Gravitational Wave Missions from LISA to Big Bang Observer

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Gravitational waves provide a new means to view the universe complimentary to the view provided by electromagnetic radiation. Because gravitational waves interact very weakly with normal matter, their detection is very difficult. Ground-based detectors have been under development since the late 1960's. Because of variations in the environment on Earth (e.g. seismic activity, varying mass distribution, etc), ground-based detectors are restricted to observation of sources with characteristic time scale of 1 second or shorter, limiting sources to those with short lifetimes. Observation of sources with longer characteristic time scales requires the use of satellites. The LISA (Laser Interferometer Space Antenna) mission, a joint ESA-NASA mission, planned for launch in the next decade, is a space version of the ground-based interferometer detectors. LISA is designed to detect compact binary start system in our galaxy and massive black hole binaries in other galaxies. Beyond LISA, gravitational wave astronomy provides the prospect for observing the universe from times before the recombination era, which are forever hidden from direct view by optical and radio telescopes. The Big Bang Observer is a Vision mission in the NASA Structure and Evolution of the Universe roadmap with the defined goal of achieving such sensitivity. Both LISA and BBO require new developments in precision laser interferometry over very large distance and in reducing disturbance forces to unprecedented levels.

Nomenclature

BBO = Big Bang Observer ESA = European Space Agency

GRACE = Gravity Recovery And Climate Experiment

GRS = Gravitational Reference Sensor
LISA = Laser Interferometer Space Antenna

LPF = LISA Pathfinder

NASA = National Aeronautics and Space Administration

I. Introduction

Gravitational waves provide a means to view the universe complimentary to the view provided by electromagnetic radiation. Because gravitational waves interact very weakly with normal matter, their detection is very difficult. The evidence for the existence of gravitational waves is, so far, indirect. The rate of decay of the Hulse-Taylor binary pulsar system¹ agrees to high accuracy with the expected decay due to the generation of gravitational waves. However direct detection of gravitational waves has not yet been accomplished.

Ground-based detectors have been under development since the late 1960's beginning with the large resonating bars of Weber through the current kilometer-scale laser interferometer detectors (LIGO, VIRGO, TAMA300 and GEO600). The prospects for direct detection on Earth by the end of the decade are considered good. However detection on Earth relies on sources expected statistically to exist but with no explicit candidate sources.

Because of variations in the environment on Earth (e.g. seismic activity, varying mass distribution, etc), ground-based detectors are restricted to observation of sources with characteristic time scale of 1 second or shorter, limiting sources to those with short lifetimes. Observation of sources with longer characteristic time scales requires the use of satellites. Experiments for detection of gravitational waves by ground tracking of single satellites have been

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performed with Pioneer 10 and 11, Mars Observer, Ulysses, and Cassini. Each successive radio tracking experiment has achieved higher precision but not yet resulted in successful detection. This is not surprising since the sensitivity of even the specially designed Cassini radio tracking experiment is limited to extremely energetic sources which are expected to occur only rarely.

The LISA² (Laser Interferometer Space Antenna) mission planned for launch in the next decade is a space version of the ground-based interferometer detectors. Using baselines defined by satellites five million kilometers apart, LISA is planned to have sufficient sensitivity to detect gravitational waves from known binary systems in our galaxy. Besides these known sources, LISA is expected to detect a large variety of galactic and extra-galactic sources, including the possibility of some not yet predicted. The LISA mission is expected to give insight into the formation of galaxies and the formation of interactions of massive black holes through the observable universe through means hidden from optical and radio telescopes by intervening matter.

Beyond LISA, gravitational wave astronomy provides the prospect for observing the universe from times before the recombination era, which are forever hidden from direct view by optical and radio telescopes. While there is some chance that LISA will see signals from the early universe, a mission with much greater sensitivity is needed to have high probability of detection of such signals. The Big Bang Observer is a Vision mission in the NASA Structure and Evolution of the Universe roadmap with the defined goal of achieving such sensitivity. A recent mission concept study has been completed to define the characteristics of one possible conceptual realization.

The LISA and BBO missions depend on technology for measurement of distance between free-flying objects with precision smaller than the size of an atom. The core new technologies are continuous-wave lasers with associated telescopes and optics and gravitational reference sensors, providing a freely floating test mass within drag-free satellites controlled to fly in formation with the test masses to shield them from non-gravitational disturbances. The drag-free satellites in turn require a new class of propulsion system capable of controlling the satellite position with a precision of less than a wavelength of light.

The technology needed for LISA is expected to be developed and validated within the next decade, culminating with the flight demonstration of the gravitational reference sensor technology on the LISA Pathfinder spacecraft scheduled to launch near the end of this decade. Technology for the BBO mission will require much more development, especially in terms of high-power short-wavelength lasers and associated optics.

II. Gravitational Waves

Rapid motion of matter is predicted to generate gravitational waves just as rapid motion of charges generate electromagnetic waves. Gravitational waves are difficult to generate and observe compared with electromagnetic waves. This is partly due to the fact the gravitational force is much smaller than the electromagnetic forces for matter (e.g. atoms). As yet gravitational waves have not been directly measured. They are predicted by Einstein's theory of General Relativity and all viable alternate theories of gravity. There is clear indirect evidence for the existence of gravitational wave. The best example is the binary pulsar PSR 1913+16, a system that has been followed in its evolution for almost 20 years³. The binary system is losing energy at exactly the rate predicted by general relativity due to the emission of gravitational waves.

The effect of a gravitational wave passing through a system of free test masses is to create a strain in space which changes the measured distances between the masses by an amount proportional to both the strength of the wave and the distance between the masses. The main problem with the detection of gravitational waves is that the relative length change they induce is exceedingly small.

The primary approach to designing detectors of gravitational waves is based on the Michelson interferometer, where the round trip light time between a reference object and a distant object is compared with the round trip light-time between the reference and an object the same distance away as the primary target but in a different direction. Several large-scale ground-based laser interferometers with arm lengths of several kilometers are now operating. These km-size ground-based laser interferometers are sensitive to gravitational waves at frequencies of 0.1 - 1000 Hz. The primary sources for these detectors will be from neutron-star binary system with orbital periods from 1 ms to 10 s. At those short periods the source will lose energy through radiation of gravitational waves that the stars will collide (coalesce) within a few seconds.

The sensitivity of the interferometers is limited by both the noise in measuring the changes in distance between the ends of the interferometer and from motions of the interferometer end mirrors from disturbances other than gravitational waves. The length change measurement is mostly limited by shot noise in the laser beams. There are many other measurement noise sources which can be made lower that the shot noise through careful engineering.

The limitation for ground detectors at low frequencies (corresponding to binary system with longer orbital periods) is primarily due to motion of the mirrors caused by seismic motion and changes in the local gravity field

(such as those caused by atmospheric tides). The interferometer cannot distinguish motion caused by these noise terms from motion due to gravitational waves. In space the displacement noise of the end mirrors can be significantly reduced. Operation in space also allows for longer distances between mirrors. However for longer arm lengths sensitivity is reduced for gravitational waves with wavelengths shorter than the length of the arms. Thus the length of the arm is one of the parameters that determines the frequencies to which the detector is most sensitive.

III. LISA Mission Concept

The three LISA spacecraft will be in individual orbit about the sun (Fig. 1). They will form a triangle with sides 5×10^6 km long. The spacecraft orbits will be chosen such that the separation between spacecraft will be nearly constant throughout the year even though each spacecraft is in its own orbit^{4,5}. The center of the triangle will be in the ecliptic plane 1 AU (150×10^6 km) from the Sun and 20° (52×10^6 km) behind the Earth. The triangle will have an apparent rotation about the center of the formation once per year due to the individual spacecraft orbits. The spacecraft positions with respect to each other will not be actively adjusted at any time after initial spacecraft deployment.

The three spacecraft will be used to form an unequal-arm Michelson interferometer. The interferometer will be formed using laser signals transmitted between spacecraft. One spacecraft will be designated the central spacecraft. The central spacecraft will transmit a continuous laser beam to the two other spacecraft. The lasers in the end spacecraft will be phase-locked to the incoming light, and thus act as amplifying mirrors. The returned beams at the central spacecraft will mixed with the transmitted light on a photo-diode. The phase shifts of the resulting signals will then be measured. The phase measurements of the two interferometer arms will be differenced to determine changes in distance between one pair of spacecraft versus the other pair. This differencing is necessary to cancel out frequency instability of the central lasers, which would otherwise mask the signal due to gravitational waves. The resulting distance accuracy will be about one picometer.

Because of the symmetry of the LISA spacecraft design, any of the three spacecraft can act as the central spacecraft of the Michelson interferometer. This allows for redundancy in the case of a partial payload failure. If all payload items are properly functioning, then it will be possible to have signals propagating between each pair of spacecraft and have two linearly-independent Michelson interferometers in action simultaneously. This allows for measurement of both of the two possible polarizations of gravitational waves and gives improved sensitivity.

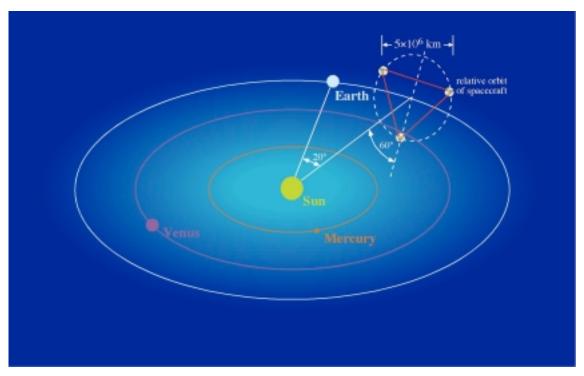


Figure 1. Schematic diagram (not to scale) of the LISA configuration. The three spacecraft will form an equilateral triangle with sides 5 million km long. The plane of the triangle will be tilted 60° out of the ecliptic.

The LISA payload is indicated schematically in Fig. 2. Each payload will contain two test masses, two 1 W lasers, and two 30 cm diameter telescopes for the transmission and reception of laser signals. The test masses will be shielded from extraneous disturbances (e.g. solar pressure) by the spacecraft. To keep the solar radiation pressure from pushing the spacecraft into contact with the test masses, the distance between the test masses and the spacecraft will be measured and thrusters fired to counteract the solar radiation pressure. This control is similar to the 'dragfree' control used to counteract atmospheric drag in some Earth-orbiting satellites. Since some of the major disturbance forces acting on the test masses are caused by variation in the spacecraft position with respect to the test masses, the spacecraft position will be controlled to an accuracy of 1 nanometer relative to the two test masses. Because the spacecraft orbits are not controlled, the distance between spacecraft will vary over the year, requiring the angle between the two telescopes to be changed by about \pm 0.5°. This angle variation is one of the major reasons that there is not a single test mass per spacecraft. The spacecraft control system is designed to minimize force noise on each test mass in the direction from it to its corresponding partner on another spacecraft. Each test mass is forced in the transverse degrees of freedom to maintain the average separation.

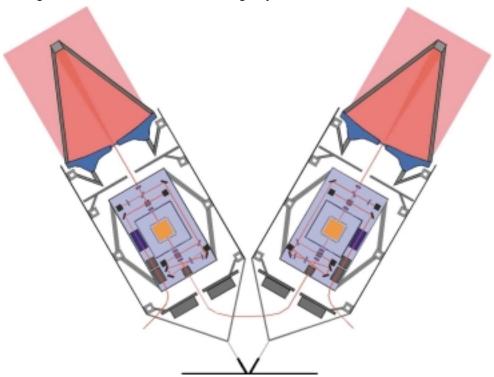


Figure 2. Cross section of the two optical assemblies comprising the main part of the payload on each LISA spacecraft. The two assemblies will be mounted from flexures at the back (bottom of figure) and from pointing actuators (not shown) at the front, near the primary mirrors.

IV. BBO Mission Concept

The BBO mission concept is based on the LISA concept with triangular formations of sets of three spacecraft. To optimize for a signal frequency of 100 mHz, the separation between spacecraft is 50,000 km, which is 100 times shorter than the 5 million km distance between spacecraft for LISA. Like LISA, BBO uses a symmetric arrangement of three spacecraft in the basic formation. Because BBO will be sensitive to binary signals that persist for only a few days, the position of the sources cannot be distinguished using a single interferometer. (In the case of LISA, most signals will persist for many months and the direction to the source can be determined by the change in the signal received by the spacecraft formation as it orbits the Sun.) Therefore BBO requires three widely separated detectors so that the source positions can be determined by difference in signal arrival time, as ground detectors do. In addition, to enable improved sensitivity to stochastic signals, cross correlation of signals from two nearby detectors is planned. This leads to the basic mission formation indicated in Fig. 3 with twelve spacecraft configured as four interferometers, each consisting of three spacecraft.

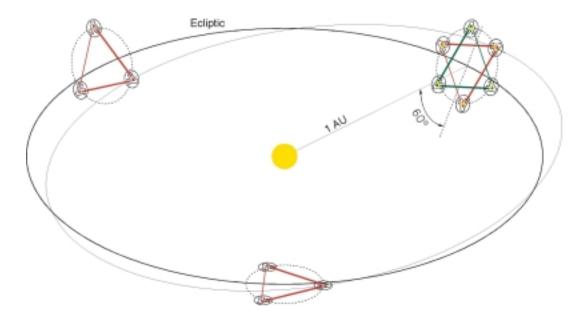


Figure 3. Schematic diagram (not to scale) of the BBO configuration. The twelve spacecraft form four interferometers consisting of three spacecraft. Each set of three spacecraft will form an equilateral triangle with sides 50,000 km long. The plane of the triangle will be tilted 60° out of the ecliptic.

The BBO sensitivity requires an improvement of 1000 in strain sensitivity compared with the LISA performance. No improvement is achieved by the smaller distance between the spacecraft for BBO than for LISA; the shorter distance does lead to a higher amount of received laser power, for a given laser power and telescope diameter, leading to a higher precision in distance measurement, but since this is divided by a shorter distance between spacecraft, the sensitivity limit for gravitational waves due to shot noise would be the same. Thus in order to reach improved sensitivity, BBO requires the use of larger telescopes and higher power lasers than for LISA. BBO will nominally use 2.5 m diameter mirrors for the telescopes and 100 W laser power. This combination will reduce the shot noise to the level required by BBO.

Unlike LISA where the light power received is much less than 1 W, the BBO received power is ~10 W. Since this is too large for practical photo detectors to handle, the BBO interferometer must be configured to maintain destructive interference between the received light and the outgoing light (i.e. operate on a dark fringe). Thus unlike LISA the BBO spacecraft (and the payload test masses) are controlled to keep the distance between them constant. Because of the shorter arm lengths, the amount of propellant needed and the magnitude of the forces required to move the test masses are much smaller than would be required for LISA.

With the distance between test masses constant a single test mass can be used. Fig. 4 shows a schematic of the notional BBO payload. The two 2.5 diameter telescopes are arranged with outgoing beams crossing each other to allow the payload to fit within a 4.6 m diameter launch vehicle fairing. The beams are combined on an optical bench containing a test mass with hexagonal cross section. The BBO test mass will be larger than the LISA test mass so that the received laser power can be spread over a larger area to reduce thermal noise effects.

V. Technology Challenges

The technology challenges are the development of a system to measure changes in distances between test masses, and to isolate the test masses from external disturbances so that changes in their separation due to gravitational waves are not masked by motions caused by other forces. The technology developments needed are divided in to the three areas of gravitational sensors, micronewton thrusters, and interferometry.

The major technology challenge for LISA is the need for test masses sufficiently isolated from non-gravitational forces. The desire to measure distance changes, due to gravitational waves, of order 10 pm, for frequencies from 10^{-4} Hz to 10^{-1} Hz, means that the non-gravitational forces on the test masses need to produce accelerations less than $3x10^{-15}$ m/s²/ $\sqrt{\text{Hz}}$ or $< 3x10^{-16}$ g. The test mass noise depends on the design of a *gravitational reference sensor* which will provide a freely floating test mass with sufficiently low disturbances. Some of the noise forces on the test mass are caused by fluctuations in the distance between the test mass and the rest of the spacecraft. For example, the spacecraft mass has a gravitational pull on the test mass and thus fluctuations in the position of the spacecraft cause

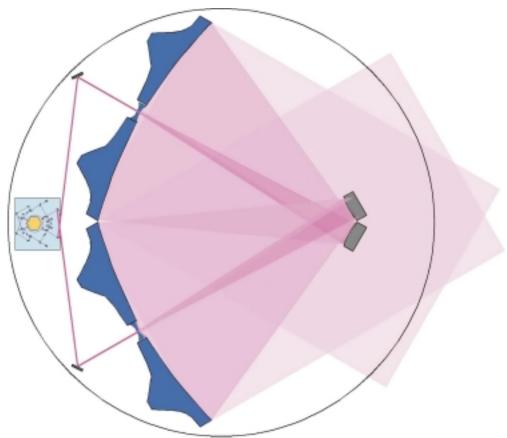


Figure 4. Cross section of the payload on each BBO spacecraft, including two 2.5 m diameter telescopes and an optical bench centered on a single test mass with hexagonal cross section.

fluctuations in the force on the test mass. Because of this the position of the spacecraft must be controlled to stay centered on the test mass. The position control requirements, derived from the inertial sensor requirements, in turn place requirements on the spacecraft thrusters. With the current mission design, the thrusters are required to have a thrust noise of about 10^{-7} N, with a continuous thrust of about 25 μ N in order to oppose the force from solar radiation pressure. A new class of *micronewton thrusters* is needed to meet these requirements.

Measurement of changes of distances between test masses is routinely done with laser interferometry. For ground-based gravitational wave detectors, techniques for measuring distance changes of order 10⁻¹⁹ m have been developed and demonstrated over the past 20 years. However, interferometers for ground-based gravitational-wave detection have been optimized for motions at much higher frequencies than desired for LISA, and at much higher laser signal powers. There are a number of error sources associated with the low-frequency *interferometry* to be used for the LISA distance measurements which much be addressed.

The BBO technology represents an extension of LISA technology. Since the primary frequency range for BBO is higher than for LISA, most noise forces acting on the test masses will naturally be reduced in the BBO measurement range. The only exceptions are those associated with the motion of the spacecraft relative to the test mass, which can be reduced by using the interferometer readouts rather than capacitance sensing for the spacecraft control. However the shorter arm lengths for BBO require much larger telescopes and lasers than LISA does, and will require major technology developments.

A. Gravitational Reference Sensor

Each LISA or BBO spacecraft includes one or two gravitational reference sensors. Each GRS consists of a freely falling test mass, an enclosing housing, a discharging subsystem, a caging mechanism, a venting mechanism and associated electronics. The design of the GRS will limit unwanted disturbances from stray forces on the test mass to an acceptable level. Capacitor plates mounted on the housing surrounding the test mass will provide attitude and displacement information, which for LISA will be used for the spacecraft for drag-free control while for BBO

will be used for 'coarse' positioning to initialize the interferometer readout. The capacitor plates will also be used to adjust the position and attitude of the test mass as necessary by appropriate applied voltages.

The GRS design for LISA is based on the heritage from the GRACE and Triad missions. The Triad mission, launched in 1972, demonstrated drag-free control of a spacecraft relative to a spherical test mass made of a gold/platinum alloy. The overall Triad system performance characterization was limited by the accuracy with which the spacecraft trajectory was measured via ground tracking. The GRACE mission, launched in 2002, includes accelerometers to measure drag forces on each of the two Earth-orbiting spacecraft. The accelerometers were tested on the ground at the level of 10^{-9} m/s²/ $\sqrt{\rm Hz}$, and are operating in orbit with about ten times better sensitivity. The GRACE test masses are rectangular parallelpipeds (4 × 4× 1 cm) made out of titanium. The LISA test mass is planned to be cubical (~4 × 4× 4 cm) and made out of gold/platinum alloy, combining aspects from Triad and GRACE.

The required performance for LISA is about 30,000 times better than the GRACE accelerometer performance. Some of the improvement is expected to come from increasing the mass of the test mass while taking more care to control thermal and magnetic variations. Another major source of improvement will be achieved by using thrusters to center the spacecraft about the test mass rather than applying electrostatic forces to center the test mass. This reduces the required electric fields at the surface of the test mass and reduces the coupling of electronic noise into force noise.

GRS development is constrained by the fact that the desired performance cannot be demonstrated on the ground. The types of noise forces that can affect the inertial sensor test mass can be identified and separately characterized in laboratory tests. Based on the noise models and tests, detailed instrument designs for a GRS meeting the LISA requirements have been completed. However the sensors cannot be operated, much less tested, in the Earth's gravity, making this a high-risk technology.

It is thus highly desirable to perform a flight demonstration of candidate inertial sensors. The LISA Pathfinder project, planned for launch in 2009, is specifically designed to test GRS performance. LPF includes four inertial GRS units, two European and two from the United States, each with a test mass with its own housing and electronics, on a single spacecraft. The GRS performance will be validated by using one test mass for spacecraft drag-free control, and measuring the position of a second test mass with respect to the first to show that both are following the same trajectory, under the influence of gravitational forces only. Any non-gravitational force would appear as a change in the position of one test mass with respect to the other. The LPF performance goal is to demonstrate performance within a factor of ten of the LISA requirement. LPF performance will be limited by noise introduced by the suspension force needed to keep the second test mass close to the first one. This suspension force will not be needed (in the direction of the interferometer beams) for LISA or BBO. another limitation of LPF is the accuracy expected for the laser interferometry. The acceleration noise on the test masses will be derived from the second derivative of the measurements of distance between spacecraft. This introduces an upper limit in in acceleration noise measurement that increases rapidly with frequency, even though most expected sources of acceleration noise will decrease at higher frequencies.

For BBO the assumed performance is a factor of ten better than the LISA performance. Assuming LPF and LISA are successful, the GRS noise terms will be well characterized. The extrapolation to the BBO requirements will largely depend on improved interferometry for measurement of the test mass position for the drag-free control. The BBO test mass will be different from LISA's because of the large amount of received laser power that will be incident on the test mass. To cope with the higher laser power, the BBO test mass needs to be larger (with characteristic dimension ~10 cm) and of a material with better thermal noise properties. Candidate materials include silica and sapphire. Some development in fabrication of the required test mass will be needed, though work for advanced ground-based detectors may produce the required development.

B. Micronewton Thrusters

LISA and BBO require thrusters with the capability of balancing the solar radiation pressure, with low enough noise such that the spacecraft can be kept centered on one of the instrument test masses to the required accuracy (~10 nm/ $\sqrt{\text{Hz}}$). The thrusters must also provide very fine spacecraft pointing control. This gives rise to the thruster requirements for thrust controllable in the range 5 μ N to 100 μ N with noise less than 0.1 μ N/ $\sqrt{\text{Hz}}$.

The best candidates for meeting these requirements are low-thrust electric-propulsion thrusters. These devices operate by ionizing and accelerating atoms or droplets of metal or other conducting fluid. The thrust can be varied precisely by control of an accelerating potential. Two types of thrusters are being developed for flight on LISA Pathfinder. NASA is funding developing thrusters based on a colloidal fluid propellant. These have demonstrated the required thrust range and noise. ESA is funding developing thrusters based on molten metal (either cesium or

indium). The thrust and thrust noise of these has not yet been directly measured but the performance estimated from measurements of current and voltage show that the requirements should be met.

Given demonstrated performance, the main technology issues remaining are related to lifetime. Lifetime issues arise from the high voltages used to ionize and accelerate the propellant to high velocity. There can be erosion of the accelerating grids and ejecting tips. Propellant reservoir volume is a related issue. The colloid thrusters propellant is a liquid at room temperature meaning that the size of the reservoir can be increased relatively easily. However the propellant is a large complex molecule that may be broken down into undesirable components due to radiation or other effects. The metallic thrusters presently use capillary action to flow the propellant, which is more difficult to extend to larger volumes.

C. Interferometry

The interferometers for LISA and BBO are similar in concept but have very different technology development requirements. Both require lasers, mirrors, beam-splitters, photo-detectors, etc. However the LISA equipment needs are generally reduced relative to ground-based detectors. This means that laboratory models largely exists, with the main challenge being engineering for flight. The BBO system requires far higher power laser and optics and much larger size, meaning much more development is needed.

The LISA interferometer design is based on a 1 W laser and 30 cm telescopes. Laboratory versions of 1 W lasers are commercially available. Flight qualified versions are not, although one company in Europe (Tesat) is selling flight-qualified 30 mW lasers of a suitable type (one of which will be flown on LPF). Either a higher power laser or a laser amplifier must be developed for flight. A flight-qualified electro-optical modulator must also be developed, for use either on the 1 W output beam or for the 30 mW beam preceding the amplifier. The light power received from a different spacecraft will be only a few nW. This power level is consistent with commercial photo-detectors, which meet LISA performance requirements but must be engineered into the required form factor and flight qualified. The other optics are generally available, although injecting the light from the laser onto the optical bench with the required wavefront quality represents an engineering challenge. Because the distance between LISA spacecraft will be continually changing, the interferometer readout electronics are more similar to the GRACE Doppler measurement system than the current ground-based interferometers. The GRACE measurement electronics performance is within a factor of ten of the LISA requirement. Modification to the GRACE electronics concept has demonstrated the required performance in the laboratory. Transition to a flight-qualified system will require moderate engineering effort.

The BBO interferometer requires much higher light levels to achieved the required measurement performance. The nominal design is based on ultraviolet lasers, operating continually at wavelength of ~300 nm with output power of 200 W. There are early laboratory experiments to show that this wavelength and power level can be achieved though only for short periods of time. An electro-optical modulator for the high laser power does not exist in any form at present. The 2.5 m diameter BBO primary mirrors are similar in scale to the Hubble Space Telescope and have similar wavefront requirements. However the mirror mass needs to be much lower to enable launch of 24 telescopes. Beryllium mirror prototypes developed for the James Webb Space Telescope have low enough mass density and fairly good optical quality. Present beryllium forming machinery limits the size of the mirrors to about 1.5 m. Either a larger scale process must be developed or an alternative mirror technology needs to be developed. The telescopes require a secondary adjustment structure with unprecedented dimensional stability which will require a significant development effort. Optics are needed to interfere UV light beams of 10 W each and 10 cm in diameter, which do not currently exist. Commercial photo-detectors meet performance requirements if the destructive interference fringe is dark enough. Achieving this will require a significant development in beam profile intensity matching optics. Since the BBO interferometer readout is similar to that for ground-based detectors, the electronics development will largely be a flight-engineering effort.

VI. Conclusion

The LISA and BBO space observatories for gravitational waves provide the prospect for major scientific advance. The technological challenges required to achieve the desired results are formidable. A reduction in force noise on test masses by a factor of 30,000 is required and is the justification for the LISA Pathfinder technology demonstration mission. While BBO is based on LISA as a precursor, its optical metrology system will present a major need for development.

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