

Supporting Information

Conditions conducive to optimal probabilistic categorization

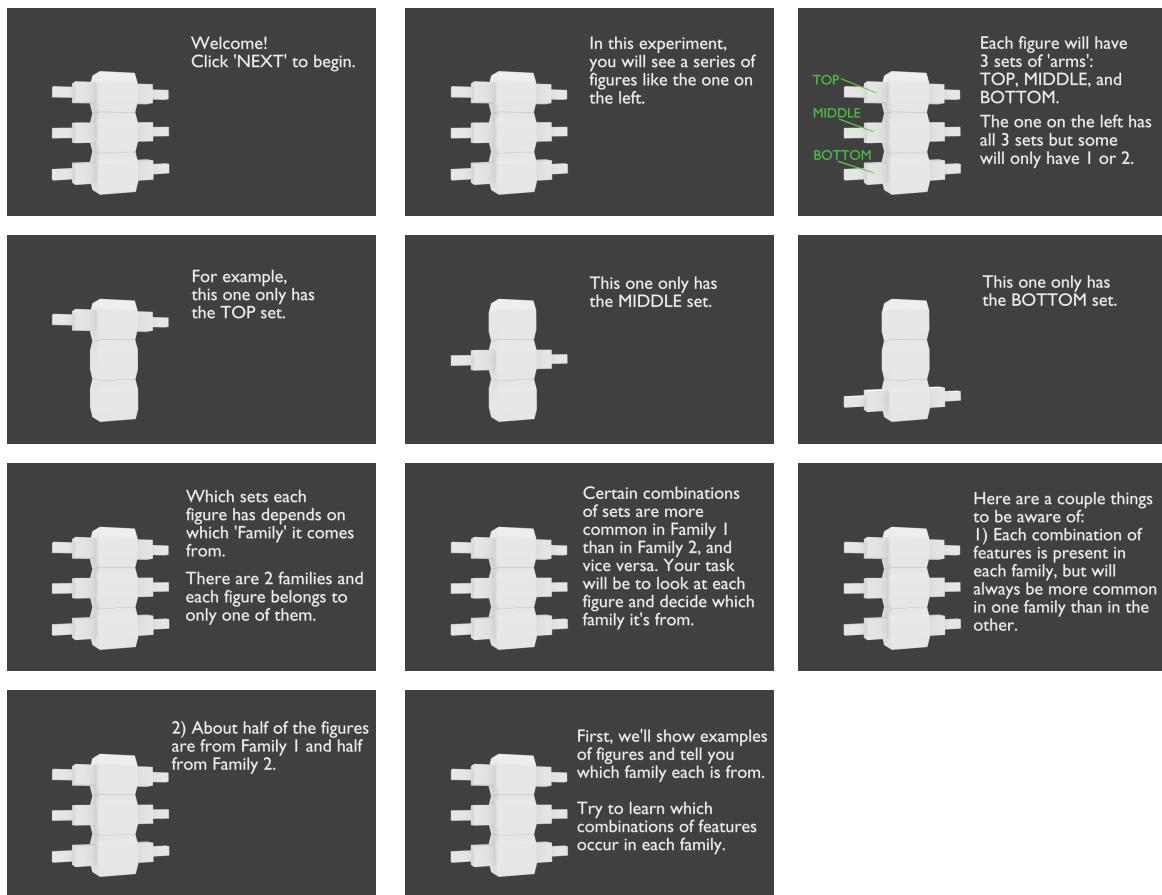
John F. Ackermann, Michelle A. Borkin, Peter J. Bex

1 Experimental instructions and trial sequence

Below are descriptions of each phase of the experiment and example stimulus screens.

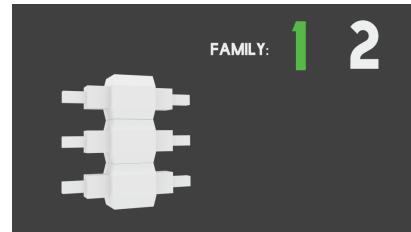
Training Instructions

Subjects viewed each instruction screen (in sequence from left to right starting at the top left) and advanced to the next screen by clicking a button labeled 'NEXT' that appeared at the lower right corner of the image.



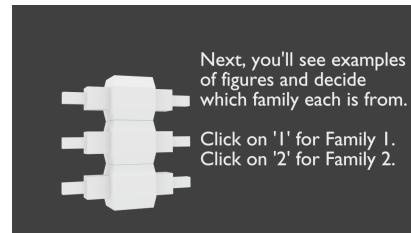
Training Block

200 trials. Subjects had unlimited time to view each stimulus and its true 'Family' label in green. They advanced to the next trial by clicking a 'NEXT' button at the lower right corner of the image.



Testing Instructions

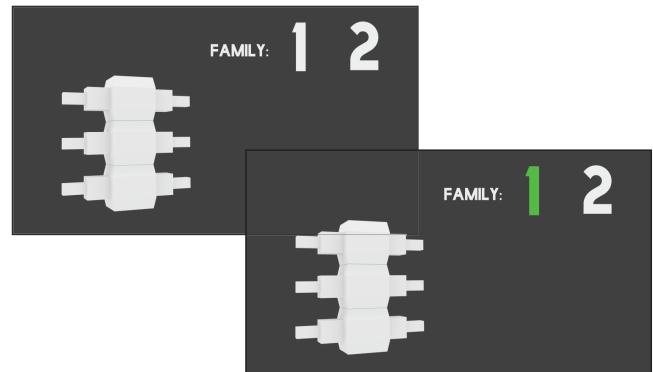
Subjects viewed a single screen and advanced to the Testing Block by clicking 'NEXT'.



Testing Block

100 trials. Subjects had unlimited time to view each stimulus. They indicated their 'Family' choice by clicking '1' or '2'.

The number turned green for 100ms before automatically advancing to the next trial.



End Survey

Subjects entered their responses and clicked 'Submit Answers' to end the experiment.

You've reached the end of the experiment!

Could you briefly describe any strategies you employed and/or any patterns you noticed that helped you categorize the images?

What is your age?

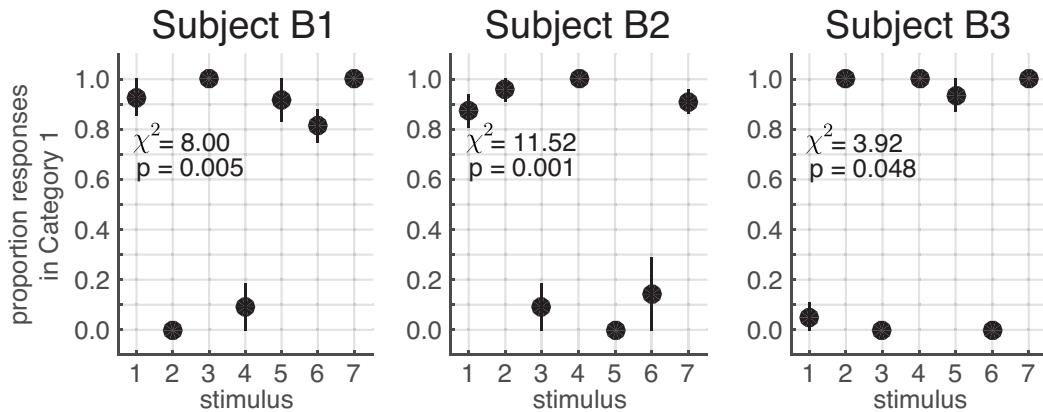
What is your gender?

Male Female

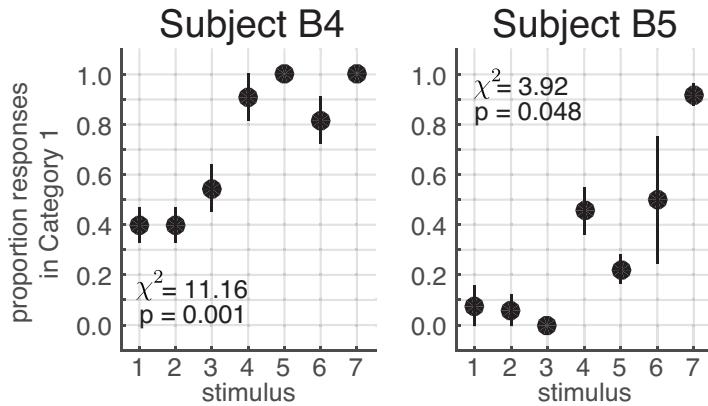
2 Response bias analyses

Our model predictions do not account for intrinsic response biases on the part of the subject. They assume that subjects' category choices in the Testing Block are based on an analyses of the stimuli in the Training Block, i.e. their probabilities, frequency, and similarity. If subjects adopted a categorization strategy which disregards these factors and instead chose categories based on a bias for selecting one category over another, this presents a potential confound to our conclusions. Thus we identified subjects with significant response biases and excluded them from subsequent analyses. Significance was determined by means of a χ^2 test comparing the proportions of trials on which each subject chose Family 1 or Family 2 to the expected proportion of .5 (i.e., the category prior probability made known explicitly to the subjects in the instructions). 33 out of 240 subjects had a significant bias. Each subject's proportion of responses in Category 1 for each stimulus and their χ^2 statistics are shown below. In each plot, the Training Block condition is indicated. "p" refers to the Probability Condition, i.e. the posterior probability of Stimulus 7 occurring in Category 1. "f" refers to the Frequency Condition, i.e. the proportion of trials in which Stimulus 7 is observed in the Training Block. Each subject is indicated by number. "B" indicates that they were one of the 33 subjects excluded for a significant bias. Error bars show the central 50th percentile of a binomial distribution over the response proportion.

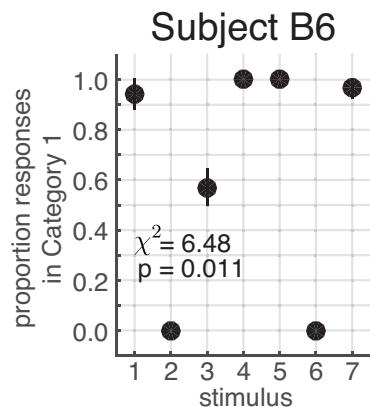
Condition 1 (p=.90; f=.05)



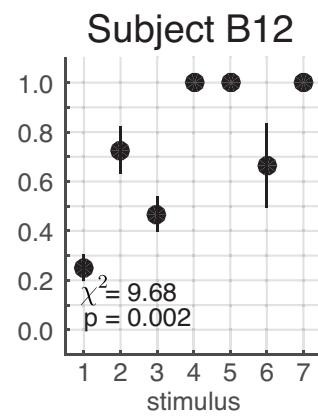
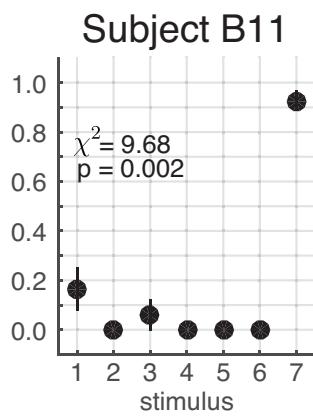
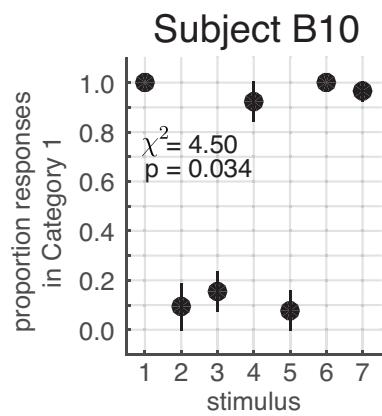
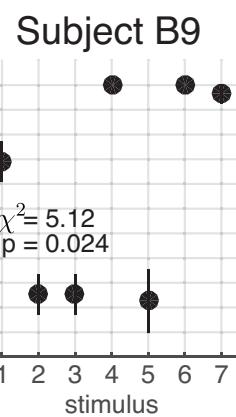
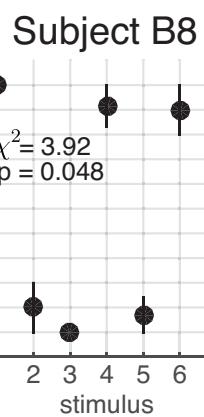
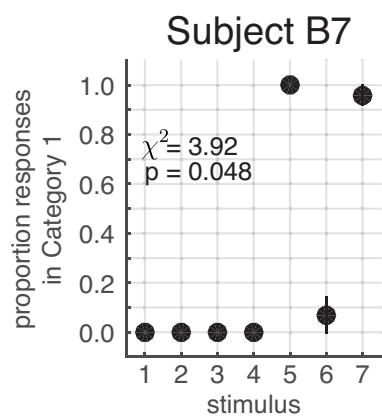
Condition 2 (p=.90; f=.10)



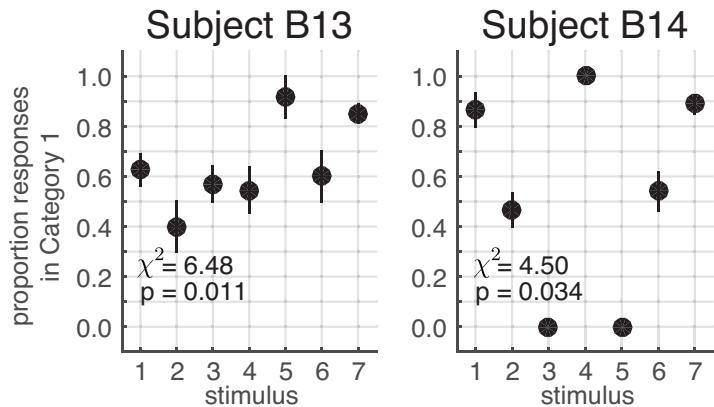
Condition 3 ($p=.90$; $f=.20$)



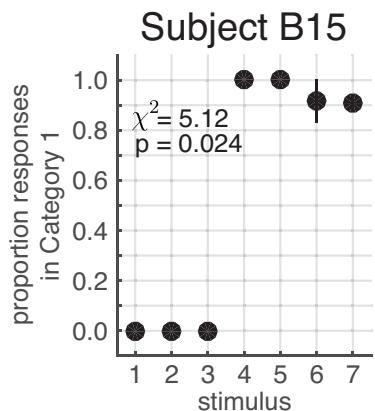
Condition 4 ($p=.90$; $f=.40$)



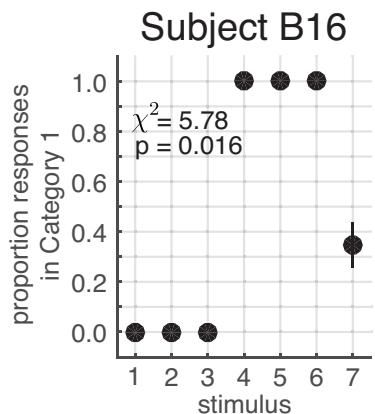
Condition 5 ($p=.75$; $f=.05$)



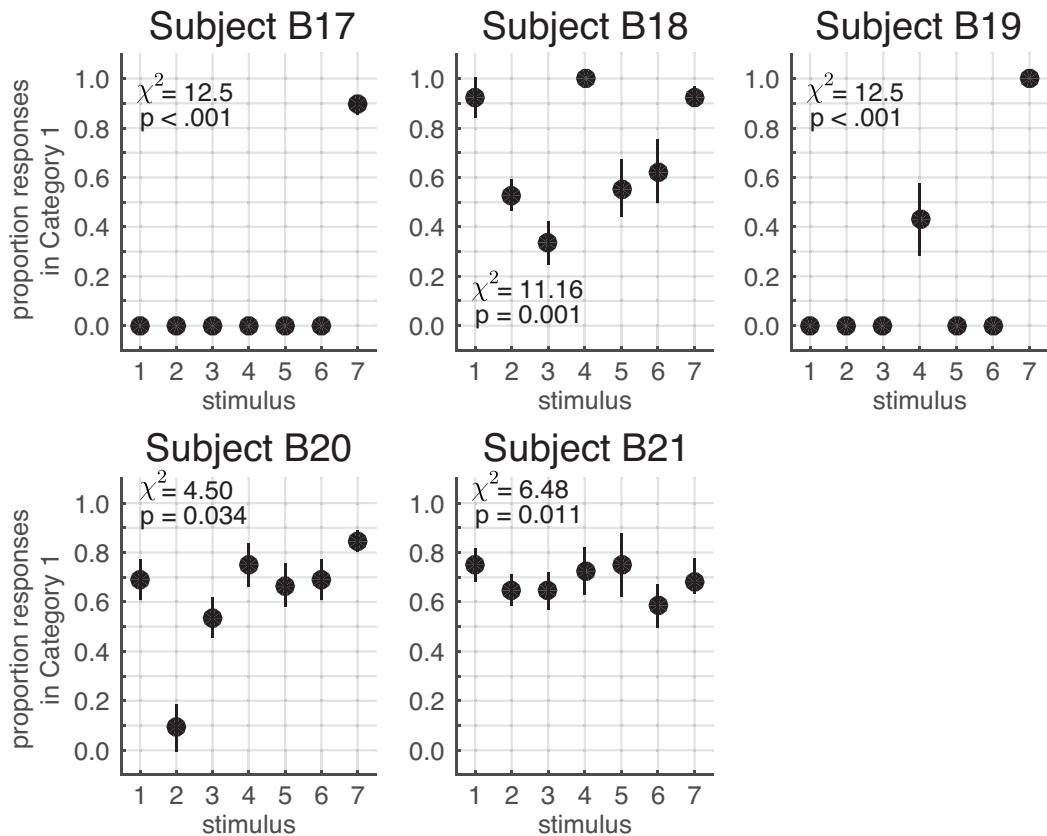
Condition 6 ($p=.75$; $f=.10$)



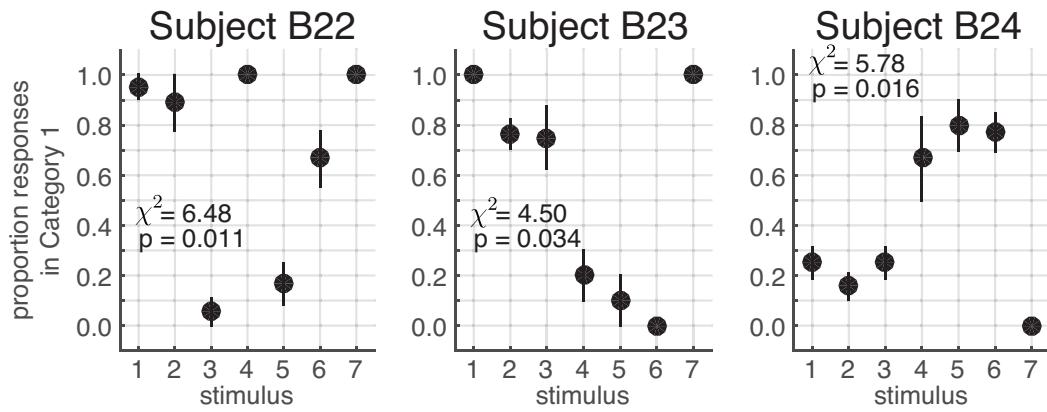
Condition 7 ($p=.75$; $f=.20$)



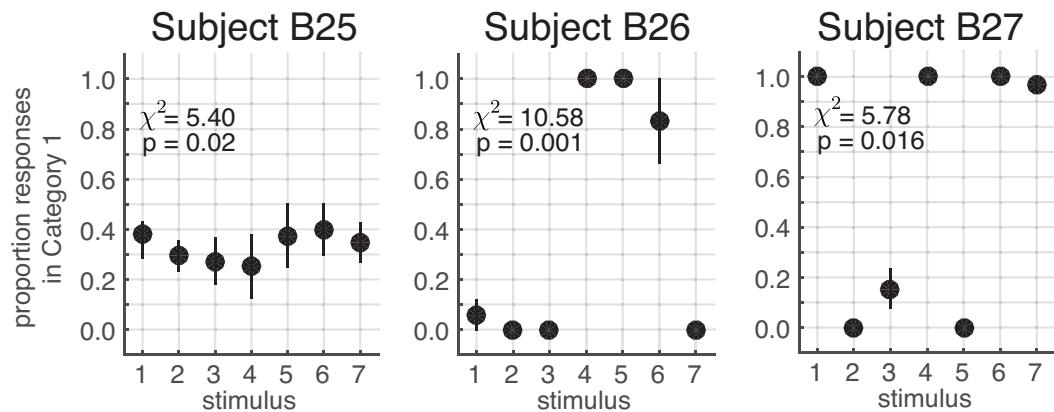
Condition 8 ($p=.75$; $f=.40$)



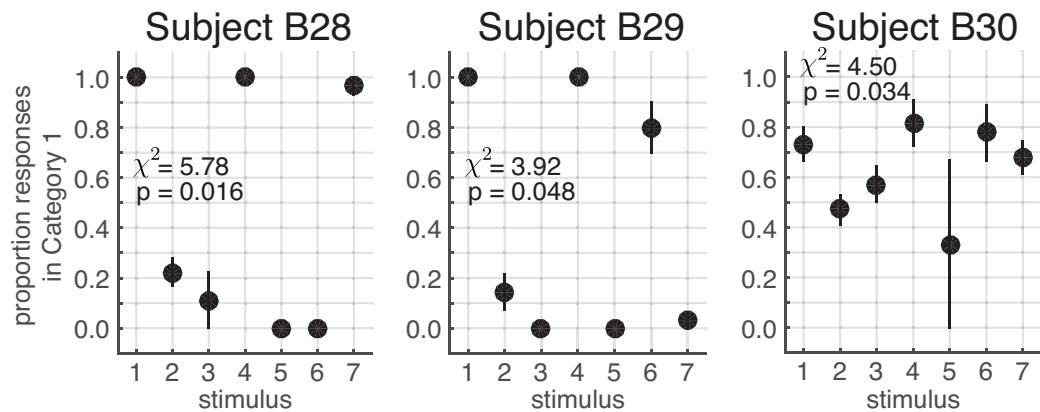
Condition 9 ($p=.60$; $f=.05$)



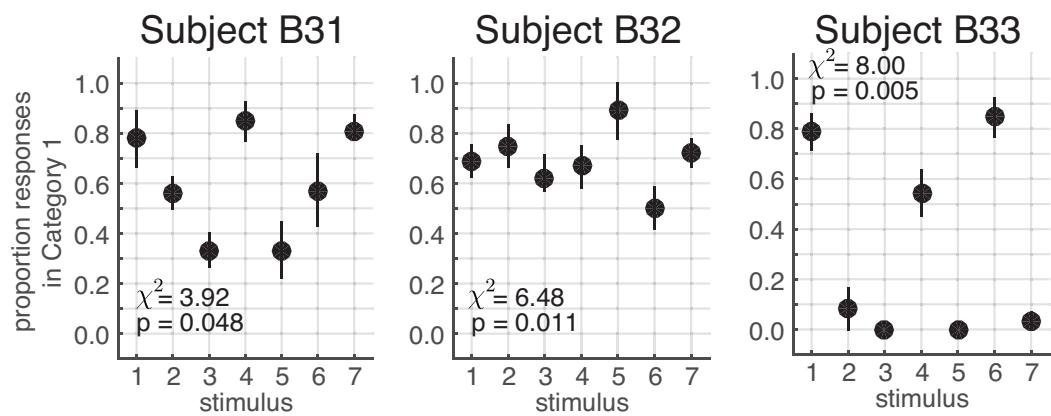
Condition 10 ($p=.60$; $f=.10$)



Condition 11 ($p=.60$; $f=.20$)



Condition 12 ($p=.60$; $f=.40$)



While category base rates were always approximately .5 in the Testing Block, altering the frequency of the stimuli and the probability of Stimulus 7 across the 12 Training Block conditions altered the category base rates in the Training Block (Fig. S1 A). We investigated whether this may have led to the significant response biases in the Testing Block. Out of the 33 subjects that had a significant response bias, 21 had a bias for choosing Category 1. The proportion of subjects in each of the 12 Training Block conditions who had a significant bias for choosing Category 1 as a function of the Category 1 base in that condition (as shown in the figures above) is shown in Fig. S1 B. The greatest proportion of subjects (4 out of 20) with a significant bias for choosing Category 1 occurred in Condition 4 ($p = .90$; $f=.40$). The regression of proportion of subjects with a significant bias onto Category 1 base rate is shown in S1 B. The slope is not significantly different from 0. Thus, while there is a trend to suggest the high base rate in that condition led to greater response, the effect is not significant. Next we examined the proportion of responses in Category 1 as a function of Category 1 base rate in the Training Block for each of the 33 subjects with significant biases. We regressed the proportion of responses in Category 1 onto the Category 1 base rates and found that the slope is not significantly different from 0 (Fig. S1 C). The bias for choosing Category 1 among these subjects does not seem to be related to an increase in the base rate.

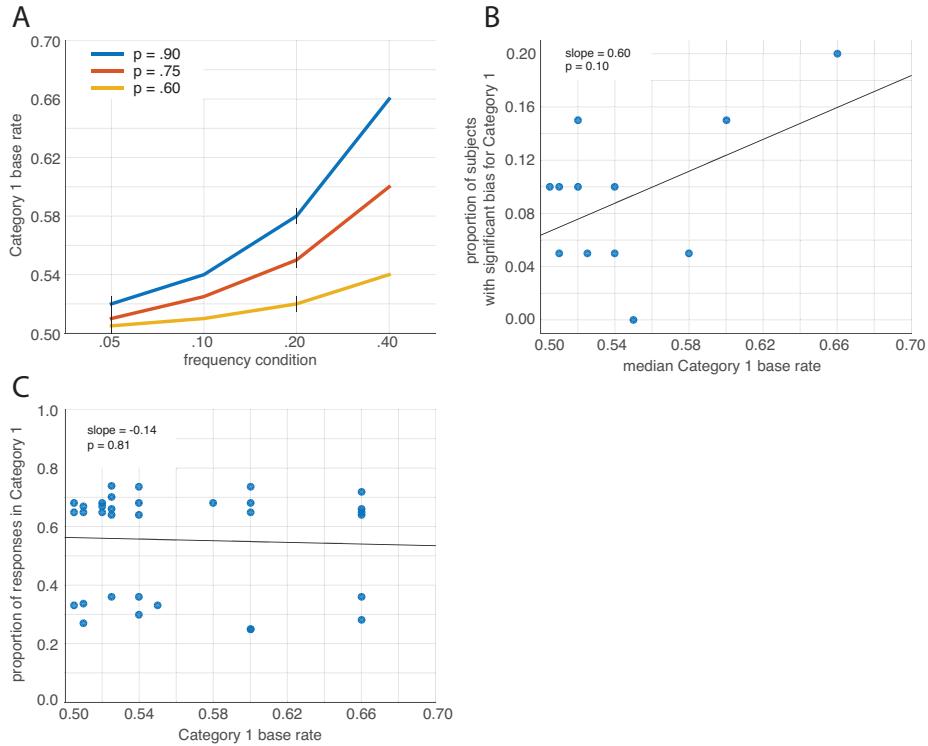


Figure S1: **A** Category 1 base rates for each Training Block condition. The horizontal axis shows the Frequency condition (i.e., the proportion of trials on which Stimulus 7 was observed). Lines show the median base rate, across subjects, for each of the Probability conditions (i.e., the probability of observing Stimulus 7 in Category 1). Error bars show the central 95th percentile. **B** The proportion of subjects in each Training Block condition that had a significant bias for choosing Category 1 as a function of the condition's median base rate (as shown in **A**). The regression line is shown in black with the slope and p value for the test for a significant difference from 0. The effect is not significant, although the trend suggests that the highest base rates (.60 and .66) may be predictive of significant response bias. **C** The proportion of responses in Category 1 for each of the 33 biased subjects as a function of Category 1 base rate in the Training Block with the regression line, slope, and p -value. The insignificant slope indicates that base rate is not predictive of response bias.

3 PFS Model implementation

As described in the text, a genetic algorithm was used to fit each subject's membership function, $m(\mathbf{s}_k, d)$ (see text Eq. 10), where $k = \{1, \dots, 7\}$ indexes each of the seven stimuli, and d indexes the d^{th} set in the domain of the subject's subjective similarity space, \mathcal{S} . For a given stimulus, m can take on a range of 7 possible vector values:

- 1) 1 0 0
- 2) 0 1 0
- 3) 0 0 1
- 4) $\frac{1}{2} \frac{1}{2} 0$
- 5) $\frac{1}{2} 0 \frac{1}{2}$
- 6) $0 \frac{1}{2} \frac{1}{2}$
- 7) $\frac{1}{3} \frac{1}{3} \frac{1}{3}$

Each 'individual' generated by the genetic algorithm consists of a 1×8 vector, \mathbf{v} , of the indices of these seven vector values, one index for each of the seven stimuli, and the probability weighting parameter γ . For example, an individual $\mathbf{v} = [1 1 1 2 2 2 3 .8]$ corresponds to membership function:

$$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and indicates that stimuli 1 - 3 are members of set \mathcal{S}_1 , stimuli 4 - 6 are members of set \mathcal{S}_2 , stimulus 7 is a member of set \mathcal{S}_3 , with a probability weighting parameter of .8. An individual $\mathbf{v} = [4 4 4 6 6 6 6 .8]$ corresponds to membership function:

$$m = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

and indicates that stimuli 1 - 3 are members of set \mathcal{S}_1 and \mathcal{S}_2 , and stimuli 4 - 7 are members of set \mathcal{S}_2 and \mathcal{S}_3 , etc.

The genetic algorithm was implemented in Matlab using function *ga*. The initial population of individuals was generated at random. The population size was 200. The algorithm created 200 generations. On each generation, offspring were generated by selecting 2 individuals that minimized the fitness function described below. The remainder of the successive population consisted of 80% crossover (combination of pairs of 'parent' individuals) and the remainder were mutations (modification of single parents). Probability weighting parameters were constrained such that $0 < \gamma \leq 1$.

The fitness function minimized the negative log likelihood of the subject's responses given the estimated membership function. On every generation, the algorithm translated the index vector \mathbf{v} into the 7×3

estimate, \hat{m} , of the membership function, m . It then derived the rule posterior, $p(R_n|\mathcal{S}, \hat{m})$, as in Eq. 11 of the text. 100 runs of the experiment were simulated by sampling from the rule posterior as described in the text. On each simulated run, the model generated category responses with respect to the stimuli viewed by the subject on each trial of the experiment by drawing a single sample from the rule posterior over decision rules and responding in accordance with the sampled rule. The result of each simulation is the counts, \hat{N}_{k,C_1} and \hat{N}_{k,C_2} , representing the number of trials on which the model selected Category 1 for the k^{th} stimulus and Category 2 for the k^{th} stimulus, respectively. We assume the response counts to follow a binomial distribution across the 100 simulated runs of the experiment and derive the maximum likelihood estimate, $\hat{p}(C_1|k, \hat{m})$, of the probability that the model will select Category 1 for the k^{th} stimulus given the estimated membership function. The fitness function's best fitting estimate of the membership function is derived by minimizing the negative log likelihood of the subject's observed category choices given the model estimates:

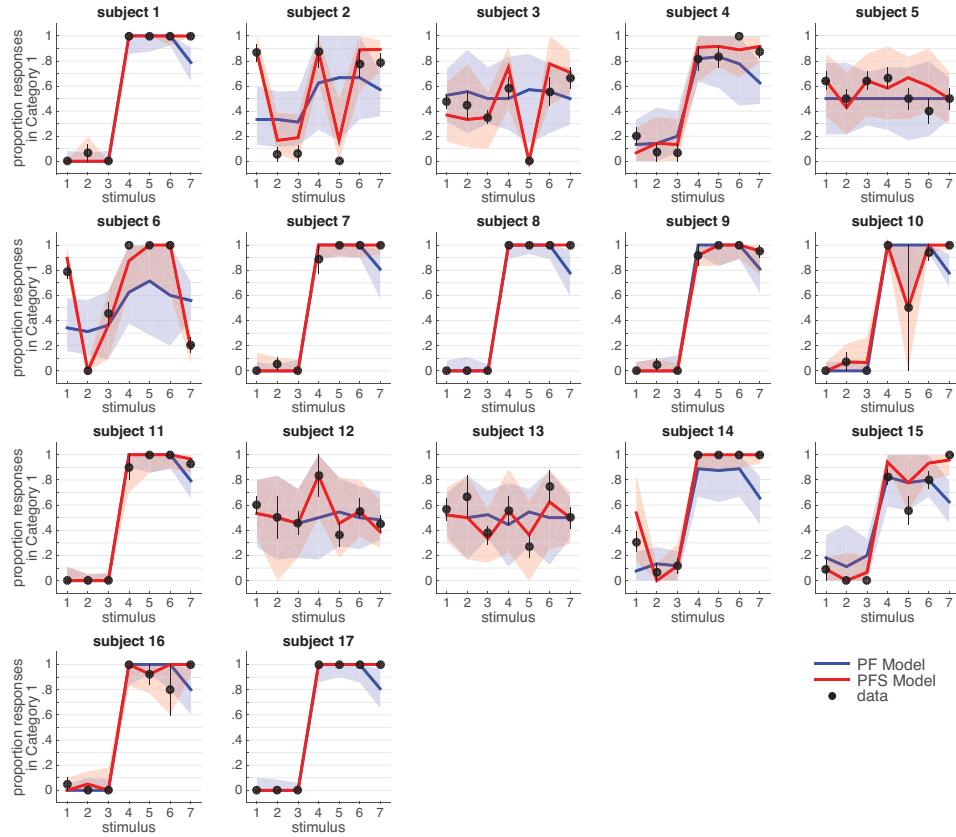
$$\arg \min_{\hat{m}} \left(- \sum_k N_{k,C_1} \ln(\hat{p}(C_1|k, \hat{m})) + N_{k,C_2} \ln(1 - \hat{p}(C_1|k, \hat{m})) \right),$$

where N_{k,C_1} and N_{k,C_2} are the number of trials on which the subject chose Category 1 for the k^{th} stimulus and Category 2 for the k^{th} stimulus, respectively.

4 Subject data and model predictions

The figures below show the proportion of trials on which each subject selected Category 1 for each of the 7 stimuli. Subjects are grouped by their respective Training Block condition. The predictions of the PF and PFS models are shown in blue and red, respectively. Error bars on the subjects' data show the central 50th percentile of a binomial distribution given their observed proportions and the number trials on which each stimulus was observed in the Testing Block. Shaded error regions on the model predictions show the central 95th percentile of the proportions derived from the model simulations. The lower part of the figure for each condition shows each subject's fitted parameters from the PFS model. m is the estimated membership function as described above. γ is the estimated probability weighting function parameter.

Condition 1 ($p=.90$; $f=.05$)



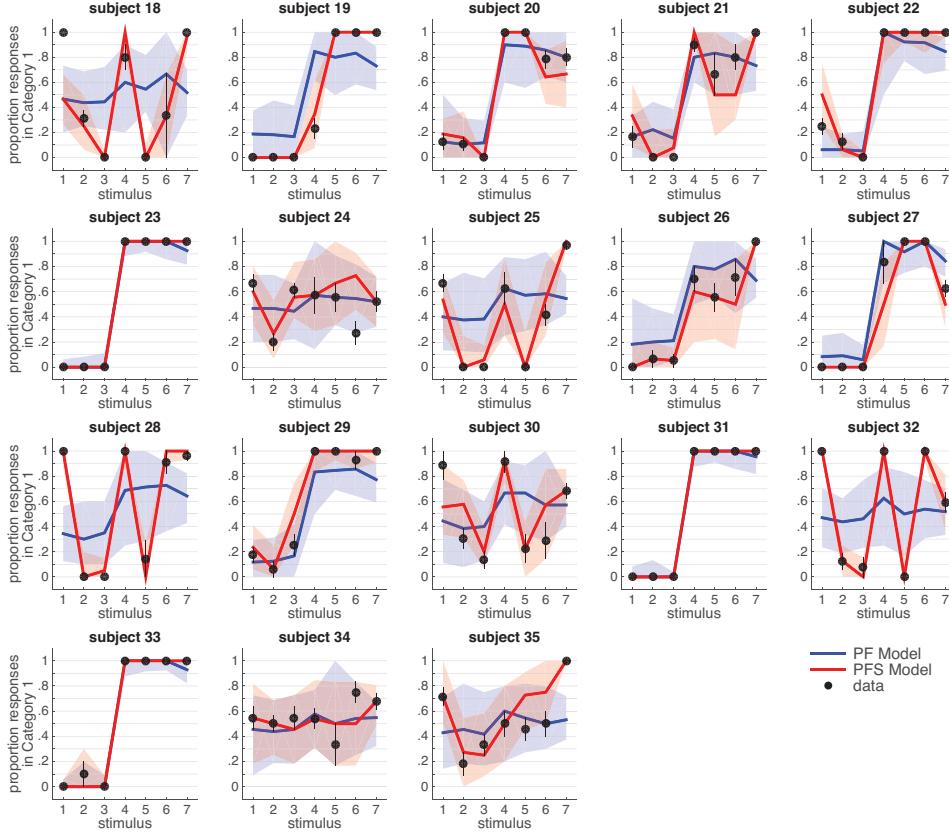
subject 1	subject 2	subject 3	subject 4	subject 5
$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}$
$\gamma = 0.85$	$\gamma = 0.85$	$\gamma = 0.92$	$\gamma = 0.78$	$\gamma = 0.82$

subject 6	subject 7	subject 8	subject 9	subject 10
$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.86$	$\gamma = 0.8$	$\gamma = 0.98$	$\gamma = 0.81$	$\gamma = 0.83$

subject 11	subject 12	subject 13	subject 14	subject 15
$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$
$\gamma = 0.83$	$\gamma = 0.83$	$\gamma = 0.79$	$\gamma = 0.84$	$\gamma = 0.79$

subject 16	subject 17
$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.8$	$\gamma = 0.97$

Condition 2 ($p=.90$; $f=.10$)



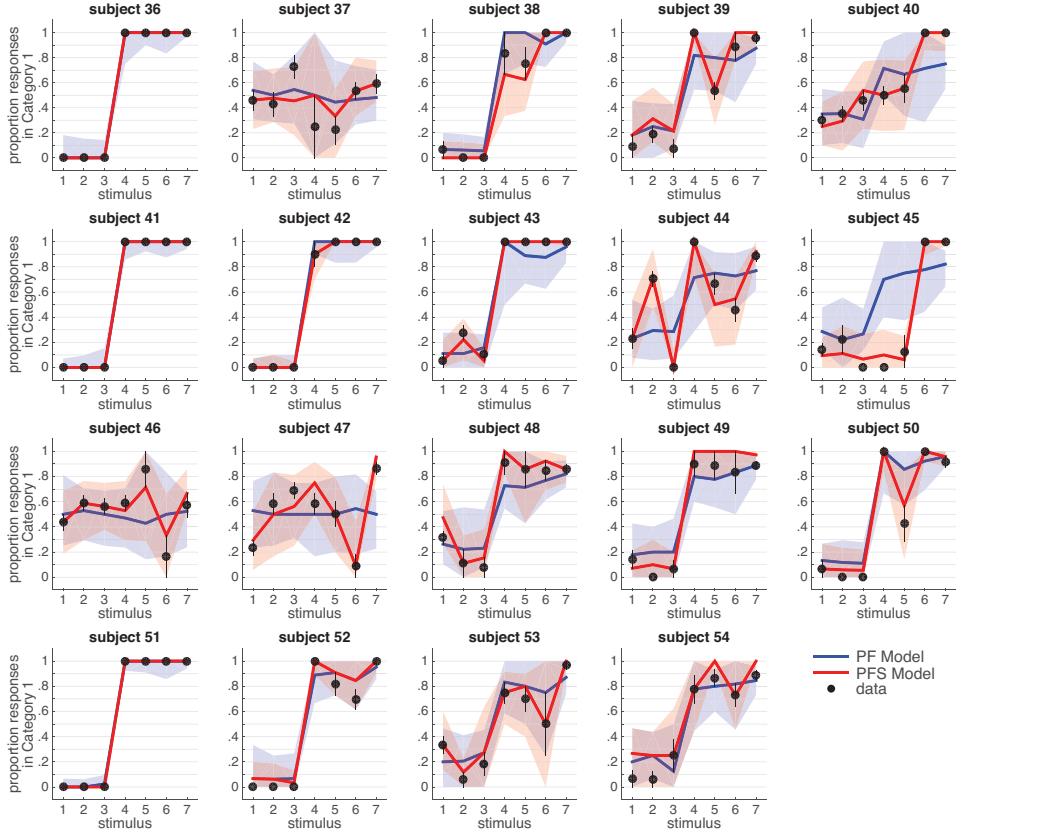
subject 18	subject 19	subject 20	subject 21	subject 22
$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.99$	$\gamma = 0.99$	$\gamma = 0.88$	$\gamma = 0.85$	$\gamma = 0.85$

subject 23	subject 24	subject 25	subject 26	subject 27
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \end{bmatrix}$
$\gamma = 0.97$	$\gamma = 0.78$	$\gamma = 0.9$	$\gamma = 0.9$	$\gamma = 0.97$

subject 28	subject 29	subject 30	subject 31	subject 32
$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0.5 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$
$\gamma = 0.87$	$\gamma = 0.81$	$\gamma = 0.98$	$\gamma = 0.94$	$\gamma = 0.95$

subject 33	subject 34	subject 35
$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 1 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.89$	$\gamma = 0.8$	$\gamma = 0.91$

Condition 3 ($p=.90$; $f=.20$)



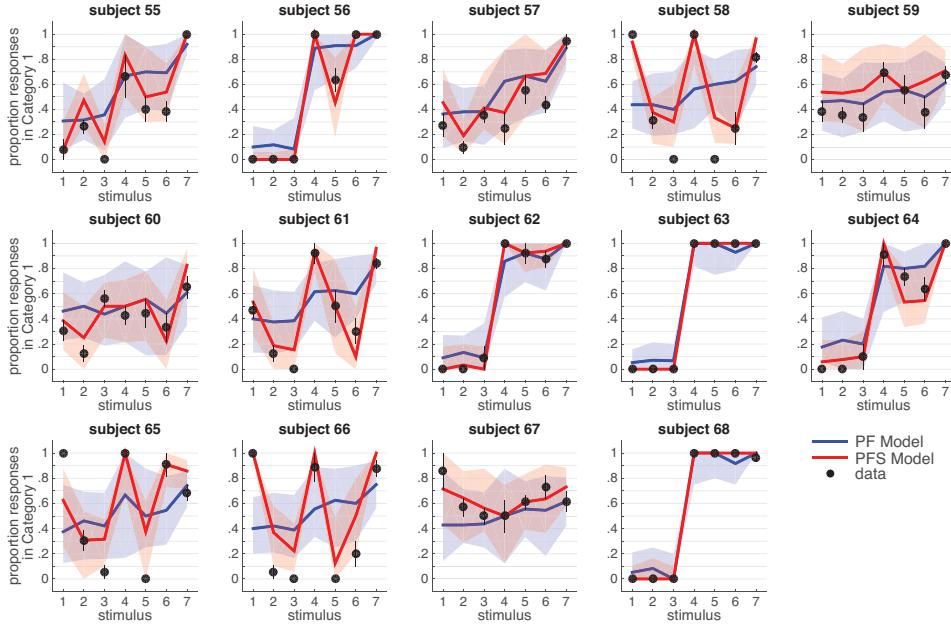
subject 36	subject 37	subject 38	subject 39	subject 40
$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.92$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.96$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.79$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.83$

subject 41	subject 42	subject 43	subject 44	subject 45
$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.92$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.85$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.82$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.85$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.89$

subject 46	subject 47	subject 48	subject 49	subject 50
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.79$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.86$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.78$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.79$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.82$

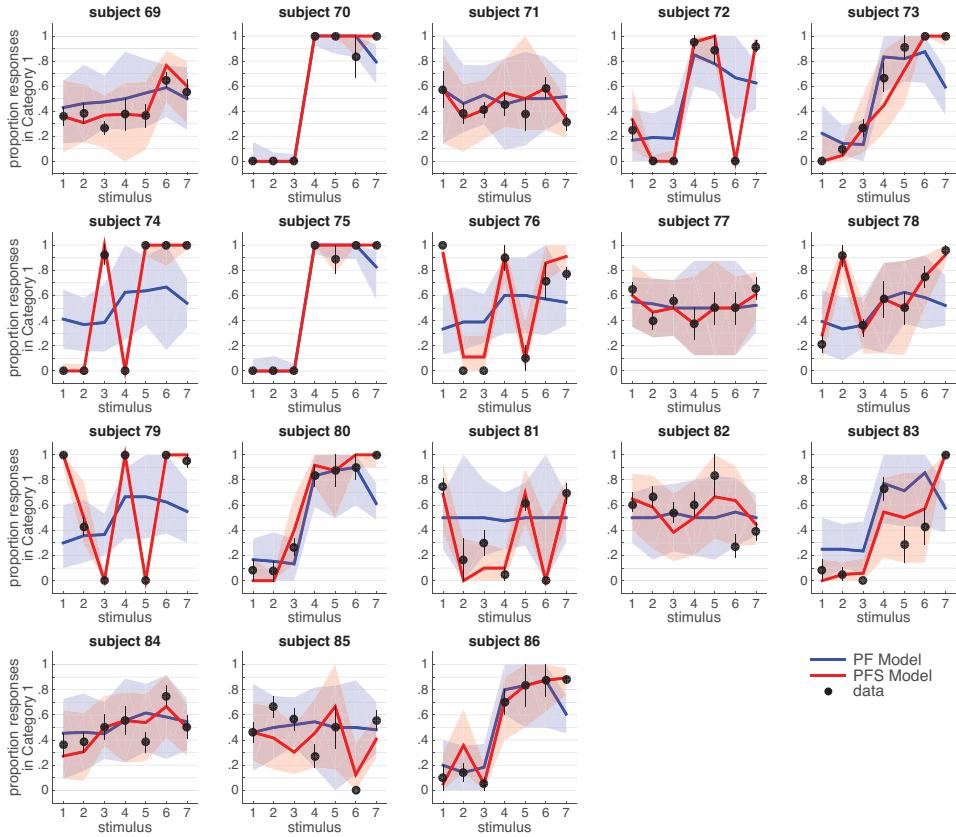
subject 51	subject 52	subject 53	subject 54
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 1$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.81$

Condition 4 ($p=.90$; $f=.40$)



subject 55	subject 56	subject 57	subject 58	subject 59
$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.97$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.99$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.96$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.77$
subject 60	subject 61	subject 62	subject 63	subject 64
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.79$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.81$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.83$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 1$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.96$
subject 65	subject 66	subject 67	subject 68	
$m = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.97$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.85$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.79$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.85$	

Condition 5 ($p=.75$; $f=.05$)



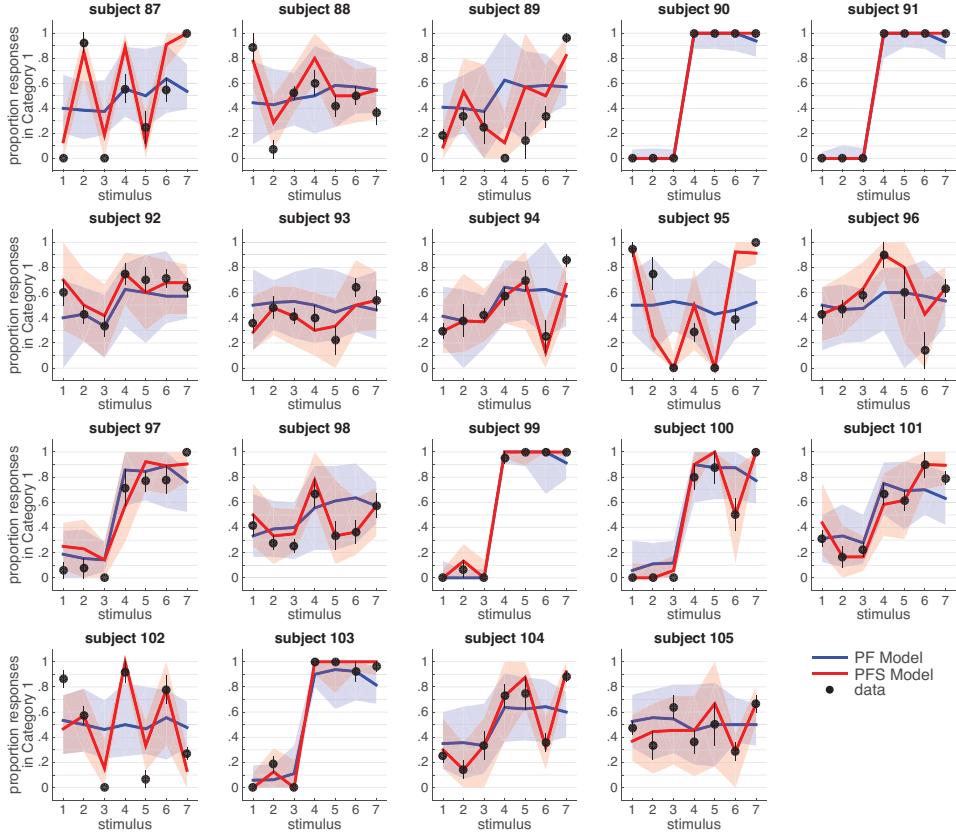
subject 69	subject 70	subject 71	subject 72	subject 73
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.84$	$\gamma = 0.88$	$\gamma = 0.8$	$\gamma = 0.85$	$\gamma = 0.82$

subject 74	subject 75	subject 76	subject 77	subject 78
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.94$	$\gamma = 0.89$	$\gamma = 0.84$	$\gamma = 0.84$	$\gamma = 0.85$

subject 79	subject 80	subject 81	subject 82	subject 83
$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.96$	$\gamma = 0.8$	$\gamma = 0.86$	$\gamma = 0.82$	$\gamma = 0.88$

subject 84	subject 85	subject 86
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.81$	$\gamma = 0.9$	$\gamma = 0.79$

Condition 6 ($p=.75$; $f=.10$)



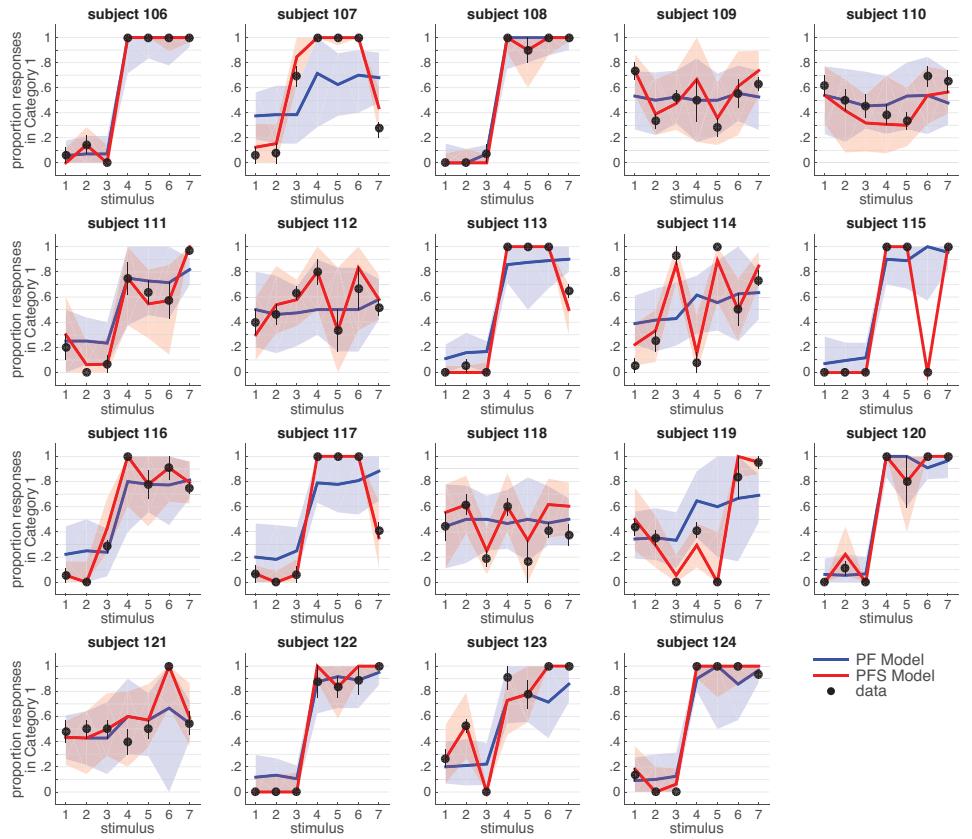
subject 87	subject 88	subject 89	subject 90	subject 91
$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0.33 & 0.33 \\ 0 & 0 & 0.33 \\ 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.83$	$\gamma = 0.88$	$\gamma = 0.88$	$\gamma = 0.99$	$\gamma = 0.95$

subject 92	subject 93	subject 94	subject 95	subject 96
$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0.33 & 0.33 \\ 0.33 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}$
$\gamma = 0.78$	$\gamma = 0.78$	$\gamma = 0.8$	$\gamma = 0.96$	$\gamma = 0.82$

subject 97	subject 98	subject 99	subject 100	subject 101
$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.78$	$\gamma = 0.83$	$\gamma = 0.82$	$\gamma = 0.83$	$\gamma = 0.79$

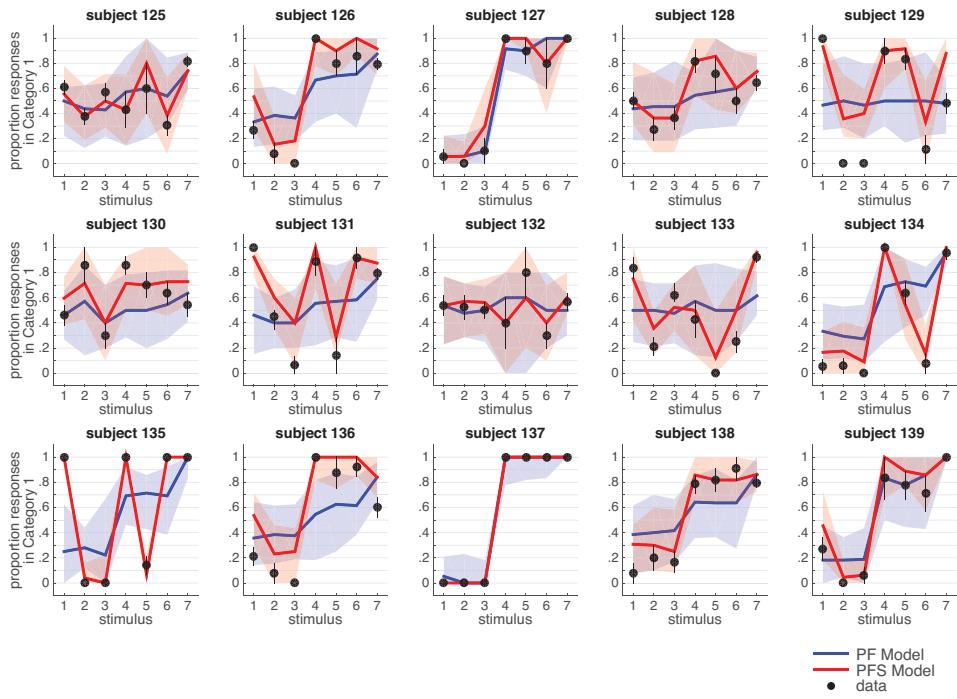
subject 102	subject 103	subject 104	subject 105
$m = \begin{bmatrix} 0.33 & 1 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 1$	$\gamma = 0.81$	$\gamma = 0.79$	$\gamma = 0.8$

Condition 7 ($p=.75$; $f=.20$)



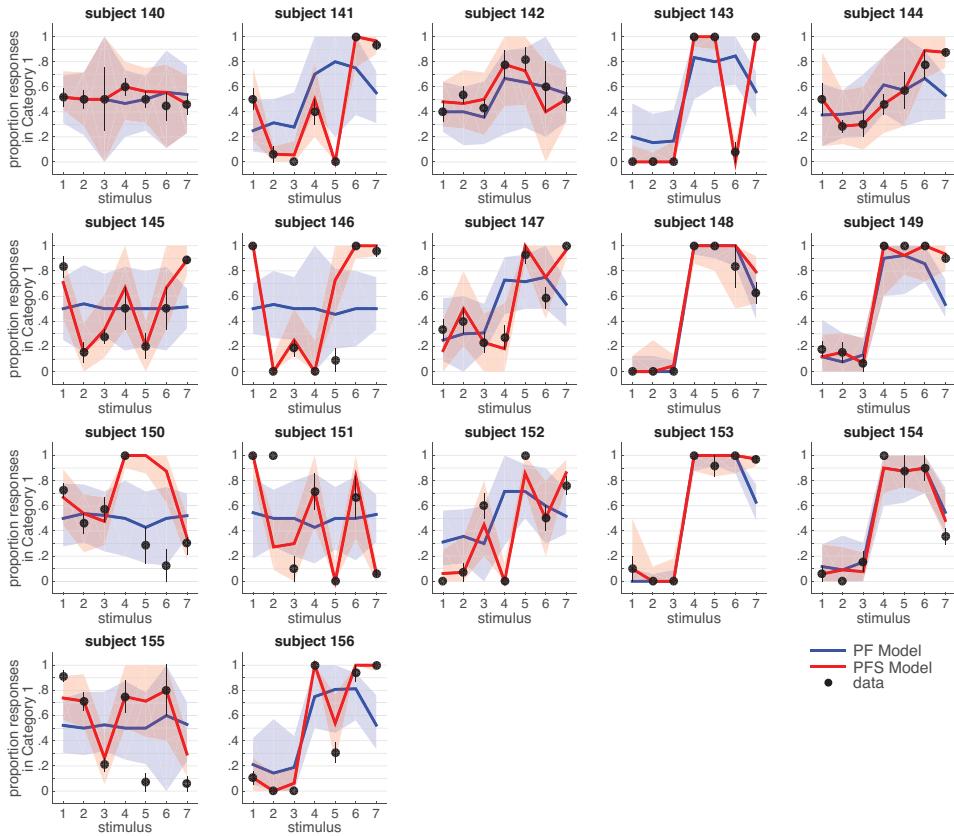
	subject 106	subject 107	subject 108	subject 109	subject 110
$m =$	$\begin{matrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$
$\gamma =$	0.82	0.93	0.82	0.79	0.79
	subject 111	subject 112	subject 113	subject 114	subject 115
$m =$	$\begin{matrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 \end{matrix}$	$m =$ $\begin{matrix} 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$
$\gamma =$	0.88	0.89	0.9	0.82	1
	subject 116	subject 117	subject 118	subject 119	subject 120
$m =$	$\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}$	$m =$ $\begin{matrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \end{matrix}$	$m =$ $\begin{matrix} 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{matrix}$	$m =$ $\begin{matrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$
$\gamma =$	0.81	0.94	0.81	0.91	1
	subject 121	subject 122	subject 123	subject 124	
$m =$	$\begin{matrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}$	$m =$ $\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$	$m =$ $\begin{matrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$
$\gamma =$	0.83	0.81	0.9	0.8	

Condition 8 ($p=.75$; $f=.40$)



subject 125	subject 126	subject 127	subject 128	subject 129
$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.33 & 0.33 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.78$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.79$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.82$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.77$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0.01 & 0.01 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.94$
subject 130	subject 131	subject 132	subject 133	subject 134
$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.77$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\gamma = 0.92$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.78$	$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.88$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.88$
subject 135	subject 136	subject 137	subject 138	subject 139
$m = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.98$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 1$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$ $\gamma = 0.77$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$ $\gamma = 0.82$

Condition 9 ($p=.60$; $f=.05$)



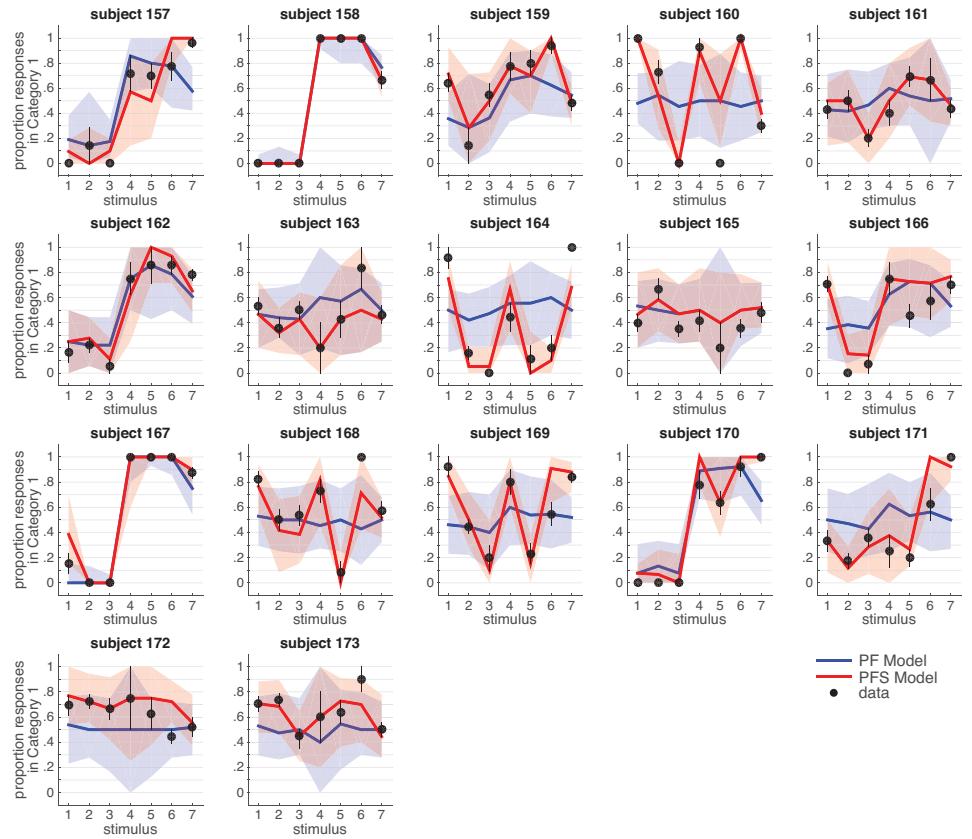
subject 140	subject 141	subject 142	subject 143	subject 144
$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 0.33 & 0.33 & 0.33 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$ $\gamma = 0.77$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.87$	$m = \begin{bmatrix} 0 & 0.5 & 0.33 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.95$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.85$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.79$

subject 145	subject 146	subject 147	subject 148	subject 149
$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.83$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.94$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.89$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.79$

subject 150	subject 151	subject 152	subject 153	subject 154
$m = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$ $\gamma = 1$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \end{bmatrix}$ $\gamma = 0.9$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.83$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$ $\gamma = 0.79$

subject 155	subject 156
$m = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\gamma = 0.8$	$m = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$ $\gamma = 0.85$

Condition 10 (p=.60; f=.10)



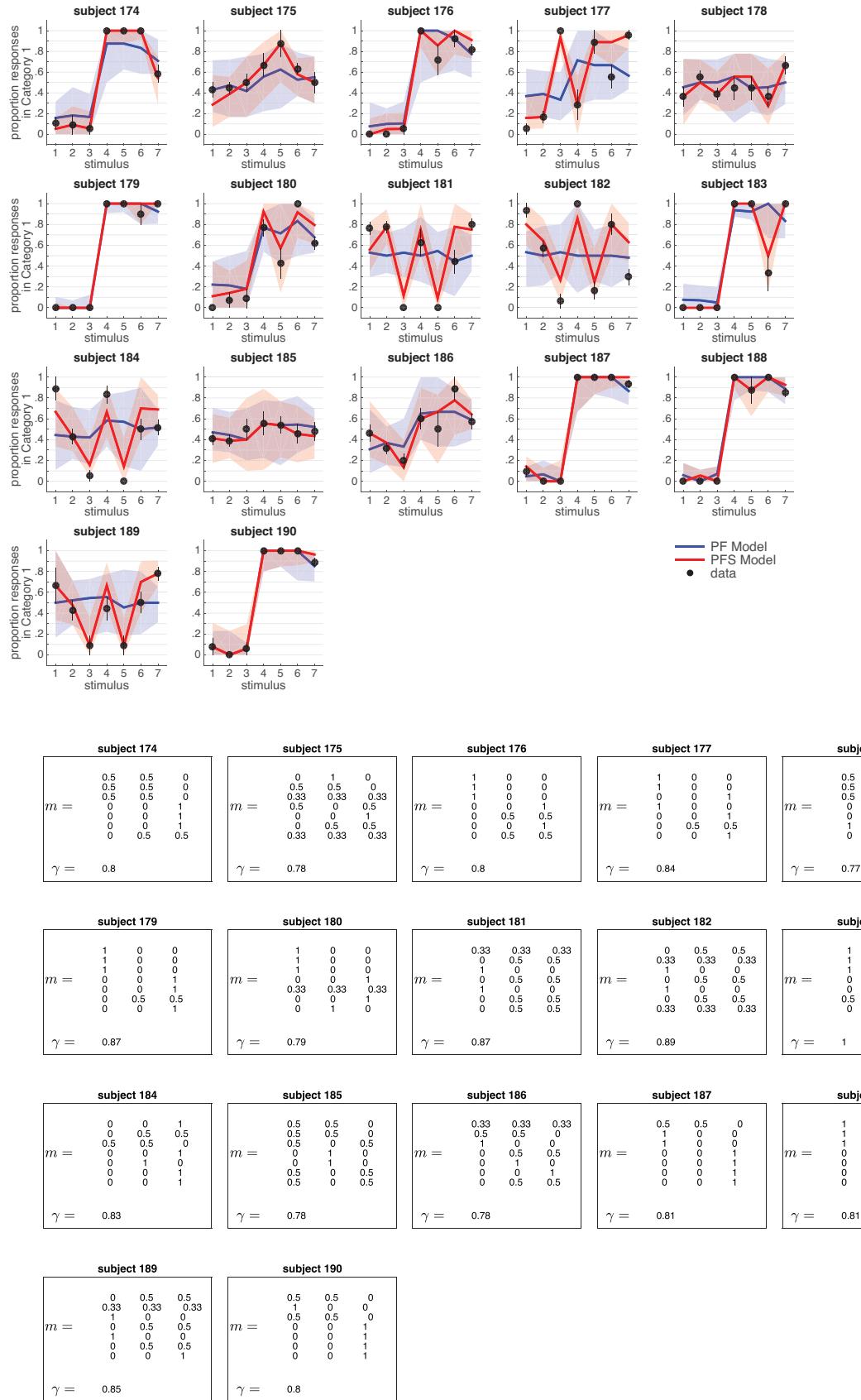
	subject 157	subject 158	subject 159	subject 160	subject 161
$m =$	$\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.1 & 0 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{matrix}$	$\begin{matrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{matrix}$
$\gamma =$	0.88	0.88	0.89	1	0.79

	subject 162	subject 163	subject 164	subject 165	subject 166
$m =$	$\begin{matrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{matrix}$	$\begin{matrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{matrix}$	$\begin{matrix} 0 & 0.5 & 1 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 0.5 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.33 & 0.33 & 0.33 \end{matrix}$	$\begin{matrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{matrix}$
$\gamma =$	0.79	0.79	0.89	0.76	0.79

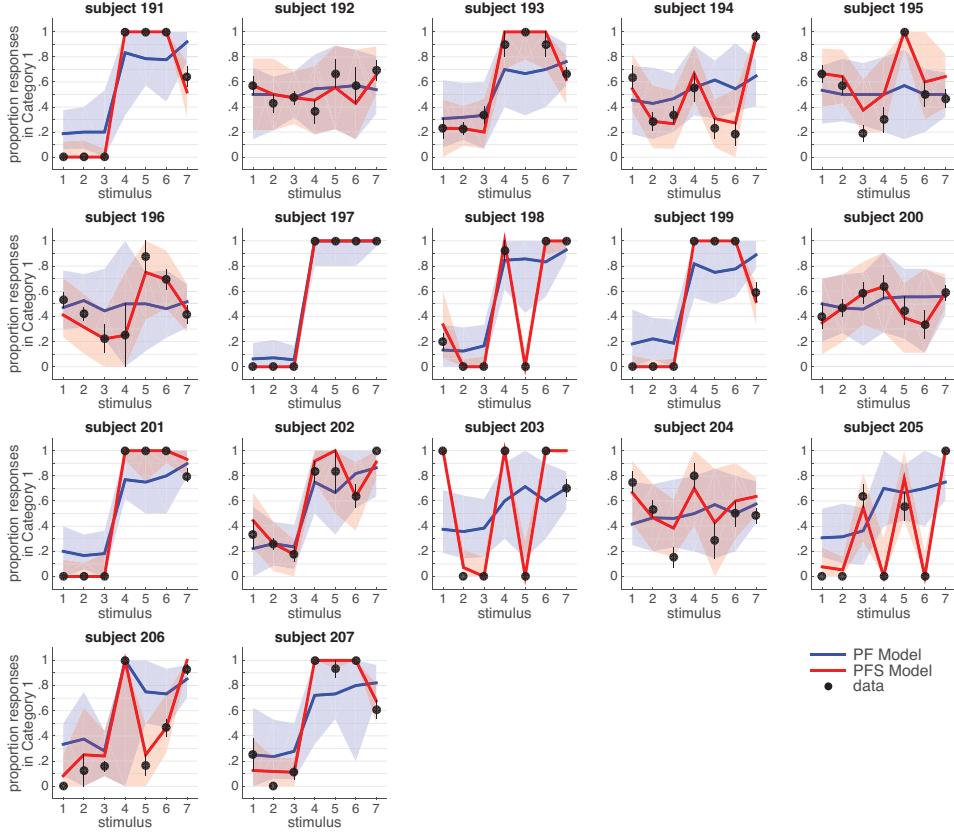
	subject 167	subject 168	subject 169	subject 170	subject 171
$m =$	$\begin{matrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{matrix}$	$\begin{matrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0.33 & 0.33 & 0.33 \end{matrix}$	$\begin{matrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.23 & 0.33 & 0.33 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{matrix}$
$\gamma =$	1	1	0.82	0.82	0.85

	subject 172	subject 173
$m =$	$\begin{matrix} 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \end{matrix}$
$\gamma =$	0.8	0.79

Condition 11 ($p=.60$; $f=.20$)



Condition 12 ($p=.60$; $f=.40$)



subject 191	subject 192	subject 193	subject 194	subject 195
$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \end{bmatrix}$
$\gamma = 1$	$\gamma = 0.78$	$\gamma = 0.79$	$\gamma = 0.93$	$\gamma = 1$

subject 196	subject 197	subject 198	subject 199	subject 200
$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
$\gamma = 0.8$	$\gamma = 1$	$\gamma = 0.89$	$\gamma = 1$	$\gamma = 0.79$

subject 201	subject 202	subject 203	subject 204	subject 205
$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.33 & 0.33 & 0.33 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$\gamma = 0.83$	$\gamma = 0.79$	$\gamma = 1$	$\gamma = 0.79$	$\gamma = 0.98$

subject 206	subject 207
$m = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}$
$\gamma = 0.86$	$\gamma = 0.87$

5 Model likelihood distributions

Below are the empirical likelihood distributions of each model's predicted proportion optimal compared to the subjects' observed proportion optimal (see text under 'Model Predictions').

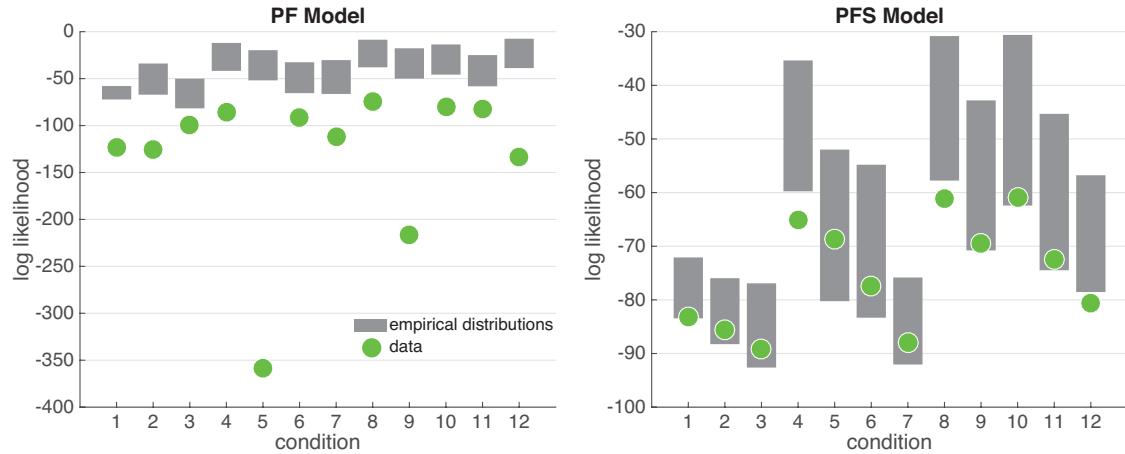


Figure S2: Log likelihood of proportion optimal for the PF (Left panel) and PFS (Right panel) models. Green circles show the likelihood of the subject data given the models. Grey regions show the upper 95th percentile of the models' empirical likelihood distributions (see text). Data points falling within the gray regions indicate that the observed data and model predictions are not significantly different.

6 Probability weighting function parameter analyses

The degree to which each subject subjectively weights the relevant probabilities of the task should not depend on the experimental condition that they participated in. If the fitted probability weighting parameters, γ , are found to vary significantly between the 12 conditions of the experiment, this would suggest that the model predictions are artifactual. The distributions of γ for each model and each condition of the experiment are shown in Fig. S3 A. As described in the text, we calculated a 1-way ANOVA comparing each model's fit values of γ across the 12 conditions of the experiment. We found no significant effect of experimental condition on the value of γ for either model (PF Model: $F(11, 195) = 1.5$, $p = .13$, mean = .83, SD = .08; PFS Model: $F(11, 195) = 1.56$, $p = .11$, mean = .86, SD = .07).

In addition, we expect each subject's two estimated values of γ (one for each model) to be significantly correlated. We calculated Pearson's ρ for the correlation of γ estimates for each model. The resulting $\rho = .23$ and the correlation is significant ($t(205) = 3.32$, $p = .0011$). Each subject's values of γ from each model are shown in Fig. S3 B.

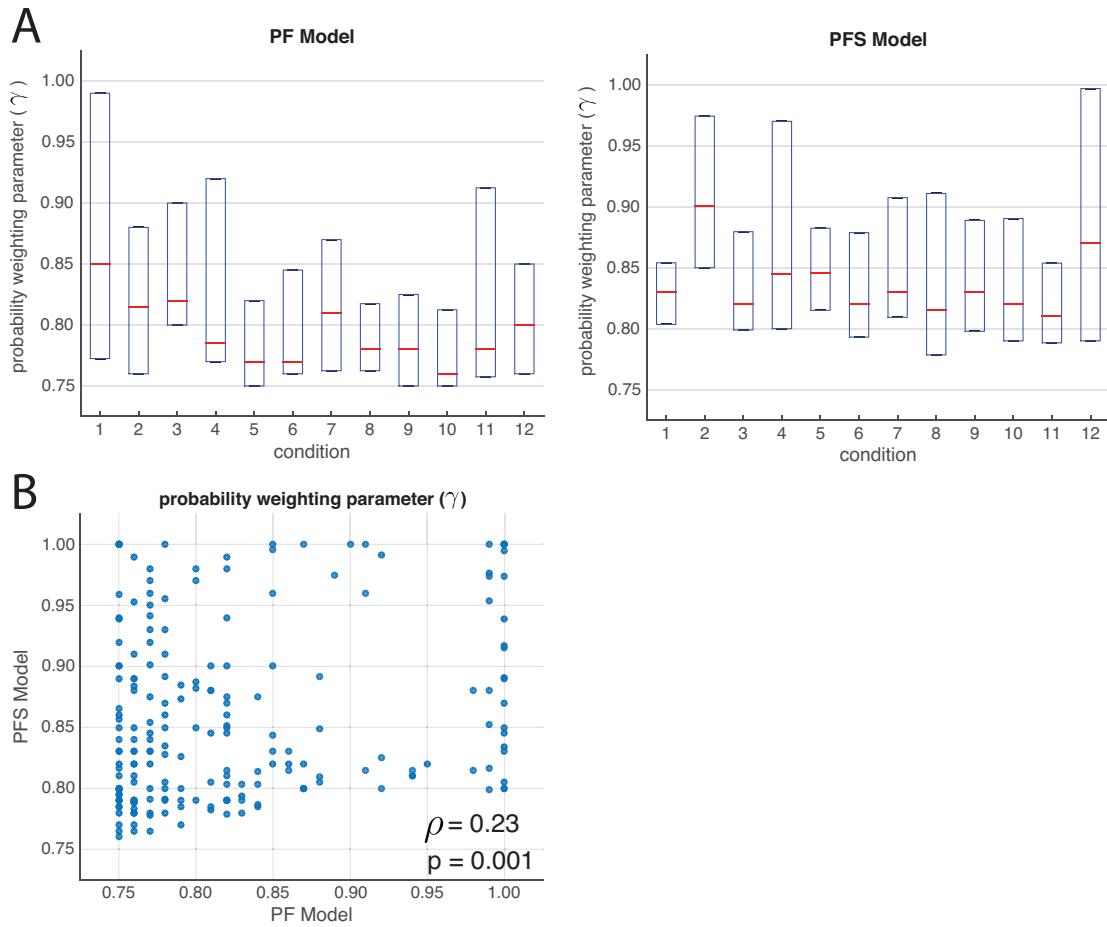


Figure S3: Probability weighting function parameter, γ . **A** Distributions of γ estimates from the PF Model (left panel) and PFS Model fits, for each Training Block condition. Red lines show the median. Boxes show the central 50th percentile. **B** γ estimates for each model. Each dot represents the pair of fitted estimates for 1 subject. Pearson's ρ and the p value for the test of a significant non-zero correlation are shown. There is a weak but significant correlation between the fitted estimates.

7 Effects of changes in probability, frequency, and similarity on Relative Uncertainty

Relative Uncertainty, U , is a measure that enables predicting changes in the likelihood that an observer will adopt an optimal categorization strategy from changes in the relevant probability, frequency, and similarity parameters (see text under “Predicting optimality using Relative Uncertainty”). Here we give examples of changes in U over a range of parameter values. In the present study, each of the 3 stimulus features, F (i.e., the top, middle, and bottom set of ‘arms’), had an independent and equal probability, $p = p(F_1|C_1) = p(F_2|C_1) = p(F_3|C_1) = .8$, of occurring in Category 1 (and a probability of $1 - p$ of occurring in Category 2). The likelihood that each stimulus, \mathbf{s}_k will occur in each category is given by:

$$p(\mathbf{s}_k|C_i) = \frac{\prod_j p(s_{k,j}|C_i)}{\sum_k \prod_j p(s_{k,j}|C_i)}$$

$$p(s_{k,j}|C_i) = \begin{cases} p(F_j|C_i) & s_{k,j} = 1 \\ 1 - p(F_j|C_i) & s_{k,j} = 0 \end{cases}$$

where $\mathbf{s}_k = [s_1 \ s_2 \ s_3]$ is a vector indicating the presence or absence of each feature, F_j , such that $s_j = 1$ when F_j is present and 0 otherwise (see text).

The posterior probability that each category is true given each stimulus is:

$$p(C_i|\mathbf{s}_k) = \frac{p(\mathbf{s}_k|C_i)p(C_i)}{\sum_i p(\mathbf{s}_k|C_i)p(C_i)}.$$

The expected frequency with which the observer will encounter each stimulus, N_k , is determined by the probability structure: $N_k = N_T \sum_i p(\mathbf{s}_k|C_i)p(C_i)$, where N_T is the total number of trials and $p(C_i) = .5$ is the prior probability of each category being the true category on a given trial.

As explained in the text, U is given by:

$$U = \frac{1}{\sum_i \sum_k E_{k,i}}$$

$$E_{k,i} = \sum_n \mathbf{D}_{k,n} \frac{\sqrt{N_{n,i}} P_{n,i}^{N_{n,i}}}{\sigma_{n,i}},$$

where $N_{n,i}$ is the expected frequency with which stimulus n will occur in category i and $P_{n,i} = p(C_i|\mathbf{s}_n)$ is the posterior probability that Category i is the true category given stimulus n .

$\mathbf{D}_{k,n}$ represents the similarity between stimuli k and n . Each element of $\mathbf{D}_{k,n}$ (for $k, n = \{1, \dots, K\}$) equals $\frac{1}{1 + \Delta_{k,n}}$, where Δ is the appropriate distance metric, in this case, the L1 distance given by:

$$\Delta_{k,n} = \sum_j |\mathbf{s}_{k,j} - \mathbf{s}_{n,j}|,$$

where the sum is over the j dimensions of the stimuli. The present study employed 7 stimuli in 3 dimensions (see Fig. 4 of the text). The associated matrix $\mathbf{D}_{k,n}$ is given by:

$$\mathbf{D}_{k,n} = \begin{bmatrix} 1 & 1 & 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 1 & 1 & 1 & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 1 & 1 & 1 & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 1 & 1 & 1 & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 1 \end{bmatrix}$$

The same 7 stimuli could be represented in a space with different dimensionality thereby changing their similarity. For example, the 7 stimuli could be represented in 1 dimension as 7 angles of a tilted line, 7 brightness levels of a single pixel, etc. The associated $\mathbf{D}_{k,n}^{1D}$ is given by:

$$\mathbf{D}_{k,n}^{1D} = \begin{bmatrix} 1 & \frac{6}{7} & \frac{5}{7} & \frac{4}{7} & \frac{3}{7} & \frac{2}{7} & \frac{1}{7} \\ \frac{6}{7} & 1 & \frac{6}{7} & \frac{5}{7} & \frac{4}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{5}{7} & \frac{6}{7} & 1 & \frac{6}{7} & \frac{5}{7} & \frac{4}{7} & \frac{3}{7} \\ \frac{4}{7} & \frac{5}{7} & \frac{6}{7} & 1 & \frac{6}{7} & \frac{5}{7} & \frac{4}{7} \\ \frac{3}{7} & \frac{4}{7} & \frac{5}{7} & \frac{6}{7} & 1 & \frac{6}{7} & \frac{5}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{4}{7} & \frac{5}{7} & \frac{6}{7} & 1 & \frac{6}{7} \\ \frac{1}{7} & \frac{2}{7} & \frac{3}{7} & \frac{4}{7} & \frac{5}{7} & \frac{6}{7} & 1 \end{bmatrix}$$

As can be seen by comparing $\mathbf{D}_{k,n}$ and $\mathbf{D}_{k,n}^{1D}$, changing the dimensionality of the stimuli changes their similarity. For example, Stimuli 1, 2 and 3 are identical when represented in 3 dimensions but dissimilar when represented in 1 dimension. The similarity between Stimulus 1 and Stimuli 6 and 7 decreases from 3D to 1D.

As the Category 1 likelihoods, $p = p(F_1|C_1) = p(F_2|C_1) = p(F_3|C_1)$, increase from .5 to 1, the MAP probabilities of each of the stimuli will increase. That is, Stimuli 1, 2, and 3 will have a higher posterior probability of occurring in Category 2 and Stimuli 4, 5, 6, and 7 will have a higher probability of occurring in Category 1. This will lead to an increase in the value of E and a decrease in U with an associated increase in the predicted optimality given the stimulus set. Likewise, increasing the total number of trials N_T will increase $N_{n,i}$ and hence E and lead to a decrease in U . In addition, representing the 7 stimuli in 1 dimension will decrease their similarity as described above leading to an increase in Relative Uncertainty with an associated decrease in the predicted optimality.

In Fig. S4 we show examples of these effects on U for a range of values of the feature likelihoods, p , the total number of training trials, N_T , and for similarity measured for 3D vs. 1D representations of the stimuli (as given by $\mathbf{D}_{k,n}$ and $\mathbf{D}_{k,n}^{1D}$, respectively). The predicted decrease in U can be seen for increasing values of p (Fig. S4 A) and N_t (Fig. S4 B). U is greater for the 1D stimulus representation but the effect, in this case, is negligible with high values of p and N_T .

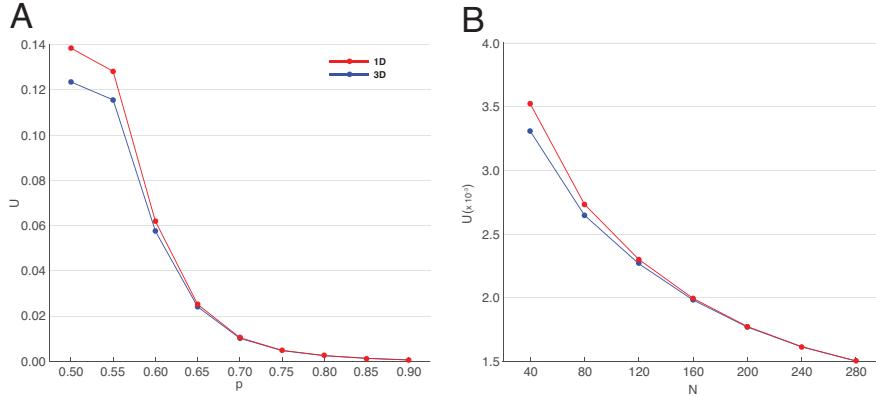


Figure S4: Relative Uncertainty, U , as a function of **A** Category 1 feature likelihood, p , and **B** total number of training trials, N_T . Blue lines represent the case in which stimuli are represented in 3D with associated similarity matrix, $\mathbf{D}_{k,n}$, and red lines represent the 1D representation of the stimuli with similarity matrix, $\mathbf{D}_{k,n}^{1D}$. Decreasing values of U are associated with a relative increase in the likelihood of an observer adopting an optimal categorization strategy for the stimuli (see text). Representing the stimuli in 1D vs 3D increases uncertainty (with a lower predicted likelihood of optimality) but the effect becomes negligible with high values of p and N_T .