

Sequential Games - Handout

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Introduction

In game theory, a sequential game is one in which players make their decisions one after another, with each player aware of the previous players' actions before deciding their own move. This awareness is crucial as it influences the strategies and potential payoffs based on the visible moves already made by others. Sequential games are typically represented using extensive form, which is a tree diagram that illustrates the order of moves, the players' choices at each decision point, and the possible outcomes and payoffs for each combination of choices.

The key feature of sequential games is the concept of perfect information, where all previous actions are known to all players, allowing them to make more informed decisions. This contrasts with imperfect information games, where some or all previous actions are not known to other players. In sequential games, strategies involve planning moves in response to all possible actions of the previous players, leading to a concept called backward induction. This method involves looking ahead to the end of the game to determine optimal moves, then sequentially deducing backwards to the start which moves will lead to an optimal final outcome.

Sequential vs. Simultaneous Games

Sequential and simultaneous games in political contexts offer different strategic advantages and challenges. In sequential games, the ability to observe and react to the actions of others can provide significant advantages or disadvantages, depending on one's position in the order of play. This is particularly relevant in politics where the timing of decisions can influence the power dynamics between actors. Conversely, simultaneous games, where decisions are made without this foresight, often require a different set of strategies focused more on predicting opponents' moves rather than reacting to them. Understanding these dynamics is crucial for political actors to effectively navigate complex strategic environments and for analysts to predict political outcomes.

Example 1: First Player Disadvantage - U.S. Presidential Nominations

In the U.S. presidential primaries, candidates from political parties announce their candidacy at different times. Those who declare early are the “first movers” and must make strategic decisions about their platform, messaging, and resources without knowing the strategies of later entrants. A later entrant, such as a strong candidate waiting to see the field before declaring, benefits from observing the positions, successes, and failures of these early entrants. They can adjust their strategies, refine their messages, and target their resources more effectively based on this information. This dynamic was evident in the 2008 Democratic primaries, where Barack Obama, entering the race after Hillary Clinton, could tailor his campaign strategies in response to the unfolding political landscape and voter reactions to Clinton’s early strategies.

Players: Candidates running for the presidential nomination (e.g., Barack Obama, Hillary Clinton).

Actions: Each candidate decides when to enter the race and what platform, messaging, and resource allocation strategy to employ.

Payoffs: Success in primaries, measured by votes, delegate counts, and eventual nomination.

Role of Time and Sequence of the Game: In principle, the timing of entry into the race is critical. Here it is the part where we must be cautious when modeling. A realistic starting point could be that early entrants must commit to a strategy without complete information about subsequent entrants, and later entrants can adapt their strategies based on the observed actions and outcomes of those who declared earlier. But, what information is on the table is central to understand their actions. Therefore, the information sets must be carefully identified.

Information Sets: Depending on the description of the case, you must identify whether early entrants have or have not complete information regarding future entrants’ strategies, positions, and resource plans. The same is true for later ones. Later entrants may have or have not more complete information, although it is fair to state that they can see the public reactions to the strategies of early entrants and any shifts in political landscape.

Why does this difference matter?: Because the strategies of the earlier entrant will differ depending on the information available and what is and what is not common knowledge for the players.

Example 2: First Player Advantage - Legislative Proposal and Voting

In parliamentary systems, the order of proposing and voting on legislation often follows a sequential game structure where the proposer has a distinct advantage. For instance, consider a legislative body discussing a budget allocation. The “first mover” advantage lies with the member or committee proposing the budget. They set the agenda and frame the initial proposal, which shapes the subsequent debate. Other members, acting as second movers, can only amend or vote on this proposal. Their ability to influence the outcome is constrained by the initial proposal, which has already set specific boundaries and priorities for the discussion. This dynamic often leads to the initial proposer having a significant advantage in directing legislative outcomes, as seen in many parliamentary democracies.

Players: Members of a legislative body (e.g., a parliament or congress), including a specific member or committee that proposes legislation. In class, we saw a reduction of the complexity of having many players through rules (median voter, pivotal voter, veto voter, etc.), but that is not always the case.

Actions: The proposer sets the initial legislative proposal. Other members react to the proposal by amending, approving, or rejecting it.

Payoffs: The proposer aims to shape the legislative outcome according to their agenda or the interests they represent. Other members seek to adjust the proposal to align with their own or their constituents’ interests, aiming for a favorable outcome or to avoid negative repercussions.

Role of Time and Sequence of the Game: The proposer acts first by setting the agenda and framing the initial proposal. Subsequent players (other legislators) act second by responding to the proposal with amendments, approvals, or rejections.

Information Sets: The initial proposer knows their own goals and strategies but does not know exactly how others will react to the proposal. Other members have the advantage of seeing the initial proposal before forming their response, allowing them to develop an strategy based on more complete information about what is being proposed and how it aligns with their own objectives.

Example 3: Applying the analysis to a real case

Now let apply our understanding of the sequential games to draw a game for the article 94 bis of the current Chilean constitution. Focus the analysis in the third paragraph:

« **Art 94 bis** An autonomous body with legal personhood and patrimony of its own, called Electoral Service will exercise the administration, supervision and control of electoral processes and plebiscites; of the compliance with rules on transparency, limit and control of electoral spending; of the norms on political parties, and the other functions that a constitutional organic law establishes.

The senior management of the Electoral Service will correspond to a Directive Council, which shall exclusively exercise the powers conferred to it by the Constitutions and the laws. This Council will be composed of five Counselors appointed by the President of the Republic, with the agreement of the Senate, adopted by two-thirds of its active members. The Counselors will serve ten years in office, may not be appointed for another term and will be partially renewed every two years.

The Counselors may only be removed by the Supreme Court, at the request of the President of the Republic or one-third of the active members of the House of Representative, on the grounds of a serious violation of the Constitution or the laws, inability, misconduct or gross negligence in the exercise of their functions. The Court will hear the case in plenum, specially convened for that purpose, and for there to be agreement on the removal there will have to be an affirmative vote of the majority of its active members.

The organization and powers of the Electoral Service shall be established by a constitutional organic law. It's organization, staffing, regime of remunerations and employment statute of its personnel will be established by a law. »

Example 4: A Model for Distribution of Benefits Among Legislators from the academic literature

So far we have connected, cases like the one in example 2 with spatial models. But that is not a requirement to find an equilibrium. Let come back to the same kind of problems in which there is a resource to be distributed (beyond ideological backgrounds). This one is based on Baron and Ferejohn (1989).

Overview This model demonstrates that legislative advantages are not always derived from special power-conferring rules (e.g., agenda setting). When considering an open rule with N individuals, Arrow's theorem indicates no equilibrium. However, by introducing sequential decision-making (an institutional feature), Baron and Ferejohn find an equilibrium.

The Game

Participants: N legislators. Objective: Decide on the distribution of a fixed sum of money (\$1 total). Draw the sequential game.

Process:

A random draw determines which legislator gets to propose a distribution of the funds.

The proposal is a distribution vector (x_1, x_2, \dots, x_n) where each component represents the amount proposed for each legislator.

Decision-making:

If you are the proposer, you choose the distribution vector.

If you are not the proposer, you vote for or against the proposal.

Voting and Outcome:

A simple majority is required for a proposal to pass.

If the proposal does not achieve a majority, a new draw occurs and the process repeats.

Legislators discount future payoffs, reflecting the urgency or preference for immediate gains over future benefits.

Dynamics of the Game

- Coalitions: Let M^* be the subset of legislators that receive something from the proposer.
- Probability and Scenarios: Each non-proposer has a probability of $\frac{(N-1)}{2N}$ of being included in M^* by the current proposer.
- Scenarios for a non-proposer ('j'):
 - Reject and Be Selected Next Round: If legislator 'j' rejects the current proposal but is selected to propose in the next round.
 - Reject and Be Chosen by Next Proponent: If 'j' rejects the offer but the next proposer includes 'j' in their distribution.
 - Reject and Be Excluded: If 'j' rejects the proposal and is not selected by subsequent proposers or part of any winning coalition, they gain nothing.
- Expected Value Calculation: Simplifying the math, the expected value for each legislator 'j' can be approximated to consider the value of immediate versus future gains, balancing the immediate payout against the potential future benefits based on the discount rate and the probability of being included in future proposals.

Implications

The introduction of sequential rules and a discount factor creates a stable equilibrium in this model. This simplified model can explain phenomena like pork-barrel politics where legislators secure budget allocations beneficial to their constituencies in exchange for their support on proposals.

Addendum

An interesting observation is that legislative votes often conclude with more than a simple majority, such as 56%, 65%, or 72% in favor. This could be due to the dynamics of negotiation, where proposers adjust offers to secure support beyond a precarious minimum, ensuring broader agreement and stability of the coalition.

Example 5: The paper scissors rock strategic game

Draw the full sequential game for the paper scissors rock strategic game.

Example 6: Let add some complexity

Imaging the following scenario: A legislative negotiation scenario that is centered around environmental policy-making.

Legislators are faced with the challenge of deciding whether to implement a new environmental policy that includes various provisions for mitigating its impact. Each participant brings their perspective, influenced by their constituency's environmental and economic concerns.

One legislator, selected at random, is tasked with drafting the initial proposal. This legislator has conducted comprehensive research and, as a result, possesses detailed information about the potential impacts—both costs and benefits—of the proposed policy. This includes data that may not be readily apparent or available to other members of the legislature, such as long-term environmental benefits or hidden implementation costs.

The other legislators must base their decisions on the proposal's visible merits, the trustworthiness of the proposer, and whatever limited information they can gather from their resources or the proposal presentation itself. Their challenge is to evaluate the proposal's potential without access to all the facts, relying on their judgment and the persuasive power of the proposer.

Your challenge: Draw a reasonable sequential game that represents this scenario in a simple but complete enough way. Discuss your proposal with your classmates.