

D[16 x^2 - 250 x + 1015, x]

- 250 + 32 x

250 / 32 // N

7.8125

16 x^2 - 250 x + 1015 /. x -> 9

61

(49 / 13) ^ 13 - 16 ^ 6 // N

1.42173 × 10⁷

(61 / 16) ^ 16 - 4 ^ 15 // N

9.18571 × 10⁸

feasiblefactorlist[41, (x + 5) ^ 25 (x - 7) ^ 12]

{ (-12 + x) (-11 + x) (-9 + x)², (-11 + x)³ (-8 + x) }

length 2

feasiblefactorlist[41, (x + 5) ^ 25 (x - 7) ^ 11]

**{ (-13 + x) (-9 + x)³ (-8 + x), (-9 + x)² (-932 + 293 x - 30 x² + x³),
(-9 + x)² (-928 + 293 x - 30 x² + x³), (-12 + x) (-11 + x) (-9 + x)² (-7 + x),
(-11 + x)³ (-8 + x) (-7 + x), (-11 + x)² (-9 + x) (68 - 17 x + x²) }**

length 6

feasiblefactorlist[41, (x + 5) ^ 25 (x - 7) ^ 10]

**{ (-13 + x) (-9 + x)³ (-8 + x) (-7 + x),
(-9 + x)² (-7 + x) (-932 + 293 x - 30 x² + x³), (-9 + x)⁴ (80 - 19 x + x²),
(-9 + x)² (-7 + x) (-928 + 293 x - 30 x² + x³), (-9 + x)² (6452 - 2975 x + 503 x² - 37 x³ + x⁴),
(-12 + x) (-11 + x) (-9 + x)² (-7 + x)², (-9 + x)³ (-712 + 251 x - 28 x² + x³),
(-11 + x) (-9 + x)² (-8 + x) (73 - 18 x + x²), (-11 + x)³ (-8 + x) (-7 + x)²,
(-11 + x) (-9 + x)² (-580 + 217 x - 26 x² + x³), (-11 + x)² (-9 + x) (-7 + x) (68 - 17 x + x²),
(-11 + x) (-9 + x)³ (64 - 17 x + x²), (-11 + x)² (-9 + x)² (52 - 15 x + x²) }**

length 13

feasiblefactorlist[41, (x + 5) ^25 (x - 7) ^9]

$$\{ (-13 + x) (-9 + x)^3 (-8 + x) (-7 + x)^2, (-9 + x)^2 (-7 + x)^2 (-932 + 293 x - 30 x^2 + x^3), \\ (-9 + x)^4 (-7 + x) (80 - 19 x + x^2), (-9 + x)^2 (-7 + x)^2 (-928 + 293 x - 30 x^2 + x^3), \\ (-9 + x)^4 (-556 + 213 x - 26 x^2 + x^3), (-9 + x)^2 (-7 + x) (6452 - 2975 x + 503 x^2 - 37 x^3 + x^4), \\ (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^3, (-9 + x)^4 (-8 + x) (69 - 18 x + x^2), \\ (-9 + x)^3 (-7 + x) (-712 + 251 x - 28 x^2 + x^3), \\ (-9 + x)^2 (-8 + x) (95 - 20 x + x^2) (59 - 16 x + x^2), \\ (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x) (73 - 18 x + x^2), \\ (-11 + x)^3 (-8 + x) (-7 + x)^3, (-9 + x)^4 (-548 + 213 x - 26 x^2 + x^3), \\ (-9 + x)^3 (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4), (-9 + x)^3 (73 - 18 x + x^2) (68 - 17 x + x^2), \\ (-11 + x) (-9 + x)^2 (-7 + x) (-580 + 217 x - 26 x^2 + x^3), \\ (-11 + x)^2 (-9 + x) (-7 + x)^2 (68 - 17 x + x^2), (-9 + x)^4 (-544 + 213 x - 26 x^2 + x^3), \\ (-9 + x)^3 (4912 - 2461 x + 447 x^2 - 35 x^3 + x^4), \\ (-9 + x)^2 (59 - 16 x + x^2) (-752 + 255 x - 28 x^2 + x^3), \\ (-11 + x) (-9 + x)^3 (-7 + x) (64 - 17 x + x^2), (-12 + x) (-9 + x)^5 (-5 + x), \\ (-9 + x)^3 (4876 - 2457 x + 447 x^2 - 35 x^3 + x^4), (-11 + x)^2 (-9 + x)^2 (-7 + x) (52 - 15 x + x^2), \\ (-9 + x)^4 (-536 + 213 x - 26 x^2 + x^3), (-11 + x)^2 (-9 + x)^3 (-8 + x) (-5 + x), \\ (-9 + x)^4 (-532 + 213 x - 26 x^2 + x^3), (-11 + x) (-9 + x)^4 (48 - 15 x + x^2) \}$$

length 28

feasiblefactorlist[41, (x + 5) ^25 (x - 7) ^8]

$$\left\{ \begin{aligned} &(-13+x)(-9+x)^3(-8+x)(-7+x)^3, \\ &(-9+x)^2(-7+x)^3(-932+293x-30x^2+x^3), (-9+x)^4(-7+x)^2(80-19x+x^2), \\ &(-9+x)^2(-7+x)^3(-928+293x-30x^2+x^3), (-9+x)^4(-7+x)(-556+213x-26x^2+x^3), \\ &(-9+x)^2(-7+x)^2(6452-2975x+503x^2-37x^3+x^4), \\ &(-12+x)(-11+x)(-9+x)^2(-7+x)^4, (-9+x)^4(-8+x)(-7+x)(69-18x+x^2), \\ &(-9+x)^3(-7+x)^2(-712+251x-28x^2+x^3), \\ &(-9+x)^2(-8+x)(-7+x)(95-20x+x^2)(59-16x+x^2), \\ &(-11+x)(-9+x)^2(-8+x)(-7+x)^2(73-18x+x^2), \\ &(-11+x)^3(-8+x)(-7+x)^4, (-9+x)^4(-7+x)(-548+213x-26x^2+x^3), \\ &(-9+x)^3(-7+x)(4948-2465x+447x^2-35x^3+x^4), \\ &(-9+x)^3(-7+x)(73-18x+x^2)(68-17x+x^2), \\ &(-11+x)(-9+x)^2(-7+x)^2(-580+217x-26x^2+x^3), \\ &(-11+x)^2(-9+x)(-7+x)^3(68-17x+x^2), (-9+x)^4(-7+x)(-544+213x-26x^2+x^3), \\ &(-9+x)^3(-7+x)(4912-2461x+447x^2-35x^3+x^4), \\ &(-9+x)^2(-7+x)(59-16x+x^2)(-752+255x-28x^2+x^3), \\ &(-11+x)(-9+x)^3(-7+x)^2(64-17x+x^2), \\ &(-12+x)(-9+x)^5(-7+x)(-5+x), (-9+x)^4(73-18x+x^2)(52-15x+x^2), \\ &(-9+x)^3(-7+x)(4876-2457x+447x^2-35x^3+x^4), \\ &(-11+x)^2(-9+x)^2(-7+x)^2(52-15x+x^2), (-9+x)^4(-8+x)(-467+195x-25x^2+x^3), \\ &(-9+x)^4(-7+x)(-536+213x-26x^2+x^3), (-9+x)^4(3768-2027x+395x^2-33x^3+x^4), \\ &(-11+x)^2(-9+x)^3(-8+x)(-7+x)(-5+x), \\ &(-11+x)(-9+x)^2(-8+x)(59-16x+x^2)^2, (-9+x)^5(-412+179x-24x^2+x^3), \\ &(-9+x)^4(-7+x)(-532+213x-26x^2+x^3), (-11+x)(-9+x)^4(-5+x)(68-17x+x^2), \\ &(-9+x)^4(3680-2019x+395x^2-33x^3+x^4), (-11+x)(-9+x)^4(-7+x)(48-15x+x^2), \\ &(-9+x)^5(-404+179x-24x^2+x^3), (-11+x)(-9+x)^4(-332+153x-22x^2+x^3), \\ &(-11+x)(-9+x)^4(-8+x)(41-14x+x^2), (-11+x)(-9+x)^6(-4+x) \end{aligned} \right\}$$

length 39

feasiblefactorlist[41, (x + 5) ^25 (x - 7) ^7]

$$\left\{ \begin{aligned} &(-13+x)(-9+x)^3(-8+x)(-7+x)^4, (-9+x)^2(-7+x)^4(-932+293x-30x^2+x^3), \\ &(-9+x)^4(-7+x)^3(80-19x+x^2), (-9+x)^2(-7+x)^4(-928+293x-30x^2+x^3), \\ &(-9+x)^4(-7+x)^2(-556+213x-26x^2+x^3), \\ &(-9+x)^2(-7+x)^3(6452-2975x+503x^2-37x^3+x^4), \\ &(-12+x)(-11+x)(-9+x)^2(-7+x)^5, (-9+x)^4(-8+x)(-7+x)^2(69-18x+x^2), \\ &(-9+x)^3(-7+x)^3(-712+251x-28x^2+x^3), \\ &(-9+x)^2(-8+x)(-7+x)^2(95-20x+x^2)(59-16x+x^2), \\ &(-11+x)(-9+x)^2(-8+x)(-7+x)^3(73-18x+x^2), \\ &(-11+x)^3(-8+x)(-7+x)^5, (-9+x)^4(-7+x)^2(-548+213x-26x^2+x^3), \end{aligned} \right\}$$

$$\begin{aligned}
& (-9+x)^3 (-7+x)^2 (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^3 (-7+x)^2 (73 - 18x + x^2) (68 - 17x + x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^3 (-580 + 217x - 26x^2 + x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^4 (68 - 17x + x^2), (-9+x)^4 (-7+x)^2 (-544 + 213x - 26x^2 + x^3), \\
& (-9+x)^3 (-7+x)^2 (4912 - 2461x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^2 (-7+x)^2 (59 - 16x + x^2) (-752 + 255x - 28x^2 + x^3), \\
& (-11+x) (-9+x)^3 (-7+x)^3 (64 - 17x + x^2), (-12+x) (-9+x)^5 (-7+x)^2 (-5+x), \\
& (-9+x)^4 (-7+x) (73 - 18x + x^2) (52 - 15x + x^2), \\
& (-9+x)^3 (-7+x)^2 (4876 - 2457x + 447x^2 - 35x^3 + x^4), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^3 (52 - 15x + x^2), \\
& (-9+x)^4 (-8+x) (-7+x) (-467 + 195x - 25x^2 + x^3), \\
& (-9+x)^5 (-8+x) (-5+x) (73 - 18x + x^2), (-9+x)^4 (-7+x)^2 (-536 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x) (3768 - 2027x + 395x^2 - 33x^3 + x^4), \\
& (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^2 (-5+x), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x) (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x) (-412 + 179x - 24x^2 + x^3), (-9+x)^5 (-5+x) (-580 + 217x - 26x^2 + x^3), \\
& (-9+x)^4 (-26084 + 17885x - 4788x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^2 (-532 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (59 - 16x + x^2) (-444 + 183x - 24x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-7+x) (-5+x) (68 - 17x + x^2), \\
& (-9+x)^3 (68 - 17x + x^2) (59 - 16x + x^2)^2, (-9+x)^5 (2864 - 1661x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^4 (-7+x) (3680 - 2019x + 395x^2 - 33x^3 + x^4), (-9+x)^6 (-5+x) (64 - 17x + x^2), \\
& (-9+x)^4 (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-25888 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^2 (48 - 15x + x^2), \\
& (-9+x)^5 (-7+x) (-404 + 179x - 24x^2 + x^3), (-9+x)^6 (-316 + 149x - 22x^2 + x^3), \\
& (-9+x)^4 (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x) (-332 + 153x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^5 (-5+x) (52 - 15x + x^2), \\
& (-9+x)^4 (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\
& (-9+x)^5 (-8+x) (-349 + 163x - 23x^2 + x^3), (-9+x)^6 (-312 + 149x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x) (41 - 14x + x^2), \\
& (-9+x)^5 (2824 - 1653x + 347x^2 - 31x^3 + x^4), (-11+x) (-9+x)^6 (-7+x) (-4+x), \\
& (-9+x)^5 (68 - 17x + x^2) (41 - 14x + x^2), (-9+x)^6 (-304 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-300 + 149x - 22x^2 + x^3), (-9+x)^6 (-8+x) (37 - 14x + x^2), \\
& (-9+x)^6 (-292 + 149x - 22x^2 + x^3), (-9+x)^7 (32 - 13x + x^2) \}
\end{aligned}$$

length 61

feasiblefactorlist[41, (x+5)^25 (x-7)^6]

$$\begin{aligned}
& \{ (-13+x) (-9+x)^3 (-8+x) (-7+x)^5, (-9+x)^2 (-7+x)^5 (-932 + 293x - 30x^2 + x^3), \\
& (-9+x)^4 (-7+x)^4 (80 - 19x + x^2), (-9+x)^2 (-7+x)^5 (-928 + 293x - 30x^2 + x^3),
\end{aligned}$$

$$\begin{aligned}
& (-9+x)^4 (-7+x)^3 (-556+213x-26x^2+x^3), \\
& (-9+x)^2 (-7+x)^4 (6452-2975x+503x^2-37x^3+x^4), \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^6, (-9+x)^4 (-8+x) (-7+x)^3 (69-18x+x^2), \\
& (-9+x)^3 (-7+x)^4 (-712+251x-28x^2+x^3), \\
& (-9+x)^2 (-8+x) (-7+x)^3 (95-20x+x^2) (59-16x+x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^4 (73-18x+x^2), \\
& (-11+x)^3 (-8+x) (-7+x)^6, (-9+x)^4 (-7+x)^3 (-548+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^3 (4948-2465x+447x^2-35x^3+x^4), \\
& (-9+x)^3 (-7+x)^3 (73-18x+x^2) (68-17x+x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^4 (-580+217x-26x^2+x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^5 (68-17x+x^2), (-9+x)^4 (-7+x)^3 (-544+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^3 (4912-2461x+447x^2-35x^3+x^4), \\
& (-9+x)^2 (-7+x)^3 (59-16x+x^2) (-752+255x-28x^2+x^3), \\
& (-11+x) (-9+x)^3 (-7+x)^4 (64-17x+x^2), (-12+x) (-9+x)^5 (-7+x)^3 (-5+x), \\
& (-9+x)^4 (-7+x)^2 (73-18x+x^2) (52-15x+x^2), \\
& (-9+x)^3 (-7+x)^3 (4876-2457x+447x^2-35x^3+x^4), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^4 (52-15x+x^2), \\
& (-9+x)^4 (-8+x) (-7+x)^2 (-467+195x-25x^2+x^3), \\
& (-9+x)^5 (-8+x) (-7+x) (-5+x) (73-18x+x^2), \\
& (-9+x)^4 (-7+x)^3 (-536+213x-26x^2+x^3), \\
& (-9+x)^4 (-7+x)^2 (3768-2027x+395x^2-33x^3+x^4), \\
& (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^3 (-5+x), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^2 (59-16x+x^2)^2, \\
& (-9+x)^5 (-7+x)^2 (-412+179x-24x^2+x^3), \\
& (-9+x)^5 (-7+x) (-5+x) (-580+217x-26x^2+x^3), \\
& (-9+x)^4 (-7+x) (-26084+17885x-4788x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4 (-7+x)^3 (-532+213x-26x^2+x^3), \\
& (-9+x)^4 (-7+x) (59-16x+x^2) (-444+183x-24x^2+x^3), \\
& (-11+x) (-9+x)^4 (-7+x)^2 (-5+x) (68-17x+x^2), \\
& (-9+x)^3 (-7+x) (68-17x+x^2) (59-16x+x^2)^2, \\
& (-9+x)^5 (-7+x) (2864-1661x+347x^2-31x^3+x^4), \\
& (-9+x)^4 (-7+x)^2 (3680-2019x+395x^2-33x^3+x^4), \\
& (-9+x)^6 (-7+x) (-5+x) (64-17x+x^2), \\
& (-9+x)^4 (-7+x) (-25904+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4 (-7+x) (-25888+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^3 (48-15x+x^2), \\
& (-9+x)^5 (-7+x)^2 (-404+179x-24x^2+x^3), (-9+x)^6 (-7+x) (-316+149x-22x^2+x^3), \\
& (-9+x)^4 (-7+x) (-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^2 (-332+153x-22x^2+x^3), \\
& (-11+x) (-9+x)^5 (-7+x) (-5+x) (52-15x+x^2), \\
& (-9+x)^4 (-7+x) (59-16x+x^2) (-436+183x-24x^2+x^3), \\
& (-9+x)^4 (59-16x+x^2)^2 (52-15x+x^2),
\end{aligned}$$

$$\begin{aligned}
& (-9+x)^5 (-8+x) (-7+x) (-349+163x-23x^2+x^3), \\
& (-9+x)^6 (-7+x) (-312+149x-22x^2+x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^2 (41-14x+x^2), \\
& (-11+x) (-9+x)^6 (-8+x) (-5+x)^2, \\
& (-9+x)^5 (-7+x) (2824-1653x+347x^2-31x^3+x^4), \\
& (-11+x) (-9+x)^6 (-7+x)^2 (-4+x), (-9+x)^6 (2172-1351x+303x^2-29x^3+x^4), \\
& (-9+x)^5 (-7+x) (68-17x+x^2) (41-14x+x^2), \\
& (-9+x)^6 (-7+x) (-304+149x-22x^2+x^3), (-9+x)^6 (2144-1347x+303x^2-29x^3+x^4), \\
& (-9+x)^7 (-5+x) (48-15x+x^2), (-9+x)^6 (-7+x) (-300+149x-22x^2+x^3), \\
& (-9+x)^6 (2116-1343x+303x^2-29x^3+x^4), \\
& (-9+x)^6 (52-15x+x^2) (41-14x+x^2), (-9+x)^6 (-8+x) (-7+x) (37-14x+x^2), \\
& (-9+x)^7 (-232+123x-20x^2+x^3), (-9+x)^6 (-7+x) (-292+149x-22x^2+x^3), \\
& (-9+x)^7 (-7+x) (32-13x+x^2), (-9+x)^8 (-8+x) (-3+x) \}
\end{aligned}$$

length 70

feasiblefactorlist[41, (x+5)^25 (x-7)^5]

$$\begin{aligned}
& \{ (-13+x) (-9+x)^3 (-8+x) (-7+x)^6, (-9+x)^2 (-7+x)^6 (-932+293x-30x^2+x^3), \\
& (-9+x)^4 (-7+x)^5 (80-19x+x^2), (-9+x)^2 (-7+x)^6 (-928+293x-30x^2+x^3), \\
& (-9+x)^4 (-7+x)^4 (-556+213x-26x^2+x^3), \\
& (-9+x)^2 (-7+x)^5 (6452-2975x+503x^2-37x^3+x^4), \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^7, (-9+x)^4 (-8+x) (-7+x)^4 (69-18x+x^2), \\
& (-9+x)^3 (-7+x)^5 (-712+251x-28x^2+x^3), \\
& (-9+x)^2 (-8+x) (-7+x)^4 (95-20x+x^2) (59-16x+x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^5 (73-18x+x^2), \\
& (-11+x)^3 (-8+x) (-7+x)^7, (-9+x)^4 (-7+x)^4 (-548+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^4 (4948-2465x+447x^2-35x^3+x^4), \\
& (-9+x)^3 (-7+x)^4 (73-18x+x^2) (68-17x+x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^5 (-580+217x-26x^2+x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^6 (68-17x+x^2), (-9+x)^4 (-7+x)^4 (-544+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^4 (4912-2461x+447x^2-35x^3+x^4), \\
& (-9+x)^2 (-7+x)^4 (59-16x+x^2) (-752+255x-28x^2+x^3), \\
& (-11+x) (-9+x)^3 (-7+x)^5 (64-17x+x^2), (-12+x) (-9+x)^5 (-7+x)^4 (-5+x), \\
& (-9+x)^4 (-7+x)^3 (73-18x+x^2) (52-15x+x^2), \\
& (-9+x)^3 (-7+x)^4 (4876-2457x+447x^2-35x^3+x^4), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^5 (52-15x+x^2), \\
& (-9+x)^4 (-8+x) (-7+x)^3 (-467+195x-25x^2+x^3), \\
& (-9+x)^5 (-8+x) (-7+x)^2 (-5+x) (73-18x+x^2), \\
& (-9+x)^4 (-7+x)^4 (-536+213x-26x^2+x^3), \\
& (-9+x)^4 (-7+x)^3 (3768-2027x+395x^2-33x^3+x^4), \\
& (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^4 (-5+x),
\end{aligned}$$

$$\begin{aligned}
& (-11+x)(-9+x)^2(-8+x)(-7+x)^3(59-16x+x^2)^2, \\
& (-9+x)^5(-7+x)^3(-412+179x-24x^2+x^3), \\
& (-9+x)^5(-7+x)^2(-5+x)(-580+217x-26x^2+x^3), \\
& (-9+x)^4(-7+x)^2(-26084+17885x-4788x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4(-7+x)^4(-532+213x-26x^2+x^3), \\
& (-9+x)^4(-7+x)^2(59-16x+x^2)(-444+183x-24x^2+x^3), \\
& (-11+x)(-9+x)^4(-7+x)^3(-5+x)(68-17x+x^2), \\
& (-9+x)^3(-7+x)^2(68-17x+x^2)(59-16x+x^2)^2, \\
& (-9+x)^5(-7+x)^2(2864-1661x+347x^2-31x^3+x^4), \\
& (-9+x)^4(-7+x)^3(3680-2019x+395x^2-33x^3+x^4), \\
& (-9+x)^6(-7+x)^2(-5+x)(64-17x+x^2), \\
& (-9+x)^4(-7+x)^2(-25904+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4(-7+x)^2(-25888+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-11+x)(-9+x)^4(-7+x)^4(48-15x+x^2), (-9+x)^5(-7+x)^3(-404+179x-24x^2+x^3), \\
& (-9+x)^6(-7+x)^2(-316+149x-22x^2+x^3), \\
& (-9+x)^4(-7+x)^2(-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\
& (-11+x)(-9+x)^4(-7+x)^3(-332+153x-22x^2+x^3), \\
& (-11+x)(-9+x)^5(-7+x)^2(-5+x)(52-15x+x^2), \\
& (-9+x)^4(-7+x)^2(59-16x+x^2)(-436+183x-24x^2+x^3), \\
& (-9+x)^4(-7+x)(59-16x+x^2)^2(52-15x+x^2), \\
& (-9+x)^5(-8+x)(-7+x)^2(-349+163x-23x^2+x^3), \\
& (-9+x)^6(-7+x)^2(-312+149x-22x^2+x^3), \\
& (-11+x)(-9+x)^4(-8+x)(-7+x)^3(41-14x+x^2), \\
& (-11+x)(-9+x)^6(-8+x)(-7+x)(-5+x)^2, \\
& (-9+x)^5(-7+x)^2(2824-1653x+347x^2-31x^3+x^4), \\
& (-9+x)^5(-8+x)(-5+x)(59-16x+x^2)^2, (-11+x)(-9+x)^6(-7+x)^3(-4+x), \\
& (-9+x)^6(-7+x)(2172-1351x+303x^2-29x^3+x^4), \\
& (-9+x)^5(-7+x)^2(68-17x+x^2)(41-14x+x^2), \\
& (-9+x)^7(-5+x)^2(68-17x+x^2), (-9+x)^6(-7+x)^2(-304+149x-22x^2+x^3), \\
& (-9+x)^6(-7+x)(2144-1347x+303x^2-29x^3+x^4), \\
& (-9+x)^7(-7+x)(-5+x)(48-15x+x^2), \\
& (-9+x)^6(59-16x+x^2)(-256+127x-20x^2+x^3), \\
& (-9+x)^6(-7+x)^2(-300+149x-22x^2+x^3), \\
& (-9+x)^6(-7+x)(2116-1343x+303x^2-29x^3+x^4), \\
& (-9+x)^7(-5+x)(-332+153x-22x^2+x^3), \\
& (-9+x)^6(-7+x)(52-15x+x^2)(41-14x+x^2), \\
& (-9+x)^6(-8+x)(-7+x)^2(37-14x+x^2), \\
& (-9+x)^6(-8+x)(59-16x+x^2)(31-12x+x^2), \\
& (-9+x)^7(-7+x)(-232+123x-20x^2+x^3), (-9+x)^7(-8+x)(-5+x)(41-14x+x^2), \\
& (-9+x)^6(-7+x)^2(-292+149x-22x^2+x^3), \\
& (-9+x)^7(1604-1089x+263x^2-27x^3+x^4), (-9+x)^9(-5+x)(-4+x), \\
& (-9+x)^7(-7+x)^2(32-13x+x^2), (-9+x)^8(-176+101x-18x^2+x^3),
\end{aligned}$$

$$\{(-9+x)^8(-172+101x-18x^2+x^3), (-9+x)^8(-8+x)(-7+x)(-3+x)\}$$

length 80

feasiblefactorlist[41, (x+5)^25(x-7)^4]

$$\begin{aligned} &\{(-13+x)(-9+x)^3(-8+x)(-7+x)^7, (-9+x)^2(-7+x)^7(-932+293x-30x^2+x^3), \\ &(-9+x)^4(-7+x)^6(80-19x+x^2), (-9+x)^2(-7+x)^7(-928+293x-30x^2+x^3), \\ &(-9+x)^4(-7+x)^5(-556+213x-26x^2+x^3), \\ &(-9+x)^2(-7+x)^6(6452-2975x+503x^2-37x^3+x^4), \\ &(-12+x)(-11+x)(-9+x)^2(-7+x)^8, (-9+x)^4(-8+x)(-7+x)^5(69-18x+x^2), \\ &(-9+x)^3(-7+x)^6(-712+251x-28x^2+x^3), \\ &(-9+x)^2(-8+x)(-7+x)^5(95-20x+x^2)(59-16x+x^2), \\ &(-11+x)(-9+x)^2(-8+x)(-7+x)^6(73-18x+x^2), \\ &(-11+x)^3(-8+x)(-7+x)^8, (-9+x)^4(-7+x)^5(-548+213x-26x^2+x^3), \\ &(-9+x)^3(-7+x)^5(4948-2465x+447x^2-35x^3+x^4), \\ &(-9+x)^3(-7+x)^5(73-18x+x^2)(68-17x+x^2), \\ &(-11+x)(-9+x)^2(-7+x)^6(-580+217x-26x^2+x^3), \\ &(-11+x)^2(-9+x)(-7+x)^7(68-17x+x^2), (-9+x)^4(-7+x)^5(-544+213x-26x^2+x^3), \\ &(-9+x)^3(-7+x)^5(4912-2461x+447x^2-35x^3+x^4), \\ &(-9+x)^2(-7+x)^5(59-16x+x^2)(-752+255x-28x^2+x^3), \\ &(-11+x)(-9+x)^3(-7+x)^6(64-17x+x^2), (-12+x)(-9+x)^5(-7+x)^5(-5+x), \\ &(-9+x)^4(-7+x)^4(73-18x+x^2)(52-15x+x^2), \\ &(-9+x)^3(-7+x)^5(4876-2457x+447x^2-35x^3+x^4), \\ &(-11+x)^2(-9+x)^2(-7+x)^6(52-15x+x^2), \\ &(-9+x)^4(-8+x)(-7+x)^4(-467+195x-25x^2+x^3), \\ &(-9+x)^5(-8+x)(-7+x)^3(-5+x)(73-18x+x^2), \\ &(-9+x)^4(-7+x)^5(-536+213x-26x^2+x^3), \\ &(-9+x)^4(-7+x)^4(3768-2027x+395x^2-33x^3+x^4), \\ &(-11+x)^2(-9+x)^3(-8+x)(-7+x)^5(-5+x), \\ &(-11+x)(-9+x)^2(-8+x)(-7+x)^4(59-16x+x^2)^2, \\ &(-9+x)^5(-7+x)^4(-412+179x-24x^2+x^3), \\ &(-9+x)^5(-7+x)^3(-5+x)(-580+217x-26x^2+x^3), \\ &(-9+x)^4(-7+x)^3(-26084+17885x-4788x^2+626x^3-40x^4+x^5), \\ &(-9+x)^4(-7+x)^5(-532+213x-26x^2+x^3), \\ &(-9+x)^4(-7+x)^3(59-16x+x^2)(-444+183x-24x^2+x^3), \\ &(-11+x)(-9+x)^4(-7+x)^4(-5+x)(68-17x+x^2), \\ &(-9+x)^3(-7+x)^3(68-17x+x^2)(59-16x+x^2)^2, \\ &(-9+x)^5(-7+x)^3(2864-1661x+347x^2-31x^3+x^4), \\ &(-9+x)^4(-7+x)^4(3680-2019x+395x^2-33x^3+x^4), \\ &(-9+x)^6(-7+x)^3(-5+x)(64-17x+x^2), \\ &(-9+x)^4(-7+x)^3(-25904+17829x-4784x^2+626x^3-40x^4+x^5), \end{aligned}$$

$$\begin{aligned}
& (-9+x)^4 (-7+x)^3 (-25888+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^5 (48-15x+x^2), (-9+x)^5 (-7+x)^4 (-404+179x-24x^2+x^3), \\
& (-9+x)^6 (-7+x)^3 (-316+149x-22x^2+x^3), \\
& (-9+x)^4 (-7+x)^3 (-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^4 (-332+153x-22x^2+x^3), \\
& (-11+x) (-9+x)^5 (-7+x)^3 (-5+x) (52-15x+x^2), \\
& (-9+x)^4 (-7+x)^3 (59-16x+x^2) (-436+183x-24x^2+x^3), \\
& (-9+x)^4 (-7+x)^2 (59-16x+x^2)^2 (52-15x+x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^3 (-349+163x-23x^2+x^3), \\
& (-9+x)^6 (-7+x)^3 (-312+149x-22x^2+x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^4 (41-14x+x^2), \\
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^2 (-5+x)^2, \\
& (-9+x)^5 (-7+x)^3 (2824-1653x+347x^2-31x^3+x^4), \\
& (-9+x)^5 (-8+x) (-7+x) (-5+x) (59-16x+x^2)^2, (-11+x) (-9+x)^6 (-7+x)^4 (-4+x), \\
& (-9+x)^6 (-7+x)^2 (2172-1351x+303x^2-29x^3+x^4), \\
& (-9+x)^5 (-7+x)^3 (68-17x+x^2) (41-14x+x^2), \\
& (-9+x)^7 (-7+x) (-5+x)^2 (68-17x+x^2), (-9+x)^6 (-7+x)^3 (-304+149x-22x^2+x^3), \\
& (-9+x)^6 (-7+x)^2 (2144-1347x+303x^2-29x^3+x^4), \\
& (-9+x)^7 (-7+x)^2 (-5+x) (48-15x+x^2), \\
& (-9+x)^6 (-7+x) (59-16x+x^2) (-256+127x-20x^2+x^3), \\
& (-9+x)^6 (-7+x)^3 (-300+149x-22x^2+x^3), \\
& (-9+x)^6 (-7+x)^2 (2116-1343x+303x^2-29x^3+x^4), \\
& (-9+x)^7 (-7+x) (-5+x) (-332+153x-22x^2+x^3), \\
& (-9+x)^6 (-7+x)^2 (52-15x+x^2) (41-14x+x^2), \\
& (-9+x)^8 (-5+x)^2 (52-15x+x^2), (-9+x)^6 (-8+x) (-7+x)^3 (37-14x+x^2), \\
& (-9+x)^6 (-8+x) (-7+x) (59-16x+x^2) (31-12x+x^2), \\
& (-9+x)^7 (-7+x)^2 (-232+123x-20x^2+x^3), \\
& (-9+x)^7 (-8+x) (-7+x) (-5+x) (41-14x+x^2), \\
& (-9+x)^6 (-7+x)^3 (-292+149x-22x^2+x^3), \\
& (-9+x)^7 (-7+x) (1604-1089x+263x^2-27x^3+x^4), (-9+x)^9 (-7+x) (-5+x) (-4+x), \\
& (-9+x)^7 (-7+x)^3 (32-13x+x^2), (-9+x)^8 (-7+x) (-176+101x-18x^2+x^3), \\
& (-9+x)^8 (-7+x) (-172+101x-18x^2+x^3), (-9+x)^8 (-8+x) (-7+x)^2 (-3+x) \}
\end{aligned}$$

length 81

feasiblefactorlist[41, (x+5)^25 (x-7)^3]

$$\begin{aligned}
& \{ (-13+x) (-9+x)^3 (-8+x) (-7+x)^8, (-9+x)^2 (-7+x)^8 (-932+293x-30x^2+x^3), \\
& (-9+x)^4 (-7+x)^7 (80-19x+x^2), (-9+x)^2 (-7+x)^8 (-928+293x-30x^2+x^3), \\
& (-9+x)^4 (-7+x)^6 (-556+213x-26x^2+x^3), \\
& (-9+x)^2 (-7+x)^7 (6452-2975x+503x^2-37x^3+x^4), \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^9, (-9+x)^4 (-8+x) (-7+x)^6 (69-18x+x^2), \}
\end{aligned}$$

$$\begin{aligned}
& (-9+x)^3 (-7+x)^7 (-712+251x-28x^2+x^3), \\
& (-9+x)^2 (-8+x) (-7+x)^6 (95-20x+x^2) (59-16x+x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^7 (73-18x+x^2), \\
& (-11+x)^3 (-8+x) (-7+x)^9, (-9+x)^4 (-7+x)^6 (-548+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^6 (4948-2465x+447x^2-35x^3+x^4), \\
& (-9+x)^3 (-7+x)^6 (73-18x+x^2) (68-17x+x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^7 (-580+217x-26x^2+x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^8 (68-17x+x^2), (-9+x)^4 (-7+x)^6 (-544+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^6 (4912-2461x+447x^2-35x^3+x^4), \\
& (-9+x)^2 (-7+x)^6 (59-16x+x^2) (-752+255x-28x^2+x^3), \\
& (-11+x) (-9+x)^3 (-7+x)^7 (64-17x+x^2), (-12+x) (-9+x)^5 (-7+x)^6 (-5+x), \\
& (-9+x)^4 (-7+x)^5 (73-18x+x^2) (52-15x+x^2), \\
& (-9+x)^3 (-7+x)^6 (4876-2457x+447x^2-35x^3+x^4), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^7 (52-15x+x^2), \\
& (-9+x)^4 (-8+x) (-7+x)^5 (-467+195x-25x^2+x^3), \\
& (-9+x)^5 (-8+x) (-7+x)^4 (-5+x) (73-18x+x^2), \\
& (-9+x)^4 (-7+x)^6 (-536+213x-26x^2+x^3), \\
& (-9+x)^4 (-7+x)^5 (3768-2027x+395x^2-33x^3+x^4), \\
& (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^6 (-5+x), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^5 (59-16x+x^2)^2, \\
& (-9+x)^5 (-7+x)^5 (-412+179x-24x^2+x^3), \\
& (-9+x)^5 (-7+x)^4 (-5+x) (-580+217x-26x^2+x^3), \\
& (-9+x)^4 (-7+x)^4 (-26084+17885x-4788x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4 (-7+x)^6 (-532+213x-26x^2+x^3), \\
& (-9+x)^4 (-7+x)^4 (59-16x+x^2) (-444+183x-24x^2+x^3), \\
& (-11+x) (-9+x)^4 (-7+x)^5 (-5+x) (68-17x+x^2), \\
& (-9+x)^3 (-7+x)^4 (68-17x+x^2) (59-16x+x^2)^2, \\
& (-9+x)^5 (-7+x)^4 (2864-1661x+347x^2-31x^3+x^4), \\
& (-9+x)^4 (-7+x)^5 (3680-2019x+395x^2-33x^3+x^4), \\
& (-9+x)^6 (-7+x)^4 (-5+x) (64-17x+x^2), \\
& (-9+x)^4 (-7+x)^4 (-25904+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4 (-7+x)^4 (-25888+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^6 (48-15x+x^2), (-9+x)^5 (-7+x)^5 (-404+179x-24x^2+x^3), \\
& (-9+x)^6 (-7+x)^4 (-316+149x-22x^2+x^3), \\
& (-9+x)^4 (-7+x)^4 (-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^5 (-332+153x-22x^2+x^3), \\
& (-11+x) (-9+x)^5 (-7+x)^4 (-5+x) (52-15x+x^2), \\
& (-9+x)^4 (-7+x)^4 (59-16x+x^2) (-436+183x-24x^2+x^3), \\
& (-9+x)^4 (-7+x)^3 (59-16x+x^2)^2 (52-15x+x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^4 (-349+163x-23x^2+x^3), \\
& (-9+x)^6 (-7+x)^4 (-312+149x-22x^2+x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^5 (41-14x+x^2),
\end{aligned}$$

$$\begin{aligned}
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^3 (-5+x)^2, \\
& (-9+x)^5 (-7+x)^4 (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^5 (-8+x) (-7+x)^2 (-5+x) (59 - 16x + x^2)^2, (-11+x) (-9+x)^6 (-7+x)^5 (-4+x), \\
& (-9+x)^6 (-7+x)^3 (2172 - 1351x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^5 (-7+x)^4 (68 - 17x + x^2) (41 - 14x + x^2), \\
& (-9+x)^7 (-7+x)^2 (-5+x)^2 (68 - 17x + x^2), (-9+x)^6 (-7+x)^4 (-304 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (2144 - 1347x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^3 (-5+x) (48 - 15x + x^2), \\
& (-9+x)^6 (-7+x)^2 (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^4 (-300 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (2116 - 1343x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^2 (-5+x) (-332 + 153x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (52 - 15x + x^2) (41 - 14x + x^2), \\
& (-9+x)^8 (-7+x) (-5+x)^2 (52 - 15x + x^2), (-9+x)^6 (-8+x) (-7+x)^4 (37 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^2 (59 - 16x + x^2) (31 - 12x + x^2), \\
& (-9+x)^7 (-7+x)^3 (-232 + 123x - 20x^2 + x^3), \\
& (-9+x)^7 (-8+x) (-7+x)^2 (-5+x) (41 - 14x + x^2), \\
& (-9+x)^9 (-8+x) (-5+x)^3, (-9+x)^6 (-7+x)^4 (-292 + 149x - 22x^2 + x^3), \\
& (-9+x)^7 (-7+x)^2 (1604 - 1089x + 263x^2 - 27x^3 + x^4), \\
& (-9+x)^9 (-7+x)^2 (-5+x) (-4+x), (-9+x)^7 (-7+x)^4 (32 - 13x + x^2), \\
& (-9+x)^8 (-7+x)^2 (-176 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-7+x)^2 (-172 + 101x - 18x^2 + x^3), (-9+x)^8 (-8+x) (-7+x)^3 (-3+x) \}
\end{aligned}$$

$$\begin{aligned}
\text{list} = & \{ (-13+x) (-9+x)^3 (-8+x) (-7+x)^{11} (5+x)^{25}, \\
& (-9+x)^2 (-7+x)^{11} (5+x)^{25} (-932 + 293x - 30x^2 + x^3), \\
& (-9+x)^4 (-7+x)^{10} (5+x)^{25} (80 - 19x + x^2), (-9+x)^2 (-7+x)^{11} (5+x)^{25} \\
& (-928 + 293x - 30x^2 + x^3), (-9+x)^4 (-7+x)^9 (5+x)^{25} (-556 + 213x - 26x^2 + x^3), \\
& (-9+x)^2 (-7+x)^{10} (5+x)^{25} (6452 - 2975x + 503x^2 - 37x^3 + x^4), \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^4 (-8+x) (-7+x)^9 (5+x)^{25} (69 - 18x + x^2), \\
& (-9+x)^3 (-7+x)^{10} (5+x)^{25} (-712 + 251x - 28x^2 + x^3), \\
& (-9+x)^2 (-8+x) (-7+x)^9 (5+x)^{25} (95 - 20x + x^2) (59 - 16x + x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^{10} (5+x)^{25} (73 - 18x + x^2), \\
& (-11+x)^3 (-8+x) (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-548 + 213x - 26x^2 + x^3), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (73 - 18x + x^2) (68 - 17x + x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^{10} (5+x)^{25} (-580 + 217x - 26x^2 + x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^{11} (5+x)^{25} (68 - 17x + x^2), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-544 + 213x - 26x^2 + x^3), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4912 - 2461x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^2 (-7+x)^9 (5+x)^{25} (59 - 16x + x^2) (-752 + 255x - 28x^2 + x^3),
\end{aligned}$$

$$\begin{aligned}
& (-11+x)(-9+x)^3(-7+x)^{10}(5+x)^{25}(64-17x+x^2), \\
& (-12+x)(-9+x)^5(-7+x)^9(-5+x)(5+x)^{25}, \\
& (-9+x)^4(-7+x)^8(5+x)^{25}(73-18x+x^2)(52-15x+x^2), \\
& (-9+x)^3(-7+x)^9(5+x)^{25}(4876-2457x+447x^2-35x^3+x^4), \\
& (-11+x)^2(-9+x)^2(-7+x)^{10}(5+x)^{25}(52-15x+x^2), \\
& (-9+x)^4(-8+x)(-7+x)^8(5+x)^{25}(-467+195x-25x^2+x^3), \\
& (-9+x)^5(-8+x)(-7+x)^7(-5+x)(5+x)^{25}(73-18x+x^2), \\
& (-9+x)^4(-7+x)^9(5+x)^{25}(-536+213x-26x^2+x^3), \\
& (-9+x)^4(-7+x)^8(5+x)^{25}(3768-2027x+395x^2-33x^3+x^4), \\
& (-11+x)^2(-9+x)^3(-8+x)(-7+x)^9(-5+x)(5+x)^{25}, \\
& (-11+x)(-9+x)^2(-8+x)(-7+x)^8(5+x)^{25}(59-16x+x^2)^2, \\
& (-9+x)^5(-7+x)^8(5+x)^{25}(-412+179x-24x^2+x^3), \\
& (-9+x)^5(-7+x)^7(-5+x)(5+x)^{25}(-580+217x-26x^2+x^3), \\
& (-9+x)^4(-7+x)^7(5+x)^{25}(-26084+17885x-4788x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4(-7+x)^9(5+x)^{25}(-532+213x-26x^2+x^3), \\
& (-9+x)^4(-7+x)^7(5+x)^{25}(59-16x+x^2)(-444+183x-24x^2+x^3), \\
& (-11+x)(-9+x)^4(-7+x)^8(-5+x)(5+x)^{25}(68-17x+x^2), \\
& (-9+x)^3(-7+x)^7(5+x)^{25}(68-17x+x^2)(59-16x+x^2)^2, \\
& (-9+x)^5(-7+x)^7(5+x)^{25}(2864-1661x+347x^2-31x^3+x^4), \\
& (-9+x)^4(-7+x)^8(5+x)^{25}(3680-2019x+395x^2-33x^3+x^4), \\
& (-9+x)^6(-7+x)^7(-5+x)(5+x)^{25}(64-17x+x^2), \\
& (-9+x)^4(-7+x)^7(5+x)^{25}(-25904+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4(-7+x)^7(5+x)^{25}(-25888+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-11+x)(-9+x)^4(-7+x)^9(5+x)^{25}(48-15x+x^2), \\
& (-9+x)^5(-7+x)^8(5+x)^{25}(-404+179x-24x^2+x^3), \\
& (-9+x)^6(-7+x)^7(5+x)^{25}(-316+149x-22x^2+x^3), \\
& (-9+x)^4(-7+x)^7(5+x)^{25}(-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\
& (-11+x)(-9+x)^4(-7+x)^8(5+x)^{25}(-332+153x-22x^2+x^3), \\
& (-11+x)(-9+x)^5(-7+x)^7(-5+x)(5+x)^{25}(52-15x+x^2), \\
& (-9+x)^4(-7+x)^7(5+x)^{25}(59-16x+x^2)(-436+183x-24x^2+x^3), \\
& (-9+x)^4(-7+x)^6(5+x)^{25}(59-16x+x^2)^2(52-15x+x^2), \\
& (-9+x)^5(-8+x)(-7+x)^7(5+x)^{25}(-349+163x-23x^2+x^3), \\
& (-9+x)^6(-7+x)^7(5+x)^{25}(-312+149x-22x^2+x^3), \\
& (-11+x)(-9+x)^4(-8+x)(-7+x)^8(5+x)^{25}(41-14x+x^2), \\
& (-11+x)(-9+x)^6(-8+x)(-7+x)^6(-5+x)^2(5+x)^{25}, \\
& (-9+x)^5(-7+x)^7(5+x)^{25}(2824-1653x+347x^2-31x^3+x^4), \\
& (-9+x)^5(-8+x)(-7+x)^5(-5+x)(5+x)^{25}(59-16x+x^2)^2, \\
& (-11+x)(-9+x)^6(-7+x)^8(-4+x)(5+x)^{25}, \\
& (-9+x)^6(-7+x)^6(5+x)^{25}(2172-1351x+303x^2-29x^3+x^4), \\
& (-9+x)^5(-7+x)^7(5+x)^{25}(68-17x+x^2)(41-14x+x^2), \\
& (-9+x)^7(-7+x)^5(-5+x)^2(5+x)^{25}(68-17x+x^2), \\
& (-9+x)^6(-7+x)^7(5+x)^{25}(-304+149x-22x^2+x^3),
\end{aligned}$$

$$\begin{aligned}
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (2144 - 1347x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^6 (-5+x) (5+x)^{25} (48 - 15x + x^2), \\
& (-9+x)^6 (-7+x)^5 (5+x)^{25} (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-300 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (2116 - 1343x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^5 (-5+x) (5+x)^{25} (-332 + 153x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (52 - 15x + x^2) (41 - 14x + x^2), \\
& (-9+x)^8 (-7+x)^4 (-5+x)^2 (5+x)^{25} (52 - 15x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^7 (5+x)^{25} (37 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^5 (5+x)^{25} (59 - 16x + x^2) (31 - 12x + x^2), \\
& (-9+x)^7 (-7+x)^6 (5+x)^{25} (-232 + 123x - 20x^2 + x^3), \\
& (-9+x)^7 (-8+x) (-7+x)^5 (-5+x) (5+x)^{25} (41 - 14x + x^2), \\
& (-9+x)^9 (-8+x) (-7+x)^3 (-5+x)^3 (5+x)^{25}, \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-292 + 149x - 22x^2 + x^3), \\
& (-9+x)^7 (-7+x)^5 (5+x)^{25} (1604 - 1089x + 263x^2 - 27x^3 + x^4), \\
& (-9+x)^9 (-7+x)^5 (-5+x) (-4+x) (5+x)^{25}, (-9+x)^7 (-7+x)^7 (5+x)^{25} \\
& (32 - 13x + x^2), (-9+x)^8 (-7+x)^5 (5+x)^{25} (-176 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-7+x)^5 (5+x)^{25} (-172 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-8+x) (-7+x)^6 (-3+x) (5+x)^{25} \};
\end{aligned}$$

Length[list]

82

listmod64 = modfilter[list, Smod64n41unrefined, 64]

$$\begin{aligned}
& \{ (-9+x)^4 (-7+x)^{10} (5+x)^{25} (80-19x+x^2), \\
& (-9+x)^2 (-7+x)^{11} (5+x)^{25} (-928+293x-30x^2+x^3), \\
& (-9+x)^2 (-7+x)^{10} (5+x)^{25} (6452-2975x+503x^2-37x^3+x^4), \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^2 (-8+x) (-7+x)^9 (5+x)^{25} (95-20x+x^2) (59-16x+x^2), \\
& (-11+x)^3 (-8+x) (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-548+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4948-2465x+447x^2-35x^3+x^4), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (73-18x+x^2) (68-17x+x^2), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-544+213x-26x^2+x^3), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4912-2461x+447x^2-35x^3+x^4), \\
& (-11+x) (-9+x)^3 (-7+x)^{10} (5+x)^{25} (64-17x+x^2), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52-15x+x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^7 (-5+x) (5+x)^{25} (73-18x+x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^8 (5+x)^{25} (59-16x+x^2)^2, \\
& (-9+x)^5 (-7+x)^8 (5+x)^{25} (-412+179x-24x^2+x^3), \\
& (-9+x)^5 (-7+x)^7 (-5+x) (5+x)^{25} (-580+217x-26x^2+x^3), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-532+213x-26x^2+x^3), \\
& (-11+x) (-9+x)^4 (-7+x)^8 (-5+x) (5+x)^{25} (68-17x+x^2), \\
& (-9+x)^4 (-7+x)^8 (5+x)^{25} (3680-2019x+395x^2-33x^3+x^4), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904+17829x-4784x^2+626x^3-40x^4+x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^9 (5+x)^{25} (48-15x+x^2), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (59-16x+x^2) (-436+183x-24x^2+x^3), \\
& (-9+x)^4 (-7+x)^6 (5+x)^{25} (59-16x+x^2)^2 (52-15x+x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349+163x-23x^2+x^3), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-312+149x-22x^2+x^3), \\
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^6 (-5+x)^2 (5+x)^{25}, \\
& (-9+x)^5 (-7+x)^7 (5+x)^{25} (2824-1653x+347x^2-31x^3+x^4), \\
& (-11+x) (-9+x)^6 (-7+x)^8 (-4+x) (5+x)^{25}, \\
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (2172-1351x+303x^2-29x^3+x^4), \\
& (-9+x)^5 (-7+x)^7 (5+x)^{25} (68-17x+x^2) (41-14x+x^2), \\
& (-9+x)^6 (-7+x)^5 (5+x)^{25} (59-16x+x^2) (-256+127x-20x^2+x^3), \\
& (-9+x)^7 (-7+x)^5 (-5+x) (5+x)^{25} (-332+153x-22x^2+x^3), \\
& (-9+x)^8 (-7+x)^4 (-5+x)^2 (5+x)^{25} (52-15x+x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^7 (5+x)^{25} (37-14x+x^2), \\
& (-9+x)^7 (-7+x)^6 (5+x)^{25} (-232+123x-20x^2+x^3), \\
& (-9+x)^7 (-8+x) (-7+x)^5 (-5+x) (5+x)^{25} (41-14x+x^2), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-292+149x-22x^2+x^3), \\
& (-9+x)^8 (-7+x)^5 (5+x)^{25} (-172+101x-18x^2+x^3), \\
& (-9+x)^8 (-8+x) (-7+x)^6 (-3+x) (5+x)^{25} \}
\end{aligned}$$

Length[listmod64]

41

listmod128 = modfilter[listmod64, Smod128n41unrefined, 128]

$$\left\{ \begin{aligned} &(-9+x)^4 (-7+x)^{10} (5+x)^{25} (80-19x+x^2), \\ &(-12+x) (-11+x) (-9+x)^2 (-7+x)^{12} (5+x)^{25}, (-11+x)^3 (-8+x) (-7+x)^{12} (5+x)^{25}, \\ &(-9+x)^3 (-7+x)^9 (5+x)^{25} (4948-2465x+447x^2-35x^3+x^4), \\ &(-9+x)^4 (-7+x)^9 (5+x)^{25} (-544+213x-26x^2+x^3), \\ &(-11+x) (-9+x)^3 (-7+x)^{10} (5+x)^{25} (64-17x+x^2), \\ &(-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52-15x+x^2), \\ &(-11+x) (-9+x)^2 (-8+x) (-7+x)^8 (5+x)^{25} (59-16x+x^2)^2, \\ &(-9+x)^5 (-7+x)^8 (5+x)^{25} (-412+179x-24x^2+x^3), \\ &(-11+x) (-9+x)^4 (-7+x)^8 (-5+x) (5+x)^{25} (68-17x+x^2), \\ &(-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904+17829x-4784x^2+626x^3-40x^4+x^5), \\ &(-11+x) (-9+x)^4 (-7+x)^9 (5+x)^{25} (48-15x+x^2), \\ &(-9+x)^4 (-7+x)^7 (5+x)^{25} (-25580+17757x-4780x^2+626x^3-40x^4+x^5), \\ &(-9+x)^4 (-7+x)^6 (5+x)^{25} (59-16x+x^2)^2 (52-15x+x^2), \\ &(-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349+163x-23x^2+x^3), \\ &(-11+x) (-9+x)^6 (-8+x) (-7+x)^6 (-5+x)^2 (5+x)^{25}, \\ &(-9+x)^5 (-7+x)^7 (5+x)^{25} (2824-1653x+347x^2-31x^3+x^4), \\ &(-11+x) (-9+x)^6 (-7+x)^8 (-4+x) (5+x)^{25}, \\ &(-9+x)^8 (-7+x)^4 (-5+x)^2 (5+x)^{25} (52-15x+x^2), \\ &(-9+x)^7 (-7+x)^6 (5+x)^{25} (-232+123x-20x^2+x^3), \\ &(-9+x)^6 (-7+x)^7 (5+x)^{25} (-292+149x-22x^2+x^3), \\ &(-9+x)^8 (-8+x) (-7+x)^6 (-3+x) (5+x)^{25} \end{aligned} \right\}$$

```
listmod128 = {(-9 + x)^4 (-7 + x)^10 (5 + x)^25 (80 - 19 x + x^2),
  (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^12 (5 + x)^25, (-11 + x)^3 (-8 + x) (-7 + x)^12 (5 + x)^25,
  (-9 + x)^3 (-7 + x)^9 (5 + x)^25 (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4),
  (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (-544 + 213 x - 26 x^2 + x^3),
  (-11 + x) (-9 + x)^3 (-7 + x)^10 (5 + x)^25 (64 - 17 x + x^2),
  (-11 + x)^2 (-9 + x)^2 (-7 + x)^10 (5 + x)^25 (52 - 15 x + x^2),
  (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^8 (5 + x)^25 (59 - 16 x + x^2)^2,
  (-9 + x)^5 (-7 + x)^8 (5 + x)^25 (-412 + 179 x - 24 x^2 + x^3),
  (-11 + x) (-9 + x)^4 (-7 + x)^8 (-5 + x) (5 + x)^25 (68 - 17 x + x^2),
  (-9 + x)^4 (-7 + x)^7 (5 + x)^25 (-25904 + 17829 x - 4784 x^2 + 626 x^3 - 40 x^4 + x^5),
  (-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (48 - 15 x + x^2),
  (-9 + x)^4 (-7 + x)^7 (5 + x)^25 (-25580 + 17757 x - 4780 x^2 + 626 x^3 - 40 x^4 + x^5),
  (-9 + x)^4 (-7 + x)^6 (5 + x)^25 (59 - 16 x + x^2)^2 (52 - 15 x + x^2),
  (-9 + x)^5 (-8 + x) (-7 + x)^7 (5 + x)^25 (-349 + 163 x - 23 x^2 + x^3),
  (-11 + x) (-9 + x)^6 (-8 + x) (-7 + x)^6 (-5 + x)^2 (5 + x)^25,
  (-9 + x)^5 (-7 + x)^7 (5 + x)^25 (2824 - 1653 x + 347 x^2 - 31 x^3 + x^4),
  (-11 + x) (-9 + x)^6 (-7 + x)^8 (-4 + x) (5 + x)^25,
  (-9 + x)^8 (-7 + x)^4 (-5 + x)^2 (5 + x)^25 (52 - 15 x + x^2),
  (-9 + x)^7 (-7 + x)^6 (5 + x)^25 (-232 + 123 x - 20 x^2 + x^3),
  (-9 + x)^6 (-7 + x)^7 (5 + x)^25 (-292 + 149 x - 22 x^2 + x^3),
  (-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^25};
```

```
Length[listmod128]
```

```
22
```

```
poly1 = listmod128[[1]]
```

```
(-9 + x)^4 (-7 + x)^10 (5 + x)^25 (80 - 19 x + x^2)
```

```
p1 = poly1 / minipoly[poly1] // Factor
```

```
(-9 + x)^3 (-7 + x)^9 (5 + x)^24
```

```
feasiblesubcharpolylist[(-9 + x)^4 (-7 + x)^10 (5 + x)^25 (80 - 19 x + x^2)]
```

```
{1815 - 1314 x + 312 x^2 - 30 x^3 + x^4,
  (-7 + x) (-249 + 151 x - 23 x^2 + x^3), 1759 - 1306 x + 312 x^2 - 30 x^3 + x^4,
  (-7 + x) (-241 + 151 x - 23 x^2 + x^3), 1703 - 1298 x + 312 x^2 - 30 x^3 + x^4,
  (-9 + x) (-191 + 123 x - 21 x^2 + x^3), (-7 + x) (-233 + 151 x - 23 x^2 + x^3),
  (-9 + x) (-183 + 123 x - 21 x^2 + x^3), (-9 + x) (-7 + x) (25 - 14 x + x^2)}
```



```

CoefficientList[{1815 - 1314 x + 312 x^2 - 30 x^3 + x^4,
  (-7 + x) (-249 + 151 x - 23 x^2 + x^3), 1759 - 1306 x + 312 x^2 - 30 x^3 + x^4,
  (-7 + x) (-241 + 151 x - 23 x^2 + x^3), 1703 - 1298 x + 312 x^2 - 30 x^3 + x^4,
  (-9 + x) (-191 + 123 x - 21 x^2 + x^3), (-7 + x) (-233 + 151 x - 23 x^2 + x^3),
  (-9 + x) (-183 + 123 x - 21 x^2 + x^3), (-9 + x) (-7 + x) (25 - 14 x + x^2)}, x]
{{1815, -1314, 312, -30, 1}, {1743, -1306, 312, -30, 1}, {1759, -1306, 312, -30, 1},
{1687, -1298, 312, -30, 1}, {1703, -1298, 312, -30, 1}, {1719, -1298, 312, -30, 1},
{1631, -1290, 312, -30, 1}, {1647, -1290, 312, -30, 1}, {1575, -1282, 312, -30, 1}}

A = {{1815, -1314, 312, -30, 1}, {1743, -1306, 312, -30, 1},
{1759, -1306, 312, -30, 1}, {1687, -1298, 312, -30, 1},
{1703, -1298, 312, -30, 1}, {1719, -1298, 312, -30, 1}, {1631, -1290,
312, -30, 1}, {1647, -1290, 312, -30, 1}, {1575, -1282, 312, -30, 1}};

MatrixRank[A]
3

CoefficientList[D[poly1, x] / p1 // Factor, x]
{72 815, -53 602, 12 792, -1230, 41}

Solve[
  Array[n, 9].{{1815, -1314, 312, -30, 1}, {1743, -1306, 312, -30, 1}, {1759, -1306,
312, -30, 1}, {1687, -1298, 312, -30, 1}, {1703, -1298, 312, -30, 1},
{1719, -1298, 312, -30, 1}, {1631, -1290, 312, -30, 1},
{1647, -1290, 312, -30, 1}, {1575, -1282, 312, -30, 1}} ==
{72 815, -53 602, 12 792, -1230, 41}, Array[n, 9]]

Solve::svars : Equations may not give solutions for all "solve" variables.      >>
{{n[7] -> 70 - 3 n[1] - 3 n[2] - 2 n[3] - 2 n[4] - n[5],
n[8] -> 60 - n[1] - n[3] - n[5] - 2 n[6],
n[9] -> -89 + 3 n[1] + 2 n[2] + 2 n[3] + n[4] + n[5] + n[6]}}

FindInstance[-n[7] + 70 - 3 n[1] - 3 n[2] - 2 n[3] - 2 n[4] - n[5] == 0 &&
-n[8] + 60 - n[1] - n[3] - n[5] - 2 n[6] == 0 &&
-n[9] - 89 + 3 n[1] + 2 n[2] + 2 n[3] + n[4] + n[5] + n[6] == 0 &&
n[1] >= 0 && n[2] >= 0 && n[3] >= 0 && n[4] >= 0 && n[5] >= 0 && n[6] >= 0 &&
n[7] >= 0 && n[8] >= 0 && n[9] >= 0, Array[n, 9], Integers]
{}

```

```

CoefficientList[{1815 - 1314 x + 312 x^2 - 30 x^3 + x^4,
  (-7 + x) (-249 + 151 x - 23 x^2 + x^3), 1759 - 1306 x + 312 x^2 - 30 x^3 + x^4,
  (-7 + x) (-241 + 151 x - 23 x^2 + x^3), 1703 - 1298 x + 312 x^2 - 30 x^3 + x^4,
  (-9 + x) (-191 + 123 x - 21 x^2 + x^3), (-7 + x) (-233 + 151 x - 23 x^2 + x^3),
  (-9 + x) (-183 + 123 x - 21 x^2 + x^3), (-9 + x) (-7 + x) (25 - 14 x + x^2)}, x] // MatrixForm

```

$$\begin{pmatrix} 1815 & -1314 & 312 & -30 & 1 \\ 1743 & -1306 & 312 & -30 & 1 \\ 1759 & -1306 & 312 & -30 & 1 \\ 1687 & -1298 & 312 & -30 & 1 \\ 1703 & -1298 & 312 & -30 & 1 \\ 1719 & -1298 & 312 & -30 & 1 \\ 1631 & -1290 & 312 & -30 & 1 \\ 1647 & -1290 & 312 & -30 & 1 \\ 1575 & -1282 & 312 & -30 & 1 \end{pmatrix}$$

```

CoefficientList[D[poly1, x] / p1 // Factor, x]

```

```

{72 815, -53 602, 12 792, -1230, 41}

```

```

Transpose[
  {
    {1815 - 1314 x + 312 x^2 - 30 x^3 + x^4,
     1743 - 1306 x + 312 x^2 - 30 x^3 + x^4,
     1759 - 1306 x + 312 x^2 - 30 x^3 + x^4,
     1687 - 1298 x + 312 x^2 - 30 x^3 + x^4,
     1703 - 1298 x + 312 x^2 - 30 x^3 + x^4,
     1719 - 1298 x + 312 x^2 - 30 x^3 + x^4,
     1631 - 1290 x + 312 x^2 - 30 x^3 + x^4,
     1647 - 1290 x + 312 x^2 - 30 x^3 + x^4,
     1575 - 1282 x + 312 x^2 - 30 x^3 + x^4}
  } // MatrixForm

```

$$\begin{pmatrix} 1815 & 1743 & 1759 & 1687 & 1703 & 1719 & 1631 & 1647 & 1575 \\ -1314 & -1306 & -1306 & -1298 & -1298 & -1298 & -1290 & -1290 & -1282 \\ 312 & 312 & 312 & 312 & 312 & 312 & 312 & 312 & 312 \\ -30 & -30 & -30 & -30 & -30 & -30 & -30 & -30 & -30 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

```

Array[m, 5].{
  {
    1815 1743 1759 1687 1703 1719 1631 1647 1575
    -1314 -1306 -1306 -1298 -1298 -1298 -1290 -1290 -1282
    312 312 312 312 312 312 312 312 312
    -30 -30 -30 -30 -30 -30 -30 -30 -30
    1 1 1 1 1 1 1 1 1
  }
}

```

```

{1815 m[1] - 1314 m[2] + 312 m[3] - 30 m[4] + m[5],
 1743 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5],
 1759 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5],
 1687 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5],
 1703 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5],
 1719 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5],
 1631 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5],
 1647 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5],
 1575 m[1] - 1282 m[2] + 312 m[3] - 30 m[4] + m[5]}

```

```

Array[m, 5].{72 815, -53 602, 12 792, -1230, 41}

```

```

72 815 m[1] - 53 602 m[2] + 12 792 m[3] - 1230 m[4] + 41 m[5]

```

```

FindInstance[1815 m[1] - 1314 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1743 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1759 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1687 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1703 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1719 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1631 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1647 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  1575 m[1] - 1282 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
  72 815 m[1] - 53 602 m[2] + 12 792 m[3] - 1230 m[4] + 41 m[5] < 0, Array[m, 5], Integers]
{{m[1] → -4390, m[2] → -26 341, m[3] → 0, m[4] → 0, m[5] → -26 642 928}}

Array[m, 5] /. {m[1] → -4390, m[2] → -26 341, m[3] → 0, m[4] → 0, m[5] → -26 642 928}
{-4390, -26 341, 0, 0, -26 642 928}

GCD[-4390, -26 341, 0, 0, -26 642 928]
1

{-4390, -26 341, 0, 0, -26 642 928}.

$$\begin{pmatrix} 1815 & 1743 & 1759 & 1687 & 1703 & 1719 & 1631 & 1647 & 1575 \\ -1314 & -1306 & -1306 & -1298 & -1298 & -1298 & -1290 & -1290 & -1282 \\ 312 & 312 & 312 & 312 & 312 & 312 & 312 & 312 & 312 \\ -30 & -30 & -30 & -30 & -30 & -30 & -30 & -30 & -30 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

{1296, 106 648, 36 408, 141 760, 71 520, 1280, 176 872, 106 632, 211 984}

{-4390, -26 341, 0, 0, -26 642 928}. {72 815, -53 602, 12 792, -1230, 41}
-87 616

```

```
poly2 = listmod128[[2]]
```

```
(-12 + x) (-11 + x) (-9 + x)2 (-7 + x)12 (5 + x)25
```

```
p2 = poly2 / minipoly[poly2] // Factor
```

```
(-9 + x) (-7 + x)11 (5 + x)24
```

```
feasiblesubcharpolylist[(-12 + x) (-11 + x) (-9 + x)2 (-7 + x)12 (5 + x)25]
```

```
{(-11 + x) (-269 + 155 x - 23 x2 + x3), (-11 + x) (-9 + x) (29 - 14 x + x2)}
```

```
CoefficientList[
```

```
{(-11 + x) (-269 + 155 x - 23 x2 + x3), (-11 + x) (-9 + x) (29 - 14 x + x2)}, x]
```

```
{{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1}}
```

```
CoefficientList[D[poly2, x] / p2 // Factor, x]
```

```
{120 135, -80 838, 16 728, -1394, 41}
```

```
Solve[Array[n, 2].{{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1}} ==
  {120 135, -80 838, 16 728, -1394, 41}, Array[n, 2]]
```

```
{}
```

```
A = {{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1}}
```

```
{{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1}}
```

```
Array[m, 5].Transpose[A]
```

```
{2959 m[1] - 1974 m[2] + 408 m[3] - 34 m[4] + m[5],
 2871 m[1] - 1966 m[2] + 408 m[3] - 34 m[4] + m[5]}
```

```
Array[m, 5].{120 135, -80 838, 16 728, -1394, 41}
```

```
120 135 m[1] - 80 838 m[2] + 16 728 m[3] - 1394 m[4] + 41 m[5]
```

```
Array[m, 5] /. FindInstance[2959 m[1] - 1974 m[2] + 408 m[3] - 34 m[4] + m[5] ≥ 0 &&
 2871 m[1] - 1966 m[2] + 408 m[3] - 34 m[4] + m[5] ≥ 0 &&
```

```
120 135 m[1] - 80 838 m[2] + 16 728 m[3] - 1394 m[4] + 41 m[5] < 0,
```

```
Array[m, 5], Integers]
```

```
{{1713, 18 832, 0, 0, 32 108 784}}
```

```
1713 * 2959 + 18 832 * (-1974) + 32 108 784
```

```
3183
```

```
GCD[1713, 18 832, 0, 0, 32 108 784]
```

```
1
```

```
{1713, 18 832, 0, 0, 32 108 784}.Transpose[A]
```

```
{3183, 3095}
```

```
{1713, 18 832, 0, 0, 32 108 784}.{120 135, -80 838, 16 728, -1394, 41}
```

```
-89 817
```

```
poly3 = listmod128[[3]]
```

```
 $(-11 + x)^3 (-8 + x) (-7 + x)^{12} (5 + x)^{25}$ 
```

```
p3 = poly3 / minipoly[poly3] // Factor
```

```
 $(-11 + x)^2 (-7 + x)^{11} (5 + x)^{24}$ 
```

```
feasiblesubcharpolylist[ $(-11 + x)^3 (-8 + x) (-7 + x)^{12} (5 + x)^{25}$ ]
```

```
{ $-223 + 131 x - 21 x^2 + x^3$ ,  $(-11 + x) (-7 + x) (-3 + x)$ }
```

```
CoefficientList[{ $-223 + 131 x - 21 x^2 + x^3$ ,  $(-11 + x) (-7 + x) (-3 + x)$ }, x]
```

```
{{-223, 131, -21, 1}, {-231, 131, -21, 1}}
```

```

{{-231, 131, -21, 1}, {-223, 131, -21, 1}} // MatrixForm

$$\begin{pmatrix} -231 & 131 & -21 & 1 \\ -223 & 131 & -21 & 1 \end{pmatrix}$$


CoefficientList[D[poly3, x] / p3 // Factor, x]
{-8895, 5371, -861, 41}

Solve[Array[n, 2].{{-223, 131, -21, 1}, {-231, 131, -21, 1}} ==
  {-8895, 5371, -861, 41}, Array[n, 2]]
{{n[1] → 72, n[2] → -31}}

Array[m, 4].Transpose[ $\begin{pmatrix} -231 & 131 & -21 & 1 \\ -223 & 131 & -21 & 1 \end{pmatrix}$ ]
{-231 m[1] + 131 m[2] - 21 m[3] + m[4], -223 m[1] + 131 m[2] - 21 m[3] + m[4]}

Array[m, 4].{-8895, 5371, -861, 41}
-8895 m[1] + 5371 m[2] - 861 m[3] + 41 m[4]

FindInstance[
  -231 m[1] + 131 m[2] - 21 m[3] + m[4] ≥ 0 && -223 m[1] + 131 m[2] - 21 m[3] + m[4] ≥ 0 &&
  -8895 m[1] + 5371 m[2] - 861 m[3] + 41 m[4] < 0, Array[m, 4], Integers]
{{m[1] → -62, m[2] → 0, m[3] → 0, m[4] → -13493}}

Array[m, 4] /. {m[1] → -62, m[2] → 0, m[3] → 0, m[4] → -13493}
{-62, 0, 0, -13493}

GCD[-62, 0, 0, -13493]
1

{-62, 0, 0, -13493} // Reverse
{-13493, 0, 0, -62}

{-62, 0, 0, -13493}.{-8895, 5371, -861, 41}
-1723

{-62, 0, 0, -13493}.Transpose[ $\begin{pmatrix} -231 & 131 & -21 & 1 \\ -223 & 131 & -21 & 1 \end{pmatrix}$ ]
{829, 333}

```

```

poly4 = listmod128[4]
(-9 + x)3 (-7 + x)9 (5 + x)25 (4948 - 2465 x + 447 x2 - 35 x3 + x4)

p4 = poly4 / minipoly[poly4] // Factor
(-9 + x)2 (-7 + x)8 (5 + x)24

```

```

feasiblesubcharpolylist[(-9 + x)3 (-7 + x)9 (5 + x)25 (4948 - 2465 x + 447 x2 - 35 x3 + x4)]
{ (-9 + x) (-7 + x) (1735 - 1298 x + 312 x2 - 30 x3 + x4),
  (-7 + x) (-15 583 + 13 417 x - 4106 x2 + 582 x3 - 39 x4 + x5),
  (-9 + x) (-7 + x) (1751 - 1298 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (1767 - 1298 x + 312 x2 - 30 x3 + x4),
  (-9 + x)2 (-7 + x) (-183 + 123 x - 21 x2 + x3), (-7 + x)2 (2113 - 1592 x + 358 x2 - 32 x3 + x4),
  (-9 + x) (-7 + x) (1663 - 1290 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (73 - 18 x + x2) (23 - 12 x + x2),
  (-9 + x) (-7 + x) (1695 - 1290 x + 312 x2 - 30 x3 + x4),
  (-9 + x)2 (-7 + x)2 (25 - 14 x + x2), (-9 + x) (-7 + x) (1591 - 1282 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (1607 - 1282 x + 312 x2 - 30 x3 + x4),
  (-9 + x)2 (-7 + x) (-167 + 123 x - 21 x2 + x3), (-9 + x) (-7 + x)2 (-217 + 151 x - 23 x2 + x3),
  (-9 + x) (-7 + x) (1535 - 1274 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (1447 - 1266 x + 312 x2 - 30 x3 + x4) }

{ (-9 + x) (-7 + x) (1735 - 1298 x + 312 x2 - 30 x3 + x4),
  (-7 + x) (-15 583 + 13 417 x - 4106 x2 + 582 x3 - 39 x4 + x5),
  (-9 + x) (-7 + x) (1751 - 1298 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (1767 - 1298 x + 312 x2 - 30 x3 + x4),
  (-9 + x)2 (-7 + x) (-183 + 123 x - 21 x2 + x3),
  (-7 + x)2 (2113 - 1592 x + 358 x2 - 32 x3 + x4), (-9 + x) (-7 + x)
  (1663 - 1290 x + 312 x2 - 30 x3 + x4), (-9 + x) (-7 + x) (73 - 18 x + x2) (23 - 12 x + x2),
  (-9 + x) (-7 + x) (1695 - 1290 x + 312 x2 - 30 x3 + x4), (-9 + x)2 (-7 + x)2 (25 - 14 x + x2),
  (-9 + x) (-7 + x) (1591 - 1282 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (1607 - 1282 x + 312 x2 - 30 x3 + x4),
  (-9 + x)2 (-7 + x) (-167 + 123 x - 21 x2 + x3), (-9 + x) (-7 + x)2 (-217 + 151 x - 23 x2 + x3),
  (-9 + x) (-7 + x) (1535 - 1274 x + 312 x2 - 30 x3 + x4),
  (-9 + x) (-7 + x) (1447 - 1266 x + 312 x2 - 30 x3 + x4) } // Length

```

```

CoefficientList[{{(-9 + x) (-7 + x) (1735 - 1298 x + 312 x^2 - 30 x^3 + x^4),
  (-7 + x) (-15 583 + 13 417 x - 4106 x^2 + 582 x^3 - 39 x^4 + x^5),
  (-9 + x) (-7 + x) (1751 - 1298 x + 312 x^2 - 30 x^3 + x^4),
  (-9 + x) (-7 + x) (1767 - 1298 x + 312 x^2 - 30 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (-183 + 123 x - 21 x^2 + x^3),
  (-7 + x)^2 (2113 - 1592 x + 358 x^2 - 32 x^3 + x^4), (-9 + x) (-7 + x)
  (1663 - 1290 x + 312 x^2 - 30 x^3 + x^4), (-9 + x) (-7 + x) (73 - 18 x + x^2) (23 - 12 x + x^2),
  (-9 + x) (-7 + x) (1695 - 1290 x + 312 x^2 - 30 x^3 + x^4), (-9 + x)^2 (-7 + x)^2 (25 - 14 x + x^2),
  (-9 + x) (-7 + x) (1591 - 1282 x + 312 x^2 - 30 x^3 + x^4),
  (-9 + x) (-7 + x) (1607 - 1282 x + 312 x^2 - 30 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (-167 + 123 x - 21 x^2 + x^3), (-9 + x) (-7 + x)^2 (-217 + 151 x - 23 x^2 + x^3),
  (-9 + x) (-7 + x) (1535 - 1274 x + 312 x^2 - 30 x^3 + x^4),
  (-9 + x) (-7 + x) (1447 - 1266 x + 312 x^2 - 30 x^3 + x^4)}, x]

{{109 305, -109 534, 42 159, -8180, 855, -46, 1},
 {109 081, -109 502, 42 159, -8180, 855, -46, 1},
 {110 313, -109 790, 42 175, -8180, 855, -46, 1},
 {111 321, -110 046, 42 191, -8180, 855, -46, 1},
 {103 761, -107 622, 41 943, -8172, 855, -46, 1},
 {103 537, -107 590, 41 943, -8172, 855, -46, 1},
 {104 769, -107 878, 41 959, -8172, 855, -46, 1},
 {105 777, -108 134, 41 975, -8172, 855, -46, 1},
 {106 785, -108 390, 41 991, -8172, 855, -46, 1},
 {99 225, -105 966, 41 743, -8164, 855, -46, 1},
 {100 233, -106 222, 41 759, -8164, 855, -46, 1},
 {101 241, -106 478, 41 775, -8164, 855, -46, 1},
 {94 689, -104 310, 41 543, -8156, 855, -46, 1},
 {95 697, -104 566, 41 559, -8156, 855, -46, 1},
 {96 705, -104 822, 41 575, -8156, 855, -46, 1},
 {91 161, -102 910, 41 359, -8148, 855, -46, 1}}

A = {{109 305, -109 534, 42 159, -8180, 855, -46, 1},
 {109 081, -109 502, 42 159, -8180, 855, -46, 1}, {110 313, -109 790, 42 175,
 -8180, 855, -46, 1}, {111 321, -110 046, 42 191, -8180, 855, -46, 1},
 {103 761, -107 622, 41 943, -8172, 855, -46, 1}, {103 537, -107 590, 41 943,
 -8172, 855, -46, 1}, {104 769, -107 878, 41 959, -8172, 855, -46, 1},
 {105 777, -108 134, 41 975, -8172, 855, -46, 1}, {106 785, -108 390, 41 991,
 -8172, 855, -46, 1}, {99 225, -105 966, 41 743, -8164, 855, -46, 1},
 {100 233, -106 222, 41 759, -8164, 855, -46, 1}, {101 241, -106 478, 41 775,
 -8164, 855, -46, 1}, {94 689, -104 310, 41 543, -8156, 855, -46, 1},
 {95 697, -104 566, 41 559, -8156, 855, -46, 1}, {96 705, -104 822, 41 575,
 -8156, 855, -46, 1}, {91 161, -102 910, 41 359, -8148, 855, -46, 1}};

CoefficientList[D[poly4, x] / p4 // Factor, x]
{4 493 145, -4 488 726, 1 727 503, -335 308, 35 055, -1886, 41}

```

Solve[

```
Array[n, 16].{{109 305, -109 534, 42 159, -8180, 855, -46, 1}, {109 081, -109 502,
  42 159, -8180, 855, -46, 1}, {110 313, -109 790, 42 175, -8180, 855, -46, 1},
  {111 321, -110 046, 42 191, -8180, 855, -46, 1}, {103 761, -107 622, 41 943,
  -8172, 855, -46, 1}, {103 537, -107 590, 41 943, -8172, 855, -46, 1},
  {104 769, -107 878, 41 959, -8172, 855, -46, 1}, {105 777, -108 134, 41 975,
  -8172, 855, -46, 1}, {106 785, -108 390, 41 991, -8172, 855, -46, 1},
  {99 225, -105 966, 41 743, -8164, 855, -46, 1}, {100 233, -106 222, 41 759,
  -8164, 855, -46, 1}, {101 241, -106 478, 41 775, -8164, 855, -46, 1},
  {94 689, -104 310, 41 543, -8156, 855, -46, 1}, {95 697, -104 566, 41 559,
  -8156, 855, -46, 1}, {96 705, -104 822, 41 575, -8156, 855, -46, 1},
  {91 161, -102 910, 41 359, -8148, 855, -46, 1}} ==
  {4 493 145, -4 488 726, 1 727 503, -335 308, 35 055, -1886, 41}, Array[n, 16]]
```

{}

MatrixForm[{{109 305, -109 534, 42 159, -8180, 855, -46, 1},

```
{109 081, -109 502, 42 159, -8180, 855, -46, 1},
{110 313, -109 790, 42 175, -8180, 855, -46, 1},
{111 321, -110 046, 42 191, -8180, 855, -46, 1},
{103 761, -107 622, 41 943, -8172, 855, -46, 1},
{103 537, -107 590, 41 943, -8172, 855, -46, 1},
{104 769, -107 878, 41 959, -8172, 855, -46, 1},
{105 777, -108 134, 41 975, -8172, 855, -46, 1},
{106 785, -108 390, 41 991, -8172, 855, -46, 1},
{99 225, -105 966, 41 743, -8164, 855, -46, 1},
{100 233, -106 222, 41 759, -8164, 855, -46, 1},
{101 241, -106 478, 41 775, -8164, 855, -46, 1},
{94 689, -104 310, 41 543, -8156, 855, -46, 1},
{95 697, -104 566, 41 559, -8156, 855, -46, 1},
{96 705, -104 822, 41 575, -8156, 855, -46, 1},
{91 161, -102 910, 41 359, -8148, 855, -46, 1}}]
```

```
( 109 305 -109 534 42 159 -8180 855 -46 1 )
( 109 081 -109 502 42 159 -8180 855 -46 1 )
( 110 313 -109 790 42 175 -8180 855 -46 1 )
( 111 321 -110 046 42 191 -8180 855 -46 1 )
( 103 761 -107 622 41 943 -8172 855 -46 1 )
( 103 537 -107 590 41 943 -8172 855 -46 1 )
( 104 769 -107 878 41 959 -8172 855 -46 1 )
( 105 777 -108 134 41 975 -8172 855 -46 1 )
( 106 785 -108 390 41 991 -8172 855 -46 1 )
( 99 225 -105 966 41 743 -8164 855 -46 1 )
( 100 233 -106 222 41 759 -8164 855 -46 1 )
( 101 241 -106 478 41 775 -8164 855 -46 1 )
( 94 689 -104 310 41 543 -8156 855 -46 1 )
( 95 697 -104 566 41 559 -8156 855 -46 1 )
( 96 705 -104 822 41 575 -8156 855 -46 1 )
( 91 161 -102 910 41 359 -8148 855 -46 1 )
```


Array[m, 7].Transpose[A]

```
{ 109 305 m[1] - 109 534 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  109 081 m[1] - 109 502 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  110 313 m[1] - 109 790 m[2] + 42 175 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  111 321 m[1] - 110 046 m[2] + 42 191 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  103 761 m[1] - 107 622 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  103 537 m[1] - 107 590 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  104 769 m[1] - 107 878 m[2] + 41 959 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  105 777 m[1] - 108 134 m[2] + 41 975 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  106 785 m[1] - 108 390 m[2] + 41 991 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  99 225 m[1] - 105 966 m[2] + 41 743 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  100 233 m[1] - 106 222 m[2] + 41 759 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  101 241 m[1] - 106 478 m[2] + 41 775 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  94 689 m[1] - 104 310 m[2] + 41 543 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  95 697 m[1] - 104 566 m[2] + 41 559 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  96 705 m[1] - 104 822 m[2] + 41 575 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ,
  91 161 m[1] - 102 910 m[2] + 41 359 m[3] - 8148 m[4] + 855 m[5] - 46 m[6] + m[7] }
```

Array[m, 7].{4 493 145, -4 488 726, 1 727 503, -335 308, 35 055, -1886, 41}

```
4 493 145 m[1] - 4 488 726 m[2] + 1 727 503 m[3] -
  335 308 m[4] + 35 055 m[5] - 1886 m[6] + 41 m[7]
```

FindInstance[

```
109 305 m[1] - 109 534 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  109 081 m[1] - 109 502 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  110 313 m[1] - 109 790 m[2] + 42 175 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  111 321 m[1] - 110 046 m[2] + 42 191 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  103 761 m[1] - 107 622 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  103 537 m[1] - 107 590 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  104 769 m[1] - 107 878 m[2] + 41 959 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  105 777 m[1] - 108 134 m[2] + 41 975 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  106 785 m[1] - 108 390 m[2] + 41 991 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  99 225 m[1] - 105 966 m[2] + 41 743 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  100 233 m[1] - 106 222 m[2] + 41 759 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  101 241 m[1] - 106 478 m[2] + 41 775 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  94 689 m[1] - 104 310 m[2] + 41 543 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  95 697 m[1] - 104 566 m[2] + 41 559 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  96 705 m[1] - 104 822 m[2] + 41 575 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  91 161 m[1] - 102 910 m[2] + 41 359 m[3] - 8148 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&
  4 493 145 m[1] - 4 488 726 m[2] + 1 727 503 m[3] - 335 308 m[4] +
    35 055 m[5] - 1886 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
```

```
{ {m[1] → -5561, m[2] → -31 263,
  m[3] → -149 845, m[4] → -427 982, m[5] → 0, m[6] → 0, m[7] → 0} }
```

```

Array[m, 7] /. {m[1] → -5561, m[2] → -31263,
  m[3] → -149845, m[4] → -427982, m[5] → 0, m[6] → 0, m[7] → 0}
{-5561, -31263, -149845, -427982, 0, 0, 0}

GCD[-5561, -31263, -149845, -427982, 0, 0, 0]
1

Solve[Array[n, 16].A == {4493145, -4488726, 1727503, -335308, 35055, -1886, 41},
  Array[n, 16]]
{}

MatrixRank[A]
4

{-5561, -31263, -149845, -427982, 0, 0, 0}.
{4493145, -4488726, 1727503, -335308, 35055, -1886, 41}
-7236986

{-5561, -31263, -149845, -427982, 0, 0, 0}.

Transpose[
  {
    109305 -109534 42159 -8180 855 -46 1
    109081 -109502 42159 -8180 855 -46 1
    110313 -109790 42175 -8180 855 -46 1
    111321 -110046 42191 -8180 855 -46 1
    103761 -107622 41943 -8172 855 -46 1
    103537 -107590 41943 -8172 855 -46 1
    104769 -107878 41959 -8172 855 -46 1
    105777 -108134 41975 -8172 855 -46 1
    106785 -108390 41991 -8172 855 -46 1
    99225 -105966 41743 -8164 855 -46 1
    100233 -106222 41759 -8164 855 -46 1
    101241 -106478 41775 -8164 855 -46 1
    94689 -104310 41543 -8156 855 -46 1
    95697 -104566 41559 -8156 855 -46 1
    96705 -104822 41575 -8156 855 -46 1
    91161 -102910 41359 -8148 855 -46 1
  }
]

{93742, 338990, 94062, 94382, 91734, 336982, 92054,
  92374, 92694, 90046, 90366, 90686, 88358, 88678, 88998, 86990}

poly5 = listmod128[[5]]
(-9 + x)4 (-7 + x)9 (5 + x)25 (-544 + 213 x - 26 x2 + x3)

p5 = poly5 / minipoly[poly5] // Factor
(-9 + x)3 (-7 + x)8 (5 + x)24

```

```

feasiblesubcharpolylist[(-9 + x)4 (-7 + x)9 (5 + x)25 (-544 + 213 x - 26 x2 + x3)]
{ (-9 + x) (1353 - 1052 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (1369 - 1052 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-183 + 123 x - 21 x2 + x3),
  (-9 + x) (1297 - 1044 x + 270 x2 - 28 x3 + x4), (-9 + x) (1313 - 1044 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (1329 - 1044 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x)2 (25 - 14 x + x2),
  (-9 + x) (73 - 18 x + x2) (17 - 10 x + x2), (-9 + x) (1257 - 1036 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (1273 - 1036 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-167 + 123 x - 21 x2 + x3),
  (-9 + x) (1185 - 1028 x + 270 x2 - 28 x3 + x4), (-9 + x) (1201 - 1028 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (-7 + x) (-159 + 123 x - 21 x2 + x3), (-9 + x) (1129 - 1020 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (1145 - 1020 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-151 + 123 x - 21 x2 + x3),
  (-9 + x) (1073 - 1012 x + 270 x2 - 28 x3 + x4), (-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2) }

{ (-9 + x) (1353 - 1052 x + 270 x2 - 28 x3 + x4), (-9 + x) (1369 - 1052 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (-7 + x) (-183 + 123 x - 21 x2 + x3), (-9 + x) (1297 - 1044 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (1313 - 1044 x + 270 x2 - 28 x3 + x4), (-9 + x) (1329 - 1044 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (-7 + x)2 (25 - 14 x + x2), (-9 + x) (73 - 18 x + x2) (17 - 10 x + x2),
  (-9 + x) (1257 - 1036 x + 270 x2 - 28 x3 + x4), (-9 + x) (1273 - 1036 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (-7 + x) (-167 + 123 x - 21 x2 + x3), (-9 + x) (1185 - 1028 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (1201 - 1028 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-159 + 123 x - 21 x2 + x3),
  (-9 + x) (1129 - 1020 x + 270 x2 - 28 x3 + x4), (-9 + x) (1145 - 1020 x + 270 x2 - 28 x3 + x4),
  (-9 + x) (-7 + x) (-151 + 123 x - 21 x2 + x3), (-9 + x) (1073 - 1012 x + 270 x2 - 28 x3 + x4),
  (-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2) } // Length

```

```

CoefficientList[
  {(-9 + x) (1353 - 1052 x + 270 x2 - 28 x3 + x4), (-9 + x) (1369 - 1052 x + 270 x2 - 28 x3 + x4),
   (-9 + x) (-7 + x) (-183 + 123 x - 21 x2 + x3), (-9 + x) (1297 - 1044 x + 270 x2 - 28 x3 + x4),
   (-9 + x) (1313 - 1044 x + 270 x2 - 28 x3 + x4), (-9 + x) (1329 - 1044 x + 270 x2 - 28 x3 + x4),
   (-9 + x) (-7 + x)2 (25 - 14 x + x2), (-9 + x) (73 - 18 x + x2) (17 - 10 x + x2),
   (-9 + x) (1257 - 1036 x + 270 x2 - 28 x3 + x4), (-9 + x) (1273 - 1036 x + 270 x2 - 28 x3 + x4),
   (-9 + x) (-7 + x) (-167 + 123 x - 21 x2 + x3), (-9 + x) (1185 - 1028 x + 270 x2 - 28 x3 + x4),
   (-9 + x) (1201 - 1028 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-159 + 123 x - 21 x2 + x3),
   (-9 + x) (1129 - 1020 x + 270 x2 - 28 x3 + x4), (-9 + x) (1145 - 1020 x + 270 x2 - 28 x3 + x4),
   (-9 + x) (-7 + x) (-151 + 123 x - 21 x2 + x3), (-9 + x) (1073 - 1012 x + 270 x2 - 28 x3 + x4),
   (-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2)}, x]

{{-12 177, 10 821, -3482, 522, -37, 1},
 {-12 321, 10 837, -3482, 522, -37, 1}, {-11 529, 10 677, -3474, 522, -37, 1},
 {-11 673, 10 693, -3474, 522, -37, 1}, {-11 817, 10 709, -3474, 522, -37, 1},
 {-11 961, 10 725, -3474, 522, -37, 1}, {-11 025, 10 549, -3466, 522, -37, 1},
 {-11 169, 10 565, -3466, 522, -37, 1}, {-11 313, 10 581, -3466, 522, -37, 1},
 {-11 457, 10 597, -3466, 522, -37, 1}, {-10 521, 10 421, -3458, 522, -37, 1},
 {-10 665, 10 437, -3458, 522, -37, 1}, {-10 809, 10 453, -3458, 522, -37, 1},
 {-10 017, 10 293, -3450, 522, -37, 1}, {-10 161, 10 309, -3450, 522, -37, 1},
 {-10 305, 10 325, -3450, 522, -37, 1}, {-9513, 10 165, -3442, 522, -37, 1},
 {-9657, 10 181, -3442, 522, -37, 1}, {-9009, 10 037, -3434, 522, -37, 1}}

A = {{-12 177, 10 821, -3482, 522, -37, 1},
      {-12 321, 10 837, -3482, 522, -37, 1}, {-11 529, 10 677, -3474, 522, -37, 1},
      {-11 673, 10 693, -3474, 522, -37, 1}, {-11 817, 10 709, -3474, 522, -37, 1},
      {-11 961, 10 725, -3474, 522, -37, 1}, {-11 025, 10 549, -3466, 522, -37, 1},
      {-11 169, 10 565, -3466, 522, -37, 1}, {-11 313, 10 581, -3466, 522, -37, 1},
      {-11 457, 10 597, -3466, 522, -37, 1}, {-10 521, 10 421, -3458, 522, -37, 1},
      {-10 665, 10 437, -3458, 522, -37, 1}, {-10 809, 10 453, -3458, 522, -37, 1},
      {-10 017, 10 293, -3450, 522, -37, 1}, {-10 161, 10 309, -3450, 522, -37, 1},
      {-10 305, 10 325, -3450, 522, -37, 1}, {-9513, 10 165, -3442, 522, -37, 1},
      {-9657, 10 181, -3442, 522, -37, 1}, {-9009, 10 037, -3434, 522, -37, 1}};

MatrixRank[A]

3

CoefficientList[D[poly5, x] / p5 // Factor, x]

{-493 225, 440 925, -142 538, 21 402, -1517, 41}

```

```

Solve[Array[n, 19].{{-12 177, 10 821, -3482, 522, -37, 1},
  {-12 321, 10 837, -3482, 522, -37, 1}, {-11 529, 10 677, -3474, 522, -37, 1},
  {-11 673, 10 693, -3474, 522, -37, 1}, {-11 817, 10 709, -3474, 522, -37, 1},
  {-11 961, 10 725, -3474, 522, -37, 1}, {-11 025, 10 549, -3466, 522, -37, 1},
  {-11 169, 10 565, -3466, 522, -37, 1}, {-11 313, 10 581, -3466, 522, -37, 1},
  {-11 457, 10 597, -3466, 522, -37, 1}, {-10 521, 10 421, -3458, 522, -37, 1},
  {-10 665, 10 437, -3458, 522, -37, 1}, {-10 809, 10 453, -3458, 522, -37, 1},
  {-10 017, 10 293, -3450, 522, -37, 1}, {-10 161, 10 309, -3450, 522, -37, 1},
  {-10 305, 10 325, -3450, 522, -37, 1}, {-9513, 10 165, -3442, 522, -37, 1},
  {-9657, 10 181, -3442, 522, -37, 1}, {-9009, 10 037, -3434, 522, -37, 1}} =
  {-493 225, 440 925, -142 538, 21 402, -1517, 41}, Array[n, 19]]

{}

```

```

Array[m, 6].Transpose[A]

```

```

{-12 177 m[1] + 10 821 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6],
 -12 321 m[1] + 10 837 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 529 m[1] + 10 677 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 673 m[1] + 10 693 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 817 m[1] + 10 709 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 961 m[1] + 10 725 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 025 m[1] + 10 549 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 169 m[1] + 10 565 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 313 m[1] + 10 581 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
 -11 457 m[1] + 10 597 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
 -10 521 m[1] + 10 421 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6],
 -10 665 m[1] + 10 437 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6],
 -10 809 m[1] + 10 453 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6],
 -10 017 m[1] + 10 293 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6],
 -10 161 m[1] + 10 309 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6],
 -10 305 m[1] + 10 325 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6],
 -9513 m[1] + 10 165 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6],
 -9657 m[1] + 10 181 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6],
 -9009 m[1] + 10 037 m[2] - 3434 m[3] + 522 m[4] - 37 m[5] + m[6]}

```

```

Array[m, 6].{-493 225, 440 925, -142 538, 21 402, -1517, 41}

```

```

-493 225 m[1] + 440 925 m[2] - 142 538 m[3] + 21 402 m[4] - 1517 m[5] + 41 m[6]

```

```

FindInstance[-12 177 m[1] + 10 821 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-12 321 m[1] + 10 837 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 529 m[1] + 10 677 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 673 m[1] + 10 693 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 817 m[1] + 10 709 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 961 m[1] + 10 725 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 025 m[1] + 10 549 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 169 m[1] + 10 565 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 313 m[1] + 10 581 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 457 m[1] + 10 597 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 521 m[1] + 10 421 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 665 m[1] + 10 437 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 809 m[1] + 10 453 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 017 m[1] + 10 293 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 161 m[1] + 10 309 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 305 m[1] + 10 325 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-9513 m[1] + 10 165 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-9657 m[1] + 10 181 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-9009 m[1] + 10 037 m[2] - 3434 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-493 225 m[1] + 440 925 m[2] - 142 538 m[3] + 21 402 m[4] - 1517 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 2454, m[2] → 22 092, m[3] → 198 820, m[4] → 0, m[5] → 0, m[6] → 483 133 968}}

Array[m, 6] /.
{m[1] → 2454, m[2] → 22 092, m[3] → 198 820, m[4] → 0, m[5] → 0, m[6] → 483 133 968}
{2454, 22 092, 198 820, 0, 0, 483 133 968}

GCD[2454, 22 092, 198 820, 0, 0, 483 133 968]
2

{1227, 11 046, 99 410, 0, 0, 241 566 984} // Reverse
{241 566 984, 0, 0, 99 410, 11 046, 1227}

{2454, 22 092, 198 820, 0, 0, 483 133 968} / 2
{1227, 11 046, 99 410, 0, 0, 241 566 984}

{1227, 11 046, 99 410, 0, 0, 241 566 984}.
{-493 225, 440 925, -142 538, 21 402, -1517, 41}
-185 761

```

```

{1227, 11046, 99410, 0, 0, 241566984}.
Transpose[{{-12177, 10821, -3482, 522, -37, 1},
  {-12321, 10837, -3482, 522, -37, 1}, {-11529, 10677, -3474, 522, -37, 1},
  {-11673, 10693, -3474, 522, -37, 1}, {-11817, 10709, -3474, 522, -37, 1},
  {-11961, 10725, -3474, 522, -37, 1}, {-11025, 10549, -3466, 522, -37, 1},
  {-11169, 10565, -3466, 522, -37, 1}, {-11313, 10581, -3466, 522, -37, 1},
  {-11457, 10597, -3466, 522, -37, 1}, {-10521, 10421, -3458, 522, -37, 1},
  {-10665, 10437, -3458, 522, -37, 1}, {-10809, 10453, -3458, 522, -37, 1},
  {-10017, 10293, -3450, 522, -37, 1}, {-10161, 10309, -3450, 522, -37, 1},
  {-10305, 10325, -3450, 522, -37, 1}, {-9513, 10165, -3442, 522, -37, 1},
  {-9657, 10181, -3442, 522, -37, 1}, {-9009, 10037, -3434, 522, -37, 1}}]
{8951, 8999, 8703, 8751, 8799, 8847, 8503, 8551, 8599,
  8647, 8303, 8351, 8399, 8103, 8151, 8199, 7903, 7951, 7703}

{2454, 22092, 198820, 0, 0, 483133968}.Transpose[A]
{17902, 17998, 17406, 17502, 17598, 17694, 17006, 17102, 17198,
  17294, 16606, 16702, 16798, 16206, 16302, 16398, 15806, 15902, 15406}

{2454, 22092, 198820, 0, 0, 483133968}.
{-493225, 440925, -142538, 21402, -1517, 41}
-371522

```

```
poly6 = listmod128[[6]]
```

```
(-11 + x) (-9 + x)3 (-7 + x)10 (5 + x)25 (64 - 17 x + x2)
```

```
p6 = poly6 / minipoly[poly6] // Factor
```

```
(-9 + x)2 (-7 + x)9 (5 + x)24
```

```
feasiblesubcharpolylist[(-11 + x) (-9 + x)3 (-7 + x)10 (5 + x)25 (64 - 17 x + x2) ]
```

```

{ (-11 + x) (1453 - 1092 x + 274 x2 - 28 x3 + x4),
  (-11 + x) (-7 + x) (-195 + 127 x - 21 x2 + x3), (-11 + x) (1381 - 1084 x + 274 x2 - 28 x3 + x4),
  (-11 + x)2 (-127 + 87 x - 17 x2 + x3), (-11 + x)2 (-7 + x) (17 - 10 x + x2),
  (-11 + x) (1325 - 1076 x + 274 x2 - 28 x3 + x4), (-11 + x) (-9 + x) (-149 + 103 x - 19 x2 + x3),
  (-9 + x) (1535 - 1274 x + 312 x2 - 30 x3 + x4), (-11 + x) (-7 + x) (-179 + 127 x - 21 x2 + x3),
  (-11 + x) (-9 + x) (-141 + 103 x - 19 x2 + x3), (-11 + x) (-9 + x) (-7 + x) (19 - 12 x + x2) }

```

```
{(-11 + x) (1453 - 1092 x + 274 x^2 - 28 x^3 + x^4), (-11 + x) (-7 + x) (-195 + 127 x - 21 x^2 + x^3),
  (-11 + x) (1381 - 1084 x + 274 x^2 - 28 x^3 + x^4), (-11 + x)^2 (-127 + 87 x - 17 x^2 + x^3),
  (-11 + x)^2 (-7 + x) (17 - 10 x + x^2), (-11 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4),
  (-11 + x) (-9 + x) (-149 + 103 x - 19 x^2 + x^3),
  (-9 + x) (1535 - 1274 x + 312 x^2 - 30 x^3 + x^4), (-11 + x) (-7 + x) (-179 + 127 x - 21 x^2 + x^3),
  (-11 + x) (-9 + x) (-141 + 103 x - 19 x^2 + x^3),
  (-11 + x) (-9 + x) (-7 + x) (19 - 12 x + x^2)} // Length
```

```
11
```

```
CoefficientList[{(-11 + x) (1453 - 1092 x + 274 x^2 - 28 x^3 + x^4),
  (-11 + x) (-7 + x) (-195 + 127 x - 21 x^2 + x^3),
  (-11 + x) (1381 - 1084 x + 274 x^2 - 28 x^3 + x^4), (-11 + x)^2 (-127 + 87 x - 17 x^2 + x^3),
  (-11 + x)^2 (-7 + x) (17 - 10 x + x^2), (-11 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4),
  (-11 + x) (-9 + x) (-149 + 103 x - 19 x^2 + x^3), (-9 + x) (1535 - 1274 x + 312 x^2 - 30 x^3 + x^4),
  (-11 + x) (-7 + x) (-179 + 127 x - 21 x^2 + x^3), (-11 + x) (-9 + x)
  (-141 + 103 x - 19 x^2 + x^3), (-11 + x) (-9 + x) (-7 + x) (19 - 12 x + x^2)}, x]
```

```
{{-15983, 13465, -4106, 582, -39, 1},
  {-15015, 13289, -4098, 582, -39, 1}, {-15191, 13305, -4098, 582, -39, 1},
  {-15367, 13321, -4098, 582, -39, 1}, {-14399, 13145, -4090, 582, -39, 1},
  {-14575, 13161, -4090, 582, -39, 1}, {-14751, 13177, -4090, 582, -39, 1},
  {-13815, 13001, -4082, 582, -39, 1}, {-13783, 13001, -4082, 582, -39, 1},
  {-13959, 13017, -4082, 582, -39, 1}, {-13167, 12857, -4074, 582, -39, 1}}
```

```
A = {{-15983, 13465, -4106, 582, -39, 1},
  {-15015, 13289, -4098, 582, -39, 1}, {-15191, 13305, -4098, 582, -39, 1},
  {-15367, 13321, -4098, 582, -39, 1}, {-14399, 13145, -4090, 582, -39, 1},
  {-14575, 13161, -4090, 582, -39, 1}, {-14751, 13177, -4090, 582, -39, 1},
  {-13815, 13001, -4082, 582, -39, 1}, {-13783, 13001, -4082, 582, -39, 1},
  {-13959, 13017, -4082, 582, -39, 1}, {-13167, 12857, -4074, 582, -39, 1}};
```

```
CoefficientList[D[poly6, x] / p6 // Factor, x]
```

```
{-639015, 548097, -168122, 23862, -1599, 41}
```

```
Solve[Array[n, 11].{{-15983, 13465, -4106, 582, -39, 1},
  {-15015, 13289, -4098, 582, -39, 1}, {-15191, 13305, -4098, 582, -39, 1},
  {-15367, 13321, -4098, 582, -39, 1}, {-14399, 13145, -4090, 582, -39, 1},
  {-14575, 13161, -4090, 582, -39, 1}, {-14751, 13177, -4090, 582, -39, 1},
  {-13815, 13001, -4082, 582, -39, 1}, {-13783, 13001, -4082, 582, -39, 1},
  {-13959, 13017, -4082, 582, -39, 1}, {-13167, 12857, -4074, 582, -39, 1}} ==
  {-639015, 548097, -168122, 23862, -1599, 41}], Array[n, 11]]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. >>
```

```
{{n[8] → 8, n[9] → 42 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - 2 n[5] - n[6],
  n[10] → 86 - 2 n[1] - n[3] - 2 n[4] - n[6] - 2 n[7],
  n[11] → -95 + 3 n[1] + 2 n[2] + 2 n[3] + 2 n[4] + n[5] + n[6] + n[7]}}
```



```
FindInstance[
  -n[8] + 8 == 0 && -n[9] + 42 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - 2 n[5] - n[6] == 0 &&
  -n[10] - 86 - 2 n[1] - n[3] - 2 n[4] - n[6] - 2 n[7] == 0 &&
  -n[11] - 95 + 3 n[1] + 2 n[2] + 2 n[3] + 2 n[4] + n[5] + n[6] + n[7] == 0 &&
  n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
  n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0, Array[n, 11], Integers]
{}
```

```
Array[m, 6].Transpose[A]
```

```
{-15 983 m[1] + 13 465 m[2] - 4106 m[3] + 582 m[4] - 39 m[5] + m[6],
 -15 015 m[1] + 13 289 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6],
 -15 191 m[1] + 13 305 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6],
 -15 367 m[1] + 13 321 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6],
 -14 399 m[1] + 13 145 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6],
 -14 575 m[1] + 13 161 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6],
 -14 751 m[1] + 13 177 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 815 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 783 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 959 m[1] + 13 017 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 167 m[1] + 12 857 m[2] - 4074 m[3] + 582 m[4] - 39 m[5] + m[6]}
```

```
Array[m, 6].{-639 015, 548 097, -168 122, 23 862, -1599, 41}
```

```
-639 015 m[1] + 548 097 m[2] - 168 122 m[3] + 23 862 m[4] - 1599 m[5] + 41 m[6]
```

```
FindInstance[-15 983 m[1] + 13 465 m[2] - 4106 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -15 015 m[1] + 13 289 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -15 191 m[1] + 13 305 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -15 367 m[1] + 13 321 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -14 399 m[1] + 13 145 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -14 575 m[1] + 13 161 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -14 751 m[1] + 13 177 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -13 815 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -13 783 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -13 959 m[1] + 13 017 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -13 167 m[1] + 12 857 m[2] - 4074 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
  -639 015 m[1] + 548 097 m[2] - 168 122 m[3] + 23 862 m[4] - 1599 m[5] + 41 m[6] < 0,
  Array[m, 6], Integers]
```

```
{m[1] → 2150, m[2] → 10 679, m[3] → 26 648, m[4] → 0, m[5] → 0, m[6] → 0}
```

```
Array[m, 6] /. {m[1] → 2150, m[2] → 10 679, m[3] → 26 648, m[4] → 0, m[5] → 0, m[6] → 0}
```

```
{2150, 10 679, 26 648, 0, 0, 0}
```

```
GCD[2150, 10 679, 26 648, 0, 0, 0]
```

```
1
```

```
{ {-15983, 13465, -4106, 582, -39, 1}, {-15015, 13289, -4098, 582, -39, 1},
  {-15191, 13305, -4098, 582, -39, 1}, {-15367, 13321, -4098, 582, -39, 1},
  {-14399, 13145, -4090, 582, -39, 1}, {-14575, 13161, -4090, 582, -39, 1},
  {-14751, 13177, -4090, 582, -39, 1}, {-13815, 13001, -4082, 582, -39, 1},
  {-13783, 13001, -4082, 582, -39, 1}, {-13959, 13017, -4082, 582, -39, 1},
  {-13167, 12857, -4074, 582, -39, 1}} // MatrixForm
```

$$\begin{pmatrix} -15983 & 13465 & -4106 & 582 & -39 & 1 \\ -15015 & 13289 & -4098 & 582 & -39 & 1 \\ -15191 & 13305 & -4098 & 582 & -39 & 1 \\ -15367 & 13321 & -4098 & 582 & -39 & 1 \\ -14399 & 13145 & -4090 & 582 & -39 & 1 \\ -14575 & 13161 & -4090 & 582 & -39 & 1 \\ -14751 & 13177 & -4090 & 582 & -39 & 1 \\ -13815 & 13001 & -4082 & 582 & -39 & 1 \\ -13783 & 13001 & -4082 & 582 & -39 & 1 \\ -13959 & 13017 & -4082 & 582 & -39 & 1 \\ -13167 & 12857 & -4074 & 582 & -39 & 1 \end{pmatrix}$$

```
{2150, 10679, 26648, 0, 0, 0}.{-639015, 548097, -168122, 23862, -1599, 41}
-869443
```

```
{2150, 10679, 26648, 0, 0, 0}.Transpose[
```

$$\begin{pmatrix} -15983 & 13465 & -4106 & 582 & -39 & 1 \\ -15015 & 13289 & -4098 & 582 & -39 & 1 \\ -15191 & 13305 & -4098 & 582 & -39 & 1 \\ -15367 & 13321 & -4098 & 582 & -39 & 1 \\ -14399 & 13145 & -4090 & 582 & -39 & 1 \\ -14575 & 13161 & -4090 & 582 & -39 & 1 \\ -14751 & 13177 & -4090 & 582 & -39 & 1 \\ -13815 & 13001 & -4082 & 582 & -39 & 1 \\ -13783 & 13001 & -4082 & 582 & -39 & 1 \\ -13959 & 13017 & -4082 & 582 & -39 & 1 \\ -13167 & 12857 & -4074 & 582 & -39 & 1 \end{pmatrix}]$$

```
{12597, 427477, 219941, 12405, 427285,
  219749, 12213, 358293, 427093, 219557, 426901}
```

```
poly7 = listmod128[[7]]
```

$$(-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52-15x+x^2)$$

```
p7 = poly7 / minipoly[poly7] // Factor
```

$$(-11+x) (-9+x) (-7+x)^9 (5+x)^{24}$$

```
feasiblesubcharpolylist[(-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52-15x+x^2)]
```

$$\begin{aligned} &\{ (-11+x) (-9+x) (-127+87x-17x^2+x^3), (-9+x)^2 (-157+103x-19x^2+x^3), \\ &(-11+x) (-9+x) (-7+x) (17-10x+x^2), (-9+x) (1325-1076x+274x^2-28x^3+x^4), \\ &(-9+x)^2 (-149+103x-19x^2+x^3), (-9+x) (-7+x) (-179+127x-21x^2+x^3), \\ &(-9+x)^2 (-141+103x-19x^2+x^3), (-9+x)^2 (-7+x) (19-12x+x^2) \} \end{aligned}$$

```
{(-11 + x) (-9 + x) (-127 + 87 x - 17 x^2 + x^3), (-9 + x)^2 (-157 + 103 x - 19 x^2 + x^3),
  (-11 + x) (-9 + x) (-7 + x) (17 - 10 x + x^2), (-9 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4),
  (-9 + x)^2 (-149 + 103 x - 19 x^2 + x^3), (-9 + x) (-7 + x) (-179 + 127 x - 21 x^2 + x^3),
  (-9 + x)^2 (-141 + 103 x - 19 x^2 + x^3), (-9 + x)^2 (-7 + x) (19 - 12 x + x^2)} // Length
```

8

```
A = {{-12 573, 11 153, -3550, 526, -37, 1}, {-12 717, 11 169, -3550, 526, -37, 1},
  {-11 781, 10 993, -3542, 526, -37, 1}, {-11 925, 11 009, -3542, 526, -37, 1},
  {-12 069, 11 025, -3542, 526, -37, 1}, {-11 277, 10 865, -3534, 526, -37, 1},
  {-11 421, 10 881, -3534, 526, -37, 1}, {-10 773, 10 737, -3526, 526, -37, 1}};
```

```
MatrixRank[A]
```

3

```
CoefficientList[
  {(-11 + x) (-9 + x) (-127 + 87 x - 17 x^2 + x^3), (-9 + x)^2 (-157 + 103 x - 19 x^2 + x^3),
    (-11 + x) (-9 + x) (-7 + x) (17 - 10 x + x^2), (-9 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4),
    (-9 + x)^2 (-149 + 103 x - 19 x^2 + x^3), (-9 + x) (-7 + x) (-179 + 127 x - 21 x^2 + x^3),
    (-9 + x)^2 (-141 + 103 x - 19 x^2 + x^3), (-9 + x)^2 (-7 + x) (19 - 12 x + x^2)}, x]
{{-12 573, 11 153, -3550, 526, -37, 1}, {-12 717, 11 169, -3550, 526, -37, 1},
  {-11 781, 10 993, -3542, 526, -37, 1}, {-11 925, 11 009, -3542, 526, -37, 1},
  {-12 069, 11 025, -3542, 526, -37, 1}, {-11 277, 10 865, -3534, 526, -37, 1},
  {-11 421, 10 881, -3534, 526, -37, 1}, {-10 773, 10 737, -3526, 526, -37, 1}}
```

```
CoefficientList[D[poly7, x] / p7 // Factor, x]
```

```
{-518 725, 457 225, -145 502, 21 566, -1517, 41}
```

```
Solve[Array[n, 8].
```

```
{{-12 573, 11 153, -3550, 526, -37, 1}, {-12 717, 11 169, -3550, 526, -37, 1},
  {-11 781, 10 993, -3542, 526, -37, 1}, {-11 925, 11 009, -3542, 526, -37, 1},
  {-12 069, 11 025, -3542, 526, -37, 1}, {-11 277, 10 865, -3534, 526, -37, 1},
  {-11 421, 10 881, -3534, 526, -37, 1}, {-10 773, 10 737, -3526, 526, -37, 1}} ==
  {-518 725, 457 225, -145 502, 21 566, -1517, 41}, Array[n, 8]]
```

```
{}
```

```
Array[m, 6].Transpose[A]
```

```
{-12 573 m[1] + 11 153 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6],
  -12 717 m[1] + 11 169 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6],
  -11 781 m[1] + 10 993 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6],
  -11 925 m[1] + 11 009 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6],
  -12 069 m[1] + 11 025 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6],
  -11 277 m[1] + 10 865 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6],
  -11 421 m[1] + 10 881 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6],
  -10 773 m[1] + 10 737 m[2] - 3526 m[3] + 526 m[4] - 37 m[5] + m[6]}
```

```
Array[m, 6].{-518 725, 457 225, -145 502, 21 566, -1517, 41}
```

```
-518 725 m[1] + 457 225 m[2] - 145 502 m[3] + 21 566 m[4] - 1517 m[5] + 41 m[6]
```

```
FindInstance[-12 573 m[1] + 11 153 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-12 717 m[1] + 11 169 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 781 m[1] + 10 993 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 925 m[1] + 11 009 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-12 069 m[1] + 11 025 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 277 m[1] + 10 865 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 421 m[1] + 10 881 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 773 m[1] + 10 737 m[2] - 3526 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-518 725 m[1] + 457 225 m[2] - 145 502 m[3] + 21 566 m[4] - 1517 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
```

```
{m[1] → 2995, m[2] → 15 614, m[3] → 38 390, m[4] → 0, m[5] → 0, m[6] → 0}
```

```
Array[m, 6] /. {m[1] → 2995, m[2] → 15 614, m[3] → 38 390, m[4] → 0, m[5] → 0, m[6] → 0}
{2995, 15 614, 38 390, 0, 0, 0}
```

```
GCD[2995, 15 614, 38 390, 0, 0, 0]
```

```
1
```

```
A.{2995, 15 614, 38 390, 0, 0, 0}
```

```
{202 307, 20 851, 383 227, 201 771, 20 315, 201 235, 19 779, 19 243}
```

```
{-518 725, 457 225, -145 502, 21 566, -1517, 41} . {2995, 15 614, 38 390, 0, 0, 0}
-292 005
```

```
{2995, 15 614, 38 390, 0, 0, 0} . {-518 725, 457 225, -145 502, 21 566, -1517, 41}
-292 005
```

```
poly8 = listmod128[[8]]
```

```
(-11 + x) (-9 + x)2 (-8 + x) (-7 + x)8 (5 + x)25 (59 - 16 x + x2)2
```

```
p8 = poly8 / minipoly[poly8] // Factor
```

```
(-9 + x) (-7 + x)7 (5 + x)24 (59 - 16 x + x2)
```

```
feasiblesubcharpolylist[(-11 + x) (-9 + x)2 (-8 + x) (-7 + x)8 (5 + x)25 (59 - 16 x + x2)2]
```

```
{(-11 + x) (-9 + x) (-7 + x) (-3 + x) (59 - 16 x + x2),
(-11 + x) (-9 + x) (-7 + x) (-169 + 107 x - 19 x2 + x3),
(-9 + x) (59 - 16 x + x2) (-223 + 131 x - 21 x2 + x3),
(-11 + x) (-9 + x) (-7 + x)2 (23 - 12 x + x2),
(-9 + x) (-12 541 + 11 153 x - 3550 x2 + 526 x3 - 37 x4 + x5),
(-11 + x) (-9 + x)2 (-7 + x) (17 - 10 x + x2),
(-9 + x) (-7 + x) (1699 - 1330 x + 316 x2 - 30 x3 + x4)}
```

```
{(-11 + x) (-9 + x) (-7 + x) (-3 + x) (59 - 16 x + x^2),
  (-11 + x) (-9 + x) (-7 + x) (-169 + 107 x - 19 x^2 + x^3),
  (-9 + x) (59 - 16 x + x^2) (-223 + 131 x - 21 x^2 + x^3),
  (-11 + x) (-9 + x) (-7 + x)^2 (23 - 12 x + x^2),
  (-9 + x) (-12 541 + 11 153 x - 3550 x^2 + 526 x^3 - 37 x^4 + x^5),
  (-11 + x) (-9 + x)^2 (-7 + x) (17 - 10 x + x^2),
  (-9 + x) (-7 + x) (1699 - 1330 x + 316 x^2 - 30 x^3 + x^4)} // Length
```

7

```
CoefficientList[{(-11 + x) (-9 + x) (-7 + x) (-3 + x) (59 - 16 x + x^2),
  (-11 + x) (-9 + x) (-7 + x) (-169 + 107 x - 19 x^2 + x^3),
  (-9 + x) (59 - 16 x + x^2) (-223 + 131 x - 21 x^2 + x^3),
  (-11 + x) (-9 + x) (-7 + x)^2 (23 - 12 x + x^2),
  (-9 + x) (-12 541 + 11 153 x - 3550 x^2 + 526 x^3 - 37 x^4 + x^5),
  (-11 + x) (-9 + x)^2 (-7 + x) (17 - 10 x + x^2),
  (-9 + x) (-7 + x) (1699 - 1330 x + 316 x^2 - 30 x^3 + x^4)}, x]
```

```
{{122 661, -116 454, 43 519, -8300, 859, -46, 1},
 {117 117, -114 542, 43 303, -8292, 859, -46, 1},
 {118 413, -114 830, 43 319, -8292, 859, -46, 1},
 {111 573, -112 630, 43 087, -8284, 859, -46, 1},
 {112 869, -112 918, 43 103, -8284, 859, -46, 1},
 {106 029, -110 718, 42 871, -8276, 859, -46, 1},
 {107 037, -110 974, 42 887, -8276, 859, -46, 1}}
```

```
A = {{122 661, -116 454, 43 519, -8300, 859, -46, 1},
      {117 117, -114 542, 43 303, -8292, 859, -46, 1}, {118 413, -114 830, 43 319,
      -8292, 859, -46, 1}, {111 573, -112 630, 43 087, -8284, 859, -46, 1},
      {112 869, -112 918, 43 103, -8284, 859, -46, 1}, {106 029, -110 718, 42 871,
      -8276, 859, -46, 1}, {107 037, -110 974, 42 887, -8276, 859, -46, 1}};
```

```
MatrixRank[A]
```

4

```
CoefficientList[D[poly8, x] / p8 // Factor, x]
```

```
{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}
```

```
Solve[
```

```
Array[n, 7].{{122 661, -116 454, 43 519, -8300, 859, -46, 1}, {117 117, -114 542,
  43 303, -8292, 859, -46, 1}, {118 413, -114 830, 43 319, -8292, 859, -46, 1},
  {111 573, -112 630, 43 087, -8284, 859, -46, 1}, {112 869, -112 918, 43 103,
  -8284, 859, -46, 1}, {106 029, -110 718, 42 871, -8276, 859, -46, 1},
  {107 037, -110 974, 42 887, -8276, 859, -46, 1}} ==
  {4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}, Array[n, 7]]
```

```
{}
```

Array[m, 7].Transpose[A]

```
{122 661 m[1] - 116 454 m[2] + 43 519 m[3] - 8300 m[4] + 859 m[5] - 46 m[6] + m[7],
 117 117 m[1] - 114 542 m[2] + 43 303 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7],
 118 413 m[1] - 114 830 m[2] + 43 319 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7],
 111 573 m[1] - 112 630 m[2] + 43 087 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7],
 112 869 m[1] - 112 918 m[2] + 43 103 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7],
 106 029 m[1] - 110 718 m[2] + 42 871 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7],
 107 037 m[1] - 110 974 m[2] + 42 887 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7]}
```

Array[m, 7].{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}

```
4 704 685 m[1] - 4 657 158 m[2] + 1 770 487 m[3] -
 339 772 m[4] + 35 219 m[5] - 1886 m[6] + 41 m[7]
```

FindInstance[

```
122 661 m[1] - 116 454 m[2] + 43 519 m[3] - 8300 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 117 117 m[1] - 114 542 m[2] + 43 303 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 118 413 m[1] - 114 830 m[2] + 43 319 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 111 573 m[1] - 112 630 m[2] + 43 087 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 112 869 m[1] - 112 918 m[2] + 43 103 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 106 029 m[1] - 110 718 m[2] + 42 871 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 107 037 m[1] - 110 974 m[2] + 42 887 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 4 704 685 m[1] - 4 657 158 m[2] + 1 770 487 m[3] - 339 772 m[4] +
 35 219 m[5] - 1886 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
```

```
{m[1] → -8185, m[2] → -53 247,
 m[3] → -295 413, m[4] → -922 821, m[5] → 0, m[6] → 0, m[7] → 0}
```

Array[m, 7] /. {m[1] → -8185, m[2] → -53 247,

```
m[3] → -295 413, m[4] → -922 821, m[5] → 0, m[6] → 0, m[7] → 0}
```

```
{-8185, -53 247, -295 413, -922 821, 0, 0, 0}
```

GCD[-8185, -53 247, -295 413, -922 821, 0, 0, 0]

1

A.{-8185, -53 247, -295 413, -922 821, 0, 0, 0}

```
{181 806, 177 822, 178 590, 173 838, 174 606, 169 854, 823 998}
```

{-8185, -53 247, -295 413, -922 821, 0, 0, 0}.Transpose[A]

```
{181 806, 177 822, 178 590, 173 838, 174 606, 169 854, 823 998}
```

{-8185, -53 247, -295 413, -922 821, 0, 0, 0}.

```
{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}
```

-4 294 018

```
poly9 = listmod128[[9]]
```

$$(-9+x)^5 (-7+x)^8 (5+x)^{25} (-412+179x-24x^2+x^3)$$

```
p9 = poly9 / minipoly[poly9] // Factor
```

$$(-9+x)^4 (-7+x)^7 (5+x)^{24}$$

```
feasiblesubcharpolylist[(-9+x)^5 (-7+x)^8 (5+x)^{25} (-412+179x-24x^2+x^3)]
```

$$\begin{aligned} &\{ (-7+x) (1361-1044x+270x^2-28x^3+x^4), (-7+x) (1273-1036x+270x^2-28x^3+x^4), \\ &(-5+x) (1811-1346x+316x^2-30x^3+x^4), (-7+x) (1289-1036x+270x^2-28x^3+x^4), \\ &-9167+8557x-2926x^2+466x^3-35x^4+x^5, (-9+x) (-7+x) (-5+x) (29-14x+x^2), \\ &(-7+x) (1185-1028x+270x^2-28x^3+x^4), (-7+x) (1201-1028x+270x^2-28x^3+x^4), \\ &(-7+x) (1217-1028x+270x^2-28x^3+x^4), -8663+8429x-2918x^2+466x^3-35x^4+x^5, \\ &(-9+x) (-7+x) (-137+99x-19x^2+x^3), (-9+x) (-5+x) (-195+127x-21x^2+x^3), \\ &(-7+x) (1129-1020x+270x^2-28x^3+x^4), (-7+x) (1145-1020x+270x^2-28x^3+x^4), \\ &-8159+8301x-2910x^2+466x^3-35x^4+x^5, (-9+x) (-7+x) (-129+99x-19x^2+x^3), \\ &(-9+x) (919-822x+232x^2-26x^3+x^4), (-7+x)^2 (-151+123x-21x^2+x^3), \\ &(-7+x) (1073-1012x+270x^2-28x^3+x^4), (-11+x) (-9+x) (-7+x) (11-8x+x^2), \\ &(-9+x) (863-814x+232x^2-26x^3+x^4), (-11+x) (-7+x)^2 (13-10x+x^2), \\ &(-9+x) (-7+x) (-113+99x-19x^2+x^3), (-7+x) (929-996x+270x^2-28x^3+x^4), \\ &(-9+x) (-7+x)^2 (15-12x+x^2), (-9+x) (-7+x) (-97+99x-19x^2+x^3) \} \end{aligned}$$

$$\begin{aligned} &\{ (-7+x) (1361-1044x+270x^2-28x^3+x^4), (-7+x) (1273-1036x+270x^2-28x^3+x^4), \\ &(-5+x) (1811-1346x+316x^2-30x^3+x^4), (-7+x) (1289-1036x+270x^2-28x^3+x^4), \\ &-9167+8557x-2926x^2+466x^3-35x^4+x^5, (-9+x) (-7+x) (-5+x) (29-14x+x^2), \\ &(-7+x) (1185-1028x+270x^2-28x^3+x^4), (-7+x) (1201-1028x+270x^2-28x^3+x^4), \\ &(-7+x) (1217-1028x+270x^2-28x^3+x^4), -8663+8429x-2918x^2+466x^3-35x^4+x^5, \\ &(-9+x) (-7+x) (-137+99x-19x^2+x^3), (-9+x) (-5+x) (-195+127x-21x^2+x^3), \\ &(-7+x) (1129-1020x+270x^2-28x^3+x^4), (-7+x) (1145-1020x+270x^2-28x^3+x^4), \\ &-8159+8301x-2910x^2+466x^3-35x^4+x^5, (-9+x) (-7+x) (-129+99x-19x^2+x^3), \\ &(-9+x) (919-822x+232x^2-26x^3+x^4), (-7+x)^2 (-151+123x-21x^2+x^3), \\ &(-7+x) (1073-1012x+270x^2-28x^3+x^4), (-11+x) (-9+x) (-7+x) (11-8x+x^2), \\ &(-9+x) (863-814x+232x^2-26x^3+x^4), (-11+x) (-7+x)^2 (13-10x+x^2), \\ &(-9+x) (-7+x) (-113+99x-19x^2+x^3), (-7+x) (929-996x+270x^2-28x^3+x^4), \\ &(-9+x) (-7+x)^2 (15-12x+x^2), (-9+x) (-7+x) (-97+99x-19x^2+x^3) \} // \text{Length} \end{aligned}$$

CoefficientList[

{(-7 + x) (1361 - 1044 x + 270 x² - 28 x³ + x⁴), (-7 + x) (1273 - 1036 x + 270 x² - 28 x³ + x⁴),
 (-5 + x) (1811 - 1346 x + 316 x² - 30 x³ + x⁴), (-7 + x) (1289 - 1036 x + 270 x² - 28 x³ + x⁴),
 -9167 + 8557 x - 2926 x² + 466 x³ - 35 x⁴ + x⁵, (-9 + x) (-7 + x) (-5 + x) (29 - 14 x + x²),
 (-7 + x) (1185 - 1028 x + 270 x² - 28 x³ + x⁴), (-7 + x) (1201 - 1028 x + 270 x² - 28 x³ + x⁴),
 (-7 + x) (1217 - 1028 x + 270 x² - 28 x³ + x⁴), -8663 + 8429 x - 2918 x² + 466 x³ - 35 x⁴ + x⁵,
 (-9 + x) (-7 + x) (-137 + 99 x - 19 x² + x³), (-9 + x) (-5 + x) (-195 + 127 x - 21 x² + x³),
 (-7 + x) (1129 - 1020 x + 270 x² - 28 x³ + x⁴), (-7 + x) (1145 - 1020 x + 270 x² - 28 x³ + x⁴),
 -8159 + 8301 x - 2910 x² + 466 x³ - 35 x⁴ + x⁵, (-9 + x) (-7 + x) (-129 + 99 x - 19 x² + x³),
 (-9 + x) (919 - 822 x + 232 x² - 26 x³ + x⁴), (-7 + x)² (-151 + 123 x - 21 x² + x³),
 (-7 + x) (1073 - 1012 x + 270 x² - 28 x³ + x⁴), (-11 + x) (-9 + x) (-7 + x) (11 - 8 x + x²),
 (-9 + x) (863 - 814 x + 232 x² - 26 x³ + x⁴), (-11 + x) (-7 + x)² (13 - 10 x + x²),
 (-9 + x) (-7 + x) (-113 + 99 x - 19 x² + x³), (-7 + x) (929 - 996 x + 270 x² - 28 x³ + x⁴),
 (-9 + x) (-7 + x)² (15 - 12 x + x²), (-9 + x) (-7 + x) (-97 + 99 x - 19 x² + x³)}, x]

{{-9527, 8669, -2934, 466, -35, 1}, {-8911, 8525, -2926, 466, -35, 1},
 {-9055, 8541, -2926, 466, -35, 1}, {-9023, 8541, -2926, 466, -35, 1},
 {-9167, 8557, -2926, 466, -35, 1}, {-9135, 8557, -2926, 466, -35, 1},
 {-8295, 8381, -2918, 466, -35, 1}, {-8407, 8397, -2918, 466, -35, 1},
 {-8519, 8413, -2918, 466, -35, 1}, {-8663, 8429, -2918, 466, -35, 1},
 {-8631, 8429, -2918, 466, -35, 1}, {-8775, 8445, -2918, 466, -35, 1},
 {-7903, 8269, -2910, 466, -35, 1}, {-8015, 8285, -2910, 466, -35, 1},
 {-8159, 8301, -2910, 466, -35, 1}, {-8127, 8301, -2910, 466, -35, 1},
 {-8271, 8317, -2910, 466, -35, 1}, {-7399, 8141, -2902, 466, -35, 1},
 {-7511, 8157, -2902, 466, -35, 1}, {-7623, 8173, -2902, 466, -35, 1},
 {-7767, 8189, -2902, 466, -35, 1}, {-7007, 8029, -2894, 466, -35, 1},
 {-7119, 8045, -2894, 466, -35, 1}, {-6503, 7901, -2886, 466, -35, 1},
 {-6615, 7917, -2886, 466, -35, 1}, {-6111, 7789, -2878, 466, -35, 1}}

A = {{-9527, 8669, -2934, 466, -35, 1}, {-8911, 8525, -2926, 466, -35, 1},
 {-9055, 8541, -2926, 466, -35, 1}, {-9023, 8541, -2926, 466, -35, 1},
 {-9167, 8557, -2926, 466, -35, 1}, {-9135, 8557, -2926, 466, -35, 1},
 {-8295, 8381, -2918, 466, -35, 1}, {-8407, 8397, -2918, 466, -35, 1},
 {-8519, 8413, -2918, 466, -35, 1}, {-8663, 8429, -2918, 466, -35, 1},
 {-8631, 8429, -2918, 466, -35, 1}, {-8775, 8445, -2918, 466, -35, 1},
 {-7903, 8269, -2910, 466, -35, 1}, {-8015, 8285, -2910, 466, -35, 1},
 {-8159, 8301, -2910, 466, -35, 1}, {-8127, 8301, -2910, 466, -35, 1},
 {-8271, 8317, -2910, 466, -35, 1}, {-7399, 8141, -2902, 466, -35, 1},
 {-7511, 8157, -2902, 466, -35, 1}, {-7623, 8173, -2902, 466, -35, 1},
 {-7767, 8189, -2902, 466, -35, 1}, {-7007, 8029, -2894, 466, -35, 1},
 {-7119, 8045, -2894, 466, -35, 1}, {-6503, 7901, -2886, 466, -35, 1},
 {-6615, 7917, -2886, 466, -35, 1}, {-6111, 7789, -2878, 466, -35, 1}};

CoefficientList[D[poly9, x] / p9 // Factor, x]

{-372 095, 350 101, -119 942, 19 106, -1435, 41}

Solve[Array[n, 26].

```
{ {-9527, 8669, -2934, 466, -35, 1}, {-8911, 8525, -2926, 466, -35, 1},
  {-9055, 8541, -2926, 466, -35, 1}, {-9023, 8541, -2926, 466, -35, 1},
  {-9167, 8557, -2926, 466, -35, 1}, {-9135, 8557, -2926, 466, -35, 1},
  {-8295, 8381, -2918, 466, -35, 1}, {-8407, 8397, -2918, 466, -35, 1},
  {-8519, 8413, -2918, 466, -35, 1}, {-8663, 8429, -2918, 466, -35, 1},
  {-8631, 8429, -2918, 466, -35, 1}, {-8775, 8445, -2918, 466, -35, 1},
  {-7903, 8269, -2910, 466, -35, 1}, {-8015, 8285, -2910, 466, -35, 1},
  {-8159, 8301, -2910, 466, -35, 1}, {-8127, 8301, -2910, 466, -35, 1},
  {-8271, 8317, -2910, 466, -35, 1}, {-7399, 8141, -2902, 466, -35, 1},
  {-7511, 8157, -2902, 466, -35, 1}, {-7623, 8173, -2902, 466, -35, 1},
  {-7767, 8189, -2902, 466, -35, 1}, {-7007, 8029, -2894, 466, -35, 1},
  {-7119, 8045, -2894, 466, -35, 1}, {-6503, 7901, -2886, 466, -35, 1},
  {-6615, 7917, -2886, 466, -35, 1}, {-6111, 7789, -2878, 466, -35, 1}} ==
{-372095, 350101, -119942, 19106, -1435, 41}, Array[n, 26]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{ {n[21] → 48 - n[3] - n[5] - n[10] - n[12] - n[15] - n[17],
  n[24] → 70 - n[1] - 2 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] -
    n[9] - n[10] - 2 n[13] - n[14] - n[15] - 2 n[18] - n[19] - n[22],
  n[25] → 29 - 6 n[1] - 4 n[2] - n[3] - 5 n[4] - 2 n[5] - 6 n[6] - 2 n[7] -
    3 n[8] - 4 n[9] - n[10] - 5 n[11] - 2 n[12] - 2 n[13] - 3 n[14] -
    4 n[16] - n[17] - n[18] - 2 n[19] - 3 n[20] - n[22] - 2 n[23],
  n[26] → -106 + 6 n[1] + 5 n[2] + 3 n[3] + 5 n[4] + 3 n[5] + 5 n[6] + 4 n[7] +
    4 n[8] + 4 n[9] + 2 n[10] + 4 n[11] + 2 n[12] + 3 n[13] + 3 n[14] +
    n[15] + 3 n[16] + n[17] + 2 n[18] + 2 n[19] + 2 n[20] + n[22] + n[23] } }
```

```
FindInstance[-n[21] + 48 - n[3] - n[5] - n[10] - n[12] - n[15] - n[17] == 0 &&
  -n[24] + 70 - n[1] - 2 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] -
    n[9] - n[10] - 2 n[13] - n[14] - n[15] - 2 n[18] - n[19] - n[22] == 0 &&
  -n[25] + 29 - 6 n[1] - 4 n[2] - n[3] - 5 n[4] - 2 n[5] - 6 n[6] -
    2 n[7] - 3 n[8] - 4 n[9] - n[10] - 5 n[11] - 2 n[12] - 2 n[13] - 3 n[14] -
    4 n[16] - n[17] - n[18] - 2 n[19] - 3 n[20] - n[22] - 2 n[23] == 0 &&
  -n[26] - 106 + 6 n[1] + 5 n[2] + 3 n[3] + 5 n[4] + 3 n[5] + 5 n[6] + 4 n[7] +
    4 n[8] + 4 n[9] + 2 n[10] + 4 n[11] + 2 n[12] + 3 n[13] + 3 n[14] + n[15] +
    3 n[16] + n[17] + 2 n[18] + 2 n[19] + 2 n[20] + n[22] + n[23] == 0
  && n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
  n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 &&
  n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0 && n[19] ≥ 0 && n[20] ≥ 0 && n[21] ≥ 0 &&
  n[22] ≥ 0 && n[23] ≥ 0 && n[24] ≥ 0 && n[25] ≥ 0 && n[26] ≥ 0, Array[n, 26], Integers]
```

{}

Array[m, 6].Transpose[A]

```
{ -9527 m[1] + 8669 m[2] - 2934 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8911 m[1] + 8525 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -9055 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -9023 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -9167 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -9135 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8295 m[1] + 8381 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8407 m[1] + 8397 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8519 m[1] + 8413 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8663 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8631 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8775 m[1] + 8445 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7903 m[1] + 8269 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8015 m[1] + 8285 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8159 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8127 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -8271 m[1] + 8317 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7399 m[1] + 8141 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7511 m[1] + 8157 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7623 m[1] + 8173 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7767 m[1] + 8189 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7007 m[1] + 8029 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -7119 m[1] + 8045 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -6503 m[1] + 7901 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -6615 m[1] + 7917 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6] ,
  -6111 m[1] + 7789 m[2] - 2878 m[3] + 466 m[4] - 35 m[5] + m[6] }
```

Array[m, 6].{-372 095, 350 101, -119 942, 19 106, -1435, 41}

```
-372 095 m[1] + 350 101 m[2] - 119 942 m[3] + 19 106 m[4] - 1435 m[5] + 41 m[6]
```

```

FindInstance[-9527 m[1] + 8669 m[2] - 2934 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8911 m[1] + 8525 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9055 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9023 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9167 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9135 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8295 m[1] + 8381 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8407 m[1] + 8397 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8519 m[1] + 8413 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8663 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8631 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8775 m[1] + 8445 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7903 m[1] + 8269 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8015 m[1] + 8285 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8159 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8127 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8271 m[1] + 8317 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7399 m[1] + 8141 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7511 m[1] + 8157 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7623 m[1] + 8173 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7767 m[1] + 8189 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7007 m[1] + 8029 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7119 m[1] + 8045 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -6503 m[1] + 7901 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -6615 m[1] + 7917 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -6111 m[1] + 7789 m[2] - 2878 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -372 095 m[1] + 350 101 m[2] - 119 942 m[3] + 19 106 m[4] - 1435 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 3851, m[2] → 26 961, m[3] → 188 720, m[4] → 0, m[5] → 0, m[6] → 356 804 550}}

Array[m, 6] /.
{m[1] → 3851, m[2] → 26 961, m[3] → 188 720, m[4] → 0, m[5] → 0, m[6] → 356 804 550}
{3851, 26 961, 188 720, 0, 0, 356 804 550}

GCD[3851, 26 961, 188 720, 0, 0, 356 804 550]

```

```

{{-9527, 8669, -2934, 466, -35, 1},
 {-8911, 8525, -2926, 466, -35, 1}, {-9055, 8541, -2926, 466, -35, 1},
 {-9023, 8541, -2926, 466, -35, 1}, {-9167, 8557, -2926, 466, -35, 1},
 {-9135, 8557, -2926, 466, -35, 1}, {-8295, 8381, -2918, 466, -35, 1},
 {-8407, 8397, -2918, 466, -35, 1}, {-8519, 8413, -2918, 466, -35, 1},
 {-8663, 8429, -2918, 466, -35, 1}, {-8631, 8429, -2918, 466, -35, 1},
 {-8775, 8445, -2918, 466, -35, 1}, {-7903, 8269, -2910, 466, -35, 1},
 {-8015, 8285, -2910, 466, -35, 1}, {-8159, 8301, -2910, 466, -35, 1},
 {-8127, 8301, -2910, 466, -35, 1}, {-8271, 8317, -2910, 466, -35, 1},
 {-7399, 8141, -2902, 466, -35, 1}, {-7511, 8157, -2902, 466, -35, 1},
 {-7623, 8173, -2902, 466, -35, 1}, {-7767, 8189, -2902, 466, -35, 1},
 {-7007, 8029, -2894, 466, -35, 1}, {-7119, 8045, -2894, 466, -35, 1},
 {-6503, 7901, -2886, 466, -35, 1}, {-6615, 7917, -2886, 466, -35, 1},
 {-6111, 7789, -2878, 466, -35, 1}} // MatrixForm

(
-9527 8669 -2934 466 -35 1
-8911 8525 -2926 466 -35 1
-9055 8541 -2926 466 -35 1
-9023 8541 -2926 466 -35 1
-9167 8557 -2926 466 -35 1
-9135 8557 -2926 466 -35 1
-8295 8381 -2918 466 -35 1
-8407 8397 -2918 466 -35 1
-8519 8413 -2918 466 -35 1
-8663 8429 -2918 466 -35 1
-8631 8429 -2918 466 -35 1
-8775 8445 -2918 466 -35 1
-7903 8269 -2910 466 -35 1
-8015 8285 -2910 466 -35 1
-8159 8301 -2910 466 -35 1
-8127 8301 -2910 466 -35 1
-8271 8317 -2910 466 -35 1
-7399 8141 -2902 466 -35 1
-7511 8157 -2902 466 -35 1
-7623 8173 -2902 466 -35 1
-7767 8189 -2902 466 -35 1
-7007 8029 -2894 466 -35 1
-7119 8045 -2894 466 -35 1
-6503 7901 -2886 466 -35 1
-6615 7917 -2886 466 -35 1
-6111 7789 -2878 466 -35 1
)

{3851, 26961, 188720, 0, 0, 356804550}.
{-372095, 350101, -119942, 19106, -1435, 41}
-332474

```

```

{3851, 26 961, 188 720, 0, 0, 356 804 550}.Transpose[
  -9527 8669 -2934 466 -35 1
  -8911 8525 -2926 466 -35 1
  -9055 8541 -2926 466 -35 1
  -9023 8541 -2926 466 -35 1
  -9167 8557 -2926 466 -35 1
  -9135 8557 -2926 466 -35 1
  -8295 8381 -2918 466 -35 1
  -8407 8397 -2918 466 -35 1
  -8519 8413 -2918 466 -35 1
  -8663 8429 -2918 466 -35 1
  -8631 8429 -2918 466 -35 1
  -8775 8445 -2918 466 -35 1
  -7903 8269 -2910 466 -35 1
  -8015 8285 -2910 466 -35 1
  -8159 8301 -2910 466 -35 1
  -8127 8301 -2910 466 -35 1
  -8271 8317 -2910 466 -35 1
  -7399 8141 -2902 466 -35 1
  -7511 8157 -2902 466 -35 1
  -7623 8173 -2902 466 -35 1
  -7767 8189 -2902 466 -35 1
  -7007 8029 -2894 466 -35 1
  -7119 8045 -2894 466 -35 1
  -6503 7901 -2886 466 -35 1
  -6615 7917 -2886 466 -35 1
  -6111 7789 -2878 466 -35 1
]

{136 502, 136 094, 12 926, 136 158, 12 990, 136 222, 135 686, 135 750,
 135 814, 12 646, 135 878, 12 710, 135 406, 135 470, 12 302, 135 534, 12 366,
 135 062, 135 126, 135 190, 12 022, 134 782, 134 846, 134 438, 134 502, 134 158}

```

```
poly10 = listmod128[[10]]
```

```
(-11 + x) (-9 + x)4 (-7 + x)8 (-5 + x) (5 + x)25 (68 - 17 x + x2)
```

```
p10 = poly10 / minipoly[poly10] // Factor
```

```
(-9 + x)3 (-7 + x)7 (5 + x)24
```

```

feasiblesubcharpolylist[(-11 + x) (-9 + x)4 (-7 + x)8 (-5 + x) (5 + x)25 (68 - 17 x + x2)]
{ (-11 + x) (-7 + x) (1059 - 870 x + 236 x2 - 26 x3 + x4),
  (-11 + x) (-7 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3),
  (-11 + x) (-7 + x) (59 - 16 x + x2) (17 - 10 x + x2),
  (-11 + x) (-7 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4),
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (23 - 12 x + x2),
  (-9 + x) (-8823 + 8461 x - 2918 x2 + 466 x3 - 35 x4 + x5),
  (-11 + x) (-7 + x) (947 - 854 x + 236 x2 - 26 x3 + x4),
  (-11 + x) (-9 + x) (-7 + x) (-107 + 83 x - 17 x2 + x3),
  (-7 + x) (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3),
  (-9 + x) (-7 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4),
  (-11 + x) (-9 + x)2 (-7 + x) (11 - 8 x + x2),
  (-9 + x) (-7 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) }

{ (-11 + x) (-7 + x) (1059 - 870 x + 236 x2 - 26 x3 + x4),
  (-11 + x) (-7 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3),
  (-11 + x) (-7 + x) (59 - 16 x + x2) (17 - 10 x + x2),
  (-11 + x) (-7 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4),
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (23 - 12 x + x2),
  (-9 + x) (-8823 + 8461 x - 2918 x2 + 466 x3 - 35 x4 + x5),
  (-11 + x) (-7 + x) (947 - 854 x + 236 x2 - 26 x3 + x4),
  (-11 + x) (-9 + x) (-7 + x) (-107 + 83 x - 17 x2 + x3),
  (-7 + x) (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3),
  (-9 + x) (-7 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4),
  (-11 + x) (-9 + x)2 (-7 + x) (11 - 8 x + x2),
  (-9 + x) (-7 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) } // Length

```

```

CoefficientList[ { (-11 + x) (-7 + x) (1059 - 870 x + 236 x^2 - 26 x^3 + x^4),
  (-11 + x) (-7 + x) (-5 + x) (-215 + 131 x - 21 x^2 + x^3),
  (-11 + x) (-7 + x) (59 - 16 x + x^2) (17 - 10 x + x^2),
  (-11 + x) (-7 + x) (1019 - 862 x + 236 x^2 - 26 x^3 + x^4),
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (23 - 12 x + x^2),
  (-9 + x) (-8823 + 8461 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-11 + x) (-7 + x) (947 - 854 x + 236 x^2 - 26 x^3 + x^4),
  (-11 + x) (-9 + x) (-7 + x) (-107 + 83 x - 17 x^2 + x^3),
  (-7 + x) (59 - 16 x + x^2) (-179 + 127 x - 21 x^2 + x^3),
  (-9 + x) (-7 + x) (1193 - 1020 x + 270 x^2 - 28 x^3 + x^4),
  (-11 + x) (-9 + x)^2 (-7 + x) (11 - 8 x + x^2),
  (-9 + x) (-7 + x) (1105 - 1012 x + 270 x^2 - 28 x^3 + x^4) }, x]

```

```

{ {81543, -86052, 34891, -7120, 781, -44, 1},
  {82775, -86340, 34907, -7120, 781, -44, 1},
  {77231, -84428, 34691, -7112, 781, -44, 1},
  {78463, -84716, 34707, -7112, 781, -44, 1},
  {79695, -85004, 34723, -7112, 781, -44, 1},
  {79407, -84972, 34723, -7112, 781, -44, 1},
  {72919, -82804, 34491, -7104, 781, -44, 1},
  {74151, -83092, 34507, -7104, 781, -44, 1},
  {73927, -83060, 34507, -7104, 781, -44, 1},
  {75159, -83348, 34523, -7104, 781, -44, 1},
  {68607, -81180, 34291, -7096, 781, -44, 1},
  {69615, -81436, 34307, -7096, 781, -44, 1} }

```

```

A = { {81543, -86052, 34891, -7120, 781, -44, 1},
  {82775, -86340, 34907, -7120, 781, -44, 1},
  {77231, -84428, 34691, -7112, 781, -44, 1}, {78463, -84716, 34707,
    -7112, 781, -44, 1}, {79695, -85004, 34723, -7112, 781, -44, 1},
  {79407, -84972, 34723, -7112, 781, -44, 1}, {72919, -82804, 34491,
    -7104, 781, -44, 1}, {74151, -83092, 34507, -7104, 781, -44, 1},
  {73927, -83060, 34507, -7104, 781, -44, 1}, {75159, -83348, 34523,
    -7104, 781, -44, 1}, {68607, -81180, 34291, -7096, 781, -44, 1},
  {69615, -81436, 34307, -7096, 781, -44, 1} };

```

```

CoefficientList[D[poly10, x] / p10 // Factor, x]

```

```

{3383255, -3537084, 1430763, -291896, 32021, -1804, 41}

```

Solve[

```
Array[n, 12].{{81543, -86052, 34891, -7120, 781, -44, 1}, {82775, -86340, 34907,
-7120, 781, -44, 1}, {77231, -84428, 34691, -7112, 781, -44, 1},
{78463, -84716, 34707, -7112, 781, -44, 1}, {79695, -85004, 34723,
-7112, 781, -44, 1}, {79407, -84972, 34723, -7112, 781, -44, 1},
{72919, -82804, 34491, -7104, 781, -44, 1}, {74151, -83092, 34507,
-7104, 781, -44, 1}, {73927, -83060, 34507, -7104, 781, -44, 1},
{75159, -83348, 34523, -7104, 781, -44, 1}, {68607, -81180, 34291,
-7096, 781, -44, 1}, {69615, -81436, 34307, -7096, 781, -44, 1}} ==
{3383255, -3537084, 1430763, -291896, 32021, -1804, 41}, Array[n, 12]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{{n[6] -> 48, n[9] -> 56 - 3 n[1] - 2 n[2] - 2 n[3] - n[4] - n[7],
n[10] -> -32 - n[2] - n[4] - 2 n[5] - n[8],
n[11] -> 5 - n[1] - n[2] - n[3] - n[4] - n[5] - n[7] - n[8],
n[12] -> -36 + 3 n[1] + 3 n[2] + 2 n[3] + 2 n[4] + 2 n[5] + n[7] + n[8]}}
```

```
FindInstance[-n[6] + 48 == 0 && -n[9] + 56 - 3 n[1] - 2 n[2] - 2 n[3] - n[4] - n[7] == 0 &&
-n[10] - 32 - n[2] - n[4] - 2 n[5] - n[8] == 0 &&
-n[11] + 5 - n[1] - n[2] - n[3] - n[4] - n[5] - n[7] - n[8] == 0 &&
-n[12] - 36 + 3 n[1] + 3 n[2] + 2 n[3] + 2 n[4] + 2 n[5] + n[7] + n[8] == 0 && n[1] >= 0 &&
n[2] >= 0 && n[3] >= 0 && n[4] >= 0 && n[5] >= 0 && n[6] >= 0 && n[7] >= 0 && n[8] >= 0 &&
n[9] >= 0 && n[10] >= 0 && n[11] >= 0 && n[12] >= 0, Array[n, 12], Integers]
```

{}

Array[m, 7].Transpose[A]

```
{81543 m[1] - 86052 m[2] + 34891 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7],
82775 m[1] - 86340 m[2] + 34907 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7],
77231 m[1] - 84428 m[2] + 34691 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
78463 m[1] - 84716 m[2] + 34707 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
79695 m[1] - 85004 m[2] + 34723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
79407 m[1] - 84972 m[2] + 34723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
72919 m[1] - 82804 m[2] + 34491 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
74151 m[1] - 83092 m[2] + 34507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
73927 m[1] - 83060 m[2] + 34507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
75159 m[1] - 83348 m[2] + 34523 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
68607 m[1] - 81180 m[2] + 34291 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7],
69615 m[1] - 81436 m[2] + 34307 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7]}
```

Array[m, 7].{3383255, -3537084, 1430763, -291896, 32021, -1804, 41}

```
3383255 m[1] - 3537084 m[2] + 1430763 m[3] -
291896 m[4] + 32021 m[5] - 1804 m[6] + 41 m[7]
```



```

FindInstance[
  81543 m[1] - 86052 m[2] + 34891 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  82775 m[1] - 86340 m[2] + 34907 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  77231 m[1] - 84428 m[2] + 34691 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  78463 m[1] - 84716 m[2] + 34707 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  79695 m[1] - 85004 m[2] + 34723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  79407 m[1] - 84972 m[2] + 34723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  72919 m[1] - 82804 m[2] + 34491 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  74151 m[1] - 83092 m[2] + 34507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  73927 m[1] - 83060 m[2] + 34507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  75159 m[1] - 83348 m[2] + 34523 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  68607 m[1] - 81180 m[2] + 34291 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  69615 m[1] - 81436 m[2] + 34307 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
  3383255 m[1] - 3537084 m[2] + 1430763 m[3] - 291896 m[4] +
    32021 m[5] - 1804 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
{{m[1] → 10481, m[2] → 50373, m[3] → 99745, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}}

Array[m, 7] /.
{m[1] → 10481, m[2] → 50373, m[3] → 99745, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}
{10481, 50373, 99745, 0, 0, 0, 0}

GCD[10481, 50373, 99745, 0, 0, 0, 0]
1

{{81543, -86052, 34891, -7120, 781, -44, 1},
 {82775, -86340, 34907, -7120, 781, -44, 1},
 {77231, -84428, 34691, -7112, 781, -44, 1},
 {78463, -84716, 34707, -7112, 781, -44, 1},
 {79695, -85004, 34723, -7112, 781, -44, 1},
 {79407, -84972, 34723, -7112, 781, -44, 1},
 {72919, -82804, 34491, -7104, 781, -44, 1},
 {74151, -83092, 34507, -7104, 781, -44, 1},
 {73927, -83060, 34507, -7104, 781, -44, 1},
 {75159, -83348, 34523, -7104, 781, -44, 1},
 {68607, -81180, 34291, -7096, 781, -44, 1},
 {69615, -81436, 34307, -7096, 781, -44, 1}} // MatrixForm
(
81543 -86052 34891 -7120 781 -44 1
82775 -86340 34907 -7120 781 -44 1
77231 -84428 34691 -7112 781 -44 1
78463 -84716 34707 -7112 781 -44 1
79695 -85004 34723 -7112 781 -44 1
79407 -84972 34723 -7112 781 -44 1
72919 -82804 34491 -7104 781 -44 1
74151 -83092 34507 -7104 781 -44 1
73927 -83060 34507 -7104 781 -44 1
75159 -83348 34523 -7104 781 -44 1
68607 -81180 34291 -7096 781 -44 1
69615 -81436 34307 -7096 781 -44 1
)

```

```
{10481, 50373, 99745, 0, 0, 0, 0}.
```

```
{3383255, -3537084, 1430763, -291896, 32021, -1804, 41}
```

```
-2181242
```

```
{10481, 50373, 99745, 0, 0, 0, 0}.
```

```
Transpose[
$$\begin{pmatrix} 81543 & -86052 & 34891 & -7120 & 781 & -44 & 1 \\ 82775 & -86340 & 34907 & -7120 & 781 & -44 & 1 \\ 77231 & -84428 & 34691 & -7112 & 781 & -44 & 1 \\ 78463 & -84716 & 34707 & -7112 & 781 & -44 & 1 \\ 79695 & -85004 & 34723 & -7112 & 781 & -44 & 1 \\ 79407 & -84972 & 34723 & -7112 & 781 & -44 & 1 \\ 72919 & -82804 & 34491 & -7104 & 781 & -44 & 1 \\ 74151 & -83092 & 34507 & -7104 & 781 & -44 & 1 \\ 73927 & -83060 & 34507 & -7104 & 781 & -44 & 1 \\ 75159 & -83348 & 34523 & -7104 & 781 & -44 & 1 \\ 68607 & -81180 & 34291 & -7096 & 781 & -44 & 1 \\ 69615 & -81436 & 34307 & -7096 & 781 & -44 & 1 \end{pmatrix}$$
]
```

```
{157582, 158670, 16820262, 16821350, 16822438, 15415846,
```

```
33482942, 33484030, 32748222, 32749310, 50145622, 49410902}
```

```
poly11 = listmod128[[11]]
```

```
 $(-9 + x)^4 (-7 + x)^7 (5 + x)^{25} (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5)$ 
```

```
p11 = poly11 / minipoly[poly11] // Factor
```

```
 $(-9 + x)^3 (-7 + x)^6 (5 + x)^{24}$ 
```

feasiblesubcharpolylist[

$$(-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5)]$$

$$\{ (-9+x) (-7+x) (-5+x) (1843 - 1346x + 316x^2 - 30x^3 + x^4), \\ (-9+x) (-7+x) (-9183 + 8573x - 2926x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x) (-7+x) (-8599 + 8429x - 2918x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x)^2 (-7+x) (-5+x) (-195 + 127x - 21x^2 + x^3), \\ (-9+x) (-7+x)^2 (1249 - 1028x + 270x^2 - 28x^3 + x^4), \\ (-9+x) (-7+x) (-8711 + 8445x - 2918x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x)^2 (-5+x) (1381 - 1084x + 274x^2 - 28x^3 + x^4), \\ (-9+x) (61921 - 68082x + 28887x^2 - 6180x^3 + 711x^4 - 42x^5 + x^6), \\ (-11+x) (-9+x) (-7+x)^2 (-5+x) (23 - 12x + x^2), \\ (-9+x) (-7+x) (-8159 + 8301x - 2910x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x)^2 (-7+x)^2 (-129 + 99x - 19x^2 + x^3), \\ (-9+x)^2 (-7+x) (919 - 822x + 232x^2 - 26x^3 + x^4), \\ (-11+x) (-9+x) (-7+x)^2 (-107 + 83x - 17x^2 + x^3), \\ (-11+x) (-9+x)^2 (-7+x) (-5+x) (17 - 10x + x^2), \\ (-9+x) (-7+x) (-8383 + 8333x - 2910x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x) (-7+x) (-8527 + 8349x - 2910x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x) (59401 - 66938x + 28719x^2 - 6172x^3 + 711x^4 - 42x^5 + x^6), \\ (-9+x) (-7+x) (-7655 + 8173x - 2902x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x)^2 (-7+x) (863 - 814x + 232x^2 - 26x^3 + x^4), \\ (-9+x)^2 (-7+x) (879 - 814x + 232x^2 - 26x^3 + x^4), \\ (-9+x) (-7+x) (-7879 + 8205x - 2902x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x) (-7+x) (-8023 + 8221x - 2902x^2 + 466x^3 - 35x^4 + x^5), \\ (-9+x)^2 (-7+x) (807 - 806x + 232x^2 - 26x^3 + x^4), \\ (-9+x)^2 (-7+x) (823 - 806x + 232x^2 - 26x^3 + x^4) \}$$

```

{ (-9 + x) (-7 + x) (-5 + x) (1843 - 1346 x + 316 x^2 - 30 x^3 + x^4),
  (-9 + x) (-7 + x) (-9183 + 8573 x - 2926 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8599 + 8429 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (-5 + x) (-195 + 127 x - 21 x^2 + x^3),
  (-9 + x) (-7 + x)^2 (1249 - 1028 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (-8711 + 8445 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-5 + x) (1381 - 1084 x + 274 x^2 - 28 x^3 + x^4),
  (-9 + x) (61921 - 68082 x + 28887 x^2 - 6180 x^3 + 711 x^4 - 42 x^5 + x^6),
  (-11 + x) (-9 + x) (-7 + x)^2 (-5 + x) (23 - 12 x + x^2),
  (-9 + x) (-7 + x) (-8159 + 8301 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x)^2 (-129 + 99 x - 19 x^2 + x^3),
  (-9 + x)^2 (-7 + x) (919 - 822 x + 232 x^2 - 26 x^3 + x^4),
  (-11 + x) (-9 + x) (-7 + x)^2 (-107 + 83 x - 17 x^2 + x^3),
  (-11 + x) (-9 + x)^2 (-7 + x) (-5 + x) (17 - 10 x + x^2),
  (-9 + x) (-7 + x) (-8383 + 8333 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6),
  (-9 + x) (-7 + x) (-7655 + 8173 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (863 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (-7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (823 - 806 x + 232 x^2 - 26 x^3 + x^4) } // Length

```

```

CoefficientList[ { (-9 + x) (-7 + x) (-5 + x) (1843 - 1346 x + 316 x^2 - 30 x^3 + x^4),
  (-9 + x) (-7 + x) (-9183 + 8573 x - 2926 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8599 + 8429 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (-5 + x) (-195 + 127 x - 21 x^2 + x^3),
  (-9 + x) (-7 + x)^2 (1249 - 1028 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (-8711 + 8445 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-5 + x) (1381 - 1084 x + 274 x^2 - 28 x^3 + x^4),
  (-9 + x) (61921 - 68082 x + 28887 x^2 - 6180 x^3 + 711 x^4 - 42 x^5 + x^6),
  (-11 + x) (-9 + x) (-7 + x)^2 (-5 + x) (23 - 12 x + x^2),
  (-9 + x) (-7 + x) (-8159 + 8301 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x)^2 (-129 + 99 x - 19 x^2 + x^3),
  (-9 + x)^2 (-7 + x) (919 - 822 x + 232 x^2 - 26 x^3 + x^4),
  (-11 + x) (-9 + x) (-7 + x)^2 (-107 + 83 x - 17 x^2 + x^3),
  (-11 + x) (-9 + x)^2 (-7 + x) (-5 + x) (17 - 10 x + x^2),
  (-9 + x) (-7 + x) (-8383 + 8333 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6),
  (-9 + x) (-7 + x) (-7655 + 8173 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (863 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (-7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (823 - 806 x + 232 x^2 - 26 x^3 + x^4) }, x]

```

```
{ {-580545, 687539, -330721, 84747, -12587, 1089, -51, 1},
  {-578529, 687027, -330689, 84747, -12587, 1089, -51, 1},
  {-541737, 668611, -327297, 84475, -12579, 1089, -51, 1},
  {-552825, 672435, -327729, 84491, -12579, 1089, -51, 1},
  {-550809, 671923, -327697, 84491, -12579, 1089, -51, 1},
  {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
  {-559305, 675171, -328097, 84507, -12579, 1089, -51, 1},
  {-557289, 674659, -328065, 84507, -12579, 1089, -51, 1},
  {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
  {-514017, 653507, -324305, 84219, -12571, 1089, -51, 1},
  {-512001, 652995, -324273, 84219, -12571, 1089, -51, 1},
  {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
  {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
  {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
  {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
  {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
  {-534609, 661843, -325409, 84267, -12571, 1089, -51, 1},
  {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
  {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
  {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
  {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
  {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
  {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
  {-466641, 627363, -318961, 83739, -12555, 1089, -51, 1}}
```

```
CoefficientList[D[poly11, x] / p11 // Factor, x]
```

```
{-23396345, 27945611, -13505961, 3469539, -515891, 44649, -2091, 41}
```

```

A = {{-580 545, 687 539, -330 721, 84 747, -12 587, 1089, -51, 1},
      {-578 529, 687 027, -330 689, 84 747, -12 587, 1089, -51, 1},
      {-541 737, 668 611, -327 297, 84 475, -12 579, 1089, -51, 1},
      {-552 825, 672 435, -327 729, 84 491, -12 579, 1089, -51, 1},
      {-550 809, 671 923, -327 697, 84 491, -12 579, 1089, -51, 1},
      {-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
      {-559 305, 675 171, -328 097, 84 507, -12 579, 1089, -51, 1},
      {-557 289, 674 659, -328 065, 84 507, -12 579, 1089, -51, 1},
      {-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
      {-514 017, 653 507, -324 305, 84 219, -12 571, 1089, -51, 1},
      {-512 001, 652 995, -324 273, 84 219, -12 571, 1089, -51, 1},
      {-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
      {-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
      {-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
      {-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
      {-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
      {-534 609, 661 843, -325 409, 84 267, -12 571, 1089, -51, 1},
      {-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
      {-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
      {-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
      {-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
      {-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
      {-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
      {-466 641, 627 363, -318 961, 83 739, -12 555, 1089, -51, 1}};

```

```
MatrixRank[A]
```

```
5
```

```

Solve[Array[n, 24].{{-580 545, 687 539, -330 721, 84 747, -12 587, 1089, -51, 1},
  {-578 529, 687 027, -330 689, 84 747, -12 587, 1089, -51, 1},
  {-541 737, 668 611, -327 297, 84 475, -12 579, 1089, -51, 1},
  {-552 825, 672 435, -327 729, 84 491, -12 579, 1089, -51, 1},
  {-550 809, 671 923, -327 697, 84 491, -12 579, 1089, -51, 1},
  {-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
  {-559 305, 675 171, -328 097, 84 507, -12 579, 1089, -51, 1},
  {-557 289, 674 659, -328 065, 84 507, -12 579, 1089, -51, 1},
  {-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
  {-514 017, 653 507, -324 305, 84 219, -12 571, 1089, -51, 1},
  {-512 001, 652 995, -324 273, 84 219, -12 571, 1089, -51, 1},
  {-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
  {-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
  {-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
  {-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
  {-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
  {-534 609, 661 843, -325 409, 84 267, -12 571, 1089, -51, 1},
  {-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
  {-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
  {-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
  {-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
  {-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
  {-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
  {-466 641, 627 363, -318 961, 83 739, -12 555, 1089, -51, 1}} =
{-23 396 345, 27 945 611, -13 505 961, 3 469 539, -515 891,
  44 649, -2091, 41}, Array[n, 24]]

```

```
{}
```

```

Solve[Array[n, 24].A == {-23 396 345, 27 945 611,
  -13 505 961, 3 469 539, -515 891, 44 649, -2091, 41}, Array[n, 24]]

```

```
{}
```


Array[m, 8].Transpose[A]

```
{-580545 m[1] + 687539 m[2] - 330721 m[3] + 84747 m[4] -
  12587 m[5] + 1089 m[6] - 51 m[7] + m[8], -578529 m[1] + 687027 m[2] -
  330689 m[3] + 84747 m[4] - 12587 m[5] + 1089 m[6] - 51 m[7] + m[8],
-541737 m[1] + 668611 m[2] - 327297 m[3] + 84475 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8], -552825 m[1] + 672435 m[2] -
  327729 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-550809 m[1] + 671923 m[2] - 327697 m[3] + 84491 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8], -548793 m[1] + 671411 m[2] -
  327665 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-559305 m[1] + 675171 m[2] - 328097 m[3] + 84507 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8], -557289 m[1] + 674659 m[2] -
  328065 m[3] + 84507 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-557865 m[1] + 674723 m[2] - 328065 m[3] + 84507 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8], -514017 m[1] + 653507 m[2] -
  324305 m[3] + 84219 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-512001 m[1] + 652995 m[2] - 324273 m[3] + 84219 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -521073 m[1] + 656307 m[2] -
  324673 m[3] + 84235 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-519057 m[1] + 655795 m[2] - 324641 m[3] + 84235 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -530145 m[1] + 659619 m[2] -
  325073 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-528129 m[1] + 659107 m[2] - 325041 m[3] + 84251 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -537201 m[1] + 662419 m[2] -
  325441 m[3] + 84267 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-534609 m[1] + 661843 m[2] - 325409 m[3] + 84267 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -482265 m[1] + 637379 m[2] -
  321249 m[3] + 83963 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-489321 m[1] + 640179 m[2] - 321617 m[3] + 83979 m[4] - 12563 m[5] +
  1089 m[6] - 51 m[7] + m[8], -498393 m[1] + 643491 m[2] -
  322017 m[3] + 83995 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-496377 m[1] + 642979 m[2] - 321985 m[3] + 83995 m[4] - 12563 m[5] +
  1089 m[6] - 51 m[7] + m[8], -505449 m[1] + 646291 m[2] -
  322385 m[3] + 84011 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-457569 m[1] + 624051 m[2] - 318561 m[3] + 83723 m[4] - 12555 m[5] +
  1089 m[6] - 51 m[7] + m[8], -466641 m[1] + 627363 m[2] -
  318961 m[3] + 83739 m[4] - 12555 m[5] + 1089 m[6] - 51 m[7] + m[8]}
```

Array[m, 8].

```
{-23396345, 27945611, -13505961, 3469539, -515891, 44649, -2091, 41}
-23396345 m[1] + 27945611 m[2] - 13505961 m[3] +
  3469539 m[4] - 515891 m[5] + 44649 m[6] - 2091 m[7] + 41 m[8]
```

```

FindInstance[
-580545 m[1] + 687539 m[2] - 330721 m[3] + 84747 m[4] - 12587 m[5] + 1089 m[6] -
  51 m[7] + m[8] ≥ 0 && -578529 m[1] + 687027 m[2] - 330689 m[3] +
  84747 m[4] - 12587 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-541737 m[1] + 668611 m[2] - 327297 m[3] + 84475 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -552825 m[1] + 672435 m[2] -
  327729 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-550809 m[1] + 671923 m[2] - 327697 m[3] + 84491 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -548793 m[1] + 671411 m[2] -
  327665 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-559305 m[1] + 675171 m[2] - 328097 m[3] + 84507 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -557289 m[1] + 674659 m[2] -
  328065 m[3] + 84507 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-557865 m[1] + 674723 m[2] - 328065 m[3] + 84507 m[4] - 12579 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -514017 m[1] + 653507 m[2] -
  324305 m[3] + 84219 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-512001 m[1] + 652995 m[2] - 324273 m[3] + 84219 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -521073 m[1] + 656307 m[2] -
  324673 m[3] + 84235 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-519057 m[1] + 655795 m[2] - 324641 m[3] + 84235 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -530145 m[1] + 659619 m[2] -
  325073 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-528129 m[1] + 659107 m[2] - 325041 m[3] + 84251 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -537201 m[1] + 662419 m[2] -
  325441 m[3] + 84267 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-534609 m[1] + 661843 m[2] - 325409 m[3] + 84267 m[4] - 12571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -482265 m[1] + 637379 m[2] -
  321249 m[3] + 83963 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-489321 m[1] + 640179 m[2] - 321617 m[3] + 83979 m[4] - 12563 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -498393 m[1] + 643491 m[2] -
  322017 m[3] + 83995 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-496377 m[1] + 642979 m[2] - 321985 m[3] + 83995 m[4] - 12563 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -505449 m[1] + 646291 m[2] -
  322385 m[3] + 84011 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-457569 m[1] + 624051 m[2] - 318561 m[3] + 83723 m[4] - 12555 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -466641 m[1] + 627363 m[2] -
  318961 m[3] + 83739 m[4] - 12555 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-23396345 m[1] + 27945611 m[2] - 13505961 m[3] + 3469539 m[4] -
  515891 m[5] + 44649 m[6] - 2091 m[7] + 41 m[8] < 0, Array[m, 8], Integers]
{{m[1] → 11780, m[2] → 28348, m[3] → 38250,
  m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0, m[8] → 0}}

Array[m, 8] /. {m[1] → 11780, m[2] → 28348,
  m[3] → 38250, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0, m[8] → 0}
{11780, 28348, 38250, 0, 0, 0, 0, 0}

```

```

GCD[11780, 28348, 38250, 0, 0, 0, 0, 0]
2

{5890, 14174, 19125, 0, 0, 0, 0, 0} // Reverse
{0, 0, 0, 0, 0, 19125, 14174, 5890}

{11780, 28348, 38250, 0, 0, 0, 0, 0} / 2
{5890, 14174, 19125, 0, 0, 0, 0, 0}

{5890, 14174, 19125, 0, 0, 0, 0, 0}.
{-23396345, 27945611, -13505961, 3469539, -515891, 44649, -2091, 41}
-4885861

{5890, 14174, 19125, 0, 0, 0, 0, 0}.
Transpose[{{-580545, 687539, -330721, 84747, -12587, 1089, -51, 1},
  {-578529, 687027, -330689, 84747, -12587, 1089, -51, 1},
  {-541737, 668611, -327297, 84475, -12579, 1089, -51, 1},
  {-552825, 672435, -327729, 84491, -12579, 1089, -51, 1},
  {-550809, 671923, -327697, 84491, -12579, 1089, -51, 1},
  {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
  {-559305, 675171, -328097, 84507, -12579, 1089, -51, 1},
  {-557289, 674659, -328065, 84507, -12579, 1089, -51, 1},
  {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
  {-514017, 653507, -324305, 84219, -12571, 1089, -51, 1},
  {-512001, 652995, -324273, 84219, -12571, 1089, -51, 1},
  {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
  {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
  {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
  {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
  {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
  {-534609, 661843, -325409, 84267, -12571, 1089, -51, 1},
  {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
  {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
  {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
  {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
  {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
  {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
  {-466641, 627363, -318961, 83739, -12555, 1089, -51, 1}}]

{728611, 5957763, 26506259, 7137315, 12366467, 17595619,
  712179, 5941331, 3455827, 32914963, 38144115, 24004323,
  29233475, 9864531, 15093683, 953891, 8668547, 49781971,
  40871331, 26731539, 31960691, 17820899, 57738339, 43598547}

{11780, 28348, 38250, 0, 0, 0, 0, 0}.
{-23396345, 27945611, -13505961, 3469539, -515891, 44649, -2091, 41}
-9771722

```

```
{11780, 28348, 38250, 0, 0, 0, 0, 0}.
```

```
Transpose[{{-580545, 687539, -330721, 84747, -12587, 1089, -51, 1},
  {-578529, 687027, -330689, 84747, -12587, 1089, -51, 1},
  {-541737, 668611, -327297, 84475, -12579, 1089, -51, 1},
  {-552825, 672435, -327729, 84491, -12579, 1089, -51, 1},
  {-550809, 671923, -327697, 84491, -12579, 1089, -51, 1},
  {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
  {-559305, 675171, -328097, 84507, -12579, 1089, -51, 1},
  {-557289, 674659, -328065, 84507, -12579, 1089, -51, 1},
  {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
  {-514017, 653507, -324305, 84219, -12571, 1089, -51, 1},
  {-512001, 652995, -324273, 84219, -12571, 1089, -51, 1},
  {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
  {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
  {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
  {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
  {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
  {-534609, 661843, -325409, 84267, -12571, 1089, -51, 1},
  {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
  {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
  {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
  {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
  {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
  {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
  {-466641, 627363, -318961, 83739, -12555, 1089, -51, 1}}]
```

```
{1457222, 11915526, 53012518, 14274630, 24732934, 35191238,
  1424358, 11882662, 6911654, 65829926, 76288230, 48008646,
  58466950, 19729062, 30187366, 1907782, 17337094, 99563942,
  81742662, 53463078, 63921382, 35641798, 115476678, 87197094}
```

```
poly12 = listmod128[[12]]
```

```
 $(-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^{25} (48 - 15x + x^2)$ 
```

```
p12 = poly12 / minipoly[poly12] // Factor
```

```
 $(-9 + x)^3 (-7 + x)^8 (5 + x)^{24}$ 
```

```

feasiblesubcharpolylist[(-11+x) (-9+x)^4 (-7+x)^9 (5+x)^25 (48-15 x+x^2)]
{(-11+x) (-5+x) (-215+131 x-21 x^2+x^3), (-11+x) (59-16 x+x^2) (17-10 x+x^2),
(-11+x) (1019-862 x+236 x^2-26 x^3+x^4), (-11+x) (-9+x) (-5+x) (23-12 x+x^2),
(-9+x) (1281-1028 x+270 x^2-28 x^3+x^4), (-11+x) (-7+x)^2 (19-12 x+x^2),
(-11+x) (947-854 x+236 x^2-26 x^3+x^4), (-11+x) (-9+x) (-107+83 x-17 x^2+x^3),
(59-16 x+x^2) (-179+127 x-21 x^2+x^3), (-9+x) (1193-1020 x+270 x^2-28 x^3+x^4),
(-11+x) (-7+x) (-125+103 x-19 x^2+x^3), (-11+x) (-9+x)^2 (11-8 x+x^2),
-9769+10197 x-3442 x^2+522 x^3-37 x^4+x^5, (-9+x) (1105-1012 x+270 x^2-28 x^3+x^4),
(-11+x) (-9+x) (-7+x) (13-10 x+x^2), (-9+x)^2 (-113+99 x-19 x^2+x^3),
(-9+x) (1033-1004 x+270 x^2-28 x^3+x^4), (-9+x)^2 (-7+x) (15-12 x+x^2)}

{(-11+x) (-5+x) (-215+131 x-21 x^2+x^3),
(-11+x) (59-16 x+x^2) (17-10 x+x^2), (-11+x) (1019-862 x+236 x^2-26 x^3+x^4),
(-11+x) (-9+x) (-5+x) (23-12 x+x^2), (-9+x) (1281-1028 x+270 x^2-28 x^3+x^4),
(-11+x) (-7+x)^2 (19-12 x+x^2), (-11+x) (947-854 x+236 x^2-26 x^3+x^4),
(-11+x) (-9+x) (-107+83 x-17 x^2+x^3), (59-16 x+x^2) (-179+127 x-21 x^2+x^3),
(-9+x) (1193-1020 x+270 x^2-28 x^3+x^4), (-11+x) (-7+x) (-125+103 x-19 x^2+x^3),
(-11+x) (-9+x)^2 (11-8 x+x^2), -9769+10197 x-3442 x^2+522 x^3-37 x^4+x^5,
(-9+x) (1105-1012 x+270 x^2-28 x^3+x^4), (-11+x) (-9+x) (-7+x) (13-10 x+x^2),
(-9+x)^2 (-113+99 x-19 x^2+x^3), (-9+x) (1033-1004 x+270 x^2-28 x^3+x^4),
(-9+x)^2 (-7+x) (15-12 x+x^2)} // Length

```

18

```

CoefficientList[{(-11+x) (-5+x) (-215+131 x-21 x^2+x^3),
(-11+x) (59-16 x+x^2) (17-10 x+x^2), (-11+x) (1019-862 x+236 x^2-26 x^3+x^4),
(-11+x) (-9+x) (-5+x) (23-12 x+x^2), (-9+x) (1281-1028 x+270 x^2-28 x^3+x^4),
(-11+x) (-7+x)^2 (19-12 x+x^2), (-11+x) (947-854 x+236 x^2-26 x^3+x^4),
(-11+x) (-9+x) (-107+83 x-17 x^2+x^3), (59-16 x+x^2) (-179+127 x-21 x^2+x^3),
(-9+x) (1193-1020 x+270 x^2-28 x^3+x^4), (-11+x) (-7+x) (-125+103 x-19 x^2+x^3),
(-11+x) (-9+x)^2 (11-8 x+x^2), -9769+10197 x-3442 x^2+522 x^3-37 x^4+x^5,
(-9+x) (1105-1012 x+270 x^2-28 x^3+x^4),
(-11+x) (-9+x) (-7+x) (13-10 x+x^2), (-9+x)^2 (-113+99 x-19 x^2+x^3),
(-9+x) (1033-1004 x+270 x^2-28 x^3+x^4), (-9+x)^2 (-7+x) (15-12 x+x^2)}, x]
{{-11825, 10645, -3466, 522, -37, 1}, {-11033, 10485, -3458, 522, -37, 1},
{-11209, 10501, -3458, 522, -37, 1}, {-11385, 10517, -3458, 522, -37, 1},
{-11529, 10533, -3458, 522, -37, 1}, {-10241, 10325, -3450, 522, -37, 1},
{-10417, 10341, -3450, 522, -37, 1}, {-10593, 10357, -3450, 522, -37, 1},
{-10561, 10357, -3450, 522, -37, 1}, {-10737, 10373, -3450, 522, -37, 1},
{-9625, 10181, -3442, 522, -37, 1}, {-9801, 10197, -3442, 522, -37, 1},
{-9769, 10197, -3442, 522, -37, 1}, {-9945, 10213, -3442, 522, -37, 1},
{-9009, 10037, -3434, 522, -37, 1}, {-9153, 10053, -3434, 522, -37, 1},
{-9297, 10069, -3434, 522, -37, 1}, {-8505, 9909, -3426, 522, -37, 1}}

```

```
CoefficientList[D[poly12, x] / p12 // Factor, x]
```

```
{-476745, 433821, -141930, 21402, -1517, 41}
```

```
Solve[Array[n, 18].
```

```
{{-11825, 10645, -3466, 522, -37, 1}, {-11033, 10485, -3458, 522, -37, 1},
{-11209, 10501, -3458, 522, -37, 1}, {-11385, 10517, -3458, 522, -37, 1},
{-11529, 10533, -3458, 522, -37, 1}, {-10241, 10325, -3450, 522, -37, 1},
{-10417, 10341, -3450, 522, -37, 1}, {-10593, 10357, -3450, 522, -37, 1},
{-10561, 10357, -3450, 522, -37, 1}, {-10737, 10373, -3450, 522, -37, 1},
{-9625, 10181, -3442, 522, -37, 1}, {-9801, 10197, -3442, 522, -37, 1},
{-9769, 10197, -3442, 522, -37, 1}, {-9945, 10213, -3442, 522, -37, 1},
{-9009, 10037, -3434, 522, -37, 1}, {-9153, 10053, -3434, 522, -37, 1},
{-9297, 10069, -3434, 522, -37, 1}, {-8505, 9909, -3426, 522, -37, 1}} ==
{-476745, 433821, -141930, 21402, -1517, 41}, Array[n, 18]]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. >>
```

```
{ {n[13] → 42 - 2 n[1] - 2 n[2] - n[3] - 2 n[6] - n[7] - n[9] - n[11], n[16] →
24 - n[3] - 2 n[4] - n[5] - n[7] - 2 n[8] - n[10] - n[11] - 2 n[12] - n[14] - 2 n[15],
n[17] → 75 - n[1] - n[3] - 2 n[4] - 3 n[5] + n[6] - n[8] - n[9] - 2 n[10] +
n[11] - n[14] + n[15], n[18] → -100 + 2 n[1] + n[2] + 2 n[3] +
3 n[4] + 3 n[5] + n[7] + 2 n[8] + n[9] + 2 n[10] + n[12] + n[14] } }
```

Array[n, 18] /.

```
Solve[-n[13] + 42 - 2 n[1] - 2 n[2] - n[3] - 2 n[6] - n[7] - n[9] - n[11] == 0 &&
  -n[16] + 24 - n[3] - 2 n[4] - n[5] -
    n[7] - 2 n[8] - n[10] - n[11] - 2 n[12] - n[14] - 2 n[15] == 0 &&
  -n[17] + 75 - n[1] - n[3] - 2 n[4] - 3 n[5] +
    n[6] - n[8] - n[9] - 2 n[10] + n[11] - n[14] + n[15] == 0 &&
  -n[18] - 100 + 2 n[1] + n[2] + 2 n[3] + 3 n[4] + 3 n[5] + n[7] +
    2 n[8] + n[9] + 2 n[10] + n[12] + n[14] == 0 && n[1] ≥ 0 && n[2] ≥ 0 &&
  n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 && n[8] ≥ 0 &&
  n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 &&
  n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0, Array[n, 18], Integers]
```

```
{ {19, 0, 4, 2, 16, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {19, 1, 2, 3, 16, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {19, 2, 0, 4, 16, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 0, 0, 4, 16, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 0, 1, 3, 16, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 0, 1, 4, 15, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 0, 2, 2, 16, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 0, 2, 3, 15, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 1, 0, 3, 16, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {20, 1, 0, 4, 15, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
  {21, 0, 0, 2, 16, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {21, 0, 0, 3, 15, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
  {21, 0, 0, 3, 16, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {21, 0, 0, 4, 14, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0},
  {21, 0, 0, 4, 15, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0} }
```

poly13 = listmod128[[13]]

$$(-9 + x)^4 (-7 + x)^7 (5 + x)^{25} (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5)$$

p13 = poly13 / minipoly[poly13] // Factor

$$(-9 + x)^3 (-7 + x)^6 (5 + x)^{24}$$

```
feasiblesubcharpolylist[
  (-9 + x)4 (-7 + x)7 (5 + x)25 (-25 580 + 17 757 x - 4780 x2 + 626 x3 - 40 x4 + x5) ]
{ (-9 + x) (-7 + x) (-9327 + 8589 x - 2926 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x) (-8711 + 8445 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x) (-8887 + 8461 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
  (-11 + x) (-9 + x) (-7 + x)2 (-5 + x) (23 - 12 x + x2) ,
  (-9 + x) (-7 + x) (-8999 + 8477 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x)2 (-7 + x) (919 - 822 x + 232 x2 - 26 x3 + x4) ,
  (-11 + x) (-9 + x) (-7 + x)2 (-107 + 83 x - 17 x2 + x3) ,
  (-11 + x) (-9 + x)2 (-7 + x) (-5 + x) (17 - 10 x + x2) ,
  (-9 + x) (-7 + x) (-8383 + 8333 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x)2 (1193 - 1020 x + 270 x2 - 28 x3 + x4) ,
  (-9 + x)2 (-7 + x) (951 - 822 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x) (-7 + x) (-8527 + 8349 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x) (23 - 12 x + x2) (-377 + 167 x - 23 x2 + x3) ,
  (-9 + x) (-7 + x) (-7655 + 8173 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
  (-11 + x) (-9 + x)2 (-7 + x)2 (11 - 8 x + x2) ,
  (-9 + x)2 (-7 + x) (863 - 814 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x) (-7 + x)2 (1105 - 1012 x + 270 x2 - 28 x3 + x4) ,
  (-9 + x)2 (-7 + x) (879 - 814 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x) (-7 + x) (-7879 + 8205 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x) (-8023 + 8221 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x)2 (-7 + x) (807 - 806 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x) (-7 + x)2 (1033 - 1004 x + 270 x2 - 28 x3 + x4) ,
  (-9 + x) (-7 + x) (59 - 16 x + x2) (-125 + 103 x - 19 x2 + x3) ,
  (-9 + x)2 (-7 + x) (751 - 798 x + 232 x2 - 26 x3 + x4) }
```


$$\{ \begin{aligned} &(-9+x)(-7+x)(-9327+8589x-2926x^2+466x^3-35x^4+x^5), \\ &(-9+x)(-7+x)(-8711+8445x-2918x^2+466x^3-35x^4+x^5), \\ &(-9+x)(-7+x)(-8887+8461x-2918x^2+466x^3-35x^4+x^5), \\ &(-11+x)(-9+x)(-7+x)^2(-5+x)(23-12x+x^2), \\ &(-9+x)(-7+x)(-8999+8477x-2918x^2+466x^3-35x^4+x^5), \\ &(-9+x)^2(-7+x)(919-822x+232x^2-26x^3+x^4), \\ &(-11+x)(-9+x)(-7+x)^2(-107+83x-17x^2+x^3), \\ &(-11+x)(-9+x)^2(-7+x)(-5+x)(17-10x+x^2), \\ &(-9+x)(-7+x)(-8383+8333x-2910x^2+466x^3-35x^4+x^5), \\ &(-9+x)(-7+x)^2(1193-1020x+270x^2-28x^3+x^4), \\ &(-9+x)^2(-7+x)(951-822x+232x^2-26x^3+x^4), \\ &(-9+x)(-7+x)(-8527+8349x-2910x^2+466x^3-35x^4+x^5), \\ &(-9+x)(-7+x)(23-12x+x^2)(-377+167x-23x^2+x^3), \\ &(-9+x)(-7+x)(-7655+8173x-2902x^2+466x^3-35x^4+x^5), \\ &(-11+x)(-9+x)^2(-7+x)^2(11-8x+x^2), \\ &(-9+x)^2(-7+x)(863-814x+232x^2-26x^3+x^4), \\ &(-9+x)(-7+x)^2(1105-1012x+270x^2-28x^3+x^4), \\ &(-9+x)^2(-7+x)(879-814x+232x^2-26x^3+x^4), \\ &(-9+x)(-7+x)(-7879+8205x-2902x^2+466x^3-35x^4+x^5), \\ &(-9+x)(-7+x)(-8023+8221x-2902x^2+466x^3-35x^4+x^5), \\ &(-9+x)^2(-7+x)(807-806x+232x^2-26x^3+x^4), \\ &(-9+x)(-7+x)^2(1033-1004x+270x^2-28x^3+x^4), \\ &(-9+x)(-7+x)(59-16x+x^2)(-125+103x-19x^2+x^3), \\ &(-9+x)^2(-7+x)(751-798x+232x^2-26x^3+x^4) \} // \text{Length} \end{aligned}$$

```

CoefficientList[ { (-9 + x) (-7 + x) (-9327 + 8589 x - 2926 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8711 + 8445 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8887 + 8461 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-11 + x) (-9 + x) (-7 + x)^2 (-5 + x) (23 - 12 x + x^2),
  (-9 + x) (-7 + x) (-8999 + 8477 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (919 - 822 x + 232 x^2 - 26 x^3 + x^4),
  (-11 + x) (-9 + x) (-7 + x)^2 (-107 + 83 x - 17 x^2 + x^3),
  (-11 + x) (-9 + x)^2 (-7 + x) (-5 + x) (17 - 10 x + x^2),
  (-9 + x) (-7 + x) (-8383 + 8333 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x)^2 (1193 - 1020 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (951 - 822 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (-8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (23 - 12 x + x^2) (-377 + 167 x - 23 x^2 + x^3),
  (-9 + x) (-7 + x) (-7655 + 8173 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-11 + x) (-9 + x)^2 (-7 + x)^2 (11 - 8 x + x^2),
  (-9 + x)^2 (-7 + x) (863 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x)^2 (1105 - 1012 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (-7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x)^2 (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x)^2 (1033 - 1004 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (59 - 16 x + x^2) (-125 + 103 x - 19 x^2 + x^3),
  (-9 + x)^2 (-7 + x) (751 - 798 x + 232 x^2 - 26 x^3 + x^4) }, x]

```

```
{ {-587601, 690339, -331089, 84763, -12587, 1089, -51, 1},
  {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
  {-559881, 675235, -328097, 84507, -12579, 1089, -51, 1},
  {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
  {-566937, 678035, -328465, 84523, -12579, 1089, -51, 1},
  {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
  {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
  {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
  {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
  {-526113, 658595, -325009, 84251, -12571, 1089, -51, 1},
  {-539217, 662931, -325473, 84267, -12571, 1089, -51, 1},
  {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
  {-546273, 665731, -325841, 84283, -12571, 1089, -51, 1},
  {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
  {-480249, 636867, -321217, 83963, -12563, 1089, -51, 1},
  {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
  {-487305, 639667, -321585, 83979, -12563, 1089, -51, 1},
  {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
  {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
  {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
  {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
  {-455553, 623539, -318529, 83723, -12555, 1089, -51, 1},
  {-464625, 626851, -318929, 83739, -12555, 1089, -51, 1},
  {-425817, 607923, -315505, 83467, -12547, 1089, -51, 1}}
```

CoefficientList[D[poly13, x] / p13 // Factor, x]

```
{ -23056145, 27727451, -13457769, 3465043, -515739, 44649, -2091, 41 }
```

```

A = {{-587 601, 690 339, -331 089, 84 763, -12 587, 1089, -51, 1},
      {-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
      {-559 881, 675 235, -328 097, 84 507, -12 579, 1089, -51, 1},
      {-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
      {-566 937, 678 035, -328 465, 84 523, -12 579, 1089, -51, 1},
      {-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
      {-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
      {-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
      {-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
      {-526 113, 658 595, -325 009, 84 251, -12 571, 1089, -51, 1},
      {-539 217, 662 931, -325 473, 84 267, -12 571, 1089, -51, 1},
      {-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
      {-546 273, 665 731, -325 841, 84 283, -12 571, 1089, -51, 1},
      {-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
      {-480 249, 636 867, -321 217, 83 963, -12 563, 1089, -51, 1},
      {-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
      {-487 305, 639 667, -321 585, 83 979, -12 563, 1089, -51, 1},
      {-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
      {-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
      {-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
      {-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
      {-455 553, 623 539, -318 529, 83 723, -12 555, 1089, -51, 1},
      {-464 625, 626 851, -318 929, 83 739, -12 555, 1089, -51, 1},
      {-425 817, 607 923, -315 505, 83 467, -12 547, 1089, -51, 1}};

```

```
MatrixRank[A]
```

```
4
```

```

Solve[Array[n, 24].{{-587 601, 690 339, -331 089, 84 763, -12 587, 1089, -51, 1},
  {-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
  {-559 881, 675 235, -328 097, 84 507, -12 579, 1089, -51, 1},
  {-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
  {-566 937, 678 035, -328 465, 84 523, -12 579, 1089, -51, 1},
  {-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
  {-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
  {-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
  {-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
  {-526 113, 658 595, -325 009, 84 251, -12 571, 1089, -51, 1},
  {-539 217, 662 931, -325 473, 84 267, -12 571, 1089, -51, 1},
  {-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
  {-546 273, 665 731, -325 841, 84 283, -12 571, 1089, -51, 1},
  {-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
  {-480 249, 636 867, -321 217, 83 963, -12 563, 1089, -51, 1},
  {-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
  {-487 305, 639 667, -321 585, 83 979, -12 563, 1089, -51, 1},
  {-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
  {-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
  {-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
  {-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
  {-455 553, 623 539, -318 529, 83 723, -12 555, 1089, -51, 1},
  {-464 625, 626 851, -318 929, 83 739, -12 555, 1089, -51, 1},
  {-425 817, 607 923, -315 505, 83 467, -12 547, 1089, -51, 1}} ==
{-23 056 145, 27 727 451, -13 457 769, 3 465 043, -515 739,
  44 649, -2091, 41}, Array[n, 24]]
{}

```

Array[m, 8].Transpose[A]

```
{-587 601 m[1] + 690 339 m[2] - 331 089 m[3] + 84 763 m[4] -
  12 587 m[5] + 1089 m[6] - 51 m[7] + m[8], -548 793 m[1] + 671 411 m[2] -
  327 665 m[3] + 84 491 m[4] - 12 579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-559 881 m[1] + 675 235 m[2] - 328 097 m[3] + 84 507 m[4] - 12 579 m[5] +
  1089 m[6] - 51 m[7] + m[8], -557 865 m[1] + 674 723 m[2] -
  328 065 m[3] + 84 507 m[4] - 12 579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-566 937 m[1] + 678 035 m[2] - 328 465 m[3] + 84 523 m[4] - 12 579 m[5] +
  1089 m[6] - 51 m[7] + m[8], -521 073 m[1] + 656 307 m[2] -
  324 673 m[3] + 84 235 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-519 057 m[1] + 655 795 m[2] - 324 641 m[3] + 84 235 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -530 145 m[1] + 659 619 m[2] -
  325 073 m[3] + 84 251 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-528 129 m[1] + 659 107 m[2] - 325 041 m[3] + 84 251 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -526 113 m[1] + 658 595 m[2] -
  325 009 m[3] + 84 251 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-539 217 m[1] + 662 931 m[2] - 325 473 m[3] + 84 267 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -537 201 m[1] + 662 419 m[2] -
  325 441 m[3] + 84 267 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-546 273 m[1] + 665 731 m[2] - 325 841 m[3] + 84 283 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8], -482 265 m[1] + 637 379 m[2] -
  321 249 m[3] + 83 963 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-480 249 m[1] + 636 867 m[2] - 321 217 m[3] + 83 963 m[4] - 12 563 m[5] +
  1089 m[6] - 51 m[7] + m[8], -489 321 m[1] + 640 179 m[2] -
  321 617 m[3] + 83 979 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-487 305 m[1] + 639 667 m[2] - 321 585 m[3] + 83 979 m[4] - 12 563 m[5] +
  1089 m[6] - 51 m[7] + m[8], -498 393 m[1] + 643 491 m[2] -
  322 017 m[3] + 83 995 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-496 377 m[1] + 642 979 m[2] - 321 985 m[3] + 83 995 m[4] - 12 563 m[5] +
  1089 m[6] - 51 m[7] + m[8], -505 449 m[1] + 646 291 m[2] -
  322 385 m[3] + 84 011 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-457 569 m[1] + 624 051 m[2] - 318 561 m[3] + 83 723 m[4] - 12 555 m[5] +
  1089 m[6] - 51 m[7] + m[8], -455 553 m[1] + 623 539 m[2] -
  318 529 m[3] + 83 723 m[4] - 12 555 m[5] + 1089 m[6] - 51 m[7] + m[8],
-464 625 m[1] + 626 851 m[2] - 318 929 m[3] + 83 739 m[4] - 12 555 m[5] +
  1089 m[6] - 51 m[7] + m[8], -425 817 m[1] + 607 923 m[2] -
  315 505 m[3] + 83 467 m[4] - 12 547 m[5] + 1089 m[6] - 51 m[7] + m[8]}
```

Array[m, 8].

```
{-23 056 145, 27 727 451, -13 457 769, 3 465 043, -515 739, 44 649, -2091, 41}
-23 056 145 m[1] + 27 727 451 m[2] - 13 457 769 m[3] +
  3 465 043 m[4] - 515 739 m[5] + 44 649 m[6] - 2091 m[7] + 41 m[8]
```

```

FindInstance[
-587 601 m[1] + 690 339 m[2] - 331 089 m[3] + 84 763 m[4] - 12 587 m[5] + 1089 m[6] -
  51 m[7] + m[8] ≥ 0 && -548 793 m[1] + 671 411 m[2] - 327 665 m[3] +
  84 491 m[4] - 12 579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-559 881 m[1] + 675 235 m[2] - 328 097 m[3] + 84 507 m[4] - 12 579 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -557 865 m[1] + 674 723 m[2] -
  328 065 m[3] + 84 507 m[4] - 12 579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-566 937 m[1] + 678 035 m[2] - 328 465 m[3] + 84 523 m[4] - 12 579 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -521 073 m[1] + 656 307 m[2] -
  324 673 m[3] + 84 235 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-519 057 m[1] + 655 795 m[2] - 324 641 m[3] + 84 235 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -530 145 m[1] + 659 619 m[2] -
  325 073 m[3] + 84 251 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-528 129 m[1] + 659 107 m[2] - 325 041 m[3] + 84 251 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -526 113 m[1] + 658 595 m[2] -
  325 009 m[3] + 84 251 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-539 217 m[1] + 662 931 m[2] - 325 473 m[3] + 84 267 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -537 201 m[1] + 662 419 m[2] -
  325 441 m[3] + 84 267 m[4] - 12 571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-546 273 m[1] + 665 731 m[2] - 325 841 m[3] + 84 283 m[4] - 12 571 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -482 265 m[1] + 637 379 m[2] -
  321 249 m[3] + 83 963 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-480 249 m[1] + 636 867 m[2] - 321 217 m[3] + 83 963 m[4] - 12 563 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -489 321 m[1] + 640 179 m[2] -
  321 617 m[3] + 83 979 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-487 305 m[1] + 639 667 m[2] - 321 585 m[3] + 83 979 m[4] - 12 563 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -498 393 m[1] + 643 491 m[2] -
  322 017 m[3] + 83 995 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-496 377 m[1] + 642 979 m[2] - 321 985 m[3] + 83 995 m[4] - 12 563 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -505 449 m[1] + 646 291 m[2] -
  322 385 m[3] + 84 011 m[4] - 12 563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-457 569 m[1] + 624 051 m[2] - 318 561 m[3] + 83 723 m[4] - 12 555 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -455 553 m[1] + 623 539 m[2] -
  318 529 m[3] + 83 723 m[4] - 12 555 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-464 625 m[1] + 626 851 m[2] - 318 929 m[3] + 83 739 m[4] - 12 555 m[5] +
  1089 m[6] - 51 m[7] + m[8] ≥ 0 && -425 817 m[1] + 607 923 m[2] -
  315 505 m[3] + 83 467 m[4] - 12 547 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-23 056 145 m[1] + 27 727 451 m[2] - 13 457 769 m[3] + 3 465 043 m[4] -
  515 739 m[5] + 44 649 m[6] - 2091 m[7] + 41 m[8] < 0, Array[m, 8], Integers]
{{m[1] → 0, m[2] → 152 389, m[3] → 2 438 199, m[4] → 29 410 745,
  m[5] → 316 965 325, m[6] → 0, m[7] → 0, m[8] → 2 198 761 371 082}}

Array[m, 8] /. {m[1] → 0, m[2] → 152 389, m[3] → 2 438 199, m[4] → 29 410 745,
  m[5] → 316 965 325, m[6] → 0, m[7] → 0, m[8] → 2 198 761 371 082}
{0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082}

```

```
GCD[0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082]
```

```
1
```

```
{0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082}.
```

```
{-23 056 145, 27 727 451, -13 457 769, 3 465 043, -515 739, 44 649, -2091, 41}  
-27 836 370
```

```
{0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082}.
```

```
Transpose[{{-587 601, 690 339, -331 089, 84 763, -12 587, 1089, -51, 1},  
{-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},  
{-559 881, 675 235, -328 097, 84 507, -12 579, 1089, -51, 1},  
{-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},  
{-566 937, 678 035, -328 465, 84 523, -12 579, 1089, -51, 1},  
{-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},  
{-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},  
{-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},  
{-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},  
{-526 113, 658 595, -325 009, 84 251, -12 571, 1089, -51, 1},  
{-539 217, 662 931, -325 473, 84 267, -12 571, 1089, -51, 1},  
{-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},  
{-546 273, 665 731, -325 841, 84 283, -12 571, 1089, -51, 1},  
{-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},  
{-480 249, 636 867, -321 217, 83 963, -12 563, 1089, -51, 1},  
{-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},  
{-487 305, 639 667, -321 585, 83 979, -12 563, 1089, -51, 1},  
{-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},  
{-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},  
{-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},  
{-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},  
{-455 553, 623 539, -318 529, 83 723, -12 555, 1089, -51, 1},  
{-464 625, 626 851, -318 929, 83 739, -12 555, 1089, -51, 1},  
{-425 817, 607 923, -315 505, 83 467, -12 547, 1089, -51, 1}}]
```

```
{1 004 902, 979 246, 984 734, 983 934, 988 622, 959 078, 958 278, 963 766,  
962 966, 962 166, 968 454, 967 654, 972 342, 933 422, 932 622, 937 310,  
936 510, 941 998, 941 198, 945 886, 915 542, 914 742, 919 430, 893 774}
```

```
poly14 = listmod128[[14]]
```

$$(-9 + x)^4 (-7 + x)^6 (5 + x)^{25} (59 - 16x + x^2)^2 (52 - 15x + x^2)$$

```
p14 = poly14 / minipoly[poly14] // Factor
```

$$(-9 + x)^3 (-7 + x)^5 (5 + x)^{24} (59 - 16x + x^2)$$


```
feasiblesubcharpolylist[ $(-9 + x)^4 (-7 + x)^6 (5 + x)^{25} (59 - 16 x + x^2)^2 (52 - 15 x + x^2)$ ]
{ $(-9 + x) (59 - 16 x + x^2) (-127 + 87 x - 17 x^2 + x^3)$ ,
 $(-9 + x) (-7 + x) (59 - 16 x + x^2) (17 - 10 x + x^2)$ }
```

```
CoefficientList[{ $(-9 + x) (59 - 16 x + x^2) (-127 + 87 x - 17 x^2 + x^3)$ ,
 $(-9 + x) (-7 + x) (59 - 16 x + x^2) (17 - 10 x + x^2)$ }, x]
{{67 437, -71 978, 29 863, -6284, 715, -42, 1},
{63 189, -70 354, 29 663, -6276, 715, -42, 1}}
```

```
CoefficientList[D[poly14, x] / p14 // Factor, x]
{2 771 285, -2 952 858, 1 224 511, -257 644, 29 315, -1722, 41}
```

```
A = {{67 437, -71 978, 29 863, -6284, 715, -42, 1},
{63 189, -70 354, 29 663, -6276, 715, -42, 1}};
```

```
Solve[Array[n, 2].{{67 437, -71 978, 29 863, -6284, 715, -42, 1},
{63 189, -70 354, 29 663, -6276, 715, -42, 1}} ==
{2 771 285, -2 952 858, 1 224 511, -257 644, 29 315, -1722, 41}, Array[n, 2]]
{}
```

```
Array[m, 7].Transpose[A]
{67 437 m[1] - 71 978 m[2] + 29 863 m[3] - 6284 m[4] + 715 m[5] - 42 m[6] + m[7],
63 189 m[1] - 70 354 m[2] + 29 663 m[3] - 6276 m[4] + 715 m[5] - 42 m[6] + m[7]}
```

```
Array[m, 7].{2 771 285, -2 952 858, 1 224 511, -257 644, 29 315, -1722, 41}
2 771 285 m[1] - 2 952 858 m[2] + 1 224 511 m[3] -
257 644 m[4] + 29 315 m[5] - 1722 m[6] + 41 m[7]
```

```
FindInstance[
67 437 m[1] - 71 978 m[2] + 29 863 m[3] - 6284 m[4] + 715 m[5] - 42 m[6] + m[7] ≥ 0 &&
63 189 m[1] - 70 354 m[2] + 29 663 m[3] - 6276 m[4] + 715 m[5] - 42 m[6] + m[7] ≥ 0 &&
2 771 285 m[1] - 2 952 858 m[2] + 1 224 511 m[3] - 257 644 m[4] +
29 315 m[5] - 1722 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
{{m[1] → 0, m[2] → 3, m[3] → -56 453,
m[4] → -889 825, m[5] → 0, m[6] → 0, m[7] → -3 905 464 984}}
```

```
Array[m, 7] /. {m[1] → 0, m[2] → 3, m[3] → -56 453,
m[4] → -889 825, m[5] → 0, m[6] → 0, m[7] → -3 905 464 984}
{0, 3, -56 453, -889 825, 0, 0, -3 905 464 984}
```

```
GCD[0, 3, -56 453, -889 825, 0, 0, -3 905 464 984]
1
```

```
{0, 3, -56 453, -889 825, 0, 0, -3 905 464 984}.
{2 771 285, -2 952 858, 1 224 511, -257 644, 29 315, -1722, 41}
-2 170 101
```

```
{0, 3, -56453, -889825, 0, 0, -3905464984}.
```

$$\text{Transpose}[\{\{67437, -71978, 29863, -6284, 715, -42, 1\},$$

$$\{63189, -70354, 29663, -6276, 715, -42, 1\}\}$$

$$\{123443, 4300315\}$$

```
poly15 = listmod128[[15]]
```

$$(-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349+163x-23x^2+x^3)$$

```
p15 = poly15 / minipoly[poly15] // Factor
```

$$(-9+x)^4 (-7+x)^6 (5+x)^{24}$$

```
feasiblesubcharpolylist[(-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349+163x-23x^2+x^3)]
```

$$\{(-9+x) (-7+x) (-3+x) (-349+163x-23x^2+x^3),$$

$$(-7+x) (-8807+8445x-2918x^2+466x^3-35x^4+x^5),$$

$$(-9+x) (-7+x) (991-830x+232x^2-26x^3+x^4),$$

$$(-9+x) (-7+x) (1007-830x+232x^2-26x^3+x^4),$$

$$(-7+x) (-8159+8301x-2910x^2+466x^3-35x^4+x^5),$$

$$(-11+x) (-9+x) (-7+x) (-5+x) (17-10x+x^2),$$

$$(-7+x) (-7655+8173x-2902x^2+466x^3-35x^4+x^5),$$

$$(-9+x) (-7+x) (879-814x+232x^2-26x^3+x^4),$$

$$(-9+x) (-7+x) (807-806x+232x^2-26x^3+x^4),$$

$$(-7+x)^2 (929-996x+270x^2-28x^3+x^4),$$

$$(-9+x) (-7+x) (751-798x+232x^2-26x^3+x^4),$$

$$(-9+x) (-7+x)^2 (-89+99x-19x^2+x^3)\}$$

$$\{(-9+x) (-7+x) (-3+x) (-349+163x-23x^2+x^3),$$

$$(-7+x) (-8807+8445x-2918x^2+466x^3-35x^4+x^5),$$

$$(-9+x) (-7+x) (991-830x+232x^2-26x^3+x^4),$$

$$(-9+x) (-7+x) (1007-830x+232x^2-26x^3+x^4),$$

$$(-7+x) (-8159+8301x-2910x^2+466x^3-35x^4+x^5),$$

$$(-11+x) (-9+x) (-7+x) (-5+x) (17-10x+x^2),$$

$$(-7+x) (-7655+8173x-2902x^2+466x^3-35x^4+x^5),$$

$$(-9+x) (-7+x) (879-814x+232x^2-26x^3+x^4),$$

$$(-9+x) (-7+x) (807-806x+232x^2-26x^3+x^4),$$

$$(-7+x)^2 (929-996x+270x^2-28x^3+x^4),$$

$$(-9+x) (-7+x) (751-798x+232x^2-26x^3+x^4),$$

$$(-9+x) (-7+x)^2 (-89+99x-19x^2+x^3)\}$$

```
// Length
```

```

CoefficientList[ { (-9 + x) (-7 + x) (-3 + x) (-349 + 163 x - 23 x^2 + x^3),
  (-7 + x) (-8807 + 8445 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (991 - 830 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (1007 - 830 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x) (-8159 + 8301 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2),
  (-7 + x) (-7655 + 8173 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x)^2 (929 - 996 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (751 - 798 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x)^2 (-89 + 99 x - 19 x^2 + x^3) }, x]

```

```

{ {65961, -69546, 29071, -6188, 711, -42, 1},
  {61649, -67922, 28871, -6180, 711, -42, 1},
  {62433, -68146, 28887, -6180, 711, -42, 1},
  {63441, -68402, 28903, -6180, 711, -42, 1},
  {57113, -66266, 28671, -6172, 711, -42, 1},
  {58905, -66746, 28703, -6172, 711, -42, 1},
  {53585, -64866, 28487, -6164, 711, -42, 1},
  {55377, -65346, 28519, -6164, 711, -42, 1},
  {50841, -63690, 28319, -6156, 711, -42, 1},
  {45521, -61810, 28103, -6148, 711, -42, 1},
  {47313, -62290, 28135, -6148, 711, -42, 1},
  {39249, -59234, 27751, -6132, 711, -42, 1} }

```

```

A = { {65961, -69546, 29071, -6188, 711, -42, 1},
  {61649, -67922, 28871, -6180, 711, -42, 1},
  {62433, -68146, 28887, -6180, 711, -42, 1}, {63441, -68402, 28903,
    -6180, 711, -42, 1}, {57113, -66266, 28671, -6172, 711, -42, 1},
  {58905, -66746, 28703, -6172, 711, -42, 1}, {53585, -64866, 28487,
    -6164, 711, -42, 1}, {55377, -65346, 28519, -6164, 711, -42, 1},
  {50841, -63690, 28319, -6156, 711, -42, 1}, {45521, -61810, 28103,
    -6148, 711, -42, 1}, {47313, -62290, 28135, -6148, 711, -42, 1},
  {39249, -59234, 27751, -6132, 711, -42, 1} };

```

```
MatrixRank[A]
```

```
4
```

```
CoefficientList[D[poly15, x] / p15 // Factor, x]
```

```
{2508625, -2769690, 1180903, -253228, 29151, -1722, 41}
```

Solve[

```
Array[n, 12].{ {65 961, -69 546, 29 071, -6188, 711, -42, 1}, {61 649, -67 922, 28 871,
-6180, 711, -42, 1}, {62 433, -68 146, 28 887, -6180, 711, -42, 1},
{63 441, -68 402, 28 903, -6180, 711, -42, 1}, {57 113, -66 266, 28 671,
-6172, 711, -42, 1}, {58 905, -66 746, 28 703, -6172, 711, -42, 1},
{53 585, -64 866, 28 487, -6164, 711, -42, 1}, {55 377, -65 346, 28 519,
-6164, 711, -42, 1}, {50 841, -63 690, 28 319, -6156, 711, -42, 1},
{45 521, -61 810, 28 103, -6148, 711, -42, 1}, {47 313, -62 290, 28 135,
-6148, 711, -42, 1}, {39 249, -59 234, 27 751, -6132, 711, -42, 1}} ==
{2 508 625, -2 769 690, 1 180 903, -253 228, 29 151, -1722, 41}, Array[n, 12]]
```

{}

Array[m, 7].Transpose[A]

```
{65 961 m[1] - 69 546 m[2] + 29 071 m[3] - 6188 m[4] + 711 m[5] - 42 m[6] + m[7],
61 649 m[1] - 67 922 m[2] + 28 871 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7],
62 433 m[1] - 68 146 m[2] + 28 887 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7],
63 441 m[1] - 68 402 m[2] + 28 903 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7],
57 113 m[1] - 66 266 m[2] + 28 671 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
58 905 m[1] - 66 746 m[2] + 28 703 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
53 585 m[1] - 64 866 m[2] + 28 487 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
45 521 m[1] - 61 810 m[2] + 28 103 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],
47 313 m[1] - 62 290 m[2] + 28 135 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],
39 249 m[1] - 59 234 m[2] + 27 751 m[3] - 6132 m[4] + 711 m[5] - 42 m[6] + m[7]}
```

Array[m, 7].{2 508 625, -2 769 690, 1 180 903, -253 228, 29 151, -1722, 41}

```
2 508 625 m[1] - 2 769 690 m[2] + 1 180 903 m[3] -
253 228 m[4] + 29 151 m[5] - 1722 m[6] + 41 m[7]
```

FindInstance[

```
65 961 m[1] - 69 546 m[2] + 29 071 m[3] - 6188 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
61 649 m[1] - 67 922 m[2] + 28 871 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
62 433 m[1] - 68 146 m[2] + 28 887 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
63 441 m[1] - 68 402 m[2] + 28 903 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
57 113 m[1] - 66 266 m[2] + 28 671 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
58 905 m[1] - 66 746 m[2] + 28 703 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
53 585 m[1] - 64 866 m[2] + 28 487 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
45 521 m[1] - 61 810 m[2] + 28 103 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
47 313 m[1] - 62 290 m[2] + 28 135 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
39 249 m[1] - 59 234 m[2] + 27 751 m[3] - 6132 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
2 508 625 m[1] - 2 769 690 m[2] + 1 180 903 m[3] - 253 228 m[4] +
29 151 m[5] - 1722 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
```

```
{ {m[1] → -8404, m[2] → -28 265, m[3] → -48 443, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0} }
```

Array[m, 7] /.

```
{m[1] → -8404, m[2] → -28265, m[3] → -48443, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}
{-8404, -28265, -48443, 0, 0, 0, 0}
```

GCD[-8404, -28265, -48443, 0, 0, 0, 0]

1

{-8404, -28265, -48443, 0, 0, 0, 0}.

```
{2508625, -2769690, 1180903, -253228, 29151, -1722, 41}
-3680679
```

{-8404, -28265, -48443, 0, 0, 0, 0}.

```
Transpose[{{65961, -69546, 29071, -6188, 711, -42, 1}, {61649, -67922,
28871, -6180, 711, -42, 1}, {62433, -68146, 28887, -6180, 711, -42, 1},
{63441, -68402, 28903, -6180, 711, -42, 1}, {57113, -66266, 28671,
-6172, 711, -42, 1}, {58905, -66746, 28703, -6172, 711, -42, 1},
{53585, -64866, 28487, -6164, 711, -42, 1}, {55377, -65346, 28519,
-6164, 711, -42, 1}, {50841, -63690, 28319, -6156, 711, -42, 1},
{45521, -61810, 28103, -6148, 711, -42, 1}, {47313, -62290, 28135,
-6148, 711, -42, 1}, {39249, -59234, 27751, -6132, 711, -42, 1}}]
{3094993, 3119281, 2086817, 76337, 4121585,
1078641, 3113409, 70465, 1072769, 3107537, 64593, 58721}
```

poly16 = listmod128[[16]]

$$(-11+x)(-9+x)^6(-8+x)(-7+x)^6(-5+x)^2(5+x)^{25}$$

p16 = poly16 / minipoly[poly16] // Factor

$$(-9+x)^5(-7+x)^5(-5+x)(5+x)^{24}$$

feasiblesubcharpolylist[(-11+x)(-9+x)^6(-8+x)(-7+x)^6(-5+x)^2(5+x)^{25}]

```
{(-11+x)(-9+x)(-7+x)(-5+x)(-3+x),
(-11+x)(-7+x)(-127+87x-17x^2+x^3), -9923+9001x-3002x^2+470x^3-35x^4+x^5,
(-9+x)(-7+x)(-157+103x-19x^2+x^3), (-9+x)(-5+x)(-223+131x-21x^2+x^3),
(-7+x)(1325-1076x+274x^2-28x^3+x^4), (-9+x)(-7+x)(-149+103x-19x^2+x^3),
(-9+x)(1059-870x+236x^2-26x^3+x^4), (-7+x)^2(-179+127x-21x^2+x^3),
(-9+x)(-7+x)(-141+103x-19x^2+x^3), (-9+x)(59-16x+x^2)(17-10x+x^2),
(-9+x)(-7+x)^2(19-12x+x^2), (-9+x)(-7+x)(-125+103x-19x^2+x^3)}
```

```
{(-11 + x) (-9 + x) (-7 + x) (-5 + x) (-3 + x), (-11 + x) (-7 + x) (-127 + 87 x - 17 x^2 + x^3),
-9923 + 9001 x - 3002 x^2 + 470 x^3 - 35 x^4 + x^5, (-9 + x) (-7 + x) (-157 + 103 x - 19 x^2 + x^3),
(-9 + x) (-5 + x) (-223 + 131 x - 21 x^2 + x^3), (-7 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4),
(-9 + x) (-7 + x) (-149 + 103 x - 19 x^2 + x^3), (-9 + x) (1059 - 870 x + 236 x^2 - 26 x^3 + x^4),
(-7 + x)^2 (-179 + 127 x - 21 x^2 + x^3), (-9 + x) (-7 + x) (-141 + 103 x - 19 x^2 + x^3),
(-9 + x) (59 - 16 x + x^2) (17 - 10 x + x^2), (-9 + x) (-7 + x)^2 (19 - 12 x + x^2),
(-9 + x) (-7 + x) (-125 + 103 x - 19 x^2 + x^3)} // Length
```

13

```
CoefficientList[{-11 + x) (-9 + x) (-7 + x) (-5 + x) (-3 + x),
(-11 + x) (-7 + x) (-127 + 87 x - 17 x^2 + x^3), -9923 + 9001 x - 3002 x^2 + 470 x^3 - 35 x^4 + x^5,
(-9 + x) (-7 + x) (-157 + 103 x - 19 x^2 + x^3), (-9 + x) (-5 + x) (-223 + 131 x - 21 x^2 + x^3),
(-7 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4), (-9 + x) (-7 + x) (-149 + 103 x - 19 x^2 + x^3),
(-9 + x) (1059 - 870 x + 236 x^2 - 26 x^3 + x^4), (-7 + x)^2 (-179 + 127 x - 21 x^2 + x^3),
(-9 + x) (-7 + x) (-141 + 103 x - 19 x^2 + x^3), (-9 + x) (59 - 16 x + x^2) (17 - 10 x + x^2),
(-9 + x) (-7 + x)^2 (19 - 12 x + x^2), (-9 + x) (-7 + x) (-125 + 103 x - 19 x^2 + x^3)}, x]
```

```
{{-10395, 9129, -3010, 470, -35, 1},
{-9779, 8985, -3002, 470, -35, 1}, {-9923, 9001, -3002, 470, -35, 1},
{-9891, 9001, -3002, 470, -35, 1}, {-10035, 9017, -3002, 470, -35, 1},
{-9275, 8857, -2994, 470, -35, 1}, {-9387, 8873, -2994, 470, -35, 1},
{-9531, 8889, -2994, 470, -35, 1}, {-8771, 8729, -2986, 470, -35, 1},
{-8883, 8745, -2986, 470, -35, 1}, {-9027, 8761, -2986, 470, -35, 1},
{-8379, 8617, -2978, 470, -35, 1}, {-7875, 8489, -2970, 470, -35, 1}}
```

```
A = {{-10395, 9129, -3010, 470, -35, 1},
{-9779, 8985, -3002, 470, -35, 1}, {-9923, 9001, -3002, 470, -35, 1},
{-9891, 9001, -3002, 470, -35, 1}, {-10035, 9017, -3002, 470, -35, 1},
{-9275, 8857, -2994, 470, -35, 1}, {-9387, 8873, -2994, 470, -35, 1},
{-9531, 8889, -2994, 470, -35, 1}, {-8771, 8729, -2986, 470, -35, 1},
{-8883, 8745, -2986, 470, -35, 1}, {-9027, 8761, -2986, 470, -35, 1},
{-8379, 8617, -2978, 470, -35, 1}, {-7875, 8489, -2970, 470, -35, 1}};
```

```
CoefficientList[D[poly16, x] / p16 // Factor, x]
```

```
{-396435, 366513, -122930, 19270, -1435, 41}
```

```
Solve[Array[n, 13].{{-10 395, 9129, -3010, 470, -35, 1},
  {-9779, 8985, -3002, 470, -35, 1}, {-9923, 9001, -3002, 470, -35, 1},
  {-9891, 9001, -3002, 470, -35, 1}, {-10 035, 9017, -3002, 470, -35, 1},
  {-9275, 8857, -2994, 470, -35, 1}, {-9387, 8873, -2994, 470, -35, 1},
  {-9531, 8889, -2994, 470, -35, 1}, {-8771, 8729, -2986, 470, -35, 1},
  {-8883, 8745, -2986, 470, -35, 1}, {-9027, 8761, -2986, 470, -35, 1},
  {-8379, 8617, -2978, 470, -35, 1}, {-7875, 8489, -2970, 470, -35, 1}} ==
  {-396 435, 366 513, -122 930, 19 270, -1435, 41}, Array[n, 13]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{ {n[9] → 42 - n[2] - n[3] - n[6], n[11] → 36 - n[3] - n[5] - n[8],
  n[12] → -11 - 5 n[1] - 2 n[2] - 4 n[4] - 2 n[5] - n[6] - 3 n[7] - n[8] - 2 n[10],
  n[13] → -26 + 4 n[1] + 2 n[2] + n[3] + 3 n[4] + 2 n[5] + n[6] + 2 n[7] + n[8] + n[10] } }
```

```
FindInstance[-n[9] + 42 - n[2] - n[3] - n[6] == 0 &&
  -n[11] + 36 - n[3] - n[5] - n[8] == 0 &&
  -n[12] - 11 - 5 n[1] - 2 n[2] - 4 n[4] - 2 n[5] - n[6] - 3 n[7] - n[8] - 2 n[10] == 0 &&
  -n[13] - 26 + 4 n[1] + 2 n[2] + n[3] + 3 n[4] + 2 n[5] + n[6] + 2 n[7] + n[8] + n[10] == 0 &&
  n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 &&
  n[6] ≥ 0 && n[7] ≥ 0 && n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 &&
  n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0, Array[n, 13], Integers]

{ }
```

```
Array[m, 6].Transpose[A]
```

```
{-10 395 m[1] + 9129 m[2] - 3010 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9779 m[1] + 8985 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9923 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9891 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
 -10 035 m[1] + 9017 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9275 m[1] + 8857 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9387 m[1] + 8873 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9531 m[1] + 8889 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6],
 -8771 m[1] + 8729 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6],
 -8883 m[1] + 8745 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6],
 -9027 m[1] + 8761 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6],
 -8379 m[1] + 8617 m[2] - 2978 m[3] + 470 m[4] - 35 m[5] + m[6],
 -7875 m[1] + 8489 m[2] - 2970 m[3] + 470 m[4] - 35 m[5] + m[6] }
```

```
Array[m, 6].{-396 435, 366 513, -122 930, 19 270, -1435, 41}
```

```
-396 435 m[1] + 366 513 m[2] - 122 930 m[3] + 19 270 m[4] - 1435 m[5] + 41 m[6]
```

```

FindInstance[-10395 m[1] + 9129 m[2] - 3010 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9779 m[1] + 8985 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9923 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9891 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -10035 m[1] + 9017 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9275 m[1] + 8857 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9387 m[1] + 8873 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9531 m[1] + 8889 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8771 m[1] + 8729 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8883 m[1] + 8745 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -9027 m[1] + 8761 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -8379 m[1] + 8617 m[2] - 2978 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -7875 m[1] + 8489 m[2] - 2970 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
  -396435 m[1] + 366513 m[2] - 122930 m[3] + 19270 m[4] - 1435 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 10302, m[2] → 82421, m[3] → 659363, m[4] → 0, m[5] → 0, m[6] → 1339775890}}

Array[m, 6] /.
{m[1] → 10302, m[2] → 82421, m[3] → 659363, m[4] → 0, m[5] → 0, m[6] → 1339775890}
{10302, 82421, 659363, 0, 0, 1339775890}

GCD[10302, 82421, 659363, 0, 0, 1339775890]
1

{10302, 82421, 659363, 0, 0, 1339775890}.
{-396435, 366513, -122930, 19270, -1435, 41}
-387497

{10302, 82421, 659363, 0, 0, 1339775890}.
Transpose[{{-10395, 9129, -3010, 470, -35, 1},
  {-9779, 8985, -3002, 470, -35, 1}, {-9923, 9001, -3002, 470, -35, 1},
  {-9891, 9001, -3002, 470, -35, 1}, {-10035, 9017, -3002, 470, -35, 1},
  {-9275, 8857, -2994, 470, -35, 1}, {-9387, 8873, -2994, 470, -35, 1},
  {-9531, 8889, -2994, 470, -35, 1}, {-8771, 8729, -2986, 470, -35, 1},
  {-8883, 8745, -2986, 470, -35, 1}, {-9027, 8761, -2986, 470, -35, 1},
  {-8379, 8617, -2978, 470, -35, 1}, {-7875, 8489, -2970, 470, -35, 1}}]
{425279, 177591, 12839, 342503, 177751, 94815,
259727, 94975, 12039, 176951, 12199, 94175, 11399}

```

```
poly17 = listmod128[[17]]
```

$$(-9 + x)^5 (-7 + x)^7 (5 + x)^{25} (2824 - 1653x + 347x^2 - 31x^3 + x^4)$$

p17 = poly17 / minipoly[poly17] // Factor

$$(-9 + x)^4 (-7 + x)^6 (5 + x)^{24}$$

feasiblesubcharpolylist[$(-9 + x)^5 (-7 + x)^7 (5 + x)^{25} (2824 - 1653 x + 347 x^2 - 31 x^3 + x^4)$]

$$\begin{aligned} & \{ (-7 + x) (-8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5), \\ & 59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6, \\ & (-9 + x) (-6737 + 6721 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5), \\ & (-7 + x) (23 - 12 x + x^2) (-377 + 167 x - 23 x^2 + x^3), \\ & (-9 + x) (-6817 + 6737 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5), \\ & (-7 + x)^2 (1105 - 1012 x + 270 x^2 - 28 x^3 + x^4), \\ & (-9 + x) (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4), \\ & (-7 + x) (-7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5), \\ & (-9 + x) (-7 + x) (-5 + x) (-179 + 127 x - 21 x^2 + x^3), \\ & (-9 + x) (-6233 + 6593 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5), \\ & (-7 + x) (-8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5), \\ & (-9 + x) (-7 + x) (911 - 814 x + 232 x^2 - 26 x^3 + x^4), \\ & (-9 + x)^2 (-5 + x) (-141 + 103 x - 19 x^2 + x^3), (-9 + x)^2 (-7 + x) (-103 + 79 x - 17 x^2 + x^3), \\ & (-9 + x) (-6457 + 6625 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5), \\ & (-9 + x) (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4), \\ & (-9 + x) (-7 + x) (823 - 806 x + 232 x^2 - 26 x^3 + x^4), \\ & (-9 + x) (-7 + x) (839 - 806 x + 232 x^2 - 26 x^3 + x^4), \\ & (-9 + x)^2 (59 - 16 x + x^2) (11 - 8 x + x^2), (-9 + x)^2 (-7 + x) (-5 + x) (19 - 12 x + x^2), \\ & (-9 + x) (-7 + x) (59 - 16 x + x^2) (13 - 10 x + x^2), \\ & (-9 + x)^2 (-7 + x) (-87 + 79 x - 17 x^2 + x^3), (-9 + x)^2 (-7 + x) (-79 + 79 x - 17 x^2 + x^3) \} \end{aligned}$$

```

{ (-7 + x) (-8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6,
  (-9 + x) (-6737 + 6721 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-7 + x) (23 - 12 x + x^2) (-377 + 167 x - 23 x^2 + x^3),
  (-9 + x) (-6817 + 6737 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-7 + x)^2 (1105 - 1012 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x) (-7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-5 + x) (-179 + 127 x - 21 x^2 + x^3),
  (-9 + x) (-6233 + 6593 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-7 + x) (-8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (911 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (-5 + x) (-141 + 103 x - 19 x^2 + x^3), (-9 + x)^2 (-7 + x) (-103 + 79 x - 17 x^2 + x^3),
  (-9 + x) (-6457 + 6625 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-9 + x) (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (823 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (839 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (59 - 16 x + x^2) (11 - 8 x + x^2), (-9 + x)^2 (-7 + x) (-5 + x) (19 - 12 x + x^2),
  (-9 + x) (-7 + x) (59 - 16 x + x^2) (13 - 10 x + x^2),
  (-9 + x)^2 (-7 + x) (-87 + 79 x - 17 x^2 + x^3),
  (-9 + x)^2 (-7 + x) (-79 + 79 x - 17 x^2 + x^3) } // Length

```

```

CoefficientList[ { (-7 + x) (-8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6,
  (-9 + x) (-6737 + 6721 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-7 + x) (23 - 12 x + x^2) (-377 + 167 x - 23 x^2 + x^3),
  (-9 + x) (-6817 + 6737 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-7 + x)^2 (1105 - 1012 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x) (-7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (-5 + x) (-179 + 127 x - 21 x^2 + x^3),
  (-9 + x) (-6233 + 6593 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-7 + x) (-8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (911 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (-5 + x) (-141 + 103 x - 19 x^2 + x^3), (-9 + x)^2 (-7 + x) (-103 + 79 x - 17 x^2 + x^3),
  (-9 + x) (-6457 + 6625 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5),
  (-9 + x) (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (823 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (839 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x)^2 (59 - 16 x + x^2) (11 - 8 x + x^2), (-9 + x)^2 (-7 + x) (-5 + x) (19 - 12 x + x^2),
  (-9 + x) (-7 + x) (59 - 16 x + x^2) (13 - 10 x + x^2),
  (-9 + x)^2 (-7 + x) (-87 + 79 x - 17 x^2 + x^3), (-9 + x)^2 (-7 + x) (-79 + 79 x - 17 x^2 + x^3) }, x]
{ {59689, -66970, 28719, -6172, 711, -42, 1},
  {59401, -66938, 28719, -6172, 711, -42, 1},
  {60633, -67226, 28735, -6172, 711, -42, 1},
  {60697, -67226, 28735, -6172, 711, -42, 1},
  {61353, -67450, 28751, -6172, 711, -42, 1},
  {54145, -65058, 28503, -6164, 711, -42, 1},
  {55377, -65346, 28519, -6164, 711, -42, 1},
  {55153, -65314, 28519, -6164, 711, -42, 1},
  {56385, -65602, 28535, -6164, 711, -42, 1},
  {56097, -65570, 28535, -6164, 711, -42, 1},
  {56161, -65570, 28535, -6164, 711, -42, 1},
  {57393, -65858, 28551, -6164, 711, -42, 1},
  {57105, -65826, 28551, -6164, 711, -42, 1},
  {58401, -66114, 28567, -6164, 711, -42, 1},
  {58113, -66082, 28567, -6164, 711, -42, 1},
  {50841, -63690, 28319, -6156, 711, -42, 1},
  {51849, -63946, 28335, -6156, 711, -42, 1},
  {52857, -64202, 28351, -6156, 711, -42, 1},
  {52569, -64170, 28351, -6156, 711, -42, 1},
  {53865, -64458, 28367, -6156, 711, -42, 1},
  {48321, -62546, 28151, -6148, 711, -42, 1},
  {49329, -62802, 28167, -6148, 711, -42, 1},
  {44793, -61146, 27967, -6140, 711, -42, 1} }

```

```
A = {{59 689, -66 970, 28 719, -6172, 711, -42, 1},
      {59 401, -66 938, 28 719, -6172, 711, -42, 1},
      {60 633, -67 226, 28 735, -6172, 711, -42, 1}, {60 697, -67 226, 28 735,
      -6172, 711, -42, 1}, {61 353, -67 450, 28 751, -6172, 711, -42, 1},
      {54 145, -65 058, 28 503, -6164, 711, -42, 1}, {55 377, -65 346, 28 519,
      -6164, 711, -42, 1}, {55 153, -65 314, 28 519, -6164, 711, -42, 1},
      {56 385, -65 602, 28 535, -6164, 711, -42, 1}, {56 097, -65 570, 28 535,
      -6164, 711, -42, 1}, {56 161, -65 570, 28 535, -6164, 711, -42, 1},
      {57 393, -65 858, 28 551, -6164, 711, -42, 1}, {57 105, -65 826, 28 551,
      -6164, 711, -42, 1}, {58 401, -66 114, 28 567, -6164, 711, -42, 1},
      {58 113, -66 082, 28 567, -6164, 711, -42, 1}, {50 841, -63 690, 28 319,
      -6156, 711, -42, 1}, {51 849, -63 946, 28 335, -6156, 711, -42, 1},
      {52 857, -64 202, 28 351, -6156, 711, -42, 1}, {52 569, -64 170, 28 351,
      -6156, 711, -42, 1}, {53 865, -64 458, 28 367, -6156, 711, -42, 1},
      {48 321, -62 546, 28 151, -6148, 711, -42, 1}, {49 329, -62 802, 28 167,
      -6148, 711, -42, 1}, {44 793, -61 146, 27 967, -6140, 711, -42, 1}};
```

```
CoefficientList[D[poly17, x] / p17 // Factor, x]
```

```
{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}
```

```
Solve[
```

```
Array[n, 23].{{59 689, -66 970, 28 719, -6172, 711, -42, 1}, {59 401, -66 938, 28 719,
      -6172, 711, -42, 1}, {60 633, -67 226, 28 735, -6172, 711, -42, 1},
      {60 697, -67 226, 28 735, -6172, 711, -42, 1}, {61 353, -67 450, 28 751, -6172,
      711, -42, 1}, {54 145, -65 058, 28 503, -6164, 711, -42, 1}, {55 377, -65 346,
      28 519, -6164, 711, -42, 1}, {55 153, -65 314, 28 519, -6164, 711, -42, 1},
      {56 385, -65 602, 28 535, -6164, 711, -42, 1}, {56 097, -65 570, 28 535,
      -6164, 711, -42, 1}, {56 161, -65 570, 28 535, -6164, 711, -42, 1},
      {57 393, -65 858, 28 551, -6164, 711, -42, 1}, {57 105, -65 826, 28 551,
      -6164, 711, -42, 1}, {58 401, -66 114, 28 567, -6164, 711, -42, 1},
      {58 113, -66 082, 28 567, -6164, 711, -42, 1}, {50 841, -63 690, 28 319,
      -6156, 711, -42, 1}, {51 849, -63 946, 28 335, -6156, 711, -42, 1},
      {52 857, -64 202, 28 351, -6156, 711, -42, 1}, {52 569, -64 170, 28 351,
      -6156, 711, -42, 1}, {53 865, -64 458, 28 367, -6156, 711, -42, 1},
      {48 321, -62 546, 28 151, -6148, 711, -42, 1}, {49 329, -62 802, 28 167,
      -6148, 711, -42, 1}, {44 793, -61 146, 27 967, -6140, 711, -42, 1}} ==
      {2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}, Array[n, 23]]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. >>
```

```
{ {n[11] → 35 - n[1] - n[2] - n[4] - n[6] - n[8],
   n[19] → 63 - n[2] - n[3] - 2 n[5] - n[10] - n[13] - n[15],
   n[21] → -23 - n[1] - n[3] + n[5] - 2 n[6] - 3 n[7] - n[8] -
       2 n[9] - n[10] - n[12] + n[15] - 3 n[16] - 2 n[17] - n[18],
   n[22] → -22 + n[2] - n[3] - n[4] - n[5] + 2 n[6] + n[8] - n[9] - 2 n[12] -
       n[13] - 3 n[14] - 2 n[15] + n[16] - n[18] - 2 n[20],
   n[23] → -12 + n[1] + 2 n[3] + n[4] + n[5] + 2 n[7] + 2 n[9] + n[10] +
       2 n[12] + n[13] + 2 n[14] + n[15] + n[16] + n[17] + n[18] + n[20] } }
```

```

FindInstance[-n[11] + 35 - n[1] - n[2] - n[4] - n[6] - n[8] == 0 &&
  -n[19] + 63 - n[2] - n[3] - 2 n[5] - n[10] - n[13] - n[15] == 0 &&
  -n[21] - 23 - n[1] - n[3] + n[5] - 2 n[6] - 3 n[7] -
    n[8] - 2 n[9] - n[10] - n[12] + n[15] - 3 n[16] - 2 n[17] - n[18] == 0 &&
  -n[22] - 22 + n[2] - n[3] - n[4] - n[5] + 2 n[6] + n[8] - n[9] -
    2 n[12] - n[13] - 3 n[14] - 2 n[15] + n[16] - n[18] - 2 n[20] == 0 &&
  -n[23] - 12 + n[1] + 2 n[3] + n[4] + n[5] + 2 n[7] + 2 n[9] + n[10] +
    2 n[12] + n[13] + 2 n[14] + n[15] + n[16] + n[17] + n[18] + n[20] == 0 &&
  n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
  n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 &&
  n[14] ≥ 0 && n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0 && n[19] ≥ 0 &&
  n[20] ≥ 0 && n[21] ≥ 0 && n[22] ≥ 0 && n[23] ≥ 0, Array[n, 23], Integers]
{ }

```

```
Array[m, 7].Transpose[A]
```

```

{ 59 689 m[1] - 66 970 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
  59 401 m[1] - 66 938 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
  60 633 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
  60 697 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
  61 353 m[1] - 67 450 m[2] + 28 751 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
  54 145 m[1] - 65 058 m[2] + 28 503 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  55 153 m[1] - 65 314 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  56 385 m[1] - 65 602 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  56 097 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  56 161 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  57 393 m[1] - 65 858 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  57 105 m[1] - 65 826 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  58 401 m[1] - 66 114 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  58 113 m[1] - 66 082 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
  50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
  51 849 m[1] - 63 946 m[2] + 28 335 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
  52 857 m[1] - 64 202 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
  52 569 m[1] - 64 170 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
  53 865 m[1] - 64 458 m[2] + 28 367 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
  48 321 m[1] - 62 546 m[2] + 28 151 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],
  49 329 m[1] - 62 802 m[2] + 28 167 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],
  44 793 m[1] - 61 146 m[2] + 27 967 m[3] - 6140 m[4] + 711 m[5] - 42 m[6] + m[7] }

```

```
Array[m, 7].{ 2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}
```

```

2 543 345 m[1] - 2 783 706 m[2] + 1 182 087 m[3] -
  253 228 m[4] + 29 151 m[5] - 1722 m[6] + 41 m[7]

```

```

FindInstance[
59 689 m[1] - 66 970 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
59 401 m[1] - 66 938 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
60 633 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
60 697 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
61 353 m[1] - 67 450 m[2] + 28 751 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
54 145 m[1] - 65 058 m[2] + 28 503 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
55 153 m[1] - 65 314 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
56 385 m[1] - 65 602 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
56 097 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
56 161 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
57 393 m[1] - 65 858 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
57 105 m[1] - 65 826 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
58 401 m[1] - 66 114 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
58 113 m[1] - 66 082 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
51 849 m[1] - 63 946 m[2] + 28 335 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
52 857 m[1] - 64 202 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
52 569 m[1] - 64 170 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
53 865 m[1] - 64 458 m[2] + 28 367 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
48 321 m[1] - 62 546 m[2] + 28 151 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
49 329 m[1] - 62 802 m[2] + 28 167 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
44 793 m[1] - 61 146 m[2] + 27 967 m[3] - 6140 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
2 543 345 m[1] - 2 783 706 m[2] + 1 182 087 m[3] - 253 228 m[4] +
29 151 m[5] - 1722 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
{{m[1] → -603, m[2] → -549, m[3] → 0, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}}

Array[m, 7] /.
{m[1] → -603, m[2] → -549, m[3] → 0, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}
{-603, -549, 0, 0, 0, 0, 0}

GCD[-603, -549, 0, 0, 0, 0, 0]
9

{-603, -549, 0, 0, 0, 0, 0} / 9
{-67, -61, 0, 0, 0, 0, 0}

{-67, -61, 0, 0, 0, 0, 0}.{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}
-598 049

```

```
{-67, -61, 0, 0, 0, 0, 0}.
```

```
Transpose[{{59689, -66970, 28719, -6172, 711, -42, 1}, {59401, -66938,
  28719, -6172, 711, -42, 1}, {60633, -67226, 28735, -6172, 711, -42, 1},
  {60697, -67226, 28735, -6172, 711, -42, 1},
  {61353, -67450, 28751, -6172, 711, -42, 1},
  {54145, -65058, 28503, -6164, 711, -42, 1}, {55377, -65346, 28519,
    -6164, 711, -42, 1}, {55153, -65314, 28519, -6164, 711, -42, 1},
  {56385, -65602, 28535, -6164, 711, -42, 1}, {56097, -65570, 28535,
    -6164, 711, -42, 1}, {56161, -65570, 28535, -6164, 711, -42, 1},
  {57393, -65858, 28551, -6164, 711, -42, 1}, {57105, -65826, 28551,
    -6164, 711, -42, 1}, {58401, -66114, 28567, -6164, 711, -42, 1},
  {58113, -66082, 28567, -6164, 711, -42, 1}, {50841, -63690, 28319,
    -6156, 711, -42, 1}, {51849, -63946, 28335, -6156, 711, -42, 1},
  {52857, -64202, 28351, -6156, 711, -42, 1}, {52569, -64170, 28351,
    -6156, 711, -42, 1}, {53865, -64458, 28367, -6156, 711, -42, 1},
  {48321, -62546, 28151, -6148, 711, -42, 1}, {49329, -62802, 28167,
    -6148, 711, -42, 1}, {44793, -61146, 27967, -6140, 711, -42, 1}}]
```

```
{86007, 103351, 38375, 34087, 3799, 340823, 275847,
  288903, 223927, 241271, 236983, 172007, 189351, 120087, 137431,
  478743, 426823, 374903, 392247, 322983, 577799, 525879, 728775}
```

```
Length[{86007, 103351, 38375, 34087, 3799, 340823, 275847,
  288903, 223927, 241271, 236983, 172007, 189351, 120087, 137431,
  478743, 426823, 374903, 392247, 322983, 577799, 525879, 728775}]
```

```
23
```

```
{-603, -549, 0, 0, 0, 0, 0}.Transpose[A]
```

```
{774063, 930159, 345375, 306783, 34191, 3067407, 2482623, 2600127, 2015343,
  2171439, 2132847, 1548063, 1704159, 1080783, 1236879, 4308687,
  3841407, 3374127, 3530223, 2906847, 5200191, 4732911, 6558975}
```

```
{-603, -549, 0, 0, 0, 0, 0}.
```

```
{2543345, -2783706, 1182087, -253228, 29151, -1722, 41}
```

```
-5382441
```

```
poly18 = listmod128[[18]]
```

```
 $(-11 + x) (-9 + x)^6 (-7 + x)^8 (-4 + x) (5 + x)^{25}$ 
```

```
p18 = poly18 / minipoly[poly18]
```

```
 $(-9 + x)^5 (-7 + x)^7 (5 + x)^{24}$ 
```

```

feasiblesubcharpolylist[(-11+x) (-9+x)6 (-7+x)8 (-4+x) (5+x)25]
{ (-11+x) (-5+x) (17-10 x+x2), 951-822 x+232 x2-26 x3+x4,
  967-822 x+232 x2-26 x3+x4, 983-822 x+232 x2-26 x3+x4,
  (-11+x) (-7+x) (11-8 x+x2), 863-814 x+232 x2-26 x3+x4,
  879-814 x+232 x2-26 x3+x4, (-5+x) (-179+127 x-21 x2+x3),
  911-814 x+232 x2-26 x3+x4, (-9+x) (-103+79 x-17 x2+x3),
  (-7+x) (-113+99 x-19 x2+x3), 807-806 x+232 x2-26 x3+x4,
  823-806 x+232 x2-26 x3+x4, 839-806 x+232 x2-26 x3+x4,
  (-9+x) (-5+x) (19-12 x+x2), (-7+x)2 (15-12 x+x2), 751-798 x+232 x2-26 x3+x4,
  (59-16 x+x2) (13-10 x+x2), (-9+x) (-87+79 x-17 x2+x3),
  (-7+x) (-97+99 x-19 x2+x3), 695-790 x+232 x2-26 x3+x4,
  (-9+x) (-79+79 x-17 x2+x3), (-7+x) (-89+99 x-19 x2+x3),
  (-9+x) (-71+79 x-17 x2+x3), (-9+x)2 (-7+x) (-1+x) }

{ (-11+x) (-5+x) (17-10 x+x2),
  951-822 x+232 x2-26 x3+x4, 967-822 x+232 x2-26 x3+x4,
  983-822 x+232 x2-26 x3+x4, (-11+x) (-7+x) (11-8 x+x2),
  863-814 x+232 x2-26 x3+x4, 879-814 x+232 x2-26 x3+x4,
  (-5+x) (-179+127 x-21 x2+x3), 911-814 x+232 x2-26 x3+x4,
  (-9+x) (-103+79 x-17 x2+x3), (-7+x) (-113+99 x-19 x2+x3),
  807-806 x+232 x2-26 x3+x4, 823-806 x+232 x2-26 x3+x4,
  839-806 x+232 x2-26 x3+x4, (-9+x) (-5+x) (19-12 x+x2),
  (-7+x)2 (15-12 x+x2), 751-798 x+232 x2-26 x3+x4,
  (59-16 x+x2) (13-10 x+x2), (-9+x) (-87+79 x-17 x2+x3),
  (-7+x) (-97+99 x-19 x2+x3), 695-790 x+232 x2-26 x3+x4,
  (-9+x) (-79+79 x-17 x2+x3), (-7+x) (-89+99 x-19 x2+x3),
  (-9+x) (-71+79 x-17 x2+x3), (-9+x)2 (-7+x) (-1+x) } // Length

```



```

CoefficientList[ { (-11 + x) (-5 + x) (17 - 10 x + x^2),
  951 - 822 x + 232 x^2 - 26 x^3 + x^4, 967 - 822 x + 232 x^2 - 26 x^3 + x^4,
  983 - 822 x + 232 x^2 - 26 x^3 + x^4, (-11 + x) (-7 + x) (11 - 8 x + x^2),
  863 - 814 x + 232 x^2 - 26 x^3 + x^4, 879 - 814 x + 232 x^2 - 26 x^3 + x^4,
  (-5 + x) (-179 + 127 x - 21 x^2 + x^3), 911 - 814 x + 232 x^2 - 26 x^3 + x^4,
  (-9 + x) (-103 + 79 x - 17 x^2 + x^3), (-7 + x) (-113 + 99 x - 19 x^2 + x^3),
  807 - 806 x + 232 x^2 - 26 x^3 + x^4, 823 - 806 x + 232 x^2 - 26 x^3 + x^4,
  839 - 806 x + 232 x^2 - 26 x^3 + x^4, (-9 + x) (-5 + x) (19 - 12 x + x^2),
  (-7 + x)^2 (15 - 12 x + x^2), 751 - 798 x + 232 x^2 - 26 x^3 + x^4,
  (59 - 16 x + x^2) (13 - 10 x + x^2), (-9 + x) (-87 + 79 x - 17 x^2 + x^3),
  (-7 + x) (-97 + 99 x - 19 x^2 + x^3), 695 - 790 x + 232 x^2 - 26 x^3 + x^4,
  (-9 + x) (-79 + 79 x - 17 x^2 + x^3), (-7 + x) (-89 + 99 x - 19 x^2 + x^3),
  (-9 + x) (-71 + 79 x - 17 x^2 + x^3), (-9 + x)^2 (-7 + x) (-1 + x) }, x]
{{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
 {967, -822, 232, -26, 1}, {983, -822, 232, -26, 1},
 {847, -814, 232, -26, 1}, {863, -814, 232, -26, 1}, {879, -814, 232, -26, 1},
 {895, -814, 232, -26, 1}, {911, -814, 232, -26, 1}, {927, -814, 232, -26, 1},
 {791, -806, 232, -26, 1}, {807, -806, 232, -26, 1}, {823, -806, 232, -26, 1},
 {839, -806, 232, -26, 1}, {855, -806, 232, -26, 1}, {735, -798, 232, -26, 1},
 {751, -798, 232, -26, 1}, {767, -798, 232, -26, 1}, {783, -798, 232, -26, 1},
 {679, -790, 232, -26, 1}, {695, -790, 232, -26, 1}, {711, -790, 232, -26, 1},
 {623, -782, 232, -26, 1}, {639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}}

A = {{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
 {967, -822, 232, -26, 1}, {983, -822, 232, -26, 1}, {847, -814, 232, -26, 1},
 {863, -814, 232, -26, 1}, {879, -814, 232, -26, 1}, {895, -814, 232, -26, 1},
 {911, -814, 232, -26, 1}, {927, -814, 232, -26, 1}, {791, -806, 232, -26, 1},
 {807, -806, 232, -26, 1}, {823, -806, 232, -26, 1}, {839, -806, 232, -26, 1},
 {855, -806, 232, -26, 1}, {735, -798, 232, -26, 1}, {751, -798, 232, -26, 1},
 {767, -798, 232, -26, 1}, {783, -798, 232, -26, 1}, {679, -790, 232, -26, 1},
 {695, -790, 232, -26, 1}, {711, -790, 232, -26, 1}, {623, -782, 232, -26, 1},
 {639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}};

CoefficientList[D[poly18, x] / p18 // Factor, x]
{39495, -33726, 9512, -1066, 41}

```

```
Solve[Array[n, 25].{{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
  {967, -822, 232, -26, 1}, {983, -822, 232, -26, 1}, {847, -814, 232, -26, 1},
  {863, -814, 232, -26, 1}, {879, -814, 232, -26, 1}, {895, -814, 232, -26, 1},
  {911, -814, 232, -26, 1}, {927, -814, 232, -26, 1}, {791, -806, 232, -26, 1},
  {807, -806, 232, -26, 1}, {823, -806, 232, -26, 1}, {839, -806, 232, -26, 1},
  {855, -806, 232, -26, 1}, {735, -798, 232, -26, 1}, {751, -798, 232, -26, 1},
  {767, -798, 232, -26, 1}, {783, -798, 232, -26, 1}, {679, -790, 232, -26, 1},
  {695, -790, 232, -26, 1}, {711, -790, 232, -26, 1}, {623, -782, 232, -26, 1},
  {639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}}} ==
  {39495, -33726, 9512, -1066, 41}, Array[n, 25]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{ {n[23] → 105 - 4 n[1] - 3 n[2] - 2 n[3] - n[4] - 5 n[5] - 4 n[6] - 3 n[7] - 2 n[8] - n[9] -
  4 n[11] - 3 n[12] - 2 n[13] - n[14] - 3 n[16] - 2 n[17] - n[18] - 2 n[20] - n[21],
  n[24] → 144 - 2 n[1] - 3 n[2] - 4 n[3] - 5 n[4] - n[6] - 2 n[7] - 3 n[8] - 4 n[9] - 5 n[10] -
  n[12] - 2 n[13] - 3 n[14] - 4 n[15] - n[17] - 2 n[18] - 3 n[19] - n[21] - 2 n[22],
  n[25] → -208 + 5 n[1] + 5 n[2] + 5 n[3] + 5 n[4] + 4 n[5] + 4 n[6] + 4 n[7] +
  4 n[8] + 4 n[9] + 4 n[10] + 3 n[11] + 3 n[12] + 3 n[13] + 3 n[14] +
  3 n[15] + 2 n[16] + 2 n[17] + 2 n[18] + 2 n[19] + n[20] + n[21] + n[22] } }
```

```
FindInstance[
  -n[23] + 105 - 4 n[1] - 3 n[2] - 2 n[3] - n[4] - 5 n[5] - 4 n[6] - 3 n[7] - 2 n[8] - n[9] -
    4 n[11] - 3 n[12] - 2 n[13] - n[14] - 3 n[16] - 2 n[17] - n[18] - 2 n[20] - n[21] == 0 &&
  -n[24] + 144 - 2 n[1] - 3 n[2] - 4 n[3] - 5 n[4] - n[6] - 2 n[7] - 3 n[8] - 4 n[9] - 5 n[10] -
    n[12] - 2 n[13] - 3 n[14] - 4 n[15] - n[17] - 2 n[18] - 3 n[19] - n[21] - 2 n[22] == 0 &&
  -n[25] - 208 + 5 n[1] + 5 n[2] + 5 n[3] + 5 n[4] + 4 n[5] + 4 n[6] +
    4 n[7] + 4 n[8] + 4 n[9] + 4 n[10] + 3 n[11] + 3 n[12] + 3 n[13] + 3 n[14] +
    3 n[15] + 2 n[16] + 2 n[17] + 2 n[18] + 2 n[19] + n[20] + n[21] + n[22] == 0
  && n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
  n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 &&
  n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0 && n[19] ≥ 0 && n[20] ≥ 0 &&
  n[21] ≥ 0 && n[22] ≥ 0 && n[23] ≥ 0 && n[24] ≥ 0 && n[25] ≥ 0, Array[n, 25], Integers]
{ }
```

Array[m, 5].Transpose[A]

```
{ 935 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  951 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  967 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  983 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  847 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  863 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  879 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  895 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  911 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  927 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  791 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  807 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  823 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  839 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  855 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  735 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  751 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  767 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  783 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  679 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  695 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  711 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  623 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  639 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ,
  567 m[1] - 774 m[2] + 232 m[3] - 26 m[4] + m[5] }
```

Array[m, 5].{39 495, -33 726, 9512, -1066, 41}

```
39 495 m[1] - 33 726 m[2] + 9512 m[3] - 1066 m[4] + 41 m[5]
```

```

FindInstance[935 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  951 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  967 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  983 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  847 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  863 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  879 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  895 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  911 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  927 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  791 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  807 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  823 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  839 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  855 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  735 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  751 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  767 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  783 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  679 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  695 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  711 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  623 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  639 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  567 m[1] - 774 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
  39495 m[1] - 33726 m[2] + 9512 m[3] - 1066 m[4] + 41 m[5] < 0, Array[m, 5], Integers]
{{m[1] → 0, m[2] → 3493, m[3] → 0, m[4] → 0, m[5] → 2871696}}

Array[m, 5] /. {m[1] → 0, m[2] → 3493, m[3] → 0, m[4] → 0, m[5] → 2871696}
{0, 3493, 0, 0, 2871696}

GCD[0, 3493, 0, 0, 2871696]
1

{0, 3493, 0, 0, 2871696} . {39495, -33726, 9512, -1066, 41}
-65382

```

```
{0, 3493, 0, 0, 2 871 696}.
```

```
Transpose[{{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
  {967, -822, 232, -26, 1}, {983, -822, 232, -26, 1}, {847, -814, 232, -26, 1},
  {863, -814, 232, -26, 1}, {879, -814, 232, -26, 1}, {895, -814, 232, -26, 1},
  {911, -814, 232, -26, 1}, {927, -814, 232, -26, 1}, {791, -806, 232, -26, 1},
  {807, -806, 232, -26, 1}, {823, -806, 232, -26, 1}, {839, -806, 232, -26, 1},
  {855, -806, 232, -26, 1}, {735, -798, 232, -26, 1}, {751, -798, 232, -26, 1},
  {767, -798, 232, -26, 1}, {783, -798, 232, -26, 1}, {679, -790, 232, -26, 1},
  {695, -790, 232, -26, 1}, {711, -790, 232, -26, 1}, {623, -782, 232, -26, 1},
  {639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}}]
```

```
{450, 450, 450, 450, 28 394, 28 394, 28 394, 28 394, 28 394,
  28 394, 56 338, 56 338, 56 338, 56 338, 56 338, 84 282, 84 282,
  84 282, 84 282, 112 226, 112 226, 112 226, 140 170, 140 170, 168 114}
```

```
poly19 = listmod128[[19]]
```

```
 $(-9 + x)^8 (-7 + x)^4 (-5 + x)^2 (5 + x)^{25} (52 - 15 x + x^2)$ 
```

```
p19 = poly19 / minipoly[poly19] // Factor
```

```
 $(-9 + x)^7 (-7 + x)^3 (-5 + x) (5 + x)^{24}$ 
```

```
feasiblesubcharpolylist[ $(-9 + x)^8 (-7 + x)^4 (-5 + x)^2 (5 + x)^{25} (52 - 15 x + x^2)$ ]
```

```
{ $(-9 + x) (-5 + x) (-127 + 87 x - 17 x^2 + x^3)$ ,  $(-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2)$ }
```

```
CoefficientList[
```

```
{ $(-9 + x) (-5 + x) (-127 + 87 x - 17 x^2 + x^3)$ ,  $(-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2)$ }, x]
```

```
{{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}}
```

```
A = {{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}};
```

```
CoefficientList[D[poly19, x] / p19 // Factor, x]
```

```
{-233 515, 233 557, -86 558, 15 170, -1271, 41}
```

```
Solve[Array[n, 2].
```

```
{{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}} ==
```

```
{-233 515, 233 557, -86 558, 15 170, -1271, 41}, Array[n, 2]]
```

```
{}
```

```
Array[m, 6].Transpose[A]
```

```
{-5715 m[1] + 5693 m[2] - 2110 m[3] + 370 m[4] - 31 m[5] + m[6],
```

```
-5355 m[1] + 5581 m[2] - 2102 m[3] + 370 m[4] - 31 m[5] + m[6]}
```

```
Array[m, 6].{-233 515, 233 557, -86 558, 15 170, -1271, 41}
```

```
-233 515 m[1] + 233 557 m[2] - 86 558 m[3] + 15 170 m[4] - 1271 m[5] + 41 m[6]
```

```

FindInstance[-5715 m[1] + 5693 m[2] - 2110 m[3] + 370 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -5355 m[1] + 5581 m[2] - 2102 m[3] + 370 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -233 515 m[1] + 233 557 m[2] - 86 558 m[3] + 15 170 m[4] - 1271 m[5] + 41 m[6] < 0,
  Array[m, 6], Integers]
{{m[1] → 14, m[2] → 1163, m[3] → 15 610, m[4] → 0, m[5] → 0, m[6] → 26 404 920}}

Array[m, 6] /.
  {m[1] → 14, m[2] → 1163, m[3] → 15 610, m[4] → 0, m[5] → 0, m[6] → 26 404 920}
{14, 1163, 15 610, 0, 0, 26 404 920}

GCD[14, 1163, 15 610, 0, 0, 26 404 920]
1

{14, 1163, 15 610, 0, 0, 26 404 920} . {-233 515, 233 557, -86 558, 15 170, -1271, 41}
-211 079

{14, 1163, 15 610, 0, 0, 26 404 920} .
  Transpose[{{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}}]
{8769, 8433}

```

```
poly20 = listmod128[[20]]
```

$$(-9+x)^7 (-7+x)^6 (5+x)^{25} (-232+123x-20x^2+x^3)$$

```
p20 = poly20 / minipoly[poly20] // Factor
```

$$(-9+x)^6 (-7+x)^5 (5+x)^{24}$$

```

feasiblesubcharpolylist[(-9+x)^7 (-7+x)^6 (5+x)^{25} (-232+123x-20x^2+x^3)]
{
  (31-12x+x^2) (-169+107x-19x^2+x^3),
  (-7+x) (681-648x+198x^2-24x^3+x^4), (-7+x) (41-14x+x^2) (17-10x+x^2),
  -4847+5233x-2034x^2+366x^3-31x^4+x^5, (-7+x) (23-12x+x^2) (31-12x+x^2),
  (-9+x) (551-522x+168x^2-22x^3+x^4), (-7+x)^2 (-87+79x-17x^2+x^3),
  (-7+x) (-5+x) (-125+103x-19x^2+x^3), (-7+x) (641-640x+198x^2-24x^3+x^4),
  (-9+x)^2 (-5+x) (11-8x+x^2), (-9+x) (-7+x) (-73+63x-15x^2+x^3),
  (-7+x)^2 (-79+79x-17x^2+x^3), (-7+x) (569-632x+198x^2-24x^3+x^4),
  (-9+x) (-7+x) (-5+x) (13-10x+x^2), (-9+x) (-7+x) (-57+63x-15x^2+x^3)}

```

```
{ (31 - 12 x + x^2) (-169 + 107 x - 19 x^2 + x^3), (-7 + x) (681 - 648 x + 198 x^2 - 24 x^3 + x^4),
  (-7 + x) (41 - 14 x + x^2) (17 - 10 x + x^2), -4847 + 5233 x - 2034 x^2 + 366 x^3 - 31 x^4 + x^5,
  (-7 + x) (23 - 12 x + x^2) (31 - 12 x + x^2), (-9 + x) (551 - 522 x + 168 x^2 - 22 x^3 + x^4),
  (-7 + x)^2 (-87 + 79 x - 17 x^2 + x^3), (-7 + x) (-5 + x) (-125 + 103 x - 19 x^2 + x^3),
  (-7 + x) (641 - 640 x + 198 x^2 - 24 x^3 + x^4), (-9 + x)^2 (-5 + x) (11 - 8 x + x^2),
  (-9 + x) (-7 + x) (-73 + 63 x - 15 x^2 + x^3), (-7 + x)^2 (-79 + 79 x - 17 x^2 + x^3),
  (-7 + x) (569 - 632 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-5 + x) (13 - 10 x + x^2),
  (-9 + x) (-7 + x) (-57 + 63 x - 15 x^2 + x^3) } // Length
```

15

```
CoefficientList[ { (31 - 12 x + x^2) (-169 + 107 x - 19 x^2 + x^3),
  (-7 + x) (681 - 648 x + 198 x^2 - 24 x^3 + x^4), (-7 + x) (41 - 14 x + x^2) (17 - 10 x + x^2),
  -4847 + 5233 x - 2034 x^2 + 366 x^3 - 31 x^4 + x^5, (-7 + x) (23 - 12 x + x^2) (31 - 12 x + x^2),
  (-9 + x) (551 - 522 x + 168 x^2 - 22 x^3 + x^4), (-7 + x)^2 (-87 + 79 x - 17 x^2 + x^3),
  (-7 + x) (-5 + x) (-125 + 103 x - 19 x^2 + x^3), (-7 + x) (641 - 640 x + 198 x^2 - 24 x^3 + x^4),
  (-9 + x)^2 (-5 + x) (11 - 8 x + x^2), (-9 + x) (-7 + x) (-73 + 63 x - 15 x^2 + x^3),
  (-7 + x)^2 (-79 + 79 x - 17 x^2 + x^3), (-7 + x) (569 - 632 x + 198 x^2 - 24 x^3 + x^4),
  (-9 + x) (-7 + x) (-5 + x) (13 - 10 x + x^2), (-9 + x) (-7 + x) (-57 + 63 x - 15 x^2 + x^3) }, x]
```

```
{ {-5239, 5345, -2042, 366, -31, 1},
  {-4767, 5217, -2034, 366, -31, 1}, {-4879, 5233, -2034, 366, -31, 1},
  {-4847, 5233, -2034, 366, -31, 1}, {-4991, 5249, -2034, 366, -31, 1},
  {-4959, 5249, -2034, 366, -31, 1}, {-4263, 5089, -2026, 366, -31, 1},
  {-4375, 5105, -2026, 366, -31, 1}, {-4487, 5121, -2026, 366, -31, 1},
  {-4455, 5121, -2026, 366, -31, 1}, {-4599, 5137, -2026, 366, -31, 1},
  {-3871, 4977, -2018, 366, -31, 1}, {-3983, 4993, -2018, 366, -31, 1},
  {-4095, 5009, -2018, 366, -31, 1}, {-3591, 4881, -2010, 366, -31, 1}}
```

```
A = { {-5239, 5345, -2042, 366, -31, 1},
  {-4767, 5217, -2034, 366, -31, 1}, {-4879, 5233, -2034, 366, -31, 1},
  {-4847, 5233, -2034, 366, -31, 1}, {-4991, 5249, -2034, 366, -31, 1},
  {-4959, 5249, -2034, 366, -31, 1}, {-4263, 5089, -2026, 366, -31, 1},
  {-4375, 5105, -2026, 366, -31, 1}, {-4487, 5121, -2026, 366, -31, 1},
  {-4455, 5121, -2026, 366, -31, 1}, {-4599, 5137, -2026, 366, -31, 1},
  {-3871, 4977, -2018, 366, -31, 1}, {-3983, 4993, -2018, 366, -31, 1},
  {-4095, 5009, -2018, 366, -31, 1}, {-3591, 4881, -2010, 366, -31, 1}};
```

```
CoefficientList[D[poly20, x] / p20 // Factor, x]
```

```
{-207175, 217305, -83618, 15006, -1271, 41}
```

```
Solve[Array[n, 15].{{-5239, 5345, -2042, 366, -31, 1},
  {-4767, 5217, -2034, 366, -31, 1}, {-4879, 5233, -2034, 366, -31, 1},
  {-4847, 5233, -2034, 366, -31, 1}, {-4991, 5249, -2034, 366, -31, 1},
  {-4959, 5249, -2034, 366, -31, 1}, {-4263, 5089, -2026, 366, -31, 1},
  {-4375, 5105, -2026, 366, -31, 1}, {-4487, 5121, -2026, 366, -31, 1},
  {-4455, 5121, -2026, 366, -31, 1}, {-4599, 5137, -2026, 366, -31, 1},
  {-3871, 4977, -2018, 366, -31, 1}, {-3983, 4993, -2018, 366, -31, 1},
  {-4095, 5009, -2018, 366, -31, 1}, {-3591, 4881, -2010, 366, -31, 1}} ==
  {-207175, 217305, -83618, 15006, -1271, 41}, Array[n, 15]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. >>

```
{ {n[10] → 36 - n[1] - n[4] - n[6],
  n[13] → 98 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] - n[9] - 2 n[12],
  n[14] → -19 - n[3] - 2 n[5] - n[6] + n[7] - n[9] - 2 n[11] + n[12],
  n[15] → -74 + 2 n[1] + 2 n[2] + 2 n[3] + n[4] + 2 n[5] + n[6] + n[7] + n[8] + n[9] + n[11] } }
```

```
FindInstance[-n[10] + 36 - n[1] - n[4] - n[6] == 0 &&
  -n[13] + 98 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] - n[9] - 2 n[12] == 0 &&
  -n[14] - 19 - n[3] - 2 n[5] - n[6] + n[7] - n[9] - 2 n[11] + n[12] == 0 &&
  -n[15] - 74 + 2 n[1] + 2 n[2] + 2 n[3] + n[4] + 2 n[5] + n[6] + n[7] + n[8] + n[9] + n[11] ==
    0 && n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 &&
  n[7] ≥ 0 && n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 &&
  n[13] ≥ 0 && n[14] ≥ 0 && n[15] ≥ 0, Array[n, 15], Integers]

{ }
```

```
Array[m, 6].Transpose[A]
```

```
{ -5239 m[1] + 5345 m[2] - 2042 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4767 m[1] + 5217 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4879 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4847 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4991 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4959 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4263 m[1] + 5089 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4375 m[1] + 5105 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4487 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4455 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4599 m[1] + 5137 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
  -3871 m[1] + 4977 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6],
  -3983 m[1] + 4993 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6],
  -4095 m[1] + 5009 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6],
  -3591 m[1] + 4881 m[2] - 2010 m[3] + 366 m[4] - 31 m[5] + m[6] }
```

```
Array[m, 6].{-207175, 217305, -83618, 15006, -1271, 41}
```

```
-207175 m[1] + 217305 m[2] - 83618 m[3] + 15006 m[4] - 1271 m[5] + 41 m[6]
```



```
FindInstance[-5239 m[1] + 5345 m[2] - 2042 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4767 m[1] + 5217 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4879 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4847 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4991 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4959 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4263 m[1] + 5089 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4375 m[1] + 5105 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4487 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4455 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4599 m[1] + 5137 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -3871 m[1] + 4977 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -3983 m[1] + 4993 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -4095 m[1] + 5009 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -3591 m[1] + 4881 m[2] - 2010 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
  -207 175 m[1] + 217 305 m[2] - 83 618 m[3] + 15 006 m[4] - 1271 m[5] + 41 m[6] < 0,
  Array[m, 6], Integers]
```

```
{ {m[1] → -1026, m[2] → -4476, m[3] → -9088, m[4] → 0, m[5] → 0, m[6] → 0} }
```

```
Array[m, 6] /.
```

```
{m[1] → -1026, m[2] → -4476, m[3] → -9088, m[4] → 0, m[5] → 0, m[6] → 0}
```

```
{-1026, -4476, -9088, 0, 0, 0}
```

```
GCD[-1026, -4476, -9088, 0, 0, 0]
```

```
2
```

```
{-1026, -4476, -9088, 0, 0, 0} / 2
```

```
{-513, -2238, -4544, 0, 0, 0}
```

```
Reverse[{-513, -2238, -4544, 0, 0, 0}]
```

```
{0, 0, 0, -4544, -2238, -513}
```

```
{-513, -2238, -4544, 0, 0, 0} . {-207 175, 217 305, -83 618, 15 006, -1271, 41}
```

```
-87 623
```

```
{-513, -2238, -4544, 0, 0, 0} .
```

```
Transpose[{{-5239, 5345, -2042, 366, -31, 1}, {-4767, 5217, -2034, 366, -31, 1},
  {-4879, 5233, -2034, 366, -31, 1}, {-4847, 5233, -2034, 366, -31, 1},
  {-4991, 5249, -2034, 366, -31, 1}, {-4959, 5249, -2034, 366, -31, 1},
  {-4263, 5089, -2026, 366, -31, 1}, {-4375, 5105, -2026, 366, -31, 1},
  {-4487, 5121, -2026, 366, -31, 1}, {-4455, 5121, -2026, 366, -31, 1},
  {-4599, 5137, -2026, 366, -31, 1}, {-3871, 4977, -2018, 366, -31, 1},
  {-3983, 4993, -2018, 366, -31, 1}, {-4095, 5009, -2018, 366, -31, 1},
  {-3591, 4881, -2010, 366, -31, 1}}]
```

```
{4345, 12 321, 33 969, 17 553, 55 617, 39 201, 3881,
```

```
25 529, 47 177, 30 761, 68 825, 17 089, 38 737, 60 385, 51 945}
```

```

Length[{4345, 12 321, 33 969, 17 553, 55 617, 39 201,
 3881, 25 529, 47 177, 30 761, 68 825, 17 089, 38 737, 60 385, 51 945}]
15
{-1026, -4476, -9088, 0, 0, 0}.{-207 175, 217 305, -83 618, 15 006, -1271, 41}
-175 246
{-1026, -4476, -9088, 0, 0, 0}.
Transpose[{{-5239, 5345, -2042, 366, -31, 1}, {-4767, 5217, -2034, 366, -31, 1},
{-4879, 5233, -2034, 366, -31, 1}, {-4847, 5233, -2034, 366, -31, 1},
{-4991, 5249, -2034, 366, -31, 1}, {-4959, 5249, -2034, 366, -31, 1},
{-4263, 5089, -2026, 366, -31, 1}, {-4375, 5105, -2026, 366, -31, 1},
{-4487, 5121, -2026, 366, -31, 1}, {-4455, 5121, -2026, 366, -31, 1},
{-4599, 5137, -2026, 366, -31, 1}, {-3871, 4977, -2018, 366, -31, 1},
{-3983, 4993, -2018, 366, -31, 1}, {-4095, 5009, -2018, 366, -31, 1},
{-3591, 4881, -2010, 366, -31, 1}}]
{8690, 24 642, 67 938, 35 106, 111 234, 78 402, 7762,
51 058, 94 354, 61 522, 137 650, 34 178, 77 474, 120 770, 103 890}

```

```
poly21 = listmod128[[21]]
```

```
(-9 + x)6 (-7 + x)7 (5 + x)25 (-292 + 149 x - 22 x2 + x3)
```

```
p21 = poly21 / minipoly[poly21] // Factor
```

```
(-9 + x)5 (-7 + x)6 (5 + x)24
```

```

feasiblesubcharpolylist[(-9 + x)6 (-7 + x)7 (5 + x)25 (-292 + 149 x - 22 x2 + x3)]
{(-9 + x) (-7 + x) (-5 + x) (19 - 12 x + x2), (-9 + x) (681 - 648 x + 198 x2 - 24 x3 + x4),
(-9 + x) (41 - 14 x + x2) (17 - 10 x + x2), (-9 + x) (23 - 12 x + x2) (31 - 12 x + x2),
(-9 + x) (-7 + x) (-87 + 79 x - 17 x2 + x3), (-9 + x) (-5 + x) (-125 + 103 x - 19 x2 + x3),
(-9 + x) (641 - 640 x + 198 x2 - 24 x3 + x4), (-9 + x) (-7 + x) (-79 + 79 x - 17 x2 + x3),
(-9 + x) (569 - 632 x + 198 x2 - 24 x3 + x4), (-9 + x) (-7 + x) (-71 + 79 x - 17 x2 + x3),
(-9 + x)2 (-57 + 63 x - 15 x2 + x3), (-9 + x)2 (-7 + x)2 (-1 + x)}

{(-9 + x) (-7 + x) (-5 + x) (19 - 12 x + x2), (-9 + x) (681 - 648 x + 198 x2 - 24 x3 + x4),
(-9 + x) (41 - 14 x + x2) (17 - 10 x + x2), (-9 + x) (23 - 12 x + x2) (31 - 12 x + x2),
(-9 + x) (-7 + x) (-87 + 79 x - 17 x2 + x3), (-9 + x) (-5 + x) (-125 + 103 x - 19 x2 + x3),
(-9 + x) (641 - 640 x + 198 x2 - 24 x3 + x4), (-9 + x) (-7 + x) (-79 + 79 x - 17 x2 + x3),
(-9 + x) (569 - 632 x + 198 x2 - 24 x3 + x4), (-9 + x) (-7 + x) (-71 + 79 x - 17 x2 + x3),
(-9 + x)2 (-57 + 63 x - 15 x2 + x3), (-9 + x)2 (-7 + x)2 (-1 + x)} // Length

```

```

CoefficientList[
  {(-9 + x) (-7 + x) (-5 + x) (19 - 12 x + x^2), (-9 + x) (681 - 648 x + 198 x^2 - 24 x^3 + x^4),
   (-9 + x) (41 - 14 x + x^2) (17 - 10 x + x^2), (-9 + x) (23 - 12 x + x^2) (31 - 12 x + x^2),
   (-9 + x) (-7 + x) (-87 + 79 x - 17 x^2 + x^3), (-9 + x) (-5 + x) (-125 + 103 x - 19 x^2 + x^3),
   (-9 + x) (641 - 640 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-79 + 79 x - 17 x^2 + x^3),
   (-9 + x) (569 - 632 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-71 + 79 x - 17 x^2 + x^3),
   (-9 + x)^2 (-57 + 63 x - 15 x^2 + x^3), (-9 + x)^2 (-7 + x)^2 (-1 + x)}, x]
{{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
 {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
 {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
 {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
 {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
 {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}}

A = {{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
      {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
      {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
      {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
      {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
      {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}};

MatrixRank[A]
3

CoefficientList[D[poly21, x] / p21 // Factor, x]
{-259665, 268537, -99702, 16974, -1353, 41}

Solve[Array[n, 12].
  {{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
   {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
   {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
   {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
   {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
   {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}} ==
  {-259665, 268537, -99702, 16974, -1353, 41}, Array[n, 12]]
{}

```

Array[m, 6].Transpose[A]

```
{-5985 m[1] + 6497 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],
-6129 m[1] + 6513 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],
-6273 m[1] + 6529 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],
-6417 m[1] + 6545 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],
-5481 m[1] + 6369 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6],
-5625 m[1] + 6385 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6],
-5769 m[1] + 6401 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6],
-4977 m[1] + 6241 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6],
-5121 m[1] + 6257 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6],
-4473 m[1] + 6113 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6],
-4617 m[1] + 6129 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6],
-3969 m[1] + 5985 m[2] - 2398 m[3] + 414 m[4] - 33 m[5] + m[6]}
```

Array[m, 6].{-259665, 268537, -99702, 16974, -1353, 41}

```
-259665 m[1] + 268537 m[2] - 99702 m[3] + 16974 m[4] - 1353 m[5] + 41 m[6]
```

FindInstance[-5985 m[1] + 6497 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&

```
-6129 m[1] + 6513 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-6273 m[1] + 6529 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-6417 m[1] + 6545 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-5481 m[1] + 6369 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-5625 m[1] + 6385 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-5769 m[1] + 6401 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-4977 m[1] + 6241 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-5121 m[1] + 6257 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-4473 m[1] + 6113 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-4617 m[1] + 6129 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-3969 m[1] + 5985 m[2] - 2398 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&
-259665 m[1] + 268537 m[2] - 99702 m[3] + 16974 m[4] - 1353 m[5] + 41 m[6] < 0,
```

Array[m, 6], Integers]

```
{m[1] → -180, m[2] → -1613, m[3] → -3873, m[4] → 0, m[5] → 0, m[6] → 0}
```

Array[m, 6] /. {m[1] → -180, m[2] → -1613, m[3] → -3873, m[4] → 0, m[5] → 0, m[6] → 0}

```
{-180, -1613, -3873, 0, 0, 0}
```

GCD[-180, -1613, -3873, 0, 0, 0]

1

{-180, -1613, -3873, 0, 0, 0}.{-259665, 268537, -99702, 16974, -1353, 41}

-264635

```
{-180, -1613, -3873, 0, 0, 0}.
Transpose[{{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
  {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
  {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
  {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
  {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
  {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}}]
{9029, 9141, 9253, 9365, 93789, 93901,
  94013, 178549, 178661, 263309, 263421, 348069}
```

```
poly22 = listmod128[[22]]
```

```
 $(-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^{25}$ 
```

```
p22 = poly22 / minipoly[poly22] // Factor
```

```
 $(-9 + x)^7 (-7 + x)^5 (5 + x)^{24}$ 
```

```
feasiblesubcharpolylist[ $(-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^{25}$ ]
```

```
{ $(-9 + x) (-7 + x) (-3 + x)^2$ ,
 $(-7 + x) (-73 + 63 x - 15 x^2 + x^3)$ ,  $(31 - 12 x + x^2) (17 - 10 x + x^2)$ ,
 $(-7 + x) (-5 + x) (13 - 10 x + x^2)$ ,  $(-7 + x) (-57 + 63 x - 15 x^2 + x^3)$ }
```

```
{ $(-9 + x) (-7 + x) (-3 + x)^2$ ,  $(-7 + x) (-73 + 63 x - 15 x^2 + x^3)$ ,
 $(31 - 12 x + x^2) (17 - 10 x + x^2)$ ,  $(-7 + x) (-5 + x) (13 - 10 x + x^2)$ ,
 $(-7 + x) (-57 + 63 x - 15 x^2 + x^3)$ } // Length
```

```
5
```

```
CoefficientList[{ $(-9 + x) (-7 + x) (-3 + x)^2$ ,
 $(-7 + x) (-73 + 63 x - 15 x^2 + x^3)$ ,  $(31 - 12 x + x^2) (17 - 10 x + x^2)$ ,
 $(-7 + x) (-5 + x) (13 - 10 x + x^2)$ ,  $(-7 + x) (-57 + 63 x - 15 x^2 + x^3)$ }, x]
```

```
{{567, -522, 168, -22, 1}, {511, -514, 168, -22, 1},
{527, -514, 168, -22, 1}, {455, -506, 168, -22, 1}, {399, -498, 168, -22, 1}}
```

```
CoefficientList[D[poly22, x] / p22 // Factor, x]
```

```
{21135, -21018, 6888, -902, 41}
```

```
Solve[Array[n, 5].{{567, -522, 168, -22, 1}, {511, -514, 168, -22, 1},
  {527, -514, 168, -22, 1}, {455, -506, 168, -22, 1}, {399, -498, 168, -22, 1}} ==
  {21135, -21018, 6888, -902, 41}, Array[n, 5]]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. >>
```

```
{{n[3] → 36, n[4] → 3 - 3 n[1] - 2 n[2], n[5] → 2 + 2 n[1] + n[2]}}
```

`Array[n, 5] /.`

`Solve[-n[3] + 36 == 0 && -n[4] + 3 - 3 n[1] - 2 n[2] == 0 && -n[5] + 2 + 2 n[1] + n[2] == 0 &&`
`n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0, Array[n, 5], Integers]`

`{{0, 0, 36, 3, 2}, {0, 1, 36, 1, 3}, {1, 0, 36, 0, 4}}`