Contents

1 Mean Filter

1.1 Moving Average

$$y_k = \frac{1}{N} \sum_{n=0}^{N-1} x_{k-n}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} - \frac{1}{N} x_{k-N}$$

$$y_k = y_{k-1} + \frac{1}{N} \left(x_k - x_{k-N} \right)$$

$$y_k = \frac{1}{N} \sum_{n=0}^{N-1} x_{k-n}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} - \frac{1}{N} x_{k-N}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} \frac{N-1}{N}$$

$$y_k = y_{k-1} + \frac{1}{N} \left(x_k - y_{k-1} \right)$$

1.2 PT1-Filter

$$Y_{(s)} = \frac{K}{1 + Ts} X_{(s)}$$

$$Y_{(s)} + T\, s\, Y_{(s)} = K\, X_{(s)} \circ\!\!-\!\!\bullet\; y_{(t)} + T\, \dot{y}_{(t)} = K\, x_{(t)}$$

$$y_k + T\, \frac{y_k - y_{k-1}}{dt} = K\, x_k$$

$$y_k + \frac{dt}{T}\, y_k = y_{k-1} + K\, \frac{dt}{T}\, x_k$$

$$y_k = \frac{T}{T + dt}\, y_{k-1} + K\, \frac{dt}{T + dt}\, x_k$$

$$\frac{T}{T + dt}\, y_{k-1} = y_{k-1} - \frac{dt}{T + dt}\, y_{k-1}$$

$$y_k = y_{k-1} + \frac{dt}{T + dt}\, \left(K\, x_k - y_{k-1} \right)$$

$$y_k = y_{k-1} + \frac{dt}{T + dt}\ (K\, x_k - y_{k-1})$$

$$K = 1$$
$$\frac{dt}{T + dt} = \frac{1}{N}$$

$$y_k = y_{k-1} + \frac{1}{N}\,(x_k - y_{k-1})$$

1.3 Low Pass Filterr

$$H_{(s)} = \frac{2\pi f_c}{2\pi f_c + s} = \frac{1}{1 + \frac{1}{2\pi f_c} s} = \frac{K}{1 + T\, s}$$

$$H_{(z)} = Z\big\{H_{(s)} H_{0(s)}\big\}$$

$$H_{(z)} = Z\left\{H_{(s)} \frac{1}{s(1 - e^{-s\,T_a})}\right\} = \frac{z-1}{z} Z\left\{\frac{H_{(s)}}{s}\right\}$$

$$Z\left\{\frac{1}{s}\right\} = \frac{z}{z-1} Z\left\{\frac{a}{a+s}\right\} = \frac{1 - e^{-a\,T_a}}{z - e^{-a\,T_a}}$$

$$H_{(z)} = \frac{z-1}{z}\,\frac{z}{z-1} K \frac{1 - e^{-\frac{T_a}{T}}}{z - e^{-\frac{T_a}{T}}} = K \frac{1 - e^{-\frac{T_a}{T}}}{z - e^{-\frac{T_a}{T}}} = \frac{K(1 - e^{-\frac{T_a}{T}}) z^{-1}}{(1 - e^{-\frac{T_a}{T}}) z^{-1}}$$

$$H_{(z)} = \frac{b_0 + b_1\, z^{-1}}{a_0 + a_1\, z^{-1}} \ \circ\!\!-\!\!\bullet\ y_k = b_1\, u_{k-1} - a_1\, y_{k-1} \approx b_1\, u_k - a_1\, y_{k-1}$$

$$b_0 = 0, b_1 = K(1 - e^{-\frac{T_a}{T}}), a_0 = 1(\rightarrow normed), a_1 = -e^{-\frac{T_a}{T}}$$