Utility Function Transformations and Money Illusion: A Further Comment

By Peter Howitt and Don Patinkin*

Our preceding comment in this issue makes two points:

1) A specific one, that the Dusansky-Kalman (D-K) papers (1974, 1976) specify utility functions which are actually monotonic transformations of standard ones (i.e., functions which are homogeneous of degree zero (*HD0*) and thus provide only a spurious generalization of the class of utility functions that generate demand functions which are free of money illusion.

2) A general one, consisting of the demonstration of the basic proposition that a necessary and sufficient condition for demand functions to be free of money illusion is that they be derivable from a standard utility function.

Ostensibly, Dusansky has provided a "reply" to our comment; actually he has not dealt with either of the preceding points. Thus, instead of defending the original D-K conditions on utility functions, he implicitly accepts our criticism of them by replacing them with yet another set of conditions. He then attempts to create the impression that this new set of conditions somehow refutes our foregoing basic proposition; actually, however, the proposition which Dusansky attributes to us and devotes his reply to refuting with the aid of some elaborate mathematics is one that we have explicitly pointed out to the reader that we have *not* made.

Specifically, Dusansky attributes to us the view that nonhomogeneous utility functions which yield illusion-free demand functions "do not differ *ordinally* from the standard utility function" (p.823). What our foregoing basic proposition actually claims and proves, however (and Dusansky has not challenged the validity of this proof), is that they do not differ *operationally* from the standard utility function, that is, they yield the same demand functions; and we explicitly emphasize in footnote 6 of our comment that our proof implies that "the class of utility functions that are operationally equivalent to *H0)* functions is broader than the class of monotonic [i.e., order-preserving] transformations" (p.821). Because of this failure on Dusansky's part to distinguish between "operationally" and "ordinally," he also fails to realize that his new conditions, though free of the specific defect that we noted in the original ones, nevertheless share with them the general defect that they provide no more than a spurious generalization of the standard class of utility functions.[1]

The basic point here (and the one essentially made in our comment, fn. 6) is that once prices enter utility functions, it is no longer true, as it is in standard consumer theory, that for two utility functions to yield the same demand function they must be positive monotonic transformations of each other. The intuitive explanation of this fact is that the entry of prices into the utility function creates an additional interdependence between it and the budget restraint that precludes certain combinations of (x, m, p) from ever being chosen by the household as optimal ones. Hence two different utility functions—$u(x, m, p)$ and $f(x, m, p)$—need not provide the same ordering of these "unchooseable" combinations in order to yield the same demand functions.

As a simple illustration of this point, consider the case of a single consumable good, where $n=1$, so that budget restraint (6) of

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[1] In fn. 2 of his reply, Dusansky effectively says that even nonoperational differences between utility functions constitute an issue for him. It is difficult to understand why.

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our note reduces to

$$x+\frac{m}{p}=\frac{W}{p}$$

It follows that every possible budget line in $(x, m/p)$ space has a slope of -1 (see Figure 1). Consider now two different utility functions which generate the same illusion-free demand functions, represented by the expansion path $0E$ in the figure (the absence of money illusion manifests itself in the uniqueness of the expansion path in $(x, m/p)$ space; i.e., in its dependence on the ratio of m to p, and not on their individual values). Let the first utility function be a nonstandard (i.e., nonhomogeneous) one, say $h(x, m, p)$, whose money illusion manifests itself in the fact that its indifference curves in the figure cannot be drawn without first specifying the price level p. (Thus all variations in m/p in the figure are now assumed to be variations in m with p held constant.) Suppose in particular that this utility function has the solid indifference curves when the price level is p_0, but has the dashed indifference curves when the price level is p_1. Since $h(x, m, p)$ is assumed to generate the expansion path $0E$, the points on both sets of these curves whose marginal rate of substitution of x for m/p is unity must all lie along $0E$; and the same would be true for the indifference curves of $h(x, m, p)$ drawn for any value of p.

Now, our basic proposition states that the illusion-free expansion path $0E$ can also be derived from a standard, illusion-free utility function, say $g(x, m/p)$. Clearly, the points on the indifference curves of this function whose marginal rate of substitution of x for m/p is unity must also all lie along $0E$. For simplicity, let these indifference curves also be represented by the solid ones in the figure. From the fact that these curves intersect with the dashed ones (at which points, of course, the marginal rates of substitution must differ) it is clear that the utility functions $g(x, m/p)$ and $h(x, m, p)$ do not represent the same system of ordinal preferences: that is, they are not positive monotonic transformations of one another. It is, however, also clear that this difference between the utility functions is not observable,

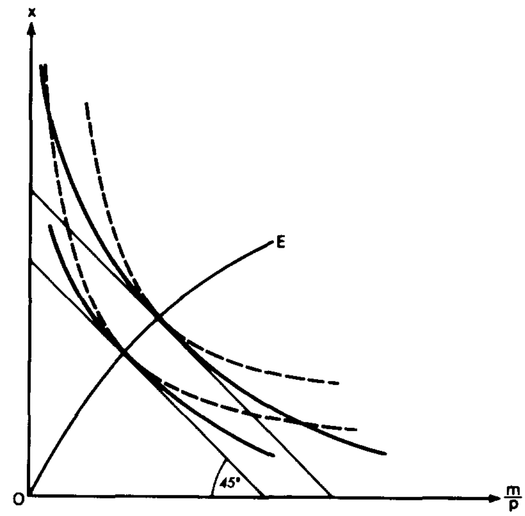

FIGURE 1

and hence not operational, for their marginal rates of substitution can differ only at points that the consumer would in any event never choose.[2]

We note finally that this figure enables us to provide an even more meaningful statement of the necessary and sufficient condition for generating illusion-free demand functions. We begin by observing that the assumption that the nonhomogeneous utility function $h(x, m, p)$ generates the illusion-free expansion path OE implies that the marginal rate of substitution of x for m/p specified by this utility function for any point on this path depends only on the ratio of m to p, and not on their individual values. This means that even if the utility function is not $HD0$ in m and p at these points, the function representing the marginal rate of substitution must be; conversely, the marginal rate of substitution function can be nonhomogeneous only at points that are off the expansion path, and hence nonobservable. Thus a necessary and sufficient condition for demand functions to be $HD0$

[2]It has not escaped our notice that this figure suggests an obvious proof that, in contrast to standard consumer theory, once prices enter utility functions there are generally no integrability conditions allowing one to infer a unique ordinal preference mapping from observed demand behavior. See also Robert Pollak, p. 70.

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in m and p is that the marginal rate of substitution function be *HD0* in these variables at all observable points.[3] This proposition is almost self-evident; we have stated it here only because of the necessity to set the record straight.[4]

[3]In the case of $n>1$, there are the usual technical problems of defining an appropriate price index p by which to deflate nominal money balances. In order to avoid such problems, this condition should then be stated in the following equivalent manner: that the *MRS* functions $(\partial h/\partial x_i)/(\partial h/\partial m)$ $(i=1,\ldots,n)$ be homogeneous of degree one in m and p at all observable points. This can readily be demonstrated from utility-maximization conditions (6)–(7) of our original comment, with $u(\ )$ replaced by $h(\ )$.

[4]In particular, Dusansky is in error when he states that the necessary and sufficient condition for commodity demand functions to be *HD0* "imposes no homogeneity requirement of any kind on the utility function, on the marginal utility functions, or on the marginal rate of substitution functions. The last is particularly important" (p. 824). Dusansky's error stems from his failure to examine the properties of the marginal rate of substitution functions at the observable points—which are, of course, the only operationally meaningful ones.

REFERENCES

R. Dusansky, "Utility Function Transformations and Money Illusion: Reply," *Amer. Econ. Rev.*, Sept. 1980, *70*, 823–25.

______ **and P. J. Kalman,** "The Foundation of Money Illusion in a Neoclassical Micro-Monetary Model," *Amer. Econ. Rev.*, Mar. 1974, *64*, 115–22.

______ **and** ______, "The Foundations of Money Illusion in a Neoclassical Micro-Monetary Model: Reply," *Amer. Econ. Rev.*, Mar. 1976, *66*, 192–95.

P. Howitt and D. Patinkin, "Utility Function Transformations and Money Illusion: Comment," *Amer. Econ. Rev.*, Sept. 1980, *70*, 819–22.

R. A. Pollak, "Price Dependent Preferences," *Amer. Econ. Rev.*, Mar. 1977, *67*, 64–75.

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