Electric Field

Refs.: Young and Freeman, University Physics, Chapter 21, © 2020

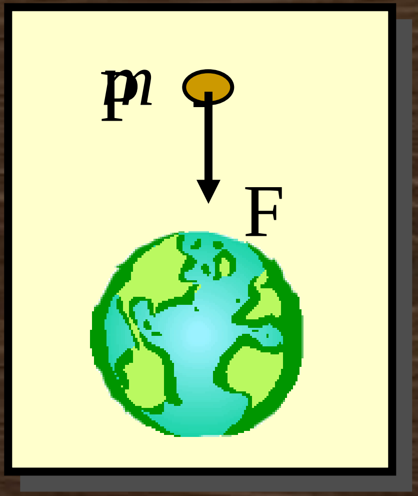
Paul E. Tippens, Southern Polytechnic State University, © 2007

Objectives: After finishing this unit you should be able to:

- Define the electric field and explain what determines its magnitude and direction.
- Write and apply formulas for the electric field intensity at known distances from point charges.
- Discuss electric field lines and the meaning of permittivity of space.

The Concept of a Field

A field is defined as a property of space in which a material object experiences a force.



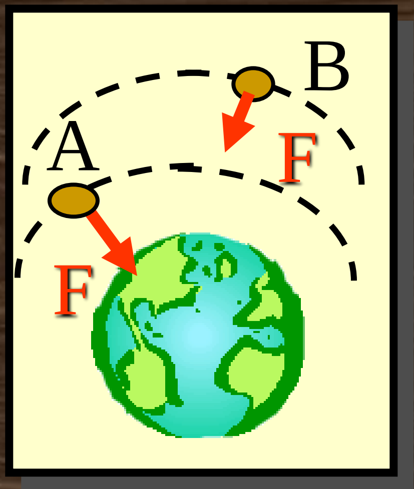
Above earth, we say there is a gravitational field at P.

Because a mass *m* experiences a downward force at that point.

No force, no field; No field, no force!

The direction of the field is determined by the force.

The Gravitational Field



Consider points A and B above the surface of the earth—just points in space.

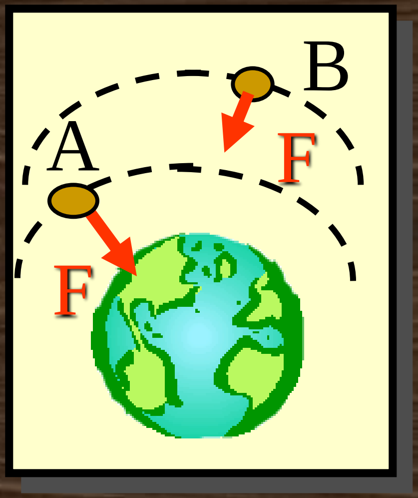
The field at points A or B might be found from:

$$\boxed{g = \frac{F}{m}}$$

If g is known at every point above the earth then the force F on a given mass can be found.

The magnitude and direction of the field g depends on the weight, which is the force F.

The Gravitational Field



Note that the force F is real, but the field is just a convenient way of describing space.

The field at points A or B might be found from:

$$g = \frac{F}{m}$$

If g is known at every point above the earth then the force F on a given mass can be found.

The magnitude and direction of the field g depends on the weight, which is the force F.

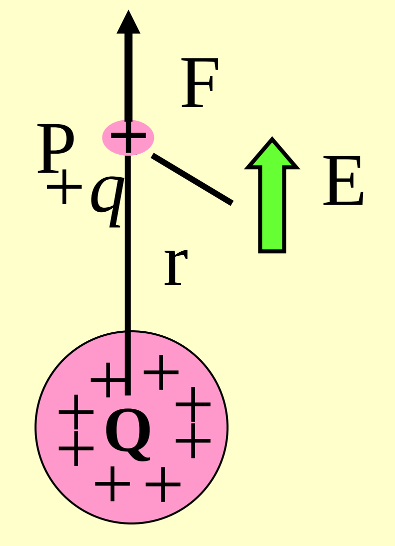
The Electric Field

1. Now, consider point P a distance r from +Q.

2. An electric field E exists at P if a test charge +q has a force F at that point.

3. The direction of the E is the same as the direction of a force on + (pos) charge.

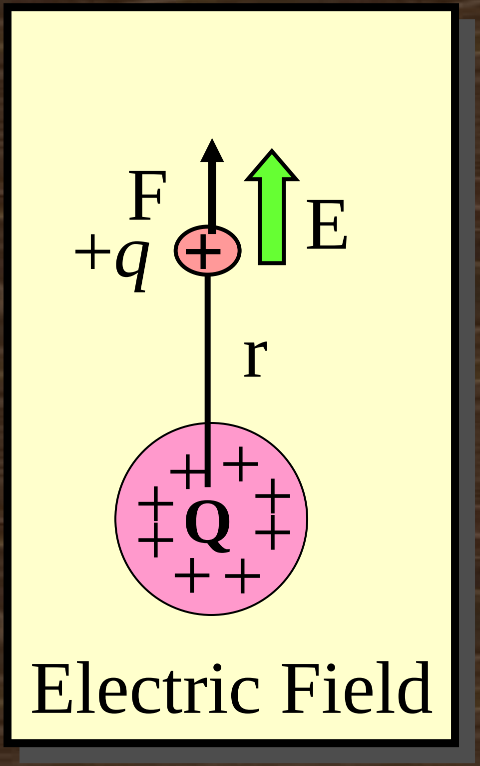
4. The magnitude of E is given by the formula:



Electric Field

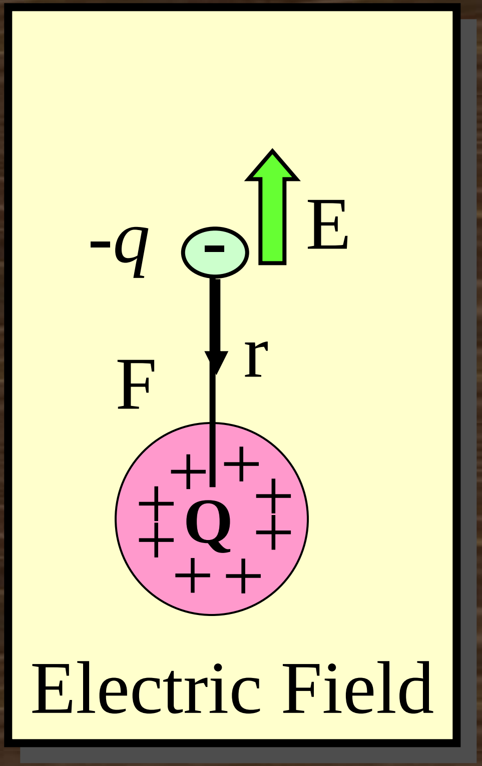
$$E = \frac{F}{q}; \text{ Units } \frac{\text{N}}{\text{C}}$$

Field is Property of Space



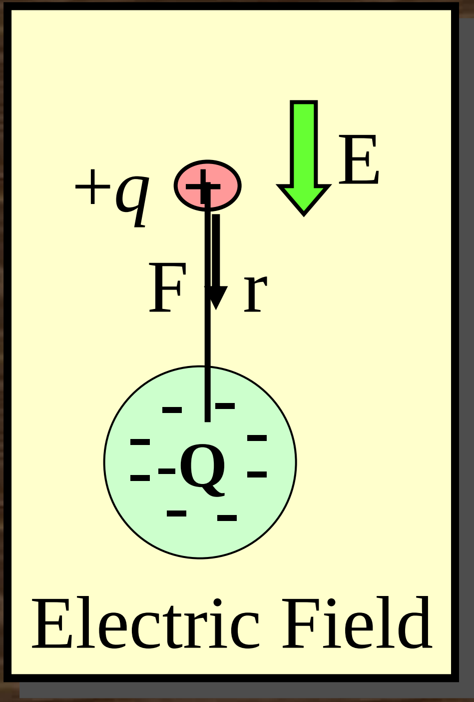
Force on +q is with field direction.

Force on -q is against field direction.



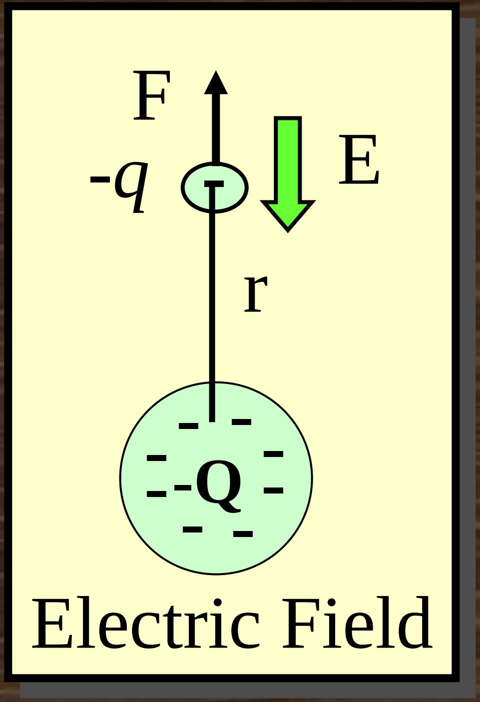
The field E at a point exists whether there is a charge at that point or not. The direction of the field is away from the +Q charge.

Field Near a Negative Charge



Force on +q is with field direction.

Force on -q is against field direction.



Note that the field E in the vicinity of a negative charge –Q is toward the charge—the direction that a +q test charge would move.

The Magnitude of E-Field

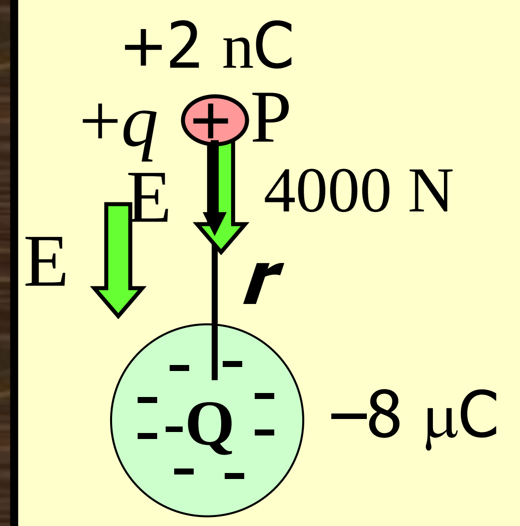
The magnitude of the electric field intensity at a point in space is defined as the force per unit charge (N/C) that would be experienced by any test charge placed at that point.

Electric Field Intensity E

$$E = \frac{F}{q}; \text{ Units } \left(\frac{\text{N}}{\text{C}}\right)$$

The direction of E at a point is the same as the direction that a positive charge would move IF placed at that point.

Example 1. A +2 nC charge is placed at a distance r from a –8 μC charge. If the charge experiences a force of 4000 N, what is the electric field intensity E at point P?



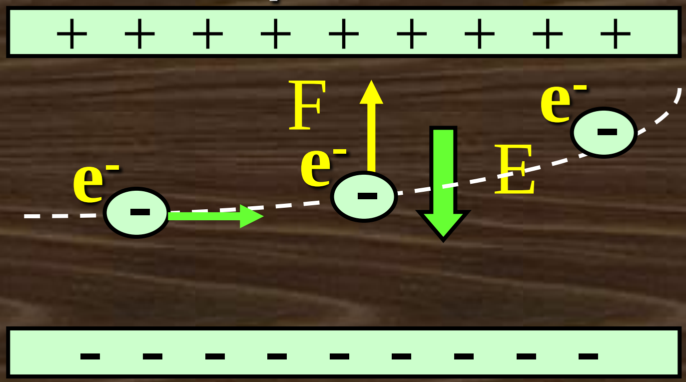
First, we note that the direction of E is toward –Q (down).

$$E = \frac{F}{q} = \frac{4000\text{ N}}{2 \text{ x } 10^{-9}\text{C}}$$

E = 2 x 10^{12} N/C Downward

Note: The field E would be the same for any charge placed at point P. It is a property of that space.

Example 2. A constant E field of 40,000 N/C is maintained between the two parallel plates. What are the magnitude and direction of the force on an electron that passes horizontally between the plates.



The E-field is downward, and the force on e⁻ is up.

$$E = \frac{F}{q}; \quad F = qE$$

$$F = qE = (1.6 \text{ x } 10^{-19}\text{C})(4 \text{ x } 10^{4}\ \tfrac{N}{C})$$

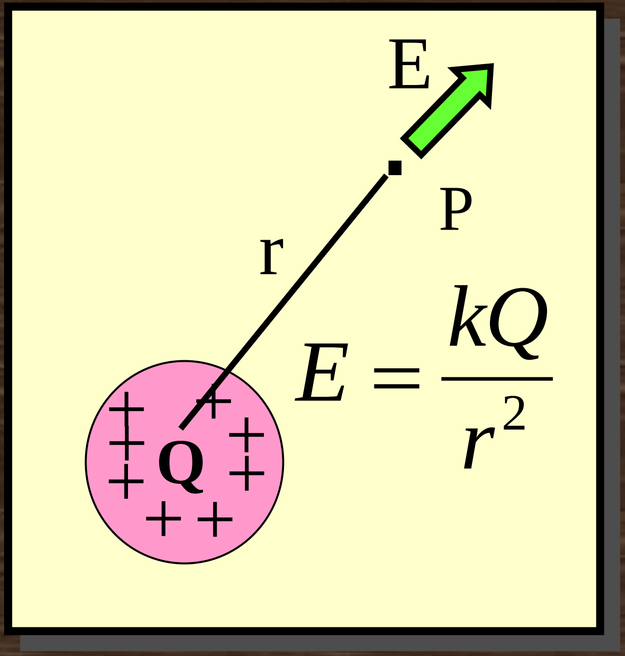
F = 6.40 x 10^{-15} N, Upward

The E-Field at a distance r from a single charge Q

Consider a test charge +q placed at P a distance r from Q.

The outward force on +q is:

$$F = \frac{kQq}{r^2}$$

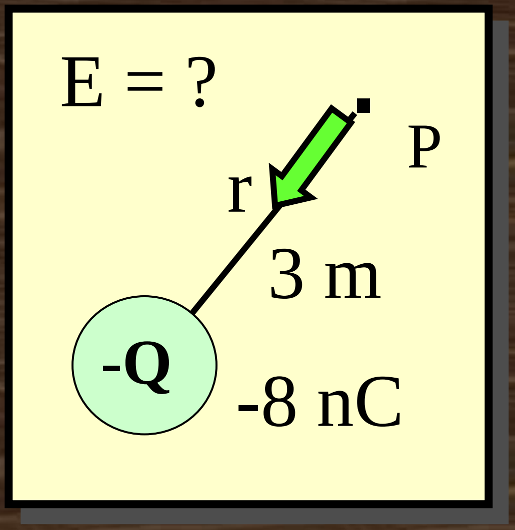


The electric field E is therefore:

$$E = \frac{F}{q} = \frac{kQ\cancel{q}/r^2}{\cancel{q}}$$

$$\boxed{E = \frac{kQ}{r^2}}$$

Example 3. What is the electric field intensity E at point P, a distance of 3 m from a negative charge of –8 nC?



First, find the magnitude:

$$E = \frac{kQ}{r^2} = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(8 \text{ x } 10^{-9}\text{C})}{(3 \text{ m})^2}$$

$\boxed{E = 8.00 \text{ N/C}}$

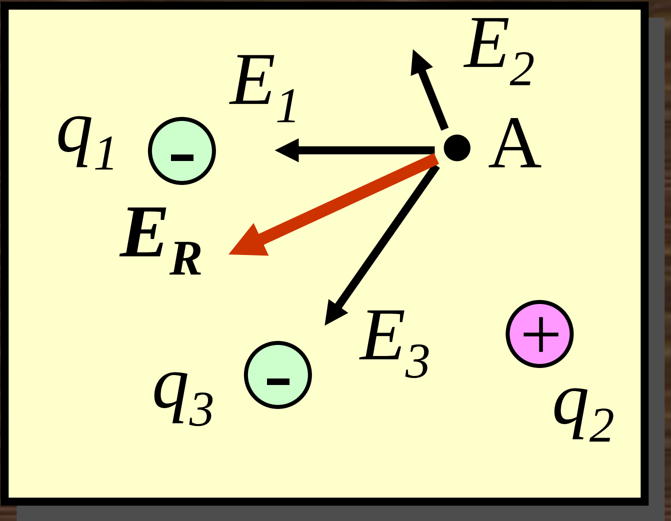
The direction is the same as the force on a positive charge if it were placed at the point P: toward –Q.

$\boxed{E = 8.00 \text{ N, toward -Q}}$

The Resultant Electric Field.

The resultant field E in the vicinity of a number of point charges is equal to the vector sum of the fields due to each charge taken individually.

Consider E for each charge.



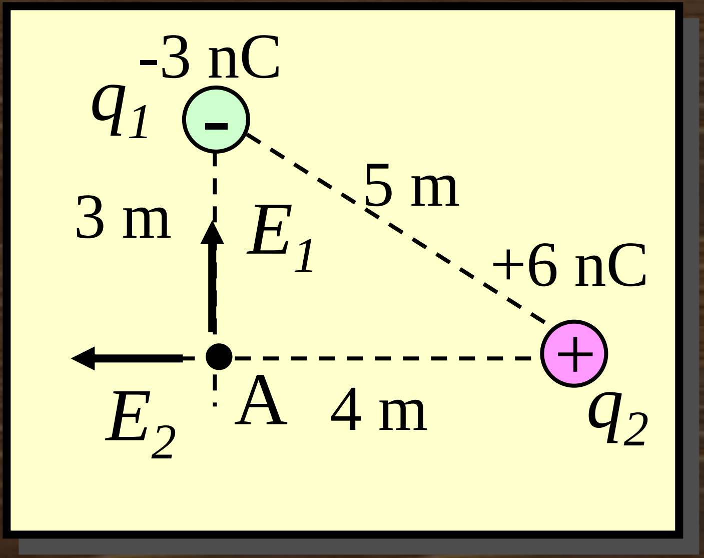
Vector Sum:
$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} + \boldsymbol{E_3}$

Magnitudes are from:

$$E = \frac{kQ}{r^2}$$

Directions are based on positive test charge.

Example 4. Find the resultant field at point A due to the –3 nC charge and the +6 nC charge arranged as shown.

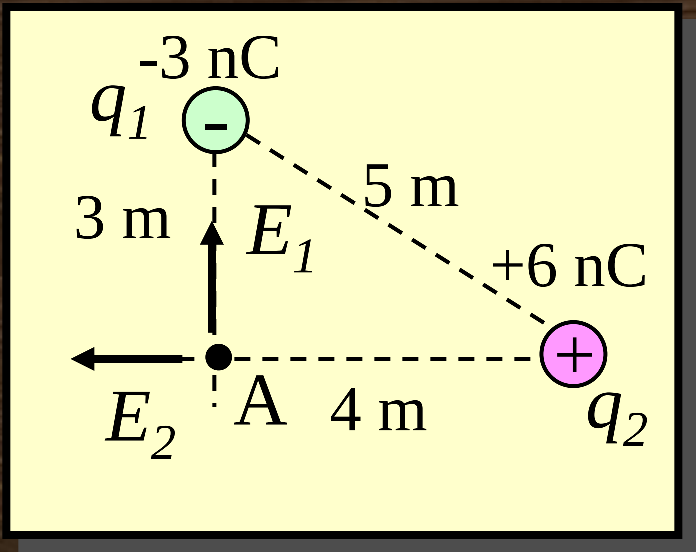


E for each q is shown with direction given.

$$E_1 = \frac{kq_1}{r_1^2}; \quad E_2 = \frac{kq_2}{r_2^2}$$

$$E_1 = \frac{(9 \text{ x } 10^9 \ \frac{\text{Nm}^2}{\text{C}^2})(3 \text{ x } 10^{-9}\text{C})}{(3 \text{ m})^2} \qquad E_2 = \frac{(9 \text{ x } 10^9 \ \frac{\text{Nm}^2}{\text{C}^2})(6 \text{ x } 10^{-9}\text{C})}{(4 \text{ m})^2}$$

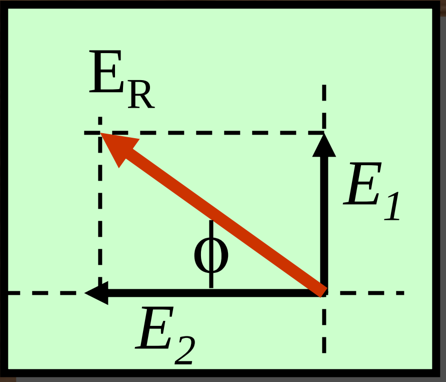
Example 4. (Cont.) Find the resultant field at point A. The magnitudes are:



$$E_1 = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(3 \text{ x } 10^{-9}\text{C})}{(3 \text{ m})^2}$$

$$E_2 = \frac{(9 \text{ x } 10^9 \frac{\text{Nm}^2}{\text{C}^2})(6 \text{ x } 10^{-9}\text{C})}{(4 \text{ m})^2}$$

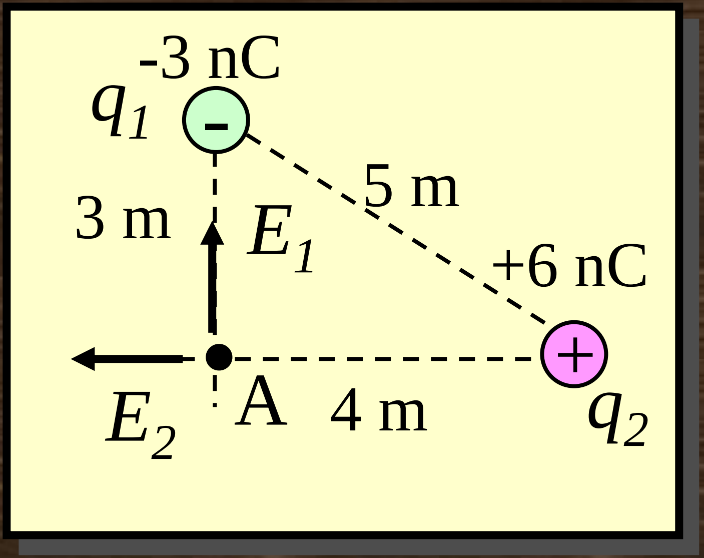
E_1 = 3.00 N, West E_2 = 3.38 N, North



Next, we find vector resultant E_R

$$E_R = \sqrt{E_2^2 + E_1^2}; \; \tan\varphi = \frac{E_1}{E_2}$$

Example 4. (Cont.) Find the resultant field at point A. The magnitudes are:



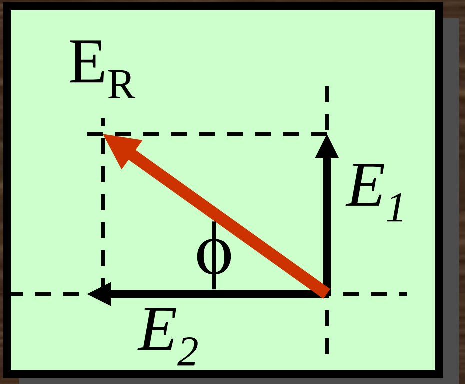
$$E_{1x} = 3.00\cos 90 = 0$$
$$E_{1y} = 3.00\sin 90 = 3.00$$

$$E_{2x} = 3.38\cos 180 = -3.38$$
$$E_{2y} = 3.38\sin 180 = 0$$

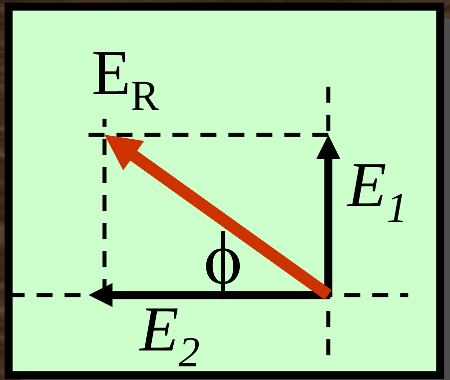
$E_1 = E_{1y}$ = 3.00 N, West $E_2 = E_{2x}$ = 3.38 N, North

Next, we find vector resultant E_R

$$E_R = \sqrt{E_2^2 + E_1^2};\ \tan\varphi = \frac{E_1}{E_2}$$



Example 4. (Cont.) Find the resultant field at point A using vector mathematics.



$E_1 = E_{1y} = 3.00$ N, West $= 3\hat{j}$

$E_2 = E_{2x} = 3.38$ N, North $= -\,3.38\hat{\imath}$

Find vector resultant E_R

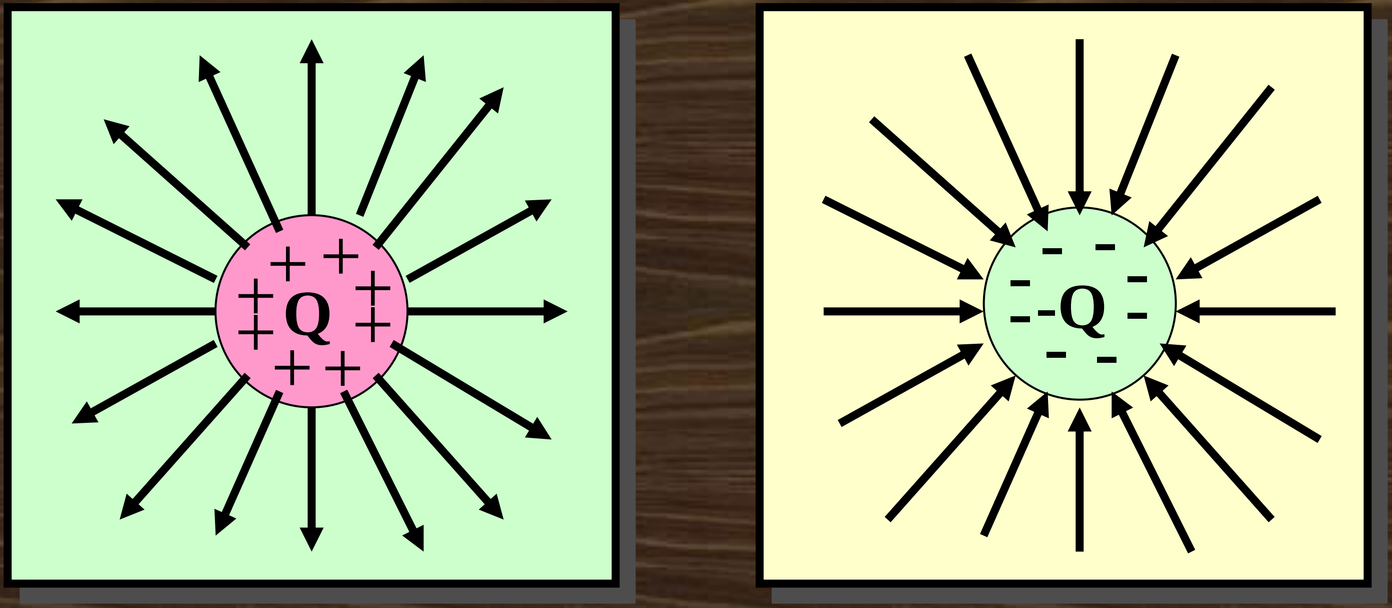
$$E = \sqrt{(-3.38\text{ N})^2 + (3.00\text{ N})^2} = 4.52\text{ N}; \quad \tan\varphi = \frac{3.00\text{ N}}{3.83\text{ N}}$$

$\phi = 41.6^0$ N of W; or $\theta = 138^0$

Resultant Field: $E_R = 4.52$ N; 138^0

Electric Field Lines

Electric Field Lines are imaginary lines drawn in such a way that their direction at any point is the same as the direction of the field at that point.

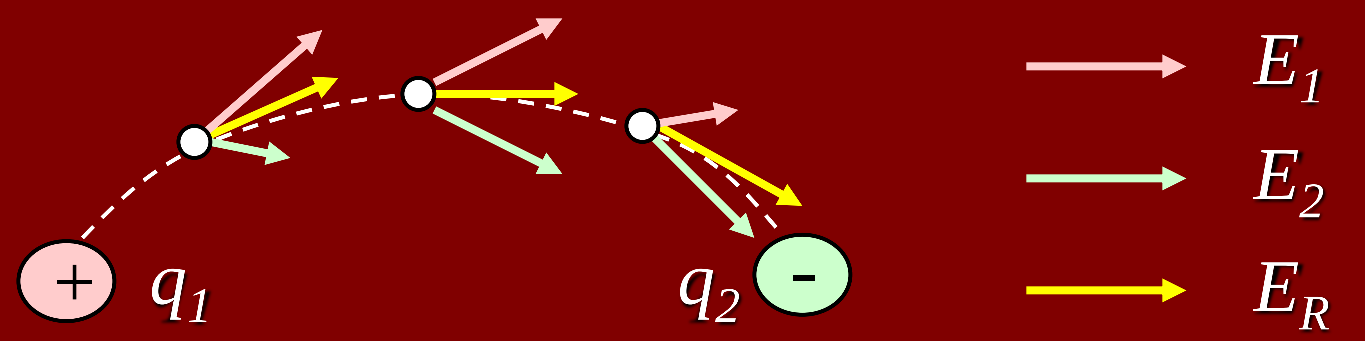


Field lines go away from positive charges and toward negative charges.

Rules for Drawing Field Lines

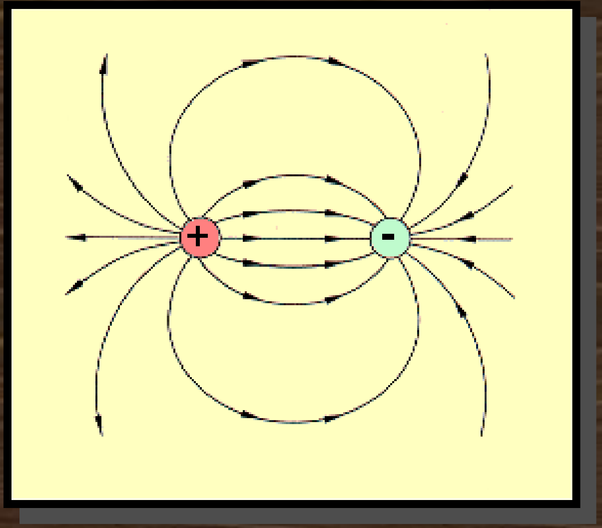
1. The direction of the field line at any point is the same as motion of +q at that point.

2. The spacing of the lines must be such that they are close together where the field is strong and far apart where the field is weak.

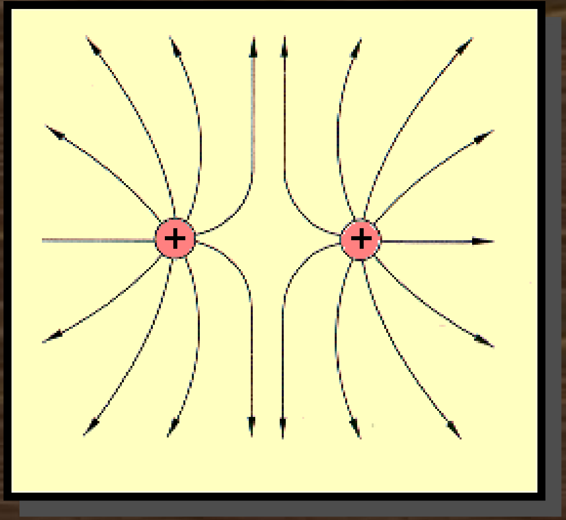


Examples of E-Field Lines

Two equal but opposite charges.



Two identical charges (both +).



Notice that lines leave + charges and enter - charges.

Also, E is strongest where field lines are most dense.

Summary of Formulas

The Electric Field Intensity *E*:

$$E = \frac{F}{q} = \frac{kQ}{r^2}$$ Units are $\frac{\text{N}}{\text{C}}$

The Electric Field Near several charges:

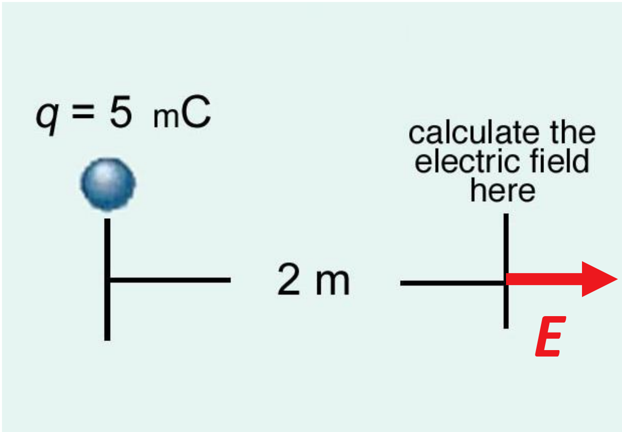
$$E = \sum \frac{kQ}{r^2}$$ Vector Sum

$$\varepsilon_0 = \frac{1}{4\pi k}$$

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \text{ x } 10^{-12} \ \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

Tutorials

EXAMPLE 1



Find the electric field strength at 2 meters from the 5 millicoulomb charge.

$$E = \frac{kq}{r^2} \qquad E = \frac{\left(9\times10^{9}\ \mathrm{Nm^2/C^2}\right)\left(5\times10^{-3}\ \mathrm{C}\right)}{(2\ \mathrm{m})^2}$$

$$E = 1.13\times10^{7}\ \text{N/C, to the right}$$

EXAMPLE 2

Find the force on a proton placed 2 meters from the 5 millicoulomb charge in the problem above.

$$E = \frac{F_e}{q} \qquad F_e = qE = \left(1.6\times10^{-19}\ \mathrm{C}\right)\left(1.13\times10^{7}\ \mathrm{N/C}\right) = 1.81\times10^{-12}\ \text{N, to the right}$$

OR

$$F_e = \frac{\left(9\times10^{9}\ \mathrm{Nm^2/C^2}\right)\left(5\times10^{-3}\ \mathrm{C}\right)\left(1.6\times10^{-19}\ \mathrm{C}\right)}{(2\ \mathrm{m})^2} = 1.8\times10^{-12}\ \text{N, to the right}$$

Example 21.5 Electric-field magnitude for a point charge

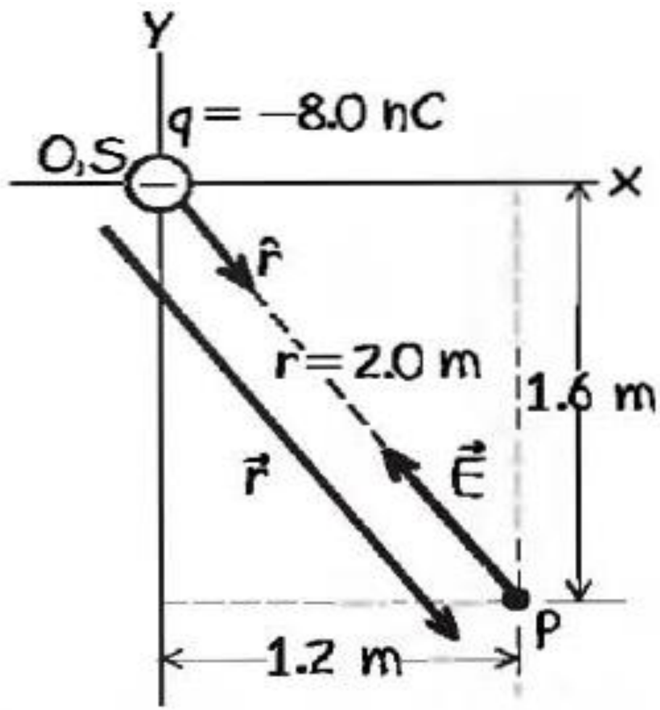
What is the magnitude of the electric field at a field point 2.0 m from a point charge $q = 4.0\ \text{nC}$? (The point charge could represent any small charged object with this value of q, provided the dimensions of the object are much less than the distance from the object to the field point.)

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.0 \times 10^{-9}\ \text{C}}{(2.0\ \text{m})^2}$$
$$= 9.0\ \text{N/C}$$

Example 21.6 Electric-field vector for a point charge

A point charge $q = -8.0\ \text{nC}$ is located at the origin. Find the electric-field vector at the field point $x = 1.2\ \text{m}$, $y = -1.6\ \text{m}$.

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\ \text{m})^2 + (-1.6\ \text{m})^2} = 2.0\ \text{m}$$



Example 21.6 Electric-field vector for a point charge

$$\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{r} = \frac{x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}}{r}$$

$$= \frac{(1.2\,\mathrm{m})\hat{\boldsymbol{\imath}} + (-1.6\,\mathrm{m})\hat{\boldsymbol{\imath}}}{2.0\,\mathrm{m}} = 0.60\hat{\boldsymbol{\imath}} - 0.80\hat{\boldsymbol{\jmath}}$$

Hence the electric-field vector is

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$

$$= (9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{(-8.0 \times 10^{-9}\,\mathrm{C})}{(2.0\,\mathrm{m})^2}(0.60\hat{\boldsymbol{\imath}} - 0.80\hat{\boldsymbol{\jmath}})$$

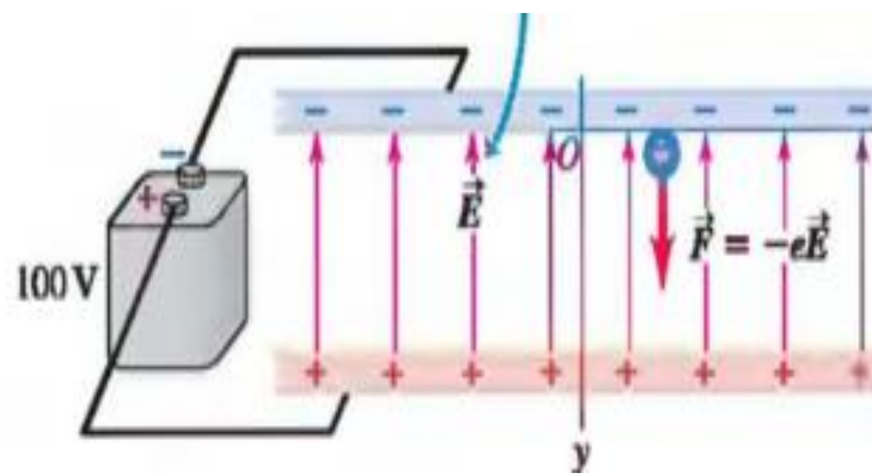
$$= (-11\,\mathrm{N/C})\hat{\boldsymbol{\imath}} + (14\,\mathrm{N/C})\hat{\boldsymbol{\jmath}}$$

Example 21.7 Electron in a uniform field

Consider an electron release from rest at the top of a pair of plates as shown.

The distance between the plates is 1 cm. Determine

a) The acceleration of the electron
b) The speed of the electron
c) The kinetic energy acquired by the electron as it hits the lower plate
d) The time it takes to reach the lower plate.



Charged Particle in a Uniform Electric Field

(a) Acceleration of the particle

$$a_y = \frac{F_y}{m} = \frac{-eE}{m} = \frac{(-1.60 \times 10^{-19}\,\mathrm{C})(1.00 \times 10^4\,\mathrm{N/C})}{9.11 \times 10^{-31}\,\mathrm{kg}}$$
$$= -1.76 \times 10^{15}\,\mathrm{m/s^2}$$

(b) Speed of particle $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

(c) Time taken $v = v_o + a_y t$ $t = \dfrac{v_y - v_{0y}}{a_y} = 3.4 \times 10^{-9}\,\mathrm{s}$

(d) Kinetic energy of the particle $K = \frac{1}{2}mv^2$

NB: When position of plates are interchanged, E is downward and F = (-e)(-E) = eE