Exponential Functions

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$a^p \times a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

$a^{-p} = \frac{1}{a^p}$

Logarithmic Functions

$(a > 0 \text{ and } a \neq 1)$

$If\ a^n = N, then\ n = log_a N.$

$log_a a = 1$

$log_a 1 = 0$

$log_a MN = log_a M + log_a N$

$log_a \frac{M}{N} = log_a M - log_a N$

$log_a M^x = x log_a M$

$a^{log_a M} = M$

$log_a b = \frac{log_c b}{log_c a}$

Equations of Straight Lines

Point-Slope form

$y - y_1 = m(x - x_1)$

Slope-Intercept form

$y = mx + c$

Two-Point form

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$, $x_1 \neq x_2$

Intercept form

$\frac{x}{a} + \frac{y}{b} = 1$, $a, b \neq 0$

General form

$Ax + By + C = 0, A \text{ or } B \neq 0$

x-intercept $= -\frac{C}{A}$ $(A \neq 0)$

y-intercept $= -\frac{B}{A}$ $(B \neq 0)$

slope $= -\frac{A}{B}$ $(B \neq 0)$

Quadratic Equation

$ax^2 + bx + c = 0, a \neq 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta = b^2 - 4ac$

$\Delta > 0$, two unequal real roots

$\Delta = 0$, repeated real roots

$\Delta < 0$, no real root

Sum of roots $= -\frac{b}{a}$

Product of roots $= \frac{c}{a}$

Vertex (h, k)

$h = -\frac{b}{2a}$ $k = \frac{4ac-b^2}{4a}$

$a > 0$, open upwards (minimum)

$a < 0$, open downwards (maximum)

Complex Numbers

$i = \sqrt{-1}$ $i^2 = -1$ $\sqrt{-a} = i\sqrt{a}, a \geq 0$

$(a + bi) + (c + di) = a + c + (b + d)i$

$(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi)(c + di) = ac - bd + (ad + bc)i$

$(a + bi)(a - bi) = a^2 + b^2$

Arithmetic Sequence

$T(n) = a + (n - 1)d$, $a = 1^{st}$ term, $l =$ last term

$S(n) = \frac{n}{2}(a + l)$, d = common difference

$S(n) = \frac{n}{2}[2a + (n - 1)d]$

Geometric Sequence

$T(n) = aR^{n-1}$, R = common ratio

$S(n) = \frac{a(1-R^n)}{1-R}$ $(R < 1)$ → last +1

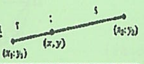
$S(\infty) = \frac{a}{1-R}$ $(-1 < R < 1)$

start from 1

Coordinates

$x = \frac{rx_2 + sx_1}{r + s}$

$y = \frac{ry_2 + sy_1}{r + s}$



distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

mid-point: $x = \frac{x_1+x_2}{2}$ $y = \frac{y_1+y_2}{2}$

$If\ l_1 // l_2, then\ m_1 = m_2.$

$If\ l_1 \perp l_2, then\ m_1 m_2 = -1.$

Permutation (with order)

$P_r^n = \frac{n!}{(n-r)!}$

Combination (without order)

$C_r^n = \frac{n!}{r!\,(n-r)!}$

Division Algorithm

$f(x) = Q(x) \cdot g(x) + R(x)$

$Dividend = Quotient \times Divisor + Remainder$

Remainder Theorem

When $f(x)$ is divided by $mx - n$, remainder is $f(\frac{n}{m})$.

Factor Theorem

If $mx - n$ is a factor of $f(x)$, $f\left(\frac{n}{m}\right) = 0$.

If $f(x)$ is a polynomial and $f\left(\frac{n}{m}\right) = 0$, $mx - n$ is a factor of $f(x)$.

Equations of Circle

Standard form

$(x - h)^2 + (y - k)^2 = r^2$

centre (h, k), radius r

General form

$x^2 + y^2 + Dx + Ey + F = 0$

centre $(-\frac{D}{2}, -\frac{E}{2})$

radius $= \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}$

Probability

$P(E) = \frac{no.\ of\ favourable\ outcomes}{Total\ no.\ of\ possible\ outcomes}$

Addition Law

If A and B are mutually exclusive $(P(A \cap B) = 0)$,

then $P(A \cup B) = P(A) + P(B)$.

If A and B are not mutually exclusive, then

(i) $P(A \cap B) > 0$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Complementary Law

(i) A and A' are mutually exclusive events

(ii) $A \cup A'$ *= all possible outcomes*

(iii) $P(A \cup A') = P(A) + P(A') = 1$

Multiplication Law

If A and B are independent, then

$P(A \cap B) = P(A) \times P(B)$

Conditional Probability

$P(B|A) = \frac{P(A \cap B)}{P(A)}$, *where* $P(A) > 0$

Area of Triangle

Area of $\Delta ABC = \frac{1}{2} absinC$

$= \frac{1}{2} bcsinA$

$= \frac{1}{2} acsinB$

Heron's Formula

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$

Sine Formula

In ΔABC, $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

Cosine Formula

In ΔABC, $a^2 = b^2 + c^2 - 2bc\ cosA$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\ cosC$

or

$cosA = \frac{b^2+c^2-a^2}{2bc}$

$cosB = \frac{a^2+c^2-b^2}{2ac}$

$cosC = \frac{a^2+b^2-c2}{2ab}$

Measures of Dispersion

Range = Max. value – Min. value

= Upper class boundary of the highest class – Lower class boundary of the lowest class

Inter-quartile range $= Q_3 - Q_1$

Mean $\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$

Median = middle term in order

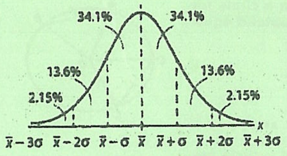
Mode = datum with highest frequency

Standard deviation $\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\cdots+(x_n-\bar{x})^2}{n}}$

Variance $= \sigma^2$

Standard score $z = \frac{x-\bar{x}}{\sigma}$

Normal Distribution



Locus

1. Maintain a fixed distance r from a fixed point G
   → circle with centre G and radius r
2. Maintain an equal distance from 2 fixed points A & B
   → perpendicular bisector of the line segment AB
3. Maintain a fixed distance d from a line L
   → 2 parallel lines to L and a distance d from it
4. Maintain a fixed distance d from a line segment AB
   → a closed curve formed by 2 semicircles of radii d and centres at A & B, and 2 line segments at a distance d from AB
5. Maintain an equal distance from 2 parallel line L_1 and L_2
   → a line parallel to L_1 and at the midway between L_1 and L_2
6. Maintain an equal distance from 2 intersecting lines L_1 and L_2
   → angle bisectors of the angles formed by L_1 and L_2

Variation

y varies directly as x

$y \propto x$ or $y = kx$

y varies inversely as x

$y \propto \frac{1}{x}$ or $y = \frac{k}{x}$

y jointly varies directly as x and z

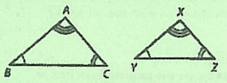
$y \propto xz$ or $y = kxz$

y is partly constant and partly varies directly as x

$y = k_1 + k_2 x$

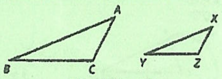
y is partly varies directly as x and partly varies inversely as z

$y = k_1 x + \frac{k_2}{z}$



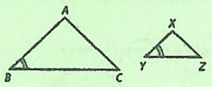
$\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$

[equiangular]



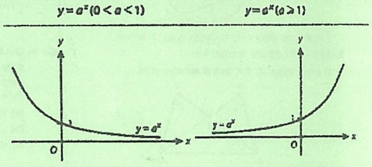
$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$

[3 sides proportional]

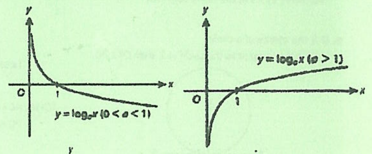


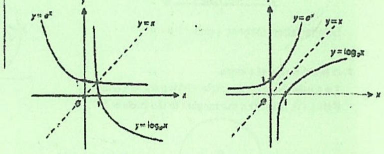
$\frac{AB}{XY} = \frac{BC}{YZ}$, $\angle B = \angle Y$

[ratio of 2 sides, inc. ∠]



$y = log_a x\ (0 < a < 1)$ $y = log_a x\ (a > 1)$





0<a<1 a>1

Function Transformations

$y = f(x) + k$ → A translation of k units upward of the graph of f(x).

$y = f(x) - k$ → A translation of k units downward of the graph of f(x).

$y = f(x - h)$ → A translation of h units to the right of the graph of f(x).

$y = f(x + h)$ → A translation of h units to the left of the graph of f(x).

$y = f(-x)$ → it is the reflected image of the graph of f(x) about y-axis.

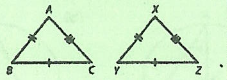
$y = -f(x)$ → it is the reflected image of the graph of f(x) about x-axis.

$y = af(x)$ → it is obtained by enlarging the graph of f(x) along the y-axis.

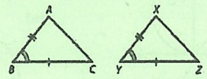
$y = \frac{1}{a}f(x)$ → it is obtained by reducing the graph of f(x) along the y-axis.

$y = f(ax)$ → it is obtained by reducing the graph of f(x) along the x-axis.

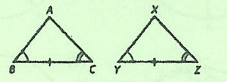
$y = f(\frac{x}{a})$ → it is obtained by enlarging the graph of f(x) along the x-axis.



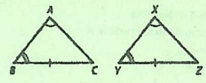
[S.S.S.]



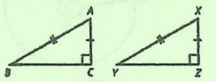
[S.A.S.]



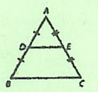
[A.S.A.]



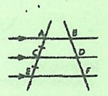
[A.A.S.]



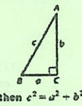
[R.H.S.]



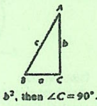
ADB and AEC are straight lines. If AD = DB and AE = EC, then
(i) DE // BC,
(ii) $DE = \frac{1}{2}BC$.
[mid-point theorem]



ACE and BDF are straight lines. If AB // CD // EF and AC = CE, then BD = DF.
[intercept theorem]

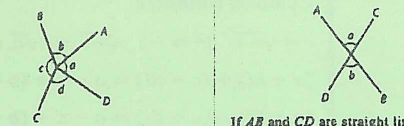


If ∠C = 90°, then $c^2 = a^2 + b^2$.
[Pyth. theorem]



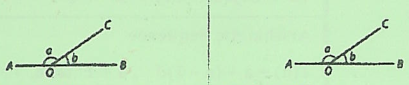
If $c^2 = a^2 + b^2$, then ∠C = 90°.
[converse of Pyth. theorem]

Area: $\pi (R+r)\sqrt{(R-r)^2+h^2}$ volume $= \frac{\pi}{3}h(r^2+R^2+Rr)$ $\angle A = \cos^{-1}\left(\frac{\cos a - \cos b \cos c}{\sin b \sin c}\right)$ $\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$



$a+b+c+d=360°$
[$\angle$s at a pt.]

If AB and CD are straight lines, then $a=b$.
[vert. opp. $\angle$s]

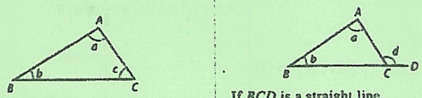


If AOB is a straight line, then $a+b=180°$.
[adj. $\angle$s on st. line]

If $a+b=180°$, then AOB is a straight line.
[adj. $\angle$s supp.]

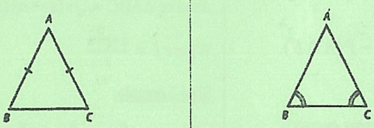
If $PQ // RS$, then
(a) $a=b$, [corr. $\angle$s, $PQ // RS$]
(b) $b=c$, [alt. $\angle$s, $PQ // RS$]
(c) $b+d=180°$. [int. $\angle$s, $PQ // RS$]

If any one of the following conditions is satisfied, then $PQ // RS$.
(a) $a=b$ [corr. $\angle$s eq.]
(b) $b=c$ [alt. $\angle$s eq.]
(c) $b+d=180°$ [int. $\angle$s supp.]



$a+b+c=180°$
[$\angle$ sum of $\triangle$]

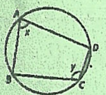
If BCD is a straight line, then $d=a+b$.
[ext. $\angle$ of $\triangle$]



If $AB=AC$, then $\angle B=\angle C$.
[base $\angle$s, isos. $\triangle$]

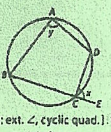
If $\angle B=\angle C$, then $AB=AC$.
[sides opp. eq. $\angle$s]

1. The opposite angles of a cyclic quadrilateral are supplementary.
$x+y=180°$



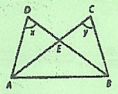
[Abbreviation: opp. $\angle$s, cyclic quad.] Section 12.2

2. For a cyclic quadrilateral, an exterior angle is equal to its interior opposite angle.
$x=y$



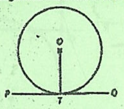
[Abbreviation: ext. $\angle$, cyclic quad.] Section 12.2

5. AEC and DEB are straight lines.
If $x=y$, then A, B, C and D are concyclic.



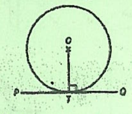
[Abbreviation: converse of $\angle$s in the same segment] Section 12.3

6. O is the centre of a circle.
If PQ is the tangent to the circle at T, then $OT \perp PQ$.



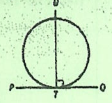
[Abbreviation: tangent $\perp$ radius] Section 12.4A

7. O is the centre of a circle.
T is a point lying on the circle. PTQ is a straight line.
If $OT \perp PQ$, then PQ is the tangent to the circle at T.



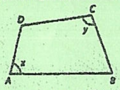
[Abbreviation: converse of tangent $\perp$ radius] Section 12.4A

8. If PQ is the tangent to a circle at T and $UT \perp PQ$, then UT passes through the centre of the circle.



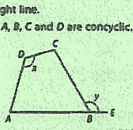
[Abbreviation: line $\perp$ tangent at pt. of contact passes through centre] Section 12.4A

3. If $x+y=180°$, then A, B, C and D are concyclic.



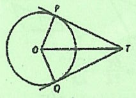
[Abbreviation: opp. $\angle$s supp.] Section 12.3

4. ABE is a straight line.
If $x=y$, then A, B, C and D are concyclic.



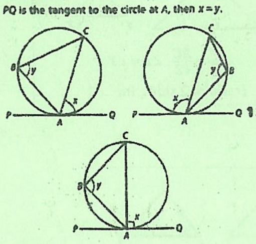
[Abbreviation: ext. $\angle$ = int. opp. $\angle$] Section 12.3

9. O is the centre of a circle.
If TP and TQ are the two tangents drawn from an external point T to the circle at P and Q respectively, then
(a) $TP=TQ$,
(b) $\angle TOP=\angle TOQ$,
(c) $\angle PTO=\angle QTO$.



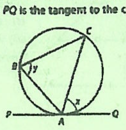
[Abbreviation: tangents from ext. pt.] Section 12.4B

10. AC is a chord.
PQ is the tangent to the circle at A, then $x=y$.



[Abbreviation: $\angle$ in alt. segment] Section 12.5

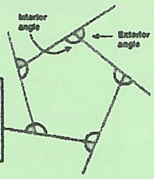
11. AC is a chord and PAQ is a straight line.
If $x=y$, then PQ is the tangent to the circle at A.



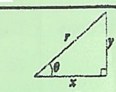
[Abbreviation: converse of $\angle$ in alt. segment] Section 12.5

The sum of the interior angles of an n-sided polygon is $(n-2)\times 180°$.
[$\angle$ sum of polygon]

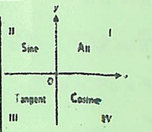
The sum of all exterior angles of a (convex) polygon is 360°.
[sum of ext. $\angle$s of polygon]

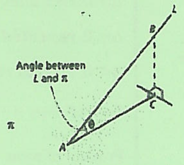


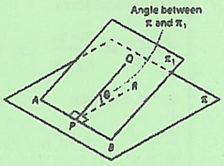
Trigonometry

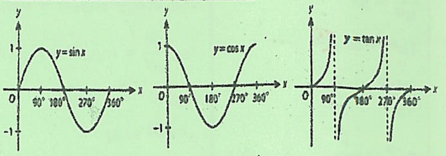


θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined



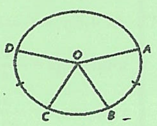




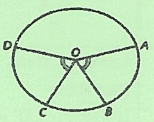


[In this summary, O is the centre of the circle in each of the figures.]

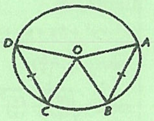
1. Equal arcs in a circle (or equal circles) subtend equal angles at the centre.
If $\overset{\frown}{AB}=\overset{\frown}{CD}$, then $\angle AOB=\angle COD$.
[Abbreviation: eq. arcs, eq. $\angle$s] Section 11.2



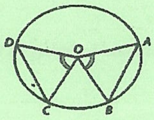
2. Equal angles at the centre of a circle (or equal circles) stand on equal arcs.
If $\angle AOB=\angle COD$, then $\overset{\frown}{AB}=\overset{\frown}{CD}$.
[Abbreviation: eq. $\angle$s, eq. arcs] Section 11.2



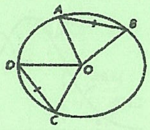
3. Equal chords in a circle (or equal circles) subtend equal angles at the centre.
If $AB=CD$, then $\angle AOB=\angle COD$.
[Abbreviation: eq. chords, eq. $\angle$s] Section 11.2



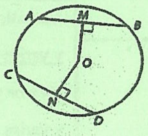
4. Equal angles at the centre of a circle (or equal circles) stand on equal chords.
If $\angle AOB=\angle COD$, then $AB=CD$.
[Abbreviation: eq. $\angle$s, eq. chords] Section 11.2



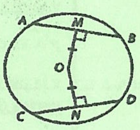
5. Equal chords in a circle (or equal circles) subtend equal arcs.
If $AB=CD$, then $\overset{\frown}{AB}=\overset{\frown}{CD}$.
[Abbreviation: eq. chords, eq. arcs] Section 11.2



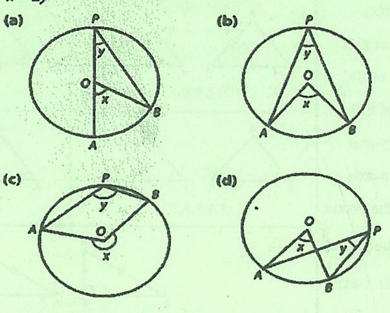
11. If two chords of a circle are equal in length, then they are equidistant from the centre.
If $AB=CD$, then $OM=ON$.
[Abbreviation: eq. chords equidistant from centre] Section 11.3B



12. If two chords of a circle are equidistant from the centre, then their lengths are equal.
If $OM=ON$, then $AB=CD$.
[Abbreviation: chords equidistant from centre eq.] Section 11.3B

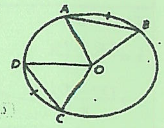


13. The angle at the centre of a circle subtended by an arc is twice the angle at the circumference subtended by the same arc.
$x=2y$

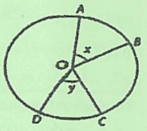


[Abbreviation: $\angle$ at centre = 2 $\angle$ at $\odot^{ce}$] Section 11.4A

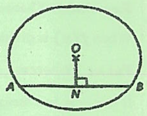
6. Equal arcs in a circle (or equal circles) subtend equal chords.
If $\overset{\frown}{AB}=\overset{\frown}{CD}$, then $AB=CD$.
[Abbreviation: eq. arcs, eq. chords] Section 11.2



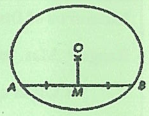
7. In a circle (or equal circles), the lengths of arcs are proportional to the sizes of angles at the centre subtended by the arcs.
$\overset{\frown}{AB}:\overset{\frown}{CD}=x:y$
[Abbreviation: arcs prop. to $\angle$s at centre] Section 11.2



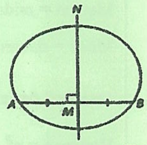
8. A perpendicular from the centre of a circle to a chord bisects the chord.
If $ON \perp AB$, then $AN=BN$.
[Abbreviation: $\perp$ from centre bisects chord] Section 11.3A



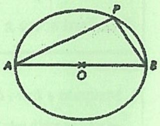
9. The line joining the centre of a circle and the mid-point of a chord (except diameters) is perpendicular to the chord.
If $AM=MB$, then $OM \perp AB$.
[Abbreviation: line joining centre and mid-pt. of chord $\perp$ chord] Section 11.3A



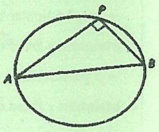
10. The perpendicular bisector of a chord passes through the centre of a circle.
[Abbreviation: $\perp$ bisector of chord passes through centre] Section 11.3A



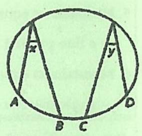
14. If AB is a diameter and P is any point lying on a circle apart from A and B, then $\angle APB=90°$.
[Abbreviation: $\angle$ in semi-circle] Section 11.4A



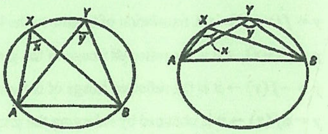
15. If A, B and P are three points lying on a circle and $\angle APB=90°$, then AB is a diameter.
[Abbreviation: converse of $\angle$ in semi-circle] Section 11.4A



16. The lengths of arcs are proportional to the sizes of angles at the circumference subtended by the arcs.
$\overset{\frown}{AB}:\overset{\frown}{CD}=x:y$
[Abbreviation: arcs prop. to $\angle$s at $\odot^{ce}$] Section 11.4A



17. Angles in the same segment are equal.
$x=y$



[Abbreviation: $\angle$s in the same segment] Section 11.4B

Matrix

Properties of Matrices Operation

$A0 = 0A = 0$

$AI = IA = A$

$A(BC) = (AB)C$

$A(B \pm C) = AB \pm AC$

$(A \pm B)C = AC \pm BC$

$(\alpha A)(\beta B) = A(\alpha\beta)B = (\alpha\beta)AB$

$(A + B)^T = A^T + B^T$

$(AB)^T = B^T A^T$.

Inverse of Matrix

$$A^{-1} = \frac{1}{|A|} adj \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

Non-singular => $\det A \neq 0$

(Properties of inverses)

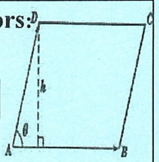
(a) $(A^{-1})^{-1} = A$, (b) $(AB)^{-1} = B^{-1}A^{-1}$.

(c) $(A^T)^{-1} = (A^{-1})^T$, (d) (A)

=

(e) $(A^n)^{-1} = (A^{-1})^n$

Application of Vectors:



Area of ABCD = $|\overrightarrow{AB} \times \overrightarrow{AD}|$

Area of ABD= $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AD}|$

Volume of Parallel pine = $(b \times c) \bullet a$

Volume of triangular prism = $\frac{1}{2}(b \times c) \bullet a$

Volume of tetrahedron = $\frac{1}{6}(b \times c) \bullet a$

Pyramid with parallelogram base =

$\frac{1}{3}(b \times c) \bullet a$

Properties of Determinants

$\det A^T = \det A$.

$\det AB = (\det A)(\det B)$.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\det A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} a_1 + ka_2 & b_1 + kb_2 & c_1 + kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$k \det A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

System of Linear Equation

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1 Cramer's Rule

$\Delta_x \rightarrow$ determinant of A with x column replaced

$$x = \frac{\Delta_x}{\det A}, \quad y = \frac{\Delta_y}{\det A}, \quad z = \frac{\Delta_z}{\det A};$$

2 Gaussian Elimination

reduce to echelon form. $\left(\begin{array}{ccc|c} 1 & a & b & e \\ 0 & 1 & d & f \\ 0 & 0 & 1 & g \end{array}\right)$

3. Use inverse of Matrix

$AX = B$

If $\det A \neq 0$, **Unique Solution** *and* $X = A^{-1}B$

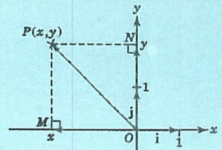
***For Homogeneous**

AX=0 ,

If $\det A \neq 0$, then X =0, trivial solution.

If det A = 0, AX = B has ∞ or no solutions (use Gaussian Elimination to Solve)

Vectors



1. $\overrightarrow{OP} = xi + yj$.

   Or $\overrightarrow{OP} = xi + yj + zk$.

2. $|\overrightarrow{OP}| = \sqrt{x^2 + y^2}$ Or $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

3. If $\overrightarrow{OP} \neq \mathbf{0}$, $\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$,

   $\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$ and $\tan\theta = \frac{y}{x}$.

4. The unit vector in the direction of **a** is $\frac{\vec{a}}{|\vec{a}|}$.

5. $m\mathbf{i} + n\mathbf{j} = x\mathbf{i} + y\mathbf{j} \Rightarrow m = x$ and $n = y$

6. $m\mathbf{i} + n\mathbf{j} = \mathbf{0} \Rightarrow m = n = 0$

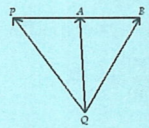
7. **a** is a unit vector iff $|a| = 1$.

1.Parallelism

If the vectors $\mathbf{p} = m\mathbf{i} + n\mathbf{j}$ and $\mathbf{q} = x\mathbf{i} + y\mathbf{j}$ are parallel, then

$\frac{x}{y} = \frac{m}{n}$ and $\frac{x}{m} = \frac{y}{n}$

2. Division of a Line Segment



AB such that AP : PB = $m : n$, then

$$\overrightarrow{QP} = \frac{m\overrightarrow{QB} + n\overrightarrow{QA}}{m+n}$$

, i.e. $\mathbf{p} = \frac{n}{n+m}a + \frac{m}{m+n}b$

3. Coplanar

Vol=0 => **Scalar Triple Product =0**

Scalar Triple Product $(b \times c) \bullet a$

If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$, $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ and

$\mathbf{c} = x_3\mathbf{i} + y_3\mathbf{j} + z_3\mathbf{k}$ then $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$.

Scalar product(Dot Product)

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between **a** and **b**.

(1) $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$.

(2) $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$.

(3) $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \Rightarrow \mathbf{b} = \mathbf{c}$

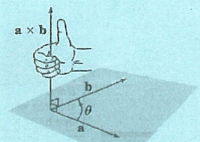
If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$, then $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2$

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}$$

If $\mathbf{a} \cdot \mathbf{b} = 0$, **a** is perpendicular to **b**.

Vector Products

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\, \mathbf{n}$, where θ is the angle between **a** and **b** and **n** is the unit vector



(a) $\mathbf{a} \times \mathbf{a} = \mathbf{0}$

(b) $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

(c) $k(\mathbf{a} \times \mathbf{b}) = (k\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (k\mathbf{b})$.

(d) $|\mathbf{a} \times \mathbf{b}| = |\mathbf{b} \times \mathbf{a}|$

(e) For non-zero vectors **a** and **b** , $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ if and only if **a** and **b** are parallel.

(h) $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$

Vectors in $\mathbf{R}^3$

(i) $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$.

(ii) $\mathbf{i} \times \mathbf{j} = \mathbf{k}$, $\mathbf{j} \times \mathbf{k} = \mathbf{i}$, $\mathbf{k} \times \mathbf{i} = \mathbf{j}$. ($\mathbf{j} \times \mathbf{i} = -\mathbf{k}$, $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$, $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$)

(iii) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \Rightarrow \mathbf{b} \neq \mathbf{c}$ (e.g. **a** and **b** are parallel, and **c** is **0**).

If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ are two non-zero vectors, and θ is the angle between them, then

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}.$$

Limit

$\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x) = L_1 \cdot L_2$;

$\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \dfrac{L_1}{L_2}$ $L_2 \neq 0$

$e^x = \lim_{n\to\infty}\left(1+\dfrac{x}{n}\right)^n = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots = \sum_{r=0}^{\infty}\dfrac{x^r}{r!}$

$\lim_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n = e$ $\lim_{\theta\to 0}\dfrac{\sin\theta}{\theta} = 1$ $\lim_{x\to 0}\dfrac{e^x - 1}{x} = 1$

First principal

$f'(x) = \lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

The Binomial Theorem

$C_r^n = \dfrac{n!}{r!(n-r)!}$ $P_r^n = (n-r)!$

$C_r^n = C_{n-r}^n$ $C_r^n + C_{r+1}^n = C_{r+1}^{n+1}$

$(x+y)^n =$
$x^n + C_1^n x^{n-1}y + C_2^n x^{n-2}y^2 + \dots + C_{n-1}^n xy^{n-1} + y^n$

S1: General term; S2, Solve for r

Trigonometry

$\sin(A\pm B) = \sin A\cos B \pm \cos A \sin B$

$\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$\tan(A\pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$ $\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Rules of differentiation

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Chain Rule

$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx}$

-----Differentiation --->

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$\dfrac{d}{dx}(\sin x) = \cos x$

$\dfrac{d}{dx}(\tan x) = \sec^2 x$

$\dfrac{d}{dx}(\sec x) = \sec x\tan x$

$\dfrac{d}{dx}(\cos x) = -\sin x$

$\dfrac{d}{dx}(\cot x) = -\csc^2 x$

$\dfrac{d}{dx}(\csc x) = -\csc x\cot x$

$\dfrac{d}{dx}(e^x) = e^x$

$\dfrac{d}{dx}a^x = a^x\ln a$

$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

$\dfrac{d}{dx}\log_a x = \dfrac{1}{x\ln a}$

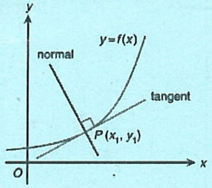
$\dfrac{d}{dx}(\ln|\sec x|) = \tan x$

$\dfrac{d}{dx}(\ln|\sec x + \tan x|) = \sec x$

<---- Integration -----

Application of Differentiation

Tangent and Normal

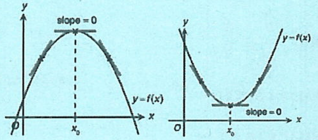


Normal : $y - y_1 = -\dfrac{1}{f'(x_1)}(x - x_1)$

Tangent : $y - y_1 = f'(x)(x - x_1)$

Second D Test

$f''(x_o)<0$ $f''(x_o)>0$



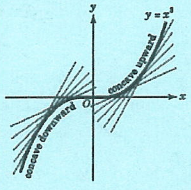
S1' set $f'(x)=0 \Rightarrow x = x_o$

S2' check $f''(x_o)$

*If $f''(x_o)<0$, $(x_o, f''(x_o))$ is a **max**. pt.*

*If $f''(x_o)>0$, $(x_o, f''(x_o))$ is a **min**. pt.*

Point of Inflexion



$f''(x_o)$ changes signs

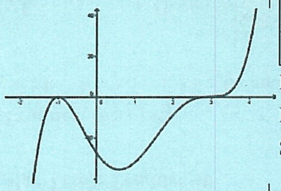
Assymptotes

e.g. $y = 2x - 3 + \dfrac{2}{[x+5]}$

Vertical: $x = -5$

Horizontal/Oblique: $y = 2x - 3$

First D Test



S1: Set $f'(x)=0$; => $x = 1, 5, or\ 7$

x	$x<1$	$x=1$	$1<x<5$	$x=5$	$5<x<7$	$x=7$	$7<x$
dy/dx	+ve	0	-ve	0	+ve	0	+ve

Maximum value: $f(1)$ Maximum point: $(1, f(1))$

Minimum value: $f(5)$ Maximum point: $(5, f(5))$

Stationary point: $(7, f(7))$

Rate of Change,

$v = \dfrac{ds}{dt}$ $a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$

S1: formulate equation i.e.

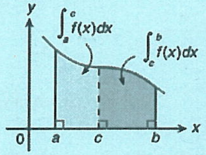
$A = 4\pi r^2$

S2: apply chain rule to diff w.r.t *t*.

i.e. $\dfrac{dA}{dt} = \dfrac{d(4\pi r^2)}{dr}\dfrac{dr}{dt}$

S3: Substitutes and solve.

Integration



$\int_a^a f(x)dx = \int_a^a f(t)dt$,

$\int_a^a f(x)dx = 0$,

$\int_b^a f(x)dx = -\int_a^b f(x)dx$,

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$,

$\int_a^b [f(x) \pm g(x)]dx$

$\int_a^b f(x)dx \pm \int_a^b g(x)dx$

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx & even \\ 0 & odd \end{cases}$

Integration by parts

$\int udv = uv - \int vdu$;

$\int u\dfrac{dv}{dx}dx = uv - \int v\dfrac{du}{dx}dx$

$xe^x -> d(e^x)$;

$x\cos x -> d(\sin x)$

$x\ln x -> d(\dfrac{x^2}{2})$;

$e^x\cos x -> d(e^x)$;

***Use by part X2 or Recurrent**

Integration Trigo-Sub

$\sqrt{a^2 - x^2}$ put $x = a\sin$ □

$\sqrt{a^2 + x^2}$, put x = a tan □

$\sqrt{x^2 - a^2}$, put $x = a\sec$

M.I.

When n=1,

…

It is true for n=1.

Assume the statement is true for n=k

i.e. …

When n=$k+1$

…

It is true for n=$k+1$

By the principle of mathematical induction, ….

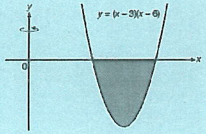
Application of Integration

Area= $\int_a^b (high - low)dx$

Shell method: $\int_a^b (2\pi R)Hdy$

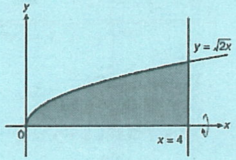
$Vol = \int_a^b 2\pi xydx$ or $\int_a^b 2\pi xydy$

(vertical->dx; horizontal ->dy)



Disc method: $\int_a^b \pi R^2 dh$

$Vol = \int_a^b \pi x^2 dy$ or $Vol = \int_a^b \pi y^2 dx$



M2

1. First Principles

a. $y = \ln(2016x + 2015)$

b. $y = 2015 \sin 2016x$

c. $y = 2015 \cos 2016x$

d. $y = 1314\sqrt{2016x}$

2. M.I.

1° when $n=1$

LHS = ...

It is true for $n=1$

2° Assume $f(x)$ is true for some +ve integer k

i.e. ______________

when $n = k+1$

.....

3. Binomial

① General term

② set eqt

③ solve for r

e.g. $(x^3 + \frac{1}{x})^4$

General term $= C^n_r (\ )^r \cdot (\ )^{n-r}$

$= C^4_r (x^3)^r (\frac{1}{x})^{4-r}$

$= C^4_r x^{3r} x^{r-4}$

$= C^4_r x^{4r-4}$

$\frac{\sin}{\cos}$

$\frac{\cos^3 x + \sin^3 x}{\cos^3 x}$

$(x^2 - \frac{1}{x})^8$

General term $= C^8_r x^{16-3r}$

Coefficient of x^4

$16 - 3r = 4$

$r = 4$

$C^8_4 = 70$

coefficient of $x^4 = 70$

$(x^2 - \frac{1}{x})^8 (2 - x + 3x^{-4})$

const. term

$\boxed{3C^8_r x^{12-3r}}$

Const. term $= 3C^8_4$

$= 210$

Hong Kong Baptist University Affiliated School
Wong Kam Fai Secondary and Primary School
G11 Mathematics (Module 2) Quiz 3

Name: Jeffrey Lee Class: 11A Class No. 11 Date:________ Marks:________

1. If $y = \ln\sqrt{\dfrac{x}{x+2}}$, find $\dfrac{dy}{dx}$.

(4 marks)

2.. If $y = (e^{2x}+1)^{10}$, find $\dfrac{dy}{dx}$.

(3 marks)

3. If $e^{y} = \dfrac{e^{x}+3y}{x^{4}}$, find $\dfrac{dy}{dx}$.

(4 marks)

4. If $y = 3^{x}(x+1)^{2}$, find $\dfrac{dy}{dx}$.

(4 marks)

5. If $y = (\tan x)e^{2x+1}$, find $\dfrac{dy}{dx}$.

(4 marks)

6. Let $f(x) = \tan^{3} x + \dfrac{1}{2}\sec 2x$. Find $f'\left(\dfrac{\pi}{6}\right)$.

(5 marks)

7. Given that $y = e^{x}\sin 2x$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

(b) If $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + ky = 0$, find the value of k.

(8 marks)