Seminar paper

Computation and analysis of structural changes

as part of the seminar
Algorithmische und statistische Methoden der Zeitreihenanalyse

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Abbreviations

BIC Bayesian Information Criterion

HAC heteroskedasticity and autocorrelation consistent

K-S test Kolmogorov-Smirnov test

LWZ modified Schwartz criterion

OLS ordinary least squares

SSR sum of squared residuals

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1 Introduction

A structural change in a time series is defined as a change of the underlying characteristics or structure of the series. This change can include a change in mean, variance or auto correlation of the series. Causes for these changes can be due to exogenous events or change in the system that is being recorded in the data.

Identifying structural changes in a time series is important for multiple reasons. In forecasting, identification of structural changes can improve the accuracy of the forecast by accounting for them in the model. Additionally, an improved understanding of the data is possible, as changes in the data become visible. Knowing that a structural break has occurred also helps in understanding the underlying system that is being recorded in the data and what might be exogenous events that affect this system. An additional possible benefit is the selection of a better suited model to describe the data, or splitting the data into different models to account for a significant change.

To summarize, structural breaks can have significant effects on model reliability, understanding of the data and forecasting accuracy.

Jushan Bai and Pierre Perron described in their paper ”Computation and analysis of multiple structural change models” (Bai and Perron 2003a) a way to compute and analyze multiple structural changes in a time series in an efficient way. The work is based on their previous publication ”Estimating and Testing Linear Models with Multiple Structural Changes” (Bai and Perron 1998).

A review paper by Achim Zeileis and Christian Kleiber ”Validating multiple structural change models-a case study” (Zeileis and Kleiber 2005) corroborated the findings by Bai and Perron by using their R package **strucchange** (Zeileis, Leisch et al. 2002) which has the method of Bai and Perron for a pure change model implemented.

The second chapter will explain the model Bai and Perron laid out in their paper. First the model in general and the algorithm to compute the minimum global sum of squared residuals, the distinction between a partial and pure change model, the parameters used and tests to compute the number of changes.

The third chapter explains the motivation for the dataset chosen and describes the statistical properties to give an overview.

The fourth chapter applies the model and algorithm as it is laid out in the paper to the dataset and identifies the number of structural changes occurring in the observation period as well their points in time.

The fifth chapter will offer a summary on the model and application to the dataset.

2 Structural change model

2.1 Model and algorithm to compute global sum of squared residuals

The model by Bai and Perron to compute the structural changes for a given number of breaks m is the multiple linear regression in the form of

$$y_t = x_t'\beta + z_t'\delta_j + u_t \tag{1}$$

with $t = T_{j-1} + 1, ..., T_j$ denoting the points in time and $j = 1, ..., m + 1$ as the counter of each regimen. Calculating the sum of squared residuals (SSR) for each regimen with

$$\sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [y_t - x_t'\beta - z_t'\delta_i] \tag{2}$$

for a given number of structural changes the break points are found where the global sum of squared residuals is minimal. Both can be expressed in Matrix form. (1) can be expressed as

$$Y = X\beta + \overline{Z}\delta + U \tag{3}$$

where $Y = (y_1, \ldots, y_T)'$, $X = (x_1, \ldots, y_T)'$, $U = (u_1, \ldots, u_T)'$, $\delta = (\delta_1, \ldots, \delta_{m+1})$ and $\overline{Z}$ that partitions Z diagonally at $(T_1, \ldots, T_m)$. (2) can be expressed as

$$(Y - X\beta - \overline{Z}\delta)'(Y - X\beta - \overline{Z}\delta) \tag{4}$$

The above linear regression is defined as a partial change model where β is estimated across all observations while δ is subject to change.

In order to provide an efficient algorithm to compute the global sum of squared residuals and find the minimum, Bai and Perron employ a dynamic programming algorithm. In their paper Bai and Perron denote that a standard grid search method to find the minimal global sum of squared residuals would require operations of order $O(T^m)$ while their method only requires operations of order $O(T^2)$.

Prerequisite to the calculation is the selection of a minimum length of sequence to calculate the residuals for. A length too large poses the risk of skipping a break and too small increases the number of sum squared residuals to calculate. The efficiency results out of identifying which residuals to calculate in the first place. Assuming a time series of a given length, a number of present breaks and a minimum length of sequence, each sequence in the time series must allow for a number of conditions to be met.

- The segment cannot be shorter than the minimal length.

- The segment must start at $t = 1$ or must allow for a segment of minimal length beforehand.
- The length of the segment must allow for a required number of segments of minimal length to follow or to precede.

2.2 Partial and pure change model

In the case of a pure change model instead of a partial change model in the form of

$$Y = \overline{Z}\delta + U \tag{5}$$

applying ordinary least squares (OLS) method to each segment, the sum of squared residuals matrix can be computed by calculating recursive residuals for each valid segment identified

$$SSR(i,j) = SSR(i,j-1) + v(i,j)^2 \tag{6}$$

where $SSR(i,j)$ applies OLS to a segment starting at time i and ends at time j, and $v(i,j)$ refers to the residual at the time j.

The computation time of the recursive residuals can be vastly improved by identifying the segments maximum length starting at time i and precalculating each residual from i to time j and then computing the cumulative sum for each combination. After obtaining the matrix of SSR for a pure change model, the global minimum of SSR can be calculated by using a dynamic programming algorithm.

The dynamic programming algorithm calculates recursively the global minimum of SSR across the observation period

$$SSR(\{T_{m,T}\}) = \min_{mh \leq j \leq T-h} [SSR(\{T_{m-1,j}\}) + SSR(j+1,T)] \tag{7}$$

Initially the possible segments of one-break are calculated from h to $T - mh$ and stored. The next step calculates the possible two-break combinations and inserts the result of the previous one-break calculation to find the global minimum across both. This continues until $m - 1$ breaks and combines then with the optimal last segment, therefore finding the global minimal SSR.

For the partial structural change model the dynamic programming algorithm cannot be applied directly because the estimate of β for the global minimum of the SSR is not known. Bai and Perron suggest an iterative approach to find the global minimum. The sum of squared residuals is rewritten as a function of β and θ, with $\theta = (\delta, T_1, \ldots, T_m)$, i.e $SSR(\beta, \theta)$. The iterative approach consists of two steps in each iteration. In the first step the dynamic programming algorithm is applied with $y_t - x_t'\beta$ as the dependent variable keeping β fixed. The result of the first step (θ^*) is then used in the second step keeping T^* fixed and maximizing β and δ at the same time. According to Bai and Perron this efficiently lead in their experiments to the global minimum in one iteration as long as the initial value of β is not far off the true value β^0. The reason being that the $\{T^*\}$ values achieved in the first step are close to $\{\hat{T}\}$ as the regimen are most influenced by changes in δ.

As per Bai and Perron a threshold model in the form of

$$y_t = x_t'\beta + z_t'\delta_j + u_t\tau_{j-1} < v_t \leq \tau_j \tag{8}$$

for $j = 1, \ldots, m+1$ with $\tau_0 = -\infty$ and $\tau_{m+1} = +\infty$ can be estimated with the algorithm as well. Some adaptions have to be made to accommodate.

2.3 Parameters of the model

The initial choice of the value for the parameter β can be achieved by treating the model as one of pure structural change and writing the model as

$$y_t = x_t'\delta_{1,j} + z_t'\delta_{2,j} + u_t \tag{9}$$

with $t = T_{j-1} + 1, \ldots, T_j$ and $j = 1, \ldots, m+1$. Applying the dynamic programming algorithm with this pure structural change model provides estimates for $\delta_{1,j}^a$ and $\delta_{2,j}^a$. For an initial value β applying OLS regression on $Y - \overline{Z}\delta_2^a = X\beta + U$. The estimate of β can then be used as a start of the iteration procedure discussed before. Bai and Perron acknowledge the difficulties in applying this method in practice when the number of breaks and/or the dimension p of β is large. An alternative would be the usage of a fixed value for β but this bears the risk of a local minimum.

Another consideration for the model is the choice of the trimming parameter $\epsilon = h/T$ which is used in the supF-test. Critical values for $\epsilon = 0.5, 0.1, 0.15, 0.2, 0.25$ are defined in Bai and Perron 1998 and Bai and Perron 2003b. The value of epsilon limits the number of breaks that are possible to test for to $m_{\text{max}} = \frac{1}{\epsilon} - 2$.

The choice of h for the minimal segment length together with number of breaks m provides the basis for the valid SSR values that need to be calculated, because the permutation of $m+1$ segment length must equal the observed number of values T. The authors advise that smaller minimal length segments may lead to tests with substantial size distortions and suggest to increase the minimal segment length with serial correlation and/or different variance in the errors.

The number of breaks m to test for with a supF-test is limited by ϵ, but also one of the limits on the number of SSR necessary to compute, as more breaks shorten the maximum length of a single segment in the SSR-matrix. If the number of breaks is unknown for the various test it would make it necessary to recalculate the SSR-matrix accordingly, therefore consideration should be given to calculate the full upper triangular matrix[1].

2.4 Test statistics

The authors introduce three separate tests:

- a test of no break versus a fixed number of breaks,
- a double maximum test,
- a test of l breaks versus $l+1$ breaks.

For the test of no break versus a fixed number of breaks, a supF type test is used where the segments calculated via the dynamic programming algorithm for a fixed number of breaks the F-statistic is calculated for the breakpoints. The breakpoint with the highest F-statistic is then compared against a critical value to decide if the break is statistically significant. H_0 in this test is no structural breaks versus H_1 the existence of $m = k$ breaks.

The test is

$$supF_T(k;q) = F_T(\hat{\lambda}_1, \ldots, \hat{\lambda}_k; q) \tag{10}$$

$$F_T(\hat{\lambda}_1, \ldots, \hat{\lambda}_k; q) = \frac{1}{T}(\frac{T-(k+1)q-p}{kq})\hat{\delta}'R'(R\hat{V}(\hat{\delta})R')^{-1}R\hat{\delta} \tag{11}$$

with $\hat{V}(\hat{\delta})$ as the estimate of the variance covariance matrix of $\hat{\delta}$ depending on the case of partial or pure change model as well as the specification of the data.

For a partial change model three specifications are defined:

- With serial correlation, different distributions of the data across segments, same distribution for the errors across segments the variance covariance matrix is estimated as
$$\hat{V}(\hat{\delta}) = (T^{-1}\overline{Z}'M_X\overline{Z})^{-1}\hat{K}_T(T^{-1}\overline{Z}'M_X\overline{Z})^{-1} \tag{12}$$
with $\hat{K}_T$ as the heteroskedasticity and autocorrelation consistent (HAC) estimator of the $(m+1)q$ vector $z_t^*\hat{u}_t$ where $z_t^* = M_X\overline{Z}$.
- Without serial correlation, different variance of the errors, different distribution of

[1]An upper triangular matrix calculation only under consideration of the minimal length h in the size of 90×90 was in R not noticeably slower than calculating only the valid segments for m breaks.

the data the variance covariance matrix is estimated as

$$\hat{V}(\hat{\delta}) = T(\overline{Z}' M_X \overline{Z})^{-1} \hat{\Upsilon} (\overline{Z}' M_X \overline{Z})^{-1} \tag{13}$$

with $\hat{\Upsilon} = \sum_{i=1}^{m+1} \hat{\sigma}_i^2 \sum_{t=\hat{T}_{i-1}+1}^{\hat{T}_i} z_t^* z_t^{*\prime}$, $\hat{\sigma}_i^2 = (\Delta \hat{T}_i)^{-1} \sum_{t=\hat{T}_{i-1}+1}^{\hat{T}_i} \hat{u}_t^2$, $Z^{*\prime} = (z_1^*, \ldots, z_T^*)$ with $Z^* = M_X \overline{Z}$.

- Without serial correlation, different distributions for the data, same distribution for the errors across segments the variance covariance matrix is estimated as

$$V(\hat{\delta}) = \sigma^2 (T^{-1} \overline{Z}' M_X \overline{Z})^{-1} \tag{14}$$

with $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2$.

For a pure change model four specifications are defined:

- Without serial correlation, different distribution for the data, identical distribution for the errors across segments, the variance covariance matrix is estimated as

$$\hat{V}(\hat{\delta}) = \hat{\sigma}^2 (T^{-1} \overline{Z}' \overline{Z})^{-1} \tag{15}$$

- Without serial correlation, different distribution for the data, different variances of the errors across segments, the variance covariance matrix is estimated as

$$\hat{V}(\hat{\delta}) = diag(\hat{V}(\hat{\delta})_1, \ldots, \hat{V}(\hat{\delta}_{m+1})) \tag{16}$$

with $\hat{V}(\hat{\delta}_i) = \hat{\sigma}_i^2 [(\Delta \hat{T}_i)^{-1} \sum_{t=\hat{T}_{i-1}+1}^{\hat{T}_i} z_t z_t']^{-1}$ and $\sigma_i^2 = (\Delta \hat{T}_i)^{-1} \sum_{t=\hat{T}_{i-1}+1}^{\hat{T}_i} \hat{u}_t^2$.

- With serial correlation, different distribution for the data, different distribution for the errors, the variance covariance matrix is estimated as

$$V(\hat{\delta}) = diag(V(\hat{\delta})_1, \ldots, V(\hat{\delta}_{m+1})) \tag{17}$$

with $V(\hat{\delta}_i) = p \lim (\Delta T_i)(Z_i' Z_i)^{-1} Z_i' \Omega_i Z_i (Z_i' Z_i)^{-1}$.

- With serial correlation, same distribution for the errors across segments, the variance covariance matrix is estimated as

$$V(\hat{\delta}) = p \lim T(\overline{Z}' \overline{Z})^{-1} (\Lambda \otimes (Z' \Omega Z))(\overline{Z}' \overline{Z})^{-1} \tag{18}$$

with $\Lambda = diag(\lambda_1 - \lambda_0, \ldots, \lambda_{m+1} - \lambda_m)$, $\hat{\lambda}_i = \hat{T}_i / T$ and a HAC estimator of $Z' \Omega Z$ based on $\{z_t \hat{u}_t\}$.

The double maximum test offers a way to test up to M breaks. Critical values are supplied for up to $M = 5$ by Bai and Perron, but supposedly are still usable for $M > 5$. The first test is an equal weighted version defined by

$$UD\max F_t(M;q) = max_{1\leq m\leq M} F_T(\hat{\lambda}_1,\ldots,\hat{\lambda}_m) \tag{19}$$

and the second version is a weighted version defined by

$$WD\max F_t(M;q) = max_{1\leq m\leq M} a_m F_T(\hat{\lambda}_1,\ldots,\hat{\lambda}_m) \tag{20}$$

where $a = \frac{c(q,\alpha,1)}{c(q,\alpha,m)})$ so that $a_1 = 1$ and for $m > 1$ the factor a weights the marginal p-values accordingly.

The third test is to test l breaks against $l + 1$ breaks. The test works by testing the model with l breaks each segment for the presence of an additional break. The test is defined as

$$F(l+1|l) = \{S_T(\hat{T}_1,\ldots,\hat{T}_l) - \min_{1\leq i\leq l+1} \inf_{\tau\in\Lambda_{i,\eta}} S_T(\hat{T}_1,\cdots,\hat{T}_{i-1},\tau,\hat{T}_i,\ldots,\hat{T}_l)\}/\hat{\sigma}^2 \tag{21}$$

where $\Lambda_{i,\eta} = \{\tau;\hat{T}_{i-1} + (\hat{T}_i - \hat{T}_{i-1})\eta \leq \tau \leq \hat{T}_i - (\hat{T}_i - \hat{T}_{i-1})\eta\}$ and η is a trimming parameter.

The methodology of $supF_T(l+1|l)$ can be used to sequentially build up the optimal number of breaks by inserting a new break until the test no longer shows significance. Bai and Perron advice that instances exist where the null hypothesis cannot be rejected and the sequential build stops prematurely.

For the estimation of the number of breaks a Bayesian Information Criterion (BIC) (Yao 1988) and a modified Schwartz criterion (LWZ) (Liu, Wu and Zidek 1997) can be considered. Bai and Perron note that both the BIC and LWZ provide a reasonable well estimate in the case of no correlation. There are shortcomings for the BIC if serial correlation or a lagged dependent variable is present. Bai and Perorn suggest the use of the sequential estimation of breaks with the application of the $supF_T(l+1|l)$ test.

The BIC is defined as

$$BIC_i = log(\frac{SSR_i}{T}) + \frac{log(T)\times i\times(q+1)}{T} \tag{22}$$

and the LWZ is defined as

$$LWZ_i = log(\frac{SSR_i}{T-(i+1)\times q - i}) + \frac{i\times(q+1)\times 0.299\times(log(T))^{2.1}}{T} \tag{23}$$

where for both i is denoting the number of breaks. The minimum BIC and LWZ respectively indicate the number of breaks present in the data.

2.5 Confidence intervals

To evaluate the found breaks, the authors describe various specifications to compute the confidence intervals for the break dates depending on the data under analysis. Bai and Perron define the limiting distribution of the break dates as

$$\frac{(\Delta)_i' Q_i \Delta_i)^2}{(\Delta_i' \Omega_i \Delta_i)}(\hat{T}_i - T_i^0) \Rightarrow \text{arg max}_s V^{(i)}(s)(i = 1, \ldots, m) \tag{24}$$

where depending on s

- $s \leq 0$: $V^{(i)}(s) = W_1^{(i)}(-s) - |s|/2$

- $s > 0$: $V^{(i)}(s) = \sqrt{\xi}(\phi_{i,2}/\phi_{i,1})W_2^{(i)}(s) - \xi_i|s|/2$

and $\xi_i = \Delta_i' Q_{i+1} \Delta_i / Delta_i' Q_i \Delta_i$, $\phi_{i,1}^2 = \Delta_i' \Omega_i \Delta_i / \Delta_i' \Omega_i \Delta_i$, $\phi_{i,2}^2 = \Delta_i \Omega_{i+1} \Delta_i / \Delta_i Q_{i+1} \Delta_i$. W_1^i and W_2^i are independent standard Weiner processes.

Δ_i can be calculated by $\hat{\Delta}_i = \hat{\delta}_{i+1} - \hat{\delta}_i$.

The following specifications exist to calculate Q_i

- if the regressors z_t are identically distributed across segments $Q_i = Q_{i+1} = Q$ with $\hat{Q} = T^{-1} \sum_{t=1}^{T} z_t z_t'$
- else $Q_i = \Delta T_i^{-1} \sum_{t=\hat{T}_{i-1}+1}^{\hat{T}_i} z_t z_t'$.

For the calculation of ϕ is a distinction if serial correlation is present in the errors. Without serial correlation the case is $\phi_{i,1}^2 = \phi_{i+1}^2 = \sigma_i^2$

- if the errors u_t are identically distributed across segments then $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t^2$ instead $\hat{\sigma}_i^2$

- else $\hat{\sigma}_i^2 = (\Delta T)^{-1} \sum_{t=T_{i-1}+1}^{T_i} \hat{u}_t^2$

and with serial correlation Ω_i and Ω_{i+1} is calculated as following to calculate ϕ as described before

- if the errors u_t are identically distributed across segments then $\Omega_i = \Omega_{i+1} = \Omega$ which can be estimated using a HAC estimator over $\{z_t, \hat{u}_t\}$ using the whole sample.

- else the HAC estimator is applied only to $\{z_t, \hat{u}_t\}$ using data of segment i .

The resulting confidence interval boundaries are then rounded to produce integer indices for the break dates by lowering the lower bound the highest smaller integer and the upper bound increased to the smallest higher integer.

3 Dataset: EU27 European Construction Cost Index

The dataset chosen for this seminar paper is the Construction Cost Index for the EU27 downloaded from Eurostat (Eurostat 2022). The Construction Cost Index is a weighted aggregate index of each contributing EU27 member state's own national construction cost index. The weight used to aggregate to a common index across member states is the respective turnover in building construction.

Further indexes based upon the Construction Cost Index are the Construction Producer Prices Index and the Construction Selling Price Index. The first shows the price the client of the builder has to pay excluding VAT and other service fees, while the second shows the final cost to the final owner of the construction, including VAT and other costs.

Input data to derive the construction costs are the cost of input factors for the construction process, mainly materials, labour, eqipment and energy. The index does not account for changes in construction method or organisation which may occur over time.

The Construction Cost Index offers insights into the development of construction costs of new residential buildings. While residential buildings have a different building materials composition than commercial or industrial buildings, for industrial property insurance it still is an important metric to understand the rise of claims payments for fire losses. Selecting the correct index to inflate past losses is most important to anticipate the next insurance years loss amounts and set prices accordingly.

Due to the fact that past losses are not inflated to current index values but need to be forecast to next years value to model next year prices, a proper forecast of the index is required. Identifying structural changes in the index and when they occurred provides insight in external influences that if taken into account improve the forecast.

Identifying the break points also provides insight into cross-validation of notification of sum insured. Usually clients notify about the change in insured values each policy year and often apply a fixed percentage on their own recorded values. Knowing that, data can be checked if values have been under reported in the past.

The used data of the Construction Cost Index for the EU27 starts in the first quarter of 2000 and ends with the second quarter of 2022. Each of the 90 data points represents a quarterly index value in reference to the base year of 2015. In figure 1 can be seen that between the start in Q1 2000 at a value of 70.2 until the end of records with a value of 130 an overall upward trend can be observed.

Construction costs have increased by 185.65 % in the 22 years of observation, so to put it into perspective a residential building build in 2000 for 200.000 EUR would cost 371.300 EUR in 2022. The current mean is at 93.23 points with a variance of 185.65. As displayed

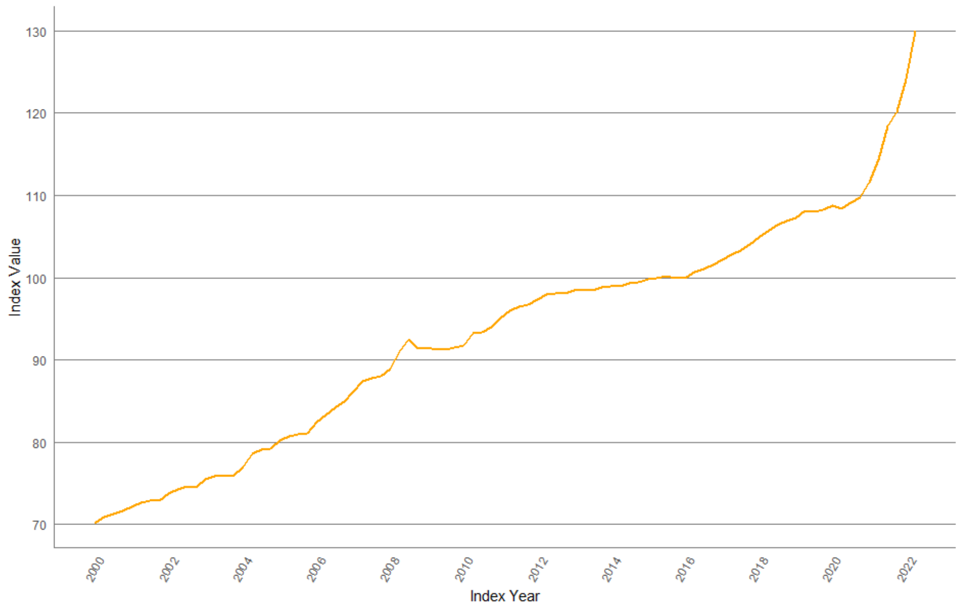

Figure 1: Construction Cost Index from 2000-Q1 to 2022-Q2

in figure 2 it can be seen that the change quarter by quarter is mostly positive with just on 6 occasions where the construction costs decrease between quarters. In the mean, construction costs increase by 0.66 points each quarter with a variance of 0.88.

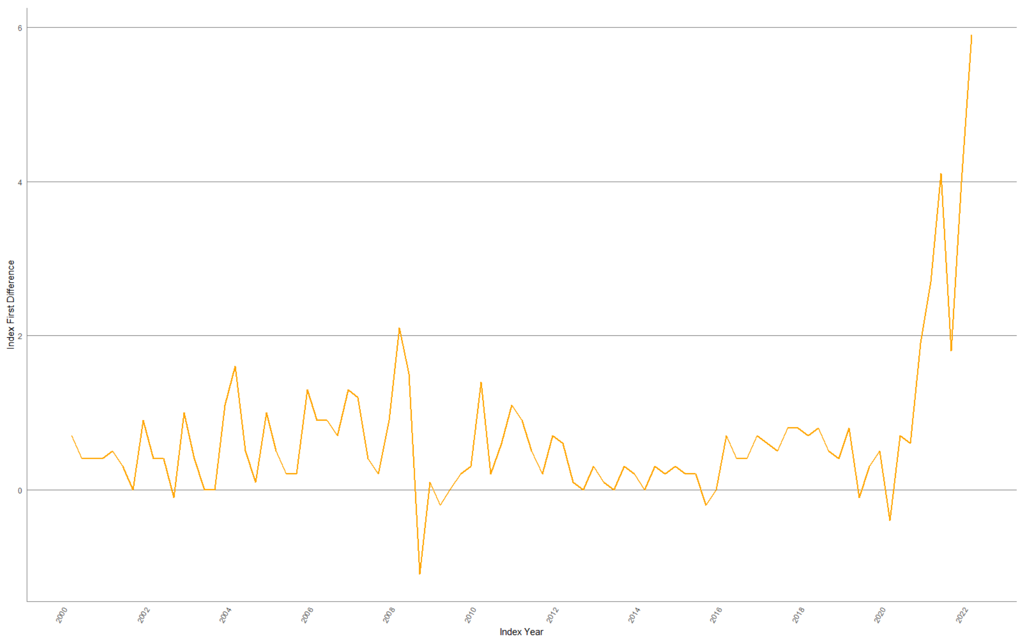

Figure 2: Construction Cost Index first difference values

The probability density function (PDF) of the construction cost data is shown in figure 3. The skewness is measured at 0.105 which means that the distribution of values is left-skewed. The kurtosis amounts to -0.52 and indicates a lower peak than a normal distributed PDF.

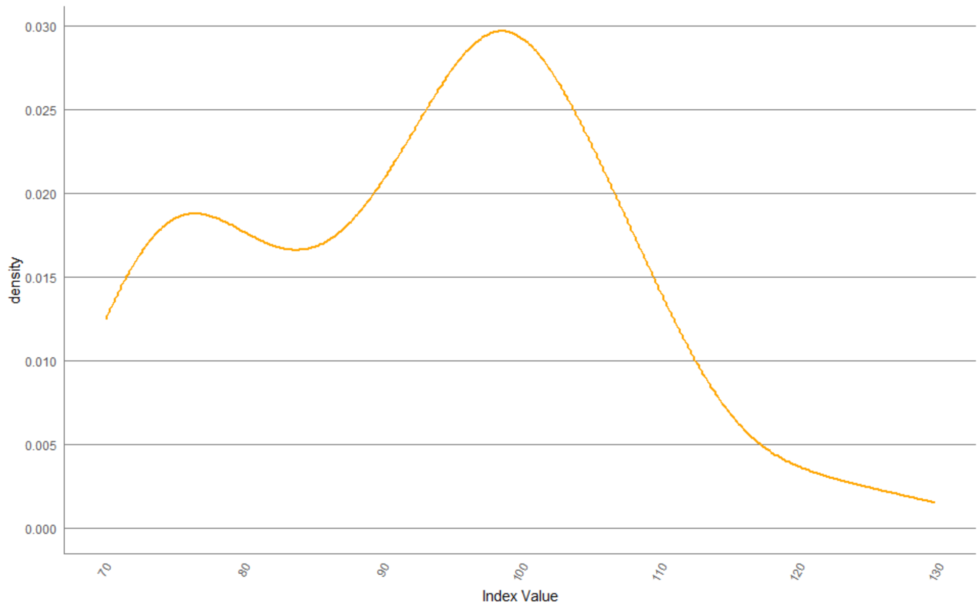

Figure 3: PDF of construction cost index values

Test for stationarity on the index values, the first difference and the respective log transformations show that the both the Augmented Dicker-Fuller Test and the Phillips-Perron Unit Root Test cannot reject the null hypothesis of non-stationarity. Only for the second difference of the values is the Phillips-Perron Unit Root Test able to reject the null hypothesis. The test results are displayed in table 1.

Stationarity Test Results (p-values)

	Augmented Dickey-Fuller Test	Phillips-Perron Unit Root Test
CCI	0.9349	0.99
First Difference CCI	0.99	0.363
Second Difference CCI	0.1185	0.01

Table 1: Augmented Dickey-Fuller Test and Phillip-Perron Unit Root Test p-value results for test of stationarity.

Measuring the autocorrelation for the Construction Cost Index values, first difference and second difference shows for the first 10 lagging values, as displayed in figure 5, that the values and the first difference values are positive autocorrelated, while the second difference shows no clear autocorrelation. The Durbin-Watson Test for autocorrelated errors in the second difference is not able to reject the null hypothesis of no autocorrelation as displayed in table 2. The time series for the second difference of the CCI is displayed in figure 4.

Considering the autocorrelation in the data comparing a model without a lagging dependent variable to a model containing a lagging variable shows that introducing a lagging regressor improves the model. Results for first and first + second difference as regressors is shown in table 3. Results show that adding the lagged variables improves both the

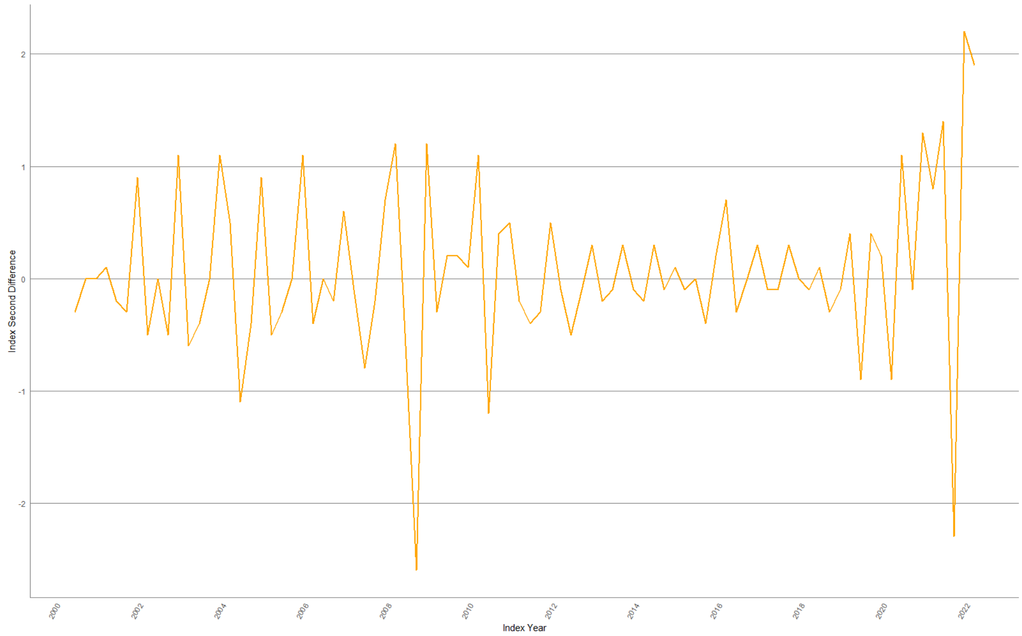

Figure 4: Construction Cost Index second difference values

Durbin-Watson Test

Lag	Autocorrelation	D-W Statistic	p-value
1	-0.22189	2.368407	0.082
2	-0.19647	2.219283	0.266
3	0.01073	1.685567	0.180

Table 2: Durbin-Watson Test for autocorrelated errors in the second difference of the Construction Cost Index.

Akaike Information Criterion and BIC in comparison to no lagged variable and decreases the deviance.

formula	R^2	adj. R^2	AIC	BIC	deviance
$value \sim 1$	0	0	709	714	15478
$value \sim diff$	0.172	0.163	694	701	12814
$value \sim diff + diff\_2$	0.181	0.162	695	705	12677

Table 3: Overview of model performance depending on lagged regressors

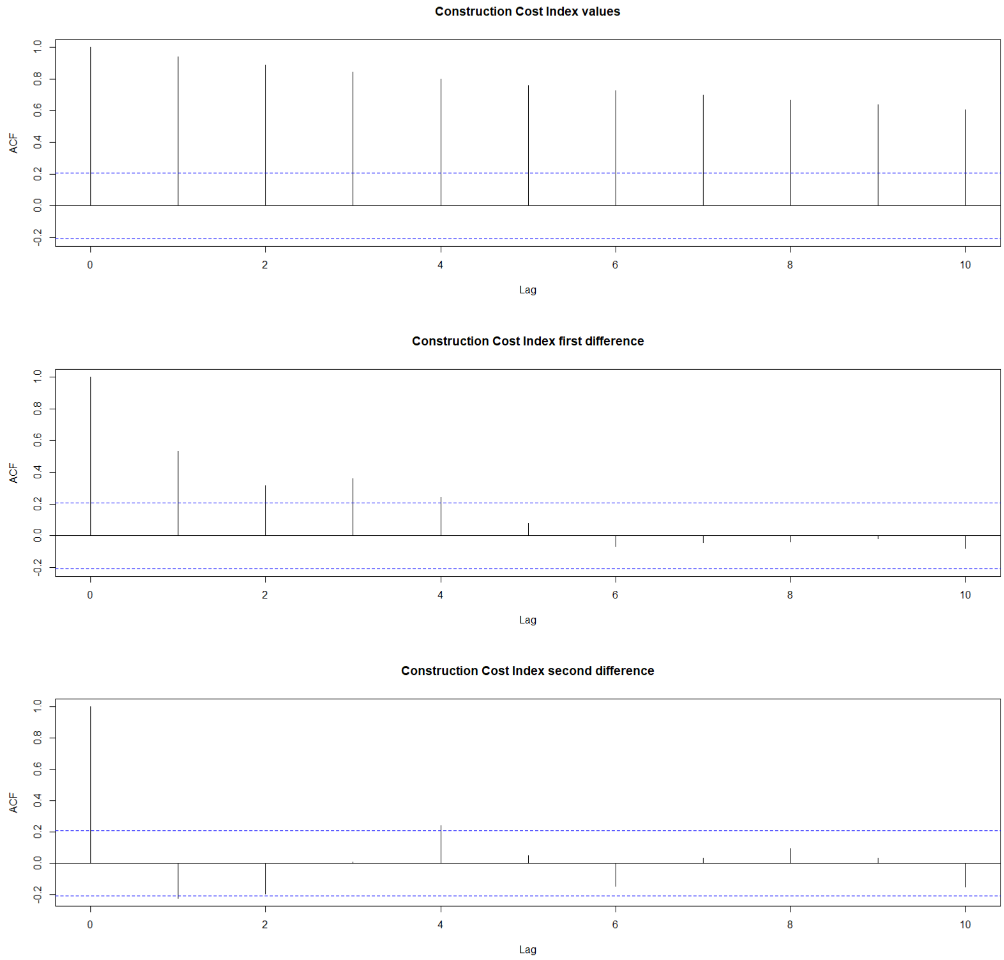

Figure 5: ACF of construction cost index values, first difference and second difference

4 Application of the structural change model

Considering the data structure in table 4, where ”time_period” contains the date begin of the quarterly value, ”value” the index value at the time, ”diff” the first difference of ”value”, and ”diff_2” the second difference.

time_period	value	diff	diff_2
...	...	...	...
2018-01-01	104.9	0.8	0.0
2018-04-01	105.6	0.7	0.1
2018-10-01	106.4	0.8	-0.1
...	...	...	...

Table 4: Data structure of Contruction Cost Index data with first and second difference values.

The time series exhibits strong autocorrelation to the lagged values. Taking this into consideration the first difference values are chosen as regressors, the pure change model henceforth is described by $y_t \sim y_{t-1}$.

From the visualised time series in figure 1 the two first observations are excluded of the data. A trimming of $\epsilon = 0.10$ is used to compute the SSR matrix with a minimal length of 8 observations in a segment. Up to 6 breaks are tested in the procedure.

For the estimation of the variance covariance matrix $\hat{V}(\hat{\delta})$ the distribution of the errors and the data of the segments are tested with a Kolmogorov-Smirnov test (K-S test) as implemented in the R-Package stats with the ks.test() function. Each segment is tested against the next, if all comparisons equal $p - value >= 0.05$ the distribution is assumed to be identical as H_0 cannot be rejected.

The result of the break change analysis is as follows. Allowing for serial correlation in the errors and testing the result of the sequential procedure with a K-S test the segments show a different distribution for the data and identical distribution for the errors, therefore a variance covariance matrix estimate as in (18) was chosen for the F-Test.

The various test results and parameter estimates are displayed in table 5. The Sup$F_T(k)$ test are significant at the 5% level for all k between 2 and 6. The Sup$F_T(l+1|l)$ test are all significant from Sup$F_T(2|1)$ up to Sup$F_T(5|4)$, with the Sup$F_T(6|5)$ test failing to be significant at the 5% level. The UD max and WD max are both significant as well.

The sequential procedure selects 3 breaks after ignoring the test for 1 break in the procedure, while the BIC selects 6 breaks and the LWZ selects 5 breaks.

The breaks selected by the sequential procedure are in 2008:Q3, 2017:Q1 and 2019:Q1. Both the break in 2008 as well as the break in 2019 have fairly narrow confidence intervals of 4 and 5 quarters respectively at a significance of 5%. The second break in 2017 has a width of 13 quarters and is faily large. The segments with the breakpoints and confidence intervals are visualised in figure 6.

There is a potential 4th break in the data, but due to the restriction of a minimal segment of $h = 8$ observations and the length of the last segment only having 13 observations it is not possible to divide the segment further. The sequential method preferred another break each time and eventually shortened the last segment below $2h$ observations.

As said before, the index does not account for change in regulation or technical advances (Eurostat 2022), so not every break can be clearly explained by exogenous effects. The first break in 2008 most likely show the effects of the global financial crisis on the construction sector where before a constant growth occurred. The second break estimated at 2017:Q1 shows the end of another growth period since the last break. Due to the large confidence interval the actual break might be earlier in 2016 already. The third break is at the end of another growth period where afterwards a decreasing growth rate can be seen before the effects of material shortages, inflation and global supply chain interrup-

Empirical results for Construction Cost Index (2000-3 - 2022-2)

Specifications					
$z_t = \{1, y_{t-1}\}$	$q = 2$	$p = 0$	$h = 8$	$M = 6$	$\epsilon = 0.10$
Tests					
Sup$F_T(1)$	Sup$F_T(2)$	Sup$F_T(3)$	Sup$F_T(4)$	Sup$F_T(5)$	Sup$F_T(6)$
10.5	12.5^*	17.4^*	21.7^*	25.1^*	30.2^*
Sup$F_T(2\|1)$	Sup$F_T(3\|2)$	Sup$F_T(4\|3)$	Sup$F_T(5\|4)$	Sup$F_T(6\|5)$	
69.6^*	18.3^*	80.2^*	79.4^*	4.12	
UD max	WD max				
30.23^*	53.66^*				
Number of breaks selected					
Sequential procedure		3			
LWZ		5			
BIC		6			
Parameter Estimates with 3 breaks					
$\hat{\delta}_{1,1}$	$\hat{\delta}_{2,1}$	$\hat{\delta}_{1,2}$	$\hat{\delta}_{2,2}$	$\hat{\delta}_{1,3}$	$\hat{\delta}_{2,3}$
75.466	6.334	96.794	1.113	106.549	-2.175
$\hat{\delta}_{1,4}$	$\hat{\delta}_{2,4}$				
107.616	3.517				
$\hat{T}_1$		$\hat{T}_2$		$\hat{T}_3$	
2008 : Q3		2017 : Q1		2019 : Q1	
(2008 : Q2 $-$ 2009 : Q2)		(2015 : Q1 $-$ 2018 : Q2)		(2018 : Q4 $-$ 2020 : Q1)	

* Statistic significant at the 5% level.

Table 5: Empirical results for the break analysis of the Construction Cost Index

tion showed their effect on the rapid growth in the index.

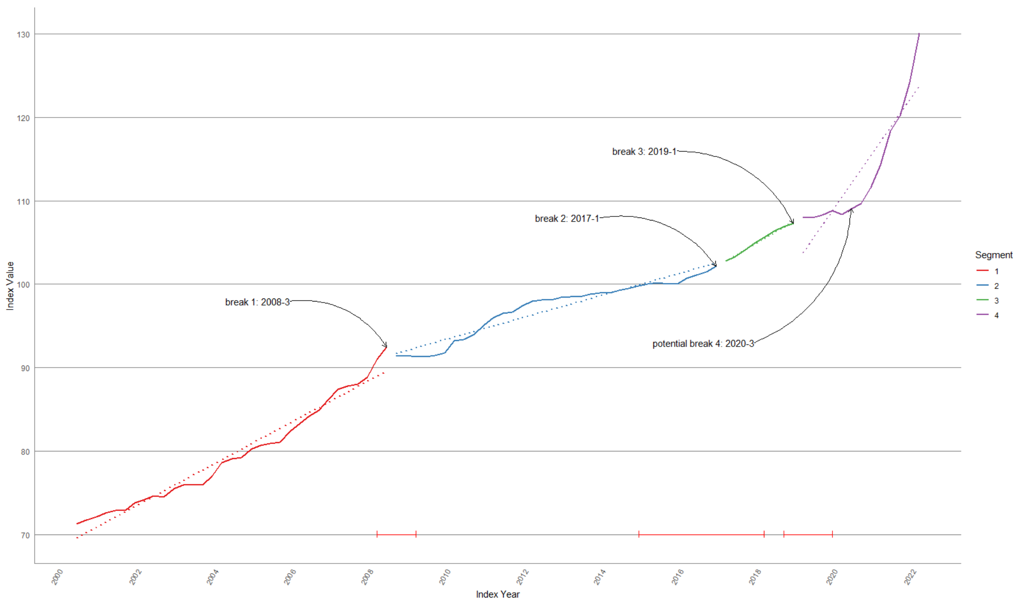

Figure 6: Result of break points estimation with linear regression line and confidence intervals

5 Results and discussion

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Eidesstattliche Erklärung