Hash Functions: Introduction

Hash function: condenses variable-length message to fixed sizes

- h = H (M)
- Usually assume hash function is public

A cryptographic hash function is:

- Computationally infeasible to find a data object that maps to a pre-specified hash result (one-way property)
- Computationally infeasible to find two data objects that map to same hash result (collision-free property)
- Used to detect changes to message (data integrity)

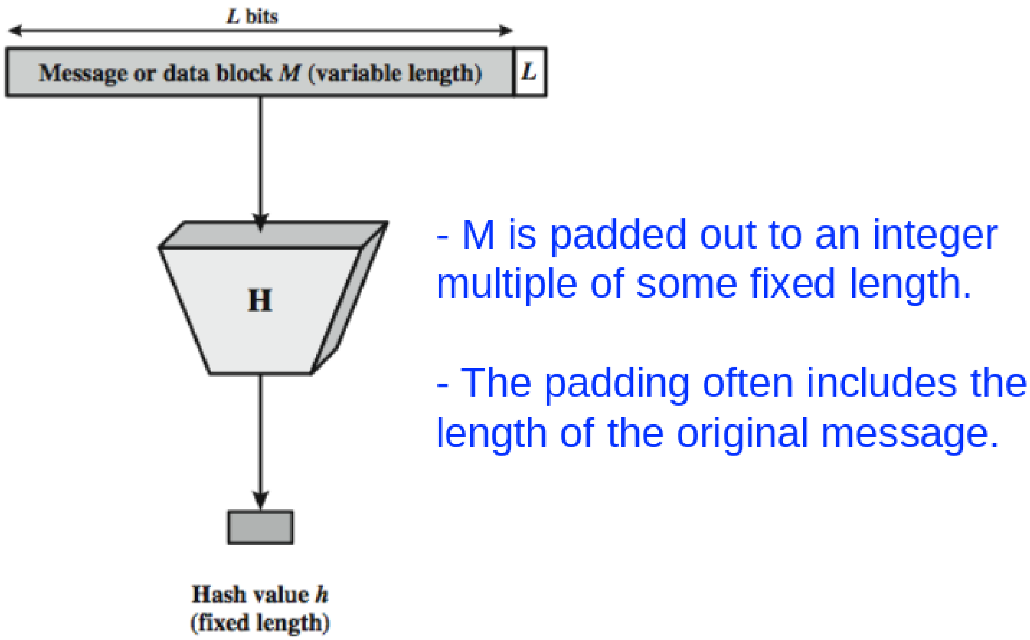


Figure 1: Cryptographic hash function

Hash Function: Used to Provide Message Authentication

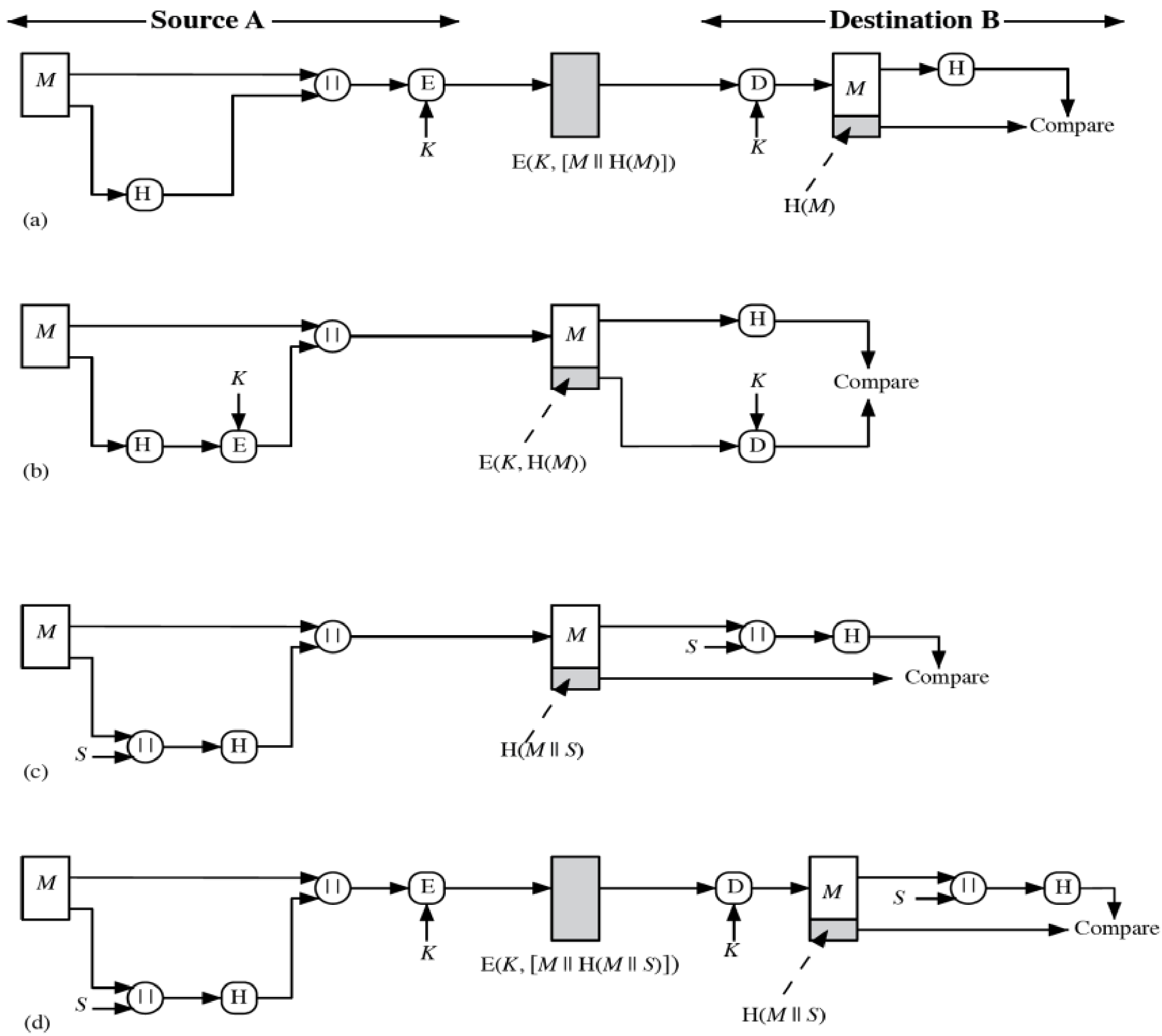


Figure 2: Hash Function Schemes for authentication

- Scheme (a) and (d) also provide confidentiality
- Scheme (a), (b), and (d) need to use encryption
- Specific MAC algorithms are normally used (chapter 12)

Hash Function: Used to Provide Digital Signature

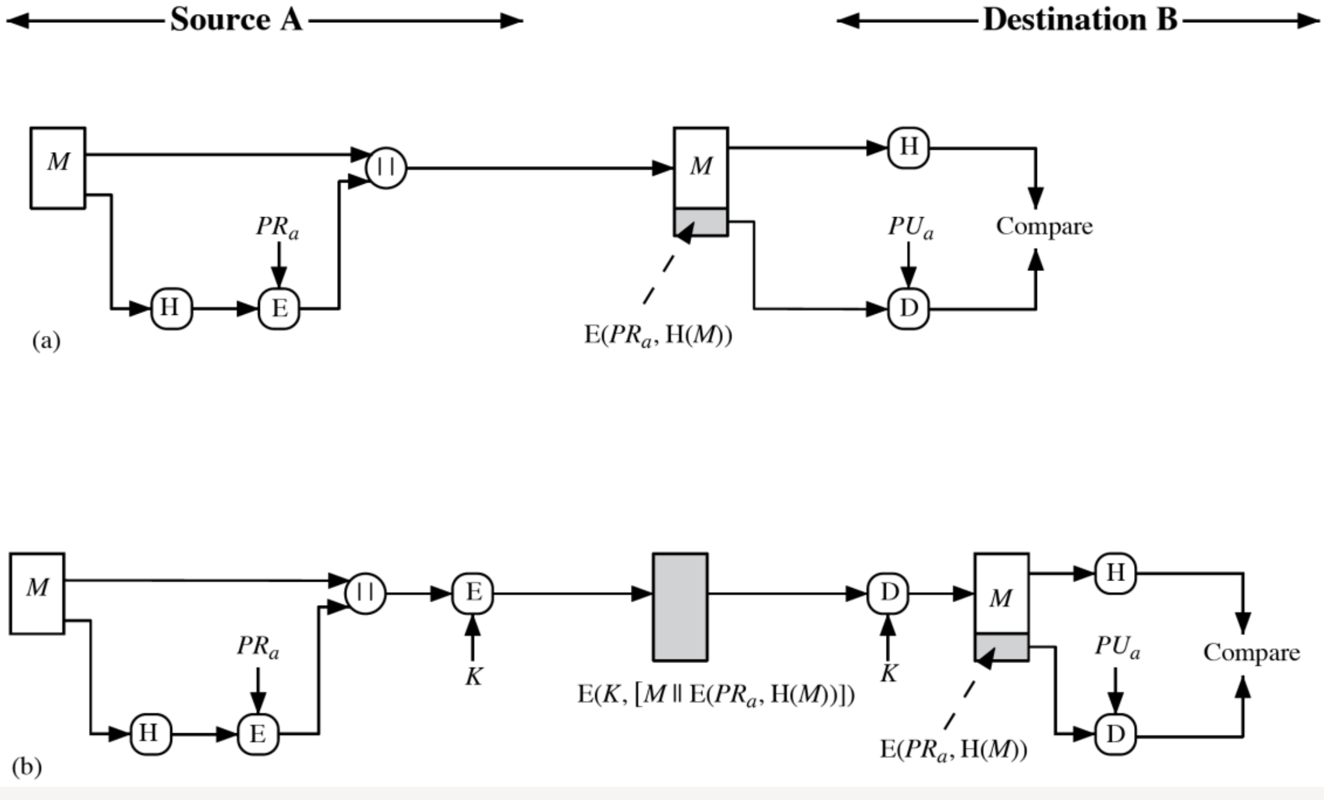


Figure 3: Hash Function Schemes for digital signatures

- Scheme (a) is similar to (b) in figure 2, but it also provides a digital signature.
- Scheme (b) also provides confidentiality
- Specific digital signature algorithms are used (chapter 13)

Other Hash Function Uses

- To create a one-way password file
  - Store hash of password not actual password
- For **intrusion detection** and **virus detection**
  - Keep and check hash of files on system
  - Keep H(F) secure, compute and compare later
- Pseudorandom function (PRF) or pseudorandom number generator (PRNG)

Two Simple Insecure Hash Functions

Bit-by-bit exclusive-OR (XOR) of every clock

- $C_i = b_{i1} \oplus b_{i2} \oplus \ldots \oplus b_{im}$ (C_i = the i^{th} bit of the hash code)
- Produces a simple parity for each bit position
- Reasonably effective as data integrity check
- Regularities in the input will reduce its effectiveness

One-bit circular shift on hash value (an improvement)

- Rotate current hash value by 1 bit then XOR each new block

Good for data integrity but useless for security

Cryptographic Hash Function Requirements

h = H(x)

- x is referred to as the **preimage of h**
- H is a many-to-one mapping
- A **collision** occurs if $x \neq y$, but H(x) = H(y) **This is undesirable (remember from algo hehe)**

Length of x: b bits

Length of H: n bits

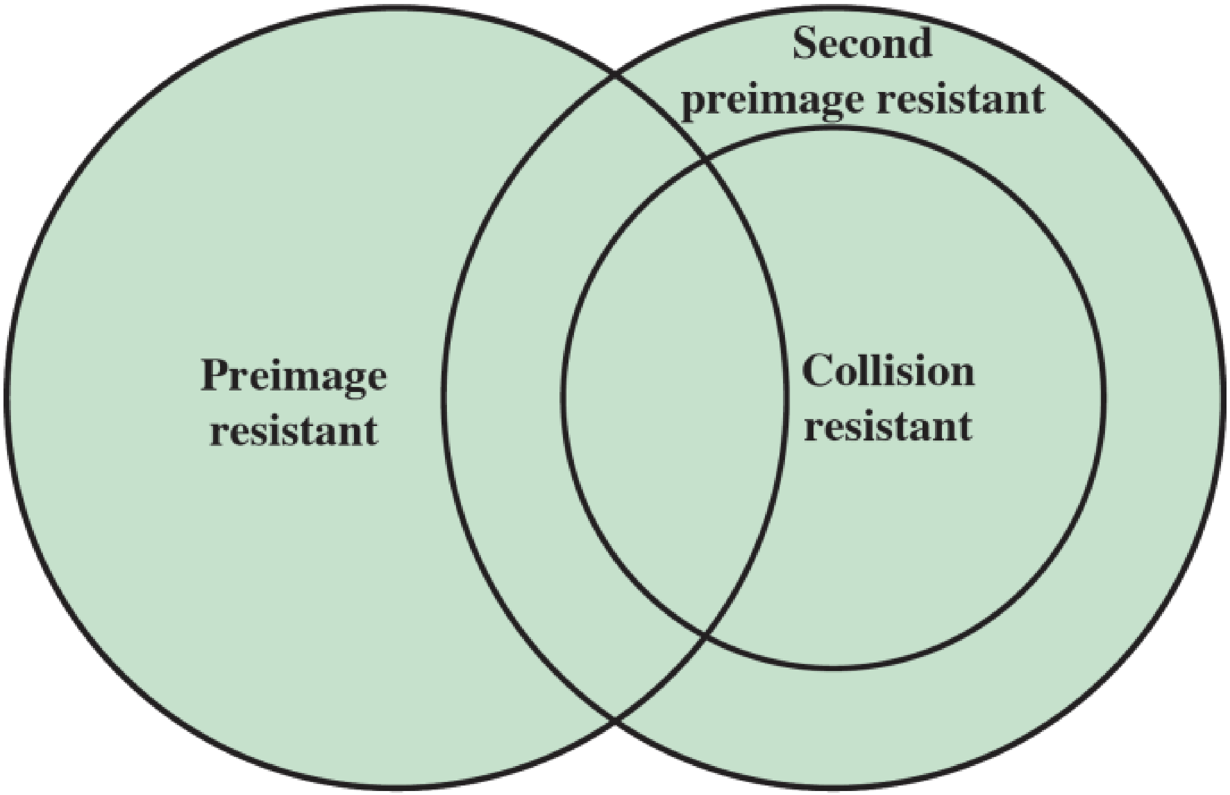
Each h, 2^{b-n} preimages

Length of x can be arbitrary, more preimages

Requirement	Description
Variable input size	H can be applied to a block of data of any size.
Fixed output size	H produces a fixed-length output.
Efficiency	H(x) is relatively easy to compute for any given x, making both hardware and software implementations practical.
Preimage resistant (one-way property)	For any given hash value h, it is computationally infeasible to find y such that H(y) = h.
Second preimage resistant (weak collision resistant)	For any given block x, it is computationally infeasible to find $y \neq x$ with H(y) = H(x).
Collision resistant (strong collision resistant)	It is computationally infeasible to find any pair (x, y) such that H(x) = H(y).
Pseudorandomness	Output of H meets standard tests for pseudorandomness

Figure 4: Hash Requirements

Relationship Among Hash Function Properties



Hash Function Resistance Properties Required for Various Data Integrity Applications

	Preimage Resistant	Second Preimage Resistant	Collision Resistant
Hash + digital signature	yes	yes	yes*
Intrusion detection and virus detection		yes	
Hash + symmetric encryption			
One-way password file	yes		
MAC	yes	yes	yes*

Figure 5: *Resistance required if attacker is able to mount a chosen message attack

Brute-Force Attacks on Hash Functions

Brute-force attacks depend on the bit length (m) of the hash value

- Attack on preimage or second preimage resistance
  - On average, 2^{m-1} attempts, (proof in appendix 11A)
- Attack on collision resistance
  - $2^{m/2}$ attempts
  - Birthday paradox (look in algo notes if you need to)
- 128-bits inadequate, 160-bits suspect

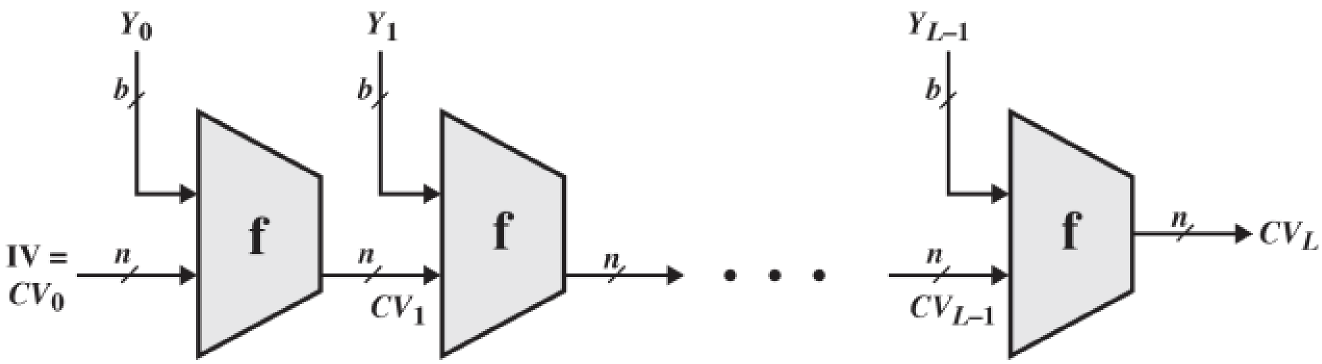
Cryptanalytic Attacks on Hash Functions

Good hash algorithms should make the effort of best cryptanalytic attacks greater than or equal to brute force

Hash functions use iterative structure processing blocks

If f is collision resistant, then so is the hash function

Attacks focus on collisions in the compression function f



Block Ciphers as Hash Functions

Can use block ciphers as hash functions

- Divide a message into N blocks $M_1, M_2, \ldots, M_N$
- Using $H_0 = 0$ and zero-pad of final block
- Compute: $H_i = E(M_i, H_{i-1})$
- $h = H_N$
- Similar to CBC but without a key

Vulnerable to attacks: small hash code if DES (64-bit) is used, birthday attack, "meet-in-the-middle" attack

- Other variants also susceptible to attacks

Secure Hash Algorithm (SHA)

The most widely used hash function

- Revised in 1995 to use 160-bit hash values
- Research on security of SHA-1 have raised concerns on its use in future applications
  - Estimated collision attack effort: 2^{69} - 2^{61}

Revised Secure Hash Standard

NIST issued another revision in 2002

Added 3 additional versions of SHA (SHA-2)

- SHA-256, SHA-384, SHA-512 with different hash value lengths in 2006
- SHA-224 in 2008