3.1 Ideal Block Cipher and Feistel Cipher Structure

Modern Symmetric Block Ciphers

- Provide confidentiality/authentication services
- Focus on data encryption standard (DES)

Block vs Stream Ciphers

- Stream ciphers: process messages a bit or byte at a time when en/decrypting.

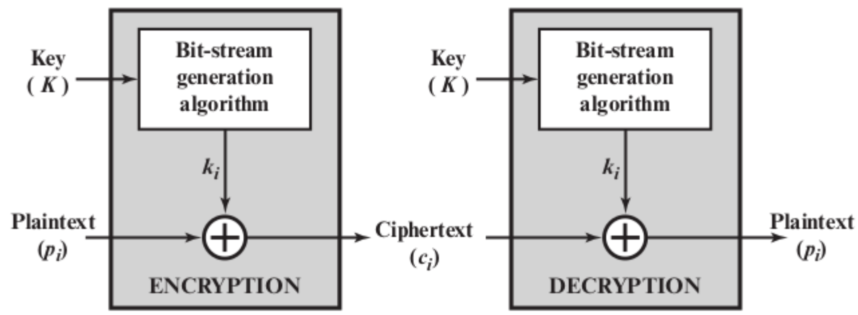


Figure 1: Example of a stream cipher

- Block ciphers: process messages in blocks each of which is then en/decrypted.

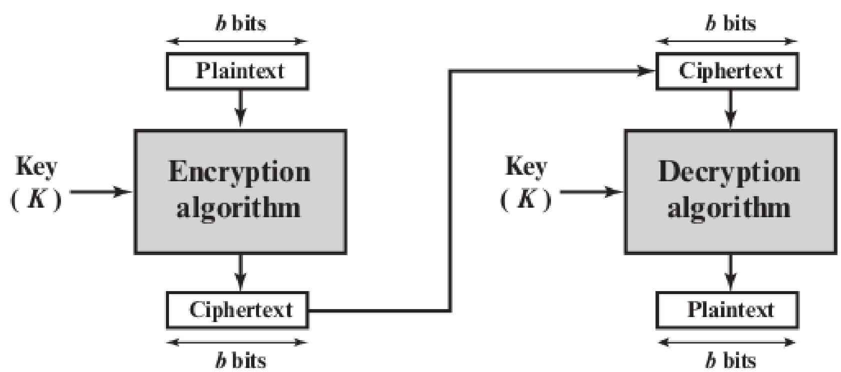


Figure 2: Example of block cipher

- Many current ciphers are block ciphers
  - Better analyzed
  - Broader range of applications

Most symmetric block ciphers are based on a Feistel Cipher Structure

Motivation for the Feistel cipher structure:

- Block ciphers:
  - n bits plaintext → n bits ciphertext
  - 2^n possible plaintext/ciphertext blocks
- Reversible (non-singular) transformation
  - Each unique plaintext block → a unique ciphertext block
  - $2^n!$ Reversible transformation
- Feistel refers to an n-bit general substitution as an *ideal block cipher*
  - It allows for the maximum number of possible encryption mappings from the plaintext to ciphertext block.

Problems of Ideal Block Cipher

The block size n must be very large to thwart attacks

- Statistical analysis of plaintext, due to small block size

Straightforward mapping are not practical from an implementation and performance point of view

- In general, key length = $n2^n$ bits
- For a 64-bit block, key length = $64*2^{64} = 2^{70} = 10^{21}$ bits
- That a lot of needed bits

Poorly confined mapping (to a subset of $2^n!$ reversible transformation) may be vulnerable to cryptanalysis.

Example of confined mapping:

- Mapping in terms of a set of linear equations

  $y_1 = k_{11}x_1 + k_{12}x_2 + k_{13}x_3 + k_{14}x_4$

  $y_1 = k_{21}x_1 + k_{22}x_2 + k_{23}x_3 + k_{24}x_4$

  $y_1 = k_{31}x_1 + k_{32}x_2 + k_{33}x_3 + k_{34}x_4$

  $y_1 = k_{41}x_1 + k_{42}x_2 + k_{43}x_3 + k_{44}x_4$

- Key length = n^2 (16 in this case)
- This is vulnerable to cryptanalysis (This is essentially a hill cipher)

Feistel Cipher

Feistel's observation: an approximation to the ideal block cipher system for large n, built up out of components that are easily realizable is needed.

Feistel Cipher idea:

- Approximate the ideal block cipher using a product cipher
  - The execution of two or more simpler ciphers in sequence
  - The final result or product is cryptographically stronger than any of the component ciphers.
- Essence of the approach
  - Key lengths k bits, block length n bits, 2^k instead of $2^n!$ transformation, k large for thwarting brute force attacks.
  - Alternate substitutions and permutations (transpositions) to defend against cryptanalytic attacks
  - Practical application of the **Claude Shannon's proposal**

Claude Shannon and Substitution-Permutation Ciphers

Claude Shannon idea: substitution-permutation (S-P) networks. S-P nets are based on the two primitive cryptographic operations:

1. Substitution (S-box)
2. Permutation (P-box)

Provide **confusion** and **diffusion** of message and key. Thwart cryptanalysis based on statistical analysis

Confusion and Diffusion

Cipher needs to completely obscure statistical properties of original message

Combining S & P elements to obtain:

- Diffusion: make the statistical relationship between the plaintext and ciphertext as complex as possible

  - Achieved by repeated permutation followed by a function (transposition, P-Box)
  - Having each ciphertext digit be affected by many plaintext digits
- Confusion: make the statistical relationship between ciphertext and key as complex as possible
  - Achieved by complex substitution (S-Box

Feistel Cipher Structure

Feistel Cipher: partition input block into two halves

- Process through multiple rounds which…
- Perform a substitution on left half data based on round function of right half & subkey
- Then have permutation swapping halves

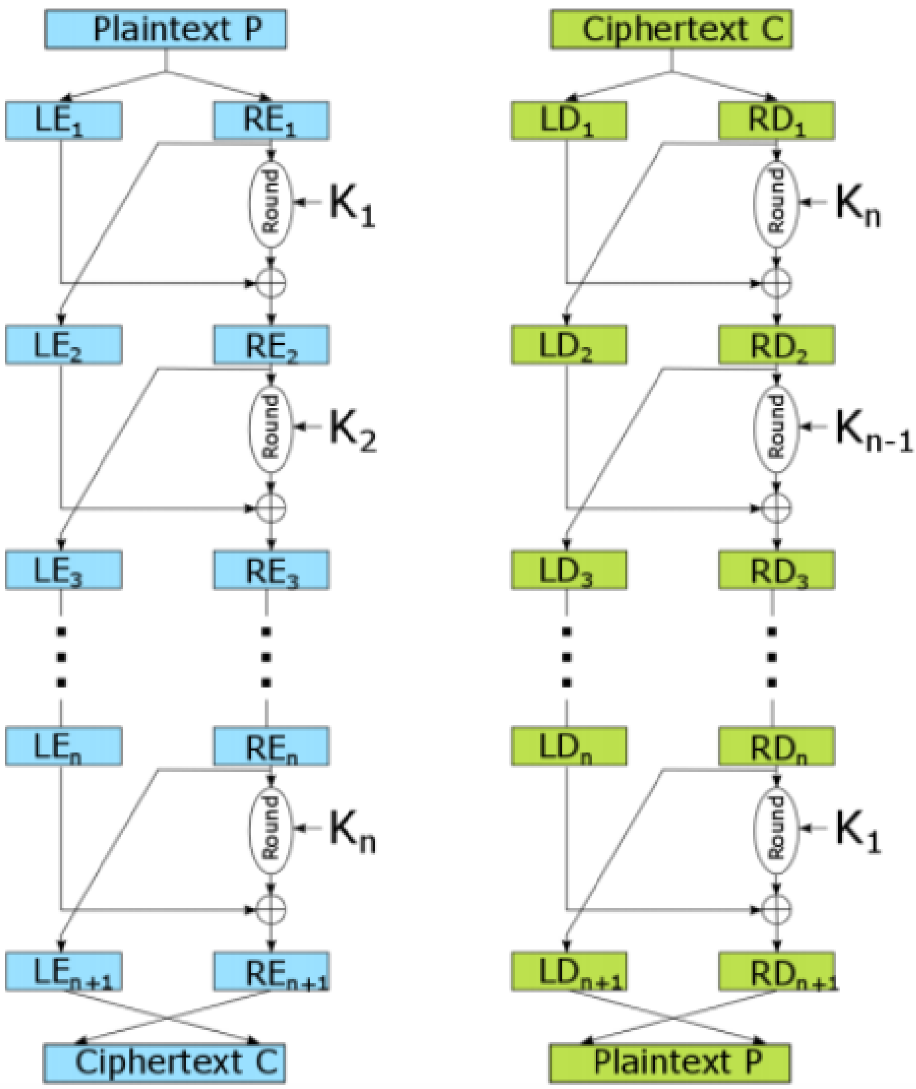


Figure 3: Example of Encryption and Decryption for Feistel Cipher

3.2 Data Encryption Standard (DES)

DES Design Controversy

- Although DES standard is public, considerable controversy over design
  - Choice of 56-bit key (vs Lucifer 128-bit)
  - Design criteria (S-boxes) were classified

DES Encryption Overview

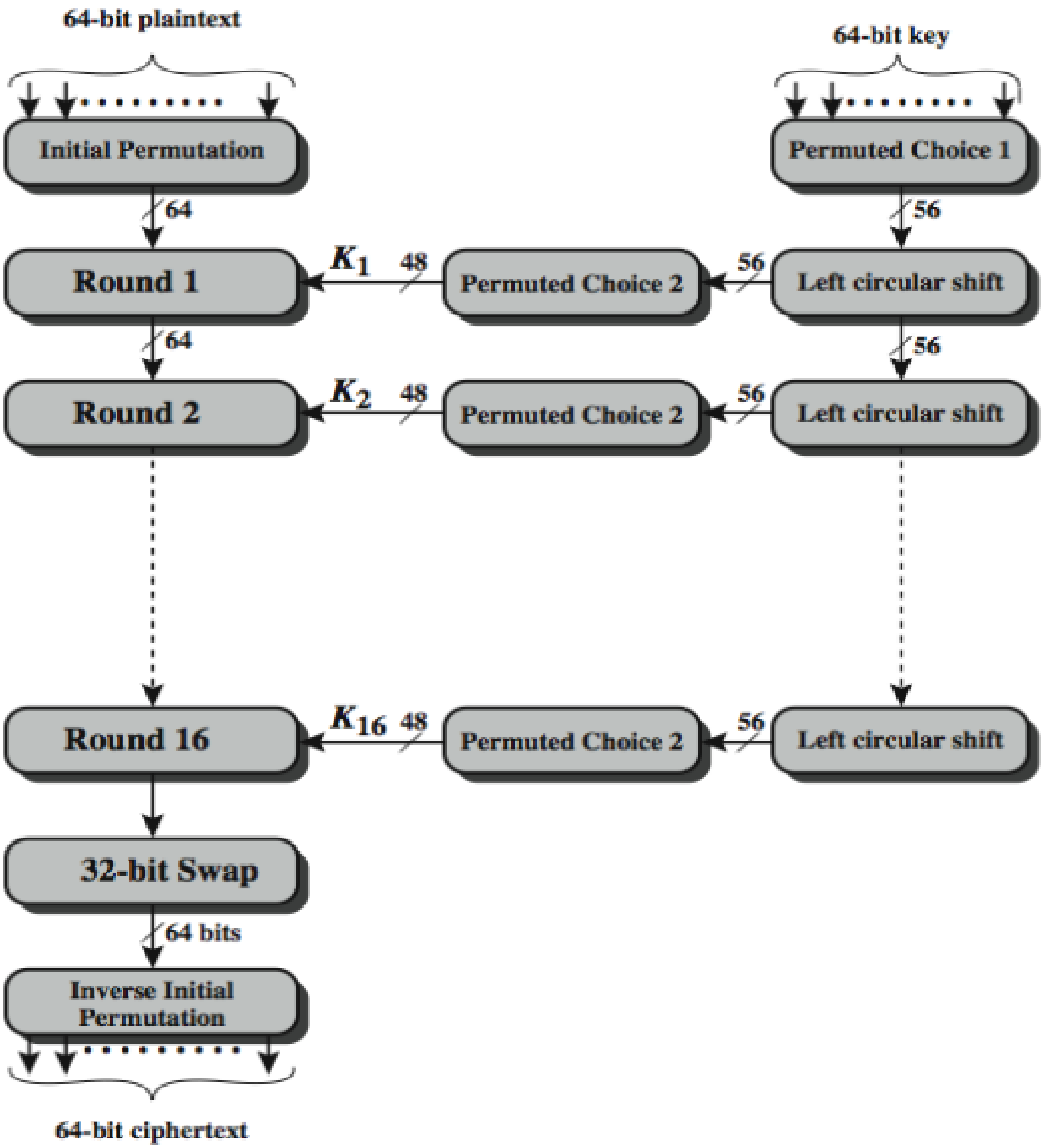


Figure 4: DES Encryption diagram

Plaintext side: Three Phases

- Initial Permutation
- 16 Rounds
  - Each round: key dependent round function with S and P
- Inverse Initial Permutation

Key function side:

- PC-1: Gets rid of 8 bits (56 bits)
- LCS: Cut key in half, perform bit shift(1 or 2 bit shift) on both half (independently)
- PC-2: Gets rid of 8 bits (48 bits)

Size of input: 64

Effective key length: 56

Initial Permutation and Inverse (IP)

IP

L_0	58	50	42	34	26	18	10	2
	60	52	44	36	28	20	12	4
	62	54	46	38	30	22	14	6
	64	56	48	40	32	24	16	8
R_0	57	49	41	33	25	17	9	1
	59	51	43	35	27	19	11	3
	61	53	45	37	29	21	13	5
	63	55	47	39	31	23	15	7

IP^{-1}

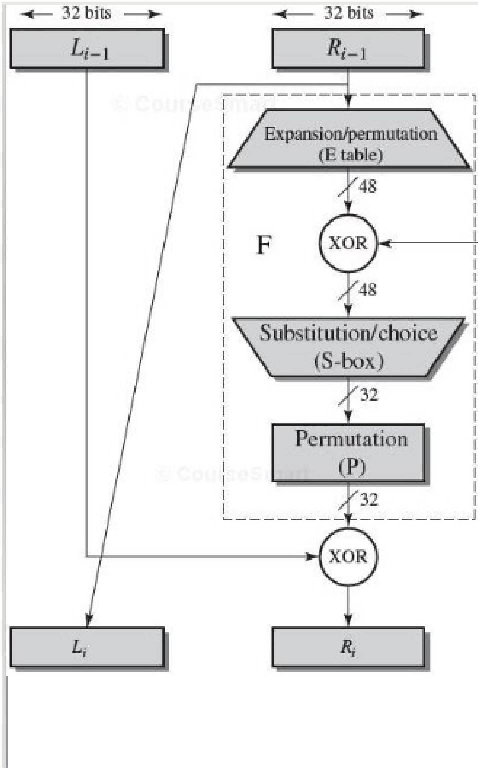
40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

Figure 5: Example of IP and IP^{-1} tables

IP (64 bits): split in half. 32 bits each.

- The 1^{st} position in the plaintext (bit 1) is now placed in the 40^{th} position as per the IP table.
- The inverse shows 40 bit = in the bit 1 position.

Single Round Function



32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

Expansion by taking the outer bits from the two adjacent groups (underlined numbers)

16	7	20	21	29	12	28	17
1	15	23	26	5	18	31	10
2	8	24	14	32	27	3	9
19	13	30	6	22	11	4	25

$LE_i = RE_{i-1}$;
$RE_i = LE_{i-1} \oplus F(RE_{i-1}, K_i)$

Figure 6: Example of a single round in the DES Algorithm

Consists of four steps:

1. Expansion (E table): take outer bits from two adjacent groups (Shown in figure 6)
   - This makes it so that the length of bits is now 48, allowing you to XOR with the key bits (also 48 bits)
2. XOR (KEY): simple XOR operation on all bits in from the E table and from the key
3. Substitution choice (S-Box): takes in 6 bits at a time and outputs given the s-box table
   - changes back to 32 bits
4. Permutation (P table): Same concept as IP except use the 8x4 table in figure 6
5. XOR (LEFT RIGHT): Take the table from step 4 and XOR with the old left side. Then take the old right side and make that the new left side.

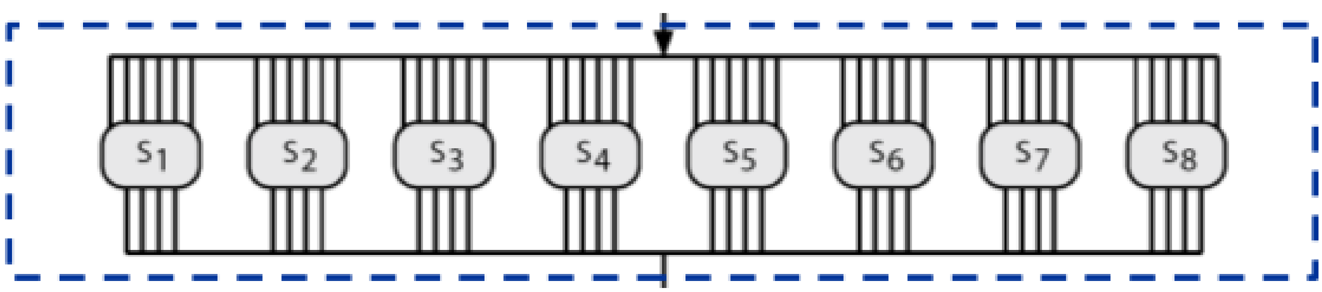


Figure 7: Example of 8 S-boxes, each accepting 6 bits, and outputs 4 bits

Substitution/choice (eight S-boxes, each accepts 6 bits, outputs 4 bits)

- Each S-box: 4 rows, from the 6 bit input, the last and first bit from input decides which row will be used. The middle 4 bits from input will determine the column

S-Boxes Example

S_1

	x0000x	x0001x	x0010x	x0011x	x0100x	x0101x	x0110x	x0111x	x1000x	x1001x	x1010x	x1011x	x1100x	x1101x	x1110x	x1111x
0yyyy0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0yyyy1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
1yyyy0	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
1yyyy1	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

Input: 011001

- 0xxxx1 = row 2
- x1100x = column 13

Each row of an S-box defines a general 4-bit reversible substitution (ideal block cipher)

- The only nonlinear part of DES

DES 32 total ideal block cipher mapping

DES Key Schedule

The steps for key scheduling encryption is as follow:

1. Permuted Choice 1: use table to reorder bits and remove 8 of them
2. Left Circular Shift: more ciphering. One copy gets sends to permuted choice 2 for a round and the other goes to another left circular shift for the next key generation for the next round.
3. Permuted Choice 2: use table to reorder bits and remove 8 more of them (create a 48 bit key for the round to use)

Permuted Choice One (PC-1)

C	57	49	41	33	25	17	9
	1	58	50	42	34	26	18
	10	2	59	51	43	35	27
	19	11	3	60	52	44	36
D	63	55	47	39	31	23	15
	7	62	54	46	38	30	22
	14	6	61	53	45	37	29
	21	13	5	28	20	12	4

Permuted Choice Two (PC-2)

14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

PC-1: Every eighth bit is discarded (8,16,32,64,24,40,48,56)

PC-2: Eight more bits are discarded

DES Decryption

Same as Feistel: DES decryption uses the **same algorithm** as encryption

Do encryption steps again using subkeys in reverse order.

Avalanche Effect

Avalanche effect: where a change of one input or key bit results in changing approximately half output bits. (DES exhibits strong avalanche)

Strength of DES - Key Size

56 bits have 2^{56} values

Strength of DES - Cryptanalytic

Utilize some deep characteristics of the cipher

- By gathering information about encryptions
- Can eventually recover some/all of the sub-key bits
- If necessary, exhaustive search the rest

Generally these are statistical attacks:

- Differential cryptanalysis: observe pairs of text blocks evolving along each round
- Linear cryptanalysis: find linear approximation to the transformations
- Related key attacks: observe the operation of a cipher under several related keys

Theoretical estimation; brute-force is practical concern

Strength of DES - Timing Attacks

Attack actual implementation of a cipher

Information about the key or the plaintext is obtained by observing how long it takes a given implementation to perform decryptions on various ciphertext.

- Specifically use fact that calculations can take varying times

DES Design Criteria

7 criteria for S-boxes provided for

- Non-linearity
- Resistance to differential cryptanalysis
- Good confusion

3 criteria for permutation P provide for

- Increase diffusion

7 Criteria for S-Boxes

3 out of 7:

1. No output bit of any S-box should be too close a linear function of the input bits.
   - Specifically if we select any output bit and any subset of the six input bits
   - Fraction of inputs for which this output bit equals the XOR of these input bits should not be close to 0 or 1, but rather should be near 1 or 2
2. Each row of an S-box (determined by a fixed value of the leftmost and rightmost input bits) should include all 16 possible output bit combinations
3. If two inputs to an S-box differ in exactly one bit, the outputs must differ in at least two bits.

3 Criteria for Permutation

2 out of 3:

1. The four output bits from each S-box at round i are distributed so that two of them affect (provide input for) "middle bits" of round (i+1) (next round) and the other two affect end bits.
   - The two middle bits of input to an S-box are not shared with adjacent S-boxes. The end bits are the two left-hand bits and the two right-hand bits, which are shared with adjacent S-boxes
2. The four output bits form each S-box affect six different S-boxes on the next round, and no two affect the same S-box

Block Cipher Design

Number of rounds: more is better

- Average Brute force time takes 2^{55}
- With DES 16 rounds it takes $2^{55.1}$ (Just beating brute force average)
- Make known cryptanalytic attack require greater effort than simple brute-force key search attacks (discourage them)

Round function f:

- Provide "confusion", nonlinear, avalanche, bit independence
- Has issues of how S-boxes are selected
- Strict avalanche criterion (SAC)
  - Any *output bit j* of an S-box should change with probability 1/2 when any single *input bit i* is inverted for all i,j.
  - A similar criterion could be applied to F as a whole
- Bit independence criterion (BIC)
  - *Output bits j and k* should change independently when any single *input bit i* is inverted for all i, j, and k
- S-Box (n x m) Design
  - Larger S-boxes are more resistant to cryptanalysis, but need larger lookup table and are more difficult to design properly
  - n limited tp 8~10

Key schedule:

- Complex subkey creation, key avalanche
- Use the main key to generate subkey for each round:
  - Maximize the difficulty of deducing individual subkeys
  - Maximize the difficulty of working back to the main key
  - Guarantee key/ciphertext meet SAC and BIC