

MODULAR BINOMIALS (80pts)

Rearrange the following equations to get the primes p,q

$$\begin{aligned}N &= p \cdot q \\c_1 &= (2 \cdot p + 3 \cdot q)^{e_1} \bmod N \\c_2 &= (5 \cdot p + 7 \cdot q)^{e_2} \bmod N\end{aligned}$$

Multiply both of the equations by each other to get similar equations

$$C1^{e2} = (2p + 3q)^{e1 \cdot e2} \bmod N$$

$$C2^{e1} = (5p + 7q)^{e2 \cdot e1} \bmod N$$

Now multiply equation C1 by 5 and C2 by 2, in order to get the same p value , u can also multiply to get same q value here but I will use p

Remember to use Power Rule: $a^b \times c^b = (a \times c)^b$

So we multiply by $5^{(e1 \cdot e2)}$ and $2^{(e1 \cdot e2)}$

$$\text{Equation1} = (10p + 15q)^{e1 \cdot e2} \bmod N = 5^{e1 \cdot e2} \times c1^{e2}$$

$$\text{Equation2} = (10p + 14q)^{e1 \cdot e2} \bmod N = 2^{e1 \cdot e2} \times c2^{e1}$$

Now subtract these equations to get $q^{(e1 \cdot e2)} \bmod N$

We know that from earlier $N = p \times q$

And we know the difference between equation 1 and equation2 is a multiple of q

And that q is a common divisor for both equations ($n = p \times q$) and the difference of equation 1 and 2 (eqn_1 - eqn_2)

Therefore q is a factor of the difference of equation 1 and 2 and its also a factor of N

So the greatest common divisor of both (eqn_1 - eqn_2) and N will give you q

Now all we have to is get the already provided values from the txt file and insert it all into a python script....