Contents

1 Mean Filter

1.1 Moving Average

$$y_k = \frac{1}{N} \sum_{n=0}^{N-1} x_{k-n}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} - \frac{1}{N} x_{k-N}$$

$$y_k = y_{k-1} + \frac{1}{N} \left(x_k - x_{k-N} \right)$$

$$y_k = \frac{1}{N} \sum_{n=0}^{N-1} x_{k-n}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} - \frac{1}{N} x_{k-N}$$

$$y_k = \frac{1}{N} x_k + y_{k-1} \frac{N-1}{N}$$

$$y_k = y_{k-1} + \frac{1}{N} \left(x_k - y_{k-1} \right)$$

1.2 PT1-Filter

$$Y_{(s)} = \frac{K}{1 + Ts} \, X_{(s)}$$

$$Y_{(s)} + T \, s \, Y_{(s)} = K \, X_{(s)} \quad \circ\!\!-\!\!\bullet \quad y_{(t)} + T \, \dot{y}_{(t)} = K \, x_{(t)}$$

$$y_k + T \, \frac{y_k - y_{k-1}}{dt} = K \, x_k$$

$$y_k + \frac{dt}{T} \, y_k = y_{k-1} + K \, \frac{dt}{T} \, x_k$$

$$y_k = \frac{T}{T + dt} \, y_{k-1} + K \, \frac{dt}{T + dt} \, x_k$$

$$\frac{T}{T + dt} \, y_{k-1} = y_{k-1} - \frac{dt}{T + dt} \, y_{k-1}$$

$$y_k = y_{k-1} + \frac{dt}{T + dt} \, \left(K \, x_k - y_{k-1} \right)$$

$$y_k = y_{k-1} + \frac{dt}{T + dt}\ (K\ x_k - y_{k-1})$$

$$K = 1$$
$$\frac{dt}{T + dt} = \frac{1}{N}$$

$$y_k = y_{k-1} + \frac{1}{N}\,(x_k - y_{k-1})$$

1.3 Low Pass Filter

$$H_{(s)} = \frac{2\pi f_c}{2\pi f_c + s} = \frac{1}{1 + \frac{1}{2\pi f_c}s} = \frac{K}{1 + T\,s}$$

$$H_{(z)} = Z\big\{H_{(s)}H_{0(s)}\big\}$$

$$H_{(z)} = Z\left\{H_{(s)}\frac{1}{s(1 - e^{-s\,T_a})}\right\} = \frac{z-1}{z}Z\left\{\frac{H_{(s)}}{s}\right\}$$

$$Z\left\{\frac{1}{s}\right\} = \frac{z}{z-1}Z\left\{\frac{a}{a+s}\right\} = \frac{1 - e^{-a\,T_a}}{z - e^{-a\,T_a}}$$

$$H_{(z)} = \frac{z-1}{z}\frac{z}{z-1}K\frac{1 - e^{-\frac{T_a}{T}}}{z - e^{-\frac{T_a}{T}}} = K\frac{1 - e^{-\frac{T_a}{T}}}{z - e^{-\frac{T_a}{T}}} = \frac{K(1 - e^{-\frac{T_a}{T}})z^{-1}}{(1 - e^{-\frac{T_a}{T}})z^{-1}}$$

$$H_{(z)} = \frac{b_0 + b_1\,z^{-1}}{a_0 + a_1\,z^{-1}} \circ\!\!-\!\!\bullet\ y_k = b_1\,u_{k-1} - a_1\,y_{k-1} \approx b_1\,u_k - a_1\,y_{k-1}$$

$$b_0 = 0, b_1 = K(1 - e^{-\frac{T_a}{T}}), a_0 = 1(\rightarrow normed), a_1 = -e^{-\frac{T_a}{T}}$$

1.4 Morphological Filter

$$Erosion \rightarrow (X \ominus B_1)[n] = \min_{m=0,..M-1} \left\{ X[n+m-M] - B_1[m] \right\}$$

$$Dilation \rightarrow (X \oplus B_2)[n] = \max_{m=0,..M-1} \left\{ X[n+m-M] - B_2[m] \right\}$$

$$X = \frac{1}{2}\left[(X \ominus B_1) + (X \oplus B_2) \right]$$