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1 Mean Filter

1.1 Moving Average

$$y_k = \frac{1}{N} \sum_{n=0}^{N-1} x_{k-n}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} - \frac{1}{N} x_{k-N}$$

$$y_k = y_{k-1} + \frac{1}{N} (x_k - x_{k-N})$$

$$y_k = \frac{1}{N} \sum_{n=0}^{N-1} x_{k-n}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} - \frac{1}{N} x_{k-N}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} \frac{N-1}{N}$$

$$y_k = y_{k-1} + \frac{1}{N} (x_k - y_{k-1})$$

1.2 PT1-Filter

$$Y_{(s)} = \frac{K}{1 + T s} X_{(s)}$$

$$Y_{(s)} + T s Y_{(s)} = K X_{(s)} \quad \circ \longrightarrow \bullet \quad y_{(t)} + T \dot{y}_{(t)} = K x_{(t)}$$

$$y_k + T \frac{y_k - y_{k-1}}{dt} = K x_k$$

$$y_k + \frac{dt}{T} y_k = y_{k-1} + K \frac{dt}{T} x_k$$

$$y_k = \frac{T}{T + dt} y_{k-1} + K \frac{dt}{T + dt} x_k$$

$$\frac{T}{T + dt} y_{k-1} = y_{k-1} - \frac{dt}{T + dt} y_{k-1}$$

$$y_k = y_{k-1} + \frac{dt}{T + dt} (K x_k - y_{k-1})$$

$$y_k = y_{k-1} + \frac{dt}{T + dt} (K x_k - y_{k-1})$$

$$K = 1$$

$$\frac{dt}{T + dt} = \frac{1}{N}$$

$$y_k = y_{k-1} + \frac{1}{N} (x_k - y_{k-1})$$

1.3 Low Pass Filter

$$H_{(s)} = \frac{2\pi f_c}{2\pi f_c + s} = \frac{1}{1 + \frac{1}{2\pi f_c} s} = \frac{K}{1 + T s}$$

$$H_{(z)} = Z\{H_{(s)}H_{0(s)}\}$$

$$H_{(z)} = Z\left\{H_{(s)}\frac{1}{s(1 - e^{-s T_a})}\right\} = \frac{z-1}{z}Z\left\{\frac{H_{(s)}}{s}\right\}$$

$$Z\left\{\frac{1}{s}\right\} = \frac{z}{z-1}Z\left\{\frac{a}{a+s}\right\} = \frac{1 - e^{-a T_a}}{z - e^{-a T_a}}$$

$$H_{(z)} = \frac{z-1}{z} \frac{z}{z-1} K \frac{1 - e^{-\frac{T_a}{T}}}{z - e^{-\frac{T_a}{T}}} = K \frac{1 - e^{-\frac{T_a}{T}}}{z - e^{-\frac{T_a}{T}}} = \frac{K(1 - e^{-\frac{T_a}{T}})z^{-1}}{(1 - e^{-\frac{T_a}{T}})z^{-1}}$$

$$H_{(z)} = \frac{b_0 + b_1 z^{-1}}{a_0 + a_1 z^{-1}} \circ \bullet y_k = b_1 u_{k-1} - a_1 y_{k-1} \approx b_1 u_k - a_1 y_{k-1}$$

$$b_0 = 0, b_1 = K(1 - e^{-\frac{T_a}{T}}), a_0 = 1(\rightarrow \text{normed}), a_1 = -e^{-\frac{T_a}{T}}$$

1.4 Morphological Filter

$$Dilation \rightarrow (X \oplus B_1)[n] = \max_{m=0, \dots, M-1} \left\{ X[n + m - M] - B_1[m] \right\}$$

$$Erosion \rightarrow (X \ominus B_2)[n] = \min_{m=0, \dots, M-1} \left\{ X[n + m - M] - B_2[m] \right\}$$

$$X = \frac{1}{2} [(X \oplus B_1) + (X \ominus B_2)]$$

$$B_1 = [10, 5, 0, 5, 10]$$

$$B_2 = [-10, -5, 0, -5, -10]$$