

d) Based on the equation given:

$$p(X=x|s) = \begin{cases} \frac{2}{4s} \left(1 - \left(\frac{x}{s}\right)^2\right) & \rightarrow \text{for } x \in (-s, s) \\ 0 & \rightarrow \text{everywhere else.} \end{cases}$$

For $p(X \leq x)$, as from $-\infty$ to $-s$ the function is equivalent to 0

then for $p(X \leq x)$ we basically have to search from $-s$ up to x .

$$\begin{aligned} \int_{-s}^x \frac{2}{4s} \left(1 - \left(\frac{x}{s}\right)^2\right) dx &= \frac{2}{4s} \cdot \int_{-s}^x \left(1 - \left(\frac{x}{s}\right)^2\right) dx \\ &= \frac{2}{4s} \cdot \int_{-s}^x 1 dx - \int_{-s}^x \left(\frac{x}{s}\right)^2 dx \\ &= \frac{2}{4s} \cdot [x]_{-s}^x - \frac{1}{s^2} \cdot \int_{-s}^x x^2 dx \\ &= \frac{2}{4s} \cdot \left((x+s) - \frac{1}{s^2} \left(\frac{1}{3} x^3\right)_{-s}^x\right) \\ &= \frac{2}{4s} \left((x+s) - \frac{x^3 + s^3}{3s^2}\right) \\ &= \frac{2}{4s} x + \frac{2}{4} - \frac{x^3 + s^3}{4s^3} \end{aligned}$$

$$e) E(\text{abs}(x)) = \int_{-s}^s \left| \frac{2}{4s} \left(1 - \left(\frac{x}{s}\right)^2\right) \right| dx$$

$$\begin{aligned} &= \int_{-s}^0 \frac{2}{4s} \left(1 - \left(\frac{x}{s}\right)^2\right) dx + \int_0^s \frac{2}{4s} \left(1 - \left(\frac{x}{s}\right)^2\right) dx \\ &= \frac{2}{4s} \int_{-s}^0 1 dx - \int_{-s}^0 \left(\frac{x}{s}\right)^2 dx + \frac{2}{4s} \int_0^s 1 dx - \int_0^s \left(\frac{x}{s}\right)^2 dx \\ &= \frac{2}{4s} \left([x]_{-s}^0 - \frac{1}{s^2} \left(\frac{1}{3} x^3\right)_{-s}^0\right) + \frac{2}{4s} \left([x]_0^s - \frac{1}{s^2} \left(\frac{1}{3} x^3\right)_0^s\right) \\ &= \frac{2}{4} \left(s - \frac{1}{s^2} \left(\frac{s^3}{3}\right)\right) + \frac{2}{4} \left(s - \frac{1}{s^2} \left(\frac{1}{3} s^3\right)\right) \\ &= 0.5 + 0.5 \\ &= 1 \end{aligned}$$