Business 4720 - Class 12

Supervised Machine Learning using R

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This Class

What You Will Learn:

- Linear Regression Models in R
  - Linear regression
  - Lasso and Ridge regression
- Classification Models in R
  - Logistic Regression
  - K-NN

Based On

Gareth James, Daniel Witten, Trevor Hastie and Robert Tibshirani: *An Introduction to Statistical Learning with Applications in R*. 2nd edition, corrected printing, June 2023. (ISLR2)

`https://www.statlearning.com`

Chapters 2, 3, 4, 5

Trevor Hastie, Robert Tibshirani, and Jerome Friedman: *The Elements of Statistical Learning*. 2nd edition, 12th corrected printing, 2017. (ESL)

`https://hastie.su.domains/ElemStatLearn/`

Chapters 2, 3, 4, 7

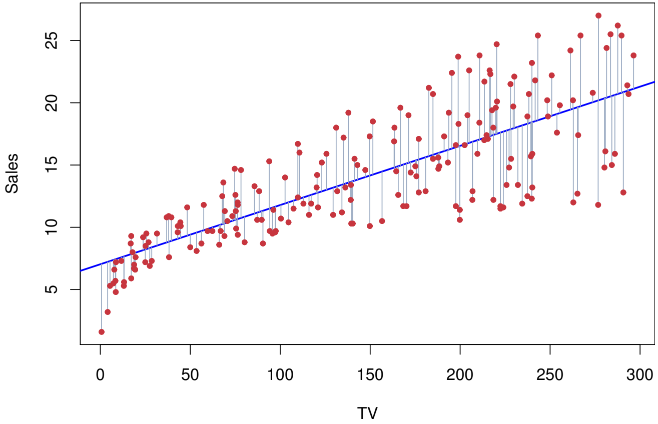
Kevin P. Murphy: *Probabilistic Machine Learning – An Introduction*. MIT Press 2022.

`https://probml.github.io/pml-book/book1.html`

Chapters 4, 6, 9, 10, 11

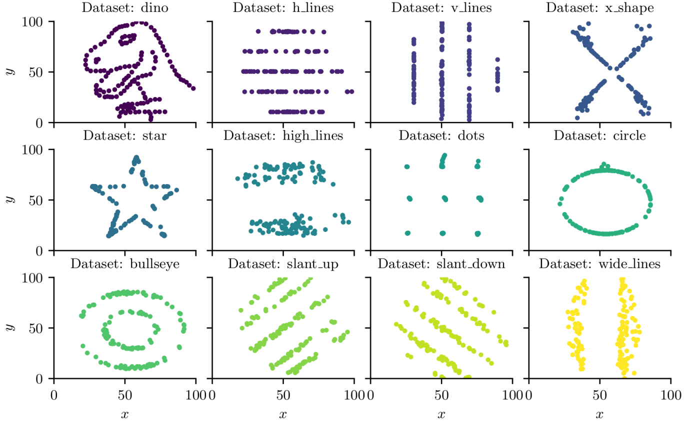
Linear Regression

$$Y = \beta_0 + \beta_1 X + \epsilon$$



Source: ISLR2 Figure 3.1

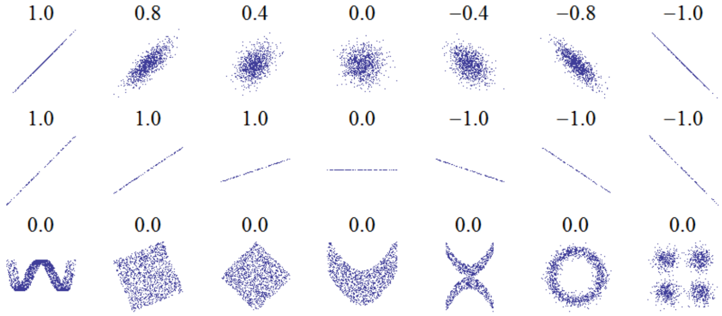
Ensure a Linear Model is Sensible



Source: Murphy Figure 2.6

The ”Datasaurus Dozen”: All datasets have the same correlation between the two variables!

Correlation versus Regression



Source: Murphy Figure 3.1

Datasets with the same correlation (as indicated above each dataset) between two variables do not need to have the same regression slope!

Estimating Linear Regression [cont'd]

- Estimate $\hat{\beta}_0$ and $\hat{\beta}_1$ to minimize the mean squared error (MSE) or residual sum of square (RSS)

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \qquad \text{Predicted/fitted values}$$

$$RSS = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right)^2$$

$$MSE = \frac{1}{n} RSS$$

- Analytically derivable *least squares estimates* are

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

where $\bar{x}$ and $\bar{y}$ are the sample means

Evaluating Linear Regression Models [cont'd]

R^2 Value:

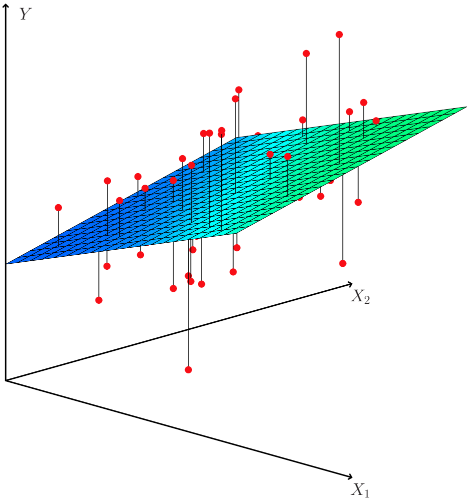
$$R^2 = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

where $TSS = \sum_i (y_i - \bar{y})^2$ is the total sum of squares

Interpretation

1. Proportion of explained variance
2. Correlation of Y and $\hat{Y}$

Generalization to Multiple Predictors



$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

Source: ISLR2 Figure 3.4

Regression with Qualitative Predictors

- Qualitative/categorical predictors (*factors* with multiple, exclusive *levels*) can be used in linear regression models using **dummy variables**:

$$x_{i1} = \begin{cases} 1 & \text{level "a"} \\ 0 & \text{else} \end{cases}$$

$$x_{i2} = \begin{cases} 1 & \text{level "b"} \\ 0 & \text{else} \end{cases}$$

$$x_{i3} = \begin{cases} 1 & \text{level "c"} \\ 0 & \text{else} \end{cases}$$

- Note: $x_{i1} = x_{i2} = x_{i3} = 0$ represents level d!

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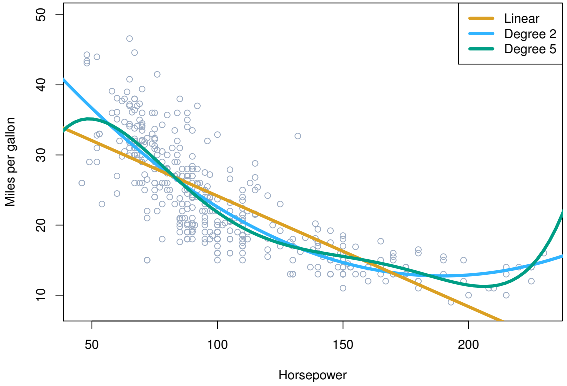
Inputs, Predictors, and Polynomials

Example:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_1 X_2 + \epsilon$$

- Still linear in β_j
- Two **inputs** X_1 and X_2
- Four **predictors** or **features**: X_1, X_1^2, X_2, $X_1 X_2$
- **Main effects** β_1 and β_3,
- **Interaction effect** β_4,
- A degree-2 **polynomial** effect β_2

Multiple Regression with Polynomials



Source: ISLR2 Figure 3.8

- Function more ”flexible”
- Fit to data improves
- Bias decreases
- Training MSE decreases (what about test MSE?)

Linear Regression in R

Load libraries for data sets:

```
# Functions and data from the textbook # 'Modern
# Applied Statistics with S'
library(MASS)
# Data sets from the textbook 'Introduction to
# Statistical Learning with Applications in R'
library(ISLR2)
```

The ’Boston’ data set contains housing values in Boston in variable `medv` (median value).

```
Examine the data:

# Get a description of the data
?Boston

# Get a summary and first few rows
summary(Boston)
head(Boston)

# Bivariate scatterplots
plot(Boston)
```

Linear Regression in R [cont'd]

Fit a simple model, examine and plot the results:

```r
# Fit a model with intercept only
fitted.model <- lm(medv ~ 1, data=Boston)
summary(fitted.model)

# Fit a model with predictor lstat
fitted.model <- lm(medv ~ lstat, data=Boston)
summary(fitted.model)

# Plot the data and the regression line
plot(medv ~ lstat, data=Boston)
abline(fitted.model, lwd=3, col='red')
```

Linear Regression in R [cont'd]

The `predict()` function calculates $\hat{y}_i$:

```
# For training data
y.hat <- predict(fitted.model, Boston)

# For new data
newData <- data.frame(lstat = c(5, 10, 15))
predict(fitted.model, newData)
```

The `residuals()` function calculates $y_i - \hat{y}_i$:

```
# Plot the residuals against predicted values
plot(predict(fitted.model), residuals(fitted.model))
```

Linear Regression in R [cont'd]

Build more complex models:

```
# Add another predictor
fitted.model <- lm(medv ~ lstat + age, data=Boston)

# Add all main effects
fitted.model <- lm(medv ~ ., data=Boston)

# Add interaction terms
fitted.model <- lm(medv ~ lstat + age + lstat:age,data=Boston)

# Shorter and equivalent
fitted.model <- lm(medv ~ lstat*age, data=Boston)
summary(fitted.model)
```

Linear Regression in R [cont'd]

Add polynomial terms:

```r
# Add a polynomial term; use the I(.) function
# for any data transformations, such as log(),
# or exp() or sqrt() as well as polynomials
fitted.model <- lm(medv ~ lstat + I(lstat^2), data=Boston)
summary(fitted.model)

# Add all polynomial terms up to degree 5
fitted.model <- lm(medv ~ poly(lstat, 5), data=Boston)

# Note the coefficients for the polynomials in the summary
summary(fitted.model)
```

Hands-On Exercises
(Source: ISLR2 Chapter 3)

Use the `Auto` data set from the ISLR2 library with `mpg` as the target.

1. Perform a linear regression with `horsepower` as predictor
2. Is there a relationship between the predictor and target? What form and how strong?
3. What is the predicted `mpg` value for a `horsepower` of 98?
4. Plot the response and predictor. Use the `abline()` function to add the regression line
5. Produce a scatterplot of all variables
6. Perform a linear regression of all main effects (except for the variable `name`)
7. Use the `*` and `:` symbols to add interaction effects.
8. Add transformations of the predictors (using the `I(.)` function) such as $\log(X)$, $\sqrt{X}$, X^2.

Cross-Validation in R – Holdout Sample

Validation set approach:

```
# Randomly use half the Auto data as training sample
train.idx <- sample(nrow(Auto), nrow(Auto)/2)
train.data <- Auto[train.idx,]
test.data <- Auto[-train.idx,]

# Fit model to (train model on) a subset
fitted.model <- lm(mpg ~ horsepower, data=train.data)

# Calculate the training data MSE
mean((train.data$mpg - predict(fitted.model, train.data))^2)

# Calculate the validation data MSE
mean((test.data$mpg - predict(fitted.model, test.data))^2)
```

Cross-Validation in R

K-Fold Cross-Validation with $K = 5$:

```
library(boot)

# Fit a model with glm and show its summary
glm.fit <- glm(mpg ~ horsepower, data=Auto)

# 5-fold CV
cv.err <- cv.glm(Auto, glm.fit, K=5)
cv.err$delta[1]
```

LOOCV is K-Fold CV with $K = N$

```
# LOOCV is k-fold CV where K equals N, num of obs
cv.err <- cv.glm(Auto, glm.fit, K=nrow(Auto))
cv.err$delta[1]
```

Hands-On Exercises – Cross-Validation

Consider the Boston housing data set `Boston`.

1. Fit a regression model using `medv` as target, and `age`, `lstat`, and `ptratio` as predictors
2. Using the validation set approach, compute the test error of this model. Perform the following steps
   - 2.1 Split the data set using 75% for training and 25% for testing
   - 2.2 Fit the model to training data
   - 2.3 Predict the target for the testing data
   - 2.4 Compute the test error
3. Repeat the previous step 2 times, using different splits. How do the results change?
4. Average the test error of the four splits.
5. Calculate the test error estimate using LOOCV. Compare your result to that of step 4.
6. Calculate the test error estimate using 10-fold cross-validation. Compare the estimate to that of step 4

Shrinkage Methods

Goals

- Avoid overfitting
- Reduce variance
- ”Shrink” regression coefficients towards zero
- Penalize coefficients that are ”too high”
- Type of ”**regularization**” (methods to prevent overfitting)

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Ridge Regression

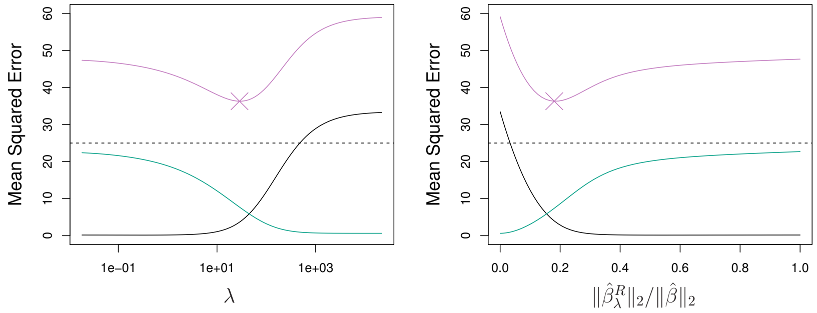
a.k.a Tikhonov Regularization

$$\text{Minimize} \quad \mathit{RSS} + \lambda \sum_{j=1}^{p} \beta_j^2 = \mathit{RSS} + \lambda ||\beta||_2^2$$

- L2 regularizer (it penalizes the L2 norm of β)
- Parameter λ controls the amount of shrinkage
- Larger λ reduce variance but increase bias
- Not scale invariant: Standardize predictors

Ridge Regression

a.k.a Tikhonov Regularization

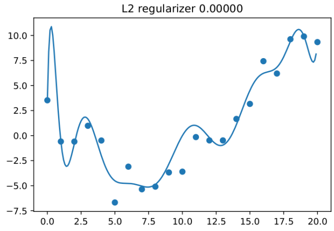


Source: ISLR2 Figure 6.5

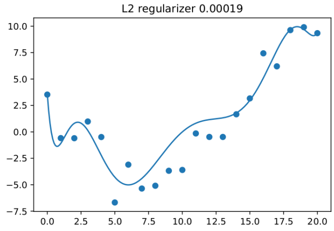
- Bias
- Variance
- MSE

Ridge Regression Example

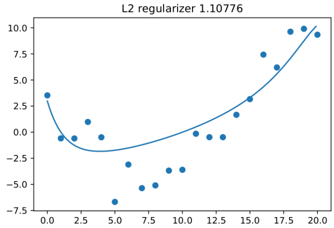
Fitting a Degree 14 Polynomial



(a)



(b)



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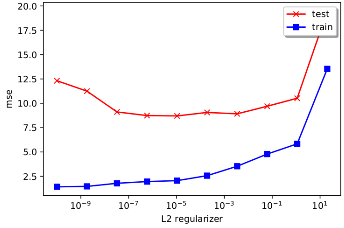


(d)

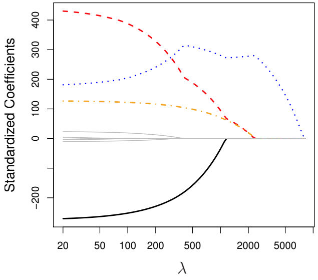
Source: Murphy Figure 4.5

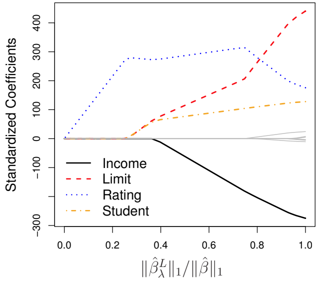
Lasso regression

"Least Absolute Shrinkage and Selection Operator"

$$\text{Minimize} \quad RSS + \lambda \sum_{j=1}^{p} |\beta_j| = RSS + \lambda ||\beta||_1$$

- L1 regularizer (it penalizes the L1 norm of β)
- Lasso may exclude variables by forcing their β_j to 0
- Parsimonious, more interpretable models than ridge regression





Source: ISLR2 Figure 6.6

Elastic Net

The Elastic Net penalty is a mix of L1-norm $||\beta||_1$ and L2-norm $||\beta||_2^2$ penalties, defined by α:

$$\lambda\left(\alpha||\beta||_1 + (1-\alpha)||\beta||_2^2\right)$$

- $\alpha = 0$: Ridge regression
- $\alpha = 1$: Lasso

The `glmnet()` function in the `glmnet` library for R uses the Elastic Net regularization method.

Ridge Regression in R

Use the `Hitters` data set to model `Salary` as outcome and other variables as predictors.

```r
library(ISLR2)
library(glmnet)

# Remove missing values
Hitters <- na.omit(ISLR2::Hitters)
```

The `glmnet()` function requires separate *x* and *y* values, instead of a formula. To create dummy variables for categorical variables, use the `model.matrix()` function:

```r
# Create dummy variables for categorical variables
# Remove intercept from model
x <- model.matrix(Salary ~ ., Hitters)[, -1]
y <- Hitters$Salary
```

Ridge Regression in R [cont'd]

Example for $\lambda = 10$ using `glmnet()` with $\alpha = 0$:

```
ridge.model <- glmnet(x, y, alpha=0, lambda=10)
```

Examine coefficients:

```
coef(ridge.model)
```

Examine L2 norm (penalty to RSS) (without intercept):

```
L2.norm = sqrt(sum(coef(ridge.model)[-1]^2))
```

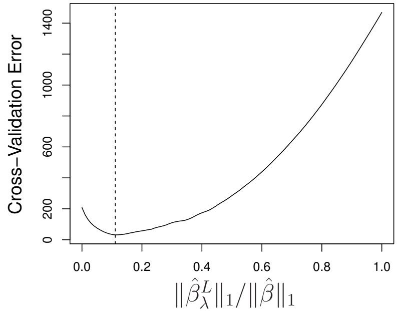
Examine MSE:

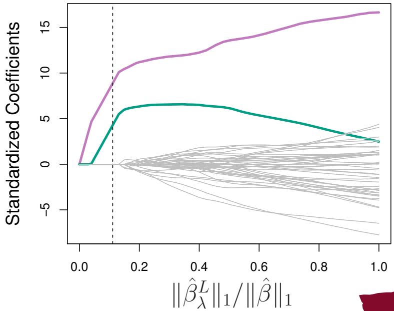
```
mean((y-predict(ridge.model, x))^2)
```

Selecting the Tuning Parameter

Grid Search

- Define a set/range of possible values for λ
- Cross-validate models for each value of λ
- Fit the final model to the optimal cross-validated error





Source: ISLR2 Figure 6.13

Ridge Regression in R – Optimal Lambda

Create holdout test data set:

```r
# Randomly split the Hitters data
train.idx <- sample(nrow(Hitters), nrow(Hitters)/2)
x.train <- x[train.idx,]
x.test <- x[-train.idx,]
y.train <- y[train.idx]
y.test <- y[-train.idx]
```

Use the training set to find optimal λ with CV:

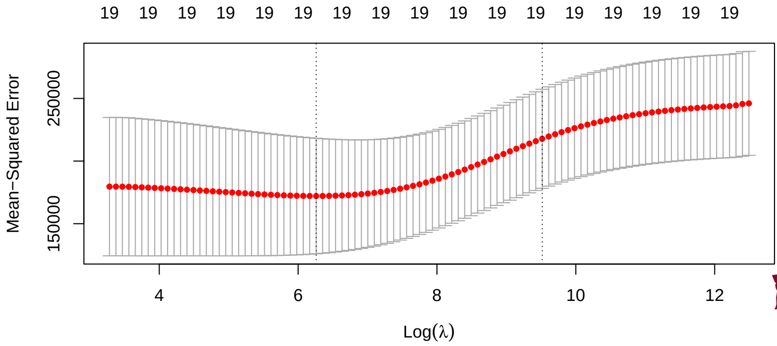
```r
# 5-fold cross-validation, use MSE as metric
cv.out <- cv.glmnet(x.train, y.train, alpha=0,
                    nfolds=5, type.measure='mse')
```

Ridge Regression in R – Optimal Lambda [cont'd]

Show the optimal lambda, the MSE and its standard error, and the number of non-zero coefficients.

```
print(cv.out)
plot(cv.out)
lambda.opt <-
  cv.out$lambda.min
```

```
> print(cv.out)
    Lambda Index Measure    SE Nonzero
min  461.6    68  102536 13016      19
1se 2244.8    51  114493 19007      19
```



Ridge Regression in R – Optimal Lambda [cont'd]

Fit test data using optimal λ:

```
ridge.model <- glmnet(x.test, y.test,
                      alpha=0, lambda=lambda.opt)
```

Compare coefficients between ridge and unpenalized least squares:

```
coef(ridge.model)
coef(lm(y.test ~ x.test + 1))
```

Ridge Regression in R – Prediction

Predict values for the test data set:

```r
y.hat.test <- predict(ridge.model, newx=x.test,
                                      type='response')
```

Calculate test MSE to compare to the CV optimal MSE above:

```r
mean((y.hat.test - y.test)^2)
```

Recall, the CV cross-validated error on the training set is an estimate/approximation of the real test error.

The Lasso in R [cont'd]

Example for $\lambda = 10$ using `glmnet()` with $\alpha = 1$:

```
lasso.model <- glmnet(x, y, alpha=1, lambda=10)
```

Examine coefficients:

```
coef(lasso.model)
```

Examine L1 norm (penalty to RSS) (without intercept):

```
L1.norm = sum(abs(coef(lasso.model)[-1]))
```

Examine MSE:

```
mean((y-predict(ridge.model, x))^2)
```

Use the training set to find optimal λ with CV:

```
# 5-fold cross-validation, use MSE as metric
cv.out <- cv.glmnet(x.train, y.train, alpha=1,
                    nfolds=5, type.measure='mse')
```

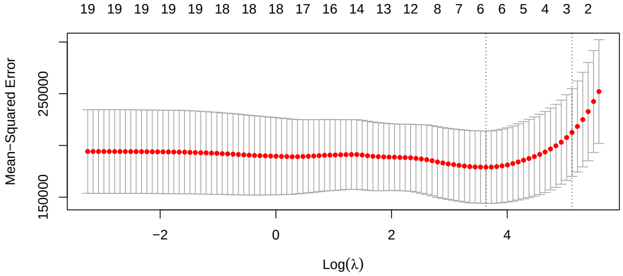
The Lasso in R – Optimal Lambda

Show the optimal lambda, the MSE and its standard error, and the number of non-zero coefficients.

```r
print(cv.out)
plot(cv.out)
lambda.opt <-
  cv.out$lambda.min
```

```
> print(cv.out)
    Lambda Index Measure    SE Nonzero
min  33.33    22  107161 17195       6
1se  92.75    11  122071 22061       4
```

Note the number of non-zero coefficients in the result:



The Lasso in R – Prediction

Fit test data using optimal λ:

```r
lasso.model <- glmnet(x.test, y.test,
                         alpha=1, lambda=lambda.opt)
```

Compare coefficients between Lasso and unpenalized least squares:

```r
coef(lasso.model)
coef(lm(y.test ~ x.test + 1))
```

Predict values for the test data set and compute test MSE

```r
y.hat.test <- predict(lasso.model, newx=x.test,
                                             type='response')
mean((y.hat.test - y.test)^2)
```

Hands-On Exercises – Shrinkage Methods

Source: ISLR2, Chapter 6

Predict the number of applications received using the other variables in the `College` dataset

1. Split the data set into a training and a test set
2. Fit an unpenalized linear model on the training set. Report the test error.
3. Fit a ridge regression model on the training set, with λ chosen by cross-validation. Report the test error.
4. Fit a lasso model on the training set, with λ chosen by cross-validation. Report the test error.
5. Compare and contrast the results

Hands-On Exercises – Shrinkage Methods

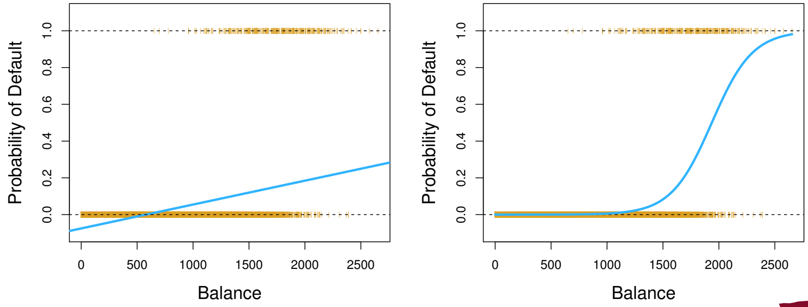
Source: ISLR2, Chapter 6

Predict the per-capita crime rate in the `Boston` data set using the other variables.

1. Split the data set into a training and a test set
2. Fit an unpenalized linear model on the training set. Report the test error.
3. Fit a ridge regression model on the training set, with λ chosen by cross-validation. Report the test error.
4. Fit a lasso model on the training set, with λ chosen by cross-validation. Report the test error.
5. Compare and conrast the results

Classification

- Qualitative (categorical) outcome
- Estimate or predict probabilities of class membership
- **Problem**: Linear combinations of predictors not in $[0, 1]$
- **Solution**: ”Link” function transforms linear combinations of predictors

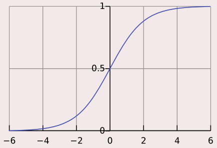


Source: ISLR2 Figure 4.2

Logistic Regression

Logistic / Sigmoid Function

$$\sigma(a) = \frac{1}{1 + e^{-a}} = \frac{e^a}{1 + e^a}$$



Binary Case (Binomial Logistic Regression)

$$p(X) = \sigma(\beta_0 + \beta_1 X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$$

where the linear predictor $\beta_0 + \beta_1 X$ are the **logits**

Logistic Regression [cont'd]

Predictions

Given estimates $\hat{\beta}_0$ and $\hat{\beta}_1$, use the link function to predict class/outcome probabilities for observation X:

$$\hat{p}(X) = \frac{e^{\hat{\beta}_0+\hat{\beta}_1 X}}{1 + e^{\hat{\beta}_0+\hat{\beta}_1 X}}$$

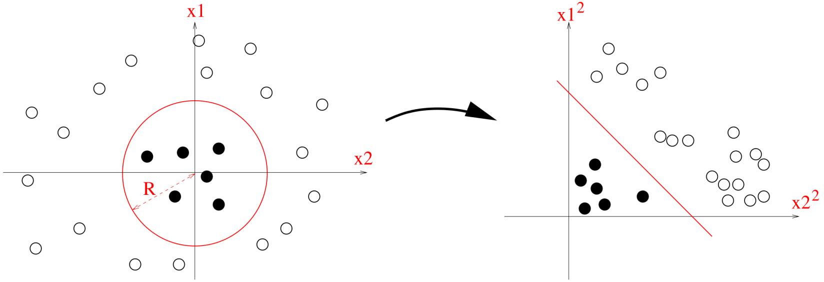
Multinomial Logistic Regression

- Same mathematical form
- Probability for class being k:

$$\Rightarrow \Pr(Y = k|X) = \frac{e^{\beta_{k0}+\beta_{k1}x_1+\cdots+\beta_{kp}x_p}}{1 + \sum_{l=1}^{K-1} e^{\beta_{l0}+\beta_{l1}x_1+\cdots+\beta_{lp}x_p}}$$

Logistic Regression with Polynomials

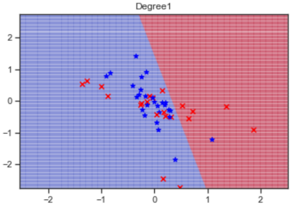
Transforming decision boundaries:



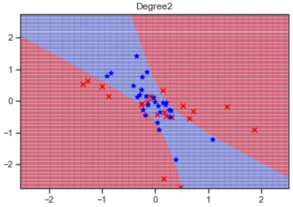
Source: Murphy Figure 10.3

Logistic Regression with Polynomials [cont'd]

But beware of overfitting:



(a)



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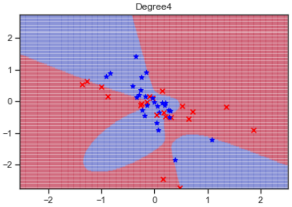

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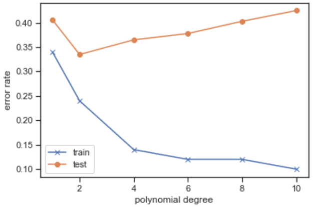

(d)

Source: Murphy Figure 10.4

Logistic Regression in R

Use the stock market data set `Smarket` to predict the binary outcome `Direction` (the direction of market changes, up or down) using prior returns as predictors:

```r
library(ISLR2)
?Smarket
```

Fit to full data set. Note the link function in the `family` argument for the `glm()` function:

```r
logreg.fitted <-
   glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume,
       data=Smarket,
       family=binomial(link='logit'))
summary(logreg.fitted)
```

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Logistic Regression in R

Use `predict()` to predict logits for data set:

```
logreg.logits <- predict(logreg.fitted, newdata = Smarket)
```

Use `predict()` to predict probabilities for data set:

```
# Predict probabilities for training test
logreg.probs <- predict(logreg.fitted, newdata = Smarket,
                     type='response')
```

Make classificiation decision using decision rule:

```
# Predict 'up' or 'down' based on probabilities
# and a fixed threshold
pred.direction <- rep(NA, nrow(Smarket))
pred.direction[logreg.probs >  .5] <- 'Up'
pred.direction[logreg.probs <= .5] <- 'Down'
```

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Logistic Regression in R

Compute confusion matrix:

```r
# Compute confusion matrix
logreg.cm <- table(pred.direction, Smarket$Direction)
print(logreg.cm)
```

Compute accuracy as mean of correct classifications:

```r
# Compute accuracy
mean(pred.direction == Smarket$Direction)
```

Logistic Regression in R – Holdout Set

Split data to train and test set. Because this is time-dependent data, split by time to avoid mixing past and future data:

```
train.data <- Smarket[Smarket$Year < 2005,]
test.data <- Smarket[!(Smarket$Year < 2005),]
```

Fit model to training set:

```
logreg.fitted <-
   glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume,
       data=train.data, family=binomial(link='logit'))
```

Predict probabilities for test data:

```
logreg.probs <- predict(logreg.fitted,
    newdata = test.data, type='response')
```

Make classification decision using decision rule:

```
pred.direction <- rep(NA, nrow(test.data))
pred.direction[logreg.probs >  .5] <- 'Up'
pred.direction[logreg.probs <= .5] <- 'Down'
```

Logistic Regression in R – Holdout Set

Compute confusion matrix for test data:

```r
logreg.cm <- table(pred.direction, test.data$Direction)
print(logreg.cm)
```

Calculate accuracy as the mean of correct classifications:

```r
mean(pred.direction == test.data$Direction)
```

Logistic Regression in R – Evaluation

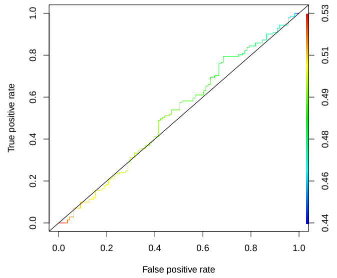
Using the `ROCR` library for classifier evaluation:

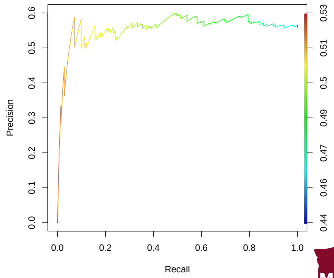
```r
library(ROCR)

# A prediction object collects predicted
# probabilities and true labels
pred.obj <- prediction(logreg.probs,
                       test.data$Direction)

# Get some classifier performance metrics
# ROCR varies the threshold.
plot(performance(pred.obj, 'acc'))
plot(performance(pred.obj, 'prec'))
plot(performance(pred.obj, 'rec'))
plot(performance(pred.obj, 'f'))
performance(pred.obj, 'auc')@y.values[[1]]
```

Logistic Regression in R – Evaluation

continued . . .

```
# ROC - True positive rate versus false positive rate
plot(performance(pred.obj, 'tpr', 'fpr'), colorize=T)
abline(0, 1)

# Precision/Recall plot
plot(performance(pred.obj, 'prec', 'rec'), colorize=T)
```





Naive Bayes Classifier

- **Bayes Theorem**:

$$\begin{aligned}\Pr(Y = c\,|X) &= \frac{p(X|Y=c)\,p(Y=c)}{p(X)} \\ &= \frac{p(X|Y=c)\,p(Y=c)}{\sum_{l=1}^{K} p(X|Y=l)\,p(Y=l)}\end{aligned}$$

- **Naive Bayes Assumption**: Within each class c, the D predictors are independent:

$$\begin{aligned}p(X|Y=c) &= p(x_1|Y=c) \times p(x_2|Y=c) \times \cdots \times p(x_D|Y=c) \\ &= \prod_{d=1}^{D} p(x_d|Y=c)\end{aligned}$$

- **Posterior probability**:

$$p(Y=c|X) = \frac{\left(\prod_{d=1}^{D} p(x_d|Y=c)\right)\, p(Y=c)}{\left(\sum_{l=1}^{K}\prod_{d=1}^{D} p(x_d|Y=l)\right)\, p(Y=l)}$$

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Naive Bayes Classifier in R

Naive Bayes using the `naiveBayes` function in the `e1071` library:

```r
library(e1071)

# Fit using same syntax as glm
nb.fitted <- naiveBayes(Direction~Lag1+Lag2, data=train.data)

# Output contains prior and conditional probabilities (and SD)
nb.fitted
```

Predict class membership and compute confusion matrix:

```r
nb.predictions <- predict(nb.fitted, test.data)
nb.cm <- table(nb.predictions, test.data$Direction)
print(nb.cm)
```

Naive Bayes Classifier in R – Evaluation

Evaluate the classifier:

```
# Predict probabilities (for use with ROCR)
nb.probs <- predict(nb.fitted, test.data, type='raw')

# Create an ROCR prediction object
nb.pred.obj <- prediction(nb.probs[,'Up'],
                          test.data$Direction)
```

Assess ROC and AUC:

```
# Generate an ROC plot
plot(performance(nb.pred.obj, 'tpr', 'fpr'), colorize=T)
abline(0, 1)

# Compute the AUC
performance(nb.pred.obj, 'auc')@y.values[[1]]
```

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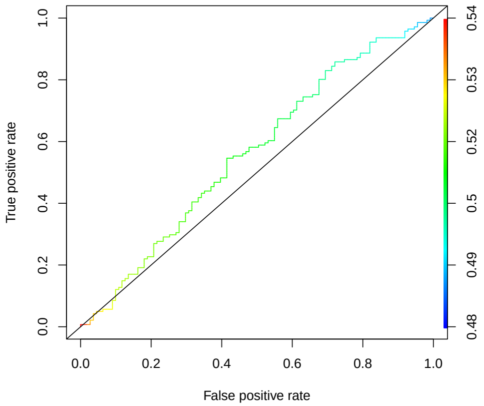
Naive Bayes Classifier in R – Evaluation



KNN Classification in R

Using the `knn()` function from the `class` library:

```
library(class)
```

Use only two predictors, Lag1 and Lag2:

```
train.x <- cbind(train.data$Lag1, train.data$Lag2)
test.x <- cbind(test.data$Lag1, test.data$Lag2)
train.y <- train.data$Direction
test.y <- test.data$Direction
```

Make predictions from training set for test set, given true classes of training set ($k = 3$ and return probabilities):

```
knn.pred <- knn(train.x, test.x, train.y, k=3, prob=T)
```

KNN Classification in R

Evaluate the classifier against test data:

```
# Confusion matrix
table(knn.pred, test.y)
# Accuracy
mean(knn.pred == test.y)
```

Save class probabilities of the majority class:

```
knn.probs <- attributes(knn.pred)$prob
```

Compute class probabilities of the minority class:

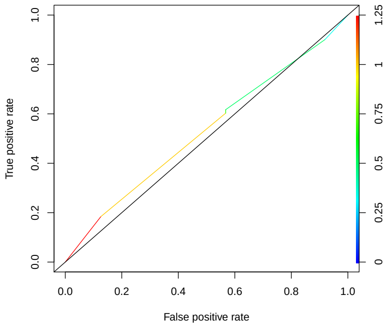
```
knn.class.probs <- knn.probs
knn.class.probs[knn.pred=='Down'] <-
                    1-knn.probs[knn.pred=='Down']
```

KNN Classification in R [cont'd]

Use ROCR functions to evaluate classifier:

```
knn.pred.obj <-
      prediction(knn.class.probs, test.data$Direction)

plot(performance(knn.pred.obj, 'tpr', 'fpr'), colorize=T)
abline(0, 1)
performance(knn.pred.obj, 'auc')@y.values[[1]]
```



Hands-On Exercises

Source: ISLR2, Chapter 4

Use the `Weekly` data set in the `ISLR2` package.

1. Use the full data set to perform a logistic regression with `Direction` as target. Which predictors are statistically significant?
2. Compute the confusion matrix and accuracy.
3. Use the 1990 to 2008 data for a training set and the 2009/2010 for a test set. Fit a logistic regression model with `Lag2` as the only predictor.
4. Repeat (3) using Naive Bayes
5. Repeat (3) using KNN with $K = 1$
6. Which model provides the best results on this data?

Hands-On Exercises
Source: ISLR2, Chapter 4

Use the `Auto` data set in the `ISLR2` package.

1. Create a binary variable, `mpg01` that contains a 1 if `mpg` is above its median, 0 otherwise. *Tip*: Use the `median()` function. Add the new variable to the data frame.
2. Split the data set into training and test set
3. Perform a logistic regression on the training data to predict `mpg01` from the other features. What is the test error of this model?
4. Repeat (3) using Naive Bayes
5. Repeat (3) using KNN with different values of K. What value of K performs best?

Hands-On Exercises

Source: ISLR2, Chapter 4

Using the `Boston` data set in the `ISLR2` library, fit classification models to predict whether a given census tract has a crime rate above or below the median.

1. Create a new binary variable `crime01` that is 1 is `crime` is above its median, and 0 otherwise. Combine this variable with the data frame. *Tip*: Use the `median()` function for this.
2. Split your data set into a training and test data set
3. Fit logistic regression, Naive Bayes, and KNN (with different K)
4. Describe your findings in terms of prediction error, precision, recall, F1 and AUC