CHAPTER 8

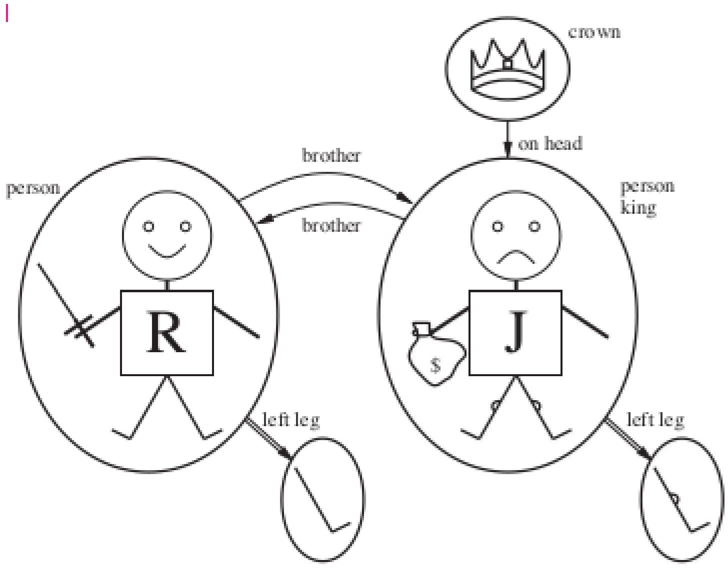
Proposition has limited expressive power. Need something better (First-order language)

First Order Logic

First-order logic (natural language) assumes the world contains...

- Objects: people, houses, numbers, theories, colors
- Relations: red, round, bogus, prime, brother of
- Functions: father of, best friend, third inning of

First Order Logic - Example



- Five objects
  1. Richard
  2. John
  3. Left leg of Richard
  4. Left leg of John
  5. Crown
- Two binary relations
  - brother(x, y)
  - onHeadOf(x, z)

- One unary relation
  - crown(x)
- One function
  - leftLegOf(x)

First Order Logic - Syntax (Basic Elements)

Constants: King John, 2, … Predicates: Brother, >, … Functions: Sqrt, LeftLegOf, … Variables: x, y, a, b Connectives: and, or, not, if, iff Equality: = Quantifiers: for all, there exist

Atomic Sentences: formed from a predicate symbol optionally followed by a parenthesized list of terms.

- Atomic sentence = predicate(term_1, … , term_n) or term_1 = term_2
  - Term = function(term_1, … ,term_n) or constant or variable
- ex. Brother(KingJohn, KingRichard)
- Simply put, an atomic sentence is a statement without using any binary connectives

Complex Sentences: formed from atomic sentences using connectives

- ex. Sibling(KingJohn, KingRichard) => Sibling(KingRichard, KingJohn)

Truth in First Order Logic

Sentences are true with respect to a *model* and an *interpretation*

Model contains >= 1 objects (**domain elements**) and relations among them

Interpretation specifies referents for

- Constant symbols -> objects
- Predicate symbols -> relations
- Function symbols -> functional relations

An atomic sentence predicate(term_1, …, term_n) is true

- iff the *objects* referred to by term_1, …, term_n
- are in the *relation* referred to by predicate

Model is basically a set of objects w/ interpretation for the constants/predicates/functions. There can be an unlimited combination of the three to form a model.

Models for First Order Logic

Entailment in propositional logic can be computed by enumerating models

Can enumerate the FOL models for a given KB vocabulary, but its not easy

Quantified Statements

Predicate logic lets us make statements about group of objects via quantified expressions

Two types of quantified statements

1. Universal: statement is true for all
2. Existential: statement is true for some

Universal Quantifier

The universal quantification of P(x) is the proposition:

- "P(x) is true for all values of *x* in the domain of discourse"
- The notation ∀x P(x) denotes the universal quantification of P(x), and is expressed as for every x, P(x)

Dummy translation of above: P is a proposition that is applied to every x person in a domain. For example p can be people at mines are smart, x represents every individual student at mines. The universal quantifier says we are all smart.

A common mistake to avoid

- Typically, -> is the connective with ∀
  - ∀x [At(x, Mines) -> Smart(x)]
  - Translation: "All students at Mines are smart"
- Common mistake: using ∧ as the main connective with ∀
  - ∀x [At(x, Mines) ∧ Smart(x)]
  - Translation: "Everyone at Mines and everyone is smart"

Existential Quantifier

The existential quantification of P(x) is the proposition:

- "There exists at least an element in the domain (universe) of discourse such that P(x) is true"
- The notation ∃x P(x) denotes the existential quantification of P(x), and is expressed as there is an x such P(x) is true

Dummy translation for above: P is a proposition applied to every x person in a domain. For example P is people at boulder are smart, and x represents every individual. The exist quantifier says there has to be at least one smart person at boulder.

A common mistake to avoid

- Typically, ∧ is the main connective with ∃
  - ∃x [At(x, Boulder) ∧ Smart(x)]
  - Translation: "Someone at Boulder is smart"
- Common mistake: using -> as the main connective with ∃
  - ∃x [At(x, Boulder) -> Smart(x)]
  - Translation: "The proposition is true if there is anyone who is not at Boulder

Quantification

Quantification converts a **propositional function** (a predicate with variables as arguments) into a **proposition** by binding a variable to a set of values from the universe of discourse.

ex.

- Let P(x) denote x > x - 1 and assume the universe of discourse of x is all real numbers
- Is P(x) a proposition? **NO** many possible substitutions
- Is ∀x P(x) a proposition? **YES** the statement is quantified in a universe of discourse. When x is defined as Real numbers, the proposition is true since all numbers is greater than itself minus 1.

Properties of quantifiers (NOT Commutative)

∀x ∀y is the same as ∀y ∀x

∃x ∃y is the same as ∃y ∃x

∃x ∀y is **NOT** the same as ∀y ∃x

Quantifier duality: each can be expressed using the other

- ∀x Likes(x, ice cream) == ¬∃x ¬Likes(x, ice cream)

Equality

$term_1 = term_2$ is true under a given interpretation iff $term_1$ and $term_2$ refer to the same object.

ex. ∀x multiply(Sqrt(x), Sqrt(x)) = x are satisfiable