

```

In[®]:= partitionsleft =
    {{8, 3, 3, 3, 3}, {7, 4, 3, 3, 3}, {6, 5, 3, 3, 3}, {6, 4, 4, 3, 3}, {5, 5, 4, 3, 3},
     {5, 4, 4, 4, 3}, {5, 3, 3, 3, 3}, {4, 4, 4, 4, 4}, {4, 4, 3, 3, 3}};

In[®]:= adjleft =
    {SparseArray[ + Specified elements: 40 Dimensions: {20, 20}], SparseArray[ + Specified elements: 40 Dimensions: {20, 20}],
     SparseArray[ + Specified elements: 40 Dimensions: {20, 20}], SparseArray[ + Specified elements: 40 Dimensions: {20, 20}],
     SparseArray[ + Specified elements: 40 Dimensions: {20, 20}], SparseArray[ + Specified elements: 40 Dimensions: {20, 20}],
     SparseArray[ + Specified elements: 40 Dimensions: {20, 20}], SparseArray[ + Specified elements: 40 Dimensions: {20, 20}]}};

In[®]:= tensubset = Subsets[Range[20], {10}];
Length[tensubset]

Out[®]= 184 756

In[®]:= growone = Array[row21, 184 756];
Length[growone]

Out[®]= 184 756

In[®]:= For[i = 1, i ≤ 184 756, i++,
  growone[[i]] = ConstantArray[0, 20];
  For[j = 1, j ≤ 10, j++,
    growone[[i, tensubset[[i, j]]]] = 1;
  ]
]

In[®]:= tensubset[[30 000]]
growone[[30 000]]

Out[®]= {1, 2, 4, 8, 11, 14, 15, 17, 19, 20}

Out[®]= {1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1}

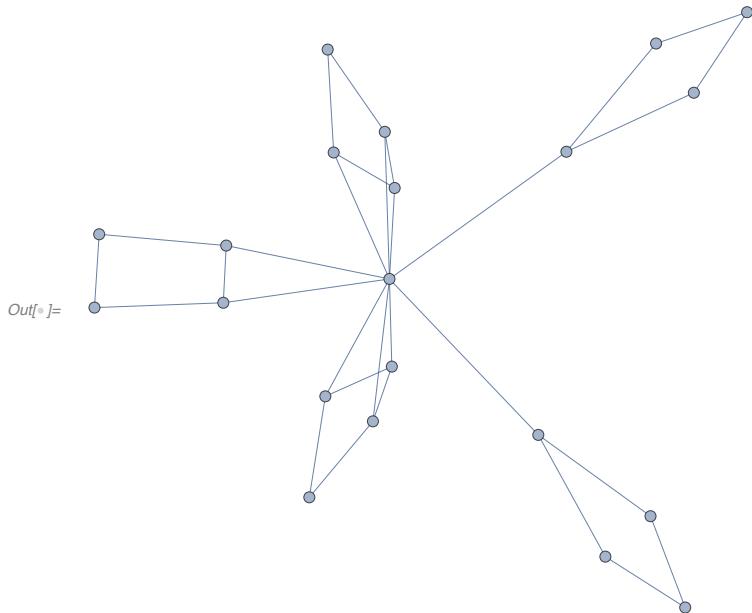
In[®]:= graphgrow21allpartitions = Array[adj21, {9, 184 756}];
graphgrow21allpartitions[[4, 30 000]]

Out[®]= adj21[4, 30 000]

```

```
In[®]:= For[i = 1, i ≤ 9, i++,
  Print[i];
  For[j = 1, j ≤ 184 756, j++,
    growto21 = ConstantArray[0, {21, 21}];
    growto21[[Range[20], Range[20]]] = adjleft[[i]];
    growto21[[21, Range[20]]] = growone[[j]];
    growto21[[Range[20], 21]] = growone[[j]];
    graphgrow21allpartitions[[i, j]] = AdjacencyGraph[growto21];
  ]
]
1
2
3
4
5
6
7
8
9
```

```
In[®]:= graphgrow21allpartitions[[8, 30 000]]
```



```
In[®]:= graphgrow21allpartitionsmodiso = {{}, {}, {}, {}, {}, {}, {}, {}, {}};
```

```
In[®]:= For[i = 1, i ≤ 9, i++,
  graphgrow21allpartitionsmodiso[[i]] =
  DeleteDuplicatesBy[graphgrow21allpartitions[[i]], CanonicalGraph];
  Print[i, " ", Length[graphgrow21allpartitionsmodiso[[i]]]]
] // AbsoluteTiming
```

```

1 122
2 266
3 254
4 352
5 272
6 222
7 46
8 30
9 83
Out[=] {74.6074, Null}

In[=]:= all = Range[184756];
indicespart1modiso = {};
While[Length[all] > 0,
  AppendTo[indicespart1modiso, all[[1]]];
  modiso = {};
  For[i = 1, i ≤ Length[all], i++,
    If[IsomorphicGraphQ[graphgrow21allpartitions[[1, all[[i]]]],
      graphgrow21allpartitions[[1, all[[i]]]], AppendTo[modiso, all[[i]]]
    ]
  ];
  Print[all[[1]], " ", Length[Complement[all, modiso]]];
  all = Complement[all, modiso];
]
1 184744
3 184690
67 184658
68 183794
88 182930
507 182642
516 182210
518 179618
603 178970
1222 178682
1231 178250
1233 175658
1318 175010
2212 174722
2214 173858
2249 171266
2263 168674

```

3224 168 386
3233 167 954
3235 165 362
3320 164 714
4214 164 138
4216 162 410
4251 157 226
4265 152 042
7085 151 994
7086 150 266
7100 149 402
7175 148 538
7177 144 650
8229 144 506
8238 144 290
8240 142 994
8325 142 670
9219 142 094
9221 140 366
9256 135 182
9270 129 998
10 506 129 710
10 508 128 846
10 543 126 254
10 557 123 662
12 090 123 566
12 091 120 110
12 105 118 382
12 180 116 654
12 182 108 878
15 093 108 830
15 094 107 102
15 108 106 238
15 183 105 374
15 185 101 486
18 657 101 198
18 663 100 334
18 665 97 742
18 799 95 150

21 946 94 862
21 948 93 998
21 983 91 406
21 997 88 814
23 530 88 766
23 531 87 038
23 545 86 174
23 620 85 310
23 622 81 422
26 533 81 326
26 534 77 870
26 548 76 142
26 623 74 414
26 625 66 638
28 249 66 590
28 250 64 862
28 264 63 998
28 339 63 134
28 341 59 246
30 097 58 670
30 103 56 942
30 105 51 758
30 239 46 574
32 968 46 550
32 969 45 686
32 983 45 254
33 058 44 822
33 060 42 878
36 532 42 302
36 538 40 574
36 540 35 390
36 674 30 206
43 264 29 918
43 266 29 486
43 280 26 894
43 413 26 246
58 994 26 234
58 995 25 802
59 009 25 586

59 084 25 370
59 086 24 398
60 842 24 110
60 848 23 246
60 850 20 654
60 984 18 062
64 274 17 774
64 280 16 910
64 282 14 318
64 416 11 726
67 574 11 438
67 576 11 006
67 590 8414
67 723 7766
80 444 7478
80 446 7046
80 460 4454
80 593 3806
86 879 3662
86 881 3446
86 895 2150
87 028 1826
92 159 1794
92 160 930
92 182 66
184 691 54
184 694 0

```

In[]:= indicespart1modiso
indicespart1modiso // Length

Out[]= {1, 3, 67, 68, 88, 507, 516, 518, 603, 1222, 1231, 1233, 1318, 2212, 2214, 2249, 2263,
3224, 3233, 3235, 3320, 4214, 4216, 4251, 4265, 7085, 7086, 7100, 7175, 7177,
8229, 8238, 8240, 8325, 9219, 9221, 9256, 9270, 10506, 10508, 10543, 10557,
12090, 12091, 12105, 12180, 12182, 15093, 15094, 15108, 15183, 15185,
18657, 18663, 18665, 18799, 21946, 21948, 21983, 21997, 23530, 23531,
23545, 23620, 23622, 26533, 26534, 26548, 26623, 26625, 28249, 28250,
28264, 28339, 28341, 30097, 30103, 30105, 30239, 32968, 32969, 32983,
33058, 33060, 36532, 36538, 36540, 36674, 43264, 43266, 43280, 43413,
58994, 58995, 59009, 59084, 59086, 60842, 60848, 60850, 60984, 64274,
64280, 64282, 64416, 67574, 67576, 67590, 67723, 80444, 80446, 80460,
80593, 86879, 86881, 86895, 87028, 92159, 92160, 92182, 184691, 184694}

Out[=] 122

In[]:= all = Range[184756];
indicespart2modiso = {};
While[Length[all] > 0,
AppendTo[indicespart2modiso, all[[1]]];
modiso = {};
For[i = 1, i ≤ Length[all], i++,
If[IsomorphicGraphQ[graphgrow21allpartitions[[2, all[[i]]]],
graphgrow21allpartitions[[2, all[[1]]]], AppendTo[modiso, all[[i]]]
]
];
Print[all[[1]], " ", Length[Complement[all, modiso]]];
all = Complement[all, modiso];
]
1 184752
3 184716
13 184698
31 184662
33 184554
203 184551
204 184497
218 184470
287 184463
288 184211
306 183959
308 183203
351 183077
353 182699
423 182615

```

424 181 103
438 180 347
876 180 221
882 180 032
884 179 465
1717 179 402
1726 179 150
1728 178 394
1798 178 310
1799 176 798
1813 176 042
1918 176 000
1919 175 244
1933 174 866
2086 174 362
2092 173 606
2094 171 338
2878 171 212
2880 171 023
2894 170 456
3719 170 393
3728 170 141
3730 169 385
3800 169 301
3801 167 789
3815 167 033
3920 166 991
3921 166 235
3935 165 857
4088 165 353
4094 164 597
4096 162 329
4880 162 203
4882 162 014
4896 161 447
6293 161 384
6295 161 195
6329 161 111
6330 159 599

6344 158 843
6497 158 339
6503 157 583
6505 155 315
6707 155 063
6713 154 685
6715 153 551
6959 153 047
6961 152 291
6975 150 023
7925 150 002
7926 149 624
7948 149 435
8724 149 372
8733 149 120
8735 148 364
8805 148 280
8806 146 768
8820 146 012
8925 145 970
8926 145 214
8940 144 836
9093 144 332
9099 143 576
9101 141 308
9885 141 182
9887 140 993
9901 140 426
11 298 140 300
11 300 139 922
11 334 139 754
11 335 136 730
11 349 135 218
11 502 134 210
11 508 132 698
11 510 128 162
11 712 127 658
11 718 126 902
11 720 124 634

11 964 123 626
11 966 122 114
11 980 117 578
12 930 117 536
12 931 116 780
12 953 116 402
17 733 116 381
17 734 116 003
17 748 115 814
17 817 115 310
17 823 114 554
17 825 112 286
18 069 111 782
18 071 111 026
18 085 108 758
18 321 108 506
18 323 108 128
18 337 106 994
18 573 106 910
18 574 105 398
18 596 104 642
19 413 104 579
19 416 104 390
22 738 104 327
22 740 104 138
22 774 104 054
22 775 102 542
22 789 101 786
22 942 101 282
22 948 100 526
22 950 98 258
23 152 98 006
23 158 97 628
23 160 96 494
23 404 95 990
23 406 95 234
23 420 92 966
24 370 92 945
24 371 92 567

24 393 92 378
25 741 92 315
25 743 92 126
25 777 92 042
25 778 90 530
25 792 89 774
25 945 89 270
25 951 88 514
25 953 86 246
26 155 85 994
26 161 85 616
26 163 84 482
26 407 83 978
26 409 83 222
26 423 80 954
27 373 80 933
27 374 80 555
27 396 80 366
29 173 80 324
29 174 79 568
29 188 79 190
29 257 78 182
29 263 76 670
29 265 72 134
29 509 71 126
29 511 69 614
29 525 65 078
29 761 64 574
29 763 63 818
29 777 61 550
30 013 61 382
30 014 58 358
30 036 56 846
30 853 56 720
30 856 56 342
35 608 56 321
35 609 55 943
35 623 55 754
35 692 55 250

35 698 54 494
35 700 52 226
35 944 51 722
35 946 50 966
35 960 48 698
36 196 48 446
36 198 48 068
36 212 46 934
36 448 46 850
36 449 45 338
36 471 44 582
37 288 44 519
37 291 44 330
42 472 44 204
42 478 44 015
42 480 43 448
42 598 42 944
42 600 42 188
42 614 39 920
42 850 39 836
42 851 38 324
42 873 37 568
43 060 37 526
43 061 36 770
43 083 36 392
43 228 36 140
43 231 35 384
43 750 35 321
59 918 35 300
59 919 34 922
59 933 34 733
60 002 34 229
60 008 33 473
60 010 31 205
60 254 30 701
60 256 29 945
60 270 27 677
60 506 27 425
60 508 27 047

60 522 25 913
60 758 25 829
60 759 24 317
60 781 23 561
61 598 23 498
61 601 23 309
66 782 23 183
66 788 22 994
66 790 22 427
66 908 21 923
66 910 21 167
66 924 18 899
67 160 18 815
67 161 17 303
67 183 16 547
67 370 16 505
67 371 15 749
67 393 15 371
67 538 15 119
67 541 14 363
68 060 14 300
79 652 14 174
79 658 13 985
79 660 13 418
79 778 12 914
79 780 12 158
79 794 9890
80 030 9806
80 031 8294
80 053 7538
80 240 7496
80 241 6740
80 263 6362
80 408 6110
80 411 5354
80 930 5291
91 664 5165
91 666 4976
91 680 4409

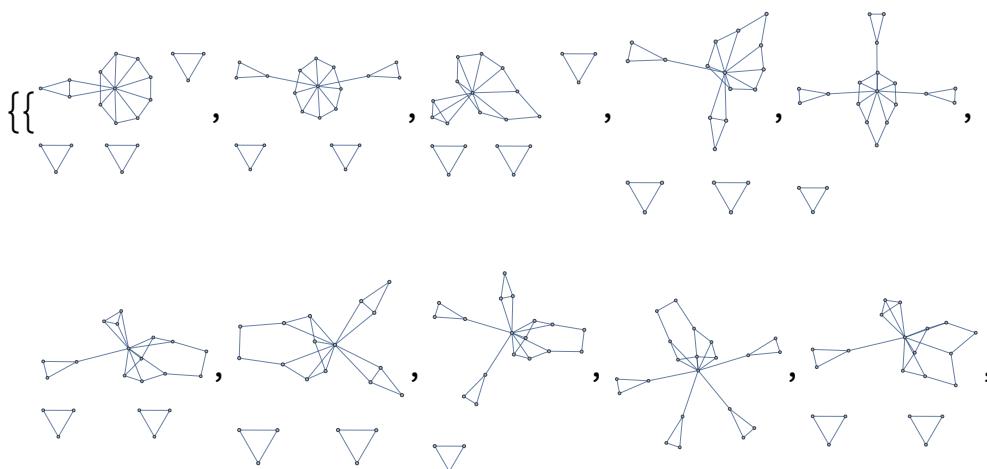
91 790 4325
91 791 2813
91 813 2057
91 958 1805
91 961 1049
92 078 923
92 081 545
92 150 293
92 378 286
184 471 283
184 472 229
184 494 202
184 555 166
184 558 58
184 627 22
184 672 4
184 690 0

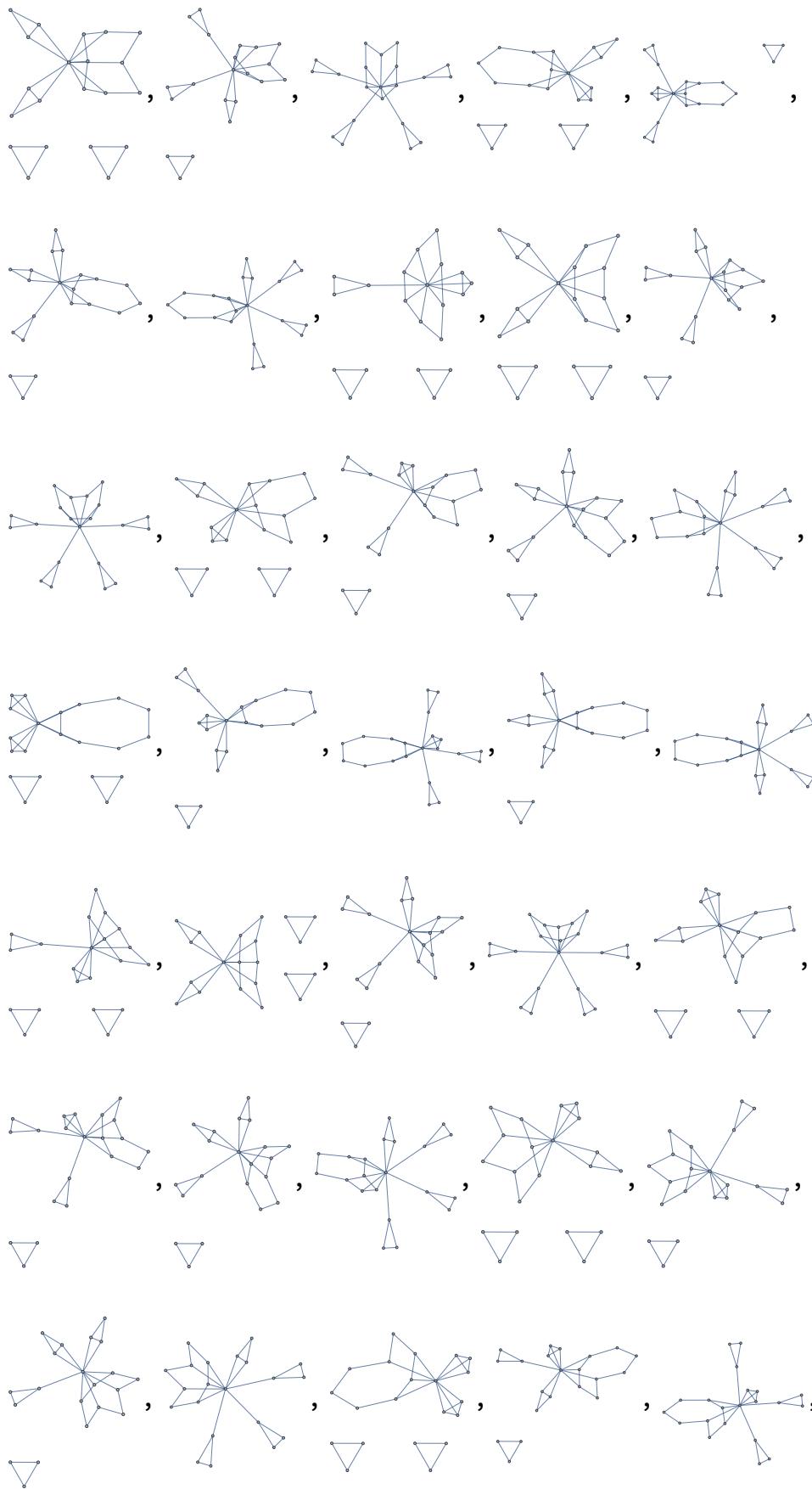
```
In[ $\circ$ ] := indicespart2modiso
indicespart2modiso // Length

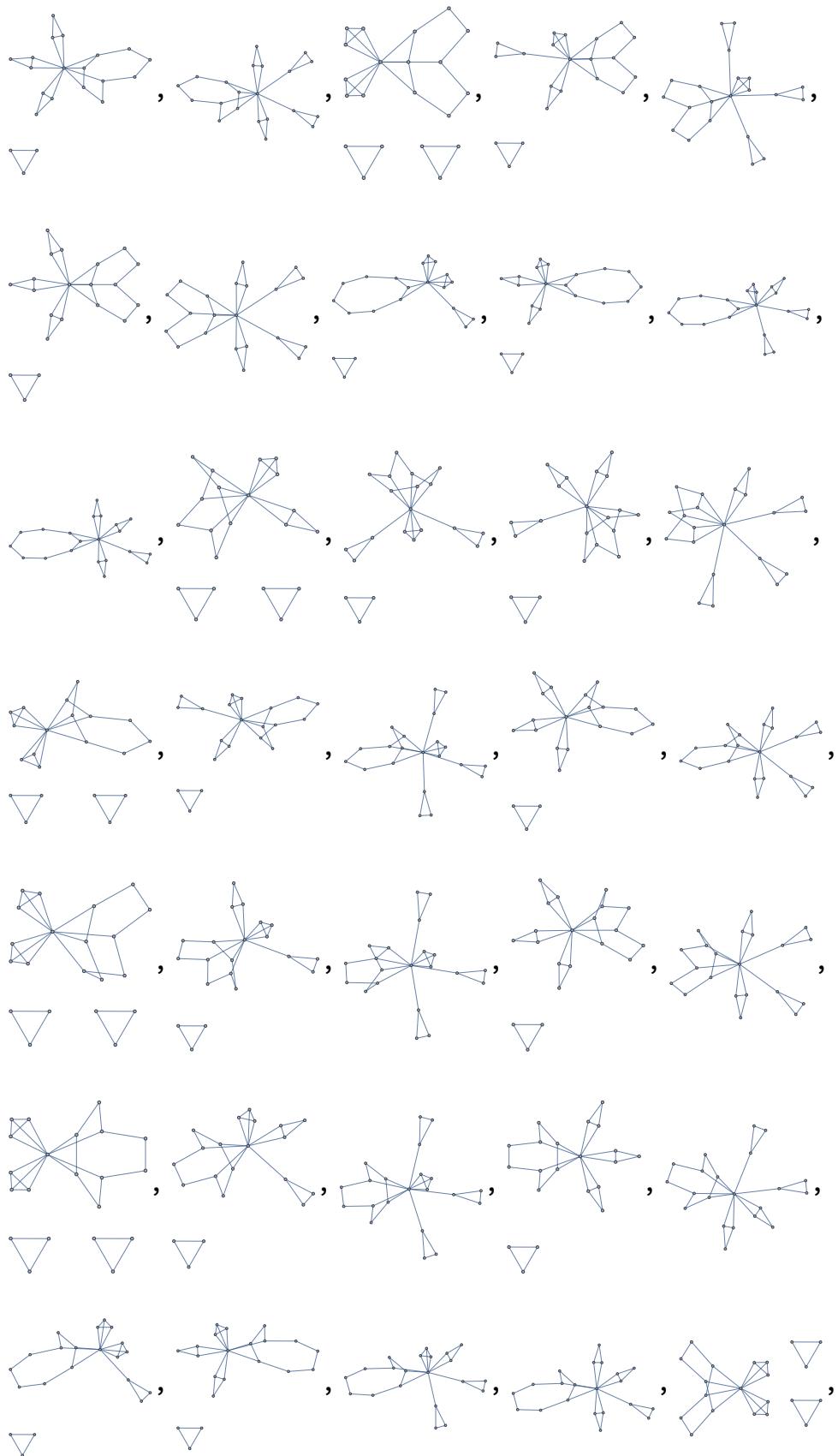
Out[ $\circ$ ] = {1, 3, 13, 31, 33, 203, 204, 218, 287, 288, 306, 308, 351, 353, 423, 424, 438, 876,
882, 884, 1717, 1726, 1728, 1798, 1799, 1813, 1918, 1919, 1933, 2086, 2092,
2094, 2878, 2880, 2894, 3719, 3728, 3730, 3800, 3801, 3815, 3920, 3921, 3935,
4088, 4094, 4096, 4880, 4882, 4896, 6293, 6295, 6329, 6330, 6344, 6497, 6503,
6505, 6707, 6713, 6715, 6959, 6961, 6975, 7925, 7926, 7948, 8724, 8733, 8735,
8805, 8806, 8820, 8925, 8926, 8940, 9093, 9099, 9101, 9885, 9887, 9901, 11298,
11300, 11334, 11335, 11349, 11502, 11508, 11510, 11712, 11718, 11720, 11964,
11966, 11980, 12930, 12931, 12953, 17733, 17734, 17748, 17817, 17823, 17825,
18069, 18071, 18085, 18321, 18323, 18337, 18573, 18574, 18596, 19413, 19416,
22738, 22740, 22774, 22775, 22789, 22942, 22948, 22950, 23152, 23158,
23160, 23404, 23406, 23420, 24370, 24371, 24393, 25741, 25743, 25777,
25778, 25792, 25945, 25951, 25953, 26155, 26161, 26163, 26407, 26409,
26423, 27373, 27374, 27396, 29173, 29174, 29188, 29257, 29263, 29265,
29509, 29511, 29525, 29761, 29763, 29777, 30013, 30014, 30036, 30853,
30856, 35608, 35609, 35623, 35692, 35698, 35700, 35944, 35946, 35960,
36196, 36198, 36212, 36448, 36449, 36471, 37288, 37291, 42472, 42478, 42480,
42598, 42600, 42614, 42850, 42851, 42873, 43060, 43061, 43083, 43228,
43231, 43750, 59918, 59919, 59933, 60002, 60008, 60010, 60254, 60256,
60270, 60506, 60508, 60522, 60758, 60759, 60781, 61598, 61601, 66782,
66788, 66790, 66908, 66910, 66924, 67160, 67161, 67183, 67370, 67371,
67393, 67538, 67541, 68060, 79652, 79658, 79660, 79778, 79780, 79794,
80030, 80031, 80053, 80240, 80241, 80263, 80408, 80411, 80930, 91664,
91666, 91680, 91790, 91791, 91813, 91958, 91961, 92078, 92081, 92150,
92378, 184471, 184472, 184494, 184555, 184558, 184627, 184672, 184690}
```

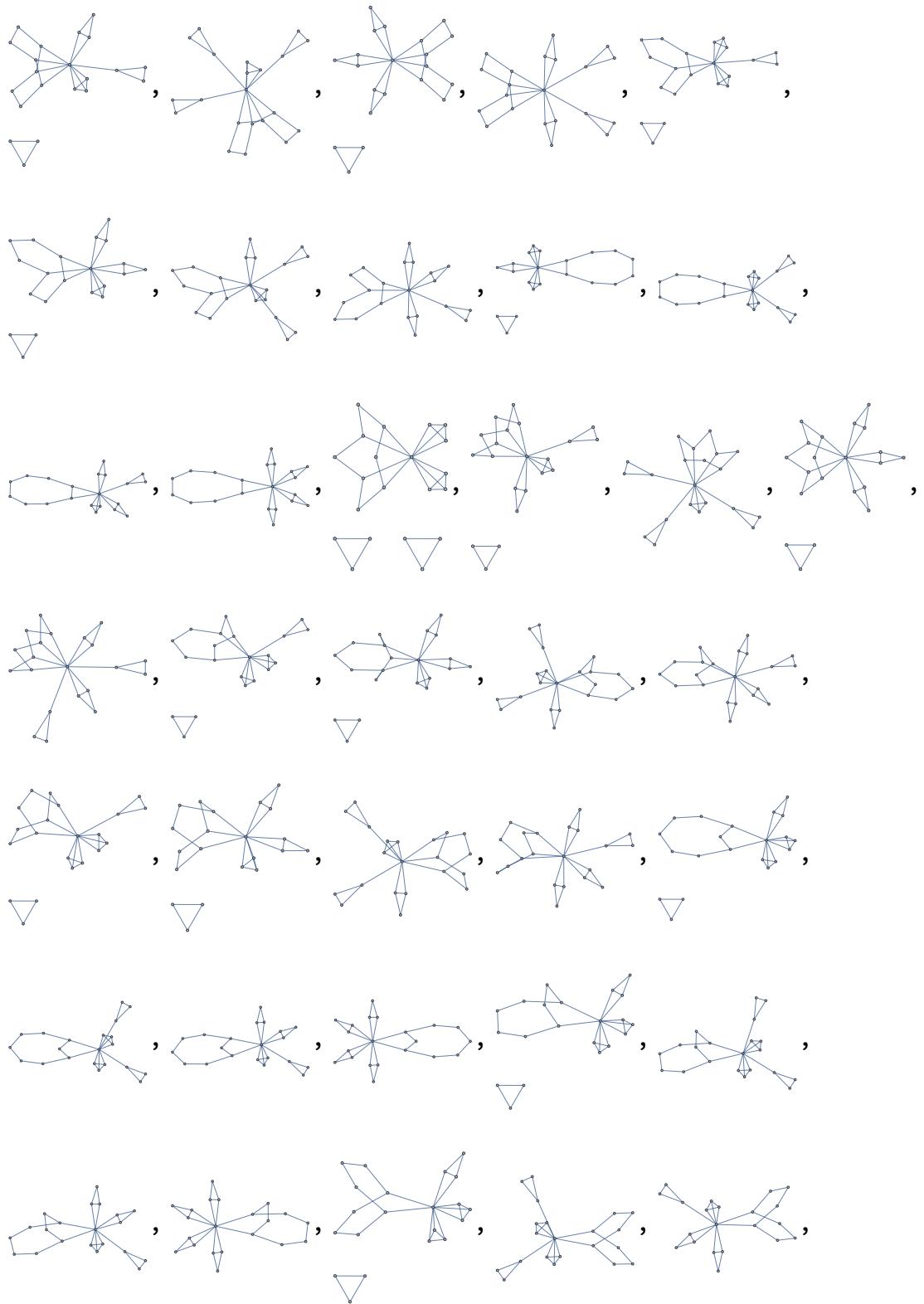
Out[\circ] = 266

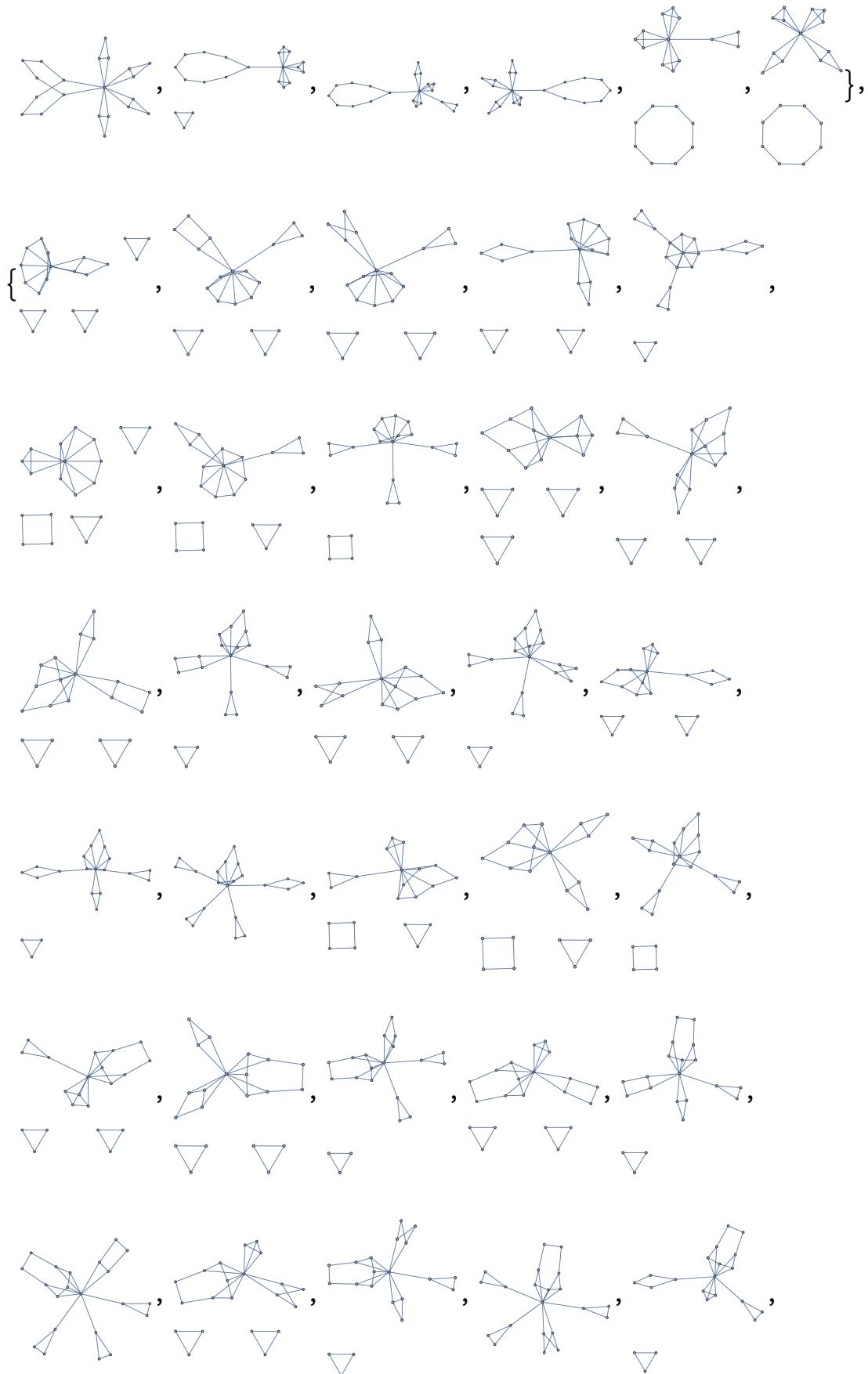
```
In[ $\circ$ ] := graphgrow21allpartitionsmodiso =
```

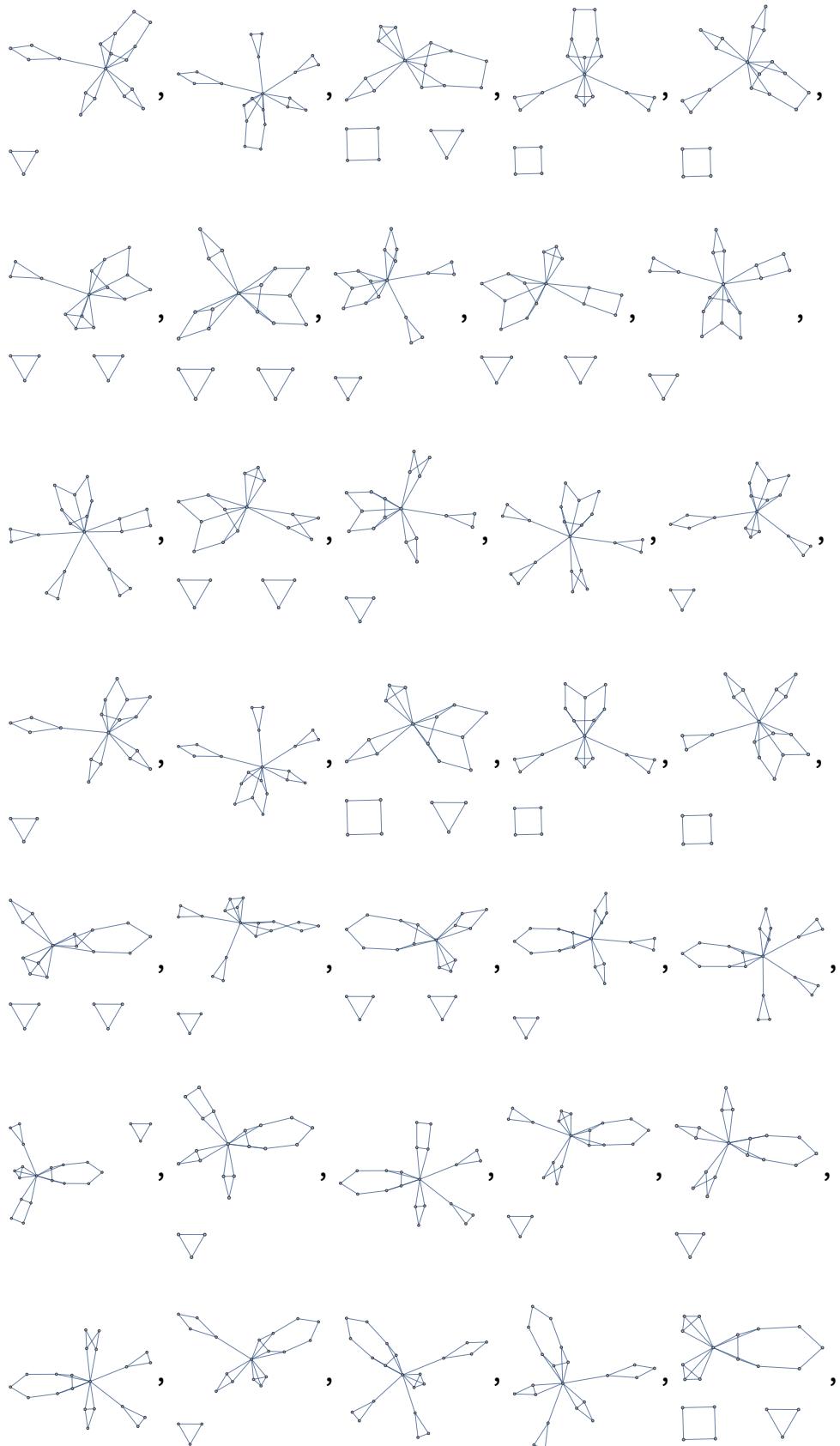


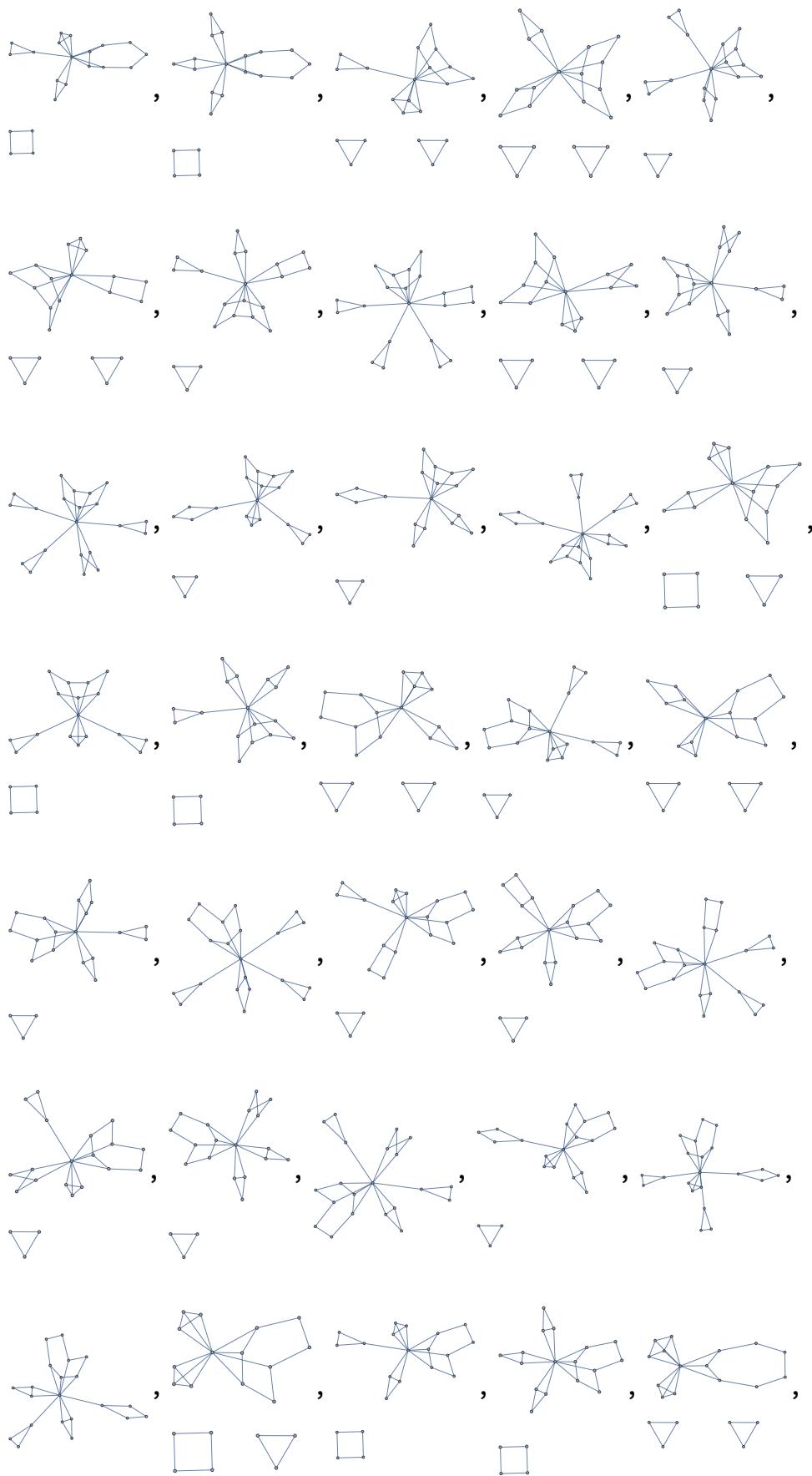


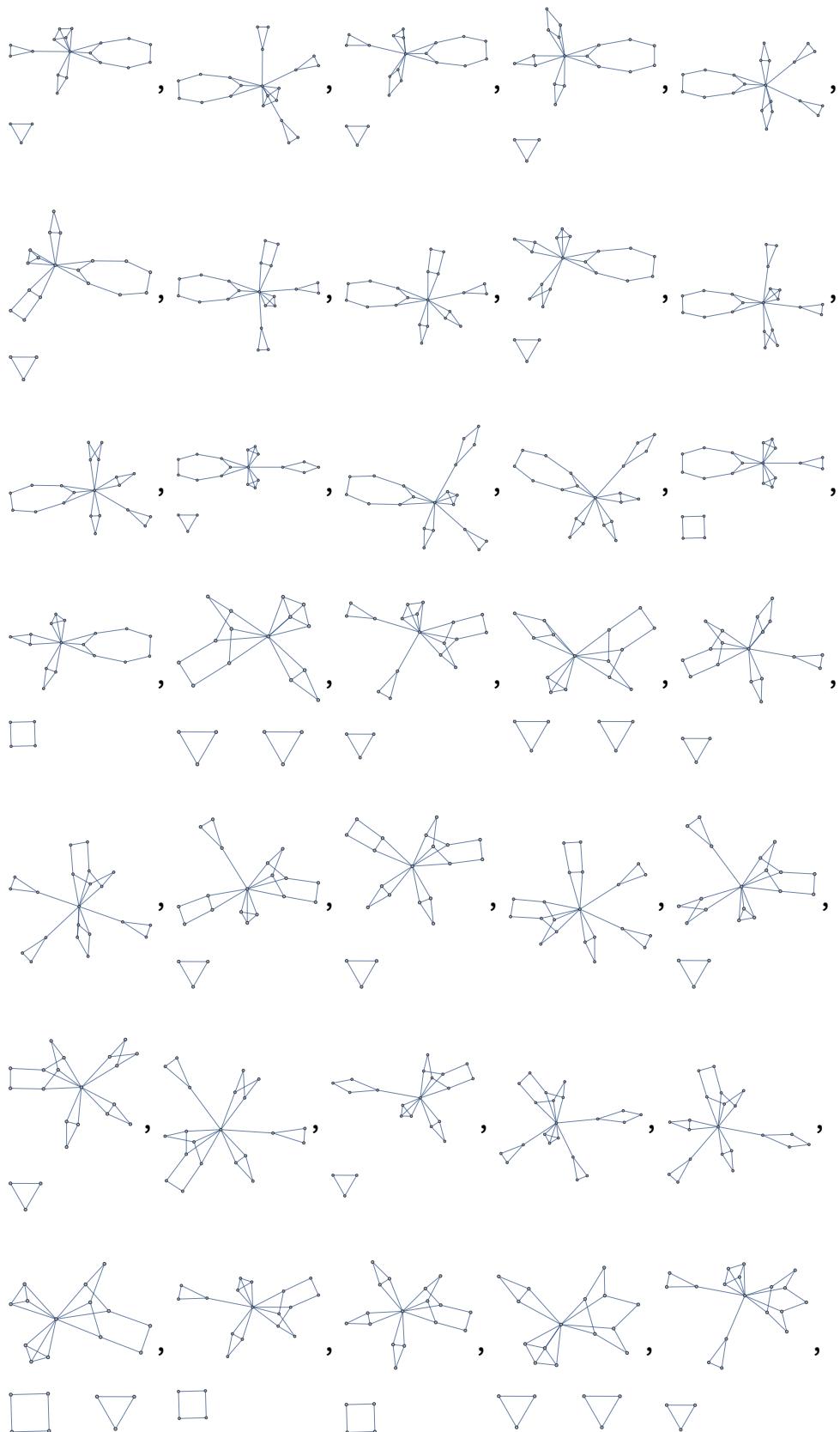


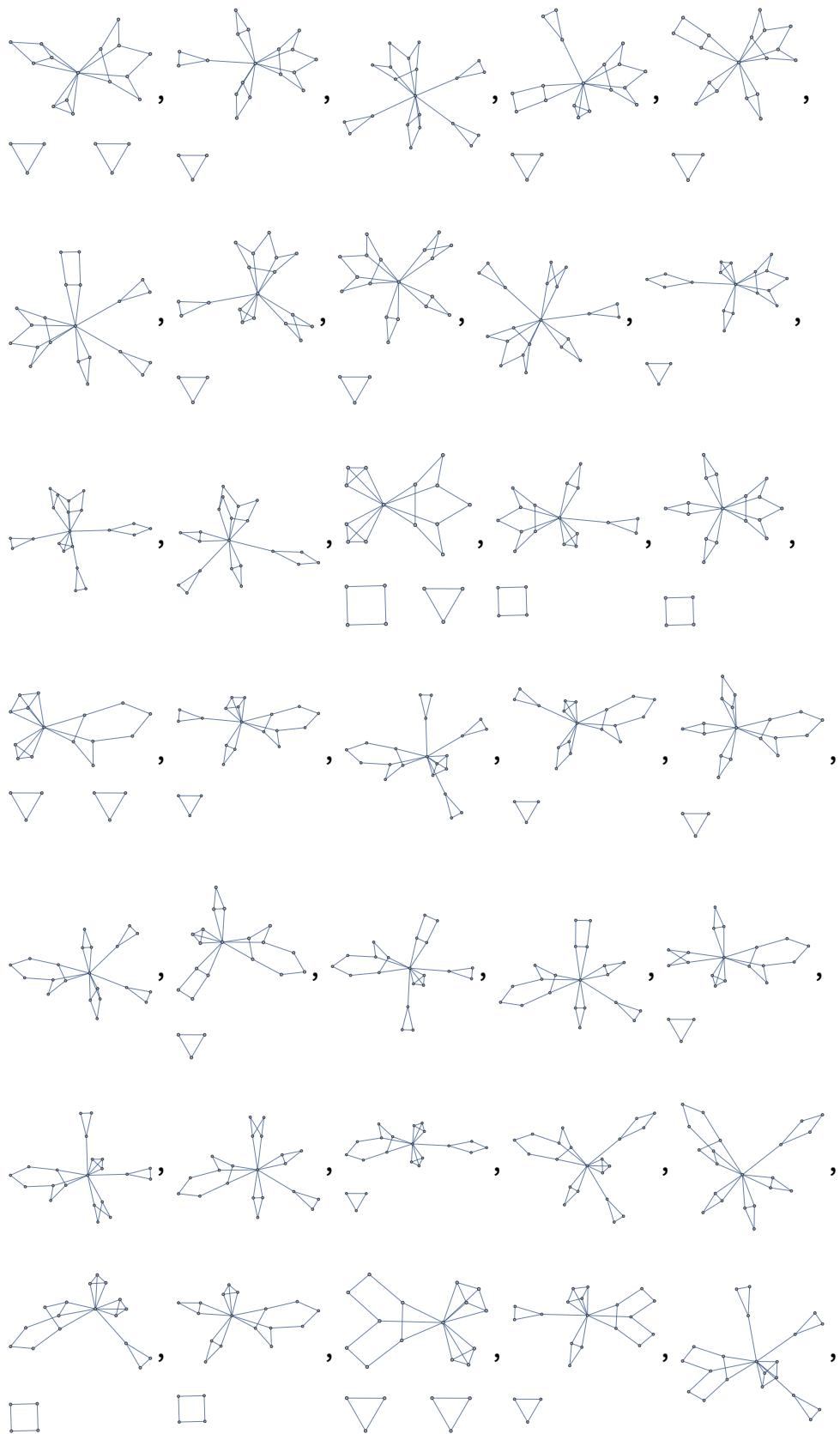


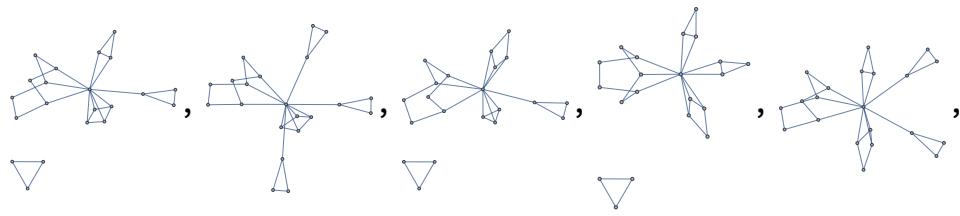
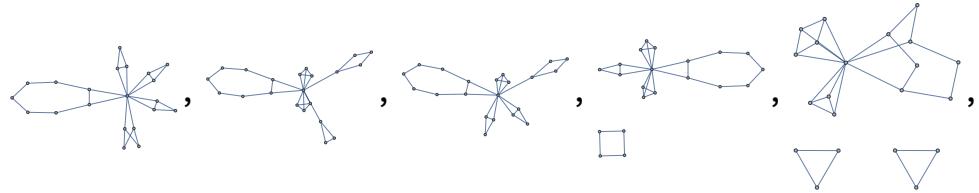
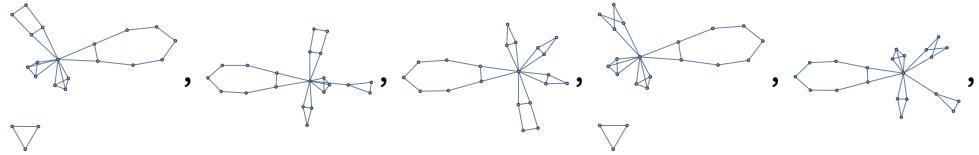
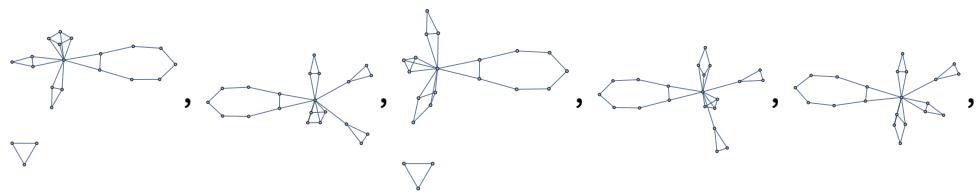
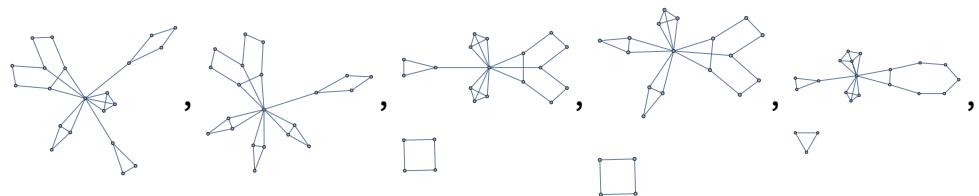
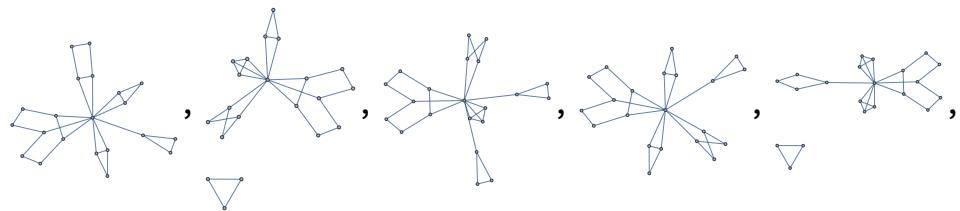
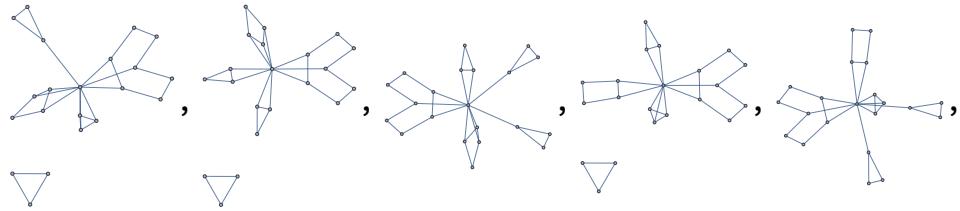


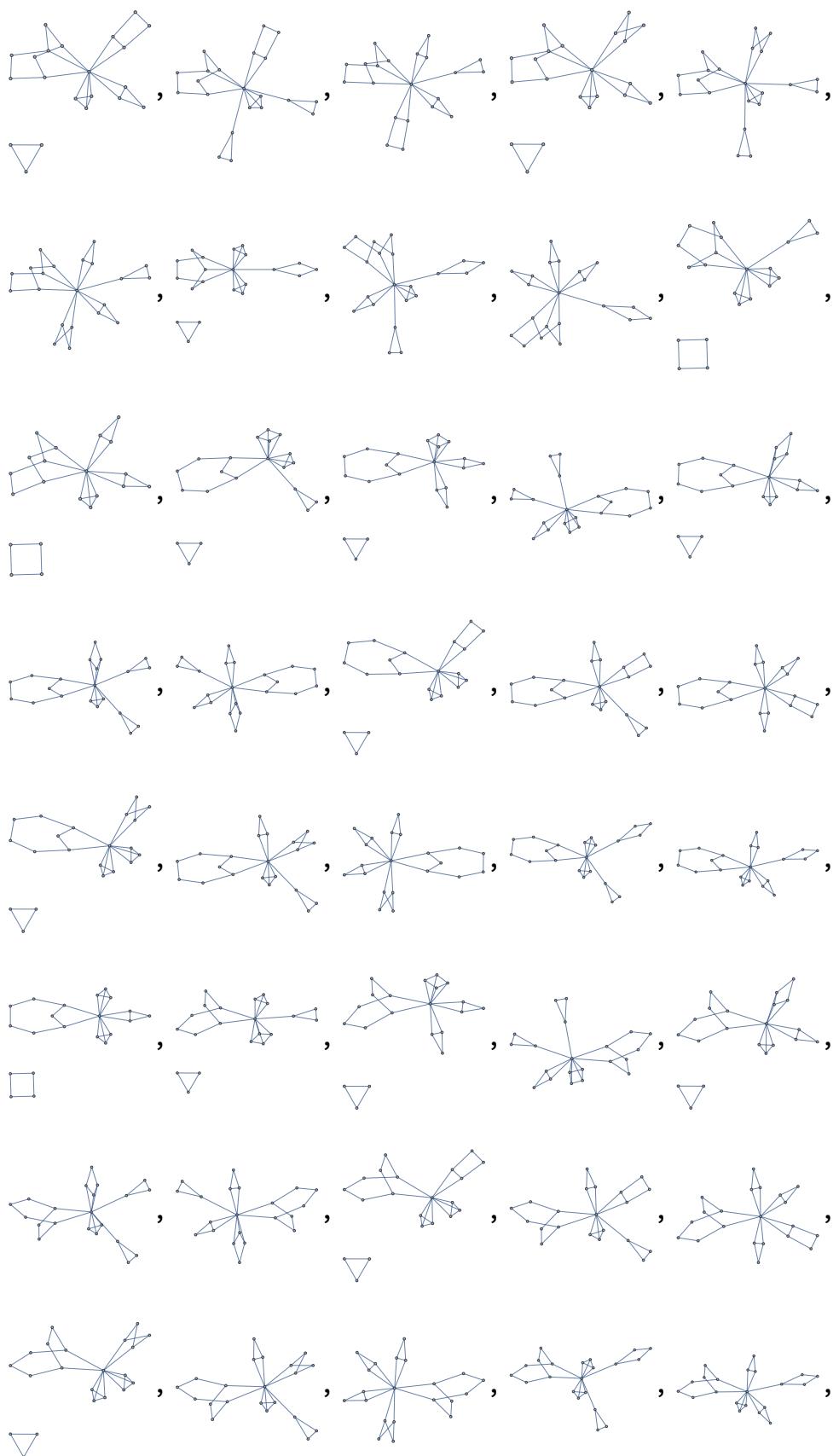


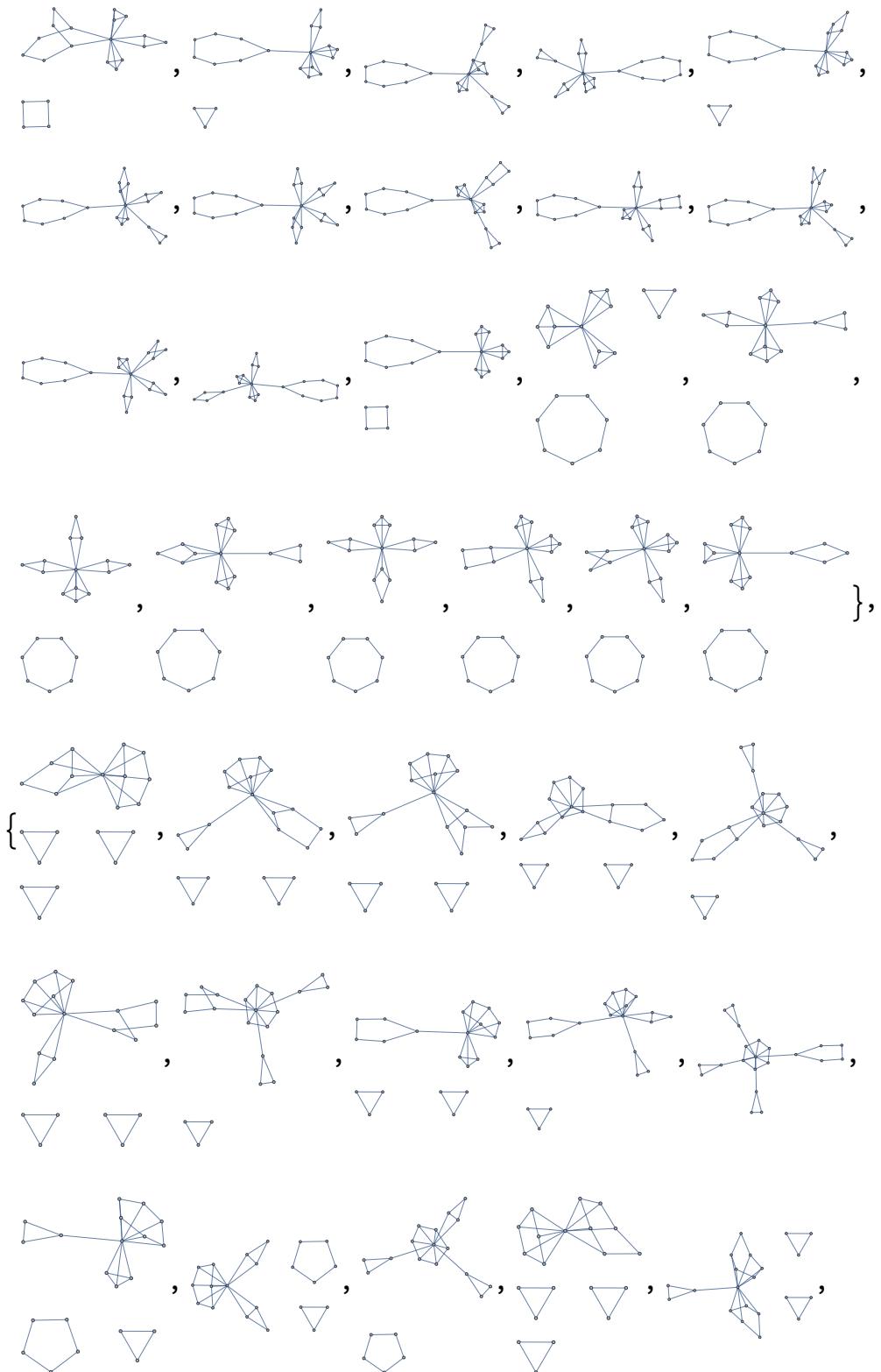


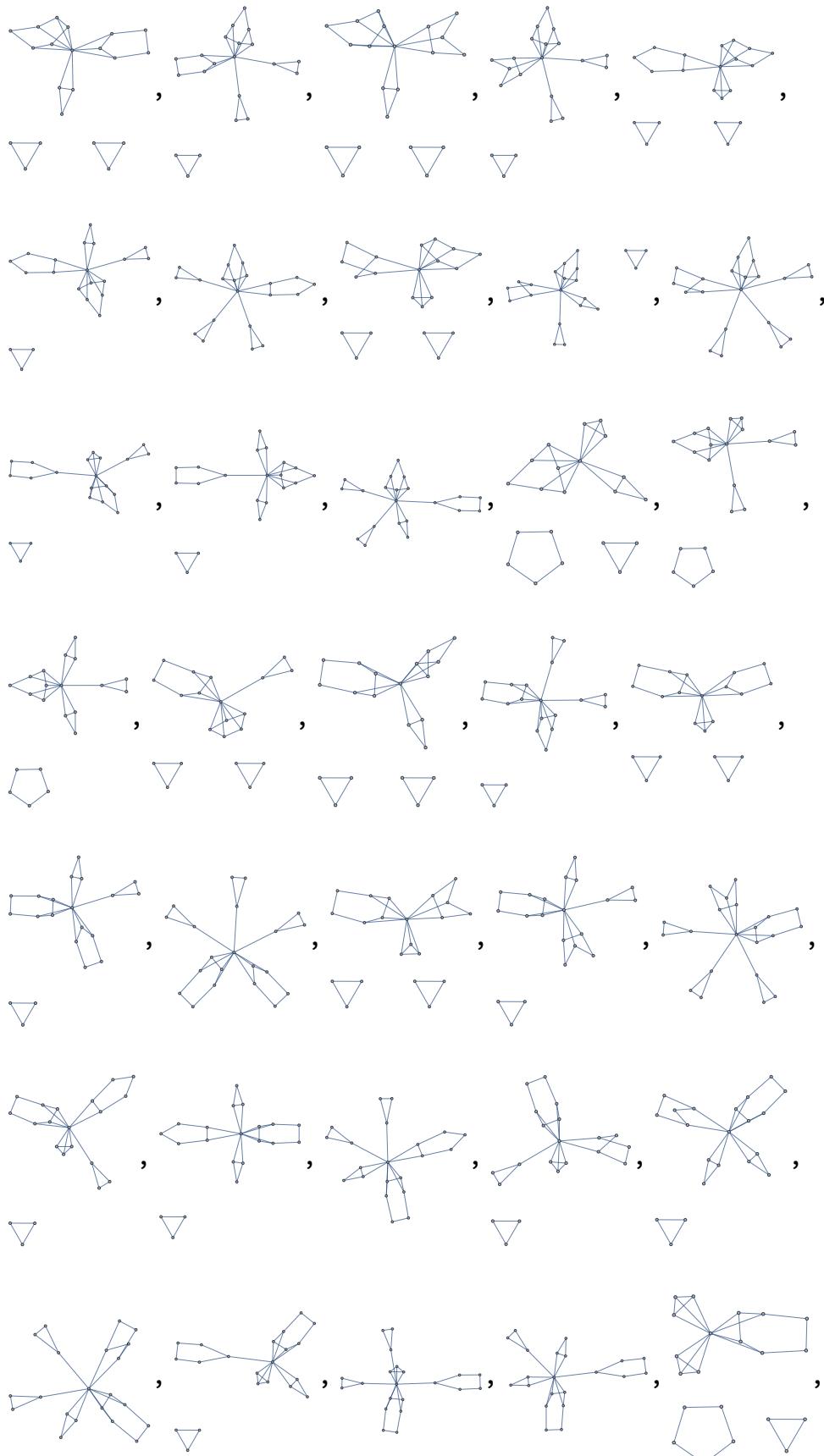


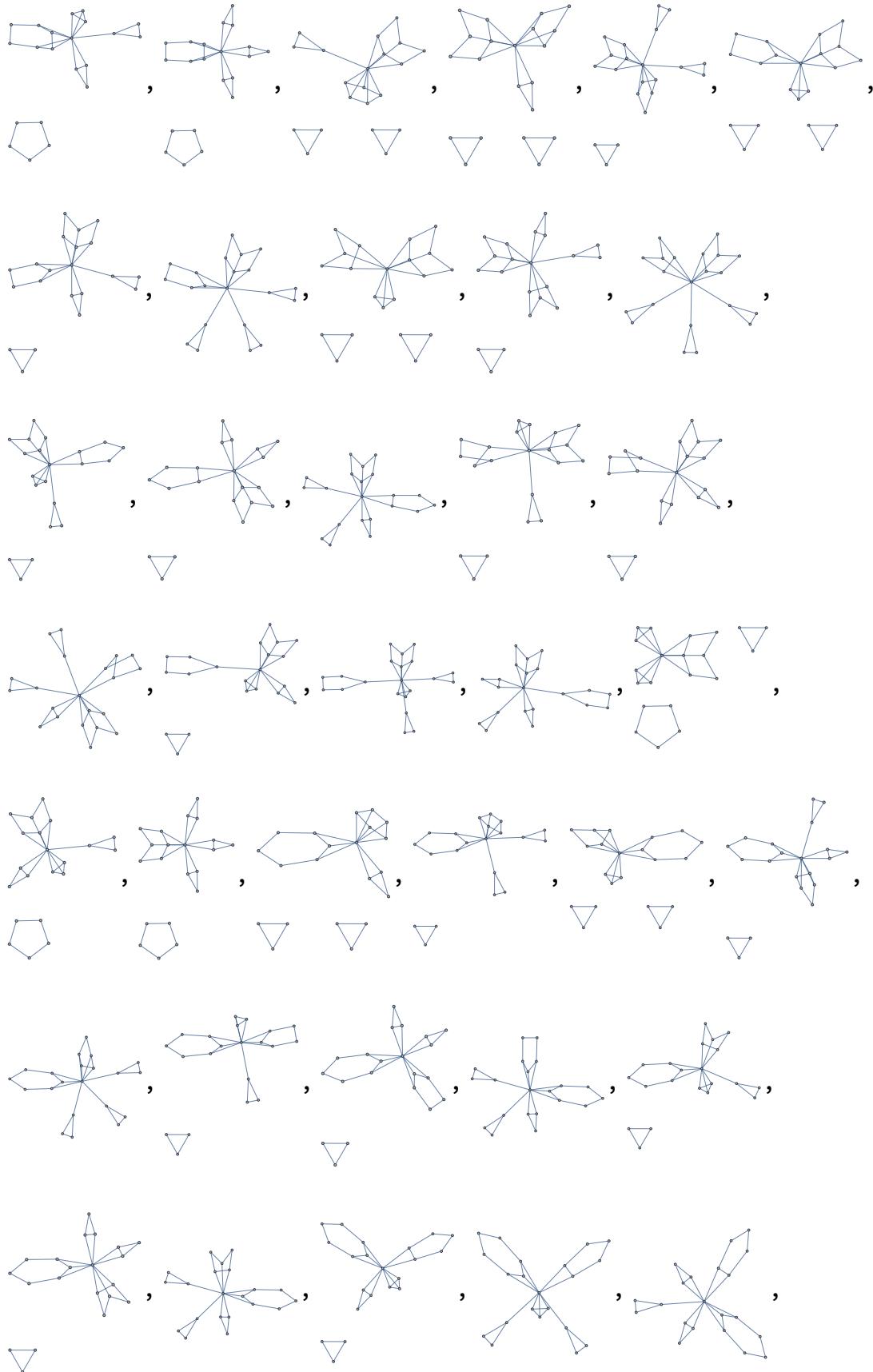


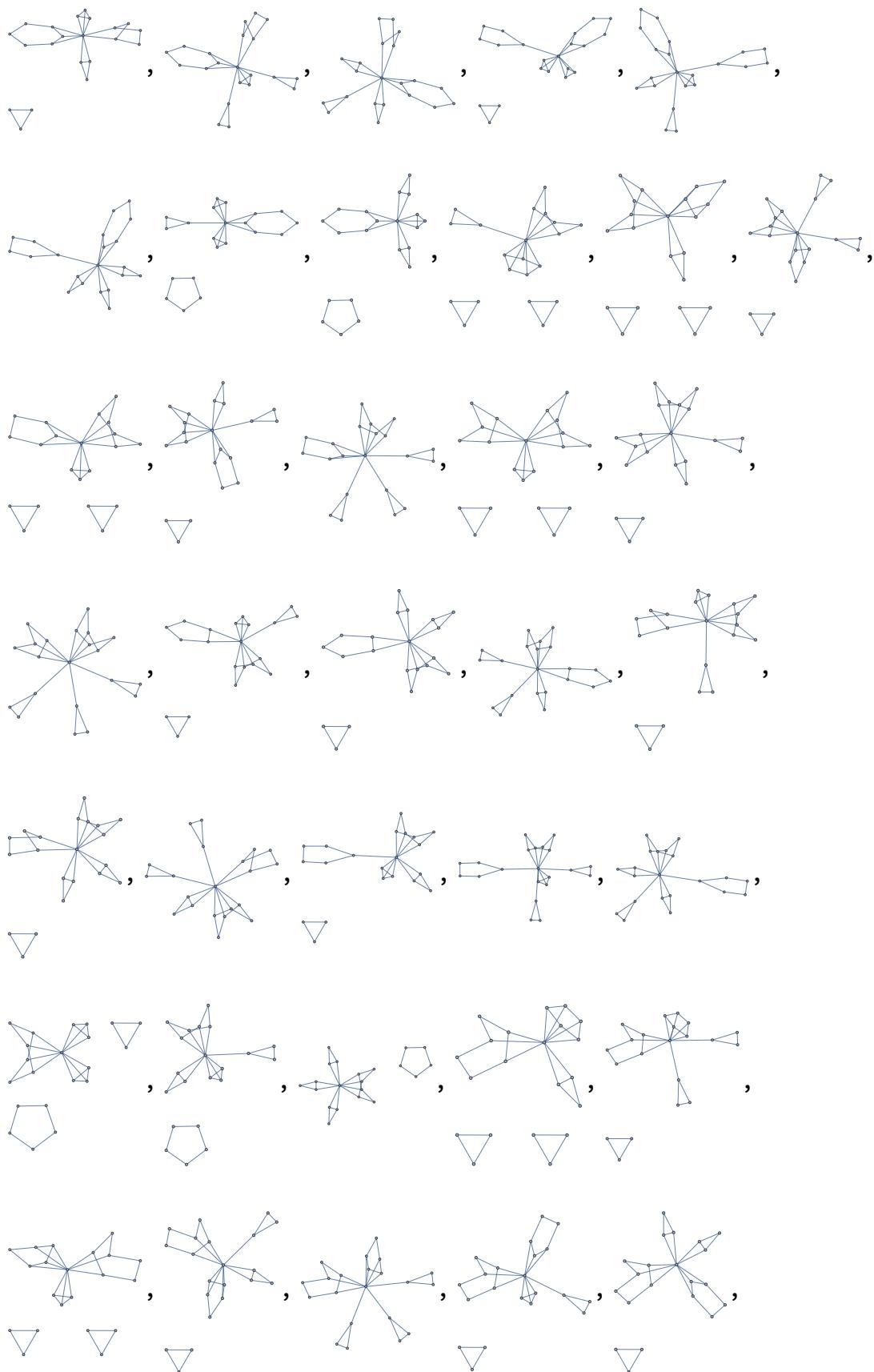


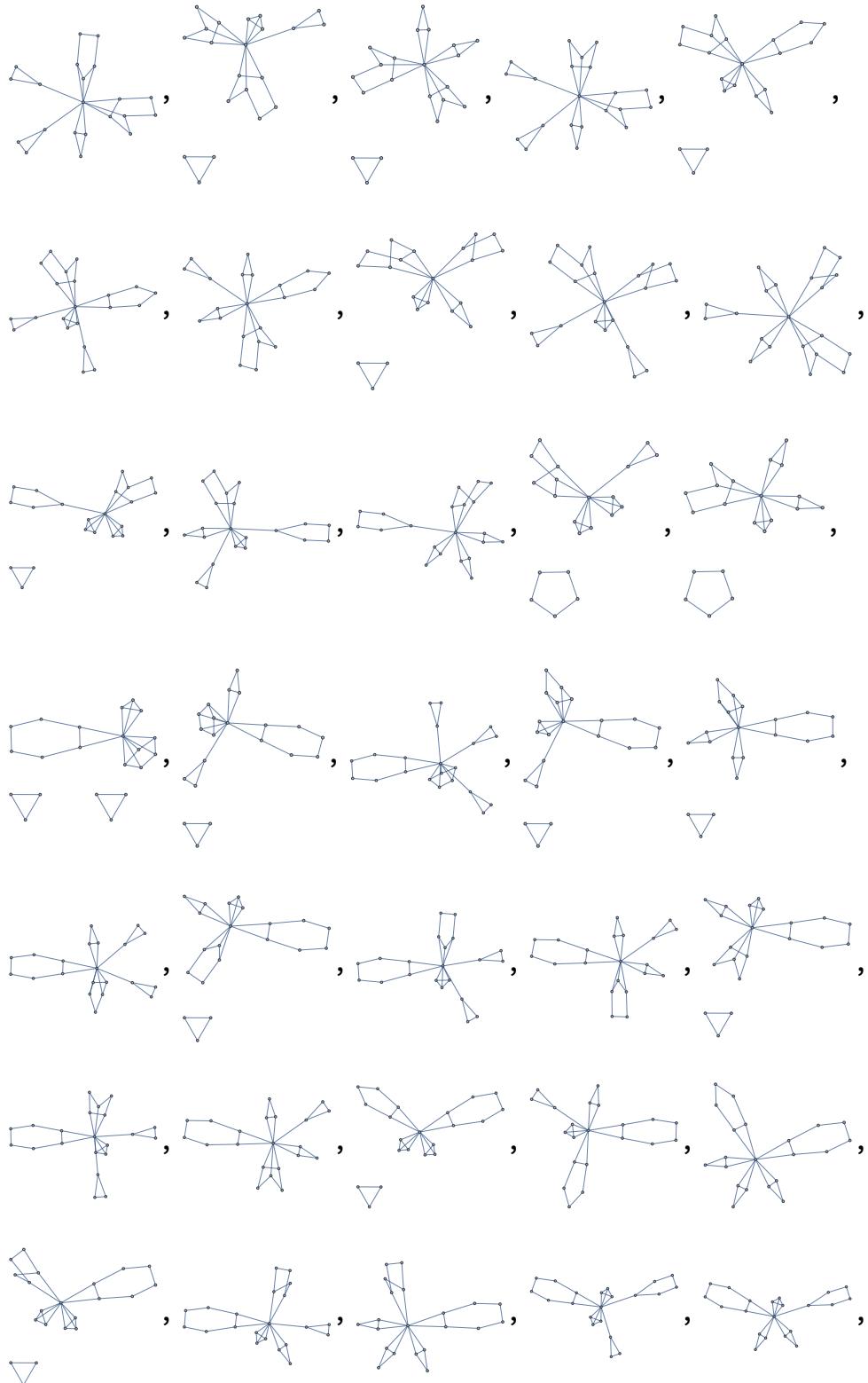


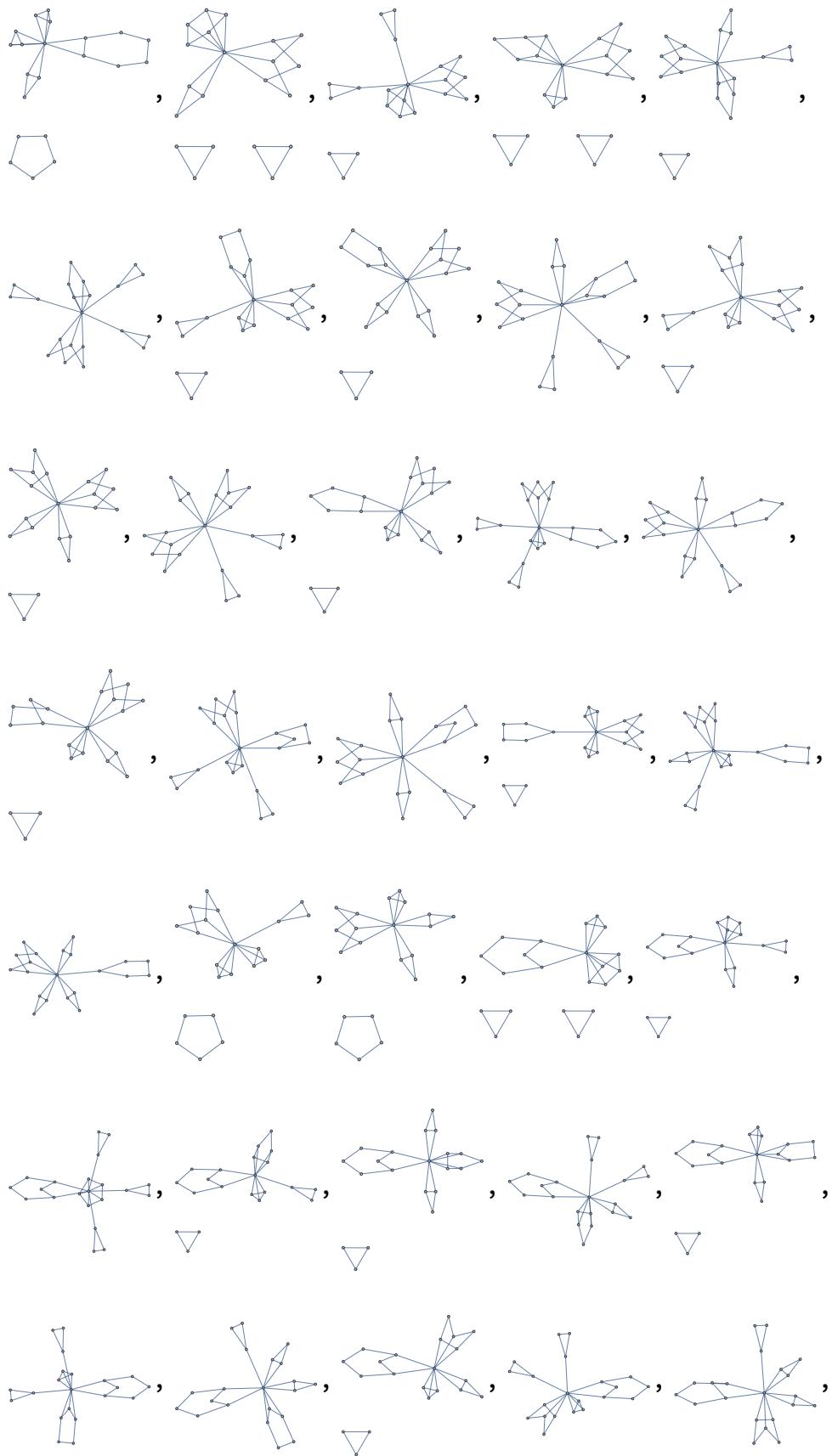


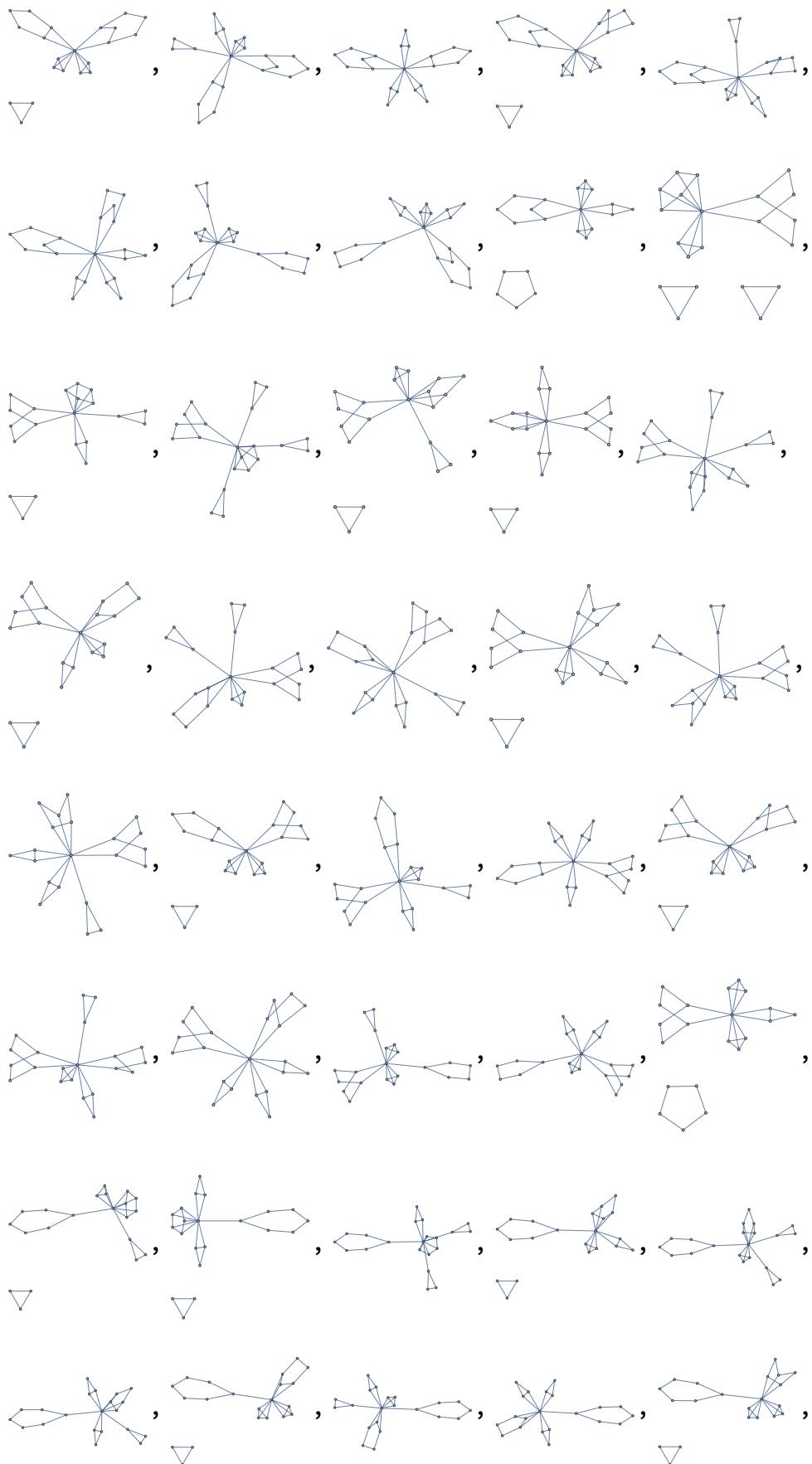


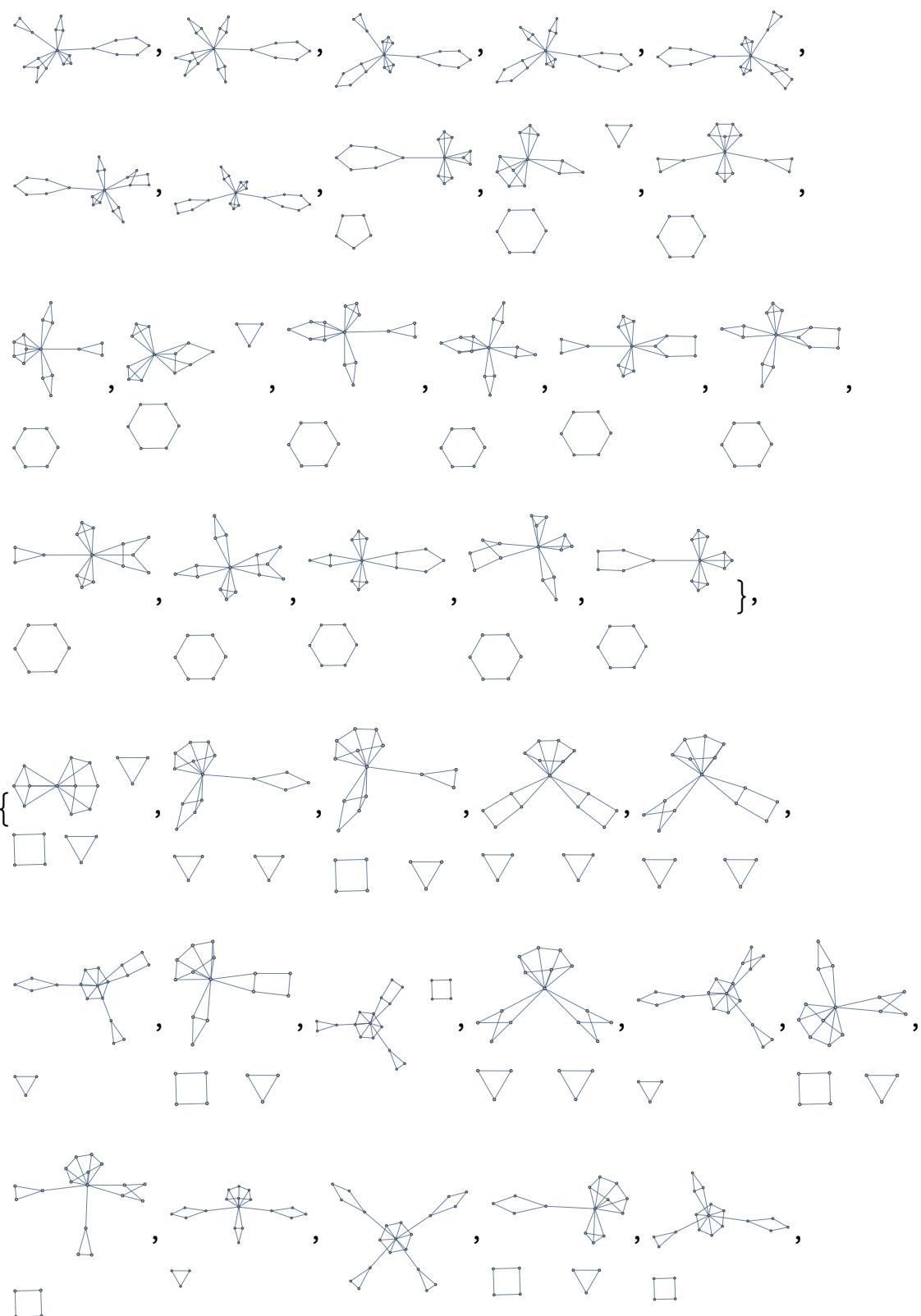


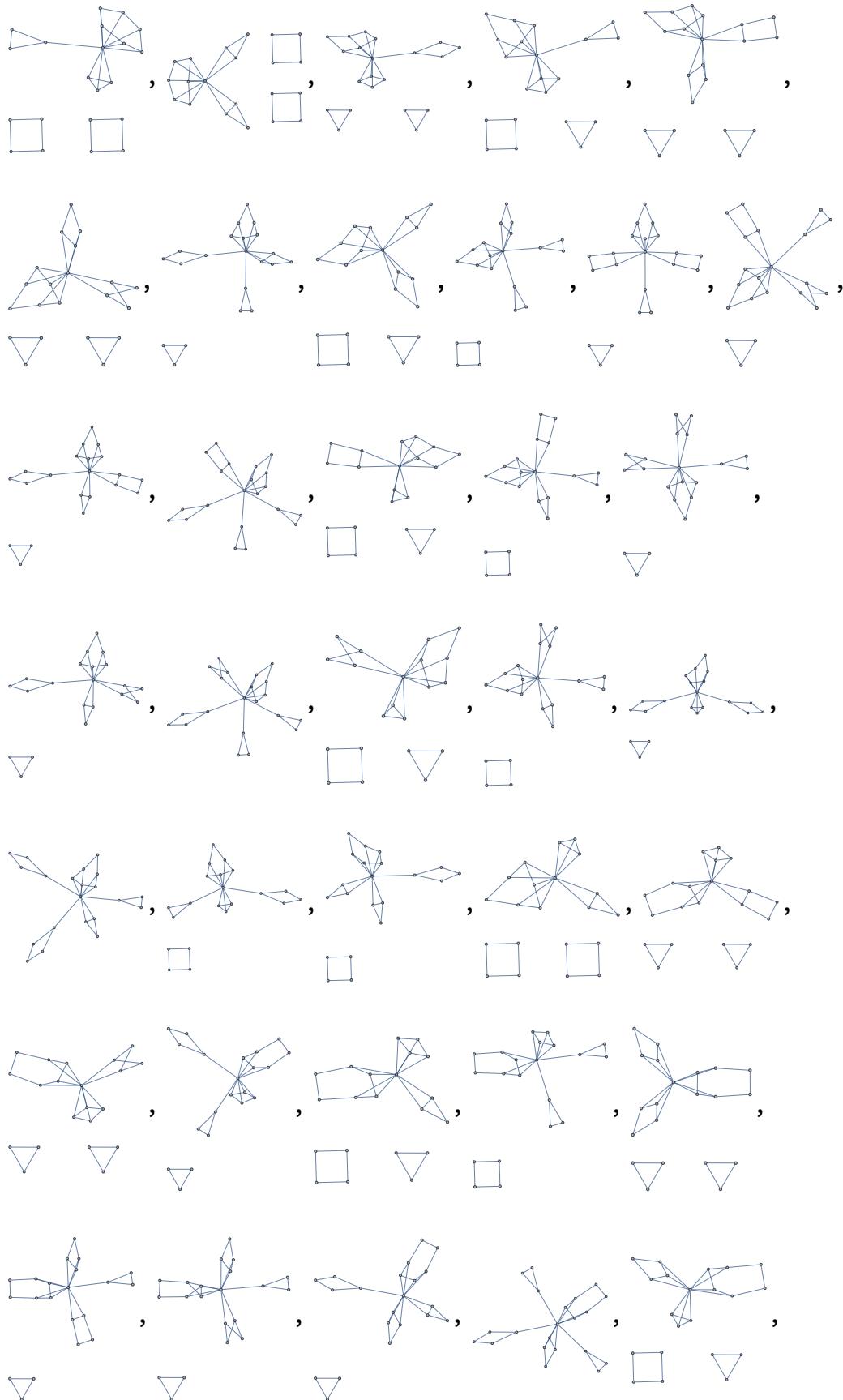


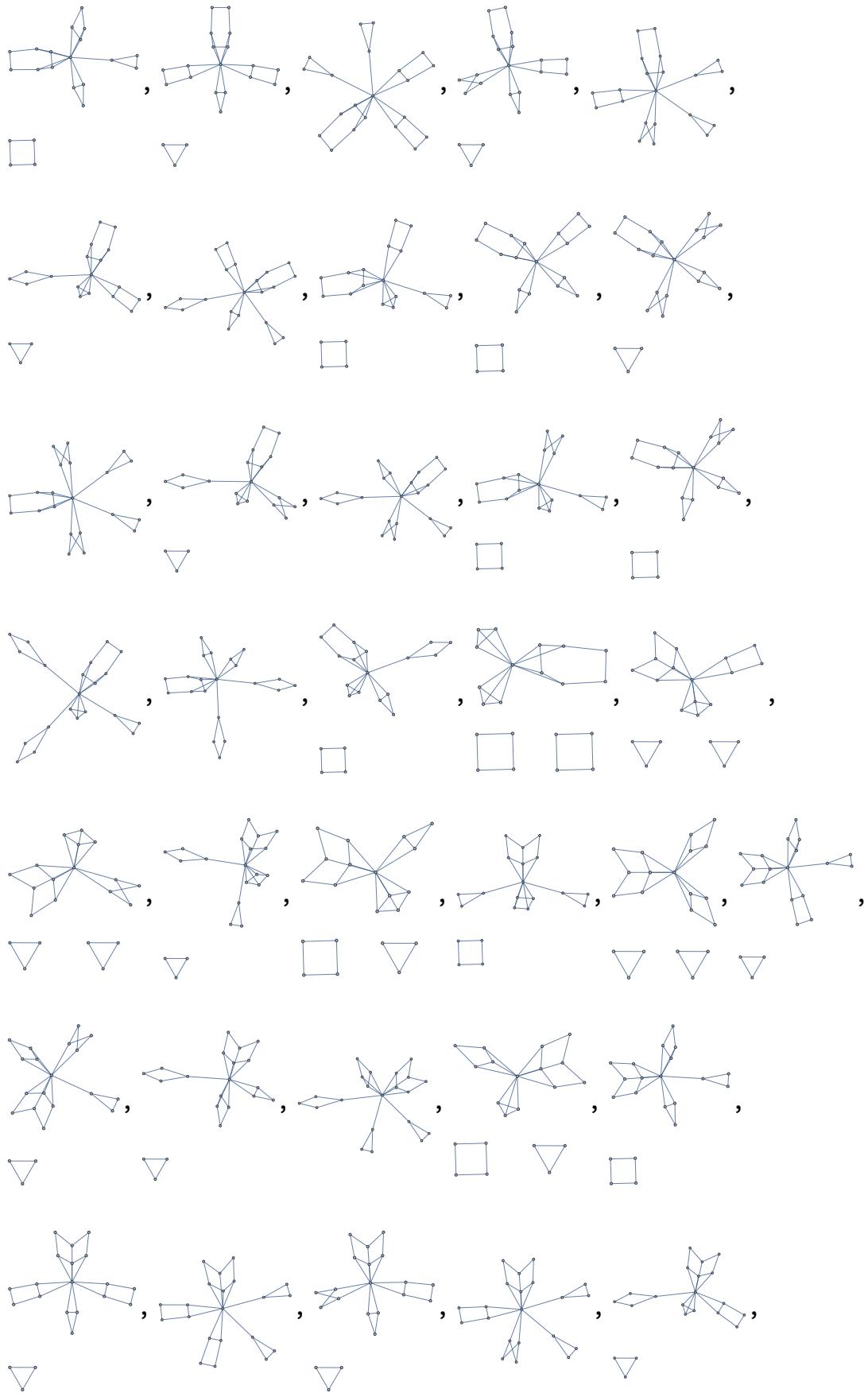


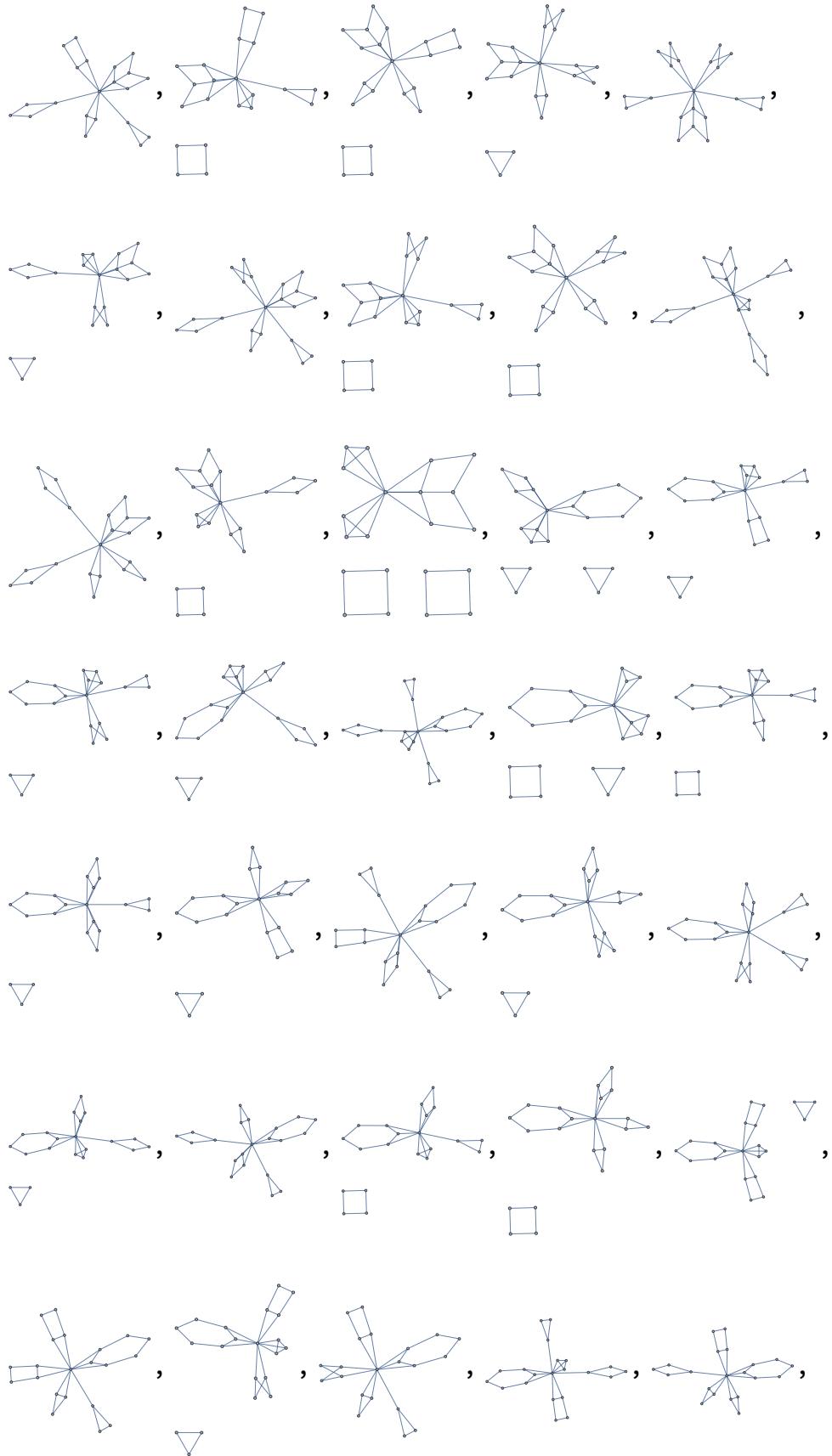


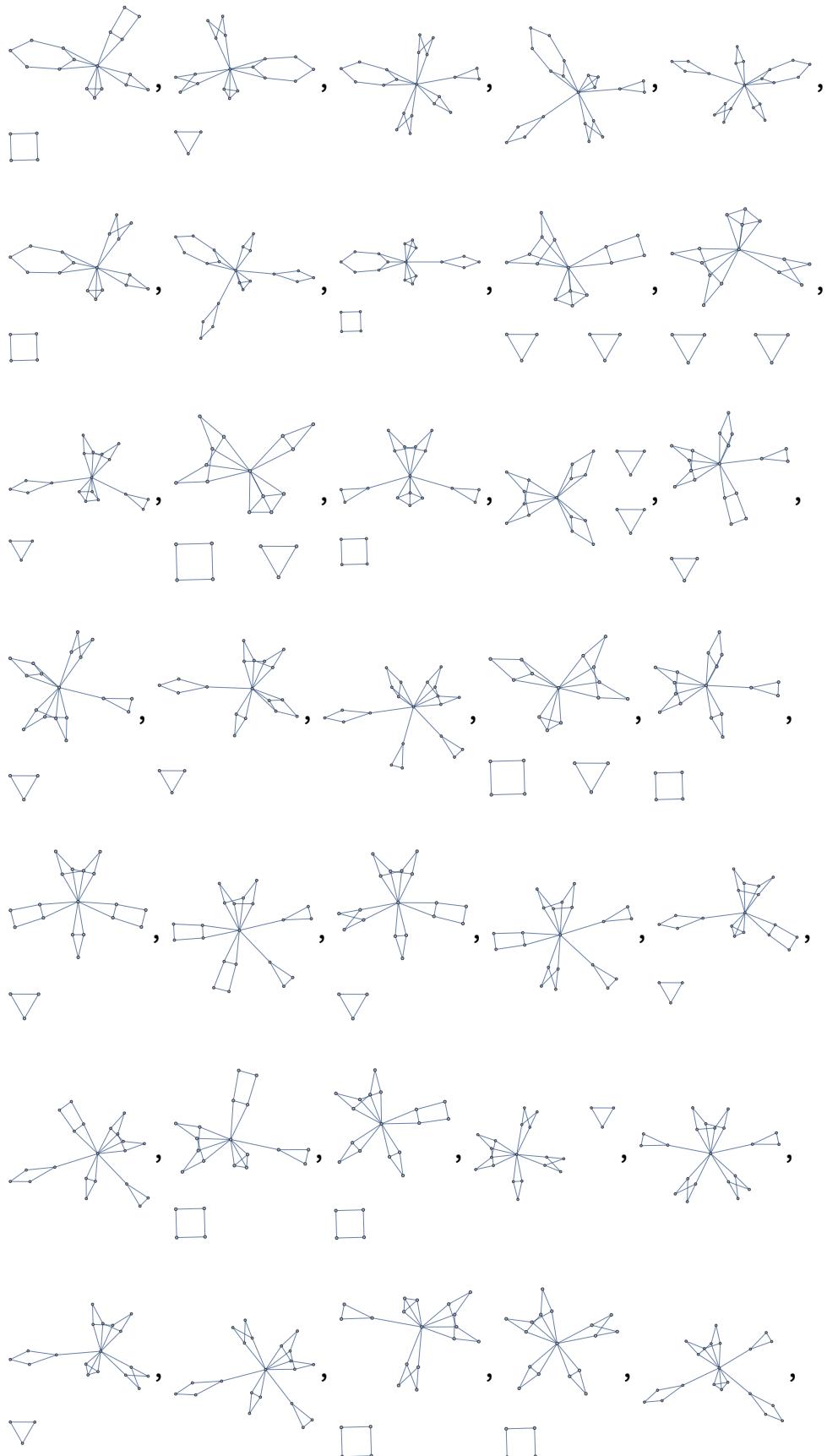


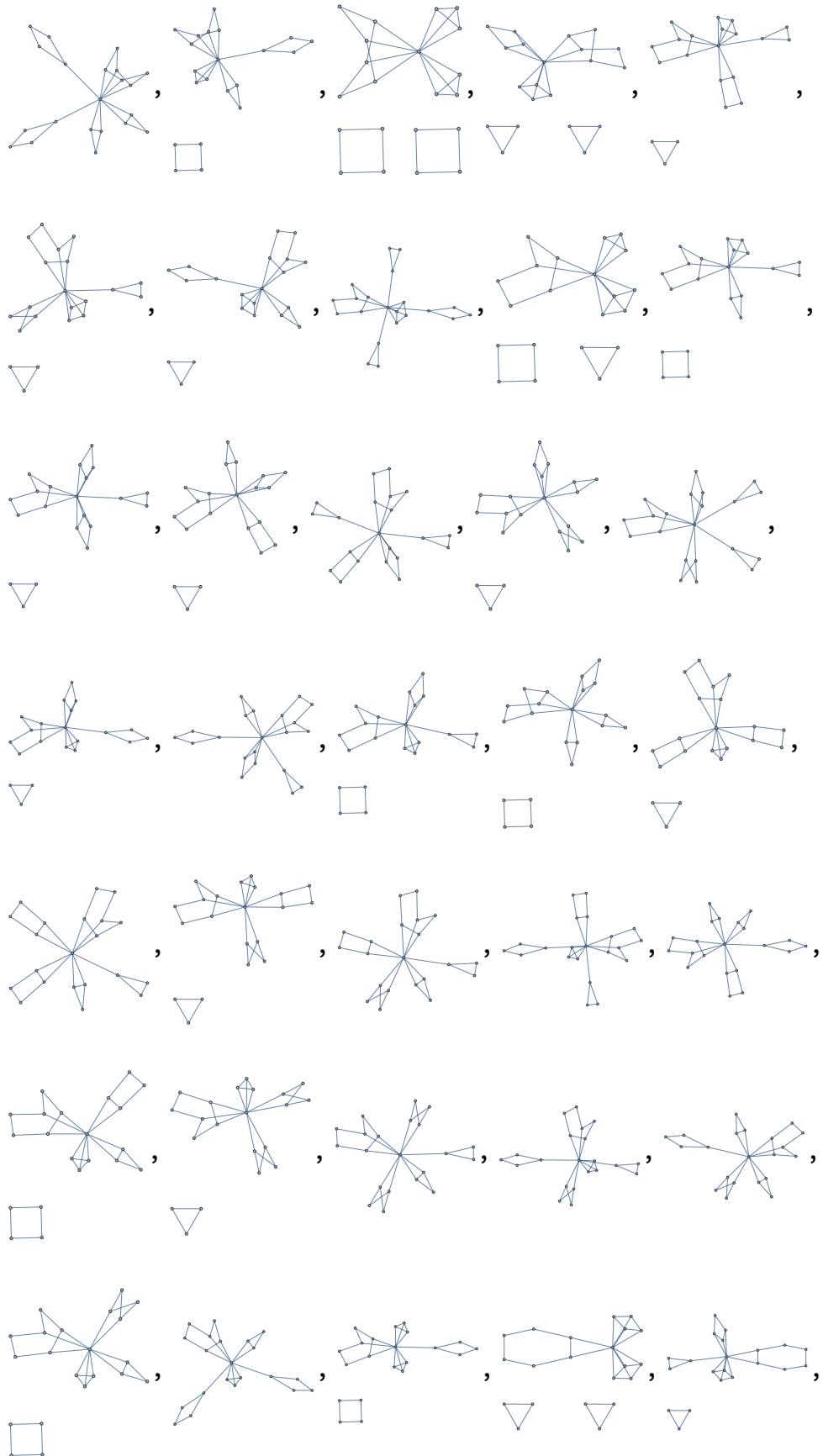


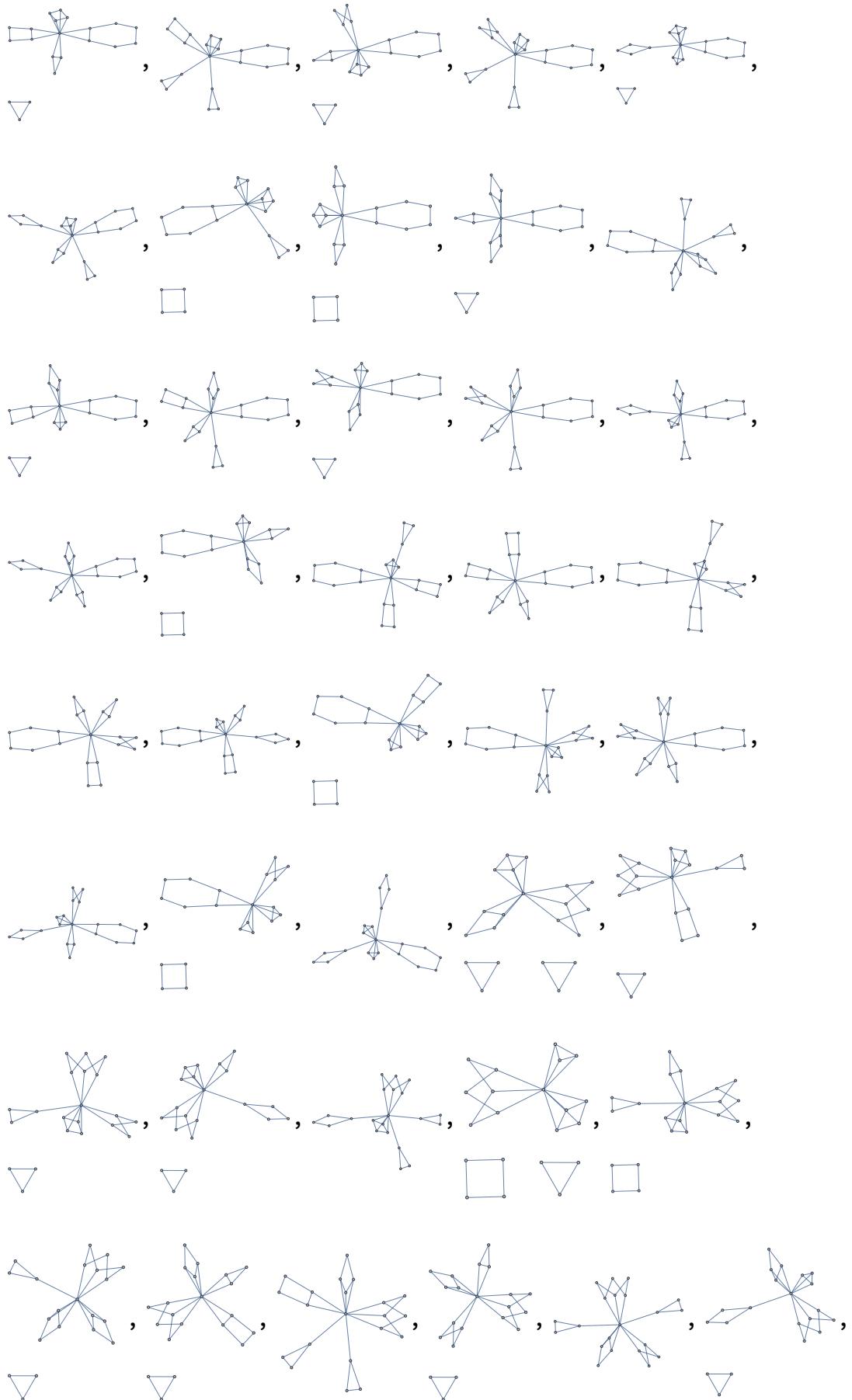


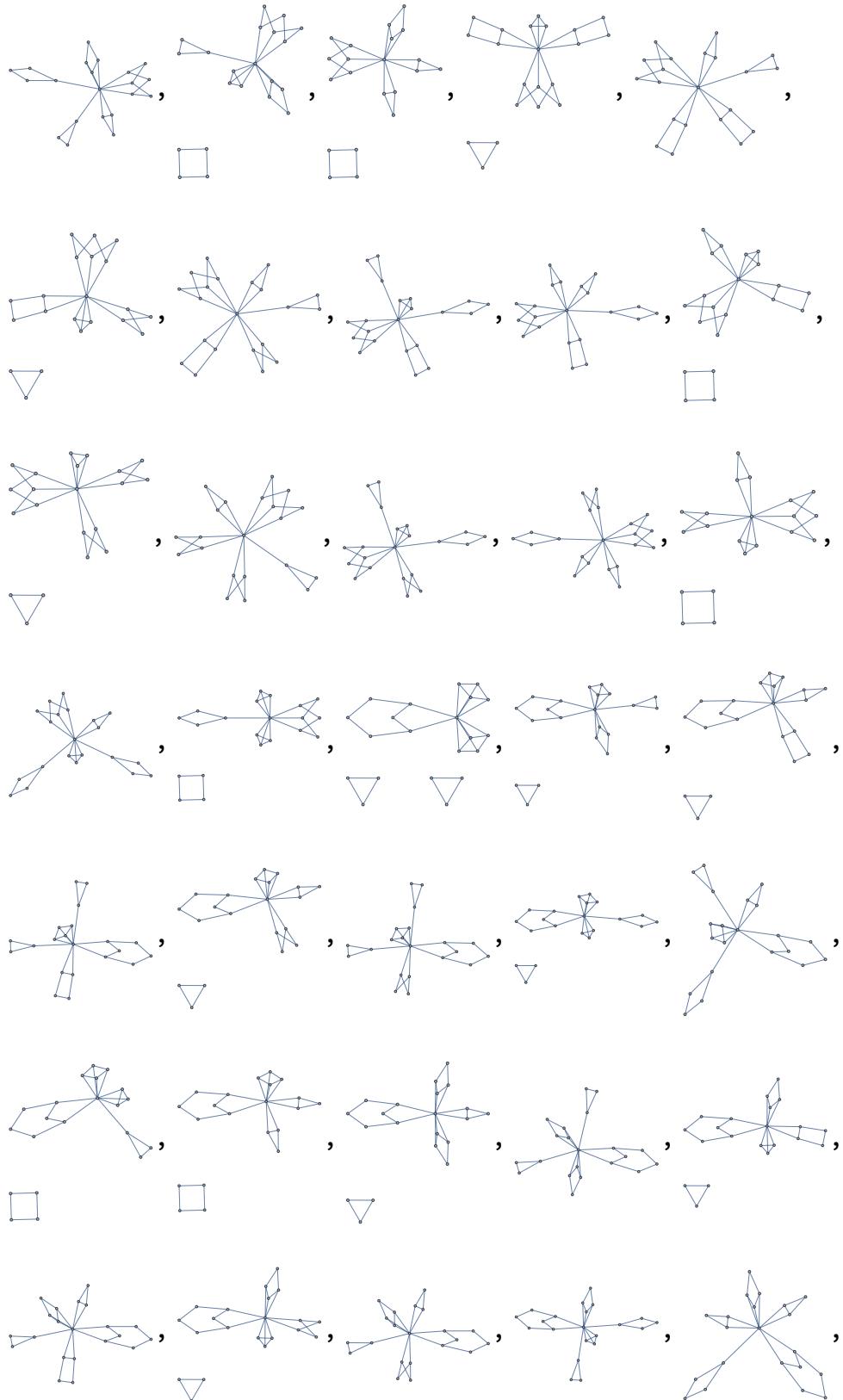


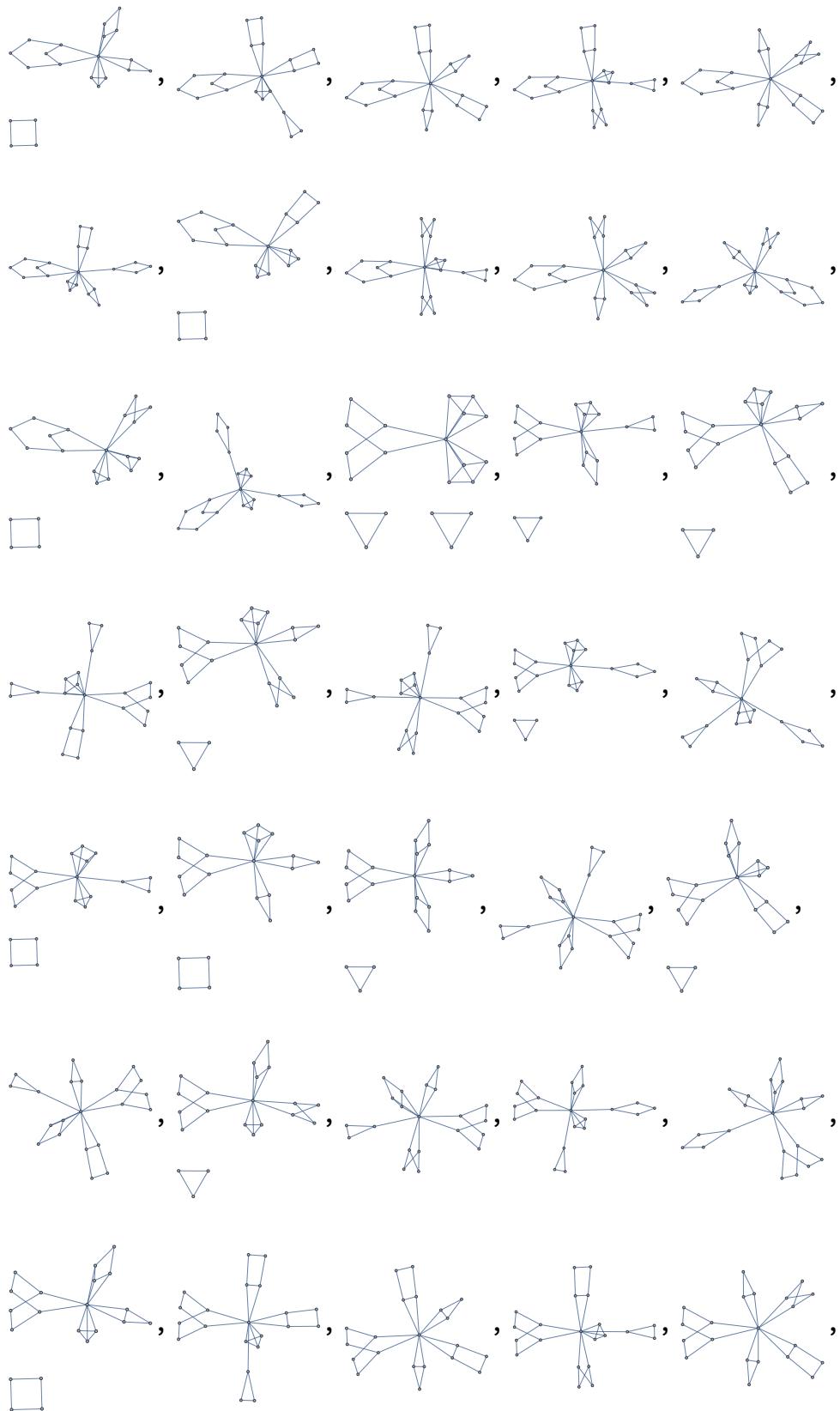


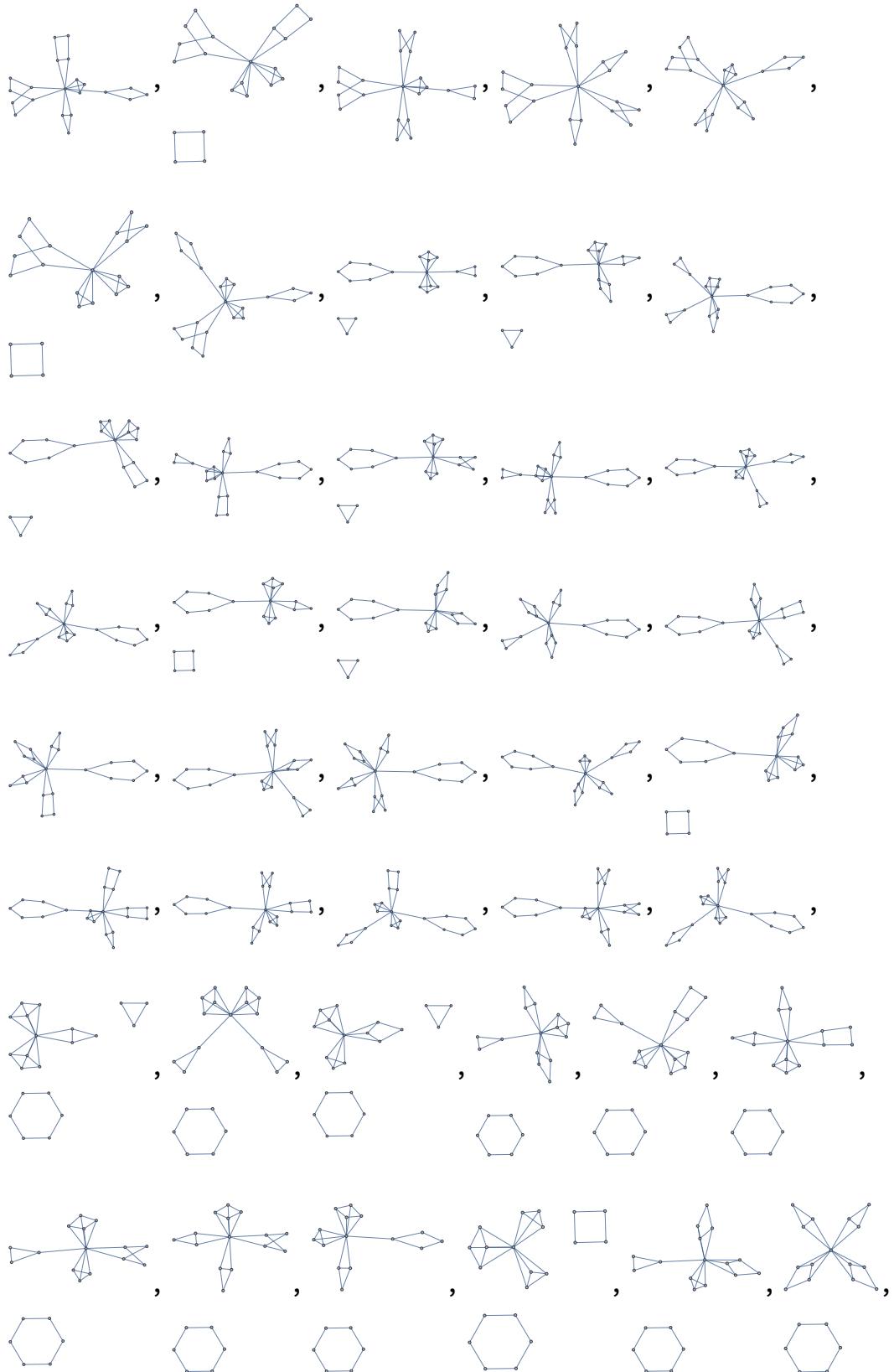


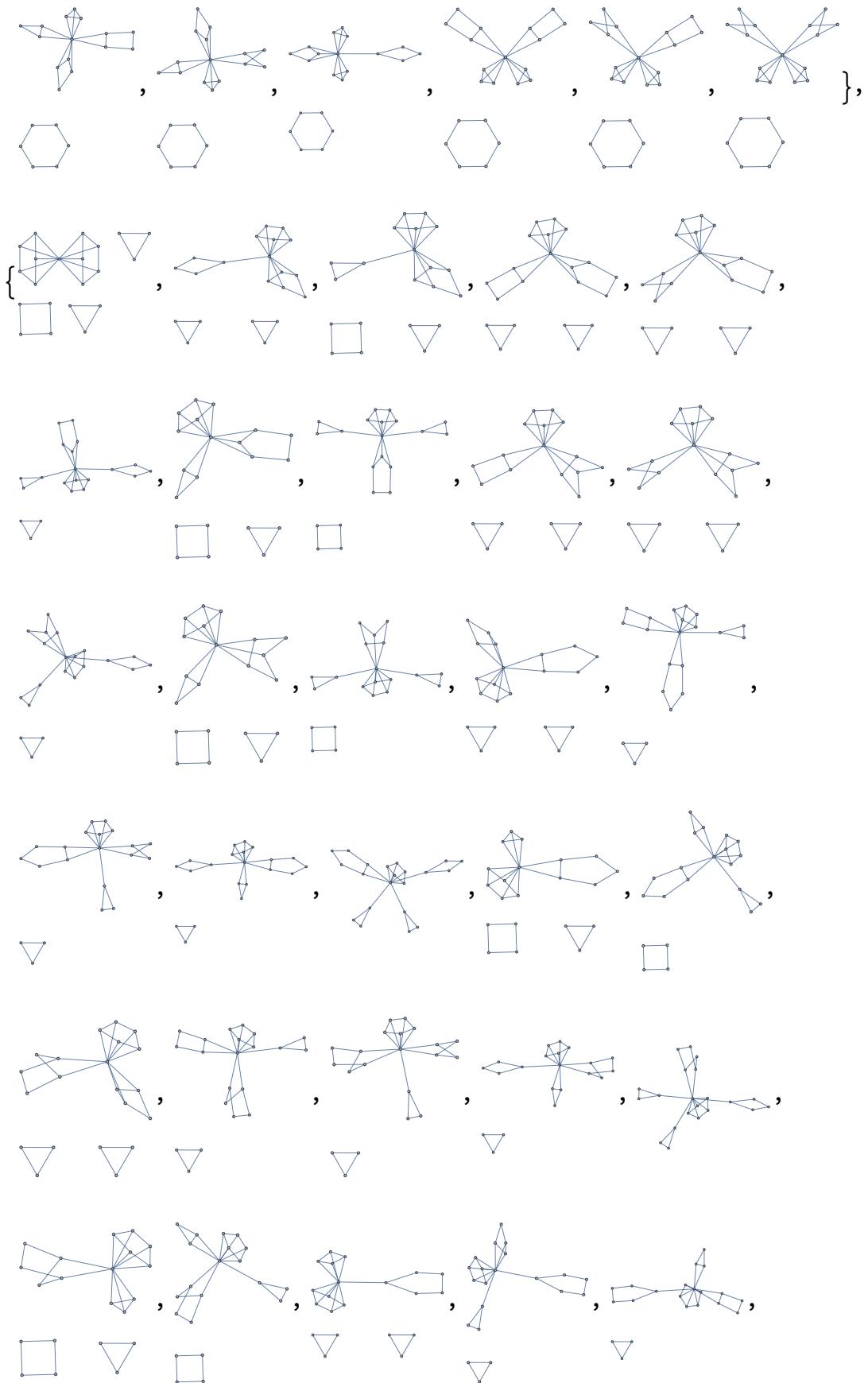


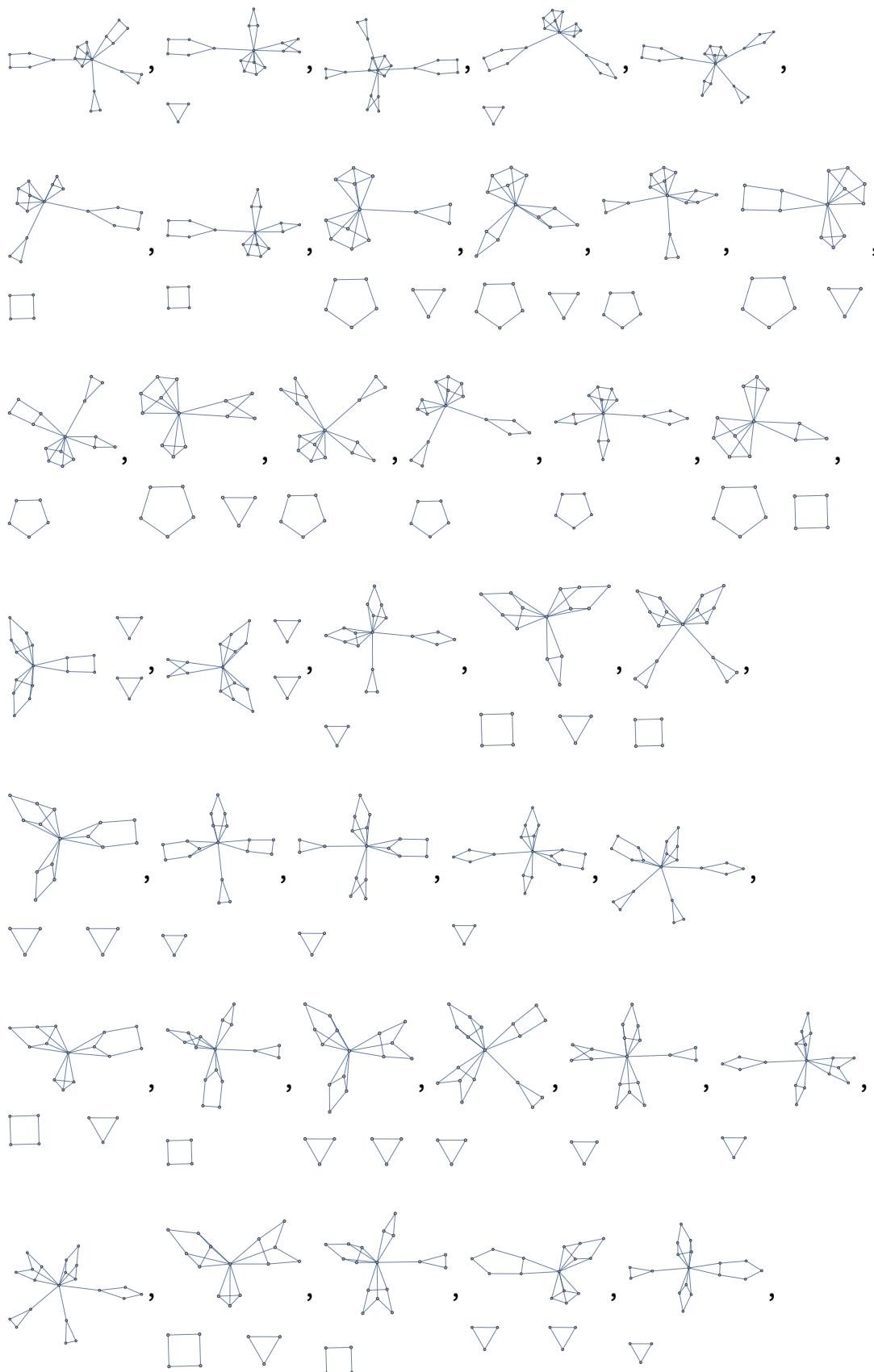


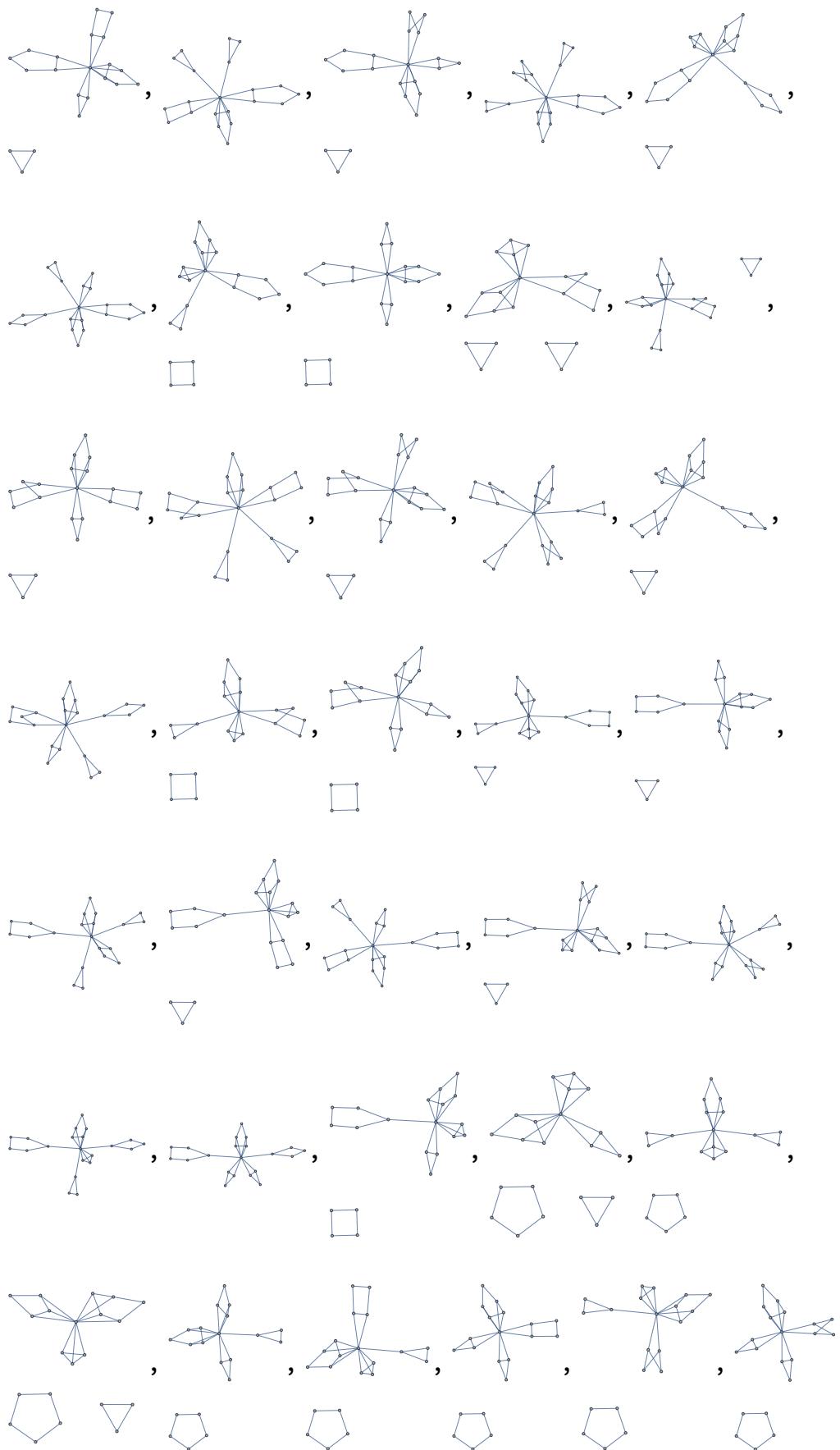


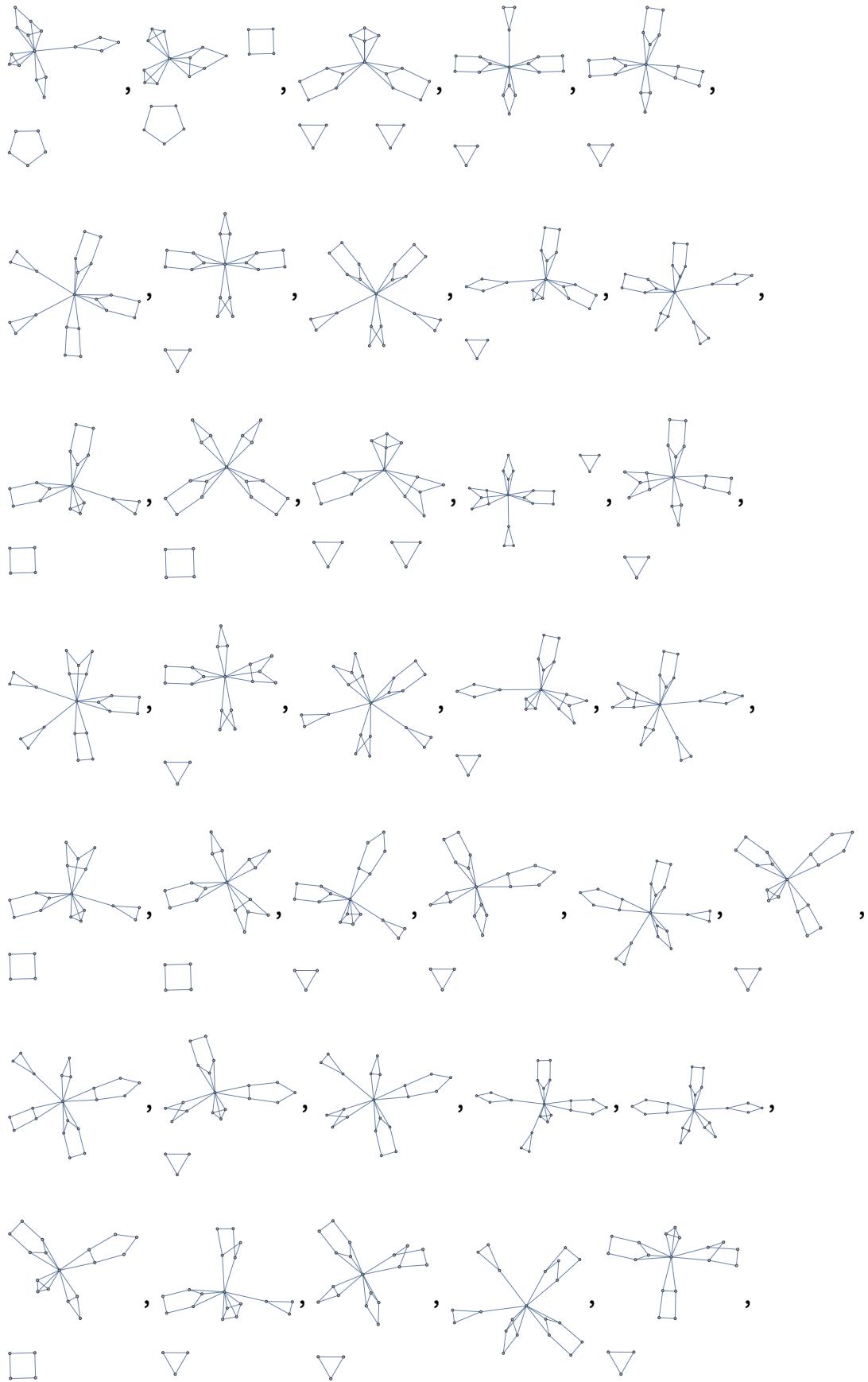


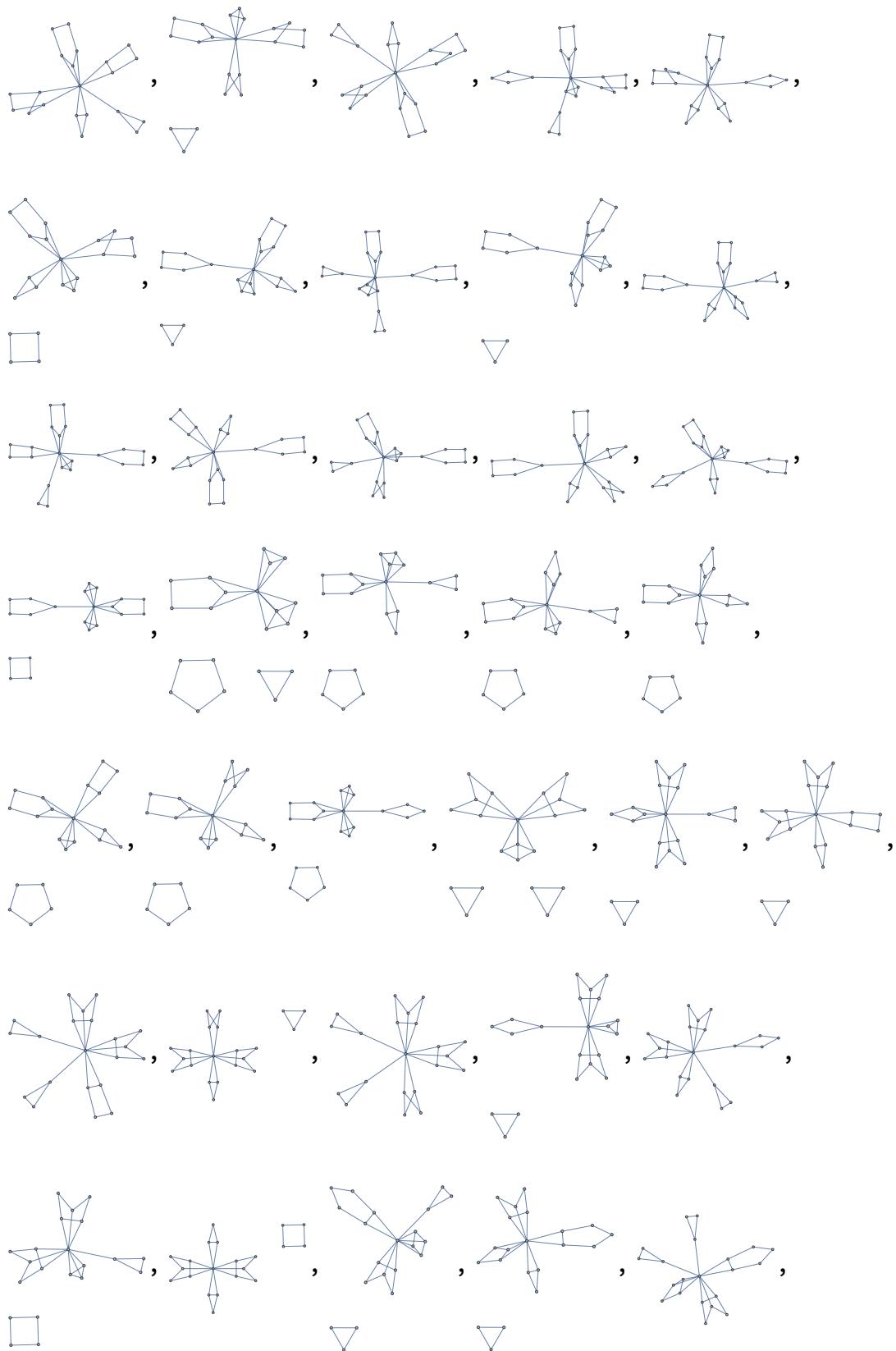


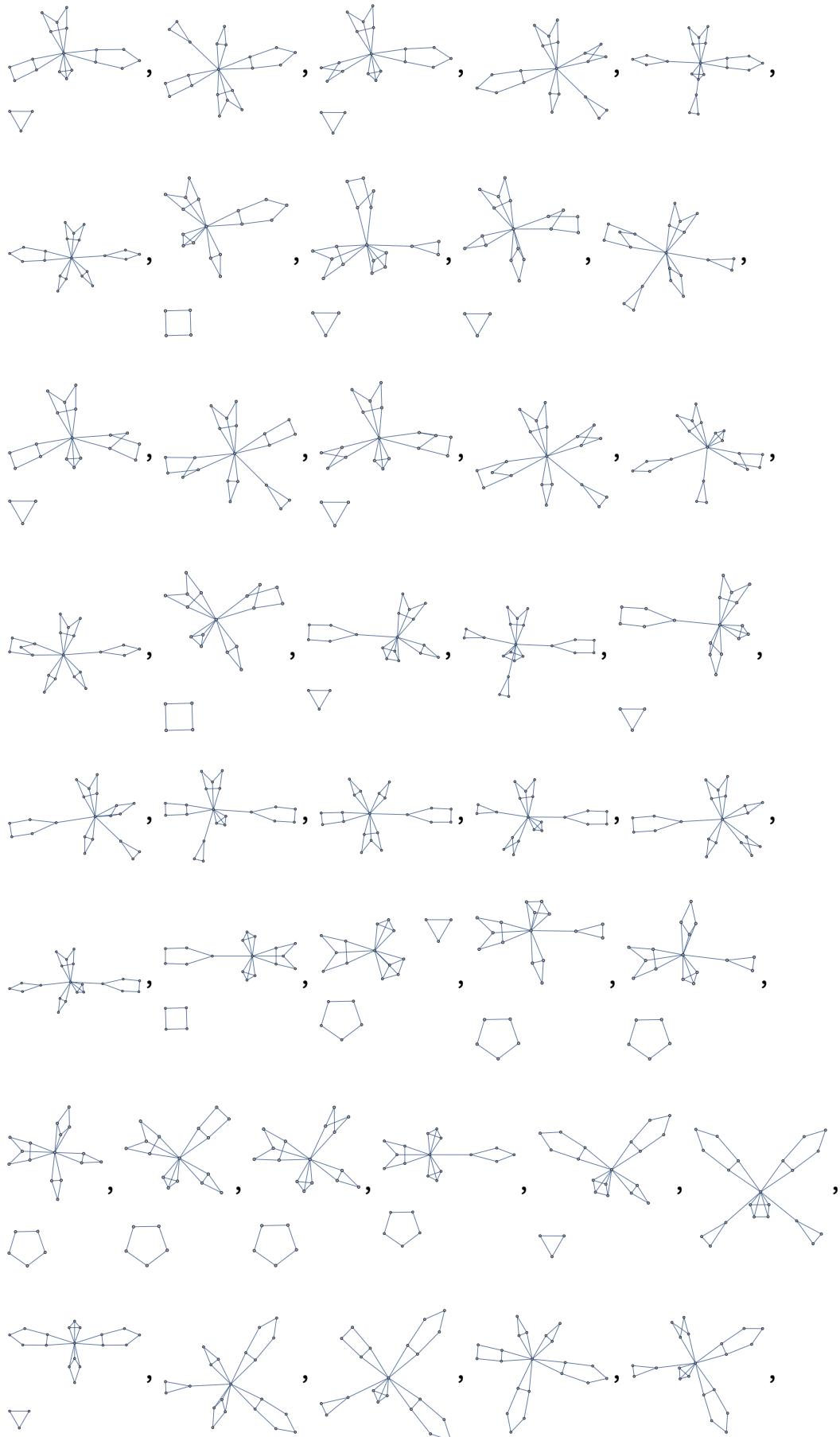


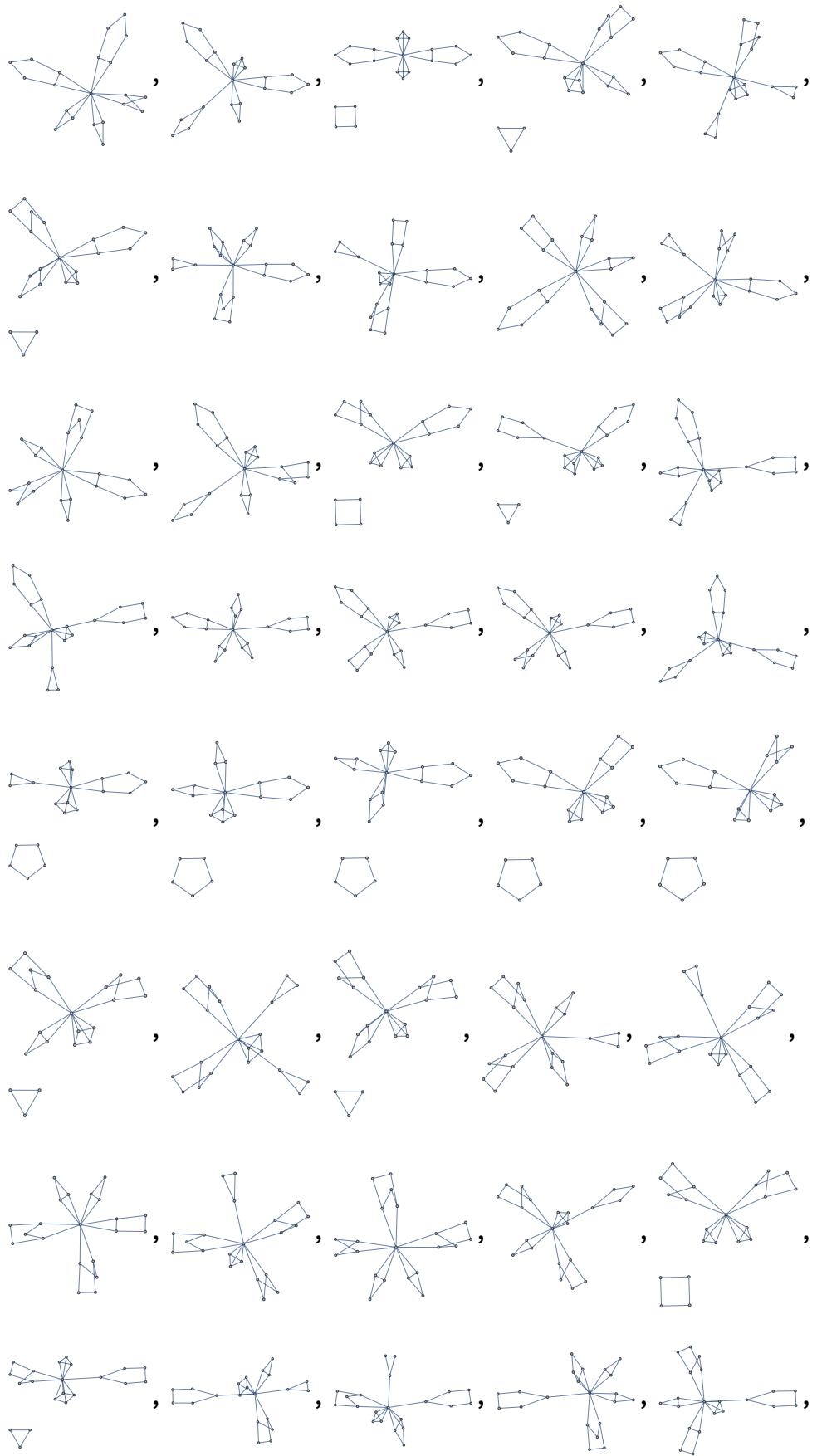


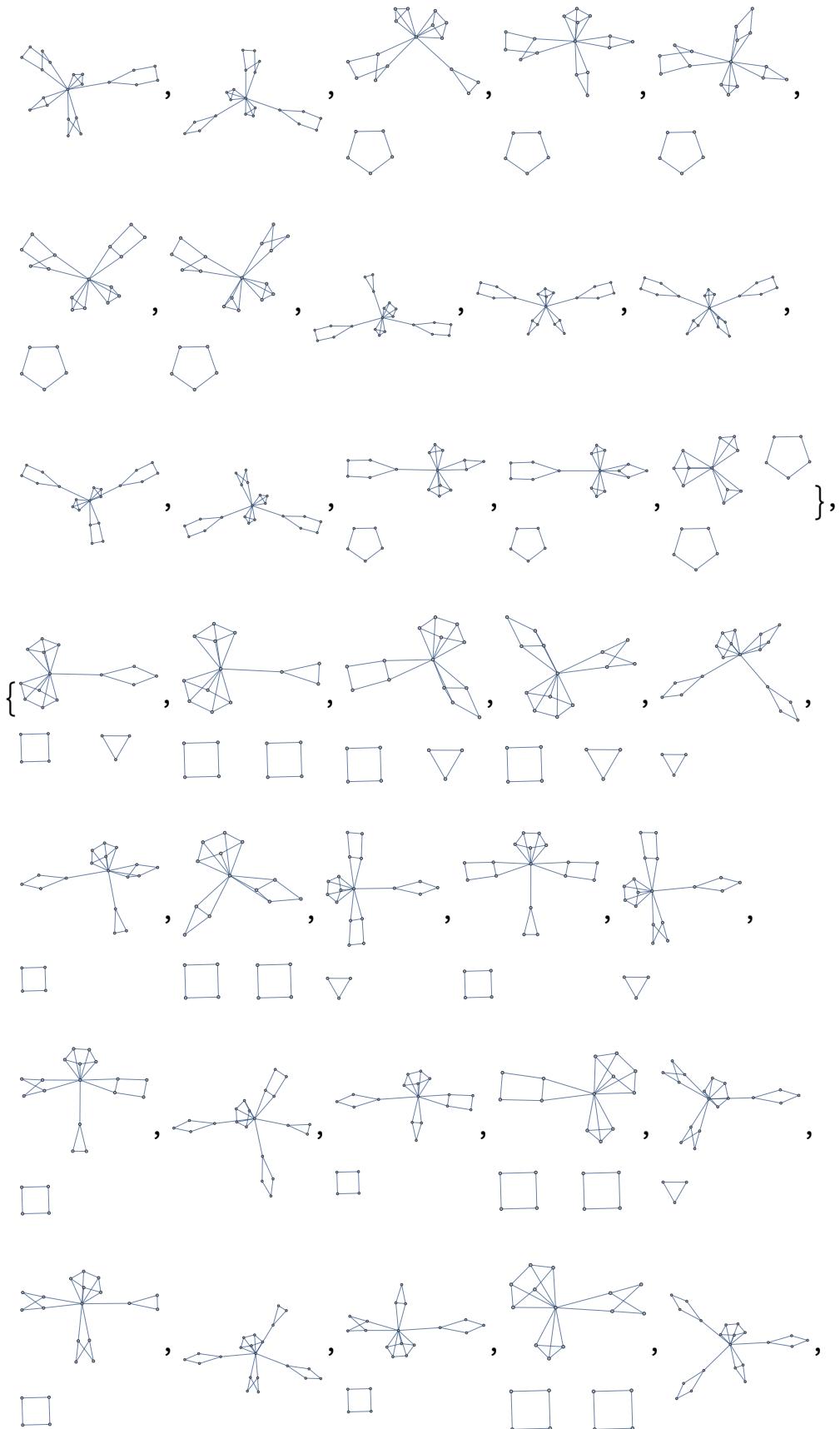


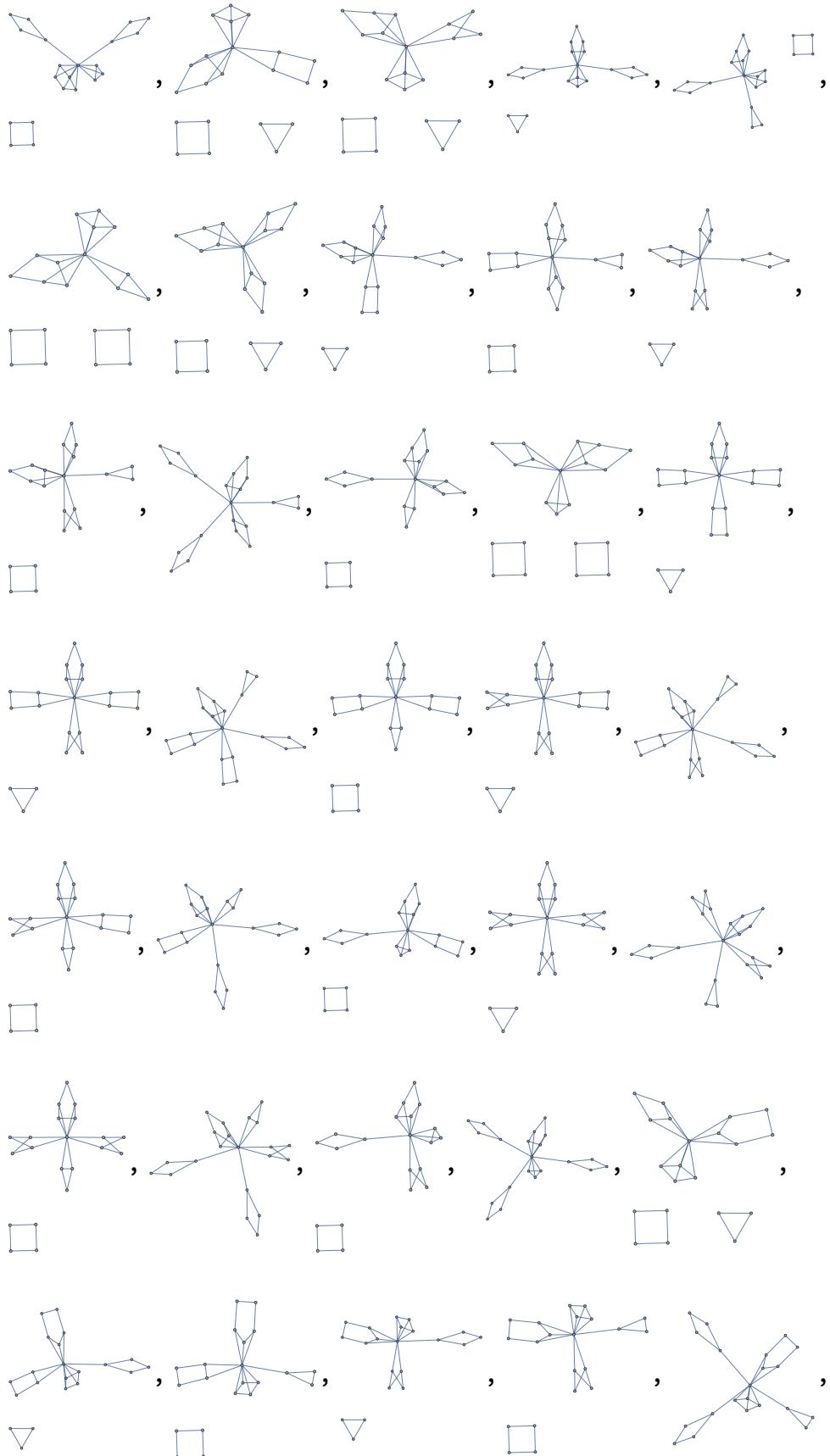


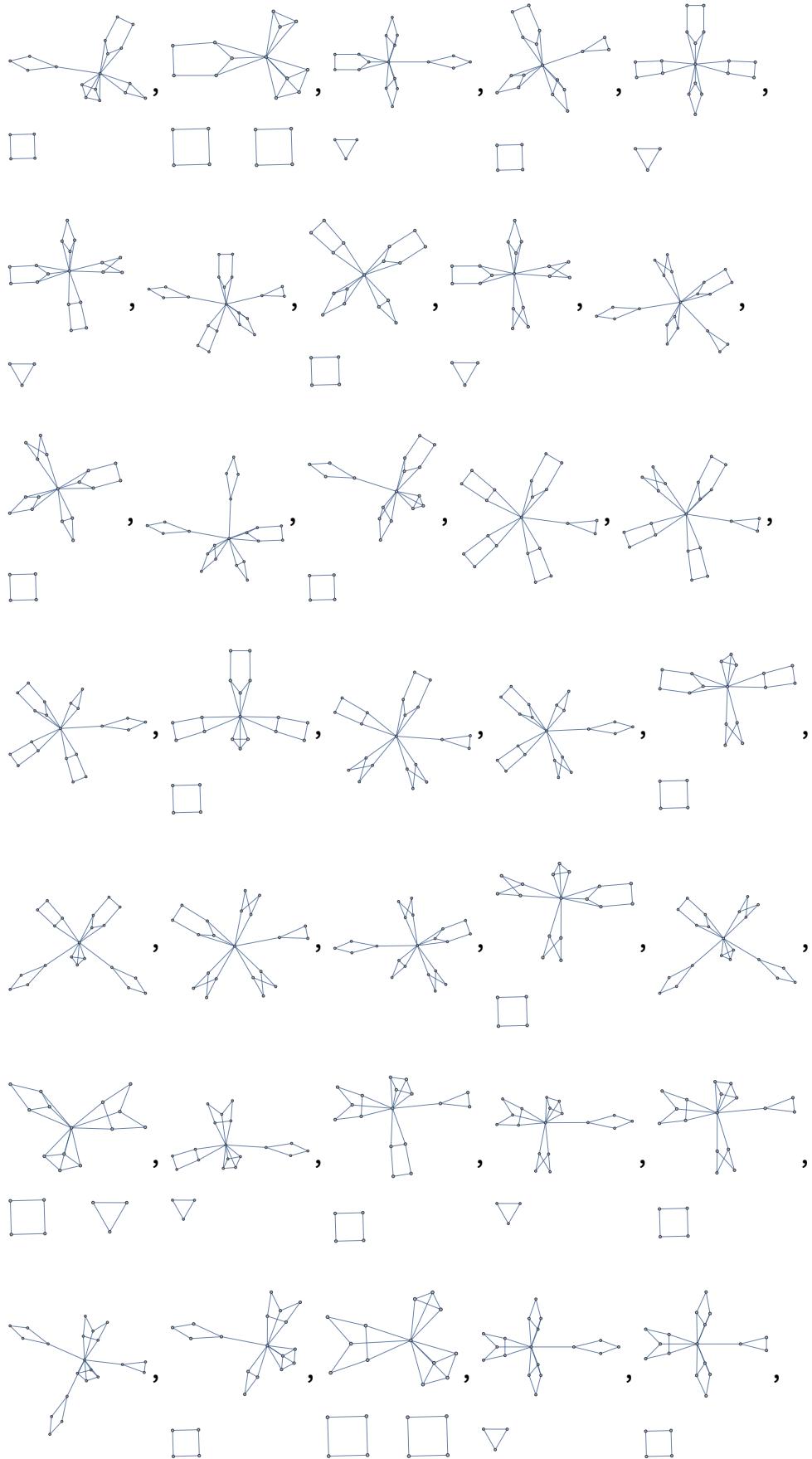


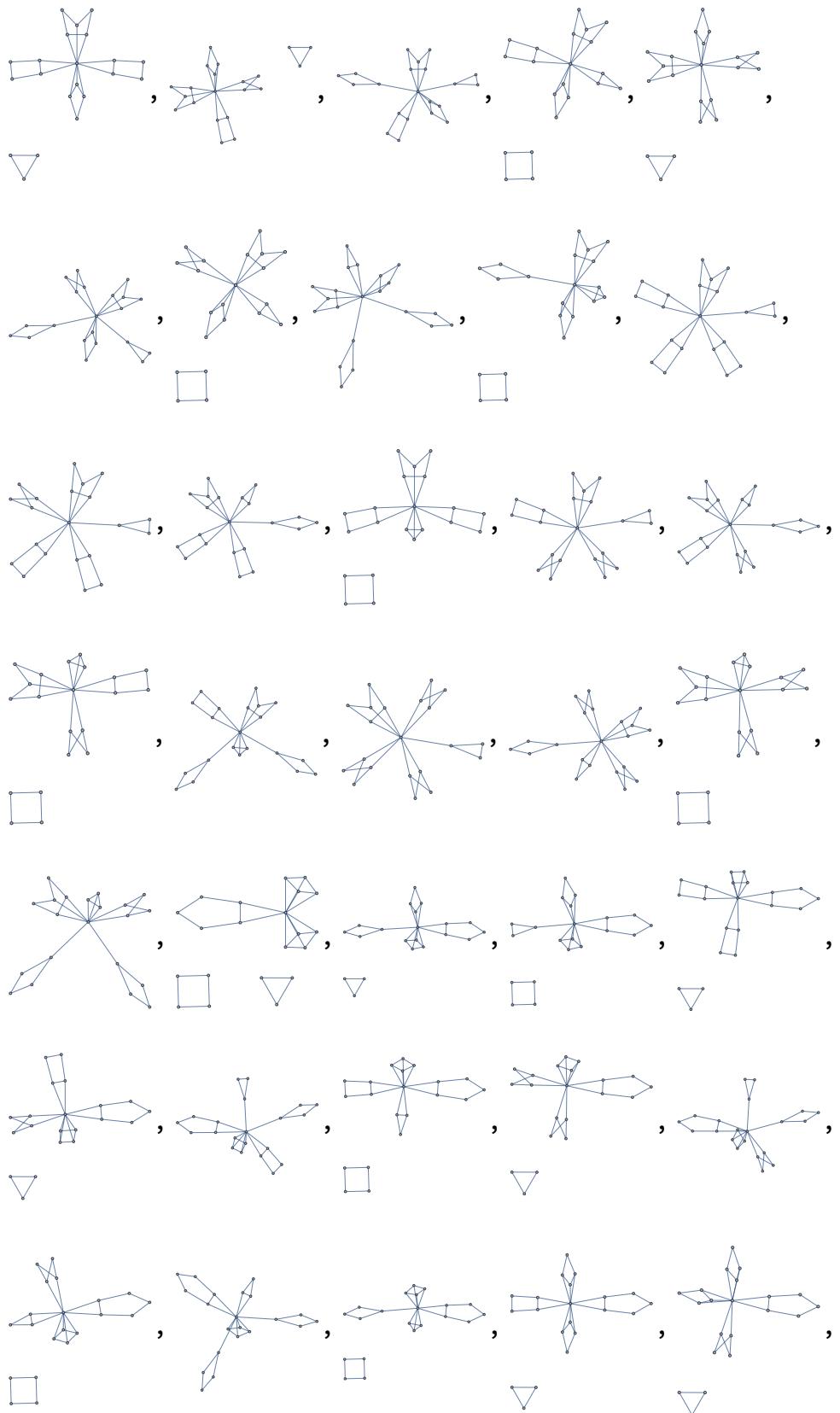


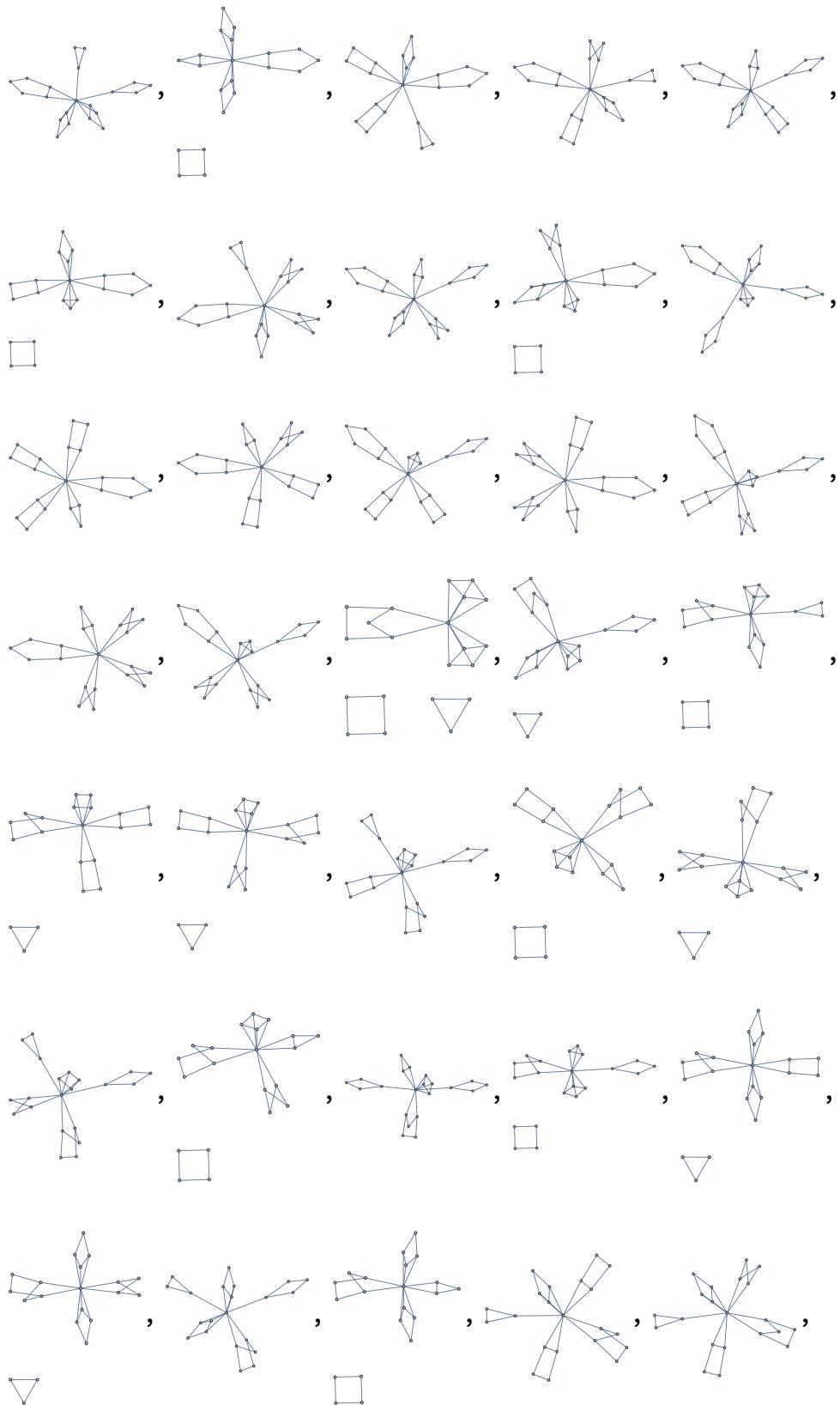


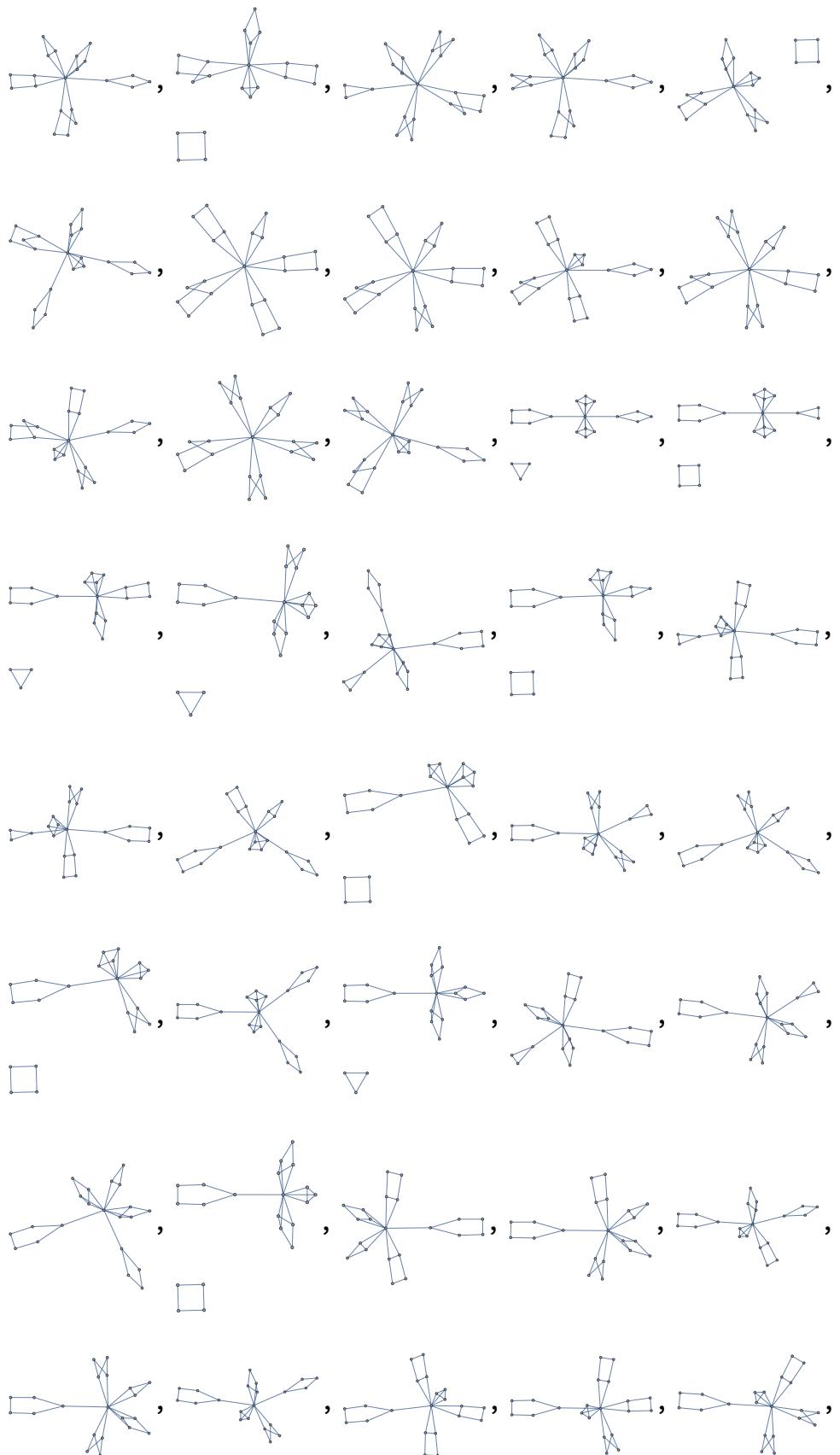


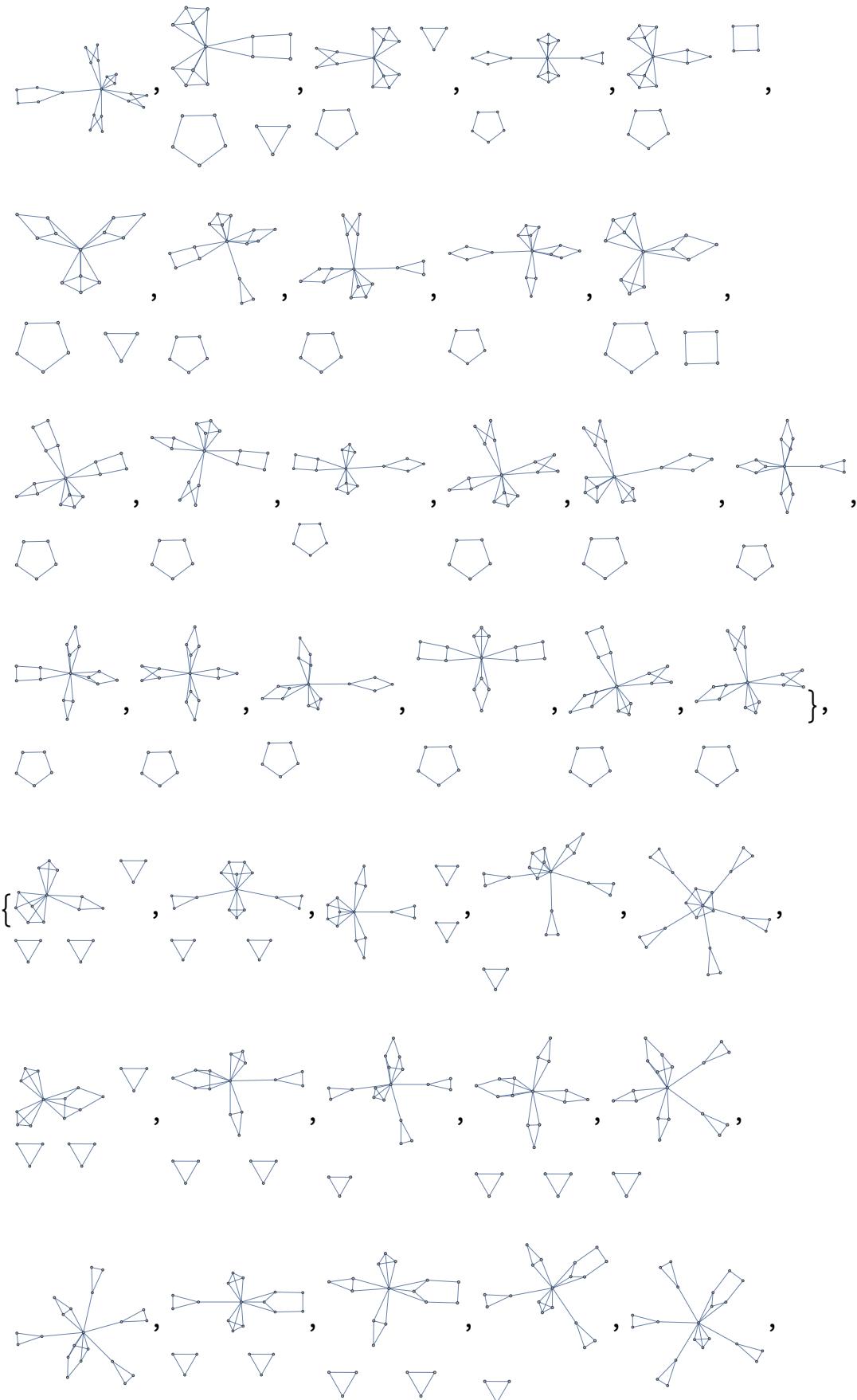


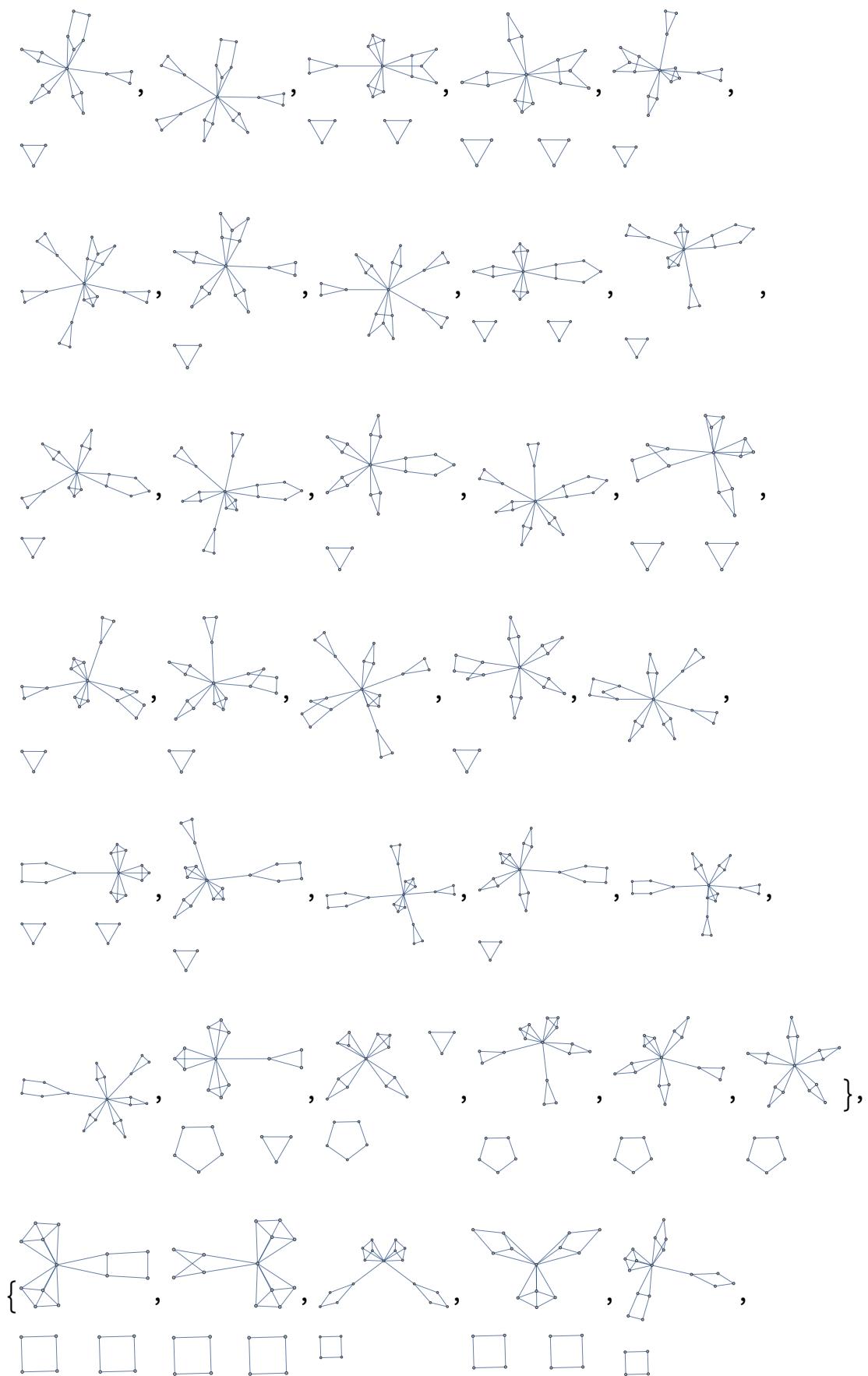


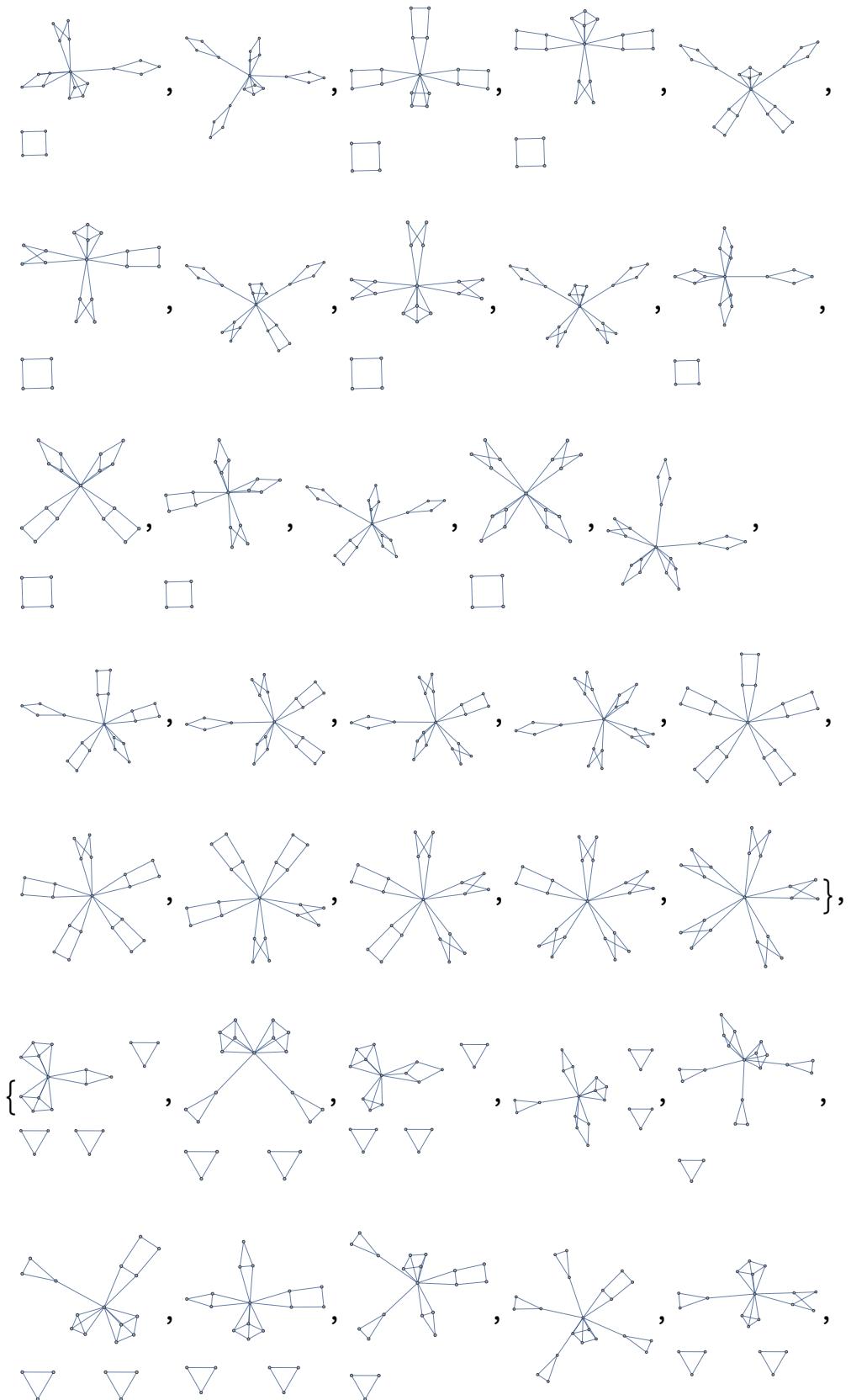


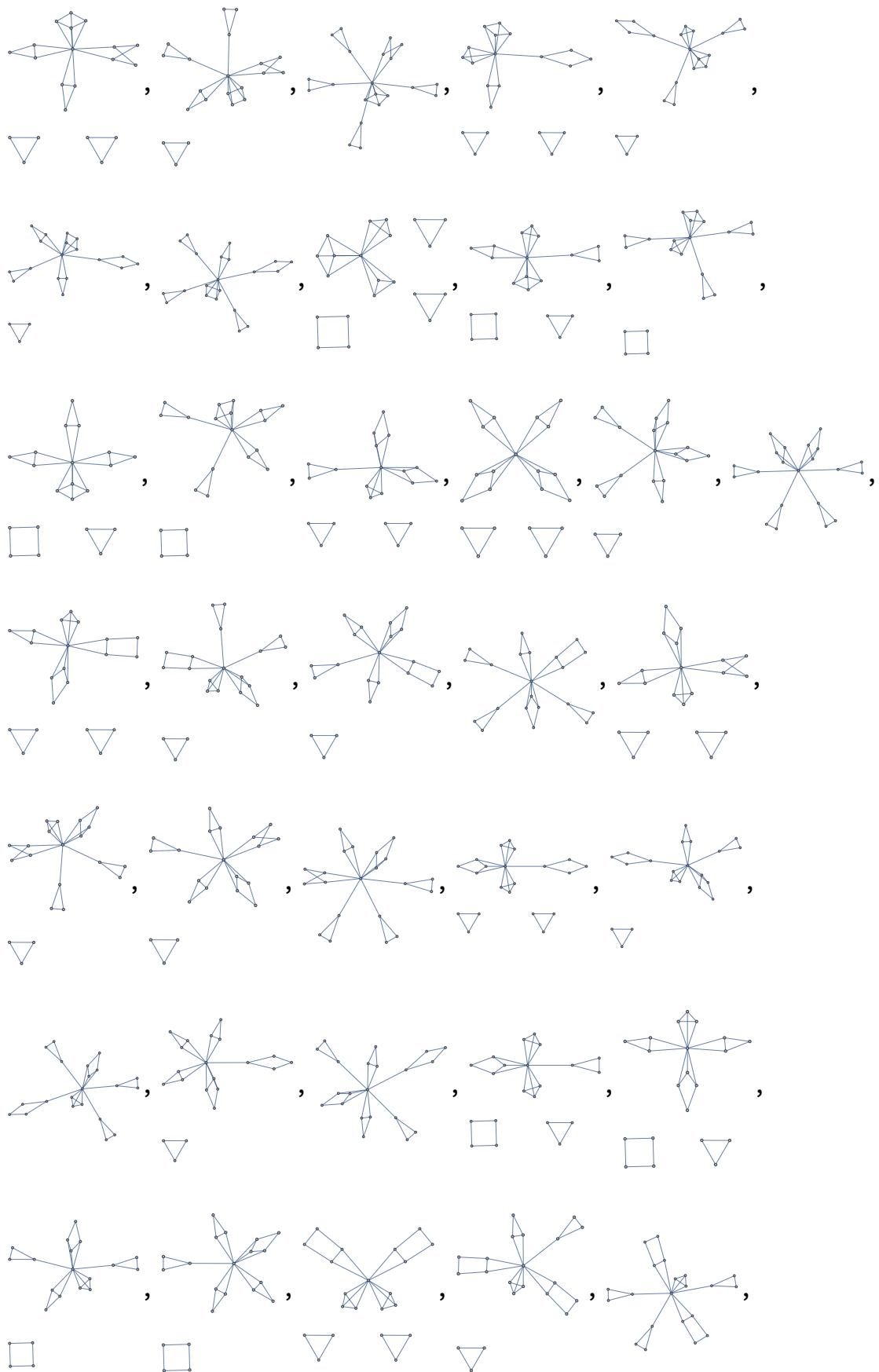


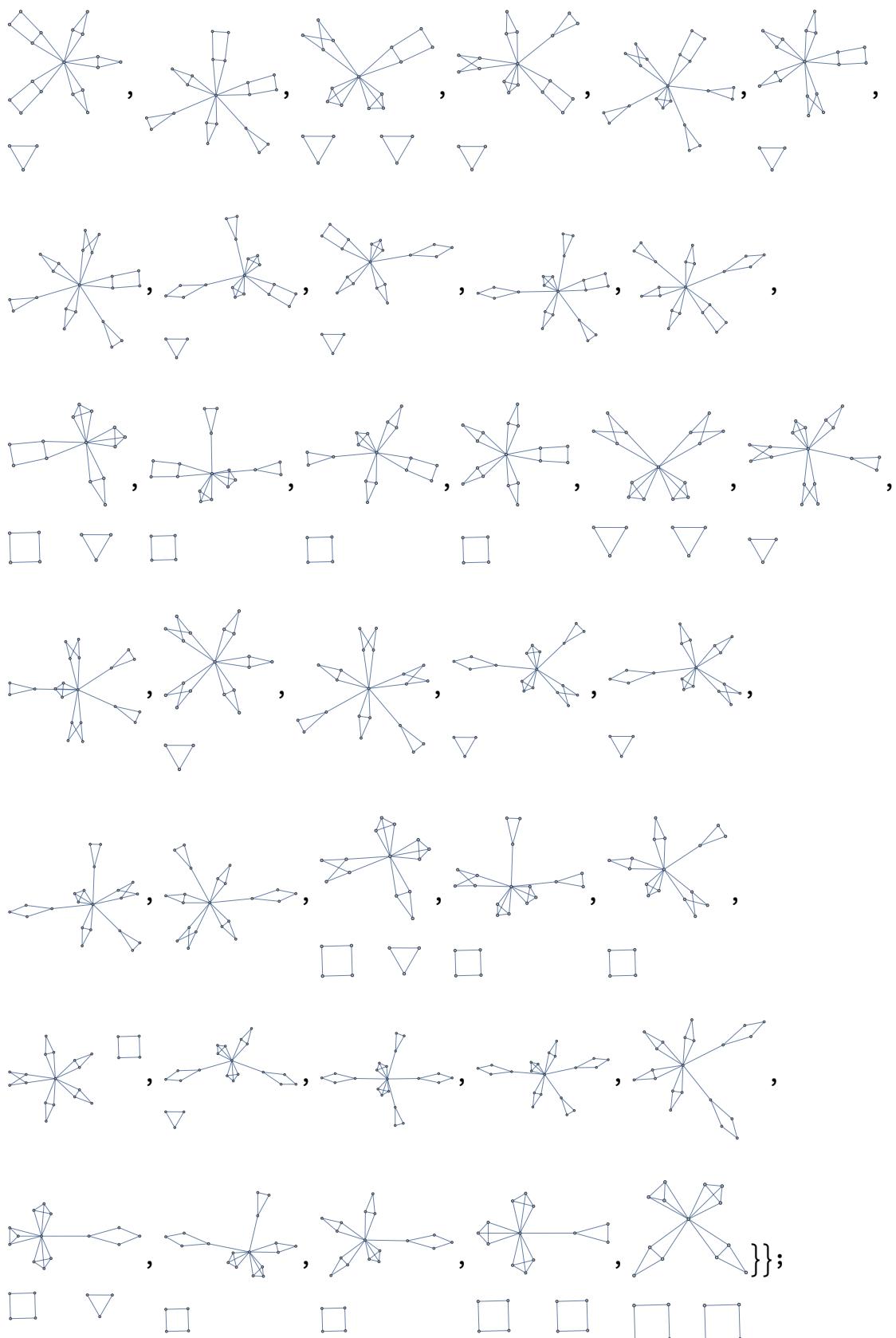












```
In[6]:= For[i = 1, i ≤ 9, i++,
  Print[i, " ", Length[graphgrow21allpartitionsmodiso[i]]];
]
```

1	122
2	266
3	254
4	352
5	272
6	222
7	46
8	30
9	83

```
In[6]:= chi60 = (x - 22) (x - 2)^42 (x + 6)^15 (x + 8)^2
```

$$Out[6]= (-22+x) (-2+x)^{42} (6+x)^{15} (8+x)^2$$

```
In[5]:= residuemat = KroneckerProduct[IdentityMatrix[3], ConstantArray[1, {20, 20}]];  
residuemat // MatrixForm
```

Out[•]//MatrixForm=

```

In[6]:= RA = 10 ConstantArray[1, {60, 60}] - 8 residuemat;

In[7]:= For[i = 1, i ≤ 9, i++,
Print[i, " ", Length[graphgrow21allpartitionsmodiso[[i]]]];
For[j = 1, j ≤ Length[graphgrow21allpartitionsmodiso[[i]]], j++,
growto21 = AdjacencyMatrix[graphgrow21allpartitionsmodiso[[i, j]]];
complchi = chi60 * Det[ $\frac{x + 4}{(x + 6)(x - 2)}$  IdentityMatrix[21] +
 $\frac{growto21}{(x + 6)(x - 2)} + \frac{9 \text{ConstantArray}[1, \{21, 21\}]}{(x + 6)(x - 2)(x - 22)} +$ 
 $\frac{(residuemat * (x - 14)) [[\text{Range}[21], \text{Range}[21]]] + RA [[\text{Range}[21], \text{Range}[21]]]}{(x + 8)(x + 6)(x - 2)(x - 22)}]$  ] // Factor;
If[PolynomialQ[complchi, x], Print[i, j]]
]
// AbsoluteTiming

```

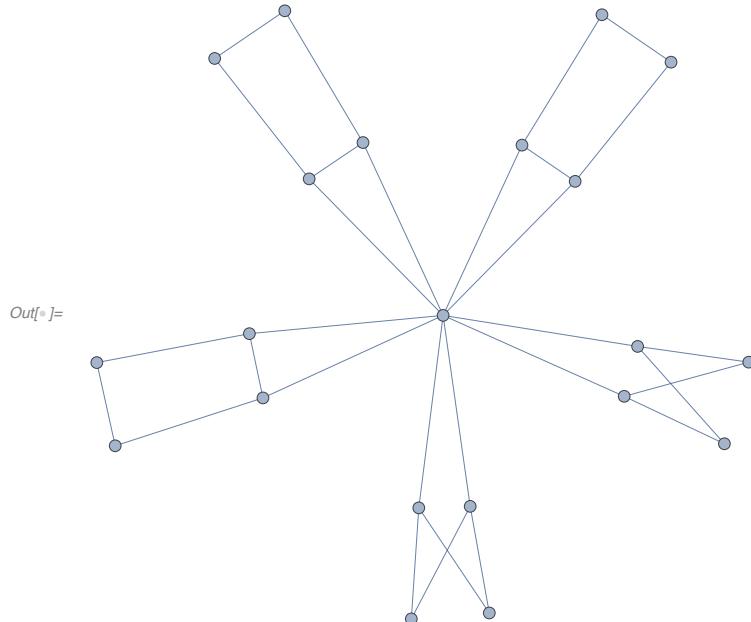
```

1 122
2 266
3 254
4 352
5 272
6 222
7 46
8 30
827
9 83

```

Out[8]:= {619.064, Null}

In[9]:= graphgrow21allpartitionsmodiso[8, 27]



```

In[10]:= growto21 = AdjacencyMatrix[graphgrow21allpartitionsmodiso[8, 27]];
Print[chi60 * Det[

$$\frac{x+4}{(x+6)(x-2)} \text{IdentityMatrix}[21] +$$


$$\frac{\text{growto21}}{(x+6)(x-2)} + \frac{9 \text{ConstantArray}[1, \{21, 21\}]}{(x+6)(x-2)(x-22)} +$$

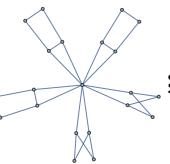

$$\frac{(\text{residue} \text{mat} * (x-14)) \text{Range}[21], \text{Range}[21]] + \text{RA}[\text{Range}[21], \text{Range}[21]]}{(x+8)(x+6)(x-2)(x-22)} \Big] //$$


$$\text{Factor}]$$


```

$$(-2+x)^{22} (2+x)^4 (4+x)^9 (-248 - 366 x - 104 x^2 + x^4)$$

```
In[6]:= theonlyvalid21 =
```



```
;
```

```
adj21 = AdjacencyMatrix[theonlyvalid21];
```