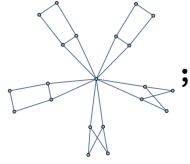


```
In[ ]:= theonlyvalid21 =
```



```
adj21 = AdjacencyMatrix[theonlyvalid21];
```

```
In[ ]:= Length[all2560]
```

```
Out[ ]:= 2560
```

```
In[ ]:= Jacobi[x_, n_, matn_] := (-22 + x) (-2 + x)42 (6 + x)15 (8 + x)2 * Det[
  
$$\frac{x + 4}{(x + 6)(x - 2)} \text{IdentityMatrix}[n] + \frac{\text{matn}}{(x + 6)(x - 2)} + \frac{9 \text{ConstantArray}[1, \{n, n\}]}{(x + 6)(x - 2)(x - 22)} +$$

  
$$\frac{(\text{residuemat} * (x - 14)) \llbracket \text{Range}[n], \text{Range}[n] \rrbracket + \text{RA} \llbracket \text{Range}[n], \text{Range}[n] \rrbracket}{(x + 8)(x + 6)(x - 2)(x - 22)} \Big];$$

```

```
In[ ]:= Jacobi[53, 21, adj21]
```

```
Out[ ]:= 16 235 399 957 685 910 857 160 339 368 675 855 206 433 975 230 193 151 623 129 978 226 875
```

```
In[ ]:= coclique22all = {};
```

```
For[j = 1, j ≤ 2560, j++,
  mat = ConstantArray[0, {22, 22}];
  mat[[Range[21], Range[21]]] = adj21;
  mat[[22, Range[20]]] = all2560[[j]];
  mat[[Range[20], 22]] = all2560[[j]];
  If[IntegerQ[Jacobi[53, 22, mat]], AppendTo[coclique22all, mat]]
] // AbsoluteTiming
coclique22all // Length
```

```
Out[ ]:= {3.10365, Null}
```

```
Out[ ]:= 454
```

```
In[ ]:= coclique22graphs = Array[coclique22g, Length[coclique22all]];
```

```
For[i = 1, i ≤ Length[coclique22all], i++,
  coclique22graphs[[i]] = AdjacencyGraph[coclique22all[[i]]]
]
coclique22 = DeleteDuplicatesBy[coclique22graphs, CanonicalGraph];
Length[coclique22]
```

```
Out[ ]:= 7
```





[illegible]



[illegible]









[illegible]

[illegible]

```
In[ ]:= Length[all454]
```

*Out*[•]= 454

```
In[*]:= coclique22all // Length
```

```
Out[*]:= 454
```

```
In[*]:= coclique23allfrom22all = {};
For[i = 1, i ≤ 454, i++,
  n = 23;
  If[Mod[i, 10] == 0, Print[i]];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = coclique22all[[i]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]]], AppendTo[coclique23allfrom22all, mat]]
  ]
] // AbsoluteTiming
coclique23allfrom22all // Length
```

```
10
```

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20
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```
30
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40
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90
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190
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200
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210
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```

220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
420
430
440
450
Out[ ]:= {300.39, Null}

Out[ ]:= 37 080

In[ ]:= coclique23graphsfrom22all =
  Array[coclique23g22all, Length[coclique23allfrom22all]];
For[i = 1, i ≤ Length[coclique23allfrom22all], i++,
  coclique23graphsfrom22all[[i]] = AdjacencyGraph[coclique23allfrom22all[[i]]]
]
coclique23from22all =
  DeleteDuplicatesBy[coclique23graphsfrom22all, CanonicalGraph];
Length[coclique23from22all]

Out[ ]:= 30

```

```

In[ ]:= coclique23all = {};
For[i = 1, i ≤ 7, i++,
  n = 23;
  Print[i];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique22[[i]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]]], AppendTo[coclique23all, mat]]
  ]
] // AbsoluteTiming
coclique23all // Length

1
2
3
4
5
6
7

Out[ ]:= {4.25689, Null}

Out[ ]:= 1556

In[ ]:= coclique23graphs = Array[coclique23g, Length[coclique23all]];
For[i = 1, i ≤ Length[coclique23all], i++,
  coclique23graphs[[i]] = AdjacencyGraph[coclique23all[[i]]
]
coclique23 = DeleteDuplicatesBy[coclique23graphs, CanonicalGraph];
Length[coclique23]

Out[ ]:= 30

```

```

In[*]:= coclique24all454 = {};
For[i = 1, i ≤ 30, i++,
  n = 24;
  If[Mod[i, 3] == 0, Print[i]];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique23[[i]]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]], AppendTo[coclique24all454, mat]]
  ]
] // AbsoluteTiming
coclique24all454 // Length
3
6
9
12
15
18
21
24
27
30

Out[*]:= {20.0645, Null}

Out[*]:= 2924

In[*]:= coclique24all2560 = {};
For[i = 1, i ≤ 30, i++,
  n = 24;
  If[Mod[i, 3] == 0, Print[i]];
  For[j = 1, j ≤ 2560, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique23[[i]]];
    mat[[n, Range[20]]] = all2560[[j]];
    mat[[Range[20], n]] = all2560[[j]];
    If[IntegerQ[Jacobi[53, n, mat]], AppendTo[coclique24all2560, mat]]
  ]
] // AbsoluteTiming
coclique24all2560 // Length

```

3

6

9

12

15

18

21

24

27

30

```
Out[*]:= {120.922, Null}
```

```
Out[*]:= 3004
```

```
In[*]:= Intersection[coclique24all454, coclique24all2560] // Length
```

```
Out[*]:= 2924
```

```
In[*]:= Complement[coclique24all2560, coclique24all454] // Length
```

```
Out[*]:= 80
```

```
In[*]:= coclique23[[9]] // AdjacencyMatrix // MatrixForm
```

```
Out[*]//MatrixForm=
```

```
( 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
  1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
  0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
  1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
  0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
  0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
  0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
  0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
  0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 1
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 1 1
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1 1 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 1
  0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 1
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
  1 1 0 0 1 1 0 0 1 1 0 0 1 0 1 0 1 0 1 0 0 0 0
  0 0 1 1 0 1 1 0 0 1 1 0 1 0 1 0 1 0 1 0 0 0
  0 0 1 1 1 0 0 1 1 1 0 0 1 0 1 1 0 1 0 0 0 0)
```



```
In[*]:= Complement[coclique24all2560, coclique24all454][[1]] // MatrixForm
```

*Out[•]//MatrixForm=*

0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	
0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	
0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	
0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	
0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	
0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	1	0	
0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1	
0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	
0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0
0	0	1	1	0	1	1	0	0	1	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0
0	0	1	1	0	1	1	0	0	1	0	1	0	0	1	1	1	0	1	0	0	0	0	0	0

[illegible]

```
In[•]:= Range[24]
```

$$Out_{\mathcal{F}}^* = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$$

```

In[*]:= ran = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 24};

In[*]:= Jacobi[53, 21, A[[ran, ran]]]

Out[*]:= 16 235 399 957 685 910 857 160 339 368 675 855 206 433 975 230 193 151 623 129 978 226 875

In[*]:= coclique24graphs454 = Array[coclique24g, Length[coclique24all454]];
For[i = 1, i ≤ Length[coclique24all454], i++,
  coclique24graphs454[[i]] = AdjacencyGraph[coclique24all454[[i]]]
]
coclique24n454 = DeleteDuplicatesBy[coclique24graphs454, CanonicalGraph];
Length[coclique24n454]

Out[*]:= 171

In[*]:= coclique24graphs2560 = Array[coclique24g, Length[coclique24all2560]];
For[i = 1, i ≤ Length[coclique24all2560], i++,
  coclique24graphs2560[[i]] = AdjacencyGraph[coclique24all2560[[i]]]
]
coclique24n2560 = DeleteDuplicatesBy[coclique24graphs2560, CanonicalGraph];
Length[coclique24n2560]

Out[*]:= 187

In[*]:= coclique24refined = {};
For[i = 1, i ≤ Length[coclique24], i++,
  If[Mod[i, 4] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 24, AdjacencyMatrix[coclique24[[i]]]] // Factor, x],
    AppendTo[coclique24refined, coclique24[[i]]]
  ] // AbsoluteTiming
coclique24refined // Length

4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64

```

68

72

76

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84

88

92

96

100

104

108

112

116

120

124

128

132

136

140

144

148

152

156

160

164

168

172

176

180

184

$Out[*]= \{152.077, \text{Null}\}$

$Out[*]= 171$

```

In[*]:= coclique25all = {};
For[i = 1, i ≤ 171, i++,
  n = 25;
  If[Mod[i, 5] == 0, Print[i]];
  For[j = 1, j ≤ 2560, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique24refined[[i]];
    mat[[n, Range[20]]] = all2560[[j]];
    mat[[Range[20], n]] = all2560[[j]];
    If[IntegerQ[Jacobi[53, n, mat]], AppendTo[coclique25all, mat]]
  ]
] // AbsoluteTiming
coclique25all // Length

```

5

10

15

20

25

30

35

40

45

50

55

60

65

70

75

80

85

90

95

100

105

110

115

120

125

130

135

140

145

150

155

160

165

170

$Out[*]= \{890.125, Null\}$

$Out[*]= 6608$

```

In[*]:= coclique25graphs = Array[coclique25g, Length[coclique25all]];
For[i = 1, i ≤ Length[coclique25all], i++,
  coclique25graphs[[i]] = AdjacencyGraph[coclique25all[[i]]]
]
coclique25 = DeleteDuplicatesBy[coclique25graphs, CanonicalGraph];
Length[coclique25]

```

```
Out[*]= 467
```

```

In[*]:= coclique25refined = {};
For[i = 1, i ≤ Length[coclique25], i++,
  If[Mod[i, 10] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 25, AdjacencyMatrix[coclique25[[i]]]] // Factor, x],
    AppendTo[coclique25refined, coclique25[[i]]]
  ] // AbsoluteTiming
coclique25refined // Length

```

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10
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50
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90
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110
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150
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160
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170
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180
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190
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200
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210
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220
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230
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240
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250
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260
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270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
420
430
440
450
460
Out[ ]:= {491.361, Null}

Out[ ]:= 467

In[ ]:= coclique26all = {};
For[i = 1, i ≤ 467, i++,
  n = 26;
  If[Mod[i, 15] == 0, Print[i]];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique25refined[[i]]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]]], AppendTo[coclique26all, mat]]
  ]
] // AbsoluteTiming
coclique26all // Length

```

15

30

45

60

75

90

105

120

135

150

165

180

195

210

225

240

255

270

285

300

315

330

345

360

375

390

405

420

435

450

465

 $Out[f_{**}] = \{461.459, \text{Null}\}$  $Out[f_{**}] = 6706$



```

In[*]:= coclique26graphs = Array[coclique26g, Length[coclique26all]];
For[i = 1, i ≤ Length[coclique26all], i++,
  coclique26graphs[[i]] = AdjacencyGraph[coclique26all[[i]]]
]
coclique26 = DeleteDuplicatesBy[coclique26graphs, CanonicalGraph];
Length[coclique26]

```

```
Out[*]:= 1007
```

```

In[*]:= coclique26refined = {};
For[i = 1, i ≤ Length[coclique26], i++,
  If[Mod[i, 20] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 26, AdjacencyMatrix[coclique26[[i]]]] // Factor, x],
    AppendTo[coclique26refined, coclique26[[i]]]
  ] // AbsoluteTiming
coclique26refined // Length

```

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100

120

140

160

180

200

220

240

260

280

300

320

340

360

380

400

420

440

460

480

500

520

540

560

580

600

620

640

660

680

700

720

740

760

780

800

820

840

860

880

900

920

940

960

980

1000

$Out[*]= \{1312.64, \text{Null}\}$

$Out[*]= 965$

```

In[*]:= coclique27all = {};
For[i = 1, i ≤ 965, i++,
  n = 27;
  If[Mod[i, 20] == 0, Print[i]];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique26refined[[i]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]], AppendTo[coclique27all, mat]]
  ]
] // AbsoluteTiming
coclique27all // Length
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
380
400
420
440
460
480
500
520
540
560

```

580

600

620

640

660

680

700

720

740

760

780

800

820

840

860

880

900

920

940

960

```
Out[8]= {985.47, Null}
```

```
Out[8]= 7140
```

```
In[8]:= coclique27graphs = Array[coclique27g, Length[coclique27all]];
For[i = 1, i ≤ Length[coclique27all], i++,
  coclique27graphs[[i]] = AdjacencyGraph[coclique27all[[i]]]
]
coclique27 = DeleteDuplicatesBy[coclique27graphs, CanonicalGraph];
Length[coclique27]
```

```
Out[8]= 1113
```

```
In[8]:= coclique27refined = {};
For[i = 1, i ≤ Length[coclique27], i++,
  If[Mod[i, 20] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 27, AdjacencyMatrix[coclique27[[i]]]] // Factor, x],
    AppendTo[coclique27refined, coclique27[[i]]]
  ] // AbsoluteTiming
coclique27refined // Length
```

20

40

60

80

100  
120  
140  
160  
180  
200  
220  
240  
260  
280  
300  
320  
340  
360  
380  
400  
420  
440  
460  
480  
500  
520  
540  
560  
580  
600  
620  
640  
660  
680  
700  
720  
740  
760  
780  
800  
820  
840  
860

```

880
900
920
940
960
980
1000
1020
1040
1060
1080
1100
Out[8]= {1721.29, Null}

Out[8]= 879

In[8]:= coclique28all = {};
For[i = 1, i ≤ 879, i++,
  n = 28;
  If[Mod[i, 20] == 0, Print[i]];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique27refined[[i]]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]], AppendTo[coclique28all, mat]]
  ]
] // AbsoluteTiming
coclique28all // Length
20
40
60
80
100
120
140
160
180
200

```

220

240

260

280

300

320

340

360

380

400

420

440

460

480

500

520

540

560

580

600

620

640

660

680

700

720

740

760

780

800

820

840

860

$Out[*]= \{991.886, Null\}$

$Out[*]= 2030$

```

In[*]:= coclique28graphs = Array[coclique28g, Length[coclique28all]];
For[i = 1, i ≤ Length[coclique28all], i++,
  coclique28graphs[[i]] = AdjacencyGraph[coclique28all[[i]]]
]
coclique28 = DeleteDuplicatesBy[coclique28graphs, CanonicalGraph];
Length[coclique28]

```

```
Out[*]= 665
```

```

In[*]:= coclique28refined = {};
For[i = 1, i ≤ Length[coclique28], i++,
  If[Mod[i, 15] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 28, AdjacencyMatrix[coclique28[[i]]]] // Factor, x],
    AppendTo[coclique28refined, coclique28[[i]]]
  ] // AbsoluteTiming
coclique28refined // Length

```

```
15
```

```
30
```

```
45
```

```
60
```

```
75
```

```
90
```

```
105
```

```
120
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135
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150
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165
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180
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195
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210
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225
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240
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255
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270
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285
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300
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315
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330
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345
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360
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375
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390
```



```

405
420
435
450
465
480
495
510
525
540
555
570
585
600
615
630
645
660

Out[*]= {1172.59, Null}

Out[*]= 417

In[*]:= deg10n29 = {};
For[i = 1, i ≤ 417, i++,
  val = AdjacencyMatrix[coclique28refined[[i]].ConstantArray[1, 28];
  For[j = 1, j ≤ 20, j++,
    If[val[[j]] == 10, AppendTo[deg10n29, i];
    Print[i, " ", val]; Break[]]
  ]
]
deg10n29 // Length
43 {4, 8, 4, 8, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}
46 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 2, 10, 6, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}
47 {4, 8, 4, 8, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}
49 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10, 10}
50 {4, 8, 4, 8, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}
53 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 8, 2, 8, 6, 10, 10, 10, 10, 10, 10, 10, 10}
61 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 2, 10, 6, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}
62 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10, 10}
63 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 2, 8, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10, 10}
66 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 2, 10, 2, 10, 10, 10, 10, 10, 10, 10, 10}
125 {4, 8, 4, 8, 6, 6, 6, 6, 6, 4, 10, 4, 6, 6, 6, 6, 6, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}

```

[illegible]

```

273 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 8, 2, 8, 6, 10, 10, 10, 10, 10, 10, 10, 10}
291 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
292 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
293 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
299 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 2, 10, 6, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10, 10}
300
    {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
301 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 2, 8, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
364
    {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 2, 10, 2, 10, 10, 10, 10, 10, 10, 10}
365 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
373 {6, 6, 6, 6, 6, 4, 8, 4, 8, 6, 8, 6, 4, 6, 6, 6, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
377 {6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 6, 4, 6, 8, 6, 8, 4, 8, 4, 10, 10, 10, 10, 10, 10, 10}
378
    {6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 6, 6, 6, 6, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
379 {6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 10, 4, 6, 6, 6, 6, 6, 8, 2, 8, 6, 10, 10, 10, 10, 10, 10, 10}
382 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 6, 4, 4, 6, 8, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
383 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 6, 4, 4, 6, 8, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
384 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 6, 4, 4, 6, 8, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
388 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 6, 4, 6, 6, 6, 6, 6, 2, 10, 6, 10, 10, 10, 10, 10, 10, 10}
389 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8, 6, 4, 4, 6, 8, 6, 6, 4, 10, 4, 10, 10, 10, 10, 10, 10, 10}
395 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
405 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
406 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
416 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}
417 {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 10, 6, 2, 6, 6, 6, 6, 6, 6, 6, 6, 10, 10, 10, 10, 10, 10, 10}

```

Out[\*]= 69

In[\*]:= deg10n29

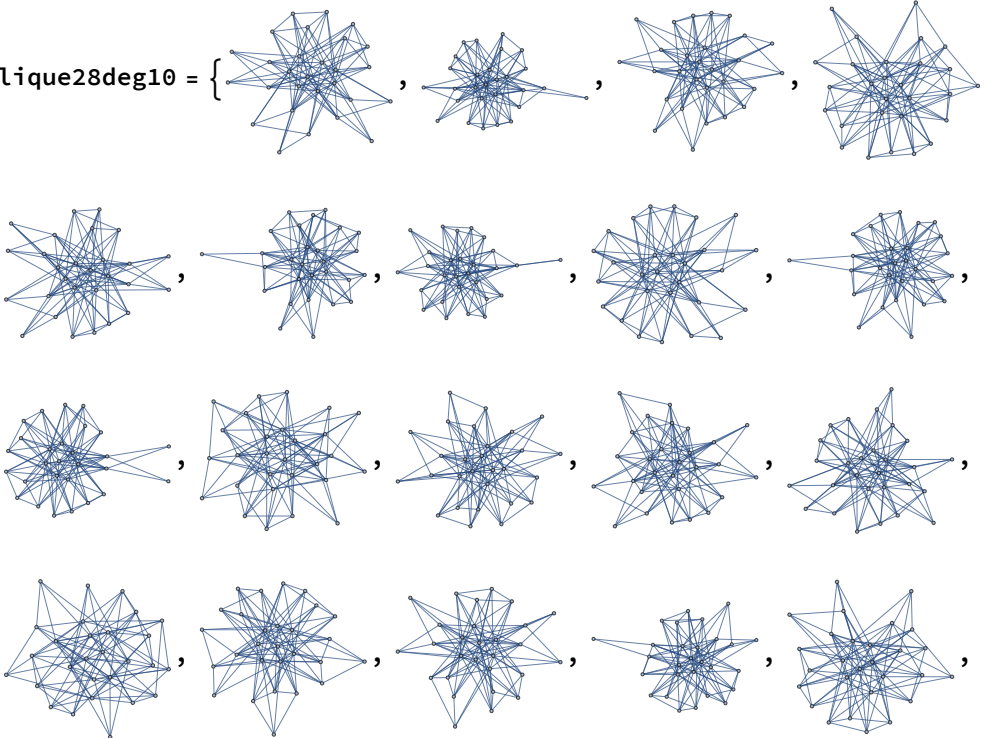
Out[\*]= {43, 46, 47, 49, 50, 53, 61, 62, 63, 66, 125, 139, 141, 152, 153, 154, 155, 156, 158,  
159, 160, 162, 165, 167, 171, 177, 178, 183, 184, 185, 193, 194, 195, 199, 201, 213,  
215, 221, 240, 241, 242, 244, 253, 256, 259, 270, 273, 291, 292, 293, 299, 300, 301,  
364, 365, 373, 377, 378, 379, 382, 383, 384, 388, 389, 395, 405, 406, 416, 417}

```
In[ ]:= coclique28refined[[405]] // AdjacencyMatrix // MatrixForm
```

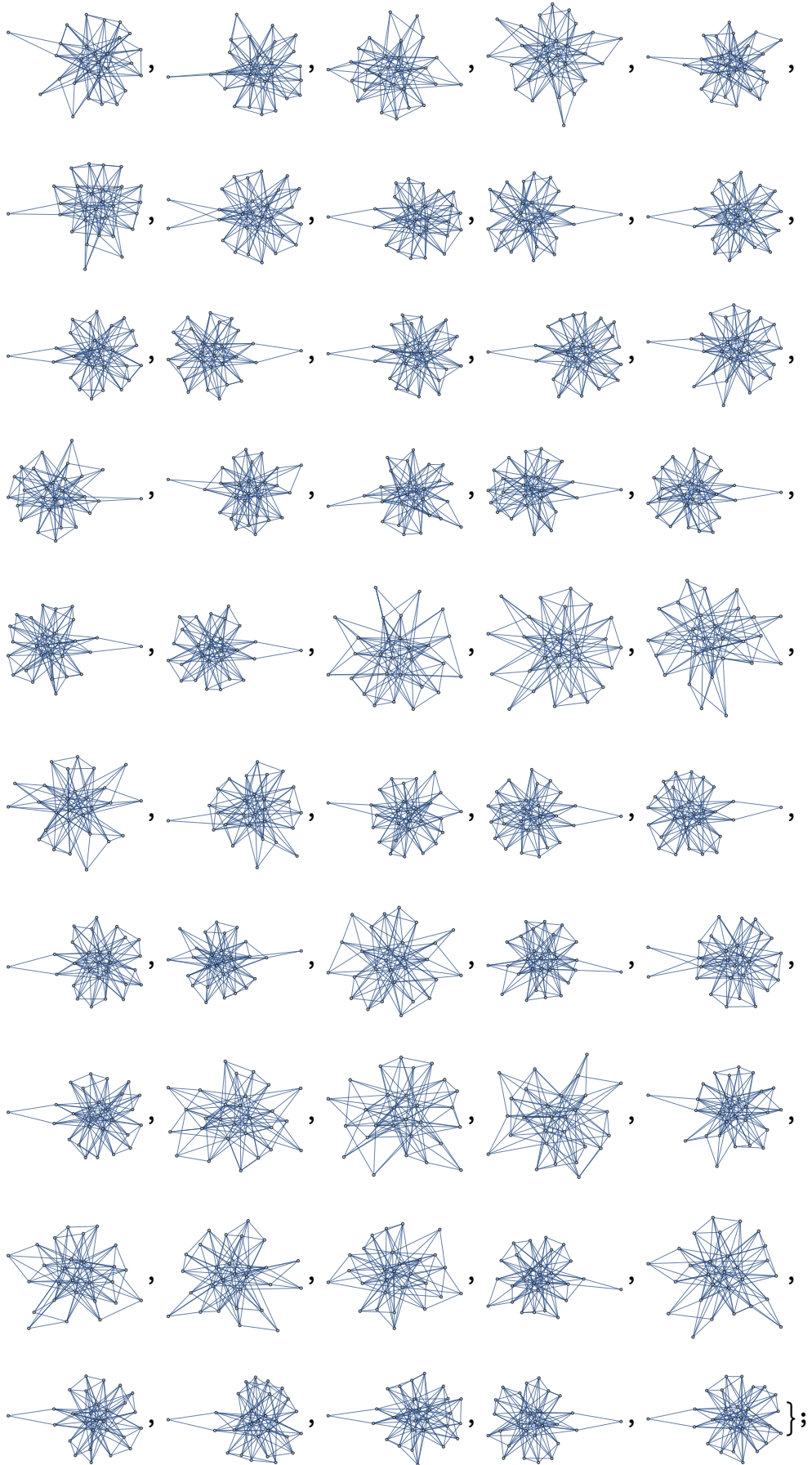
```
Out[ ]:= //MatrixForm=
```

```
(
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 1
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 0 0 1
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 1 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 1 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 1 1
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 1
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 0 1 0 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 1 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 0 0 1 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 0 1 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 1 0 0 1
1 1 0 0 1 1 0 0 1 1 0 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0
0 0 1 1 0 1 1 0 1 1 0 0 0 1 0 1 1 0 1 0 0 0 0 0 0 0 0
0 0 1 1 1 0 0 1 1 1 0 0 1 0 1 0 0 1 0 1 0 0 0 0 0 0 0
0 1 0 1 0 1 0 1 0 1 1 0 0 0 1 1 0 0 1 1 0 0 0 0 0 0 0
0 1 0 1 1 0 1 0 0 1 1 0 0 1 1 0 1 1 0 0 0 0 0 0 0 0 0
1 0 1 0 0 1 0 1 0 1 1 0 1 0 0 1 1 1 0 0 0 0 0 0 0 0 0
1 0 1 0 1 0 1 0 0 1 1 0 1 1 0 0 0 0 1 1 0 0 0 0 0 0 0
1 1 0 0 0 0 1 1 1 1 0 0 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0
)
```

```
In[ ]:= coclique28deg10 = {
```



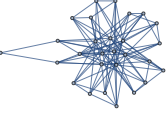
```
,
```



```
In[*]:= AdjacencyMatrix[] // MatrixForm
```

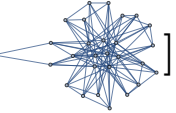
```
Out[*] // MatrixForm =
```

```
( 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 1
  1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 0 0 0
  0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 1 1
  1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0
  0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 1 0
  0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 1
  0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 0 0 1
  0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 0
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 1 0 0 1 1 0
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 1 0 0 1 1
  0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 0 1 1 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 1 1 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 1 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 1 0 0 1 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 0 0 1
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 1 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 1 1 0
  1 1 0 0 1 1 0 0 1 1 0 0 1 0 1 0 1 0 1 0 0 0 0 0 0
  0 0 1 1 0 1 1 0 1 1 0 0 0 1 0 1 1 0 1 0 0 0 0 0
  0 1 0 1 0 1 0 1 0 1 1 0 0 0 1 1 0 0 1 1 0 0 0
  0 1 0 1 1 0 1 0 0 1 1 0 1 1 0 0 0 1 1 0 0 0
  0 1 1 0 0 0 1 1 1 1 0 0 1 0 1 0 0 1 0 1 0 0
  1 0 0 1 1 0 0 1 1 1 0 0 0 1 0 1 0 1 0 1 0 0
  1 0 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 1 0 0 0
  1 0 1 0 1 0 1 0 0 1 1 0 1 0 0 1 1 0 0 0 0)
```

```
In[*]:= FindGraphIsomorphism[, , All]
```

```
Out[*] = { <| 1 → 1, 2 → 2, 3 → 3, 4 → 4, 5 → 5, 6 → 6, 7 → 7, 8 → 8, 9 → 9, 10 → 10, 11 → 11,
  12 → 12, 13 → 13, 14 → 14, 15 → 15, 16 → 16, 17 → 17, 18 → 18, 19 → 19, 20 → 20,
  21 → 21, 22 → 22, 23 → 23, 24 → 24, 25 → 25, 26 → 26, 27 → 27, 28 → 28 |>,
  <| 1 → 5, 2 → 6, 3 → 7, 4 → 8, 5 → 1, 6 → 2, 7 → 3, 8 → 4, 9 → 9, 10 → 10, 11 → 11,
  12 → 12, 13 → 17, 14 → 18, 15 → 19, 16 → 20, 17 → 13, 18 → 14, 19 → 15, 20 → 16,
  21 → 21, 22 → 25, 23 → 23, 24 → 27, 25 → 22, 26 → 26, 27 → 24, 28 → 28 |>,
  <| 1 → 16, 2 → 15, 3 → 14, 4 → 13, 5 → 20, 6 → 19, 7 → 18, 8 → 17, 9 → 11, 10 → 10,
  11 → 9, 12 → 12, 13 → 8, 14 → 7, 15 → 6, 16 → 5, 17 → 4, 18 → 3, 19 → 2, 20 → 1,
  21 → 23, 22 → 24, 23 → 21, 24 → 25, 25 → 27, 26 → 28, 27 → 22, 28 → 26 |>,
  <| 1 → 20, 2 → 19, 3 → 18, 4 → 17, 5 → 16, 6 → 15, 7 → 14, 8 → 13, 9 → 11, 10 → 10,
  11 → 9, 12 → 12, 13 → 4, 14 → 3, 15 → 2, 16 → 1, 17 → 8, 18 → 7, 19 → 6, 20 → 5,
  21 → 23, 22 → 27, 23 → 21, 24 → 22, 25 → 24, 26 → 28, 27 → 25, 28 → 26 |> }
```

```
In[ ]:= GraphAutomorphismGroup[
```



```
Out[ ]:= PermutationGroup[
  {Cycles[{{1, 5}, {2, 6}, {3, 7}, {4, 8}, {13, 17}, {14, 18}, {15, 19}, {16, 20},
    {22, 25}, {24, 27}}], Cycles[{{1, 16, 5, 20}, {2, 15, 6, 19}, {3, 14, 7, 18},
    {4, 13, 8, 17}, {9, 11}, {21, 23}, {22, 24, 25, 27}, {26, 28}}]}]
```

```
In[ ]:= GroupOrder[%6]
```

```
Out[ ]:= 4
```

```
In[ ]:= coclique29all = {};
For[i = 1, i ≤ 417, i++,
  n = 29;
  If[Mod[i, 15] == 0, Print[i]];
  For[j = 1, j ≤ 454, j++,
    mat = ConstantArray[0, {n, n}];
    mat[[Range[n - 1], Range[n - 1]]] = AdjacencyMatrix[coclique28refined[[i]]];
    mat[[n, Range[20]]] = all454[[j]];
    mat[[Range[20], n]] = all454[[j]];
    If[IntegerQ[Jacobi[53, n, mat]], AppendTo[coclique29all, mat]]
  ]
] // AbsoluteTiming
coclique29all // Length
```

```

15
30
45
60
75
90
105
120
135
150
165
180
195
210
225
240
255
270
285
300
315
330
345
360
375
390
405
Out[ ]:= {516.183, Null}

Out[ ]:= 128

In[ ]:= coclique29graphs = Array[coclique29g, Length[coclique29all]];
For[i = 1, i ≤ Length[coclique29all], i++,
  coclique29graphs[[i]] = AdjacencyGraph[coclique29all[[i]]]
]
coclique29 = DeleteDuplicatesBy[coclique29graphs, CanonicalGraph];
Length[coclique29]

Out[ ]:= 56

In[ ]:= AdjacencyMatrix[coclique29[[56]]].ConstantArray[1, 29]
Out[ ]:= {4, 7, 5, 10, 7, 8, 5, 6, 8, 7, 6, 5, 8, 4,
  7, 7, 7, 5, 8, 6, 10, 10, 10, 10, 10, 10, 10, 10}

```



```

In[*]:= Jacobi[53, 29, AdjacencyMatrix[coclique29[[56]]]
        Jacobi[x, 29, AdjacencyMatrix[coclique29[[56]]] // Factor
Out[*]:= 270 260 461 526 483 871 266 743 056 343 762 377 305 573 244 790 078 125

Out[*]:= 
$$\frac{1}{(6+x)^2} (-2+x)^{14} (2+x)^3 (4+x) (2+4x+x^2)^2$$


$$(36\,096 + 139\,200x + 72\,352x^2 - 207\,392x^3 - 336\,000x^4 -$$


$$218\,340x^5 - 74\,364x^6 - 12\,986x^7 - 686x^8 + 128x^9 + 22x^{10} + x^{11})$$


In[*]:= coclique29refined = {};
For[i = 1, i ≤ Length[coclique29], i++,
  If[Mod[i, 4] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 29, AdjacencyMatrix[coclique29[[i]]] // Factor, x],
    AppendTo[coclique29refined, coclique29[[i]]]
  ] // AbsoluteTiming
coclique29refined // Length
4
8
12
16
20
24
28
32
36
40
44
48
52
56

Out[*]:= {113.802, Null}

Out[*]:= 0

In[*]:= Length[coclique29all]
Out[*]:= 128

In[*]:= coclique29nomodiso = {};
For[i = 1, i ≤ Length[coclique29all], i++,
  If[Mod[i, 4] == 0, Print[i]];
  If[PolynomialQ[Jacobi[x, 29, coclique29all[[i]]] // Factor, x],
    AppendTo[coclique29nomodiso, coclique29all[[i]]]
  ] // AbsoluteTiming
coclique29nomodiso // Length

```

4

8

12

16

20

24

28

32

36

40

44

48

52

56

60

64

68

72

76

80

84

88

92

96

100

104

108

112

116

120

124

128

 $Out[*]= \{260.534, \text{Null}\}$  $Out[*]= 0$