Finite Automata

Introduction

Finite Automata: a model of computation with finite state (memory)

- Application: text processing, program verification, compilers

Outline

- Languages and Automata
  - Definitions
  - Example
- Deterministic Finite Automata (DFA)
- Nondeterministic Finite Automata (NFA)

Language and Automata

Definitions

Symbol: An abstract, primitive, atomic "thing"

Set: An unordered collection, without repetition

Alphabet: A non-empty, finite set of symbols

- Ex. $\sum_B = \{0, 1\}$ alphabet of booleans

Sequence: An ordered list of objects

- Ex. $(1, 2, 3, 5, 8, \dots)$

String: A sequence over some alphabet

- Ex. hello (if the alphabet is our actual alphabet)

Languages: A set of strings

Example Automata

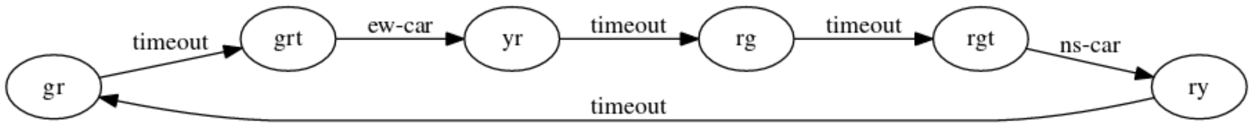


Figure 1: Example Automata of traffic light system

States:

- NS = {red, yellow, green} (cars going north and south)
- EW = {red, yellow, green} (cars going east and west)
- timeout = {0, 1} (timer on the traffic light)

Events:

- Q = {timeout, ns-car, ew-car}

Transition Table:

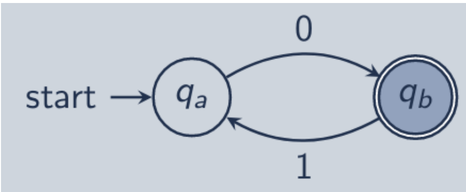
State	timeout	ns-car	ew-car
gr	grt	-	-
grt	-	-	yr
yr	rg	-	-
rg	rgt	-	-
rgt	-	ry	-
ry	gr	-	-

Deterministic Finite Automata (DFA)

Deterministic Finite Automata: a 5-tuple: $M = (Q, \sum, \delta, q_0, F)$, where:

- Q is a finite set called the **states**
- $\sum$ is a finite set called the **alphabet**
- $\delta : Q \times \sum \rightarrow Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subset Q$ is the set of accept states

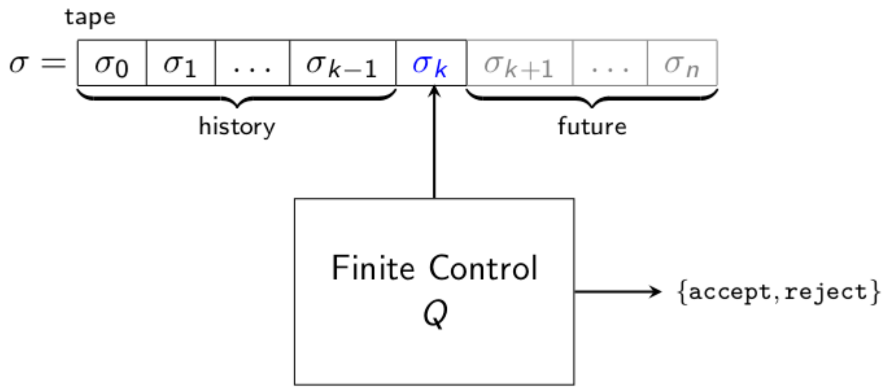
Example:



- $Q = \{q_a, q_b\}$
- $\sum = \{0, 1\}$
- $\delta(q_a, 0) = q_b$, $\delta(q_b, 1) = q_a$
- $q_0 = q_a$
- $F = \{q_b\}$

Conceptual DFA Operation

M : $\Sigma^* \to \{\text{accept, reject}\}$



Can represent a DFA as a machine with

- Finite Control state Q
- σ which is an input string (tape)
- At the end of the string two things can happen:
  1. **Accept** reach end of tape (string) with control in accept state q $\in$ F
  2. **Reject** reach end of tape (string) with control not in accept state q $\notin$ F

With this representation we could say the **language** of M is the set of strings accepted by M.

Meaning a DFA can produce a set of strings that end with an accepting state

Symbolic DFA Semantics

Transition Functions: transition from state q_{pred} to q_{succ} on symbol σ

$\delta(q_{\text{pred}}, \sigma) = q_{\text{succ}}$

Extended Transition Function: transition from state q_0 to q_n on string w

base: $\delta(q, \epsilon) = q$

recursive: For a $\in \Sigma$ and $B \in \Sigma^*$,

$$\hat{\delta}(q, aB) = \hat{\delta}(\delta(q, a), B)$$

DFA Simulation

Input: DFA M and Input String w

Find: Does M accept the input string

Algorithm:

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1. Evaluate the extended transition function on input string *w*
2. At the end of the input string:
    - If the resulting state is an accept state, return accept
    - Otherwise, return reject

Dumdum: Go through the states given the string transition, if it ends with accept then we good.
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DFA Language Definitions

Acceptance: DFA M **accepts** string w when the extended transition function results in an accept state:

$\hat{\delta}(q_0, w) \in F$

Rejection: DFA M **rejects** string w when the extended transition function results in an non-accept state:

$\hat{\delta}(q_0, w) \notin F$

Recognition: DFA M **recognizes** the language $L(M)$ consisting of the set of strings accept by M:

$L(M) = \{w \mid \hat{\delta}(q_0, w) \in F\}$

Regular Languages

Regular Languages (R): are languages that can be recognized by DFAs

$R = \{L \mid \text{for some DFA M, } L = L(M)\}$

A language (L) is regular if and only if there exists a DFA (M) that recognizes it:

$(L \in R) \leftrightarrow \exists M, (L = L(M))$

Nondeterministic Finite Automata (NFA)

Nondeterministic Finite Automata: a 5-tuple: $N = (Q, \sum, \delta, q_0, F)$, where:

- Q is a finite set called the **states**
- Σ is a finite set called the **alphabet**
- δ: $Q \times \Sigma \to P(Q)$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

NFA VS DFA

- DFA can only transition to a single state, NFA can transition to multiple states
- NFA cannot represent more languages than DFA

Proof that NFA cannot represent more languages than DFA:

1. Each nondeterministic step of an NFA, we are in a set of states
2. All such sets are the powerset of NFA states $P(Q_{\text{NFA}})$
3. The powerset of a finite set is still a finite set
4. We can create a DFA who's states correspond to $P(Q_{\text{NFA}})$