Handout - Information

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The Role of Information

The Problem to Solve: Impact of Information Asymmetries

Motivating examples:

- *Politician and Tax Law*: A politician may know that a new tax law will negatively impact businesses but may choose not to share this information with voters to avoid damaging his popularity.

- *Political Party and Polls*: A political party might have access to private opinion polls showing that a certain idea lacks popular support, so they decide not to support it to enhance their standing against a political ally who is pushing the idea but lacks this information.

- *Government and Military Intelligence*: A government may have military intelligence revealing the plans of a potential enemy attack, but the enemy may have anticipated this and created false information to deceive the spies.

- *Candidate and Promises*: A candidate may promise policies that are popular among voters but has no intention of implementing them.

In all these examples, there is incomplete information in the game. The standard practice (Harsanyi) is to convert the game of incomplete information into one of complete but imperfect information, usually by considering a fictitious player (called "Nature") who acts by giving information to some players, while the rest only know a probability distribution of the possible actions.

Classic Examples of Political Authorities and Collegial Bodies

A. Principal-Agent Problems

Temptation for Opportunistic Behavior: An elected representative might be tempted to behave opportunistically, using their position for personal gain rather than acting in the best interest of their constituents.
Incentivizing Private Information Disclosure: A president might want to incentivize their ministers to disclose crucial information for making effective political decisions.

B. Credible Threats with Uncertainty
Preventive Actions: A country might take preventive measures based on the threat from another country, even if the threat is not certain.
Cheap Talk Games: During an election campaign, candidates might make promises they have no intention of keeping to win votes.

A useful classification of situations with asymmetric information

- Asymmetry based on previous characteristics –> *Adverse Selection*
- Asymmetry based on current or future behaviors –> *Moral Hazard*

Formal Definition

A game with asymmetric information is a strategic interaction where at least one player has private information that other players do not know. This private information typically pertains to the player's type, which influences their preferences, strategies, and payoffs. To formally define such a game, we use the framework of Bayesian games.

So far, we talk about the part of a game listing the following: players, actions, rules of the game, and strategies and their payoffs. Now, we need to add something else to take care of the information asymmetry.

Components of a Bayesian Game

1. Players: A finite number of players
2. Types: Each player has a finite set of possible types. The type of the player represents his private information. If a given player has only one type means that he doesn't have private information because all other players know his type.
3. Probability Distribution: The probability of each type (this is common knowledge).
4. Actions: A given set of possible actions
5. Payoff Functions: Payoffs vary according with the actions and the types.
6. Strategies: The strategies represent the way that each player convert their type into action

In a game with asymmetric information, each player knows their own type but not the types of others. The common prior distribution of probabilities represents the players' beliefs about the likelihood of different type profiles. Players choose strategies that maximize their expected utility, considering both their own type and their beliefs about the types of others. The Bayesian Nash Equilibrium ensures that no player can improve their expected utility by unilaterally deviating from their strategy, given their beliefs and the strategies of others.

Application: Simultaneous Game

Trade War between the US and China
In the current global geopolitical context, the US and China face off as the superpower and the rising challenger. This has led to a series of trade war strategies between the two. For example, during the Trump era, the US imposed tariffs on Chinese goods, and China retaliated with tariffs on US products. These actions have continued beyond Trump's presidency, reflecting ongoing tensions.

To understand this interaction using a formal political economy model, we need to identify the actors, their actions, the payoffs for those actions, the rules of interaction, and the information each actor possesses. Simplifying, we focus on two types of actors: those inclined to impose unilateral, type u, trade barriers and those inclined to reciprocity, or type b (i.e. they sanction if sanctioned and not otherwise).

The actors are $N = \{E, C\}$.
Their possible preferences are $\theta_E = \{Unilateral, Bilateral\}$ and $\theta_C = \{Unilateral, Bilateral\}$.
We consider the existence of a third actor ("**Nature**") who randomly chooses a type of actor for each country. Both know their type, but only the probability of the other country's type.

Each country is free to choose between two actions: a) impose trade limits (l) or b) allow free trade (f). The payoffs are defined as follows:

If the other country is unilateral, u:

$\pi_i(l_i, f_j) = 3$
$\pi_i(f_i, f_j) = 2$
$\pi_i(l_i, l_j) = 1$
$\pi_i(f_i, l_j) = 0$

If the other country is bilateral:

$\pi_i(l_i, f_j) = 2$
$\pi_i(f_i, f_j) = 3$
$\pi_i(l_i, l_j) = 1$
$\pi_i(f_i, l_j) = 0$

A strategy in this game maps the types the actor can have into the actions they can take.

Look that when an actor is unilateral, they have a dominant strategy in l (limit trade). This allows us to discard certain equilibria, leaving only two possible scenarios:

Scenario 1: Both countries limit trade (strategy l).
Scenario 2: Unilateral limits trade and Bilateral does not.

Spoiler: For a country to choose f (free trade), the probability that the other country is unilateral must be less than 1/2. Otherwise, both countries will impose sanctions.

Proof

Let p be the probability that the other player chooses l.

If the rival j follows the strategy in scenario 2, then country's i payoffs are:

- If nature told him that its type is u:

$$EU_E(s_E = l|s_C, \theta_E = u) = p + (1-p) * 3$$

$$EU_E(s_E = f|s_C, \theta_E = u) = (1-p) * 2$$

Therefore:

$$EU_E(s_E = l|s_C, \theta_E = u) > EU_E(s_i = f|s_C, \theta_E = u)$$

$$p + (1-p) * 3 > (1-p) * 2$$

$$1 > 0$$

- If nature told him that its type is b:

$$EU_E(s_i = l|s_C, \theta_E = b) = p + (1-p) * 2$$

$$EU_E(s_i = f|s_C, \theta_E = b) = (1-p) * 3$$

Hence:

$$EU_E(s_E = l|s_C, \theta_E = b) > EU_E(s_i = f|s_C, \theta_E = b) \qquad sii \qquad p > 1/2$$

$$p + (1-p) * 2 > (1-p) * 3$$
$$p > (1-p)$$
$$p > 1/2$$

As the problem is symmetrical, the same applies to China with probabilities expressed in terms of q or the probability that USA imposes limit trades.

Therefore, taking action f is only possible when the probability that the other country is of the unilateral type is less than 1/2. Consequently, if each country expects the other to be of the unilateral type with probability $p > 1/2$, they will impose sanctions regardless of their own type and will only refrain from doing so if they are of the bilateral type and believe that the probability of the other country being unilateral is less than 1/2.

Therefore, the probability of a scenario with free trade is $(1-p)(1-q)$. In this case, give that it is a symmetruc game that happens with probability $(1-p)^2$

Application: Sequential Game

Let consider now a game with an additional complexity: beliefs. Once you have sequential games, the first player is sending information or signals (with or without intention) to the other players. How those signals are interpreted by the later is a matter of beliefs. Therefore, a concept of equilibrium now must consider that the beliefs of the second players must be coherent with respect to the behaviors of both players.

Challenger and Incumbent in Elections

$(based\ on\ Gibbons, A\ Primer\ on\ Game\ Theory,\ chapter\ 4)$

Consider a political challenger deciding whether to run for office and an incumbent who wants to be reelected and must decide whether to take actions to dissuade the challenger. The incumbent acts first, influencing the challenger's decision.

There are two types of incumbents (strong and weak) with two possible actions (pre-campaign or not).

The challenger must decide whether to compete or not. The associated payoffs could be:

To find one or more equilibria, it is necessary to incorporate the fact that the challenger must make a conjecture (or interpretation) of the signal received from the incumbent and analyze whether, given that conjecture, it is advantageous for the incumbent to deviate (considering they have private information).

Suppose that **Nature*“** delivers the signal that there is a 0.5 probability that the incumbent is strong.

There are 4 pure strategies for the incumbent:

$(s(\theta_i = F), s(\theta_i = D)) = (G, G)$ <- you read this: Both, a strong (F) & a weak (D) players will play G.
$(s(\theta_i = F), s(\theta_i = D)) = (G, NG)$
$(s(\theta_i = F), s(\theta_i = D)) = (NG, G)$
$(s(\theta_i = F), s(\theta_i = D)) = (NG, NG)$

First, let's consider the alternative where the *incumbent* decides (G, G): In this case, according with the diagram, if they choose to enter, the *challenger* gets:

$$EU_d(compete|G) = q * 1 + (1-q)0$$

and if they choose not to enter, they get:

$$EU_d(no\ compete|G) = q0 + (1-q) * 2$$

.

So, it is only beneficial to enter if $q * 1 > (1-q) * 2$ or when:

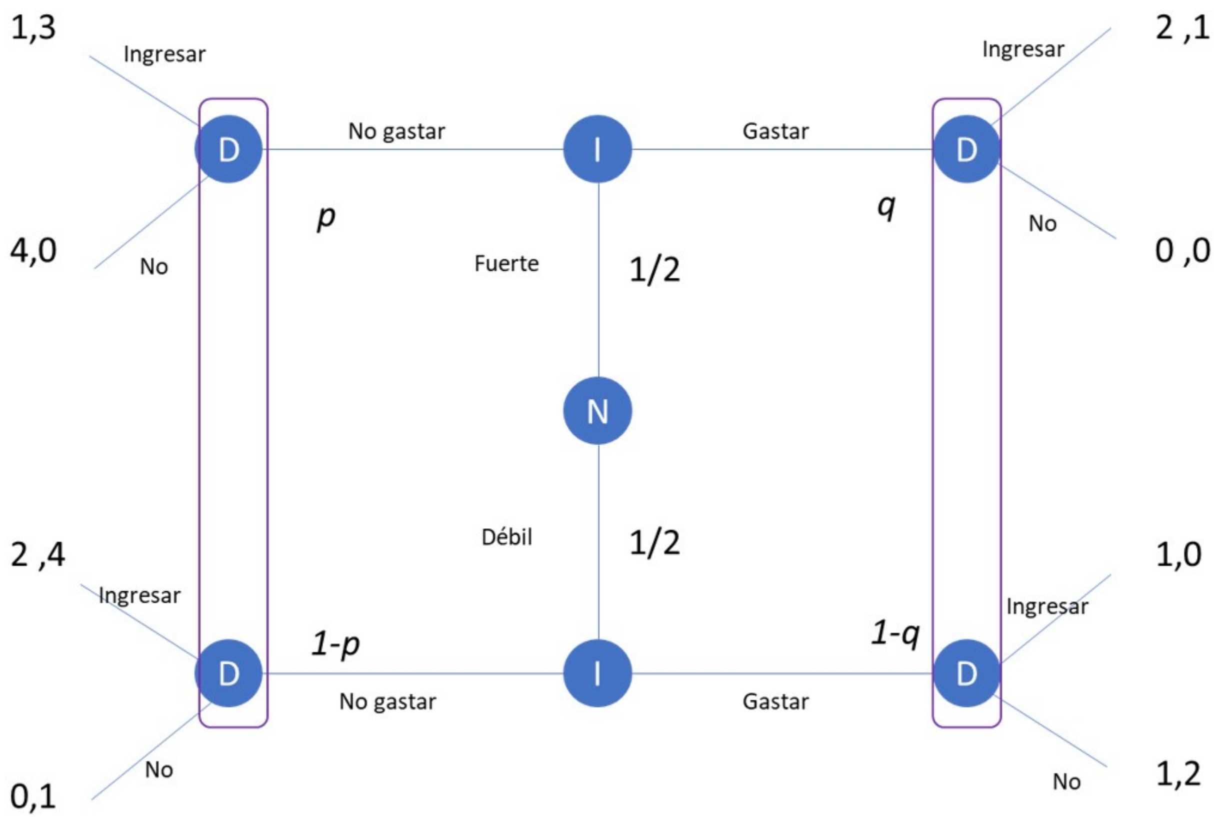

Figure 1: Pre-entry

$$q > 2/3$$

Can we say that the equilibrium in this case would be that the incumbent always spends and the challenger only enters when $q > 2/3$?

Not yet.

We only know the best response of the *challenger* against an incumbent who always chooses G regardless of their type. It is necessary to know whether, given this response, a strong or weak incumbent would prefer to take another action unilaterally and if, in response to that other action (signal), the *challenger* has incentives to act differently.

Let's first consider the decision of the *incumbent*: Given $s_d\,(compete|G, compete|G, q > 2/3)$, is it better for the *incumbent* to choose NG instead of G?

We need to examine case by case.

For a *strong incumbent*, changing from G to NG would mean going from gaining 2 to gaining 1 (not beneficial), but for a *weak incumbent*, it would mean going from gaining 1 to gaining 2 (beneficial). But if they send the signal NG, what does the *challenger* do? If they persist in entering, their utility of entering $EU_D(compete|NG)$ is always greater than not entering, $EU_D(not\ compete|NG)$. Therefore, we can rule out that the *incumbent* always chooses G regardless of their type when facing a *challenger* who always enters.

Case A:

let's consider the alternative where the *incumbent* decides (NG, NG):
In this case, as we saw in 1, the *challenger* always benefits from entering. We should then ask whether the *incumbent* would maintain his intention to play NG. For the *strong incumbent*, changing from NG to G would mean going from gaining 1 to gaining 2. Therefore, if he sends the signal G in response to the *challenger*'s decision to enter, his utility improves. But, would the *challenger* maintain the decision to enter seeing a G signal? As we saw in (1), the *challenger* would only do so when $q > 2/3$ and would change to *not compete* if $q \leq 2/3$. Hence, there is an equilibrium where the *incumbent* always chooses NG regardless of whether they are *strong* or *weak*, and the *challenger* chooses to enter when $p = 0.5$ and $q \leq 2/3$. This can be summarized as follows:

$$(s_I(\theta_I = Fuerte), (s_I(\theta_I = Débil), (s_D(señal = NG), (s_D(señal = G), p, q)) =$$

$$((NG, NG), (compete, not\ compete), p = 0.5, q =\leq 2/3)$$

This type of equilibrium is called *pooling equilibria* because all types (in this case: 2) choose the same type of action in the path of equilibrium (note that they also act off the equilibrium path!).

Case B:

Let's consider the case where the *incumbent* decides (NG, G):
In this case, the only consistent probabilities with that behavior are that the *challenger* conjectures that $p = 1$ and $q = 0$. If they believe that $p = 1$, then they choose to enter because $3 > 0$ and we have already seen that it benefits the *incumbent* to switch to G because $2 > 1$. Since in the conjecture $q = 0$, when faced with G, the *challenger* chooses not to enter. Therefore, when facing (NG, G), the *challenger* chooses $(compete, not\ compete)$. It would be beneficial for the *weak incumbent* to switch to G. Therefore, there is no equilibrium.

Finally, when the *incumbent* plays (G, NG): It can be shown that there is an equilibrium in this case with:

$$((G, NG), (compete, compete), p = 0, q = 1)$$

A strong incumbent pre-campaigns (G): The challenger competes if they believe the incumbent is weak.
A weak incumbent does not pre-campaign (NG): The challenger might compete if they believe the incumbent is weak. Evaluating the possible strategies and conjectures, we find:

If the incumbent always chooses to pre-campaign (G), the challenger only competes if they believe the incumbent is weak with a probability greater than 1/3.
If the incumbent never pre-campaigns (NG), the challenger always competes.
If the incumbent chooses NG if strong and G if weak, the challenger can adjust their strategy based on the received signal.
A possible equilibrium is that a strong incumbent pre-campaigns and a weak incumbent does not, while the challenger decides to compete or not based on their conjecture of the incumbent's strength.

This type of analysis shows how signals and asymmetric information can influence strategic decisions in political contexts.

Finally, an application of signaling on a spatial model

[Figures in the blackboard]

Consider a case of budget fixation

A player M has a preference regarding the magnitude of real expenditure during the next period. His ideal is to keep the actual expenditure as closer as possible to the planned budget (i.e. his ideal is 0 difference with respect to the planned budget). But M is aware that there are "imponderables" (identified as w) that can increase or decrease the effective expenditure. He also knows that w can be in the range [-1,1].

1) An expert, A, has a better signal about the value of w. He knows that w is either positive or negative. And he can tells M what he thinks.

Let assume that A says that w will be negative. if w is a uniform random variable, what should M does with that information?

BUT, should M believe on A? When? When not?

2) Let consider now the case in which A is not only an expert, but also is a policymaker in charge of applying the policies covered by the budget. In this case, we move from an expert's trustworthyness problem to a delegation problem. Here the question would be how much discretion give to A. There is a trade off between giving less restriction and moral hazard temptations.