Exercises on Subgame Perfection

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Before we move on, you have a task to do:

The Cuba Missile Crisis, 1962.

The Cuban Missile Crisis: Video –> here

The Cuban Missile Crisis, also known as the Caribbean Crisis or the October Crisis, was a 13-day confrontation between the United States (Country A) and the Soviet Union (Country B) that took place in October 1962.

During the crisis, the United States discovered Soviet ballistic missile installations in Cuba. The presence of these missiles represented a direct and immediate threat to American national security, prompting the United States to consider a range of responses, from diplomatic negotiations to direct military action.

Exercise Task:

a) Watch the Video: Before proceeding with the exercise, watch the provided video detailing the events and timeline of the Cuban Missile Crisis.

b) Develop a Sequential Game: After watching the video, construct a sequential game to represent the situation starting from the moment the crisis became explicit. This means your game should begin at the point where the United States must decide how to react to the Soviet Union's installation of missiles in Cuba.

c) Considering that the crisis ended in negotiations between the parties, consider payoffs (numbers) for each country that would be consistent with that outcome of the game.

d) Replace the payoffs (numbers) in question (c) by letters and take in account any relationship between those letters that are consistent with the Nash Equilibrium found in (c).

Do these at home

1. Bargaining the budget

In Chile, the budget approval process begins in September and must conclude by the last day of November. If the legislative assembly fails to approve a new budget, the default outcome is the adoption of the previous year's budget. This rule was implemented following the 1891 institutional crisis, a pivotal moment that significantly shaped Chile's political and legislative landscape.

Historical Context: The 1891 crisis in Chile, marked by a violent conflict between President José Manuel Balmaceda and the Congress, resulted in a civil war. The conflict centered around disputes over the constitutional powers of the president versus Congress, particularly in fiscal policy and budget control. Balmaceda's administration attempted to centralize authority and control over budgetary decisions, which led to fierce opposition from Congress. The crisis highlighted the need for mechanisms to ensure fiscal governance and prevent such disputes from crippling government operations. The fallback rule of reverting to the previous year's budget if a new one isn't agreed upon was one of the reforms instituted to provide stability and prevent governmental deadlock.

A subgame perfect equilibrium in this context ensures that the strategies by all parties (support, oppose, negotiate amendments) are optimal at every voting stage. Draw the game before the institutional crisis and the change after it. Select payoffs that are consistent with the historical background.

2. Election Campaign: Two-Party Electoral Competition

Two political parties, Red and Blue, compete for votes by choosing a position on a linear spectrum from 0 (extreme left) to 10 (extreme right). Red moves first and selects a position, followed by Blue. Voters have ideal points distributed uniformly across the spectrum. The party closest to a voter's ideal point wins that voter's vote. The party with the most votes wins the election.

Using subgame perfection, determine the optimal strategies for Red and Blue. How does the first-mover advantage influence the outcome?

3. Issues with the distribution of resources

In a legislative body, three legislators need to decide on the allocation of a budget of $100. Each legislator wants to secure as much of the budget as possible for their district. The game proceeds as follows: Legislator 1 proposes a distribution of the budget. Legislators 2 and 3 can either Accept or Reject the proposal. If the proposal is rejected, Legislator 2 then gets to make a proposal, followed by Legislator 3 if the second proposal is also rejected. If all proposals are rejected, the budget is equally divided among all three districts.

Analyze this game using subgame perfection. What distribution does each legislator propose when it is their turn? Assume each legislator prefers to maximize their district's share over ending the game early.

Subgame Perfection

In the previous session, we examined how various exercises can be structured or presented as strategic games with perfect information. In the extensive form, each node represented an individual information point, technically known as a *singleton*, and the solution to these games are by **backward induction**.

Today, we will move forward to games in which payoffs are already defined, allowing us to observe various outcomes. The technique to solve them, **subgame perfection** (SPNE), is a refinement of Nash models. Backward induction is a subset of subgame perfection for dynamic games of complete information; once we open the possibilities to **imperfect information** we moves to the broader concept of SPNE.

In a SPNE, strategies form a Nash equilibrium in every subgame, essentially requiring strategies to be optimal not only for the game as a whole but also for every possible point at which decisions are made within the game. The key point here is that each possible point may or may not be a singleton.

Romer and Rosenthal (1979) This paper is an application of sequential game with perfect information

Topic: Budget Approval Agenda power under certainty and the importance of the reversion point

Context: Direct democracy

Reversion point: What happens when the budget is not approved

The game:
Two players: A bureaucrat (B) and an assembly of N citizens who decide by simple majority
If it is an assembly, why two players?
Two types of goods: A collectively decided good (G) and a private consumption good (C)
$\forall i \in N : U_i(C_i, G)$ is quasi-concave.
The size of G is proportional to the expenditure E made on it.
$G_i = f(E), f' > 0, f'' \leq 0$
Each individual has an income Y_i
$E = \tau Y$
For each citizen, the problem to maximize is:
$\max U_i(C_i, G_i)$ subject to $C_i \leq Y_i^0 - \tau Y_i$
Which is equal to: $\max U_i \left(Y_i^0 - \frac{E}{Y} Y_i, f(E)\right)$

If the voter were free to choose the level of expenditure, he would choose his *ideal expenditure* $\overline{E}^i$. (See Figure I.) An essential feature of the political allocation mechanism, of course, is that the individual is not free to choose the value of E. Rather, the decision must be made in a referendum in which the voter faces a choice between an expenditure level proposed by the setter and some predetermined expenditure that we shall call the *reversion point.* This reversion point may correspond to the current level of expenditure, or it may be defined by a legally prescribed "reversion rule" that specifies the level of expenditure that occurs if the *alternative* proposed by the setter is voted down.

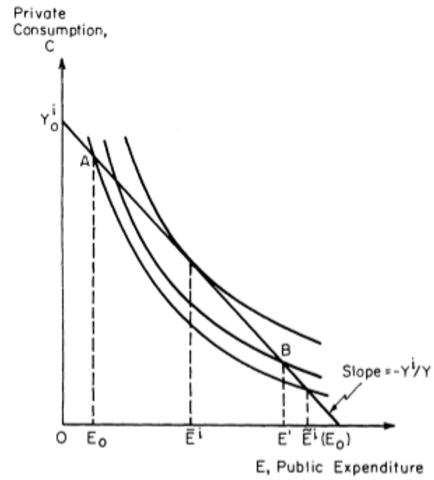


FIGURE I

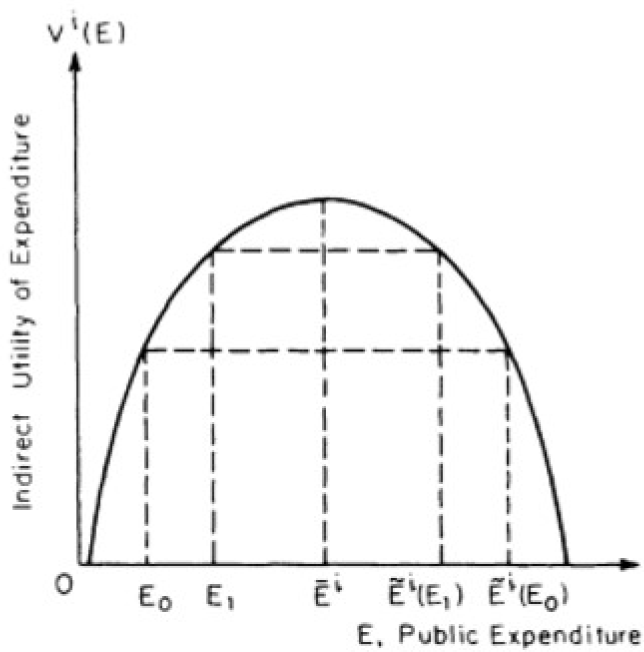
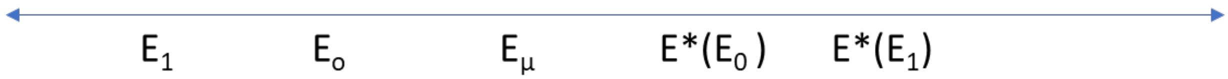


FIGURE II
$\tilde{E}^i(E_0)$ and $\tilde{E}^i(E_1)$ are the largest expenditures receiving a Yea vote against reversion expenditures E_0 and E_1, respectively.

- For the bureaucrat, the goal is to maximize E such that it is approved. That is:
- Let $b_i(E) = \{0$ if votes 'Against', 1 if votes 'In favor' $\}$
- For each reversion point E_0 of an individual i there exists a maximum amount $E_i^{max}(E_0)$ for which $b_i(E_i^{max}) = 1$
- Therefore, the equilibrium depends on the rule and what the status quo is.



Implication:

In government programs, spending often exceeds what the median voter would desire.

A model of benefit distribution among legislators
Baron and Ferejohn (1989)

Advantages are not always obtained from special rules that grant power (e.g., agenda setting).

Open rule, N individuals: Under Arrow there is no equilibrium

Adding sequentiality (institution) and B&F find equilibrium.

The game:

N legislators

Decide how to distribute a good ($1)

Lottery: Everyone has a probability of 1/N of being the one who proposes a distribution

A proposition is a distribution vector of the good $(x_1, x_2, \ldots, x_n)$

Decision:
If you are the proposer $\rightarrow$ which vector
If you are not $\rightarrow$ in favor or against

Simple majority:
If there is a majority, it is approved, end.
If there is no majority, new lottery.
Discount rate

Formalization:

Let M^* be the group of people who receive something from the proposer

Probability of being in M^* : $\frac{(N-1)}{2N}$

If in period t the proposer offers j to be in M^* by giving s^j, will j accept?

Scenarios:
Not accepting and being selected in the next round
Not accepting and the proposer in $t+1$ selects them
Not accepting and being out of everything in the next round

Scenario 1:
Not accepting and being selected in the next round

$$\left(\frac{1}{N}(1 - \sum s_j)\right)$$

$$\forall j \in M^*$$

Members of M^* would accept if and only if:

$$\delta s_j^{t+1} \leq s_j^t$$

Hence, j would get: $s_j = \frac{1}{N}\left(1 - \frac{(N-1)}{2}\right)\delta s_j^{t+1}$

Scenario 2:
Not accepting and the proposer in $t+1$ selects them to distribute:
$s_j = \frac{(N-1)}{2N}\delta s_j^{t+1}$

Scenario 3:
Not accepting, not being selected, not being part of the winning coalition:
Obtains 0.

Therefore, the expected value $E(s_j) = \frac{1}{N}(1 - \frac{(N-1)}{2})\delta s_j^{t+1} + \frac{(N-1)}{2N}\delta s_j^{t+1}$
$E(s_j) = \frac{1}{N}\delta s_j^{t+1}$

Individuals in M^* will accept if $s_j^t > \frac{\delta^{t+1}}{N} * s_j^{t+1}$

Observation:

Now, an observation: Typically, voting does not end with a simple majority in favor, but rather 56%, 65%, 72%, etc. What could explain this phenomenon?