Draft

- Temperature (mathematically)
- Thermal Expansion
  - Linear
  - Volume
- Heat Capacity
- Latent Heat
- Heat Transfer
  - Conduction: Heat Equation
  - Convection
  - Radiation: Stefan-Boltzmann Law

    Temperature of the Earth
- Equation of State
- Ideal Gas

  Atmospheric Pressure at diff heights
  - pV diagram
  - Kinetic Theory: Model KE, v_{rms}
- Mean-free path
- Heat Capacity of Ideal Gas
  - Equipartition Theorem
- Maxwell-Boltzmann Distribution

  Derivation
- Phases of Matter
  - Clausius-Clapeyron Relation

- 1st Law of Thermodynamics: $Q = \Delta U + W$
  - Internal Energy $U(p,V,T)$ path-ind
  - Work $W = \int p\,dV$ path-dep
- Thermodynamic Processes
  - Isochoric
  - Isobaric
  - Isothermal
  - Free expansion
  - Heat Capacity
  - Adiabatic
    Speed of sound in air
- van der Waals gas
- Heat Engine
  - Thermal Efficiency
  - Otto Cycle
  - Diesel Cycle
  - Carnot Cycle
- Refrigerator
- 2nd Law of Thermodynamics
- Carnot Efficiency
- Entropy
  - 2nd Law of Thermodynamics rephrased
  - State Function
  - Microscopic Interpretation: Probability & Multiplicity
- Re-defining Temperature

Thermodynamics

Temperature

Thermal eqm.

- A and A are always in thermal eqm
- A and B are in thermal eqm ⇒ so are B and A
- (0th law of thermodynamics) A and B in thermal eqm, B and C in thermal eqm
  ⇒ A and C are in thermal eqm

⇒ equivalence relation
labelled by temperature T: measure of hotness

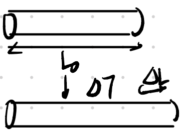
If not in thermal eqm.: energy flow from hotter to colder object

$T_A > T_B$
heat/energy flow
$T_A' = T_B'$

Temperature: Celsius, $\underbrace{\text{Kelvin K}}_{\text{SI Unit}}$

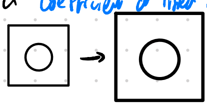
Thermal Expansion

Low T High T



$$\underbrace{\frac{\Delta L}{L_0}}_{\text{fraction of length change}} = \alpha \Delta T$$

α coefficient of linear expansion $[K^{-1}]$



$$\underbrace{\frac{\Delta V}{V_0}}_{\text{fractional change in vol.}} = \beta \Delta T$$

β coeff. of vol. Δ

$V_0 \mapsto V_0 + \Delta V$
$L_0^3 \mapsto (L_0 + \Delta L)^3 \approx L_0^3 + 3 L_0^2 \Delta L$ $\qquad \beta = \frac{1}{\Delta T}\frac{\Delta V}{V_0} = \frac{1}{\Delta T}\frac{3L_0^2 \Delta L}{L_0^3} = 3\alpha$

Heat Capacity

$\Delta Q = C \Delta T$

ΔQ Δ in heat $[Q] = L^2 M T^{-2}$ unit J

C Heat capacity (of an object) $[C] = L^2 M T^{-2} \Theta^{-1}$ unit JK^{-1}

ΔT change in temperature

e.g. metal $C(T) = \alpha T + \beta T^3$

$dQ = C(T)\, dT$

$Q = \int C\, dT$

$\Delta Q = mc \Delta T$

m mass of object

c specific heat capacity (of a material) $[c] = L^2 T^{-2} \Theta^{-1}$

$dQ = mc\, dT$

$Q = m \int c\, dT$

Latent Heat

Phase e.g. water steam ice

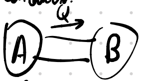
solid, liquid gas

$\Delta Q = L \Delta m$

L latent heat $[L] = L^2 T^{-2}$

Heat Transfer

Conduction



$$\dot{Q} = kA \frac{T_H - T_C}{L}$$

T_A T_B

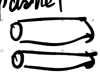
k thermal conductivity $[k] = ?$ $W m^{-1} K^{-1}$

A cross-sectional area $[A] = L^2$ m^2

$\frac{T_H - T_C}{L}$ temperature gradient $[\cdot] = \Theta L^{-1}$ $K m^{-1}$

k_1, A_1, L_1 k_2, A_2, L_2

parallel



$$\dot{Q} = \left(\frac{k_1 A_1}{L_1} + \frac{k_2 A_2}{L_2}\right)(T_H - T_C)$$

series

$$\dot{Q} = \frac{k_1 A_1 (T_H - T)}{L_1} = \frac{k_2 A_2 (T - T_C)}{L_2}$$

Exercise: find T hence $\dot{Q}$

General Case: heat equation

$$\frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial T}{\partial t}$$

Convection

$\dot{Q} = hA(T_H - T_C)$

exponent: 1 or 5/4?

Radiation

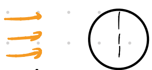
Stefan-Boltzmann Law $H = Ae\sigma T^4$

A surface area

e emissivity

σ Stefan-Boltzmann constant $\approx 5.6704\times10^{-8}\ W m^{-2} K^{-4}$

Example: find temperature of Earth

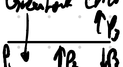


$I \cdot \pi R_E^2 = 4\pi R_E^2 \sigma T_E^4$

$I = 0.7 \times 1370\ Wm^{-2}$ Solar intensity

$T_E \approx 255K = -18°C$

Greenhouse effect



$P_3 = P_1$

$P_1 + P_3 = 2P_1$

$T \sim P^{\frac{1}{4}}$

$T_E = 255 \cdot 2^{\frac{1}{4}} \approx 303K$

Equation of state

macroscopic ⟷ microscopic

p, V, T, n $\quad v_i^2, \ldots$

$f(p,V,T) = 0$

e.g. ideal gas

- gas particles are point masses i.e. size=0
- no interaction between particles
- elastic, instantaneous collision

$pV = nRT$

p Pressure $[p] = L^{-1} M T^{-2}$ Pa

V Volume $[V] = L^3$ m^3

n # of moles of gas $[n] = N$ mol $\quad N = nN_A$, $N_A \approx 6.02\times10^{23}\ mol^{-1}$ Avogadro's Constant

R universal gas constant $\approx 8.314\ J mol^{-1} K^{-1}$

T temperature K

Two gas in thermal eqm. ⇒ pV/n are same

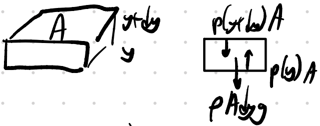
Consider a gas, $pV/T = nR$

M molar mass $\quad kg\,mol^{-1}$

$pV=\frac{m}{M}RT \Rightarrow p=\frac{\rho}{M}RT$

Standard Temperature and Pressure $T=0°C=273.15K$, $p=1atm=1.013\times10^5\,Pa$

Example: Assume Temperature is constant, find $p(y)$, y is height from the ground in terms of molar mass M, gravity g, universal gas constant R, temperature T



$(p(y+dy)-p(y))A=-\rho A\,dy\,g$

$\frac{dp}{dy}dy\cdot A=-\frac{pM}{RT}A\,dy\,g$

$\frac{dp}{dy}=-p\frac{Mg}{RT}$

$\frac{dp}{p}=-\frac{Mg}{RT}y$

$\ln\left(\frac{p}{p_0}\right)=-\frac{Mg}{RT}y$

$p=p_0\exp\left(-\frac{Mgy}{RT}\right)$

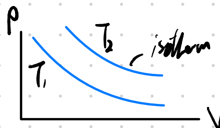
$M=m_{tot}/n=Nm/n=m\,N_A$

$R=N_A k_B$

$p=p_0\exp\left(-\frac{mgy}{k_BT}\right)$

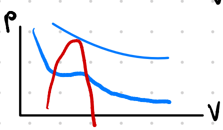
$=p_0\boxed{\exp(-U/k_BT)}$

p-V diagram



$p=\frac{nRT}{V}\sim\frac{1}{V}$

ideal gas



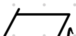
real gas

Kinetic Theory

$K=\frac{3}{2}nRT=\frac{3}{2}NkT=\sum_i\frac{1}{2}mv_i^2=\frac{1}{2}Nmv_{rms}^2$, $v_{rms}=\sqrt{\frac{\sum_{i=1}^N v_i^2}{N}}$

$v_{rms}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3RT}{M}}$

Equipartition Thm.: $K=\frac{f}{2}NkT$, f = degree of freedom



$f=2$

Translation o—o d Dof
Rotation o-o $k_B T \gtrsim \hbar\omega_r$ $\omega_r = \hbar/I$
Vibration o~o 2 Dof $k_0 T \geq \hbar\omega_0$, $\omega_0 = \sqrt{k/\mu}$
e.g. monoatomic $f=3$ He, Ar
diatomic $f=5$ $N_2, O_2, \ldots$

Mean-free path

Assume particles have size as sphere of radius

r $2r$

collision rate $= \frac{\pi (2r)^2 \cdot v\,dt \cdot \frac{N}{V}}{dt} = \frac{4\pi r^2 v N}{V}$

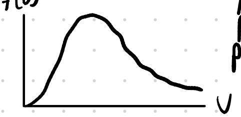
Collision rate actual collision rate $= \frac{4\sqrt{2}\pi r^2 v N}{V}$

mean free time $t_{mean} = \frac{V}{4\sqrt{2}\pi r^2 v N}$, mean-free path $\lambda = v t_{mean} = \frac{V}{4\sqrt{2}\pi r^2 N} = \frac{kT}{4\sqrt{2}\pi r^2 p}$

Heat Capacity of Ideal Gas

constant volume
$dE = nC_V dT = dK = \frac{f}{2} nR\,dT$
molar heat capacity
$C_V = \frac{f}{2}R$ molar heat capacity at const vol.

Maxwell-Boltzmann Distribution



Normalization: $\int_0^{+\infty} f(v)\,dv = 1$
$P(v) = 0$
$P([v_1, v_2]) = \int_{v_1}^{v_2} f(v)\,dv$
probability of finding particle at speed range $[v_1, v_2]$

Maxwell-Boltzmann distribution
$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-mv^2/2kT}$
Derive phase space

$P(\vec{v}) \sim \exp(-E/kT)\,d\vec{v} = \exp(-mv^2/2kT)$
$P(v) = 4\pi v^2 P(\vec{v})$
$= 4\pi v^2 \exp(-mv^2/2kT)\,dv$
Exercise: find $\langle v \rangle = \int_0^{+\infty} v f(v)\,dv$
find $v_{rms} = \sqrt{\int_0^{+\infty} v^2 f(v)\,dv}$

Hint: $\int_0^{+\infty} x^2 e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \frac{1}{4\alpha}$ $\int_0^{+\infty} x^4 e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \frac{3}{8\alpha^2}$

$\langle v \rangle = 4\pi \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \underbrace{\int_0^{+\infty} v^3 e^{-mv^2/2kT}\,dv}$

$\frac{1}{2}\int_0^{+\infty} u\, e^{-\frac{mu}{2kT}}\,du$

$u = v^2$
$du = 2v\,dv$

D	I
u	$e^{-\frac{mu}{2kT}}$
1	$-\frac{2kT}{m} e^{-\frac{mu}{2kT}}$
0	$+\left(\frac{2kT}{m}\right)^2 e^{-\frac{mu}{2kT}}$

$\langle v\rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\left(\frac{2kT}{m}\right)^2$

$= \sqrt{\frac{8kT}{\pi m}}$

$\int_0^{+\infty} v^2 f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\int_0^{+\infty} v^4 e^{-\frac{mv^2}{2kT}}\,dv \qquad w = \sqrt{\frac{m}{2kT}}\,v$

$= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\int_0^{+\infty}\left(\frac{2kT}{m}\right)^2 w^4 e^{-w^2}\sqrt{\frac{2kT}{m}}\,dw$

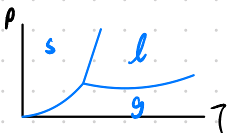
$= 4\pi\cdot\frac{2}{\pi^{\frac{3}{2}}}\cdot\frac{kT}{m}\int_0^{+\infty} w^4 e^{-w^2}\,dw$

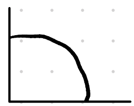
$= \frac{8}{\sqrt{\pi}}\frac{kT}{m}\cdot\sqrt{\pi}\cdot\frac{3}{8}$

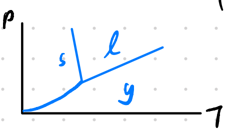
$= \frac{3kT}{m}$

$v_{rms} = \sqrt{\frac{3kT}{m}}$

Phases of Matter







implication: $\rho_{ice} < \rho_{water}$, ice floats on water

Clausius-Clapeyron Relation

$\frac{dP}{dT} = \frac{L}{T\Delta V}, \quad V = V_l - V_g$

Ideal Gas

1st law of thermodynamics $\Delta Q = U + \Delta W$

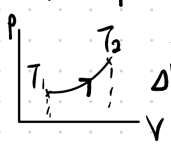
ΔQ heat flow (>0 into, <0 out of)

U total energy state function $U(p,V,T)$

ΔW work done (>0 work done by the object)

$dQ = dU + dW$

$dW = \vec{F}\cdot d\vec{v} = p\,dV$



$U = \frac{f}{2}nR\Delta T = \frac{f}{2}nR(T_2 - T_1)$

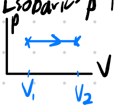
$\Delta W = \int_p dV = \int_{V_1}^{V_2} p(V)\,dV$

$\Delta Q = U + \Delta W$

In general, set of $p(t), V(t)$?

$W = \int p(t)\,dV = \int p(t)V'(t)\,dt$

Isobaric: p is constant

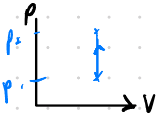


$\Delta W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$

$\Delta U = \frac{f}{2}nR\Delta T = \frac{f}{2}p\Delta V = \frac{f}{2}p(V_2 - V_1)$

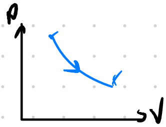
$\Delta Q = \Delta U + \Delta W = \frac{f+2}{2}p(V_2 - V_1)$

Isochoric: V is constant



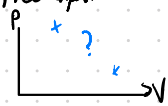
$$\Delta W = 0$$
$$\Delta U = \frac{f}{2}(p_2 - p_1)V \qquad \frac{f}{2}nR(T_2 - T_1)$$
$$= \Delta Q$$

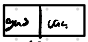
Isothermal: T is const



$$\Delta W = \int p\, dV$$
$$= \int \frac{nRT}{V} dV$$
$$= nRT \ln(V_2/V_1) = \Delta Q$$
$$\Delta U = 0$$

Free expansion





$$\Delta W = 0$$
$$\Delta U = 0$$
$$Q = 0$$

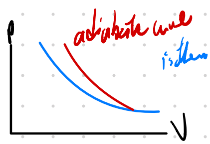
Heat Capacities of Ideal Gas

const.! volume / pressure

$$dQ_V = nC_V dT$$
$$dQ_p = nC_p dT$$
$$dQ = dU + dW$$
$$nC_p dV = nC_V dT + nR dT$$
$$\Rightarrow C_p = C_V + R$$
$$= \left(\frac{f+2}{2}\right)R$$

adiabatic ratio $\gamma = C_p/C_V = (f+2)/f$

Adiabatic Process: $\Delta Q = 0$



$$dU + dW = 0$$
$$d\left(\frac{f}{2}pV\right) + p\,dV = 0$$
$$\frac{f}{2}V\,dp + \frac{f}{2}p\,dV + p\,dV = 0$$
$$\frac{f+2}{2}p\,dV = -\frac{f}{2}V\,dp$$
$$\gamma\frac{dV}{V} = -\frac{dp}{p}$$
$$pV^\gamma = \text{const.} \qquad p_1V_1^\gamma = p_2V_2^\gamma \qquad p \sim \frac{T}{V}$$
$$TV^{\gamma-1} = \text{const.} \qquad T V_1^{\gamma-1} = T V_2^{\gamma-1}$$
$$p^{\gamma-1} \sim T^\gamma$$
$$\Delta U = \frac{f}{2}nR(T_2 - T_1)$$
$$= \frac{f}{2}nR\left(\frac{p_2V_2}{nR} - \frac{p_1V_1}{nR}\right)$$
$$= (p_2V_2 - p_1V_1)/2/f$$
$$= \frac{1}{\gamma - 1}(p_2V_2 - p_1V_1)$$
$$\Delta W = -\Delta U$$
$$= \frac{1}{\gamma-1}(p_1V_1 - p_2V_2)$$

Heat Engine

heat → work

T_H

$\downarrow Q_H$

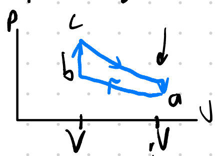
$\to W$

$\downarrow Q_c$

T_c

thermal efficiency $e = W/|Q_H|$

Example Otto Cycle



$a \to b$: adiabatic compression
$b \to c$: isochoric heating
$c \to d$: adiabatic expansion
$d \to a$: isochoric cooling

Exercise find thermal efficiency e in terms of r and other coeffs

$a \to b: \Delta W = \frac{1}{\gamma - 1}(p_bV_b - p_aV_a)$

$pV^\gamma = const \quad p_aV_a^\gamma = p_bV_b^\gamma \quad p_a r^\gamma = p_b$

$\Delta W = \frac{1}{\gamma-1}(p_a r^\gamma V - p_a r V)$

$= \frac{r^\gamma - r}{\gamma - 1} p_a V$

$b \to c: \Delta U = \frac{f}{2}nR(T_c - T_b)$
$= Q_H$

$d \to a: Q_c = \Delta U = \frac{f}{2}nR(T_a - T_d)$

$e = 1 - \left|\frac{Q_c}{Q_H}\right|$

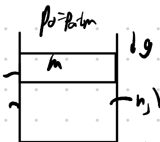
$= \frac{Q_H + Q_c}{Q_H} \qquad TV^{\gamma-1} = const$

$= 1 - \frac{T_d - T_a}{T_c - T_b}$

$= 1 - \frac{1}{r^{\gamma-1}}$

Exercise:

insulating



if the slab is slightly displaced, find the period of oscillation

$T = T_0$ at eqm. $V = A \times h_0$ at eqm.

Solution $p_{eqm} = p_0 + mg/A$

$$m\ddot{y} = (p - p_0)A - mg$$

adiabatic: $pV^\gamma = \text{const}$

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0$$

$$dp/p + \gamma\, dy/y = 0$$

Assume slightly displaced by dy (upward)

$$dF = (dp)A$$

$$= -\gamma p \cdot \frac{dy}{y} A$$

$$= -\gamma \frac{p_0}{h_0} A\, dy \qquad p_0 = \frac{nRT_0}{Ah_0}$$

$$= -\gamma \frac{nRT_0}{h_0^2}\, dy$$

$$k = \gamma \frac{nRT_0}{h_0^2}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mh_0^2}{\gamma nRT_0}} \,//$$