MATH 2014 Ch.1 Partial Differentiation (1)

1.1: continuity:

$$f(x,y) = \begin{cases} ? & (x,y) \neq (0,0) \\ \text{const.} & (x,y) = (0,0) \end{cases}$$

prove continuous: 1. change to polar $\quad x = r\cos\theta$, $y = r\sin\theta$

2. prove $\sin\theta$ / $\cos\theta$ bounded, sandwich thm. $\lim_{r\to 0}$

$|\,?\,| \leq |c|$ "bounded"

3. conclusion

prove discontinuous: A: change to polar, $\lim_{r\to 0}$, choose 2 θ, $f(x,y)$ different

B: fix x or $y=0$, then $\lim_{(x,y)\to(0,0)}$ "along half line $y=0$ $x>0$"

compare with $f(0,0)$.

1.2. partial derivatives $\quad W_x \quad \frac{\partial w}{\partial x}$: treat other variables as const.

mixed order: $\quad W_{xy} = (W_x)_y = \frac{\partial^2 w}{\partial y \partial x}$ ← inverted!

C^2 same

1.3 Chain rule

$$\frac{\partial z}{\partial ?} = \frac{\partial z}{\partial i_1}\frac{\partial i_1}{\partial ?} + \frac{\partial z}{\partial i_2}\frac{\partial i_2}{\partial ?}$$

$(a^x)' = a^x \cdot \ln a$

each input one by one
(u,v) (x)

recommend: calculate $\frac{\partial z}{\partial i}$ individually, then combine and sub.

1.4 Total differential : $\Delta w \approx dw = w_x(x_0, y_0)\, \Delta x + w_y(x_0,y_0)\, \Delta y$

$\left|\frac{\Delta Q}{Q}\right| \le \left|\frac{\Delta r}{r}\right| + \dots$ $\quad w(x+\Delta x, y+\Delta y) = \Delta w + w(x,y)$

$\Delta w \le |\Delta x\, w_x| + |\Delta y\, w_y|$

1.5 Taylor's formula : $f(x,y) \approx f(x_0,y_0)$

$+ [\Delta x\, f_x(x_0,y_0) + \Delta y\, f_y(x_0,y_0)]$

calculate partials separately first!

$+ \frac{1}{2!}\left[(\Delta x)^2 f_{xx}(x_0,y_0) + 2\Delta x \Delta y\, f_{xy}(x_0,y_0) + (\Delta y)^2 f_{yy}(x_0,y_0)\right]$

→ slightly $>/<$. x_0, y_0

1.6 Relative extrema : $f_x(x_0,y_0) = 0$, $f_y(x_0,y_0) = 0$

↳ use this first if fail, lagrange

$A = f_{xx}$ $\quad B = f_{xy}$ $\quad C = f_{yy}$

range: try x axis, y axis, 0, 0, $\boxed{\infty, \infty}$

✩ missing roots ! $\ne 0$ $\pm^2$

$H = AC - B^2$

divide by variable :

Draw a table : A B C H

case 1 : normal

$H > 0$, $A > 0$: min $\quad H < 0$: saddle

"boundary" case 2 : $\div 0$

$H > 0$, $A < 0$: max $\quad H = 0$: inconclusive

1.7 Lagrange

$f_x = \lambda g_x$
$f_y = \lambda g_y$
$f_z = \lambda g_z$
$g = 0$

multiple:

$f_x = \lambda g_x + \mu h_x$
$f_y = \lambda g_y + \mu h_y$
$f_z = \lambda g_z + \mu h_z$
$g = 0 \quad h = 0$

constraint min/max value

distance: remove $\sqrt{\ }$ first, add later.

solve λ first by divide

min/max: compare roots.

has max? : put c_1 into c_2 (sub) let $x = a$, $\lim_{a \to n}$

use 2 eqt at a time

✩ divide by variable! consider $= 0$, $\ne 0$.

range / boundary ! range: ABCH, lagrange: boundary

$(x-u)^2 + (y-v)^2$

1.8 Numerical methods: ① root estimation

Newton-Raphson method $x^2 - 2 = 0$ given $x_0 = 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

② simul. eqt.

$$\begin{cases} f(x,y) = \cdots = 0 \\ g(x,y) = \cdots = 0 \end{cases}$$

$$\begin{cases} f_x(x_0,y_0)\,h + f_y(x_0,y_0)\,k = -f(x_0,y_0) \\ g_x(x_0,y_0)\,h + g_y(x_0,y_0)\,k = -g(x_0,y_0) \end{cases}$$

$$x_1 = x_0 + h \qquad y_1 = y_0 + k$$