

Linear Algebra ASG 1

$$1.) \left[\begin{array}{cccc|c} 3 & -2 & 4 & -1 & 0 \\ -2 & 3 & -2 & -2 & a \\ 0 & 5 & 8 & -7 & b \end{array} \right]$$

$$R_1 + R_2 \rightarrow R_1$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -3 & a \\ -2 & 3 & -2 & -2 & a \\ 0 & 5 & 8 & -7 & b \end{array} \right]$$

$$2R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -3 & a \\ 0 & 5 & 2 & -8 & 3a \\ 0 & 5 & 8 & -7 & b \end{array} \right]$$

$$\frac{1}{5}R_2 \rightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -3 & a \\ 0 & 1 & \frac{2}{5} & -\frac{8}{5} & \frac{3a}{5} \\ 0 & 5 & 8 & -7 & b \end{array} \right]$$

$$5R_2 - R_3 \rightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -3 & a \\ 0 & 1 & \frac{2}{5} & -\frac{8}{5} & \frac{3a}{5} \\ 0 & 0 & -6 & -1 & 15a - b \end{array} \right]$$

$$-\frac{1}{6}R_3 \rightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -3 & a \\ 0 & 1 & \frac{2}{5} & -\frac{8}{5} & \frac{3a}{5} \\ 0 & 0 & 1 & \frac{1}{6} & \frac{15a - b}{6} \end{array} \right]$$

$$R_1 - R_2 \rightarrow R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{8}{5} & -\frac{7}{5} & \frac{2a}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{8}{5} & \frac{3a}{5} \\ 0 & 0 & 1 & \frac{1}{6} & \frac{15a - b}{6} \end{array} \right]$$

$$R_1 - \frac{8}{5}R_3 \rightarrow R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{5}{2} & \frac{-108a - 8b}{30} \\ 0 & 1 & \frac{2}{5} & -\frac{8}{5} & \frac{3a}{5} \\ 0 & 0 & 1 & \frac{1}{6} & \frac{-15a - b}{6} \end{array} \right]$$

$$R_2 - \frac{2}{5}R_3 \rightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{5}{2} & \frac{-108a - 8b}{30} \\ 0 & 1 & 0 & -\frac{5}{2} & \frac{-30a - 2b}{30} \\ 0 & 0 & 1 & \frac{1}{6} & \frac{-15a - b}{6} \end{array} \right]$$

$$① \quad w - \frac{5}{2}z = \frac{-108a - 8b}{30}$$

$$② \quad x - \frac{5}{2}z = \frac{-30a - 2b}{30}$$

$$③ \quad y + \frac{1}{6}z = \frac{-15a - b}{6}$$

$$① \text{ dan } ②$$

$$w - \frac{5}{2}z = \frac{-108a - 8b}{30}$$

$$x - \frac{5}{2}z = \frac{-30a - 2b}{30}$$

$$w - x = \frac{-108a - 8b + 30a + 2b}{30}$$

$$w - x = \frac{-78a - 6b}{30} \quad (4)$$

$$y = \frac{-78a - 6b}{30} + x \quad (5)$$

$$③$$

$$y + \frac{1}{6}z = \frac{-15a - b}{6}$$

$$6y + z = -15a - b$$

$$b = -6y - z - 15a \quad (5)$$

$$④$$

$$w - x = \frac{-78a - 6b}{30}$$

$$30w - 30x = -78a - 6(-6y - z - 15a)$$

$$= -78a + 36y + 6z + 90a$$

$$90a - 78a = 30w - 30x - 36y - 6z$$

$$12a = 30w - 30x - 36y - 6z$$

$$2a = 5w - 5x - 6y - z$$

$$a = \frac{5w - 5x - 6y - z}{2}$$

$$b = -6y - z - 15x$$

$$b = -6y - z - 15 \left(\frac{75}{2}w - \frac{75}{2}x - 6y - z \right)$$

$$b = -6y - z - \frac{75}{2}w + \frac{75}{2}x + \frac{90}{2}y + \frac{15}{2}z$$

$$b = -\frac{12}{2}y + \frac{90}{2}y - \frac{2}{2}z + \frac{15}{2}z - \frac{75}{2}w + \frac{75}{2}x$$

$$= \frac{78}{2}y + \frac{13}{2}z - \frac{75}{2}w + \frac{75}{2}x$$

$$b = \frac{-75w + 75x + 78y + 13z}{2}$$

$$\begin{bmatrix} 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & \frac{7}{3} & \frac{28}{3} \\ 0 & -2 & -10 \\ 0 & -\frac{23}{3} & -\frac{8}{3} \end{bmatrix}$$

$$R_2 \cdot \frac{3}{7} R_2 \rightarrow R_2$$

$$\begin{bmatrix} 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & 1 & 4 \\ 0 & -2 & -10 \\ 0 & -\frac{23}{3} & -\frac{8}{3} \end{bmatrix}$$

$$R_1 - \frac{2}{3} R_2 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & -2 & -10 \\ 0 & -\frac{23}{3} & -\frac{8}{3} \end{bmatrix}$$

$$R_3 + 2R_2 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \\ 0 & -\frac{23}{3} & -\frac{8}{3} \end{bmatrix}$$

$$R_4 + \frac{23}{3} R_2 \rightarrow R_4$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 28 \end{bmatrix}$$

$$-\frac{1}{2} R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 28 \end{bmatrix}$$

$$R_4 - 28R_3 \rightarrow R_4$$

$$R_1 + 2R_3 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 28 \end{bmatrix}$$

$$R_2 - 4R_3 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 28 \end{bmatrix}$$

$$R_4 - 28R_3 \rightarrow R_4$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2.a) \begin{bmatrix} 3 & 2 & 2 \\ -2 & 1 & 8 \\ 3 & 0 & -8 \\ 7 & -3 & 2 \end{bmatrix}$$

$$\frac{1}{3} R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & \frac{2}{3} & \frac{2}{3} \\ -2 & 1 & 8 \\ 3 & 0 & -8 \\ 7 & -3 & 2 \end{bmatrix}$$

$$R_2 + 2R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & \frac{7}{3} & \frac{28}{3} \\ 3 & 0 & -8 \\ 7 & -3 & 2 \end{bmatrix}$$

$$R_3 - 3R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & \frac{7}{3} & \frac{28}{3} \\ 0 & -2 & -10 \\ 7 & -3 & 2 \end{bmatrix}$$

$$R_4 - 7R_1 \rightarrow R_4$$



$$2.b) \begin{bmatrix} 2 & 4 & 2 & -1 & 3 \\ 2 & -1 & 3 & 1 & 2 \\ 3 & -2 & 3 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{2} R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 2 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 2 & -1 & 3 & 1 & 2 \\ 3 & -2 & 3 & 2 & 1 \end{bmatrix}$$

$$R_2 - 2R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & -5 & 1 & 2 & -1 \\ 3 & -2 & 3 & 2 & 1 \end{bmatrix}$$

$$R_3 - 3R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & -5 & 1 & 2 & -1 \\ 0 & -8 & 0 & \frac{7}{2} & -\frac{7}{2} \end{bmatrix}$$

$$-\frac{1}{5} R_2 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ 0 & -8 & 0 & \frac{7}{2} & -\frac{7}{2} \end{bmatrix}$$

$$R_3 + 8R_2 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ 0 & 0 & -\frac{8}{5} & \frac{3}{5} & -\frac{19}{5} \end{bmatrix}$$

$$-\frac{5}{8} R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 1 & -\frac{3}{16} & \frac{19}{16} \end{bmatrix}$$

$$R_1 - 2R_2 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & \frac{7}{5} & \frac{3}{10} & \frac{11}{10} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 1 & -\frac{3}{16} & \frac{19}{16} \end{bmatrix}$$

$$R_1 - \frac{7}{5} R_3 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{9}{16} & -\frac{9}{16} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 1 & -\frac{3}{16} & \frac{19}{16} \end{bmatrix}$$

$$R_2 + \frac{1}{5} R_3 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{9}{16} & -\frac{9}{16} \\ 0 & 1 & 0 & -\frac{7}{16} & \frac{7}{16} \\ 0 & 0 & 1 & -\frac{3}{16} & \frac{19}{16} \end{bmatrix}$$

$$2.c) \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{\cos \theta} R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & \frac{\sin \theta}{\cos \theta} & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 + \sin \theta R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & \frac{\sin \theta}{\cos \theta} & 0 \\ 0 & \cos^2 \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 + \sin \theta R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & \frac{\sin \theta}{\cos \theta} & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\cos \theta}{\cos^2 \theta + \sin^2 \theta} R_2 \rightarrow R_2$$



$$\begin{bmatrix} 1 & \frac{\sin 0}{\cos 0} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 - \frac{\sin 0}{\cos 0} R_2 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$8x + 22y + 16z = -14$$

$$8x + 22\left(-\frac{13}{31}\right) + 16(1) = -14$$

$$8x = \frac{286}{31} - 16 - 14$$

$$= \frac{286 - 930}{31}$$

$$8x = \frac{-644}{31}$$

$$x = \frac{-644}{31}$$

$$= \frac{-644}{31} \cdot \frac{1}{8}$$

$$= \frac{-644}{248}$$

$$x = \frac{-161}{62}$$

$$3) \begin{bmatrix} 1 & 9 & 7 & 0 & 0 \\ 2 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 7 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -14 \\ -13 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 22 & 16 \\ 16 & 12 & 18 \\ 0 & 0 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -14 \\ -13 \\ 12 \end{bmatrix}$$

$$8x + 22y + 16z = -14$$

$$16x + 12y + 18z = -13$$

$$12z = 12$$

$$z = 1$$

$$8x + 22y + 16z = -14 \quad \times 5$$

$$16x + 12y + 18z = -13 \quad \times 4$$

$$40x + 110y + 80z = -70$$

$$40x + 48y + 72z = -52$$

$$62y + 8z = -18$$

$$62y = -18 - 8z$$

$$= -18 - 8(1)$$

$$62y = -26$$

$$y = \frac{-26}{62}$$

$$y = \frac{-13}{31}$$

