Simple linear regression I

EC 320, Set 04

Andrew Dickinson

Spring 2023

Housekeeping

PS02:

- Due Tuesday at 11:59p
- Covers remaining topics from the review and from lecture Wednesday

LA03:

- Due Friday at 5:00p

Reading: *(up to this point)*

ItE: R, **1** MM: 1, **2**

Regression logic

Regression models

Modeling is about reducing something ***really complicated*** into something *simple* that represents some part of the complicated reality.

- Try to tell stories that are easy to understand, and easy to learn from
- Model *toy versions* of reality

Economists *often* rely on **linear regression** for statistical comparisons.

- Describes the relationship between a dependent (endogenous) variable and one or more explanatory (exogenous) variable(s)
- *"Linear"* is more flexible than you think

Regression models

Regression analysis helps us make *all else equal* comparisons

Running regressions provide correlative (*and even causal*) information between two variables

Ex. By how much does Y change when X increases by one unit?

Regression models

Modelling forces us to be explicit about the potential sources of selection bias

- Model the effect of X on Y while **controlling** for *potential confounders* that may muddy the water
- Failure to account for sources of selection bias, leads to biased estimates.

Ex. Not controlling for confounding variables, leads to *omitted-variable bias*, a close cousin of selection bias

- *Why?* Omitted variables that correlate with our covariate of interest, hides within the model, distorting our results.

Returns to education

Research Question: By how much does an additional year of schooling increase future earnings?

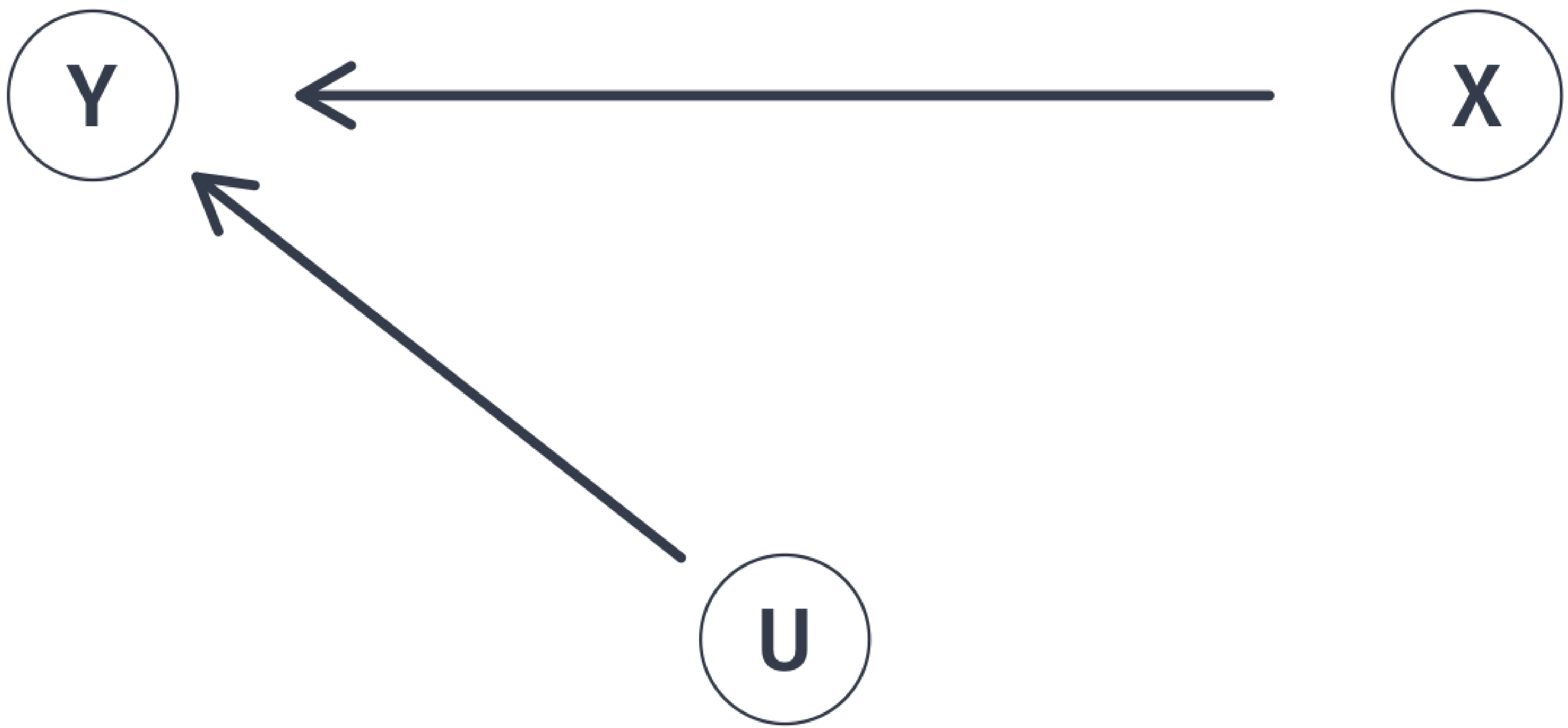
- Dependent variable: Earnings
- Independent variable: An additional year of school

Q. How might education increase earnings?

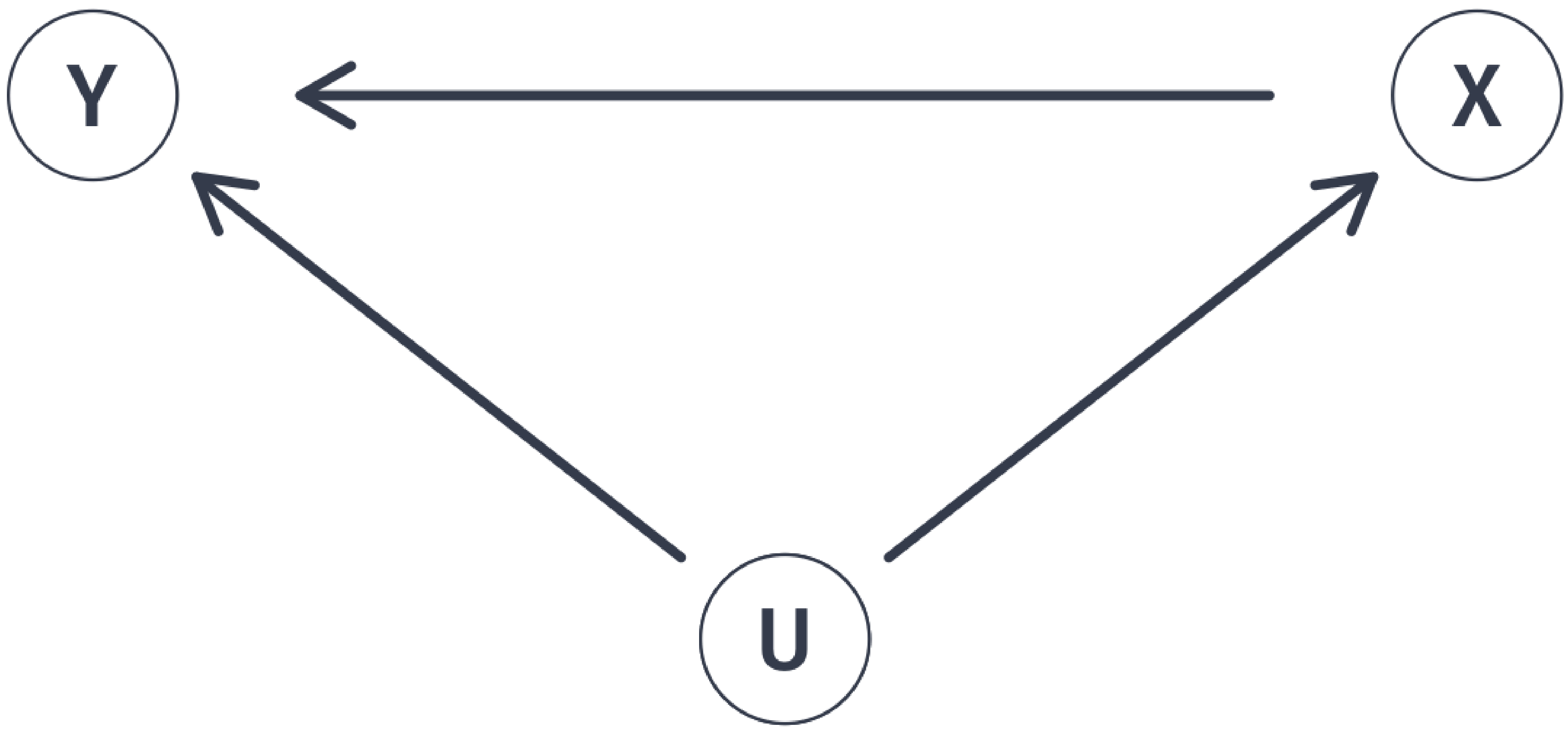
Q. Why might a simple comparison between high and low educated not isolate the economic returns to education?

More education (**X**) increases lifetime earnings (**Y**)

More education (**X**) increases lifetime earnings (**Y**) along with a lot of other things (**U**).



More education (**X**) increases lifetime earnings (**Y**) along with a lot of other things (**U**). But a lot of other things (**U**) also impact education (**X**).



Any unobserved variable that connects a *backdoor path* between education (**X**) and earnings (**Y**) is called a **confounder**

Returns to education

How might we estimate the causal effect of an additional year of schooling on earnings?

Approach 1: Compare average earnings of private college graduates with those of public college graduates.

- Prone to selection bias by variety of confounding variables

Approach 2: Estimate a regression that compares the earnings of individuals with the same admissions profiles.

- *Try to* **control** for *confounders* by including them in the model

But before taking on *confounders* and using regression to link causal relationships… let's breakdown the anatomy of the *simple regression model*

Simple regression model

The regression model

We can estimate the effect of X on Y by estimating a **regression model:**

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

- Y_i is the outcome variable.
- X_i is the treatment variable (continuous).
- β_0 is the **intercept** parameter. $\mathbb{E}[Y_i | X_i = 0] = \beta_0$
- β_1 is the **slope** parameter, which under the correct causal setting represents marginal change in X_i's effect on Y_i. $\frac{\partial Y_i}{\partial X_i} = \beta_1$
- u_i is an error term including all other (omitted) factors affecting Y_i.

The error term

u_i is quite special

Consider the *data generating process* of variable Y_i,

- u_i captures all unobserved variables that explain variation in Y_i.

Some error will exist in all models, our aim is to minimize error under a set of constraints

- Error is the price we are willing to accept for simplified model

The error term

Five items contribute to the existence of the disturbance term:

1. ***Omission of explanatory variables***

- Our description (model) of the relationship between $\mathbf{Y}$ and $\mathbf{X}$ is a simplification
- Other variables have been left out (omitted)

The error term

Five items contribute to the existence of the disturbance term:

1. Omission of explanatory variables

2. ***Aggregation of Variables***

- Microeconomic relationships are often summarized
- *Ex.* Housing prices (X) are described by county-level median home value data

The error term

Five items contribute to the existence of the disturbance term:

1. Omission of explanatory variables

2. Aggregation of Variables

3. ***Model misspecificiation***

- Model structure is incorrectly specified
- *Ex.* Y depends on the anticipated value of X in the previous period, not X

The error term

Five items contribute to the existence of the disturbance term:

1. Omission of explanatory variables

2. Aggregation of Variables

3. Model misspecificiation

4. ***Functional misspecificiation***

- The functional relationship is specified incorrectly
- True relationship is nonlinear, not linear

The error term

Five items contribute to the existence of the disturbance term:

1. Omission of explanatory variables

2. Aggregation of Variables

3. Model misspecificiation

4. Functional misspecificiation

5. ***Measurement error***

- Measurement of the variables in the data is just wrong
- Y or X

The error term

Five items contribute to the existence of the disturbance term:

1. Omission of explanatory variables
2. Aggregation of Variables
3. Model misspecificiation
4. Functional misspecificiation
5. Measurement error

Running regressions

Using an estimator with data on X_i and Y_i, we can estimate a **fitted regression line:**

$$\hat{Y_i} = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

- $\hat{Y_i}$ is the **fitted value** of Y_i.
- $\hat{\beta}_0$ is the **estimated intercept**.
- $\hat{\beta}_1$ is the **estimated slope**.

This procedure produces misses, known as **residuals**, $Y_i - \hat{Y_i}$

I think it would be easier to think about regression with a concrete example.

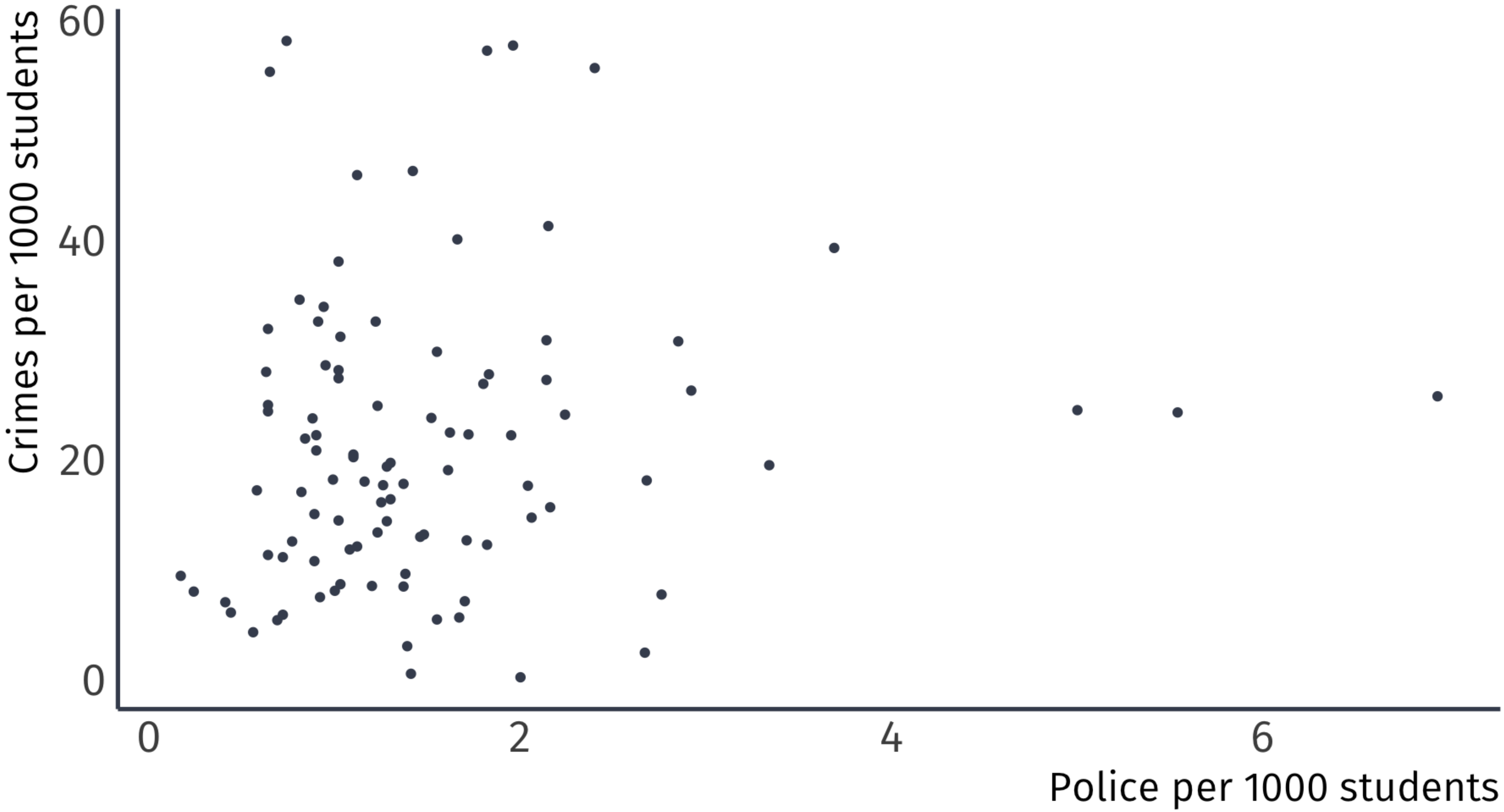
Ex. Effect of police on crime

Ex. Effect of police on crime

Empirical question:

> Does the number of on-campus police officers affect campus crime rates? If so, by how much?

Always plot your data first



Crimes per 1000 students
60
40
20
0
0
2
4
6
Police per 1000 students

Ex. **Effect of police on crime**

The scatter plot suggest that a *weak* positive relationship exists

- A sample correlation of 0.14 confirms this

But correlation does not imply causation

Lets estimate a statistical model

Ex. Effect of police on crime

We express the relationship between a **explained variable** and an **explanatory variable** as linear:

{\color{#81A1C1} \text{Crime}_i} = \beta_1 + \beta_2 {\color{#B48EAD} \text{Police}_i}

- β_1 is the *intercept* or constant.
- β_2 is the *slope coefficient*.
- u_i is an *error term* or disturbance term.

Ex. Effect of police on crime

The **intercept** tells us the expected value of Crime_i when $\text{Police}_i = 0$.

$$\text{Crime}_i = \beta_1 + \beta_2 \text{Police}_i + u_i$$

Usually not the focus of an analysis.

Ex. Effect of police on crime

The **slope coefficient** tells us the expected change in Crime_i when Police_i increases by one.

$$\text{Crime}_i = \beta_1 + \beta_2 \text{Police}_i + u_i$$

"A one-unit increase in Police_i is associated with a β_2-unit increase in Crime_i."

Interpretation of this parameter is **crucial**

Under certain (strong) assumptions[1], β_2 is the *effect of* X_i *on* Y_i.

- Otherwise, it's the *association of* X_i *with* Y_i.

1. Assumptions regarding the error term

Ex. Effect of police on crime

The **error term** reminds us that Police_i does not perfectly explain Y_i.

$$\text{Crime}_i = \beta_1 + \beta_2 \text{Police}_i + u_i$$

Represents all other factors that explain Crime_i.

- Useful mnemonic: pretend that u stands for *"unobserved"* or *"unexplained."*

Ex. Effect of police on crime

How might we apply the simple linear regression model to our question about the effect of on-campus police on campus crime?

$$\text{Crime}_i = \beta_1 + \beta_2 \text{Police}_i + u_i.$$

- β_1 is the crime rate for colleges without police.
- β_2 is the increase in the crime rate for an additional police officer per 1000 students.

Ex. Effect of police on crime

How might we apply the simple linear regression model to our question?

$$\text{Crime}_i = \beta_1 + \beta_2 \text{Police}_i + u_i$$

β_1 and β_2 are the unobserved population parameters we want

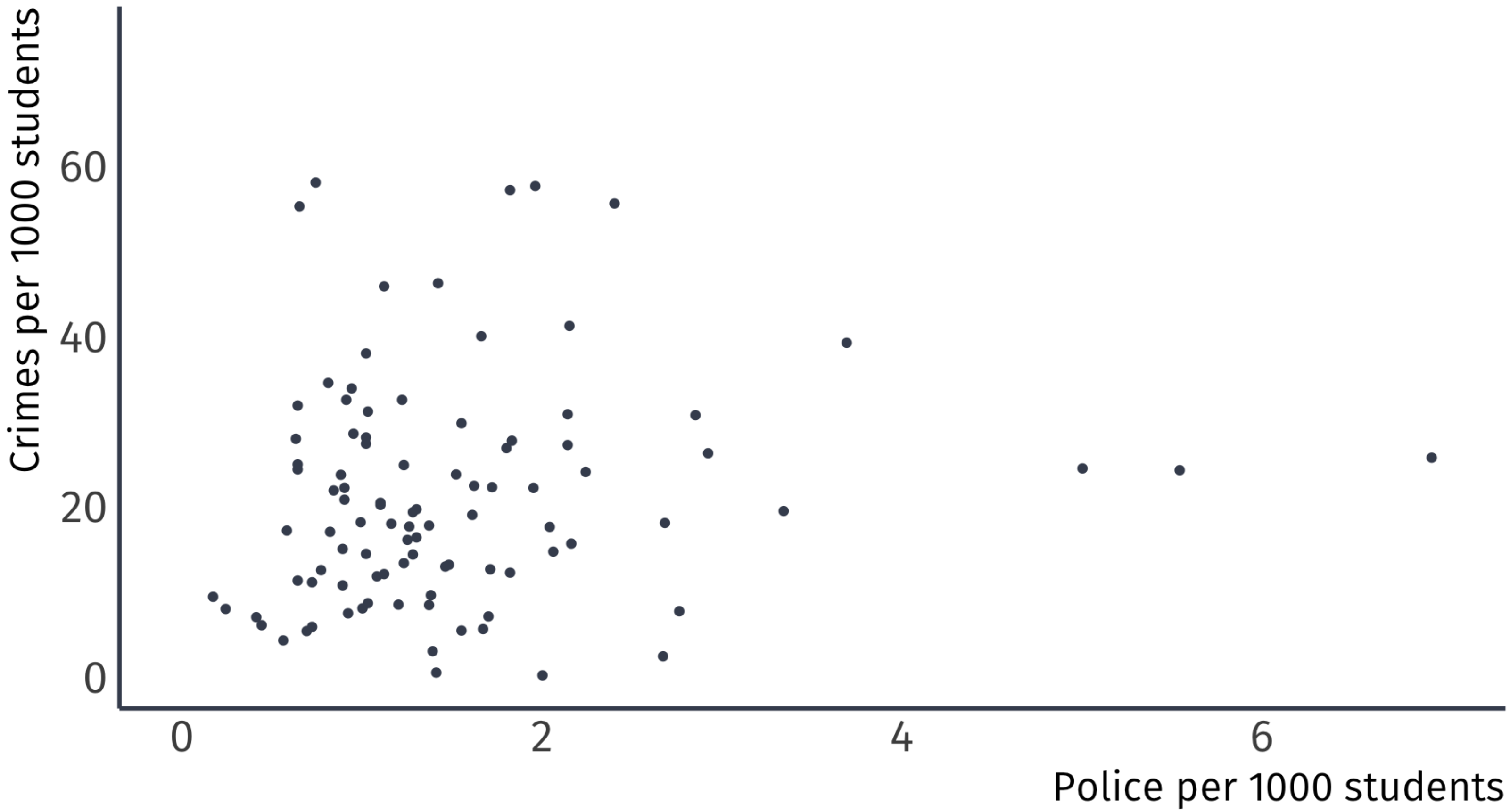
We estimate

- $\hat{\beta}_1$ and $\hat{\beta}_2$ generate predictions of Crime_i called $\widehat{\text{Crime}_i}$.
- We call the predictions of the dependent variable **fitted values.**

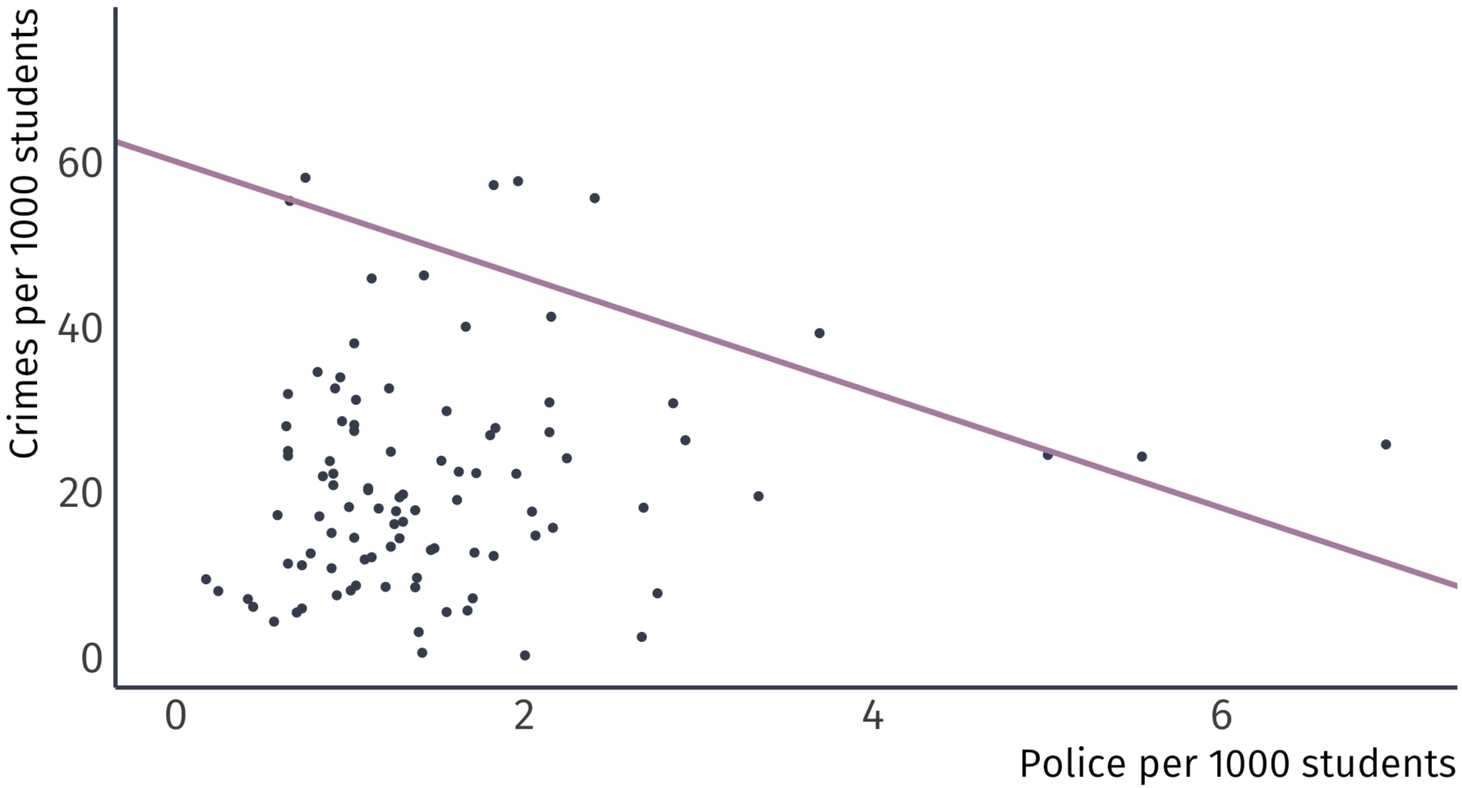
- Together, these trace a line: $\widehat{\text{Crime}_i} = \hat{\beta}_1 + \hat{\beta}_2 \text{Police}_i$.

So, the question becomes, *how do I pick* $\hat{\beta}_1$ *and* $\hat{\beta}_2$

Let's take some guesses:

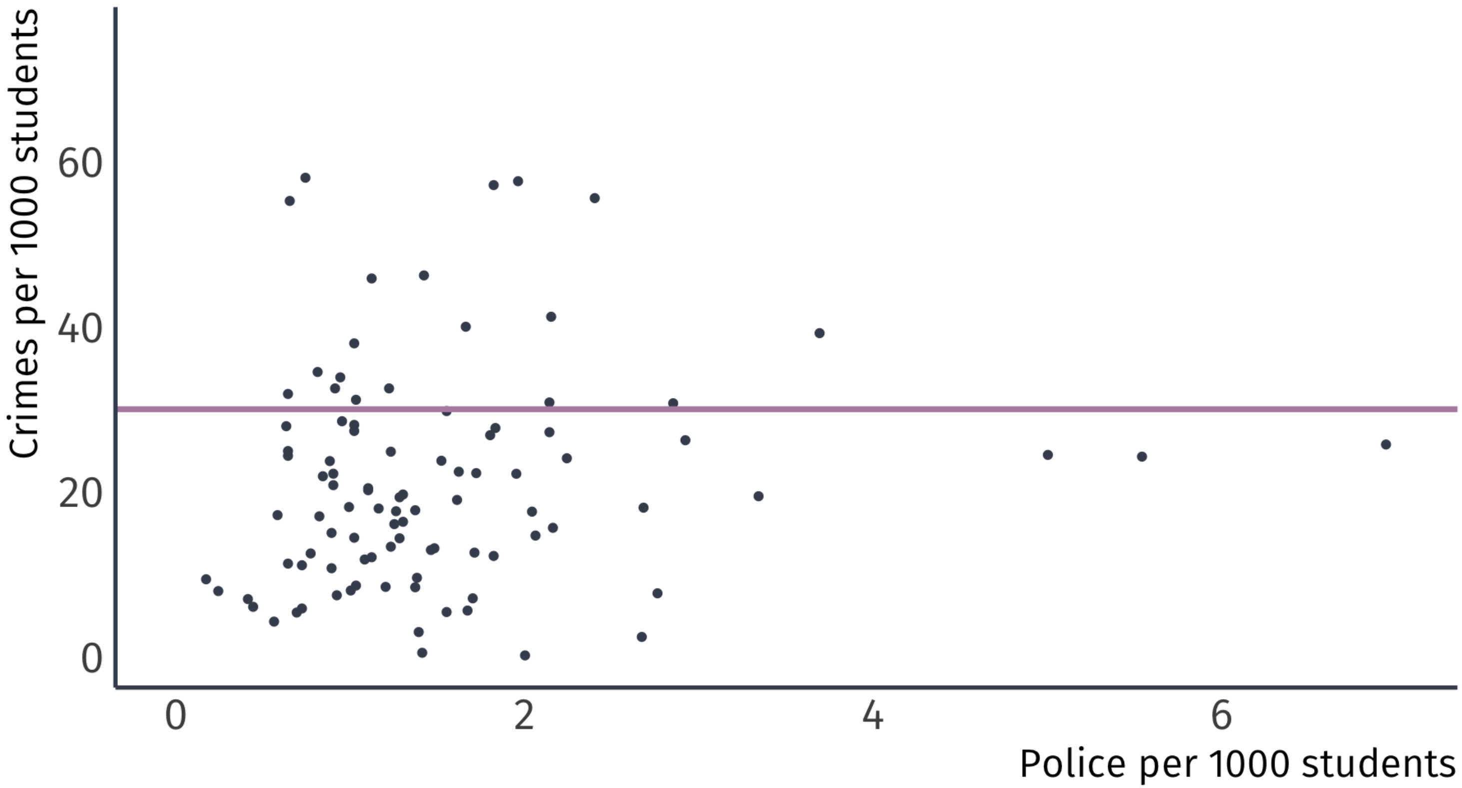


Let's take some guesses: $\hat{\beta}_1 = 60$ and $\hat{\beta}_2 = -7$



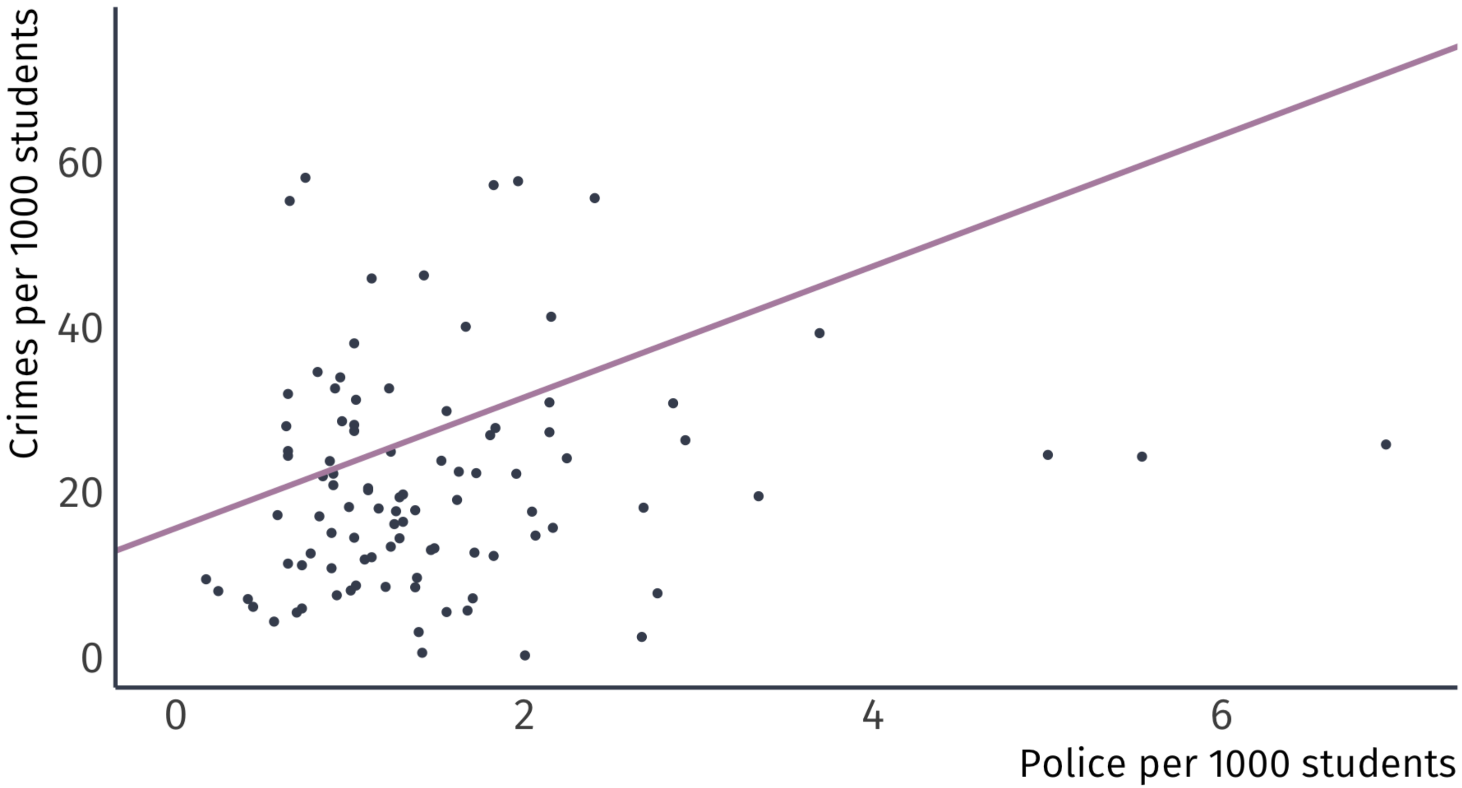
Does this line represent the data well?

Let's take some guesses: $\hat{\beta}_1 = 60$ and $\hat{\beta}_2 = -7$



What about this one?

Let's take some guesses: $\hat{\beta}_1 = 60$ and $\hat{\beta}_2 = -7$



Or this one?

Residuals

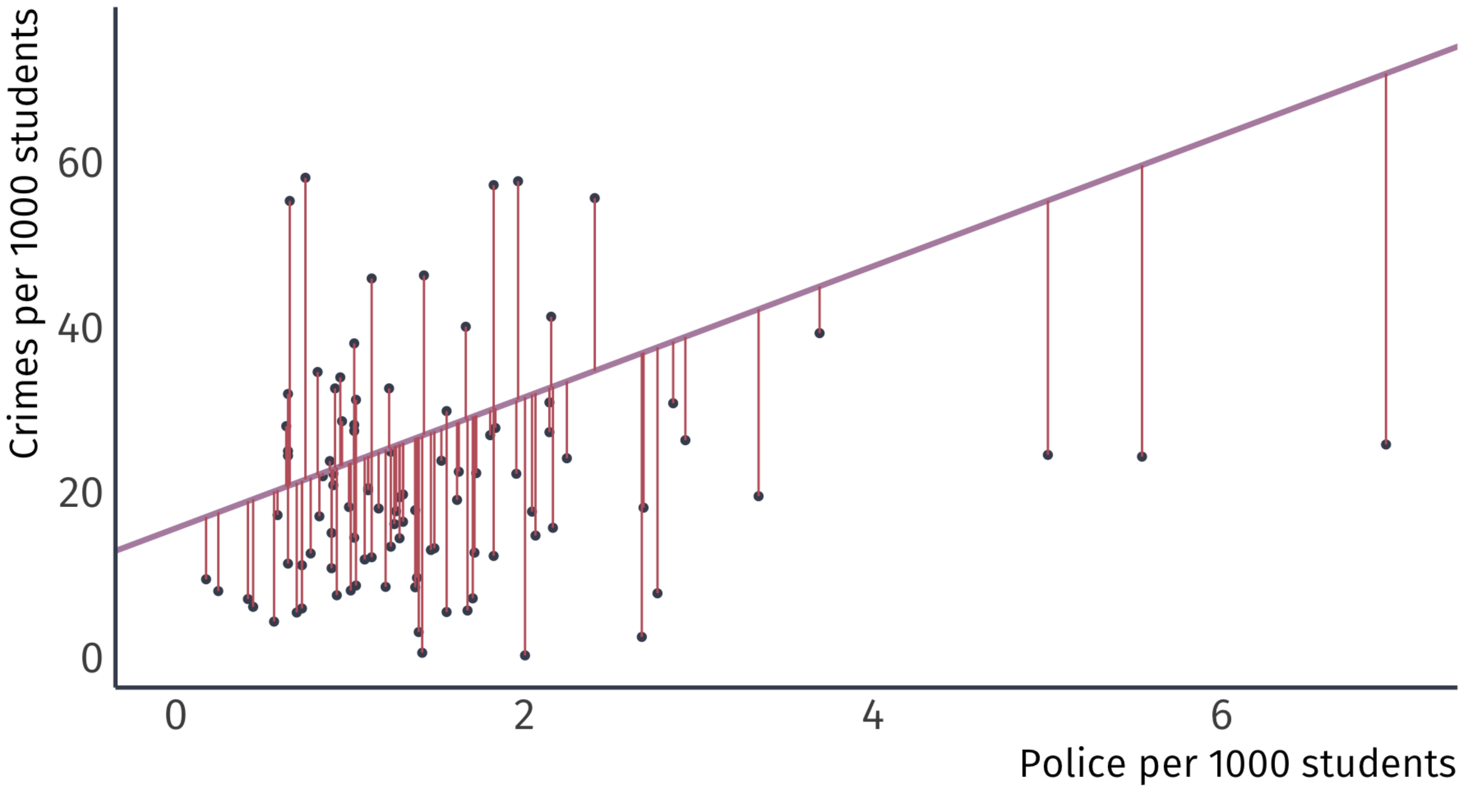
Using $\hat{\beta}_1$ and $\hat{\beta}_2$ to make $\hat{Y}_i$ generates misses.

We call these misses **residuals:**

{\color{#BF616A} \hat{u}_i} = {\color{#BF616A}Y_i - \hat{Y_i}}.

AKA e_i.

$\hat{\beta}_1 = 60$ and $\hat{\beta}_2 = -7$



Does this line represent the data well?

Residuals

What is we picked an estimator that minimizes the residuals?

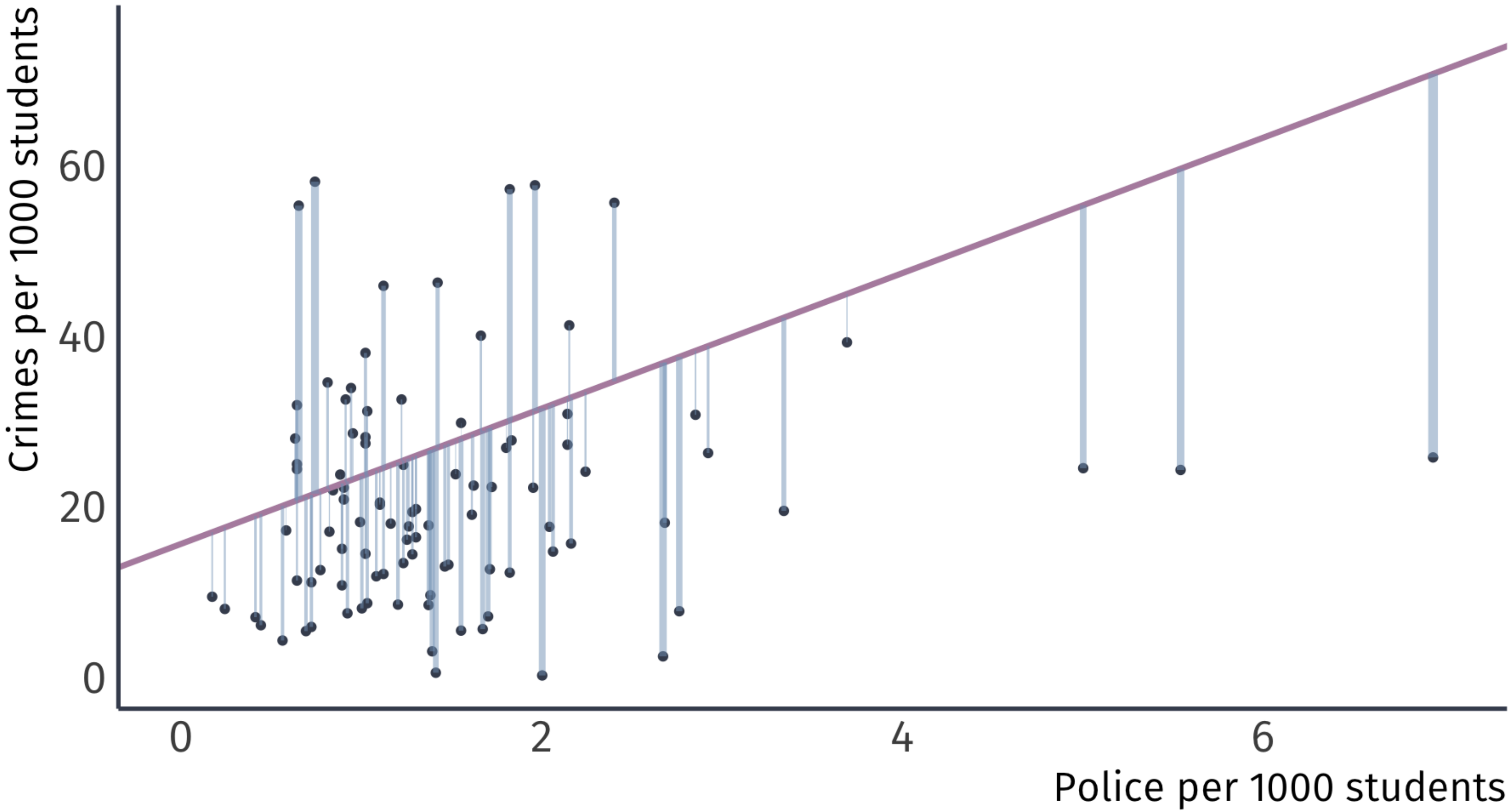
Why not minimize

$$\sum_{i=1}^{n} \hat{u}_i^2$$

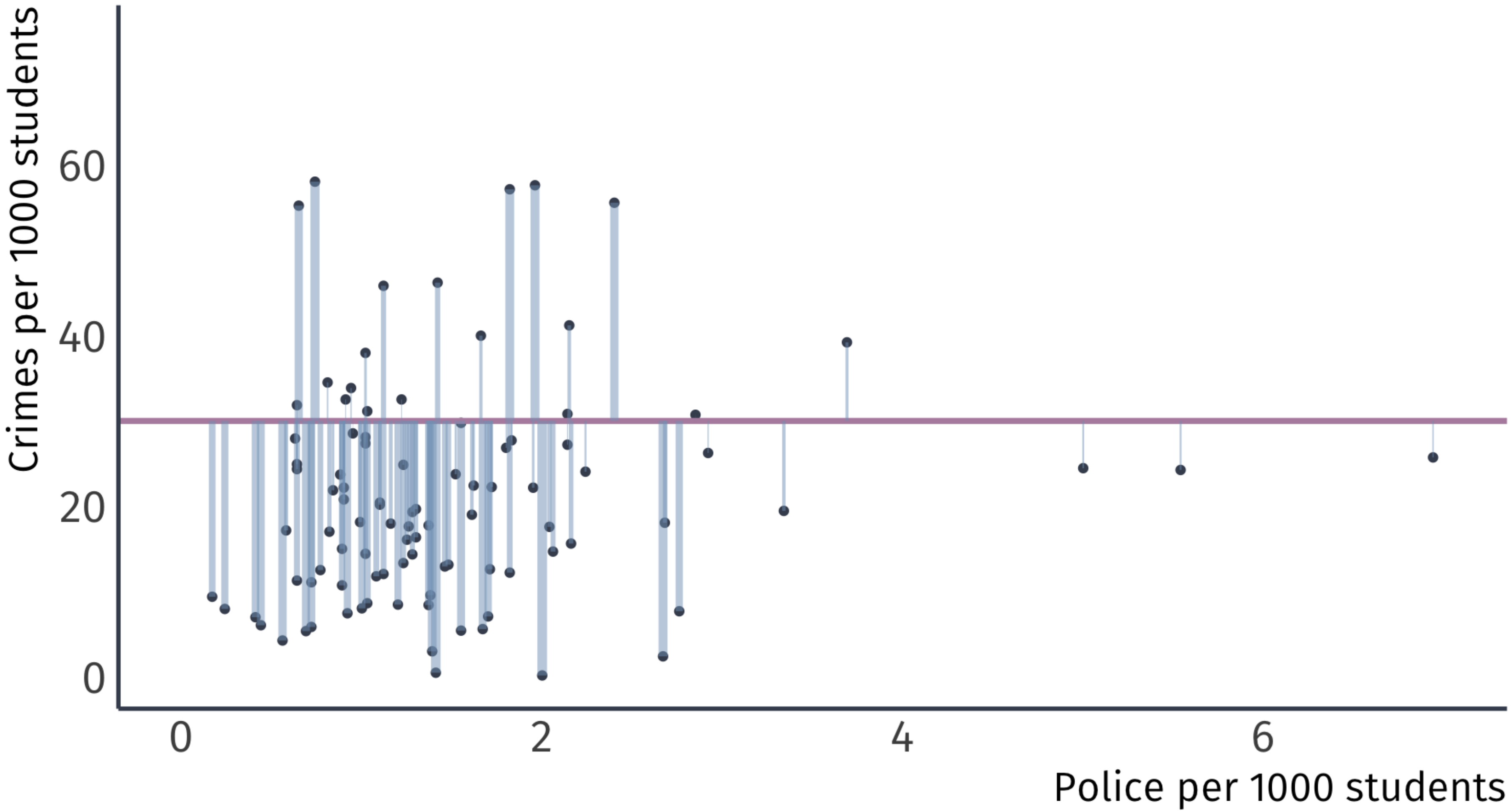
so that the estimator makes fewer big misses?

This estimator, the *residual sum of squares* (**RSS**), is convenient because *squared numbers are never negative* so we can minimize an absolute sum of the residuals

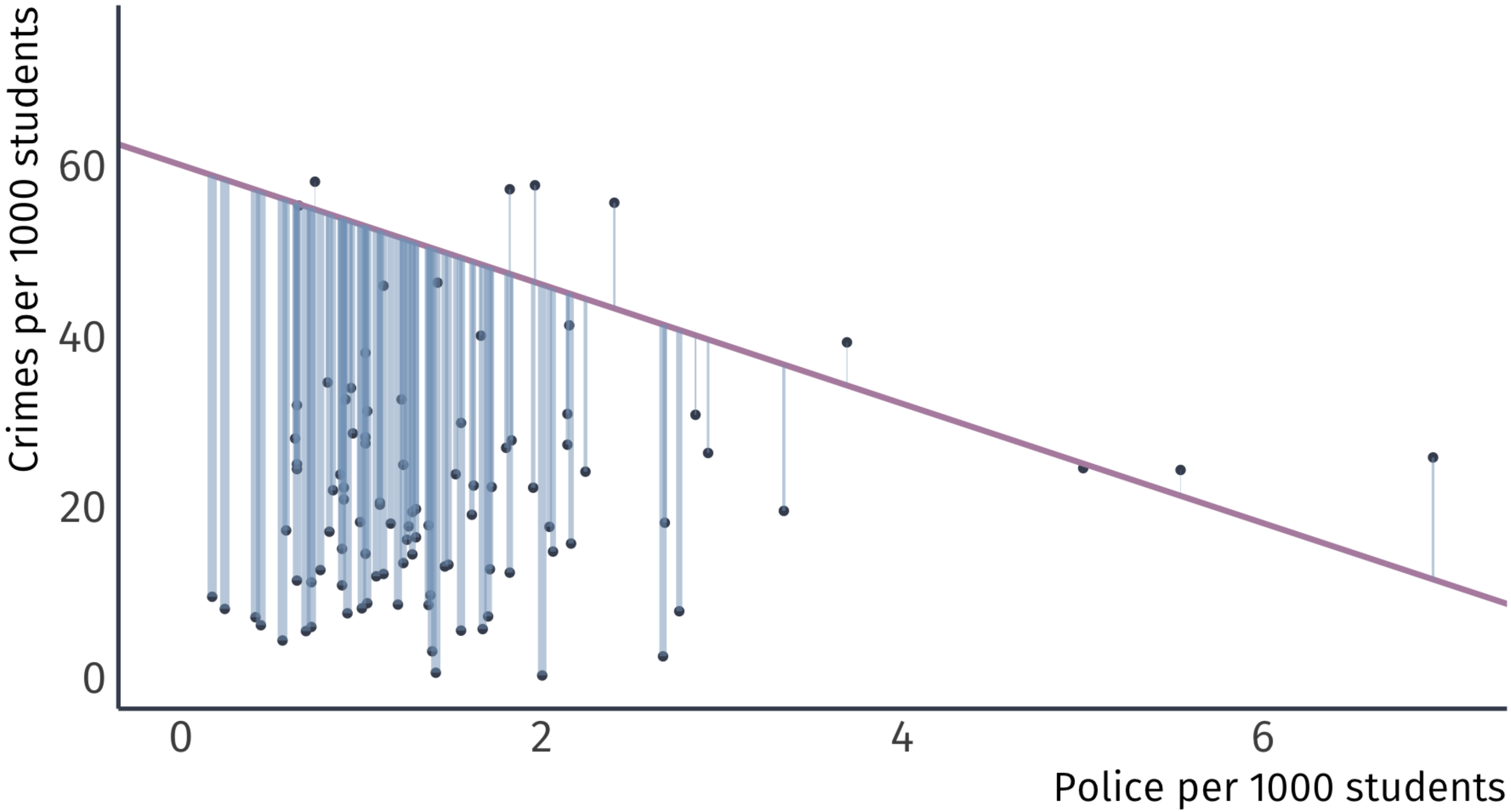
RSS gives bigger penalties to bigger residuals.



RSS gives bigger penalties to bigger residuals.



RSS gives bigger penalties to bigger residuals.



Minimizing RSS

We could test thousands of guesses of β_1 and β_2 and pick the pair the has the smallest RSS

Or... We could just do a little math

Ordinary least squares

OLS

The *OLS estimator* chooses the parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ that minimize the **residual sum of squares (RSS):**

$$\min_{\hat{\beta}_1,\, \hat{\beta}_2} \quad \sum_{i=1}^{n} \hat{u}_i^2$$

This is why we call the estimator ordinary **least squares.**

OLS Formulas

For details, see the handout posted on Canvas.

Slope coefficient

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n} (Y_i - Y)(X_i - X)}{\sum_{i=1}^{n} (X_i - X)^2}$$

Intercept

$$\hat{\beta}_1 = Y - \hat{\beta}_2 X$$

Slope coefficient

The slope estimator is equal to the sample covariance divided by the sample variance of X:

$$
\begin{aligned}
\hat{\beta}_2 &= \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} \\
&= \frac{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2} \\
&= \frac{S_{XY}}{S_X^2}.
\end{aligned}
$$

Continue with OLS next time