

```

In[®]:= tupn9 = Tuples[{0, -1, 1}, 9];
bivareps10 = Array[bve10, Length[tupn9]];
For[i = 1, i ≤ Length[tupn9], i++,
  bivareps10[[i]] =
    conedeletionthreshold[adjsignedthreshold[tupn9[[i]]], λ, u, "even"]
] // AbsoluteTiming
Length[tupn9]
Length[DeleteDuplicates[bivareps10 // Factor]]

```

Out[®]= {232.083, Null}

Out[®]= 19 683

Out[®]= 19 683

```

In[®]:= tupn10 = Tuples[{0, -1, 1}, 10];
bivareps11 = Array[bve11, Length[tupn10]];
For[i = 1, i ≤ Length[tupn10], i++,
  If[Mod[i, 10 000] == 0, Print[i]];
  If[Mod[i + 2, 3] == 0, bivareps11[[i]] = Factor[λ bivareps10[[(i + 2)/3]]];
  If[Mod[i + 1, 3] == 0, poly = bivareps10[[(i + 1)/3]];
  bivareps11[[i]] = Factor[u poly + (λ - u) (poly /. {λ → λ - 1, u → u + 1})];
  If[Mod[i, 3] == 0, poly = bivareps10[[(i)/3]];
  bivareps11[[i]] =
    Factor[u (poly /. {λ → λ - 1, u → u - 1}) + (λ - u) (poly /. {λ → λ - 1, u → u + 1})]
  ] // AbsoluteTiming

```

10 000

20 000

30 000

40 000

50 000

Out[®]= {46.1626, Null}

```

In[®]:= Length[bivareps11]
Length[DeleteDuplicates[bivareps11]] // AbsoluteTiming

```

Out[®]= 59 049

Out[®]= {0.315306, 59 049}

```

In[®]:= ind = RandomInteger[{1, 3^10}]
tup = tupn10[[ind]]
bivareps11[[ind]]
bivareps = conedeletionthreshold[adjsignedthreshold[tup], λ, u, "even"]
Factor[bivareps - bivareps11[[ind]]]

Out[®]= 58817

Out[®]= {1, 1, 1, 1, 1, 0, 0, -1, 0, -1}

Out[®]= (-5 + λ) (-4 + λ) (-3 + λ) (-2 + λ) (-1 + λ)
(330 u + 162 u2 - 330 λ - 751 u λ - 195 u2 λ + 589 λ2 +
590 u λ2 + 64 u2 λ2 - 395 λ3 - 187 u λ3 + 123 λ4 + 18 u λ4 - 18 λ5 + λ6)

Out[®]= (-5 + λ) (-4 + λ) (-3 + λ) (-2 + λ) (-1 + λ)
(330 u + 162 u2 - 330 λ - 751 u λ - 195 u2 λ + 589 λ2 +
590 u λ2 + 64 u2 λ2 - 395 λ3 - 187 u λ3 + 123 λ4 + 18 u λ4 - 18 λ5 + λ6)

Out[®]= 0

In[®]:= tupn11 = Tuples[{0, -1, 1}, 11];
bivareps12 = Array[bve12, Length[tupn11]];
For[i = 1, i ≤ Length[tupn11], i++,
If[Mod[i, 10 000] == 0, Print[i]];
If[Mod[i + 2, 3] == 0, bivareps12[[i]] = Factor[λ bivareps11[[(i + 2)/3]]];
If[Mod[i + 1, 3] == 0, poly = bivareps11[[(i + 1)/3]];
bivareps12[[i]] = Factor[u poly + (λ - u) (poly /. {λ → λ - 1, u → u + 1})];
If[Mod[i, 3] == 0, poly = bivareps11[[(i)/3]];
bivareps12[[i]] =
Factor[u (poly /. {λ → λ - 1, u → u - 1}) + (λ - u) (poly /. {λ → λ - 1, u → u + 1})]
]
// AbsoluteTiming

```

```

10 000
20 000
30 000
40 000
50 000
60 000
70 000
80 000
90 000
100 000
110 000
120 000
130 000
140 000
150 000
160 000
170 000
Out[=] {237.016, Null}

In[]:= Length[bivareps12]
Length[DeleteDuplicates[bivareps12]] // AbsoluteTiming
Out[=] 177 147

Out[=] {1.4751, 177 147}

In[]:= ind = RandomInteger[{1, 3^11}]
tup = tupn11[[ind]]
bivareps12[[ind]]
bivareps = conedeletionthreshold[adjsignedthreshold[tup], λ, u, "even"]
Factor[bivareps - bivareps12[[ind]]]
Out[=] 24 703

Out[=] {0, -1, 0, 1, 0, 1, -1, 1, 1, 1, 0}

Out[=] (-5 + λ)^2 (-4 + λ) λ
(55 296 u + 11 736 u^2 - 57 096 λ - 91 980 u λ - 8016 u^2 λ + 85 314 λ^2 + 54 982 u λ^2 +
2054 u^2 λ^2 - 52 653 λ^3 - 16 560 u λ^3 - 234 u^2 λ^3 + 17 855 λ^4 +
2750 u λ^4 + 10 u^2 λ^4 - 3646 λ^5 - 243 u λ^5 + 453 λ^6 + 9 u λ^6 - 32 λ^7 + λ^8)

Out[=] (-5 + λ)^2 (-4 + λ) λ
(55 296 u + 11 736 u^2 - 57 096 λ - 91 980 u λ - 8016 u^2 λ + 85 314 λ^2 + 54 982 u λ^2 +
2054 u^2 λ^2 - 52 653 λ^3 - 16 560 u λ^3 - 234 u^2 λ^3 + 17 855 λ^4 +
2750 u λ^4 + 10 u^2 λ^4 - 3646 λ^5 - 243 u λ^5 + 453 λ^6 + 9 u λ^6 - 32 λ^7 + λ^8)

Out[=] 0

```

```

In[6]:= tupn12 = Tuples[{0, -1, 1}, 12];
bivareps13 = Array[bve13, Length[tupn12]];
For[i = 1, i ≤ Length[tupn12], i++,
  If[Mod[i, 10 000] == 0, Print[i]];
  If[Mod[i + 2, 3] == 0, bivareps13[[i]] = Factor[λ bivareps12[[(i + 2)/3]]];
  If[Mod[i + 1, 3] == 0, poly = bivareps12[[(i + 1)/3]];
  bivareps13[[i]] = Factor[u poly + (λ - u) (poly /. {λ → λ - 1, u → u + 1})];
  If[Mod[i, 3] == 0, poly = bivareps12[[(i)/3]];
  bivareps13[[i]] =
    Factor[u (poly /. {λ → λ - 1, u → u - 1}) + (λ - u) (poly /. {λ → λ - 1, u → u + 1})];
  ] // AbsoluteTiming
10 000
20 000
30 000
40 000
50 000
60 000
70 000
80 000
90 000
100 000
110 000
120 000
130 000
140 000
150 000
160 000
170 000
180 000
190 000
200 000
210 000

```

```

220 000
230 000
240 000
250 000
260 000
270 000
280 000
290 000
300 000
310 000
320 000
330 000
340 000
350 000
360 000
370 000
380 000
390 000
400 000
410 000
420 000
430 000
440 000
450 000
460 000
470 000
480 000
490 000
500 000
510 000
520 000
530 000
Out[=] {628.623, Null}

In[=] Length[bivareps13]
Length[DeleteDuplicates[bivareps13]] // AbsoluteTiming
Out[=] 531 441
Out[=] {10.9251, 531 441}

```

```

In[®]:= ind = RandomInteger[{1, 3^12}]
tup = tupn12[[ind]]
bivareps13[[ind]]
bivareps = conedeletionthreshold[adjsignedthreshold[tup], λ, u, "even"]
Factor[bivareps - bivareps13[[ind]]]

Out[®]= 281424

Out[®]= {-1, -1, 1, 0, 1, 1, 0, 0, -1, 0, 0, 1}

Out[®]= (-4 + λ) (-1 + λ)^2
(91194 u + 31860 u^2 - 91482 λ - 275573 u λ - 61317 u^2 λ + 245345 λ^2 + 342209 u λ^2 +
47334 u^2 λ^2 - 284854 λ^3 - 230425 u λ^3 - 18819 u^2 λ^3 + 188480 λ^4 +
93004 u λ^4 + 4080 u^2 λ^4 - 78697 λ^5 - 23309 u λ^5 - 459 u^2 λ^5 + 21636 λ^6 +
3585 u λ^6 + 21 u^2 λ^6 - 3944 λ^7 - 313 u λ^7 + 463 λ^8 + 12 u λ^8 - 32 λ^9 + λ^10)

Out[®]= (-4 + λ) (-1 + λ)^2
(91194 u + 31860 u^2 - 91482 λ - 275573 u λ - 61317 u^2 λ + 245345 λ^2 + 342209 u λ^2 +
47334 u^2 λ^2 - 284854 λ^3 - 230425 u λ^3 - 18819 u^2 λ^3 + 188480 λ^4 +
93004 u λ^4 + 4080 u^2 λ^4 - 78697 λ^5 - 23309 u λ^5 - 459 u^2 λ^5 + 21636 λ^6 +
3585 u λ^6 + 21 u^2 λ^6 - 3944 λ^7 - 313 u λ^7 + 463 λ^8 + 12 u λ^8 - 32 λ^9 + λ^10)

Out[®]= 0

```