

**D**[ $16x^2 - 250x + 1015$ ,  $x$ ]

$-250 + 32x$

$250 / 32 // N$

7.8125

$16x^2 - 250x + 1015 /. x \rightarrow 9$

61

$(49 / 13)^{13} - 16^6 // N$

$1.42173 \times 10^7$

$(61 / 16)^{16} - 4^{15} // N$

$9.18571 \times 10^8$

**feasiblefactorlist**[41,  $(x + 5)^{25}(x - 7)^{12}$ ]

$\{(-12 + x)(-11 + x)(-9 + x)^2, (-11 + x)^3(-8 + x)\}$

**length** 2

**feasiblefactorlist**[41,  $(x + 5)^{25}(x - 7)^{11}$ ]

$\{(-13 + x)(-9 + x)^3(-8 + x), (-9 + x)^2(-932 + 293x - 30x^2 + x^3), (-9 + x)^2(-928 + 293x - 30x^2 + x^3), (-12 + x)(-11 + x)(-9 + x)^2(-7 + x), (-11 + x)^3(-8 + x)(-7 + x), (-11 + x)^2(-9 + x)(68 - 17x + x^2)\}$

**length** 6

**feasiblefactorlist**[41,  $(x + 5)^{25}(x - 7)^{10}$ ]

$\{(-13 + x)(-9 + x)^3(-8 + x)(-7 + x), (-9 + x)^2(-7 + x)(-932 + 293x - 30x^2 + x^3), (-9 + x)^4(80 - 19x + x^2), (-9 + x)^2(-7 + x)(-928 + 293x - 30x^2 + x^3), (-9 + x)^2(6452 - 2975x + 503x^2 - 37x^3 + x^4), (-12 + x)(-11 + x)(-9 + x)^2(-7 + x)^2, (-9 + x)^3(-712 + 251x - 28x^2 + x^3), (-11 + x)(-9 + x)^2(-8 + x)(73 - 18x + x^2), (-11 + x)^3(-8 + x)(-7 + x)^2, (-11 + x)(-9 + x)^2(-580 + 217x - 26x^2 + x^3), (-11 + x)^2(-9 + x)(-7 + x)(68 - 17x + x^2), (-11 + x)(-9 + x)^3(64 - 17x + x^2), (-11 + x)^2(-9 + x)^2(52 - 15x + x^2)\}$

**length** 13

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feasiblefactorlist[41, (x + 5)^25 (x - 7)^9]
{ (-13 + x) (-9 + x)^3 (-8 + x) (-7 + x)^2, (-9 + x)^2 (-7 + x)^2 (-932 + 293 x - 30 x^2 + x^3),
  (-9 + x)^4 (-7 + x) (80 - 19 x + x^2), (-9 + x)^2 (-7 + x)^2 (-928 + 293 x - 30 x^2 + x^3),
  (-9 + x)^4 (-556 + 213 x - 26 x^2 + x^3), (-9 + x)^2 (-7 + x) (6452 - 2975 x + 503 x^2 - 37 x^3 + x^4),
  (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^3, (-9 + x)^4 (-8 + x) (69 - 18 x + x^2),
  (-9 + x)^3 (-7 + x) (-712 + 251 x - 28 x^2 + x^3),
  (-9 + x)^2 (-8 + x) (95 - 20 x + x^2) (59 - 16 x + x^2),
  (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x) (73 - 18 x + x^2),
  (-11 + x)^3 (-8 + x) (-7 + x)^3, (-9 + x)^4 (-548 + 213 x - 26 x^2 + x^3),
  (-9 + x)^3 (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4), (-9 + x)^3 (73 - 18 x + x^2) (68 - 17 x + x^2),
  (-11 + x) (-9 + x)^2 (-7 + x) (-580 + 217 x - 26 x^2 + x^3),
  (-11 + x)^2 (-9 + x) (-7 + x)^2 (68 - 17 x + x^2), (-9 + x)^4 (-544 + 213 x - 26 x^2 + x^3),
  (-9 + x)^3 (4912 - 2461 x + 447 x^2 - 35 x^3 + x^4),
  (-9 + x)^2 (59 - 16 x + x^2) (-752 + 255 x - 28 x^2 + x^3),
  (-11 + x) (-9 + x)^3 (-7 + x) (64 - 17 x + x^2), (-12 + x) (-9 + x)^5 (-5 + x),
  (-9 + x)^3 (4876 - 2457 x + 447 x^2 - 35 x^3 + x^4), (-11 + x)^2 (-9 + x)^2 (-7 + x) (52 - 15 x + x^2),
  (-9 + x)^4 (-536 + 213 x - 26 x^2 + x^3), (-11 + x)^2 (-9 + x)^3 (-8 + x) (-5 + x),
  (-9 + x)^4 (-532 + 213 x - 26 x^2 + x^3), (-11 + x) (-9 + x)^4 (48 - 15 x + x^2) }

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**length 28**

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feasiblefactorlist[41, (x + 5) ^ 25 (x - 7) ^ 8]
{ (-13 + x) (-9 + x)^3 (-8 + x) (-7 + x)^3,
  (-9 + x)^2 (-7 + x)^3 (-932 + 293 x - 30 x^2 + x^3), (-9 + x)^4 (-7 + x)^2 (80 - 19 x + x^2),
  (-9 + x)^2 (-7 + x)^3 (-928 + 293 x - 30 x^2 + x^3), (-9 + x)^4 (-7 + x) (-556 + 213 x - 26 x^2 + x^3),
  (-9 + x)^2 (-7 + x)^2 (6452 - 2975 x + 503 x^2 - 37 x^3 + x^4),
  (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^4, (-9 + x)^4 (-8 + x) (-7 + x) (69 - 18 x + x^2),
  (-9 + x)^3 (-7 + x)^2 (-712 + 251 x - 28 x^2 + x^3),
  (-9 + x)^2 (-8 + x) (-7 + x) (95 - 20 x + x^2) (59 - 16 x + x^2),
  (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^2 (73 - 18 x + x^2),
  (-11 + x)^3 (-8 + x) (-7 + x)^4, (-9 + x)^4 (-7 + x) (-548 + 213 x - 26 x^2 + x^3),
  (-9 + x)^3 (-7 + x) (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4),
  (-9 + x)^3 (-7 + x) (73 - 18 x + x^2) (68 - 17 x + x^2),
  (-11 + x) (-9 + x)^2 (-7 + x)^2 (-580 + 217 x - 26 x^2 + x^3),
  (-11 + x)^2 (-9 + x) (-7 + x)^3 (68 - 17 x + x^2), (-9 + x)^4 (-7 + x) (-544 + 213 x - 26 x^2 + x^3),
  (-9 + x)^3 (-7 + x) (4912 - 2461 x + 447 x^2 - 35 x^3 + x^4),
  (-9 + x)^2 (-7 + x) (59 - 16 x + x^2) (-752 + 255 x - 28 x^2 + x^3),
  (-11 + x) (-9 + x)^3 (-7 + x)^2 (64 - 17 x + x^2),
  (-12 + x) (-9 + x)^5 (-7 + x) (-5 + x), (-9 + x)^4 (73 - 18 x + x^2) (52 - 15 x + x^2),
  (-9 + x)^3 (-7 + x) (4876 - 2457 x + 447 x^2 - 35 x^3 + x^4),
  (-11 + x)^2 (-9 + x)^2 (-7 + x)^2 (52 - 15 x + x^2), (-9 + x)^4 (-8 + x) (-467 + 195 x - 25 x^2 + x^3),
  (-9 + x)^4 (-7 + x) (-536 + 213 x - 26 x^2 + x^3), (-9 + x)^4 (3768 - 2027 x + 395 x^2 - 33 x^3 + x^4),
  (-11 + x)^2 (-9 + x)^3 (-8 + x) (-7 + x) (-5 + x),
  (-11 + x) (-9 + x)^2 (-8 + x) (59 - 16 x + x^2)^2, (-9 + x)^5 (-412 + 179 x - 24 x^2 + x^3),
  (-9 + x)^4 (-7 + x) (-532 + 213 x - 26 x^2 + x^3), (-11 + x) (-9 + x)^4 (-5 + x) (68 - 17 x + x^2),
  (-9 + x)^4 (3680 - 2019 x + 395 x^2 - 33 x^3 + x^4), (-11 + x) (-9 + x)^4 (-7 + x) (48 - 15 x + x^2),
  (-9 + x)^5 (-404 + 179 x - 24 x^2 + x^3), (-11 + x) (-9 + x)^4 (-332 + 153 x - 22 x^2 + x^3),
  (-11 + x) (-9 + x)^4 (-8 + x) (41 - 14 x + x^2), (-11 + x) (-9 + x)^6 (-4 + x) }

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**length 39**

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feasiblefactorlist[41, (x + 5) ^ 25 (x - 7) ^ 7]
{ (-13 + x) (-9 + x)^3 (-8 + x) (-7 + x)^4, (-9 + x)^2 (-7 + x)^4 (-932 + 293 x - 30 x^2 + x^3),
  (-9 + x)^4 (-7 + x)^3 (80 - 19 x + x^2), (-9 + x)^2 (-7 + x)^4 (-928 + 293 x - 30 x^2 + x^3),
  (-9 + x)^4 (-7 + x)^2 (-556 + 213 x - 26 x^2 + x^3),
  (-9 + x)^2 (-7 + x)^3 (6452 - 2975 x + 503 x^2 - 37 x^3 + x^4),
  (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^5, (-9 + x)^4 (-8 + x) (-7 + x)^2 (69 - 18 x + x^2),
  (-9 + x)^3 (-7 + x)^3 (-712 + 251 x - 28 x^2 + x^3),
  (-9 + x)^2 (-8 + x) (-7 + x)^2 (95 - 20 x + x^2) (59 - 16 x + x^2),
  (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^3 (73 - 18 x + x^2),
  (-11 + x)^3 (-8 + x) (-7 + x)^5, (-9 + x)^4 (-7 + x)^2 (-548 + 213 x - 26 x^2 + x^3),

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$$\begin{aligned}
& (-9+x)^3 (-7+x)^2 (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^3 (-7+x)^2 (73 - 18x + x^2) (68 - 17x + x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^3 (-580 + 217x - 26x^2 + x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^4 (68 - 17x + x^2), (-9+x)^4 (-7+x)^2 (-544 + 213x - 26x^2 + x^3), \\
& (-9+x)^3 (-7+x)^2 (4912 - 2461x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^2 (-7+x)^2 (59 - 16x + x^2) (-752 + 255x - 28x^2 + x^3), \\
& (-11+x) (-9+x)^3 (-7+x)^3 (64 - 17x + x^2), (-12+x) (-9+x)^5 (-7+x)^2 (-5+x), \\
& (-9+x)^4 (-7+x) (73 - 18x + x^2) (52 - 15x + x^2), \\
& (-9+x)^3 (-7+x)^2 (4876 - 2457x + 447x^2 - 35x^3 + x^4), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^3 (52 - 15x + x^2), \\
& (-9+x)^4 (-8+x) (-7+x) (-467 + 195x - 25x^2 + x^3), \\
& (-9+x)^5 (-8+x) (-5+x) (73 - 18x + x^2), (-9+x)^4 (-7+x)^2 (-536 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x) (3768 - 2027x + 395x^2 - 33x^3 + x^4), \\
& (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^2 (-5+x), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x) (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x) (-412 + 179x - 24x^2 + x^3), (-9+x)^5 (-5+x) (-580 + 217x - 26x^2 + x^3), \\
& (-9+x)^4 (-26084 + 17885x - 4788x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^2 (-532 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (59 - 16x + x^2) (-444 + 183x - 24x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-7+x) (-5+x) (68 - 17x + x^2), \\
& (-9+x)^3 (68 - 17x + x^2) (59 - 16x + x^2)^2, (-9+x)^5 (2864 - 1661x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^4 (-7+x) (3680 - 2019x + 395x^2 - 33x^3 + x^4), (-9+x)^6 (-5+x) (64 - 17x + x^2), \\
& (-9+x)^4 (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-25888 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^2 (48 - 15x + x^2), \\
& (-9+x)^5 (-7+x) (-404 + 179x - 24x^2 + x^3), (-9+x)^6 (-316 + 149x - 22x^2 + x^3), \\
& (-9+x)^4 (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x) (-332 + 153x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^5 (-5+x) (52 - 15x + x^2), \\
& (-9+x)^4 (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\
& (-9+x)^5 (-8+x) (-349 + 163x - 23x^2 + x^3), (-9+x)^6 (-312 + 149x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x) (41 - 14x + x^2), \\
& (-9+x)^5 (2824 - 1653x + 347x^2 - 31x^3 + x^4), (-11+x) (-9+x)^6 (-7+x) (-4+x), \\
& (-9+x)^5 (68 - 17x + x^2) (41 - 14x + x^2), (-9+x)^6 (-304 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-300 + 149x - 22x^2 + x^3), (-9+x)^6 (-8+x) (37 - 14x + x^2), \\
& (-9+x)^6 (-292 + 149x - 22x^2 + x^3), (-9+x)^7 (32 - 13x + x^2) \}
\end{aligned}$$

**length 61**

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feasiblefactorlist[41, (x + 5)^25 (x - 7)^6]
{(-13+x) (-9+x)^3 (-8+x) (-7+x)^5, (-9+x)^2 (-7+x)^5 (-932 + 293x - 30x^2 + x^3),
 (-9+x)^4 (-7+x)^4 (80 - 19x + x^2), (-9+x)^2 (-7+x)^5 (-928 + 293x - 30x^2 + x^3),

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$$\begin{aligned}
& (-9 + x)^4 (-7 + x)^3 (-556 + 213 x - 26 x^2 + x^3), \\
& (-9 + x)^2 (-7 + x)^4 (6452 - 2975 x + 503 x^2 - 37 x^3 + x^4), \\
& (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^6, (-9 + x)^4 (-8 + x) (-7 + x)^3 (69 - 18 x + x^2), \\
& (-9 + x)^3 (-7 + x)^4 (-712 + 251 x - 28 x^2 + x^3), \\
& (-9 + x)^2 (-8 + x) (-7 + x)^3 (95 - 20 x + x^2) (59 - 16 x + x^2), \\
& (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^4 (73 - 18 x + x^2), \\
& (-11 + x)^3 (-8 + x) (-7 + x)^6, (-9 + x)^4 (-7 + x)^3 (-548 + 213 x - 26 x^2 + x^3), \\
& (-9 + x)^3 (-7 + x)^3 (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4), \\
& (-9 + x)^3 (-7 + x)^3 (73 - 18 x + x^2) (68 - 17 x + x^2), \\
& (-11 + x) (-9 + x)^2 (-7 + x)^4 (-580 + 217 x - 26 x^2 + x^3), \\
& (-11 + x)^2 (-9 + x) (-7 + x)^5 (68 - 17 x + x^2), (-9 + x)^4 (-7 + x)^3 (-544 + 213 x - 26 x^2 + x^3), \\
& (-9 + x)^3 (-7 + x)^3 (4912 - 2461 x + 447 x^2 - 35 x^3 + x^4), \\
& (-9 + x)^2 (-7 + x)^3 (59 - 16 x + x^2) (-752 + 255 x - 28 x^2 + x^3), \\
& (-11 + x) (-9 + x)^3 (-7 + x)^4 (64 - 17 x + x^2), (-12 + x) (-9 + x)^5 (-7 + x)^3 (-5 + x), \\
& (-9 + x)^4 (-7 + x)^2 (73 - 18 x + x^2) (52 - 15 x + x^2), \\
& (-9 + x)^3 (-7 + x)^3 (4876 - 2457 x + 447 x^2 - 35 x^3 + x^4), \\
& (-11 + x)^2 (-9 + x)^2 (-7 + x)^4 (52 - 15 x + x^2), \\
& (-9 + x)^4 (-8 + x) (-7 + x)^2 (-467 + 195 x - 25 x^2 + x^3), \\
& (-9 + x)^5 (-8 + x) (-7 + x) (-5 + x) (73 - 18 x + x^2), \\
& (-9 + x)^4 (-7 + x)^3 (-536 + 213 x - 26 x^2 + x^3), \\
& (-9 + x)^4 (-7 + x)^2 (3768 - 2027 x + 395 x^2 - 33 x^3 + x^4), \\
& (-11 + x)^2 (-9 + x)^3 (-8 + x) (-7 + x)^3 (-5 + x), \\
& (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^2 (59 - 16 x + x^2)^2, \\
& (-9 + x)^5 (-7 + x)^2 (-412 + 179 x - 24 x^2 + x^3), \\
& (-9 + x)^5 (-7 + x) (-5 + x) (-580 + 217 x - 26 x^2 + x^3), \\
& (-9 + x)^4 (-7 + x) (-26084 + 17885 x - 4788 x^2 + 626 x^3 - 40 x^4 + x^5), \\
& (-9 + x)^4 (-7 + x)^3 (-532 + 213 x - 26 x^2 + x^3), \\
& (-9 + x)^4 (-7 + x) (59 - 16 x + x^2) (-444 + 183 x - 24 x^2 + x^3), \\
& (-11 + x) (-9 + x)^4 (-7 + x)^2 (-5 + x) (68 - 17 x + x^2), \\
& (-9 + x)^3 (-7 + x) (68 - 17 x + x^2) (59 - 16 x + x^2)^2, \\
& (-9 + x)^5 (-7 + x) (2864 - 1661 x + 347 x^2 - 31 x^3 + x^4), \\
& (-9 + x)^4 (-7 + x)^2 (3680 - 2019 x + 395 x^2 - 33 x^3 + x^4), \\
& (-9 + x)^6 (-7 + x) (-5 + x) (64 - 17 x + x^2), \\
& (-9 + x)^4 (-7 + x) (-25904 + 17829 x - 4784 x^2 + 626 x^3 - 40 x^4 + x^5), \\
& (-9 + x)^4 (-7 + x) (-25888 + 17829 x - 4784 x^2 + 626 x^3 - 40 x^4 + x^5), \\
& (-11 + x) (-9 + x)^4 (-7 + x)^3 (48 - 15 x + x^2), \\
& (-9 + x)^5 (-7 + x)^2 (-404 + 179 x - 24 x^2 + x^3), (-9 + x)^6 (-7 + x) (-316 + 149 x - 22 x^2 + x^3), \\
& (-9 + x)^4 (-7 + x) (-25580 + 17757 x - 4780 x^2 + 626 x^3 - 40 x^4 + x^5), \\
& (-11 + x) (-9 + x)^4 (-7 + x)^2 (-332 + 153 x - 22 x^2 + x^3), \\
& (-11 + x) (-9 + x)^5 (-7 + x) (-5 + x) (52 - 15 x + x^2), \\
& (-9 + x)^4 (-7 + x) (59 - 16 x + x^2) (-436 + 183 x - 24 x^2 + x^3), \\
& (-9 + x)^4 (59 - 16 x + x^2)^2 (52 - 15 x + x^2),
\end{aligned}$$

$$\begin{aligned}
& (-9+x)^5 (-8+x) (-7+x) (-349+163x-23x^2+x^3), \\
& (-9+x)^6 (-7+x) (-312+149x-22x^2+x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^2 (41-14x+x^2), \\
& (-11+x) (-9+x)^6 (-8+x) (-5+x)^2, \\
& (-9+x)^5 (-7+x) (2824-1653x+347x^2-31x^3+x^4), \\
& (-11+x) (-9+x)^6 (-7+x)^2 (-4+x), \quad (-9+x)^6 (2172-1351x+303x^2-29x^3+x^4), \\
& (-9+x)^5 (-7+x) (68-17x+x^2) (41-14x+x^2), \\
& (-9+x)^6 (-7+x) (-304+149x-22x^2+x^3), \quad (-9+x)^6 (2144-1347x+303x^2-29x^3+x^4), \\
& (-9+x)^7 (-5+x) (48-15x+x^2), \quad (-9+x)^6 (-7+x) (-300+149x-22x^2+x^3), \\
& (-9+x)^6 (2116-1343x+303x^2-29x^3+x^4), \\
& (-9+x)^6 (52-15x+x^2) (41-14x+x^2), \quad (-9+x)^6 (-8+x) (-7+x) (37-14x+x^2), \\
& (-9+x)^7 (-232+123x-20x^2+x^3), \quad (-9+x)^6 (-7+x) (-292+149x-22x^2+x^3), \\
& (-9+x)^7 (-7+x) (32-13x+x^2), \quad (-9+x)^8 (-8+x) (-3+x) \}
\end{aligned}$$

**length** 70

$$\begin{aligned}
& \text{feasiblefactorlist}[41, (x+5)^{25} (x-7)^5] \\
& \left\{ (-13+x) (-9+x)^3 (-8+x) (-7+x)^6, \quad (-9+x)^2 (-7+x)^6 (-932+293x-30x^2+x^3), \right. \\
& \quad (-9+x)^4 (-7+x)^5 (80-19x+x^2), \quad (-9+x)^2 (-7+x)^6 (-928+293x-30x^2+x^3), \\
& \quad (-9+x)^4 (-7+x)^4 (-556+213x-26x^2+x^3), \\
& \quad (-9+x)^2 (-7+x)^5 (6452-2975x+503x^2-37x^3+x^4), \\
& \quad (-12+x) (-11+x) (-9+x)^2 (-7+x)^7, \quad (-9+x)^4 (-8+x) (-7+x)^4 (69-18x+x^2), \\
& \quad (-9+x)^3 (-7+x)^5 (-712+251x-28x^2+x^3), \\
& \quad (-9+x)^2 (-8+x) (-7+x)^4 (95-20x+x^2) (59-16x+x^2), \\
& \quad (-11+x) (-9+x)^2 (-8+x) (-7+x)^5 (73-18x+x^2), \\
& \quad (-11+x)^3 (-8+x) (-7+x)^7, \quad (-9+x)^4 (-7+x)^4 (-548+213x-26x^2+x^3), \\
& \quad (-9+x)^3 (-7+x)^4 (4948-2465x+447x^2-35x^3+x^4), \\
& \quad (-9+x)^3 (-7+x)^4 (73-18x+x^2) (68-17x+x^2), \\
& \quad (-11+x) (-9+x)^2 (-7+x)^5 (-580+217x-26x^2+x^3), \\
& \quad (-11+x)^2 (-9+x) (-7+x)^6 (68-17x+x^2), \quad (-9+x)^4 (-7+x)^4 (-544+213x-26x^2+x^3), \\
& \quad (-9+x)^3 (-7+x)^4 (4912-2461x+447x^2-35x^3+x^4), \\
& \quad (-9+x)^2 (-7+x)^4 (59-16x+x^2) (-752+255x-28x^2+x^3), \\
& \quad (-11+x) (-9+x)^3 (-7+x)^5 (64-17x+x^2), \quad (-12+x) (-9+x)^5 (-7+x)^4 (-5+x), \\
& \quad (-9+x)^4 (-7+x)^3 (73-18x+x^2) (52-15x+x^2), \\
& \quad (-9+x)^3 (-7+x)^4 (4876-2457x+447x^2-35x^3+x^4), \\
& \quad (-11+x)^2 (-9+x)^2 (-7+x)^5 (52-15x+x^2), \\
& \quad (-9+x)^4 (-8+x) (-7+x)^3 (-467+195x-25x^2+x^3), \\
& \quad (-9+x)^5 (-8+x) (-7+x)^2 (-5+x) (73-18x+x^2), \\
& \quad (-9+x)^4 (-7+x)^4 (-536+213x-26x^2+x^3), \\
& \quad (-9+x)^4 (-7+x)^3 (3768-2027x+395x^2-33x^3+x^4), \\
& \quad \left. (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^4 (-5+x) \right\}
\end{aligned}$$

$$\begin{aligned}
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^3 (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x)^3 (-412 + 179x - 24x^2 + x^3), \\
& (-9+x)^5 (-7+x)^2 (-5+x) (-580 + 217x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x)^2 (-26084 + 17885x - 4788x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^4 (-532 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x)^2 (59 - 16x + x^2) (-444 + 183x - 24x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-7+x)^3 (-5+x) (68 - 17x + x^2), \\
& (-9+x)^3 (-7+x)^2 (68 - 17x + x^2) (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x)^2 (2864 - 1661x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^4 (-7+x)^3 (3680 - 2019x + 395x^2 - 33x^3 + x^4), \\
& (-9+x)^6 (-7+x)^2 (-5+x) (64 - 17x + x^2), \\
& (-9+x)^4 (-7+x)^2 (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^2 (-25888 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^4 (48 - 15x + x^2), (-9+x)^5 (-7+x)^3 (-404 + 179x - 24x^2 + x^3), \\
& (-9+x)^6 (-7+x)^2 (-316 + 149x - 22x^2 + x^3), \\
& (-9+x)^4 (-7+x)^2 (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^3 (-332 + 153x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^5 (-7+x)^2 (-5+x) (52 - 15x + x^2), \\
& (-9+x)^4 (-7+x)^2 (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\
& (-9+x)^4 (-7+x) (59 - 16x + x^2)^2 (52 - 15x + x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^2 (-349 + 163x - 23x^2 + x^3), \\
& (-9+x)^6 (-7+x)^2 (-312 + 149x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^3 (41 - 14x + x^2), \\
& (-11+x) (-9+x)^6 (-8+x) (-7+x) (-5+x)^2, \\
& (-9+x)^5 (-7+x)^2 (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^5 (-8+x) (-5+x) (59 - 16x + x^2)^2, (-11+x) (-9+x)^6 (-7+x)^3 (-4+x), \\
& (-9+x)^6 (-7+x) (2172 - 1351x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^5 (-7+x)^2 (68 - 17x + x^2) (41 - 14x + x^2), \\
& (-9+x)^7 (-5+x)^2 (68 - 17x + x^2), (-9+x)^6 (-7+x)^2 (-304 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x) (2144 - 1347x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x) (-5+x) (48 - 15x + x^2), \\
& (-9+x)^6 (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^2 (-300 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x) (2116 - 1343x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-5+x) (-332 + 153x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x) (52 - 15x + x^2) (41 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^2 (37 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (59 - 16x + x^2) (31 - 12x + x^2), \\
& (-9+x)^7 (-7+x) (-232 + 123x - 20x^2 + x^3), (-9+x)^7 (-8+x) (-5+x) (41 - 14x + x^2), \\
& (-9+x)^6 (-7+x)^2 (-292 + 149x - 22x^2 + x^3), \\
& (-9+x)^7 (1604 - 1089x + 263x^2 - 27x^3 + x^4), (-9+x)^9 (-5+x) (-4+x), \\
& (-9+x)^7 (-7+x)^2 (32 - 13x + x^2), (-9+x)^8 (-176 + 101x - 18x^2 + x^3),
\end{aligned}$$

$$\left. \begin{aligned} & (-9+x)^8 (-172 + 101x - 18x^2 + x^3), \quad (-9+x)^8 (-8+x) (-7+x) (-3+x) \end{aligned} \right\}$$

**length** 80

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feasiblefactorlist[41, (x+5)^25 (x-7)^4]
{(-13+x) (-9+x)^3 (-8+x) (-7+x)^7, (-9+x)^2 (-7+x)^7 (-932 + 293x - 30x^2 + x^3),
 (-9+x)^4 (-7+x)^6 (80 - 19x + x^2), (-9+x)^2 (-7+x)^7 (-928 + 293x - 30x^2 + x^3),
 (-9+x)^4 (-7+x)^5 (-556 + 213x - 26x^2 + x^3),
 (-9+x)^2 (-7+x)^6 (6452 - 2975x + 503x^2 - 37x^3 + x^4),
 (-12+x) (-11+x) (-9+x)^2 (-7+x)^8, (-9+x)^4 (-8+x) (-7+x)^5 (69 - 18x + x^2),
 (-9+x)^3 (-7+x)^6 (-712 + 251x - 28x^2 + x^3),
 (-9+x)^2 (-8+x) (-7+x)^5 (95 - 20x + x^2) (59 - 16x + x^2),
 (-11+x) (-9+x)^2 (-8+x) (-7+x)^6 (73 - 18x + x^2),
 (-11+x)^3 (-8+x) (-7+x)^8, (-9+x)^4 (-7+x)^5 (-548 + 213x - 26x^2 + x^3),
 (-9+x)^3 (-7+x)^5 (4948 - 2465x + 447x^2 - 35x^3 + x^4),
 (-9+x)^3 (-7+x)^5 (73 - 18x + x^2) (68 - 17x + x^2),
 (-11+x) (-9+x)^2 (-7+x)^6 (-580 + 217x - 26x^2 + x^3),
 (-11+x)^2 (-9+x) (-7+x)^7 (68 - 17x + x^2), (-9+x)^4 (-7+x)^5 (-544 + 213x - 26x^2 + x^3),
 (-9+x)^3 (-7+x)^5 (4912 - 2461x + 447x^2 - 35x^3 + x^4),
 (-9+x)^2 (-7+x)^5 (59 - 16x + x^2) (-752 + 255x - 28x^2 + x^3),
 (-11+x) (-9+x)^3 (-7+x)^6 (64 - 17x + x^2), (-12+x) (-9+x)^5 (-7+x)^5 (-5+x),
 (-9+x)^4 (-7+x)^4 (73 - 18x + x^2) (52 - 15x + x^2),
 (-9+x)^3 (-7+x)^5 (4876 - 2457x + 447x^2 - 35x^3 + x^4),
 (-11+x)^2 (-9+x)^2 (-7+x)^6 (52 - 15x + x^2),
 (-9+x)^4 (-8+x) (-7+x)^4 (-467 + 195x - 25x^2 + x^3),
 (-9+x)^5 (-8+x) (-7+x)^3 (-5+x) (73 - 18x + x^2),
 (-9+x)^4 (-7+x)^5 (-536 + 213x - 26x^2 + x^3),
 (-9+x)^4 (-7+x)^4 (3768 - 2027x + 395x^2 - 33x^3 + x^4),
 (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^5 (-5+x),
 (-11+x) (-9+x)^2 (-8+x) (-7+x)^4 (59 - 16x + x^2)^2,
 (-9+x)^5 (-7+x)^4 (-412 + 179x - 24x^2 + x^3),
 (-9+x)^5 (-7+x)^3 (-5+x) (-580 + 217x - 26x^2 + x^3),
 (-9+x)^4 (-7+x)^3 (-26084 + 17885x - 4788x^2 + 626x^3 - 40x^4 + x^5),
 (-9+x)^4 (-7+x)^5 (-532 + 213x - 26x^2 + x^3),
 (-9+x)^4 (-7+x)^3 (59 - 16x + x^2) (-444 + 183x - 24x^2 + x^3),
 (-11+x) (-9+x)^4 (-7+x)^4 (-5+x) (68 - 17x + x^2),
 (-9+x)^3 (-7+x)^3 (68 - 17x + x^2) (59 - 16x + x^2)^2,
 (-9+x)^5 (-7+x)^3 (2864 - 1661x + 347x^2 - 31x^3 + x^4),
 (-9+x)^4 (-7+x)^4 (3680 - 2019x + 395x^2 - 33x^3 + x^4),
 (-9+x)^6 (-7+x)^3 (-5+x) (64 - 17x + x^2),
 (-9+x)^4 (-7+x)^3 (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5),

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$$\begin{aligned}
& (-9+x)^4 (-7+x)^3 (-25888 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^5 (48 - 15x + x^2), \quad (-9+x)^5 (-7+x)^4 (-404 + 179x - 24x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (-316 + 149x - 22x^2 + x^3), \\
& (-9+x)^4 (-7+x)^3 (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^4 (-332 + 153x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^5 (-7+x)^3 (-5+x) (52 - 15x + x^2), \\
& (-9+x)^4 (-7+x)^3 (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\
& (-9+x)^4 (-7+x)^2 (59 - 16x + x^2)^2 (52 - 15x + x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^3 (-349 + 163x - 23x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (-312 + 149x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^4 (41 - 14x + x^2), \\
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^2 (-5+x)^2, \\
& (-9+x)^5 (-7+x)^3 (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^5 (-8+x) (-7+x) (-5+x) (59 - 16x + x^2)^2, \quad (-11+x) (-9+x)^6 (-7+x)^4 (-4+x), \\
& (-9+x)^6 (-7+x)^2 (2172 - 1351x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^5 (-7+x)^3 (68 - 17x + x^2) (41 - 14x + x^2), \\
& (-9+x)^7 (-7+x) (-5+x) (68 - 17x + x^2), \quad (-9+x)^6 (-7+x)^3 (-304 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^2 (2144 - 1347x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^2 (-5+x) (48 - 15x + x^2), \\
& (-9+x)^6 (-7+x) (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (-300 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^2 (2116 - 1343x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x) (-5+x) (-332 + 153x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^2 (52 - 15x + x^2) (41 - 14x + x^2), \\
& (-9+x)^8 (-5+x)^2 (52 - 15x + x^2), \quad (-9+x)^6 (-8+x) (-7+x)^3 (37 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x) (59 - 16x + x^2) (31 - 12x + x^2), \\
& (-9+x)^7 (-7+x)^2 (-232 + 123x - 20x^2 + x^3), \\
& (-9+x)^7 (-8+x) (-7+x) (-5+x) (41 - 14x + x^2), \\
& (-9+x)^6 (-7+x)^3 (-292 + 149x - 22x^2 + x^3), \\
& (-9+x)^7 (-7+x) (1604 - 1089x + 263x^2 - 27x^3 + x^4), \quad (-9+x)^9 (-7+x) (-5+x) (-4+x), \\
& (-9+x)^7 (-7+x)^3 (32 - 13x + x^2), \quad (-9+x)^8 (-7+x) (-176 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-7+x) (-172 + 101x - 18x^2 + x^3), \quad (-9+x)^8 (-8+x) (-7+x)^2 (-3+x) \}
\end{aligned}$$

**length 81**

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feasiblefactorlist[41, (x + 5)^25 (x - 7)^3]
{(-13+x) (-9+x)^3 (-8+x) (-7+x)^8, (-9+x)^2 (-7+x)^8 (-932 + 293x - 30x^2 + x^3),
 (-9+x)^4 (-7+x)^7 (80 - 19x + x^2), (-9+x)^2 (-7+x)^8 (-928 + 293x - 30x^2 + x^3),
 (-9+x)^4 (-7+x)^6 (-556 + 213x - 26x^2 + x^3),
 (-9+x)^2 (-7+x)^7 (6452 - 2975x + 503x^2 - 37x^3 + x^4),
 (-12+x) (-11+x) (-9+x)^2 (-7+x)^9, (-9+x)^4 (-8+x) (-7+x)^6 (69 - 18x + x^2),

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$$\begin{aligned}
& (-9 + x)^3 (-7 + x)^7 (-712 + 251x - 28x^2 + x^3), \\
& (-9 + x)^2 (-8 + x) (-7 + x)^6 (95 - 20x + x^2) (59 - 16x + x^2), \\
& (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^7 (73 - 18x + x^2), \\
& (-11 + x)^3 (-8 + x) (-7 + x)^9, (-9 + x)^4 (-7 + x)^6 (-548 + 213x - 26x^2 + x^3), \\
& (-9 + x)^3 (-7 + x)^6 (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\
& (-9 + x)^3 (-7 + x)^6 (73 - 18x + x^2) (68 - 17x + x^2), \\
& (-11 + x) (-9 + x)^2 (-7 + x)^7 (-580 + 217x - 26x^2 + x^3), \\
& (-11 + x)^2 (-9 + x) (-7 + x)^8 (68 - 17x + x^2), (-9 + x)^4 (-7 + x)^6 (-544 + 213x - 26x^2 + x^3), \\
& (-9 + x)^3 (-7 + x)^6 (4912 - 2461x + 447x^2 - 35x^3 + x^4), \\
& (-9 + x)^2 (-7 + x)^6 (59 - 16x + x^2) (-752 + 255x - 28x^2 + x^3), \\
& (-11 + x) (-9 + x)^3 (-7 + x)^7 (64 - 17x + x^2), (-12 + x) (-9 + x)^5 (-7 + x)^6 (-5 + x), \\
& (-9 + x)^4 (-7 + x)^5 (73 - 18x + x^2) (52 - 15x + x^2), \\
& (-9 + x)^3 (-7 + x)^6 (4876 - 2457x + 447x^2 - 35x^3 + x^4), \\
& (-11 + x)^2 (-9 + x)^2 (-7 + x)^7 (52 - 15x + x^2), \\
& (-9 + x)^4 (-8 + x) (-7 + x)^5 (-467 + 195x - 25x^2 + x^3), \\
& (-9 + x)^5 (-8 + x) (-7 + x)^4 (-5 + x) (73 - 18x + x^2), \\
& (-9 + x)^4 (-7 + x)^6 (-536 + 213x - 26x^2 + x^3), \\
& (-9 + x)^4 (-7 + x)^5 (3768 - 2027x + 395x^2 - 33x^3 + x^4), \\
& (-11 + x)^2 (-9 + x)^3 (-8 + x) (-7 + x)^6 (-5 + x), \\
& (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^5 (59 - 16x + x^2)^2, \\
& (-9 + x)^5 (-7 + x)^5 (-412 + 179x - 24x^2 + x^3), \\
& (-9 + x)^5 (-7 + x)^4 (-5 + x) (-580 + 217x - 26x^2 + x^3), \\
& (-9 + x)^4 (-7 + x)^4 (-26084 + 17885x - 4788x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9 + x)^4 (-7 + x)^6 (-532 + 213x - 26x^2 + x^3), \\
& (-9 + x)^4 (-7 + x)^4 (59 - 16x + x^2) (-444 + 183x - 24x^2 + x^3), \\
& (-11 + x) (-9 + x)^4 (-7 + x)^5 (-5 + x) (68 - 17x + x^2), \\
& (-9 + x)^3 (-7 + x)^4 (68 - 17x + x^2) (59 - 16x + x^2)^2, \\
& (-9 + x)^5 (-7 + x)^4 (2864 - 1661x + 347x^2 - 31x^3 + x^4), \\
& (-9 + x)^4 (-7 + x)^5 (3680 - 2019x + 395x^2 - 33x^3 + x^4), \\
& (-9 + x)^6 (-7 + x)^4 (-5 + x) (64 - 17x + x^2), \\
& (-9 + x)^4 (-7 + x)^4 (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9 + x)^4 (-7 + x)^4 (-25888 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11 + x) (-9 + x)^4 (-7 + x)^6 (48 - 15x + x^2), (-9 + x)^5 (-7 + x)^5 (-404 + 179x - 24x^2 + x^3), \\
& (-9 + x)^6 (-7 + x)^4 (-316 + 149x - 22x^2 + x^3), \\
& (-9 + x)^4 (-7 + x)^4 (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11 + x) (-9 + x)^4 (-7 + x)^5 (-332 + 153x - 22x^2 + x^3), \\
& (-11 + x) (-9 + x)^5 (-7 + x)^4 (-5 + x) (52 - 15x + x^2), \\
& (-9 + x)^4 (-7 + x)^4 (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\
& (-9 + x)^4 (-7 + x)^3 (59 - 16x + x^2)^2 (52 - 15x + x^2), \\
& (-9 + x)^5 (-8 + x) (-7 + x)^4 (-349 + 163x - 23x^2 + x^3), \\
& (-9 + x)^6 (-7 + x)^4 (-312 + 149x - 22x^2 + x^3), \\
& (-11 + x) (-9 + x)^4 (-8 + x) (-7 + x)^5 (41 - 14x + x^2),
\end{aligned}$$

$$\begin{aligned}
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^3 (-5+x)^2, \\
& (-9+x)^5 (-7+x)^4 (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^5 (-8+x) (-7+x)^2 (-5+x) (59 - 16x + x^2)^2, (-11+x) (-9+x)^6 (-7+x)^5 (-4+x), \\
& (-9+x)^6 (-7+x)^3 (2172 - 1351x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^5 (-7+x)^4 (68 - 17x + x^2) (41 - 14x + x^2), \\
& (-9+x)^7 (-7+x)^2 (-5+x)^2 (68 - 17x + x^2), (-9+x)^6 (-7+x)^4 (-304 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (2144 - 1347x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^3 (-5+x) (48 - 15x + x^2), \\
& (-9+x)^6 (-7+x)^2 (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^4 (-300 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (2116 - 1343x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^2 (-5+x) (-332 + 153x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^3 (52 - 15x + x^2) (41 - 14x + x^2), \\
& (-9+x)^8 (-7+x)^2 (52 - 15x + x^2), (-9+x)^6 (-8+x) (-7+x)^4 (37 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^2 (59 - 16x + x^2) (31 - 12x + x^2), \\
& (-9+x)^7 (-7+x)^3 (-232 + 123x - 20x^2 + x^3), \\
& (-9+x)^7 (-8+x) (-7+x)^2 (-5+x) (41 - 14x + x^2), \\
& (-9+x)^9 (-8+x) (-5+x)^3, (-9+x)^6 (-7+x)^4 (-292 + 149x - 22x^2 + x^3), \\
& (-9+x)^7 (-7+x)^2 (1604 - 1089x + 263x^2 - 27x^3 + x^4), \\
& (-9+x)^9 (-7+x)^2 (-5+x) (-4+x), (-9+x)^7 (-7+x)^4 (32 - 13x + x^2), \\
& (-9+x)^8 (-7+x)^2 (-176 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-7+x)^2 (-172 + 101x - 18x^2 + x^3), (-9+x)^8 (-8+x) (-7+x)^3 (-3+x) \}
\end{aligned}$$

$$\begin{aligned}
\text{list} = & \left\{ (-13+x) (-9+x)^3 (-8+x) (-7+x)^{11} (5+x)^{25}, \right. \\
& (-9+x)^2 (-7+x)^{11} (5+x)^{25} (-932 + 293x - 30x^2 + x^3), \\
& (-9+x)^4 (-7+x)^{10} (5+x)^{25} (80 - 19x + x^2), (-9+x)^2 (-7+x)^{11} (5+x)^{25} \\
& (-928 + 293x - 30x^2 + x^3), (-9+x)^4 (-7+x)^9 (5+x)^{25} (-556 + 213x - 26x^2 + x^3), \\
& (-9+x)^2 (-7+x)^{10} (5+x)^{25} (6452 - 2975x + 503x^2 - 37x^3 + x^4), \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^4 (-8+x) (-7+x)^9 (5+x)^{25} (69 - 18x + x^2), \\
& (-9+x)^3 (-7+x)^{10} (5+x)^{25} (-712 + 251x - 28x^2 + x^3), \\
& (-9+x)^2 (-8+x) (-7+x)^9 (5+x)^{25} (95 - 20x + x^2) (59 - 16x + x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^{10} (5+x)^{25} (73 - 18x + x^2), \\
& (-11+x)^3 (-8+x) (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-548 + 213x - 26x^2 + x^3), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (73 - 18x + x^2) (68 - 17x + x^2), \\
& (-11+x) (-9+x)^2 (-7+x)^{10} (5+x)^{25} (-580 + 217x - 26x^2 + x^3), \\
& (-11+x)^2 (-9+x) (-7+x)^{11} (5+x)^{25} (68 - 17x + x^2), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-544 + 213x - 26x^2 + x^3), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4912 - 2461x + 447x^2 - 35x^3 + x^4), \\
& \left. (-9+x)^2 (-7+x)^9 (5+x)^{25} (59 - 16x + x^2) (-752 + 255x - 28x^2 + x^3) \right\}
\end{aligned}$$

$$\begin{aligned}
& (-11+x) (-9+x)^3 (-7+x)^{10} (5+x)^{25} (64 - 17x + x^2), \\
& (-12+x) (-9+x)^5 (-7+x)^9 (-5+x) (5+x)^{25}, \\
& (-9+x)^4 (-7+x)^8 (5+x)^{25} (73 - 18x + x^2) (52 - 15x + x^2), \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4876 - 2457x + 447x^2 - 35x^3 + x^4), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52 - 15x + x^2), \\
& (-9+x)^4 (-8+x) (-7+x)^8 (5+x)^{25} (-467 + 195x - 25x^2 + x^3), \\
& (-9+x)^5 (-8+x) (-7+x)^7 (-5+x) (5+x)^{25} (73 - 18x + x^2), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-536 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x)^8 (5+x)^{25} (3768 - 2027x + 395x^2 - 33x^3 + x^4), \\
& (-11+x)^2 (-9+x)^3 (-8+x) (-7+x)^9 (-5+x) (5+x)^{25}, \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^8 (5+x)^{25} (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x)^8 (5+x)^{25} (-412 + 179x - 24x^2 + x^3), \\
& (-9+x)^5 (-7+x)^7 (-5+x) (5+x)^{25} (-580 + 217x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-26084 + 17885x - 4788x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-532 + 213x - 26x^2 + x^3), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (59 - 16x + x^2) (-444 + 183x - 24x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-7+x)^8 (-5+x) (5+x)^{25} (68 - 17x + x^2), \\
& (-9+x)^3 (-7+x)^7 (5+x)^{25} (68 - 17x + x^2) (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x)^7 (5+x)^{25} (2864 - 1661x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^4 (-7+x)^8 (5+x)^{25} (3680 - 2019x + 395x^2 - 33x^3 + x^4), \\
& (-9+x)^6 (-7+x)^7 (-5+x) (5+x)^{25} (64 - 17x + x^2), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25888 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^9 (5+x)^{25} (48 - 15x + x^2), \\
& (-9+x)^5 (-7+x)^8 (5+x)^{25} (-404 + 179x - 24x^2 + x^3), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-316 + 149x - 22x^2 + x^3), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^8 (5+x)^{25} (-332 + 153x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^5 (-7+x)^7 (-5+x) (5+x)^{25} (52 - 15x + x^2), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\
& (-9+x)^4 (-7+x)^6 (5+x)^{25} (59 - 16x + x^2)^2 (52 - 15x + x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349 + 163x - 23x^2 + x^3), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-312 + 149x - 22x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-8+x) (-7+x)^8 (5+x)^{25} (41 - 14x + x^2), \\
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^6 (-5+x)^2 (5+x)^{25}, \\
& (-9+x)^5 (-7+x)^7 (5+x)^{25} (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\
& (-9+x)^5 (-8+x) (-7+x)^5 (-5+x) (5+x)^{25} (59 - 16x + x^2)^2, \\
& (-11+x) (-9+x)^6 (-7+x)^8 (-4+x) (5+x)^{25}, \\
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (2172 - 1351x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^5 (-7+x)^7 (5+x)^{25} (68 - 17x + x^2) (41 - 14x + x^2), \\
& (-9+x)^7 (-7+x)^5 (-5+x)^2 (5+x)^{25} (68 - 17x + x^2), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-304 + 149x - 22x^2 + x^3),
\end{aligned}$$

$$\begin{aligned}
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (2144 - 1347x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^6 (-5+x) (5+x)^{25} (48 - 15x + x^2), \\
& (-9+x)^6 (-7+x)^5 (5+x)^{25} (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-300 + 149x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (2116 - 1343x + 303x^2 - 29x^3 + x^4), \\
& (-9+x)^7 (-7+x)^5 (-5+x) (5+x)^{25} (-332 + 153x - 22x^2 + x^3), \\
& (-9+x)^6 (-7+x)^6 (5+x)^{25} (52 - 15x + x^2) (41 - 14x + x^2), \\
& (-9+x)^8 (-7+x)^4 (-5+x)^2 (5+x)^{25} (52 - 15x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^7 (5+x)^{25} (37 - 14x + x^2), \\
& (-9+x)^6 (-8+x) (-7+x)^5 (5+x)^{25} (59 - 16x + x^2) (31 - 12x + x^2), \\
& (-9+x)^7 (-7+x)^6 (5+x)^{25} (-232 + 123x - 20x^2 + x^3), \\
& (-9+x)^7 (-8+x) (-7+x)^5 (-5+x) (5+x)^{25} (41 - 14x + x^2), \\
& (-9+x)^9 (-8+x) (-7+x)^3 (-5+x)^3 (5+x)^{25}, \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-292 + 149x - 22x^2 + x^3), \\
& (-9+x)^7 (-7+x)^5 (5+x)^{25} (1604 - 1089x + 263x^2 - 27x^3 + x^4), \\
& (-9+x)^9 (-7+x)^5 (-5+x) (-4+x) (5+x)^{25}, (-9+x)^7 (-7+x)^7 (5+x)^{25} \\
& \quad (32 - 13x + x^2), (-9+x)^8 (-7+x)^5 (5+x)^{25} (-176 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-7+x)^5 (5+x)^{25} (-172 + 101x - 18x^2 + x^3), \\
& (-9+x)^8 (-8+x) (-7+x)^6 (-3+x) (5+x)^{25} \}; 
\end{aligned}$$

`Length[list]`

82

`listmod64 = modfilter[list, Smod64n41unrefined, 64]`

$$\left\{ \begin{aligned} & (-9+x)^4 (-7+x)^{10} (5+x)^{25} (80 - 19x + x^2), \\ & (-9+x)^2 (-7+x)^{11} (5+x)^{25} (-928 + 293x - 30x^2 + x^3), \\ & (-9+x)^2 (-7+x)^{10} (5+x)^{25} (6452 - 2975x + 503x^2 - 37x^3 + x^4), \\ & (-12+x) (-11+x) (-9+x)^2 (-7+x)^{12} (5+x)^{25}, \\ & (-9+x)^2 (-8+x) (-7+x)^9 (5+x)^{25} (95 - 20x + x^2) (59 - 16x + x^2), \\ & (-11+x)^3 (-8+x) (-7+x)^{12} (5+x)^{25}, \\ & (-9+x)^4 (-7+x)^9 (5+x)^{25} (-548 + 213x - 26x^2 + x^3), \\ & (-9+x)^3 (-7+x)^9 (5+x)^{25} (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\ & (-9+x)^3 (-7+x)^9 (5+x)^{25} (73 - 18x + x^2) (68 - 17x + x^2), \\ & (-9+x)^4 (-7+x)^9 (5+x)^{25} (-544 + 213x - 26x^2 + x^3), \\ & (-9+x)^3 (-7+x)^9 (5+x)^{25} (4912 - 2461x + 447x^2 - 35x^3 + x^4), \\ & (-11+x) (-9+x)^3 (-7+x)^{10} (5+x)^{25} (64 - 17x + x^2), \\ & (-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52 - 15x + x^2), \\ & (-9+x)^5 (-8+x) (-7+x)^7 (-5+x) (5+x)^{25} (73 - 18x + x^2), \\ & (-11+x) (-9+x)^2 (-8+x) (-7+x)^8 (5+x)^{25} (59 - 16x + x^2)^2, \\ & (-9+x)^5 (-7+x)^8 (5+x)^{25} (-412 + 179x - 24x^2 + x^3), \\ & (-9+x)^5 (-7+x)^7 (-5+x) (5+x)^{25} (-580 + 217x - 26x^2 + x^3), \\ & (-9+x)^4 (-7+x)^9 (5+x)^{25} (-532 + 213x - 26x^2 + x^3), \\ & (-11+x) (-9+x)^4 (-7+x)^8 (-5+x) (5+x)^{25} (68 - 17x + x^2), \\ & (-9+x)^4 (-7+x)^8 (5+x)^{25} (3680 - 2019x + 395x^2 - 33x^3 + x^4), \\ & (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\ & (-11+x) (-9+x)^4 (-7+x)^9 (5+x)^{25} (48 - 15x + x^2), \\ & (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\ & (-9+x)^4 (-7+x)^7 (5+x)^{25} (59 - 16x + x^2) (-436 + 183x - 24x^2 + x^3), \\ & (-9+x)^4 (-7+x)^6 (5+x)^{25} (59 - 16x + x^2)^2 (52 - 15x + x^2), \\ & (-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349 + 163x - 23x^2 + x^3), \\ & (-9+x)^6 (-7+x)^7 (5+x)^{25} (-312 + 149x - 22x^2 + x^3), \\ & (-11+x) (-9+x)^6 (-8+x) (-7+x)^6 (-5+x)^2 (5+x)^{25}, \\ & (-9+x)^5 (-7+x)^7 (5+x)^{25} (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\ & (-11+x) (-9+x)^6 (-7+x)^8 (-4+x) (5+x)^{25}, \\ & (-9+x)^6 (-7+x)^6 (5+x)^{25} (2172 - 1351x + 303x^2 - 29x^3 + x^4), \\ & (-9+x)^5 (-7+x)^7 (5+x)^{25} (68 - 17x + x^2) (41 - 14x + x^2), \\ & (-9+x)^6 (-7+x)^5 (5+x)^{25} (59 - 16x + x^2) (-256 + 127x - 20x^2 + x^3), \\ & (-9+x)^7 (-7+x)^5 (-5+x) (5+x)^{25} (-332 + 153x - 22x^2 + x^3), \\ & (-9+x)^8 (-7+x)^4 (-5+x)^2 (5+x)^{25} (52 - 15x + x^2), \\ & (-9+x)^6 (-8+x) (-7+x)^7 (5+x)^{25} (37 - 14x + x^2), \\ & (-9+x)^7 (-7+x)^6 (5+x)^{25} (-232 + 123x - 20x^2 + x^3), \\ & (-9+x)^7 (-8+x) (-7+x)^5 (-5+x) (5+x)^{25} (41 - 14x + x^2), \\ & (-9+x)^6 (-7+x)^7 (5+x)^{25} (-292 + 149x - 22x^2 + x^3), \\ & (-9+x)^8 (-7+x)^5 (5+x)^{25} (-172 + 101x - 18x^2 + x^3), \\ & (-9+x)^8 (-8+x) (-7+x)^6 (-3+x) (5+x)^{25} \end{aligned} \right\}$$

```

Length[listmod64]
41

listmod128 = modfilter[listmod64, Smod128n41unrefined, 128]
{ (-9 + x)^4 (-7 + x)^10 (5 + x)^25 (80 - 19 x + x^2),
  (-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^12 (5 + x)^25, (-11 + x)^3 (-8 + x) (-7 + x)^12 (5 + x)^25,
  (-9 + x)^3 (-7 + x)^9 (5 + x)^25 (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4),
  (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (-544 + 213 x - 26 x^2 + x^3),
  (-11 + x) (-9 + x)^3 (-7 + x)^10 (5 + x)^25 (64 - 17 x + x^2),
  (-11 + x)^2 (-9 + x)^2 (-7 + x)^10 (5 + x)^25 (52 - 15 x + x^2),
  (-11 + x) (-9 + x)^2 (-8 + x) (-7 + x)^8 (5 + x)^25 (59 - 16 x + x^2)^2,
  (-9 + x)^5 (-7 + x)^8 (5 + x)^25 (-412 + 179 x - 24 x^2 + x^3),
  (-11 + x) (-9 + x)^4 (-7 + x)^8 (-5 + x) (5 + x)^25 (68 - 17 x + x^2),
  (-9 + x)^4 (-7 + x)^7 (5 + x)^25 (-25904 + 17829 x - 4784 x^2 + 626 x^3 - 40 x^4 + x^5),
  (-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (48 - 15 x + x^2),
  (-9 + x)^4 (-7 + x)^7 (5 + x)^25 (-25580 + 17757 x - 4780 x^2 + 626 x^3 - 40 x^4 + x^5),
  (-9 + x)^4 (-7 + x)^6 (5 + x)^25 (59 - 16 x + x^2)^2 (52 - 15 x + x^2),
  (-9 + x)^5 (-8 + x) (-7 + x)^7 (5 + x)^25 (-349 + 163 x - 23 x^2 + x^3),
  (-11 + x) (-9 + x)^6 (-8 + x) (-7 + x)^6 (-5 + x)^2 (5 + x)^25,
  (-9 + x)^5 (-7 + x)^7 (5 + x)^25 (2824 - 1653 x + 347 x^2 - 31 x^3 + x^4),
  (-11 + x) (-9 + x)^6 (-7 + x)^8 (-4 + x) (5 + x)^25,
  (-9 + x)^8 (-7 + x)^4 (-5 + x)^2 (5 + x)^25 (52 - 15 x + x^2),
  (-9 + x)^7 (-7 + x)^6 (5 + x)^25 (-232 + 123 x - 20 x^2 + x^3),
  (-9 + x)^6 (-7 + x)^7 (5 + x)^25 (-292 + 149 x - 22 x^2 + x^3),
  (-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^25}

```

$$\begin{aligned}
\text{listmod128} = & \left\{ (-9+x)^4 (-7+x)^{10} (5+x)^{25} (80 - 19x + x^2), \right. \\
& (-12+x) (-11+x) (-9+x)^2 (-7+x)^{12} (5+x)^{25}, (-11+x)^3 (-8+x) (-7+x)^{12} (5+x)^{25}, \\
& (-9+x)^3 (-7+x)^9 (5+x)^{25} (4948 - 2465x + 447x^2 - 35x^3 + x^4), \\
& (-9+x)^4 (-7+x)^9 (5+x)^{25} (-544 + 213x - 26x^2 + x^3), \\
& (-11+x) (-9+x)^3 (-7+x)^{10} (5+x)^{25} (64 - 17x + x^2), \\
& (-11+x)^2 (-9+x)^2 (-7+x)^{10} (5+x)^{25} (52 - 15x + x^2), \\
& (-11+x) (-9+x)^2 (-8+x) (-7+x)^8 (5+x)^{25} (59 - 16x + x^2)^2, \\
& (-9+x)^5 (-7+x)^8 (5+x)^{25} (-412 + 179x - 24x^2 + x^3), \\
& (-11+x) (-9+x)^4 (-7+x)^8 (-5+x) (5+x)^{25} (68 - 17x + x^2), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5), \\
& (-11+x) (-9+x)^4 (-7+x)^9 (5+x)^{25} (48 - 15x + x^2), \\
& (-9+x)^4 (-7+x)^7 (5+x)^{25} (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5), \\
& (-9+x)^4 (-7+x)^6 (5+x)^{25} (59 - 16x + x^2)^2 (52 - 15x + x^2), \\
& (-9+x)^5 (-8+x) (-7+x)^7 (5+x)^{25} (-349 + 163x - 23x^2 + x^3), \\
& (-11+x) (-9+x)^6 (-8+x) (-7+x)^6 (-5+x)^2 (5+x)^{25}, \\
& (-9+x)^5 (-7+x)^7 (5+x)^{25} (2824 - 1653x + 347x^2 - 31x^3 + x^4), \\
& (-11+x) (-9+x)^6 (-7+x)^8 (-4+x) (5+x)^{25}, \\
& (-9+x)^8 (-7+x)^4 (-5+x)^2 (5+x)^{25} (52 - 15x + x^2), \\
& (-9+x)^7 (-7+x)^6 (5+x)^{25} (-232 + 123x - 20x^2 + x^3), \\
& (-9+x)^6 (-7+x)^7 (5+x)^{25} (-292 + 149x - 22x^2 + x^3), \\
& \left. (-9+x)^8 (-8+x) (-7+x)^6 (-3+x) (5+x)^{25} \right\};
\end{aligned}$$

**Length[listmod128]**

22

$$\begin{aligned}
\text{poly1} = & \text{listmod128}[1] \\
& (-9+x)^4 (-7+x)^{10} (5+x)^{25} (80 - 19x + x^2) \\
\text{p1} = & \text{poly1} / \text{minipoly}[\text{poly1}] // \text{Factor} \\
& (-9+x)^3 (-7+x)^9 (5+x)^{24} \\
\text{feasible} = & \text{subcharpolylevelist}[( -9+x)^4 (-7+x)^{10} (5+x)^{25} (80 - 19x + x^2)] \\
& \{1815 - 1314x + 312x^2 - 30x^3 + x^4, \\
& (-7+x) (-249 + 151x - 23x^2 + x^3), 1759 - 1306x + 312x^2 - 30x^3 + x^4, \\
& (-7+x) (-241 + 151x - 23x^2 + x^3), 1703 - 1298x + 312x^2 - 30x^3 + x^4, \\
& (-9+x) (-191 + 123x - 21x^2 + x^3), (-7+x) (-233 + 151x - 23x^2 + x^3), \\
& (-9+x) (-183 + 123x - 21x^2 + x^3), (-9+x) (-7+x) (25 - 14x + x^2) \}
\end{aligned}$$

```

CoefficientList[{1815 - 1314 x + 312 x2 - 30 x3 + x4,
  (-7 + x) (-249 + 151 x - 23 x2 + x3), 1759 - 1306 x + 312 x2 - 30 x3 + x4,
  (-7 + x) (-241 + 151 x - 23 x2 + x3), 1703 - 1298 x + 312 x2 - 30 x3 + x4,
  (-9 + x) (-191 + 123 x - 21 x2 + x3), (-7 + x) (-233 + 151 x - 23 x2 + x3),
  (-9 + x) (-183 + 123 x - 21 x2 + x3), (-9 + x) (-7 + x) (25 - 14 x + x2)}, x]

{{1815, -1314, 312, -30, 1}, {1743, -1306, 312, -30, 1}, {1759, -1306, 312, -30, 1},
 {1687, -1298, 312, -30, 1}, {1703, -1298, 312, -30, 1}, {1719, -1298, 312, -30, 1},
 {1631, -1290, 312, -30, 1}, {1647, -1290, 312, -30, 1}, {1575, -1282, 312, -30, 1}};

A = {{1815, -1314, 312, -30, 1}, {1743, -1306, 312, -30, 1},
  {1759, -1306, 312, -30, 1}, {1687, -1298, 312, -30, 1},
  {1703, -1298, 312, -30, 1}, {1719, -1298, 312, -30, 1}, {1631, -1290,
  312, -30, 1}, {1647, -1290, 312, -30, 1}, {1575, -1282, 312, -30, 1}};

MatrixRank[A]
3

CoefficientList[D[poly1, x] / p1 // Factor, x]
{72815, -53602, 12792, -1230, 41}

Solve[
  Array[n, 9].{{1815, -1314, 312, -30, 1}, {1743, -1306, 312, -30, 1}, {1759, -1306,
  312, -30, 1}, {1687, -1298, 312, -30, 1}, {1703, -1298, 312, -30, 1},
  {1719, -1298, 312, -30, 1}, {1631, -1290, 312, -30, 1},
  {1647, -1290, 312, -30, 1}, {1575, -1282, 312, -30, 1}} ==
  {72815, -53602, 12792, -1230, 41}, Array[n, 9]]

Solve::svars : Equations may not give solutions for all "solve" variables. >>>
{{n[7] → 70 - 3 n[1] - 3 n[2] - 2 n[3] - 2 n[4] - n[5],
  n[8] → 60 - n[1] - n[3] - n[5] - 2 n[6],
  n[9] → -89 + 3 n[1] + 2 n[2] + 2 n[3] + n[4] + n[5] + n[6]}}

FindInstance[-n[7] + 70 - 3 n[1] - 3 n[2] - 2 n[3] - 2 n[4] - n[5] == 0 &&
  -n[8] + 60 - n[1] - n[3] - n[5] - 2 n[6] == 0 &&
  -n[9] - 89 + 3 n[1] + 2 n[2] + 2 n[3] + n[4] + n[5] + n[6] == 0 &&
  n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 &&
  n[7] ≥ 0 && n[8] ≥ 0 && n[9] ≥ 0, Array[n, 9], Integers]
{ }

```

```

CoefficientList[{1815 - 1314 x + 312 x2 - 30 x3 + x4,
  (-7 + x) (-249 + 151 x - 23 x2 + x3) , 1759 - 1306 x + 312 x2 - 30 x3 + x4,
  (-7 + x) (-241 + 151 x - 23 x2 + x3) , 1703 - 1298 x + 312 x2 - 30 x3 + x4,
  (-9 + x) (-191 + 123 x - 21 x2 + x3) , (-7 + x) (-233 + 151 x - 23 x2 + x3) ,
  (-9 + x) (-183 + 123 x - 21 x2 + x3) , (-9 + x) (-7 + x) (25 - 14 x + x2) }, x] // MatrixForm

{1815 -1314 312 -30 1
 1743 -1306 312 -30 1
 1759 -1306 312 -30 1
 1687 -1298 312 -30 1
 1703 -1298 312 -30 1
 1719 -1298 312 -30 1
 1631 -1290 312 -30 1
 1647 -1290 312 -30 1
 1575 -1282 312 -30 1}

CoefficientList[D[poly1, x] / p1 // Factor, x]
{72815, -53602, 12792, -1230, 41}

Transpose[{1815 -1314 312 -30 1
 1743 -1306 312 -30 1
 1759 -1306 312 -30 1
 1687 -1298 312 -30 1
 1703 -1298 312 -30 1
 1719 -1298 312 -30 1
 1631 -1290 312 -30 1
 1647 -1290 312 -30 1
 1575 -1282 312 -30 1}] // MatrixForm

{1815 1743 1759 1687 1703 1719 1631 1647 1575
 -1314 -1306 -1306 -1298 -1298 -1298 -1290 -1290 -1282
 312 312 312 312 312 312 312 312 312
 -30 -30 -30 -30 -30 -30 -30 -30 -30
 1 1 1 1 1 1 1 1 1}

Array[m, 5].{{1815 1743 1759 1687 1703 1719 1631 1647 1575
 -1314 -1306 -1306 -1298 -1298 -1298 -1290 -1290 -1282
 312 312 312 312 312 312 312 312 312
 -30 -30 -30 -30 -30 -30 -30 -30 -30
 1 1 1 1 1 1 1 1 1}};

{1815 m[1] - 1314 m[2] + 312 m[3] - 30 m[4] + m[5],
 1743 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5],
 1759 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5],
 1687 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5],
 1703 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5],
 1719 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5],
 1631 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5],
 1647 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5],
 1575 m[1] - 1282 m[2] + 312 m[3] - 30 m[4] + m[5]}

Array[m, 5].{72815, -53602, 12792, -1230, 41}
72815 m[1] - 53602 m[2] + 12792 m[3] - 1230 m[4] + 41 m[5]

```

```

FindInstance[1815 m[1] - 1314 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1743 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1759 m[1] - 1306 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1687 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1703 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1719 m[1] - 1298 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1631 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1647 m[1] - 1290 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
1575 m[1] - 1282 m[2] + 312 m[3] - 30 m[4] + m[5] ≥ 0 &&
72815 m[1] - 53602 m[2] + 12792 m[3] - 1230 m[4] + 41 m[5] < 0, Array[m, 5], Integers]

{{m[1] → -4390, m[2] → -26341, m[3] → 0, m[4] → 0, m[5] → -26642928} }

Array[m, 5] /. {m[1] → -4390, m[2] → -26341, m[3] → 0, m[4] → 0, m[5] → -26642928}

{-4390, -26341, 0, 0, -26642928}

GCD[-4390, -26341, 0, 0, -26642928]

1

{-4390, -26341, 0, 0, -26642928}.


$$\begin{pmatrix} 1815 & 1743 & 1759 & 1687 & 1703 & 1719 & 1631 & 1647 & 1575 \\ -1314 & -1306 & -1306 & -1298 & -1298 & -1298 & -1290 & -1290 & -1282 \\ 312 & 312 & 312 & 312 & 312 & 312 & 312 & 312 & 312 \\ -30 & -30 & -30 & -30 & -30 & -30 & -30 & -30 & -30 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$


{1296, 106648, 36408, 141760, 71520, 1280, 176872, 106632, 211984}

{-4390, -26341, 0, 0, -26642928}.{72815, -53602, 12792, -1230, 41}

-87616

```

```

poly2 = listmod128[[2]]

(-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^12 (5 + x)^25

p2 = poly2 / minipoly[poly2] // Factor

(-9 + x) (-7 + x)^11 (5 + x)^24

feasiblesubcharpolylist[(-12 + x) (-11 + x) (-9 + x)^2 (-7 + x)^12 (5 + x)^25]
{(-11 + x) (-269 + 155 x - 23 x^2 + x^3), (-11 + x) (-9 + x) (29 - 14 x + x^2)}

CoefficientList[
{(-11 + x) (-269 + 155 x - 23 x^2 + x^3), (-11 + x) (-9 + x) (29 - 14 x + x^2)}, x]

{{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1} }

CoefficientList[D[poly2, x] / p2 // Factor, x]

{120135, -80838, 16728, -1394, 41}

```

```

Solve[Array[n, 2].{{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1}} ==
{120135, -80838, 16728, -1394, 41}, Array[n, 2]]
{ }

A = {{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1}}
{{2959, -1974, 408, -34, 1}, {2871, -1966, 408, -34, 1} }

Array[m, 5].Transpose[A]
{2959 m[1] - 1974 m[2] + 408 m[3] - 34 m[4] + m[5],
2871 m[1] - 1966 m[2] + 408 m[3] - 34 m[4] + m[5]}

Array[m, 5].{120135, -80838, 16728, -1394, 41}
120135 m[1] - 80838 m[2] + 16728 m[3] - 1394 m[4] + 41 m[5]

Array[m, 5] /. FindInstance[2959 m[1] - 1974 m[2] + 408 m[3] - 34 m[4] + m[5] ≥ 0 &&
2871 m[1] - 1966 m[2] + 408 m[3] - 34 m[4] + m[5] ≥ 0 &&
120135 m[1] - 80838 m[2] + 16728 m[3] - 1394 m[4] + 41 m[5] < 0,
Array[m, 5], Integers]
{{1713, 18832, 0, 0, 32108784} }

1713 * 2959 + 18832 * (-1974) + 32108784
3183

GCD[1713, 18832, 0, 0, 32108784]
1

{1713, 18832, 0, 0, 32108784}.Transpose[A]
{3183, 3095}

{1713, 18832, 0, 0, 32108784}.{120135, -80838, 16728, -1394, 41}
-89817

```

```

poly3 = listmod128[[3]]
(-11 + x)^3 (-8 + x) (-7 + x)^12 (5 + x)^25

p3 = poly3 / minipoly[poly3] // Factor
(-11 + x)^2 (-7 + x)^11 (5 + x)^24

feasiblesubcharpolylist[(-11 + x)^3 (-8 + x) (-7 + x)^12 (5 + x)^25]
{-223 + 131 x - 21 x^2 + x^3, (-11 + x) (-7 + x) (-3 + x) }

CoefficientList[{-223 + 131 x - 21 x^2 + x^3, (-11 + x) (-7 + x) (-3 + x)}, x]
{{-223, 131, -21, 1}, {-231, 131, -21, 1}}

```

```

{{{-231, 131, -21, 1}, {-223, 131, -21, 1}} // MatrixForm

$$\begin{pmatrix} -231 & 131 & -21 & 1 \\ -223 & 131 & -21 & 1 \end{pmatrix}$$


CoefficientList[D[poly3, x] / p3 // Factor, x]
{-8895, 5371, -861, 41}

Solve[Array[n, 2].{{-223, 131, -21, 1}, {-231, 131, -21, 1}} ==
{-8895, 5371, -861, 41}, Array[n, 2]]
{{n[1] → 72, n[2] → -31} }

Array[m, 4].Transpose[{{-231, 131, -21, 1}, {-223, 131, -21, 1}}]
{-231 m[1] + 131 m[2] - 21 m[3] + m[4], -223 m[1] + 131 m[2] - 21 m[3] + m[4]}

Array[m, 4].{-8895, 5371, -861, 41}
-8895 m[1] + 5371 m[2] - 861 m[3] + 41 m[4]

FindInstance[
-231 m[1] + 131 m[2] - 21 m[3] + m[4] ≥ 0 && -223 m[1] + 131 m[2] - 21 m[3] + m[4] ≥ 0 &&
-8895 m[1] + 5371 m[2] - 861 m[3] + 41 m[4] < 0, Array[m, 4], Integers]
{{m[1] → -62, m[2] → 0, m[3] → 0, m[4] → -13493} }

Array[m, 4] /. {m[1] → -62, m[2] → 0, m[3] → 0, m[4] → -13493}
{-62, 0, 0, -13493}

GCD[-62, 0, 0, -13493]
1

{-62, 0, 0, -13493} // Reverse
{-13493, 0, 0, -62}

{-62, 0, 0, -13493}.{-8895, 5371, -861, 41}
-1723

{-62, 0, 0, -13493}.Transpose[{{-231, 131, -21, 1}, {-223, 131, -21, 1}}]
{829, 333}

poly4 = listmod128[[4]]
(-9 + x)^3 (-7 + x)^9 (5 + x)^25 (4948 - 2465 x + 447 x^2 - 35 x^3 + x^4)

p4 = poly4 / minipoly[poly4] // Factor
(-9 + x)^2 (-7 + x)^8 (5 + x)^24

```

```

feasiblesubcharpolylist[ (-9 + x)3 (-7 + x)9 (5 + x)25 (4948 - 2465 x + 447 x2 - 35 x3 + x4) ]
{ (-9 + x) (-7 + x) (1735 - 1298 x + 312 x2 - 30 x3 + x4) ,
(-7 + x) (-15 583 + 13 417 x - 4106 x2 + 582 x3 - 39 x4 + x5) ,
(-9 + x) (-7 + x) (1751 - 1298 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (1767 - 1298 x + 312 x2 - 30 x3 + x4) ,
(-9 + x)2 (-7 + x) (-183 + 123 x - 21 x2 + x3) , (-7 + x)2 (2113 - 1592 x + 358 x2 - 32 x3 + x4) ,
(-9 + x) (-7 + x) (1663 - 1290 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (73 - 18 x + x2) (23 - 12 x + x2) ,
(-9 + x) (-7 + x) (1695 - 1290 x + 312 x2 - 30 x3 + x4) ,
(-9 + x)2 (-7 + x)2 (25 - 14 x + x2) , (-9 + x) (-7 + x) (1591 - 1282 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (1607 - 1282 x + 312 x2 - 30 x3 + x4) ,
(-9 + x)2 (-7 + x) (-167 + 123 x - 21 x2 + x3) , (-9 + x) (-7 + x)2 (-217 + 151 x - 23 x2 + x3) ,
(-9 + x) (-7 + x) (1535 - 1274 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (1447 - 1266 x + 312 x2 - 30 x3 + x4) }

{ (-9 + x) (-7 + x) (1735 - 1298 x + 312 x2 - 30 x3 + x4) ,
(-7 + x) (-15 583 + 13 417 x - 4106 x2 + 582 x3 - 39 x4 + x5) ,
(-9 + x) (-7 + x) (1751 - 1298 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (1767 - 1298 x + 312 x2 - 30 x3 + x4) ,
(-9 + x)2 (-7 + x) (-183 + 123 x - 21 x2 + x3) ,
(-7 + x)2 (2113 - 1592 x + 358 x2 - 32 x3 + x4) , (-9 + x) (-7 + x)
(1663 - 1290 x + 312 x2 - 30 x3 + x4) , (-9 + x) (-7 + x) (73 - 18 x + x2) (23 - 12 x + x2) ,
(-9 + x) (-7 + x) (1695 - 1290 x + 312 x2 - 30 x3 + x4) , (-9 + x)2 (-7 + x)2 (25 - 14 x + x2) ,
(-9 + x) (-7 + x) (1591 - 1282 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (1607 - 1282 x + 312 x2 - 30 x3 + x4) ,
(-9 + x)2 (-7 + x) (-167 + 123 x - 21 x2 + x3) , (-9 + x) (-7 + x)2 (-217 + 151 x - 23 x2 + x3) ,
(-9 + x) (-7 + x) (1535 - 1274 x + 312 x2 - 30 x3 + x4) ,
(-9 + x) (-7 + x) (1447 - 1266 x + 312 x2 - 30 x3 + x4) } // Length

```

```

CoefficientList[{\{(-9+x) (-7+x) (1735 - 1298 x + 312 x2 - 30 x3 + x4), 
  (-7+x) (-15583 + 13417 x - 4106 x2 + 582 x3 - 39 x4 + x5), 
  (-9+x) (-7+x) (1751 - 1298 x + 312 x2 - 30 x3 + x4), 
  (-9+x) (-7+x) (1767 - 1298 x + 312 x2 - 30 x3 + x4), 
  (-9+x)2 (-7+x) (-183 + 123 x - 21 x2 + x3), 
  (-7+x)2 (2113 - 1592 x + 358 x2 - 32 x3 + x4), (-9+x) (-7+x) 
  (1663 - 1290 x + 312 x2 - 30 x3 + x4), (-9+x) (-7+x) (73 - 18 x + x2) (23 - 12 x + x2), 
  (-9+x) (-7+x) (1695 - 1290 x + 312 x2 - 30 x3 + x4), (-9+x)2 (-7+x)2 (25 - 14 x + x2), 
  (-9+x) (-7+x) (1591 - 1282 x + 312 x2 - 30 x3 + x4), 
  (-9+x) (-7+x) (1607 - 1282 x + 312 x2 - 30 x3 + x4), 
  (-9+x)2 (-7+x) (-167 + 123 x - 21 x2 + x3), (-9+x) (-7+x)2 (-217 + 151 x - 23 x2 + x3), 
  (-9+x) (-7+x) (1535 - 1274 x + 312 x2 - 30 x3 + x4), 
  (-9+x) (-7+x) (1447 - 1266 x + 312 x2 - 30 x3 + x4)\}, x]

{{109305, -109534, 42159, -8180, 855, -46, 1}, 
 {109081, -109502, 42159, -8180, 855, -46, 1}, 
 {110313, -109790, 42175, -8180, 855, -46, 1}, 
 {111321, -110046, 42191, -8180, 855, -46, 1}, 
 {103761, -107622, 41943, -8172, 855, -46, 1}, 
 {103537, -107590, 41943, -8172, 855, -46, 1}, 
 {104769, -107878, 41959, -8172, 855, -46, 1}, 
 {105777, -108134, 41975, -8172, 855, -46, 1}, 
 {106785, -108390, 41991, -8172, 855, -46, 1}, 
 {99225, -105966, 41743, -8164, 855, -46, 1}, 
 {100233, -106222, 41759, -8164, 855, -46, 1}, 
 {101241, -106478, 41775, -8164, 855, -46, 1}, 
 {94689, -104310, 41543, -8156, 855, -46, 1}, 
 {95697, -104566, 41559, -8156, 855, -46, 1}, 
 {96705, -104822, 41575, -8156, 855, -46, 1}, 
 {91161, -102910, 41359, -8148, 855, -46, 1}\}

A = {{109305, -109534, 42159, -8180, 855, -46, 1}, 
 {109081, -109502, 42159, -8180, 855, -46, 1}, {110313, -109790, 42175, 
 -8180, 855, -46, 1}, {111321, -110046, 42191, -8180, 855, -46, 1}, 
 {103761, -107622, 41943, -8172, 855, -46, 1}, {103537, -107590, 41943, 
 -8172, 855, -46, 1}, {104769, -107878, 41959, -8172, 855, -46, 1}, 
 {105777, -108134, 41975, -8172, 855, -46, 1}, {106785, -108390, 41991, 
 -8172, 855, -46, 1}, {99225, -105966, 41743, -8164, 855, -46, 1}, 
 {100233, -106222, 41759, -8164, 855, -46, 1}, {101241, -106478, 41775, 
 -8164, 855, -46, 1}, {94689, -104310, 41543, -8156, 855, -46, 1}, 
 {95697, -104566, 41559, -8156, 855, -46, 1}, {96705, -104822, 41575, 
 -8156, 855, -46, 1}, {91161, -102910, 41359, -8148, 855, -46, 1}\};

CoefficientList[D[poly4, x] / p4 // Factor, x]
{4493145, -4488726, 1727503, -335308, 35055, -1886, 41}

```

```

Solve[
  Array[n, 16].{{109 305, -109 534, 42 159, -8180, 855, -46, 1}, {109 081, -109 502,
    42 159, -8180, 855, -46, 1}, {110 313, -109 790, 42 175, -8180, 855, -46, 1},
    {111 321, -110 046, 42 191, -8180, 855, -46, 1}, {103 761, -107 622, 41 943,
      -8172, 855, -46, 1}, {103 537, -107 590, 41 943, -8172, 855, -46, 1},
      {104 769, -107 878, 41 959, -8172, 855, -46, 1}, {105 777, -108 134, 41 975,
        -8172, 855, -46, 1}, {106 785, -108 390, 41 991, -8172, 855, -46, 1},
        {99 225, -105 966, 41 743, -8164, 855, -46, 1}, {100 233, -106 222, 41 759,
          -8164, 855, -46, 1}, {101 241, -106 478, 41 775, -8164, 855, -46, 1},
          {94 689, -104 310, 41 543, -8156, 855, -46, 1}, {95 697, -104 566, 41 559,
            -8156, 855, -46, 1}, {96 705, -104 822, 41 575, -8156, 855, -46, 1},
            {91 161, -102 910, 41 359, -8148, 855, -46, 1}} =
  {4 493 145, -4 488 726, 1 727 503, -335 308, 35 055, -1886, 41}, Array[n, 16]]
{ }

MatrixForm[{{109 305, -109 534, 42 159, -8180, 855, -46, 1},
  {109 081, -109 502, 42 159, -8180, 855, -46, 1},
  {110 313, -109 790, 42 175, -8180, 855, -46, 1},
  {111 321, -110 046, 42 191, -8180, 855, -46, 1},
  {103 761, -107 622, 41 943, -8172, 855, -46, 1},
  {103 537, -107 590, 41 943, -8172, 855, -46, 1},
  {104 769, -107 878, 41 959, -8172, 855, -46, 1},
  {105 777, -108 134, 41 975, -8172, 855, -46, 1},
  {106 785, -108 390, 41 991, -8172, 855, -46, 1},
  {99 225, -105 966, 41 743, -8164, 855, -46, 1},
  {100 233, -106 222, 41 759, -8164, 855, -46, 1},
  {101 241, -106 478, 41 775, -8164, 855, -46, 1},
  {94 689, -104 310, 41 543, -8156, 855, -46, 1},
  {95 697, -104 566, 41 559, -8156, 855, -46, 1},
  {96 705, -104 822, 41 575, -8156, 855, -46, 1},
  {91 161, -102 910, 41 359, -8148, 855, -46, 1}]

{{109 305, -109 534, 42 159, -8180, 855, -46, 1},
  {109 081, -109 502, 42 159, -8180, 855, -46, 1},
  {110 313, -109 790, 42 175, -8180, 855, -46, 1},
  {111 321, -110 046, 42 191, -8180, 855, -46, 1},
  {103 761, -107 622, 41 943, -8172, 855, -46, 1},
  {103 537, -107 590, 41 943, -8172, 855, -46, 1},
  {104 769, -107 878, 41 959, -8172, 855, -46, 1},
  {105 777, -108 134, 41 975, -8172, 855, -46, 1},
  {106 785, -108 390, 41 991, -8172, 855, -46, 1},
  {99 225, -105 966, 41 743, -8164, 855, -46, 1},
  {100 233, -106 222, 41 759, -8164, 855, -46, 1},
  {101 241, -106 478, 41 775, -8164, 855, -46, 1},
  {94 689, -104 310, 41 543, -8156, 855, -46, 1},
  {95 697, -104 566, 41 559, -8156, 855, -46, 1},
  {96 705, -104 822, 41 575, -8156, 855, -46, 1},
  {91 161, -102 910, 41 359, -8148, 855, -46, 1}}

```

```
Array[m, 7].Transpose[A]
```

```
{109 305 m[1] - 109 534 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7],  
109 081 m[1] - 109 502 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7],  
110 313 m[1] - 109 790 m[2] + 42 175 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7],  
111 321 m[1] - 110 046 m[2] + 42 191 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7],  
103 761 m[1] - 107 622 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7],  
103 537 m[1] - 107 590 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7],  
104 769 m[1] - 107 878 m[2] + 41 959 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7],  
105 777 m[1] - 108 134 m[2] + 41 975 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7],  
106 785 m[1] - 108 390 m[2] + 41 991 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7],  
99 225 m[1] - 105 966 m[2] + 41 743 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7],  
100 233 m[1] - 106 222 m[2] + 41 759 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7],  
101 241 m[1] - 106 478 m[2] + 41 775 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7],  
94 689 m[1] - 104 310 m[2] + 41 543 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7],  
95 697 m[1] - 104 566 m[2] + 41 559 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7],  
96 705 m[1] - 104 822 m[2] + 41 575 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7],  
91 161 m[1] - 102 910 m[2] + 41 359 m[3] - 8148 m[4] + 855 m[5] - 46 m[6] + m[7]}
```

```
Array[m, 7].{4 493 145, -4 488 726, 1 727 503, -335 308, 35 055, -1886, 41}
```

```
4 493 145 m[1] - 4 488 726 m[2] + 1 727 503 m[3] -  
335 308 m[4] + 35 055 m[5] - 1886 m[6] + 41 m[7]
```

```
FindInstance[
```

```
109 305 m[1] - 109 534 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
109 081 m[1] - 109 502 m[2] + 42 159 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
110 313 m[1] - 109 790 m[2] + 42 175 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
111 321 m[1] - 110 046 m[2] + 42 191 m[3] - 8180 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
103 761 m[1] - 107 622 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
103 537 m[1] - 107 590 m[2] + 41 943 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
104 769 m[1] - 107 878 m[2] + 41 959 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
105 777 m[1] - 108 134 m[2] + 41 975 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
106 785 m[1] - 108 390 m[2] + 41 991 m[3] - 8172 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
99 225 m[1] - 105 966 m[2] + 41 743 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
100 233 m[1] - 106 222 m[2] + 41 759 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
101 241 m[1] - 106 478 m[2] + 41 775 m[3] - 8164 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
94 689 m[1] - 104 310 m[2] + 41 543 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
95 697 m[1] - 104 566 m[2] + 41 559 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
96 705 m[1] - 104 822 m[2] + 41 575 m[3] - 8156 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
91 161 m[1] - 102 910 m[2] + 41 359 m[3] - 8148 m[4] + 855 m[5] - 46 m[6] + m[7] ≥ 0 &&  
4 493 145 m[1] - 4 488 726 m[2] + 1 727 503 m[3] - 335 308 m[4] +  
35 055 m[5] - 1886 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
```

```
{ {m[1] → -5561, m[2] → -31 263,  
m[3] → -149 845, m[4] → -427 982, m[5] → 0, m[6] → 0, m[7] → 0} }
```

```

Array[m, 7] /. {m[1] → -5561, m[2] → -31263,
  m[3] → -149845, m[4] → -427982, m[5] → 0, m[6] → 0, m[7] → 0}
{-5561, -31263, -149845, -427982, 0, 0, 0}

GCD[-5561, -31263, -149845, -427982, 0, 0, 0]
1

Solve[Array[n, 16].A == {4493145, -4488726, 1727503, -335308, 35055, -1886, 41},
  Array[n, 16]]
{ }

MatrixRank[A]
4

{-5561, -31263, -149845, -427982, 0, 0, 0}.

{4493145, -4488726, 1727503, -335308, 35055, -1886, 41}

-7236986

{-5561, -31263, -149845, -427982, 0, 0, 0}.

Transpose[{{109305, -109534, 42159, -8180, 855, -46, 1},
  {109081, -109502, 42159, -8180, 855, -46, 1},
  {110313, -109790, 42175, -8180, 855, -46, 1},
  {111321, -110046, 42191, -8180, 855, -46, 1},
  {103761, -107622, 41943, -8172, 855, -46, 1},
  {103537, -107590, 41943, -8172, 855, -46, 1},
  {104769, -107878, 41959, -8172, 855, -46, 1},
  {105777, -108134, 41975, -8172, 855, -46, 1},
  {106785, -108390, 41991, -8172, 855, -46, 1},
  {99225, -105966, 41743, -8164, 855, -46, 1},
  {100233, -106222, 41759, -8164, 855, -46, 1},
  {101241, -106478, 41775, -8164, 855, -46, 1},
  {94689, -104310, 41543, -8156, 855, -46, 1},
  {95697, -104566, 41559, -8156, 855, -46, 1},
  {96705, -104822, 41575, -8156, 855, -46, 1},
  {91161, -102910, 41359, -8148, 855, -46, 1}]]

{93742, 338990, 94062, 94382, 91734, 336982, 92054,
  92374, 92694, 90046, 90366, 90686, 88358, 88678, 88998, 86990}

```

```

poly5 = listmod128[[5]]
(-9 + x)^4 (-7 + x)^9 (5 + x)^25 (-544 + 213 x - 26 x^2 + x^3)

p5 = poly5 / minipoly[poly5] // Factor
(-9 + x)^3 (-7 + x)^8 (5 + x)^24

```

```

feasiblesubcharpolylist[ (-9 + x)4 (-7 + x)9 (5 + x)25 (-544 + 213 x - 26 x2 + x3) ]
```

$$\left\{ (-9 + x) \left(1353 - 1052 x + 270 x^2 - 28 x^3 + x^4\right), \right.$$

$$(-9 + x) \left(1369 - 1052 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) (-7 + x) \left(-183 + 123 x - 21 x^2 + x^3\right),$$

$$(-9 + x) \left(1297 - 1044 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) \left(1313 - 1044 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) \left(1329 - 1044 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) (-7 + x)^2 \left(25 - 14 x + x^2\right),$$

$$(-9 + x) \left(73 - 18 x + x^2\right) \left(17 - 10 x + x^2\right), (-9 + x) \left(1257 - 1036 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) \left(1273 - 1036 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) (-7 + x) \left(-167 + 123 x - 21 x^2 + x^3\right),$$

$$(-9 + x) \left(1185 - 1028 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) \left(1201 - 1028 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) (-7 + x) \left(-159 + 123 x - 21 x^2 + x^3\right), (-9 + x) \left(1129 - 1020 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) \left(1145 - 1020 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) (-7 + x) \left(-151 + 123 x - 21 x^2 + x^3\right),$$

$$\left. (-9 + x) \left(1073 - 1012 x + 270 x^2 - 28 x^3 + x^4\right), (-11 + x) (-9 + x) (-7 + x) \left(13 - 10 x + x^2\right) \right\}$$

$$\left\{ (-9 + x) \left(1353 - 1052 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) \left(1369 - 1052 x + 270 x^2 - 28 x^3 + x^4\right), \right.$$

$$(-9 + x) (-7 + x) \left(-183 + 123 x - 21 x^2 + x^3\right), (-9 + x) \left(1297 - 1044 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) \left(1313 - 1044 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) \left(1329 - 1044 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) (-7 + x)^2 \left(25 - 14 x + x^2\right), (-9 + x) \left(73 - 18 x + x^2\right) \left(17 - 10 x + x^2\right),$$

$$(-9 + x) \left(1257 - 1036 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) \left(1273 - 1036 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) (-7 + x) \left(-167 + 123 x - 21 x^2 + x^3\right), (-9 + x) \left(1185 - 1028 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) \left(1201 - 1028 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) (-7 + x) \left(-159 + 123 x - 21 x^2 + x^3\right),$$

$$(-9 + x) \left(1129 - 1020 x + 270 x^2 - 28 x^3 + x^4\right), (-9 + x) \left(1145 - 1020 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$(-9 + x) (-7 + x) \left(-151 + 123 x - 21 x^2 + x^3\right), (-9 + x) \left(1073 - 1012 x + 270 x^2 - 28 x^3 + x^4\right),$$

$$\left. (-11 + x) (-9 + x) (-7 + x) \left(13 - 10 x + x^2\right) \right\} // \text{Length}$$

```

CoefficientList[
{(-9 + x) (1353 - 1052 x + 270 x2 - 28 x3 + x4), (-9 + x) (1369 - 1052 x + 270 x2 - 28 x3 + x4),
(-9 + x) (-7 + x) (-183 + 123 x - 21 x2 + x3), (-9 + x) (1297 - 1044 x + 270 x2 - 28 x3 + x4),
(-9 + x) (1313 - 1044 x + 270 x2 - 28 x3 + x4), (-9 + x) (1329 - 1044 x + 270 x2 - 28 x3 + x4),
(-9 + x) (-7 + x)2 (25 - 14 x + x2), (-9 + x) (73 - 18 x + x2) (17 - 10 x + x2),
(-9 + x) (1257 - 1036 x + 270 x2 - 28 x3 + x4), (-9 + x) (1273 - 1036 x + 270 x2 - 28 x3 + x4),
(-9 + x) (-7 + x) (-167 + 123 x - 21 x2 + x3), (-9 + x) (1185 - 1028 x + 270 x2 - 28 x3 + x4),
(-9 + x) (1201 - 1028 x + 270 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-159 + 123 x - 21 x2 + x3),
(-9 + x) (1129 - 1020 x + 270 x2 - 28 x3 + x4), (-9 + x) (1145 - 1020 x + 270 x2 - 28 x3 + x4),
(-9 + x) (-7 + x) (-151 + 123 x - 21 x2 + x3), (-9 + x) (1073 - 1012 x + 270 x2 - 28 x3 + x4),
(-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2)}, x]

{{-12177, 10821, -3482, 522, -37, 1},
{-12321, 10837, -3482, 522, -37, 1}, {-11529, 10677, -3474, 522, -37, 1},
{-11673, 10693, -3474, 522, -37, 1}, {-11817, 10709, -3474, 522, -37, 1},
{-11961, 10725, -3474, 522, -37, 1}, {-11025, 10549, -3466, 522, -37, 1},
{-11169, 10565, -3466, 522, -37, 1}, {-11313, 10581, -3466, 522, -37, 1},
{-11457, 10597, -3466, 522, -37, 1}, {-10521, 10421, -3458, 522, -37, 1},
{-10665, 10437, -3458, 522, -37, 1}, {-10809, 10453, -3458, 522, -37, 1},
{-10017, 10293, -3450, 522, -37, 1}, {-10161, 10309, -3450, 522, -37, 1},
{-10305, 10325, -3450, 522, -37, 1}, {-9513, 10165, -3442, 522, -37, 1},
{-9657, 10181, -3442, 522, -37, 1}, {-9009, 10037, -3434, 522, -37, 1}};

A = {{-12177, 10821, -3482, 522, -37, 1},
{-12321, 10837, -3482, 522, -37, 1}, {-11529, 10677, -3474, 522, -37, 1},
{-11673, 10693, -3474, 522, -37, 1}, {-11817, 10709, -3474, 522, -37, 1},
{-11961, 10725, -3474, 522, -37, 1}, {-11025, 10549, -3466, 522, -37, 1},
{-11169, 10565, -3466, 522, -37, 1}, {-11313, 10581, -3466, 522, -37, 1},
{-11457, 10597, -3466, 522, -37, 1}, {-10521, 10421, -3458, 522, -37, 1},
{-10665, 10437, -3458, 522, -37, 1}, {-10809, 10453, -3458, 522, -37, 1},
{-10017, 10293, -3450, 522, -37, 1}, {-10161, 10309, -3450, 522, -37, 1},
{-10305, 10325, -3450, 522, -37, 1}, {-9513, 10165, -3442, 522, -37, 1},
{-9657, 10181, -3442, 522, -37, 1}, {-9009, 10037, -3434, 522, -37, 1}};

MatrixRank[A]
3

CoefficientList[D[poly5, x] / p5 // Factor, x]
{-493225, 440925, -142538, 21402, -1517, 41}

```

```

Solve[Array[n, 19].{{-12177, 10821, -3482, 522, -37, 1},
{-12321, 10837, -3482, 522, -37, 1}, {-11529, 10677, -3474, 522, -37, 1},
{-11673, 10693, -3474, 522, -37, 1}, {-11817, 10709, -3474, 522, -37, 1},
{-11961, 10725, -3474, 522, -37, 1}, {-11025, 10549, -3466, 522, -37, 1},
{-11169, 10565, -3466, 522, -37, 1}, {-11313, 10581, -3466, 522, -37, 1},
{-11457, 10597, -3466, 522, -37, 1}, {-10521, 10421, -3458, 522, -37, 1},
{-10665, 10437, -3458, 522, -37, 1}, {-10809, 10453, -3458, 522, -37, 1},
{-10017, 10293, -3450, 522, -37, 1}, {-10161, 10309, -3450, 522, -37, 1},
{-10305, 10325, -3450, 522, -37, 1}, {-9513, 10165, -3442, 522, -37, 1},
{-9657, 10181, -3442, 522, -37, 1}, {-9009, 10037, -3434, 522, -37, 1}} ==
{-493225, 440925, -142538, 21402, -1517, 41}, Array[n, 19]]

{ }

Array[m, 6].Transpose[A]

{-12177 m[1] + 10821 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6],
-12321 m[1] + 10837 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6],
-11529 m[1] + 10677 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
-11673 m[1] + 10693 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
-11817 m[1] + 10709 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
-11961 m[1] + 10725 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6],
-11025 m[1] + 10549 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
-11169 m[1] + 10565 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
-11313 m[1] + 10581 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
-11457 m[1] + 10597 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6],
-10521 m[1] + 10421 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6],
-10665 m[1] + 10437 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6],
-10809 m[1] + 10453 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6],
-10017 m[1] + 10293 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6],
-10161 m[1] + 10309 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6],
-10305 m[1] + 10325 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6],
-9513 m[1] + 10165 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6],
-9657 m[1] + 10181 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6],
-9009 m[1] + 10037 m[2] - 3434 m[3] + 522 m[4] - 37 m[5] + m[6]}

Array[m, 6].{-493225, 440925, -142538, 21402, -1517, 41}

-493225 m[1] + 440925 m[2] - 142538 m[3] + 21402 m[4] - 1517 m[5] + 41 m[6]

```

```

FindInstance[-12 177 m[1] + 10 821 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-12 321 m[1] + 10 837 m[2] - 3482 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 529 m[1] + 10 677 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 673 m[1] + 10 693 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 817 m[1] + 10 709 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 961 m[1] + 10 725 m[2] - 3474 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 025 m[1] + 10 549 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 169 m[1] + 10 565 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 313 m[1] + 10 581 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 457 m[1] + 10 597 m[2] - 3466 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 521 m[1] + 10 421 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 665 m[1] + 10 437 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 809 m[1] + 10 453 m[2] - 3458 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 017 m[1] + 10 293 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 161 m[1] + 10 309 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 305 m[1] + 10 325 m[2] - 3450 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-9513 m[1] + 10 165 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-9657 m[1] + 10 181 m[2] - 3442 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-9009 m[1] + 10 037 m[2] - 3434 m[3] + 522 m[4] - 37 m[5] + m[6] ≥ 0 &&
-493 225 m[1] + 440 925 m[2] - 142 538 m[3] + 21 402 m[4] - 1517 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 2454, m[2] → 22 092, m[3] → 198 820, m[4] → 0, m[5] → 0, m[6] → 483 133 968}}

```

**Array[m, 6] /.**

```

{m[1] → 2454, m[2] → 22 092, m[3] → 198 820, m[4] → 0, m[5] → 0, m[6] → 483 133 968}
{2454, 22 092, 198 820, 0, 0, 483 133 968}

```

**GCD[2454, 22 092, 198 820, 0, 0, 483 133 968]**

```

2

```

```

{1227, 11 046, 99 410, 0, 0, 241 566 984} // Reverse
{241 566 984, 0, 0, 99 410, 11 046, 1227}

```

```

{2454, 22 092, 198 820, 0, 0, 483 133 968} / 2
{1227, 11 046, 99 410, 0, 0, 241 566 984}

```

```

{1227, 11 046, 99 410, 0, 0, 241 566 984}.
{-493 225, 440 925, -142 538, 21 402, -1517, 41}
-185 761

```

```

{1227, 11046, 99410, 0, 0, 241566984}.

Transpose[{{{-12177, 10821, -3482, 522, -37, 1},
  {-12321, 10837, -3482, 522, -37, 1}, {-11529, 10677, -3474, 522, -37, 1},
  {-11673, 10693, -3474, 522, -37, 1}, {-11817, 10709, -3474, 522, -37, 1},
  {-11961, 10725, -3474, 522, -37, 1}, {-11025, 10549, -3466, 522, -37, 1},
  {-11169, 10565, -3466, 522, -37, 1}, {-11313, 10581, -3466, 522, -37, 1},
  {-11457, 10597, -3466, 522, -37, 1}, {-10521, 10421, -3458, 522, -37, 1},
  {-10665, 10437, -3458, 522, -37, 1}, {-10809, 10453, -3458, 522, -37, 1},
  {-10017, 10293, -3450, 522, -37, 1}, {-10161, 10309, -3450, 522, -37, 1},
  {-10305, 10325, -3450, 522, -37, 1}, {-9513, 10165, -3442, 522, -37, 1},
  {-9657, 10181, -3442, 522, -37, 1}, {-9009, 10037, -3434, 522, -37, 1}}]

{8951, 8999, 8703, 8751, 8799, 8847, 8503, 8551, 8599,
  8647, 8303, 8351, 8399, 8103, 8151, 8199, 7903, 7951, 7703}

{2454, 22092, 198820, 0, 0, 483133968}.Transpose[A]

{17902, 17998, 17406, 17502, 17598, 17694, 17006, 17102, 17198,
  17294, 16606, 16702, 16798, 16206, 16302, 16398, 15806, 15902, 15406}

{2454, 22092, 198820, 0, 0, 483133968}.
{-493225, 440925, -142538, 21402, -1517, 41}

-371522

```

```

poly6 = listmod128[6]
(-11 + x) (-9 + x)^3 (-7 + x)^10 (5 + x)^25 (64 - 17 x + x^2)

p6 = poly6 / minipoly[poly6] // Factor
(-9 + x)^2 (-7 + x)^9 (5 + x)^24

feasiblesubcharpolylist[(-11 + x) (-9 + x)^3 (-7 + x)^10 (5 + x)^25 (64 - 17 x + x^2)]
{(-11 + x) (1453 - 1092 x + 274 x^2 - 28 x^3 + x^4),
 (-11 + x) (-7 + x) (-195 + 127 x - 21 x^2 + x^3), (-11 + x) (1381 - 1084 x + 274 x^2 - 28 x^3 + x^4),
 (-11 + x)^2 (-127 + 87 x - 17 x^2 + x^3), (-11 + x)^2 (-7 + x) (17 - 10 x + x^2),
 (-11 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4), (-11 + x) (-9 + x) (-149 + 103 x - 19 x^2 + x^3),
 (-9 + x) (1535 - 1274 x + 312 x^2 - 30 x^3 + x^4), (-11 + x) (-7 + x) (-179 + 127 x - 21 x^2 + x^3),
 (-11 + x) (-9 + x) (-141 + 103 x - 19 x^2 + x^3), (-11 + x) (-9 + x) (-7 + x) (19 - 12 x + x^2)}

```

```

{(-11+x) (1453 - 1092 x + 274 x2 - 28 x3 + x4), (-11+x) (-7+x) (-195 + 127 x - 21 x2 + x3),
 (-11+x) (1381 - 1084 x + 274 x2 - 28 x3 + x4), (-11+x)2 (-127 + 87 x - 17 x2 + x3),
 (-11+x)2 (-7+x) (17 - 10 x + x2), (-11+x) (1325 - 1076 x + 274 x2 - 28 x3 + x4),
 (-11+x) (-9+x) (-149 + 103 x - 19 x2 + x3),
 (-9+x) (1535 - 1274 x + 312 x2 - 30 x3 + x4), (-11+x) (-7+x) (-179 + 127 x - 21 x2 + x3),
 (-11+x) (-9+x) (-141 + 103 x - 19 x2 + x3),
 (-11+x) (-9+x) (-7+x) (19 - 12 x + x2)} // Length

```

11

```

CoefficientList[{(-11+x) (1453 - 1092 x + 274 x2 - 28 x3 + x4),
 (-11+x) (-7+x) (-195 + 127 x - 21 x2 + x3),
 (-11+x) (1381 - 1084 x + 274 x2 - 28 x3 + x4), (-11+x)2 (-127 + 87 x - 17 x2 + x3),
 (-11+x)2 (-7+x) (17 - 10 x + x2), (-11+x) (1325 - 1076 x + 274 x2 - 28 x3 + x4),
 (-11+x) (-9+x) (-149 + 103 x - 19 x2 + x3), (-9+x) (1535 - 1274 x + 312 x2 - 30 x3 + x4),
 (-11+x) (-7+x) (-179 + 127 x - 21 x2 + x3), (-11+x) (-9+x)
 (-141 + 103 x - 19 x2 + x3), (-11+x) (-9+x) (-7+x) (19 - 12 x + x2)}, x]

```

```

{{-15 983, 13 465, -4106, 582, -39, 1},
 {-15 015, 13 289, -4098, 582, -39, 1}, {-15 191, 13 305, -4098, 582, -39, 1},
 {-15 367, 13 321, -4098, 582, -39, 1}, {-14 399, 13 145, -4090, 582, -39, 1},
 {-14 575, 13 161, -4090, 582, -39, 1}, {-14 751, 13 177, -4090, 582, -39, 1},
 {-13 815, 13 001, -4082, 582, -39, 1}, {-13 783, 13 001, -4082, 582, -39, 1},
 {-13 959, 13 017, -4082, 582, -39, 1}, {-13 167, 12 857, -4074, 582, -39, 1}}

```

```

A = {{-15 983, 13 465, -4106, 582, -39, 1},
 {-15 015, 13 289, -4098, 582, -39, 1}, {-15 191, 13 305, -4098, 582, -39, 1},
 {-15 367, 13 321, -4098, 582, -39, 1}, {-14 399, 13 145, -4090, 582, -39, 1},
 {-14 575, 13 161, -4090, 582, -39, 1}, {-14 751, 13 177, -4090, 582, -39, 1},
 {-13 815, 13 001, -4082, 582, -39, 1}, {-13 783, 13 001, -4082, 582, -39, 1},
 {-13 959, 13 017, -4082, 582, -39, 1}, {-13 167, 12 857, -4074, 582, -39, 1}};

```

```

CoefficientList[D[poly6, x] / p6 // Factor, x]
{-639 015, 548 097, -168 122, 23 862, -1599, 41}

```

```

Solve[Array[n, 11].{{-15 983, 13 465, -4106, 582, -39, 1},
 {-15 015, 13 289, -4098, 582, -39, 1}, {-15 191, 13 305, -4098, 582, -39, 1},
 {-15 367, 13 321, -4098, 582, -39, 1}, {-14 399, 13 145, -4090, 582, -39, 1},
 {-14 575, 13 161, -4090, 582, -39, 1}, {-14 751, 13 177, -4090, 582, -39, 1},
 {-13 815, 13 001, -4082, 582, -39, 1}, {-13 783, 13 001, -4082, 582, -39, 1},
 {-13 959, 13 017, -4082, 582, -39, 1}, {-13 167, 12 857, -4074, 582, -39, 1}} ==
 {-639 015, 548 097, -168 122, 23 862, -1599, 41}, Array[n, 11]]

```

Solve::svrs : Equations may not give solutions for all "solve" variables. >>

```

{{n[8] → 8, n[9] → 42 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - 2 n[5] - n[6],
 n[10] → 86 - 2 n[1] - n[3] - 2 n[4] - n[6] - 2 n[7],
 n[11] → -95 + 3 n[1] + 2 n[2] + 2 n[3] + 2 n[4] + n[5] + n[6] + n[7]}}

```

```

FindInstance[
 -n[8] + 8 == 0 && -n[9] + 42 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - 2 n[5] - n[6] == 0 &&
 -n[10] - 86 - 2 n[1] - n[3] - 2 n[4] - n[6] - 2 n[7] == 0 &&
 -n[11] - 95 + 3 n[1] + 2 n[2] + 2 n[3] + 2 n[4] + n[5] + n[6] + n[7] == 0 &&
 n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
 n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0, Array[n, 11], Integers]
{ }

Array[m, 6].Transpose[A]
{-15 983 m[1] + 13 465 m[2] - 4106 m[3] + 582 m[4] - 39 m[5] + m[6],
 -15 015 m[1] + 13 289 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6],
 -15 191 m[1] + 13 305 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6],
 -15 367 m[1] + 13 321 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6],
 -14 399 m[1] + 13 145 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6],
 -14 575 m[1] + 13 161 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6],
 -14 751 m[1] + 13 177 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 815 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 783 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 959 m[1] + 13 017 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6],
 -13 167 m[1] + 12 857 m[2] - 4074 m[3] + 582 m[4] - 39 m[5] + m[6]}

Array[m, 6].{-639 015, 548 097, -168 122, 23 862, -1599, 41}
-639 015 m[1] + 548 097 m[2] - 168 122 m[3] + 23 862 m[4] - 1599 m[5] + 41 m[6]

FindInstance[-15 983 m[1] + 13 465 m[2] - 4106 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -15 015 m[1] + 13 289 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -15 191 m[1] + 13 305 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -15 367 m[1] + 13 321 m[2] - 4098 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -14 399 m[1] + 13 145 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -14 575 m[1] + 13 161 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -14 751 m[1] + 13 177 m[2] - 4090 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -13 815 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -13 783 m[1] + 13 001 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -13 959 m[1] + 13 017 m[2] - 4082 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -13 167 m[1] + 12 857 m[2] - 4074 m[3] + 582 m[4] - 39 m[5] + m[6] ≥ 0 &&
 -639 015 m[1] + 548 097 m[2] - 168 122 m[3] + 23 862 m[4] - 1599 m[5] + 41 m[6] < 0,
 Array[m, 6], Integers]
{{m[1] → 2150, m[2] → 10 679, m[3] → 26 648, m[4] → 0, m[5] → 0, m[6] → 0} }

Array[m, 6] /. {m[1] → 2150, m[2] → 10 679, m[3] → 26 648, m[4] → 0, m[5] → 0, m[6] → 0}
{2150, 10 679, 26 648, 0, 0, 0}

GCD[2150, 10 679, 26 648, 0, 0, 0]
1

```

```

{{{-15983, 13465, -4106, 582, -39, 1}, {-15015, 13289, -4098, 582, -39, 1},
  {-15191, 13305, -4098, 582, -39, 1}, {-15367, 13321, -4098, 582, -39, 1},
  {-14399, 13145, -4090, 582, -39, 1}, {-14575, 13161, -4090, 582, -39, 1},
  {-14751, 13177, -4090, 582, -39, 1}, {-13815, 13001, -4082, 582, -39, 1},
  {-13783, 13001, -4082, 582, -39, 1}, {-13959, 13017, -4082, 582, -39, 1},
  {-13167, 12857, -4074, 582, -39, 1}} // MatrixForm

{{-15983 13465 -4106 582 -39 1}
 {-15015 13289 -4098 582 -39 1}
 {-15191 13305 -4098 582 -39 1}
 {-15367 13321 -4098 582 -39 1}
 {-14399 13145 -4090 582 -39 1}
 {-14575 13161 -4090 582 -39 1}
 {-14751 13177 -4090 582 -39 1}
 {-13815 13001 -4082 582 -39 1}
 {-13783 13001 -4082 582 -39 1}
 {-13959 13017 -4082 582 -39 1}
 {-13167 12857 -4074 582 -39 1}]

{2150, 10679, 26648, 0, 0, 0}.{-639015, 548097, -168122, 23862, -1599, 41}

-869443

{2150, 10679, 26648, 0, 0, 0}.Transpose[{{-15983 13465 -4106 582 -39 1}
 {-15015 13289 -4098 582 -39 1}
 {-15191 13305 -4098 582 -39 1}
 {-15367 13321 -4098 582 -39 1}
 {-14399 13145 -4090 582 -39 1}
 {-14575 13161 -4090 582 -39 1}
 {-14751 13177 -4090 582 -39 1}
 {-13815 13001 -4082 582 -39 1}
 {-13783 13001 -4082 582 -39 1}
 {-13959 13017 -4082 582 -39 1}
 {-13167 12857 -4074 582 -39 1}]]

{12597, 427477, 219941, 12405, 427285,
 219749, 12213, 358293, 427093, 219557, 426901}

```

```

poly7 = listmod128[[7]]

$$(-11 + x)^2 (-9 + x)^2 (-7 + x)^{10} (5 + x)^{25} (52 - 15 x + x^2)$$


p7 = poly7 / minipoly[poly7] // Factor

$$(-11 + x) (-9 + x) (-7 + x)^9 (5 + x)^{24}$$


feasiblesubcharpolylist[(-11 + x)^2 (-9 + x)^2 (-7 + x)^{10} (5 + x)^{25} (52 - 15 x + x^2)]

$$\left\{ (-11 + x) (-9 + x) (-127 + 87 x - 17 x^2 + x^3), (-9 + x)^2 (-157 + 103 x - 19 x^2 + x^3), \right.$$


$$(-11 + x) (-9 + x) (-7 + x) (17 - 10 x + x^2), (-9 + x) (1325 - 1076 x + 274 x^2 - 28 x^3 + x^4),$$


$$(-9 + x)^2 (-149 + 103 x - 19 x^2 + x^3), (-9 + x) (-7 + x) (-179 + 127 x - 21 x^2 + x^3),$$


$$\left. (-9 + x)^2 (-141 + 103 x - 19 x^2 + x^3), (-9 + x)^2 (-7 + x) (19 - 12 x + x^2) \right\}$$


```

```

 $\{ (-11+x) (-9+x) (-127+87x-17x^2+x^3), (-9+x)^2 (-157+103x-19x^2+x^3),$ 
 $(-11+x) (-9+x) (-7+x) (17-10x+x^2), (-9+x) (1325-1076x+274x^2-28x^3+x^4),$ 
 $(-9+x)^2 (-149+103x-19x^2+x^3), (-9+x) (-7+x) (-179+127x-21x^2+x^3),$ 
 $(-9+x)^2 (-141+103x-19x^2+x^3), (-9+x)^2 (-7+x) (19-12x+x^2) \} // Length$ 
8

A = {{-12573, 11153, -3550, 526, -37, 1}, {-12717, 11169, -3550, 526, -37, 1},
{-11781, 10993, -3542, 526, -37, 1}, {-11925, 11009, -3542, 526, -37, 1},
{-12069, 11025, -3542, 526, -37, 1}, {-11277, 10865, -3534, 526, -37, 1},
{-11421, 10881, -3534, 526, -37, 1}, {-10773, 10737, -3526, 526, -37, 1}};

MatrixRank[A]
3

CoefficientList[
{{(-11+x) (-9+x) (-127+87x-17x^2+x^3), (-9+x)^2 (-157+103x-19x^2+x^3),
(-11+x) (-9+x) (-7+x) (17-10x+x^2), (-9+x) (1325-1076x+274x^2-28x^3+x^4),
(-9+x)^2 (-149+103x-19x^2+x^3), (-9+x) (-7+x) (-179+127x-21x^2+x^3),
(-9+x)^2 (-141+103x-19x^2+x^3), (-9+x)^2 (-7+x) (19-12x+x^2)}, x],
{{-12573, 11153, -3550, 526, -37, 1}, {-12717, 11169, -3550, 526, -37, 1},
{-11781, 10993, -3542, 526, -37, 1}, {-11925, 11009, -3542, 526, -37, 1},
{-12069, 11025, -3542, 526, -37, 1}, {-11277, 10865, -3534, 526, -37, 1},
{-11421, 10881, -3534, 526, -37, 1}, {-10773, 10737, -3526, 526, -37, 1}}]

CoefficientList[D[poly7, x] / p7 // Factor, x]
{-518725, 457225, -145502, 21566, -1517, 41}

Solve[Array[n, 8].
{{-12573, 11153, -3550, 526, -37, 1}, {-12717, 11169, -3550, 526, -37, 1},
{-11781, 10993, -3542, 526, -37, 1}, {-11925, 11009, -3542, 526, -37, 1},
{-12069, 11025, -3542, 526, -37, 1}, {-11277, 10865, -3534, 526, -37, 1},
{-11421, 10881, -3534, 526, -37, 1}, {-10773, 10737, -3526, 526, -37, 1}} ==
{-518725, 457225, -145502, 21566, -1517, 41}, Array[n, 8]]
{}

Array[m, 6].Transpose[A]
{-12573 m[1] + 11153 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6],
-12717 m[1] + 11169 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6],
-11781 m[1] + 10993 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6],
-11925 m[1] + 11009 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6],
-12069 m[1] + 11025 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6],
-11277 m[1] + 10865 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6],
-11421 m[1] + 10881 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6],
-10773 m[1] + 10737 m[2] - 3526 m[3] + 526 m[4] - 37 m[5] + m[6]}

Array[m, 6].{-518725, 457225, -145502, 21566, -1517, 41}
-518725 m[1] + 457225 m[2] - 145502 m[3] + 21566 m[4] - 1517 m[5] + 41 m[6]

```

```

FindInstance[-12 573 m[1] + 11 153 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-12 717 m[1] + 11 169 m[2] - 3550 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 781 m[1] + 10 993 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 925 m[1] + 11 009 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-12 069 m[1] + 11 025 m[2] - 3542 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 277 m[1] + 10 865 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-11 421 m[1] + 10 881 m[2] - 3534 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-10 773 m[1] + 10 737 m[2] - 3526 m[3] + 526 m[4] - 37 m[5] + m[6] ≥ 0 &&
-518 725 m[1] + 457 225 m[2] - 145 502 m[3] + 21 566 m[4] - 1517 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]

{{m[1] → 2995, m[2] → 15 614, m[3] → 38 390, m[4] → 0, m[5] → 0, m[6] → 0} }

Array[m, 6] /. {m[1] → 2995, m[2] → 15 614, m[3] → 38 390, m[4] → 0, m[5] → 0, m[6] → 0}
{2995, 15 614, 38 390, 0, 0, 0}

GCD[2995, 15 614, 38 390, 0, 0, 0]
1

A.{2995, 15 614, 38 390, 0, 0, 0}
{202 307, 20 851, 383 227, 201 771, 20 315, 201 235, 19 779, 19 243}

{-518 725, 457 225, -145 502, 21 566, -1517, 41}.{2995, 15 614, 38 390, 0, 0, 0}
-292 005

{2995, 15 614, 38 390, 0, 0, 0}.{-518 725, 457 225, -145 502, 21 566, -1517, 41}
-292 005

```

```

poly8 = listmod128[8]
(-11 + x) (-9 + x)2 (-8 + x) (-7 + x)8 (5 + x)25 (59 - 16 x + x2)2

p8 = poly8 / minipoly[poly8] // Factor
(-9 + x) (-7 + x)7 (5 + x)24 (59 - 16 x + x2)

feasiblesubcharpolylist[(-11 + x) (-9 + x)2 (-8 + x) (-7 + x)8 (5 + x)25 (59 - 16 x + x2)2]
{(-11 + x) (-9 + x) (-7 + x) (-3 + x) (59 - 16 x + x2),
 (-11 + x) (-9 + x) (-7 + x) (-169 + 107 x - 19 x2 + x3),
 (-9 + x) (59 - 16 x + x2) (-223 + 131 x - 21 x2 + x3),
 (-11 + x) (-9 + x) (-7 + x)2 (23 - 12 x + x2),
 (-9 + x) (-12 541 + 11 153 x - 3550 x2 + 526 x3 - 37 x4 + x5),
 (-11 + x) (-9 + x)2 (-7 + x) (17 - 10 x + x2),
 (-9 + x) (-7 + x) (1699 - 1330 x + 316 x2 - 30 x3 + x4)}

```

```

{ (-11 + x) (-9 + x) (-7 + x) (-3 + x) (59 - 16 x + x2) ,
  (-11 + x) (-9 + x) (-7 + x) (-169 + 107 x - 19 x2 + x3) ,
  (-9 + x) (59 - 16 x + x2) (-223 + 131 x - 21 x2 + x3) ,
  (-11 + x) (-9 + x) (-7 + x)2 (23 - 12 x + x2) ,
  (-9 + x) (-12 541 + 11 153 x - 3550 x2 + 526 x3 - 37 x4 + x5) ,
  (-11 + x) (-9 + x)2 (-7 + x) (17 - 10 x + x2) ,
  (-9 + x) (-7 + x) (1699 - 1330 x + 316 x2 - 30 x3 + x4) } // Length

```

7

```

CoefficientList[{(-11 + x) (-9 + x) (-7 + x) (-3 + x) (59 - 16 x + x2) ,
  (-11 + x) (-9 + x) (-7 + x) (-169 + 107 x - 19 x2 + x3) ,
  (-9 + x) (59 - 16 x + x2) (-223 + 131 x - 21 x2 + x3) ,
  (-11 + x) (-9 + x) (-7 + x)2 (23 - 12 x + x2) ,
  (-9 + x) (-12 541 + 11 153 x - 3550 x2 + 526 x3 - 37 x4 + x5) ,
  (-11 + x) (-9 + x)2 (-7 + x) (17 - 10 x + x2) ,
  (-9 + x) (-7 + x) (1699 - 1330 x + 316 x2 - 30 x3 + x4) }, x]

```

```

{{122 661, -116 454, 43 519, -8300, 859, -46, 1},
 {117 117, -114 542, 43 303, -8292, 859, -46, 1},
 {118 413, -114 830, 43 319, -8292, 859, -46, 1},
 {111 573, -112 630, 43 087, -8284, 859, -46, 1},
 {112 869, -112 918, 43 103, -8284, 859, -46, 1},
 {106 029, -110 718, 42 871, -8276, 859, -46, 1},
 {107 037, -110 974, 42 887, -8276, 859, -46, 1}}

```

```

A = {{122 661, -116 454, 43 519, -8300, 859, -46, 1},
  {117 117, -114 542, 43 303, -8292, 859, -46, 1}, {118 413, -114 830, 43 319,
  -8292, 859, -46, 1}, {111 573, -112 630, 43 087, -8284, 859, -46, 1},
  {112 869, -112 918, 43 103, -8284, 859, -46, 1}, {106 029, -110 718, 42 871,
  -8276, 859, -46, 1}, {107 037, -110 974, 42 887, -8276, 859, -46, 1}};

```

```
MatrixRank[A]
```

4

```

CoefficientList[D[poly8, x] / p8 // Factor, x]
{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}

```

```
Solve[
```

```

Array[n, 7].{{122 661, -116 454, 43 519, -8300, 859, -46, 1}, {117 117, -114 542,
  43 303, -8292, 859, -46, 1}, {118 413, -114 830, 43 319, -8292, 859, -46, 1},
  {111 573, -112 630, 43 087, -8284, 859, -46, 1}, {112 869, -112 918, 43 103,
  -8284, 859, -46, 1}, {106 029, -110 718, 42 871, -8276, 859, -46, 1},
  {107 037, -110 974, 42 887, -8276, 859, -46, 1}} =
{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}, Array[n, 7]]

```

```
{}
```

```

Array[m, 7].Transpose[A]
{122 661 m[1] - 116 454 m[2] + 43 519 m[3] - 8300 m[4] + 859 m[5] - 46 m[6] + m[7],
 117 117 m[1] - 114 542 m[2] + 43 303 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7],
 118 413 m[1] - 114 830 m[2] + 43 319 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7],
 111 573 m[1] - 112 630 m[2] + 43 087 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7],
 112 869 m[1] - 112 918 m[2] + 43 103 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7],
 106 029 m[1] - 110 718 m[2] + 42 871 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7],
 107 037 m[1] - 110 974 m[2] + 42 887 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7]}

Array[m, 7].{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}
4 704 685 m[1] - 4 657 158 m[2] + 1 770 487 m[3] -
 339 772 m[4] + 35 219 m[5] - 1886 m[6] + 41 m[7]

FindInstance[
 122 661 m[1] - 116 454 m[2] + 43 519 m[3] - 8300 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 117 117 m[1] - 114 542 m[2] + 43 303 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 118 413 m[1] - 114 830 m[2] + 43 319 m[3] - 8292 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 111 573 m[1] - 112 630 m[2] + 43 087 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 112 869 m[1] - 112 918 m[2] + 43 103 m[3] - 8284 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 106 029 m[1] - 110 718 m[2] + 42 871 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 107 037 m[1] - 110 974 m[2] + 42 887 m[3] - 8276 m[4] + 859 m[5] - 46 m[6] + m[7] ≥ 0 &&
 4 704 685 m[1] - 4 657 158 m[2] + 1 770 487 m[3] - 339 772 m[4] +
 35 219 m[5] - 1886 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
{{m[1] → -8185, m[2] → -53 247,
  m[3] → -295 413, m[4] → -922 821, m[5] → 0, m[6] → 0, m[7] → 0} }

Array[m, 7] /. {m[1] → -8185, m[2] → -53 247,
  m[3] → -295 413, m[4] → -922 821, m[5] → 0, m[6] → 0, m[7] → 0}
{-8185, -53 247, -295 413, -922 821, 0, 0, 0}

GCD[-8185, -53 247, -295 413, -922 821, 0, 0, 0]
1

A.{-8185, -53 247, -295 413, -922 821, 0, 0, 0}
{181 806, 177 822, 178 590, 173 838, 174 606, 169 854, 823 998}

{-8185, -53 247, -295 413, -922 821, 0, 0, 0}.Transpose[A]
{181 806, 177 822, 178 590, 173 838, 174 606, 169 854, 823 998}

{-8185, -53 247, -295 413, -922 821, 0, 0, 0}.
{4 704 685, -4 657 158, 1 770 487, -339 772, 35 219, -1886, 41}
-4 294 018

```

```

poly9 = listmod128[[9]]

$$(-9+x)^5 (-7+x)^8 (5+x)^{25} (-412+179x-24x^2+x^3)$$


p9 = poly9 / minipoly[poly9] // Factor

$$(-9+x)^4 (-7+x)^7 (5+x)^{24}$$


feasiblesubcharpolylist[(-9+x)^5 (-7+x)^8 (5+x)^{25} (-412+179x-24x^2+x^3)]

$$\left\{ (-7+x) \left(1361 - 1044x + 270x^2 - 28x^3 + x^4\right), (-7+x) \left(1273 - 1036x + 270x^2 - 28x^3 + x^4\right), \right.$$


$$(-5+x) \left(1811 - 1346x + 316x^2 - 30x^3 + x^4\right), (-7+x) \left(1289 - 1036x + 270x^2 - 28x^3 + x^4\right),$$


$$-9167 + 8557x - 2926x^2 + 466x^3 - 35x^4 + x^5, (-9+x) (-7+x) (-5+x) \left(29 - 14x + x^2\right),$$


$$(-7+x) \left(1185 - 1028x + 270x^2 - 28x^3 + x^4\right), (-7+x) \left(1201 - 1028x + 270x^2 - 28x^3 + x^4\right),$$


$$(-7+x) \left(1217 - 1028x + 270x^2 - 28x^3 + x^4\right), -8663 + 8429x - 2918x^2 + 466x^3 - 35x^4 + x^5,$$


$$(-9+x) (-7+x) \left(-137 + 99x - 19x^2 + x^3\right), (-9+x) (-5+x) \left(-195 + 127x - 21x^2 + x^3\right),$$


$$(-7+x) \left(1129 - 1020x + 270x^2 - 28x^3 + x^4\right), (-7+x) \left(1145 - 1020x + 270x^2 - 28x^3 + x^4\right),$$


$$-8159 + 8301x - 2910x^2 + 466x^3 - 35x^4 + x^5, (-9+x) (-7+x) \left(-129 + 99x - 19x^2 + x^3\right),$$


$$(-9+x) \left(919 - 822x + 232x^2 - 26x^3 + x^4\right), (-7+x)^2 \left(-151 + 123x - 21x^2 + x^3\right),$$


$$(-7+x) \left(1073 - 1012x + 270x^2 - 28x^3 + x^4\right), (-11+x) (-9+x) (-7+x) \left(11 - 8x + x^2\right),$$


$$(-9+x) \left(863 - 814x + 232x^2 - 26x^3 + x^4\right), (-11+x) (-7+x)^2 \left(13 - 10x + x^2\right),$$


$$(-9+x) (-7+x) \left(-113 + 99x - 19x^2 + x^3\right), (-7+x) \left(929 - 996x + 270x^2 - 28x^3 + x^4\right),$$


$$(-9+x) (-7+x)^2 \left(15 - 12x + x^2\right), (-9+x) (-7+x) \left(-97 + 99x - 19x^2 + x^3\right) \}$$


$$\left\{ (-7+x) \left(1361 - 1044x + 270x^2 - 28x^3 + x^4\right), (-7+x) \left(1273 - 1036x + 270x^2 - 28x^3 + x^4\right), \right.$$


$$(-5+x) \left(1811 - 1346x + 316x^2 - 30x^3 + x^4\right), (-7+x) \left(1289 - 1036x + 270x^2 - 28x^3 + x^4\right),$$


$$-9167 + 8557x - 2926x^2 + 466x^3 - 35x^4 + x^5, (-9+x) (-7+x) (-5+x) \left(29 - 14x + x^2\right),$$


$$(-7+x) \left(1185 - 1028x + 270x^2 - 28x^3 + x^4\right), (-7+x) \left(1201 - 1028x + 270x^2 - 28x^3 + x^4\right),$$


$$(-7+x) \left(1217 - 1028x + 270x^2 - 28x^3 + x^4\right), -8663 + 8429x - 2918x^2 + 466x^3 - 35x^4 + x^5,$$


$$(-9+x) (-7+x) \left(-137 + 99x - 19x^2 + x^3\right), (-9+x) (-5+x) \left(-195 + 127x - 21x^2 + x^3\right),$$


$$(-7+x) \left(1129 - 1020x + 270x^2 - 28x^3 + x^4\right), (-7+x) \left(1145 - 1020x + 270x^2 - 28x^3 + x^4\right),$$


$$-8159 + 8301x - 2910x^2 + 466x^3 - 35x^4 + x^5, (-9+x) (-7+x) \left(-129 + 99x - 19x^2 + x^3\right),$$


$$(-9+x) \left(919 - 822x + 232x^2 - 26x^3 + x^4\right), (-7+x)^2 \left(-151 + 123x - 21x^2 + x^3\right),$$


$$(-7+x) \left(1073 - 1012x + 270x^2 - 28x^3 + x^4\right), (-11+x) (-9+x) (-7+x) \left(11 - 8x + x^2\right),$$


$$(-9+x) \left(863 - 814x + 232x^2 - 26x^3 + x^4\right), (-11+x) (-7+x)^2 \left(13 - 10x + x^2\right),$$


$$(-9+x) (-7+x) \left(-113 + 99x - 19x^2 + x^3\right), (-7+x) \left(929 - 996x + 270x^2 - 28x^3 + x^4\right),$$


$$(-9+x) (-7+x)^2 \left(15 - 12x + x^2\right), (-9+x) (-7+x) \left(-97 + 99x - 19x^2 + x^3\right) \} // Length$$


```

```

CoefficientList[
{(-7+x) (1361 - 1044 x + 270 x2 - 28 x3 + x4), (-7+x) (1273 - 1036 x + 270 x2 - 28 x3 + x4),
(-5+x) (1811 - 1346 x + 316 x2 - 30 x3 + x4), (-7+x) (1289 - 1036 x + 270 x2 - 28 x3 + x4),
-9167 + 8557 x - 2926 x2 + 466 x3 - 35 x4 + x5, (-9+x) (-7+x) (-5+x) (29 - 14 x + x2),
(-7+x) (1185 - 1028 x + 270 x2 - 28 x3 + x4), (-7+x) (1201 - 1028 x + 270 x2 - 28 x3 + x4),
(-7+x) (1217 - 1028 x + 270 x2 - 28 x3 + x4), -8663 + 8429 x - 2918 x2 + 466 x3 - 35 x4 + x5,
(-9+x) (-7+x) (-137 + 99 x - 19 x2 + x3), (-9+x) (-5+x) (-195 + 127 x - 21 x2 + x3),
(-7+x) (1129 - 1020 x + 270 x2 - 28 x3 + x4), (-7+x) (1145 - 1020 x + 270 x2 - 28 x3 + x4),
-8159 + 8301 x - 2910 x2 + 466 x3 - 35 x4 + x5, (-9+x) (-7+x) (-129 + 99 x - 19 x2 + x3),
(-9+x) (919 - 822 x + 232 x2 - 26 x3 + x4), (-7+x)2 (-151 + 123 x - 21 x2 + x3),
(-7+x) (1073 - 1012 x + 270 x2 - 28 x3 + x4), (-11+x) (-9+x) (-7+x) (11 - 8 x + x2),
(-9+x) (863 - 814 x + 232 x2 - 26 x3 + x4), (-11+x) (-7+x)2 (13 - 10 x + x2),
(-9+x) (-7+x) (-113 + 99 x - 19 x2 + x3), (-7+x) (929 - 996 x + 270 x2 - 28 x3 + x4),
(-9+x) (-7+x)2 (15 - 12 x + x2), (-9+x) (-7+x) (-97 + 99 x - 19 x2 + x3)}, x]

{{-9527, 8669, -2934, 466, -35, 1}, {-8911, 8525, -2926, 466, -35, 1},
{-9055, 8541, -2926, 466, -35, 1}, {-9023, 8541, -2926, 466, -35, 1},
{-9167, 8557, -2926, 466, -35, 1}, {-9135, 8557, -2926, 466, -35, 1},
{-8295, 8381, -2918, 466, -35, 1}, {-8407, 8397, -2918, 466, -35, 1},
{-8519, 8413, -2918, 466, -35, 1}, {-8663, 8429, -2918, 466, -35, 1},
{-8631, 8429, -2918, 466, -35, 1}, {-8775, 8445, -2918, 466, -35, 1},
{-7903, 8269, -2910, 466, -35, 1}, {-8015, 8285, -2910, 466, -35, 1},
{-8159, 8301, -2910, 466, -35, 1}, {-8127, 8301, -2910, 466, -35, 1},
{-8271, 8317, -2910, 466, -35, 1}, {-7399, 8141, -2902, 466, -35, 1},
{-7511, 8157, -2902, 466, -35, 1}, {-7623, 8173, -2902, 466, -35, 1},
{-7767, 8189, -2902, 466, -35, 1}, {-7007, 8029, -2894, 466, -35, 1},
{-7119, 8045, -2894, 466, -35, 1}, {-6503, 7901, -2886, 466, -35, 1},
{-6615, 7917, -2886, 466, -35, 1}, {-6111, 7789, -2878, 466, -35, 1}},

A = {{-9527, 8669, -2934, 466, -35, 1}, {-8911, 8525, -2926, 466, -35, 1},
{-9055, 8541, -2926, 466, -35, 1}, {-9023, 8541, -2926, 466, -35, 1},
{-9167, 8557, -2926, 466, -35, 1}, {-9135, 8557, -2926, 466, -35, 1},
{-8295, 8381, -2918, 466, -35, 1}, {-8407, 8397, -2918, 466, -35, 1},
{-8519, 8413, -2918, 466, -35, 1}, {-8663, 8429, -2918, 466, -35, 1},
{-8631, 8429, -2918, 466, -35, 1}, {-8775, 8445, -2918, 466, -35, 1},
{-7903, 8269, -2910, 466, -35, 1}, {-8015, 8285, -2910, 466, -35, 1},
{-8159, 8301, -2910, 466, -35, 1}, {-8127, 8301, -2910, 466, -35, 1},
{-8271, 8317, -2910, 466, -35, 1}, {-7399, 8141, -2902, 466, -35, 1},
{-7511, 8157, -2902, 466, -35, 1}, {-7623, 8173, -2902, 466, -35, 1},
{-7767, 8189, -2902, 466, -35, 1}, {-7007, 8029, -2894, 466, -35, 1},
{-7119, 8045, -2894, 466, -35, 1}, {-6503, 7901, -2886, 466, -35, 1},
{-6615, 7917, -2886, 466, -35, 1}};

CoefficientList[D[poly9, x] / p9 // Factor, x]
{-372 095, 350 101, -119 942, 19 106, -1435, 41}

```

```

Solve[Array[n, 26].
{{{-9527, 8669, -2934, 466, -35, 1}, {-8911, 8525, -2926, 466, -35, 1},
{-9055, 8541, -2926, 466, -35, 1}, {-9023, 8541, -2926, 466, -35, 1},
{-9167, 8557, -2926, 466, -35, 1}, {-9135, 8557, -2926, 466, -35, 1},
{-8295, 8381, -2918, 466, -35, 1}, {-8407, 8397, -2918, 466, -35, 1},
{-8519, 8413, -2918, 466, -35, 1}, {-8663, 8429, -2918, 466, -35, 1},
{-8631, 8429, -2918, 466, -35, 1}, {-8775, 8445, -2918, 466, -35, 1},
{-7903, 8269, -2910, 466, -35, 1}, {-8015, 8285, -2910, 466, -35, 1},
{-8159, 8301, -2910, 466, -35, 1}, {-8127, 8301, -2910, 466, -35, 1},
{-8271, 8317, -2910, 466, -35, 1}, {-7399, 8141, -2902, 466, -35, 1},
{-7511, 8157, -2902, 466, -35, 1}, {-7623, 8173, -2902, 466, -35, 1},
{-7767, 8189, -2902, 466, -35, 1}, {-7007, 8029, -2894, 466, -35, 1},
{-7119, 8045, -2894, 466, -35, 1}, {-6503, 7901, -2886, 466, -35, 1},
{-6615, 7917, -2886, 466, -35, 1}, {-6111, 7789, -2878, 466, -35, 1}} ==
{-372095, 350101, -119942, 19106, -1435, 41}, Array[n, 26]]]

Solve::svrs : Equations may not give solutions for all "solve" variables.    >>
{{n[21] == 48 - n[3] - n[5] - n[10] - n[12] - n[15] - n[17],
n[24] == 70 - n[1] - 2 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] -
n[9] - n[10] - 2 n[13] - n[14] - n[15] - 2 n[18] - n[19] - n[22],
n[25] == 29 - 6 n[1] - 4 n[2] - n[3] - 5 n[4] - 2 n[5] - 6 n[6] - 2 n[7] -
3 n[8] - 4 n[9] - n[10] - 5 n[11] - 2 n[12] - 2 n[13] - 3 n[14] -
4 n[16] - n[17] - n[18] - 2 n[19] - 3 n[20] - n[22] - 2 n[23],
n[26] == -106 + 6 n[1] + 5 n[2] + 3 n[3] + 5 n[4] + 3 n[5] + 5 n[6] + 4 n[7] +
4 n[8] + 4 n[9] + 2 n[10] + 4 n[11] + 2 n[12] + 3 n[13] + 3 n[14] +
n[15] + 3 n[16] + n[17] + 2 n[18] + 2 n[19] + 2 n[20] + n[22] + n[23]}}

FindInstance[-n[21] + 48 - n[3] - n[5] - n[10] - n[12] - n[15] - n[17] == 0 &&
-n[24] + 70 - n[1] - 2 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] -
n[9] - n[10] - 2 n[13] - n[14] - n[15] - 2 n[18] - n[19] - n[22] == 0 &&
-n[25] + 29 - 6 n[1] - 4 n[2] - n[3] - 5 n[4] - 2 n[5] - 6 n[6] -
2 n[7] - 3 n[8] - 4 n[9] - n[10] - 5 n[11] - 2 n[12] - 2 n[13] - 3 n[14] -
4 n[16] - n[17] - n[18] - 2 n[19] - 3 n[20] - n[22] - 2 n[23] == 0 &&
-n[26] - 106 + 6 n[1] + 5 n[2] + 3 n[3] + 5 n[4] + 3 n[5] + 5 n[6] + 4 n[7] +
4 n[8] + 4 n[9] + 2 n[10] + 4 n[11] + 2 n[12] + 3 n[13] + 3 n[14] + n[15] +
3 n[16] + n[17] + 2 n[18] + 2 n[19] + 2 n[20] + n[22] + n[23] == 0
&& n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 &&
n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0 && n[19] ≥ 0 && n[20] ≥ 0 && n[21] ≥ 0 &&
n[22] ≥ 0 && n[23] ≥ 0 && n[24] ≥ 0 && n[25] ≥ 0 && n[26] ≥ 0, Array[n, 26], Integers]
{ }

```

```

Array[m, 6].Transpose[A]
{-9527 m[1] + 8669 m[2] - 2934 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8911 m[1] + 8525 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6],
 -9055 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6],
 -9023 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6],
 -9167 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6],
 -9135 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8295 m[1] + 8381 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8407 m[1] + 8397 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8519 m[1] + 8413 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8663 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8631 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8775 m[1] + 8445 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7903 m[1] + 8269 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8015 m[1] + 8285 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8159 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8127 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6],
 -8271 m[1] + 8317 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7399 m[1] + 8141 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7511 m[1] + 8157 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7623 m[1] + 8173 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7767 m[1] + 8189 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7007 m[1] + 8029 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6],
 -7119 m[1] + 8045 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6],
 -6503 m[1] + 7901 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6],
 -6615 m[1] + 7917 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6],
 -6111 m[1] + 7789 m[2] - 2878 m[3] + 466 m[4] - 35 m[5] + m[6]}

```

**Array[m, 6].{-372 095, 350 101, -119 942, 19 106, -1435, 41}**

```

-372 095 m[1] + 350 101 m[2] - 119 942 m[3] + 19 106 m[4] - 1435 m[5] + 41 m[6]

```

```

FindInstance[-9527 m[1] + 8669 m[2] - 2934 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8911 m[1] + 8525 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9055 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9023 m[1] + 8541 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9167 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9135 m[1] + 8557 m[2] - 2926 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8295 m[1] + 8381 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8407 m[1] + 8397 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8519 m[1] + 8413 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8663 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8631 m[1] + 8429 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8775 m[1] + 8445 m[2] - 2918 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7903 m[1] + 8269 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8015 m[1] + 8285 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8159 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8127 m[1] + 8301 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8271 m[1] + 8317 m[2] - 2910 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7399 m[1] + 8141 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7511 m[1] + 8157 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7623 m[1] + 8173 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7767 m[1] + 8189 m[2] - 2902 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7007 m[1] + 8029 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7119 m[1] + 8045 m[2] - 2894 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-6503 m[1] + 7901 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-6615 m[1] + 7917 m[2] - 2886 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-6111 m[1] + 7789 m[2] - 2878 m[3] + 466 m[4] - 35 m[5] + m[6] ≥ 0 &&
-372 095 m[1] + 350 101 m[2] - 119 942 m[3] + 19 106 m[4] - 1435 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 3851, m[2] → 26 961, m[3] → 188 720, m[4] → 0, m[5] → 0, m[6] → 356 804 550} }

Array[m, 6] /.
{m[1] → 3851, m[2] → 26 961, m[3] → 188 720, m[4] → 0, m[5] → 0, m[6] → 356 804 550}
{3851, 26 961, 188 720, 0, 0, 356 804 550}

GCD[3851, 26 961, 188 720, 0, 0, 356 804 550]
1

```

```

{{{-9527, 8669, -2934, 466, -35, 1},
  {-8911, 8525, -2926, 466, -35, 1}, {-9055, 8541, -2926, 466, -35, 1},
  {-9023, 8541, -2926, 466, -35, 1}, {-9167, 8557, -2926, 466, -35, 1},
  {-9135, 8557, -2926, 466, -35, 1}, {-8295, 8381, -2918, 466, -35, 1},
  {-8407, 8397, -2918, 466, -35, 1}, {-8519, 8413, -2918, 466, -35, 1},
  {-8663, 8429, -2918, 466, -35, 1}, {-8631, 8429, -2918, 466, -35, 1},
  {-8775, 8445, -2918, 466, -35, 1}, {-7903, 8269, -2910, 466, -35, 1},
  {-8015, 8285, -2910, 466, -35, 1}, {-8159, 8301, -2910, 466, -35, 1},
  {-8127, 8301, -2910, 466, -35, 1}, {-8271, 8317, -2910, 466, -35, 1},
  {-7399, 8141, -2902, 466, -35, 1}, {-7511, 8157, -2902, 466, -35, 1},
  {-7623, 8173, -2902, 466, -35, 1}, {-7767, 8189, -2902, 466, -35, 1},
  {-7007, 8029, -2894, 466, -35, 1}, {-7119, 8045, -2894, 466, -35, 1},
  {-6503, 7901, -2886, 466, -35, 1}, {-6615, 7917, -2886, 466, -35, 1},
  {-6111, 7789, -2878, 466, -35, 1}} // MatrixForm

(-9527 8669 -2934 466 -35 1)
(-8911 8525 -2926 466 -35 1)
(-9055 8541 -2926 466 -35 1)
(-9023 8541 -2926 466 -35 1)
(-9167 8557 -2926 466 -35 1)
(-9135 8557 -2926 466 -35 1)
(-8295 8381 -2918 466 -35 1)
(-8407 8397 -2918 466 -35 1)
(-8519 8413 -2918 466 -35 1)
(-8663 8429 -2918 466 -35 1)
(-8631 8429 -2918 466 -35 1)
(-8775 8445 -2918 466 -35 1)
(-7903 8269 -2910 466 -35 1)
(-8015 8285 -2910 466 -35 1)
(-8159 8301 -2910 466 -35 1)
(-8127 8301 -2910 466 -35 1)
(-8271 8317 -2910 466 -35 1)
(-7399 8141 -2902 466 -35 1)
(-7511 8157 -2902 466 -35 1)
(-7623 8173 -2902 466 -35 1)
(-7767 8189 -2902 466 -35 1)
(-7007 8029 -2894 466 -35 1)
(-7119 8045 -2894 466 -35 1)
(-6503 7901 -2886 466 -35 1)
(-6615 7917 -2886 466 -35 1)
(-6111 7789 -2878 466 -35 1)

{3851, 26961, 188720, 0, 0, 356804550}.
{-372095, 350101, -119942, 19106, -1435, 41}
-332474

```

```

{3851, 26961, 188720, 0, 0, 356804550}.Transpose[ ]
```

$$\begin{pmatrix} -9527 & 8669 & -2934 & 466 & -35 & 1 \\ -8911 & 8525 & -2926 & 466 & -35 & 1 \\ -9055 & 8541 & -2926 & 466 & -35 & 1 \\ -9023 & 8541 & -2926 & 466 & -35 & 1 \\ -9167 & 8557 & -2926 & 466 & -35 & 1 \\ -9135 & 8557 & -2926 & 466 & -35 & 1 \\ -8295 & 8381 & -2918 & 466 & -35 & 1 \\ -8407 & 8397 & -2918 & 466 & -35 & 1 \\ -8519 & 8413 & -2918 & 466 & -35 & 1 \\ -8663 & 8429 & -2918 & 466 & -35 & 1 \\ -8631 & 8429 & -2918 & 466 & -35 & 1 \\ -8775 & 8445 & -2918 & 466 & -35 & 1 \\ -7903 & 8269 & -2910 & 466 & -35 & 1 \\ -8015 & 8285 & -2910 & 466 & -35 & 1 \\ -8159 & 8301 & -2910 & 466 & -35 & 1 \\ -8127 & 8301 & -2910 & 466 & -35 & 1 \\ -8271 & 8317 & -2910 & 466 & -35 & 1 \\ -7399 & 8141 & -2902 & 466 & -35 & 1 \\ -7511 & 8157 & -2902 & 466 & -35 & 1 \\ -7623 & 8173 & -2902 & 466 & -35 & 1 \\ -7767 & 8189 & -2902 & 466 & -35 & 1 \\ -7007 & 8029 & -2894 & 466 & -35 & 1 \\ -7119 & 8045 & -2894 & 466 & -35 & 1 \\ -6503 & 7901 & -2886 & 466 & -35 & 1 \\ -6615 & 7917 & -2886 & 466 & -35 & 1 \\ -6111 & 7789 & -2878 & 466 & -35 & 1 \end{pmatrix}$$

{136502, 136094, 12926, 136158, 12990, 136222, 135686, 135750,  
 135814, 12646, 135878, 12710, 135406, 135470, 12302, 135534, 12366,  
 135062, 135126, 135190, 12022, 134782, 134846, 134438, 134502, 134158}

```

poly10 = listmod128[[10]]
(-11 + x) (-9 + x)^4 (-7 + x)^8 (-5 + x) (5 + x)^25 (68 - 17 x + x^2)

p10 = poly10 / minipoly[poly10] // Factor
(-9 + x)^3 (-7 + x)^7 (5 + x)^24
```

```

feasiblesubcharpolylist[ (-11 + x) (-9 + x)4 (-7 + x)8 (-5 + x) (5 + x)25 (68 - 17 x + x2) ]
{ (-11 + x) (-7 + x) (1059 - 870 x + 236 x2 - 26 x3 + x4) ,
  (-11 + x) (-7 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3) ,
  (-11 + x) (-7 + x) (59 - 16 x + x2) (17 - 10 x + x2) ,
  (-11 + x) (-7 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4) ,
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (23 - 12 x + x2) ,
  (-9 + x) (-8823 + 8461 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
  (-11 + x) (-7 + x) (947 - 854 x + 236 x2 - 26 x3 + x4) ,
  (-11 + x) (-9 + x) (-7 + x) (-107 + 83 x - 17 x2 + x3) ,
  (-7 + x) (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3) ,
  (-9 + x) (-7 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4) ,
  (-11 + x) (-9 + x)2 (-7 + x) (11 - 8 x + x2) ,
  (-9 + x) (-7 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) }

{ (-11 + x) (-7 + x) (1059 - 870 x + 236 x2 - 26 x3 + x4) ,
  (-11 + x) (-7 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3) ,
  (-11 + x) (-7 + x) (59 - 16 x + x2) (17 - 10 x + x2) ,
  (-11 + x) (-7 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4) ,
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (23 - 12 x + x2) ,
  (-9 + x) (-8823 + 8461 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
  (-11 + x) (-7 + x) (947 - 854 x + 236 x2 - 26 x3 + x4) ,
  (-11 + x) (-9 + x) (-7 + x) (-107 + 83 x - 17 x2 + x3) ,
  (-7 + x) (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3) ,
  (-9 + x) (-7 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4) ,
  (-11 + x) (-9 + x)2 (-7 + x) (11 - 8 x + x2) ,
  (-9 + x) (-7 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) } // Length

```

```

CoefficientList[{\{(-11+x) (-7+x) (1059 - 870 x + 236 x2 - 26 x3 + x4), 
  (-11+x) (-7+x) (-5+x) (-215 + 131 x - 21 x2 + x3), 
  (-11+x) (-7+x) (59 - 16 x + x2) (17 - 10 x + x2), 
  (-11+x) (-7+x) (1019 - 862 x + 236 x2 - 26 x3 + x4), 
  (-11+x) (-9+x) (-7+x) (-5+x) (23 - 12 x + x2), 
  (-9+x) (-8823 + 8461 x - 2918 x2 + 466 x3 - 35 x4 + x5), 
  (-11+x) (-7+x) (947 - 854 x + 236 x2 - 26 x3 + x4), 
  (-11+x) (-9+x) (-7+x) (-107 + 83 x - 17 x2 + x3), 
  (-7+x) (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3), 
  (-9+x) (-7+x) (1193 - 1020 x + 270 x2 - 28 x3 + x4), 
  (-11+x) (-9+x)2 (-7+x) (11 - 8 x + x2), 
  (-9+x) (-7+x) (1105 - 1012 x + 270 x2 - 28 x3 + x4)\}, x]

{{81543, -86052, 34891, -7120, 781, -44, 1}, 
 {82775, -86340, 34907, -7120, 781, -44, 1}, 
 {77231, -84428, 34691, -7112, 781, -44, 1}, 
 {78463, -84716, 34707, -7112, 781, -44, 1}, 
 {79695, -85004, 34723, -7112, 781, -44, 1}, 
 {79407, -84972, 34723, -7112, 781, -44, 1}, 
 {72919, -82804, 34491, -7104, 781, -44, 1}, 
 {74151, -83092, 34507, -7104, 781, -44, 1}, 
 {73927, -83060, 34507, -7104, 781, -44, 1}, 
 {75159, -83348, 34523, -7104, 781, -44, 1}, 
 {68607, -81180, 34291, -7096, 781, -44, 1}, 
 {69615, -81436, 34307, -7096, 781, -44, 1}};

A = {{81543, -86052, 34891, -7120, 781, -44, 1}, 
 {82775, -86340, 34907, -7120, 781, -44, 1}, 
 {77231, -84428, 34691, -7112, 781, -44, 1}, 
 {78463, -84716, 34707, -7112, 781, -44, 1}, 
 {79695, -85004, 34723, -7112, 781, -44, 1}, 
 {79407, -84972, 34723, -7112, 781, -44, 1}, 
 {72919, -82804, 34491, -7104, 781, -44, 1}, 
 {74151, -83092, 34507, -7104, 781, -44, 1}, 
 {73927, -83060, 34507, -7104, 781, -44, 1}, 
 {75159, -83348, 34523, -7104, 781, -44, 1}, 
 {68607, -81180, 34291, -7096, 781, -44, 1}, 
 {69615, -81436, 34307, -7096, 781, -44, 1}};

CoefficientList[D[poly10, x] / p10 // Factor, x]
{3383255, -3537084, 1430763, -291896, 32021, -1804, 41}

```

```

Solve[
  Array[n, 12].{{81543, -86052, 34891, -7120, 781, -44, 1}, {82775, -86340, 34907,
    -7120, 781, -44, 1}, {77231, -84428, 34691, -7112, 781, -44, 1},
    {78463, -84716, 34707, -7112, 781, -44, 1}, {79695, -85004, 34723,
    -7112, 781, -44, 1}, {79407, -84972, 34723, -7112, 781, -44, 1},
    {72919, -82804, 34491, -7104, 781, -44, 1}, {74151, -83092, 34507,
    -7104, 781, -44, 1}, {73927, -83060, 34507, -7104, 781, -44, 1},
    {75159, -83348, 34523, -7104, 781, -44, 1}, {68607, -81180, 34291,
    -7096, 781, -44, 1}, {69615, -81436, 34307, -7096, 781, -44, 1}} ==
  {3383255, -3537084, 1430763, -291896, 32021, -1804, 41}, Array[n, 12]]]

Solve::svrs : Equations may not give solutions for all "solve" variables. >>>

{{n[6] → 48, n[9] → 56 - 3 n[1] - 2 n[2] - 2 n[3] - n[4] - n[7],
  n[10] → -32 - n[2] - n[4] - 2 n[5] - n[8],
  n[11] → 5 - n[1] - n[2] - n[3] - n[4] - n[5] - n[7] - n[8],
  n[12] → -36 + 3 n[1] + 3 n[2] + 2 n[3] + 2 n[4] + 2 n[5] + n[7] + n[8]}}

FindInstance[-n[6] + 48 == 0 && -n[9] + 56 - 3 n[1] - 2 n[2] - 2 n[3] - n[4] - n[7] == 0 &&
  -n[10] - 32 - n[2] - n[4] - 2 n[5] - n[8] == 0 &&
  -n[11] + 5 - n[1] - n[2] - n[3] - n[4] - n[5] - n[7] - n[8] == 0 &&
  -n[12] - 36 + 3 n[1] + 3 n[2] + 2 n[3] + 2 n[4] + 2 n[5] + n[7] + n[8] == 0 && n[1] ≥ 0 &&
  n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 && n[8] ≥ 0 &&
  n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0, Integer]
{ }

Array[m, 7].Transpose[A]

{81543 m[1] - 86052 m[2] + 34891 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7],
  82775 m[1] - 86340 m[2] + 34907 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7],
  77231 m[1] - 84428 m[2] + 34691 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
  78463 m[1] - 84716 m[2] + 34707 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
  79695 m[1] - 85004 m[2] + 34723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
  79407 m[1] - 84972 m[2] + 34723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7],
  72919 m[1] - 82804 m[2] + 34491 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
  74151 m[1] - 83092 m[2] + 34507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
  73927 m[1] - 83060 m[2] + 34507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
  75159 m[1] - 83348 m[2] + 34523 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7],
  68607 m[1] - 81180 m[2] + 34291 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7],
  69615 m[1] - 81436 m[2] + 34307 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7]}

Array[m, 7].{3383255, -3537084, 1430763, -291896, 32021, -1804, 41}

3383255 m[1] - 3537084 m[2] + 1430763 m[3] -
  291896 m[4] + 32021 m[5] - 1804 m[6] + 41 m[7]

```

```

FindInstance[
81 543 m[1] - 86 052 m[2] + 34 891 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
82 775 m[1] - 86 340 m[2] + 34 907 m[3] - 7120 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
77 231 m[1] - 84 428 m[2] + 34 691 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
78 463 m[1] - 84 716 m[2] + 34 707 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
79 695 m[1] - 85 004 m[2] + 34 723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
79 407 m[1] - 84 972 m[2] + 34 723 m[3] - 7112 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
72 919 m[1] - 82 804 m[2] + 34 491 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
74 151 m[1] - 83 092 m[2] + 34 507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
73 927 m[1] - 83 060 m[2] + 34 507 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
75 159 m[1] - 83 348 m[2] + 34 523 m[3] - 7104 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
68 607 m[1] - 81 180 m[2] + 34 291 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
69 615 m[1] - 81 436 m[2] + 34 307 m[3] - 7096 m[4] + 781 m[5] - 44 m[6] + m[7] ≥ 0 &&
3 383 255 m[1] - 3 537 084 m[2] + 1 430 763 m[3] - 291 896 m[4] +
32 021 m[5] - 1804 m[6] + 41 m[7] < 0, Array[m, 7], Integers]

{{m[1] → 10 481, m[2] → 50 373, m[3] → 99 745, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0} }

Array[m, 7] /.
{m[1] → 10 481, m[2] → 50 373, m[3] → 99 745, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}
{10 481, 50 373, 99 745, 0, 0, 0, 0}

GCD[10 481, 50 373, 99 745, 0, 0, 0, 0]
1

{{81 543, -86 052, 34 891, -7120, 781, -44, 1},
 {82 775, -86 340, 34 907, -7120, 781, -44, 1},
 {77 231, -84 428, 34 691, -7112, 781, -44, 1},
 {78 463, -84 716, 34 707, -7112, 781, -44, 1},
 {79 695, -85 004, 34 723, -7112, 781, -44, 1},
 {79 407, -84 972, 34 723, -7112, 781, -44, 1},
 {72 919, -82 804, 34 491, -7104, 781, -44, 1},
 {74 151, -83 092, 34 507, -7104, 781, -44, 1},
 {73 927, -83 060, 34 507, -7104, 781, -44, 1},
 {75 159, -83 348, 34 523, -7104, 781, -44, 1},
 {68 607, -81 180, 34 291, -7096, 781, -44, 1},
 {69 615, -81 436, 34 307, -7096, 781, -44, 1}} // MatrixForm

81 543 -86 052 34 891 -7120 781 -44 1
82 775 -86 340 34 907 -7120 781 -44 1
77 231 -84 428 34 691 -7112 781 -44 1
78 463 -84 716 34 707 -7112 781 -44 1
79 695 -85 004 34 723 -7112 781 -44 1
79 407 -84 972 34 723 -7112 781 -44 1
72 919 -82 804 34 491 -7104 781 -44 1
74 151 -83 092 34 507 -7104 781 -44 1
73 927 -83 060 34 507 -7104 781 -44 1
75 159 -83 348 34 523 -7104 781 -44 1
68 607 -81 180 34 291 -7096 781 -44 1
69 615 -81 436 34 307 -7096 781 -44 1

```

```

{10 481, 50 373, 99 745, 0, 0, 0, 0}.

{3 383 255, -3 537 084, 1 430 763, -291 896, 32 021, -1804, 41}

-2 181 242

{10 481, 50 373, 99 745, 0, 0, 0, 0}.

Transpose[{{81 543, -86 052, 34 891, -7120, 781, -44, 1},
           {82 775, -86 340, 34 907, -7120, 781, -44, 1},
           {77 231, -84 428, 34 691, -7112, 781, -44, 1},
           {78 463, -84 716, 34 707, -7112, 781, -44, 1},
           {79 695, -85 004, 34 723, -7112, 781, -44, 1},
           {79 407, -84 972, 34 723, -7112, 781, -44, 1},
           {72 919, -82 804, 34 491, -7104, 781, -44, 1},
           {74 151, -83 092, 34 507, -7104, 781, -44, 1},
           {73 927, -83 060, 34 507, -7104, 781, -44, 1},
           {75 159, -83 348, 34 523, -7104, 781, -44, 1},
           {68 607, -81 180, 34 291, -7096, 781, -44, 1},
           {69 615, -81 436, 34 307, -7096, 781, -44, 1}}]

{157 582, 158 670, 16 820 262, 16 821 350, 16 822 438, 15 415 846,
 33 482 942, 33 484 030, 32 748 222, 32 749 310, 50 145 622, 49 410 902}

```

```

poly11 = listmod128[[11]]
(-9 + x)^4 (-7 + x)^7 (5 + x)^25 (-25 904 + 17 829 x - 4784 x^2 + 626 x^3 - 40 x^4 + x^5)

p11 = poly11 / minipoly[poly11] // Factor
(-9 + x)^3 (-7 + x)^6 (5 + x)^24

```

```

feasiblesubcharpolylist[

$$(-9+x)^4 (-7+x)^7 (5+x)^{25} (-25904 + 17829x - 4784x^2 + 626x^3 - 40x^4 + x^5) ]$$


$$\{ (-9+x) (-7+x) (-5+x) (1843 - 1346x + 316x^2 - 30x^3 + x^4),$$


$$(-9+x) (-7+x) (-9183 + 8573x - 2926x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x) (-8599 + 8429x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-7+x) (-5+x) (-195 + 127x - 21x^2 + x^3),$$


$$(-9+x) (-7+x)^2 (1249 - 1028x + 270x^2 - 28x^3 + x^4),$$


$$(-9+x) (-7+x) (-8711 + 8445x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-5+x) (1381 - 1084x + 274x^2 - 28x^3 + x^4),$$


$$(-9+x) (61921 - 68082x + 28887x^2 - 6180x^3 + 711x^4 - 42x^5 + x^6),$$


$$(-11+x) (-9+x) (-7+x)^2 (-5+x) (23 - 12x + x^2),$$


$$(-9+x) (-7+x) (-8159 + 8301x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-7+x)^2 (-129 + 99x - 19x^2 + x^3),$$


$$(-9+x)^2 (-7+x) (919 - 822x + 232x^2 - 26x^3 + x^4),$$


$$(-11+x) (-9+x) (-7+x)^2 (-107 + 83x - 17x^2 + x^3),$$


$$(-11+x) (-9+x)^2 (-7+x) (-5+x) (17 - 10x + x^2),$$


$$(-9+x) (-7+x) (-8383 + 8333x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x) (-8527 + 8349x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (59401 - 66938x + 28719x^2 - 6172x^3 + 711x^4 - 42x^5 + x^6),$$


$$(-9+x) (-7+x) (-7655 + 8173x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-7+x) (863 - 814x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x)^2 (-7+x) (879 - 814x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x) (-7+x) (-7879 + 8205x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x) (-8023 + 8221x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-7+x) (807 - 806x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x)^2 (-7+x) (823 - 806x + 232x^2 - 26x^3 + x^4) \}$$


```

$$\left\{ (-9+x) (-7+x) (-5+x) (1843 - 1346x + 316x^2 - 30x^3 + x^4), \right.$$

$$(-9+x) (-7+x) (-9183 + 8573x - 2926x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x) (-7+x) (-8599 + 8429x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x)^2 (-7+x) (-5+x) (-195 + 127x - 21x^2 + x^3),$$

$$(-9+x) (-7+x)^2 (1249 - 1028x + 270x^2 - 28x^3 + x^4),$$

$$(-9+x) (-7+x) (-8711 + 8445x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x)^2 (-5+x) (1381 - 1084x + 274x^2 - 28x^3 + x^4),$$

$$(-9+x) (61921 - 68082x + 28887x^2 - 6180x^3 + 711x^4 - 42x^5 + x^6),$$

$$(-11+x) (-9+x) (-7+x)^2 (-5+x) (23 - 12x + x^2),$$

$$(-9+x) (-7+x) (-8159 + 8301x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x)^2 (-7+x)^2 (-129 + 99x - 19x^2 + x^3),$$

$$(-9+x)^2 (-7+x) (919 - 822x + 232x^2 - 26x^3 + x^4),$$

$$(-11+x) (-9+x) (-7+x)^2 (-107 + 83x - 17x^2 + x^3),$$

$$(-11+x) (-9+x)^2 (-7+x) (-5+x) (17 - 10x + x^2),$$

$$(-9+x) (-7+x) (-8383 + 8333x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x) (-7+x) (-8527 + 8349x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x) (59401 - 66938x + 28719x^2 - 6172x^3 + 711x^4 - 42x^5 + x^6),$$

$$(-9+x) (-7+x) (-7655 + 8173x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x)^2 (-7+x) (863 - 814x + 232x^2 - 26x^3 + x^4),$$

$$(-9+x)^2 (-7+x) (879 - 814x + 232x^2 - 26x^3 + x^4),$$

$$(-9+x) (-7+x) (-7879 + 8205x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x) (-7+x) (-8023 + 8221x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$

$$(-9+x)^2 (-7+x) (807 - 806x + 232x^2 - 26x^3 + x^4),$$

$$\left. (-9+x)^2 (-7+x) (823 - 806x + 232x^2 - 26x^3 + x^4) \right\} // \text{Length}$$

```

CoefficientList[{\{(-9+x) (-7+x) (-5+x) (1843 - 1346 x + 316 x2 - 30 x3 + x4),
(-9+x) (-7+x) (-9183 + 8573 x - 2926 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x) (-7+x) (-8599 + 8429 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x)2 (-7+x) (-5+x) (-195 + 127 x - 21 x2 + x3) ,
(-9+x) (-7+x)2 (1249 - 1028 x + 270 x2 - 28 x3 + x4) ,
(-9+x) (-7+x) (-8711 + 8445 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x)2 (-5+x) (1381 - 1084 x + 274 x2 - 28 x3 + x4) ,
(-9+x) (61921 - 68082 x + 28887 x2 - 6180 x3 + 711 x4 - 42 x5 + x6) ,
(-11+x) (-9+x) (-7+x)2 (-5+x) (23 - 12 x + x2) ,
(-9+x) (-7+x) (-8159 + 8301 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x)2 (-7+x)2 (-129 + 99 x - 19 x2 + x3) ,
(-9+x)2 (-7+x) (919 - 822 x + 232 x2 - 26 x3 + x4) ,
(-11+x) (-9+x) (-7+x)2 (-107 + 83 x - 17 x2 + x3) ,
(-11+x) (-9+x)2 (-7+x) (-5+x) (17 - 10 x + x2) ,
(-9+x) (-7+x) (-8383 + 8333 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x) (-7+x) (-8527 + 8349 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x) (59401 - 66938 x + 28719 x2 - 6172 x3 + 711 x4 - 42 x5 + x6) ,
(-9+x) (-7+x) (-7655 + 8173 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x)2 (-7+x) (863 - 814 x + 232 x2 - 26 x3 + x4) ,
(-9+x)2 (-7+x) (879 - 814 x + 232 x2 - 26 x3 + x4) ,
(-9+x) (-7+x) (-7879 + 8205 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x) (-7+x) (-8023 + 8221 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
(-9+x)2 (-7+x) (807 - 806 x + 232 x2 - 26 x3 + x4) ,
(-9+x)2 (-7+x) (823 - 806 x + 232 x2 - 26 x3 + x4) \}, x]

```

```

{ {-580 545, 687 539, -330 721, 84 747, -12 587, 1089, -51, 1},
  {-578 529, 687 027, -330 689, 84 747, -12 587, 1089, -51, 1},
  {-541 737, 668 611, -327 297, 84 475, -12 579, 1089, -51, 1},
  {-552 825, 672 435, -327 729, 84 491, -12 579, 1089, -51, 1},
  {-550 809, 671 923, -327 697, 84 491, -12 579, 1089, -51, 1},
  {-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
  {-559 305, 675 171, -328 097, 84 507, -12 579, 1089, -51, 1},
  {-557 289, 674 659, -328 065, 84 507, -12 579, 1089, -51, 1},
  {-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
  {-514 017, 653 507, -324 305, 84 219, -12 571, 1089, -51, 1},
  {-512 001, 652 995, -324 273, 84 219, -12 571, 1089, -51, 1},
  {-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
  {-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
  {-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
  {-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
  {-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
  {-534 609, 661 843, -325 409, 84 267, -12 571, 1089, -51, 1},
  {-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
  {-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
  {-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
  {-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
  {-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
  {-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
  {-466 641, 627 363, -318 961, 83 739, -12 555, 1089, -51, 1} }

CoefficientList[D[poly11, x] / p11 // Factor, x]
{-23 396 345, 27 945 611, -13 505 961, 3 469 539, -515 891, 44 649, -2091, 41}

```

```

A = {{-580545, 687539, -330721, 84747, -12587, 1089, -51, 1},
      {-578529, 687027, -330689, 84747, -12587, 1089, -51, 1},
      {-541737, 668611, -327297, 84475, -12579, 1089, -51, 1},
      {-552825, 672435, -327729, 84491, -12579, 1089, -51, 1},
      {-550809, 671923, -327697, 84491, -12579, 1089, -51, 1},
      {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
      {-559305, 675171, -328097, 84507, -12579, 1089, -51, 1},
      {-557289, 674659, -328065, 84507, -12579, 1089, -51, 1},
      {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
      {-514017, 653507, -324305, 84219, -12571, 1089, -51, 1},
      {-512001, 652995, -324273, 84219, -12571, 1089, -51, 1},
      {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
      {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
      {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
      {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
      {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
      {-534609, 661843, -325409, 84267, -12571, 1089, -51, 1},
      {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
      {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
      {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
      {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
      {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
      {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
      {-466641, 627363, -318961, 83739, -12555, 1089, -51, 1}};

MatrixRank[A]
5

```

```

Solve[Array[n, 24].{{-580545, 687539, -330721, 84747, -12587, 1089, -51, 1},
{-578529, 687027, -330689, 84747, -12587, 1089, -51, 1},
{-541737, 668611, -327297, 84475, -12579, 1089, -51, 1},
{-552825, 672435, -327729, 84491, -12579, 1089, -51, 1},
{-550809, 671923, -327697, 84491, -12579, 1089, -51, 1},
{-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
{-559305, 675171, -328097, 84507, -12579, 1089, -51, 1},
{-557289, 674659, -328065, 84507, -12579, 1089, -51, 1},
{-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
{-514017, 653507, -324305, 84219, -12571, 1089, -51, 1},
{-512001, 652995, -324273, 84219, -12571, 1089, -51, 1},
{-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
{-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
{-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
{-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
{-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
{-534609, 661843, -325409, 84267, -12571, 1089, -51, 1},
{-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
{-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
{-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
{-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
{-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
{-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
{-466641, 627363, -318961, 83739, -12555, 1089, -51, 1}} ==
{-23396345, 27945611, -13505961, 3469539, -515891,
44649, -2091, 41}, Array[n, 24]]
{ }

Solve[Array[n, 24].A == {-23396345, 27945611,
-13505961, 3469539, -515891, 44649, -2091, 41}, Array[n, 24]]
{ }

```

**Array[m, 8].Transpose[A]**

```
{-580545 m[1] + 687539 m[2] - 330721 m[3] + 84747 m[4] -
12587 m[5] + 1089 m[6] - 51 m[7] + m[8], -578529 m[1] + 687027 m[2] -
330689 m[3] + 84747 m[4] - 12587 m[5] + 1089 m[6] - 51 m[7] + m[8],
-541737 m[1] + 668611 m[2] - 327297 m[3] + 84475 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8], -552825 m[1] + 672435 m[2] -
327729 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-550809 m[1] + 671923 m[2] - 327697 m[3] + 84491 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8], -548793 m[1] + 671411 m[2] -
327665 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-559305 m[1] + 675171 m[2] - 328097 m[3] + 84507 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8], -557289 m[1] + 674659 m[2] -
328065 m[3] + 84507 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-557865 m[1] + 674723 m[2] - 328065 m[3] + 84507 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8], -514017 m[1] + 653507 m[2] -
324305 m[3] + 84219 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-512001 m[1] + 652995 m[2] - 324273 m[3] + 84219 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -521073 m[1] + 656307 m[2] -
324673 m[3] + 84235 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-519057 m[1] + 655795 m[2] - 324641 m[3] + 84235 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -530145 m[1] + 659619 m[2] -
325073 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-528129 m[1] + 659107 m[2] - 325041 m[3] + 84251 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -537201 m[1] + 662419 m[2] -
325441 m[3] + 84267 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-534609 m[1] + 661843 m[2] - 325409 m[3] + 84267 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -482265 m[1] + 637379 m[2] -
321249 m[3] + 83963 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-489321 m[1] + 640179 m[2] - 321617 m[3] + 83979 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8], -498393 m[1] + 643491 m[2] -
322017 m[3] + 83995 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-496377 m[1] + 642979 m[2] - 321985 m[3] + 83995 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8], -505449 m[1] + 646291 m[2] -
322385 m[3] + 84011 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-457569 m[1] + 624051 m[2] - 318561 m[3] + 83723 m[4] - 12555 m[5] +
1089 m[6] - 51 m[7] + m[8], -466641 m[1] + 627363 m[2] -
318961 m[3] + 83739 m[4] - 12555 m[5] + 1089 m[6] - 51 m[7] + m[8] }
```

**Array[m, 8].**

```
{-23396345, 27945611, -13505961, 3469539, -515891, 44649, -2091, 41}
-23396345 m[1] + 27945611 m[2] - 13505961 m[3] +
3469539 m[4] - 515891 m[5] + 44649 m[6] - 2091 m[7] + 41 m[8]
```

```

FindInstance[
-580545 m[1] + 687539 m[2] - 330721 m[3] + 84747 m[4] - 12587 m[5] + 1089 m[6] -
51 m[7] + m[8] ≥ 0 && -578529 m[1] + 687027 m[2] - 330689 m[3] +
84747 m[4] - 12587 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-541737 m[1] + 668611 m[2] - 327297 m[3] + 84475 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -552825 m[1] + 672435 m[2] -
327729 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-550809 m[1] + 671923 m[2] - 327697 m[3] + 84491 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -548793 m[1] + 671411 m[2] -
327665 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-559305 m[1] + 675171 m[2] - 328097 m[3] + 84507 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -557289 m[1] + 674659 m[2] -
328065 m[3] + 84507 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-557865 m[1] + 674723 m[2] - 328065 m[3] + 84507 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -514017 m[1] + 653507 m[2] -
324305 m[3] + 84219 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-512001 m[1] + 652995 m[2] - 324273 m[3] + 84219 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -521073 m[1] + 656307 m[2] -
324673 m[3] + 84235 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-519057 m[1] + 655795 m[2] - 324641 m[3] + 84235 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -530145 m[1] + 659619 m[2] -
325073 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-528129 m[1] + 659107 m[2] - 325041 m[3] + 84251 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -537201 m[1] + 662419 m[2] -
325441 m[3] + 84267 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-534609 m[1] + 661843 m[2] - 325409 m[3] + 84267 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -482265 m[1] + 637379 m[2] -
321249 m[3] + 83963 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-489321 m[1] + 640179 m[2] - 321617 m[3] + 83979 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -498393 m[1] + 643491 m[2] -
322017 m[3] + 83995 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-496377 m[1] + 642979 m[2] - 321985 m[3] + 83995 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -505449 m[1] + 646291 m[2] -
322385 m[3] + 84011 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-457569 m[1] + 624051 m[2] - 318561 m[3] + 83723 m[4] - 12555 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -466641 m[1] + 627363 m[2] -
318961 m[3] + 83739 m[4] - 12555 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-23396345 m[1] + 27945611 m[2] - 13505961 m[3] + 3469539 m[4] -
515891 m[5] + 44649 m[6] - 2091 m[7] + 41 m[8] < 0, Array[m, 8], Integers]

{{m[1] → 11780, m[2] → 28348, m[3] → 38250,
m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0, m[8] → 0} }

Array[m, 8] /. {m[1] → 11780, m[2] → 28348,
m[3] → 38250, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0, m[8] → 0}
{11780, 28348, 38250, 0, 0, 0, 0, 0}

```

```

GCD[11 780, 28 348, 38 250, 0, 0, 0, 0, 0]
2

{5890, 14 174, 19 125, 0, 0, 0, 0, 0} // Reverse
{0, 0, 0, 0, 0, 19 125, 14 174, 5890}

{11 780, 28 348, 38 250, 0, 0, 0, 0, 0} / 2
{5890, 14 174, 19 125, 0, 0, 0, 0, 0}

{5890, 14 174, 19 125, 0, 0, 0, 0, 0}.
{-23 396 345, 27 945 611, -13 505 961, 3 469 539, -515 891, 44 649, -2091, 41}
-4 885 861

{5890, 14 174, 19 125, 0, 0, 0, 0, 0}.

Transpose[{{{-580 545, 687 539, -330 721, 84 747, -12 587, 1089, -51, 1},
{-578 529, 687 027, -330 689, 84 747, -12 587, 1089, -51, 1},
{-541 737, 668 611, -327 297, 84 475, -12 579, 1089, -51, 1},
{-552 825, 672 435, -327 729, 84 491, -12 579, 1089, -51, 1},
{-550 809, 671 923, -327 697, 84 491, -12 579, 1089, -51, 1},
{-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
{-559 305, 675 171, -328 097, 84 507, -12 579, 1089, -51, 1},
{-557 289, 674 659, -328 065, 84 507, -12 579, 1089, -51, 1},
{-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
{-514 017, 653 507, -324 305, 84 219, -12 571, 1089, -51, 1},
{-512 001, 652 995, -324 273, 84 219, -12 571, 1089, -51, 1},
{-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
{-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
{-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
{-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
{-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
{-534 609, 661 843, -325 409, 84 267, -12 571, 1089, -51, 1},
{-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
{-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
{-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
{-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
{-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
{-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
{-466 641, 627 363, -318 961, 83 739, -12 555, 1089, -51, 1}}]

{728 611, 5 957 763, 26 506 259, 7 137 315, 12 366 467, 17 595 619,
712 179, 5 941 331, 3 455 827, 32 914 963, 38 144 115, 24 004 323,
29 233 475, 9 864 531, 15 093 683, 953 891, 8 668 547, 49 781 971,
40 871 331, 26 731 539, 31 960 691, 17 820 899, 57 738 339, 43 598 547}

{11 780, 28 348, 38 250, 0, 0, 0, 0, 0}.

{-23 396 345, 27 945 611, -13 505 961, 3 469 539, -515 891, 44 649, -2091, 41}
-9 771 722

```

```

{11780, 28348, 38250, 0, 0, 0, 0, 0}.

Transpose[{{{-580545, 687539, -330721, 84747, -12587, 1089, -51, 1},
{-578529, 687027, -330689, 84747, -12587, 1089, -51, 1},
{-541737, 668611, -327297, 84475, -12579, 1089, -51, 1},
{-552825, 672435, -327729, 84491, -12579, 1089, -51, 1},
{-550809, 671923, -327697, 84491, -12579, 1089, -51, 1},
{-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
{-559305, 675171, -328097, 84507, -12579, 1089, -51, 1},
{-557289, 674659, -328065, 84507, -12579, 1089, -51, 1},
{-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
{-514017, 653507, -324305, 84219, -12571, 1089, -51, 1},
{-512001, 652995, -324273, 84219, -12571, 1089, -51, 1},
{-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
{-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
{-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
{-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
{-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
{-534609, 661843, -325409, 84267, -12571, 1089, -51, 1},
{-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
{-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
{-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
{-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
{-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
{-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
{-466641, 627363, -318961, 83739, -12555, 1089, -51, 1}}]

{1457222, 11915526, 53012518, 14274630, 24732934, 35191238,
1424358, 11882662, 6911654, 65829926, 76288230, 48008646,
58466950, 19729062, 30187366, 1907782, 17337094, 99563942,
81742662, 53463078, 63921382, 35641798, 115476678, 87197094}

```

```

poly12 = listmod128[[12]]
(-11 + x) (-9 + x)^4 (-7 + x)^9 (5 + x)^25 (48 - 15 x + x^2)

p12 = poly12 / minipoly[poly12] // Factor
(-9 + x)^3 (-7 + x)^8 (5 + x)^24

```

```

feasiblesubcharpolylist[ (-11 + x) (-9 + x)4 (-7 + x)9 (5 + x)25 (48 - 15 x + x2) ]
{ (-11 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3) , (-11 + x) (59 - 16 x + x2) (17 - 10 x + x2) ,
(-11 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4) , (-11 + x) (-9 + x) (-5 + x) (23 - 12 x + x2) ,
(-9 + x) (1281 - 1028 x + 270 x2 - 28 x3 + x4) , (-11 + x) (-7 + x)2 (19 - 12 x + x2) ,
(-11 + x) (947 - 854 x + 236 x2 - 26 x3 + x4) , (-11 + x) (-9 + x) (-107 + 83 x - 17 x2 + x3) ,
(59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3) , (-9 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4) ,
(-11 + x) (-7 + x) (-125 + 103 x - 19 x2 + x3) , (-11 + x) (-9 + x)2 (11 - 8 x + x2) ,
-9769 + 10197 x - 3442 x2 + 522 x3 - 37 x4 + x5 , (-9 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) ,
(-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2) , (-9 + x)2 (-113 + 99 x - 19 x2 + x3) ,
(-9 + x) (1033 - 1004 x + 270 x2 - 28 x3 + x4) , (-9 + x)2 (-7 + x) (15 - 12 x + x2) }

{ (-11 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3) ,
(-11 + x) (59 - 16 x + x2) (17 - 10 x + x2) , (-11 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4) ,
(-11 + x) (-9 + x) (-5 + x) (23 - 12 x + x2) , (-9 + x) (1281 - 1028 x + 270 x2 - 28 x3 + x4) ,
(-11 + x) (-7 + x)2 (19 - 12 x + x2) , (-11 + x) (947 - 854 x + 236 x2 - 26 x3 + x4) ,
(-11 + x) (-9 + x) (-107 + 83 x - 17 x2 + x3) , (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3) ,
(-9 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4) , (-11 + x) (-7 + x) (-125 + 103 x - 19 x2 + x3) ,
(-11 + x) (-9 + x)2 (11 - 8 x + x2) , -9769 + 10197 x - 3442 x2 + 522 x3 - 37 x4 + x5 ,
(-9 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) , (-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2) ,
(-9 + x)2 (-113 + 99 x - 19 x2 + x3) , (-9 + x) (1033 - 1004 x + 270 x2 - 28 x3 + x4) ,
(-9 + x)2 (-7 + x) (15 - 12 x + x2) } // Length

```

18

```

CoefficientList[{ (-11 + x) (-5 + x) (-215 + 131 x - 21 x2 + x3) ,
(-11 + x) (59 - 16 x + x2) (17 - 10 x + x2) , (-11 + x) (1019 - 862 x + 236 x2 - 26 x3 + x4) ,
(-11 + x) (-9 + x) (-5 + x) (23 - 12 x + x2) , (-9 + x) (1281 - 1028 x + 270 x2 - 28 x3 + x4) ,
(-11 + x) (-7 + x)2 (19 - 12 x + x2) , (-11 + x) (947 - 854 x + 236 x2 - 26 x3 + x4) ,
(-11 + x) (-9 + x) (-107 + 83 x - 17 x2 + x3) , (59 - 16 x + x2) (-179 + 127 x - 21 x2 + x3) ,
(-9 + x) (1193 - 1020 x + 270 x2 - 28 x3 + x4) , (-11 + x) (-7 + x) (-125 + 103 x - 19 x2 + x3) ,
(-11 + x) (-9 + x)2 (11 - 8 x + x2) , -9769 + 10197 x - 3442 x2 + 522 x3 - 37 x4 + x5 ,
(-9 + x) (1105 - 1012 x + 270 x2 - 28 x3 + x4) ,
(-11 + x) (-9 + x) (-7 + x) (13 - 10 x + x2) , (-9 + x)2 (-113 + 99 x - 19 x2 + x3) ,
(-9 + x) (1033 - 1004 x + 270 x2 - 28 x3 + x4) , (-9 + x)2 (-7 + x) (15 - 12 x + x2) }, x]
{ {-11825, 10645, -3466, 522, -37, 1}, {-11033, 10485, -3458, 522, -37, 1},
{-11209, 10501, -3458, 522, -37, 1}, {-11385, 10517, -3458, 522, -37, 1},
{-11529, 10533, -3458, 522, -37, 1}, {-10241, 10325, -3450, 522, -37, 1},
{-10417, 10341, -3450, 522, -37, 1}, {-10593, 10357, -3450, 522, -37, 1},
{-10561, 10357, -3450, 522, -37, 1}, {-10737, 10373, -3450, 522, -37, 1},
{-9625, 10181, -3442, 522, -37, 1}, {-9801, 10197, -3442, 522, -37, 1},
{-9769, 10197, -3442, 522, -37, 1}, {-9945, 10213, -3442, 522, -37, 1},
{-9009, 10037, -3434, 522, -37, 1}, {-9153, 10053, -3434, 522, -37, 1},
{-9297, 10069, -3434, 522, -37, 1}, {-8505, 9909, -3426, 522, -37, 1} }

```

```

CoefficientList[D[poly12, x] / p12 // Factor, x]
{-476 745, 433 821, -141 930, 21 402, -1517, 41}

Solve[Array[n, 18].
{{{-11 825, 10 645, -3466, 522, -37, 1}, {-11 033, 10 485, -3458, 522, -37, 1},
{-11 209, 10 501, -3458, 522, -37, 1}, {-11 385, 10 517, -3458, 522, -37, 1},
{-11 529, 10 533, -3458, 522, -37, 1}, {-10 241, 10 325, -3450, 522, -37, 1},
{-10 417, 10 341, -3450, 522, -37, 1}, {-10 593, 10 357, -3450, 522, -37, 1},
{-10 561, 10 357, -3450, 522, -37, 1}, {-10 737, 10 373, -3450, 522, -37, 1},
{-9625, 10 181, -3442, 522, -37, 1}, {-9801, 10 197, -3442, 522, -37, 1},
{-9769, 10 197, -3442, 522, -37, 1}, {-9945, 10 213, -3442, 522, -37, 1},
{-9009, 10 037, -3434, 522, -37, 1}, {-9153, 10 053, -3434, 522, -37, 1},
{-9297, 10 069, -3434, 522, -37, 1}, {-8505, 9909, -3426, 522, -37, 1}} ==
{-476 745, 433 821, -141 930, 21 402, -1517, 41}, Array[n, 18]]

Solve::svars : Equations may not give solutions for all "solve" variables. >>
{{n[13] → 42 - 2 n[1] - 2 n[2] - n[3] - 2 n[6] - n[7] - n[9] - n[11], n[16] →
24 - n[3] - 2 n[4] - n[5] - n[7] - 2 n[8] - n[10] - n[11] - 2 n[12] - n[14] - 2 n[15],
n[17] → 75 - n[1] - n[3] - 2 n[4] - 3 n[5] + n[6] - n[8] - n[9] - 2 n[10] +
n[11] - n[14] + n[15], n[18] → -100 + 2 n[1] + n[2] + 2 n[3] +
3 n[4] + 3 n[5] + n[7] + 2 n[8] + n[9] + 2 n[10] + n[12] + n[14]}}

```

```

Array[n, 18] /.

Solve[-n[13] + 42 - 2 n[1] - 2 n[2] - n[3] - 2 n[6] - n[7] - n[9] - n[11] == 0 &&
-n[16] + 24 - n[3] - 2 n[4] - n[5] -
n[7] - 2 n[8] - n[10] - n[11] - 2 n[12] - n[14] - 2 n[15] == 0 &&
-n[17] + 75 - n[1] - n[3] - 2 n[4] - 3 n[5] +
n[6] - n[8] - n[9] - 2 n[10] + n[11] - n[14] + n[15] == 0 &&
-n[18] - 100 + 2 n[1] + n[2] + 2 n[3] + 3 n[4] + 3 n[5] + n[7] +
2 n[8] + n[9] + 2 n[10] + n[12] + n[14] == 0 && n[1] ≥ 0 && n[2] ≥ 0 &&
n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 && n[8] ≥ 0 &&
n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 &&
n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0, Array[n, 18], Integers]

{{19, 0, 4, 2, 16, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {19, 1, 2, 3, 16, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {19, 2, 0, 4, 16, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 0, 0, 4, 16, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 0, 1, 3, 16, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 0, 1, 4, 15, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 0, 2, 2, 16, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 0, 2, 3, 15, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 1, 0, 3, 16, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {20, 1, 0, 4, 15, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {21, 0, 0, 2, 16, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {21, 0, 0, 3, 15, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {21, 0, 0, 3, 16, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {21, 0, 0, 4, 14, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0}, {21, 0, 0, 4, 15, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}

```

```

poly13 = listmod128[[13]]
(-9 + x)^4 (-7 + x)^7 (5 + x)^25 (-25580 + 17757 x - 4780 x^2 + 626 x^3 - 40 x^4 + x^5)

p13 = poly13 / minipoly[poly13] // Factor
(-9 + x)^3 (-7 + x)^6 (5 + x)^24

```

```

feasiblesubcharpololist[

$$(-9+x)^4 (-7+x)^7 (5+x)^{25} (-25580 + 17757x - 4780x^2 + 626x^3 - 40x^4 + x^5) ]$$


$$\left\{ (-9+x) (-7+x) (-9327 + 8589x - 2926x^2 + 466x^3 - 35x^4 + x^5), \right.$$


$$(-9+x) (-7+x) (-8711 + 8445x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x) (-8887 + 8461x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-11+x) (-9+x) (-7+x)^2 (-5+x) (23 - 12x + x^2),$$


$$(-9+x) (-7+x) (-8999 + 8477x - 2918x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-7+x) (919 - 822x + 232x^2 - 26x^3 + x^4),$$


$$(-11+x) (-9+x)^2 (-7+x)^2 (-107 + 83x - 17x^2 + x^3),$$


$$(-11+x) (-9+x)^2 (-7+x) (-5+x) (17 - 10x + x^2),$$


$$(-9+x) (-7+x) (-8383 + 8333x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x)^2 (1193 - 1020x + 270x^2 - 28x^3 + x^4),$$


$$(-9+x)^2 (-7+x) (951 - 822x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x) (-7+x) (-8527 + 8349x - 2910x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x) (23 - 12x + x^2) (-377 + 167x - 23x^2 + x^3),$$


$$(-9+x) (-7+x) (-7655 + 8173x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-11+x) (-9+x)^2 (-7+x)^2 (11 - 8x + x^2),$$


$$(-9+x)^2 (-7+x) (863 - 814x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x) (-7+x)^2 (1105 - 1012x + 270x^2 - 28x^3 + x^4),$$


$$(-9+x)^2 (-7+x) (879 - 814x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x) (-7+x) (-7879 + 8205x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x) (-7+x) (-8023 + 8221x - 2902x^2 + 466x^3 - 35x^4 + x^5),$$


$$(-9+x)^2 (-7+x) (807 - 806x + 232x^2 - 26x^3 + x^4),$$


$$(-9+x) (-7+x)^2 (1033 - 1004x + 270x^2 - 28x^3 + x^4),$$


$$\left. (-9+x) (-7+x) (59 - 16x + x^2) (-125 + 103x - 19x^2 + x^3), \right.$$


$$\left. (-9+x)^2 (-7+x) (751 - 798x + 232x^2 - 26x^3 + x^4) \right\}$$


```

$$\left\{ (-9+x) (-7+x) \left(-9327 + 8589x - 2926x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x) (-7+x) \left(-8711 + 8445x - 2918x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x) (-7+x) \left(-8887 + 8461x - 2918x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-11+x) (-9+x) (-7+x)^2 (-5+x) \left(23 - 12x + x^2\right), \right. \\ \left. (-9+x) (-7+x) \left(-8999 + 8477x - 2918x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x)^2 (-7+x) \left(919 - 822x + 232x^2 - 26x^3 + x^4\right), \right. \\ \left. (-11+x) (-9+x) (-7+x)^2 \left(-107 + 83x - 17x^2 + x^3\right), \right. \\ \left. (-11+x) (-9+x)^2 (-7+x) (-5+x) \left(17 - 10x + x^2\right), \right. \\ \left. (-9+x) (-7+x) \left(-8383 + 8333x - 2910x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x) (-7+x)^2 \left(1193 - 1020x + 270x^2 - 28x^3 + x^4\right), \right. \\ \left. (-9+x)^2 (-7+x) \left(951 - 822x + 232x^2 - 26x^3 + x^4\right), \right. \\ \left. (-9+x) (-7+x) \left(-8527 + 8349x - 2910x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x) (-7+x) \left(23 - 12x + x^2\right) \left(-377 + 167x - 23x^2 + x^3\right), \right. \\ \left. (-9+x) (-7+x) \left(-7655 + 8173x - 2902x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-11+x) (-9+x)^2 (-7+x)^2 \left(11 - 8x + x^2\right), \right. \\ \left. (-9+x)^2 (-7+x) \left(863 - 814x + 232x^2 - 26x^3 + x^4\right), \right. \\ \left. (-9+x) (-7+x)^2 \left(1105 - 1012x + 270x^2 - 28x^3 + x^4\right), \right. \\ \left. (-9+x)^2 (-7+x) \left(879 - 814x + 232x^2 - 26x^3 + x^4\right), \right. \\ \left. (-9+x) (-7+x) \left(-7879 + 8205x - 2902x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x) (-7+x) \left(-8023 + 8221x - 2902x^2 + 466x^3 - 35x^4 + x^5\right), \right. \\ \left. (-9+x)^2 (-7+x) \left(807 - 806x + 232x^2 - 26x^3 + x^4\right), \right. \\ \left. (-9+x) (-7+x)^2 \left(1033 - 1004x + 270x^2 - 28x^3 + x^4\right), \right. \\ \left. (-9+x) (-7+x) \left(59 - 16x + x^2\right) \left(-125 + 103x - 19x^2 + x^3\right), \right. \\ \left. \left. (-9+x)^2 (-7+x) \left(751 - 798x + 232x^2 - 26x^3 + x^4\right)\right\} // \text{Length}$$

```

CoefficientList[{\{ (-9+x) (-7+x) \((-9327+8589 x-2926 x^2+466 x^3-35 x^4+x^5)\),
(-9+x) (-7+x) \((-8711+8445 x-2918 x^2+466 x^3-35 x^4+x^5)\),
(-9+x) (-7+x) \((-8887+8461 x-2918 x^2+466 x^3-35 x^4+x^5)\),
(-11+x) (-9+x) \((-7+x)^2 (-5+x) \((23-12 x+x^2)\),
(-9+x) (-7+x) \((-8999+8477 x-2918 x^2+466 x^3-35 x^4+x^5)\),
(-9+x)^2 (-7+x) \((919-822 x+232 x^2-26 x^3+x^4)\),
(-11+x) (-9+x) \((-7+x)^2 \(-107+83 x-17 x^2+x^3\),
(-11+x) (-9+x)^2 (-7+x) \((-5+x) \((17-10 x+x^2)\),
(-9+x) (-7+x) \((-8383+8333 x-2910 x^2+466 x^3-35 x^4+x^5)\),
(-9+x) (-7+x)^2 \((1193-1020 x+270 x^2-28 x^3+x^4)\),
(-9+x)^2 (-7+x) \((951-822 x+232 x^2-26 x^3+x^4)\),
(-9+x) (-7+x) \((-8527+8349 x-2910 x^2+466 x^3-35 x^4+x^5)\),
(-9+x) (-7+x) \((23-12 x+x^2) \((-377+167 x-23 x^2+x^3)\),
(-9+x) (-7+x) \((-7655+8173 x-2902 x^2+466 x^3-35 x^4+x^5)\),
(-11+x) (-9+x)^2 (-7+x)^2 \((11-8 x+x^2)\),
(-9+x)^2 (-7+x) \((863-814 x+232 x^2-26 x^3+x^4)\),
(-9+x) (-7+x)^2 \((1105-1012 x+270 x^2-28 x^3+x^4)\),
(-9+x)^2 (-7+x) \((879-814 x+232 x^2-26 x^3+x^4)\),
(-9+x) (-7+x) \((-7879+8205 x-2902 x^2+466 x^3-35 x^4+x^5)\),
(-9+x) (-7+x) \((-8023+8221 x-2902 x^2+466 x^3-35 x^4+x^5)\),
(-9+x)^2 (-7+x) \((807-806 x+232 x^2-26 x^3+x^4)\),
(-9+x) (-7+x)^2 \((1033-1004 x+270 x^2-28 x^3+x^4)\),
(-9+x) (-7+x) \((59-16 x+x^2) \((-125+103 x-19 x^2+x^3)\),
(-9+x)^2 (-7+x) \((751-798 x+232 x^2-26 x^3+x^4)\}, x]

```

```

{ {-587601, 690339, -331089, 84763, -12587, 1089, -51, 1},
  {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
  {-559881, 675235, -328097, 84507, -12579, 1089, -51, 1},
  {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
  {-566937, 678035, -328465, 84523, -12579, 1089, -51, 1},
  {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
  {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
  {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
  {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
  {-526113, 658595, -325009, 84251, -12571, 1089, -51, 1},
  {-539217, 662931, -325473, 84267, -12571, 1089, -51, 1},
  {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
  {-546273, 665731, -325841, 84283, -12571, 1089, -51, 1},
  {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
  {-480249, 636867, -321217, 83963, -12563, 1089, -51, 1},
  {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
  {-487305, 639667, -321585, 83979, -12563, 1089, -51, 1},
  {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
  {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
  {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
  {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
  {-455553, 623539, -318529, 83723, -12555, 1089, -51, 1},
  {-464625, 626851, -318929, 83739, -12555, 1089, -51, 1},
  {-425817, 607923, -315505, 83467, -12547, 1089, -51, 1} }

CoefficientList[D[poly13, x] / p13 // Factor, x]
{-23056145, 27727451, -13457769, 3465043, -515739, 44649, -2091, 41}

```

```

A = {{-587601, 690339, -331089, 84763, -12587, 1089, -51, 1},
      {-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
      {-559881, 675235, -328097, 84507, -12579, 1089, -51, 1},
      {-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
      {-566937, 678035, -328465, 84523, -12579, 1089, -51, 1},
      {-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
      {-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
      {-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
      {-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
      {-526113, 658595, -325009, 84251, -12571, 1089, -51, 1},
      {-539217, 662931, -325473, 84267, -12571, 1089, -51, 1},
      {-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
      {-546273, 665731, -325841, 84283, -12571, 1089, -51, 1},
      {-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
      {-480249, 636867, -321217, 83963, -12563, 1089, -51, 1},
      {-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
      {-487305, 639667, -321585, 83979, -12563, 1089, -51, 1},
      {-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
      {-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
      {-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
      {-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
      {-455553, 623539, -318529, 83723, -12555, 1089, -51, 1},
      {-464625, 626851, -318929, 83739, -12555, 1089, -51, 1},
      {-425817, 607923, -315505, 83467, -12547, 1089, -51, 1}};

MatrixRank[A]

```

```

Solve[Array[n, 24].{{-587601, 690339, -331089, 84763, -12587, 1089, -51, 1},
{-548793, 671411, -327665, 84491, -12579, 1089, -51, 1},
{-559881, 675235, -328097, 84507, -12579, 1089, -51, 1},
{-557865, 674723, -328065, 84507, -12579, 1089, -51, 1},
{-566937, 678035, -328465, 84523, -12579, 1089, -51, 1},
{-521073, 656307, -324673, 84235, -12571, 1089, -51, 1},
{-519057, 655795, -324641, 84235, -12571, 1089, -51, 1},
{-530145, 659619, -325073, 84251, -12571, 1089, -51, 1},
{-528129, 659107, -325041, 84251, -12571, 1089, -51, 1},
{-526113, 658595, -325009, 84251, -12571, 1089, -51, 1},
{-539217, 662931, -325473, 84267, -12571, 1089, -51, 1},
{-537201, 662419, -325441, 84267, -12571, 1089, -51, 1},
{-546273, 665731, -325841, 84283, -12571, 1089, -51, 1},
{-482265, 637379, -321249, 83963, -12563, 1089, -51, 1},
{-480249, 636867, -321217, 83963, -12563, 1089, -51, 1},
{-489321, 640179, -321617, 83979, -12563, 1089, -51, 1},
{-487305, 639667, -321585, 83979, -12563, 1089, -51, 1},
{-498393, 643491, -322017, 83995, -12563, 1089, -51, 1},
{-496377, 642979, -321985, 83995, -12563, 1089, -51, 1},
{-505449, 646291, -322385, 84011, -12563, 1089, -51, 1},
{-457569, 624051, -318561, 83723, -12555, 1089, -51, 1},
{-455553, 623539, -318529, 83723, -12555, 1089, -51, 1},
{-464625, 626851, -318929, 83739, -12555, 1089, -51, 1},
{-425817, 607923, -315505, 83467, -12547, 1089, -51, 1}} ==
{-23056145, 27727451, -13457769, 3465043, -515739,
44649, -2091, 41}, Array[n, 24]]
{ }

```

```
Array[m, 8].Transpose[A]
```

```
{-587601 m[1] + 690339 m[2] - 331089 m[3] + 84763 m[4] -
12587 m[5] + 1089 m[6] - 51 m[7] + m[8], -548793 m[1] + 671411 m[2] -
327665 m[3] + 84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-559881 m[1] + 675235 m[2] - 328097 m[3] + 84507 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8], -557865 m[1] + 674723 m[2] -
328065 m[3] + 84507 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8],
-566937 m[1] + 678035 m[2] - 328465 m[3] + 84523 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8], -521073 m[1] + 656307 m[2] -
324673 m[3] + 84235 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-519057 m[1] + 655795 m[2] - 324641 m[3] + 84235 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -530145 m[1] + 659619 m[2] -
325073 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-528129 m[1] + 659107 m[2] - 325041 m[3] + 84251 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -526113 m[1] + 658595 m[2] -
325009 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-539217 m[1] + 662931 m[2] - 325473 m[3] + 84267 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -537201 m[1] + 662419 m[2] -
325441 m[3] + 84267 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8],
-546273 m[1] + 665731 m[2] - 325841 m[3] + 84283 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8], -482265 m[1] + 637379 m[2] -
321249 m[3] + 83963 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-480249 m[1] + 636867 m[2] - 321217 m[3] + 83963 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8], -489321 m[1] + 640179 m[2] -
321617 m[3] + 83979 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-487305 m[1] + 639667 m[2] - 321585 m[3] + 83979 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8], -498393 m[1] + 643491 m[2] -
322017 m[3] + 83995 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-496377 m[1] + 642979 m[2] - 321985 m[3] + 83995 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8], -505449 m[1] + 646291 m[2] -
322385 m[3] + 84011 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8],
-457569 m[1] + 624051 m[2] - 318561 m[3] + 83723 m[4] - 12555 m[5] +
1089 m[6] - 51 m[7] + m[8], -455553 m[1] + 623539 m[2] -
318529 m[3] + 83723 m[4] - 12555 m[5] + 1089 m[6] - 51 m[7] + m[8],
-464625 m[1] + 626851 m[2] - 318929 m[3] + 83739 m[4] - 12555 m[5] +
1089 m[6] - 51 m[7] + m[8], -425817 m[1] + 607923 m[2] -
315505 m[3] + 83467 m[4] - 12547 m[5] + 1089 m[6] - 51 m[7] + m[8]}
```

```
Array[m, 8].
```

```
{-23056145, 27727451, -13457769, 3465043, -515739, 44649, -2091, 41}
-23056145 m[1] + 27727451 m[2] - 13457769 m[3] +
3465043 m[4] - 515739 m[5] + 44649 m[6] - 2091 m[7] + 41 m[8]
```

```

FindInstance[
-587601 m[1] + 690339 m[2] - 331089 m[3] + 84763 m[4] - 12587 m[5] + 1089 m[6] -
51 m[7] + m[8] ≥ 0 && -548793 m[1] + 671411 m[2] - 327665 m[3] +
84491 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-559881 m[1] + 675235 m[2] - 328097 m[3] + 84507 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -557865 m[1] + 674723 m[2] -
328065 m[3] + 84507 m[4] - 12579 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-566937 m[1] + 678035 m[2] - 328465 m[3] + 84523 m[4] - 12579 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -521073 m[1] + 656307 m[2] -
324673 m[3] + 84235 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-519057 m[1] + 655795 m[2] - 324641 m[3] + 84235 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -530145 m[1] + 659619 m[2] -
325073 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-528129 m[1] + 659107 m[2] - 325041 m[3] + 84251 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -526113 m[1] + 658595 m[2] -
325009 m[3] + 84251 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-539217 m[1] + 662931 m[2] - 325473 m[3] + 84267 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -537201 m[1] + 662419 m[2] -
325441 m[3] + 84267 m[4] - 12571 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-546273 m[1] + 665731 m[2] - 325841 m[3] + 84283 m[4] - 12571 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -482265 m[1] + 637379 m[2] -
321249 m[3] + 83963 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-480249 m[1] + 636867 m[2] - 321217 m[3] + 83963 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -489321 m[1] + 640179 m[2] -
321617 m[3] + 83979 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-487305 m[1] + 639667 m[2] - 321585 m[3] + 83979 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -498393 m[1] + 643491 m[2] -
322017 m[3] + 83995 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-496377 m[1] + 642979 m[2] - 321985 m[3] + 83995 m[4] - 12563 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -505449 m[1] + 646291 m[2] -
322385 m[3] + 84011 m[4] - 12563 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-457569 m[1] + 624051 m[2] - 318561 m[3] + 83723 m[4] - 12555 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -455553 m[1] + 623539 m[2] -
318529 m[3] + 83723 m[4] - 12555 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-464625 m[1] + 626851 m[2] - 318929 m[3] + 83739 m[4] - 12555 m[5] +
1089 m[6] - 51 m[7] + m[8] ≥ 0 && -425817 m[1] + 607923 m[2] -
315505 m[3] + 83467 m[4] - 12547 m[5] + 1089 m[6] - 51 m[7] + m[8] ≥ 0 &&
-23056145 m[1] + 27727451 m[2] - 13457769 m[3] + 3465043 m[4] -
515739 m[5] + 44649 m[6] - 2091 m[7] + 41 m[8] < 0, Array[m, 8], Integers]
{{m[1] → 0, m[2] → 152389, m[3] → 2438199, m[4] → 29410745,
m[5] → 316965325, m[6] → 0, m[7] → 0, m[8] → 2198761371082} }

Array[m, 8] /. {m[1] → 0, m[2] → 152389, m[3] → 2438199, m[4] → 29410745,
m[5] → 316965325, m[6] → 0, m[7] → 0, m[8] → 2198761371082}

{0, 152389, 2438199, 29410745, 316965325, 0, 0, 2198761371082}

```

```

GCD[0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082]
1

{0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082}.
{-23 056 145, 27 727 451, -13 457 769, 3 465 043, -515 739, 44 649, -2091, 41}
-27 836 370

{0, 152 389, 2 438 199, 29 410 745, 316 965 325, 0, 0, 2 198 761 371 082}.
Transpose[{{{-587 601, 690 339, -331 089, 84 763, -12 587, 1089, -51, 1},
{-548 793, 671 411, -327 665, 84 491, -12 579, 1089, -51, 1},
{-559 881, 675 235, -328 097, 84 507, -12 579, 1089, -51, 1},
{-557 865, 674 723, -328 065, 84 507, -12 579, 1089, -51, 1},
{-566 937, 678 035, -328 465, 84 523, -12 579, 1089, -51, 1},
{-521 073, 656 307, -324 673, 84 235, -12 571, 1089, -51, 1},
{-519 057, 655 795, -324 641, 84 235, -12 571, 1089, -51, 1},
{-530 145, 659 619, -325 073, 84 251, -12 571, 1089, -51, 1},
{-528 129, 659 107, -325 041, 84 251, -12 571, 1089, -51, 1},
{-526 113, 658 595, -325 009, 84 251, -12 571, 1089, -51, 1},
{-539 217, 662 931, -325 473, 84 267, -12 571, 1089, -51, 1},
{-537 201, 662 419, -325 441, 84 267, -12 571, 1089, -51, 1},
{-546 273, 665 731, -325 841, 84 283, -12 571, 1089, -51, 1},
{-482 265, 637 379, -321 249, 83 963, -12 563, 1089, -51, 1},
{-480 249, 636 867, -321 217, 83 963, -12 563, 1089, -51, 1},
{-489 321, 640 179, -321 617, 83 979, -12 563, 1089, -51, 1},
{-487 305, 639 667, -321 585, 83 979, -12 563, 1089, -51, 1},
{-498 393, 643 491, -322 017, 83 995, -12 563, 1089, -51, 1},
{-496 377, 642 979, -321 985, 83 995, -12 563, 1089, -51, 1},
{-505 449, 646 291, -322 385, 84 011, -12 563, 1089, -51, 1},
{-457 569, 624 051, -318 561, 83 723, -12 555, 1089, -51, 1},
{-455 553, 623 539, -318 529, 83 723, -12 555, 1089, -51, 1},
{-464 625, 626 851, -318 929, 83 739, -12 555, 1089, -51, 1},
{-425 817, 607 923, -315 505, 83 467, -12 547, 1089, -51, 1}}]

{1 004 902, 979 246, 984 734, 983 934, 988 622, 959 078, 958 278, 963 766,
962 966, 962 166, 968 454, 967 654, 972 342, 933 422, 932 622, 937 310,
936 510, 941 998, 941 198, 945 886, 915 542, 914 742, 919 430, 893 774}

```

```

poly14 = listmod128[[14]]
(-9 + x)^4 (-7 + x)^6 (5 + x)^25 (59 - 16 x + x^2)^2 (52 - 15 x + x^2)

p14 = poly14 / minipoly[poly14] // Factor
(-9 + x)^3 (-7 + x)^5 (5 + x)^24 (59 - 16 x + x^2)

```

```

feasiblesubcharpolylist $\left[ \left( -9 + x \right)^4 \left( -7 + x \right)^6 \left( 5 + x \right)^{25} \left( 59 - 16 x + x^2 \right)^2 \left( 52 - 15 x + x^2 \right) \right]$ 
 $\left\{ \left( -9 + x \right) \left( 59 - 16 x + x^2 \right) \left( -127 + 87 x - 17 x^2 + x^3 \right),$ 
 $\left( -9 + x \right) \left( -7 + x \right) \left( 59 - 16 x + x^2 \right) \left( 17 - 10 x + x^2 \right) \right\}$ 

CoefficientList $\left[ \left\{ \left( -9 + x \right) \left( 59 - 16 x + x^2 \right) \left( -127 + 87 x - 17 x^2 + x^3 \right),$ 
 $\left( -9 + x \right) \left( -7 + x \right) \left( 59 - 16 x + x^2 \right) \left( 17 - 10 x + x^2 \right) \right\}, x \right]$ 
 $\left\{ \left\{ 67437, -71978, 29863, -6284, 715, -42, 1 \right\},$ 
 $\left\{ 63189, -70354, 29663, -6276, 715, -42, 1 \right\} \right\}$ 

CoefficientList $\left[ D[poly14, x] / p14 // Factor, x \right]$ 
 $\left\{ 2771285, -2952858, 1224511, -257644, 29315, -1722, 41 \right\}$ 

A =  $\left\{ \left\{ 67437, -71978, 29863, -6284, 715, -42, 1 \right\},$ 
 $\left\{ 63189, -70354, 29663, -6276, 715, -42, 1 \right\} \right\};$ 

Solve $\left[ Array[n, 2].\left\{ \left\{ 67437, -71978, 29863, -6284, 715, -42, 1 \right\},$ 
 $\left\{ 63189, -70354, 29663, -6276, 715, -42, 1 \right\} \right\} =$ 
 $\left\{ 2771285, -2952858, 1224511, -257644, 29315, -1722, 41 \right\}, Array[n, 2] \right]$ 
 $\left\{ \right\}$ 

Array[m, 7].Transpose[A]
 $\left\{ 67437 m[1] - 71978 m[2] + 29863 m[3] - 6284 m[4] + 715 m[5] - 42 m[6] + m[7],$ 
 $63189 m[1] - 70354 m[2] + 29663 m[3] - 6276 m[4] + 715 m[5] - 42 m[6] + m[7] \right\}$ 

Array[m, 7].{2771285, -2952858, 1224511, -257644, 29315, -1722, 41}
 $2771285 m[1] - 2952858 m[2] + 1224511 m[3] -$ 
 $257644 m[4] + 29315 m[5] - 1722 m[6] + 41 m[7]$ 

FindInstance[  

 $67437 m[1] - 71978 m[2] + 29863 m[3] - 6284 m[4] + 715 m[5] - 42 m[6] + m[7] \geq 0 \&&$ 
 $63189 m[1] - 70354 m[2] + 29663 m[3] - 6276 m[4] + 715 m[5] - 42 m[6] + m[7] \geq 0 \&&$ 
 $2771285 m[1] - 2952858 m[2] + 1224511 m[3] - 257644 m[4] +$ 
 $29315 m[5] - 1722 m[6] + 41 m[7] < 0, Array[m, 7], \text{Integers}]$ 
 $\left\{ \left\{ m[1] \rightarrow 0, m[2] \rightarrow 3, m[3] \rightarrow -56453,$ 
 $m[4] \rightarrow -889825, m[5] \rightarrow 0, m[6] \rightarrow 0, m[7] \rightarrow -3905464984 \right\} \right\}$ 

Array[m, 7] /. {m[1] → 0, m[2] → 3, m[3] → -56453,
 $m[4] \rightarrow -889825, m[5] \rightarrow 0, m[6] \rightarrow 0, m[7] \rightarrow -3905464984}$ 
 $\left\{ 0, 3, -56453, -889825, 0, 0, -3905464984 \right\}$ 

GCD[0, 3, -56453, -889825, 0, 0, -3905464984]
1

 $\left\{ 0, 3, -56453, -889825, 0, 0, -3905464984 \right\}.$ 
 $\left\{ 2771285, -2952858, 1224511, -257644, 29315, -1722, 41 \right\}$ 
-2170101

```

```
{0, 3, -56453, -889825, 0, 0, -3905464984}.

Transpose[{{67437, -71978, 29863, -6284, 715, -42, 1},
{63189, -70354, 29663, -6276, 715, -42, 1}}]

{123443, 4300315}
```

```
poly15 = listmod128[[15]]

(-9 + x)^5 (-8 + x) (-7 + x)^7 (5 + x)^25 (-349 + 163 x - 23 x^2 + x^3)

p15 = poly15 / minipoly[poly15] // Factor

(-9 + x)^4 (-7 + x)^6 (5 + x)^24

feasiblesubcharpolylist[(-9 + x)^5 (-8 + x) (-7 + x)^7 (5 + x)^25 (-349 + 163 x - 23 x^2 + x^3) ]

{ (-9 + x) (-7 + x) (-3 + x) (-349 + 163 x - 23 x^2 + x^3),
  (-7 + x) (-8807 + 8445 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (991 - 830 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (1007 - 830 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x) (-8159 + 8301 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2),
  (-7 + x) (-7655 + 8173 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x)^2 (929 - 996 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (751 - 798 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x)^2 (-89 + 99 x - 19 x^2 + x^3) }

{ (-9 + x) (-7 + x) (-3 + x) (-349 + 163 x - 23 x^2 + x^3),
  (-7 + x) (-8807 + 8445 x - 2918 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (991 - 830 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (1007 - 830 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x) (-8159 + 8301 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-11 + x) (-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2),
  (-7 + x) (-7655 + 8173 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5),
  (-9 + x) (-7 + x) (879 - 814 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x) (807 - 806 x + 232 x^2 - 26 x^3 + x^4),
  (-7 + x)^2 (929 - 996 x + 270 x^2 - 28 x^3 + x^4),
  (-9 + x) (-7 + x) (751 - 798 x + 232 x^2 - 26 x^3 + x^4),
  (-9 + x) (-7 + x)^2 (-89 + 99 x - 19 x^2 + x^3) } // Length
```

```

CoefficientList[{\{ (-9 + x) (-7 + x) (-3 + x) (-349 + 163 x - 23 x2 + x3) ,
(-7 + x) (-8807 + 8445 x - 2918 x2 + 466 x3 - 35 x4 + x5) ,
(-9 + x) (-7 + x) (991 - 830 x + 232 x2 - 26 x3 + x4) ,
(-9 + x) (-7 + x) (1007 - 830 x + 232 x2 - 26 x3 + x4) ,
(-7 + x) (-8159 + 8301 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
(-11 + x) (-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x2) ,
(-7 + x) (-7655 + 8173 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
(-9 + x) (-7 + x) (879 - 814 x + 232 x2 - 26 x3 + x4) ,
(-9 + x) (-7 + x) (807 - 806 x + 232 x2 - 26 x3 + x4) ,
(-7 + x)2 (929 - 996 x + 270 x2 - 28 x3 + x4) ,
(-9 + x) (-7 + x) (751 - 798 x + 232 x2 - 26 x3 + x4) ,
(-9 + x) (-7 + x)2 (-89 + 99 x - 19 x2 + x3) \}, x]

```

```

{{65 961, -69 546, 29 071, -6188, 711, -42, 1},
{61 649, -67 922, 28 871, -6180, 711, -42, 1},
{62 433, -68 146, 28 887, -6180, 711, -42, 1},
{63 441, -68 402, 28 903, -6180, 711, -42, 1},
{57 113, -66 266, 28 671, -6172, 711, -42, 1},
{58 905, -66 746, 28 703, -6172, 711, -42, 1},
{53 585, -64 866, 28 487, -6164, 711, -42, 1},
{55 377, -65 346, 28 519, -6164, 711, -42, 1},
{50 841, -63 690, 28 319, -6156, 711, -42, 1},
{45 521, -61 810, 28 103, -6148, 711, -42, 1},
{47 313, -62 290, 28 135, -6148, 711, -42, 1},
{39 249, -59 234, 27 751, -6132, 711, -42, 1}}

```

```

A = {{65 961, -69 546, 29 071, -6188, 711, -42, 1},
{61 649, -67 922, 28 871, -6180, 711, -42, 1},
{62 433, -68 146, 28 887, -6180, 711, -42, 1}, {63 441, -68 402, 28 903,
-6180, 711, -42, 1}, {57 113, -66 266, 28 671, -6172, 711, -42, 1},
{58 905, -66 746, 28 703, -6172, 711, -42, 1}, {53 585, -64 866, 28 487,
-6164, 711, -42, 1}, {55 377, -65 346, 28 519, -6164, 711, -42, 1},
{50 841, -63 690, 28 319, -6156, 711, -42, 1}, {45 521, -61 810, 28 103,
-6148, 711, -42, 1}, {47 313, -62 290, 28 135, -6148, 711, -42, 1},
{39 249, -59 234, 27 751, -6132, 711, -42, 1}};

```

```
MatrixRank[A]
```

4

```

CoefficientList[D[poly15, x] / p15 // Factor, x]
{2 508 625, -2 769 690, 1 180 903, -253 228, 29 151, -1722, 41}

```

```

Solve[
  Array[n, 12].{{65 961, -69 546, 29 071, -6188, 711, -42, 1}, {61 649, -67 922, 28 871,
    -6180, 711, -42, 1}, {62 433, -68 146, 28 887, -6180, 711, -42, 1},
    {63 441, -68 402, 28 903, -6180, 711, -42, 1}, {57 113, -66 266, 28 671,
    -6172, 711, -42, 1}, {58 905, -66 746, 28 703, -6172, 711, -42, 1},
    {53 585, -64 866, 28 487, -6164, 711, -42, 1}, {55 377, -65 346, 28 519,
    -6164, 711, -42, 1}, {50 841, -63 690, 28 319, -6156, 711, -42, 1},
    {45 521, -61 810, 28 103, -6148, 711, -42, 1}, {47 313, -62 290, 28 135,
    -6148, 711, -42, 1}, {39 249, -59 234, 27 751, -6132, 711, -42, 1}} =
  {2 508 625, -2 769 690, 1 180 903, -253 228, 29 151, -1722, 41}, Array[n, 12]]
  }

Array[m, 7].Transpose[A]
{65 961 m[1] - 69 546 m[2] + 29 071 m[3] - 6188 m[4] + 711 m[5] - 42 m[6] + m[7],  

 61 649 m[1] - 67 922 m[2] + 28 871 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7],  

 62 433 m[1] - 68 146 m[2] + 28 887 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7],  

 63 441 m[1] - 68 402 m[2] + 28 903 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7],  

 57 113 m[1] - 66 266 m[2] + 28 671 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],  

 58 905 m[1] - 66 746 m[2] + 28 703 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],  

 53 585 m[1] - 64 866 m[2] + 28 487 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],  

 55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],  

 50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],  

 45 521 m[1] - 61 810 m[2] + 28 103 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],  

 47 313 m[1] - 62 290 m[2] + 28 135 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],  

 39 249 m[1] - 59 234 m[2] + 27 751 m[3] - 6132 m[4] + 711 m[5] - 42 m[6] + m[7]}

Array[m, 7].{2 508 625, -2 769 690, 1 180 903, -253 228, 29 151, -1722, 41}
2 508 625 m[1] - 2 769 690 m[2] + 1 180 903 m[3] -  

 253 228 m[4] + 29 151 m[5] - 1722 m[6] + 41 m[7]

FindInstance[
  65 961 m[1] - 69 546 m[2] + 29 071 m[3] - 6188 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  61 649 m[1] - 67 922 m[2] + 28 871 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  62 433 m[1] - 68 146 m[2] + 28 887 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  63 441 m[1] - 68 402 m[2] + 28 903 m[3] - 6180 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  57 113 m[1] - 66 266 m[2] + 28 671 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  58 905 m[1] - 66 746 m[2] + 28 703 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  53 585 m[1] - 64 866 m[2] + 28 487 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  45 521 m[1] - 61 810 m[2] + 28 103 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  47 313 m[1] - 62 290 m[2] + 28 135 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  39 249 m[1] - 59 234 m[2] + 27 751 m[3] - 6132 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&  

  2 508 625 m[1] - 2 769 690 m[2] + 1 180 903 m[3] - 253 228 m[4] +  

  29 151 m[5] - 1722 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
{m[1] → -8404, m[2] → -28 265, m[3] → -48 443, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}

```

```

Array[m, 7] /.
{m[1] → -8404, m[2] → -28 265, m[3] → -48 443, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}
{-8404, -28 265, -48 443, 0, 0, 0, 0}

GCD[-8404, -28 265, -48 443, 0, 0, 0, 0]
1

{-8404, -28 265, -48 443, 0, 0, 0, 0}.

{2 508 625, -2 769 690, 1 180 903, -253 228, 29 151, -1722, 41}
-3 680 679

{-8404, -28 265, -48 443, 0, 0, 0, 0}.

Transpose[{{65 961, -69 546, 29 071, -6188, 711, -42, 1}, {61 649, -67 922,
28 871, -6180, 711, -42, 1}, {62 433, -68 146, 28 887, -6180, 711, -42, 1},
{63 441, -68 402, 28 903, -6180, 711, -42, 1}, {57 113, -66 266, 28 671,
-6172, 711, -42, 1}, {58 905, -66 746, 28 703, -6172, 711, -42, 1},
{53 585, -64 866, 28 487, -6164, 711, -42, 1}, {55 377, -65 346, 28 519,
-6164, 711, -42, 1}, {50 841, -63 690, 28 319, -6156, 711, -42, 1},
{45 521, -61 810, 28 103, -6148, 711, -42, 1}, {47 313, -62 290, 28 135,
-6148, 711, -42, 1}, {39 249, -59 234, 27 751, -6132, 711, -42, 1}]]

{3 094 993, 3 119 281, 2 086 817, 76 337, 4 121 585,
1 078 641, 3 113 409, 70 465, 1 072 769, 3 107 537, 64 593, 58 721}

```

```

poly16 = listmod128[[16]]
(-11 + x) (-9 + x)6 (-8 + x) (-7 + x)6 (-5 + x)2 (5 + x)25

p16 = poly16 / minipoly[poly16] // Factor
(-9 + x)5 (-7 + x)5 (-5 + x) (5 + x)24

feasiblesubcharpolylist[(-11 + x) (-9 + x)6 (-8 + x) (-7 + x)6 (-5 + x)2 (5 + x)25]
{(-11 + x) (-9 + x) (-7 + x) (-5 + x) (-3 + x),
(-11 + x) (-7 + x) (-127 + 87 x - 17 x2 + x3), -9923 + 9001 x - 3002 x2 + 470 x3 - 35 x4 + x5,
(-9 + x) (-7 + x) (-157 + 103 x - 19 x2 + x3), (-9 + x) (-5 + x) (-223 + 131 x - 21 x2 + x3),
(-7 + x) (1325 - 1076 x + 274 x2 - 28 x3 + x4), (-9 + x) (-7 + x) (-149 + 103 x - 19 x2 + x3),
(-9 + x) (1059 - 870 x + 236 x2 - 26 x3 + x4), (-7 + x)2 (-179 + 127 x - 21 x2 + x3),
(-9 + x) (-7 + x) (-141 + 103 x - 19 x2 + x3), (-9 + x) (59 - 16 x + x2) (17 - 10 x + x2),
(-9 + x) (-7 + x)2 (19 - 12 x + x2), (-9 + x) (-7 + x) (-125 + 103 x - 19 x2 + x3)}

```

$$\left\{ (-11+x) (-9+x) (-7+x) (-5+x) (-3+x), (-11+x) (-7+x) (-127+87x-17x^2+x^3), -9923+9001x-3002x^2+470x^3-35x^4+x^5, (-9+x) (-7+x) (-157+103x-19x^2+x^3), (-9+x) (-5+x) (-223+131x-21x^2+x^3), (-7+x) (1325-1076x+274x^2-28x^3+x^4), (-9+x) (-7+x) (-149+103x-19x^2+x^3), (-9+x) (1059-870x+236x^2-26x^3+x^4), (-7+x)^2 (-179+127x-21x^2+x^3), (-9+x) (-7+x) (-141+103x-19x^2+x^3), (-9+x) (59-16x+x^2) (17-10x+x^2), (-9+x) (-7+x)^2 (19-12x+x^2), (-9+x) (-7+x) (-125+103x-19x^2+x^3) \right\} // \text{Length}$$

13

```
CoefficientList[{\{(-11+x) (-9+x) (-7+x) (-5+x) (-3+x),
(-11+x) (-7+x) (-127+87x-17x^2+x^3), -9923+9001x-3002x^2+470x^3-35x^4+x^5,
(-9+x) (-7+x) (-157+103x-19x^2+x^3), (-9+x) (-5+x) (-223+131x-21x^2+x^3),
(-7+x) (1325-1076x+274x^2-28x^3+x^4), (-9+x) (-7+x) (-149+103x-19x^2+x^3),
(-9+x) (1059-870x+236x^2-26x^3+x^4), (-7+x)^2 (-179+127x-21x^2+x^3),
(-9+x) (-7+x) (-141+103x-19x^2+x^3), (-9+x) (59-16x+x^2) (17-10x+x^2),
(-9+x) (-7+x)^2 (19-12x+x^2), (-9+x) (-7+x) (-125+103x-19x^2+x^3)\}, x],
{\{-10395, 9129, -3010, 470, -35, 1\},
{-9779, 8985, -3002, 470, -35, 1\}, {-9923, 9001, -3002, 470, -35, 1\},
{-9891, 9001, -3002, 470, -35, 1\}, {-10035, 9017, -3002, 470, -35, 1\},
{-9275, 8857, -2994, 470, -35, 1\}, {-9387, 8873, -2994, 470, -35, 1\},
{-9531, 8889, -2994, 470, -35, 1\}, {-8771, 8729, -2986, 470, -35, 1\},
{-8883, 8745, -2986, 470, -35, 1\}, {-9027, 8761, -2986, 470, -35, 1\},
{-8379, 8617, -2978, 470, -35, 1\}, {-7875, 8489, -2970, 470, -35, 1\}}
```

```
A = {\{-10395, 9129, -3010, 470, -35, 1\},
{-9779, 8985, -3002, 470, -35, 1\}, {-9923, 9001, -3002, 470, -35, 1\},
{-9891, 9001, -3002, 470, -35, 1\}, {-10035, 9017, -3002, 470, -35, 1\},
{-9275, 8857, -2994, 470, -35, 1\}, {-9387, 8873, -2994, 470, -35, 1\},
{-9531, 8889, -2994, 470, -35, 1\}, {-8771, 8729, -2986, 470, -35, 1\},
{-8883, 8745, -2986, 470, -35, 1\}, {-9027, 8761, -2986, 470, -35, 1\},
{-8379, 8617, -2978, 470, -35, 1\}, {-7875, 8489, -2970, 470, -35, 1\}};
```

```
CoefficientList[D[poly16, x] / p16 // Factor, x]
```

```
{-396435, 366513, -122930, 19270, -1435, 41}
```

```

Solve[Array[n, 13].{{-10395, 9129, -3010, 470, -35, 1},
{-9779, 8985, -3002, 470, -35, 1}, {-9923, 9001, -3002, 470, -35, 1},
{-9891, 9001, -3002, 470, -35, 1}, {-10035, 9017, -3002, 470, -35, 1},
{-9275, 8857, -2994, 470, -35, 1}, {-9387, 8873, -2994, 470, -35, 1},
{-9531, 8889, -2994, 470, -35, 1}, {-8771, 8729, -2986, 470, -35, 1},
{-8883, 8745, -2986, 470, -35, 1}, {-9027, 8761, -2986, 470, -35, 1},
{-8379, 8617, -2978, 470, -35, 1}, {-7875, 8489, -2970, 470, -35, 1}} ==
{-396435, 366513, -122930, 19270, -1435, 41}, Array[n, 13]]

Solve::svars : Equations may not give solutions for all "solve" variables. >>>
{{n[9] → 42 - n[2] - n[3] - n[6], n[11] → 36 - n[3] - n[5] - n[8],
n[12] → -11 - 5 n[1] - 2 n[2] - 4 n[4] - 2 n[5] - n[6] - 3 n[7] - n[8] - 2 n[10],
n[13] → -26 + 4 n[1] + 2 n[2] + n[3] + 3 n[4] + 2 n[5] + n[6] + 2 n[7] + n[8] + n[10]}}

FindInstance[-n[9] + 42 - n[2] - n[3] - n[6] == 0 &&
-n[11] + 36 - n[3] - n[5] - n[8] == 0 &&
-n[12] - 11 - 5 n[1] - 2 n[2] - 4 n[4] - 2 n[5] - n[6] - 3 n[7] - n[8] - 2 n[10] == 0 &&
-n[13] - 26 + 4 n[1] + 2 n[2] + n[3] + 3 n[4] + 2 n[5] + n[6] + 2 n[7] + n[8] + n[10] == 0 &&
n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 &&
n[6] ≥ 0 && n[7] ≥ 0 && n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 &&
n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0, Array[n, 13], Integers]
{ }

Array[m, 6].Transpose[A]

{-10395 m[1] + 9129 m[2] - 3010 m[3] + 470 m[4] - 35 m[5] + m[6],
-9779 m[1] + 8985 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
-9923 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
-9891 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
-10035 m[1] + 9017 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6],
-9275 m[1] + 8857 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6],
-9387 m[1] + 8873 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6],
-9531 m[1] + 8889 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6],
-8771 m[1] + 8729 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6],
-8883 m[1] + 8745 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6],
-9027 m[1] + 8761 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6],
-8379 m[1] + 8617 m[2] - 2978 m[3] + 470 m[4] - 35 m[5] + m[6],
-7875 m[1] + 8489 m[2] - 2970 m[3] + 470 m[4] - 35 m[5] + m[6]}

Array[m, 6].{-396435, 366513, -122930, 19270, -1435, 41}
-396435 m[1] + 366513 m[2] - 122930 m[3] + 19270 m[4] - 1435 m[5] + 41 m[6]

```

```

FindInstance[-10395 m[1] + 9129 m[2] - 3010 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9779 m[1] + 8985 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9923 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9891 m[1] + 9001 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-10035 m[1] + 9017 m[2] - 3002 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9275 m[1] + 8857 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9387 m[1] + 8873 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9531 m[1] + 8889 m[2] - 2994 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8771 m[1] + 8729 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8883 m[1] + 8745 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-9027 m[1] + 8761 m[2] - 2986 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-8379 m[1] + 8617 m[2] - 2978 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-7875 m[1] + 8489 m[2] - 2970 m[3] + 470 m[4] - 35 m[5] + m[6] ≥ 0 &&
-396435 m[1] + 366513 m[2] - 122930 m[3] + 19270 m[4] - 1435 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 10302, m[2] → 82421, m[3] → 659363, m[4] → 0, m[5] → 0, m[6] → 1339775890} }

Array[m, 6] /.
{m[1] → 10302, m[2] → 82421, m[3] → 659363, m[4] → 0, m[5] → 0, m[6] → 1339775890}
{10302, 82421, 659363, 0, 0, 1339775890}

GCD[10302, 82421, 659363, 0, 0, 1339775890]
1

{10302, 82421, 659363, 0, 0, 1339775890}.
{-396435, 366513, -122930, 19270, -1435, 41}
-387497

{10302, 82421, 659363, 0, 0, 1339775890}.
Transpose[{{{-10395, 9129, -3010, 470, -35, 1},
{-9779, 8985, -3002, 470, -35, 1}, {-9923, 9001, -3002, 470, -35, 1},
{-9891, 9001, -3002, 470, -35, 1}, {-10035, 9017, -3002, 470, -35, 1},
{-9275, 8857, -2994, 470, -35, 1}, {-9387, 8873, -2994, 470, -35, 1},
{-9531, 8889, -2994, 470, -35, 1}, {-8771, 8729, -2986, 470, -35, 1},
{-8883, 8745, -2986, 470, -35, 1}, {-9027, 8761, -2986, 470, -35, 1},
{-8379, 8617, -2978, 470, -35, 1}, {-7875, 8489, -2970, 470, -35, 1}}]

{425279, 177591, 12839, 342503, 177751, 94815,
259727, 94975, 12039, 176951, 12199, 94175, 11399}

```

```

poly17 = listmod128[[17]]

$$(-9 + x)^5 (-7 + x)^7 (5 + x)^{25} (2824 - 1653 x + 347 x^2 - 31 x^3 + x^4)$$


```

```

p17 = poly17 / minipoly[poly17] // Factor

$$(-9 + x)^4 (-7 + x)^6 (5 + x)^{24}$$


feasiblesubcharpolylist[  $(-9 + x)^5 (-7 + x)^7 (5 + x)^{25} (2824 - 1653 x + 347 x^2 - 31 x^3 + x^4)$  ]

$$\left\{ (-7 + x) \left( -8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5 \right), \right.$$


$$59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6,$$


$$(-9 + x) \left( -6737 + 6721 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$


$$(-7 + x) \left( 23 - 12 x + x^2 \right) \left( -377 + 167 x - 23 x^2 + x^3 \right),$$


$$(-9 + x) \left( -6817 + 6737 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$


$$(-7 + x)^2 \left( 1105 - 1012 x + 270 x^2 - 28 x^3 + x^4 \right),$$


$$(-9 + x) (-7 + x) \left( 879 - 814 x + 232 x^2 - 26 x^3 + x^4 \right),$$


$$(-7 + x) \left( -7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5 \right),$$


$$(-9 + x) (-7 + x) (-5 + x) \left( -179 + 127 x - 21 x^2 + x^3 \right),$$


$$(-9 + x) \left( -6233 + 6593 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$


$$(-7 + x) \left( -8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5 \right),$$


$$(-9 + x) (-7 + x) \left( 911 - 814 x + 232 x^2 - 26 x^3 + x^4 \right),$$


$$(-9 + x)^2 (-5 + x) \left( -141 + 103 x - 19 x^2 + x^3 \right), (-9 + x)^2 (-7 + x) \left( -103 + 79 x - 17 x^2 + x^3 \right),$$


$$(-9 + x) \left( -6457 + 6625 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$


$$(-9 + x) (-7 + x) \left( 807 - 806 x + 232 x^2 - 26 x^3 + x^4 \right),$$


$$(-9 + x) (-7 + x) \left( 823 - 806 x + 232 x^2 - 26 x^3 + x^4 \right),$$


$$(-9 + x) (-7 + x) \left( 839 - 806 x + 232 x^2 - 26 x^3 + x^4 \right),$$


$$(-9 + x)^2 \left( 59 - 16 x + x^2 \right) \left( 11 - 8 x + x^2 \right), (-9 + x)^2 (-7 + x) (-5 + x) \left( 19 - 12 x + x^2 \right),$$


$$(-9 + x) (-7 + x) \left( 59 - 16 x + x^2 \right) \left( 13 - 10 x + x^2 \right),$$


$$\left. (-9 + x)^2 (-7 + x) \left( -87 + 79 x - 17 x^2 + x^3 \right), (-9 + x)^2 (-7 + x) \left( -79 + 79 x - 17 x^2 + x^3 \right) \right\}$$


```

$$\left\{ (-7 + x) \left( -8527 + 8349 x - 2910 x^2 + 466 x^3 - 35 x^4 + x^5 \right), \right.$$

$$59401 - 66938 x + 28719 x^2 - 6172 x^3 + 711 x^4 - 42 x^5 + x^6,$$

$$(-9 + x) \left( -6737 + 6721 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$

$$(-7 + x) \left( 23 - 12 x + x^2 \right) \left( -377 + 167 x - 23 x^2 + x^3 \right),$$

$$(-9 + x) \left( -6817 + 6737 x - 2446 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$

$$(-7 + x)^2 \left( 1105 - 1012 x + 270 x^2 - 28 x^3 + x^4 \right),$$

$$(-9 + x) (-7 + x) \left( 879 - 814 x + 232 x^2 - 26 x^3 + x^4 \right),$$

$$(-7 + x) \left( -7879 + 8205 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5 \right),$$

$$(-9 + x) (-7 + x) (-5 + x) \left( -179 + 127 x - 21 x^2 + x^3 \right),$$

$$(-9 + x) \left( -6233 + 6593 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$

$$(-7 + x) \left( -8023 + 8221 x - 2902 x^2 + 466 x^3 - 35 x^4 + x^5 \right),$$

$$(-9 + x) (-7 + x) \left( 911 - 814 x + 232 x^2 - 26 x^3 + x^4 \right),$$

$$(-9 + x)^2 (-5 + x) \left( -141 + 103 x - 19 x^2 + x^3 \right), (-9 + x)^2 (-7 + x) \left( -103 + 79 x - 17 x^2 + x^3 \right),$$

$$(-9 + x) \left( -6457 + 6625 x - 2438 x^2 + 414 x^3 - 33 x^4 + x^5 \right),$$

$$(-9 + x) (-7 + x) \left( 807 - 806 x + 232 x^2 - 26 x^3 + x^4 \right),$$

$$(-9 + x) (-7 + x) \left( 823 - 806 x + 232 x^2 - 26 x^3 + x^4 \right),$$

$$(-9 + x) (-7 + x) \left( 839 - 806 x + 232 x^2 - 26 x^3 + x^4 \right),$$

$$(-9 + x)^2 \left( 59 - 16 x + x^2 \right) \left( 11 - 8 x + x^2 \right), (-9 + x)^2 (-7 + x) (-5 + x) \left( 19 - 12 x + x^2 \right),$$

$$(-9 + x) (-7 + x) \left( 59 - 16 x + x^2 \right) \left( 13 - 10 x + x^2 \right),$$

$$(-9 + x)^2 (-7 + x) \left( -87 + 79 x - 17 x^2 + x^3 \right),$$

$$\left. (-9 + x)^2 (-7 + x) \left( -79 + 79 x - 17 x^2 + x^3 \right) \right\} // \text{Length}$$

```

CoefficientList[{\{ (-7 + x) (-8527 + 8349 x - 2910 x2 + 466 x3 - 35 x4 + x5) ,
  59 401 - 66 938 x + 28 719 x2 - 6172 x3 + 711 x4 - 42 x5 + x6 ,
  (-9 + x) (-6737 + 6721 x - 2446 x2 + 414 x3 - 33 x4 + x5) ,
  (-7 + x) (23 - 12 x + x2) (-377 + 167 x - 23 x2 + x3) ,
  (-9 + x) (-6817 + 6737 x - 2446 x2 + 414 x3 - 33 x4 + x5) ,
  (-7 + x)2 (1105 - 1012 x + 270 x2 - 28 x3 + x4) ,
  (-9 + x) (-7 + x) (879 - 814 x + 232 x2 - 26 x3 + x4) ,
  (-7 + x) (-7879 + 8205 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x) (-5 + x) (-179 + 127 x - 21 x2 + x3) ,
  (-9 + x) (-6233 + 6593 x - 2438 x2 + 414 x3 - 33 x4 + x5) ,
  (-7 + x) (-8023 + 8221 x - 2902 x2 + 466 x3 - 35 x4 + x5) ,
  (-9 + x) (-7 + x) (911 - 814 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x)2 (-5 + x) (-141 + 103 x - 19 x2 + x3) , (-9 + x)2 (-7 + x) (-103 + 79 x - 17 x2 + x3) ,
  (-9 + x) (-6457 + 6625 x - 2438 x2 + 414 x3 - 33 x4 + x5) ,
  (-9 + x) (-7 + x) (807 - 806 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x) (-7 + x) (823 - 806 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x) (-7 + x) (839 - 806 x + 232 x2 - 26 x3 + x4) ,
  (-9 + x)2 (59 - 16 x + x2) (11 - 8 x + x2) , (-9 + x)2 (-7 + x) (-5 + x) (19 - 12 x + x2) ,
  (-9 + x) (-7 + x) (59 - 16 x + x2) (13 - 10 x + x2) ,
  (-9 + x)2 (-7 + x) (-87 + 79 x - 17 x2 + x3) , (-9 + x)2 (-7 + x) (-79 + 79 x - 17 x2 + x3) \}, x]
{{59 689, -66 970, 28 719, -6172, 711, -42, 1},
 {59 401, -66 938, 28 719, -6172, 711, -42, 1},
 {60 633, -67 226, 28 735, -6172, 711, -42, 1},
 {60 697, -67 226, 28 735, -6172, 711, -42, 1},
 {61 353, -67 450, 28 751, -6172, 711, -42, 1},
 {54 145, -65 058, 28 503, -6164, 711, -42, 1},
 {55 377, -65 346, 28 519, -6164, 711, -42, 1},
 {55 153, -65 314, 28 519, -6164, 711, -42, 1},
 {56 385, -65 602, 28 535, -6164, 711, -42, 1},
 {56 097, -65 570, 28 535, -6164, 711, -42, 1},
 {56 161, -65 570, 28 535, -6164, 711, -42, 1},
 {57 393, -65 858, 28 551, -6164, 711, -42, 1},
 {57 105, -65 826, 28 551, -6164, 711, -42, 1},
 {58 401, -66 114, 28 567, -6164, 711, -42, 1},
 {58 113, -66 082, 28 567, -6164, 711, -42, 1},
 {50 841, -63 690, 28 319, -6156, 711, -42, 1},
 {51 849, -63 946, 28 335, -6156, 711, -42, 1},
 {52 857, -64 202, 28 351, -6156, 711, -42, 1},
 {52 569, -64 170, 28 351, -6156, 711, -42, 1},
 {53 865, -64 458, 28 367, -6156, 711, -42, 1},
 {48 321, -62 546, 28 151, -6148, 711, -42, 1},
 {49 329, -62 802, 28 167, -6148, 711, -42, 1},
 {44 793, -61 146, 27 967, -6140, 711, -42, 1}}

```

```

A = {{59 689, -66 970, 28 719, -6172, 711, -42, 1},
      {59 401, -66 938, 28 719, -6172, 711, -42, 1},
      {60 633, -67 226, 28 735, -6172, 711, -42, 1}, {60 697, -67 226, 28 735,
      -6172, 711, -42, 1}, {61 353, -67 450, 28 751, -6172, 711, -42, 1},
      {54 145, -65 058, 28 503, -6164, 711, -42, 1}, {55 377, -65 346, 28 519,
      -6164, 711, -42, 1}, {55 153, -65 314, 28 519, -6164, 711, -42, 1},
      {56 385, -65 602, 28 535, -6164, 711, -42, 1}, {56 097, -65 570, 28 535,
      -6164, 711, -42, 1}, {56 161, -65 570, 28 535, -6164, 711, -42, 1},
      {57 393, -65 858, 28 551, -6164, 711, -42, 1}, {57 105, -65 826, 28 551,
      -6164, 711, -42, 1}, {58 401, -66 114, 28 567, -6164, 711, -42, 1},
      {58 113, -66 082, 28 567, -6164, 711, -42, 1}, {50 841, -63 690, 28 319,
      -6156, 711, -42, 1}, {51 849, -63 946, 28 335, -6156, 711, -42, 1},
      {52 857, -64 202, 28 351, -6156, 711, -42, 1}, {52 569, -64 170, 28 351,
      -6156, 711, -42, 1}, {53 865, -64 458, 28 367, -6156, 711, -42, 1},
      {48 321, -62 546, 28 151, -6148, 711, -42, 1}, {49 329, -62 802, 28 167,
      -6148, 711, -42, 1}, {44 793, -61 146, 27 967, -6140, 711, -42, 1}};

CoefficientList[D[poly17, x] / p17 // Factor, x]
{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}

Solve[
Array[n, 23].{{59 689, -66 970, 28 719, -6172, 711, -42, 1}, {59 401, -66 938, 28 719,
-6172, 711, -42, 1}, {60 633, -67 226, 28 735, -6172, 711, -42, 1},
{60 697, -67 226, 28 735, -6172, 711, -42, 1}, {61 353, -67 450, 28 751, -6172,
711, -42, 1}, {54 145, -65 058, 28 503, -6164, 711, -42, 1}, {55 377, -65 346,
28 519, -6164, 711, -42, 1}, {55 153, -65 314, 28 519, -6164, 711, -42, 1},
{56 385, -65 602, 28 535, -6164, 711, -42, 1}, {56 097, -65 570, 28 535,
-6164, 711, -42, 1}, {56 161, -65 570, 28 535, -6164, 711, -42, 1},
{57 393, -65 858, 28 551, -6164, 711, -42, 1}, {57 105, -65 826, 28 551,
-6164, 711, -42, 1}, {58 401, -66 114, 28 567, -6164, 711, -42, 1},
{58 113, -66 082, 28 567, -6164, 711, -42, 1}, {50 841, -63 690, 28 319,
-6156, 711, -42, 1}, {51 849, -63 946, 28 335, -6156, 711, -42, 1},
{52 857, -64 202, 28 351, -6156, 711, -42, 1}, {52 569, -64 170, 28 351,
-6156, 711, -42, 1}, {53 865, -64 458, 28 367, -6156, 711, -42, 1},
{48 321, -62 546, 28 151, -6148, 711, -42, 1}, {49 329, -62 802, 28 167,
-6148, 711, -42, 1}, {44 793, -61 146, 27 967, -6140, 711, -42, 1}} =
{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}, Array[n, 23]]

Solve::svrs : Equations may not give solutions for all "solve" variables. >>
{{n[11] → 35 - n[1] - n[2] - n[4] - n[6] - n[8],
n[19] → 63 - n[2] - n[3] - 2 n[5] - n[10] - n[13] - n[15],
n[21] → -23 - n[1] - n[3] + n[5] - 2 n[6] - 3 n[7] - n[8] -
2 n[9] - n[10] - n[12] + n[15] - 3 n[16] - 2 n[17] - n[18],
n[22] → -22 + n[2] - n[3] - n[4] - n[5] + 2 n[6] + n[8] - n[9] - 2 n[12] -
n[13] - 3 n[14] - 2 n[15] + n[16] - n[18] - 2 n[20],
n[23] → -12 + n[1] + 2 n[3] + n[4] + n[5] + 2 n[7] + 2 n[9] + n[10] +
2 n[12] + n[13] + 2 n[14] + n[15] + n[16] + n[17] + n[18] + n[20]}}

```

```

FindInstance[-n[11] + 35 - n[1] - n[2] - n[4] - n[6] - n[8] == 0 &&
-n[19] + 63 - n[2] - n[3] - 2 n[5] - n[10] - n[13] - n[15] == 0 &&
-n[21] - 23 - n[1] - n[3] + n[5] - 2 n[6] - 3 n[7] -
n[8] - 2 n[9] - n[10] - n[12] + n[15] - 3 n[16] - 2 n[17] - n[18] == 0 &&
-n[22] - 22 + n[2] - n[3] - n[4] - n[5] + 2 n[6] + n[8] - n[9] -
2 n[12] - n[13] - 3 n[14] - 2 n[15] + n[16] - n[18] - 2 n[20] == 0 &&
-n[23] - 12 + n[1] + 2 n[3] + n[4] + n[5] + 2 n[7] + 2 n[9] + n[10] +
2 n[12] + n[13] + 2 n[14] + n[15] + n[16] + n[17] + n[18] + n[20] == 0 &&
n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 &&
n[14] ≥ 0 && n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0 && n[19] ≥ 0 &&
n[20] ≥ 0 && n[21] ≥ 0 && n[22] ≥ 0 && n[23] ≥ 0, Array[n, 23], Integers]
{ }

Array[m, 7].Transpose[A]

{59 689 m[1] - 66 970 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
59 401 m[1] - 66 938 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
60 633 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
60 697 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
61 353 m[1] - 67 450 m[2] + 28 751 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7],
54 145 m[1] - 65 058 m[2] + 28 503 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
55 153 m[1] - 65 314 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
56 385 m[1] - 65 602 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
56 097 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
56 161 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
57 393 m[1] - 65 858 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
57 105 m[1] - 65 826 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
58 401 m[1] - 66 114 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
58 113 m[1] - 66 082 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7],
50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
51 849 m[1] - 63 946 m[2] + 28 335 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
52 857 m[1] - 64 202 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
52 569 m[1] - 64 170 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
53 865 m[1] - 64 458 m[2] + 28 367 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7],
48 321 m[1] - 62 546 m[2] + 28 151 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],
49 329 m[1] - 62 802 m[2] + 28 167 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7],
44 793 m[1] - 61 146 m[2] + 27 967 m[3] - 6140 m[4] + 711 m[5] - 42 m[6] + m[7]}

```

Array[m, 7].{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}

2 543 345 m[1] - 2 783 706 m[2] + 1 182 087 m[3] -
253 228 m[4] + 29 151 m[5] - 1722 m[6] + 41 m[7]

```

FindInstance[
 59 689 m[1] - 66 970 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 59 401 m[1] - 66 938 m[2] + 28 719 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 60 633 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 60 697 m[1] - 67 226 m[2] + 28 735 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 61 353 m[1] - 67 450 m[2] + 28 751 m[3] - 6172 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 54 145 m[1] - 65 058 m[2] + 28 503 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 55 377 m[1] - 65 346 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 55 153 m[1] - 65 314 m[2] + 28 519 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 56 385 m[1] - 65 602 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 56 097 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 56 161 m[1] - 65 570 m[2] + 28 535 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 57 393 m[1] - 65 858 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 57 105 m[1] - 65 826 m[2] + 28 551 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 58 401 m[1] - 66 114 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 58 113 m[1] - 66 082 m[2] + 28 567 m[3] - 6164 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 50 841 m[1] - 63 690 m[2] + 28 319 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 51 849 m[1] - 63 946 m[2] + 28 335 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 52 857 m[1] - 64 202 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 52 569 m[1] - 64 170 m[2] + 28 351 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 53 865 m[1] - 64 458 m[2] + 28 367 m[3] - 6156 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 48 321 m[1] - 62 546 m[2] + 28 151 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 49 329 m[1] - 62 802 m[2] + 28 167 m[3] - 6148 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 44 793 m[1] - 61 146 m[2] + 27 967 m[3] - 6140 m[4] + 711 m[5] - 42 m[6] + m[7] ≥ 0 &&
 2 543 345 m[1] - 2 783 706 m[2] + 1 182 087 m[3] - 253 228 m[4] +
 29 151 m[5] - 1722 m[6] + 41 m[7] < 0, Array[m, 7], Integers]
{ {m[1] → -603, m[2] → -549, m[3] → 0, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0} }

Array[m, 7] /.
{m[1] → -603, m[2] → -549, m[3] → 0, m[4] → 0, m[5] → 0, m[6] → 0, m[7] → 0}
{-603, -549, 0, 0, 0, 0, 0}

GCD[-603, -549, 0, 0, 0, 0, 0]
9

{-603, -549, 0, 0, 0, 0, 0} / 9
{-67, -61, 0, 0, 0, 0, 0}

{-67, -61, 0, 0, 0, 0, 0}.{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}
-598 049

```

```

{-67, -61, 0, 0, 0, 0, 0}.

Transpose[{{59 689, -66 970, 28 719, -6172, 711, -42, 1}, {59 401, -66 938,
28 719, -6172, 711, -42, 1}, {60 633, -67 226, 28 735, -6172, 711, -42, 1},
{60 697, -67 226, 28 735, -6172, 711, -42, 1},
{61 353, -67 450, 28 751, -6172, 711, -42, 1},
{54 145, -65 058, 28 503, -6164, 711, -42, 1}, {55 377, -65 346, 28 519,
-6164, 711, -42, 1}, {55 153, -65 314, 28 519, -6164, 711, -42, 1},
{56 385, -65 602, 28 535, -6164, 711, -42, 1}, {56 097, -65 570, 28 535,
-6164, 711, -42, 1}, {56 161, -65 570, 28 535, -6164, 711, -42, 1},
{57 393, -65 858, 28 551, -6164, 711, -42, 1}, {57 105, -65 826, 28 551,
-6164, 711, -42, 1}, {58 401, -66 114, 28 567, -6164, 711, -42, 1},
{58 113, -66 082, 28 567, -6164, 711, -42, 1}, {50 841, -63 690, 28 319,
-6156, 711, -42, 1}, {51 849, -63 946, 28 335, -6156, 711, -42, 1},
{52 857, -64 202, 28 351, -6156, 711, -42, 1}, {52 569, -64 170, 28 351,
-6156, 711, -42, 1}, {53 865, -64 458, 28 367, -6156, 711, -42, 1},
{48 321, -62 546, 28 151, -6148, 711, -42, 1}, {49 329, -62 802, 28 167,
-6148, 711, -42, 1}, {44 793, -61 146, 27 967, -6140, 711, -42, 1}}]

{86 007, 103 351, 38 375, 34 087, 3799, 340 823, 275 847,
288 903, 223 927, 241 271, 236 983, 172 007, 189 351, 120 087, 137 431,
478 743, 426 823, 374 903, 392 247, 322 983, 577 799, 525 879, 728 775}

Length[{86 007, 103 351, 38 375, 34 087, 3799, 340 823, 275 847,
288 903, 223 927, 241 271, 236 983, 172 007, 189 351, 120 087, 137 431,
478 743, 426 823, 374 903, 392 247, 322 983, 577 799, 525 879, 728 775}]
```

23

```

{-603, -549, 0, 0, 0, 0, 0}.Transpose[A]
{774 063, 930 159, 345 375, 306 783, 34 191, 3 067 407, 2 482 623, 2 600 127, 2 015 343,
2 171 439, 2 132 847, 1 548 063, 1 704 159, 1 080 783, 1 236 879, 4 308 687,
3 841 407, 3 374 127, 3 530 223, 2 906 847, 5 200 191, 4 732 911, 6 558 975}

{-603, -549, 0, 0, 0, 0, 0}.
{2 543 345, -2 783 706, 1 182 087, -253 228, 29 151, -1722, 41}
-5 382 441

```

```

poly18 = listmod128[[18]]

$$(-11 + x) (-9 + x)^6 (-7 + x)^8 (-4 + x) (5 + x)^{25}$$

p18 = poly18 / minipoly[poly18]

$$(-9 + x)^5 (-7 + x)^7 (5 + x)^{24}$$


```

```

feasiblesubcharpolylist[ (-11 + x) (-9 + x)6 (-7 + x)8 (-4 + x) (5 + x)25 ]
{ (-11 + x) (-5 + x) (17 - 10 x + x2) , 951 - 822 x + 232 x2 - 26 x3 + x4 ,
967 - 822 x + 232 x2 - 26 x3 + x4 , 983 - 822 x + 232 x2 - 26 x3 + x4 ,
(-11 + x) (-7 + x) (11 - 8 x + x2) , 863 - 814 x + 232 x2 - 26 x3 + x4 ,
879 - 814 x + 232 x2 - 26 x3 + x4 , (-5 + x) (-179 + 127 x - 21 x2 + x3) ,
911 - 814 x + 232 x2 - 26 x3 + x4 , (-9 + x) (-103 + 79 x - 17 x2 + x3) ,
(-7 + x) (-113 + 99 x - 19 x2 + x3) , 807 - 806 x + 232 x2 - 26 x3 + x4 ,
823 - 806 x + 232 x2 - 26 x3 + x4 , 839 - 806 x + 232 x2 - 26 x3 + x4 ,
(-9 + x) (-5 + x) (19 - 12 x + x2) , (-7 + x)2 (15 - 12 x + x2) , 751 - 798 x + 232 x2 - 26 x3 + x4 ,
(59 - 16 x + x2) (13 - 10 x + x2) , (-9 + x) (-87 + 79 x - 17 x2 + x3) ,
(-7 + x) (-97 + 99 x - 19 x2 + x3) , 695 - 790 x + 232 x2 - 26 x3 + x4 ,
(-9 + x) (-79 + 79 x - 17 x2 + x3) , (-7 + x) (-89 + 99 x - 19 x2 + x3) ,
(-9 + x) (-71 + 79 x - 17 x2 + x3) , (-9 + x)2 (-7 + x) (-1 + x) }

{ (-11 + x) (-5 + x) (17 - 10 x + x2) ,
951 - 822 x + 232 x2 - 26 x3 + x4 , 967 - 822 x + 232 x2 - 26 x3 + x4 ,
983 - 822 x + 232 x2 - 26 x3 + x4 , (-11 + x) (-7 + x) (11 - 8 x + x2) ,
863 - 814 x + 232 x2 - 26 x3 + x4 , 879 - 814 x + 232 x2 - 26 x3 + x4 ,
(-5 + x) (-179 + 127 x - 21 x2 + x3) , 911 - 814 x + 232 x2 - 26 x3 + x4 ,
(-9 + x) (-103 + 79 x - 17 x2 + x3) , (-7 + x) (-113 + 99 x - 19 x2 + x3) ,
807 - 806 x + 232 x2 - 26 x3 + x4 , 823 - 806 x + 232 x2 - 26 x3 + x4 ,
839 - 806 x + 232 x2 - 26 x3 + x4 , (-9 + x) (-5 + x) (19 - 12 x + x2) ,
(-7 + x)2 (15 - 12 x + x2) , 751 - 798 x + 232 x2 - 26 x3 + x4 ,
(59 - 16 x + x2) (13 - 10 x + x2) , (-9 + x) (-87 + 79 x - 17 x2 + x3) ,
(-7 + x) (-97 + 99 x - 19 x2 + x3) , 695 - 790 x + 232 x2 - 26 x3 + x4 ,
(-9 + x) (-79 + 79 x - 17 x2 + x3) , (-7 + x) (-89 + 99 x - 19 x2 + x3) ,
(-9 + x) (-71 + 79 x - 17 x2 + x3) , (-9 + x)2 (-7 + x) (-1 + x) } // Length

```

```

CoefficientList[ { (-11 + x) (-5 + x) (17 - 10 x + x^2),
  951 - 822 x + 232 x^2 - 26 x^3 + x^4, 967 - 822 x + 232 x^2 - 26 x^3 + x^4,
  983 - 822 x + 232 x^2 - 26 x^3 + x^4, (-11 + x) (-7 + x) (11 - 8 x + x^2),
  863 - 814 x + 232 x^2 - 26 x^3 + x^4, 879 - 814 x + 232 x^2 - 26 x^3 + x^4,
  (-5 + x) (-179 + 127 x - 21 x^2 + x^3), 911 - 814 x + 232 x^2 - 26 x^3 + x^4,
  (-9 + x) (-103 + 79 x - 17 x^2 + x^3), (-7 + x) (-113 + 99 x - 19 x^2 + x^3),
  807 - 806 x + 232 x^2 - 26 x^3 + x^4, 823 - 806 x + 232 x^2 - 26 x^3 + x^4,
  839 - 806 x + 232 x^2 - 26 x^3 + x^4, (-9 + x) (-5 + x) (19 - 12 x + x^2),
  (-7 + x)^2 (15 - 12 x + x^2), 751 - 798 x + 232 x^2 - 26 x^3 + x^4,
  (59 - 16 x + x^2) (13 - 10 x + x^2), (-9 + x) (-87 + 79 x - 17 x^2 + x^3),
  (-7 + x) (-97 + 99 x - 19 x^2 + x^3), 695 - 790 x + 232 x^2 - 26 x^3 + x^4,
  (-9 + x) (-79 + 79 x - 17 x^2 + x^3), (-7 + x) (-89 + 99 x - 19 x^2 + x^3),
  (-9 + x) (-71 + 79 x - 17 x^2 + x^3), (-9 + x)^2 (-7 + x) (-1 + x)}, x]

{{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
 {967, -822, 232, -26, 1}, {983, -822, 232, -26, 1},
 {847, -814, 232, -26, 1}, {863, -814, 232, -26, 1}, {879, -814, 232, -26, 1},
 {895, -814, 232, -26, 1}, {911, -814, 232, -26, 1}, {927, -814, 232, -26, 1},
 {791, -806, 232, -26, 1}, {807, -806, 232, -26, 1}, {823, -806, 232, -26, 1},
 {839, -806, 232, -26, 1}, {855, -806, 232, -26, 1}, {735, -798, 232, -26, 1},
 {751, -798, 232, -26, 1}, {767, -798, 232, -26, 1}, {783, -798, 232, -26, 1},
 {679, -790, 232, -26, 1}, {695, -790, 232, -26, 1}, {711, -790, 232, -26, 1},
 {623, -782, 232, -26, 1}, {639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}};

A = {{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
 {967, -822, 232, -26, 1}, {983, -822, 232, -26, 1}, {847, -814, 232, -26, 1},
 {863, -814, 232, -26, 1}, {879, -814, 232, -26, 1}, {895, -814, 232, -26, 1},
 {911, -814, 232, -26, 1}, {927, -814, 232, -26, 1}, {791, -806, 232, -26, 1},
 {807, -806, 232, -26, 1}, {823, -806, 232, -26, 1}, {839, -806, 232, -26, 1},
 {855, -806, 232, -26, 1}, {735, -798, 232, -26, 1}, {751, -798, 232, -26, 1},
 {767, -798, 232, -26, 1}, {783, -798, 232, -26, 1}, {679, -790, 232, -26, 1},
 {695, -790, 232, -26, 1}, {711, -790, 232, -26, 1}, {623, -782, 232, -26, 1},
 {639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}};

CoefficientList[D[poly18, x] / p18 // Factor, x]
{39495, -33726, 9512, -1066, 41}

```

```

Solve[Array[n, 25].{{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
{967, -822, 232, -26, 1}, {983, -822, 232, -26, 1}, {847, -814, 232, -26, 1},
{863, -814, 232, -26, 1}, {879, -814, 232, -26, 1}, {895, -814, 232, -26, 1},
{911, -814, 232, -26, 1}, {927, -814, 232, -26, 1}, {791, -806, 232, -26, 1},
{807, -806, 232, -26, 1}, {823, -806, 232, -26, 1}, {839, -806, 232, -26, 1},
{855, -806, 232, -26, 1}, {735, -798, 232, -26, 1}, {751, -798, 232, -26, 1},
{767, -798, 232, -26, 1}, {783, -798, 232, -26, 1}, {679, -790, 232, -26, 1},
{695, -790, 232, -26, 1}, {711, -790, 232, -26, 1}, {623, -782, 232, -26, 1},
{639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}} ==
{39495, -33726, 9512, -1066, 41}, Array[n, 25]]

Solve::svrs : Equations may not give solutions for all "solve" variables.    >>
{{n[23] → 105 - 4 n[1] - 3 n[2] - 2 n[3] - n[4] - 5 n[5] - 4 n[6] - 3 n[7] - 2 n[8] - n[9] -
4 n[11] - 3 n[12] - 2 n[13] - n[14] - 3 n[16] - 2 n[17] - n[18] - 2 n[20] - n[21],
n[24] → 144 - 2 n[1] - 3 n[2] - 4 n[3] - 5 n[4] - n[6] - 2 n[7] - 3 n[8] - 4 n[9] - 5 n[10] -
n[12] - 2 n[13] - 3 n[14] - 4 n[15] - n[17] - 2 n[18] - 3 n[19] - n[21] - 2 n[22],
n[25] → -208 + 5 n[1] + 5 n[2] + 5 n[3] + 5 n[4] + 4 n[5] + 4 n[6] + 4 n[7] +
4 n[8] + 4 n[9] + 4 n[10] + 3 n[11] + 3 n[12] + 3 n[13] + 3 n[14] +
3 n[15] + 2 n[16] + 2 n[17] + 2 n[18] + 2 n[19] + n[20] + n[21] + n[22]}}

FindInstance[
-n[23] + 105 - 4 n[1] - 3 n[2] - 2 n[3] - n[4] - 5 n[5] - 4 n[6] - 3 n[7] - 2 n[8] - n[9] -
4 n[11] - 3 n[12] - 2 n[13] - n[14] - 3 n[16] - 2 n[17] - n[18] - 2 n[20] - n[21] == 0 &&
-n[24] + 144 - 2 n[1] - 3 n[2] - 4 n[3] - 5 n[4] - n[6] - 2 n[7] - 3 n[8] - 4 n[9] - 5 n[10] -
n[12] - 2 n[13] - 3 n[14] - 4 n[15] - n[17] - 2 n[18] - 3 n[19] - n[21] - 2 n[22] == 0 &&
-n[25] - 208 + 5 n[1] + 5 n[2] + 5 n[3] + 5 n[4] + 4 n[5] + 4 n[6] +
4 n[7] + 4 n[8] + 4 n[9] + 4 n[10] + 3 n[11] + 3 n[12] + 3 n[13] + 3 n[14] +
3 n[15] + 2 n[16] + 2 n[17] + 2 n[18] + 2 n[19] + n[20] + n[21] + n[22] == 0
&& n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 && n[7] ≥ 0 &&
n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 && n[13] ≥ 0 && n[14] ≥ 0 &&
n[15] ≥ 0 && n[16] ≥ 0 && n[17] ≥ 0 && n[18] ≥ 0 && n[19] ≥ 0 && n[20] ≥ 0 &&
n[21] ≥ 0 && n[22] ≥ 0 && n[23] ≥ 0 && n[24] ≥ 0 && n[25] ≥ 0, Array[n, 25], Integers]
{ }

```

```
Array[m, 5].Transpose[A]
```

```
{935 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 951 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 967 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 983 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 847 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 863 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 879 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 895 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 911 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 927 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 791 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 807 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 823 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 839 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 855 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 735 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 751 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 767 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 783 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 679 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 695 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 711 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 623 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 639 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ,
 567 m[1] - 774 m[2] + 232 m[3] - 26 m[4] + m[5] }
```

```
Array[m, 5].{39 495, -33 726, 9512, -1066, 41}
```

```
39 495 m[1] - 33 726 m[2] + 9512 m[3] - 1066 m[4] + 41 m[5]
```

```

FindInstance[935 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
951 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
967 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
983 m[1] - 822 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
847 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
863 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
879 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
895 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
911 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
927 m[1] - 814 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
791 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
807 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
823 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
839 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
855 m[1] - 806 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
735 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
751 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
767 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
783 m[1] - 798 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
679 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
695 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
711 m[1] - 790 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
623 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
639 m[1] - 782 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
567 m[1] - 774 m[2] + 232 m[3] - 26 m[4] + m[5] ≥ 0 &&
39 495 m[1] - 33 726 m[2] + 9512 m[3] - 1066 m[4] + 41 m[5] < 0, Array[m, 5], Integers]
{{m[1] → 0, m[2] → 3493, m[3] → 0, m[4] → 0, m[5] → 2 871 696} }

Array[m, 5] /. {m[1] → 0, m[2] → 3493, m[3] → 0, m[4] → 0, m[5] → 2 871 696}
{0, 3493, 0, 0, 2 871 696}

GCD[0, 3493, 0, 0, 2 871 696]
1

{0, 3493, 0, 0, 2 871 696}.{39 495, -33 726, 9512, -1066, 41}
-65 382

```

```

{0, 3493, 0, 0, 2871696}.

Transpose[{{935, -822, 232, -26, 1}, {951, -822, 232, -26, 1},
{967, -822, 232, -26, 1}, {983, -822, 232, -26, 1}, {847, -814, 232, -26, 1},
{863, -814, 232, -26, 1}, {879, -814, 232, -26, 1}, {895, -814, 232, -26, 1},
{911, -814, 232, -26, 1}, {927, -814, 232, -26, 1}, {791, -806, 232, -26, 1},
{807, -806, 232, -26, 1}, {823, -806, 232, -26, 1}, {839, -806, 232, -26, 1},
{855, -806, 232, -26, 1}, {735, -798, 232, -26, 1}, {751, -798, 232, -26, 1},
{767, -798, 232, -26, 1}, {783, -798, 232, -26, 1}, {679, -790, 232, -26, 1},
{695, -790, 232, -26, 1}, {711, -790, 232, -26, 1}, {623, -782, 232, -26, 1},
{639, -782, 232, -26, 1}, {567, -774, 232, -26, 1}]]

{450, 450, 450, 450, 28394, 28394, 28394, 28394, 28394,
28394, 56338, 56338, 56338, 56338, 56338, 84282, 84282,
84282, 84282, 112226, 112226, 112226, 140170, 140170, 168114}

```

```

poly19 = listmod128[[19]]

$$(-9 + x)^8 (-7 + x)^4 (-5 + x)^2 (5 + x)^{25} (52 - 15 x + x^2)$$


p19 = poly19 / minipoly[poly19] // Factor

$$(-9 + x)^7 (-7 + x)^3 (-5 + x) (5 + x)^{24}$$


feasiblesubcharpolylelist[ (-9 + x)^8 (-7 + x)^4 (-5 + x)^2 (5 + x)^{25} (52 - 15 x + x^2) ]

$$\{ (-9 + x) (-5 + x) (-127 + 87 x - 17 x^2 + x^3), (-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2) \}$$


CoefficientList[

$$\{ (-9 + x) (-5 + x) (-127 + 87 x - 17 x^2 + x^3), (-9 + x) (-7 + x) (-5 + x) (17 - 10 x + x^2) \}, x]$$

{{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}}
```

A = {{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}};

```

CoefficientList[D[poly19, x] / p19 // Factor, x]
{-233515, 233557, -86558, 15170, -1271, 41}

Solve[Array[n, 2].
{{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}} ==
{-233515, 233557, -86558, 15170, -1271, 41}, Array[n, 2]]
{ }

Array[m, 6].Transpose[A]
{-5715 m[1] + 5693 m[2] - 2110 m[3] + 370 m[4] - 31 m[5] + m[6],
-5355 m[1] + 5581 m[2] - 2102 m[3] + 370 m[4] - 31 m[5] + m[6]}

Array[m, 6].{-233515, 233557, -86558, 15170, -1271, 41}
-233515 m[1] + 233557 m[2] - 86558 m[3] + 15170 m[4] - 1271 m[5] + 41 m[6]

```

```

FindInstance[-5715 m[1] + 5693 m[2] - 2110 m[3] + 370 m[4] - 31 m[5] + m[6] ≥ 0 &&
-5355 m[1] + 5581 m[2] - 2102 m[3] + 370 m[4] - 31 m[5] + m[6] ≥ 0 &&
-233515 m[1] + 233557 m[2] - 86558 m[3] + 15170 m[4] - 1271 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]
{{m[1] → 14, m[2] → 1163, m[3] → 15610, m[4] → 0, m[5] → 0, m[6] → 26404920} }

Array[m, 6] /.
{m[1] → 14, m[2] → 1163, m[3] → 15610, m[4] → 0, m[5] → 0, m[6] → 26404920}
{14, 1163, 15610, 0, 0, 26404920}

GCD[14, 1163, 15610, 0, 0, 26404920]
1

{14, 1163, 15610, 0, 0, 26404920}.{-233515, 233557, -86558, 15170, -1271, 41}
-211079

{14, 1163, 15610, 0, 0, 26404920}.
Transpose[{{{-5715, 5693, -2110, 370, -31, 1}, {-5355, 5581, -2102, 370, -31, 1}}]
{8769, 8433}

```

```

poly20 = listmod128[20]
(-9 + x)^7 (-7 + x)^6 (5 + x)^25 (-232 + 123 x - 20 x^2 + x^3)

p20 = poly20 / minipoly[poly20] // Factor
(-9 + x)^6 (-7 + x)^5 (5 + x)^24

feasiblesubcharpolylist[(-9 + x)^7 (-7 + x)^6 (5 + x)^25 (-232 + 123 x - 20 x^2 + x^3)]
{ (31 - 12 x + x^2) (-169 + 107 x - 19 x^2 + x^3),
  (-7 + x) (681 - 648 x + 198 x^2 - 24 x^3 + x^4), (-7 + x) (41 - 14 x + x^2) (17 - 10 x + x^2),
  -4847 + 5233 x - 2034 x^2 + 366 x^3 - 31 x^4 + x^5, (-7 + x) (23 - 12 x + x^2) (31 - 12 x + x^2),
  (-9 + x) (551 - 522 x + 168 x^2 - 22 x^3 + x^4), (-7 + x)^2 (-87 + 79 x - 17 x^2 + x^3),
  (-7 + x) (-5 + x) (-125 + 103 x - 19 x^2 + x^3), (-7 + x) (641 - 640 x + 198 x^2 - 24 x^3 + x^4),
  (-9 + x)^2 (-5 + x) (11 - 8 x + x^2), (-9 + x) (-7 + x) (-73 + 63 x - 15 x^2 + x^3),
  (-7 + x)^2 (-79 + 79 x - 17 x^2 + x^3), (-7 + x) (569 - 632 x + 198 x^2 - 24 x^3 + x^4),
  (-9 + x) (-7 + x) (-5 + x) (13 - 10 x + x^2), (-9 + x) (-7 + x) (-57 + 63 x - 15 x^2 + x^3) }

```

```
{(31 - 12 x + x2) (-169 + 107 x - 19 x2 + x3), (-7 + x) (681 - 648 x + 198 x2 - 24 x3 + x4),
(-7 + x) (41 - 14 x + x2) (17 - 10 x + x2), -4847 + 5233 x - 2034 x2 + 366 x3 - 31 x4 + x5,
(-7 + x) (23 - 12 x + x2) (31 - 12 x + x2), (-9 + x) (551 - 522 x + 168 x2 - 22 x3 + x4),
(-7 + x)2 (-87 + 79 x - 17 x2 + x3), (-7 + x) (-5 + x) (-125 + 103 x - 19 x2 + x3),
(-7 + x) (641 - 640 x + 198 x2 - 24 x3 + x4), (-9 + x)2 (-5 + x) (11 - 8 x + x2),
(-9 + x) (-7 + x) (-73 + 63 x - 15 x2 + x3), (-7 + x)2 (-79 + 79 x - 17 x2 + x3),
(-7 + x) (569 - 632 x + 198 x2 - 24 x3 + x4), (-9 + x) (-7 + x) (-5 + x) (13 - 10 x + x2),
(-9 + x) (-7 + x) (-57 + 63 x - 15 x2 + x3)} // Length
```

15

```
CoefficientList[{(31 - 12 x + x2) (-169 + 107 x - 19 x2 + x3),
(-7 + x) (681 - 648 x + 198 x2 - 24 x3 + x4), (-7 + x) (41 - 14 x + x2) (17 - 10 x + x2),
-4847 + 5233 x - 2034 x2 + 366 x3 - 31 x4 + x5, (-7 + x) (23 - 12 x + x2) (31 - 12 x + x2),
(-9 + x) (551 - 522 x + 168 x2 - 22 x3 + x4), (-7 + x)2 (-87 + 79 x - 17 x2 + x3),
(-7 + x) (-5 + x) (-125 + 103 x - 19 x2 + x3), (-7 + x) (641 - 640 x + 198 x2 - 24 x3 + x4),
(-9 + x)2 (-5 + x) (11 - 8 x + x2), (-9 + x) (-7 + x) (-73 + 63 x - 15 x2 + x3),
(-7 + x)2 (-79 + 79 x - 17 x2 + x3), (-7 + x) (569 - 632 x + 198 x2 - 24 x3 + x4),
(-9 + x) (-7 + x) (-5 + x) (13 - 10 x + x2), (-9 + x) (-7 + x) (-57 + 63 x - 15 x2 + x3)}, x]
```

```
{ {-5239, 5345, -2042, 366, -31, 1},
{-4767, 5217, -2034, 366, -31, 1}, {-4879, 5233, -2034, 366, -31, 1},
{-4847, 5233, -2034, 366, -31, 1}, {-4991, 5249, -2034, 366, -31, 1},
{-4959, 5249, -2034, 366, -31, 1}, {-4263, 5089, -2026, 366, -31, 1},
{-4375, 5105, -2026, 366, -31, 1}, {-4487, 5121, -2026, 366, -31, 1},
{-4455, 5121, -2026, 366, -31, 1}, {-4599, 5137, -2026, 366, -31, 1},
{-3871, 4977, -2018, 366, -31, 1}, {-3983, 4993, -2018, 366, -31, 1},
{-4095, 5009, -2018, 366, -31, 1}, {-3591, 4881, -2010, 366, -31, 1}}
```

```
A = {{-5239, 5345, -2042, 366, -31, 1},
{-4767, 5217, -2034, 366, -31, 1}, {-4879, 5233, -2034, 366, -31, 1},
{-4847, 5233, -2034, 366, -31, 1}, {-4991, 5249, -2034, 366, -31, 1},
{-4959, 5249, -2034, 366, -31, 1}, {-4263, 5089, -2026, 366, -31, 1},
{-4375, 5105, -2026, 366, -31, 1}, {-4487, 5121, -2026, 366, -31, 1},
{-4455, 5121, -2026, 366, -31, 1}, {-4599, 5137, -2026, 366, -31, 1},
{-3871, 4977, -2018, 366, -31, 1}, {-3983, 4993, -2018, 366, -31, 1},
{-4095, 5009, -2018, 366, -31, 1}, {-3591, 4881, -2010, 366, -31, 1}};
```

```
CoefficientList[D[poly20, x] / p20 // Factor, x]
```

```
{-207175, 217305, -83618, 15006, -1271, 41}
```

```

Solve[Array[n, 15].{{-5239, 5345, -2042, 366, -31, 1},
{-4767, 5217, -2034, 366, -31, 1}, {-4879, 5233, -2034, 366, -31, 1},
{-4847, 5233, -2034, 366, -31, 1}, {-4991, 5249, -2034, 366, -31, 1},
{-4959, 5249, -2034, 366, -31, 1}, {-4263, 5089, -2026, 366, -31, 1},
{-4375, 5105, -2026, 366, -31, 1}, {-4487, 5121, -2026, 366, -31, 1},
{-4455, 5121, -2026, 366, -31, 1}, {-4599, 5137, -2026, 366, -31, 1},
{-3871, 4977, -2018, 366, -31, 1}, {-3983, 4993, -2018, 366, -31, 1},
{-4095, 5009, -2018, 366, -31, 1}, {-3591, 4881, -2010, 366, -31, 1}} ==
{-207175, 217305, -83618, 15006, -1271, 41}, Array[n, 15]]

Solve::svrs : Equations may not give solutions for all "solve" variables.    >>>

{{n[10] → 36 - n[1] - n[4] - n[6],
n[13] → 98 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] - n[9] - 2 n[12],
n[14] → -19 - n[3] - 2 n[5] - n[6] + n[7] - n[9] - 2 n[11] + n[12],
n[15] → -74 + 2 n[1] + 2 n[2] + 2 n[3] + n[4] + 2 n[5] + n[6] + n[7] + n[8] + n[9] + n[11]}}

FindInstance[-n[10] + 36 - n[1] - n[4] - n[6] == 0 &&
-n[13] + 98 - 2 n[1] - 3 n[2] - 2 n[3] - n[4] - n[5] - 3 n[7] - 2 n[8] - n[9] - 2 n[12] == 0 &&
-n[14] - 19 - n[3] - 2 n[5] - n[6] + n[7] - n[9] - 2 n[11] + n[12] == 0 &&
-n[15] - 74 + 2 n[1] + 2 n[2] + 2 n[3] + n[4] + 2 n[5] + n[6] + n[7] + n[8] + n[9] + n[11] ==
0 && n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0 && n[6] ≥ 0 &&
n[7] ≥ 0 && n[8] ≥ 0 && n[9] ≥ 0 && n[10] ≥ 0 && n[11] ≥ 0 && n[12] ≥ 0 &&
n[13] ≥ 0 && n[14] ≥ 0 && n[15] ≥ 0, Array[n, 15], Integers]

{ }

Array[m, 6].Transpose[A]

{-5239 m[1] + 5345 m[2] - 2042 m[3] + 366 m[4] - 31 m[5] + m[6],
-4767 m[1] + 5217 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
-4879 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
-4847 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
-4991 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
-4959 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6],
-4263 m[1] + 5089 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
-4375 m[1] + 5105 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
-4487 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
-4455 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
-4599 m[1] + 5137 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6],
-3871 m[1] + 4977 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6],
-3983 m[1] + 4993 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6],
-4095 m[1] + 5009 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6],
-3591 m[1] + 4881 m[2] - 2010 m[3] + 366 m[4] - 31 m[5] + m[6]}

Array[m, 6].{-207175, 217305, -83618, 15006, -1271, 41}
-207175 m[1] + 217305 m[2] - 83618 m[3] + 15006 m[4] - 1271 m[5] + 41 m[6]

```

```

FindInstance[-5239 m[1] + 5345 m[2] - 2042 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4767 m[1] + 5217 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4879 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4847 m[1] + 5233 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4991 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4959 m[1] + 5249 m[2] - 2034 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4263 m[1] + 5089 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4375 m[1] + 5105 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4487 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4455 m[1] + 5121 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4599 m[1] + 5137 m[2] - 2026 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-3871 m[1] + 4977 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-3983 m[1] + 4993 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-4095 m[1] + 5009 m[2] - 2018 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-3591 m[1] + 4881 m[2] - 2010 m[3] + 366 m[4] - 31 m[5] + m[6] ≥ 0 &&
-207175 m[1] + 217305 m[2] - 83618 m[3] + 15006 m[4] - 1271 m[5] + 41 m[6] < 0,
Array[m, 6], Integers]

{{m[1] → -1026, m[2] → -4476, m[3] → -9088, m[4] → 0, m[5] → 0, m[6] → 0} }

Array[m, 6] /.
{m[1] → -1026, m[2] → -4476, m[3] → -9088, m[4] → 0, m[5] → 0, m[6] → 0}
{-1026, -4476, -9088, 0, 0, 0}

GCD[-1026, -4476, -9088, 0, 0, 0]
2

{-1026, -4476, -9088, 0, 0, 0}/2
{-513, -2238, -4544, 0, 0, 0}

Reverse[{-513, -2238, -4544, 0, 0, 0}]
{0, 0, 0, -4544, -2238, -513}

{-513, -2238, -4544, 0, 0, 0}.{-207175, 217305, -83618, 15006, -1271, 41}
-87623

{-513, -2238, -4544, 0, 0, 0}.
Transpose[{{{-5239, 5345, -2042, 366, -31, 1}, {-4767, 5217, -2034, 366, -31, 1},
{-4879, 5233, -2034, 366, -31, 1}, {-4847, 5233, -2034, 366, -31, 1},
{-4991, 5249, -2034, 366, -31, 1}, {-4959, 5249, -2034, 366, -31, 1},
{-4263, 5089, -2026, 366, -31, 1}, {-4375, 5105, -2026, 366, -31, 1},
{-4487, 5121, -2026, 366, -31, 1}, {-4455, 5121, -2026, 366, -31, 1},
{-4599, 5137, -2026, 366, -31, 1}, {-3871, 4977, -2018, 366, -31, 1},
{-3983, 4993, -2018, 366, -31, 1}, {-4095, 5009, -2018, 366, -31, 1},
{-3591, 4881, -2010, 366, -31, 1}}]

{4345, 12321, 33969, 17553, 55617, 39201, 3881,
25529, 47177, 30761, 68825, 17089, 38737, 60385, 51945}

```

```

Length[{4345, 12321, 33969, 17553, 55617, 39201,
 3881, 25529, 47177, 30761, 68825, 17089, 38737, 60385, 51945}]

15

{-1026, -4476, -9088, 0, 0, 0}.{-207175, 217305, -83618, 15006, -1271, 41}
-175246

{-1026, -4476, -9088, 0, 0, 0}.
Transpose[{{{-5239, 5345, -2042, 366, -31, 1}, {-4767, 5217, -2034, 366, -31, 1},
  {-4879, 5233, -2034, 366, -31, 1}, {-4847, 5233, -2034, 366, -31, 1},
  {-4991, 5249, -2034, 366, -31, 1}, {-4959, 5249, -2034, 366, -31, 1},
  {-4263, 5089, -2026, 366, -31, 1}, {-4375, 5105, -2026, 366, -31, 1},
  {-4487, 5121, -2026, 366, -31, 1}, {-4455, 5121, -2026, 366, -31, 1},
  {-4599, 5137, -2026, 366, -31, 1}, {-3871, 4977, -2018, 366, -31, 1},
  {-3983, 4993, -2018, 366, -31, 1}, {-4095, 5009, -2018, 366, -31, 1},
  {-3591, 4881, -2010, 366, -31, 1}}]

{8690, 24642, 67938, 35106, 111234, 78402, 7762,
 51058, 94354, 61522, 137650, 34178, 77474, 120770, 103890}

```

```

poly21 = listmod128[[21]]
(-9 + x)^6 (-7 + x)^7 (5 + x)^25 (-292 + 149 x - 22 x^2 + x^3)

p21 = poly21 / minipoly[poly21] // Factor
(-9 + x)^5 (-7 + x)^6 (5 + x)^24

feasiblesubcharpolylist[(-9 + x)^6 (-7 + x)^7 (5 + x)^25 (-292 + 149 x - 22 x^2 + x^3)]
{(-9 + x) (-7 + x) (-5 + x) (19 - 12 x + x^2), (-9 + x) (681 - 648 x + 198 x^2 - 24 x^3 + x^4),
 (-9 + x) (41 - 14 x + x^2) (17 - 10 x + x^2), (-9 + x) (23 - 12 x + x^2) (31 - 12 x + x^2),
 (-9 + x) (-7 + x) (-87 + 79 x - 17 x^2 + x^3), (-9 + x) (-5 + x) (-125 + 103 x - 19 x^2 + x^3),
 (-9 + x) (641 - 640 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-79 + 79 x - 17 x^2 + x^3),
 (-9 + x) (569 - 632 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-71 + 79 x - 17 x^2 + x^3),
 (-9 + x)^2 (-57 + 63 x - 15 x^2 + x^3), (-9 + x)^2 (-7 + x)^2 (-1 + x) }

{(-9 + x) (-7 + x) (-5 + x) (19 - 12 x + x^2), (-9 + x) (681 - 648 x + 198 x^2 - 24 x^3 + x^4),
 (-9 + x) (41 - 14 x + x^2) (17 - 10 x + x^2), (-9 + x) (23 - 12 x + x^2) (31 - 12 x + x^2),
 (-9 + x) (-7 + x) (-87 + 79 x - 17 x^2 + x^3), (-9 + x) (-5 + x) (-125 + 103 x - 19 x^2 + x^3),
 (-9 + x) (641 - 640 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-79 + 79 x - 17 x^2 + x^3),
 (-9 + x) (569 - 632 x + 198 x^2 - 24 x^3 + x^4), (-9 + x) (-7 + x) (-71 + 79 x - 17 x^2 + x^3),
 (-9 + x)^2 (-57 + 63 x - 15 x^2 + x^3), (-9 + x)^2 (-7 + x)^2 (-1 + x)} // Length

```

```

CoefficientList[
{(-9+x) (-7+x) (-5+x) (19-12x+x2), (-9+x) (681-648x+198x2-24x3+x4),
 (-9+x) (41-14x+x2) (17-10x+x2), (-9+x) (23-12x+x2) (31-12x+x2),
 (-9+x) (-7+x) (-87+79x-17x2+x3), (-9+x) (-5+x) (-125+103x-19x2+x3),
 (-9+x) (641-640x+198x2-24x3+x4), (-9+x) (-7+x) (-79+79x-17x2+x3),
 (-9+x) (569-632x+198x2-24x3+x4), (-9+x) (-7+x) (-71+79x-17x2+x3),
 (-9+x)2 (-57+63x-15x2+x3), (-9+x)2 (-7+x)2 (-1+x)}, x]

{{{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
 {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
 {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
 {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
 {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
 {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}}}

A = {{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
 {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
 {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
 {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
 {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
 {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}};

MatrixRank[A]
3

CoefficientList[D[poly21, x] / p21 // Factor, x]
{-259 665, 268 537, -99 702, 16 974, -1353, 41}

Solve[Array[n, 12].
{{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
 {-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
 {-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
 {-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
 {-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
 {-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}} =
{-259 665, 268 537, -99 702, 16 974, -1353, 41}, Array[n, 12]]
{ }

```

```

Array[m, 6].Transpose[A]
{-5985 m[1] + 6497 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -6129 m[1] + 6513 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -6273 m[1] + 6529 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -6417 m[1] + 6545 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -5481 m[1] + 6369 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -5625 m[1] + 6385 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -5769 m[1] + 6401 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -4977 m[1] + 6241 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -5121 m[1] + 6257 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -4473 m[1] + 6113 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -4617 m[1] + 6129 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6],  

 -3969 m[1] + 5985 m[2] - 2398 m[3] + 414 m[4] - 33 m[5] + m[6]}

Array[m, 6].{-259 665, 268 537, -99 702, 16 974, -1353, 41}
-259 665 m[1] + 268 537 m[2] - 99 702 m[3] + 16 974 m[4] - 1353 m[5] + 41 m[6]

FindInstance[-5985 m[1] + 6497 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-6129 m[1] + 6513 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-6273 m[1] + 6529 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-6417 m[1] + 6545 m[2] - 2430 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-5481 m[1] + 6369 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-5625 m[1] + 6385 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-5769 m[1] + 6401 m[2] - 2422 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-4977 m[1] + 6241 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-5121 m[1] + 6257 m[2] - 2414 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-4473 m[1] + 6113 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-4617 m[1] + 6129 m[2] - 2406 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-3969 m[1] + 5985 m[2] - 2398 m[3] + 414 m[4] - 33 m[5] + m[6] ≥ 0 &&  

-259 665 m[1] + 268 537 m[2] - 99 702 m[3] + 16 974 m[4] - 1353 m[5] + 41 m[6] < 0,  

Array[m, 6], Integers]  

{{m[1] → -180, m[2] → -1613, m[3] → -3873, m[4] → 0, m[5] → 0, m[6] → 0}}
```

**Array[m, 6] /. {m[1] → -180, m[2] → -1613, m[3] → -3873, m[4] → 0, m[5] → 0, m[6] → 0}**

{-180, -1613, -3873, 0, 0, 0}

**GCD[-180, -1613, -3873, 0, 0, 0]**

1

{-180, -1613, -3873, 0, 0, 0}.{-259 665, 268 537, -99 702, 16 974, -1353, 41}

-264 635

```

{-180, -1613, -3873, 0, 0, 0}.

Transpose[{{{-5985, 6497, -2430, 414, -33, 1}, {-6129, 6513, -2430, 414, -33, 1},
{-6273, 6529, -2430, 414, -33, 1}, {-6417, 6545, -2430, 414, -33, 1},
{-5481, 6369, -2422, 414, -33, 1}, {-5625, 6385, -2422, 414, -33, 1},
{-5769, 6401, -2422, 414, -33, 1}, {-4977, 6241, -2414, 414, -33, 1},
{-5121, 6257, -2414, 414, -33, 1}, {-4473, 6113, -2406, 414, -33, 1},
{-4617, 6129, -2406, 414, -33, 1}, {-3969, 5985, -2398, 414, -33, 1}}]

{9029, 9141, 9253, 9365, 93789, 93901,
94013, 178549, 178661, 263309, 263421, 348069}

poly22 = listmod128[[22]]

(-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^25

p22 = poly22 / minipoly[poly22] // Factor
(-9 + x)^7 (-7 + x)^5 (5 + x)^24

feasiblesubcharpolylist[(-9 + x)^8 (-8 + x) (-7 + x)^6 (-3 + x) (5 + x)^25]
{(-9 + x) (-7 + x) (-3 + x)^2,
(-7 + x) (-73 + 63 x - 15 x^2 + x^3), (31 - 12 x + x^2) (17 - 10 x + x^2),
(-7 + x) (-5 + x) (13 - 10 x + x^2), (-7 + x) (-57 + 63 x - 15 x^2 + x^3) }

{(-9 + x) (-7 + x) (-3 + x)^2, (-7 + x) (-73 + 63 x - 15 x^2 + x^3),
(31 - 12 x + x^2) (17 - 10 x + x^2), (-7 + x) (-5 + x) (13 - 10 x + x^2),
(-7 + x) (-57 + 63 x - 15 x^2 + x^3)} // Length
5

CoefficientList[{(-9 + x) (-7 + x) (-3 + x)^2,
(-7 + x) (-73 + 63 x - 15 x^2 + x^3), (31 - 12 x + x^2) (17 - 10 x + x^2),
(-7 + x) (-5 + x) (13 - 10 x + x^2), (-7 + x) (-57 + 63 x - 15 x^2 + x^3)}, x]
{{567, -522, 168, -22, 1}, {511, -514, 168, -22, 1},
{527, -514, 168, -22, 1}, {455, -506, 168, -22, 1}, {399, -498, 168, -22, 1} }

CoefficientList[D[poly22, x] / p22 // Factor, x]
{21135, -21018, 6888, -902, 41}

Solve[Array[n, 5].{{567, -522, 168, -22, 1}, {511, -514, 168, -22, 1},
{527, -514, 168, -22, 1}, {455, -506, 168, -22, 1}, {399, -498, 168, -22, 1}} ==
{21135, -21018, 6888, -902, 41}, Array[n, 5]]
Solve::svrs : Equations may not give solutions for all "solve" variables.    >>
{{n[3] → 36, n[4] → 3 - 3 n[1] - 2 n[2], n[5] → 2 + 2 n[1] + n[2]}}

```

```
Array[n, 5] /.
Solve[-n[3] + 36 == 0 && -n[4] + 3 - 3 n[1] - 2 n[2] == 0 && -n[5] + 2 + 2 n[1] + n[2] == 0 &&
n[1] ≥ 0 && n[2] ≥ 0 && n[3] ≥ 0 && n[4] ≥ 0 && n[5] ≥ 0, Array[n, 5], Integers]
{{0, 0, 36, 3, 2}, {0, 1, 36, 1, 3}, {1, 0, 36, 0, 4}}
```