Calculus

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That famous Oxymoron

'Instantaneous rate of change'
In calculus, we do not measure an instantaneous rate of change; we measure what a rate approaches, with dx approaching 0. Seeing what this rate approaches shows what rate is tangent to the curve.

1 Derivative

1. Here is the general equation for the derivative of a point on a function. It shows the rise over run using a value of dx which approaches 0

$$\frac{df(x)}{dx} = \frac{f(x+dt) - f(x)}{dx}$$

To work through an example, let's take the derivative of $y = x^3$ while $x = 4$:

$$\lim_{dx \to 0} \frac{df(x)}{dx} = \lim_{dx \to 0} \frac{(x+dx)^2 - x^2}{dx}$$
$$= \lim_{dx \to 0} \frac{x^2 + dx^3 - x^2}{dx}$$
$$= \lim_{dx \to 0} 3x^2 + 3xdx + dx^2$$

as dx approaches 0, all parts with dx as a coefficient will approach 0:

$$= 3x^2$$

2. Let's find a general formula for derivatives of polynomials:

$$f(x) = x^n$$

$$(x+dx)^n = \overbrace{(x+dx)(x+dx)(x+dx)\ldots(x+dx)}^{n \text{ times}}$$
$$= x^n + n\color{green}{x^{n-1}}\color{red}{dx}\color{black}\cdots + \text{Multiples of } dx^2$$

$$\frac{dy}{dx} = \frac{(x+dx)^n - x^n}{dx}$$