$$x_{n+1} = x_n + h v_n + \frac{1}{2} h^2 a_n + \frac{1}{6} a'_n h^3 + O(h^4)$$

Usando la fórmula de derivada central: $a'_n = \dfrac{a_{n+1} - a_{n-1}}{2h}$

$$\Rightarrow x_{n+1} = x_n + h v_n + \frac{1}{2} h^2 a_n + \frac{h^3}{6}\left(\frac{a_{n+1} - a_{n-1}}{2h}\right)$$

Expandiendo a_{n+1} en Taylor se obtiene: (primer orden)

$$a_{n+1} = a_n + h a'_n = a_n + h\left(\frac{a_{n+1} - a_{n-1}}{2h}\right)$$

$$\Rightarrow 2a_{n+1} = 2a_n + a_{n+1} - a_{n-1} \Rightarrow a_{n+1} = 2a_n - a_{n-1}$$

$$\Rightarrow x_{n+1} = x_n + h v_n + \frac{1}{2} h^2 a_n + \frac{h^2}{6}\left(\frac{2a_n - 2a_{n-1}}{2}\right)$$

$$\Rightarrow x_{n+1} = x_n + h v_n + \frac{1}{2} h^2 a_n + \frac{h^2}{6}(a_n - a_{n-1})$$

$$\Rightarrow x_{n+1} = x_n + h v_n + \frac{4}{6} h^2 a_n - \frac{h^2}{6} a_{n-1}$$
$$= x_n + h v_n + \frac{h^2}{6}(4a_n - a_{n-1})$$

Note que si $a_{n+1} = 2a_n - a_{n-1}$ se reorganiza a:

$$a_{n-1} = 2a_n - a_{n+1}$$

y se reemplaza en la ecuación anterior, se obtiene:

$$x_{n+1} = x_n + h v_n + \frac{h^2}{6}(4a_n - (2a_n - a_{n+1}))$$
$$= x_n + h v_n + \frac{h^2}{6}(2a_n + a_{n+1})$$

Por último, igualamos las siguientes expresiones para x_{n+1}:

$$2x_n - x_{n-1} = x_{n+1} = x_n + hV_n + \frac{h^2}{6}(4a_n - a_{n-1})$$

$$\Rightarrow x_n - x_{n-1} - \frac{h^2}{6}(4a_n - a_{n-1})$$

Por último, usamos la fórmula de Störmer - Verlet y la igualamos a la primera expresión encontrada para x_{n+1}

$$\Rightarrow 2x_n - x_{n-1} + a_n h^2 = x_{n+1} = x_n + hV_n + \frac{h^2}{6}(4a_n - a_{n-1})$$

$$\Rightarrow hV_n = x_n - x_{n-1} + \frac{h^2}{6}(2a_n + a_{n-1})$$

Realizando un cambio de variable se obtiene:

$n = k+1$:

$$\boxed{hV_{k+1} = x_{k+1} - x_k + \frac{h^2}{6}(2a_{k+1} + a_k)}$$