

# MODULAR BINOMIALS (80pts)

Rearrange the following equations to get the primes  $p, q$

$$\begin{aligned}N &= p \cdot q \\c_1 &= (2 \cdot p + 3 \cdot q)^{e_1} \bmod N \\c_2 &= (5 \cdot p + 7 \cdot q)^{e_2} \bmod N\end{aligned}$$

Multiply both of the equations by each other to get similar equations

$$C1^{e_2} = (2p + 3q)^{e_1 e_2} \bmod N$$

$$C2^{e_1} = (5p + 7q)^{e_2 e_1} \bmod N$$

Now multiply equation C1 by 5 and C2 by 2, in order to get the same  $p$  value, you can also multiply to get same  $q$  value here but I will use  $p$

Remember to use Power Rule:  $a^b \times c^b = (a \times c)^b$

So we multiply by  $5^{(e_1 \wedge e_2)}$  and  $2^{(e_1 \wedge e_2)}$

$$\text{Equation1} = (10p + 15q)^{e_1 e_2} \bmod N = 5^{e_1 e_2} \times c1^{e_2}$$

$$\text{Equation2} = (10p + 14q)^{e_1 e_2} \bmod N = 2^{e_2 e_1} \times c2^{e_1}$$

Now subtract these equations to get  $q^{(e_1 e_2)} \bmod N$

We know that from earlier  $N = p \times q$

And we know the difference between equation 1 and equation 2 is a multiple of  $q$

And that  $q$  is a common divisor for both equations ( $n = p \times q$ ) and the difference of equation 1 and 2 ( $\text{eqn\_1} - \text{eqn\_2}$ )

Therefore  $q$  is a factor of the difference of equation 1 and 2 and its also a factor of  $N$

So the greatest common divisor of both ( $\text{eqn\_1} - \text{eqn\_2}$ ) and  $N$  will give you  $q$

Now all we have to is get the already provided values from the txt file and insert it all into a python script....