CHAPTER 7

Knowledge-Based Agents and Inference Engine

Knowledge Base (KB): Knowledge (*a set of sentences*) that describe facts about the world in some formal (*representational*) language. **Domain specific**

Inference Engine: A set of procedures that use the representational language to infer new facts from known ones, or answer a variety of KB queries. *Inferences typically require search.* **Domain independent**

MYCIN Example

MYCIN: an expert system for diagnosis of bacterial infections

Knowledge base represents…

- Facts about a specific patient case
- Rules describing relations between entities in the bacterial infection domain
  - **IF**
    1. The strain of the organism is gram-positive, and
    2. The morphology of the organism is coccus, and
    3. The growth conformation of the organism is chains
  - **THEN**
    * The identity of the organism is streptococcus

Inference engine:

- Manipulates the facts and known relations to answer diagnostic queries (consistent with findings and rules)

Knowledge Representation

Objective: express the knowledge about the world in a computer-tractable form

Knowledge representational languages (KRL) *key aspects*:

- Syntax: describe how sentences in KRL are formed in the language
- Semantics: describe the meaning of sentences, what is it the sentence refers to in the real world
- Computational aspect: describe how sentences and objects in KRL are manipulated in concordance with semantic conventions

Many KB systems rely on some variant of logics

Logic

Logic: a formal language for expressing knowledge and for making logical inferences, which is defined by…

- A set of sentences: a sentence is constructed from a set of *primitives* according to syntactic rules
- A set of interpretations: an interpretation I gives a semantic to primitives. It associates primitives with objects or values.
  - I: primitives -> objects/values
- The valuation (meaning) function V: Assigns a values (typically the truth value) to a given sentence under some interpretation
  - V: sentence x interpretation -> {True, False}

Propositional Logic - The Simplest Logic

Proposition: is a statement that is either true or false

Propositional Logic - Syntax

P: syntax + interpretation + semantics

- ex. P represents: "Golden is part of the Jefferson Country"

Connectives: and, or, if, if then, not

Atomic sentences: constructed from constants and propositional symbols - True, False are atomic sentences - P is an atomic sentences

Composite sentences: constructed from valid sentences via logical connectives - (P and Q)

Propositional Logic - Semantics

Semantic: semantic of a language gives the meaning to a sentence

The semantics in a propositional logic is defined by

- Interpretation of propositional symbols and constants
  - Semantics of atomic sentences
- Through the meaning of logical connectives
  - Meaning (semantics) of composite sentences

In a propositional symbol…

- Start with a statement about the world that is true or false
- Interpretation maps symbol to one of the two values (True or False)

- The meaning (value) of the propositional symbol for a given interpretation is given by its interpretation. False interpretation = false value.

Truth table for composite sentences

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Translation

Translation of English sentences to propositional logics

- Identify atomic sentences that are propositions
- Use logical connectives to translate more complex composite sentences that consist of many atomic sentences
- ex. “It is not sunny this afternoon and it is colder than yesterday
  - p = it is sunny this afternoon
  - q = it is colder than yesterday
  - not p and q

Contradiction and Tautology

Contradiction: always false

Tautology: always true

Model, Validity, and Satisfiability

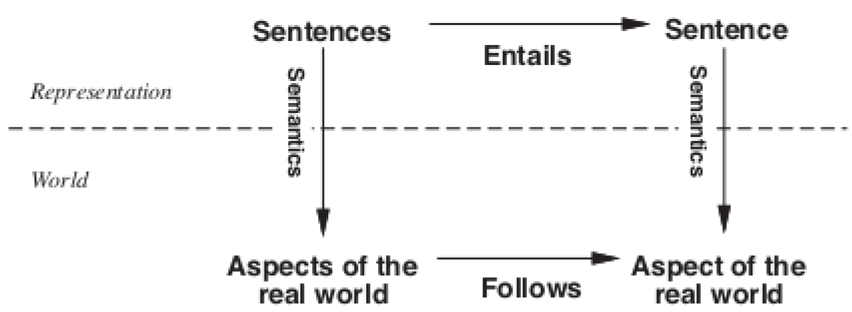
Model: an interpretation is **a model for a set of sentences** if it assigns true to each sentence in the set

Satisfiable: a sentence is satisfiable if it has a model

Validity: a sentence is valid if it is true in ALL interpretations

Entailment

Entailment: reflects the relation of one fact in the world following from the others



Knowledge base *KB* entails sentence P if and only if P is True in all worlds where KB is true

Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated:

- We say *m* is a model of sentence P if P is true in *m*
- M(P) is the set of all models of P
- Then KB entails P if and only if M(KB) is in the set of M(P)
- ex. KB = giants and red won, P = giants won

Sound and Complete Inference

Inference: inference is the process by which new sentences are derived from existing sentences in the KB

- The inference process is implemented on a computer
- Assume an inference procedure *i* that
  - derives a sentences a from the KB: KB provable$_i$ a

Soundness: an inference procedure is sound...

- If KB provable$_i$ a then it is true that KB entails a

Completeness: an inference procedure is complete

- If KB entails a then it is true that KB provable$_i$ a

Logical Inference Problem

Logical Inference Problem:

- Given:
  - A knowledge base KB (a set of sentences) and
  - A sentence *a* (called a theorem)
- Does a KB semantically entail *a*?

– In other words: In all interpretations in which sentences in the KB are true, is a also true?

How to design procedure that answer KB entails a? Three approaches…

1. Truth-table approach
2. Inference rules
3. Conversion to the inverse SAT problem (resolution-refutation)

Truth-Table Approach

Problem: KB entails a?

- Need to check all possible interpretations for which the KB is true (models of KB) whether a is true for each of them

Truth Table:

- Enumerate truth values of sentences for all possible interpretations (assignments of True/False to propositional symbols)

Two step procedure

1. Generate table for all possible interpretations
2. Check whether the sentence a evaluates to true whenever KB evaluates to true

Example: $KB = (A \vee C) \wedge (B \vee \neg C)$ $\quad \alpha = (A \vee B)$

A	B	C	$A \vee C$	$(B \vee \neg C)$	KB	α	
True	*True*	*True*	*True*	*True*	*True*	*True*	✓
True	*True*	*False*	*True*	*True*	*True*	*True*	✓
True	*False*	*True*	*True*	*False*	*False*	*True*	
True	*False*	*False*	*True*	*True*	*True*	*True*	✓
False	*True*	*True*	*True*	*True*	*True*	*True*	✓
False	*True*	*False*	*False*	*True*	*False*	*True*	
False	*False*	*True*	*True*	*False*	*False*	*False*	
False	*False*	*False*	*False*	*True*	*False*	*False*	

Limitations of truth table approach: computational complexity is 2^n rows in the table has to be filled

How to make process more efficient?

- We only have to check entries of KB that are True (this is the idea behind *inference rules approach*)

Inference Rules for Logic

Inference Rules:

- Represent sound inference patterns repeated in inferences
- Can be used to generate new (sound) sentences from the existing ones

Modus ponens:

$$\begin{array}{c} A \Rightarrow B, \quad A \\ \hline B \end{array} \quad \begin{array}{l} \leftarrow \textbf{premise} \\ \leftarrow \textbf{conclusion} \end{array}$$

- If both sentences in the premis are true, then the conclusion is true

List of inference rules for logic:

- And-elimination: $\dfrac{A_1 \wedge A_2 \wedge \quad A_n}{A_i}$
- And-introduction: $\dfrac{A_1, A_2, \quad A_n}{A_1 \wedge A_2 \wedge \quad A_n}$
- Or-introduction: $\dfrac{A_i}{A_1 \vee A_2 \vee \ldots A_i \vee \quad A_n}$
- Elimination of double negation: $\dfrac{\neg\neg A}{A}$
- Unit resolution (a special case of resolution): $\dfrac{A \vee B, \quad \neg A}{B}$
- Resolution: $\dfrac{A \vee B, \quad \neg B \vee C}{A \vee C}$

Logical Equivalence

Logical Equivalence: two sentences are **logically equivalent** iff true in same models:

A logically equivalent B iff A entails B and B entails A

Logical Inferences & Search

For each instance of list of rules so far, many possible rules can be applied next (THIS IS LIKE A SEARCH PROBLEM)

Inference rule method as a search problem

- State: a set of sentences that are known to be true
- Initial State: a set of sentences in the KB
- Operators: applications of inference rules (allow us to add new sound sentences to old ones)
- Goal state: a theorem a is derived from KB

Logical inference:

- Proof: a sequence of sentences that are immediate consequences of applied inference rules
- Theorem proving: process of finding a proof of theorem

Problem: too many inference rules (big branching factor)

Solution: Simplify inferences using on of the normal forms

1. Conjunctive normal form (CNF): conjunction of clauses

$$(A \vee B) \wedge (\neg A \vee \neg C \vee D)$$

2. Disjunctive normal form (DNF): disjunction of terms

$$(A \wedge \neg B) \vee (\neg A \wedge C) \vee (C \wedge \neg D)$$

Conversion to CNF

1. Eliminate => and <=>
2. Reduce the scope of signs through De Morgan laws and double negation
3. Convert to CNF using the associative and distributive laws

Resolution Rule

Resolution rule: sound inference rule that fits the CNF

$$\frac{A \vee B, \quad \neg B \vee C}{A \vee C}$$

When applied directly to KB in CNF to infer a:

- Incomplete: repeated application of the resolution rule to a KB in CNF may fail to derive new valid sentences

Trick to make it work:

- Proof by contradiction:
  - Disprove KB and not a
  - Proves entailment KB entails a
- **Resolution is refutation complete**

Resolution Algorithm

- Convert KB to CNF form
- Apply iteratively the resolution rule starting from KB, not A
- Stop when:
  - Contradiction (empty clause) is reached:
    - A, not A -> 0
  - No more new sentences can be derived
    - Disproved

Properties of Inference Solutions

Truth-table:

- Blind
- Exponential in the numbers of variables

Inference rules:

- More efficient
- Many inference rules to cover logic

Conversion to SAT - Resolution refutation

- More efficient lol
- Sentences must be converted into CNF
- One rule (**RESOLUTION RULE**) is sufficient to perform all inferences

The Wumpus World

Performance measure

- gold: +1000
- death: -1000
- per step: -1
- using arrow: -10

Environment

- Squares adjacent to Wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter iff gold is in the same square
- Shooting kills Wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square

Actuators

- Left turn, right turn, forward, grab, release, shoot

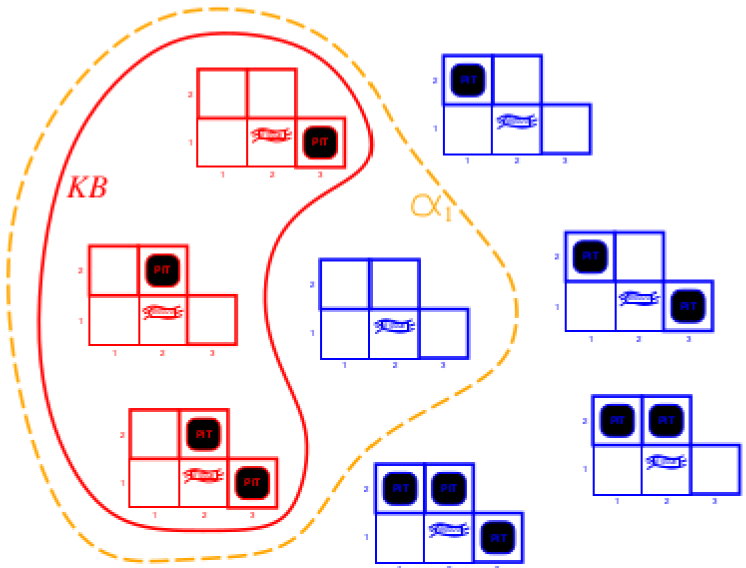
Sensors

- Breeze, glitter, smell

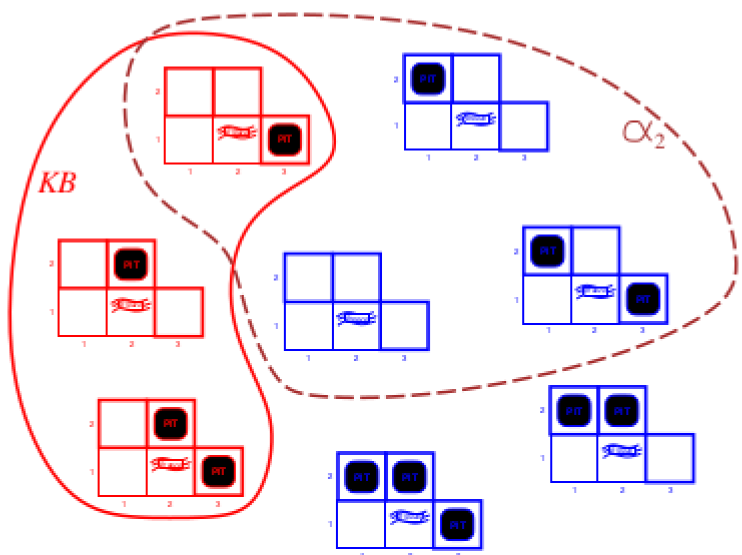
Wumpus Models

KB = Wumpus world rules + observations

a_1 = "[1,2] is safe", KB entails a_1 proved by model checking



a_2 = "[2,2] is safe", KB entails a_2



Stuff I didn't know instantly

- Knowledge Representation Language (KRL)
- Model, Validity, and Satisfiability
- Sound and completeness of inference
- Inference rules (remember them)
- Logic Inferences as a search problem
- Normal Forms
  - CNF
  - DNF
- Resolution Rule

Stuff that will probably be on the test

- Knowledge Based
- Entailment
- Proposition stuff (maybe, might be too easy)
- Logical inference (3 approaches)
  - Truth table
  - Inference rule
  - Conversion to the inverse SAT problem (Resolution)
- Wumpus