Designing a Microstrip Line with a Quarter Wave Transformer

ECE 3317

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Academic Honesty Statement

The author of this paper made sure to adhere to the University of Houston's Academic Honesty Policy. This means that the author of this paper did not discuss this project's content with anyone other than Professor Jackson.

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Abstract

This project demonstrates how a transformer can be used to match the load on a microstrip line in order eliminate reflection in the system at a specific design frequency. For this project, the specified design frequency was at 2.5 [GHz], which is where the microstrip line is meant to have a perfect match. The program TXLINE 2003 was used to determine the length of the transformer, l_T = 2.093 [cm], the width of the transformer, w_T = 0.7405 [cm], the width of the mainline, w_M = 0.4521 [cm], and the distance from the right side of the transformer to the load, d_1 = 4.256 [cm]. The characteristic impedance of the transformer was determined to be Z_0^T = 35.36 [Ω]. The guided wavelength of the mainline and transformer were calculated to be λ_g^M = 8.52 [cm] and λ_g^T = 8.38 [cm] respectively at the design frequency. The characteristics of the mainline and transformer were then used to calculate the SWR of the system at various frequencies ranging from 1.5 [GHz] to 3.5 [GHz], which showed how the reflection of the system continued to grow the more the frequency deviated from the design frequency of 2.5 [GHz]. The plotting was done using MATLAB.

Introduction

In a world filled with technology, it is no surprise that transmission lines are seen in everyday life. This paper discusses the implementation of a microstrip transmission line, which usually consists of a microstrip copper line on top of a ground plane, with a dielectric substrate separating the two. A transformer was also placed onto the microstrip line to match the impedances at the design frequency. The microstrip transmission line is often seen radio frequency and microwave designs [1].

In general transmission lines have a characteristic impedance denoted by Z_0, which is a value derived from the physical characteristics of the transmission line. When a transmission line is connected to a load Z_L, a reflected wave that travels back towards the generator can result if $Z_0 \neq Z_L$. In most transmission lines, including the one that will be analyzed in this paper, this reflection backs toward the generator should be eliminated as it could damage the generator [2]. This reflection will be eliminated through the implementation of a transformer.

The microstrip line to be analyzed in this paper is that of Figure 1. It consists of a microstrip line with a load connected to the right side of the mainline. The load consists of two 50 [Ω] impedances connected in parallel, meaning that the right end of the mainline will always see a load of 25 [Ω]. The characteristic impedance of the mainline is defined as $Z_0^M = 50$ [Ω]. The goal of this project is to place a transformer with width w_T and length l_T a distance d_1 away from the load so that the left end of the mainline will match the impedance of the load at the design frequency of 2.5 [GHz]. This means at 2.5 [GHz] there will be no reflection on the line.

Task 1: Calculating Microstrip Dimensions

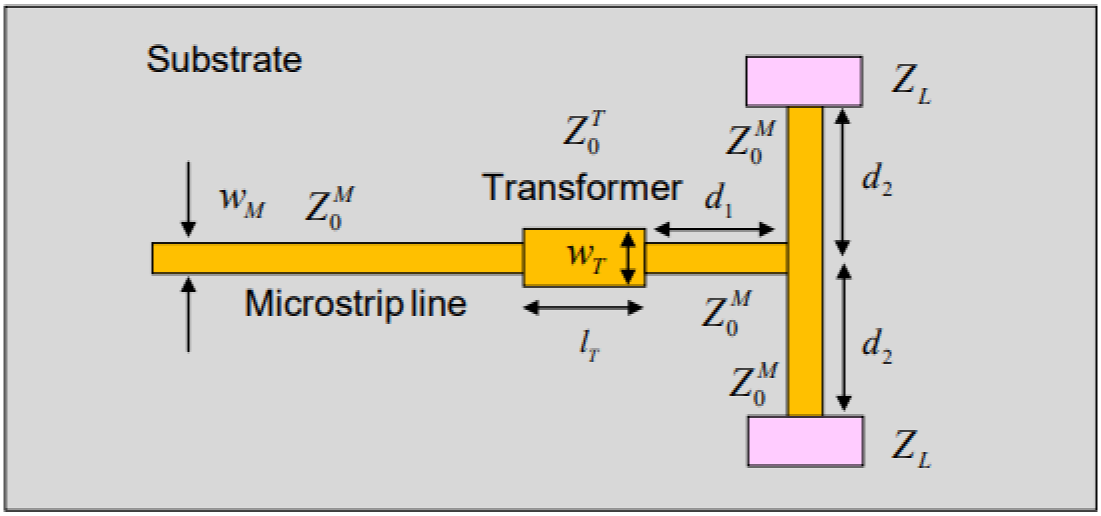

Figure 1: top down view of microstrip design (taken from ECE 3317 Fall 2020 project desc. by Professor Jackson)

Figure 1 shows the microstrip that this whole project focuses on. The goal of this first task is to calculate the dimensions w_M, w_T, l_T, and d_1 at the specified design frequency of 2.5 [GHz]. TXLINE 2003 was used to calculate these dimensions as seen in Figure 2. The first step was to enter the information we already knew into TXLINE. All of the parameters that are boxed in red in Figure 2 were given characteristics of this microstrip line. In the Material Parameters section, the conductivity of the mainline was chosen to be an arbitrarily large number since this microstrip was assumed to be lossless, which is also why the loss tangent value is 0 as well.

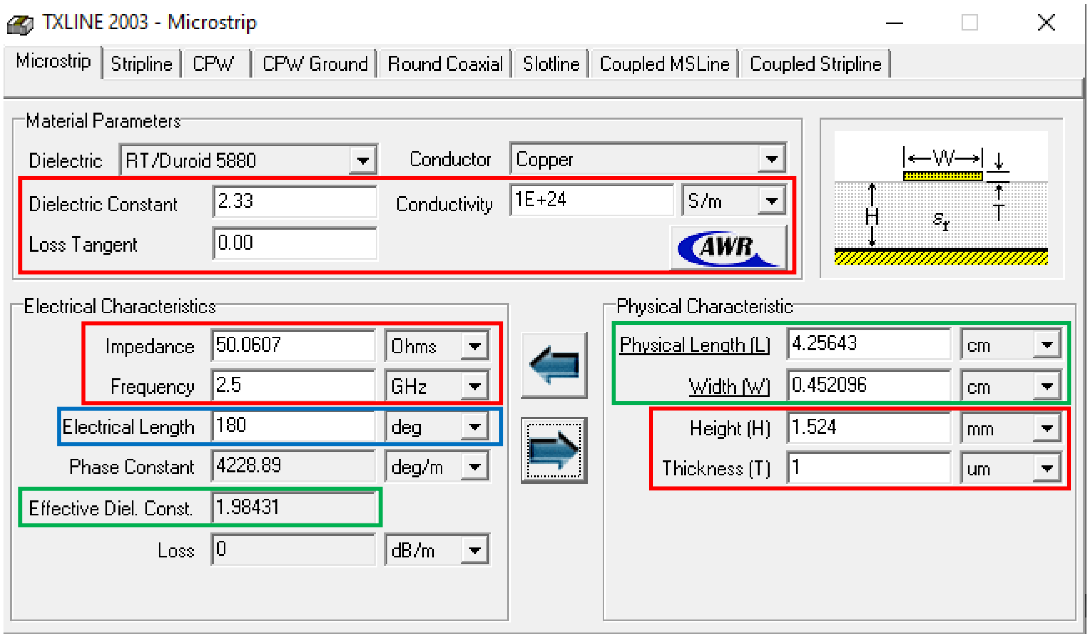

Figure 2: calculations of the mainline dimensions using TXLINE 2003

In the Electrical Characteristics section of Figure 2, the frequency is that of the specified design frequency and the impedance is the characteristic impedance of the mainline, which was defined as $Z_0^M = 50$ [Ω]. The last input parameter needed is the electrical length.

$$\beta = \frac{2\pi}{\lambda_g^M} \qquad (1)$$

$$d_1 = \frac{\lambda_g^M}{2} \quad (2)$$

$$\beta d_1 = \pi \ (3)$$

The electrical length is equivalent to equation (3), which is derived by multiplying equation (1) by equation (2). Equation (3) is only specific to the design frequency of 2.5 [GHz] because equation (2) is specifically stated to be true at a frequency of 2.5 [GHz]. The value of π = 180 degrees, which was entered into the electrical length section of Figure 2. Pressing the right arrow button then calculated the values boxed in green in Figure 2. This means that the effective dielectric constant of the mainline $\varepsilon_r^{eff,M} = 1.98431$, $d_1 = 4.256$ [cm], and $w_M = 0.4521$ [cm].

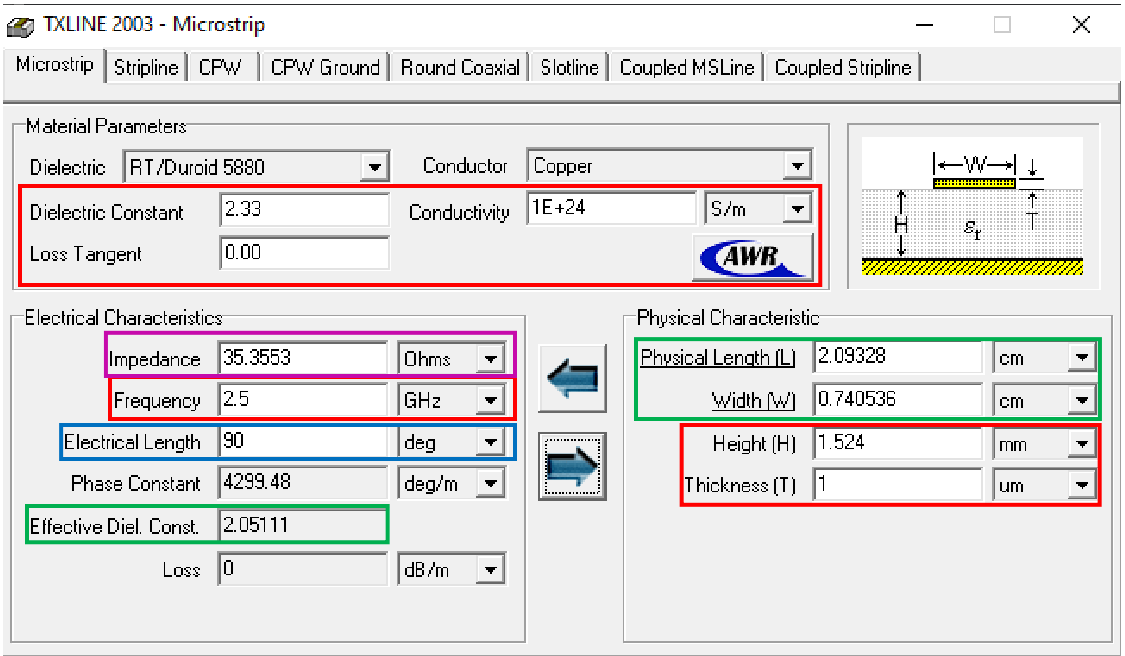

Figure 3: calculations for the transformer dimensions using TXLINE

TXLINE can again be used to calculate w_T and l_t of the transformer on the microstrip, but some changes need to be made to the input parameters. The parameters that stayed the same are boxed in red in Figure 3, since the properties of the substrate and thickness of the microstrip do not change. The design frequency is also still at 2.5 [GHz].

$$\beta = \frac{2\pi}{\lambda_g^T} \quad (4)$$

$$l_T = \frac{\lambda_g^T}{4} \quad (5)$$

$$\beta l_T = \frac{\pi}{2} \quad (6)$$

The electrical length in Figure 3 changed as well, since the properties of the guided wave in the transformer are defined by equation (5) at the design frequency at 2.5 [GHz]. The new electrical length is equal to 90 degrees, which is seen in equation (6), which was calculated by multiplying equation (4) by equation (5).

There is still one more parameter that needs to be changed, which is the characteristic impedance Z_0^T, which is denoted by the magenta box in Figure 3. Unlike Z_0^M, Z_0^T has to be calculated, since it was not given.

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0\, tan(\beta\, d)}{Z_0 + jZ_L\, tan(\beta d)}\right) \; (7)$$

In order to determine Z_0^T, equation (7) will have to be used twice. This equation will first be used to calculate the input impedance (Z_{in1}) seen by the right end of the transformer a distance d_1 away from the load. In this case, the load Z_L = 25 [Ω], $Z_0 = Z_0^M$ = 50 [Ω], and $\beta d = \beta d_1 = \pi$ from equation (3). Plugging those values into equation (7) yields the result of Z_{in1} = 25 [Ω].

A variation of equation (7) must be used again to calculate the input impedance seen by the left side of the transformer (Z_{in2}) a distance l_T away. Since we are designing these parameters at the design frequency of 2.5 [GHz], we want the load that is seen by the left section of the mainline to match the characteristic impedance of the mainline (Z_0^M) so that there is no reflection. This means that $Z_0^M = Z_{in2}$ = 50 [Ω]. For this case, $\beta d = \beta l_T = \frac{\pi}{2}$. However, $\tan(\frac{\pi}{2})$ is undefined, which means we need to use equation (8).

$$z_{in} = \frac{(Z_0)^2}{Z_L} \quad (8)$$

This equation is normally used for quarter wave transformers, and from equation (5), we know that we are designing a quarter wave transformer and can use equation (8). $Z_0 = Z_0^T$, which is what we are solving for. $Z_L = Z_{in1}$ = 25 [Ω], and $Z_{in} = Z_{in2} = Z_0^M$ = 50 [Ω]. This means that Z_0^T = 35.36 [Ω]. This value was then inputted into the magenta box in Figure 3. Clicking the right arrow then calculated the values of l_T = 2.093 [cm] and w_T = 0.7405 [cm]. All of the calculated values can be seen in Table 1.

Table 1: calculated characteristics of the microstrip line at 2.5 [GHz]

Characteristics of the Microstrip
w_M = 0.4521 [cm]
w_T = 0.7405 [cm]
l_T = 2.093 [cm]
d_1 = 4.256 [cm],
$\varepsilon_r^{eff,M}$ = 1.984
$\varepsilon_r^{eff,T}$ = 2.051
λ_g^M = 8.52 [cm]
λ_g^T = 8.38 [cm]

The values of λ_g^M and λ_g^T were calculated using equations (10) and (11) respectively. The variable λ_0 was calculated using equation (9), where *c* is the speed of light and *f* = 2.5 [GHz].

$$\lambda_0 = \frac{c}{f} \quad (9)$$

$$\lambda_g^M = \frac{\lambda_0}{\sqrt{\varepsilon_r^{eff,M}}} \quad (10)$$

$$\lambda_g^T = \frac{\lambda_0}{\sqrt{\varepsilon_r^{eff,T}}} \quad (11)$$

Task 2: Check Values Using the Design Formulas

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_r^{eff}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right); & \text{for } \dfrac{w}{h} \le 1 \\ \dfrac{120\pi}{\sqrt{\varepsilon_r^{eff}}\left(\dfrac{w}{h} + 1.393 + 0.667\ln\left(\dfrac{w}{h} + 1.444\right)\right)}; & \text{for } \dfrac{w}{h} \ge 1 \end{cases} \qquad (12)$$

$$\varepsilon_r^{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(\frac{1}{\sqrt{1 + 12\left(\frac{h}{w}\right)}}\right) \qquad (13)$$

To make sure that the values calculated by TXLINE are correct, equations (12) and (13) were used to double check. The height is a constant h = 1.524 [mm] and the dielectric constant of the substrate stays at ε_r = 2.33. When solving for Z_0^T, equation (13) was first used to find $\varepsilon_r^{eff,T}$, where $w = w_T$ from Table 1. The value of $\varepsilon_r^{eff,T}$ was then plugged into equation (12) along with w_T to solve for Z_0^T. A similar process was used to calculate Z_0^M, except that $w = w_M$. The ε_r^{eff} 's that were calculated using equation (13) were then plugged into equations (9-11) to obtain λ_g^T and λ_g^M. All of these values are recorded in Table 2. These values are very similar to those in Table 1, meaning that the calculations are correct.

Table 2: characteristic impedances and guided wavelengths from design formulas

Design Formula Values
Z_0^M = 50.32 [Ω]
Z_0^T = 35.44 [Ω]
λ_g^M = 8.569 [cm]
λ_g^T = 8.439 [cm]
$\varepsilon_r^{eff,M}$ = 1.961
$\varepsilon_r^{eff,T}$ = 2.022

Task 3: Making an SWR Plot with MATLAB

This task entails making an SWR plot of the mainline to the left of the transformer. The plot contains the frequencies from 1.5 [GHz] to 3.5 [GHz] along the X-axis, and the corresponding SWR from 1 to 3 along the Y-axis. The complete plot is shown in Figure 4.

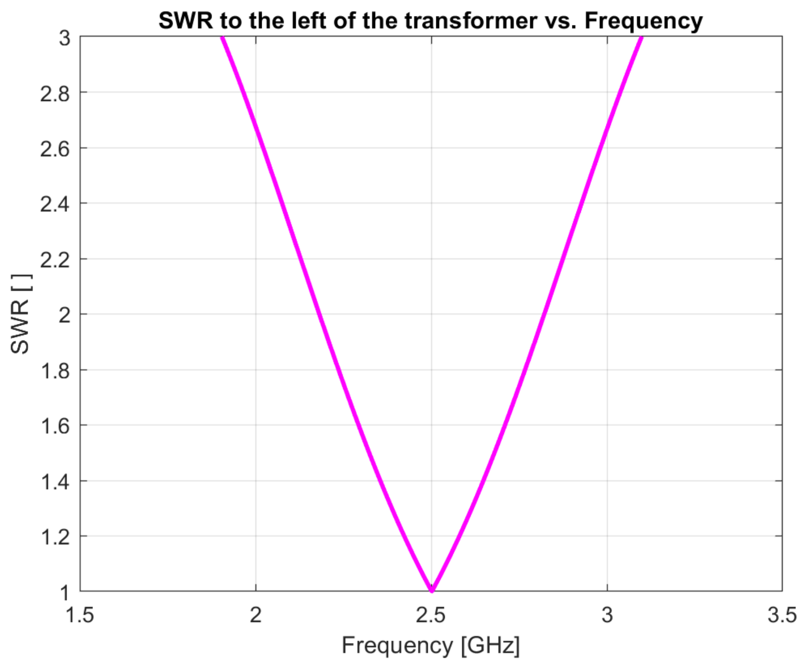

Figure 4: SWR to the left of the transformer vs. frequency

The process of calculating the data points for this plot was automated using MATLAB. An array of 1000 elements was created that started at 1.5 [GHz] and slowly counted up to 3.5 [GHz] at the 1000th spot. This array holds all of the input frequency values that was used to calculate the corresponding SWR value at each frequency (meaning there is also an output SWR array of 1000 elements). MATLAB was then used to plot the array frequencies along the X-axis versus the array of SWR values on the Y-Axis, which resulted in the graph of Figure 4.

The method of calculating each SWR at each frequency involved using the microstrip characteristics from Table 1 as well as the many of the equations from Task 1. The process starts by using the input frequency f to calculate λ_0 from equation (9). The values of λ_g^M and λ_g^T were then calculated from λ_0 and the corresponding dielectric constant from Table 1 by using equations (10) and (11) respectively.

The input impedance as seen by the right side of the transformer a distance d_1 away from the 25 [Ω] load (called Z_{in1}) was calculated using equation (7), where β was calculated using equation (1), $d = d_1$ was taken from Table 1, the load Z_L = 25 [Ω], and $Z_0 = Z_0^M$ = 50 [Ω]. After calculating the value Z_{in1}, equation (7) was used again to calculate the input impedance (Z_{in2}) at the left end of the transformer a distance of l_T away from the load. This means when solving for Z_{in2} with equation (7), $Z_L = Z_{in1}$, $Z_0 = Z_0^T$ = 35.36 [Ω], β was calculated from equation (4), and $d = l_T$ from Table 1.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad (14)$$

$$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \qquad (15)$$

After using equation (7) twice, we are now at the left part of the mainline of Figure 1. Equation (14) is used to calculate the reflection coefficient of the left part of the mainline, Γ_L. For equation (14), $Z_0 = Z_0^M$ = 50 [Ω] and $Z_L = Z_{in2}$ (because the input impedance seen by the left end of the transformer is the same as the load impedance seen by the left section of the mainline). Lastly, after using MATLAB's absolute value function (abs(Γ_L) = $|\Gamma_L|$) to obtain $|\Gamma_L|$, equation (15) was used to find the SWR. MATLAB then repeated these calculations 999 more times to gather enough data points to construct the graph in Figure 4.

Task 4: Percent Bandwidth of the System

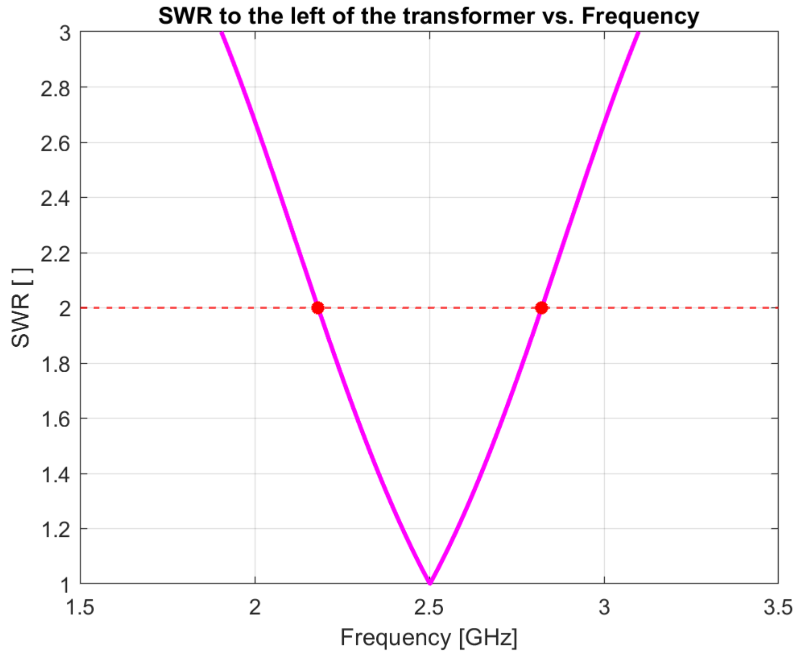

Figure 5: SWR vs. frequency with SWR = 2 line

The percent bandwidth equation is shown in equation (16), where f_1 is the lower frequency and f_2 is the upper frequency for when SWR = 2. The variable f_0 = 2.5 [GHz], which is the design frequency.

$$BW\% \equiv \left(\frac{f_2 - f_1}{f_0}\right) 100 \qquad (16)$$

The values f_1 = 2.182 [GHz] and f_2 = 2.822 [GHz] were taken from the intersections of the SWR = 2 line and the main plot from Figure 5. With these values the bandwidth percentage calculated out to BW% = 25.6 %.

Conclusion

Overall, this project shows how important transformers can be in eliminating reflection on a transmission line. More importantly, this project shows why it is necessary to operate the microstrip line with quarter wave transformer only at its specific design frequency, which in this case is 2.5 [GHz]. Figure 4 shows how the slightest change in frequency away from 2.5 [GHz] leads to an SWR greater than 1, which means that the microstrip system is experiencing reflection. The further one deviates from 2.5 [GHz], the worse the reflection gets. The reason why the SWR varies by frequency is because λ_0 is dependent on the frequency, and that in turn affects the rest of the calculations. When designing a microstrip line, we do not want reflection as that can harm the source generator.

References

[1] Parise, B., 2020. *Microstrip Transmission Lines In RF PCB Design*. [online] Blog.optimumdesign.com. Available at: <http://blog.optimumdesign.com/microstrip> [Accessed 13 December 2020].

[2]L. Shen and J. Kong, *Applied Electromagnetism*. Stamford, CT: PWS Publishers, 1995, pp. 189-190.