UNIVERSIDAD CENTROAMERICANA “JOSÉ SIMEÓN CAÑAS”
FACULTAD DE INGENIERÍA Y ARQUITECTURA

MEF 4D

Asignatura:
Simulación de Computadoras

Sección 01

Equipo de Proyecto: **Los Simuladores**

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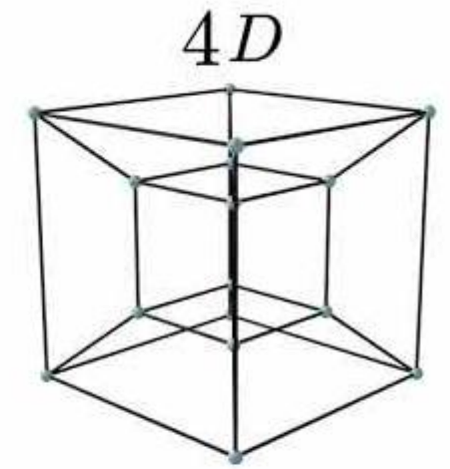
Definición del Modelo



$$-\frac{d}{dx}(K\frac{d}{dx}T) = Q$$

$$-\frac{\partial}{\partial x}(K\frac{\partial}{\partial x}T) = Q$$

$$\nabla f = \frac{\mathrm{df}}{\mathrm{df}x}\mathrm{i} + \frac{\mathrm{df}}{\mathrm{d}y}\mathrm{j} + \frac{\mathrm{df}}{\mathrm{d}z}\mathrm{k} + \frac{\mathrm{df}}{\mathrm{d}s}\mathrm{s}$$

$$-\frac{\partial}{\partial x}(K\frac{\partial}{\partial x}T) - \frac{\partial}{\partial y}(K\frac{\partial}{\partial y}T) - \frac{\partial}{\partial z}(K\frac{\partial}{\partial z}T) + \frac{\partial}{\partial s}(K\frac{\partial}{\partial s}T) = Q$$

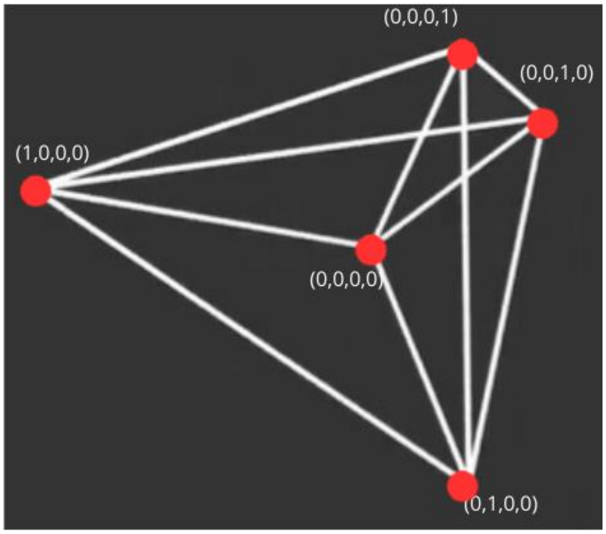
$$\nabla f = \begin{bmatrix} \frac{\mathrm{d}f}{\mathrm{d}x} \\ \frac{\mathrm{d}f}{\mathrm{d}y} \\ \frac{\mathrm{d}f}{\mathrm{d}z} \\ \frac{\mathrm{d}f}{\mathrm{d}s} \end{bmatrix} \qquad - \begin{bmatrix} \frac{\mathrm{d}f}{\mathrm{d}x} \\ \frac{\mathrm{d}f}{\mathrm{d}y} \\ \frac{\mathrm{d}f}{\mathrm{d}z} \\ \frac{\mathrm{d}f}{\mathrm{d}s} \end{bmatrix} * (K \begin{bmatrix} \frac{\mathrm{d}f}{\mathrm{d}x} \\ \frac{\mathrm{d}f}{\mathrm{d}y} \\ \frac{\mathrm{d}f}{\mathrm{d}z} \\ \frac{\mathrm{d}f}{\mathrm{d}s} \end{bmatrix} T) = Q$$

Ecuación del Calor Final y considerando las 4 Dimensiones:

$$-\nabla(K\nabla T) = Q$$

Algoritmo del método de los elementos Finitos.

1) Localización (Pentacoro)



1.5) Isoparametrizacion

$$N1 = 1 - \varepsilon - \eta - \phi - \alpha$$
$$N2 = \varepsilon$$
$$N3 = \eta$$
$$N4 = \phi$$
$$N5 = \alpha$$

2) Interpolación

$$T \approx N1 * T1 + N2 * T2 + N3 * T3 + N4 * T4 + N5 * T5$$

$$T \approx \begin{bmatrix} N1 & N2 & N3 & N4 & N5 \end{bmatrix} * \begin{bmatrix} T1 \\ T2 \\ T3 \\ T4 \\ T5 \end{bmatrix}$$

$$\mathrm{T} \approx N * T$$

3)Aproximación del Modelo.

$$-\nabla(K\nabla T) = Q$$
$$-\nabla\big(K\nabla(N * T)\big) = Q$$

3.5) Definición del Residual

$$R = Q + \nabla\big(K\nabla(N * T)\big)$$

4) Método de los Residuos Ponderados.

$$\int_{v_4} w * R\mathrm{d}v = 0$$

$$\int_{v_4} \begin{bmatrix} w1 \\ w2 \\ w3 \\ w4 \end{bmatrix} * R\mathrm{d}v = 0$$

$$\int_{v_4} \mathrm{W} * \Big(Q + \nabla\big(K\nabla(N * T)\big)\Big)\,\mathrm{d}v = 0$$

5) Método de Galerkin

$$\int_{v_4} w * \Big(Q + \nabla\big(K\nabla(N * T)\big)\Big)\,\mathrm{d}v = 0$$

$$\int_{v_4} N^T\,(Q + \nabla\big(K\nabla(\boldsymbol{N} * \boldsymbol{T})\big))\mathrm{d}v = 0$$

$$\int_{v_4} N^T\,Q + \Big(\nabla\big(K\nabla(N * T)\big)\Big)\,\mathrm{d}v = 0$$

$$\int_{v_4} N^T\,Q\,\mathrm{d}v + \;N^T(\nabla * \big(K\nabla(N * T)\big))\mathrm{d}v = 0$$

$$\int_{v_4} N^T * Q\mathrm{d}v + \int_{v_4} \boldsymbol{N^T} * \nabla\big(K\nabla(N * T)\big)\mathrm{d}v = 0$$

$$\int_{v_4} N^T * Q\mathrm{d}v + \left(\int_{v_4} N^T * \nabla\big(K\nabla(N)\big)\mathrm{d}v\right)T = 0$$

$$-\left(\int_{v_4} N^T * \nabla\big(K\nabla(N)\big)\mathrm{d}v\right)T = \int_{v_4} N^T * Q\mathrm{d}v$$

6) Resolución de Integrales

Resolviendo Termino Independiente (Lado Derecho).

$$Q\int_{v_4} N^T \, \mathrm{d}v$$

$$Q\int\int\int\int \begin{bmatrix} 1-\varepsilon-\eta-\phi-\alpha \\ \varepsilon \\ \eta \\ \phi \\ \alpha \end{bmatrix} \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}s$$

$$JQ\int\int\int\int \begin{bmatrix} 1-\varepsilon-\eta-\phi-\theta \\ \varepsilon \\ \eta \\ \phi \\ \alpha \end{bmatrix} \mathrm{d}\varepsilon\mathrm{d}\eta\mathrm{d}\phi\mathrm{d}\alpha$$

Para Calcular los Limites de Integración, Usamos la Ecuación del Hiperplano $Ax + By + Cz + Du \ + E = 0$, y en base al plano original podemos ya asumir los valores de algunos límites de las integrales.

$\varepsilon = (1,0,0,0)$ $\eta = (0,1,0,0)$ $\phi = (0,0,1,0)$ $\alpha = (0,0,0,1)$

Sustituimos los valores de sus coordenadas en las ecuaciones :

$$A\varepsilon + B\eta + C\phi + D\alpha + E = 0$$

$$A + E = 0$$

$$B + E = 0$$

$$C + E = 0$$

$$D + E = 0$$

$$E = -A \qquad B = A \qquad C = A \qquad D = -E$$

$$Sustituyendo\ en\ la\ Ecuacion\ Oriignal\ Nos\ Quedaria$$

$$A\varepsilon + A\eta + A\phi + A\alpha - A = 0$$

$$\frac{A\varepsilon}{A} + \frac{A\eta}{A} + \frac{A\phi}{A} + \frac{A\alpha}{A} - \frac{A}{A} = \frac{0}{A}$$

$$1 - \varepsilon + \eta + \phi + \alpha = 0$$

$$\varepsilon = 1 - \eta - \phi - \alpha$$

Por lo que nos quedaría de la siguiente Forma.

$$Q \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} \begin{bmatrix} 1-\varepsilon-\eta-\phi-\alpha \\ \varepsilon \\ \eta \\ \phi \\ \alpha \end{bmatrix} \mathrm{d}\varepsilon \mathrm{d}\eta \mathrm{d}\phi \mathrm{d}\alpha$$

$$QJ \begin{bmatrix} \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} (1-\varepsilon-\eta-\phi-\alpha) \\ \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} (\varepsilon) \\ \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} (\eta) \\ \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} (\phi) \\ \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} (\alpha) \end{bmatrix} \mathrm{d}\varepsilon \mathrm{d}\eta \mathrm{d}\phi \mathrm{d}\alpha$$

$$\int_0^{1-\eta-\phi-\alpha} \epsilon \, d\epsilon$$

$$\int \epsilon \, d\epsilon = \frac{\epsilon^2}{2} + C = \left[\frac{\epsilon^2}{2}\right]_0^{1-\eta-\phi-\alpha}$$

$$= \frac{(1-\eta-\phi-\alpha)^2}{2} - \frac{0^2}{2}$$

$$\int_0^{1-\eta-\phi-\alpha} \epsilon \, d\epsilon = \frac{(1-\eta-\phi-\alpha)^2}{2}$$

$$\int_0^{1-\phi-\alpha} \frac{(1-\eta-\phi-\alpha)^2}{2}\, d\eta$$

$$\int_0^{1-\phi-\alpha} \frac{((1-\phi-\alpha)^2 - 2(1-\phi-\alpha)\eta + \eta^2)}{2}\, d\eta$$

$$\int_0^{1-\phi-\alpha} \frac{(1-\phi-\alpha)^2}{2}\, d\eta - \int_0^{1-\phi-\alpha} (1-\phi-\alpha)\eta\, d\eta + \int_0^{1-\phi-\alpha} \frac{\eta^2}{2}$$

$$\int_0^{1-\phi-\alpha} \frac{(1-\phi-\alpha)^2}{2} d\eta = \frac{(1-\phi-\alpha)^2}{2} \int_0^{1-\phi-\alpha} d\eta$$

$$= \frac{(1-\phi-\alpha)^2}{2} [\eta]_0^{1-\phi-\alpha}$$

$$= \frac{(1-\phi-\alpha)^2}{2} \cdot (1-\phi-\alpha) = \frac{(1-\phi-\alpha)^3}{2}$$

$$\int_0^{1-\phi-\alpha} \frac{\eta^2}{2} d\eta = \frac{1}{2} \int_0^{1-\phi-\alpha} \eta^2 d\eta = \frac{1}{2} [\frac{\eta^3}{3}]_0^{1-\phi-\alpha} = \frac{1}{2} \cdot \frac{(1-\phi-\alpha)^3}{3} = \frac{(1-\phi-\alpha)^3}{6}$$

$$\frac{(1-\phi-\alpha)^3}{2} - \frac{(1-\phi-\alpha)^3}{2} + \frac{(1-\phi-\alpha)^3}{6} = \frac{(1-\phi-\alpha)^3}{6}$$

$$\frac{(1-\phi-\alpha)^3}{6} \times \frac{1}{2} = \frac{(1-\phi-\alpha)^3}{12}$$

$$\int_0^{1-\alpha} \frac{(1-\phi-\alpha)^3}{12} d\phi [(1-\phi-\alpha)^3 = (1-\phi-\alpha)(1-\phi-\alpha)(1-\phi-\alpha)]$$

$$(1-\phi-\alpha)^3 = 1 - 3\phi + 3\phi^2 + 6\phi\alpha + 3\alpha^2 - \phi^3 - 3\phi^2\alpha - 3\phi\alpha^2 - \alpha^3$$

$$[\frac{1}{12}\left(\int_0^{1-\alpha} 1\,d\phi - 3\int_0^{1-\alpha} \phi\,d\phi - 3\alpha\int_0^{1-\alpha} 1\,d\phi + 3\int_0^{1-\alpha} \phi^2\,d\phi + 6\alpha\int_0^{1-\alpha} \phi\,d\phi\cdot\right.$$

$$\left. + 3\alpha^2\int_0^{1-\alpha} 1\,d\phi - \int_0^{1-\alpha} \phi^3\,d\phi - 3\alpha\int_0^{1-\alpha} \phi^2\,d\phi - 3\alpha^2\int_0^{1-\alpha} \phi\,d\phi - \alpha^3\int_0^{1-\alpha} 1\,d\phi\right)]$$

$$[\frac{1}{12}\left[(1-\alpha) - 3\cdot\frac{(1-\alpha)^2}{2} - 3\alpha(1-\alpha) + 3\cdot\frac{(1-\alpha)^3}{3} + 6\alpha\cdot\frac{(1-\alpha)^2}{2} + 3\alpha^2(1-\alpha)\right.$$

$$\left. - \frac{(1-\alpha)^4}{4} - 3\alpha\cdot\frac{(1-\alpha)^3}{3} - 3\alpha^2\cdot\frac{(1-\alpha)^2}{2} - \alpha^3(1-\alpha)\right] =$$

$$[= \frac{(1-\alpha)^4}{48}]$$

$$\int_0^{1-\alpha} \frac{(1-\phi-\alpha)^3}{12}\,d\phi = \frac{(1-\alpha)^4}{48}$$

$$\int_0^1 \frac{(1-\alpha)^4}{48}\,d\alpha = \frac{1}{48}\int_0^1 (1-\alpha)^4\,d\alpha$$

$$(1-\alpha)^4 = \sum_{k=0}^{4}\binom{4}{k}(1)^{4-k}(-\alpha)^k$$

$$(1-\alpha)^4 = 1 - 4\alpha + 6\alpha^2 - 4\alpha^3 + \alpha^4$$

$$\frac{1}{48}\int_0^1 (1 - 4\alpha + 6\alpha^2 - 4\alpha^3 + \alpha^4)\,d\alpha$$

$$\frac{1}{48}(\int_0^1 1\ d\alpha - 4\int_0^1 \alpha\ d\alpha + 6\int_0^1 \alpha^2 d\alpha - 4\int_0^1 \alpha^3 d\alpha + \int_0^1 \alpha^4 d\alpha)$$

$$140 - 10 - 1 - 0 - 1 = \frac{1}{48}(128) = \frac{128}{48} = \frac{8}{3}$$

$$\int_0^1 \alpha^2 d\alpha = [\frac{\alpha^3}{3}]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

$$\int_0^1 \alpha^2 d\alpha = [\frac{\alpha^3}{3}]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

$$\int_0^1 \alpha^4 d\alpha = [\frac{\alpha^5}{5}]_0^1 = \frac{1^5}{5} - \frac{0^5}{5} = \frac{1}{5}$$

$$\frac{1}{6}(1 - 2 + 2 - 1 + \frac{1}{5}) = \frac{1}{6}(1 - 2 + 2 - 1 + \frac{1}{5}) = \frac{1}{6}(\frac{1}{5}) = \frac{1}{24}$$

$$\int_0^1 \frac{(1-\alpha)^4}{48} d\alpha = \frac{1}{24}$$

$$\int_0^{1-\eta-\phi-\alpha} \epsilon \ d\epsilon = \frac{1}{24}$$

Cuando Cambiamos los Limites de integración , e incluimos a Épsilon en el espacio, cambiando el orden de Integración , entonces decimos que eta, phi y Alpha se encuentran en el mismo espacio que épsilon por lo que la respuesta es la misma para todas las Variables de las funciones de Forma. Ya que se aplica la regla vista en clase de:

$$\int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\varepsilon-\phi-\alpha} \eta \ \mathrm{d}\eta \mathrm{d}\varepsilon \mathrm{d}\phi \mathrm{d}\alpha = \frac{1}{24}$$

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \int_0^{1-\varepsilon-\eta-\phi} \alpha \ \mathrm{d}\alpha \mathrm{d}\varepsilon \mathrm{d}\eta \mathrm{d}\phi = \frac{1}{24}$$

$$\int_0^1 \int_0^{1-\alpha} \int_0^{1-\eta-\alpha} \int_0^{1-\varepsilon-\eta-\alpha} \phi \ \mathrm{d}\phi \mathrm{d}\varepsilon \mathrm{d}\eta \mathrm{d}\alpha = \frac{1}{24}$$

$$\int_0^1 \int_0^{1-C} \int_0^{1-B-C} \int_0^{1-B-C-D} \mathrm{A}\mathrm{d}A\mathrm{d}B\mathrm{d}C\mathrm{d}D = \quad \frac{1}{24}$$

Sin Embargo, falta calcular la integral de:

$$\int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} 1 - \varepsilon - \eta - \phi - \theta \; \mathrm{d}\varepsilon \mathrm{d}\eta \mathrm{d}\phi \mathrm{d}\alpha$$

$$= \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} 1 \; d\epsilon d\eta d\phi d\alpha \quad - \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} \varepsilon \; d\epsilon d\eta d\phi d\alpha$$

$$- \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} \eta \; d\epsilon d\eta d\phi d\alpha \quad - \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} \phi \; d\epsilon d\eta d\phi d\alpha$$

$$- \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} \alpha \; d\epsilon d\eta d\phi d\alpha$$

$$= \int_0^1 \int_0^{1-\alpha} \int_0^{1-\phi-\alpha} \int_0^{1-\eta-\phi-\alpha} 1 \; d\epsilon d\eta d\phi d\alpha - \frac{1}{24} - \frac{1}{24} - \frac{1}{24} - \frac{1}{24}$$

$$\int_0^{1-\eta-\phi-\alpha} 1 \, d\epsilon = 1 \cdot ((1-\eta-\phi-\alpha) - 0)$$

$$1-\eta-\phi-\alpha$$

$$\int_0^{1-\phi-\alpha} 1-\eta-\phi-\alpha \, d\eta$$

$$\int (1-\phi-\alpha-\eta) \, d\eta = \int 1 \, d\eta - \phi \int d\eta - \alpha \int d\eta - \int \eta \, d\eta$$

$$\int 1 \, d\eta = \eta$$

$$\int \eta \, d\eta = \frac{\eta^2}{2}$$

$$\int (1-\phi-\alpha-\eta) \, d\eta = \eta - \phi\eta - \alpha\eta - \frac{\eta^2}{2}$$

$$\left[\eta - \phi\eta - \alpha\eta - \frac{\eta^2}{2}\right]_0^{1-\phi-\alpha}$$

$$\left[(1-\phi-\alpha)-\phi(1-\phi-\alpha)-\alpha(1-\phi-\alpha)-\frac{(1-\phi-\alpha)^2}{2}\right]$$

$$\phi^2+2\phi\alpha-2\phi-\frac{(\phi-\alpha+1)^2}{2}+\alpha^2-2\alpha+1$$

$$\int_0^{1-\phi}\phi^2+2\phi\alpha-2\phi-\frac{(\phi-\alpha+1)^2}{2}+\alpha^2-2\alpha+1\,d\alpha$$

$$\int_0^{1-\phi}\alpha^2 d\alpha+\int_0^{1-\phi}2\phi\alpha d\alpha-\int_0^{1-\phi}2\phi\,d\alpha-\int_0^{1-\phi}\frac{(\phi-\alpha+1)^2}{2}d\alpha+\int_0^{1-\phi}\alpha^2\,d\alpha-\int_0^{1-\phi}2\alpha\,d\alpha+\int_0^{1-\phi}1\,d\alpha$$

$$\int\alpha^2 d\alpha=\frac{\alpha^3}{3}$$

$$\int 2\phi\alpha d\alpha=2\phi\int\alpha d\alpha=2\phi\cdot\frac{\alpha^2}{2}=\phi\alpha^2$$

$$\int 2\phi d\alpha=2\phi\alpha$$

$$\int\frac{(\phi-\alpha+1)^2}{2}d\alpha=\frac{1}{2}\int(\phi^2-2\phi\alpha+\alpha^2+2\phi-2\alpha+1)d\alpha$$

$$=\frac{1}{2}\left(\phi^2\alpha-\phi\alpha^2+\frac{\alpha^3}{3}+2\phi\alpha-\alpha^2+\alpha\right)$$

$$\frac{1}{2}\left((\phi^2+2(\phi-1)+(\alpha-1)^3\right)d\alpha=\frac{1}{2}\left(\frac{\alpha^3}{3}+2(\phi-1)\alpha^2+(\alpha-1)^3\right)$$

$$= \frac{\phi^3}{3} + \alpha\phi^2 - \phi^2 = \frac{1}{2}\left[\frac{\phi^3}{3} + 2(\alpha-1)\phi^2 + (\alpha-1)^2\phi\right] + \frac{\alpha^2}{2} - 2\alpha\phi + \alpha\Big|_0^{1-\alpha}$$

$$\int_0^{1-\alpha}\left(\phi^2 + 2\phi\alpha - 2\phi - \frac{(\phi-\alpha+1)^2}{2} + \alpha^2 - 2\alpha + 1\right)d\phi = \frac{(\alpha+1)^3}{3} - \frac{(\alpha+1)^3}{6}$$

$$\int_0^1\left[\frac{(\alpha+1)^3}{3} - \frac{(\alpha+1)^3}{6}\right]d\alpha$$

$$= \left[\frac{1}{3}\left(\frac{\alpha^4}{4} + \frac{3\alpha^3}{2} - \alpha\right) - \frac{1}{6}\left(\alpha^4 + \alpha^3 - 2\alpha\right)\right]_0^1$$

$$= \frac{1}{3}\left(\frac{1^4}{4} + \frac{3\cdot 1^3}{2} - 1\right) - \frac{1}{6}\left(1^4 + 1^3 - 2\cdot 1\right)$$

$$= -\frac{1}{3}\left(\frac{1}{4} + \frac{3\cdot 0^2}{2} - 0\right) - \frac{1}{6}\left(0^4 + 0^3 - 2\cdot 0\right)$$

$$= \left(0^4\cdot\frac{3}{4} + 0^3\cdot\frac{3\cdot 0^2}{2} - 0\right)$$

$$\int_0^1\left[-\frac{(\alpha+1)^3}{3} - \left(-\frac{(\alpha+1)^3}{6}\right)\right]d\alpha = \frac{1}{6}$$

$$\int_0^1\int_0^{1-\alpha}\int_0^{1-\phi-\alpha}\int_0^{1-\eta-\phi-\alpha} 1\ d\epsilon d\eta d\phi d\alpha = \frac{1}{6}$$

Entonces sustituyendo nos da los valores de

$$= \frac{1}{6} - \frac{1}{24} - \frac{1}{24} - \frac{1}{24} - \frac{1}{24} = 0$$

Por lo que la Resolución al lado derecho de las integrales

$$Q\int_0^1\int_0^{1-\alpha}\int_0^{1-\phi-\alpha}\int_0^{1-\eta-\phi-\alpha}\begin{bmatrix}1-\varepsilon-\eta-\phi-\alpha\\ \varepsilon\\ \eta\\ \phi\\ \alpha\end{bmatrix}\mathrm{d}\varepsilon\mathrm{d}\eta\mathrm{d}\phi\mathrm{d}\alpha$$

$$=QJ\begin{bmatrix}0\\ \frac{1}{24}\\ \frac{1}{24}\\ \frac{1}{24}\\ \frac{1}{24}\end{bmatrix}=\frac{QJ}{24}\begin{bmatrix}0\\1\\1\\1\\1\end{bmatrix}=\mathbf{b}$$

Procedemos a Resolver el lado Izquierdo de la Ecuación:

$$-\left(\int_{v_4} N^T * \nabla\big(K\nabla(N)\big)\mathrm{d}v\right)T$$

Aplicamos integración por Partes

$$\int udv \; - \; [uv]| - \int duv$$

$$U = \; \boldsymbol{N^T} \; dv = \nabla\big(K\nabla(\boldsymbol{N})\big)$$

$$du = \nabla \; \boldsymbol{N^T} \; v = \; K\nabla(\boldsymbol{N})$$

$$-[\nabla\mathbf{N}^{\mathbf{T}}K\nabla(\mathbf{N})]|v-(\int_{v4}\nabla\mathbf{N}^{\mathbf{T}}(K\nabla(\mathbf{N}))\mathrm{d}v)$$

$$\nabla \mathbf{N} = \begin{bmatrix} \frac{\mathrm{d}f}{\mathrm{d}x} \\ \frac{\mathrm{d}f}{\mathrm{d}y} \\ \frac{\mathrm{d}f}{\mathrm{d}z} \\ \frac{\mathrm{d}f}{\mathrm{d}s} \end{bmatrix} * \eta[1-\varepsilon-\eta-\phi-\alpha \quad \varepsilon \quad \eta \quad \phi \quad \alpha]$$

$$\nabla \mathbf{N} = J^{-1} * \begin{bmatrix} \frac{\partial}{\partial \varepsilon}(1-\varepsilon-\eta-\phi-\alpha) & \frac{\partial}{\partial \varepsilon}\varepsilon & \frac{\partial}{\partial \varepsilon}\eta & \frac{\partial}{\partial \varepsilon}\phi & \frac{\partial}{\partial \varepsilon}\alpha \\ \frac{\partial}{\partial \eta}(1-\varepsilon-\eta-\phi-\alpha) & \frac{\partial}{\partial \eta}\varepsilon & \frac{\partial}{\partial \eta}\eta & \frac{\partial}{\partial \eta}\phi & \frac{\partial}{\partial \eta}\alpha \\ \frac{\partial}{\partial \phi}(1-\varepsilon-\eta-\phi-\alpha) & \frac{\partial}{\partial \phi}\varepsilon & \frac{\partial}{\partial \phi}\eta & \frac{\partial}{\partial \phi}\phi & \frac{\partial}{\partial \phi}\alpha \\ \frac{\partial}{\partial \alpha}(1-\varepsilon-\eta-\phi-\alpha) & \frac{\partial}{\partial \alpha}\varepsilon & \frac{\partial}{\partial \alpha}\eta & \frac{\partial}{\partial \alpha}\phi & \frac{\partial}{\partial \alpha}\alpha \end{bmatrix}$$

$$\nabla \mathbf{N} = \mathbf{J}^{-1} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\nabla \mathbf{N} = \mathbf{J}^{-1}\mathbf{B}$$

Es Necesario Calcular también el Jacobiano:

$$J = \begin{bmatrix} \frac{\partial \mathrm{x}}{\partial \varepsilon} & \frac{\partial \mathrm{x}}{\partial \eta} & \frac{\partial \mathrm{x}}{\partial \phi} & \frac{\partial \mathrm{x}}{\partial \alpha} \\ \frac{\partial \mathrm{y}}{\partial \varepsilon} & \frac{\partial \mathrm{y}}{\partial \eta} & \frac{\partial \mathrm{y}}{\partial \phi} & \frac{\partial \mathrm{y}}{\partial \alpha} \\ \frac{\partial \mathrm{z}}{\partial \varepsilon} & \frac{\partial \mathrm{z}}{\partial \eta} & \frac{\partial \mathrm{z}}{\partial \phi} & \frac{\partial \mathrm{z}}{\partial \alpha} \\ \frac{\partial \mathrm{s}}{\partial \varepsilon} & \frac{\partial \mathrm{s}}{\partial \eta} & \frac{\partial \mathrm{s}}{\partial \phi} & \frac{\partial \mathrm{s}}{\partial \alpha} \end{bmatrix}$$

Necesitamos Calcular los Datos del Jacobiano.

$$x = N1 * x1 + N2 * x2 + N3 * x3 + N4 * x4 + N5 * x5$$

$$x = (1 - \varepsilon - \eta - \phi - \theta) * x1 + (\varepsilon) * x2 + (\eta) * x3 + (\phi) * x4 + (\theta) * x5$$

$$x = x1 - \varepsilon(x1) - \eta(x1) - \phi(x1) - \theta(x1) + \varepsilon(x2) + \eta(x3) + \phi(x4) + \theta(x5)$$

$$x = (x2 - x1)\varepsilon + (x3 - x1)\eta + (x4 - x1)\phi + (x5 - x1)\theta + x1$$

$$y = (y2 - y1)\varepsilon + (y3 - y1)\eta + (y4 - y1)\phi + (y5 - y1)\theta + y1$$

$$z = (z2 - z1)\varepsilon + (z3 - z1)\eta + (z4 - z1)\phi + (z5 - z1)\theta + z1$$

$$u = (s2 - s1)\varepsilon + (s3 - s1)\eta + (s4 - s1)\phi + (s5 - s1)\theta + s1$$

$$j = \begin{bmatrix} (x2 - x1) & (x3 - x1) & (x4 - x1) & (x5 - x1) \\ (y2 - y1) & (y3 - y1) & (y4 - y1) & (y5 - y1) \\ (z2 - z1) & (z3 - z1) & (z4 - z1) & (z5 - z1) \\ (s2 - s1) & (s3 - s1) & (s4 - s1) & (s5 - s1) \end{bmatrix}$$

Donde el Termino de A y A^T se calculan con la inversa del Jacobiano y con con la transpuesta de la inversa , debido a ser un proceso muy complejo y largo por ser en 4 dimensiones, se hace la observación , de que A y A^T , se dejan indicadas.

$$\int_{v_4} N^T * \nabla\big(K\nabla(N)\big)\mathrm{d}v$$

$$N^T * \nabla\big(K\nabla(N)\big)\int_{v_4} \mathrm{d}v$$

$$\frac{k}{j^2} B^T A^T A B \int_{v_4} \mathrm{d}v$$

$$\frac{kV}{j^2} B^T A^T A B$$

$$= K$$

Por lo que Uniendo todas las Respuestas quedaría de la siguiente Forma:

$$\left(\frac{KV_4}{j^2}\mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}\mathbf{A}\mathbf{B}\right)\begin{bmatrix} T1 \\ T2 \\ T3 \\ T4 \\ T5 \end{bmatrix} = \frac{QJ}{24}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Sistema de Ecuaciones Finales

$$\left(\frac{KV_4}{j^2}\mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}\mathbf{A}\mathbf{B}\right)\mathbf{T} = \frac{QJ}{24}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{K} * \mathbf{T} = \mathbf{B}$$