## Exponential Functions

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$a^p \times a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

$a^{-p} = \frac{1}{a^p}$

## Logarithmic Functions

$(a > 0 \; and \; a \neq 1)$

$If \; a^n = N, then \; n = log_a N.$

$log_a a = 1$

$log_a 1 = 0$

$log_a MN = log_a M + log_a N$

$log_a \frac{M}{N} = log_a M - log_a N$

$log_a M^x = x log_a M$

$a^{log_a M} = M$

$log_a b = \frac{log_c b}{log_c a}$

## Equations of Straight Lines

Point-Slope form

$y - y_1 = m(x - x_1)$

Slope-Intercept form

$y = mx + c$

Two-Point form

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \;, x_1 \neq x_2$

Intercept form

$\frac{x}{a} + \frac{y}{b} = 1 \,, a, b \neq 0$

General form

$Ax + By + C = 0, A \; or \; B \neq 0$

x-intercept $= -\frac{C}{A}$ $(A \neq 0)$

y-intercept $= -\frac{B}{A}$ $(B \neq 0)$

slope $= -\frac{A}{B}$ $(B \neq 0)$

## Quadratic Equation

$ax^2 + bx + c = 0, a \neq 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta = b^2 - 4ac$

$\Delta > 0, two \; unequal \; real \; roots$

$\Delta = 0, repeated \; real \; roots$

$\Delta < 0, no \; real \; root$

Sum of roots $= -\frac{b}{a}$

Product of roots $= \frac{c}{a}$

Vertex $(h, k)$

$h = -\frac{b}{2a} \quad k = \frac{4ac-b^2}{4a}$

$a > 0$, open upwards (minimum)

$a < 0$, open downwards (maximum)

## Complex Numbers

$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, a \geq 0$

$(a + bi) + (c + di) = a + c + (b + d)i$

$(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi)(c + di) = ac - bd + (ad + bc)i$

$(a + bi)(a - bi) = a^2 + b^2$

## Arithmetic Sequence

$T(n) = a + (n - 1)d$ , a = 1st term, $l$ = last term

$S(n) = \frac{n}{2}(a + l)$ , d = common difference

$S(n) = \frac{n}{2}[2a + (n - 1)d]$

## Geometric Sequence

$T(n) = aR^{n-1}$ , $R$ = common ratio

$S(n) = \frac{a(1-R^n)}{1-R} (R < 1)$ → last +1

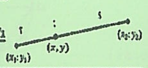
$S(\infty) = \frac{a}{1-R} (-1 < R < 1)$

start from 1

## Coordinates

$x = \frac{rx_2 + sx_1}{r + s}$

$y = \frac{ry_2 + sy_1}{r + s}$




distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

mid-point: $x = \frac{x_1+x_2}{2} \quad y = \frac{y_1+y_2}{2}$

$If \; l_1 // l_2, then \; m_1 = m_2.$

$If \; l_1 \perp l_2, then \; m_1 m_2 = -1.$

## Permutation (with order)

$P_r^n = \frac{n!}{(n-r)!}$

## Combination (without order)

$C_r^n = \frac{n!}{r!\,(n-r)!}$

## Division Algorithm

$f(x) = Q(x) \cdot g(x) + R(x)$

$Dividend = Quotient \times Divisor + Remainder$

**Remainder Theorem**

When $f(x)$ is divided by $mx - n$, remainder is $f\left(\frac{n}{m}\right)$.

**Factor Theorem**

If $mx - n$ is a factor of $f(x)$, $f\left(\frac{n}{m}\right) = 0$.

If $f(x)$ is a polynomial and $f\left(\frac{n}{m}\right) = 0$, $mx - n$ is a factor of $f(x)$.

## Equations of Circle

Standard form

$(x - h)^2 + (y - k)^2 = r^2$

centre $(h, k)$, radius $r$

General form

$x^2 + y^2 + Dx + Ey + F = 0$

centre $\left(-\frac{D}{2}, -\frac{E}{2}\right)$

radius $= \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}$

## Probability

$P(E) = \frac{no. of \; favourable \; outcomes}{Total \; no. of \; possible \; outcomes}$

**Addition Law**

*If A and B are mutually exclusive* $(P(A \cap B) = 0)$,

*then* $P(A \cup B) = P(A) + P(B)$.

*If A and B are not mutually exclusive, then*

*(i)* $P(A \cap B) > 0$

*(ii)* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Complementary Law**

*(i) A and A' are mutually exclusive events*

*(ii)* $A \cup A'$ *= all possible outcomes*

*(iii)* $P(A \cup A') = P(A) + P(A') = 1$

**Multiplication Law**

*If A and B are independent, then*

$P(A \cap B) = P(A) \times P(B)$

**Conditional Probability**

$P(B|A) = \frac{P(A \cap B)}{P(A)}, where \; P(A) > 0$

## Area of Triangle

Area of $\Delta ABC = \frac{1}{2} absinC$

$= \frac{1}{2} bcsinA$

$= \frac{1}{2} acsinB$

**Heron's Formula**

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$

**Sine Formula**

In $\Delta ABC$, $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

**Cosine Formula**

In $\Delta ABC$, $a^2 = b^2 + c^2 - 2bc \; cosA$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \; cosC$

or $cosA = \frac{b^2+c^2-a^2}{2bc}$

$cosB = \frac{a^2+c^2-b^2}{2ac}$

$cosC = \frac{a^2+b^2-c2}{2ab}$

## Measures of Dispersion

Range = Max. value – Min. value

= Upper class boundary of the highest class – Lower class boundary of the lowest class

Inter-quartile range $= Q_3 - Q_1$

Mean $\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$

Median = middle term in order

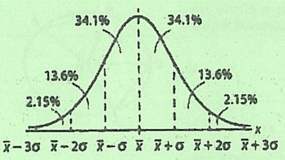
Mode = datum with highest frequency

Standard deviation $\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\cdots+(x_n-\bar{x})^2}{n}}$

Variance $= \sigma^2$

Standard score $z = \frac{x-\bar{x}}{\sigma}$

Normal Distribution




## Locus

1. Maintain a fixed distance r from a fixed point G
   → circle with centre G and radius r
2. Maintain an equal distance from 2 fixed points A & B
   → perpendicular bisector of the line segment AB
3. Maintain a fixed distance d from a line L
   → 2 parallel lines to L and a distance d from it
4. Maintain a fixed distance d from a line segment AB
   → a closed curve formed by 2 semicircles of radii d and centres at A & B, and 2 line segments at a distance d from AB
5. Maintain an equal distance from 2 parallel line $L_1$ and $L_2$
   → a line parallel to $L_1$ and at the midway between $L_1$ and $L_2$
6. Maintain an equal distance from 2 intersecting lines $L_1$ and $L_2$
   → angle bisectors of the angles formed by $L_1$ and $L_2$

## Variation

**y varies directly as x**

$y \propto x \; or \; y = kx$

**y varies inversely as x**

$y \propto \frac{1}{x} \; or \; y = \frac{k}{x}$

**y jointly varies directly as x and z**

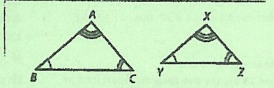
$y \propto xz \; or \; y = kxz$

**y is partly constant and partly varies directly as x**

$y = k_1 + k_2 x$

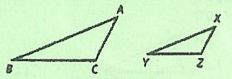
**y is partly varies directly as x and partly varies inversely as z**

$y = k_1 x + \frac{k_2}{z}$



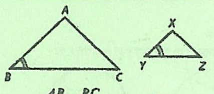

$\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$

[equiangular]



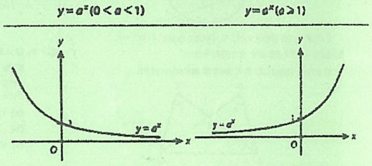

$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$

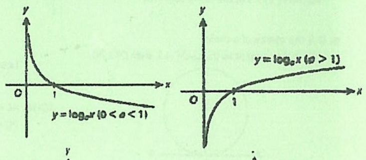
[3 sides proportional]

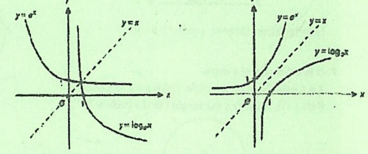



$\frac{AB}{XY} = \frac{BC}{YZ}, \angle B = \angle Y$

[ratio of 2 sides, inc. ∠]




## Function Transformations

$y = f(x) + k$ → A translation of k units upward of the graph of f(x).

$y = f(x) - k$ → A translation of k units downward of the graph of f(x).

$y = f(x - h)$ → A translation of h units to the right of the graph of f(x).

$y = f(x + h)$ → A translation of h units to the left of the graph of f(x).

$y = f(-x)$ → it is the reflected image of the graph of f(x) about y-axis.

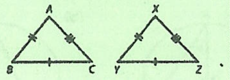
$y = -f(x)$ → it is the reflected image of the graph of f(x) about x-axis.

$y = af(x)$ → it is obtained by enlarging the graph of f(x) along the y-axis.

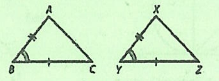
$y = \frac{1}{a}f(x)$ → it is obtained by reducing the graph of f(x) along the y-axis.

$y = f(ax)$ → it is obtained by reducing the graph of f(x) along the x-axis.

$y = f(\frac{x}{a})$ → it is obtained by enlarging the graph of f(x) along the x-axis.




[S.S.S.]

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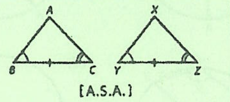


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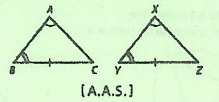
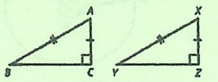
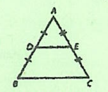



[A.A.S.]




[R.H.S.]

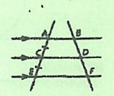



ADB and AEC are straight lines. If AD = DB and AE = EC, then

(i) DE // BC,

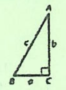
(ii) $DE = \frac{1}{2} BC$.

[mid-point theorem]



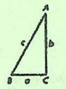

ACE and BDF are straight lines. If AB // CD // EF and AC = CE, then BD = DF.

[intercept theorem]




If ∠C = 90°, then $c^2 = a^2 + b^2$.

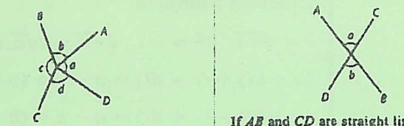
[Pyth. theorem]




If $c^2 = a^2 + b^2$, then ∠C = 90°.

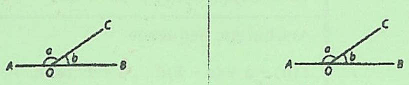
[converse of Pyth. theorem]

Area: $\pi(R+r)\sqrt{(R-r)^2+h^2}$ volume $= \frac{\pi}{3}h(r^2+R^2+Rr)$ $\angle A = \cos^{-1}\left(\frac{\cos a - \cos b\cos c}{\sin b \sin c}\right)$ $\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$



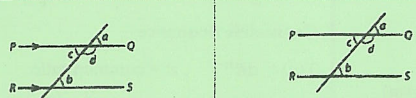
$a+b+c+d=360°$
[$\angle$s at a pt.]

If $AB$ and $CD$ are straight lines, then $a=b$.
[vert. opp. $\angle$s]



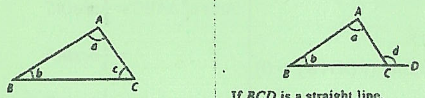
If $AOB$ is a straight line, then $a+b=180°$.
[adj. $\angle$s on st. line]

If $a+b=180°$, then $AOB$ is a straight line.
[adj. $\angle$s supp.]



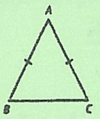
If $PQ // RS$, then
(a) $a=b$, [corr. $\angle$s, $PQ // RS$]
(b) $b=c$, [alt. $\angle$s, $PQ // RS$]
(c) $b+d=180°$. [int. $\angle$s, $PQ // RS$]

If any one of the following conditions is satisfied, then $PQ // RS$.
(a) $a=b$ [corr. $\angle$s eq.]
(b) $b=c$ [alt. $\angle$s eq.]
(c) $b+d=180°$ [int. $\angle$s supp.]

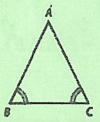


$a+b+c=180°$
[$\angle$ sum of $\Delta$]

If $BCD$ is a straight line, then $d=a+b$.
[ext. $\angle$ of $\Delta$]



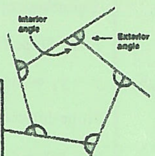
If $AB=AC$, then $\angle B=\angle C$.
[base $\angle$s, isos. $\Delta$]



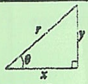
If $\angle B=\angle C$, then $AB=AC$.
[sides opp. eq. $\angle$s]

The sum of the interior angles of an n-sided polygon is $(n-2) \times 180°$.
[$\angle$ sum of polygon]

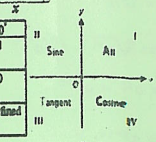
The sum of all exterior angles of a (convex) polygon is $360°$.
[sum of ext. $\angle$s of polygon]

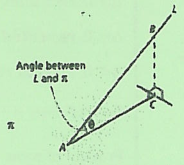


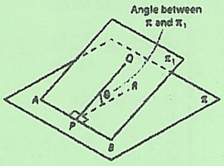

## Trigonometry

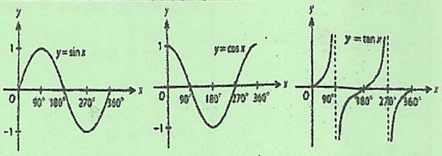


| $\theta$ | $0°$ | $30°$ | $45°$ | $60°$ | $90°$ |
|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\tan\theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | undefined |

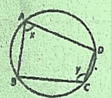


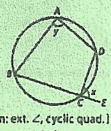




1. The opposite angles of a cyclic quadrilateral are supplementary.
$x+y=180°$



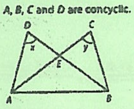
[Abbreviation: opp. $\angle$s, cyclic quad.] Section 12.2

2. For a cyclic quadrilateral, an exterior angle is equal to its interior opposite angle.
$x=y$



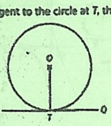
[Abbreviation: ext. $\angle$, cyclic quad.] Section 12.2

5. $AEC$ and $DEB$ are straight lines.
If $x=y$, then $A$, $B$, $C$ and $D$ are concyclic.



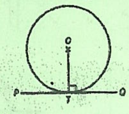
[Abbreviation:
converse of $\angle$s in the same segment] Section 12.3

6. $O$ is the centre of a circle.
If $PQ$ is the tangent to the circle at $T$, then $OT \perp PQ$.



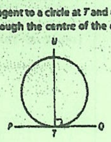
[Abbreviation: tangent $\perp$ radius] Section 12.4A

7. $O$ is the centre of a circle.
$T$ is a point lying on the circle. $PTQ$ is a straight line.
If $OT \perp PQ$, then $PQ$ is the tangent to the circle at $T$.



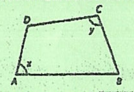
[Abbreviation:
converse of tangent $\perp$ radius] Section 12.4A

8. If $PQ$ is the tangent to a circle at $T$ and $UT \perp PQ$, then $UT$ passes through the centre of the circle.



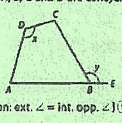
[Abbreviation:
line $\perp$ tangent at pt. of contact passes through centre] Section 12.4B

3. If $x+y=180°$, then $A$, $B$, $C$ and $D$ are concyclic.



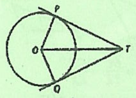
[Abbreviation: opp. $\angle$s supp.] Section 12.3

4. $ABE$ is a straight line.
If $x=y$, then $A$, $B$, $C$ and $D$ are concyclic.



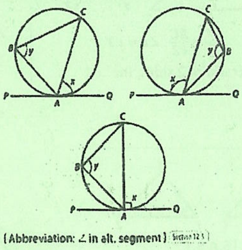
[Abbreviation: ext. $\angle$ = int. opp. $\angle$] Section 12.3

9. $O$ is the centre of a circle.
If $TP$ and $TQ$ are the two tangents drawn from an external point $T$ to the circle at $P$ and $Q$ respectively, then
(a) $TP=TQ$,
(b) $\angle TOP=\angle TOQ$,
(c) $\angle PTO=\angle QTO$.



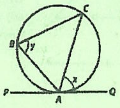
[Abbreviation: tangents from ext. pt.] Section 12.4B

10. $AC$ is a chord.
$PQ$ is the tangent to the circle at $A$, then $x=y$.



[Abbreviation: $\angle$ in alt. segment] Section 12.5

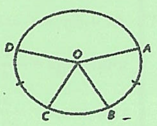
11. $AC$ is a chord and $PAQ$ is a straight line.
If $x=y$, then $PQ$ is the tangent to the circle at $A$.



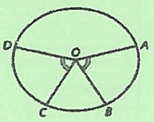
[Abbreviation:
converse of $\angle$ in alt. segment] Section 12.5

[In this summary, $O$ is the centre of the circle in each of the figures.]

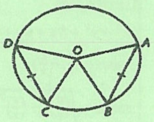
1. Equal arcs in a circle (or equal circles) subtend equal angles at the centre.
If $\overset{\frown}{AB}=\overset{\frown}{CD}$,
then $\angle AOB=\angle COD$.
[Abbreviation:
eq. arcs, eq. $\angle$s] Section 11.2



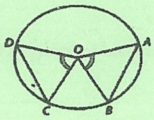
2. Equal angles at the centre of a circle (or equal circles) stand on equal arcs.
If $\angle AOB=\angle COD$,
then $\overset{\frown}{AB}=\overset{\frown}{CD}$.
[Abbreviation:
eq. $\angle$s, eq. arcs] Section 11.2



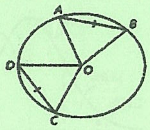
3. Equal chords in a circle (or equal circles) subtend equal angles at the centre.
If $AB=CD$,
then $\angle AOB=\angle COD$.
[Abbreviation:
eq. chords, eq. $\angle$s] Section 11.2



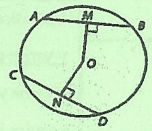
4. Equal angles at the centre of a circle (or equal circles) stand on equal chords.
If $\angle AOB=\angle COD$,
then $AB=CD$.
[Abbreviation:
eq. $\angle$s, eq. chords] Section 11.2



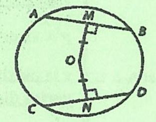
5. Equal chords in a circle (or equal circles) subtend equal arcs.
If $AB=CD$,
then $\overset{\frown}{AB}=\overset{\frown}{CD}$.
[Abbreviation:
eq. chords, eq. arcs] Section 11.2



11. If two chords of a circle are equal in length, then they are equidistant from the centre.
If $AB=CD$,
then $OM=ON$.
[Abbreviation:
eq. chords equidistant from centre] Section 11.3B

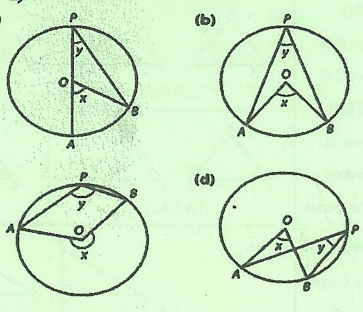


12. If two chords of a circle are equidistant from the centre, then their lengths are equal.
If $OM=ON$,
then $AB=CD$.
[Abbreviation:
chords equidistant from centre eq.] Section 11.3B



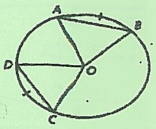
13. The angle at the centre of a circle subtended by an arc is twice the angle at the circumference subtended by the same arc.
$x=2y$

(a) (b) (c) (d)

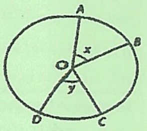


[Abbreviation: $\angle$ at centre $= 2\,\angle$ at $\odot^{ce}$] Section 11.4A

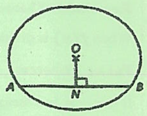
6. Equal arcs in a circle (or equal circles) subtend equal chords.
If $\overset{\frown}{AB}=\overset{\frown}{CD}$,
then $AB=CD$.
[Abbreviation:
eq. arcs, eq. chords] Section 11.2



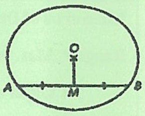
7. In a circle (or equal circles), the lengths of arcs are proportional to the sizes of angles at the centre subtended by the arcs.
$\overset{\frown}{AB} : \overset{\frown}{CD} = x : y$
[Abbreviation:
arcs prop. to $\angle$s at centre] Section 11.2



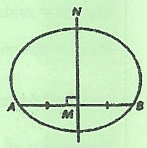
8. A perpendicular from the centre of a circle to a chord bisects the chord.
If $ON \perp AB$,
then $AN=BN$.
[Abbreviation:
$\perp$ from centre bisects chord] Section 11.3A



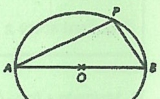
9. The line joining the centre of a circle and the mid-point of a chord (except diameters) is perpendicular to the chord.
If $AM=MB$,
then $OM \perp AB$.
[Abbreviation:
line joining centre and mid-pt. of chord $\perp$ chord] Section 11.3A



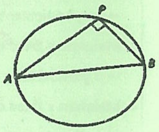
10. The perpendicular bisector of a chord passes through the centre of a circle.
[Abbreviation:
$\perp$ bisector of chord passes through centre] Section 11.3A



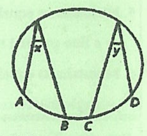
14. If $AB$ is a diameter and $P$ is any point lying on a circle apart from $A$ and $B$, then $\angle APB=90°$.
[Abbreviation:
$\angle$ in semi-circle] Section 11.4A



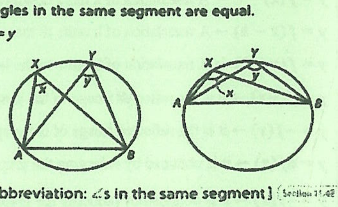
15. If $A$, $B$ and $P$ are three points lying on a circle and $\angle APB=90°$, then $AB$ is a diameter.
[Abbreviation:
converse of $\angle$ in semi-circle] Section 11.4A



16. The lengths of arcs are proportional to the sizes of angles at the circumference subtended by the arcs.
$\overset{\frown}{AB} : \overset{\frown}{CD} = x : y$
[Abbreviation:
arcs prop. to $\angle$s at $\odot^{ce}$] Section 11.4A



17. Angles in the same segment are equal.
$x=y$



[Abbreviation: $\angle$s in the same segment] Section 11.4B

## Matrix

**Properties of Matrices Operation**

$A0 = 0A = 0$

$AI = IA = A$

$A(BC) = (AB)C$

$A(B \pm C) = AB \pm AC$

$(A \pm B)C = AC \pm BC$

$(\alpha A)(\beta B) = A(\alpha\beta)B = (\alpha\beta)AB$

$(A + B)^T = A^T + B^T$

$(AB)^T = B^T A^T$.

**Inverse of Matrix**

$$A^{-1} = \frac{1}{|A|} adj \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

Non-singular => $\det A \neq 0$

**(Properties of inverses)**

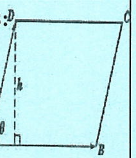
(a) $(A^{-1})^{-1} = A$, (b) $(AB)^{-1} = B^{-1}A^{-1}$.

(c) $(A^T)^{-1} = (A^{-1})^T$, (d) ( $\square$A)

= $\square$

(e) $(A^n)^{-1} = (A^{-1})^n$

## Application of Vectors:




Area of ABCD = $|\overrightarrow{AB} \times \overrightarrow{AD}|$

Area of ABD= $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AD}|$

Volume of Parallel pine = $(b \times c) \bullet a$

Volume of triangular prism = $\frac{1}{2}(b \times c) \bullet a$

Volume of tetrahedron = $\frac{1}{6}(b \times c) \bullet a$

Pyramid with parallelogram base =

$\frac{1}{3}(b \times c) \bullet a$

**Properties of Determinants**

$\det A^T = \det A$.

$\det AB = (\det A)(\det B)$.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\det A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} a_1 + ka_2 & b_1 + kb_2 & c_1 + kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$k \det A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

**System of Linear Equation**

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

**1 Cramer's Rule**

$\Delta_x \rightarrow$ determinant of A with x column replaced

$$x = \frac{\Delta_x}{\det A}, \quad y = \frac{\Delta_y}{\det A}, \quad z = \frac{\Delta_z}{\det A};$$

**2 Gaussian Elimination**

reduce to echelon form. $\left(\begin{array}{ccc|c} 1 & a & b & e \\ 0 & 1 & d & f \\ 0 & 0 & 1 & g \end{array}\right)$

**3. Use inverse of Matrix**

$AX = B$

If $\det A \neq 0$, **Unique Solution** *and* $X = A^{-1}B$

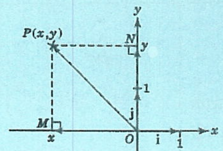
***For Homogeneous**

AX=0 ,

If $\det A \neq 0$, then X =0, trivial solution.

If det A = 0, AX = B has $\infty$ or no solutions (use Gaussian Elimination to Solve)

## Vectors




1. $\overrightarrow{OP} = xi + yj$.

Or $\overrightarrow{OP} = xi + y\,j + zk$.

2. $|\overrightarrow{OP}| = \sqrt{x^2 + y^2}$ Or $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

3. If $\overrightarrow{OP} \neq 0$, $\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$,

$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$ and $\tan\theta = \frac{y}{x}$ .

4. The unit vector in the direction of **a** is $\frac{\vec{a}}{|\vec{a}|}$.

5. $mi + nj = xi + yj \Rightarrow m = x$ and $n = y$

6. $mi + nj = 0 \Rightarrow m = n = 0$

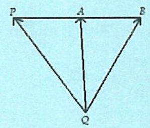
7. **a** is a unit vector iff $|a| = 1$.

**1.Parallelism**

If the vectors $\mathbf{p} = m\mathbf{i} + n\mathbf{j}$ and $\mathbf{q} = x\mathbf{i} + y\mathbf{j}$ are parallel, then

$\frac{x}{y} = \frac{m}{n}$ and $\frac{x}{m} = \frac{y}{n}$

**2. Division of a Line Segment**




AB such that AP : PB = $m : n$, then

$$\overrightarrow{QP} = \frac{m\overrightarrow{QB} + n\overrightarrow{QA}}{m + n}$$

, i.e. $\mathbf{p} = \frac{n}{n+m}a + \frac{m}{m+n}b$

**3. Coplanar**

Vol=0 => **Scalar Triple Product =0**

**Scalar Triple Product** $(b \times c) \bullet a$

If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$, $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ and

$\mathbf{c} = x_3\mathbf{i} + y_3\mathbf{j} + z_3\mathbf{k}$ then $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$.

**Scalar product(Dot Product)**

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $\theta$ is the angle between **a** and **b**.

(1) $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$.

(2) $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$.

(3) $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \Rightarrow \mathbf{b} = \mathbf{c}$

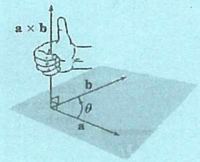
If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$, then $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2$

$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}$$

If $\mathbf{a} \cdot \mathbf{b} = 0$, **a** is perpendicular to **b**.

**Vector Products**

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\mathbf{n}$, where $\theta$ is the angle between **a** and **b** and **n** is the unit vector




(a) $\mathbf{a} \times \mathbf{a} = \mathbf{0}$

(b) $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

(c) $k(\mathbf{a} \times \mathbf{b}) = (k\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (k\mathbf{b})$ .

(d) $|\mathbf{a} \times \mathbf{b}| = |\mathbf{b} \times \mathbf{a}|$

(e) For non-zero vectors **a** and **b** , $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ if and only if **a** and **b** are parallel.

(h) $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$

**Vectors in $R^3$**

(i) $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$.

(ii) $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ , $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ , $\mathbf{k} \times \mathbf{i} = \mathbf{j}$. ($\mathbf{j} \times \mathbf{i} = -\mathbf{k}$ , $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ , $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$)

(iii) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \Rightarrow \mathbf{b} \neq \mathbf{c}$ (e.g. **a** and **b** are parallel, and **c** is **0**).

If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ are two non-zero vectors, and $\theta$ is the angle between them, then

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}.$$

## Limit

$\lim_{x\to a}[f(x)\cdot g(x)]=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x)=L_1\cdot L_2$;

$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}=\frac{L_1}{L_2}\quad L_2\neq 0$

$e^x=\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots=\sum_{r=0}^{\infty}\frac{x^r}{r!}$

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e\quad \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\quad \lim_{x\to 0}\frac{e^x-1}{x}=1$

**First principal**

$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

## The Binomial Theorem

$C_r^n=\frac{n!}{r!(n-r)!}\qquad P_r^n=(n-r)!$

$C_r^n=C_{n-r}^n\qquad C_r^n+C_{r+1}^n=C_{r+1}^{n+1}$

$(x+y)^n=$

$x^n+C_1^n x^{n-1}y+C_2^n x^{n-2}y^2+\dots+C_{n-1}^n xy^{n-1}+y^n$

S1: General term; S2, Solve for r

## Trigonometry

$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$

$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$2\sin A\cos B=\sin(A+B)+\sin(A-B)$

$2\cos A\cos B=\cos(A+B)+\cos(A-B)$

$2\sin A\sin B=\cos(A-B)-\cos(A+B)$

$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$

$\sin^2 x=\frac{1-\cos 2x}{2}\quad \cos^2 x=\frac{1+\cos 2x}{2}$

$\tan^2 x+1=\sec^2 x$

$\cot^2 x+1=\csc^2 x$

## Rules of differentiation

$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

**Chain Rule**

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

**-----Differentiation --->**

$\frac{d}{dx}(x^n)=nx^{n-1}$

$\frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\sec x)=\sec x\tan x$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(\cot x)=-\csc^2 x$

$\frac{d}{dx}(\csc x)=-\csc x\cot x$

$\frac{d}{dx}(e^x)=e^x$

$\frac{d}{dx}a^x=a^x\ln a$

$\frac{d}{dx}(\ln x)=\frac{1}{x}$

$\frac{d}{dx}\log_a x=\frac{1}{x\ln a}$

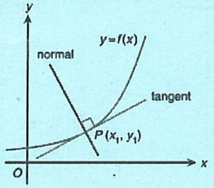
$\frac{d}{dx}(\ln|\sec x|)=\tan x$

$\frac{d}{dx}(\ln|\sec x+\tan x|)=\sec x$

**<---- Integration -----**

## Application of Differentiation

Tangent and Normal

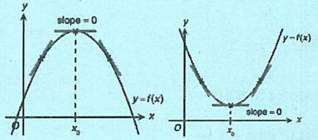



Normal : $y-y_1=-\frac{1}{f'(x_1)}(x-x_1)$

Tangent : $y-y_1=f'(x)(x-x_1)$

**Second D Test**

$f''(x_o)<0\qquad f''(x_o)>0$



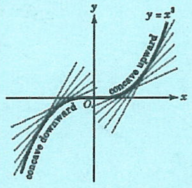

S1' set $f'(x)=0 \Rightarrow x=x_o$

S2'check $f''(x_o)$

*If $f''(x_o)<0$, $(x_o, f''(x_o))$ is a **max**. pt.*

*If $f''(x_o)>0$, $(x_o, f''(x_o))$ is a **min**. pt.*

**Point of Inflexion**




*$f''(x_o)$ changes signs*

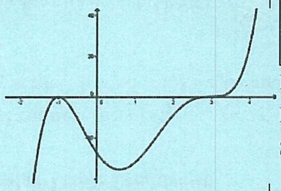
**Assymptotes**

e.g. $y=2x-3+\frac{2}{[x+5]}$

Vertical: $x=-5$

Horizontal/Oblique: $y=2x-3$

## First D Test



S1: Set $f'(x)=0$; => $x=1,5,or\ 7$

| $x$ | $x<1$ | $x=1$ | $1<x<5$ | $x=5$ | $5<x<7$ | $x=7$ | $7<$ |
|---|---|---|---|---|---|---|---|
| $dy/dx$ | +ve | 0 | -ve | 0 | +ve | 0 | +ve |

Maximum value: $f(1)$ Maximum point: $(1,f(1))$

Minimum value: $f(5)$ Maximum point: $(5,f(5))$

Stationary point: $(7,f(7))$

**Rate of Change,**

$v=\frac{ds}{dt}\qquad a=\frac{dv}{dt}=\frac{d^2s}{dt^2}$

S1: formulate equation i.e.

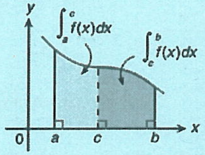
$A=4\pi r^2$

S2: apply chain rule to diff w.r.t *t*.

i.e. $\frac{dA}{dt}=\frac{d(4\pi r^2)}{dr}\frac{dr}{dt}$

S3: Substitutes and solve.

**Integration**




$\int_a^a f(x)dx=\int_a^a f(t)dt$,

$\int_a^a f(x)dx=0$,

$\int_b^a f(x)dx=-\int_a^b f(x)dx$,

$\int_a^b kf(x)dx=k\int_a^b f(x)dx$,

$\int_a^b[f(x)\pm g(x)]dx$

$\int_a^b f(x)dx\pm\int_a^b g(x)dx$

$\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$

$\int_{-a}^a f(x)dx=\begin{cases}2\int_0^a f(x)dx & even\\ 0 & odd\end{cases}$

**Integration by parts**

$\int u\,dv=uv-\int v\,du$ ;

$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$

$xe^x->d(e^x)$;

$x\cos x->d(\sin x)$

$x\ln x->d(\frac{x^2}{2})$;

$e^x\cos x->d(e^x)$;

***Use by part X2 or Recurrent**

**Integration Trigo-Sub**

$\sqrt{a^2-x^2}$ put $x=a\sin\square$

$\sqrt{a^2+x^2}$, put $x=a\tan\ \square$

$\sqrt{x^2-a^2}$, put $x=a\sec$

**M.I.**

When $n$=1,

...

It is true for $n$=1.

Assume the statement is true for

$n$=$k$

*i.e.* ...

When $n$=$k$+1

...

It is true for $n$=$k$+1

By the principle of mathematical induction, ....

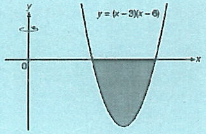
## Application of Integration

**Area=** $\int_a^b(high-low)dx$

**Shell method:** $\int_a^b(2\pi R)H\,dy$

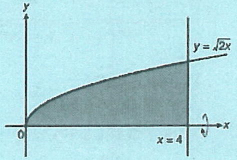
$Vol=\int_a^b 2\pi xy\,dx\ or\int_a^b 2\pi xy\,dy$

(vertical->$dx$; horizontal ->$dy$)




**Disc method:** $\int_a^b\pi R^2\,dh$

$Vol=\int_a^b\pi x^2dy$ or $Vol=\int_a^b\pi y^2dx$



M2

1. First Principles

a. $y = \ln(2016x + 2015)$

b. $y = 2015 \sin 2016x$

c. $y = 2015 \cos 2016x$

d. $y = 1314\sqrt{2016x}$

2. M.I.

1° when $n=1$

LHS = . . .

It is true for $n=1$

2° Assume $f(x)$ is true for some +ve integer $k$

i.e. ____________

When $n=k+1$

. . . . .

3. Binomial

① General term

② set eqt

③ solve for $r$

e.g. $(x^3 + \frac{1}{x})^4$

$$\text{General term} = C_r^n (\ )^r \cdot (\ )^{n-r}$$

$$= C_r^4 (x^3)^r (\tfrac{1}{x})^{4-r}$$

$$= C_r^4 x^{3r} x^{r-4}$$

$$= C_r^4 x^{4r-4}$$

$\frac{\sin}{\cos}$

$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$(x^2 - \frac{1}{x})^8$

$$\text{General term} = C_r^8 x^{16-3r}$$

Coefficient of $x^4$

$16 - 3r = 4$

$r = 4$

$C_4^8 = 70$

coefficient of $x^4 = 70$

$(x^2 - \frac{1}{x})^8 (2 - x + 3x^{-4})$ const. term

$$\boxed{3 C_r^8 x^{12-3r}}$$

Const. term $= 3C_4^8$

$= 210$