```mathematica
dim16last2 = {(-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (48 - 15 x + x^2),
   (-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^25};
```

```mathematica
chi = (-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (48 - 15 x + x^2)
```

$(-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^{25} (48 - 15 x + x^2)$

```mathematica
CoefficientList[feasibleinterlacingpolylist[chi], x]
```

{{-11825, 10645, -3466, 522, -37, 1}, {-11033, 10485, -3458, 522, -37, 1},
 {-11209, 10501, -3458, 522, -37, 1}, {-11385, 10517, -3458, 522, -37, 1},
 {-11529, 10533, -3458, 522, -37, 1}, {-10241, 10325, -3450, 522, -37, 1},
 {-10417, 10341, -3450, 522, -37, 1}, {-10593, 10357, -3450, 522, -37, 1},
 {-10561, 10357, -3450, 522, -37, 1}, {-10737, 10373, -3450, 522, -37, 1},
 {-9625, 10181, -3442, 522, -37, 1}, {-9801, 10197, -3442, 522, -37, 1},
 {-9769, 10197, -3442, 522, -37, 1}, {-9945, 10213, -3442, 522, -37, 1},
 {-9009, 10037, -3434, 522, -37, 1}, {-9153, 10053, -3434, 522, -37, 1},
 {-9297, 10069, -3434, 522, -37, 1}, {-8505, 9909, -3426, 522, -37, 1}}

```mathematica
A = {{-11825, 10645, -3466, 522, -37, 1}, {-11033, 10485, -3458, 522, -37, 1},
   {-11209, 10501, -3458, 522, -37, 1}, {-11385, 10517, -3458, 522, -37, 1},
   {-11529, 10533, -3458, 522, -37, 1}, {-10241, 10325, -3450, 522, -37, 1},
   {-10417, 10341, -3450, 522, -37, 1}, {-10593, 10357, -3450, 522, -37, 1},
   {-10561, 10357, -3450, 522, -37, 1}, {-10737, 10373, -3450, 522, -37, 1},
   {-9625, 10181, -3442, 522, -37, 1}, {-9801, 10197, -3442, 522, -37, 1},
   {-9769, 10197, -3442, 522, -37, 1}, {-9945, 10213, -3442, 522, -37, 1},
   {-9009, 10037, -3434, 522, -37, 1}, {-9153, 10053, -3434, 522, -37, 1},
   {-9297, 10069, -3434, 522, -37, 1}, {-8505, 9909, -3426, 522, -37, 1}};
```

```
A // MatrixForm
```

$$\begin{pmatrix}
-11825 & 10645 & -3466 & 522 & -37 & 1 \\
-11033 & 10485 & -3458 & 522 & -37 & 1 \\
-11209 & 10501 & -3458 & 522 & -37 & 1 \\
-11385 & 10517 & -3458 & 522 & -37 & 1 \\
-11529 & 10533 & -3458 & 522 & -37 & 1 \\
-10241 & 10325 & -3450 & 522 & -37 & 1 \\
-10417 & 10341 & -3450 & 522 & -37 & 1 \\
-10593 & 10357 & -3450 & 522 & -37 & 1 \\
-10561 & 10357 & -3450 & 522 & -37 & 1 \\
-10737 & 10373 & -3450 & 522 & -37 & 1 \\
-9625 & 10181 & -3442 & 522 & -37 & 1 \\
-9801 & 10197 & -3442 & 522 & -37 & 1 \\
-9769 & 10197 & -3442 & 522 & -37 & 1 \\
-9945 & 10213 & -3442 & 522 & -37 & 1 \\
-9009 & 10037 & -3434 & 522 & -37 & 1 \\
-9153 & 10053 & -3434 & 522 & -37 & 1 \\
-9297 & 10069 & -3434 & 522 & -37 & 1 \\
-8505 & 9909 & -3426 & 522 & -37 & 1
\end{pmatrix}$$

```
g = CoefficientList[D[chi, x] / mu[chi] // Factor, x]
```

$\{-476745, 433821, -141930, 21402, -1517, 41\}$

```
FindInstance[n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 &&
   n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 && n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 &&
   n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 && n[15] ≥ 0 && n[16] ≥ 0 &&
   n[17] ≥ 0 && n[18] ≥ 0 && Array[n, 18].A == g, Array[n, 18], Integers]
```

$\{\{n[1] \to 21, n[2] \to 0, n[3] \to 0, n[4] \to 4, n[5] \to 14,$
$n[6] \to 0, n[7] \to 0, n[8] \to 0, n[9] \to 0, n[10] \to 2, n[11] \to 0, n[12] \to 0,$
$n[13] \to 0, n[14] \to 0, n[15] \to 0, n[16] \to 0, n[17] \to 0, n[18] \to 0\}\}$

```
Array[c, 6].Transpose[A]
```

$\{-11\,825\,c[1] + 10\,645\,c[2] - 3466\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-11\,033\,c[1] + 10\,485\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-11\,209\,c[1] + 10\,501\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-11\,385\,c[1] + 10\,517\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-11\,529\,c[1] + 10\,533\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-10\,241\,c[1] + 10\,325\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-10\,417\,c[1] + 10\,341\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-10\,593\,c[1] + 10\,357\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-10\,561\,c[1] + 10\,357\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-10\,737\,c[1] + 10\,373\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9625\,c[1] + 10\,181\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9801\,c[1] + 10\,197\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9769\,c[1] + 10\,197\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9945\,c[1] + 10\,213\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9009\,c[1] + 10\,037\,c[2] - 3434\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9153\,c[1] + 10\,053\,c[2] - 3434\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-9297\,c[1] + 10\,069\,c[2] - 3434\,c[3] + 522\,c[4] - 37\,c[5] + c[6],$
$-8505\,c[1] + 9909\,c[2] - 3426\,c[3] + 522\,c[4] - 37\,c[5] + c[6]\}$

```
Array[c, 6].g
```

$-476\,745\,c[1] + 433\,821\,c[2] - 141\,930\,c[3] + 21\,402\,c[4] - 1517\,c[5] + 41\,c[6]$

```
warrant1 = Flatten[Array[c, 6] /. FindInstance[
```

$-476\,745\,c[1] + 433\,821\,c[2] - 141\,930\,c[3] + 21\,402\,c[4] - 1517\,c[5] + 41\,c[6] < 0\ \&\&$
$-11\,825\,c[1] + 10\,645\,c[2] - 3466\,c[3] + 522\,c[4] - 37\,c[5] + c[6] < 0\ \&\&$
$-11\,033\,c[1] + 10\,485\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-11\,209\,c[1] + 10\,501\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-11\,385\,c[1] + 10\,517\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-11\,529\,c[1] + 10\,533\,c[2] - 3458\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-10\,241\,c[1] + 10\,325\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-10\,417\,c[1] + 10\,341\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-10\,593\,c[1] + 10\,357\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-10\,561\,c[1] + 10\,357\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-10\,737\,c[1] + 10\,373\,c[2] - 3450\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9625\,c[1] + 10\,181\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9801\,c[1] + 10\,197\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9769\,c[1] + 10\,197\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9945\,c[1] + 10\,213\,c[2] - 3442\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9009\,c[1] + 10\,037\,c[2] - 3434\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9153\,c[1] + 10\,053\,c[2] - 3434\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-9297\,c[1] + 10\,069\,c[2] - 3434\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0\ \&\&$
$-8505\,c[1] + 9909\,c[2] - 3426\,c[3] + 522\,c[4] - 37\,c[5] + c[6] \geq 0,$

```
     Array[c, 6], Integers]]
```

$\{565, 620, 0, 0, 0, 0\}$

```
GCD[565, 620, 0, 0, 0, 0]
```
5

```
warrant1 = warrant1 / 5
```
{113, 124, 0, 0, 0, 0}

```
Reverse[warrant1]
```
{0, 0, 0, 0, 124, 113}

```
warrant1.g
```
-78 381

```
warrant1.Transpose[A]
```
{-16 245, 53 411, 35 507, 17 603, 3315, 123 067, 105 163, 87 259, 90 875,
 72 971, 174 819, 156 915, 160 531, 142 627, 226 571, 212 283, 197 995, 267 651}

```
warrant2 = Flatten[Array[c, 6] /. FindInstance[
    -476 745 c[1] + 433 821 c[2] - 141 930 c[3] + 21 402 c[4] - 1517 c[5] + 41 c[6] < 0 &&
      -11 825 c[1] + 10 645 c[2] - 3466 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 033 c[1] + 10 485 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 209 c[1] + 10 501 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 385 c[1] + 10 517 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] < 0 &&
      -11 529 c[1] + 10 533 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 241 c[1] + 10 325 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 417 c[1] + 10 341 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 593 c[1] + 10 357 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 561 c[1] + 10 357 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 737 c[1] + 10 373 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9625 c[1] + 10 181 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9801 c[1] + 10 197 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9769 c[1] + 10 197 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9945 c[1] + 10 213 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9009 c[1] + 10 037 c[2] - 3434 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9153 c[1] + 10 053 c[2] - 3434 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9297 c[1] + 10 069 c[2] - 3434 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -8505 c[1] + 9909 c[2] - 3426 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0,
    Array[c, 6], Integers]]
```
{33 608, 336 081, 3 461 627, 0, 0, 8 817 840 108}

```
GCD[33 608, 336 081, 3 461 627, 0, 0, 8 817 840 108]
```
1

```
Reverse[warrant2]
```
{8 817 840 108, 0, 0, 3 461 627, 336 081, 33 608}

```
warrant2.g
```

$-726\,141$

```
warrant2.Transpose[A]
```

{8571, 546 163, 8451, −529 261, 8483, 1 083 755, 546 043, 8331, 1 083 787, 546 075,
 1 083 635, 545 923, 1 621 379, 1 083 667, 1 083 515, 1 621 259, 2 159 003, 2 696 595}

```
warrant3 = Flatten[Array[c, 6] /. FindInstance[
     -476 745 c[1] + 433 821 c[2] - 141 930 c[3] + 21 402 c[4] - 1517 c[5] + 41 c[6] < 0 &&
      -11 825 c[1] + 10 645 c[2] - 3466 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 033 c[1] + 10 485 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 209 c[1] + 10 501 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 385 c[1] + 10 517 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -11 529 c[1] + 10 533 c[2] - 3458 c[3] + 522 c[4] - 37 c[5] + c[6] < 0 &&
      -10 241 c[1] + 10 325 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 417 c[1] + 10 341 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 593 c[1] + 10 357 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 561 c[1] + 10 357 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -10 737 c[1] + 10 373 c[2] - 3450 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9625 c[1] + 10 181 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9801 c[1] + 10 197 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9769 c[1] + 10 197 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9945 c[1] + 10 213 c[2] - 3442 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9009 c[1] + 10 037 c[2] - 3434 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9153 c[1] + 10 053 c[2] - 3434 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -9297 c[1] + 10 069 c[2] - 3434 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0 &&
      -8505 c[1] + 9909 c[2] - 3426 c[3] + 522 c[4] - 37 c[5] + c[6] ≥ 0,
     Array[c, 6], Integers]]
```

{1289, 5147, 11 405, 0, 0, 0}

```
GCD[1289, 5147, 11 405, 0, 0, 0]
```

1

```
Reverse[warrant3]
```

{0, 0, 0, 11 405, 5147, 1289}

```
warrant3.g
```

$-359\,268$

```
warrant3.Transpose[A]
```

{17 660, 306 268, 161 756, 17 244, −86 020, 594 876, 450 364, 305 852, 347 100,
 202 588, 738 972, 594 460, 635 708, 491 196, 883 068, 779 804, 676 540, 965 148}

```
feasibleinterlacingpolylist[chi]
```

$\{(-11+x)(-5+x)(-215+131x-21x^2+x^3), (-11+x)(59-16x+x^2)(17-10x+x^2),$
$(-11+x)(1019-862x+236x^2-26x^3+x^4), (-11+x)(-9+x)(-5+x)(23-12x+x^2),$
$(-9+x)(1281-1028x+270x^2-28x^3+x^4), (-11+x)(-7+x)^2(19-12x+x^2),$
$(-11+x)(947-854x+236x^2-26x^3+x^4), (-11+x)(-9+x)(-107+83x-17x^2+x^3),$
$(59-16x+x^2)(-179+127x-21x^2+x^3), (-9+x)(1193-1020x+270x^2-28x^3+x^4),$
$(-11+x)(-7+x)(-125+103x-19x^2+x^3), (-11+x)(-9+x)^2(11-8x+x^2),$
$-9769+10197x-3442x^2+522x^3-37x^4+x^5, (-9+x)(1105-1012x+270x^2-28x^3+x^4),$
$(-11+x)(-9+x)(-7+x)(13-10x+x^2), (-9+x)^2(-113+99x-19x^2+x^3),$
$(-9+x)(1033-1004x+270x^2-28x^3+x^4), (-9+x)^2(-7+x)(15-12x+x^2)\}$

```
list =
```

$\{(-11+x)(-5+x)(-215+131x-21x^2+x^3),$
$(-11+x)(59-16x+x^2)(17-10x+x^2),$
$(-11+x)(1019-862x+236x^2-26x^3+x^4), (-11+x)(-9+x)(-5+x)(23-12x+x^2),$
$(-9+x)(1281-1028x+270x^2-28x^3+x^4), (-11+x)(-7+x)^2(19-12x+x^2),$
$(-11+x)(947-854x+236x^2-26x^3+x^4), (-11+x)(-9+x)(-107+83x-17x^2+x^3),$
$(59-16x+x^2)(-179+127x-21x^2+x^3), (-9+x)(1193-1020x+270x^2-28x^3+x^4),$
$(-11+x)(-7+x)(-125+103x-19x^2+x^3), (-11+x)(-9+x)^2(11-8x+x^2),$
$-9769+10197x-3442x^2+522x^3-37x^4+x^5,$
$(-9+x)(1105-1012x+270x^2-28x^3+x^4),$
$(-11+x)(-9+x)(-7+x)(13-10x+x^2), (-9+x)^2(-113+99x-19x^2+x^3),$
$(-9+x)(1033-1004x+270x^2-28x^3+x^4), (-9+x)^2(-7+x)(15-12x+x^2)\};$

```
warrantpoly =
```
$\{(-11+x)(-5+x)(-215+131x-21x^2+x^3),$
$(-11+x)(-9+x)(-5+x)(23-12x+x^2), (-9+x)(1281-1028x+270x^2-28x^3+x^4)\}$

$\{(-11+x)(-5+x)(-215+131x-21x^2+x^3),$
$(-11+x)(-9+x)(-5+x)(23-12x+x^2), (-9+x)(1281-1028x+270x^2-28x^3+x^4)\}$

```
anglesquared = anglesquaredmat[chi, list] // FullSimplify;
```

anglesquared // MatrixForm

$$
\begin{pmatrix}
\frac{475}{777} & \frac{77+9\sqrt{33}}{4884} & \frac{1}{6} & \frac{4}{21} & \frac{77-9\sqrt{33}}{4884} & 0 \\[4pt]
\frac{943}{1554} & \frac{583+47\sqrt{33}}{9768} & \frac{1}{12} & \frac{4}{21} & \frac{583-47\sqrt{33}}{9768} & 0 \\[4pt]
\frac{472}{777} & \frac{319-5\sqrt{33}}{4884} & \frac{1}{6} & \frac{2}{21} & \frac{319+5\sqrt{33}}{4884} & 0 \\[4pt]
\frac{45}{74} & \frac{693-67\sqrt{33}}{9768} & \frac{1}{4} & 0 & \frac{693+67\sqrt{33}}{9768} & 0 \\[4pt]
\frac{1081}{1776} & \frac{121-7\sqrt{33}}{6512} & \frac{7}{24} & 0 & \frac{121+7\sqrt{33}}{6512} & \frac{1}{16} \\[4pt]
\frac{156}{259} & \frac{253+19\sqrt{33}}{2442} & 0 & \frac{4}{21} & \frac{253-19\sqrt{33}}{2442} & 0 \\[4pt]
\frac{937}{1554} & \frac{1067+19\sqrt{33}}{9768} & \frac{1}{12} & \frac{2}{21} & \frac{1067-19\sqrt{33}}{9768} & 0 \\[4pt]
\frac{67}{111} & \frac{561-19\sqrt{33}}{4884} & \frac{1}{6} & 0 & \frac{561+19\sqrt{33}}{4884} & 0 \\[4pt]
\frac{2501}{4144} & \frac{1111+151\sqrt{33}}{19\,536} & \frac{1}{8} & \frac{2}{21} & \frac{1111-151\sqrt{33}}{19\,536} & \frac{1}{16} \\[4pt]
\frac{29}{48} & \frac{1}{528}\left(33+\sqrt{33}\right) & \frac{5}{24} & 0 & \frac{1}{528}\left(33-\sqrt{33}\right) & \frac{1}{16} \\[4pt]
\frac{155}{259} & \frac{187+6\sqrt{33}}{1221} & 0 & \frac{2}{21} & \frac{187-6\sqrt{33}}{1221} & 0 \\[4pt]
\frac{133}{222} & \frac{517-3\sqrt{33}}{3256} & \frac{1}{12} & 0 & \frac{517+3\sqrt{33}}{3256} & 0 \\[4pt]
\frac{7447}{12\,432} & \frac{179+19\sqrt{33}}{1776} & \frac{1}{24} & \frac{2}{21} & \frac{179-19\sqrt{33}}{1776} & \frac{1}{16} \\[4pt]
\frac{355}{592} & \frac{63}{592}+\frac{95}{592\sqrt{33}} & \frac{1}{8} & 0 & \frac{63}{592}-\frac{95}{592\sqrt{33}} & \frac{1}{16} \\[4pt]
\frac{22}{37} & \frac{5\left(99+\sqrt{33}\right)}{2442} & 0 & 0 & -\frac{5\left(-99+\sqrt{33}\right)}{2442} & 0 \\[4pt]
\frac{1057}{1776} & \frac{979+51\sqrt{33}}{6512} & \frac{1}{24} & 0 & \frac{979-51\sqrt{33}}{6512} & \frac{1}{16} \\[4pt]
\frac{529}{888} & \frac{957+133\sqrt{33}}{9768} & \frac{1}{12} & 0 & \frac{957-133\sqrt{33}}{9768} & \frac{1}{8} \\[4pt]
\frac{175}{296} & \frac{3\left(77+9\sqrt{33}\right)}{1628} & 0 & 0 & -\frac{3\left(-77+9\sqrt{33}\right)}{1628} & \frac{1}{8}
\end{pmatrix}
$$

combinations[chi, (x + 5) (x - 7) (x - 9), warrantpoly〚1〛,
  warrantpoly〚2〛, list, anglesquared] // FullSimplify

$$
\left\{ \sqrt{\frac{1}{33}\left(369+32\sqrt{102}\right)}\,,\ \sqrt{\frac{1}{33}\left(369+32\sqrt{102}\right)}\,,\right.
$$

$$
\left. \text{Root}\left[961-738\,\#1^2+33\,\#1^4\,\&,\ 3\right],\ \text{Root}\left[961-738\,\#1^2+33\,\#1^4\,\&,\ 3\right] \right\}
$$

compatible[combinations[chi, (x + 5) (x - 7) (x - 9),
  warrantpoly〚1〛, warrantpoly〚2〛, list, anglesquared] // FullSimplify]

0

chi = (-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^{25}

$(-9 + x)^8\ (-8 + x)\ (-7 + x)^6\ (-3 + x)\ (5 + x)^{25}$

```
CoefficientList[feasibleinterlacingpolylist[chi], x]
```

{{567, -522, 168, -22, 1}, {511, -514, 168, -22, 1},
 {527, -514, 168, -22, 1}, {455, -506, 168, -22, 1}, {399, -498, 168, -22, 1}}

```
A = {{567, -522, 168, -22, 1}, {511, -514, 168, -22, 1},
    {527, -514, 168, -22, 1}, {455, -506, 168, -22, 1}, {399, -498, 168, -22, 1}};
```

```
A // MatrixForm
```

$$\begin{pmatrix} 567 & -522 & 168 & -22 & 1 \\ 511 & -514 & 168 & -22 & 1 \\ 527 & -514 & 168 & -22 & 1 \\ 455 & -506 & 168 & -22 & 1 \\ 399 & -498 & 168 & -22 & 1 \end{pmatrix}$$

```
g = CoefficientList[D[chi, x] / mu[chi] // Factor, x]
```

{21 135, -21 018, 6888, -902, 41}

```
Solve[n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && Array[n, 5].A == g,
 Array[n, 5], Integers]
```

{{n[1] → 0, n[2] → 0, n[3] → 36, n[4] → 3, n[5] → 2},
 {n[1] → 0, n[2] → 1, n[3] → 36, n[4] → 1, n[5] → 3},
 {n[1] → 1, n[2] → 0, n[3] → 36, n[4] → 0, n[5] → 4}}

```
Array[c, 5].Transpose[A]
```

{567 c[1] - 522 c[2] + 168 c[3] - 22 c[4] + c[5],
 511 c[1] - 514 c[2] + 168 c[3] - 22 c[4] + c[5],
 527 c[1] - 514 c[2] + 168 c[3] - 22 c[4] + c[5],
 455 c[1] - 506 c[2] + 168 c[3] - 22 c[4] + c[5],
 399 c[1] - 498 c[2] + 168 c[3] - 22 c[4] + c[5]}

```
Array[c, 5].g
```

21 135 c[1] - 21 018 c[2] + 6888 c[3] - 902 c[4] + 41 c[5]

```
warrant1 = Flatten[Array[c, 5] /.
    FindInstance[21 135 c[1] - 21 018 c[2] + 6888 c[3] - 902 c[4] + 41 c[5] < 0 &&
      567 c[1] - 522 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      511 c[1] - 514 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      527 c[1] - 514 c[2] + 168 c[3] - 22 c[4] + c[5] < 0 &&
      455 c[1] - 506 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      399 c[1] - 498 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0, Array[c, 5], Integers]]
```

{-86, -608, 0, 0, -267 792}

```
GCD[-86, -608, 0, 0, -267 792]
```

2

```
warrant1 = warrant1 / 2
```

{-43, -304, 0, 0, -133 896}

```
Reverse[warrant1]
```

$\{-133896, 0, 0, -304, -43\}$

```
warrant1.g
```

-9069

```
warrant1.Transpose[A]
```

$\{411, 387, -301, 363, 339\}$

```
warrant2 = Flatten[Array[c, 5] /.
    FindInstance[21135 c[1] - 21018 c[2] + 6888 c[3] - 902 c[4] + 41 c[5] < 0 &&
      567 c[1] - 522 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      511 c[1] - 514 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      527 c[1] - 514 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      455 c[1] - 506 c[2] + 168 c[3] - 22 c[4] + c[5] ≥ 0 &&
      399 c[1] - 498 c[2] + 168 c[3] - 22 c[4] + c[5] < 0, Array[c, 5], Integers]]
```

$\{-1562, -14059, 0, 0, -6402648\}$

```
GCD[-1562, -14059, 0, 0, -6402648]
```

1

```
Reverse[warrant2]
```

$\{-6402648, 0, 0, -14059, -1562\}$

```
warrant2.g
```

-29376

```
warrant2.Transpose[A]
```

$\{50496, 25496, 504, 496, -24504\}$

```
feasibleinterlacingpolylist[chi]
```

$\left\{ (-9+x) (-7+x) (-3+x)^2, \right.$
  $\left(-7+x\right) \left(-73+63 x-15 x^2+x^3\right), \left(31-12 x+x^2\right) \left(17-10 x+x^2\right),$
  $\left. (-7+x) (-5+x) \left(13-10 x+x^2\right), (-7+x) \left(-57+63 x-15 x^2+x^3\right) \right\}$

```
list = {(-9 + x) (-7 + x) (-3 + x)²,
```
    $\left(-7+x\right) \left(-73+63 x-15 x^2+x^3\right), \left(31-12 x+x^2\right) \left(17-10 x+x^2\right),$
    $\left. (-7+x) (-5+x) \left(13-10 x+x^2\right), (-7+x) \left(-57+63 x-15 x^2+x^3\right) \right\};$

```
warrantpoly = {(31 - 12 x + x²) (17 - 10 x + x²), (-7 + x) (-57 + 63 x - 15 x² + x³)}
```

$\left\{ \left(31-12 x+x^2\right) \left(17-10 x+x^2\right), (-7+x) \left(-57+63 x-15 x^2+x^3\right) \right\}$

```
anglesquared = anglesquaredmat[chi, list] // FullSimplify;
```

anglesquared // MatrixForm

$$
\begin{pmatrix}
\frac{8}{13} & 0 & 0 & \frac{5}{13} & 0 \\[4pt]
\frac{111}{182} & \frac{1}{30} & 0 & \frac{17}{65} & \frac{2}{21} \\[4pt]
\frac{667}{1092} & \frac{1}{60} & \frac{1}{6} & \frac{1}{65} & \frac{4}{21} \\[4pt]
\frac{55}{91} & \frac{1}{15} & 0 & \frac{9}{65} & \frac{4}{21} \\[4pt]
\frac{109}{182} & \frac{1}{10} & 0 & \frac{1}{65} & \frac{2}{7}
\end{pmatrix}
$$

combinations[chi, (x + 5) (x – 7) (x – 9), warrantpoly〚1〛,
 warrantpoly〚2〛, list, anglesquared] // FullSimplify

$$
\left\{ \frac{1}{5} \left(-1 + 16 \sqrt{6} \right), \ \frac{1}{5} \left(1 + 16 \sqrt{6} \right) \right\}
$$

compatible[combinations[chi, (x + 5) (x – 7) (x – 9),
 warrantpoly〚1〛, warrantpoly〚2〛, list, anglesquared] // FullSimplify]

0