

Example: Expand the following using the power rule for logarithms.

a. $log_2(x^5)$

b. $log_3(25)$

Solutions:

a. $log_2(x^5) = \boldsymbol{5log_2 x}$

b. $log_3(25) = log_3(5^2)$
$= \boldsymbol{2log_3(5)}$

Property 6: Let b and M be positive real numbers with b ≠ 1.

$$\boldsymbol{log_b \frac{1}{M} = -log_b M}$$

That is, the **logarithm of the reciprocal of a number** is the **negative of the logarithm of the number.**

Example:
a. $log\ \frac{2}{5} = -\boldsymbol{log}\ \frac{5}{2}$

Change-of-Base Formula
If $a \neq 1, b \neq 1$ and M are positive real numbers, then
$log_b\text{M}=\frac{\log M}{\log b}$ and $log_b\text{M}=\frac{\ln M}{\ln b}$.

Example: Use the change-of-base formula to find an approximation up to four decimal places for each of the logarithm expressions.

a. $log_5 17$

b. $log_2 0.1$

Solutions:

a. We use natural logarithms.

$log_5 17 = \frac{\ln 17}{\ln 5} \approx \frac{2.833213344}{1.609437192} \approx \mathbf{1.760374428}$

b. Here, we use common logarithms.

$log_2 0.1 = \frac{\log 0.1}{\log 2} = \frac{-1}{0.301029995} \approx \mathbf{-3.321928095}$

Solving Logarithmic Equations

When asked to solve a logarithmic equation such as log_2 (5x + 7) = 5 or log_3 (7x + 3) = log_3(5x + 9), the first thing we need to decide is how to solve the problem. Some logarithmic problems are solved by simply dropping the logarithms while others are solved by rewriting the logarithmic problem in exponential form.

How do we decide what is the correct way to solve a logarithmic problem? The key is to look at the problem and decide if the problem contains only logarithms or if the problem has terms without logarithms.

If we consider the problem log_2 (5x + 7) = 5, this problem contains a term, 5, that does not have a logarithm. So, the correct way to solve this type of logarithmic problem is to rewrite the logarithmic problem in exponential form. For the case of log_3 (7x + 3) = log_3 (5x + 9), this problem contains only logarithms, therefore the correct way to solve this type of logarithmic problem is to simply drop the logarithms.

One-to-One Property of Logarithms

If $log_b M = log_b N$, then M=N.

This statement says that if an equation contains only two logarithms, on opposite sides of the equal sign, with the same base then the problem can be solved by simply dropping the logarithms.

Example 1: Solve $\boldsymbol{log_3(7x+3) = log_3(5x+9)}$

Solution:

$log_3(7x+3) = log_3(5x+9)$	Given
(7x + 3) = (5x + 9)	Drop the logarithms.
7x + 3 = 5x + 9	
7x – 5x = 9 - 3	Combine like terms
2x = 6	Divide both sides by 2.
x = 3	

Checking:

We can check our answer by simply substituting the value of x to the given equation.

log_3 (7x + 3) = log_3 (5x + 9) ; where x = 3

log_3 (7(3) + 3) = log_3 (5(3) + 9)

log_3 (24) = log_3 (24)

Therefore, the solution to the problem log_3 (7x + 3) = log3 (5x + 9) is **x = 3**.

Example 2: Solve $\boldsymbol{log_5}$(18 - $\boldsymbol{x^2}$) = $\boldsymbol{log_5}$(6 - x)

Solution:

log_5(18 - x^2) = log_5(6 - x)	Given
(18 - x^2) = (6 – x)	Drop the logarithms.
18 - x^2 = 6 – x	
-x^2 + x +18 – 6 = 0	Combine terms on the left side and

equate to zero

$-x^2 + x + 12 = 0$ — Simplify and divide the equation by -1

$x^2 - x - 12 = 0$ — Apply quadratic factoring

$(x - 4)(x + 3) = 0$ — Equate the two factors to zero and solve for x

x = 4 and **x = -3**

Checking:

We can check our answer by simply substituting the values of x to the given equation.

x = 4

$$log_5(18 - x^2) = log_5(6 - x)$$
$$log_5(18 - 4^2) = log_5(6 - 4)$$
$$log_5(18 - 16) = log_5(6 - 4)$$
$$log_5 2 = log_5 2$$

x = -3

$$log_5(18 - x^2) = log_5(6 - x)$$
$$log_5(18 - (-3)^2) = log_5(6 - (-3))$$
$$log_5(18 - 9) = log_5 9$$
$$log_5 9 = log_5 9$$

Therefore, the solution to the problem $log_5(18 - x^2) = log_5(6 - x)$ are **x = 4** and **x = - 3.**

Example 3: Solve log_2(5x + 7) = 5

Solution:

$log_2(5x + 7) = 5$	Given
$5x + 7 = 2^5$	Rewrite the equation to exponential form
$5x + 7 = 32$	Simplify
$5x = 32 - 7$	Apply Subraction Property of Equality
$5x = 25$	Divide both sides by 5
x = 5	

Therefore, the solution to the problem log_2 (5x + 7) = 5 is **5.**

Example 4: Solve $log\ x - log\ 2 = log\ (x + 8) - log\ (x + 2)$

Solution:

$log\ x - log\ 2 = log\ (x + 8) - log\ (x + 2)$	Given
$log\frac{x}{2} = log\ \frac{x+8}{x+2}$	Apply Quotient Property of Logarithm
$\frac{x}{2} = \frac{x+8}{x+2}$	Drop the logarithms
$x^2 + 2x = 2x + 16$	Apply Cross products
$x^2 + 2x - 2x - 16 = 0$	Simplify
$x^2 - 16 = 0$	Factor
$(x + 4)(x - 4) = 0$	Equate the two factors to zero and solve for x
$x + 4 = 0 \quad x - 4 = 0$	
x = -4 and x = 4	

Checking:
We can check our answer by simply substituting the values of x to the given equation.

x = -4

$$log\ x - log\ 2 = log\ (\ x + 8) - log\ (x + 2)$$
$$log\ (-4) - log\ 2 = log\ (-4 + 8) - log\ (-4 + 2)$$
$$log\ (-4) - log\ 2 = log\ (4) - log\ (-2)$$
$$log \frac{-4}{2} = log\ \frac{4}{-2}$$
$$log - 2 = log - 2$$

x = 4

$$log\ x - log\ 2 = log\ (\ x + 8) - log\ (x + 2)$$
$$log\ (4) - log\ 2 = log\ (4 + 8) - log\ (4 + 2)$$
$$log\ (4) - log\ 2 = log\ (12) - log\ (6)$$
$$log \frac{4}{2} = log\ \frac{12}{6}$$
$$log\ 2 = log\ \ \ 2$$

Note: In the logarithmic form $log_b x$, x cannot be negative.

Therefore, the solution to the problem $log\ x - log\ 2 = log\ (\ x + 8) - log\ (x + 2)$
Is only **x = 4.**

Example 5: Solve ln(x -5) + ln(10 – x) = ln(x – 6) + ln(x – 1)

Solution:

ln(x -5) + ln(10 – x) = ln(x – 6) + ln(x – 1)	Given
ln(x -5) (10 – x) = ln(x -6) (x -1)	Combining the logarithmic terms.
(x – 5) (10 – x) = (x -6) (x – 1)	Drop the natural logarithm
10x – x^2 – 50 + 5x = x^2 – x – 6x + 6	Expand each side.
$-x^2$ + 15x – 50 = x^2 – 7x + 6	Simplify
$-x^2$ - x^2 + 15 x + 7x -50 – 6 = 0	Arrange terms in descending order and equate to zero
-2x^2 + 22x - 56 = 0	Simplify
2x^2 – 22x + 56 = 0	
x^2 – 11x + 28 = 0	Divide the equation by -2
(x – 7) (x – 4) = 0	Apply quadratic factoring
x – 7 = 0 x – 4 = 0	Equate the two factors to zero
x = 7 and x = 4	

Checking:
We can check our answer by simply substituting the values of x to the given equation

x = 7

ln(x -5) + ln(10 – x) = ln(x – 6) + ln(x – 1)
ln(7-5) + ln(10 – 7) = ln(7 – 6) + ln(7 – 1)
ln(2) + ln(3) = ln(1) + ln(6)
ln(6) = ln(6)
OKAY

x = 4

ln(x -5) + ln(10 – x) = ln(x – 6) + ln(x – 1)
ln(4 -5) + ln(10 – 4) = ln(4 – 6) + ln(4 – 1)
ln(-1) + ln(6) = ln(-2) + ln(3)
NOT OKAY

Therefore, the only solution to the problem ln(x -5) + ln(10 – x) = ln(x – 6) + ln(x – 1) is **x = 7.**

Solving Logarithmic Inequalities

The key to working with logarithmic inequalities is the following fact:

If $a > 1$ and x> $y,$ then $log_a x > log_a y$. Otherwise, if 0<a<1, then $log_a x < log_a y$.

Of course, the base of a logarithm cannot be 1 or nonpositive. More importantly, the converse is true as well:

If a > 1 and $log_a x$ > $log_a y$, then x > y. Otherwise, if 0 < a < 1, then x < y.

In more formal terms, the logarithmic function f(x) = $log_a x$ is monotonically increasing (increasing x always increases f(x) for a > 1), and monotonically decreasing (increasing x always decreases f(x) for 0 < a < 1).

It is also important to keep in mind the following fact:

The argument of the logarithm **must** be positive!

Thus, it is also necessary to take into account any inequalities resulting from the arguments being positive; for example, an inequality involving the term log_2(2x– 3) > 0 immediately requires x > $\frac{3}{2}$.

When both sides of an inequality have the same base, the key facts from the introduction can be applied directly.

Example 1: What values of x satisfy the inequality log_2(2x + 3) > log_2(3x)?

Solution:

Ensure that the logarithms are defined. 2x + 3 > 0 and x + 2 > 0 must be satisfied. 2x + 3 > 0 implies x > $\frac{3}{2}$ and 3x > 0 implies x > 0. To make both logarithms defined then x is greater than 0.

log_2 (2x + 3) > log_2(3x)	Given
2x + 3 > 3x	Drop the logarithms
2x – 3x > -3	Combine like terms
-x > -3	Apply rule for operation in inequality (reverse the direction of inequality symbol in dividing each side by negative number)
***x* < 3**	

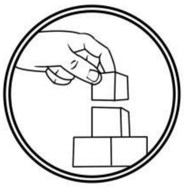
Therefore, the values of x that satisfy the inequality log_2(2x + 3) > log_2(3x) is ***x* < 3** or can be described as the solution set **0 < x < 3** or **(0, 3)**

Example 2: What values of x satisfy the inequality $log_3(4x + 1) > log_3(2x + 3)$?
Solution:

Ensure that the logarithms are defined. $4x + 1 > 0$ and $2x + 3 > 0$ must be satisfied. $4x + 1 > 0$ implies $x > -\frac{1}{4}$ and $2x + 3 > 0$ implies $x > -1.5$. To make both logarithms defined then x is greater than $-\frac{1}{4}$.

$log_3(4x + 1) > log_3(2x + 3)$	Given
$4x + 1 > 2x + 3$	Drop the logarithms
$4x - 2x > 3 - 1$	Combine like terms
$2x > 2$	Divide both sides by 2
$x > 1$	

Therefore, the values of x that satisfy the inequality $log_3(4x + 1) > log_3(2x + 3)$ is $\boldsymbol{x > 1}$ or can be described as the solution set **$(1, +\infty)$.**



Explore

Here are some enrichment activities for you to work on to master and strengthen the basic concepts you have learned from this lesson.

Activity 1: Logarithmic Function vs Logarithmic Equation vs Logarithmic Inequality

Directions: Determine whether the given is a logarithmic function, logarithmic equation, or logarithmic inequality. Write **LF** for logarithmic function, **LE** for logarithmic equation and **LI** for logarithmic inequality. Use separate sheet of paper for you answers.

1. $log_3(2x - 1) > log_3 x + 2$
2. $h(x) = log_{0.25} x$
3. $2 + y = log_3 x$
4. $log_3(2x - 1) = 2$
5. $\log x^2 = 2$
6. $log_4(2x) = log_4 10$
7. $log_x 16 = 2$
8. $log_8(3x - 5) < 2$
9. $log_4(x + 1) < log_4 2x$
10. $f(x) = log_{0.5}(x - 3)$

Activity 2: Check Me!

Directions: Below are word problems depicting representations of functions in real-life situations. Check the word problems that represent a logarithmic function in real life situation. (Hint: You have to check five boxes .)

- [ ] A 1-liter solution contains 10–8 moles of hydrogen ions. Determine whether the solution is acidic, neutral, or basic.
- [ ] A barangay has 1,000 individuals and its population doubles every 60 years. What is the population of the barangay in 20 years?
- [ ] If the temperature in a thermometer reads 101.3°F, what is that in °C?
- [ ] In an inter-barangay basketball league, the team from Barangay Quara has won 18 out of 30 games, a winning percentage of 52%. We have seen that they need to win 90 games consecutively to raise their percentage to at least 60%. What will be their winning percentage if they win 10 games in a row?
- [ ] Suppose the intensity of the sound of a jet during take-off is 100 watts/m2. What is the corresponding sound intensity in decibels?
- [ ] The half-life of a radioactive substance is 1500 years. If the initial amount of the substance is 500 grams, what amount of substance remains after 1000 years?
- [ ] The maximum sound intensity of Paul's iPod player is 115 dB. What is the maximum sound intensity in watts per square meter?
- [ ] The world's largest and strongest earthquake with an instrumentally documented magnitude of 9.5 happened in 1960 near Chile. Philippines, on the other hand recorded a magnitude of 8 in 1976 at Mindanao as its strongest earthquake. How much more energy was released by the world's strongest earthquake compared to that by the Philippines strongest earthquake?
- [ ] Two ships traveling from Cebu to Bohol differ in average speed by 10 kph. The slower ship takes 3 hours longer to travel a 75-kilometer route than for the faster ship to travel a 70-kilometer route. What is the speed of the faster ship?
- [ ] What is the magnitude in the Richter scale of an earthquake that released 1014 joules of energy?

Activity 3: Complete Me!

Directions: Answer the following logarithmic equations and complete the table. Use a separate sheet for your solution. You can use your calculator.

Logarithmic Equations	**Answer**
1. log 4 + log 25	
2. log 10 – log 5	
3. 3log(5)	
4. $\log_5(5)$	
5. $\log_5(1)$	
6. log 2 + log 12	
7. log 20 – log 5	
8. 4log(2)	
9. $\log_2(2)$	
10. log 4 + log 5	

Activity 4: Find My X!

Directions: Solve for the value/s of x. Use a separate sheet for your solutions. Always check your answer.

1. $log_3(x+4) = log_3(2x-4)$
2. $log_3(2x-1) > log_3(x+2)$
3. $(log_2 x)^2 - 4 = 0$
4. $log_8(3x-5) < 2$
5. $log_2(x-2) \leq 4$

Assessment Rubric

Score	**Indicators**
5	Both the solution and final answer is correct.
3	Solution and final answer have minimal error
1	Solutions and final answer are erroneous.
0	Did not attempt to answer.

Great job! You are almost done with this module!

Deepen

To accentuate your understanding of the lesson, here is another activity for you to work on. Good luck and have fun!

Directions: Solve the given word problem below. Use a separate sheet for your solutions.

Problem

Using the formula A=$P(1+r)^n$, where A is the future value, P is the principal, r is the fixed annual interest rate, and n is the number of years, how many years will it take an investment amounting to P100,000.00 to double if the interest rate per annum is 2.5%?

Assessment Rubric for Problem Solving

Area of Assessment	Proficient 5	Apprentice 3	Novice 1
Applies Appropriate Procedures	Applies completely appropriate procedures	Applies some appropriate procedures	Applies inappropriate procedures
Uses Representations	Uses a representation that clearly depicts the problem	Uses a representation that gives some important information about the problem	Uses a representation that gives little or no significant information about the problem
Answers the Problem	Correct solution	Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly	No answer or wrong answer based upon an inappropriate plan

Gauge

I. Multiple Choices

Directions: Read and understand the following questions carefully. Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which is defined as the exponent that indicates the power to which a base Number b is raised to produce a given number?
   A. Logarithmic equation   B. Logarithmic function
   C. Logarithmic expression   D. Logarithmic Inequality
2. Which of the following is a logarithmic function?
   A. $\log_2(2) + \log_2(3x - 5) = 3$ B. $g(x) = \log_2(x + 1)$
   C. $\log(3^x) = \log 5$ D. $\log_{\frac{1}{2}}(3x) > \log_{\frac{1}{2}}(2x + 3)$
3. Which of the following is a logarithmic inequality?
   A. $log_2(2) + log_2(3x - 5) = 3$ B. $g(x) = log_2(x + 1)$
   C. $\log(3^x) = \log 5$ D. $log_{\frac{1}{2}}(3x) > log_{\frac{1}{2}}(2x + 3)$
4. Which of the following is a logarithmic equation?
   A. $log_2(2) + log_2(3x - 5) = 3$ B. $g(x) = log_2(x + 1)$
   C. $\log(3^x) < \log 5$ D. $log_{\frac{1}{2}}(3x) > log_{\frac{1}{2}}(2x + 3)$
5. Which is equivalent to $2^6 = 64$?
   A. $log_2 64 = 6$ B. $log_6 2 = 64$ C. $log_6 64 = 2$ D. $log_2 6 = 64$
6. What is $log_3 \frac{1}{9} = -2$ in exponential form?
   A. $\left(\frac{1}{9}\right)^{-2} = 3$ B. $\left(\frac{1}{9}\right)^{3} = 2$ C. $(3)^{-2} = \frac{1}{9}$ D. $(-2)^3 = \frac{1}{9}$
7. What is the value of x in the equation $log_4(2x) = log_4 10$?
   A. 3   B. 5   C. 7   D. 11
8. What is the value of x in the equation $log_3(2x - 1) = 2$?
   A. 1   B. 3   C. 5   D. 7
9. What is the value of x in the inequality $log_4(x + 1) < log_4 2x$?
   A. $(1, -\infty)$ B. $(2, +\infty)$ C. $(-2, +\infty)$ D. $(1, +\infty)$
10. What is the value of x in $log3(2x - 1) > log3(x + 2)$?
   A. $(3, +\infty)$ B. $(-3, -\infty)$ C. $(3, -\infty)$ D. $(-3, +\infty)$

II. Identification

Directions: Distinguish the given below whether **logarithmic function, logarithmic equation,** or **logarithmic inequality**. Write your answer on a separate sheet of paper.

1. $log_2(2) + log_2(3x - 5) = 3$ **2.** $h(x) = log_2(x) + \left(log_2(x)\right)^2$ **3.** $log_2(x^2 - 5x + 6) > 1$

4. $log_2(2x + 3) > log_2(3x)$ **5.** $log_{\frac{1}{2}}(3x) > log_{\frac{1}{2}}(2x + 3)$

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