In[ ]:=
```
chilist = {(-13 + x)^3 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2)^3,
    (-13 + x)^5 (-11 + x)^11 (5 + x)^42 (131 - 24 x + x^2),
    (-13 + x)^4 (-11 + x)^10 (5 + x)^42 (18 749 - 6576 x + 850 x^2 - 48 x^3 + x^4),
    (-15 + x) (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^42 (139 - 24 x + x^2),
    (-13 + x)^3 (-11 + x)^12 (5 + x)^42 (-2049 + 495 x - 39 x^2 + x^3),
    (-13 + x)^2 (-11 + x)^13 (5 + x)^42 (-2423 + 551 x - 41 x^2 + x^3),
    (-15 + x) (-13 + x) (-11 + x)^14 (5 + x)^42 (191 - 28 x + x^2),
    (-13 + x)^4 (-11 + x)^12 (5 + x)^42 (157 - 26 x + x^2),
    (-13 + x)^2 (-11 + x)^12 (5 + x)^42 (191 - 28 x + x^2) (139 - 24 x + x^2),
    (-15 + x) (-13 + x)^2 (-11 + x)^13 (5 + x)^42 (161 - 26 x + x^2),
    (-13 + x)^4 (-11 + x)^11 (-9 + x) (5 + x)^42 (191 - 28 x + x^2),
    (-13 + x)^3 (-11 + x)^12 (5 + x)^42 (-2033 + 495 x - 39 x^2 + x^3),
    (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (161 - 26 x + x^2) (139 - 24 x + x^2),
    (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^42 (-1763 + 447 x - 37 x^2 + x^3),
    (-15 + x)^2 (-13 + x) (-11 + x)^13 (5 + x)^42 (139 - 24 x + x^2),
    (-13 + x)^4 (-11 + x)^11 (5 + x)^42 (-1711 + 443 x - 37 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^10 (-9 + x) (5 + x)^42 (161 - 26 x + x^2),
    (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (22 259 - 7470 x + 924 x^2 - 50 x^3 + x^4),
    (-13 + x)^3 (-11 + x)^10 (5 + x)^42 (139 - 24 x + x^2) (-1763 + 447 x - 37 x^2 + x^3),
    (-15 + x)^2 (-13 + x)^3 (-11 + x)^12 (-9 + x) (5 + x)^42,
    (-15 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^42 (139 - 24 x + x^2)^2,
    (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^42 (-1763 + 447 x - 37 x^2 + x^3),
    (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (-1477 + 399 x - 35 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^10 (5 + x)^42 (-1433 + 395 x - 35 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^9 (5 + x)^42 (15 779 - 5778 x + 780 x^2 - 46 x^3 + x^4),
    (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^42 (113 - 22 x + x^2),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2) (-1477 + 399 x - 35 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^8 (-9 + x) (5 + x)^42 (139 - 24 x + x^2)^2,
    (-15 + x) (-13 + x)^5 (-11 + x)^10 (5 + x)^42 (95 - 20 x + x^2),
    (-13 + x)^6 (-11 + x)^9 (5 + x)^42 (-1207 + 351 x - 33 x^2 + x^3),
    (-13 + x)^6 (-11 + x)^8 (-9 + x) (5 + x)^42 (-1477 + 399 x - 35 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2) (113 - 22 x + x^2),
    (-13 + x)^7 (-11 + x)^7 (-9 + x)^2 (5 + x)^42 (139 - 24 x + x^2),
    (-13 + x)^6 (-11 + x)^8 (5 + x)^42 (139 - 24 x + x^2) (95 - 20 x + x^2),
    (-13 + x)^7 (-11 + x)^8 (-9 + x) (5 + x)^42 (113 - 22 x + x^2),
    (-13 + x)^6 (-11 + x)^9 (5 + x)^42 (-1191 + 351 x - 33 x^2 + x^3),
    (-13 + x)^7 (-11 + x)^8 (5 + x)^42 (-1009 + 311 x - 31 x^2 + x^3), (-13 + x)^8 (-11 + x)^7
     (-9 + x) (5 + x)^42 (95 - 20 x + x^2), (-13 + x)^8 (-11 + x)^9 (-7 + x) (5 + x)^42};
Length[chilist]
```

Out[ ]= 39

```
In[ ]:= last5 = {(-15 + x)^3 (-11 + x)^15 (5 + x)^42, (-13 + x)^9 (-11 + x)^6 (-9 + x)^3 (5 + x)^42,
    (-17 + x) (-13 + x)^3 (-11 + x)^14 (5 + x)^42, (-15 + x) (-13 + x)^6 (-11 + x)^9
     (-9 + x)^2 (5 + x)^42, (-13 + x)^6 (-11 + x)^10 (5 + x)^42 (109 - 22 x + x^2)};

In[ ]:= interlacingpolylist = Array[intpoly, 39];
Coeffmatlist = Array[Coeff, 39];
glist = Array[g, 39];
certlist = Array[cert, 39];
```

```
i =;
chilist[[i]]
interlacingpolylist[[i]] =;
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];

Coeffmatlist[[i]] // MatrixForm

glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]] // Factor, x]

Array[c,].Transpose[Coeffmatlist[[i]]

Array[c,].glist[[i]

certlist[[i]] = Flatten[Array[c,] /. FindInstance[ < 0 &&, Array[c,], Integers]]

certlist[[i]] = certlist[[i]] / GCD[]

certlist[[i]].glist[[i]
certlist[[i]].Transpose[Coeffmatlist[[i]]

i
chilist[[i]]
Reverse[certlist[[i]]
```

```
In[•]:= i = 1;
        chilist⟦i⟧
        interlacingpolylist⟦i⟧ =
          {(-13 + x)^2 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2)^2 (-1018 + 325 x - 32 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^9 (-6 + x) (5 + x)^41 (139 - 24 x + x^2)^2};
        Coeffmatlist⟦i⟧ =
          CoefficientList[Factor[interlacingpolylist⟦i⟧] / mu[chilist⟦i⟧], x];
Out[•]= (-13 + x)^3 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2)^3

In[•]:= Coeffmatlist⟦i⟧ // MatrixForm
Out[•]//MatrixForm=
        ( 11198  -4593  677  -43  1 )
        ( 11154  -4589  677  -43  1 )

In[•]:= glist⟦i⟧ = CoefficientList[D[chilist⟦i⟧, x] / mu[chilist⟦i⟧] // Factor, x]
Out[•]= {679104, -276132, 40620, -2580, 60}

In[•]:= Array[c, 5].Transpose[Coeffmatlist⟦i⟧]
Out[•]= {11198 c[1] - 4593 c[2] + 677 c[3] - 43 c[4] + c[5],
         11154 c[1] - 4589 c[2] + 677 c[3] - 43 c[4] + c[5]}

In[•]:= Array[c, 5].glist⟦i⟧
Out[•]= 679104 c[1] - 276132 c[2] + 40620 c[3] - 2580 c[4] + 60 c[5]

In[•]:= certlist⟦i⟧ = Flatten[Array[c, 5] /.
          FindInstance[679104 c[1] - 276132 c[2] + 40620 c[3] - 2580 c[4] + 60 c[5] < 0 &&
            11198 c[1] - 4593 c[2] + 677 c[3] - 43 c[4] + c[5] ≥ 0 &&
            11154 c[1] - 4589 c[2] + 677 c[3] - 43 c[4] + c[5] ≥ 0, Array[c, 5], Integers]]
Out[•]= {-169, -415, 0, 0, 0}

In[•]:= certlist⟦i⟧ = certlist⟦i⟧ / GCD[-169, -415, 0, 0, 0]
Out[•]= {-169, -415, 0, 0, 0}

In[•]:= certlist⟦i⟧.glist⟦i⟧
        certlist⟦i⟧.Transpose[Coeffmatlist⟦i⟧]
Out[•]= -173796

Out[•]= {13633, 19409}

In[•]:= i
        chilist⟦i⟧
        Reverse[certlist⟦i⟧]
Out[•]= 1

Out[•]= (-13 + x)^3 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2)^3

Out[•]= {0, 0, 0, -415, -169}
```

In[ ]:=
```
i = 2;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10646 - 4449 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10570 - 4445 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^41 (-1178 + 363 x - 34 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-818 + 279 x - 30 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^12 (5 + x)^41 (86 - 21 x + x^2),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10438 - 4433 x + 669 x^2 - 43 x^3 + x^4),
   (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-942 + 317 x - 32 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10394 - 4429 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
   (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (62 - 17 x + x^2),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10262 - 4417 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-926 + 317 x - 32 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-786 + 279 x - 30 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-914 + 317 x - 32 x^2 + x^3),
   (-14 + x) (-13 + x)^5 (-11 + x)^11 (-5 + x) (5 + x)^41};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
```

Out[ ]=
$$(-13+x)^5\,(-11+x)^{11}\,(5+x)^{42}\left(131-24\,x+x^2\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=
$$\begin{pmatrix}
10646 & -4449 & 669 & -43 & 1\\
10570 & -4445 & 669 & -43 & 1\\
10602 & -4445 & 669 & -43 & 1\\
10634 & -4445 & 669 & -43 & 1\\
10406 & -4433 & 669 & -43 & 1\\
10438 & -4433 & 669 & -43 & 1\\
10470 & -4433 & 669 & -43 & 1\\
10362 & -4429 & 669 & -43 & 1\\
10394 & -4429 & 669 & -43 & 1\\
10426 & -4429 & 669 & -43 & 1\\
10230 & -4417 & 669 & -43 & 1\\
10262 & -4417 & 669 & -43 & 1\\
10186 & -4413 & 669 & -43 & 1\\
10218 & -4413 & 669 & -43 & 1\\
10054 & -4401 & 669 & -43 & 1\\
10010 & -4397 & 669 & -43 & 1
\end{pmatrix}$$

```
In[•]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[•]= {639 936, -267 012, 40 140, -2580, 60}

In[•]:= Array[c, 5].Transpose[Coeffmatlist[[i]]]
Out[•]= {10 646 c[1] - 4449 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 570 c[1] - 4445 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 602 c[1] - 4445 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 634 c[1] - 4445 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 406 c[1] - 4433 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 438 c[1] - 4433 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 470 c[1] - 4433 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 362 c[1] - 4429 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 394 c[1] - 4429 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 426 c[1] - 4429 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 230 c[1] - 4417 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 262 c[1] - 4417 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 186 c[1] - 4413 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 218 c[1] - 4413 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 054 c[1] - 4401 c[2] + 669 c[3] - 43 c[4] + c[5],
  10 010 c[1] - 4397 c[2] + 669 c[3] - 43 c[4] + c[5]}

In[•]:= Array[c, 5].glist[[i]]
Out[•]= 639 936 c[1] - 267 012 c[2] + 40 140 c[3] - 2580 c[4] + 60 c[5]

In[•]:= certlist[[i]] = Flatten[Array[c, 5] /.
    FindInstance[639 936 c[1] - 267 012 c[2] + 40 140 c[3] - 2580 c[4] + 60 c[5] < 0 &&
      10 646 c[1] - 4449 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 570 c[1] - 4445 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 602 c[1] - 4445 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 634 c[1] - 4445 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 406 c[1] - 4433 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 438 c[1] - 4433 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 470 c[1] - 4433 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 362 c[1] - 4429 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 394 c[1] - 4429 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 426 c[1] - 4429 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 230 c[1] - 4417 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 262 c[1] - 4417 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 186 c[1] - 4413 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 218 c[1] - 4413 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 054 c[1] - 4401 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0 &&
      10 010 c[1] - 4397 c[2] + 669 c[3] - 43 c[4] + c[5] ≥ 0, Array[c, 5], Integers]]
Out[•]= {-946, -2265, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[-946, -2265, 0, 0, 0]
Out[•]= {-946, -2265, 0, 0, 0}
```

In[ ]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[ ]= $-597\,276$

Out[ ]= $\{5869, 68\,705, 38\,433, 8161, 196\,669, 166\,397, 136\,125, 229\,233, 198\,961, 168\,689, 326\,925, 296\,653, 359\,489, 329\,217, 457\,181, 489\,745\}$

In[ ]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[ ]= 2

Out[ ]= $(-13+x)^5\,(-11+x)^{11}\,(5+x)^{42}\,\left(131-24\,x+x^2\right)$

Out[ ]= $\{0, 0, 0, -2265, -946\}$

In[•]:=
```
i = 3;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^3 (-11 + x)^10 (-9 + x) (5 + x)^41 (15374 - 5901 x + 805 x^2 - 47 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^41 (-1178 + 363 x - 34 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116590 + 59497 x - 11804 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-137794 + 68387 x - 13142 x^2 + 1228 x^3 - 56 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116942 + 59529 x - 11804 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115946 + 59393 x - 11800 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^3 (-11 + x)^9 (5 + x)^41
    (1506946 - 888023 x + 212793 x^2 - 26646 x^3 + 1844 x^4 - 67 x^5 + x^6),
   (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-994 + 321 x - 32 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115638 + 59321 x - 11796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115990 + 59353 x - 11796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10438 - 4433 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-135662 + 68067 x - 13130 x^2 + 1228 x^3 - 56 x^4 + x^5),
   (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115138 + 59233 x - 11792 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10502 - 4433 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10394 - 4429 x + 669 x^2 - 43 x^3 + x^4),
   (-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9006 - 3931 x + 613 x^2 - 41 x^3 + x^4),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^9 (-10 + x) (5 + x)^41 (-877 + 299 x - 31 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10262 - 4417 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-786 + 279 x - 30 x^2 + x^3)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

Out[•]=
$$(-13+x)^4\,(-11+x)^{10}\,(5+x)^{42}\,\left(18749-6576\,x+850\,x^2-48\,x^3+x^4\right)$$

In[ ]:= **Coeffmatlist[[i]] // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1\,522\,026 & -891\,679 & 213\,089 & -26\,654 & 1844 & -67 & 1 \\
1\,516\,086 & -890\,083 & 212\,949 & -26\,650 & 1844 & -67 & 1 \\
1\,515\,670 & -890\,051 & 212\,949 & -26\,650 & 1844 & -67 & 1 \\
1\,515\,734 & -890\,051 & 212\,949 & -26\,650 & 1844 & -67 & 1 \\
1\,520\,246 & -890\,819 & 212\,981 & -26\,650 & 1844 & -67 & 1 \\
1\,507\,298 & -888\,055 & 212\,793 & -26\,646 & 1844 & -67 & 1 \\
1\,506\,946 & -888\,023 & 212\,793 & -26\,646 & 1844 & -67 & 1 \\
1\,511\,874 & -888\,823 & 212\,825 & -26\,646 & 1844 & -67 & 1 \\
1\,503\,294 & -886\,811 & 212\,669 & -26\,642 & 1844 & -67 & 1 \\
1\,507\,870 & -887\,579 & 212\,701 & -26\,642 & 1844 & -67 & 1 \\
1\,492\,634 & -884\,431 & 212\,497 & -26\,638 & 1844 & -67 & 1 \\
1\,492\,282 & -884\,399 & 212\,497 & -26\,638 & 1844 & -67 & 1 \\
1\,497\,210 & -885\,199 & 212\,529 & -26\,638 & 1844 & -67 & 1 \\
1\,496\,794 & -885\,167 & 212\,529 & -26\,638 & 1844 & -67 & 1 \\
1\,501\,786 & -885\,967 & 212\,561 & -26\,638 & 1844 & -67 & 1 \\
1\,486\,342 & -882\,803 & 212\,357 & -26\,634 & 1844 & -67 & 1 \\
1\,485\,990 & -882\,771 & 212\,357 & -26\,634 & 1844 & -67 & 1 \\
1\,490\,918 & -883\,571 & 212\,389 & -26\,634 & 1844 & -67 & 1 \\
1\,482\,130 & -881\,543 & 212\,233 & -26\,630 & 1844 & -67 & 1 \\
1\,467\,466 & -877\,919 & 211\,937 & -26\,622 & 1844 & -67 & 1 \\
1\,461\,174 & -876\,291 & 211\,797 & -26\,618 & 1844 & -67 & 1
\end{pmatrix}$$

In[ ]:= **glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]**

Out[ ]= {91 593 024, −53 558 972, 12 789 140, −1 599 312, 110 640, −4020, 60}

In[ ]:= **Array[c, 7].Transpose[Coeffmatlist[[i]]]**

Out[ ]= {1 522 026 c[1] − 891 679 c[2] + 213 089 c[3] − 26 654 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 516 086 c[1] − 890 083 c[2] + 212 949 c[3] − 26 650 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 515 670 c[1] − 890 051 c[2] + 212 949 c[3] − 26 650 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 515 734 c[1] − 890 051 c[2] + 212 949 c[3] − 26 650 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 520 246 c[1] − 890 819 c[2] + 212 981 c[3] − 26 650 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 507 298 c[1] − 888 055 c[2] + 212 793 c[3] − 26 646 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 506 946 c[1] − 888 023 c[2] + 212 793 c[3] − 26 646 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 511 874 c[1] − 888 823 c[2] + 212 825 c[3] − 26 646 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 503 294 c[1] − 886 811 c[2] + 212 669 c[3] − 26 642 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 507 870 c[1] − 887 579 c[2] + 212 701 c[3] − 26 642 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 492 634 c[1] − 884 431 c[2] + 212 497 c[3] − 26 638 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 492 282 c[1] − 884 399 c[2] + 212 497 c[3] − 26 638 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 497 210 c[1] − 885 199 c[2] + 212 529 c[3] − 26 638 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 496 794 c[1] − 885 167 c[2] + 212 529 c[3] − 26 638 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 501 786 c[1] − 885 967 c[2] + 212 561 c[3] − 26 638 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 486 342 c[1] − 882 803 c[2] + 212 357 c[3] − 26 634 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 485 990 c[1] − 882 771 c[2] + 212 357 c[3] − 26 634 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 490 918 c[1] − 883 571 c[2] + 212 389 c[3] − 26 634 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 482 130 c[1] − 881 543 c[2] + 212 233 c[3] − 26 630 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 467 466 c[1] − 877 919 c[2] + 211 937 c[3] − 26 622 c[4] + 1844 c[5] − 67 c[6] + c[7],
1 461 174 c[1] − 876 291 c[2] + 211 797 c[3] − 26 618 c[4] + 1844 c[5] − 67 c[6] + c[7]}

```
In[•]:= Array[c, 7].glist[[i]]
Out[•]= 91 593 024 c[1] - 53 558 972 c[2] + 12 789 140 c[3] -
         1 599 312 c[4] + 110 640 c[5] - 4020 c[6] + 60 c[7]

In[•]:= certlist[[i]] = Flatten[
          Array[c, 7] /. FindInstance[91 593 024 c[1] - 53 558 972 c[2] + 12 789 140 c[3] -
                1 599 312 c[4] + 110 640 c[5] - 4020 c[6] + 60 c[7] < 0 &&
              1 522 026 c[1] - 891 679 c[2] + 213 089 c[3] - 26 654 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 516 086 c[1] - 890 083 c[2] + 212 949 c[3] -
                26 650 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 && 1 515 670 c[1] -
                890 051 c[2] + 212 949 c[3] - 26 650 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 515 734 c[1] - 890 051 c[2] + 212 949 c[3] - 26 650 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 520 246 c[1] - 890 819 c[2] + 212 981 c[3] -
                26 650 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 && 1 507 298 c[1] -
                888 055 c[2] + 212 793 c[3] - 26 646 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 506 946 c[1] - 888 023 c[2] + 212 793 c[3] - 26 646 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 511 874 c[1] - 888 823 c[2] + 212 825 c[3] -
                26 646 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 && 1 503 294 c[1] -
                886 811 c[2] + 212 669 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 507 870 c[1] - 887 579 c[2] + 212 701 c[3] - 26 642 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 492 634 c[1] - 884 431 c[2] +
                212 497 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 492 282 c[1] - 884 399 c[2] + 212 497 c[3] - 26 638 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 497 210 c[1] - 885 199 c[2] +
                212 529 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 496 794 c[1] - 885 167 c[2] + 212 529 c[3] - 26 638 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 501 786 c[1] - 885 967 c[2] +
                212 561 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 486 342 c[1] - 882 803 c[2] + 212 357 c[3] - 26 634 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 485 990 c[1] - 882 771 c[2] +
                212 357 c[3] - 26 634 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 490 918 c[1] - 883 571 c[2] + 212 389 c[3] - 26 634 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 482 130 c[1] - 881 543 c[2] +
                212 233 c[3] - 26 630 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 467 466 c[1] - 877 919 c[2] + 211 937 c[3] - 26 622 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 461 174 c[1] - 876 291 c[2] + 211 797 c[3] -
                26 618 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
Out[•]= {-927, -1584, 0, 0, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[-927, -1584, 0, 0, 0, 0, 0]
Out[•]= {-103, -176, 0, 0, 0, 0, 0}
```

In[ ]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[ ]= $-7\,702\,400$

Out[ ]= $\{166\,826, 497\,750, 534\,966, 528\,374, 198\,806, 1\,045\,986, 1\,076\,610, 709\,826, 1\,239\,454, 903\,294, 1\,918\,554, 1\,949\,178, 1\,582\,394, 1\,619\,610, 1\,246\,234, 2\,280\,102, 2\,310\,726, 1\,943\,942, 2\,492\,178, 3\,364\,746, 3\,726\,294\}$

In[ ]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[ ]= 3

Out[ ]= $(-13+x)^4\,(-11+x)^{10}\,(5+x)^{42}\,\left(18\,749-6576\,x+850\,x^2-48\,x^3+x^4\right)$

Out[ ]= $\{0, 0, 0, 0, 0, -176, -103\}$

In[ ]:=
```
i = 4;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^3 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (11 018 - 4529 x + 673 x^2 - 43 x^3 + x^4),
   (-15 + x) (-13 + x)^3 (-11 + x)^9 (-9 + x) (5 + x)^41 (11 302 - 4601 x + 677 x^2 - 43 x^3 + x^4),
   (-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9182 - 3947 x + 613 x^2 - 41 x^3 + x^4),
   (-15 + x) (-13 + x)^4 (-11 + x)^9 (-9 + x) (5 + x)^41 (-866 + 287 x - 30 x^2 + x^3), (-13 + x)^3
    (-11 + x)^9 (5 + x)^41 (1 519 478 - 890 755 x + 212 981 x^2 - 26 650 x^3 + 1844 x^4 - 67 x^5 + x^6),
   (-13 + x)^3 (-11 + x)^9 (5 + x)^41 (1 511 522 - 888 791 x + 212 825 x^2 -
      26 646 x^3 + 1844 x^4 - 67 x^5 + x^6), (-13 + x)^3 (-11 + x)^9 (5 + x)^41
    (1 511 106 - 888 759 x + 212 825 x^2 - 26 646 x^3 + 1844 x^4 - 67 x^5 + x^6), (-13 + x)^3
    (-11 + x)^9 (5 + x)^41 (1 502 942 - 886 779 x + 212 669 x^2 - 26 642 x^3 + 1844 x^4 - 67 x^5 + x^6),
   (-13 + x)^3 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (10 810 - 4513 x + 673 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 958 + 59 353 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (-6 + x) (5 + x)^41 (139 - 24 x + x^2)^2,
   (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 138 + 59 233 x - 11 792 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-136 078 + 68 099 x - 13 130 x^2 + 1228 x^3 - 56 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 490 + 59 265 x - 11 792 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
   (-14 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9679 - 4166 x + 640 x^2 - 42 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (-818 + 283 x - 30 x^2 + x^3)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
```

Out[ ]=
$$(-15+x)\,(-13+x)^4\,(-11+x)^{10}\,(-9+x)\,(5+x)^{42}\,\left(139-24\,x+x^2\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

1 531 502	−893 963	213 261	−26 658	1844	−67	1
1 525 770	−892 383	213 121	−26 654	1844	−67	1
1 515 030	−889 987	212 949	−26 650	1844	−67	1
1 519 830	−890 787	212 981	−26 650	1844	−67	1
1 519 478	−890 755	212 981	−26 650	1844	−67	1
1 511 522	−888 791	212 825	−26 646	1844	−67	1
1 511 106	−888 759	212 825	−26 646	1844	−67	1
1 502 942	−886 779	212 669	−26 642	1844	−67	1
1 502 590	−886 747	212 669	−26 642	1844	−67	1
1 507 454	−887 547	212 701	−26 642	1844	−67	1
1 507 038	−887 515	212 701	−26 642	1844	−67	1
1 497 210	−885 199	212 529	−26 638	1844	−67	1
1 496 794	−885 167	212 529	−26 638	1844	−67	1
1 496 858	−885 167	212 529	−26 638	1844	−67	1
1 501 370	−885 935	212 561	−26 638	1844	−67	1
1 490 918	−883 571	212 389	−26 634	1844	−67	1
1 490 566	−883 539	212 389	−26 634	1844	−67	1
1 478 126	−880 299	212 109	−26 626	1844	−67	1

In[ ]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[ ]=
```
{91 675 200, -53 576 764, 12 790 036, -1 599 312, 110 640, -4020, 60}
```

In[ ]:=
```
Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

Out[ ]=
```
{1 531 502 c[1] - 893 963 c[2] + 213 261 c[3] - 26 658 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 525 770 c[1] - 892 383 c[2] + 213 121 c[3] - 26 654 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 515 030 c[1] - 889 987 c[2] + 212 949 c[3] - 26 650 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 519 830 c[1] - 890 787 c[2] + 212 981 c[3] - 26 650 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 519 478 c[1] - 890 755 c[2] + 212 981 c[3] - 26 650 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 511 522 c[1] - 888 791 c[2] + 212 825 c[3] - 26 646 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 511 106 c[1] - 888 759 c[2] + 212 825 c[3] - 26 646 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 502 942 c[1] - 886 779 c[2] + 212 669 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 502 590 c[1] - 886 747 c[2] + 212 669 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 507 454 c[1] - 887 547 c[2] + 212 701 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 507 038 c[1] - 887 515 c[2] + 212 701 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 497 210 c[1] - 885 199 c[2] + 212 529 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 496 794 c[1] - 885 167 c[2] + 212 529 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 496 858 c[1] - 885 167 c[2] + 212 529 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 501 370 c[1] - 885 935 c[2] + 212 561 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 490 918 c[1] - 883 571 c[2] + 212 389 c[3] - 26 634 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 490 566 c[1] - 883 539 c[2] + 212 389 c[3] - 26 634 c[4] + 1844 c[5] - 67 c[6] + c[7],
 1 478 126 c[1] - 880 299 c[2] + 212 109 c[3] - 26 626 c[4] + 1844 c[5] - 67 c[6] + c[7]}
```

In[ ]:=
```
Array[c, 7].glist[[i]]
```

Out[ ]=
```
91 675 200 c[1] - 53 576 764 c[2] + 12 790 036 c[3] -
 1 599 312 c[4] + 110 640 c[5] - 4020 c[6] + 60 c[7]
```

```
In[•]:= certlist[[i]] = Flatten[
          Array[c, 7] /. FindInstance[91 675 200 c[1] - 53 576 764 c[2] + 12 790 036 c[3] -
                1 599 312 c[4] + 110 640 c[5] - 4020 c[6] + 60 c[7] < 0 &&
              1 531 502 c[1] - 893 963 c[2] + 213 261 c[3] - 26 658 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 525 770 c[1] - 892 383 c[2] + 213 121 c[3] -
                26 654 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 && 1 515 030 c[1] -
                889 987 c[2] + 212 949 c[3] - 26 650 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 519 830 c[1] - 890 787 c[2] + 212 981 c[3] - 26 650 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 519 478 c[1] - 890 755 c[2] + 212 981 c[3] -
                26 650 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 && 1 511 522 c[1] -
                888 791 c[2] + 212 825 c[3] - 26 646 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 511 106 c[1] - 888 759 c[2] + 212 825 c[3] - 26 646 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 502 942 c[1] - 886 779 c[2] +
                212 669 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 502 590 c[1] - 886 747 c[2] + 212 669 c[3] - 26 642 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 507 454 c[1] - 887 547 c[2] +
                212 701 c[3] - 26 642 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 507 038 c[1] - 887 515 c[2] + 212 701 c[3] - 26 642 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 497 210 c[1] - 885 199 c[2] +
                212 529 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 496 794 c[1] - 885 167 c[2] + 212 529 c[3] - 26 638 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 496 858 c[1] - 885 167 c[2] +
                212 529 c[3] - 26 638 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 501 370 c[1] - 885 935 c[2] + 212 561 c[3] - 26 638 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 490 918 c[1] - 883 571 c[2] +
                212 389 c[3] - 26 634 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0 &&
              1 490 566 c[1] - 883 539 c[2] + 212 389 c[3] - 26 634 c[4] + 1844 c[5] -
                67 c[6] + c[7] ≥ 0 && 1 478 126 c[1] - 880 299 c[2] + 212 109 c[3] -
                26 626 c[4] + 1844 c[5] - 67 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]

Out[•]= {-43 657, -207 778, -557 458, 0, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[-43 657, -207 778, -557 458, 0, 0, 0, 0]

Out[•]= {-43 657, -207 778, -557 458, 0, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[•]= -99 224 496

Out[•]= {1 010 862, 1 007 666, 67 930 534, 6 760 678, 15 479 046, 41 701 594,
         53 214 010, 85 192 766, 93 911 134, 29 947 230, 41 459 646, 85 189 570,
         96 701 986, 93 907 938, 38 662 402, 99 660 950, 108 379 318, 134 359 918}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 4
```

$$\text{Out[ ]= } (-15+x)\,(-13+x)^4\,(-11+x)^{10}\,(-9+x)\,(5+x)^{42}\left(139-24\,x+x^2\right)$$

```
Out[ ]= {0, 0, 0, 0, -557 458, -207 778, -43 657}
```

```
In[ ]:= i = 5;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^2 (-11 + x)^13 (5 + x)^41 (-1382 + 405 x - 36 x^2 + x^3),
            (-13 + x)^3 (-11 + x)^13 (5 + x)^41 (106 - 23 x + x^2),
            (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-166 094 + 79 217 x - 14 640 x^2 + 1318 x^3 - 58 x^4 + x^5),
            (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 806 - 5111 x + 733 x^2 - 45 x^3 + x^4),
            (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (14 994 - 5821 x + 801 x^2 - 47 x^3 + x^4),
            (-13 + x)^3 (-11 + x)^12 (5 + x)^41 (-1150 + 359 x - 34 x^2 + x^3),
            (-13 + x)^3 (-11 + x)^12 (5 + x)^41 (-1134 + 359 x - 34 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
```

$$\text{Out[ ]= } (-13+x)^3\,(-11+x)^{12}\,(5+x)^{42}\left(-2049+495\,x-39\,x^2+x^3\right)$$

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

$$\begin{pmatrix} -167\,222 & 79\,409 & -14\,648 & 1318 & -58 & 1 \\ -166\,738 & 79\,321 & -14\,644 & 1318 & -58 & 1 \\ -166\,094 & 79\,217 & -14\,640 & 1318 & -58 & 1 \\ -166\,478 & 79\,249 & -14\,640 & 1318 & -58 & 1 \\ -164\,934 & 79\,025 & -14\,632 & 1318 & -58 & 1 \\ -164\,450 & 78\,937 & -14\,628 & 1318 & -58 & 1 \\ -162\,162 & 78\,553 & -14\,612 & 1318 & -58 & 1 \end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]] // Factor, x]
```

```
Out[ ]= {-10 016 064, 4 759 788, -878 616, 79 080, -3480, 60}
```

```
In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= {-167 222 c[1] + 79 409 c[2] - 14 648 c[3] + 1318 c[4] - 58 c[5] + c[6],
         -166 738 c[1] + 79 321 c[2] - 14 644 c[3] + 1318 c[4] - 58 c[5] + c[6],
         -166 094 c[1] + 79 217 c[2] - 14 640 c[3] + 1318 c[4] - 58 c[5] + c[6],
         -166 478 c[1] + 79 249 c[2] - 14 640 c[3] + 1318 c[4] - 58 c[5] + c[6],
         -164 934 c[1] + 79 025 c[2] - 14 632 c[3] + 1318 c[4] - 58 c[5] + c[6],
         -164 450 c[1] + 78 937 c[2] - 14 628 c[3] + 1318 c[4] - 58 c[5] + c[6],
         -162 162 c[1] + 78 553 c[2] - 14 612 c[3] + 1318 c[4] - 58 c[5] + c[6]}
```

```
In[•]:= Array[c, 6].glist[[i]]
```

```
Out[•]= -10 016 064 c[1] + 4 759 788 c[2] - 878 616 c[3] + 79 080 c[4] - 3480 c[5] + 60 c[6]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -10 016 064 c[1] + 4 759 788 c[2] - 878 616 c[3] + 79 080 c[4] - 3480 c[5] + 60 c[6] <
            0 && -167 222 c[1] + 79 409 c[2] - 14 648 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -166 738 c[1] + 79 321 c[2] - 14 644 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -166 094 c[1] + 79 217 c[2] - 14 640 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -166 478 c[1] + 79 249 c[2] - 14 640 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -164 934 c[1] + 79 025 c[2] - 14 632 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -164 450 c[1] + 78 937 c[2] - 14 628 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -162 162 c[1] + 78 553 c[2] - 14 612 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]
```

```
Out[•]= {3624, 20 286, 68 588, 0, 0, 0}
```

```
In[•]:= certlist[[i]] = certlist[[i]] / GCD[3624, 20 286, 68 588, 0, 0, 0]
```

```
Out[•]= {1812, 10 143, 34 294, 0, 0, 0}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -1 835 388
```

```
Out[•]= {100 711, 222 311, 471 543, 100 311, 900 359, 1 021 959, 1 821 607}
```

```
In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[•]= 5
```

```
Out[•]= (-13 + x)^3 (-11 + x)^12 (5 + x)^42 (-2049 + 495 x - 39 x^2 + x^3)
```

```
Out[•]= {0, 0, 0, 34 294, 10 143, 1812}
```

```
In[•]:= i = 6;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^2 (-11 + x)^13 (5 + x)^41 (-1366 + 405 x - 36 x^2 + x^3),
            (-13 + x)^2 (-11 + x)^13 (5 + x)^41 (-1362 + 405 x - 36 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
```

```
Out[•]= (-13 + x)^2 (-11 + x)^13 (5 + x)^42 (-2423 + 551 x - 41 x^2 + x^3)
```

```
In[•]:= Coeffmatlist[[i]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -195\,338 & 90\,699 & -16\,234 & 1412 & -60 & 1 \\ -194\,766 & 90\,603 & -16\,230 & 1412 & -60 & 1 \end{pmatrix}$$

```
In[•]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[•]= {-11 844 608, 5 460 596, -974 736, 84 720, -3600, 60}

In[•]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[•]= {-195 338 c[1] + 90 699 c[2] - 16 234 c[3] + 1412 c[4] - 60 c[5] + c[6],
          -194 766 c[1] + 90 603 c[2] - 16 230 c[3] + 1412 c[4] - 60 c[5] + c[6]}

In[•]:= Array[c, 6].glist[[i]]
Out[•]= -11 844 608 c[1] + 5 460 596 c[2] - 974 736 c[3] + 84 720 c[4] - 3600 c[5] + 60 c[6]

In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -11 844 608 c[1] + 5 460 596 c[2] - 974 736 c[3] + 84 720 c[4] - 3600 c[5] + 60 c[6] <
            0 && -195 338 c[1] + 90 699 c[2] - 16 234 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0 &&
           -194 766 c[1] + 90 603 c[2] - 16 230 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]
Out[•]= {35, -1854, -29 631, 0, 0, -305 901 333}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[35, -1854, -29 631, 0, 0, -305 901 333]
Out[•]= {35, -1854, -29 631, 0, 0, -305 901 333}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[•]= -10 183 828

Out[•]= {135 545, 215 025}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[•]= 6

Out[•]= (-13 + x)^2 (-11 + x)^13 (5 + x)^42 (-2423 + 551 x - 41 x^2 + x^3)

Out[•]= {-305 901 333, 0, 0, -29 631, -1854, 35}
```

```
In[ ]:= i = 7;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-15 + x) (-13 + x) (-11 + x)^13 (5 + x)^41 (-1202 + 363 x - 34 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
Out[ ]= (-15 + x) (-13 + x) (-11 + x)^14 (5 + x)^42 (191 - 28 x + x^2)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        (-234 390  104 441  -17 996  1510  -62  1)

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-14 005 600, 6 258 604, -1 079 496, 90 600, -3720, 60}

In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[ ]= {-234 390 c[1] + 104 441 c[2] - 17 996 c[3] + 1510 c[4] - 62 c[5] + c[6]}

In[ ]:= Array[c, 6].glist[[i]]
Out[ ]= -14 005 600 c[1] + 6 258 604 c[2] - 1 079 496 c[3] + 90 600 c[4] - 3720 c[5] + 60 c[6]

In[ ]:= certlist[[i]] =
         Flatten[Array[c, 6] /. FindInstance[-14 005 600 c[1] + 6 258 604 c[2] - 1 079 496 c[3] +
                 90 600 c[4] - 3720 c[5] + 60 c[6] < 0 && -234 390 c[1] + 104 441 c[2] -
                 17 996 c[3] + 1510 c[4] - 62 c[5] + c[6] ≥ 0, Array[c, 6], Integers]]
Out[ ]= {0, 9, -81 026, 0, 0, -1 458 935 482}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[0, 9, -81 026, 0, 0, -1 458 935 482]
Out[ ]= {0, 9, -81 026, 0, 0, -1 458 935 482}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[ ]= -12 558 588

Out[ ]= {148 383}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[ ]= 7

Out[ ]= (-15 + x) (-13 + x) (-11 + x)^14 (5 + x)^42 (191 - 28 x + x^2)

Out[ ]= {-1 458 935 482, 0, 0, -81 026, 9, 0}
```

```
In[ ]:= i = 8;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^3 (-11 + x)^12 (5 + x)^41 (-1150 + 359 x - 34 x^2 + x^3),
            (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 682 - 5099 x + 733 x^2 - 45 x^3 + x^4),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-978 + 317 x - 32 x^2 + x^3),
            (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 630 - 5095 x + 733 x^2 - 45 x^3 + x^4),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-974 + 317 x - 32 x^2 + x^3),
            (-13 + x)^3 (-11 + x)^12 (5 + x)^41 (-1134 + 359 x - 34 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-962 + 317 x - 32 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-958 + 317 x - 32 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^12 (5 + x)^41 (86 - 21 x + x^2)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^4 (-11 + x)^12 (5 + x)^42 (157 - 26 x + x^2)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( 12 650  -5099  733  -45  1 )
        ( 12 682  -5099  733  -45  1 )
        ( 12 714  -5099  733  -45  1 )
        ( 12 630  -5095  733  -45  1 )
        ( 12 662  -5095  733  -45  1 )
        ( 12 474  -5083  733  -45  1 )
        ( 12 506  -5083  733  -45  1 )
        ( 12 454  -5079  733  -45  1 )
        ( 12 298  -5067  733  -45  1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {767 352, -306 420, 43 980, -2700, 60}

In[ ]:= Array[c, 5].Transpose[Coeffmatlist[[i]]]
Out[ ]= {12 650 c[1] - 5099 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 682 c[1] - 5099 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 714 c[1] - 5099 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 630 c[1] - 5095 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 662 c[1] - 5095 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 474 c[1] - 5083 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 506 c[1] - 5083 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 454 c[1] - 5079 c[2] + 733 c[3] - 45 c[4] + c[5],
         12 298 c[1] - 5067 c[2] + 733 c[3] - 45 c[4] + c[5]}

In[ ]:= Array[c, 5].glist[[i]]
Out[ ]= 767 352 c[1] - 306 420 c[2] + 43 980 c[3] - 2700 c[4] + 60 c[5]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 5] /.
          FindInstance[767 352 c[1] - 306 420 c[2] + 43 980 c[3] - 2700 c[4] + 60 c[5] < 0 &&
            12 650 c[1] - 5099 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 682 c[1] - 5099 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 714 c[1] - 5099 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 630 c[1] - 5095 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 662 c[1] - 5095 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 474 c[1] - 5083 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 506 c[1] - 5083 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 454 c[1] - 5079 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0 &&
            12 298 c[1] - 5067 c[2] + 733 c[3] - 45 c[4] + c[5] ≥ 0, Array[c, 5], Integers]]
Out[•]= {-338, -843, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[-338, -843, 0, 0, 0]
Out[•]= {-338, -843, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[•]= -1 052 916

Out[•]= {22 757, 11 941, 1125, 26 145, 15 329, 68 757, 57 941, 72 145, 114 757}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[•]= 8

Out[•]= (-13 + x)^4 (-11 + x)^12 (5 + x)^42 (157 - 26 x + x^2)

Out[•]= {0, 0, 0, -843, -338}
```

In[ ]:=
```
i = 9;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-166 798 + 79 281 x - 14 640 x^2 + 1318 x^3 - 58 x^4 + x^5),
   (-13 + x)^2 (-11 + x)^11 (-10 + x) (5 + x)^41 (191 - 28 x + x^2) (87 - 20 x + x^2),
   (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-166 138 + 79 177 x - 14 636 x^2 + 1318 x^3 - 58 x^4 + x^5),
   (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (139 - 24 x + x^2) (-1198 + 363 x - 34 x^2 + x^3),
   (-13 + x) (-11 + x)^12 (5 + x)^41 (-195 306 + 90 699 x - 16 234 x^2 + 1412 x^3 - 60 x^4 + x^5),
   (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-165 670 + 79 089 x - 14 632 x^2 + 1318 x^3 - 58 x^4 + x^5),
   (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (15 058 - 5821 x + 801 x^2 - 47 x^3 + x^4),
   (-13 + x)^2 (-11 + x)^13 (5 + x)^41 (-1362 + 405 x - 36 x^2 + x^3),
   (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (161 - 26 x + x^2) (-1026 + 325 x - 32 x^2 + x^3),
   (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-164 158 + 78 865 x - 14 624 x^2 + 1318 x^3 - 58 x^4 + x^5),
   (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-163 850 + 78 793 x - 14 620 x^2 + 1318 x^3 - 58 x^4 + x^5),
   (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (14 818 - 5805 x + 801 x^2 - 47 x^3 + x^4),
   (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (178 - 27 x + x^2) (83 - 20 x + x^2)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

Out[ ]=
$$(-13+x)^2\,(-11+x)^{12}\,(5+x)^{42}\,\left(191-28\,x+x^2\right)\,\left(139-24\,x+x^2\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
2\,168\,374 & -1\,197\,451 & 269\,601 & -31\,774 & 2072 & -71 & 1\\
2\,160\,210 & -1\,195\,471 & 269\,445 & -31\,770 & 2072 & -71 & 1\\
2\,159\,794 & -1\,195\,439 & 269\,445 & -31\,770 & 2072 & -71 & 1\\
2\,164\,786 & -1\,196\,239 & 269\,477 & -31\,770 & 2072 & -71 & 1\\
2\,148\,366 & -1\,192\,995 & 269\,273 & -31\,766 & 2072 & -71 & 1\\
2\,153\,710 & -1\,193\,827 & 269\,305 & -31\,766 & 2072 & -71 & 1\\
2\,153\,294 & -1\,193\,795 & 269\,305 & -31\,766 & 2072 & -71 & 1\\
2\,142\,426 & -1\,191\,399 & 269\,133 & -31\,762 & 2072 & -71 & 1\\
2\,147\,418 & -1\,192\,199 & 269\,165 & -31\,762 & 2072 & -71 & 1\\
2\,134\,054 & -1\,189\,403 & 268\,977 & -31\,758 & 2072 & -71 & 1\\
2\,130\,050 & -1\,188\,159 & 268\,853 & -31\,754 & 2072 & -71 & 1\\
2\,118\,974 & -1\,185\,747 & 268\,681 & -31\,750 & 2072 & -71 & 1\\
2\,112\,682 & -1\,184\,119 & 268\,541 & -31\,746 & 2072 & -71 & 1
\end{pmatrix}$$

In[ ]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[ ]=
$$\{129\,764\,344,\ -71\,753\,988,\ 16\,167\,788,\ -1\,906\,200,\ 124\,320,\ -4260,\ 60\}$$

```
In[ ]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= {2 168 374 c[1] - 1 197 451 c[2] + 269 601 c[3] - 31 774 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 160 210 c[1] - 1 195 471 c[2] + 269 445 c[3] - 31 770 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 159 794 c[1] - 1 195 439 c[2] + 269 445 c[3] - 31 770 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 164 786 c[1] - 1 196 239 c[2] + 269 477 c[3] - 31 770 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 148 366 c[1] - 1 192 995 c[2] + 269 273 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 153 710 c[1] - 1 193 827 c[2] + 269 305 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 153 294 c[1] - 1 193 795 c[2] + 269 305 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 142 426 c[1] - 1 191 399 c[2] + 269 133 c[3] - 31 762 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 147 418 c[1] - 1 192 199 c[2] + 269 165 c[3] - 31 762 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 134 054 c[1] - 1 189 403 c[2] + 268 977 c[3] - 31 758 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 130 050 c[1] - 1 188 159 c[2] + 268 853 c[3] - 31 754 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 118 974 c[1] - 1 185 747 c[2] + 268 681 c[3] - 31 750 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 112 682 c[1] - 1 184 119 c[2] + 268 541 c[3] - 31 746 c[4] + 2072 c[5] - 71 c[6] + c[7]}
```

```
In[ ]:= Array[c, 7].glist[[i]]
```

```
Out[ ]= 129 764 344 c[1] - 71 753 988 c[2] + 16 167 788 c[3] -
 1 906 200 c[4] + 124 320 c[5] - 4260 c[6] + 60 c[7]
```

```
In[ ]:= certlist[[i]] = Flatten[
  Array[c, 7] /. FindInstance[129 764 344 c[1] - 71 753 988 c[2] + 16 167 788 c[3] -
      1 906 200 c[4] + 124 320 c[5] - 4260 c[6] + 60 c[7] < 0 &&
    2 168 374 c[1] - 1 197 451 c[2] + 269 601 c[3] - 31 774 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 160 210 c[1] - 1 195 471 c[2] + 269 445 c[3] -
      31 770 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 && 2 159 794 c[1] -
      1 195 439 c[2] + 269 445 c[3] - 31 770 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 164 786 c[1] - 1 196 239 c[2] + 269 477 c[3] - 31 770 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 148 366 c[1] - 1 192 995 c[2] +
      269 273 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 153 710 c[1] - 1 193 827 c[2] + 269 305 c[3] - 31 766 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 153 294 c[1] - 1 193 795 c[2] +
      269 305 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 142 426 c[1] - 1 191 399 c[2] + 269 133 c[3] - 31 762 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 147 418 c[1] - 1 192 199 c[2] +
      269 165 c[3] - 31 762 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 134 054 c[1] - 1 189 403 c[2] + 268 977 c[3] - 31 758 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 130 050 c[1] - 1 188 159 c[2] +
      268 853 c[3] - 31 754 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 118 974 c[1] - 1 185 747 c[2] + 268 681 c[3] - 31 750 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 112 682 c[1] - 1 184 119 c[2] + 268 541 c[3] -
      31 746 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
```

```
Out[ ]= {-235 607, -2 928 267, -36 586 502, -458 256 882, 0, 0, -7 692 466 694 472}
```

```
In[ ]:= certlist[[i]] = certlist[[i]] /
  GCD[-235 607, -2 928 267, -36 586 502, -458 256 882, 0, 0, -7 692 466 694 472]
```

```
Out[ ]= {-235 607, -2 928 267, -36 586 502, -458 256 882, 0, 0, -7 692 466 694 472}
```

In[•]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[•]= $-93\,815\,508$

Out[•]= $\{2\,102\,893, 2\,096\,565, 6\,404\,533, 2\,099\,925, 2\,087\,597, 8\,553\,869, 12\,861\,837, 17\,161\,797, 12\,857\,189, 19\,309\,453, 23\,614\,453, 30\,068\,397, 34\,371\,717\}$

In[•]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[•]= 9

Out[•]= $(-13+x)^2\,(-11+x)^{12}\,(5+x)^{42}\,\left(191-28\,x+x^2\right)\,\left(139-24\,x+x^2\right)$

Out[•]= $\{-7\,692\,466\,694\,472, 0, 0, -458\,256\,882, -36\,586\,502, -2\,928\,267, -235\,607\}$

In[•]:=
```
i = 10;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-15 + x) (-13 + x) (-11 + x)^13 (5 + x)^41 (-1186 + 363 x - 34 x^2 + x^3),
   (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (-1006 + 321 x - 32 x^2 + x^3),
   (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (15 014 - 5817 x + 801 x^2 - 47 x^3 + x^4),
   (-15 + x) (-13 + x) (-11 + x)^12 (5 + x)^41 (13 010 - 5175 x + 737 x^2 - 45 x^3 + x^4),
   (-15 + x)^2 (-13 + x)^2 (-11 + x)^13 (-6 + x) (5 + x)^41};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
```

Out[•]= $(-15+x)\,(-13+x)^2\,(-11+x)^{13}\,(5+x)^{42}\,\left(161-26\,x+x^2\right)$

In[•]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -195\,690 & 90\,731 & -16\,234 & 1412 & -60 & 1 \\ -196\,170 & 90\,763 & -16\,234 & 1412 & -60 & 1 \\ -195\,182 & 90\,635 & -16\,230 & 1412 & -60 & 1 \\ -195\,150 & 90\,635 & -16\,230 & 1412 & -60 & 1 \\ -193\,050 & 90\,315 & -16\,218 & 1412 & -60 & 1 \end{pmatrix}$$

In[•]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[•]= $\{-11\,804\,200, 5\,451\,604, -974\,280, 84\,720, -3600, 60\}$

```
In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= {-195 690 c[1] + 90 731 c[2] - 16 234 c[3] + 1412 c[4] - 60 c[5] + c[6],
         -196 170 c[1] + 90 763 c[2] - 16 234 c[3] + 1412 c[4] - 60 c[5] + c[6],
         -195 182 c[1] + 90 635 c[2] - 16 230 c[3] + 1412 c[4] - 60 c[5] + c[6],
         -195 150 c[1] + 90 635 c[2] - 16 230 c[3] + 1412 c[4] - 60 c[5] + c[6],
         -193 050 c[1] + 90 315 c[2] - 16 218 c[3] + 1412 c[4] - 60 c[5] + c[6]}
```

```
In[ ]:= Array[c, 6].glist[[i]]
```

```
Out[ ]= -11 804 200 c[1] + 5 451 604 c[2] - 974 280 c[3] + 84 720 c[4] - 3600 c[5] + 60 c[6]
```

```
In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -11 804 200 c[1] + 5 451 604 c[2] - 974 280 c[3] + 84 720 c[4] - 3600 c[5] + 60 c[6] <
            0 && -195 690 c[1] + 90 731 c[2] - 16 234 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0 &&
           -196 170 c[1] + 90 763 c[2] - 16 234 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0 &&
           -195 182 c[1] + 90 635 c[2] - 16 230 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0 &&
           -195 150 c[1] + 90 635 c[2] - 16 230 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0 &&
           -193 050 c[1] + 90 315 c[2] - 16 218 c[3] + 1412 c[4] - 60 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]
```

```
Out[ ]= {855, 1850, 0, 0, 0, 0}
```

```
In[ ]:= certlist[[i]] = certlist[[i]] / GCD[855, 1850, 0, 0, 0, 0]
```

```
Out[ ]= {171, 370, 0, 0, 0, 0}
```

```
In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= -1 424 720
```

```
Out[ ]= {107 480, 37 240, 158 828, 164 300, 405 000}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 10
```

```
Out[ ]= (-15 + x) (-13 + x)^2 (-11 + x)^13 (5 + x)^42 (161 - 26 x + x^2)
```

```
Out[ ]= {0, 0, 0, 0, 370, 171}
```

```
In[ ]:= i = 11;
chilist[[i]]
interlacingpolylist[[i]] = {(-13 + x)^4 (-11 + x)^12 (5 + x)^41 (86 - 21 x + x^2),
    (-13 + x)^4 (-11 + x)^11 (-9 + x) (5 + x)^41 (110 - 23 x + x^2),
    (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 682 - 5099 x + 733 x^2 - 45 x^3 + x^4),
    (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-139 886 + 68 803 x - 13 162 x^2 + 1228 x^3 - 56 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-978 + 317 x - 32 x^2 + x^3),
    (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 630 - 5095 x + 733 x^2 - 45 x^3 + x^4),
    (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-139 314 + 68 707 x - 13 158 x^2 + 1228 x^3 - 56 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-974 + 317 x - 32 x^2 + x^3),
    (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^41 (-1194 + 363 x - 34 x^2 + x^3),
    (-13 + x)^3 (-11 + x)^12 (5 + x)^41 (-1134 + 359 x - 34 x^2 + x^3),
    (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-962 + 317 x - 32 x^2 + x^3),
    (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 422 - 5079 x + 733 x^2 - 45 x^3 + x^4),
    (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-958 + 317 x - 32 x^2 + x^3),
    (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-942 + 317 x - 32 x^2 + x^3),
    (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (62 - 17 x + x^2)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

```
Out[ ]= (-13 + x)^4 (-11 + x)^11 (-9 + x) (5 + x)^42 (191 - 28 x + x^2)
```

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -135\,278 & 68\,035 & -13\,130 & 1228 & -56 & 1 \\ -141\,570 & 69\,091 & -13\,174 & 1228 & -56 & 1 \\ -139\,502 & 68\,771 & -13\,162 & 1228 & -56 & 1 \\ -139\,886 & 68\,803 & -13\,162 & 1228 & -56 & 1 \\ -139\,854 & 68\,803 & -13\,162 & 1228 & -56 & 1 \\ -138\,930 & 68\,675 & -13\,158 & 1228 & -56 & 1 \\ -139\,314 & 68\,707 & -13\,158 & 1228 & -56 & 1 \\ -139\,282 & 68\,707 & -13\,158 & 1228 & -56 & 1 \\ -139\,698 & 68\,739 & -13\,158 & 1228 & -56 & 1 \\ -137\,214 & 68\,387 & -13\,146 & 1228 & -56 & 1 \\ -137\,566 & 68\,419 & -13\,146 & 1228 & -56 & 1 \\ -136\,642 & 68\,291 & -13\,142 & 1228 & -56 & 1 \\ -136\,994 & 68\,323 & -13\,142 & 1228 & -56 & 1 \\ -134\,706 & 67\,939 & -13\,126 & 1228 & -56 & 1 \\ -132\,990 & 67\,651 & -13\,114 & 1228 & -56 & 1 \end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

```
Out[ ]= {-8 400 304, 4 129 012, -789 744, 73 680, -3360, 60}
```

In[ ]:=
```
Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

Out[ ]=
```
{-135 278 c[1] + 68 035 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -141 570 c[1] + 69 091 c[2] - 13 174 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 502 c[1] + 68 771 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 886 c[1] + 68 803 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 854 c[1] + 68 803 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -138 930 c[1] + 68 675 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 314 c[1] + 68 707 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 282 c[1] + 68 707 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 698 c[1] + 68 739 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 214 c[1] + 68 387 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 566 c[1] + 68 419 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -136 642 c[1] + 68 291 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -136 994 c[1] + 68 323 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -134 706 c[1] + 67 939 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -132 990 c[1] + 67 651 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6]}
```

In[ ]:=
```
Array[c, 6].glist[[i]]
```

Out[ ]=
```
-8 400 304 c[1] + 4 129 012 c[2] - 789 744 c[3] + 73 680 c[4] - 3360 c[5] + 60 c[6]
```

In[ ]:=
```
certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
    -8 400 304 c[1] + 4 129 012 c[2] - 789 744 c[3] + 73 680 c[4] - 3360 c[5] + 60 c[6] < 0 &&
     -135 278 c[1] + 68 035 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -141 570 c[1] + 69 091 c[2] - 13 174 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -139 502 c[1] + 68 771 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -139 886 c[1] + 68 803 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -139 854 c[1] + 68 803 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -138 930 c[1] + 68 675 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -139 314 c[1] + 68 707 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -139 282 c[1] + 68 707 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -139 698 c[1] + 68 739 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -137 214 c[1] + 68 387 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -137 566 c[1] + 68 419 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -136 642 c[1] + 68 291 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -136 994 c[1] + 68 323 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -134 706 c[1] + 67 939 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
     -132 990 c[1] + 67 651 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0,
    Array[c, 6], Integers]]
```

Out[ ]=
```
{16 971, 198 001, 2 370 351, 0, 0, 19 949 599 256}
```

In[ ]:=
```
certlist[[i]] = certlist[[i]] / GCD[16 971, 198 001, 2 370 351, 0, 0, 19 949 599 256]
```

Out[ ]=
```
{16 971, 198 001, 2 370 351, 0, 0, 19 949 599 256}
```

In[•]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[•]= $-7\,578\,956$

Out[•]= $\{2\,085\,723, 97\,803, 277\,723, 96\,891, 639\,963, 458\,443, 277\,611, 820\,683, 96\,779, 1\,000\,603, 1\,362\,843, 1\,181\,323, 1\,543\,563, 2\,266\,443, 2\,808\,603\}$

In[•]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[•]= 11

Out[•]= $(-13+x)^4\,(-11+x)^{11}\,(-9+x)\,(5+x)^{42}\,\left(191-28\,x+x^2\right)$

Out[•]= $\{19\,949\,599\,256, 0, 0, 2\,370\,351, 198\,001, 16\,971\}$

In[•]:=
```
i = 12;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13+x)^2 (-11+x)^11 (5+x)^41 (-166150+79121 x-14632 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^4 (-11+x)^11 (5+x)^41 (-978+317 x-32 x^2+x^3),
   (-13+x)^2 (-11+x)^11 (5+x)^41 (-165250+79001 x-14628 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^4 (-11+x)^11 (5+x)^41 (-974+317 x-32 x^2+x^3),
   (-13+x)^3 (-11+x)^11 (5+x)^41 (12694-5095 x+733 x^2-45 x^3+x^4),
   (-13+x)^2 (-11+x)^11 (5+x)^41 (-163498+78761 x-14620 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^2 (-11+x)^11 (5+x)^41 (-163914+78793 x-14620 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^2 (-11+x)^11 (5+x)^41 (-163030+78673 x-14616 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^2 (-11+x)^11 (5+x)^41 (-163446+78705 x-14616 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^3 (-11+x)^12 (5+x)^41 (-1134+359 x-34 x^2+x^3),
   (-13+x)^4 (-11+x)^11 (5+x)^41 (-962+317 x-32 x^2+x^3),
   (-13+x)^3 (-11+x)^11 (5+x)^41 (12538-5083 x+733 x^2-45 x^3+x^4),
   (-13+x)^4 (-11+x)^11 (5+x)^41 (-958+317 x-32 x^2+x^3),
   (-13+x)^3 (-11+x)^11 (5+x)^41 (12486-5079 x+733 x^2-45 x^3+x^4),
   (-13+x)^2 (-11+x)^11 (5+x)^41 (-161210+78377 x-14604 x^2+1318 x^3-58 x^4+x^5),
   (-13+x)^4 (-11+x)^12 (5+x)^41 (86-21 x+x^2)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]]/mu[chilist[[i]]], x];
```

Out[•]= $(-13+x)^3\,(-11+x)^{12}\,(5+x)^{42}\,\left(-2033+495\,x-39\,x^2+x^3\right)$

In[ ]:= **Coeffmatlist[[i]] // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} -166\,150 & 79\,121 & -14\,632 & 1318 & -58 & 1 \\ -165\,282 & 79\,001 & -14\,628 & 1318 & -58 & 1 \\ -165\,250 & 79\,001 & -14\,628 & 1318 & -58 & 1 \\ -164\,606 & 78\,897 & -14\,624 & 1318 & -58 & 1 \\ -165\,022 & 78\,929 & -14\,624 & 1318 & -58 & 1 \\ -163\,498 & 78\,761 & -14\,620 & 1318 & -58 & 1 \\ -163\,914 & 78\,793 & -14\,620 & 1318 & -58 & 1 \\ -163\,030 & 78\,673 & -14\,616 & 1318 & -58 & 1 \\ -163\,446 & 78\,705 & -14\,616 & 1318 & -58 & 1 \\ -162\,162 & 78\,553 & -14\,612 & 1318 & -58 & 1 \\ -162\,578 & 78\,585 & -14\,612 & 1318 & -58 & 1 \\ -162\,994 & 78\,617 & -14\,612 & 1318 & -58 & 1 \\ -161\,902 & 78\,481 & -14\,608 & 1318 & -58 & 1 \\ -162\,318 & 78\,513 & -14\,608 & 1318 & -58 & 1 \\ -161\,210 & 78\,377 & -14\,604 & 1318 & -58 & 1 \\ -159\,874 & 78\,169 & -14\,596 & 1318 & -58 & 1 \end{pmatrix}$$

In[ ]:= **glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]] // Factor, x]**

Out[ ]= {−9 935 088, 4 741 836, −877 704, 79 080, −3480, 60}

In[ ]:= **Array[c, 6].Transpose[Coeffmatlist[[i]]]**

Out[ ]= {−166 150 c[1] + 79 121 c[2] − 14 632 c[3] + 1318 c[4] − 58 c[5] + c[6],
−165 282 c[1] + 79 001 c[2] − 14 628 c[3] + 1318 c[4] − 58 c[5] + c[6],
−165 250 c[1] + 79 001 c[2] − 14 628 c[3] + 1318 c[4] − 58 c[5] + c[6],
−164 606 c[1] + 78 897 c[2] − 14 624 c[3] + 1318 c[4] − 58 c[5] + c[6],
−165 022 c[1] + 78 929 c[2] − 14 624 c[3] + 1318 c[4] − 58 c[5] + c[6],
−163 498 c[1] + 78 761 c[2] − 14 620 c[3] + 1318 c[4] − 58 c[5] + c[6],
−163 914 c[1] + 78 793 c[2] − 14 620 c[3] + 1318 c[4] − 58 c[5] + c[6],
−163 030 c[1] + 78 673 c[2] − 14 616 c[3] + 1318 c[4] − 58 c[5] + c[6],
−163 446 c[1] + 78 705 c[2] − 14 616 c[3] + 1318 c[4] − 58 c[5] + c[6],
−162 162 c[1] + 78 553 c[2] − 14 612 c[3] + 1318 c[4] − 58 c[5] + c[6],
−162 578 c[1] + 78 585 c[2] − 14 612 c[3] + 1318 c[4] − 58 c[5] + c[6],
−162 994 c[1] + 78 617 c[2] − 14 612 c[3] + 1318 c[4] − 58 c[5] + c[6],
−161 902 c[1] + 78 481 c[2] − 14 608 c[3] + 1318 c[4] − 58 c[5] + c[6],
−162 318 c[1] + 78 513 c[2] − 14 608 c[3] + 1318 c[4] − 58 c[5] + c[6],
−161 210 c[1] + 78 377 c[2] − 14 604 c[3] + 1318 c[4] − 58 c[5] + c[6],
−159 874 c[1] + 78 169 c[2] − 14 596 c[3] + 1318 c[4] − 58 c[5] + c[6]}

In[ ]:= **Array[c, 6].glist[[i]]**

Out[ ]= −9 935 088 c[1] + 4 741 836 c[2] − 877 704 c[3] + 79 080 c[4] − 3480 c[5] + 60 c[6]

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -9 935 088 c[1] + 4 741 836 c[2] - 877 704 c[3] + 79 080 c[4] - 3480 c[5] + 60 c[6] < 0 &&
           -166 150 c[1] + 79 121 c[2] - 14 632 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -165 282 c[1] + 79 001 c[2] - 14 628 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -165 250 c[1] + 79 001 c[2] - 14 628 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -164 606 c[1] + 78 897 c[2] - 14 624 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -165 022 c[1] + 78 929 c[2] - 14 624 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -163 498 c[1] + 78 761 c[2] - 14 620 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -163 914 c[1] + 78 793 c[2] - 14 620 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -163 030 c[1] + 78 673 c[2] - 14 616 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -163 446 c[1] + 78 705 c[2] - 14 616 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -162 162 c[1] + 78 553 c[2] - 14 612 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -162 578 c[1] + 78 585 c[2] - 14 612 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -162 994 c[1] + 78 617 c[2] - 14 612 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -161 902 c[1] + 78 481 c[2] - 14 608 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -162 318 c[1] + 78 513 c[2] - 14 608 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -161 210 c[1] + 78 377 c[2] - 14 604 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0 &&
           -159 874 c[1] + 78 169 c[2] - 14 596 c[3] + 1318 c[4] - 58 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]

Out[•]= {-162 889, -2 280 433, -31 763 185, 0, 0, -311 392 576 602}

In[•]:= certlist[[i]] =
         certlist[[i]] / GCD[-162 889, -2 280 433, -31 763 185, 0, 0, -311 392 576 602]

Out[•]= {-162 889, -2 280 433, -31 763 185, 0, 0, -311 392 576 602}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[•]= -2 814 636

Out[•]= {214 275, 5 425 843, 213 395, 5 425 171, 213 139, 8 030 307, 2 818 275, 5 423 619,
         211 587, 10 635 187, 5 423 155, 211 123, 5 422 483, 210 451, 2 815 587, 5 420 467}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[•]= 12

Out[•]= (-13 + x)^3 (-11 + x)^12 (5 + x)^42 (-2033 + 495 x - 39 x^2 + x^3)

Out[•]= {-311 392 576 602, 0, 0, -31 763 185, -2 280 433, -162 889}
```

```
In[•]:= i = 13;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^2 (-11 + x)^11 (5 + x)^41 (161 - 26 x + x^2) (-1026 + 325 x - 32 x^2 + x^3),
           (-14 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (161 - 26 x + x^2) (73 - 18 x + x^2),
           (-13 + x)^2 (-11 + x)^10 (5 + x)^41
            (1814474 - 1033177 x + 239793 x^2 - 29122 x^3 + 1956 x^4 - 69 x^5 + x^6),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (139 - 24 x + x^2) (-1182 + 363 x - 34 x^2 + x^3),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-163446 + 78705 x - 14616 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12538 - 5083 x + 733 x^2 - 45 x^3 + x^4),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-162962 + 78617 x - 14612 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12486 - 5079 x + 733 x^2 - 45 x^3 + x^4)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
Out[•]= (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (161 - 26 x + x^2) (139 - 24 x + x^2)

In[•]:= Coeffmatlist[[i]] // MatrixForm
Out[•]//MatrixForm=
        ( 1817046  -1034197  239909  -29126  1956  -69  1 )
        ( 1809962  -1032409  239761  -29122  1956  -69  1 )
        ( 1814474  -1033177  239793  -29122  1956  -69  1 )
        ( 1807278  -1031373  239645  -29118  1956  -69  1 )
        ( 1797906  -1029201  239481  -29114  1956  -69  1 )
        ( 1792934  -1027781  239349  -29110  1956  -69  1 )
        ( 1792582  -1027749  239349  -29110  1956  -69  1 )
        ( 1785498  -1025961  239201  -29106  1956  -69  1 )

In[•]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[•]= {109367984, -62113108, 14397476, -1747584, 117360, -4140, 60}

In[•]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
Out[•]= {1817046 c[1] - 1034197 c[2] + 239909 c[3] - 29126 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1809962 c[1] - 1032409 c[2] + 239761 c[3] - 29122 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1814474 c[1] - 1033177 c[2] + 239793 c[3] - 29122 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1807278 c[1] - 1031373 c[2] + 239645 c[3] - 29118 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1797906 c[1] - 1029201 c[2] + 239481 c[3] - 29114 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1792934 c[1] - 1027781 c[2] + 239349 c[3] - 29110 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1792582 c[1] - 1027749 c[2] + 239349 c[3] - 29110 c[4] + 1956 c[5] - 69 c[6] + c[7],
         1785498 c[1] - 1025961 c[2] + 239201 c[3] - 29106 c[4] + 1956 c[5] - 69 c[6] + c[7]}

In[•]:= Array[c, 7].glist[[i]]
Out[•]= 109367984 c[1] - 62113108 c[2] + 14397476 c[3] -
         1747584 c[4] + 117360 c[5] - 4140 c[6] + 60 c[7]
```

```
In[ ]:= certlist[[i]] = Flatten[
          Array[c, 7] /. FindInstance[109 367 984 c[1] - 62 113 108 c[2] + 14 397 476 c[3] -
                1 747 584 c[4] + 117 360 c[5] - 4140 c[6] + 60 c[7] < 0 &&
              1 817 046 c[1] - 1 034 197 c[2] + 239 909 c[3] - 29 126 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 809 962 c[1] - 1 032 409 c[2] + 239 761 c[3] -
                29 122 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 && 1 814 474 c[1] -
                1 033 177 c[2] + 239 793 c[3] - 29 122 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 807 278 c[1] - 1 031 373 c[2] + 239 645 c[3] - 29 118 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 797 906 c[1] - 1 029 201 c[2] + 239 481 c[3] -
                29 114 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 && 1 792 934 c[1] -
                1 027 781 c[2] + 239 349 c[3] - 29 110 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 792 582 c[1] - 1 027 749 c[2] + 239 349 c[3] - 29 110 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 785 498 c[1] - 1 025 961 c[2] + 239 201 c[3] -
                29 106 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]

Out[ ]= {-790, -1389, 0, 0, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-790, -1389, 0, 0, 0, 0, 0]

Out[ ]= {-790, -1389, 0, 0, 0, 0, 0}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[ ]= -125 600 348

Out[ ]= {1 033 293, 4 146 121, 1 648 393, 4 827 477, 9 214 449, 11 169 949, 11 403 581, 14 516 409}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[ ]= 13

Out[ ]= (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (161 - 26 x + x^2) (139 - 24 x + x^2)

Out[ ]= {0, 0, 0, 0, 0, -1389, -790}
```

```
In[ ]:= i = 14;
chilist[[i]]
interlacingpolylist[[i]] = {(-15 + x) (-13 + x) (-11 + x)^11
     (5 + x)^41 (-143 890 + 70 047 x - 13 286 x^2 + 1232 x^3 - 56 x^4 + x^5),
    (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (-1006 + 321 x - 32 x^2 + x^3),
    (-15 + x) (-13 + x) (-11 + x)^12 (5 + x)^41 (13 010 - 5175 x + 737 x^2 - 45 x^3 + x^4),
    (-15 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (11 038 - 4533 x + 673 x^2 - 43 x^3 + x^4),
    (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-164 926 + 78 929 x - 14 624 x^2 + 1318 x^3 - 58 x^4 + x^5),
    (-13 + x) (-11 + x)^11 (5 + x)^41
     (2 129 698 - 1 188 127 x + 268 853 x^2 - 31 754 x^3 + 2072 x^4 - 71 x^5 + x^6),
    (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-164 266 + 78 825 x - 14 620 x^2 + 1318 x^3 - 58 x^4 + x^5),
    (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-163 382 + 78 705 x - 14 616 x^2 + 1318 x^3 - 58 x^4 + x^5),
    (-15 + x)^2 (-13 + x)^2 (-11 + x)^13 (-6 + x) (5 + x)^41,
    (-15 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (10 862 - 4517 x + 673 x^2 - 43 x^3 + x^4),
    (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (-974 + 321 x - 32 x^2 + x^3)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

```
Out[ ]= (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^42 (-1763 + 447 x - 37 x^2 + x^3)
```

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

```
Out[ ]//MatrixForm=
(2 158 350  -1 194 595  269 337  -31 766  2072  -71  1
 2 157 870  -1 194 563  269 337  -31 766  2072  -71  1
 2 146 650  -1 192 135  269 165  -31 762  2072  -71  1
 2 152 410  -1 192 999  269 197  -31 762  2072  -71  1
 2 144 038  -1 191 003  269 041  -31 758  2072  -71  1
 2 129 698  -1 188 127  268 853  -31 754  2072  -71  1
 2 135 458  -1 188 991  268 885  -31 754  2072  -71  1
 2 123 966  -1 186 547  268 713  -31 750  2072  -71  1
 2 123 550  -1 186 515  268 713  -31 750  2072  -71  1
 2 118 090  -1 184 951  268 573  -31 746  2072  -71  1
 2 089 230  -1 178 467  268 089  -31 734  2072  -71  1)
```

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

```
Out[ ]= {129 238 000, -71 596 812, 16 152 884, -1 905 744, 124 320, -4260, 60}
```

```
In[•]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= {2 158 350 c[1] - 1 194 595 c[2] + 269 337 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 157 870 c[1] - 1 194 563 c[2] + 269 337 c[3] - 31 766 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 146 650 c[1] - 1 192 135 c[2] + 269 165 c[3] - 31 762 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 152 410 c[1] - 1 192 999 c[2] + 269 197 c[3] - 31 762 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 144 038 c[1] - 1 191 003 c[2] + 269 041 c[3] - 31 758 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 129 698 c[1] - 1 188 127 c[2] + 268 853 c[3] - 31 754 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 135 458 c[1] - 1 188 991 c[2] + 268 885 c[3] - 31 754 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 123 966 c[1] - 1 186 547 c[2] + 268 713 c[3] - 31 750 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 123 550 c[1] - 1 186 515 c[2] + 268 713 c[3] - 31 750 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 118 090 c[1] - 1 184 951 c[2] + 268 573 c[3] - 31 746 c[4] + 2072 c[5] - 71 c[6] + c[7],
 2 089 230 c[1] - 1 178 467 c[2] + 268 089 c[3] - 31 734 c[4] + 2072 c[5] - 71 c[6] + c[7]}
```

```
In[•]:= Array[c, 7].glist[[i]]
```

```
Out[•]= 129 238 000 c[1] - 71 596 812 c[2] + 16 152 884 c[3] -
 1 905 744 c[4] + 124 320 c[5] - 4260 c[6] + 60 c[7]
```

```
In[•]:= certlist[[i]] = Flatten[
  Array[c, 7] /. FindInstance[129 238 000 c[1] - 71 596 812 c[2] + 16 152 884 c[3] -
      1 905 744 c[4] + 124 320 c[5] - 4260 c[6] + 60 c[7] < 0 &&
    2 158 350 c[1] - 1 194 595 c[2] + 269 337 c[3] - 31 766 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 157 870 c[1] - 1 194 563 c[2] + 269 337 c[3] -
      31 766 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 && 2 146 650 c[1] -
      1 192 135 c[2] + 269 165 c[3] - 31 762 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 152 410 c[1] - 1 192 999 c[2] + 269 197 c[3] - 31 762 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 144 038 c[1] - 1 191 003 c[2] +
      269 041 c[3] - 31 758 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 129 698 c[1] - 1 188 127 c[2] + 268 853 c[3] - 31 754 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 135 458 c[1] - 1 188 991 c[2] +
      268 885 c[3] - 31 754 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 123 966 c[1] - 1 186 547 c[2] + 268 713 c[3] - 31 750 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 123 550 c[1] - 1 186 515 c[2] +
      268 713 c[3] - 31 750 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0 &&
    2 118 090 c[1] - 1 184 951 c[2] + 268 573 c[3] - 31 746 c[4] + 2072 c[5] -
      71 c[6] + c[7] ≥ 0 && 2 089 230 c[1] - 1 178 467 c[2] + 268 089 c[3] -
      31 734 c[4] + 2072 c[5] - 71 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
```

```
Out[•]= {-8823, -43 601, -122 674, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]] = certlist[[i]] / GCD[-8823, -43 601, -122 674, 0, 0, 0, 0]
```

```
Out[•]= {-8823, -43 601, -122 674, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -113 165 804
```

```
Out[•]= {1 767 407, 4 607 215, 18 837 975, 1 763 191, 7 738 895,
 31 926 951, 14 852 167, 30 785 167, 33 060 303, 30 216 279, 61 513 391}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[ ]= 14
Out[ ]= (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^42 (-1763 + 447 x - 37 x^2 + x^3)
Out[ ]= {0, 0, 0, 0, -122 674, -43 601, -8823}
```

```
In[ ]:= i = 15;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-15 + x)^2 (-11 + x)^13 (5 + x)^41 (-1018 + 325 x - 32 x^2 + x^3),
            (-15 + x)^2 (-13 + x)^2 (-11 + x)^13 (-6 + x) (5 + x)^41,
            (-15 + x) (-13 + x) (-11 + x)^12 (5 + x)^41 (12 834 - 5159 x + 737 x^2 - 45 x^3 + x^4)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
Out[ ]= (-15 + x)^2 (-13 + x) (-11 + x)^13 (5 + x)^42 (139 - 24 x + x^2)
```

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -167 970  80 093  -14 748  1322  -58  1 )
        ( -167 310  79 989  -14 744  1322  -58  1 )
        ( -166 842  79 901  -14 740  1322  -58  1 )
```

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-10 189 840, 4 820 668, -885 384, 79 320, -3480, 60}
```

```
In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[ ]= {-167 970 c[1] + 80 093 c[2] - 14 748 c[3] + 1322 c[4] - 58 c[5] + c[6],
         -167 310 c[1] + 79 989 c[2] - 14 744 c[3] + 1322 c[4] - 58 c[5] + c[6],
         -166 842 c[1] + 79 901 c[2] - 14 740 c[3] + 1322 c[4] - 58 c[5] + c[6]}
```

```
In[ ]:= Array[c, 6].glist[[i]]
Out[ ]= -10 189 840 c[1] + 4 820 668 c[2] - 885 384 c[3] + 79 320 c[4] - 3480 c[5] + 60 c[6]
```

```
In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
            -10 189 840 c[1] + 4 820 668 c[2] - 885 384 c[3] + 79 320 c[4] - 3480 c[5] + 60 c[6] <
              0 && -167 970 c[1] + 80 093 c[2] - 14 748 c[3] + 1322 c[4] - 58 c[5] + c[6] ≥ 0 &&
             -167 310 c[1] + 79 989 c[2] - 14 744 c[3] + 1322 c[4] - 58 c[5] + c[6] ≥ 0 &&
             -166 842 c[1] + 79 901 c[2] - 14 740 c[3] + 1322 c[4] - 58 c[5] + c[6] ≥ 0,
            Array[c, 6], Integers]]
Out[ ]= {199, 420, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]] = certlist[[i]] / GCD[199, 420, 0, 0, 0, 0]
Out[•]= {199, 420, 0, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[•]= -3 097 600

Out[•]= {213 030, 300 690, 356 862}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[•]= 15

Out[•]= (-15 + x)^2 (-13 + x) (-11 + x)^13 (5 + x)^42 (139 - 24 x + x^2)

Out[•]= {0, 0, 0, 0, 420, 199}
```

```
In[•]:= i = 16;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^4 (-11 + x)^12 (5 + x)^41 (86 - 21 x + x^2),
            (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (82 - 19 x + x^2) (-1711 + 443 x - 37 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 790 - 4465 x + 669 x^2 - 43 x^3 + x^4),
            (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^41 (-1194 + 363 x - 34 x^2 + x^3),
            (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-137 598 + 68 419 x - 13 146 x^2 + 1228 x^3 - 56 x^4 + x^5),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-962 + 317 x - 32 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 614 - 4449 x + 669 x^2 - 43 x^3 + x^4),
            (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-137 026 + 68 323 x - 13 142 x^2 + 1228 x^3 - 56 x^4 + x^5),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-958 + 317 x - 32 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 570 - 4445 x + 669 x^2 - 43 x^3 + x^4),
            (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 438 - 4433 x + 669 x^2 - 43 x^3 + x^4),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-942 + 317 x - 32 x^2 + x^3),
            (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 394 - 4429 x + 669 x^2 - 43 x^3 + x^4),
            (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (62 - 17 x + x^2),
            (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-926 + 317 x - 32 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
Out[•]= (-13 + x)^4 (-11 + x)^11 (5 + x)^42 (-1711 + 443 x - 37 x^2 + x^3)
```

In[•]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -135\,278 & 68\,035 & -13\,130 & 1228 & -56 & 1 \\ -140\,302 & 68\,835 & -13\,162 & 1228 & -56 & 1 \\ -140\,270 & 68\,835 & -13\,162 & 1228 & -56 & 1 \\ -139\,698 & 68\,739 & -13\,158 & 1228 & -56 & 1 \\ -137\,598 & 68\,419 & -13\,146 & 1228 & -56 & 1 \\ -137\,566 & 68\,419 & -13\,146 & 1228 & -56 & 1 \\ -137\,982 & 68\,451 & -13\,146 & 1228 & -56 & 1 \\ -137\,026 & 68\,323 & -13\,142 & 1228 & -56 & 1 \\ -136\,994 & 68\,323 & -13\,142 & 1228 & -56 & 1 \\ -137\,410 & 68\,355 & -13\,142 & 1228 & -56 & 1 \\ -135\,694 & 68\,067 & -13\,130 & 1228 & -56 & 1 \\ -134\,706 & 67\,939 & -13\,126 & 1228 & -56 & 1 \\ -135\,122 & 67\,971 & -13\,126 & 1228 & -56 & 1 \\ -132\,990 & 67\,651 & -13\,114 & 1228 & -56 & 1 \\ -132\,418 & 67\,555 & -13\,110 & 1228 & -56 & 1 \end{pmatrix}$$

In[•]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[•]=
```
{-8 359 736, 4 120 052, -789 288, 73 680, -3360, 60}
```

In[•]:=
```
Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

Out[•]=
```
{-135 278 c[1] + 68 035 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -140 302 c[1] + 68 835 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -140 270 c[1] + 68 835 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -139 698 c[1] + 68 739 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 598 c[1] + 68 419 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 566 c[1] + 68 419 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 982 c[1] + 68 451 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 026 c[1] + 68 323 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -136 994 c[1] + 68 323 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -137 410 c[1] + 68 355 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -135 694 c[1] + 68 067 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -134 706 c[1] + 67 939 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -135 122 c[1] + 67 971 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -132 990 c[1] + 67 651 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -132 418 c[1] + 67 555 c[2] - 13 110 c[3] + 1228 c[4] - 56 c[5] + c[6]}
```

In[•]:=
```
Array[c, 6].glist[[i]]
```

Out[•]=
```
-8 359 736 c[1] + 4 120 052 c[2] - 789 288 c[3] + 73 680 c[4] - 3360 c[5] + 60 c[6]
```

```
In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -8 359 736 c[1] + 4 120 052 c[2] - 789 288 c[3] + 73 680 c[4] - 3360 c[5] + 60 c[6] < 0 &&
           -135 278 c[1] + 68 035 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -140 302 c[1] + 68 835 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -140 270 c[1] + 68 835 c[2] - 13 162 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -139 698 c[1] + 68 739 c[2] - 13 158 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -137 598 c[1] + 68 419 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -137 566 c[1] + 68 419 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -137 982 c[1] + 68 451 c[2] - 13 146 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -137 026 c[1] + 68 323 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -136 994 c[1] + 68 323 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -137 410 c[1] + 68 355 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -135 694 c[1] + 68 067 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -134 706 c[1] + 67 939 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -135 122 c[1] + 67 971 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -132 990 c[1] + 67 651 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -132 418 c[1] + 67 555 c[2] - 13 110 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]
Out[ ]= {8869, 62 958, 234 708, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[8869, 62 958, 234 708, 0, 0, 0]
Out[ ]= {8869, 62 958, 234 708, 0, 0, 0}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[ ]= -4 472 672

Out[ ]= {1 850 908, 148 796, 432 604, 400 536, 1 695 372, 1 979 180, 304 332,
         1 663 304, 1 947 112, 272 264, 176 060, 1 818 840, 143 992, 1 722 636, 1 690 568}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[ ]= 16

Out[ ]= (-13 + x)^4 (-11 + x)^11 (5 + x)^42 (-1711 + 443 x - 37 x^2 + x^3)

Out[ ]= {0, 0, 0, 234 708, 62 958, 8869}
```

In[ ]:=
```
i = 17;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^4 (-11 + x)^9 (5 + x)^41 (161 - 26 x + x^2) (-734 + 253 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (-10 + x) (5 + x)^41 (161 - 26 x + x^2) (73 - 18 x + x^2),
   (-13 + x)^4 (-11 + x)^9 (-9 + x) (5 + x)^41 (13 098 - 5183 x + 737 x^2 - 45 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-117 222 + 59 641 x - 11 812 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-117 542 + 59 673 x - 11 812 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 614 - 4449 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-117 074 + 59 585 x - 11 808 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 570 - 4445 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^41 (-1178 + 363 x - 34 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116 590 + 59 497 x - 11 804 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 946 + 59 393 x - 11 800 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-994 + 321 x - 32 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 638 + 59 321 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 438 - 4433 x + 669 x^2 - 43 x^3 + x^4),
   (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 394 - 4429 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^9 (-10 + x) (5 + x)^41 (-877 + 299 x - 31 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 262 - 4417 x + 669 x^2 - 43 x^3 + x^4)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
```

Out[ ]=
$$(-13+x)^5\,(-11+x)^{10}\,(-9+x)\,(5+x)^{42}\,\left(161-26\,x+x^2\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

-118 174	59 817	-11 820	1142	-54	1
-117 530	59 713	-11 816	1142	-54	1
-117 882	59 745	-11 816	1142	-54	1
-117 222	59 641	-11 812	1142	-54	1
-117 542	59 673	-11 812	1142	-54	1
-116 754	59 553	-11 808	1142	-54	1
-117 074	59 585	-11 808	1142	-54	1
-116 270	59 465	-11 804	1142	-54	1
-116 622	59 497	-11 804	1142	-54	1
-116 590	59 497	-11 804	1142	-54	1
-115 946	59 393	-11 800	1142	-54	1
-116 298	59 425	-11 800	1142	-54	1
-115 638	59 321	-11 796	1142	-54	1
-114 818	59 201	-11 792	1142	-54	1
-115 170	59 233	-11 792	1142	-54	1
-114 334	59 113	-11 788	1142	-54	1
-114 686	59 145	-11 788	1142	-54	1
-114 010	59 041	-11 784	1142	-54	1
-112 882	58 849	-11 776	1142	-54	1

In[•]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[•]=
```
{-7 079 944, 3 586 012, -709 008, 68 520, -3240, 60}
```

In[•]:=
```
Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

Out[•]=
```
{-118 174 c[1] + 59 817 c[2] - 11 820 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 530 c[1] + 59 713 c[2] - 11 816 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 882 c[1] + 59 745 c[2] - 11 816 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 222 c[1] + 59 641 c[2] - 11 812 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 542 c[1] + 59 673 c[2] - 11 812 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 754 c[1] + 59 553 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 074 c[1] + 59 585 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 270 c[1] + 59 465 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 622 c[1] + 59 497 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 590 c[1] + 59 497 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -115 946 c[1] + 59 393 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 298 c[1] + 59 425 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -115 638 c[1] + 59 321 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 818 c[1] + 59 201 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -115 170 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 334 c[1] + 59 113 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 686 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 010 c[1] + 59 041 c[2] - 11 784 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -112 882 c[1] + 58 849 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6]}
```

In[•]:=
```
Array[c, 6].glist[[i]]
```

Out[•]=
```
-7 079 944 c[1] + 3 586 012 c[2] - 709 008 c[3] + 68 520 c[4] - 3240 c[5] + 60 c[6]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -7 079 944 c[1] + 3 586 012 c[2] - 709 008 c[3] + 68 520 c[4] - 3240 c[5] + 60 c[6] < 0 &&
           -118 174 c[1] + 59 817 c[2] - 11 820 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 530 c[1] + 59 713 c[2] - 11 816 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 882 c[1] + 59 745 c[2] - 11 816 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 222 c[1] + 59 641 c[2] - 11 812 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 542 c[1] + 59 673 c[2] - 11 812 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 754 c[1] + 59 553 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 074 c[1] + 59 585 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 270 c[1] + 59 465 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 622 c[1] + 59 497 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 590 c[1] + 59 497 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 946 c[1] + 59 393 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 298 c[1] + 59 425 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 638 c[1] + 59 321 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 818 c[1] + 59 201 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 170 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 334 c[1] + 59 113 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 686 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 010 c[1] + 59 041 c[2] - 11 784 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 882 c[1] + 58 849 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]

Out[•]= {8550, 41 635, 125 208, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[8550, 41 635, 125 208, 0, 0, 0]

Out[•]= {8550, 41 635, 125 208, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[•]= -3 385 244

Out[•]= {134 535, 1 811 527, 134 247, 1 948 039, 544 359, 2 786 391,
         1 382 711, 3 761 543, 2 084 263, 2 357 863, 4 034 855, 2 357 575, 4 171 367,
         6 686 999, 5 009 719, 7 662 151, 5 984 871, 7 935 463, 10 587 607}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[•]= 17

Out[•]= (-13 + x)^5 (-11 + x)^10 (-9 + x) (5 + x)^42 (161 - 26 x + x^2)

Out[•]= {0, 0, 0, 125 208, 41 635, 8550}
```

```
In[ ]:= i = 18;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-140 018 + 68 771 x - 13 158 x^2 + 1228 x^3 - 56 x^4 + x^5),
           (-13 + x)^2 (-11 + x)^10 (5 + x)^41
            (1 808 046 - 1 031 437 x + 239 645 x^2 - 29 118 x^3 + 1956 x^4 - 69 x^5 + x^6),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-164 330 + 78 825 x - 14 620 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-163 446 + 78 705 x - 14 616 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-162 546 + 78 585 x - 14 612 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (178 - 27 x + x^2) (-775 + 267 x - 29 x^2 + x^3),
           (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 538 - 5083 x + 733 x^2 - 45 x^3 + x^4),
           (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-958 + 317 x - 32 x^2 + x^3),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-161 870 + 78 481 x - 14 608 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-137 378 + 68 355 x - 13 142 x^2 + 1228 x^3 - 56 x^4 + x^5),
           (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (12 486 - 5079 x + 733 x^2 - 45 x^3 + x^4),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-161 210 + 78 377 x - 14 604 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-160 742 + 78 289 x - 14 600 x^2 + 1318 x^3 - 58 x^4 + x^5),
           (-13 + x)^4 (-11 + x)^12 (5 + x)^41 (86 - 21 x + x^2),
           (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (-822 + 283 x - 30 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-942 + 317 x - 32 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
Out[ ]= (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (22 259 - 7470 x + 924 x^2 - 50 x^3 + x^4)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( 1 820 234  -1 034 041  239 825  -29 122  1956  -69  1 )
        ( 1 808 046  -1 031 437  239 645  -29 118  1956  -69  1 )
        ( 1 807 630  -1 031 405  239 645  -29 118  1956  -69  1 )
        ( 1 797 906  -1 029 201  239 481  -29 114  1956  -69  1 )
        ( 1 788 006  -1 026 981  239 317  -29 110  1956  -69  1 )
        ( 1 793 350  -1 027 813  239 349  -29 110  1956  -69  1 )
        ( 1 792 934  -1 027 781  239 349  -29 110  1956  -69  1 )
        ( 1 780 922  -1 025 193  239 169  -29 106  1956  -69  1 )
        ( 1 780 570  -1 025 161  239 169  -29 106  1956  -69  1 )
        ( 1 785 914  -1 025 993  239 201  -29 106  1956  -69  1 )
        ( 1 785 498  -1 025 961  239 201  -29 106  1956  -69  1 )
        ( 1 773 310  -1 023 357  239 021  -29 102  1956  -69  1 )
        ( 1 768 162  -1 021 921  238 889  -29 098  1956  -69  1 )
        ( 1 758 614  -1 019 733  238 725  -29 094  1956  -69  1 )
        ( 1 763 190  -1 020 501  238 757  -29 094  1956  -69  1 )
        ( 1 751 178  -1 017 913  238 577  -29 090  1956  -69  1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {108 758 584, -61 938 236, 14 381 692, -1 747 128, 117 360, -4140, 60}
```

```
In[•]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= {1 820 234 c[1] - 1 034 041 c[2] + 239 825 c[3] - 29 122 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 808 046 c[1] - 1 031 437 c[2] + 239 645 c[3] - 29 118 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 807 630 c[1] - 1 031 405 c[2] + 239 645 c[3] - 29 118 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 797 906 c[1] - 1 029 201 c[2] + 239 481 c[3] - 29 114 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 788 006 c[1] - 1 026 981 c[2] + 239 317 c[3] - 29 110 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 793 350 c[1] - 1 027 813 c[2] + 239 349 c[3] - 29 110 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 792 934 c[1] - 1 027 781 c[2] + 239 349 c[3] - 29 110 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 780 922 c[1] - 1 025 193 c[2] + 239 169 c[3] - 29 106 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 780 570 c[1] - 1 025 161 c[2] + 239 169 c[3] - 29 106 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 785 914 c[1] - 1 025 993 c[2] + 239 201 c[3] - 29 106 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 785 498 c[1] - 1 025 961 c[2] + 239 201 c[3] - 29 106 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 773 310 c[1] - 1 023 357 c[2] + 239 021 c[3] - 29 102 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 768 162 c[1] - 1 021 921 c[2] + 238 889 c[3] - 29 098 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 758 614 c[1] - 1 019 733 c[2] + 238 725 c[3] - 29 094 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 763 190 c[1] - 1 020 501 c[2] + 238 757 c[3] - 29 094 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1 751 178 c[1] - 1 017 913 c[2] + 238 577 c[3] - 29 090 c[4] + 1956 c[5] - 69 c[6] + c[7]}
```

```
In[•]:= Array[c, 7].glist[[i]]
```

```
Out[•]= 108 758 584 c[1] - 61 938 236 c[2] + 14 381 692 c[3] -
 1 747 128 c[4] + 117 360 c[5] - 4140 c[6] + 60 c[7]
```

```
In[ ]:= certlist[[i]] = Flatten[
          Array[c, 7] /. FindInstance[108 758 584 c[1] - 61 938 236 c[2] + 14 381 692 c[3] -
                1 747 128 c[4] + 117 360 c[5] - 4140 c[6] + 60 c[7] < 0 &&
              1 820 234 c[1] - 1 034 041 c[2] + 239 825 c[3] - 29 122 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 808 046 c[1] - 1 031 437 c[2] + 239 645 c[3] -
                29 118 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 && 1 807 630 c[1] -
                1 031 405 c[2] + 239 645 c[3] - 29 118 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 797 906 c[1] - 1 029 201 c[2] + 239 481 c[3] - 29 114 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 788 006 c[1] - 1 026 981 c[2] + 239 317 c[3] -
                29 110 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 && 1 793 350 c[1] -
                1 027 813 c[2] + 239 349 c[3] - 29 110 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 792 934 c[1] - 1 027 781 c[2] + 239 349 c[3] - 29 110 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 780 922 c[1] - 1 025 193 c[2] +
                239 169 c[3] - 29 106 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 780 570 c[1] - 1 025 161 c[2] + 239 169 c[3] - 29 106 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 785 914 c[1] - 1 025 993 c[2] +
                239 201 c[3] - 29 106 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 785 498 c[1] - 1 025 961 c[2] + 239 201 c[3] - 29 106 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 773 310 c[1] - 1 023 357 c[2] +
                239 021 c[3] - 29 102 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 768 162 c[1] - 1 021 921 c[2] + 238 889 c[3] - 29 098 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 758 614 c[1] - 1 019 733 c[2] +
                238 725 c[3] - 29 094 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 763 190 c[1] - 1 020 501 c[2] + 238 757 c[3] - 29 094 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 751 178 c[1] - 1 017 913 c[2] + 238 577 c[3] -
                29 090 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
Out[ ]= {-85 438, -747 551, -5 579 245, -24 742 927, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-85 438, -747 551, -5 579 245, -24 742 927, 0, 0, 0]
Out[ ]= {-85 438, -747 551, -5 579 245, -24 742 927, 0, 0, 0}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[ ]= -48 258 640

Out[ ]= {2 319 068, 2 307 000, 13 927 576, 13 148 756, 15 446 208,
         2 292 128, 13 912 704, 10 824 364, 16 976 908, 3 822 828, 15 443 404,
         15 431 336, 19 271 556, 15 416 464, 20 035 504, 16 947 164}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[ ]= 18

Out[ ]= (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (22 259 - 7470 x + 924 x^2 - 50 x^3 + x^4)

Out[ ]= {0, 0, 0, -24 742 927, -5 579 245, -747 551, -85 438}
```

```
In[ ]:= i = 19;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^2 (-11 + x)^9 (5 + x)^41
             (-19 909 034 + 13 159 597 x - 3 668 060 x^2 + 559 959 x^3 - 50 634 x^4 + 2715 x^5 - 80 x^6 + x^7),
            (-13 + x)^2 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2)
             (-143 406 + 69 959 x - 13 282 x^2 + 1232 x^3 - 56 x^4 + x^5),
            (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (-163 862 + 78 737 x - 14 616 x^2 + 1318 x^3 - 58 x^4 + x^5),
            (-13 + x)^3 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (10 986 - 4529 x + 673 x^2 - 43 x^3 + x^4),
            (-13 + x)^2 (-11 + x)^10 (5 + x)^41
             (1 792 998 - 1 027 781 x + 239 349 x^2 - 29 110 x^3 + 1956 x^4 - 69 x^5 + x^6), (-13 + x)^3
             (-11 + x)^9 (5 + x)^41 (1 521 258 - 891 615 x + 213 089 x^2 - 26 654 x^3 + 1844 x^4 - 67 x^5 + x^6),
            (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-137 378 + 68 355 x - 13 142 x^2 + 1228 x^3 - 56 x^4 + x^5),
            (-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9182 - 3947 x + 613 x^2 - 41 x^3 + x^4),
            (-13 + x)^3 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (10 810 - 4513 x + 673 x^2 - 43 x^3 + x^4)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^3 (-11 + x)^10 (5 + x)^42 (139 - 24 x + x^2) (-1763 + 447 x - 37 x^2 + x^3)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -19 909 034  13 159 597  -3 668 060  559 959  -50 634  2715  -80  1
          -19 933 434  13 166 045  -3 668 620  559 975  -50 634  2715  -80  1
          -19 827 302  13 132 141  -3 664 612  559 767  -50 630  2715  -80  1
          -19 851 702  13 138 589  -3 665 172  559 783  -50 630  2715  -80  1
          -19 722 978  13 098 589  -3 660 620  559 559  -50 626  2715  -80  1
          -19 776 354  13 112 253  -3 661 772  559 591  -50 626  2715  -80  1
          -19 645 054  13 071 837  -3 657 204  559 367  -50 622  2715  -80  1
          -19 695 390  13 084 861  -3 658 324  559 399  -50 622  2715  -80  1
          -19 533 670  13 030 301  -3 651 444  559 015  -50 614  2715  -80  1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-1 197 412 712, 790 392 652, -220 166 288, 33 600 996, -3 038 088, 162 900, -4800, 60}
```

```
In[ ]:= Array[c, 8].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= {-19 909 034 c[1] + 13 159 597 c[2] - 3 668 060 c[3] +
    559 959 c[4] - 50 634 c[5] + 2715 c[6] - 80 c[7] + c[8],
   -19 933 434 c[1] + 13 166 045 c[2] - 3 668 620 c[3] + 559 975 c[4] - 50 634 c[5] +
    2715 c[6] - 80 c[7] + c[8], -19 827 302 c[1] + 13 132 141 c[2] -
    3 664 612 c[3] + 559 767 c[4] - 50 630 c[5] + 2715 c[6] - 80 c[7] + c[8],
   -19 851 702 c[1] + 13 138 589 c[2] - 3 665 172 c[3] + 559 783 c[4] - 50 630 c[5] +
    2715 c[6] - 80 c[7] + c[8], -19 722 978 c[1] + 13 098 589 c[2] -
    3 660 620 c[3] + 559 559 c[4] - 50 626 c[5] + 2715 c[6] - 80 c[7] + c[8],
   -19 776 354 c[1] + 13 112 253 c[2] - 3 661 772 c[3] + 559 591 c[4] - 50 626 c[5] +
    2715 c[6] - 80 c[7] + c[8], -19 645 054 c[1] + 13 071 837 c[2] -
    3 657 204 c[3] + 559 367 c[4] - 50 622 c[5] + 2715 c[6] - 80 c[7] + c[8],
   -19 695 390 c[1] + 13 084 861 c[2] - 3 658 324 c[3] + 559 399 c[4] - 50 622 c[5] +
    2715 c[6] - 80 c[7] + c[8], -19 533 670 c[1] + 13 030 301 c[2] -
    3 651 444 c[3] + 559 015 c[4] - 50 614 c[5] + 2715 c[6] - 80 c[7] + c[8]}
```

```
In[ ]:= Array[c, 8].glist[[i]]
```

```
Out[ ]= -1 197 412 712 c[1] + 790 392 652 c[2] - 220 166 288 c[3] +
   33 600 996 c[4] - 3 038 088 c[5] + 162 900 c[6] - 4800 c[7] + 60 c[8]
```

```
In[ ]:= certlist[[i]] = Flatten[
   Array[c, 8] /. FindInstance[-1 197 412 712 c[1] + 790 392 652 c[2] - 220 166 288 c[3] +
        33 600 996 c[4] - 3 038 088 c[5] + 162 900 c[6] - 4800 c[7] + 60 c[8] < 0 &&
      -19 909 034 c[1] + 13 159 597 c[2] - 3 668 060 c[3] + 559 959 c[4] - 50 634 c[5] +
        2715 c[6] - 80 c[7] + c[8] ≥ 0 && -19 933 434 c[1] + 13 166 045 c[2] -
        3 668 620 c[3] + 559 975 c[4] - 50 634 c[5] + 2715 c[6] - 80 c[7] + c[8] ≥ 0 &&
      -19 827 302 c[1] + 13 132 141 c[2] - 3 664 612 c[3] + 559 767 c[4] - 50 630 c[5] +
        2715 c[6] - 80 c[7] + c[8] ≥ 0 && -19 851 702 c[1] + 13 138 589 c[2] -
        3 665 172 c[3] + 559 783 c[4] - 50 630 c[5] + 2715 c[6] - 80 c[7] + c[8] ≥ 0 &&
      -19 722 978 c[1] + 13 098 589 c[2] - 3 660 620 c[3] + 559 559 c[4] - 50 626 c[5] +
        2715 c[6] - 80 c[7] + c[8] ≥ 0 && -19 776 354 c[1] + 13 112 253 c[2] -
        3 661 772 c[3] + 559 591 c[4] - 50 626 c[5] + 2715 c[6] - 80 c[7] + c[8] ≥ 0 &&
      -19 645 054 c[1] + 13 071 837 c[2] - 3 657 204 c[3] + 559 367 c[4] - 50 622 c[5] +
        2715 c[6] - 80 c[7] + c[8] ≥ 0 && -19 695 390 c[1] + 13 084 861 c[2] -
        3 658 324 c[3] + 559 399 c[4] - 50 622 c[5] + 2715 c[6] - 80 c[7] + c[8] ≥ 0 &&
      -19 533 670 c[1] + 13 030 301 c[2] - 3 651 444 c[3] + 559 015 c[4] -
        50 614 c[5] + 2715 c[6] - 80 c[7] + c[8] ≥ 0, Array[c, 8], Integers]]
```

```
Out[ ]= {2970, 4498, 0, 0, 0, 0, 0, 0}
```

```
In[ ]:= certlist[[i]] = certlist[[i]] / GCD[2970, 4498, 0, 0, 0, 0, 0, 0]
```

```
Out[ ]= {1485, 2249, 0, 0, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -564 802 972
```

```
Out[•]= {31 018 163, 9 285 715, 90 641 639, 68 909 191,
         170 104 331, 121 571 307, 225 656 223, 180 198 239, 297 646 999}
```

```
In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[•]= 19
```

$$\text{Out[•]=}\ (-13+x)^3\,(-11+x)^{10}\,(5+x)^{42}\,\left(139-24\,x+x^2\right)\,\left(-1763+447\,x-37\,x^2+x^3\right)$$

```
Out[•]= {0, 0, 0, 0, 0, 0, 2249, 1485}
```

```
In[•]:= i = 20;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-15 + x)^2 (-13 + x)^2 (-11 + x)^11 (-9 + x) (5 + x)^41 (82 - 19 x + x^2),
           (-15 + x)^2 (-13 + x)^2 (-11 + x)^13 (-6 + x) (5 + x)^41,
           (-15 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (10 922 - 4521 x + 673 x^2 - 43 x^3 + x^4),
           (-15 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (10 862 - 4517 x + 673 x^2 - 43 x^3 + x^4),
           (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (-838 + 283 x - 30 x^2 + x^3),
           (-15 + x) (-13 + x)^2 (-11 + x)^12 (5 + x)^41 (-974 + 321 x - 32 x^2 + x^3),
           (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (-822 + 283 x - 30 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
```

$$\text{Out[•]=}\ (-15+x)^2\,(-13+x)^3\,(-11+x)^{12}\,(-9+x)\,(5+x)^{42}$$

```
In[•]:= Coeffmatlist[[i]] // MatrixForm
```

```
Out[•]//MatrixForm=
⎛11 070  -4533  673  -43  1⎞
⎜10 890  -4521  673  -43  1⎟
⎜10 922  -4521  673  -43  1⎟
⎜10 862  -4517  673  -43  1⎟
⎜10 894  -4517  673  -43  1⎟
⎜10 714  -4505  673  -43  1⎟
⎝10 686  -4501  673  -43  1⎠
```

```
In[•]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

```
Out[•]= {659 640, -271 548, 40 380, -2580, 60}
```

```
In[•]:= Array[c, 5].Transpose[Coeffmatlist[[i]]]
Out[•]= {11 070 c[1] - 4533 c[2] + 673 c[3] - 43 c[4] + c[5],
         10 890 c[1] - 4521 c[2] + 673 c[3] - 43 c[4] + c[5],
         10 922 c[1] - 4521 c[2] + 673 c[3] - 43 c[4] + c[5],
         10 862 c[1] - 4517 c[2] + 673 c[3] - 43 c[4] + c[5],
         10 894 c[1] - 4517 c[2] + 673 c[3] - 43 c[4] + c[5],
         10 714 c[1] - 4505 c[2] + 673 c[3] - 43 c[4] + c[5],
         10 686 c[1] - 4501 c[2] + 673 c[3] - 43 c[4] + c[5]}

In[•]:= Array[c, 5].glist[[i]]
Out[•]= 659 640 c[1] - 271 548 c[2] + 40 380 c[3] - 2580 c[4] + 60 c[5]

In[•]:= certlist[[i]] = Flatten[Array[c, 5] /.
          FindInstance[659 640 c[1] - 271 548 c[2] + 40 380 c[3] - 2580 c[4] + 60 c[5] < 0 &&
            11 070 c[1] - 4533 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0 &&
            10 890 c[1] - 4521 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0 &&
            10 922 c[1] - 4521 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0 &&
            10 862 c[1] - 4517 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0 &&
            10 894 c[1] - 4517 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0 &&
            10 714 c[1] - 4505 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0 &&
            10 686 c[1] - 4501 c[2] + 673 c[3] - 43 c[4] + c[5] ≥ 0, Array[c, 5], Integers]]
Out[•]= {-5074, -55 815, 0, 0, -196 837 694}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[-5074, -55 815, 0, 0, -196 837 694]
Out[•]= {-5074, -55 815, 0, 0, -196 837 694}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[•]= -823 380

Out[•]= {2521, 246 061, 83 693, 164 873, 2505, 246 045, 164 857}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[•]= 20

Out[•]= (-15 + x)^2 (-13 + x)^3 (-11 + x)^12 (-9 + x) (5 + x)^42

Out[•]= {-196 837 694, 0, 0, -55 815, -5074}
```

```
In[ ]:= i = 21;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-15 + x) (-13 + x) (-11 + x)^10
             (5 + x)^41 (139 - 24 x + x^2) (11 294 - 4601 x + 677 x^2 - 43 x^3 + x^4),
            (-13 + x)^2 (-11 + x)^10 (5 + x)^41 (139 - 24 x + x^2)^2 (94 - 21 x + x^2),
            (-13 + x)^2 (-11 + x)^11 (5 + x)^41 (139 - 24 x + x^2) (-1166 + 363 x - 34 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
Out[ ]= (-15 + x) (-13 + x)^2 (-11 + x)^11 (5 + x)^42 (139 - 24 x + x^2)^2

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -169 410   80 309   -14 756   1322   -58   1 )
        ( -169 858   80 341   -14 756   1322   -58   1 )
        ( -166 738   79 893   -14 740   1322   -58   1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-10 188 200, 4 820 876, -885 408, 79 320, -3480, 60}

In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[ ]= {-169 410 c[1] + 80 309 c[2] - 14 756 c[3] + 1322 c[4] - 58 c[5] + c[6],
         -169 858 c[1] + 80 341 c[2] - 14 756 c[3] + 1322 c[4] - 58 c[5] + c[6],
         -166 738 c[1] + 79 893 c[2] - 14 740 c[3] + 1322 c[4] - 58 c[5] + c[6]}

In[ ]:= Array[c, 6].glist[[i]]
Out[ ]= -10 188 200 c[1] + 4 820 876 c[2] - 885 408 c[3] + 79 320 c[4] - 3480 c[5] + 60 c[6]

In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
            -10 188 200 c[1] + 4 820 876 c[2] - 885 408 c[3] + 79 320 c[4] - 3480 c[5] + 60 c[6] <
              0 && -169 410 c[1] + 80 309 c[2] - 14 756 c[3] + 1322 c[4] - 58 c[5] + c[6] ≥ 0 &&
             -169 858 c[1] + 80 341 c[2] - 14 756 c[3] + 1322 c[4] - 58 c[5] + c[6] ≥ 0 &&
             -166 738 c[1] + 79 893 c[2] - 14 740 c[3] + 1322 c[4] - 58 c[5] + c[6] ≥ 0,
            Array[c, 6], Integers]]
Out[ ]= {-2664, -37 294, -172 391, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-2664, -37 294, -172 391, 0, 0, 0]
Out[ ]= {-2664, -37 294, -172 391, 0, 0, 0}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[ ]= -12 014 216

Out[ ]= {65 990, 66 054, 5 703 830}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 21
```

$$\text{Out[ ]=}\ (-15+x)\,(-13+x)^2\,(-11+x)^{11}\,(5+x)^{42}\left(139-24\,x+x^2\right)^2$$

```
Out[ ]= {0, 0, 0, -172 391, -37 294, -2664}
```

```
In[ ]:= i = 22;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^5 (-11 + x)^9 (-10 + x) (-9 + x) (5 + x)^41 (101 - 22 x + x^2),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-117 074 + 59 585 x - 11 808 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^4 (-11 + x)^10 (-9 + x) (5 + x)^41 (-1178 + 363 x - 34 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116 590 + 59 497 x - 11 804 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^5 (-11 + x)^8 (-9 + x) (5 + x)^41 (10 994 - 4529 x + 673 x^2 - 43 x^3 + x^4),
           (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-994 + 321 x - 32 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 670 + 59 321 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 638 + 59 321 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 202 + 59 233 x - 11 792 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114 718 + 59 145 x - 11 788 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-978 + 321 x - 32 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113 766 + 58 969 x - 11 780 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8678 - 3855 x + 609 x^2 - 41 x^3 + x^4)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
```

$$\text{Out[ ]=}\ (-13+x)^5\,(-11+x)^9\,(-9+x)\,(5+x)^{42}\left(-1763+447\,x-37\,x^2+x^3\right)$$

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1\,299\,870 & -775\,717 & 189\,753 & -24\,378 & 1736 & -65 & 1 \\
1\,287\,814 & -772\,509 & 189\,473 & -24\,370 & 1736 & -65 & 1 \\
1\,282\,842 & -771\,089 & 189\,341 & -24\,366 & 1736 & -65 & 1 \\
1\,282\,490 & -771\,057 & 189\,341 & -24\,366 & 1736 & -65 & 1 \\
1\,286\,298 & -771\,761 & 189\,373 & -24\,366 & 1736 & -65 & 1 \\
1\,279\,278 & -769\,973 & 189\,225 & -24\,362 & 1736 & -65 & 1 \\
1\,272\,370 & -768\,201 & 189\,077 & -24\,358 & 1736 & -65 & 1 \\
1\,272\,018 & -768\,169 & 189\,077 & -24\,358 & 1736 & -65 & 1 \\
1\,267\,222 & -766\,765 & 188\,945 & -24\,354 & 1736 & -65 & 1 \\
1\,266\,870 & -766\,733 & 188\,945 & -24\,354 & 1736 & -65 & 1 \\
1\,261\,898 & -765\,313 & 188\,813 & -24\,350 & 1736 & -65 & 1 \\
1\,258\,686 & -764\,229 & 188\,697 & -24\,346 & 1736 & -65 & 1 \\
1\,251\,426 & -762\,425 & 188\,549 & -24\,342 & 1736 & -65 & 1 \\
1\,240\,954 & -759\,537 & 188\,285 & -24\,334 & 1736 & -65 & 1
\end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {77 514 592, -46 404 804, 11 372 036, -1 462 272, 104 160, -3900, 60}

In[ ]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
Out[ ]= {1 299 870 c[1] - 775 717 c[2] + 189 753 c[3] - 24 378 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 287 814 c[1] - 772 509 c[2] + 189 473 c[3] - 24 370 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 282 842 c[1] - 771 089 c[2] + 189 341 c[3] - 24 366 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 282 490 c[1] - 771 057 c[2] + 189 341 c[3] - 24 366 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 286 298 c[1] - 771 761 c[2] + 189 373 c[3] - 24 366 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 279 278 c[1] - 769 973 c[2] + 189 225 c[3] - 24 362 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 272 370 c[1] - 768 201 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 272 018 c[1] - 768 169 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 267 222 c[1] - 766 765 c[2] + 188 945 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 266 870 c[1] - 766 733 c[2] + 188 945 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 261 898 c[1] - 765 313 c[2] + 188 813 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 258 686 c[1] - 764 229 c[2] + 188 697 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 251 426 c[1] - 762 425 c[2] + 188 549 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 240 954 c[1] - 759 537 c[2] + 188 285 c[3] - 24 334 c[4] + 1736 c[5] - 65 c[6] + c[7]}

In[ ]:= Array[c, 7].glist[[i]]
Out[ ]= 77 514 592 c[1] - 46 404 804 c[2] + 11 372 036 c[3] -
  1 462 272 c[4] + 104 160 c[5] - 3900 c[6] + 60 c[7]
```

In[ ]:=
```
certlist[[i]] = Flatten[
  Array[c, 7] /. FindInstance[77 514 592 c[1] - 46 404 804 c[2] + 11 372 036 c[3] -
      1 462 272 c[4] + 104 160 c[5] - 3900 c[6] + 60 c[7] < 0 &&
    1 299 870 c[1] - 775 717 c[2] + 189 753 c[3] - 24 378 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 287 814 c[1] - 772 509 c[2] + 189 473 c[3] -
      24 370 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 282 842 c[1] -
      771 089 c[2] + 189 341 c[3] - 24 366 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 282 490 c[1] - 771 057 c[2] + 189 341 c[3] - 24 366 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 286 298 c[1] - 771 761 c[2] + 189 373 c[3] -
      24 366 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 279 278 c[1] -
      769 973 c[2] + 189 225 c[3] - 24 362 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 272 370 c[1] - 768 201 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 272 018 c[1] - 768 169 c[2] +
      189 077 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 267 222 c[1] - 766 765 c[2] + 188 945 c[3] - 24 354 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 266 870 c[1] - 766 733 c[2] +
      188 945 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 261 898 c[1] - 765 313 c[2] + 188 813 c[3] - 24 350 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 258 686 c[1] - 764 229 c[2] +
      188 697 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 251 426 c[1] - 762 425 c[2] + 188 549 c[3] - 24 342 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 240 954 c[1] - 759 537 c[2] + 188 285 c[3] -
      24 334 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
```

Out[ ]= {-190 406, -1 403 713, -8 914 066, -34 871 227, 0, 0, 0}

In[ ]:=
```
certlist[[i]] = certlist[[i]] / GCD[-190 406, -1 403 713, -8 914 066, -34 871 227, 0, 0, 0]
```

Out[ ]= {-190 406, -1 403 713, -8 914 066, -34 871 227, 0, 0, 0}

In[ ]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[ ]= -77 377 732

Out[ ]= {996 109, 10 388 205, 986 181, 23 090 277, 988 069, 7 596 205, 15 338 277,
37 442 373, 16 988 301, 39 092 397, 29 690 373, 14 196 301, 44 042 469, 58 394 565}

In[ ]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[ ]= 22

Out[ ]= $(-13+x)^5\,(-11+x)^9\,(-9+x)\,(5+x)^{42}\left(-1763+447\,x-37\,x^2+x^3\right)$

Out[ ]= {0, 0, 0, -34 871 227, -8 914 066, -1 403 713, -190 406}

```
In[ ]:= i = 23;
chilist[[i]]
interlacingpolylist[[i]] = {(-15 + x) (-13 + x)^2 (-11 + x)^10
     (5 + x)^41 (-120526 + 60685 x - 11924 x^2 + 1146 x^3 - 54 x^4 + x^5),
   (-15 + x) (-13 + x)^2 (-11 + x)^10 (5 + x)^41 (73 - 18 x + x^2) (-1642 + 425 x - 36 x^2 + x^3),
   (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (-838 + 283 x - 30 x^2 + x^3),
   (-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9182 - 3947 x + 613 x^2 - 41 x^3 + x^4),
   (-13 + x)^2 (-11 + x)^10 (5 + x)^41
    (1773154 - 1022721 x + 238921 x^2 - 29098 x^3 + 1956 x^4 - 69 x^5 + x^6),
   (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (-822 + 283 x - 30 x^2 + x^3),
   (-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9006 - 3931 x + 613 x^2 - 41 x^3 + x^4),
   (-14 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^41 (-877 + 299 x - 31 x^2 + x^3),
   (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (62 - 17 x + x^2)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

Out[ ]= $(-15+x)\,(-13+x)^3\,(-11+x)^{11}\,(5+x)^{42}\,\left(-1477+399\,x-35\,x^2+x^3\right)$

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1807890 & -1030801 & 239545 & -29114 & 1956 & -69 & 1\\
1797990 & -1028581 & 239381 & -29110 & 1956 & -69 & 1\\
1797510 & -1028549 & 239381 & -29110 & 1956 & -69 & 1\\
1790490 & -1026761 & 239233 & -29106 & 1956 & -69 & 1\\
1773154 & -1022721 & 238921 & -29098 & 1956 & -69 & 1\\
1763190 & -1020501 & 238757 & -29094 & 1956 & -69 & 1\\
1756170 & -1018713 & 238609 & -29090 & 1956 & -69 & 1\\
1755754 & -1018681 & 238609 & -29090 & 1956 & -69 & 1\\
1728870 & -1012453 & 238133 & -29078 & 1956 & -69 & 1
\end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[ ]= {108231200, -61781188, 14366804, -1746672, 117360, -4140, 60}

```
In[ ]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

Out[ ]= {1807890 c[1] - 1030801 c[2] + 239545 c[3] - 29114 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1797990 c[1] - 1028581 c[2] + 239381 c[3] - 29110 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1797510 c[1] - 1028549 c[2] + 239381 c[3] - 29110 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1790490 c[1] - 1026761 c[2] + 239233 c[3] - 29106 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1773154 c[1] - 1022721 c[2] + 238921 c[3] - 29098 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1763190 c[1] - 1020501 c[2] + 238757 c[3] - 29094 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1756170 c[1] - 1018713 c[2] + 238609 c[3] - 29090 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1755754 c[1] - 1018681 c[2] + 238609 c[3] - 29090 c[4] + 1956 c[5] - 69 c[6] + c[7],
 1728870 c[1] - 1012453 c[2] + 238133 c[3] - 29078 c[4] + 1956 c[5] - 69 c[6] + c[7]}

```
In[ ]:= Array[c, 7].glist[[i]]
```

Out[ ]= 108231200 c[1] - 61781188 c[2] + 14366804 c[3] -
 1746672 c[4] + 117360 c[5] - 4140 c[6] + 60 c[7]

```
In[ ]:= certlist[[i]] = Flatten[
          Array[c, 7] /. FindInstance[108 231 200 c[1] - 61 781 188 c[2] + 14 366 804 c[3] -
                1 746 672 c[4] + 117 360 c[5] - 4140 c[6] + 60 c[7] < 0 &&
              1 807 890 c[1] - 1 030 801 c[2] + 239 545 c[3] - 29 114 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 797 990 c[1] - 1 028 581 c[2] + 239 381 c[3] -
                29 110 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 && 1 797 510 c[1] -
                1 028 549 c[2] + 239 381 c[3] - 29 110 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 790 490 c[1] - 1 026 761 c[2] + 239 233 c[3] - 29 106 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 773 154 c[1] - 1 022 721 c[2] +
                238 921 c[3] - 29 098 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 763 190 c[1] - 1 020 501 c[2] + 238 757 c[3] - 29 094 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 756 170 c[1] - 1 018 713 c[2] +
                238 609 c[3] - 29 090 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0 &&
              1 755 754 c[1] - 1 018 681 c[2] + 238 609 c[3] - 29 090 c[4] + 1956 c[5] -
                69 c[6] + c[7] ≥ 0 && 1 728 870 c[1] - 1 012 453 c[2] + 238 133 c[3] -
                29 078 c[4] + 1956 c[5] - 69 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]

Out[ ]= {-5780, -32 024, -94 177, 0, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-5780, -32 024, -94 177, 0, 0, 0, 0]

Out[ ]= {-5780, -32 024, -94 177, 0, 0, 0, 0}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[ ]= -118 071 796

Out[ ]= {1 137 559, 2 711 307, 4 460 939, 1 715 823,
         1 924 167, 3 867 835, 1 122 719, 2 502 431, 3 274 731}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[ ]= 23

Out[ ]= (-15 + x) (-13 + x)^3 (-11 + x)^11 (5 + x)^42 (-1477 + 399 x - 35 x^2 + x^3)

Out[ ]= {0, 0, 0, 0, -94 177, -32 024, -5780}
```

```
In[•]:= i = 24;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-117490 + 59617 x - 11808 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10678 - 4449 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-117006 + 59529 x - 11804 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-818 + 279 x - 30 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116330 + 59425 x - 11800 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8978 - 3883 x + 609 x^2 - 41 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115670 + 59321 x - 11796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116054 + 59353 x - 11796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10438 - 4433 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115202 + 59233 x - 11792 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10394 - 4429 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114718 + 59145 x - 11788 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114042 + 59041 x - 11784 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113382 + 58937 x - 11780 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (62 - 17 x + x^2),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10262 - 4417 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-926 + 317 x - 32 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-786 + 279 x - 30 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-914 + 317 x - 32 x^2 + x^3),
   (-14 + x) (-13 + x)^5 (-11 + x)^11 (-5 + x) (5 + x)^41};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
Out[•]= (-13 + x)^5 (-11 + x)^10 (5 + x)^42 (-1433 + 395 x - 35 x^2 + x^3)
```

In[•]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -117490 & 59617 & -11808 & 1142 & -54 & 1 \\ -117458 & 59617 & -11808 & 1142 & -54 & 1 \\ -117006 & 59529 & -11804 & 1142 & -54 & 1 \\ -116974 & 59529 & -11804 & 1142 & -54 & 1 \\ -116330 & 59425 & -11800 & 1142 & -54 & 1 \\ -116714 & 59457 & -11800 & 1142 & -54 & 1 \\ -115670 & 59321 & -11796 & 1142 & -54 & 1 \\ -116054 & 59353 & -11796 & 1142 & -54 & 1 \\ -114818 & 59201 & -11792 & 1142 & -54 & 1 \\ -115202 & 59233 & -11792 & 1142 & -54 & 1 \\ -115170 & 59233 & -11792 & 1142 & -54 & 1 \\ -114334 & 59113 & -11788 & 1142 & -54 & 1 \\ -114718 & 59145 & -11788 & 1142 & -54 & 1 \\ -114686 & 59145 & -11788 & 1142 & -54 & 1 \\ -114042 & 59041 & -11784 & 1142 & -54 & 1 \\ -113382 & 58937 & -11780 & 1142 & -54 & 1 \\ -112530 & 58817 & -11776 & 1142 & -54 & 1 \\ -112882 & 58849 & -11776 & 1142 & -54 & 1 \\ -112046 & 58729 & -11772 & 1142 & -54 & 1 \\ -112398 & 58761 & -11772 & 1142 & -54 & 1 \\ -110594 & 58465 & -11760 & 1142 & -54 & 1 \\ -110110 & 58377 & -11756 & 1142 & -54 & 1 \end{pmatrix}$$

In[•]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]] // Factor, x]
```

Out[•]=
```
{-6998648, 3568124, -708096, 68520, -3240, 60}
```

In[•]:=
```
Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

Out[•]=
```
{-117 490 c[1] + 59 617 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 458 c[1] + 59 617 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -117 006 c[1] + 59 529 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 974 c[1] + 59 529 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 330 c[1] + 59 425 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 714 c[1] + 59 457 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -115 670 c[1] + 59 321 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -116 054 c[1] + 59 353 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 818 c[1] + 59 201 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -115 202 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -115 170 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 334 c[1] + 59 113 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 718 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 686 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -114 042 c[1] + 59 041 c[2] - 11 784 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -113 382 c[1] + 58 937 c[2] - 11 780 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -112 530 c[1] + 58 817 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -112 882 c[1] + 58 849 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -112 046 c[1] + 58 729 c[2] - 11 772 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -112 398 c[1] + 58 761 c[2] - 11 772 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -110 594 c[1] + 58 465 c[2] - 11 760 c[3] + 1142 c[4] - 54 c[5] + c[6],
 -110 110 c[1] + 58 377 c[2] - 11 756 c[3] + 1142 c[4] - 54 c[5] + c[6]}
```

In[•]:=
```
Array[c, 6].glist[[i]]
```

Out[•]=
```
-6 998 648 c[1] + 3 568 124 c[2] - 708 096 c[3] + 68 520 c[4] - 3240 c[5] + 60 c[6]
```

```
In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -6 998 648 c[1] + 3 568 124 c[2] - 708 096 c[3] + 68 520 c[4] - 3240 c[5] + 60 c[6] < 0 &&
           -117 490 c[1] + 59 617 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 458 c[1] + 59 617 c[2] - 11 808 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -117 006 c[1] + 59 529 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 974 c[1] + 59 529 c[2] - 11 804 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 330 c[1] + 59 425 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 714 c[1] + 59 457 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 670 c[1] + 59 321 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 054 c[1] + 59 353 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 818 c[1] + 59 201 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 202 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 170 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 334 c[1] + 59 113 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 718 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 686 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 042 c[1] + 59 041 c[2] - 11 784 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -113 382 c[1] + 58 937 c[2] - 11 780 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 530 c[1] + 58 817 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 882 c[1] + 58 849 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 046 c[1] + 58 729 c[2] - 11 772 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 398 c[1] + 58 761 c[2] - 11 772 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -110 594 c[1] + 58 465 c[2] - 11 760 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -110 110 c[1] + 58 377 c[2] - 11 756 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]

Out[ ]= {26 593, 310 262, 3 643 351, 0, 0, 27 648 369 302}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[26 593, 310 262, 3 643 351, 0, 0, 27 648 369 302]

Out[ ]= {26 593, 310 262, 3 643 351, 0, 0, 27 648 369 302}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[ ]= -1 869 352

Out[ ]= {158 778, 1 009 754, 300 138, 1 151 114, 583 162, 299 834, 440 698,
         157 370, 439 898, 156 570, 1 007 546, 581 258, 297 930, 1 148 906, 580 954,
         438 490, 437 690, 1 005 338, 579 050, 1 146 698, 1 003 130, 1 144 490}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[ ]= 24

Out[ ]= (-13 + x)^5 (-11 + x)^10 (5 + x)^42 (-1433 + 395 x - 35 x^2 + x^3)

Out[ ]= {27 648 369 302, 0, 0, 3 643 351, 310 262, 26 593}
```

In[ ]:=
```
i = 25;
chilist[[i]]
interlacingpolylist[[i]] = {(-13 + x)^4 (-11 + x)^8 (5 + x)^41
     (1 286 650 - 771 793 x + 189 373 x^2 - 24 366 x^3 + 1736 x^4 - 65 x^5 + x^6),
    (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-818 + 279 x - 30 x^2 + x^3),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116 330 + 59 425 x - 11 800 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-994 + 321 x - 32 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^8 (5 + x)^41 (-98 726 + 51 691 x - 10 582 x^2 + 1060 x^3 - 52 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 670 + 59 321 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 638 + 59 321 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116 054 + 59 353 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^8 (5 + x)^41
     (1 276 178 - 768 905 x + 189 109 x^2 - 24 358 x^3 + 1736 x^4 - 65 x^5 + x^6),
    (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 586 + 59 265 x - 11 792 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^4 (-11 + x)^8 (5 + x)^41
     (1 271 030 - 767 469 x + 188 977 x^2 - 24 354 x^3 + 1736 x^4 - 65 x^5 + x^6),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114 718 + 59 145 x - 11 788 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8854 - 3871 x + 609 x^2 - 41 x^3 + x^4),
    (-13 + x)^5 (-11 + x)^8 (-9 + x) (5 + x)^41 (10 818 - 4513 x + 673 x^2 - 43 x^3 + x^4),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114 042 + 59 041 x - 11 784 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^5 (-11 + x)^9 (-10 + x) (5 + x)^41 (-877 + 299 x - 31 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-978 + 321 x - 32 x^2 + x^3),
    (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8834 - 3867 x + 609 x^2 - 41 x^3 + x^4),
    (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113 766 + 58 969 x - 11 780 x^2 + 1142 x^3 - 54 x^4 + x^5),
    (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8678 - 3855 x + 609 x^2 - 41 x^3 + x^4),
    (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8626 - 3851 x + 609 x^2 - 41 x^3 + x^4)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

Out[ ]=
$$(-13+x)^5\,(-11+x)^9\,(5+x)^{42}\left(15\,779-5778\,x+780\,x^2-46\,x^3+x^4\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

1 286 650	−771 793	189 373	−24 366	1736	−65	1
1 286 714	−771 793	189 373	−24 366	1736	−65	1
1 279 630	−770 005	189 225	−24 362	1736	−65	1
1 279 278	−769 973	189 225	−24 362	1736	−65	1
1 283 438	−770 709	189 257	−24 362	1736	−65	1
1 272 370	−768 201	189 077	−24 358	1736	−65	1
1 272 018	−768 169	189 077	−24 358	1736	−65	1
1 276 594	−768 937	189 109	−24 358	1736	−65	1
1 276 178	−768 905	189 109	−24 358	1736	−65	1
1 266 870	−766 733	188 945	−24 354	1736	−65	1
1 271 446	−767 501	188 977	−24 354	1736	−65	1
1 271 030	−767 469	188 977	−24 354	1736	−65	1
1 261 898	−765 313	188 813	−24 350	1736	−65	1
1 261 546	−765 281	188 813	−24 350	1736	−65	1
1 266 122	−766 049	188 845	−24 350	1736	−65	1
1 265 706	−766 017	188 845	−24 350	1736	−65	1
1 254 462	−763 493	188 665	−24 346	1736	−65	1
1 254 110	−763 461	188 665	−24 346	1736	−65	1
1 258 686	−764 229	188 697	−24 346	1736	−65	1
1 263 262	−764 997	188 729	−24 346	1736	−65	1
1 251 426	−762 425	188 549	−24 342	1736	−65	1
1 240 954	−759 537	188 285	−24 334	1736	−65	1
1 233 518	−757 717	188 137	−24 330	1736	−65	1

In[ ]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[ ]=
```
{77 067 464, −46 265 772, 11 358 076, −1 461 816, 104 160, −3900, 60}
```

```
In[•]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= {1 286 650 c[1] - 771 793 c[2] + 189 373 c[3] - 24 366 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 286 714 c[1] - 771 793 c[2] + 189 373 c[3] - 24 366 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 279 630 c[1] - 770 005 c[2] + 189 225 c[3] - 24 362 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 279 278 c[1] - 769 973 c[2] + 189 225 c[3] - 24 362 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 283 438 c[1] - 770 709 c[2] + 189 257 c[3] - 24 362 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 272 370 c[1] - 768 201 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 272 018 c[1] - 768 169 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 276 594 c[1] - 768 937 c[2] + 189 109 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 276 178 c[1] - 768 905 c[2] + 189 109 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 266 870 c[1] - 766 733 c[2] + 188 945 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 271 446 c[1] - 767 501 c[2] + 188 977 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 271 030 c[1] - 767 469 c[2] + 188 977 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 261 898 c[1] - 765 313 c[2] + 188 813 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 261 546 c[1] - 765 281 c[2] + 188 813 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 266 122 c[1] - 766 049 c[2] + 188 845 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 265 706 c[1] - 766 017 c[2] + 188 845 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 254 462 c[1] - 763 493 c[2] + 188 665 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 254 110 c[1] - 763 461 c[2] + 188 665 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 258 686 c[1] - 764 229 c[2] + 188 697 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 263 262 c[1] - 764 997 c[2] + 188 729 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 251 426 c[1] - 762 425 c[2] + 188 549 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 240 954 c[1] - 759 537 c[2] + 188 285 c[3] - 24 334 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 233 518 c[1] - 757 717 c[2] + 188 137 c[3] - 24 330 c[4] + 1736 c[5] - 65 c[6] + c[7]}
```

```
In[•]:= Array[c, 7].glist[[i]]
```

```
Out[•]= 77 067 464 c[1] - 46 265 772 c[2] + 11 358 076 c[3] -
  1 461 816 c[4] + 104 160 c[5] - 3900 c[6] + 60 c[7]
```

In[•]:=
```
certlist[[i]] = Flatten[
  Array[c, 7] /. FindInstance[77 067 464 c[1] - 46 265 772 c[2] + 11 358 076 c[3] -
      1 461 816 c[4] + 104 160 c[5] - 3900 c[6] + 60 c[7] < 0 &&
    1 286 650 c[1] - 771 793 c[2] + 189 373 c[3] - 24 366 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 286 714 c[1] - 771 793 c[2] + 189 373 c[3] -
      24 366 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 279 630 c[1] -
      770 005 c[2] + 189 225 c[3] - 24 362 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 279 278 c[1] - 769 973 c[2] + 189 225 c[3] - 24 362 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 283 438 c[1] - 770 709 c[2] + 189 257 c[3] -
      24 362 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 272 370 c[1] -
      768 201 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 272 018 c[1] - 768 169 c[2] + 189 077 c[3] - 24 358 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 276 594 c[1] - 768 937 c[2] + 189 109 c[3] -
      24 358 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 276 178 c[1] -
      768 905 c[2] + 189 109 c[3] - 24 358 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 266 870 c[1] - 766 733 c[2] + 188 945 c[3] - 24 354 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 271 446 c[1] - 767 501 c[2] +
      188 977 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 271 030 c[1] - 767 469 c[2] + 188 977 c[3] - 24 354 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 261 898 c[1] - 765 313 c[2] +
      188 813 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 261 546 c[1] - 765 281 c[2] + 188 813 c[3] - 24 350 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 266 122 c[1] - 766 049 c[2] +
      188 845 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 265 706 c[1] - 766 017 c[2] + 188 845 c[3] - 24 350 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 254 462 c[1] - 763 493 c[2] +
      188 665 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 254 110 c[1] - 763 461 c[2] + 188 665 c[3] - 24 346 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 258 686 c[1] - 764 229 c[2] +
      188 697 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 263 262 c[1] - 764 997 c[2] + 188 729 c[3] - 24 346 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 251 426 c[1] - 762 425 c[2] +
      188 549 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
    1 240 954 c[1] - 759 537 c[2] + 188 285 c[3] - 24 334 c[4] + 1736 c[5] -
      65 c[6] + c[7] ≥ 0 && 1 233 518 c[1] - 757 717 c[2] + 188 137 c[3] -
      24 330 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
```

Out[•]=
```
{292 487, 2 380 728, 16 503 986, 68 304 472, 0, 0, 0}
```

In[•]:=
```
certlist[[i]] = certlist[[i]] / GCD[292 487, 2 380 728, 16 503 986, 68 304 472, 0, 0, 0]
```

Out[•]=
```
{292 487, 2 380 728, 16 503 986, 68 304 472, 0, 0, 0}
```

```
In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= -30 249 264
```

```
Out[ ]= {1 769 272, 20 488 440, 35 880 156, 9 108 028, 1 765 692,
         37 885 808, 11 113 680, 49 262 640, 3 771 344, 18 807 748, 56 956 708,
         11 465 412, 39 887 880, 13 115 752, 51 264 712, 5 773 416, 28 507 468,
         1 735 340, 39 884 300, 78 033 260, 41 889 952, 43 892 024, 32 511 612}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 25
```

$$\text{Out[ ]= } (-13+x)^5\,(-11+x)^9\,(5+x)^{42}\left(15\,779-5778\,x+780\,x^2-46\,x^3+x^4\right)$$

```
Out[ ]= {0, 0, 0, 68 304 472, 16 503 986, 2 380 728, 292 487}
```

```
In[ ]:= i = 26;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9214 - 3947 x + 613 x^2 - 41 x^3 + x^4),
           (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
           (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (-136 078 + 68 099 x - 13 130 x^2 + 1228 x^3 - 56 x^4 + x^5),
           (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 502 - 4433 x + 669 x^2 - 43 x^3 + x^4),
           (-15 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9006 - 3931 x + 613 x^2 - 41 x^3 + x^4),
           (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
           (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (62 - 17 x + x^2),
           (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 262 - 4417 x + 669 x^2 - 43 x^3 + x^4),
           (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-926 + 317 x - 32 x^2 + x^3),
           (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-786 + 279 x - 30 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-914 + 317 x - 32 x^2 + x^3),
           (-14 + x) (-13 + x)^5 (-11 + x)^11 (-5 + x) (5 + x)^41};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

$$\text{Out[ ]= } (-15+x)\,(-13+x)^4\,(-11+x)^{11}\,(5+x)^{42}\left(113-22\,x+x^2\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -138\,210 & 68\,419 & -13\,142 & 1228 & -56 & 1 \\ -136\,110 & 68\,099 & -13\,130 & 1228 & -56 & 1 \\ -136\,078 & 68\,099 & -13\,130 & 1228 & -56 & 1 \\ -136\,526 & 68\,131 & -13\,130 & 1228 & -56 & 1 \\ -135\,090 & 67\,971 & -13\,126 & 1228 & -56 & 1 \\ -135\,538 & 68\,003 & -13\,126 & 1228 & -56 & 1 \\ -132\,990 & 67\,651 & -13\,114 & 1228 & -56 & 1 \\ -133\,406 & 67\,683 & -13\,114 & 1228 & -56 & 1 \\ -132\,418 & 67\,555 & -13\,110 & 1228 & -56 & 1 \\ -132\,834 & 67\,587 & -13\,110 & 1228 & -56 & 1 \\ -130\,702 & 67\,267 & -13\,098 & 1228 & -56 & 1 \\ -130\,130 & 67\,171 & -13\,094 & 1228 & -56 & 1 \end{pmatrix}$$

In[ ]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[ ]=
```
{-8 278 600, 4 102 132, -788 376, 73 680, -3360, 60}
```

In[ ]:=
```
Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

Out[ ]=
```
{-138 210 c[1] + 68 419 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -136 110 c[1] + 68 099 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -136 078 c[1] + 68 099 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -136 526 c[1] + 68 131 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -135 090 c[1] + 67 971 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -135 538 c[1] + 68 003 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -132 990 c[1] + 67 651 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -133 406 c[1] + 67 683 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -132 418 c[1] + 67 555 c[2] - 13 110 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -132 834 c[1] + 67 587 c[2] - 13 110 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -130 702 c[1] + 67 267 c[2] - 13 098 c[3] + 1228 c[4] - 56 c[5] + c[6],
 -130 130 c[1] + 67 171 c[2] - 13 094 c[3] + 1228 c[4] - 56 c[5] + c[6]}
```

In[ ]:=
```
Array[c, 6].glist[[i]]
```

Out[ ]=
```
-8 278 600 c[1] + 4 102 132 c[2] - 788 376 c[3] + 73 680 c[4] - 3360 c[5] + 60 c[6]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -8 278 600 c[1] + 4 102 132 c[2] - 788 376 c[3] + 73 680 c[4] - 3360 c[5] + 60 c[6] < 0 &&
           -138 210 c[1] + 68 419 c[2] - 13 142 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -136 110 c[1] + 68 099 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -136 078 c[1] + 68 099 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -136 526 c[1] + 68 131 c[2] - 13 130 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -135 090 c[1] + 67 971 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -135 538 c[1] + 68 003 c[2] - 13 126 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -132 990 c[1] + 67 651 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -133 406 c[1] + 67 683 c[2] - 13 114 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -132 418 c[1] + 67 555 c[2] - 13 110 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -132 834 c[1] + 67 587 c[2] - 13 110 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -130 702 c[1] + 67 267 c[2] - 13 098 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0 &&
           -130 130 c[1] + 67 171 c[2] - 13 094 c[3] + 1228 c[4] - 56 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]

Out[•]= {4254, 29 388, 108 251, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[4254, 29 388, 108 251, 0, 0, 0]

Out[•]= {4254, 29 388, 108 251, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[•]= -6 199 560

Out[•]= {117 590, 945 842, 1 081 970, 116 594, 1 956 262, 990 886,
         2 784 514, 1 955 266, 2 829 558, 2 000 310, 2 964 690, 3 009 734}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[•]= 26

Out[•]= (-15 + x) (-13 + x)^4 (-11 + x)^11 (5 + x)^42 (113 - 22 x + x^2)

Out[•]= {0, 0, 0, 108 251, 29 388, 4254}
```

```
In[ ]:= i = 27;
chilist[[i]]
interlacingpolylist[[i]] = {(-13 + x)^3 (-11 + x)^8 (5 + x)^41 (139 - 24 x + x^2)
    (-120422 + 60677 x - 11924 x^2 + 1146 x^3 - 54 x^4 + x^5), (-13 + x)^4 (-11 + x)^8
    (5 + x)^41 (1289818 - 772433 x + 189405 x^2 - 24366 x^3 + 1736 x^4 - 65 x^5 + x^6),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116618 + 59457 x - 11800 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^3 (-11 + x)^9 (5 + x)^41
    (1507934 - 887579 x + 212701 x^2 - 26642 x^3 + 1844 x^4 - 67 x^5 + x^6),
   (-13 + x)^4 (-11 + x)^9 (-6 + x) (5 + x)^41 (139 - 24 x + x^2)^2, (-13 + x)^3 (-11 + x)^8
    (5 + x)^41 (-1477 + 399 x - 35 x^2 + x^3) (11250 - 4597 x + 677 x^2 - 43 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115490 + 59265 x - 11792 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-14 + x) (-13 + x)^3 (-11 + x)^10 (5 + x)^41 (9679 - 4166 x + 640 x^2 - 42 x^3 + x^4),
   (-13 + x)^3 (-11 + x)^8 (5 + x)^41 (139 - 24 x + x^2)
    (-118134 + 60293 x - 11908 x^2 + 1146 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115070 + 59177 x - 11788 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^8 (5 + x)^41
    (1265354 - 765985 x + 188845 x^2 - 24350 x^3 + 1736 x^4 - 65 x^5 + x^6),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114394 + 59073 x - 11784 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113734 + 58969 x - 11780 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (-818 + 283 x - 30 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113266 + 58881 x - 11776 x^2 + 1142 x^3 - 54 x^4 + x^5)};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^4 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2) (-1477 + 399 x - 35 x^2 + x^3)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix}
-16738658 & 11324231 & -3234106 & 506147 & -46934 & 2581 & -78 & 1\\
-16767634 & 11331447 & -3234698 & 506163 & -46934 & 2581 & -78 & 1\\
-16676374 & 11301183 & -3230986 & 505963 & -46930 & 2581 & -78 & 1\\
-16587274 & 11271303 & -3227290 & 505763 & -46926 & 2581 & -78 & 1\\
-16577418 & 11269703 & -3227226 & 505763 & -46926 & 2581 & -78 & 1\\
-16616250 & 11278519 & -3227882 & 505779 & -46926 & 2581 & -78 & 1\\
-16515070 & 11246655 & -3224106 & 505579 & -46922 & 2581 & -78 & 1\\
-16396226 & 11209495 & -3219818 & 505363 & -46918 & 2581 & -78 & 1\\
-16420626 & 11215943 & -3220378 & 505379 & -46918 & 2581 & -78 & 1\\
-16455010 & 11223991 & -3221002 & 505395 & -46918 & 2581 & -78 & 1\\
-16449602 & 11223159 & -3220970 & 505395 & -46918 & 2581 & -78 & 1\\
-16358342 & 11192895 & -3217258 & 505195 & -46914 & 2581 & -78 & 1\\
-16263962 & 11162183 & -3213530 & 504995 & -46910 & 2581 & -78 & 1\\
-16259386 & 11161415 & -3213498 & 504995 & -46910 & 2581 & -78 & 1\\
-16197038 & 11138367 & -3210378 & 504811 & -46906 & 2581 & -78 & 1
\end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-1002780968, 678800388, -193948408, 30362580, -2815896, 154860, -4680, 60}
```

In[•]:=
```
Array[c, 8].Transpose[Coeffmatlist[[i]]]
```

Out[•]=
```
{-16 738 658 c[1] + 11 324 231 c[2] - 3 234 106 c[3] +
  506 147 c[4] - 46 934 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 767 634 c[1] + 11 331 447 c[2] - 3 234 698 c[3] + 506 163 c[4] - 46 934 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 676 374 c[1] + 11 301 183 c[2] -
  3 230 986 c[3] + 505 963 c[4] - 46 930 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 587 274 c[1] + 11 271 303 c[2] - 3 227 290 c[3] + 505 763 c[4] - 46 926 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 577 418 c[1] + 11 269 703 c[2] -
  3 227 226 c[3] + 505 763 c[4] - 46 926 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 616 250 c[1] + 11 278 519 c[2] - 3 227 882 c[3] + 505 779 c[4] - 46 926 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 515 070 c[1] + 11 246 655 c[2] -
  3 224 106 c[3] + 505 579 c[4] - 46 922 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 396 226 c[1] + 11 209 495 c[2] - 3 219 818 c[3] + 505 363 c[4] - 46 918 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 420 626 c[1] + 11 215 943 c[2] -
  3 220 378 c[3] + 505 379 c[4] - 46 918 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 455 010 c[1] + 11 223 991 c[2] - 3 221 002 c[3] + 505 395 c[4] - 46 918 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 449 602 c[1] + 11 223 159 c[2] -
  3 220 970 c[3] + 505 395 c[4] - 46 918 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 358 342 c[1] + 11 192 895 c[2] - 3 217 258 c[3] + 505 195 c[4] - 46 914 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 263 962 c[1] + 11 162 183 c[2] -
  3 213 530 c[3] + 504 995 c[4] - 46 910 c[5] + 2581 c[6] - 78 c[7] + c[8],
 -16 259 386 c[1] + 11 161 415 c[2] - 3 213 498 c[3] + 504 995 c[4] - 46 910 c[5] +
  2581 c[6] - 78 c[7] + c[8], -16 197 038 c[1] + 11 138 367 c[2] -
  3 210 378 c[3] + 504 811 c[4] - 46 906 c[5] + 2581 c[6] - 78 c[7] + c[8]}
```

In[•]:=
```
Array[c, 8].glist[[i]]
```

Out[•]=
```
-1 002 780 968 c[1] + 678 800 388 c[2] - 193 948 408 c[3] +
 30 362 580 c[4] - 2 815 896 c[5] + 154 860 c[6] - 4680 c[7] + 60 c[8]
```

```
In[•]:= certlist[[i]] = Flatten[
          Array[c, 8] /. FindInstance[-1 002 780 968 c[1] + 678 800 388 c[2] - 193 948 408 c[3] +
                30 362 580 c[4] - 2 815 896 c[5] + 154 860 c[6] - 4680 c[7] + 60 c[8] < 0 &&
              -16 738 658 c[1] + 11 324 231 c[2] - 3 234 106 c[3] + 506 147 c[4] - 46 934 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 767 634 c[1] + 11 331 447 c[2] -
                3 234 698 c[3] + 506 163 c[4] - 46 934 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 676 374 c[1] + 11 301 183 c[2] - 3 230 986 c[3] + 505 963 c[4] - 46 930 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 587 274 c[1] + 11 271 303 c[2] -
                3 227 290 c[3] + 505 763 c[4] - 46 926 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 577 418 c[1] + 11 269 703 c[2] - 3 227 226 c[3] + 505 763 c[4] - 46 926 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 616 250 c[1] + 11 278 519 c[2] -
                3 227 882 c[3] + 505 779 c[4] - 46 926 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 515 070 c[1] + 11 246 655 c[2] - 3 224 106 c[3] + 505 579 c[4] - 46 922 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 396 226 c[1] + 11 209 495 c[2] -
                3 219 818 c[3] + 505 363 c[4] - 46 918 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 420 626 c[1] + 11 215 943 c[2] - 3 220 378 c[3] + 505 379 c[4] - 46 918 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 455 010 c[1] + 11 223 991 c[2] -
                3 221 002 c[3] + 505 395 c[4] - 46 918 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 449 602 c[1] + 11 223 159 c[2] - 3 220 970 c[3] + 505 395 c[4] - 46 918 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 358 342 c[1] + 11 192 895 c[2] -
                3 217 258 c[3] + 505 195 c[4] - 46 914 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 263 962 c[1] + 11 162 183 c[2] - 3 213 530 c[3] + 504 995 c[4] - 46 910 c[5] +
                2581 c[6] - 78 c[7] + c[8] ≥ 0 && -16 259 386 c[1] + 11 161 415 c[2] -
                3 213 498 c[3] + 504 995 c[4] - 46 910 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0 &&
              -16 197 038 c[1] + 11 138 367 c[2] - 3 210 378 c[3] + 504 811 c[4] -
                46 906 c[5] + 2581 c[6] - 78 c[7] + c[8] ≥ 0, Array[c, 8], Integers]]

Out[•]= {-539 442, -5 293 439, -48 329 261, -379 343 200, -1 845 487 303, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] /
          GCD[-539 442, -5 293 439, -48 329 261, -379 343 200, -1 845 487 303, 0, 0, 0]

Out[•]= {-539 442, -5 293 439, -48 329 261, -379 343 200, -1 845 487 303, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[•]= -988 235 500

Out[•]= {40 532 695, 15 379 575, 75 014 507, 40 431 759,
         100 121 103, 15 278 639, 100 078 627, 15 137 639, 40 322 727,
         74 867 047, 15 169 607, 74 804 539, 49 572 927, 99 911 135, 99 868 659}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[•]= 27

Out[•]= (-13 + x)^4 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2) (-1477 + 399 x - 35 x^2 + x^3)

Out[•]= {0, 0, 0, -1 845 487 303, -379 343 200, -48 329 261, -5 293 439, -539 442}
```

```
In[ ]:= i = 28;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^4 (-11 + x)^7 (-9 + x) (5 + x)^41 (139 - 24 x + x^2)^2 (82 - 19 x + x^2),
           (-13 + x)^4 (-11 + x)^9 (-6 + x) (5 + x)^41 (139 - 24 x + x^2)^2,
           (-13 + x)^4 (-11 + x)^7 (5 + x)^41 (139 - 24 x + x^2)
            (-101 234 + 52 623 x - 10 690 x^2 + 1064 x^3 - 52 x^4 + x^5), (-13 + x)^4 (-11 + x)^7
            (5 + x)^41 (139 - 24 x + x^2) (-100 678 + 52 527 x - 10 686 x^2 + 1064 x^3 - 52 x^4 + x^5)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
Out[ ]= (-13 + x)^5 (-11 + x)^8 (-9 + x) (5 + x)^42 (139 - 24 x + x^2)^2

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -102 582   52 879   -10 702   1064   -52   1
          -100 914   52 591   -10 690   1064   -52   1
          -101 234   52 623   -10 690   1064   -52   1
          -100 678   52 527   -10 686   1064   -52   1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-6 110 696, 3 164 420, -641 736, 63 840, -3120, 60}

In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[ ]= {-102 582 c[1] + 52 879 c[2] - 10 702 c[3] + 1064 c[4] - 52 c[5] + c[6],
         -100 914 c[1] + 52 591 c[2] - 10 690 c[3] + 1064 c[4] - 52 c[5] + c[6],
         -101 234 c[1] + 52 623 c[2] - 10 690 c[3] + 1064 c[4] - 52 c[5] + c[6],
         -100 678 c[1] + 52 527 c[2] - 10 686 c[3] + 1064 c[4] - 52 c[5] + c[6]}

In[ ]:= Array[c, 6].glist[[i]]
Out[ ]= -6 110 696 c[1] + 3 164 420 c[2] - 641 736 c[3] + 63 840 c[4] - 3120 c[5] + 60 c[6]

In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
            -6 110 696 c[1] + 3 164 420 c[2] - 641 736 c[3] + 63 840 c[4] - 3120 c[5] + 60 c[6] < 0 &&
             -102 582 c[1] + 52 879 c[2] - 10 702 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0 &&
             -100 914 c[1] + 52 591 c[2] - 10 690 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0 &&
             -101 234 c[1] + 52 623 c[2] - 10 690 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0 &&
             -100 678 c[1] + 52 527 c[2] - 10 686 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0,
            Array[c, 6], Integers]]
Out[ ]= {51 999, 519 994, 5 251 934, 0, 0, 34 043 665 264}
```

In[ ]:=
```
certlist[[i]] = certlist[[i]] / GCD[51 999, 519 994, 5 251 934, 0, 0, 34 043 665 264]
```

Out[ ]=
```
{51 999, 519 994, 5 251 934, 0, 0, 34 043 665 264}
```

In[ ]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[ ]=
```
-5 869 408
```

Out[ ]=
```
{68 904, 68 172, 68 300, 68 056}
```

In[ ]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[ ]=
```
28
```

Out[ ]=
$$(-13+x)^5\,(-11+x)^8\,(-9+x)\,(5+x)^{42}\left(139-24\,x+x^2\right)^2$$

Out[ ]=
```
{34 043 665 264, 0, 0, 5 251 934, 519 994, 51 999}
```

In[ ]:=
```
i = 29;
chilist[[i]]
interlacingpolylist[[i]] =
  {(-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8978 - 3883 x + 609 x^2 - 41 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-116 054 + 59 353 x - 11 796 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (95 - 20 x + x^2) (-1226 + 367 x - 34 x^2 + x^3),
   (-15 + x) (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (-698 + 249 x - 28 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-115 586 + 59 265 x - 11 792 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114 718 + 59 145 x - 11 788 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-802 + 279 x - 30 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8854 - 3871 x + 609 x^2 - 41 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114 042 + 59 041 x - 11 784 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-978 + 321 x - 32 x^2 + x^3),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113 382 + 58 937 x - 11 780 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-113 766 + 58 969 x - 11 780 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^4 (-11 + x)^10 (5 + x)^41 (10 262 - 4417 x + 669 x^2 - 43 x^3 + x^4),
   (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-786 + 279 x - 30 x^2 + x^3),
   (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8678 - 3855 x + 609 x^2 - 41 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-111 754 + 58 657 x - 11 768 x^2 + 1142 x^3 - 54 x^4 + x^5),
   (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8626 - 3851 x + 609 x^2 - 41 x^3 + x^4),
   (-13 + x)^4 (-11 + x)^11 (5 + x)^41 (-914 + 317 x - 32 x^2 + x^3),
   (-14 + x) (-13 + x)^5 (-11 + x)^11 (-5 + x) (5 + x)^41};
Coeffmatlist[[i]] =
  CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

Out[ ]=
$$(-15+x)\,(-13+x)^5\,(-11+x)^{10}\,(5+x)^{42}\,\left(95-20\,x+x^2\right)$$

In[ ]:=
```
Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

-116 714	59 457	-11 800	1142	-54	1
-116 054	59 353	-11 796	1142	-54	1
-116 470	59 385	-11 796	1142	-54	1
-115 170	59 233	-11 792	1142	-54	1
-115 586	59 265	-11 792	1142	-54	1
-114 718	59 145	-11 788	1142	-54	1
-114 686	59 145	-11 788	1142	-54	1
-115 102	59 177	-11 788	1142	-54	1
-114 042	59 041	-11 784	1142	-54	1
-114 426	59 073	-11 784	1142	-54	1
-113 382	58 937	-11 780	1142	-54	1
-113 766	58 969	-11 780	1142	-54	1
-112 882	58 849	-11 776	1142	-54	1
-112 398	58 761	-11 772	1142	-54	1
-112 814	58 793	-11 772	1142	-54	1
-111 754	58 657	-11 768	1142	-54	1
-112 138	58 689	-11 768	1142	-54	1
-110 594	58 465	-11 760	1142	-54	1
-110 110	58 377	-11 756	1142	-54	1

In[•]:=
```
glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

Out[•]=
$$\{-6\,958\,000,\ 3\,559\,180,\ -707\,640,\ 68\,520,\ -3240,\ 60\}$$

In[•]:=
```
Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

Out[•]=
$$\begin{aligned}
&\{-116\,714\,c[1] + 59\,457\,c[2] - 11\,800\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -116\,054\,c[1] + 59\,353\,c[2] - 11\,796\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -116\,470\,c[1] + 59\,385\,c[2] - 11\,796\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -115\,170\,c[1] + 59\,233\,c[2] - 11\,792\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -115\,586\,c[1] + 59\,265\,c[2] - 11\,792\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -114\,718\,c[1] + 59\,145\,c[2] - 11\,788\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -114\,686\,c[1] + 59\,145\,c[2] - 11\,788\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -115\,102\,c[1] + 59\,177\,c[2] - 11\,788\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -114\,042\,c[1] + 59\,041\,c[2] - 11\,784\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -114\,426\,c[1] + 59\,073\,c[2] - 11\,784\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -113\,382\,c[1] + 58\,937\,c[2] - 11\,780\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -113\,766\,c[1] + 58\,969\,c[2] - 11\,780\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -112\,882\,c[1] + 58\,849\,c[2] - 11\,776\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -112\,398\,c[1] + 58\,761\,c[2] - 11\,772\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -112\,814\,c[1] + 58\,793\,c[2] - 11\,772\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -111\,754\,c[1] + 58\,657\,c[2] - 11\,768\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -112\,138\,c[1] + 58\,689\,c[2] - 11\,768\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -110\,594\,c[1] + 58\,465\,c[2] - 11\,760\,c[3] + 1142\,c[4] - 54\,c[5] + c[6],\\
&\ -110\,110\,c[1] + 58\,377\,c[2] - 11\,756\,c[3] + 1142\,c[4] - 54\,c[5] + c[6]\}
\end{aligned}$$

In[•]:=
```
Array[c, 6].glist[[i]]
```

Out[•]=
$$-6\,958\,000\,c[1] + 3\,559\,180\,c[2] - 707\,640\,c[3] + 68\,520\,c[4] - 3240\,c[5] + 60\,c[6]$$

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -6 958 000 c[1] + 3 559 180 c[2] - 707 640 c[3] + 68 520 c[4] - 3240 c[5] + 60 c[6] < 0 &&
           -116 714 c[1] + 59 457 c[2] - 11 800 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 054 c[1] + 59 353 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -116 470 c[1] + 59 385 c[2] - 11 796 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 170 c[1] + 59 233 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 586 c[1] + 59 265 c[2] - 11 792 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 718 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 686 c[1] + 59 145 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -115 102 c[1] + 59 177 c[2] - 11 788 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 042 c[1] + 59 041 c[2] - 11 784 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -114 426 c[1] + 59 073 c[2] - 11 784 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -113 382 c[1] + 58 937 c[2] - 11 780 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -113 766 c[1] + 58 969 c[2] - 11 780 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 882 c[1] + 58 849 c[2] - 11 776 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 398 c[1] + 58 761 c[2] - 11 772 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 814 c[1] + 58 793 c[2] - 11 772 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -111 754 c[1] + 58 657 c[2] - 11 768 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -112 138 c[1] + 58 689 c[2] - 11 768 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -110 594 c[1] + 58 465 c[2] - 11 760 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0 &&
           -110 110 c[1] + 58 377 c[2] - 11 756 c[3] + 1142 c[4] - 54 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]

Out[•]= {47 069, 611 903, 8 048 870, 0, 0, 64 088 824 162}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[47 069, 611 903, 8 048 870, 0, 0, 64 088 824 162]

Out[•]= {47 069, 611 903, 8 048 870, 0, 0, 64 088 824 162}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]

Out[•]= -6 099 540

Out[•]= {463 567, 86 675, 86 867, 462 791, 462 983, 85 995,
         1 592 203, 1 592 395, 462 207, 1 968 607, 85 315, 1 591 715, 1 967 831,
         3 097 243, 3 097 435, 1 967 247, 3 473 647, 3 472 871, 4 602 283}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]

Out[•]= 29

Out[•]= (-15 + x) (-13 + x)^5 (-11 + x)^10 (5 + x)^42 (95 - 20 x + x^2)

Out[•]= {64 088 824 162, 0, 0, 8 048 870, 611 903, 47 069}
```

```
In[ ]:= i = 30;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8854 - 3871 x + 609 x^2 - 41 x^3 + x^4),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8886 - 3871 x + 609 x^2 - 41 x^3 + x^4),
           (-13 + x)^5 (-11 + x)^8 (5 + x)^41 (-97 714 + 51 467 x - 10 570 x^2 + 1060 x^3 - 52 x^4 + x^5),
           (-13 + x)^5 (-11 + x)^9 (-9 + x) (5 + x)^41 (-978 + 321 x - 32 x^2 + x^3),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8834 - 3867 x + 609 x^2 - 41 x^3 + x^4),
           (-13 + x)^5 (-11 + x)^8 (5 + x)^41 (-97 142 + 51 371 x - 10 566 x^2 + 1060 x^3 - 52 x^4 + x^5),
           (-13 + x)^5 (-11 + x)^10 (5 + x)^41 (-786 + 279 x - 30 x^2 + x^3),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8678 - 3855 x + 609 x^2 - 41 x^3 + x^4),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-670 + 245 x - 28 x^2 + x^3),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8626 - 3851 x + 609 x^2 - 41 x^3 + x^4),
           (-13 + x)^6 (-11 + x)^9 (-9 + x) (5 + x)^41 (74 - 19 x + x^2),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-654 + 245 x - 28 x^2 + x^3),
           (-13 + x)^7 (-11 + x)^9 (-10 + x) (-5 + x) (5 + x)^41};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^6 (-11 + x)^9 (5 + x)^42 (-1207 + 351 x - 33 x^2 + x^3)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ⎛ -97 394  51 435  -10 570  1060  -52  1 ⎞
        ⎜ -97 746  51 467  -10 570  1060  -52  1 ⎟
        ⎜ -97 714  51 467  -10 570  1060  -52  1 ⎟
        ⎜ -96 822  51 339  -10 566  1060  -52  1 ⎟
        ⎜ -97 174  51 371  -10 566  1060  -52  1 ⎟
        ⎜ -97 142  51 371  -10 566  1060  -52  1 ⎟
        ⎜ -95 106  51 051  -10 554  1060  -52  1 ⎟
        ⎜ -95 458  51 083  -10 554  1060  -52  1 ⎟
        ⎜ -95 810  51 115  -10 554  1060  -52  1 ⎟
        ⎜ -94 886  50 987  -10 550  1060  -52  1 ⎟
        ⎜ -95 238  51 019  -10 550  1060  -52  1 ⎟
        ⎜ -93 522  50 731  -10 538  1060  -52  1 ⎟
        ⎝ -92 950  50 635  -10 534  1060  -52  1 ⎠

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-5 893 872, 3 094 836, -634 560, 63 600, -3120, 60}
```

```
In[•]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= {-97 394 c[1] + 51 435 c[2] - 10 570 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -97 746 c[1] + 51 467 c[2] - 10 570 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -97 714 c[1] + 51 467 c[2] - 10 570 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -96 822 c[1] + 51 339 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -97 174 c[1] + 51 371 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -97 142 c[1] + 51 371 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -95 106 c[1] + 51 051 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -95 458 c[1] + 51 083 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -95 810 c[1] + 51 115 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -94 886 c[1] + 50 987 c[2] - 10 550 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -95 238 c[1] + 51 019 c[2] - 10 550 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -93 522 c[1] + 50 731 c[2] - 10 538 c[3] + 1060 c[4] - 52 c[5] + c[6],
  -92 950 c[1] + 50 635 c[2] - 10 534 c[3] + 1060 c[4] - 52 c[5] + c[6]}
```

```
In[•]:= Array[c, 6].glist[[i]]
```

```
Out[•]= -5 893 872 c[1] + 3 094 836 c[2] - 634 560 c[3] + 63 600 c[4] - 3120 c[5] + 60 c[6]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
      -5 893 872 c[1] + 3 094 836 c[2] - 634 560 c[3] + 63 600 c[4] - 3120 c[5] + 60 c[6] < 0 &&
       -97 394 c[1] + 51 435 c[2] - 10 570 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -97 746 c[1] + 51 467 c[2] - 10 570 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -97 714 c[1] + 51 467 c[2] - 10 570 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -96 822 c[1] + 51 339 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -97 174 c[1] + 51 371 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -97 142 c[1] + 51 371 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -95 106 c[1] + 51 051 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -95 458 c[1] + 51 083 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -95 810 c[1] + 51 115 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -94 886 c[1] + 50 987 c[2] - 10 550 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -95 238 c[1] + 51 019 c[2] - 10 550 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -93 522 c[1] + 50 731 c[2] - 10 538 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
       -92 950 c[1] + 50 635 c[2] - 10 534 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0,
      Array[c, 6], Integers]]
```

```
Out[•]= {599, 1139, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]] = certlist[[i]] / GCD[599, 1139, 0, 0, 0, 0]
```

```
Out[•]= {599, 1139, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -5 411 124
```

```
Out[•]= {245 459, 71 059, 90 227, 478 743, 304 343, 323 511, 1 178 595,
  1 004 195, 829 795, 1 237 479, 1 063 079, 1 762 931, 1 996 215}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 30
```

$$\text{Out[ ]= } (-13+x)^6\,(-11+x)^9\,(5+x)^{42}\left(-1207+351\,x-33\,x^2+x^3\right)$$

```
Out[ ]= {0, 0, 0, 0, 1139, 599}
```

```
In[ ]:= i = 31;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^6 (-11 + x)^8 (-9 + x)^2 (5 + x)^41 (94 - 21 x + x^2),
            (-13 + x)^6 (-11 + x)^7 (-9 + x)^2 (5 + x)^41 (-1030 + 325 x - 32 x^2 + x^3),
            (-13 + x)^5 (-11 + x)^8 (-9 + x) (5 + x)^41 (10 818 - 4513 x + 673 x^2 - 43 x^3 + x^4), (-13 + x)^5
             (-11 + x)^7 (-9 + x) (5 + x)^41 (-119 382 + 60 493 x - 11 916 x^2 + 1146 x^3 - 54 x^4 + x^5),
            (-13 + x)^6 (-11 + x)^8 (5 + x)^41 (7514 - 3381 x + 553 x^2 - 39 x^3 + x^4),
            (-13 + x)^5 (-11 + x)^8 (5 + x)^41 (-97 142 + 51 371 x - 10 566 x^2 + 1060 x^3 - 52 x^4 + x^5),
            (-13 + x)^6 (-11 + x)^8 (-10 + x) (-9 + x) (5 + x)^41 (83 - 20 x + x^2), (-13 + x)^5 (-11 + x)^7
             (5 + x)^41 (1 064 350 - 660 963 x + 167 473 x^2 - 22 222 x^3 + 1632 x^4 - 63 x^5 + x^6),
            (-13 + x)^6 (-11 + x)^7 (-9 + x) (5 + x)^41 (9094 - 3939 x + 613 x^2 - 41 x^3 + x^4),
            (-13 + x)^6 (-11 + x)^7 (5 + x)^41 (59 - 16 x + x^2) (-1382 + 379 x - 34 x^2 + x^3),
            (-13 + x)^5 (-11 + x)^8 (5 + x)^41 (-95 842 + 51 115 x - 10 554 x^2 + 1060 x^3 - 52 x^4 + x^5),
            (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-670 + 245 x - 28 x^2 + x^3),
            (-13 + x)^6 (-11 + x)^9 (-9 + x) (5 + x)^41 (74 - 19 x + x^2)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
```

$$\text{Out[ ]= } (-13+x)^6\,(-11+x)^8\,(-9+x)\,(5+x)^{42}\left(-1477+399\,x-35\,x^2+x^3\right)$$

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix}
1\,088\,802 & -667\,935 & 168\,125 & -22\,242 & 1632 & -63 & 1\\
1\,084\,590 & -666\,675 & 168\,001 & -22\,238 & 1632 & -63 & 1\\
1\,070\,982 & -663\,147 & 167\,705 & -22\,230 & 1632 & -63 & 1\\
1\,074\,438 & -663\,819 & 167\,737 & -22\,230 & 1632 & -63 & 1\\
1\,074\,502 & -663\,819 & 167\,737 & -22\,230 & 1632 & -63 & 1\\
1\,068\,562 & -662\,223 & 167\,597 & -22\,226 & 1632 & -63 & 1\\
1\,068\,210 & -662\,191 & 167\,597 & -22\,226 & 1632 & -63 & 1\\
1\,064\,350 & -660\,963 & 167\,473 & -22\,222 & 1632 & -63 & 1\\
1\,063\,998 & -660\,931 & 167\,473 & -22\,222 & 1632 & -63 & 1\\
1\,059\,994 & -659\,687 & 167\,349 & -22\,218 & 1632 & -63 & 1\\
1\,054\,262 & -658\,107 & 167\,209 & -22\,214 & 1632 & -63 & 1\\
1\,053\,910 & -658\,075 & 167\,209 & -22\,214 & 1632 & -63 & 1\\
1\,047\,618 & -656\,447 & 167\,069 & -22\,210 & 1632 & -63 & 1
\end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {64 915 088, -39 954 796, 10 075 892, -1 334 160, 97 920, -3780, 60}

In[ ]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
Out[ ]= {1 088 802 c[1] - 667 935 c[2] + 168 125 c[3] - 22 242 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 084 590 c[1] - 666 675 c[2] + 168 001 c[3] - 22 238 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 070 982 c[1] - 663 147 c[2] + 167 705 c[3] - 22 230 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 074 438 c[1] - 663 819 c[2] + 167 737 c[3] - 22 230 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 074 502 c[1] - 663 819 c[2] + 167 737 c[3] - 22 230 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 068 562 c[1] - 662 223 c[2] + 167 597 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 068 210 c[1] - 662 191 c[2] + 167 597 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 064 350 c[1] - 660 963 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 063 998 c[1] - 660 931 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 059 994 c[1] - 659 687 c[2] + 167 349 c[3] - 22 218 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 054 262 c[1] - 658 107 c[2] + 167 209 c[3] - 22 214 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 053 910 c[1] - 658 075 c[2] + 167 209 c[3] - 22 214 c[4] + 1632 c[5] - 63 c[6] + c[7],
  1 047 618 c[1] - 656 447 c[2] + 167 069 c[3] - 22 210 c[4] + 1632 c[5] - 63 c[6] + c[7]}

In[ ]:= Array[c, 7].glist[[i]]
Out[ ]= 64 915 088 c[1] - 39 954 796 c[2] + 10 075 892 c[3] -
  1 334 160 c[4] + 97 920 c[5] - 3780 c[6] + 60 c[7]

In[ ]:= certlist[[i]] = Flatten[
   Array[c, 7] /. FindInstance[64 915 088 c[1] - 39 954 796 c[2] + 10 075 892 c[3] -
        1 334 160 c[4] + 97 920 c[5] - 3780 c[6] + 60 c[7] < 0 &&
      1 088 802 c[1] - 667 935 c[2] + 168 125 c[3] - 22 242 c[4] + 1632 c[5] -
        63 c[6] + c[7] ≥ 0 && 1 084 590 c[1] - 666 675 c[2] + 168 001 c[3] -
        22 238 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 && 1 070 982 c[1] -
        663 147 c[2] + 167 705 c[3] - 22 230 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
      1 074 438 c[1] - 663 819 c[2] + 167 737 c[3] - 22 230 c[4] + 1632 c[5] -
        63 c[6] + c[7] ≥ 0 && 1 074 502 c[1] - 663 819 c[2] + 167 737 c[3] -
        22 230 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 && 1 068 562 c[1] -
        662 223 c[2] + 167 597 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
      1 068 210 c[1] - 662 191 c[2] + 167 597 c[3] - 22 226 c[4] + 1632 c[5] -
        63 c[6] + c[7] ≥ 0 && 1 064 350 c[1] - 660 963 c[2] + 167 473 c[3] -
        22 222 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 && 1 063 998 c[1] -
        660 931 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
      1 059 994 c[1] - 659 687 c[2] + 167 349 c[3] - 22 218 c[4] + 1632 c[5] -
        63 c[6] + c[7] ≥ 0 && 1 054 262 c[1] - 658 107 c[2] +
        167 209 c[3] - 22 214 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
      1 053 910 c[1] - 658 075 c[2] + 167 209 c[3] - 22 214 c[4] + 1632 c[5] -
        63 c[6] + c[7] ≥ 0 && 1 047 618 c[1] - 656 447 c[2] + 167 069 c[3] -
        22 210 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
Out[ ]= {-68 325, -316 527, -814 967, 0, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-68 325, -316 527, -814 967, 0, 0, 0, 0]
Out[ ]= {-68 325, -316 527, -814 967, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -71 149 672
```

```
Out[•]= {10 738 220, 755 008, 55 044 584, 5 540 584, 1 167 784, 15 936 572,
          29 858 108, 5 953 360, 19 874 896, 744 516, 6 366 136, 20 287 672, 48 977 996}
```

```
In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[•]= 31
```

$$\text{Out[•]= } (-13+x)^6\,(-11+x)^8\,(-9+x)\,(5+x)^{42}\left(-1477+399\,x-35\,x^2+x^3\right)$$

```
Out[•]= {0, 0, 0, 0, -814 967, -316 527, -68 325}
```

```
In[•]:= i = 32;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^4 (-11 + x)^9 (5 + x)^41 (113 - 22 x + x^2) (-1026 + 325 x - 32 x^2 + x^3),
           (-14 + x) (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (113 - 22 x + x^2) (73 - 18 x + x^2),
           (-13 + x)^6 (-11 + x)^8 (5 + x)^41 (7514 - 3381 x + 553 x^2 - 39 x^3 + x^4),
           (-13 + x)^6 (-11 + x)^8 (-10 + x) (-9 + x) (5 + x)^41 (83 - 20 x + x^2),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (139 - 24 x + x^2) (-818 + 283 x - 30 x^2 + x^3),
           (-13 + x)^4 (-11 + x)^9 (5 + x)^41 (-114 150 + 59 001 x - 11 780 x^2 + 1142 x^3 - 54 x^4 + x^5),
           (-13 + x)^4 (-11 + x)^8 (5 + x)^41
            (1 255 234 - 763 129 x + 188 581 x^2 - 24 342 x^3 + 1736 x^4 - 65 x^5 + x^6),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8678 - 3855 x + 609 x^2 - 41 x^3 + x^4),
           (-13 + x)^5 (-11 + x)^9 (5 + x)^41 (8626 - 3851 x + 609 x^2 - 41 x^3 + x^4)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

$$\text{Out[•]= } (-13+x)^5\,(-11+x)^9\,(5+x)^{42}\left(139-24\,x+x^2\right)\left(113-22\,x+x^2\right)$$

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1\,275\,318 & -768\,205 & 189\,009 & -24\,354 & 1736 & -65 & 1 \\ 1\,270\,346 & -766\,785 & 188\,877 & -24\,350 & 1736 & -65 & 1 \\ 1\,269\,866 & -766\,753 & 188\,877 & -24\,350 & 1736 & -65 & 1 \\ 1\,262\,430 & -764\,933 & 188\,729 & -24\,346 & 1736 & -65 & 1 \\ 1\,250\,722 & -762\,361 & 188\,549 & -24\,342 & 1736 & -65 & 1 \\ 1\,255\,650 & -763\,161 & 188\,581 & -24\,342 & 1736 & -65 & 1 \\ 1\,255\,234 & -763\,129 & 188\,581 & -24\,342 & 1736 & -65 & 1 \\ 1\,240\,954 & -759\,537 & 188\,285 & -24\,334 & 1736 & -65 & 1 \\ 1\,233\,518 & -757\,717 & 188\,137 & -24\,330 & 1736 & -65 & 1 \end{pmatrix}$$

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {76 702 672, -46 144 500, 11 345 012, -1 461 360, 104 160, -3900, 60}

In[ ]:= Array[c, 7].Transpose[Coeffmatlist[[i]]]
Out[ ]= {1 275 318 c[1] - 768 205 c[2] + 189 009 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 270 346 c[1] - 766 785 c[2] + 188 877 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 269 866 c[1] - 766 753 c[2] + 188 877 c[3] - 24 350 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 262 430 c[1] - 764 933 c[2] + 188 729 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 250 722 c[1] - 762 361 c[2] + 188 549 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 255 650 c[1] - 763 161 c[2] + 188 581 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 255 234 c[1] - 763 129 c[2] + 188 581 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 240 954 c[1] - 759 537 c[2] + 188 285 c[3] - 24 334 c[4] + 1736 c[5] - 65 c[6] + c[7],
  1 233 518 c[1] - 757 717 c[2] + 188 137 c[3] - 24 330 c[4] + 1736 c[5] - 65 c[6] + c[7]}

In[ ]:= Array[c, 7].glist[[i]]
Out[ ]= 76 702 672 c[1] - 46 144 500 c[2] + 11 345 012 c[3] -
  1 461 360 c[4] + 104 160 c[5] - 3900 c[6] + 60 c[7]

In[ ]:= certlist[[i]] = Flatten[
    Array[c, 7] /. FindInstance[76 702 672 c[1] - 46 144 500 c[2] + 11 345 012 c[3] -
        1 461 360 c[4] + 104 160 c[5] - 3900 c[6] + 60 c[7] < 0 && 1 275 318 c[1] -
        768 205 c[2] + 189 009 c[3] - 24 354 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
      1 270 346 c[1] - 766 785 c[2] + 188 877 c[3] - 24 350 c[4] + 1736 c[5] -
        65 c[6] + c[7] ≥ 0 && 1 269 866 c[1] - 766 753 c[2] + 188 877 c[3] -
        24 350 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 262 430 c[1] -
        764 933 c[2] + 188 729 c[3] - 24 346 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
      1 250 722 c[1] - 762 361 c[2] + 188 549 c[3] - 24 342 c[4] + 1736 c[5] -
        65 c[6] + c[7] ≥ 0 && 1 255 650 c[1] - 763 161 c[2] + 188 581 c[3] -
        24 342 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 && 1 255 234 c[1] -
        763 129 c[2] + 188 581 c[3] - 24 342 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0 &&
      1 240 954 c[1] - 759 537 c[2] + 188 285 c[3] - 24 334 c[4] + 1736 c[5] -
        65 c[6] + c[7] ≥ 0 && 1 233 518 c[1] - 757 717 c[2] + 188 137 c[3] -
        24 330 c[4] + 1736 c[5] - 65 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
Out[ ]= {-1400, -2326, 0, 0, 0, 0, 0}

In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-1400, -2326, 0, 0, 0, 0, 0]
Out[ ]= {-700, -1163, 0, 0, 0, 0, 0}
```

```
In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= -25 816 900
```

```
Out[ ]= {699 815, 2 528 755, 2 827 539, 5 916 079,
          11 120 443, 8 601 243, 8 855 227, 14 673 731, 17 762 271}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 32
```

```
Out[ ]= (-13 + x)^5 (-11 + x)^9 (5 + x)^42 (139 - 24 x + x^2) (113 - 22 x + x^2)
```

```
Out[ ]= {0, 0, 0, 0, 0, -1163, -700}
```

```
In[ ]:= i = 33;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^6 (-11 + x)^7 (-9 + x)^2 (5 + x)^41 (-1018 + 325 x - 32 x^2 + x^3),
           (-13 + x)^7 (-11 + x)^6 (-10 + x) (-9 + x) (5 + x)^41 (-775 + 267 x - 29 x^2 + x^3),
           (-13 + x)^8 (-11 + x)^7 (-9 + x)^2 (-6 + x) (5 + x)^41};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
```

```
Out[ ]= (-13 + x)^7 (-11 + x)^7 (-9 + x)^2 (5 + x)^42 (139 - 24 x + x^2)
```

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
```

```
Out[ ]//MatrixForm=
( -100 782   52 535   -10 686   1064   -52   1 )
( -100 750   52 535   -10 686   1064   -52   1 )
( -100 386   52 455   -10 682   1064   -52   1 )
```

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]] // Factor, x]
```

```
Out[ ]= {-6 109 456, 3 164 548, -641 760, 63 840, -3120, 60}
```

```
In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= {-100 782 c[1] + 52 535 c[2] - 10 686 c[3] + 1064 c[4] - 52 c[5] + c[6],
         -100 750 c[1] + 52 535 c[2] - 10 686 c[3] + 1064 c[4] - 52 c[5] + c[6],
         -100 386 c[1] + 52 455 c[2] - 10 682 c[3] + 1064 c[4] - 52 c[5] + c[6]}
```

```
In[ ]:= Array[c, 6].glist[[i]]
```

```
Out[ ]= -6 109 456 c[1] + 3 164 548 c[2] - 641 760 c[3] + 63 840 c[4] - 3120 c[5] + 60 c[6]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -6 109 456 c[1] + 3 164 548 c[2] - 641 760 c[3] + 63 840 c[4] - 3120 c[5] + 60 c[6] < 0 &&
           -100 782 c[1] + 52 535 c[2] - 10 686 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -100 750 c[1] + 52 535 c[2] - 10 686 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -100 386 c[1] + 52 455 c[2] - 10 682 c[3] + 1064 c[4] - 52 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]
```

```
Out[•]= {258, 496, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]] = certlist[[i]] / GCD[258, 496, 0, 0, 0, 0]
```

```
Out[•]= {129, 248, 0, 0, 0, 0}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -3 311 920
```

```
Out[•]= {27 802, 31 930, 59 046}
```

```
In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[•]= 33
```

```
Out[•]= (-13 + x)^7 (-11 + x)^7 (-9 + x)^2 (5 + x)^42 (139 - 24 x + x^2)
```

```
Out[•]= {0, 0, 0, 0, 248, 129}
```

```
In[ ]:= i = 34;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^6 (-11 + x)^7 (5 + x)^41 (139 - 24 x + x^2) (-598 + 219 x - 26 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^8 (5 + x)^41 (7514 - 3381 x + 553 x^2 - 39 x^3 + x^4),
           (-13 + x)^6 (-11 + x)^8 (-10 + x) (-9 + x) (5 + x)^41 (83 - 20 x + x^2),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-682 + 245 x - 28 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^7 (-10 + x) (5 + x)^41 (113 - 22 x + x^2) (73 - 18 x + x^2), (-13 + x)^5
            (-11 + x)^7 (5 + x)^41 (1064350 - 660963 x + 167473 x^2 - 22222 x^3 + 1632 x^4 - 63 x^5 + x^6),
           (-13 + x)^6 (-11 + x)^7 (-9 + x) (5 + x)^41 (9094 - 3939 x + 613 x^2 - 41 x^3 + x^4),
           (-13 + x)^6 (-11 + x)^7 (5 + x)^41 (73 - 18 x + x^2) (-1126 + 333 x - 32 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^7 (5 + x)^41 (59 - 16 x + x^2) (-1382 + 379 x - 34 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^7 (5 + x)^41 (95 - 20 x + x^2) (-862 + 287 x - 30 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-670 + 245 x - 28 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^8 (5 + x)^41 (7402 - 3365 x + 553 x^2 - 39 x^3 + x^4),
           (-13 + x)^6 (-11 + x)^9 (-9 + x) (5 + x)^41 (74 - 19 x + x^2),
           (-13 + x)^7 (-11 + x)^8 (5 + x)^41 (-566 + 215 x - 26 x^2 + x^3),
           (-13 + x)^7 (-11 + x)^7 (-9 + x) (5 + x)^41 (-686 + 249 x - 28 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-654 + 245 x - 28 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^6 (-11 + x)^8 (5 + x)^42 (139 - 24 x + x^2) (95 - 20 x + x^2)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( 1080586  -665431  167877  -22234  1632  -63  1 )
        ( 1074502  -663819  167737  -22230  1632  -63  1 )
        ( 1068210  -662191  167597  -22226  1632  -63  1 )
        ( 1072786  -662959  167629  -22226  1632  -63  1 )
        ( 1072370  -662927  167629  -22226  1632  -63  1 )
        ( 1064350  -660963  167473  -22222  1632  -63  1 )
        ( 1063998  -660931  167473  -22222  1632  -63  1 )
        ( 1068574  -661699  167505  -22222  1632  -63  1 )
        ( 1059994  -659687  167349  -22218  1632  -63  1 )
        ( 1064570  -660455  167381  -22218  1632  -63  1 )
        ( 1053910  -658075  167209  -22214  1632  -63  1 )
        ( 1058486  -658843  167241  -22214  1632  -63  1 )
        ( 1047618  -656447  167069  -22210  1632  -63  1 )
        ( 1052194  -657215  167101  -22210  1632  -63  1 )
        ( 1043406  -655187  166945  -22206  1632  -63  1 )
        ( 1028742  -651563  166649  -22198  1632  -63  1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {64467080, -39815860, 10061948, -1333704, 97920, -3780, 60}
```

In[•]:=
```
Array[c, 7].Transpose[Coeffmatlist[[i]]]
```

Out[•]=
```
{1 080 586 c[1] - 665 431 c[2] + 167 877 c[3] - 22 234 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 074 502 c[1] - 663 819 c[2] + 167 737 c[3] - 22 230 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 068 210 c[1] - 662 191 c[2] + 167 597 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 072 786 c[1] - 662 959 c[2] + 167 629 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 072 370 c[1] - 662 927 c[2] + 167 629 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 064 350 c[1] - 660 963 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 063 998 c[1] - 660 931 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 068 574 c[1] - 661 699 c[2] + 167 505 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 059 994 c[1] - 659 687 c[2] + 167 349 c[3] - 22 218 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 064 570 c[1] - 660 455 c[2] + 167 381 c[3] - 22 218 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 053 910 c[1] - 658 075 c[2] + 167 209 c[3] - 22 214 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 058 486 c[1] - 658 843 c[2] + 167 241 c[3] - 22 214 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 047 618 c[1] - 656 447 c[2] + 167 069 c[3] - 22 210 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 052 194 c[1] - 657 215 c[2] + 167 101 c[3] - 22 210 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 043 406 c[1] - 655 187 c[2] + 166 945 c[3] - 22 206 c[4] + 1632 c[5] - 63 c[6] + c[7],
 1 028 742 c[1] - 651 563 c[2] + 166 649 c[3] - 22 198 c[4] + 1632 c[5] - 63 c[6] + c[7]}
```

In[•]:=
```
Array[c, 7].glist[[i]]
```

Out[•]=
```
64 467 080 c[1] - 39 815 860 c[2] + 10 061 948 c[3] -
 1 333 704 c[4] + 97 920 c[5] - 3780 c[6] + 60 c[7]
```

```
In[ ]:= certlist[[i]] = Flatten[
          Array[c, 7] /. FindInstance[64 467 080 c[1] - 39 815 860 c[2] + 10 061 948 c[3] -
                1 333 704 c[4] + 97 920 c[5] - 3780 c[6] + 60 c[7] < 0 &&
              1 080 586 c[1] - 665 431 c[2] + 167 877 c[3] - 22 234 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 074 502 c[1] - 663 819 c[2] + 167 737 c[3] -
                22 230 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 && 1 068 210 c[1] -
                662 191 c[2] + 167 597 c[3] - 22 226 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
              1 072 786 c[1] - 662 959 c[2] + 167 629 c[3] - 22 226 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 072 370 c[1] - 662 927 c[2] + 167 629 c[3] -
                22 226 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 && 1 064 350 c[1] -
                660 963 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
              1 063 998 c[1] - 660 931 c[2] + 167 473 c[3] - 22 222 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 068 574 c[1] - 661 699 c[2] +
                167 505 c[3] - 22 222 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
              1 059 994 c[1] - 659 687 c[2] + 167 349 c[3] - 22 218 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 064 570 c[1] - 660 455 c[2] +
                167 381 c[3] - 22 218 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
              1 053 910 c[1] - 658 075 c[2] + 167 209 c[3] - 22 214 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 058 486 c[1] - 658 843 c[2] +
                167 241 c[3] - 22 214 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
              1 047 618 c[1] - 656 447 c[2] + 167 069 c[3] - 22 210 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 052 194 c[1] - 657 215 c[2] +
                167 101 c[3] - 22 210 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0 &&
              1 043 406 c[1] - 655 187 c[2] + 166 945 c[3] - 22 206 c[4] + 1632 c[5] -
                63 c[6] + c[7] ≥ 0 && 1 028 742 c[1] - 651 563 c[2] + 166 649 c[3] -
                22 198 c[4] + 1632 c[5] - 63 c[6] + c[7] ≥ 0, Array[c, 7], Integers]]
Out[ ]= {39 328, 511 258, 6 567 704, 83 964 354, 0, 0, 1 062 007 428 672}

In[ ]:= certlist[[i]] =
         certlist[[i]] / GCD[39 328, 511 258, 6 567 704, 83 964 354, 0, 0, 1 062 007 428 672]
Out[ ]= {19 664, 255 629, 3 283 852, 41 982 177, 0, 0, 531 003 714 336}

In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[ ]= -34 284 572

Out[ ]= {395 127, 1 022 727, 1 650 279, 392 935, 392 839, 390 711, 1 649 111, 391 767,
         1 647 991, 390 647, 2 275 591, 1 018 247, 2 903 143, 1 645 799, 2 901 975, 4 785 799}

In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[ ]= 34

Out[ ]= (-13 + x)^6 (-11 + x)^8 (5 + x)^42 (139 - 24 x + x^2) (95 - 20 x + x^2)

Out[ ]= {531 003 714 336, 0, 0, 41 982 177, 3 283 852, 255 629, 19 664}
```

```
In[ ]:= i = 35;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^6 (-11 + x)^7 (5 + x)^41 (113 - 22 x + x^2) (-734 + 253 x - 28 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^7 (-10 + x) (5 + x)^41 (113 - 22 x + x^2) (73 - 18 x + x^2),
           (-13 + x)^6 (-11 + x)^7 (-9 + x)^2 (5 + x)^41 (-1018 + 325 x - 32 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^7 (5 + x)^41 (-82 166 + 44 577 x - 9456 x^2 + 982 x^3 - 50 x^4 + x^5),
           (-13 + x)^8 (-11 + x)^7 (-9 + x)^2 (-6 + x) (5 + x)^41,
           (-13 + x)^6 (-11 + x)^7 (5 + x)^41 (59 - 16 x + x^2) (-1382 + 379 x - 34 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-670 + 245 x - 28 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^9 (-9 + x) (5 + x)^41 (74 - 19 x + x^2),
           (-13 + x)^7 (-11 + x)^8 (5 + x)^41 (-566 + 215 x - 26 x^2 + x^3),
           (-13 + x)^7 (-11 + x)^7 (-9 + x) (5 + x)^41 (-686 + 249 x - 28 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^7 (-11 + x)^8 (-9 + x) (5 + x)^42 (113 - 22 x + x^2)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -82 942  44 737  -9464  982  -50  1
          -82 490  44 649  -9460  982  -50  1
          -82 458  44 649  -9460  982  -50  1
          -82 166  44 577  -9456  982  -50  1
          -82 134  44 577  -9456  982  -50  1
          -81 538  44 473  -9452  982  -50  1
          -81 070  44 385  -9448  982  -50  1
          -80 586  44 297  -9444  982  -50  1
          -80 938  44 329  -9444  982  -50  1
          -80 262  44 225  -9440  982  -50  1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-4 965 352, 2 682 172, -567 744, 58 920, -3000, 60}
```

```
In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= {-82 942 c[1] + 44 737 c[2] - 9464 c[3] + 982 c[4] - 50 c[5] + c[6],
  -82 490 c[1] + 44 649 c[2] - 9460 c[3] + 982 c[4] - 50 c[5] + c[6],
  -82 458 c[1] + 44 649 c[2] - 9460 c[3] + 982 c[4] - 50 c[5] + c[6],
  -82 166 c[1] + 44 577 c[2] - 9456 c[3] + 982 c[4] - 50 c[5] + c[6],
  -82 134 c[1] + 44 577 c[2] - 9456 c[3] + 982 c[4] - 50 c[5] + c[6],
  -81 538 c[1] + 44 473 c[2] - 9452 c[3] + 982 c[4] - 50 c[5] + c[6],
  -81 070 c[1] + 44 385 c[2] - 9448 c[3] + 982 c[4] - 50 c[5] + c[6],
  -80 586 c[1] + 44 297 c[2] - 9444 c[3] + 982 c[4] - 50 c[5] + c[6],
  -80 938 c[1] + 44 329 c[2] - 9444 c[3] + 982 c[4] - 50 c[5] + c[6],
  -80 262 c[1] + 44 225 c[2] - 9440 c[3] + 982 c[4] - 50 c[5] + c[6]}
```

```
In[ ]:= Array[c, 6].glist[[i]]
```

```
Out[ ]= -4 965 352 c[1] + 2 682 172 c[2] - 567 744 c[3] + 58 920 c[4] - 3000 c[5] + 60 c[6]
```

```
In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
      -4 965 352 c[1] + 2 682 172 c[2] - 567 744 c[3] + 58 920 c[4] - 3000 c[5] + 60 c[6] < 0 &&
       -82 942 c[1] + 44 737 c[2] - 9464 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -82 490 c[1] + 44 649 c[2] - 9460 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -82 458 c[1] + 44 649 c[2] - 9460 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -82 166 c[1] + 44 577 c[2] - 9456 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -82 134 c[1] + 44 577 c[2] - 9456 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -81 538 c[1] + 44 473 c[2] - 9452 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -81 070 c[1] + 44 385 c[2] - 9448 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -80 586 c[1] + 44 297 c[2] - 9444 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -80 938 c[1] + 44 329 c[2] - 9444 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -80 262 c[1] + 44 225 c[2] - 9440 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0,
      Array[c, 6], Integers]]
```

```
Out[ ]= {-43 111, -431 106, -4 267 959, 0, 0, -24 681 212 876}
```

```
In[ ]:= certlist[[i]] = certlist[[i]] / GCD[-43 111, -431 106, -4 267 959, 0, 0, -24 681 212 876]
```

```
Out[ ]= {-43 111, -431 106, -4 267 959, 0, 0, -24 681 212 876}
```

```
In[ ]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[ ]= -3 810 224
```

```
Out[ ]= {74 540, 1 453 860, 74 308, 1 453 692, 74 140,
  2 143 172, 2 832 716, 2 832 484, 4 212 164, 2 832 316}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[ ]= 35
```

```
Out[ ]= (-13 + x)^7 (-11 + x)^8 (-9 + x) (5 + x)^42 (113 - 22 x + x^2)
```

```
Out[ ]= {-24 681 212 876, 0, 0, -4 267 959, -431 106, -43 111}
```

```
In[ ]:= i = 36;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-682 + 245 x - 28 x^2 + x^3),
            (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-670 + 245 x - 28 x^2 + x^3),
            (-13 + x)^6 (-11 + x)^9 (-9 + x) (5 + x)^41 (74 - 19 x + x^2),
            (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-654 + 245 x - 28 x^2 + x^3),
            (-13 + x)^7 (-11 + x)^9 (-10 + x) (-5 + x) (5 + x)^41,
            (-13 + x)^6 (-11 + x)^10 (5 + x)^41 (58 - 17 x + x^2)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
Out[ ]= (-13 + x)^6 (-11 + x)^9 (5 + x)^42 (-1191 + 351 x - 33 x^2 + x^3)

In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -97 526  51 403  -10 566  1060  -52  1
          -95 810  51 115  -10 554  1060  -52  1
          -95 238  51 019  -10 550  1060  -52  1
          -93 522  50 731  -10 538  1060  -52  1
          -92 950  50 635  -10 534  1060  -52  1
          -91 234  50 347  -10 522  1060  -52  1 )

In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]] // Factor, x]
Out[ ]= {-5 812 416, 3 076 980, -633 648, 63 600, -3120, 60}

In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[ ]= {-97 526 c[1] + 51 403 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6],
         -95 810 c[1] + 51 115 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6],
         -95 238 c[1] + 51 019 c[2] - 10 550 c[3] + 1060 c[4] - 52 c[5] + c[6],
         -93 522 c[1] + 50 731 c[2] - 10 538 c[3] + 1060 c[4] - 52 c[5] + c[6],
         -92 950 c[1] + 50 635 c[2] - 10 534 c[3] + 1060 c[4] - 52 c[5] + c[6],
         -91 234 c[1] + 50 347 c[2] - 10 522 c[3] + 1060 c[4] - 52 c[5] + c[6]}

In[ ]:= Array[c, 6].glist[[i]]
Out[ ]= -5 812 416 c[1] + 3 076 980 c[2] - 633 648 c[3] + 63 600 c[4] - 3120 c[5] + 60 c[6]
```

```
In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
          -5 812 416 c[1] + 3 076 980 c[2] - 633 648 c[3] + 63 600 c[4] - 3120 c[5] + 60 c[6] < 0 &&
           -97 526 c[1] + 51 403 c[2] - 10 566 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -95 810 c[1] + 51 115 c[2] - 10 554 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -95 238 c[1] + 51 019 c[2] - 10 550 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -93 522 c[1] + 50 731 c[2] - 10 538 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -92 950 c[1] + 50 635 c[2] - 10 534 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0 &&
           -91 234 c[1] + 50 347 c[2] - 10 522 c[3] + 1060 c[4] - 52 c[5] + c[6] ≥ 0,
          Array[c, 6], Integers]]
Out[•]= {0, -33 284, -798 929, 0, 0, -6 730 490 844}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[0, -33 284, -798 929, 0, 0, -6 730 490 844]
Out[•]= {0, -33 284, -798 929, 0, 0, -6 730 490 844}

In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[•]= -3 889 968

Out[•]= {95 518, 94 162, 93 710, 92 354, 91 902, 90 546}

In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[•]= 36

Out[•]= (-13 + x)^6 (-11 + x)^9 (5 + x)^42 (-1191 + 351 x - 33 x^2 + x^3)

Out[•]= {-6 730 490 844, 0, 0, -798 929, -33 284, 0}

In[•]:= i = 37;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^6 (-11 + x)^7 (5 + x)^41 (95 - 20 x + x^2) (-862 + 287 x - 30 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^8 (5 + x)^41 (7402 - 3365 x + 553 x^2 - 39 x^3 + x^4),
           (-13 + x)^7 (-11 + x)^8 (5 + x)^41 (-566 + 215 x - 26 x^2 + x^3),
           (-13 + x)^7 (-11 + x)^7 (-9 + x) (5 + x)^41 (-686 + 249 x - 28 x^2 + x^3),
           (-13 + x)^6 (-11 + x)^9 (5 + x)^41 (-654 + 245 x - 28 x^2 + x^3),
           (-13 + x)^7 (-11 + x)^9 (-10 + x) (-5 + x) (5 + x)^41};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]], x];
Out[•]= (-13 + x)^7 (-11 + x)^8 (5 + x)^42 (-1009 + 311 x - 31 x^2 + x^3)
```

```
In[•]:= Coeffmatlist[[i]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -81\,890 & 44\,505 & -9452 & 982 & -50 & 1 \\ -81\,422 & 44\,417 & -9448 & 982 & -50 & 1 \\ -80\,938 & 44\,329 & -9444 & 982 & -50 & 1 \\ -80\,262 & 44\,225 & -9440 & 982 & -50 & 1 \\ -79\,134 & 44\,033 & -9432 & 982 & -50 & 1 \\ -78\,650 & 43\,945 & -9428 & 982 & -50 & 1 \end{pmatrix}$$

```
In[•]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[•]= {-4 924 544, 2 673 260, -567 288, 58 920, -3000, 60}

In[•]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[•]= {-81 890 c[1] + 44 505 c[2] - 9452 c[3] + 982 c[4] - 50 c[5] + c[6],
  -81 422 c[1] + 44 417 c[2] - 9448 c[3] + 982 c[4] - 50 c[5] + c[6],
  -80 938 c[1] + 44 329 c[2] - 9444 c[3] + 982 c[4] - 50 c[5] + c[6],
  -80 262 c[1] + 44 225 c[2] - 9440 c[3] + 982 c[4] - 50 c[5] + c[6],
  -79 134 c[1] + 44 033 c[2] - 9432 c[3] + 982 c[4] - 50 c[5] + c[6],
  -78 650 c[1] + 43 945 c[2] - 9428 c[3] + 982 c[4] - 50 c[5] + c[6]}

In[•]:= Array[c, 6].glist[[i]]
Out[•]= -4 924 544 c[1] + 2 673 260 c[2] - 567 288 c[3] + 58 920 c[4] - 3000 c[5] + 60 c[6]

In[•]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
      -4 924 544 c[1] + 2 673 260 c[2] - 567 288 c[3] + 58 920 c[4] - 3000 c[5] + 60 c[6] < 0 &&
       -81 890 c[1] + 44 505 c[2] - 9452 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -81 422 c[1] + 44 417 c[2] - 9448 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -80 938 c[1] + 44 329 c[2] - 9444 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -80 262 c[1] + 44 225 c[2] - 9440 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -79 134 c[1] + 44 033 c[2] - 9432 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0 &&
       -78 650 c[1] + 43 945 c[2] - 9428 c[3] + 982 c[4] - 50 c[5] + c[6] ≥ 0,
      Array[c, 6], Integers]]
Out[•]= {1444, 2658, 0, 0, 0, 0}

In[•]:= certlist[[i]] = certlist[[i]] / GCD[1444, 2658, 0, 0, 0, 0]
Out[•]= {722, 1329, 0, 0, 0, 0}

In[•]:= certlist[[i]].glist[[i]]
    certlist[[i]].Transpose[Coeffmatlist[[i]]]
Out[•]= -2 758 228

Out[•]= {22 565, 243 509, 476 005, 825 861, 1 385 109, 1 617 605}
```

```
In[ ]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
Out[ ]= 37
Out[ ]= (-13 + x)^7 (-11 + x)^8 (5 + x)^42 (-1009 + 311 x - 31 x^2 + x^3)
Out[ ]= {0, 0, 0, 0, 1329, 722}
```

```
In[ ]:= i = 38;
        chilist[[i]]
        interlacingpolylist[[i]] =
          {(-13 + x)^7 (-11 + x)^6 (-9 + x) (5 + x)^41 (7782 - 3445 x + 557 x^2 - 39 x^3 + x^4),
           (-13 + x)^8 (-11 + x)^7 (-9 + x)^2 (-6 + x) (5 + x)^41};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
Out[ ]= (-13 + x)^8 (-11 + x)^7 (-9 + x) (5 + x)^42 (95 - 20 x + x^2)
```

```
In[ ]:= Coeffmatlist[[i]] // MatrixForm
Out[ ]//MatrixForm=
        ( -70 038  38 787  -8458  908  -48  1 )
        ( -69 498  38 691  -8454  908  -48  1 )
```

```
In[ ]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
Out[ ]= {-4 173 280, 2 321 140, -507 168, 54 480, -2880, 60}
```

```
In[ ]:= Array[c, 6].Transpose[Coeffmatlist[[i]]]
Out[ ]= {-70 038 c[1] + 38 787 c[2] - 8458 c[3] + 908 c[4] - 48 c[5] + c[6],
         -69 498 c[1] + 38 691 c[2] - 8454 c[3] + 908 c[4] - 48 c[5] + c[6]}
```

```
In[ ]:= Array[c, 6].glist[[i]]
Out[ ]= -4 173 280 c[1] + 2 321 140 c[2] - 507 168 c[3] + 54 480 c[4] - 2880 c[5] + 60 c[6]
```

```
In[ ]:= certlist[[i]] = Flatten[Array[c, 6] /. FindInstance[
            -4 173 280 c[1] + 2 321 140 c[2] - 507 168 c[3] + 54 480 c[4] - 2880 c[5] + 60 c[6] < 0 &&
             -70 038 c[1] + 38 787 c[2] - 8458 c[3] + 908 c[4] - 48 c[5] + c[6] ≥ 0 && -69 498 c[1] +
               38 691 c[2] - 8454 c[3] + 908 c[4] - 48 c[5] + c[6] ≥ 0, Array[c, 6], Integers]]
Out[ ]= {14, -4880, -119 057, 0, 0, -816 690 681}
```

```
In[ ]:= certlist[[i]] = certlist[[i]] / GCD[14, -4880, -119 057, 0, 0, -816 690 681]
Out[ ]= {14, -4880, -119 057, 0, 0, -816 690 681}
```

```
In[•]:= certlist[[i]].glist[[i]]
        certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= -5 129 404
```

```
Out[•]= {32 333, 32 145}
```

```
In[•]:= i
        chilist[[i]]
        Reverse[certlist[[i]]]
```

```
Out[•]= 38
```

$$\text{Out[•]= } (-13+x)^8\,(-11+x)^7\,(-9+x)\,(5+x)^{42}\left(95-20\,x+x^2\right)$$

```
Out[•]= {-816 690 681, 0, 0, -119 057, -4880, 14}
```

```
In[•]:= i = 39;
        chilist[[i]]
        interlacingpolylist[[i]] = {(-13 + x)^7 (-11 + x)^9 (-10 + x) (-5 + x) (5 + x)^41,
            (-13 + x)^7 (-11 + x)^8 (5 + x)^41 (-566 + 215 x - 26 x^2 + x^3)};
        Coeffmatlist[[i]] =
          CoefficientList[Factor[interlacingpolylist[[i]] / mu[chilist[[i]]], x];
```

$$\text{Out[•]= } (-13+x)^8\,(-11+x)^9\,(-7+x)\,(5+x)^{42}$$

```
In[•]:= Coeffmatlist[[i]] // MatrixForm
```

```
Out[•]//MatrixForm=
        ( -550  215  -26  1 )
        ( -566  215  -26  1 )
```

```
In[•]:= glist[[i]] = CoefficientList[D[chilist[[i]], x] / mu[chilist[[i]]] // Factor, x]
```

```
Out[•]= {-34 152, 12 900, -1560, 60}
```

```
In[•]:= Array[c, 4].Transpose[Coeffmatlist[[i]]]
```

```
Out[•]= {-550 c[1] + 215 c[2] - 26 c[3] + c[4], -566 c[1] + 215 c[2] - 26 c[3] + c[4]}
```

```
In[•]:= Array[c, 4].glist[[i]]
```

```
Out[•]= -34 152 c[1] + 12 900 c[2] - 1560 c[3] + 60 c[4]
```

```
In[•]:= certlist[[i]] = Flatten[
          Array[c, 4] /. FindInstance[-34 152 c[1] + 12 900 c[2] - 1560 c[3] + 60 c[4] < 0 &&
              -550 c[1] + 215 c[2] - 26 c[3] + c[4] ≥ 0 &&
              -566 c[1] + 215 c[2] - 26 c[3] + c[4] ≥ 0, Array[c, 4], Integers]]
```

```
Out[•]= {253, 0, 0, 143 620}
```

In[ ]:=
```
certlist[[i]] = certlist[[i]] / GCD[253, 0, 0, 143 620]
```

Out[ ]=
```
{253, 0, 0, 143 620}
```

In[ ]:=
```
certlist[[i]].glist[[i]]
certlist[[i]].Transpose[Coeffmatlist[[i]]]
```

Out[ ]=
```
-23 256
```

Out[ ]=
```
{4470, 422}
```

In[ ]:=
```
i
chilist[[i]]
Reverse[certlist[[i]]]
```

Out[ ]=
```
39
```

Out[ ]=
```
(-13 + x)^8 (-11 + x)^9 (-7 + x) (5 + x)^42
```

Out[ ]=
```
{143 620, 0, 0, 253}
```