

```

In[*]:= tupn9 = Tuples[{0, -1, 1}, 9];
bivareps10 = Array[bve10, Length[tupn9]];
For[i = 1, i ≤ Length[tupn9], i++,
  bivareps10[[i]] =
    conedeletionthreshold[adjsignedthreshold[tupn9[[i]], λ, u, "even"]
] // AbsoluteTiming
Length[tupn9]
Length[DeleteDuplicates[bivareps10 // Factor]]

Out[*]:= {232.083, Null}

Out[*]:= 19 683

Out[*]:= 19 683

```

```

In[*]:= tupn10 = Tuples[{0, -1, 1}, 10];
bivareps11 = Array[bve11, Length[tupn10]];
For[i = 1, i ≤ Length[tupn10], i++,
  If[Mod[i, 10 000] == 0, Print[i]];
  If[Mod[i + 2, 3] == 0, bivareps11[[i]] = Factor[λ bivareps10[[ $\frac{i+2}{3}$ ]]];
  If[Mod[i + 1, 3] == 0, poly = bivareps10[[ $\frac{i+1}{3}$ ]];
  bivareps11[[i]] = Factor[u poly + (λ - u) (poly /. {λ → λ - 1, u → u + 1})];
  If[Mod[i, 3] == 0, poly = bivareps10[[ $\frac{i}{3}$ ]];
  bivareps11[[i]] =
    Factor[u (poly /. {λ → λ - 1, u → u - 1}) + (λ - u) (poly /. {λ → λ - 1, u → u + 1})]
] // AbsoluteTiming

10 000
20 000
30 000
40 000
50 000

Out[*]:= {46.1626, Null}

In[*]:= Length[bivareps11]
Length[DeleteDuplicates[bivareps11]] // AbsoluteTiming

Out[*]:= 59 049

Out[*]:= {0.315306, 59 049}

```

```

In[*]:= ind = RandomInteger[{1, 310}]
tup = tupn10[[ind]]
bivareps11[[ind]]
bivareps = condeletionthreshold[adjsignedthreshold[tup], λ, u, "even"]
Factor[bivareps - bivareps11[[ind]]]

Out[*]:= 58817

Out[*]:= {1, 1, 1, 1, 1, 0, 0, -1, 0, -1}

Out[*]:= (-5 + λ) (-4 + λ) (-3 + λ) (-2 + λ) (-1 + λ)
(330 u + 162 u2 - 330 λ - 751 u λ - 195 u2 λ + 589 λ2 +
590 u λ2 + 64 u2 λ2 - 395 λ3 - 187 u λ3 + 123 λ4 + 18 u λ4 - 18 λ5 + λ6)

Out[*]:= (-5 + λ) (-4 + λ) (-3 + λ) (-2 + λ) (-1 + λ)
(330 u + 162 u2 - 330 λ - 751 u λ - 195 u2 λ + 589 λ2 +
590 u λ2 + 64 u2 λ2 - 395 λ3 - 187 u λ3 + 123 λ4 + 18 u λ4 - 18 λ5 + λ6)

Out[*]:= 0

In[*]:= tupn11 = Tuples[{0, -1, 1}, 11];
bivareps12 = Array[bve12, Length[tupn11]];
For[i = 1, i ≤ Length[tupn11], i++,
If[Mod[i, 10000] == 0, Print[i]];
If[Mod[i + 2, 3] == 0, bivareps12[[i]] = Factor[λ bivareps11[[ $\frac{i+2}{3}$ ]]]];
If[Mod[i + 1, 3] == 0, poly = bivareps11[[ $\frac{i+1}{3}$ ]];
bivareps12[[i]] = Factor[u poly + (λ - u) (poly /. {λ → λ - 1, u → u + 1})]];
If[Mod[i, 3] == 0, poly = bivareps11[[ $\frac{i}{3}$ ]];
bivareps12[[i]] =
Factor[u (poly /. {λ → λ - 1, u → u - 1}) + (λ - u) (poly /. {λ → λ - 1, u → u + 1})]]
] // AbsoluteTiming

```

10 000  
 20 000  
 30 000  
 40 000  
 50 000  
 60 000  
 70 000  
 80 000  
 90 000  
 100 000  
 110 000  
 120 000  
 130 000  
 140 000  
 150 000  
 160 000  
 170 000

Out[ ]:= {237.016, Null}

In[ ]:= Length[bivareps12]  
 Length[DeleteDuplicates[bivareps12]] // AbsoluteTiming

Out[ ]:= 177 147

Out[ ]:= {1.4751, 177 147}

In[ ]:= ind = RandomInteger[{1, 3<sup>11</sup>}]  
 tup = tupn11[[ind]]  
 bivareps12[[ind]]  
 bivareps = conedeletionthreshold[adjsignedthreshold[tup], λ, u, "even"]  
 Factor[bivareps - bivareps12[[ind]]]

Out[ ]:= 24 703

Out[ ]:= {0, -1, 0, 1, 0, 1, -1, 1, 1, 1, 0}

Out[ ]:=  $(-5 + \lambda)^2 (-4 + \lambda) \lambda$   
 $(55\,296\,u + 11\,736\,u^2 - 57\,096\,\lambda - 91\,980\,u\,\lambda - 8016\,u^2\,\lambda + 85\,314\,\lambda^2 + 54\,982\,u\,\lambda^2 +$   
 $2054\,u^2\,\lambda^2 - 52\,653\,\lambda^3 - 16\,560\,u\,\lambda^3 - 234\,u^2\,\lambda^3 + 17\,855\,\lambda^4 +$   
 $2750\,u\,\lambda^4 + 10\,u^2\,\lambda^4 - 3646\,\lambda^5 - 243\,u\,\lambda^5 + 453\,\lambda^6 + 9\,u\,\lambda^6 - 32\,\lambda^7 + \lambda^8)$

Out[ ]:=  $(-5 + \lambda)^2 (-4 + \lambda) \lambda$   
 $(55\,296\,u + 11\,736\,u^2 - 57\,096\,\lambda - 91\,980\,u\,\lambda - 8016\,u^2\,\lambda + 85\,314\,\lambda^2 + 54\,982\,u\,\lambda^2 +$   
 $2054\,u^2\,\lambda^2 - 52\,653\,\lambda^3 - 16\,560\,u\,\lambda^3 - 234\,u^2\,\lambda^3 + 17\,855\,\lambda^4 +$   
 $2750\,u\,\lambda^4 + 10\,u^2\,\lambda^4 - 3646\,\lambda^5 - 243\,u\,\lambda^5 + 453\,\lambda^6 + 9\,u\,\lambda^6 - 32\,\lambda^7 + \lambda^8)$

Out[ ]:= 0

```

In[ ]:= tupn12 = Tuples[{0, -1, 1}, 12];
bivareps13 = Array[bve13, Length[tupn12]];
For[i = 1, i ≤ Length[tupn12], i++,
  If[Mod[i, 10 000] == 0, Print[i]];
  If[Mod[i + 2, 3] == 0, bivareps13[[i]] = Factor[λ bivareps12[[ $\frac{i+2}{3}$ ]]];
  If[Mod[i + 1, 3] == 0, poly = bivareps12[[ $\frac{i+1}{3}$ ]];
  bivareps13[[i]] = Factor[u poly + (λ - u) (poly /. {λ → λ - 1, u → u + 1})];
  If[Mod[i, 3] == 0, poly = bivareps12[[ $\frac{i}{3}$ ]];
  bivareps13[[i]] =
    Factor[u (poly /. {λ → λ - 1, u → u - 1}) + (λ - u) (poly /. {λ → λ - 1, u → u + 1})]
] // AbsoluteTiming
10 000
20 000
30 000
40 000
50 000
60 000
70 000
80 000
90 000
100 000
110 000
120 000
130 000
140 000
150 000
160 000
170 000
180 000
190 000
200 000
210 000

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220 000  
 230 000  
 240 000  
 250 000  
 260 000  
 270 000  
 280 000  
 290 000  
 300 000  
 310 000  
 320 000  
 330 000  
 340 000  
 350 000  
 360 000  
 370 000  
 380 000  
 390 000  
 400 000  
 410 000  
 420 000  
 430 000  
 440 000  
 450 000  
 460 000  
 470 000  
 480 000  
 490 000  
 500 000  
 510 000  
 520 000  
 530 000

`Out[ ]:= {628.623, Null}`

`In[ ]:= Length[bivareps13]`  
`Length[DeleteDuplicates[bivareps13]] // AbsoluteTiming`

`Out[ ]:= 531 441`

`Out[ ]:= {10.9251, 531 441}`

```

In[*]:= ind = RandomInteger[{1, 312}]
tup = tupn12[[ind]]
bivareps13[[ind]]
bivareps = conedletionthreshold[adjsignedthreshold[tup], λ, u, "even"]
Factor[bivareps - bivareps13[[ind]]]

Out[*]= 281424

Out[*]= {-1, -1, 1, 0, 1, 1, 0, 0, -1, 0, 0, 1}

Out[*]= (-4 + λ) (-1 + λ)2
(91194 u + 31860 u2 - 91482 λ - 275573 u λ - 61317 u2 λ + 245345 λ2 + 342209 u λ2 +
47334 u2 λ2 - 284854 λ3 - 230425 u λ3 - 18819 u2 λ3 + 188480 λ4 +
93004 u λ4 + 4080 u2 λ4 - 78697 λ5 - 23309 u λ5 - 459 u2 λ5 + 21636 λ6 +
3585 u λ6 + 21 u2 λ6 - 3944 λ7 - 313 u λ7 + 463 λ8 + 12 u λ8 - 32 λ9 + λ10)

Out[*]= (-4 + λ) (-1 + λ)2
(91194 u + 31860 u2 - 91482 λ - 275573 u λ - 61317 u2 λ + 245345 λ2 + 342209 u λ2 +
47334 u2 λ2 - 284854 λ3 - 230425 u λ3 - 18819 u2 λ3 + 188480 λ4 +
93004 u λ4 + 4080 u2 λ4 - 78697 λ5 - 23309 u λ5 - 459 u2 λ5 + 21636 λ6 +
3585 u λ6 + 21 u2 λ6 - 3944 λ7 - 313 u λ7 + 463 λ8 + 12 u λ8 - 32 λ9 + λ10)

Out[*]= 0

```