2.

a. $x(t+h) = 2x(t) - x(t-h) + a(t)\cdot h^2$

$$\Rightarrow \frac{\partial x(t+h)}{\partial x} = 2\frac{\partial x(t)}{\partial x} - \frac{\partial x(t-h)}{\partial x} + h^2\frac{\partial a(t)}{\partial x}$$

$$\Rightarrow \epsilon_{n+1} = 2\epsilon_n - \epsilon_{n-1} + h^2 a_n' \epsilon_n$$

$$\Rightarrow \epsilon_{n+1} - (2 + h^2 a_n')\epsilon_n + \epsilon_{n-1} = 0$$

b. En un oscilador armónico se tiene:

$$F = -kx \quad \Rightarrow \quad a = \frac{F}{m} = \frac{-kx}{m}$$

Teniendo en cuenta que $\omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 m$, obtenemos

$$a = -\omega^2 x \quad \Rightarrow \quad a' = \frac{\partial a}{\partial x} = -\omega^2$$

$$\Rightarrow \epsilon_{n+1} - (2 + h^2(-\omega^2))\epsilon_n + \epsilon_{n-1} = 0$$

$$\Rightarrow \epsilon_{n+1} - 2\left(1 - \frac{h^2\omega^2}{2}\right)\epsilon_n + \epsilon_{n-1} = 0$$

$$\Rightarrow \epsilon_{n+1} - 2(1-R)\epsilon_n + \epsilon_{n-1} = 0$$

donde $2R = h^2\omega^2 \Leftrightarrow R = \frac{h^2\omega^2}{2}$

c. Reemplazamos $\epsilon_n = \epsilon_0 \lambda^n$:

$$\epsilon_0\lambda^{n+1} - 2(1-R)\epsilon_0\lambda^n + \epsilon_0\lambda^{n-1} = 0$$

$$\Rightarrow \lambda^2 - 2(1-R)\lambda + 1 = 0$$

Usando la ecuación cuadrática, obtenemos:

$$\frac{2(1-R) \pm \sqrt{4(1-R)^2 - 4}}{2} = 1-R \pm \sqrt{(1-R)^2 - 1}$$

$$= 1-R \pm \sqrt{R^2 - 2R} = \lambda_\pm$$

d. $\left|1-R \pm \sqrt{R^2-2R}\right| \leq 1$

$\hookrightarrow 1-R \pm \sqrt{R^2-2R} \leq 1$ (caso 1)

$\pm\sqrt{R^2-2R} \leq R \Rightarrow R^2-2R \leq R^2 \Rightarrow R \leq 0$ No sirve para simular

$-\left(1-R \pm \sqrt{R^2-2R}\right) \leq 1 \Rightarrow 1-R \pm \sqrt{R^2-2R} \geq -1$ (caso 2)

$\Rightarrow \pm\sqrt{R^2-2R} \leq R-2 \Rightarrow R^2-2R \leq R^2-4R+4$

$\Rightarrow 2R \leq 4 \Rightarrow h^2\omega^2 \leq 4 \Rightarrow \boxed{h \leq \frac{2}{\omega}}$