### Business 4720 - Class 22

#### Reinforcement Learning - Function Approximation

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### This Class

#### What You Will Learn:

- ▶ Reinforcement Learning
  - ► Action-value approximation
  - Policy approximation



## Based On

Richard S. Sutton and Andrew G. Barto (2018) *Reinforcement Learning – An Introduction*. 2nd edition, The MIT Press, Cambridge, MA. (SB)

http://incompleteideas.net/book/the-book.html

Chapters 9–13

Sudharsan Ravichandiran (2020) *Deep Reinforcement Learning with Python*. 2nd edition. Packt Publishing, Birmingham, UK.

Chapters 9-11



### Resources

#### Implementations are available on the following GitHub repo:

https://github.com/jevermann/busi4720-rl

#### The project can be cloned from this URL:

https://github.com/jevermann/busi4720-rl.git



# **Function Approximation**

## Previously

 Tabular methods only suitable for small state space and discrete actions

#### Now

Approximate the state-value function v by parameterized function  $\hat{v}$ :

$$\hat{\mathbf{v}}(\mathbf{s}) = \hat{\mathbf{v}}(\mathbf{s}, \theta) \approx \mathbf{v}_{\pi}(\mathbf{s})$$

► Approximate the action-value function *q* by a parameterized function  $\hat{q}$ :

$$\hat{q}(s,a) = \hat{q}(s,a, heta) pprox q_{\pi}(s,a)$$

▶ Approximate policy  $\pi$  by a parameterized function  $\hat{\pi}$ :

$$\hat{\pi}(a|s) = \hat{\pi}(a|s,\theta) \approx \pi(a|s)$$



# **Function Approximation**

### Advantages

- Continuous states and/or actions
- Tractable problems despite large state space
- Flexible functions (linear, trees, neural networks)
- Generalization to related states
  - Changing  $\theta$  changes the values of multiple states
- Applicable to partially observable problems
  - State function may not depend on complete state information



## Stochastic Gradient Methods

Assume a MSE value error:

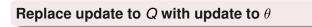
$$ar{\mathit{VE}} = \sum_{m{s} \in \mathcal{S}} \mu(m{s}) \left[ m{q}_{\pi}(m{s}, m{a}) - \hat{m{q}}(m{s}, m{a}, m{ heta}) 
ight]^2$$

Follow the steepest slope ("gradient"; vector of partial derivatives) of the function to update parameters:

$$\theta_{t+1} = \theta_t - \frac{1}{2} \alpha \nabla \left[ q_{\pi}(S_t, A_t) - \hat{q}(S_t, A_t, \theta_t) \right]^2$$
  
=  $\theta_t + \alpha \left[ q_{\pi}(S_t, A_t) - \hat{q}(S_t, A_t, \theta_t) \right] \nabla \hat{q}(S_t, A_t, \theta_t)$ 

True value  $q_{\pi}(S_t, A_t)$  generally unknown; use an unbiased estimate  $U_t = R_t + \gamma \hat{q}(S_{t+1}, A_{t+1}, \theta)$  instead:

$$\theta_{t+1} = \theta_t + \alpha \left[ U_t - \hat{q}(S_t, A_t, \theta_t) \right] \nabla \hat{q}(S_t, A_t, \theta_t)$$





## Example – Semi-gradient SARSA

until S is terminal

```
Initialize \theta \in \mathbb{R}^d arbitrarily
Loop for each episode:
   Initialize S<sub>∩</sub>
   Choose A as a function of \hat{q}(S_0,..,\theta) e.g., \epsilon-greedy
   Loop for each step of episode:
      Take action A. observe R, S'
       Choose A' as a function of \hat{q}(S',..,\theta) e.g., \epsilon-greedy
      \theta \leftarrow \theta + \alpha [R + \gamma \hat{q}(S', A', \theta) - \hat{q}(S, A, \theta)] \nabla \hat{q}(S, A, \theta)
       S \leftarrow S' \cdot A \leftarrow A'
```

## "The Deadly Triad"

**Instability** and **Divergence** arise when combining all three elements:

- ► Function approximation: Generalizing from a state space using linear functions or neural networks
- Bootstrapping: Targets include existing estimates (e.g. SARSA) rather than actual rewards only (e.g. MC methods)
- Off-policy training: Training on a distribution of transitions other than that produced by the target policy



### DQN – Double Q Network

### **Experience Replay**

- ► Store sequences *S*, *A*, *R*, *S'*, *A'* in replay buffer
- ► FIFO gueue of limited size
- Sample from replay buffer for each batch
- Removes/limits correlation of states within batches
- Smoothes data distribution changes

## **Target Network**

- Maintain stable targets during updates
- ► Periodic update from "main" network



## DQN - Algorithm

Init replay buffer  $D \leftarrow \emptyset$ 

Init main action-value function approximation  $\hat{q}_M$  with random parameters  $\theta_M$  Init target action-value function approximation  $\hat{q}_T$  with parameters  $\theta_T = \theta_M$  Loop for each episode:

Initialize S

For each step of the episode:

Select action A using an  $\epsilon$ -greedy policy based on  $\hat{q_M}$ 

Take action A and observe R,  $S_{t+1}$ 

Store transition  $(S_t, A_t, R_t, S_{t+1})$  in D

Sample minibatch  $(S_j, A_j, R_j, S_{j+1})$  from D

Target 
$$y_j \leftarrow \begin{cases} r_j \\ r_j + \gamma \max_{A'} \hat{q}_T(S_{j+1}, A'; \theta_T) \end{cases}$$

if  $S_{j+1}$  is terminal otherwise

$$\theta_{M} \leftarrow \theta_{M} + \alpha[y_{j} - \hat{q}_{M}(S_{j}, A_{j}, \theta_{M})]\nabla\hat{q}_{M}(S_{j}, A_{j}, \theta_{M})$$

Every *C* steps, update  $\hat{q}_T \leftarrow \hat{q}_M$  by setting  $\theta_T \leftarrow \theta_M$ 

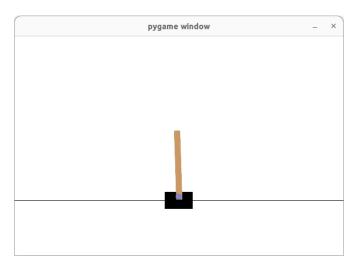


## DQN – Algorithm

- ▶ In practice, S is a function  $\phi(X)$  of inputs X through feature-extraction and pre-processing
- ▶ In practice, the update  $y_i Q(S_i, a_i, \theta)$  is clipped to [-1, 1]



# DQN Example – Cart Pole





## DQN Example – Cart Pole

#### Action Space:

0	Push cart to the left
1	Push cart to the right

#### State/feature space:

Num	Observation	Min	Max
0	Cart position	-4.8	4.8
1	Cart velocity	-Inf	Inf
2	Pole angle	-24 deg	24 deg
3	Pole angular velocity	-Inf	Inf

Rewards are +1 for every step taken

#### Termination occurs either:

- ► Pole angle is greater than ±12 deg
- ► Cart position is greater than  $\pm 2.4$
- Episode length is grater than 200



# DQN in Python

#### Use the "CartPole" environment from

https://gymnasium.farama.org/:

```
import math
import random
import keras
from keras import layers
import gymnasium as gym
import tensorflow as tf
import numpy as np
import pygame
env = gym.make("CartPole-v1", render_mode="human")
Actions = range(0, env.action_space.n)
Ssize = env.observation_space.shape[0]
```



### Neural network and RL hyperparameters:

```
# Neural net parameters
batch_size = 8
dropout = 0.25
activation = 'relu'

epsilon = 0.8 # initial epsilon
gamma = 0.9 # discount factor
neps = 1000 # dicreasing epsilon factor
C = 50*batch_size # When to update weights

# Replay buffer D
D = collections.deque(maxlen=5000)
```

#### Define the neural networks:

```
# Main network, used to select actions
0 = keras.Sequential([
    lavers.InputLayer(input_shape=(Ssize+1),
                      batch size=batch size,
                      dtvpe=tf.float32).
    layers.Dense(Ssize*4, activation=activation),
    layers.Dropout (rate=dropout),
    layers.Dense(Ssize*2, activation=activation),
    layers.Dropout (rate=dropout),
    layers.Dense(1, activation='linear')
1)
O.compile(loss='huber', optimizer='adam')
# Target network, used to compute targets
Ohat = keras.models.clone model(0)
Qhat.compile(loss='huber', optimizer='adam')
Ohat.set weights(O.get weights())
```

Getting a Q(s, a) value involves predicting from the neural net:

```
def getQ(Q, s, a):
    return Q.predict(np.expand_dims(np.array( \
        s.tolist()+[a]), axis=0), verbose=0)[0][0]
```

Max/Argmax operator for Q(s, a) using prediction from main or target network:

```
def maxQ(Q, s, arg):
    maxq = -np.inf
    maxa = None
    for a in Actions:
        q = getQ(Q, s, a)
        if q > maxq:
            maxq = q
            maxa = a
    return maxa if arg else maxq
```

### $\epsilon$ -greedy policy $pi_{\epsilon}(s)$

```
def pi(s, epsilon):
    if random.random() < epsilon:
        return random.choice(Actions)
    else:
        return maxQ(Q, s, True)</pre>
```

#### Update target for DQN:

```
def target_DQN(Q, Qhat, a, r, sprime):
    return r + gamma * maxQ(Qhat, sprime, False)
```

#### Update target for DDQN:



### Creating x and y data for training:

```
def training_xy(batch, ddqn=False):
    x = np.zeros((batch_size, Ssize+1))
    y = np.zeros(batch_size)
    for i, (s, a, r, t, sprime) in enumerate(batch):
        x[i] = list(s) + [a]
        if t == 1:
            y[i] = r
        else:
            if ddqn:
                 y[i]=target_DDQN(Q, Qhat, a, r, sprime)
        else:
            y[i]=target_DQN(Q, Qhat, a, r, sprime)
        return x, y
```

#### DQN/DDQN code:

```
for t in range (max steps):
    s = env.reset()[0]
    a = pi(s, epsilon*math.exp(-t/neps))
    sprime, r, terminal, , = env.step(a)
    G += r
    D.append((s, a, r, int(terminal), sprime))
    s = sprime
    if t >= batch size:
        batch = random.sample(D, batch size)
        x, y = training_xy(batch, ddgn=True)
        loss = 0.train on batch(x=x, v=v)
    if t % C == 0:
        Qhat.set_weights(Q.get_weights())
```

### Complete code at

https://evermann.ca/busi4720/DDQN\_tuples.py



## DQN Extensions

### Prioritized Experience Replay (PEX)

- Important actions are sampled with higher probability
- Use absolute TD error as priorities
- Faster learning

#### Double DQN

- Based on Double Q-Learning
- Uses target network Q as second Q function
- Removes upwards bias from using max() functions as estimator



# Advantage Function

$$A(s,a) = Q(s,a) - V(s)$$

Advantage of action a in state s over the average action in state s

Rewrite as:

$$Q(s,a) = V(s) + A(s,a)$$



# **Dueling DQN**

- Neural network from features x for value function V(s) ("value stream")
- Neural network from features x for advantage function A(s, a) ("advantage stream")
- Typically, value stream and advantage stream follow one or more common layers
- Aggregate to compute Q(s, a)

$$Q(s, a, \theta, \alpha, \beta) = V(s, \theta, \beta) + \left(A(s, a, \theta, \alpha) - \frac{a}{|A|}A(s, a', \theta, \alpha)\right)$$

Where  $\theta$  are shared neural-network parameters,  $\beta$  are parameters only for the "value-stream" neural network, and  $\alpha$  are parameters only for the "advantage-stream" neural network

#### Idea

Learn a parameterized policy:

$$\pi(s, a) = \pi(s, a, \theta) = \Pr(A_t = a | S_t = s, \theta_t = \theta)$$

Optimize:

$$J( heta) = extstyle v_{\pi_{ heta}}( extstyle s_o)$$



### Advantages

- Simpler to approximate than action-value function
- Selection of actions with arbitrary probabilities
- ▶ Can better approach deterministic policy than  $\epsilon$ -greedy action selection over action values
- Suitable for large and continuous action spaces



#### REINFORCE update:

$$\theta_{t+1} = \theta_t + \alpha G_t \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta_t)}$$

- Update proportional to return G<sub>t</sub>
- lacktriangle Update inversely proportional to action probability  $\pi$



### REINFORCE: Monte-Carlo Control (episodic)

Input: A differentiable policy  $\pi(a|s,\theta)$ ; step size  $\alpha>0$ Initialize policy parameters  $\theta\in\mathbb{R}^d$  arbitrarily Loop forever (for each episode):

Generate an episode  $S_0, A_0, R_1, \dots S_{T-1}, A_{T-1}, R_T$ , Loop for each step of the episode  $t = 0, 1, \dots, T-1$ :

$$G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$
$$\theta \leftarrow \theta + \alpha G \nabla \ln \pi (A_t | S_t, \theta)$$



## REINFORCE in Python

### Policy function/network for discrete actions:

### Action selection using policy network probabilities:

```
def select_action(state):
    probs = p_net(np.expand_dims(state, axis=0))[0].numpy()
    action = np.random.choice(Actions, size=1, p=probs)[0]
    return action
```

## REINFORCE in Python [cont'd]

#### Generate an episode:

```
# Do this for each episode
for e in range (neps):
    # Initialize variables and lists
    T = 0
    rewards, states, actions = [], [], []
    # Reset environment
    s = env.reset()[0]
    terminal = False
    # Generate an episode and keep track
    # of states, actions, rewards
    while (T < max_steps) and not terminal:</pre>
        a = select_action(s)
        sprime, r, terminal, _, _ = env.step(a)
        states.append(s)
        actions.append(a)
        rewards.append(r)
        s = sprime
        T += 1
    print(f'Episode {e:5} goes to step {T:3}')
    # Compute discounted returns
    returns = discounted_returns(rewards, gamma)
```

## REINFORCE in Python [cont'd]

### Learning from episode:

```
# Learn for each step of the episode
for t in range(len(returns)):
    with tf.GradientTape() as tape:
        # Action probabilities
        pi = p_net(np.expand_dims(states[t], axis=0))
        # Action index
        action_idx = np.array(actions[t], dtype=np.int32)
        # Return
        G = np.array(returns[t])
        # LOSS
        loss = - G * gamma**t *
            tf.math.log(tf.reduce_sum(tf.math.multiply(pi,
                tf.one_hot(action_idx, env.action_space.n)),
                    axis=1))
    # Calculate gradients and update parameters
    grads = tape.gradient(loss, p_net.trainable_variables)
    optimizer.apply_gradients(zip(grads,
        p_net.trainable_variables))
```

REINFORCE with a baseline  $b(S_t)$ :

$$heta_{t+1} = heta_t + lpha(G_t - b(S_t)) rac{
abla \pi(A_t | S_t, heta)}{\pi(A_t | S_t, heta_t)}$$

Choose  $b(S_t) = \hat{v}(S_t)$  the state-value function:

$$egin{aligned} heta_{t+1} &= heta_t + lpha(G_t - \hat{oldsymbol{v}}(S_t)) rac{
abla \pi(A_t | S_t, heta)}{\pi(A_t | S_t, heta_t)} \end{aligned}$$

- Baseline leaves the expected value unchanged (unbiased)
- Can significantly reduce the variance
- Can improve speed of learning



### REINFORCE with Baseline (episodic)

Input: A policy  $\pi(a|s,\theta)$ ; step size  $\alpha_{\theta} > 0$ 

Input: A state-value function  $\hat{v}(s, w)$ ; step size  $\alpha_w > 0$ 

Initialize parameters  $\theta \in \mathbb{R}^d$ ,  $w \in \mathbb{R}^d$  arbitrarily

Loop forever (for each episode):

Generate an episode  $S_0, A_0, R_1, \dots S_{T-1}, A_{T-1}, R_T$ , Loop for each step of the episode  $t = 0, 1, \dots, T-1$ :

$$G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$
  
$$\delta \leftarrow G - \hat{v}(S_t, w)$$
  
$$w \leftarrow w + \alpha_w \delta \nabla \hat{v}(S_t, w)$$
  
$$\theta \leftarrow \theta + \alpha_\theta \gamma^t G \nabla \ln \pi(A_t | S_t, \theta)$$



# REINFORCE with Baseline in Python [cont'd]

#### Learning from episode:

```
for t in range(len(returns)):
    with tf.GradientTape() as p_tape:
        with tf.GradientTape() as v_tape:
            pi = policy_network(np.expand_dims(states[t], axis=0))
            action_idx = np.array(actions[t], dtype=np.int32)
            # Value function
            v = value network(np.expand dims(states[t], axis=0))
            # Return
            G = np.array(returns[t])
            delta = G - v
            # Losses for policy and value networks
            p_loss = -delta * gamma**t * tf.math.log(tf.reduce_sum(tf
            v loss = -delta * v
    # Calculate gradients and update parameters for policy network
    p_grads = p_tape.gradient(p_loss,p_net.trainable_variables)
    p_optimizer.apply_gradients(zip(p_grads,
        p_net.trainable_variables))
    # Calculate gradients and update parameters for value network
    v_grads = v_tape.gradient(v_loss,v_net.trainable_variables)
    v optimizer.apply gradients(zip(v grads,
        v network.trainable variables))
```

#### **One-Step Actor-Critic:**

$$\theta_{t+1} = \theta_t + \alpha (G_t - \hat{v}(S_t)) \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta_t)}$$

$$= \theta_t + \alpha (R_{t+1} + \gamma \hat{v}(S_{t+1}, w) - \hat{v}(S_t, w)) \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta_t)}$$

$$= \theta_t + \alpha \delta_t \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta_t)}$$

- Analogous to TD, SARSA and Q-Learning for tabular methods
- Improve on slow learning of MC methods
- Useful for non-episodic, continuous problems



## One-Step Actor-Critic

```
Input: A policy \pi(a|s,\theta); step size \alpha_{\theta} > 0
Input: A state-value function \hat{v}(s, w); step size \alpha_w > 0
Initialize parameters \theta \in \mathbb{R}^d, \mathbf{w} \in \mathbb{R}^d arbitrarily
Loop forever (for each episode):
    Initialize S (first state of episode); I \leftarrow 1
   Loop while S not terminal (for each time step):
        Sample A from \pi(.|S,\theta)
       Take action A. observe S', R
       \delta \leftarrow R + \gamma \hat{\mathbf{v}}(S', \mathbf{w}) - \hat{\mathbf{v}}(S, \mathbf{w})
        \mathbf{W} \leftarrow \mathbf{W} + \alpha_{\mathbf{W}} \delta \nabla \hat{\mathbf{v}}(\mathbf{S}_t, \mathbf{W})
       \theta \leftarrow \theta + \alpha_{\theta} G \nabla \ln \pi (A_t | S_t, \theta)
        S \leftarrow S' : I \leftarrow \gamma I
```

## Reference Implementations

#### Stable Baselines

- Reference Python implementation of RL algorithms
- Pre-trained agents ("Baselines Zoo")
- Originally developed at OpenAl
- Pointers to additional learning materials

https://stable-baselines.readthedocs.io/en/master/

## Gymnasium

- ► A standard programming interface (API) for RL environments
- Collection of reference environments
- Originally developed at OpenAl

https://gymnasium.farama.org/index.html



## AlphaGo



https://www.alphagomovie.com

Google's DeepMind division became famous in 2017 when it trained a computer to beat the human world champion at the game of Go. An award-winning full-length documentary has been made about this achievement.

https://www.alphagomovie.com https://www.youtube.com/watch? v=WXuK6gekU1Y

The introductory paper by David Silver and others in Nature should be easy to understand: "Mastering the game of Go without human knowledge". Nature. 550 (7676): 354–359

https://www.nature.com/articles/nature24270



### Additional Materials I

### David Silver, UCL and Google DeepMind

Dr. Silver (https://www.davidsilver.uk/) of University College London has an excellent introductory course on reinforcement learning with class materials (from 2015) and lectures in a YouTube playlist. Updated courses (2018, 2021) are available on the DeepMind YouTube channel. The 2021 course include topics on deep reinforcement learning.

```
https://www.davidsilver.uk/teaching/
```

https://www.youtube.com/playlist?list= PLqYmG7hTraZDM-OYHWqPebj2MfCFzFObQ.

https://www.youtube.com/@Google\_DeepMind/playlists.



### Additional Materials II

## **UC** Berkeley

UC Berkeley hosted a Deep RL Bootcamp in 2017 with slides and lecture videos available online. Additionally, UC Berkeley's course on Deep RL is available online, with lecture slides and videos of past years.

```
https://sites.google.com/view/deep-rl-bootcamp/lectures
```

```
https://rail.eecs.berkeley.edu/deeprlcourse/
```

### **Denny Britz**

Formerly at the Google AI team, Denny Britz applied RL algorithms to financial markets and trading. He has a interesting blog, and a GitHub repository with resources and algorithm implementations of popular RL algorithms.

```
https://dennybritz.com/
https://github.com/dennybritz/reinforcement-learning
```

### Additional Materials III

## Massimiliano Patacchiola, Cambridge University

Dr. Patacchiola is a postdoc at Cambridge University. He has written a series of excellent blog posts on reinforcement based on the book "Artificial Intelligence – A Modern Approach" by Russell and Norvig. There are lots of illustrations and pointers to implementation and code in multiple languages.

```
https://github.com/mpatacchiola/dissecting-reinforcement-learning
```

## Pascal Poupart, University of Waterloo

Dr. Poupart has made available videos and all course materials for all lectures for a course on reinforcement learning at UWaterloo.

```
https://www.youtube.com/playlist?list=
PLdAoL1zKcqTXFJniO3Tqqn6xMBBL07EDc
https://cs.uwaterloo.ca/~ppoupart/teaching/
cs885-spring18/schedule.html
```

### Additional Materials IV

## Andrew Ng, Stanford University

Dr. Ng (https://www.andrewng.org/) has taught an introductory class on reinforcement learning, as part of a broader course on machine learning.

```
https://www.youtube.com/watch?v=RtxI449ZjSc
```

https://www.youtube.com/playlist?list=PLA89DCFA6ADACE599

## Andrei Karpathy, OpenAI, formerly Tesla

Andrei Karpathy (https://karpathy.ai/ was a founding member of OpenAI (makers of ChatGPT and Dall-E) and later became the Tesla lead for their Autopilot program. An early blog post by Andrei Karpathy on RL is at the introductory level.

https://karpathy.github.io/2016/05/31/rl/



### Additional Materials V

### Lilian Weng, OpenAl

Dr. Weng (https://lilianweng.github.io/) is a lead researchers at OpenAI (makers of ChatGPT and Dall-E). She has written an early blog post on RL and another one on policy gradient algorithms.

```
https:
//lilianweng.github.io/posts/2018-02-19-rl-overview/
https:
//lilianweng.github.io/posts/2018-04-08-policy-gradient/
```



### Additional Materials VI

### **OpenAl**

OpenAI (https://openai.com; makers of ChatGPT and Dall-E) post regularly on their blog, on all things deep learning and also reinforcement learning. The blog posts are easy introduction to a variety of analytics topics.

```
https://openai.com/blog/openai-baselines-ppo/
https://openai.com/blog/evolved-policy-gradients/
https://openai.com/blog/evolution-strategies/
```

