$x^3 y' = x^4 y^2 - 2x^2 y - 1$

Proponemos $y = z^{\lambda}$ y $x = z$

$\frac{dy}{dz} = \lambda z^{\lambda-1}$ $\quad dx = dz$

$\lambda z^{\lambda+2} = z^{2\lambda+4} - 2z^{\lambda+2} - 1$

Quitamos termino cuadratico

$z^{2\lambda+4} = 1 \rightarrow 2\lambda + 4 = 0 \rightarrow \lambda = -2$

Ahora con $y = z^{-2}$ sustituimos en la eq. original

$dy = -2z^{-3}\, dz$

$x^3 (-2z^{-3}) \frac{dz}{dx} = x^4 z^{-4} - 2x^2 z^{-2} - 1$

$x^{-1} z = u$; $z = ux$; $dz = u\,dx + x\,du$

$\underbrace{-2u^{-3}\, dz}_{-2u^{-2}dx \;\; -2u^{-3} x\, du} = (u^{-4} - 2u^{-2} - 1)\,dx$

$-2u^{-3} x\, du = u^{-4} dx - dx$

$-2 \frac{1}{u^{-1} - u^{3}} du = \frac{1}{x} dx$

$\frac{u}{u^4 - 1} du = \frac{1}{2x} dx$

$u^2 = s$

$ds = 2u\, du$

$$\frac{1}{2}\int \frac{1}{1-s^2}\,ds = \frac{1}{2}\ln(x) + c$$

$$\frac{1}{2}\operatorname{arctanh}(s) = \frac{1}{2}\ln(x) + c$$

$$\operatorname{arctanh}(s) = \frac{1}{2}\ln\left(\frac{1+s}{1-s}\right)$$

$$\frac{1}{4}\ln\left(\frac{1+u^2}{1-u^2}\right) = \frac{1}{2}\ln(x) + c \qquad ; \; B = e^c$$

$$\left(\frac{1+u^2}{1-u^2}\right)^{1/4} = x^{1/2}B$$

$$\left(\frac{1+(\frac{z}{x})^2}{1-(\frac{z}{x})^2}\right)^{1/4} = x^{1/2}B \qquad , \; A = B^4$$

$$\frac{1 + x^{-2}y^{-1}}{1 - x^{-2}y^{-1}} = x^2 A$$

$$1 + x^{-2}y^{-1} = x^2A(1 - x^{-2}y^{-1})$$

$$1 + x^{-2}y^{-1} = x^2A - Ay^{-1}$$

$$y^{-1}(x^{-2} + A) = x^2A - 1$$

$$y = \frac{x^{-2}+A}{x^2A-1}$$

Para $y(\sqrt{2}) = 0 \rightarrow \quad 0 = \dfrac{\frac{1}{2} + A}{\frac{1}{2}A - 1} \rightarrow A = -1/2$

$$y = \frac{x^{-2}+\frac{1}{2}}{\frac{1}{2}x^2 - 1}$$