# Characterizing Left-Right Gait Balance Using Footstep-Induced Structural Vibrations

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### ABSTRACT

In this paper, we introduce a method for estimating human left/right walking gait balance using footstep-induced structural vibrations. Understanding human gait balance is an integral component of assessing gait, neurological and musculoskeletal conditions, overall health status, and risk of falls. Existing techniques utilize pressuresensing mats, wearable devices, and human observation-based assessment by healthcare providers. These existing methods are collectively limited in their operation and deployment; often requiring dense sensor deployment or direct user interaction. To address these limitations, we utilize footstep-induced structural vibration responses. Based on the physical insight that the vibration energy is a function of the force exerted by a footstep, we calculate the vibration signal energy due to a footstep and use it to estimate the footstep force. By comparing the footstep forces while walking, we determine balance. This approach enables non-intrusive gait balance assessment using sparsely deployed sensors. The primary research challenge is that the floor vibration signal energy is also significantly affected by the distance between the footstep location and the vibration sensor; this function is unclear in real-world scenarios and is a mixed function of wave propagation and structure-dependent properties. We overcome this challenge through footstep localization and incorporating structural factors into an analytical force-energy-distance function. This function is estimated through a nonlinear least squares regression analysis. We evaluate the performance of our method with a real-world deployment in a campus building. Our approach estimates footstep forces with a RMSE of 61.0N (8% of participant's body weight), representing a 1.54X improvement over the baseline.

**Keywords:** Occupant Characterization, Smart Structures, Footstep-induced Structural Vibration, Human-Structure Interaction, Gait Health, Feature Extraction

# 1. INTRODUCTION

Human gait balance is an integral aspect of health assessment in a variety of applications. For the general population, assessing walking gait balance can assist with diagnosis of balance disorders, neurological conditions, and musculoskeletal impairments.<sup>1</sup> Among the elderly population, gait and balance disorders are the second highest cause of falls and elders exhibiting gait imbalance have been shown to increase likelihood of falls by 2.9X.<sup>2–5</sup> Having a quantitative assessment of walking gait balance can assist medical personnel with assessment and help identify elders with higher risk of falling.

Several techniques have been developed to characterize gait balance. Traditionally, gait balance assessment is conducted visually, through observation-based testing by medical personnel and/or physical therapists. The most prominent of these assessments are the Tinetti Balance and Gait Test, Berg Balance Scale, The "Timed Up and Go Test", and the "Dynamic Gait Index". 6–8 While these traditional approaches have been successful over time, they are often subjective, and require a significant amount of time and attention from the caregivers. 7 Recently, medical personnel have turned to emerging sensing technologies including pressure sensing mats, 3-D video capture, and wearable devices. 7–10 Collectively, these sensing approaches have addressed many of the limitations of traditional approaches, but they too can be limited in their application and deployment. For instance, these methods may restrict assessment to specialized rooms or areas that have densely populated sensors, may only provide qualitative balance assessment, or may require direct interaction from the users. 10

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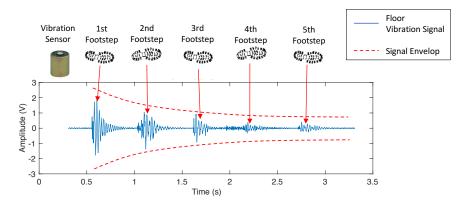


Figure 1: Raw Floor Vibration Signal with footsteps increasingly far from the sensor. Note the decrease in signal amplitude (energy) with increasing footstep-sensor distance.

In this paper, we present a system for monitoring walking gait balance using human footstep-induced floor vibrations. As footsteps hit the floor, they cause vibrations in the floor structure. At the point of impact, the energy of these vibrations changes with varying footstep force (i.e., the higher the footstep force, the higher the floor vibration amplitude/energy). By quantifying each footstep force during a walking trace, we enable objective measurement of walking gait balance. While passively recording floor vibrations, our approach utilizes wave propagation through the structure medium to achieve sparse sensor deployment, and does not require visual observation by medical staff nor the use of a wearable.

The research challenges with our approach are: 1) the recorded vibration signal is largely affected by the distance between the footstep location and the vibration sensor and 2) footstep-induced floor vibrations exhibit a low signal-to-noise ratio, and the recorded signals are significantly affected by environmental noise. Each of these challenges causes a change in the vibration signal characteristics, and makes it difficult to estimate walking gait balance. We address the first research challenge through footstep localization and by combining analytical and experimental wave propagation and structural damping models to relate distance and vibration signal energy loss. To address the second challenge, we minimize the effects of environmental noise on individual sensor estimations by performing sensor fusion and combining estimation from multiple sensors.

The core contributions of this paper are:

- 1. We utilize footstep-induced structural floor vibrations to passively estimate left/right walking gait balance in a non-intrusive manner with sparse sensor deployment.
- 2. We incorporate structural properties and wave propagation characteristics to formulate the signal energy loss with distance for more accurate footstep force and balance estimation.
- 3. We evaluate our approach using real-world human walking experiments on a concrete floor structure.

In the paper that follows, we present the physical intuition that supports our method in Section 2. Then, in Section 3, we elaborate on the specific method and algorithm utilized to quantify footstep forces using footstep-induced structural floor vibrations. Following this, we present our evaluation technique, results and observations in Section 4. Finally, Section 5 provides conclusions and discusses future work.

## 2. PHYSICAL INTUITION

The physical intuitions that enable our method involve principles of energy conservation and wave propagation. We can describe the force from a footstep-induced excitation as the transfer of kinetic and potential energy from the footstep into the structure. The measured vibration signal energy therefore contains information about this input footstep force, and we can establish a relationship between the two. For example, if a person walks with a force of 400N (90 lbs.) in their right foot, but a force of only 300N (67 lbs.) in their left foot, we can anticipate

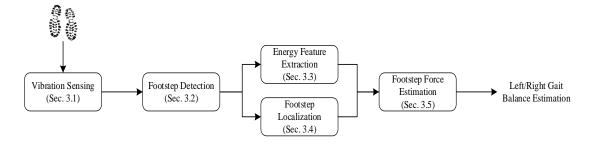


Figure 2: Left/Right Gait Balance Estimation Overview

that the energy transferred to the structure due to the right foot will be greater than that in the left foot. From this observed relationship, we can conclude that observed higher energies in the vibration signal are a result of higher input energies (forces), and use this information to establish a relationship between the vibration signal energy, the footstep forces, and the walking gait balance.

In addition, we observe that the measured signal energy is not only a function of the footstep force, but is also a function of the distance between the footstep location and the sensor location. This observation exemplifies some of the principles of wave propagation. When a vibration wave travels through a medium (the floor structure), it experiences energy loss through attenuation and damping. This relationship will be further explored in Section 3.5, and the key intuition is that the greater the distance the footstep occurs from the sensor, the greater the vibration signal energy loss. Thus, measured vibration signal energy is inversely proportional to the distance between the sensor and the footstep location given the same footstep force. A visual representation of this effect can be seen in Figure 1. In this figure, the sensor is located nearest to the first footstep, and each subsequent footstep is increasingly far from the sensor. With this plot, it is clear that the amplitude (and energy) of the vibration signal due to each footstep decreases as the footsteps are increasingly far from the vibration sensor. In the following section we will elaborate on this relationship and use it to establish our force-energy-distance function.

## 3. LEFT/RIGHT GAIT BALANCE ESTIMATION

To enable left/right gait balance estimation, our approach focuses on quantifying footstep forces during a walking trace. Figure 2 provides an overview of our method. Our method of quantifying footstep force using footstep-induced floor vibrations contains five distinct modules: 1) our system senses the floor vibration, 2) footstep-induced signals are detected and isolated, 3) our system calculates the vibration signal energy, 4) each footstep location and footstep-sensor distance is calculated, and 5) the footstep forces are estimated considering the effect of energy dissipation with distance using a nonlinear least squares regression. Using the estimated footstep forces, we can provide a quantitative measurement of left/right gait balance.

#### 3.1 Vibration Sensing

The first step in force-energy-distance training module is to collect the footstep-induced structural vibrations. In our method, this sensing module consists of a series of geophones<sup>11</sup> whose signal is amplified and then digitized using an analog-to-digital converter (ADC). To enable calibration-free localization, at least 4 geophones are required in the sensing area.<sup>12</sup> In real-world scenarios, the amplitude of footstep-induced floor vibrations is low and the raw signal is difficult to interpret. Therefore, to improve the resolution of the vibration signal, we amplify the signal using an operational amplifier with a range of 200-2000X, and then adjust the gain to produce the highest amplification for normal walking while minimizing/eliminating clipping of the signal.

## 3.2 Footstep Detection

Once the signal has been digitized, we detect each footstep using an anomaly detection and classification method with on-line noise modeling. <sup>13, 14</sup> Footstep induced signals have peak amplitudes that are larger than environmental noise, therefore we use anomaly detection to identify footsteps. Our anomaly detection and classification

approach characterizes footstep impulses and distinguishes them from background noise and other impulses (e.g., items dropping). Once the footsteps have been detected, they are isolated from the overall signal and stored for energy feature extraction.

## 3.3 Energy Feature Extraction

In this step, we extract the vibration signal energy from the detected footsteps, which is a key feature for our footstep force estimation. Through conservation of energy during the footstep event, we infer that as the footstep force of an individual step increases, so does the vibration signal energy (as mentioned in Section 2). We calculate the vibration signal energy using the following expression:<sup>15</sup>

$$E_{ij} = \frac{1}{Z} \sum_{n=1}^{N} \|a_n\|^2 \tag{1}$$

where  $E_{ij}$  represents the vibration signal energy (in Joules) of step i as measured by sensor j, Z is a constant representing the gain and voltage-to-velocity conversion factor for the analog-to-digital converter,  $a_n$  is the amplitude (Volts) of the vibration signal, and N is the length of each footstep signal. Using this formulation, we calculate the energy of each footstep within the walking trace as measured by each sensor.

## 3.4 Footstep Localization

We localize each footstep by adapting a wavelet-based multilateration approach, as introduced in our previous work.<sup>12</sup> By localizing each footstep, we learn the footstep-sensor distance for each footstep, and can use this information to understand the vibration signal energy loss associated with that distance. To localize the footsteps, we utilize the vibration signal measurements from four sensors and perform multilateration with the time difference of arrival (TDoA) of the first peak for each footstep signal relative to each sensor. We observe that environmental noise may result in false detection of the first peak for the TDoA estimations. To reduce this effect, we first apply a low-pass filter with a cutoff frequency of 150 Hz, based on observing that most of the signal energy is within 0-150Hz. With footstep-induced floor vibrations, dispersion affects the wave propagation velocity of each frequency component differently, resulting in inaccurate TDoA measurements.<sup>12</sup> To minimize this effect, we use a wavelet-based decomposition to isolate each frequency component and mitigate the effect of dispersion. In particular, we choose the frequency component with the highest energy to maximize the signal-to-noise ratio. Using this frequency component, we determine TDoAs for multilateration and calculate the distance between the footstep location and each sensor (the footstep-sensor distance).

#### 3.5 Footstep Force Estimation

Once we have obtained each footstep signal energy and footstep-sensor distance, we use them to quantify the footstep force using a nonlinear least squares regression. From the physical intuition discussed in Section 2, we know that the signal energy changes with distance and magnitude of the footstep force. Using this intuition, we can conclude that the unknown footstep force is a function of the measured signal energy and the footstep-sensor distance. To develop this function, we consider an analytical model that incorporates wave attenuation and structure-specific dampening and stiffness properties. Henceforth, we will refer to this function as the force-distance-energy function.

Each term in our force-energy-distance model has been analytically derived to represent the principles of wave attenuation and structural dynamics. As described by Chopra, it is not possible to identify all factors that contribute to vibration energy loss in structural floor vibrations. As such, we focus on four of the largest effects: wave path loss, structural damping, and body wave spreading and surface wave spreading (attenuation). The Path loss is used to define the loss of vibration wave energy with distance travelled through a medium. For our method, we consider the path loss due to the vibration wave travelling through the structural floor medium. In structural dynamics, damping describes the wave amplitude loss due to the oscillation (vibration) of the floor structure. For our formulation, we observe that damping ratios for a given structure are relatively constant, and assume a constant damping ratio for the sensing area in our method. Next, our model incorporates wave

attenuation in the form of vibration wave spreading. Wave spreading is typically described as a function of Body and Raleigh (surface) wave spreading as a function of the distance travelled.<sup>17,19</sup> With footstep-induced structural floor vibrations, we assume that the total footstep event is a combination of an impulse-like heel strike and a friction force from the toe push-off, generating a linear combination of body and surface waves. The body waves spreading results in a proportional decrease in amplitude (squared distance for energy) with distance, while the surface wave amplitude spreads proportional to the square root of the distance (distance for energy). Finally, we approximate the remaining sensor- and structure-specific effects with a constant value. In our formulation, we assume that each of the aforementioned energy dissipation terms affect the wave concurrently, and we represent the total energy loss as a linear combination of their reciprocals. The resulting function is provided by the following expression:

$$F_{ij}^2 = \frac{KE_{ij}(x)}{\left[\frac{A_1}{log_{10}x_{ij}} + \frac{A_2}{e^{2\alpha x_{ij}}} + \frac{A_3}{x_{ij}^2} + \frac{A_4}{x_{ij}} + A_5\right]}$$
(2)

where  $F_{ij}$  represents the unknown footstep force of step i as estimated by sensor j,  $x_{ij}$  represents the distance from sensor j to the footstep i location,  $E_{ij}(x)$  represents the vibration signal energy of step i as measured by sensor j, K represents the stiffness of the floor structure,  $A_1$  is the coefficient for the path loss term,  $A_2$  is the coefficient for the structure damping term,  $\alpha$  is the structure-specific damping coefficient,  $A_3$  is the coefficient for the body wave spreading,  $A_4$  is the coefficient for the surface wave spreading, and  $A_5$  represents the remaining energy loss effects.

To determine the coefficients  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $A_5$ , and K we use known values of footstep-sensor distance, vibration signal energy, and footstep force to perform a nonlinear least squares regression. The damping coefficient,  $\alpha$ , is estimated based on the structure type (i.e. concrete, steel, wood). This regression is completed independently for each sensor in the sensing area in order to account for location- and sensor-specific properties that are dependent on the specific sensor and sensor location. With the trained nonlinear least squares regression model, we estimate the footstep force for new footstep signals.

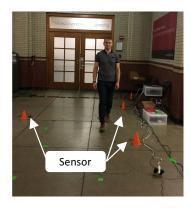
Using the multiple force estimations from individual sensors for each footstep, we address the challenge of low signal-to-noise ratio for structural floor vibrations by performing sensor fusion. In our method, we accomplish sensor fusion by finding the average footstep force value between all of the sensors in the sensing area. By fusing the results of all of the sensors, we reduce the possibility of force estimation outliers from environmental noise sources on individual sensors, thus mitigating the effect of environmental noise and low signal-to-noise ratio for our method.

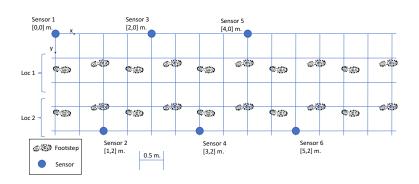
#### 4. EVALUATION

To evaluate our method, we have conducted a real-world walking experiment on the ground floor of Porter Hall at Carnegie Mellon University (CMU) in Pittsburgh, PA. Our objective is to evaluate the accuracy of our system in calculating the walking footstep force, validate our force-energy-distance function formulation, and infer the ability of our method to assess left/right walking gait balance. Our evaluation involves training and testing phases. For comparison purposes, we consider two approaches: 1) a "baseline" that does not account for the distance dissipation effect on vibration signal energy, and 2) our force estimation approach with sensor fusion of all of the sensors in the experimental setup.

#### 4.1 Experimental Setup

We conducted our experimental evaluation in a CMU campus building in a hallway with an exposed concrete slab-on-grade floor. Figure 3a shows the participant walking during the experiment. We instrumented a 7m x 2m area with six geophone sensors spaced at 2m on center at each side of the walking area (marked by orange cones in Figure 3a) and performed a series of 30 walking traces (15 for training and 15 evaluation) with one individual (weight 175 lbs (79kg)), consisting of 10 consecutive footsteps in each trace. To obtain a full range of distances for each sensor, the walking traces were divided into two locations (as shown in Figure 3b), with 20 traces occurring in Location "1" and 10 traces occurring in Location "2". The participant was instructed





(a) Experimental Setup

(b) Sensor and Footstep Location Layout

Figure 3: Experimental setup: (a) shows the experimental setup in the CMU campus building. The typical sensor layout and footstep locations are depicted in (b).

to walk at a natural pace for each trace to maintain consistency in footstep forces/balance. Figure 3b shows the experimental setup for the evaluation. In this figure, footstep locations are indicated by the shoe print, and sensor locations are indicated by blue dots. Each trace began with a right footstep and terminated with a left footstep, with an average stride length of 0.8m.

For each footstep, ground truth footstep forces were recorded using FlexiForce A401 pressure sensors.<sup>21</sup> Pressure readings are multiplied by the pressure sensor sensing area to determine force values. The pressure sensors were placed at the center of the heel on sole of the shoe to obtain a proportional measure of the peak heel force, which is a good representation of the first peak of the total footstep ground reaction force.<sup>22</sup> In our evaluation, we removed any outlier pressure sensor readings (due to pressure sensor limitations), defined as being greater than 315N (70 lbs.) or less than 220N (50 lbs.), resulting in a total of 69 steps used for training, and 63 steps used for evaluation.

## 4.2 Force-Energy-Distance Training

In order to accurately assess the force-energy-distance function described in Equation 2, we train our system using 15 traces of 10 footsteps with known footstep forces and locations. With this training exercise, we estimate the force-energy-distance term coefficients K, A1, A2, A3, A4, and A5 through the nonlinear regression procedure outlined in Section 3.5. This enables estimation of the wave propagation and structural damping/stiffness properties for the experimental location. For the structural damping ratio  $\alpha$ , we assume a constant of 0.02 based on the testing occurring on a concrete structural floor.<sup>23</sup> Footsteps were all located as described in Section 4.1. With our training data, we noted that the force-energy-distance function is primarily dominated by the path loss term for the majority of the sensors, with an average coefficient value of 4,300. We also noted that the body wave spreading term had the smallest effect for the majority of the sensors, with coefficient values near zero for 5 of the 6 sensors.

## 4.3 Footstep Force and Balance Estimation

We evaluate the performance of our footstep force and balance estimation method in a real-world setting. With the two evaluation approaches, the force estimation accuracy is evaluated with root mean squared error (RMSE) of the footstep force estimation for the total dataset (63 footsteps). In addition, we consider the footstep force estimation error,  $F_{ij} - \hat{F}_{ij}$ , where  $F_{ij}$  represents the ground truth footstep force and  $\hat{F}_{ij}$  represents the estimated force.

For the baseline approach, we consider the footstep force estimation results without accounting for the energy loss with increasing distance from the vibration sensor. This baseline approach assumes that there is not any energy loss with distance, and directly relates the footstep forces to the vibration signal energy. In this way,

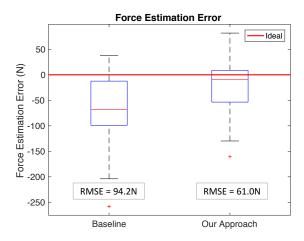


Figure 4: Force estimation error for the two evaluation approaches: 1) baseline and 2) our force estimation approach with sensor fusion of all of the sensors in the experimental setup. The blue boxes represent the 25th and 75th quantiles, the small red lines represent the median estimation error, and the whiskers represent the estimation error ranges. Estimation error outliers are represented by red crosses, and the solid red line represents the "ideal" error (0N). Our approach shows a RMSE improvement of 1.54X over the baseline.)

Equation 2 is evaluated with the terms corresponding to  $A_1$  - $A_5$  eliminated. To determine the effective stiffness coefficient, K, the footstep force estimation is trained with known footstep forces, each occurring at the locations shown in Figure 3b and described in Section 4.2. Then, an average K is determined for each sensor from each of the training footsteps and used for estimating footstep forces of new footsteps.

With estimated locations, our approach exhibits a significant improvement over the baseline. To quantify our accuracy, we consider the ratio of the footstep force estimation RMSE to the root mean squared average force for all of the footsteps (266N). Figure 4 shows the error distribution for each of the two approaches. In this figure, the blue boxes represent the 25th and 75th quantiles, the small red line represents the median estimation error, and the whiskers represent the estimation error range. Estimation error outliers are represented by red crosses, and the solid red line represents the "ideal" error (0N). For the baseline, footstep forces were estimated with a RMSE of 94.2N (21.2 lbs.; 12% of the participant's weight), and exhibit a mean of -70.9N (15.9 lbs) and a standard deviation of error of 67.8N (15.2 lbs). In comparison, our approach exhibits a RMSE of 61.0N (13.7 lbs.; 8% of the participant's weight). This results in a 1.54X improvement in RMSE from the baseline. In addition, our approach exhibits a mean of -19.4N (4.4 lbs.) and a standard deviation of estimation error of 56.8N (12.8 lbs.). This also results in an improvement of the mean and standard deviation of error from the baseline. Based on these force estimations, we can compute the left-right gait balance.

## 5. CONCLUSION

We present our system for quantifying walking footstep force and left/right gait balance using human footstep-induced structural floor vibrations. Our system accomplishes this passively in a non-intrusive and sparse sensing environment and does not require the direct observation of medical personnel. We overcome the system challenges of 1) unknown footstep-sensor distance energy dissipation effect and 2) noisy sensing environment by estimating footstep locations, incorporating wave attenuation and structural damping effects into a combined analytical and experimental force-energy-distance model, and through sensor fusion of footstep force estimation. Our approach achieves a RMSE of 61.0N (approximately 8% of the participants body weight), which is a 1.54X improvement from the baseline.

In our future work, we will further explore the effects of in-foot footstep pressure distribution and will evaluate the robustness of our approach with respect to different structures and different shoe types. We will consider placement of additional pressure sensors during training to better understand the effects of in-foot pressure distribution and establish a clearer means of mapping the ground reaction forces to the floor vibration signal.

Beyond this, we will explore the performance of our system with a larger range of footstep forces and different balance/imbalance scenarios (e.g., "healthy", "left side imbalance", history of falls, etc.).

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